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Modelling the heuristic dynamics of the wage and price curve model of equilibrium unemployment

Abstract:
A standard model of equilibrium unemployment consists of static equations for real wage ambitions (wage curve) and real wage scope (price curve), which jointly determine the NAIRU. The heuristics of the model states that unless the rate of unemployment approaches the NAIRU from any given initial value, inflation will be increasing or decreasing over time. We formalize this influential heuristic argument with the aid of a dynamic model of the wage-price spiral where the static theory's equations are re-interpretated as attractor relationships. We show that NAIRU unemployment dynamics are sufficient but not necessary for inflation stabilization, and that the dynamic wage-price spiral model generally has a dynamically stable solution for any pre-determined rate of unemployment. We also discuss a restricted version of the model that conforms to the accelerationist view that inflation increases/falls if unemployment is not at its ‘natural rate’.

Keywords: AS-AD, equilibrium-correction, imperfect competition, macroeconomics, NAIRU, Phillips curve, unemployment, wage-price spiral.

JEL classification: E24, E30, J50.

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Sammendrag

En standardmodell for eksistensen av et likevektsnivå på arbeidsledighetsraten består av to statiske likninger for reallønn som funksjon av blant annet ledighetsraten. Ledigheten virker dempende på lønnstakernes reallønnsambisjon (ved å moderere nominelt lønnskrav) og stimulerende på bedriftenes reallønnsmål (siden ledigheten samvarierer negativt med aktivitetsnivået og ressursutnyttelsen i økonomien, og dermed virker dempende på produsentprisen). Der lønnstakernes fallende reallønnskurve og bedriftenes stigende reallønnskurve, begge som funksjon av ledighetsraten, krysser hverandre defineres en likevektsledighet kalt Non-Accelerating Inflation Rate of Unemployment eller NAIRU. Modellframstillingen resonnerer således at hvis ledighetsraten ikke konvergerer mot NAIRU over tid, så vil inflasjonen tilta hvis ledigheten ligger under NAIRU og avta hvis ledigheten ligger over.

Vi formaliserer denne heuristikken bak NAIRU i en liten makromodell, og avviser at det bare er en eneste ledighetsrate som leder til en stabil og dermed "naturlig" inflasjonsrate. De to statiske reallønnslikningene bygges inn i en dynamisk modell for en åpen økonomi, der ingen av partene har makt til å tvinge igjennom sin reallønnsmålsetting. I stedet justeres nominell lønn og produsentpris skrittvis, simultant og relativt til hverandre mot henholdsvis lønnstakernes og bedriftenes reallønnsmål. De to statiske reallønnslikningene i standardmodellen fungerer dermed som attraktorer på de faktiske lønns- og prisjusteringene i vår dynamiske modell. I denne justeringsprosessen er ikke ledighetsnivået avgjørende for hvorvidt lønns- og prisveksten konvergerer slik at både inflasjonen og reallønnen blir stabil. I stedet er det attraktorene og selve justeringsprosessen som sørger for at både lønns- og prisveksten tilpasser seg inflasjonen i utlandet. (Importprisen vektes sammen med produsent¬prisen til en konsumpris som påvirker lønnkravet). Dermed sikres en stabil reallønn og inflasjon for ethvert konstant nivå på en eksogent gitt ledighet. Når vi endogeniserer ledigheten blir den også stabil, men selve nivået avhenger av alle parametre og eksogene variable i modellen. Dermed er ledighetsnivået ikke naturlig, men en følge av dynamisk interaksjon så vel som statiske reallønnsmål.

Den statiske standardmodellens heuristikk holder ikke i vår dynamiske modell, som dermed er ikke-aksellerasjonistisk. Ved å la produsentenes justering av prisen påvirkes av tidligere prisaksellerasjon samt pålegge bestemte ekskluderende restriksjoner på en attraktor og på dynamikken får vi fram et spesialtilfelle av modellen der inflasjonen bare er stabil for et bestemt ledighetsnivå, et NAIRU. Grunnen til det er at ledigheten i denne modellversjonen må ta over (stabiliseringsfunksjonen) for visse variable som restriksjonene har eliminert fra modellen. Med begrenset kausalitet og redusert dynamik "gjenopp-står" NAIRU. Denne artikkelen viser at dynamikk kan være kvalitativt og ikke bare kvantitativt viktig. En mer tolkningsbasert konklusjon er at dynamisk interaksjon mellom økonomiske aktører kan fortjene like mye oppmerksomhet som aktørenes egenskaper og rammebetingelser i økonomisk modellering.

1 Introduction

Late in last millennium there was an interesting development in macroeconomic modelling of equilibrium unemployment. First, macroeconomic implications of imperfect competition with price-setting firms were developed in several papers and books, see e.g., Bruno (1979), Blanchard and Kiyotaki (1987), Bruno and Sachs (1984) and Blanchard and Fisher (1989, Chapter 8). Second, the economic theory of labour unions, pioneered by Dunlop (1944), was extended and formalized in a game theoretic framework, see e.g. Nickell and Andrews (1983), Oswald (1986), Hoel and Nymoen (1988). Models of European unemployment, that incorporated elements from both these developments, appeared in Layard and Nickell (1986), Carlin and Soskice (1990), Lindbeck (1993) and Layard et al. (2005).

The new macro model of unemployment is incontestably linked to Layard and Nickell and their coauthors, but the framework is also often referred to as the Incomplete Competition Model (ICM) or, interchangeably, the *wage-curve* framework, to help distinguish it from the earlier Phillips Curve Model (PCM)[1]. The ICM label seems apt since the model's defining characteristic is imperfect competition in both product and labour markets, see e.g. Bårdsen and Nymoen (2003). The ICM model is included in leading textbooks in macroeconomics, for example Sørensen and Whitta-Jacobsen (2010, Ch.13), Blanchard (2009, Ch. 6), Burda and Wyplosz (2006, Ch. 12.4) and Miles and Scott (2002, Ch. 8).

Despite the differences, the ICM/wage-curve model and the PCM share an important underlying premise, namely that it is the equilibrium rate of unemployment that reconciles the conflict about the division of value-added between wage earners and firms. Both models take the view that the equilibrium or steady-state rate of unemployment is determined by a limited number of factors that reflect structural aspects such as production technology, union preferences and institutional factors (characteristics of the bargaining system, the unemployment insurance system). Thus, in both theories, demand management and monetary policy have only a short-term effect on the rate of unemployment. In the (hypothetical) situation in which all shocks are switched off, the rate of unemployment would return to a unique equilibrium rate, called the natural rate or the NAIRU.[2] In particular, the ICM is a model of the natural rate both in its motivation and in its implications: «In the long run, unemployment is determined entirely by long-run supply factors and equals the NAIRU» (Layard et al. (1994, p 23)).

Both the ICM and the PCM represent the *accelerationist view*, that inflation increases when actual unemployment is higher than the supply-side determined equilibrium rate, and decreases when it is below that 'natural' rate. The accelerationist view has become widely accepted in macroeconomic thinking. Specifically, it represents the main premise for monetary policy, see for example King (1998). The formalization of the accelerationist view is straight-forward in the PCM version of the theory. It is given by the restriction known as dynamic homogeneity, and it implies that the long-run Phillips curve is vertical. It is a different matter for the ICM. Unlike the PCM, this is a static model from the outset, and there is a gap between the strong heuristics of this model and the corresponding explicit dynamic model of the wage-price spiral with accelerationist properties. In this paper we attempt to bridge that gap.

We formalize the heuristics of the NAIRU in a dynamic version of the ICM model, and refute the claim that only a certain level of unemployment secures a stable rate of inflation. In our small and dynamic open-economy macro model, the two static real-wage equations of the basic ICM serve as attractors in a wage-price spiral where neither the wage earners (or unions) nor the firms have sufficient power to force through their real-wage

[1] The ICM acrynom may be confusing, in particular if taken to imply that the alternative Phillips curve model represents perfect competition, cf. the New Keynesian Phillips curve (Gali and Gertler (1999)).

