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# Correlated Measurement Errors, Bounds on Parameters, and a Model of Producer Behavior

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**Abstract:**
We examine estimation of a model of producer behavior in the presence of correlated measurement errors in the regressors. Scale economies and price-cost margins are estimated from a set of panel data for manufacturing plants. The paper presents a somewhat new model for estimation of these parameters which is highly flexible but with a simple regression structure. Perhaps the most important contribution of the paper is some new results on deriving parameter bounds for a regression model with errors in variables. In particular, we consider the case where the measurement errors might be correlated. We derive asymptotic standard errors for the parameter bounds. These asymptotic standard errors are compared to bootstrap estimates. Our new results on parameter bounds are applied to the estimation of the model of producer behavior.

**Keywords:** Estimation, errors-in-variables, parameter bounds, imperfect competition, scale economies.

**JEL classification:** C13, C29, C43

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# 1 Introduction

Imperfect competition and scale economies have been the focus of a large amount of economic research for decades. Still, the empirical significance of these two phenomena remain controversial questions. There is not even a settled approach to study the empirical importance of imperfect competition and scale economies. Hall (1988, 1990) has presented a new approach to estimating the magnitude of the margin between price and marginal costs, as well as scale economies. This paper merges Hall's framework with the literature on Divisia-index numbers[1] and productivity measurement. The result is an extremely flexible framework with a very simple linear regression structure containing only a few parameters to be estimated. The structural parameters in the final regression equation are directly interpretable as the parameters of our primary interest; the price cost margin and the scale elasticity. The linear and simple structure greatly facilitates the statistical analysis of these parameters.

While Hall (1988, 1990) examined price-cost margins on the assumption of constant returns to scale, our model integrates the study of price-cost margins and scale economies. This is highly desirable, given the close links between the estimates of scale economies and markups. For instance, with price and average costs as the observable point of departure, underestimating the scale economies will imply overestimated marginal costs, providing an underestimated price-marginal cost ratio. Given the large magnitude of the scale economies estimated in Hall's separate study (Hall, 1990), it is somewhat discomforting that Hall retains constant returns to scale as a maintained hypothesis in his parallel study of price-cost margins[2].

Our model is estimated on a set of plant level data. It is by now widely recognized that there is a lot of heterogeneity in the technology and behavior of firms even within narrow industry groups. Griliches and Mairesse (1990) have documented the large amount of heterogeneity and instability in production function parameters. This heterogeneity and instability makes it desirable to move beyond rigid parametric functional forms in studies of firm behavior. The framework presented in this paper is ideally suited to this task. By transforming our variables in the appropriate way, the model is fully consistent with a situation where each plant has its *individual* translog production technology and separate productivity levels. Persistent differences

[1] See e.g. Diewert (1976, 1980).

[2] Given that Hall has used highly aggregate data in his study, the scale economies relevant for the pricing decision might differ from the scale economies at the industry level, due to externalities, entry and exit.

in productivity levels are found in most micro level data sets on plants and firms.

The framework presented below is consistent with capital inputs being adjusted in a suboptimal way, due to e.g. adjustment costs, installation lags and expectational errors. An excessive capital stock, say, will imply low marginal costs. Our estimates of the ratio between price and marginal costs takes into considerations such effects of capital adjustments being out off long run equilibrium.

It is noteworthy that we can obtain such a flexible model within a regression model with only two structural parameters. It is also a desirable property, in our opinion, that the coefficients of scale economies and markups appear as the parameters assumed to be fixed across our sample, rather than, say, parameters in the translog production function which are harder to interpret and which perhaps vary more across firms and over time.

The model has been implemented on a panel data set covering 3101 plants from nine (2.5 digit ISIC) manufacturing industries. We have a balanced panel for two cross sections from 1983 and 1990. Estimating our model on this sample gave us an initial set of estimates which are not implausible, but too low to be entirely acceptable without further inquiry. That is to say, the initial results suggested decreasing returns to scale in the clear majority of the industries examined. None of the industries revealed results suggesting pricing beyond marginal costs. A common explanation for (too) low parameter estimates based on panel data estimation with fixed effects, is that the data are contaminated with measurement errors[3]. Allowing for fixed effects removes one source of bias - the endogeneity of the input variables in the regression - but at the expense of augmenting the problem of measurement errors. The result of measurement errors in the regressors is typically parameter estimates biased towards zero.

To examine the question of biases due to measurement errors, we have used as our point of departure some basic ideas from the literature on identification of parameter bounds in the presence of errors in variables. The basic result in this literature was derived by Reiersøl (1941). He proved that the parameters in a regression model with measurement errors in the regressors, could be bounded by the coefficients from the set of the direct and the reversed regressions[4]. This result has been derived under the assumption of *no correlation* among the measurement errors.