[2] In this paper, the asymptotically stable equilibrium rate of unemployment, which is determined on the supply side, correspond to a natural rate or to a NAIRU. The NAIRU depends on the (foreign) steady-state rate of inflation, while a natural rate does not. Often we will simply use the term natural rate for brevity.

target. Rather, the nominal wage and the producer price are both subject to simultaneous, partial and reciprocal adjustments toward the their respective real-wage targets. In this adjustment process no rate of unemployment is decisive for the convergence of the wage and price inflation in order to make the real wage stable. It is not unemployment that secures the coordination of wage and price growth, but the wage-price spiral. The attractors, i.e. the real-wage targets, and the small and reciprocal adjustments cause the wage and price growth to adapt to foreign inflation. (Foreign price is imported and weighted together with domestic producer price into a consumer price which affects the wage claim). This secures a stable real-wage and a stable inflation for any rate of exogenous unemployment. When we endogenize unemployment, it also becomes stable, but its stable rate depends on all parameters and exogenous variables in the model. Hence, the unemployment level is not "natural", but a result of the static real-wage goals and the dynamic interactions of variables in the wage-price spiral.

The heuristics of the standard ICM does not apply to our dynamic model. Our model is non-accelerationistic. For our model to reflect the accelerationist view, we have to impose certain restrictions on both causality and dynamics. First, we let the price adjustment be influenced by previous price acceleration. Second, we impose certain excluding restrictions on an attractor and on the dynamics. Then we get a special case of our model where inflation is stable only for a certain rate of unemployment, i.e. a NAIRU. With limited causality and reduced dynamics the NAIRU "rises again". The reason is that unemployment in this version of the model has to take over the stabilizing function of the variables that the restrictions have excluded. This paper shows that dynamics can be of qualitative as well as quantitative importance. A more interpretative conclusion is that dynamic interaction between economic agents may deserve as much attention as their individual and the economic framework.

The rest of the paper is organized as follows. In section 2 we review the heuristics of the ICM model, while section 3 contains our formalization of the dynamics and the analysis of the stability properties of our dynamic ICM. The accelerationistic version of the model is presented in section 3.5. In the concluding section 4 we briefly discuss the relevance of our formalization for empirical work, which we think may be quite large.

2 The heuristic argument

The basic argument for the NAIRU is that unemployment serves as an arbiter between conflicting real-wage claims from wage-earners and firms. The wage-price spiral will lead to increasing inflation if unemployment is too low or falling inflation if unemployment is too high. The NAIRU is the particular unemployment rate that equilibrates the real-wage claims and stabilizes inflation. This was succinctly expressed by Layard et al. (1994, p. 18, authors' italics):

> Only if the real wage (W/P) desired by wage setters is the same as that desired by price setters will inflation be stable. *And, the variable that brings about this consistency is the level of unemployment. ... Thus, inflation will be stable only if unemployment is at the appropriate equilibrium level.*

The economic argument is developed from two theoretical propositions about price and wage setting, often referred to as the price-curve and the wage-curve. In log-linearized form, the price setting equation is

$$q^f = m_q + w - a - \vartheta\, u, \tag{1}$$

with $m_q > 0$ and $\vartheta \geq 0$. The variable q^f in (1) refers to the theoretical price index determined by monopolistic firms in a situation characterized by known and stable growth in the hourly wage w, and in average labour productivity a. The profit maximizing conditions imply that the mark-up coefficient m_q is positive. We follow custom and approximate marginal labour costs with $w - a - \vartheta u$. With reference to Okun's law[3], we interpret the rate of unemployment u as a proxy for capacity utilization. The often relevant case of $\vartheta = 0$ is called *normal cost pricing.*

Turning to the wage formation, we first note that there are several theoretical models that have an important common implication: workers' real-wage claims are negatively related to the rate of unemployment. Such wage aspirations can result from aggregation from individual wage agreements. But the institutional framework that we have in mind is one where both claims and decisions about the real wage are moulded through a process of collective bargaining. Therefore we define w^b as the 'bargained wage', which is

$$w^b = m_w + q + a - \varpi\, u + \omega\,(p - q)\,, \tag{2}$$

with $m_w > 0$, $\varpi \geq 0$ and $0 \leq \omega \leq 1$. The right hand side contains the variables that are expected to systematically influence on the bargained wage. The producer price q and productivity a are central variables in the model of wage formation. This is well established theoretically, by e.g., Nymoen and Rødseth (2003) and Forslund et al. (2008). These variables are also found to be the main empirical determinants of the secular growth in wages in bargaining based systems. The elasticity of q and a have been set to 1 in (2), with reference to homogeneity of degree 1 in nominal variables.

The impact of unemployment on the bargained wage is given by the elasticity $-\varpi \leq 0$. Blanchflower and Oswald (1994) provide evidence for the existence of an empirical law that the value of ϖ is 0.1, which is the slope coefficient of their wage-curve. Other authors instead emphasize that the slope of the wage-curve is likely to depend on the level of aggregation and on institutional factors. For example, one influential view holds that economies with a high level of coordination and centralization is expected to be characterized by a more sensitive responsiveness to unemployment (a higher ϖ) than uncoordinated systems, which give little incentive to solidarity in wage bargaining, cf. Layard et al. (2005, Ch. 8).

Finally, equation (2) is seen to include the variable $p - q$, where p is the (log of) the consumer price index. Because $p - q$ is the difference between producer real wage $w - q$ and the consumer real wage $w - p$, this variable is referred to as the *wedge:* $we \equiv p - q$. The elasticity of the wedge is ω. The theoretical status of the wedge is not entirely clear. For example, one implication of the theory of collective bargaining (between labour unions and profit maximizing firms) is that the consumer price p plays no role in determining the bargaining outcome, see Forslund et al. (2008). The crux of the argument is that wage bargaining is first and foremost about sharing of the value-added created by capital and labour. All other considerations are of secondary importance in that theory. That be so implies $\omega = 0$ in (2). However, we shall see that the wedge ($\omega > 0$) may serve a similar arbiter role between conflicting real-wage aspirations as the unemployment rate.

It is not clear that the same bargaining model is equally relevant for understanding wage setting in all sectors of the economy. In the service sectors, where unions may have little bargaining power, wage setting may be dominated by so called efficiency wage considerations. Interestingly, efficiency wage theory has qualitatively the same implications as the bargaining model. Equation (2) is consistent with both theories, but the hypothesized magnitude of the coefficients are different: The efficiency wage model predicts a larger role for cost-of-living considerations. Thus $\omega > 0$ may be used as one characteristic of efficiency wage models. In summary, since we have in mind a model of the total economy it is relevant to consider the behaviour of the model both when $\omega = 0$, and $0 < \omega < 1$.[4]

[3] Okun's "law" refers to the observed regularity that a change in unemployment is inversely proportional to a change in real output, cf. e.g. Prachowny (1993).

[4] In empirical studies of wage setting in manufacturing in the Nordic counrties, where union-firm bargaining dominates, the hypothesis of $\omega = 0$, is typically not rejected, see e.g., Nymoen and Rødseth (2003).

The bargaining model interpretation also tells us that the mark-up term m_w is conditioned by different institutional features of the wage-setting system. For example, a highly coordinated or centralized system may be associated with a lower m_w, all things equal, than in a system with little coordination between the different industries of the economy. Hence, institutional changes can lead to shifts in the optimal wage level.

Since both (1) and (2) are homogenous of degree one in the nominal variables, we can use them to define the real wage consistent with wage setting:

$$rw^b \equiv w^b - q = m_w + \omega\, we + a - \varpi\, u \tag{3}$$

where $we \equiv p - q$, and price setting:

$$rw^f \equiv w - q^f = -m_q + a + \vartheta\, u. \tag{4}$$

Equation (3) represents the algebraic counterpart to the downward sloping curve in the first graph in Figure 1, while (4) represents the upward sloping curve. For a given rate of unemployment u, and without making further assumptions, there is no reason why $rw^b = rw^f$. However, there are two additional elements in the Layard-Nickell model. First, that no equilibrium with a constant rate of inflation is possible without the condition $rw_t^b = rw_t^f$. The equilibrium rate of unemployment is found as u^* in Figure 1. Second, the adjustment of the unemployment rate is the singular equilibrating mechanism that brings about the necessary equalization of the competing claims.

Even though it is the wage and price *levels* that appear as variables in the model, the heuristical explanation for the behaviour outside equilibrium is in terms of *changes* in these two nominal variables: When the unemployment rate is too low for equilibrium to exist: $u < u^*$ and $rw^b > rw^f$, then wage and price *growth* will be increasing. Hence, inflation will be rising. Conversely, $u < u^*$ is assumed to concur with falling inflation. Only when $u = u^*$ and $rw^b = rw^f$ will the rates of change in wages and prices be constant from one time period to the next. A common interpretation is that inconsistent wage and price *expectations* of wage and price setters result in (de)accelerating wage and prices, and that expectations remain inconstent as long as $rw_t^b \neq rw_t^f$. Heuristically, the only way of maintaining a steady-state with constant inflation (non-accelerating price level) is by securing that $rw^b = rw_t^f$. It is unemployment that reconciles the claims, see Layard et al. (1994, Ch. 3), and brings about the equlibrium.

The "regions" with changing inflation are indicated in Figure 1. Since wage and price inflation are both constant when $u = u^*$, the equilibrium unemployment rate u^* is often referred to as the NAIRU level of unemployment. NAIRU stands for the Non Accelerating Inflation Rate of Unemployment.[5] The lower part of Figure 1 illustrates the dynamic argument in the familiar Phillips-curve diagram, and this suggests that the main contribution from the ICM is that it enriches the theory of the determination of u^*, while the implied dynamics is exactly as in the "old" Phillips-curve. However, as we shall show below, when we formulate an explicitly dynamic version of the ICM, there is no one-to-one relationship between ICM dynamics and Phillips curve dynamics. They are in general quite different models of inflation.

The process leading to rising or falling inflation outside equilibrium is called the *wage-price* spiral. When the expected real wage from wage-setting is higher than what the firms' expect (at the given level of unemployment), firms will increase their nominal prices in order to reduce the actual real wage in an attempt to restore profitability. As wage setters

However, in empirical studies that use aggregate (nation wide) data, a positive coefficient of the wedge is typically reported. This may be because other considerations than profit-sharing play an important role in the publics sector and in some private sectors, i.e. efficiency wages, or because product prices and productivity may be poorly measured.

[5] As many writers have commented, NAIRU is still a misnomer, since it is prices and wages that (de)accelerate outside equilibrium, not inflation.

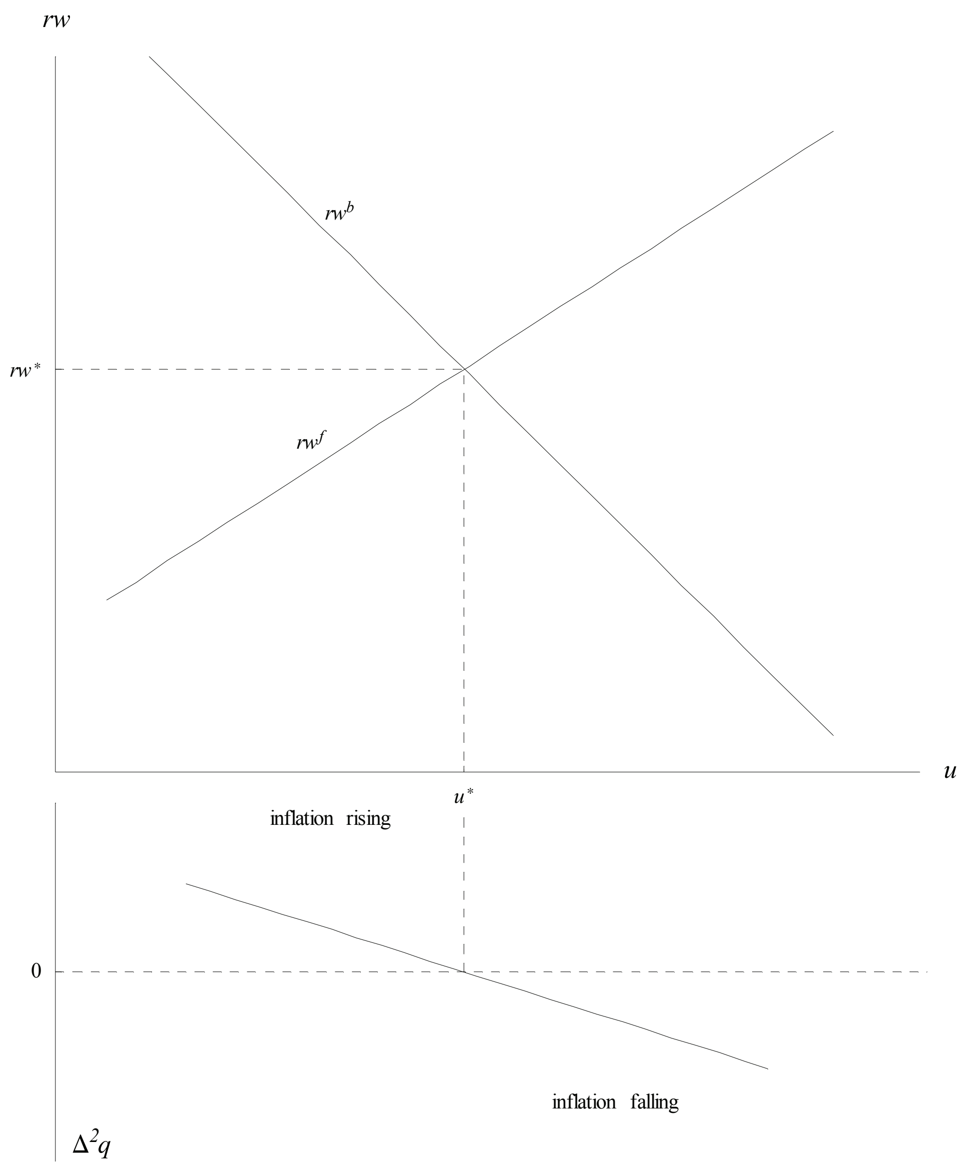

Figure 1: Static determination of equilibrium real wage rw^*, unemployment u^* and change in inflation $\Delta^2 q^*$ $(= 0)$.

seek compensation by pressing for higher nominal wages, a wage-price spiral develops with wages and prices chasing each other upwards. It is an important part of the heuristics that wages and prices increase at a faster rate when $u_t < u^*$, which motivates rising inflation in the region of the figure where $u_t < u^*$. However logical this heuristic may appear, it is not inherent in a wage-price spiral. On the contrary, we shall show that there may be alternative equilibrating mechanisms in the wage-price spiral that exempt unemployment from a unique equilibrating rate u^*.

The two real-wage equations can be reformulated in terms of competing wage shares:

$$ws^b \equiv rw^b - a = m_w + \omega\, we - \varpi\, u \quad \text{and} \quad ws^f \equiv rw^f - a = -m_q + \vartheta\, u, \tag{5}$$

which show that the model can be interpreted in terms of competing claims on GDP. In this interpretation, the labelling of the vertical axis in the first graph in Figure 1 can be changed from real wage to wage share. One benefit of this interpretation is that the wage-setting curve ws^b and the price-setting curve ws^f are invariant or Hicks-neutral: they do not shift when productivity a changes over time. Consequently, the equilibrium rate of unemployment is also independent of the productivity level.

According to the heuristics of the theory there is another equally important spiral which equilibrium unemployment eliminates: the wage-wage spiral. If unemployment is too low (below u^*), wage-setters will try to raise their relative wage (rw or ws), and only if unemployment is high enough will this wage-contamination process be halted. In equilibrium, unemployment must be high enough to balance both types of competing claims.

The heuristics have been successful in shaping the belief that European unemployment is mainly an equilibrium phenomenon and that policies to reduce unemployment should target the socioeconomic determinants of the slopes of the wage and price curves (ϑ and in particular ϖ) and the mark-up coefficients (m_q and in particular m_w). This has given an important rationale for policies that take a structural and more long-term view on the unemployment problem than the earlier policies that analysed unemployment within a more short-term Keynesian framework. By the same argument, any concerns for negative long-term effects of real-demand shocks on unemployment have been seen as largely misplaced.

3 Modelling the heuristic dynamics of the wage-curve model

3.1 Equilibrium unemployment

The heuristics of the ICM equilibrium unemployment theory is dynamics, although the two central equations (3) and (4) are static. Hence, the relevant interpretation of the model is not that $rw_t^b = rw_t^f$ in each time period, denoted by subscript t. Rather, if all temporary shocks were switched off, then

$$u_t \underset{t\to\infty}{\longrightarrow} u^* \Longrightarrow \left| rw_t^b - rw_t^f \right| \underset{t\to\infty}{\longrightarrow} 0$$

Unemployment moving toward its natural rate causes the two real-wage claims to converge.

A sensible interpretation of (3)-(4) is that they represent steady-state relationships:

$$rw_{ss}^b(t) = m_w + \omega\, we_{ss}(t) + a_{ss}(t) - \varpi u^* \quad \text{and} \quad rw_{ss}^f(t) = -m_q + a_{ss}(t) + \vartheta u^*,$$

where $p_{ss}(t)$, $q_{ss}(t)$ and $a_{ss}(t)$ are smooth and trending deterministic steady-state growth paths. Since productivity is exogenous and assumed to be integrated of order 1, we can write $a_{ss}(t) = g_a t$, where g_a is the exogenous productivity growth rate, and a_0 has been set to zero. To preclude from the outset a deterministic trend in the steady-state wage-share: $ws_{ss}(t) \equiv rw_{ss}(t) - a_{ss}(t)$ or steady-state wedge $we_{ss}(t) \equiv p_{ss}(t) - q_{ss}(t)$, we impose $ws_{ss}(t) = ws^*$. That implies $rw_{ss}(t) - rw_{ss}(t-1) = g_a$, and $we_{ss}(t) = we^*$. The model of the steady state is now

$$ws^{b*} = m_w + \omega\, we^* - \varpi u^* \quad \text{and} \quad ws^{f*} \equiv -m_q + \vartheta u^*. \tag{6}$$

Setting $ws^{b*} = ws^{f*} \equiv ws^*$, we now have three steady-state variables ws^*, we^* and u^* and two equations. Without further assumptions, the steady-state is undetermined.

There are two solution to this problem. First, one can invoke the idea that the steady-state value of the wedge is determined 'from outside'. Usually this is done by requiring that the current account has to be in balance in a macroeconomic equilibrium situation, and assuming that this requirement determines an equilibrium real exchange rate, and thus a unique steady-state wedge we^* that can be taken as an exogenous variable in (6), see Wright (1992). The expression for the NAIRU then becomes:

$$u^* = \frac{m_q + m_w}{\vartheta + \varpi} + \frac{\omega}{\vartheta + \varpi} we^* \tag{7}$$

As mentioned above there are disagreement about the relevance of the wedge variable, and subject to $\omega = 0$, the equilibrium rate from the wage-price curve model becomes:

$$u^* = (m_q + m_w) \,/\, (\vartheta + \varpi) \,. \tag{8}$$

In the next sections we investigate whether a dynamic wage-price model implies a steady-state rate of unemployment which is equal to the ICM/wage-curve NAIRU (7) or (8), and we will refer to both expressions as we proceed.

3.2 The accelerationist view and a non-accelerationist alternative

Before we move to the model of the wage-price spiral, it can perhaps help to fix ideas that the heuristics of the ICM-NAIRU model is consistent with the *accelerationist view*, which means, following Solow (1999), that

> ...there is a degree of supply-demand balance of the economy as a whole, measured by the unemployment rate although capacity utilization or output-gap can also be used, with the property that inflation speeds up if the economy is tighter and decelerates if the economy is slacker. That special state of the real economy is usually called the 'natural rate' of unemployment, or the NAIRU.[6]

Clearly, this view captures the gist of the ICM model to the point. It motivates the formulation of an *accelerationist* model of the wage-price spiral. That model should have the properties

$$u_t \underset{t\to\infty}{\longrightarrow} u^N \Longleftrightarrow \Delta^2 q_t \underset{t\to\infty}{\longrightarrow} 0, \tag{9}$$

where $\Delta q_t = q_t - q_{t-1}$ and where $\Delta^2 q_t = \Delta\Delta q_t = \Delta q_t - \Delta q_{t-1}$ represents inflation. In words: a stable dynamic process for unemployment is impossible without constant inflation, or conversely "inflation will be stable only if unemployment is at the appropriate equilibrium level" (Layard et al. (1994, p 18)). We present such a model in section 3.5.

However, the problem with only analyzing the accelerationist version of the wage-price spiral is that other relevant equilibrium paths for the rate of uenmployment may be overlooked. As shown in Kolsrud and Nymoen (1998), a model of the wage price spiral can be formulated where the existence of a steady-state for the wage share and constancy of inflation does not depend on a particular long-run level of unemployment. Hence, the more general property contained in that dynamic model of the wage-price spiral is not (9), but instead

$$u_t \underset{t\to\infty}{\longrightarrow} \overline{u} \Longrightarrow \Delta^2 q_t \underset{t\to\infty}{\longrightarrow} 0. \tag{10}$$

where $\overline{u}$ denotes any constant rate of unemployment. With reference to (10), the accelerationist view that the dynamic stability of the wage-price spiral requires that unemployment has "natural rate dynamics" is refuted. Hence, instead of asserting either the non-accelerationist or the accelerationist version from the outset, we next formulate a dynamic model of the wage-price spiral that accommodates both views.

3.3 A dynamic model of the wage-price spiral

In this section we incorporate the two real-wage goals (3) and (4) in a dynamic model that describe adjustments to the nominal wage w_t and to the producer price q_t, and thus adjustments to the real wage $wq_t = w_t - q_t$. The upper panel in Figure 1 shows the two real-wage goals as long-run or static functions of unemployment u. In that static picture the dynamic versions of the real wage can be thought of as a time path for the realized real wage superimposed on the two real-wage lines already drawn. The question is then: does the dynamics reflect the heuristics? Do the model dynamics converge to the static solution (u^*, rw^*)? Is there a unique NAIRU $= u^*$, or does the dynamic model possess other mechanisms for reconciling the real-wage conflict?

[6] Solow is not comfortable with the accelerationist model. He argued that the evidence of the model is extremely limited.

The model of the wage-price spiral we have in mind is a simultaneous equations model for the nominal wage and price adjustments. Simultaneity is used as a convenient way of representing the two-way causation between wage and price. For simplicity we ignore error terms since they serve no purpose in this paper. The model is

$$\Delta q_t = c_q + \psi_{qw}\Delta w_t + \psi_{qpi}\Delta pi_t - \psi_{qu}u_t + \theta_q ecm^f_{t-1} + \psi_{qq}\Delta^2 q_{t-1}, \quad (11)$$

$$\Delta w_t = c_w + \psi_{wq}\Delta q_t + \psi_{wp}\Delta p_t - \psi_{wu}u_t - \theta_w ecm^b_{t-1}, \quad (12)$$

$$\Delta p_t = \phi\Delta q_t + (1-\phi)\Delta pi_t, \quad (13)$$

where pi is an import price index. All parameters are non-negative, and $0 < \phi < 1$ reflects the openness of the economy[7]. To keep the model small and focused we do not include an equation for pi. We assume that pi_t, like productivity a, follows a random walk with a positive drift parameter g_{pi}. Hence both import prices and productivity are exogenous and non-stationary variables in the model.

To keep the model simple and analytically tractable we have limited the dynamics to a single lag, with one exception. We represent price-acceleration explicitly by including $\Delta^2 q_{t-1} \equiv \Delta q_{t-1} - \Delta q_{t-2}$, so that previous period's price acceleration may stimulate the current price change (11). A more general model could include both leads and more lags of the variables due to expectations and overlapping contracts, but such a model would be analytically too complicated.

The wage- and price-setting curves in Figure 1 are affecting the actual wage and price. The curves express workers' and firms' real-wage goals, rw^f_t and rw^b_t, and serve as attractors in the growth process of nominal price (11) and nominal wage (12) through the equilibrium correcting terms

$$ecm^f_t \equiv rw_t - rw^f_t = q^f_t - q_t = (w_t - q_t - a_t) - \vartheta u_t + m_q, \quad (14)$$

$$ecm^b_t \equiv rw_t - rw^b_t = w_t - w^b_t = (w_t - q_t - \iota a_t) - \omega(p_t - q_t) + \varpi u_t - m_w. \quad (15)$$

If $\theta_w = \theta_q = 0$, we have a Phillips curve model (PCM). If $\theta_w > 0$ or $\theta_q > 0$, one or both of the theoretical variables rw^b_t and rw^f_t of the Layard-Nickell ICM enter as attractors in an equilibrium correction formulation.

The model (11)-(13) has four contemporaneous endogenous variables: Δw_t, Δp_t, Δq_t and u_t, but only three equations. Without further assumptions or restrictions, the model is undetermined. Next, we discuss two determined models. The first represents the non-accelerationist view, while the second is an attempt to represent the accelerationist view which is central in the heuristics of the natural rate.

3.4 A non-accelerationist model

In a stable non-accelerationist model there is per definition no need of a term including accelerating prices $\Delta^2 q_{t-1}$, and we set $\psi_{qq} = 0$. For simplicity, and with no loss of generality, we also set $\psi_{wu} = \psi_{qu} = 0$, so that only the lagged unemployment rate enters as a variable in (11)-(13) via the equilibrium correction terms ecm^f_{t-1} and ecm^b_{t-1}. We first consider the case where unemployment follows a process which is exogenous with respect to the outcome of the wage-price spiral: unemployment is not Granger-caused by inflation or any derived variables such as the wage share. This is not meant as a model that we would preferer to use for forecasting or policy analysis, since there are so many channels through which unemployment becomes linked to real wages and the real excange rate. We will use it to show by a *counter example* that there is in general no reason to believe that there is a singular natural rate of unemployment which is consistent with stable inflation in an acceptable model of the wage-price spiral.

[7] Note that, due to the log-form, $\phi = im/(1 - im)$ where im the import share in private consumption.

We also review the non-accelerationist solution when unemployment is Granger-caused in the sense just mentioned, see Kolsrud and Nymoen (2010).[8]

3.4.1 The solution with exogenous (targeted) unemployment

The key to the full dynamic solution of the wage-price inflation model (11)-(13) is to re-express the model as a dynamic system that determines time paths for two *real* variables: the wage-share ws and the real exchange rate $re \equiv pi - q$. When re is determined, the wedge variable $we = p - q$ is also determined, since $we = (1-\phi)\, re$. In the Layard-Nickell ICM, the parameter restriction $\iota = 1$ eliminates a deterministic trend from the equilibrium rate of unemployment. The restriction $\iota = 1$ plays an important role in the wage-price spiral model (11)-(13) as well, and that is to eliminate a deterministic trend in the wage share ws_t. The trend would result if the wage and price adjustments were influenced by two different productivity corrected real-wages, i.e. Δq being influenced by $w - q - a$ in (11) while Δw were influenced by $w - q - \iota\, a$ (with $i < 1$) in (12). Hence, in the following, we condition on $\iota = 1$, and (11) and (12) become

$$\begin{aligned} \Delta q_t &= (c_q + \theta_q m_q) + \psi_{qw}\,\Delta w_t + \psi_{qpi}\,\Delta pi_t - \theta_q \vartheta\, u_{t-1} + \theta_q\, ws_{t-1}, && (16)\\ \Delta w_t &= (c_w + \theta_w m_w) + \psi_{wq}\,\Delta q_t + \psi_{wp}\,\Delta p_t - \theta_w\, u_{t-1} - \theta_w \varpi\, ws_{t-1} + \theta_w\,\omega\, we_{t-1}, && (17) \end{aligned}$$

Importantly, u_{t-1} is a predetermined and exogenous variable in the model. If the model was accelerationist, the system would only be stable if (by coincidence) $u_t \rightarrow u^N$ as $t \rightarrow \infty$. But, as the analysis in Kolsrud and Nymoen (2010) shows, the solutions for ws_t and re_t are dynamically stable for any stationary rate of unemployment. Their conditional expectations converge to constant levels that are independent of starting values, as long as the following sufficient conditions hold simultaneously: (i) wage growth is influenced by the wage share: $\theta_w > 0$, and the wedge: $\omega > 0$, (ii) price adjustments are influenced by the wage-share: $\theta_q > 0$, or the wage growth: $\psi_{qw} > 0$, (iii) there is *not* both a price Phillips curve ($\theta_q = 0$) and full pass-through of wage growth ($\psi_{qw} = 1$), hence $\theta_q + (1 - \psi_{qw}) > 0$, and (iv) there is *not* a certain kind of dynamic wage and price homogeneity, hence $\psi_{qw}(\phi\psi_{wp} + \psi_{wq}) < 1$. These stability restrictions can be combined and simplified to a single inequality:

$$0 < \theta_w\,\omega\,\left[\theta_q + \psi_{qw}(1-\psi_{qw})\right]\left(1 - \psi_{qw}[\phi\psi_{wp0} + \psi_{wq}]\right), \tag{18}$$

which does not place any restrictions on the path followed by u_t. As long as u_t follows *any* linear stochastic process that is stationary, the system (11)-(13) is stable in the sense that the steady-state wage and price growth rates align with the steady-state growth rates of productivity (g_a) and foreign prices (g_{pi}) to make the steady-state real exchange rate and the wage share stable:

$$\begin{aligned} re_{ss} &= e_{ss}\, g_{pi} + b_{ss}\, g_a + \frac{\vartheta + \varpi}{\omega(1-\phi)}\, u^* - d_{ss}, && (19)\\ ws_{ss} &= \xi_{ss}\, g_{pi} - \beta_{ss}\, g_a + \vartheta\, u^* - \delta_{ss}. && (20) \end{aligned}$$

The composite coefficients and constant terms (with subscript ss) are all positive except b_{ss}, which may be zero or negative for non-realistical parameter values. It also follows from the definition of the real exchange rate and the wage share that the dynamically stable steady-state values of price and wage inflation are $\mathsf{E}[\Delta p_t] = \mathsf{E}[\Delta q_t] = \mathsf{E}[\Delta pi_t] = g_{pi}$, and $\mathsf{E}[\Delta w_t] = \mathsf{E}[\Delta q_t] + \mathsf{E}[\Delta a_t] = g_{pi} + g_a$. Different stable unemployment rates cause different stable real exchange rates and different stable wage shares, but steady-state domestic inflation remains equal to steady-state foreign inflation.

[8] They extend the results in Bårdsen et al. (2005, Ch. 6) and Kolsrud and Nymoen (1998)

In the wage-price spiral (16)-(17) it is not the unemployment level that checks the wage and price growth rates so that the wage share ws and the real exchange rate $re \equiv pi - q$ are stable. The stabilizing mechanism is the wage share $w - q - a$ and the wedge $p - q$ (which is proportional to the real exchange rate $re = we/(1 - \phi)$) influencing the growth in their constituent variables w and q. The wage share affects the growth in both w and q, while the real exchange rate influences the growth in w and *through simultaneity* also the growth in q. Since the levels of the two real variables (ws, re) affect the growth of their constituent endogenous variables (w, q) the real variables themselves become stable. Hence, it is the dynamic nominal adjustment process informed by (whatever) real levels that is the stabilizing mechanism. The actual levels of the stable ws and rw depend on the exogenous growth rates g_{pi} and g_a, and on all parameters in the wage-price spiral (11)-(13). Stability in a dynamic model is clearly a more complicated and less intuitive concept than stability in a static model, reflected by Figure 1 That is illustrated by Figure 2, which shows dynamic simulations of the non-accelerationsist model with exogenous unemployment.

A no-wedge restriction $\omega = 0$ in the wage-price spiral prevents domestic inflation from aligning with foreign inflation, hence $\mathsf{E}[\Delta p_t] \neq \mathsf{E}[\Delta q_t] \neq \mathsf{E}[\Delta pi_t] = g_{pi}$. This makes the real exchange rate re_t trend and the steady-state expression (19) invalid, while the wage share is stable.

3.4.2 Solution with endogenous unemployment

Since equations (3)-(4) correspond to the wage and price setting curves in Figure 1, it is tempting to extend the system (11)-(13) with the unemployment equation

$$u_t = c_u + \alpha\, u_{t-1} + \theta_u(rw^b_{t-1} - rw^f_{t-1}), \quad \text{with } 0 < \theta_u < 1, \tag{21}$$

since this seems to capture the heuristics: u_t increases when $rw^b_{t-1} > rw^f_{t-1}$, reduces when $rw^b_{t-1} < rw^f_{t-1}$, and is stable if $rw^b_{t-1} = rw^f_{t-1}$. However, after substituting for the two real-wage goals, the resulting dynamic equation $u_t = [\alpha - \theta_u(\varpi + \vartheta)]\, u_{t-1} + \theta_u \omega\, we_{t-1} + \theta_u(m_w + m_q)$ shows a positive effect of the wedge on unemployment. It reflects directly that the wedge increases the workers' real-wage goal rw^b_{t-1} but does not affect the producers' real-wage goal rw^f_{t-1}. This property of the attractors (the static real-wage goals) should not dominate the dynamics of unemployment in a way which does not make economic sense. The wedge is proportional to the real exchange rate or price competitiveness: $we = (1 - \phi)re$, and a real depreciation should reduce rather than increase unemployment. This example suggests that heuristics (as in section 2) may be misleading in dynamic models.

Simultaneity in the wage-price spiral (16)-(17) makes the wedge influence both the nominal wage w_t and the producer price q_t, and thus both real-wage goals $rw^b_t \equiv w^b_t - q_t$ and $rw^f_t \equiv w_t - q^f_t$ (the time subscript makes either goal a dynamic variable which changes with the development of the whole dynamic system). Instead of closing the model with (21), we use the following very simple unemployment equation where the real exchange rate has the "right" sign:

$$u_t = c_u + \alpha\, u_{t-1} - \rho\, re_{t-1} + \varepsilon_{u,t}, \quad \text{with } 0 < \alpha < 1 \text{ and } \rho > 0. \tag{22}$$

Kolsrud and Nymoen (2010) rationalize this intuitively plausible reduced form equation by job search theory and the concepts of matching and job destruction. They calculate the following steady-state when (18) holds:

$$
\begin{aligned}
re &= e'_{ss}\, g_{pi} + \underset{\pm}{b'_{ss}}\, g_a + \underset{\pm}{d'_{ss}}, &(23)\\
ws &= \underset{\pm}{\xi'_{ss}}\, g_{pi} - \beta'_{ss}\, g_a - \underset{\pm}{\delta'_{ss}}, &(24)\\
u &= -\,\mathfrak{e}_{ss}\, g_{pi} - \underset{\pm}{\mathfrak{b}_{ss}}\, g_a + \mathfrak{d}_{ss}. &(25)
\end{aligned}
$$

Again, all composite coefficients are positive, except where the label $\pm$ indicates that a zero value or a negative sign is numerically possible. In the unemployment equation (25) the coefficients are

$$
\begin{aligned}
\mathfrak{e}_{ss} &= \rho\,[\theta_q(1-\psi_{wq}-\psi_{wp}) + \theta_w(1-\psi_{qw}-\psi_{qpi})]/(\theta_q\,\theta_w\,\Omega),\\
\mathfrak{b}_{ss} &= \rho\,(\theta_q - \theta_w\,\psi_{qw})/(\theta_q\,\theta_w\,\Omega),\\
\mathfrak{d}_{ss} &= [c_u\,\omega(1-\phi) + \rho\,(m_w + m_q + c_w/\theta_w + c_q/\theta_q)]/\Omega,
\end{aligned}
$$

and $\Omega = \omega\,(1-\phi)\,(1-\alpha) + \rho\,(\varpi + \vartheta)$. Contrary to the case with exogenous unemployment, a no-wedge restriction $\omega = 0$ does not make the real exchange rate trend. The reason is that the real exchange rate influences producer price growth through endogenous unemployment (22).

When unemployment (22) interacts with the wage and price formation, there is an equilibrium rate of unemployment given by (25). It depends on the trends in foreign inflation and domestic productivity, and *all the* parameters of the model. The latter means that short-term dynamics and long-term stability are not independent. Rigidities or frictions that cause partial and lagged adjustments of wage and prices and unemployment in the short-run also influence the long-run levels of the real variables (23)-(25). In the dynamic model (13), (16), (17) and (22), there is no "natural" level of unemployment that depends only on structural elements like technology and preferences, represented by the parameters in the price and wage goals (1)-(2): m_q, ϑ and $m_w, \iota = 1, \varpi, \omega$. The equilibrium or long-run level of unemployment also depends on the dynamic interaction of the economic agents (producers, workers/unions, etc.), represented by parameters of change-variables, like $\theta_q, \psi_{wq}, \psi_{wp}$ and $\theta_w, \psi_{qw}, \psi_{qpi}$. Finally, the long-run depends on the openness of the economy ϕ, and on the unemployment process c_u, α and ρ.

Figure 2 shows four dynamic simulations of the non-accelerationsist model: with exogenous and endogenous unemployment, and without and with random shocks. All simulations start from the same equilibrium, and are subject to the same permanent exogenous shock to unemployment, by a change in the constant c_u. The panels show bivariate time-trajectories for (u_t, ws_t) and $(u_t, \Delta^2 q_t)$. All models simulations use the same parameterization, which is guided by econometric results in Bårdsen et al. (2005). The parameter values are: $m_q = 0.31$, $\vartheta = 0.065$, $m_w = 0.46$, $\varpi = 0.1$, $\omega = 0.5$, $\theta_q = 0.13, \psi_{qw} = 0.4, \psi_{qpi} = 0.4, \theta_w = 0.12$, $\psi_{wq} = 0.5$, $\psi_{wp} = 0.2$, $\phi = 0.6$, $\alpha = 0.85$ and $\rho = 0.1$. The permanent shock to unemployment is of size 0.1, and shifts c_u from one value to another. The values of the constants c_q, c_w and c_u differ between the simulations with exogenous unemployment and the simulations with exogenous unemployment in order for all simulations to start from the same equilibrium. Figure 2 clearly illustrates that there is no singular NAIRU.

3.5 An accelerationist model of the wage-price spiral

The previous section showed that a model of the wage-price spiral that incorporates the wage and price setting curves of the ICM as attractors for the adjustments does not have accelerationistic properties: the rate of unemployment may follow any stable dynamic process with a long-run mean $\bar{u}$. A structural break that changes the steady-state level

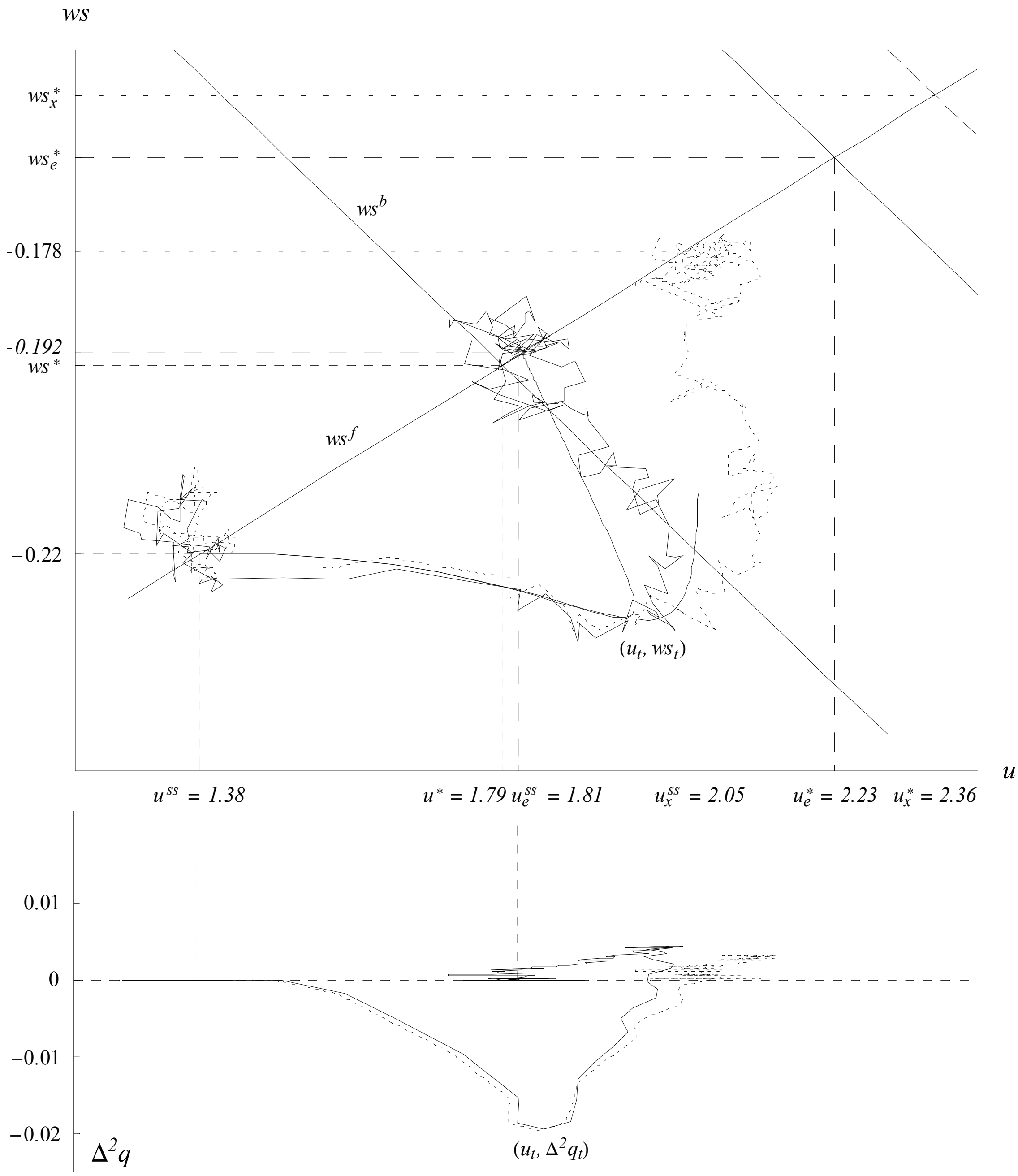

Figure 2: Phase diagrams of dynamic versus static determination of equilibria in the non-accelerationist model. Figure 2 *is* Figure 1 with model simulations superimposed. In the upper panel the static wage curve ws^b is shifted rightward all the way to the dashed line by a permanent shock to exogenous unemployment (22) ($\rho = 0$ makes u_t autonomous and thus exogenous, denoted by subscript x). With endogenous unemployment (subscript e) the same shift to c_u in (22) (now with $\rho > 0$) is partly countered by a change in the equilibrium wedge we, hence ws^b shifts less, to the solid line.

The upper panel shows four dynamic simulations, all with the same plausible parameterization guided by econometric results. The simulations differ by exogenous vs. endogenous unemployment, and none vs. random shocks. All simulations start in the steady-state $(u^*, ws^*) = (1.38, -0.22)$. The simulations are displayed as curves traced by the pair (u_t, ws_t) through time as the variables move from one steady-state to another in response to the shock to unemployment. The smooth curves are simulations without random shocks, while the ragged graphs are simulations with shocks. The leftmost and heavier graphs are simulations with endogenous unemployment, while the rightmost and lighter graphs are simulations with exogenous unemployment. Note that the dynamic equilibrium (u^{ss}, ws^{ss}) is considerably less than the static equilibrium (u^*, ws^*), and that the dynamic equilibrium is close to the price curve ws^f and far from the wage curve ws^b, both with endogenous and exogenous unemployment. Note that the static equilibrium is never an attractor for the dynamic process.

The lower panel shows the corresponding time-trajectories of the unemployment-inflation pair $(u_t, \Delta^2 q_t)$. Inflation is unstable only during the transition between the equilibria.

of unemployment will influence the solution and the steady state values of the wage share and the real exchange rate, but the steady-state rate of inflation will not be subject to a structural break[9]. The interactive wage and price adjustment prosesses (16)-(17) relieve unemployment of mitigating the conflict between the real-wage goals.
In this section we still incorporate the real-wage conflict into the model, but not only in the form of competing real-wage goals. We also let the real-wage conflict be expressed by the dynamics. In order to do that and at the same time keep the model simple and managable, we work only with the wage-price spiral (11)-(13), and include no separate unemployment equation like (22). Instead we consider $\psi_{qu} > 0$ and/or $\psi_{wu} > 0$ in (11)-(13). Then the current rate of unemployment enters as a variable in the wage-price spiral in addition to the lagged rate.

The model of the wage price spiral then has four contemporaneous endogenous variables: Δw_t, $\Delta p_t, \Delta q_t$ and u_t, but only three equations. Hence, the model is undetermined. Formalization of the heuristics of the ICM therefore requires restrictions on the wage-price spiral. One possibility is to set $\theta_q = \theta_w = 0$ which gives a Phillips Curve Model (PCM). If the PCM is restricted so that the implied long-run Phillips curve is vertical, and unemployment is postulated to be a function of the lagged wage share, then the rate of unemployment will have natural-rate dynamics, see Bårdsen et al. (2005, Ch. 4.2) This gives an accelerationist model which corresponds to the standard AD-AS model in macroeconomic textbooks, and also the north-American triangular model of inflation Gordon (1997). Intermediate cases are also possible. For example, Bårdsen and Nymoen (2009) show that with $\theta_q = 0$ but $\theta_w > 0$, the wage-price spiral is stable, and the steady-state rate of unemployment may be affected by permanent demand shocks.

However, in this paper we seek an accelerationist model with both $\theta_q > 0$ and $\theta_w > 0$, since this ensures that the two wage claims equations are fully integrated in the wage-price spiral, in accordance with the heuristical explanation of the natural rate dynamics. We obtain that accelerationist model by imposing the following restrictions: $\psi_{wq} = \psi_{qw} = 1$ make domestic price and wage changes have simultaneous and full impact on each other, while $\psi_{qpi} = \psi_{wp} = 0$ eliminate direct effects of import-price inflation and consumer-price inflation on the wage inflation and producer-price inflation. These restrictions suffice for the real-wage conflict to bear directly on the dynamics.

The parameter restrictions change the dynamic wage-price spiral to

$$\Delta q_t = -\theta_q(rw^f_{t-1} - rw_{t-1}) + \psi_{qq}\Delta^2 q_{t-1} + \Delta w_t - \psi_{qu}u_t + c_q + \varepsilon_{q,t}, \quad (26)$$
$$\Delta w_t = \theta_w(rw^b_{t-1} - rw_{t-1}) + \Delta q_t - \psi_{wu}u_t + c_w + \varepsilon_{w,t}. \quad (27)$$

Writing the change in the real wage as $\Delta rw_t = \Delta w_t - \Delta q_t$, we obtain:

$$\Delta rw_t = \theta_q(rw^f_{t-1} - rw_{t-1}) - \psi_{qq}\Delta^2 q_{t-1} + \psi_{qu}u_t - c_q - \varepsilon_{q,t}, \quad (28)$$
$$\Delta rw_t = \theta_w(rw^b_{t-1} - rw_{t-1}) - \psi_{wu}u_t + c_w + \varepsilon_{w,t}. \quad (29)$$

Substituting (3) and (4) for the real-wage goals rw^b_{t-1} and rw^f_{t-1}, and ignoring the residual terms, yield

$$rw_t = (1-\theta_q)\, rw_{t-1} + \theta_q\,(-m_q + a_{t-1} + \vartheta u_{t-1}) - \psi_{qq}\Delta^2 q_{t-1} + \psi_{qu}u_t - c_q, \quad (30)$$
$$rw_t = (1-\theta_w)\, rw_{t-1} + \theta_w(m_w + \omega we_{t-1} + a_{t-1} - \varpi u_{t-1}) - \psi_{wu}u_t + c_w. \quad (31)$$

These equations express a conflict between real-wage dynamics as well as a conflict between real-wage claims. The two equations provide solutions for the two contemporaneous

[9] In case of no wedge and exogenous unemployment, the steady-state inflation rate will shift when unemployment does. The no-wedge restriction $\omega = 0$ cancels information about price levels in the wage-price spiral (16)-(17). Then the domestic price q no longer gets leveled with the import price pi, hence the real exchange rate $pi - q$ becomes a trending (non-stationary) variable.

variables, rw_t and u_t given previous period values for all variables. Since price acceleration $\Delta^2 q_{t-1}$ is lagged and thus predetermined, we may say that it is the current rate of unemployment u_t which solves the conflicts.

The model (30)-(31) can be written as a first order vector autoregression (VAR) for ws_t and u_t. The productivity growth Δa_t is exogenous, and the wedge we_{t-1} and the price acceleration $\Delta^2 q_{t-1}$ are predetermined variables in the VAR. Its reduced form is

$$\begin{aligned} ws_t &= r_{11} ws_{t-1} + r_{12} u_{t-1} + p_{11} we_{t-1} - \Delta a_t - p_{12} \Delta^2 q_{t-1} + p_{13}, && (32)\\ u_t &= r_{21} ws_{t-1} - r_{22} u_{t-1} + p_{21} we_{t-1} \qquad + p_{22} \Delta^2 q_{t-1} + p_{23}, && (33) \end{aligned}$$

with coefficients:

$$\begin{array}{ll} r_{11} = 1 - \left[\theta_q \psi_{wu} + \theta_w \psi_{qu}\right]/\Psi > 0, & r_{21} = [\theta_q - \theta_w]/\Psi, \\ r_{12} = \left[\vartheta\, \theta_q \psi_{wu} - \varpi\, \theta_w \psi_{qu}\right]/\Psi, & r_{22} = [\vartheta\, \theta_q + \varpi\, \theta_w]/\Psi > 0, \\ p_{11} = \omega\, \theta_w \psi_{wu}/\Psi \geq 0, & p_{21} = \omega\, \theta_w/\Psi \geq 0, \\ p_{12} = \psi_{wu}\psi_{qq}/\Psi > 0, & p_{22} = \psi_{qq}/\Psi > 0, \\ p_{13} = \left[(c_w + \theta_w m_w)\psi_{qu} - (c_q + \theta_q m_q)\psi_{wu}\right]/\Psi, & p_{23} = [c_w + \theta_w m_w + c_q + \theta_q m_q]/\Psi > 0, \end{array}$$

and $\Psi = \psi_{qu} + \psi_{wu}$. The ICM-restrictions reduce the wage-price spiral (11)-(13) to a model of the wage share ws (32) and the unemployment rate u (33) conditional on exogenous productivity growth Δa and possibly two quasi-exogenous variables, the wedge we and price acceleration $\Delta^2 q$, which are not being determined by the ICM, but are nevertheless closely linked to it. If $\psi_{qq} \Delta^2 q_{t-1} \neq 0$ then $\mathsf{E}[\Delta q_t] \neq \mathsf{E}[pi_t] = g_{pi}$, and the wedge becomes unstable. If $\omega > 0$ the wedge (or the proportional real exchange rate $re = we/(1-\phi)$) also affects the wage share and the unemployment rate, and both become unstable. In the case of no wedge ($p_{11} = 0$), the wage share $ws \equiv w - q - a$ might be stable if nominal wage growth follows price growth, such that $\mathsf{E}[\Delta w_t] = \mathsf{E}[\Delta q_t] + g_a$ and $E\left[\Delta^2 w_t\right] = E\left[\Delta^2 q_t\right]$.

Stability of the wage share and unemployment thus requires $\psi_{qq} = 0$, which is not an option in this accelerationist case, or no wedge[10] ($\omega = 0 \Rightarrow p_{11} = p_{21} = 0$) and $\mathsf{E}[\Delta w_t] = \mathsf{E}[\Delta q_t] + g_a$ and $E\left[\Delta^2 w_t\right] = E\left[\Delta^2 q_t\right]$. These nominal variables are not variables in the ICM (32)-(33). In addition, the eigenvalues of the dynamic mapping of the wage share and unemployment rate,

$$\lambda = \frac{1}{2}\left(r_{11} - r_{22}\right) \pm \frac{1}{2}\left(r_{11} + r_{22}\right)\sqrt{1 + 4\, r_{12} r_{21} / \left(r_{11} + r_{22}\right)^2},$$

both have to be less than 1 in magnitude. That might well be the case for realistic parameter values. If so, the steady-state solutions are

$$ws^* = \beta\, g_a + \gamma\, \Delta^2 q^* - \delta \qquad \text{and} \qquad u^* = b\, g_a - c\, \Delta^2 q^* - d, \tag{34}$$

with $g_a \equiv \mathsf{E}[\Delta a_t]$ and

$$\begin{aligned} \beta &= \left(1 + r_{22}\right)/R = \left(\psi_{qu} + \psi_{wu} + \vartheta\, \theta_q + \varpi\, \theta_w\right)/(R\Psi), \\ \gamma &= [p_{12}(1 + r_{22}) - p_{22} r_{12}]/R = \psi_{qq}\left(\psi_{wu} + \varpi\, \theta_w\right)/(R\Psi); \\ \delta &= \left(p_{13}(1 + r_{22}) + p_{23} r_{12}\right)/R = [(c_w + \theta_w m_w)\left(\psi_{qu} + \vartheta\, \theta_q\right)\Psi \\ &\qquad - \left(c_q + \theta_q m_q\right)\left(\psi_{wu}\Psi + 2\vartheta\, \theta_q \psi_{wu} + \varpi\, \theta_w \left[\psi_{wu} - \psi_{qu}\right]\right)] / \left(R\Psi^2\right), \\ b &= r_{21}/R = \left(\theta_q - \theta_w\right)/(R\Psi), \\ c &= [p_{22}(1 - r_{11}) - p_{12} r_{21}]/R = \psi_{qq}\theta_w/(R\Psi), \\ d &= [p_{23}(1 - r_{11}) + p_{13} r_{21}]/R = [(c_w + \theta_w m_w)\, \theta_q + (c_q + \theta_q m_q)\, \theta_w]/R\Psi, \end{aligned}$$

[10] If wage bargaining is first and foremost about sharing of the value-added created by capital and labour then $\omega = 0$ is a logical implication, see Forslund et al. (2008).

where $R = r_{12}r_{21} - (1 + r_{22})(1 - r_{11}) = 1 - \theta_q\theta_w(\varpi + \vartheta)$. Note that $\Delta^2 q^* \neq 0 \Rightarrow \Delta^2 p^* \neq 0$ according to the consumer price definition (13). Equation (34) says that there is a steady-state rate of unemployment for any constant price acceleration $\Delta^2 q^*$ or $\Delta^2 p^*$. When there is no price acceleration the equation defines what can be interpreted as the NAIRU: $u^N = b\, g_a - d$.

From the inverse relationship between unemployment and change in inflation implied by the unemployment equation (34) we extract the following formalization of the accelerationist view:

$$\Delta^2 q^* = \frac{1}{c}\,(b\, g_a - u^* - d) = -\frac{1}{c}\left(u^* - u^N\right) \begin{cases} > 0 & \text{if } u^* < u^N, \\ = 0 & \text{if } u^* = u^N, \\ < 0 & \text{if } u^* > u^N. \end{cases} \tag{35}$$

4 Discussion

We have seen that care must be taken when formalizing the heuristics of the ICM/wage-price model of unemployment (*aka* Layard-Nickell model). The formalization requires specific restrictions on the model of the wage-price spiral that completely remove nominal rigidity. Without these restrictions, it no longer follows logically that the wage-price spiral model is accelerationistic, and that the rate of unemployment logically has to adjust to a NAIRU in order to stabilize inflation. Adding an independent dynamic unemployment equation to the wage-price spiral gives a model of equilibrium unemployment, but not an accelerationist one. To arrive at an accelerationsist model, the unemployment dynamics must be implied in full by the equations of the (restricted) model of the wage-price spiral.

Our analysis implies that care must be taken when the wage curve theory of equilibrium unemployment is taken to data. The approach that uses time series data to estimate the wage and price schedules in Figure 1, and then proceeds to solve those estimated equations for the NAIRU, rests on bold assumptions about the properties of the model of the wage price spiral, and also on the exact form of how unemployment reacts to the real-wage gaps. Often in empirical studies of this type, the dynamic process for the rate of unemployment is not formulated explicitly, and the necessary conditions for the empirical relevance of natural rate dynamics are therefore not evaluated. For example, finding that the rate of unemployment is weakly exogenous with respect to the parameters of the wage and price equations will refute the accelerationist interpretation. More specifically, the role of the estimated natural rate as an attractor for actual unemployment is often not tested. Since such approaches do not question the nature of the underlying equilibrium mechanism, it is difficult to assess precisely what they find out about equilibrium unemployment.

Another branch of the empirical literature aims at estimating a reduced form of unemployment which is consistent with the heuristics of the model, preferably with macro panel data, in order to have variation in institutional indicators that may influence the wage and price mark-ups and therefore also unemployment. The above analysis is relevant for the specification of these models. For example, the order of dynamics should at least be of second order, since this is implied by first order dynamics in the wage and price equations. We also note that according to our formalization, there is no way to rationalize the inclusion of demand-shocks variables in these econometric equations. This is because the shock terms in the two equations of the wage-price spiral have the interpretation as supply shocks. In the final equation for unemployment, there is no room for other shocks. It is interesting to note that in the existing literature we find that demand shocks are included alongside variables that represent supply shocks, see Nickell et al. (2005). In one sense this only confirms that it is difficult to formulate dynamic models that are consistent with the static ICM framework.

Finally, the analysis has implications for macroeconomic model building. Specifically, we show that the long-run equilibrium unemployment rate is *not* determined by the wage and price setting curves as long as the wage-price spiral is non-accelerationist. That is a good reason for building larger systems of equations, even if the first objective and primary concern is the analysis of wages, prices and unemployment, see Bårdsen et al. (2005) for discussions. Conversely, no inconsistencies or issues about "overdetermination" arise from adding a separate equation for the rate of unemployment, where demand variables may enter, to the wage-price setting equations. The enlarged model will have a steady state, subject to conditions that can be tested. The equilibrium rate of unemployment implied by this type of model is not of the natural rate type, since factors from the demand side may have lasting effects (depending on the nature of the shocks and on policy responses). Apart from in the , The equilibrium rate of unemployment is a system property (except in the accelerationist model of the wage price spiral), and it seems worthwhile to develop that perspective in models of the macroeconomy.

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