[3]See e.g. Griliches (1986) and Mairesse (1990).

[4]This useful result have been derived in alternative ways by Klepper and Leamer (1984) and Willassen (1987), among others.

No correlation of the measurement errors in the regressors is an untenable assumption in our case. Given the way the model is set up, and the way the variables have been transformed, measurement errors in one regressor will surely affect also the other regressor, as the second regressor is a function of the first regressor and some other variables. Such a situation is clearly not uncommon in applied work. Trivial variable transformations will often create *correlation* in the measurement errors between regressors, given that measurement errors are present in the untransformed variables[5].

A primary contribution of our paper is to prove that parameter bounds can still be identified in the presence of correlated measurement errors. In fact, we show that the ("standard") parameter bounds identified by Reiersøl are still valid when we allow for correlation in the measurement errors. The paper specifies the conditions required for the bounds identified by Reiersøl's to be valid in the more general case when the measurement errors are correlated across regressors. Furthermore, we show that these conditions are empirically testable. Such a test is carried out for our empirical model by menas of a bootstrap procedure. Our test results show that the standard parameter bounds are valid in our case.

It turns out that the standard parameter bounds are too wide to be of much interest in our case. In our data we have access to (only) one instrument, i.e. a variable which can be taken to be uncorrelated with the measurement errors in the regressors, but correlated with at least one of the regressors. Using an idea from Willassen (1987), we show how to combine the information in this single instrument with the "standard" parameter bounds to narrow down the parameter bounds further. We also present the asymptotic theory required to derive standard errors on these new set of (narrower) parameter bounds. Bootstrap estimates of these standard errors are also presented to check the validity of the asymptotic approximations.

Adding the information in our instrumental variable narrows down the parameter bounds substantially. The final set of estimated parameter bounds and their standard errors show that our method comes close to identify the parameter values. More substantially, our parameter bounds show that market power and scale economies of the magnitude identified by Hall (1988, 1990) do not prevail in the industries considered in our study. The results suggest that measurement errors in our regressors gave a downward bias of the ordinary regression results by

[5]Lach (1993) has made a similar point. He examined various implications of correlated measurement errors in the regressors, introduced by one particular variable transformation.

(slightly) less than 5 percent for both of the parameters of interest. Our findings indicate that price taking behavior and constant returns to scale are adequate approximations for the majority of the industries considered. This result is discussed further in the concluding section.

Our paper is organized as follows: The next section presents the theoretical framework and the basic stochastic model which is the basis for our empirical analysis. This section illustrates how simple transformations of the basic variables may induce *correlation* in the measurement errors of the regressors. Section 3, the main statistical section of our paper, proves that the "standard" parameter bounds bracket the true parameter values under more general conditions than stated by Reiersøl (1941). A sufficient condition - more general than no correlation in the measurement errors in the variables - for the validity of the "standard" parameter bounds is presented. The analysis outlines the ideas and results in general terms to reveal the wider usefulness beyond our specific empirical model. Section 4 discusses how to carry out a formal statistical test of the condition required for the parameter bounds to be valid. How to combine the "standard" parameter bounds with the information in the available instruments is also the topic of this section. Section 5 examines the possibilities for testing hypotheses on the unidentified structural parameters. Our samples and details on the estimation procedures are presented in section 6. Section 7 exhibits and discusses our empirical results. Final remarks are added in section 8.

## 2 The model of producer behavior

This section presents our framework for production analysis[6]. The framework can be considered a hybrid of non-parametric productivity analysis and the traditional econometrics of producer behavior. The analysis presented below consider an imperfectly competitive firm in temporary equilibrium (i.e. with quasi-fixed capital) which uses a technology with scale economies.

The starting point of this analysis is a differentiable production function for a technology with one output and a number of inputs. Let us denote the production function as $y = F(x)$, where $x$ denotes the vector of inputs and $y$ is output. Assume that the production function is a second order polynomial in log of the inputs, such as for the general translog production function. We do not assume constant returns to scale or a homothetic technology. Diewert (1976) has proved that in this case, for *any* two input-output vectors; $(y_1, x_1)$ and $(y_2, x_2)$, we

[6]This section builds on Klette (1993) and Hall (1988, 1990).

have that[7]

$$\hat{y} + \sum_{l \in I} \alpha^l \hat{x}^l = 0, \tag{2.1}$$

where we use the notation that a hat above a variable represents the logarithmic changes, e.g. $\hat{y} = \ln(y_2/y_1)$ etc., and

$$\alpha^l \equiv \frac{1}{2}\left(\frac{x_1^l}{F(x_1)}\frac{\partial F(x_1)}{\partial x_1^l} + \frac{x_2^l}{F(x_2)}\frac{\partial F(x_2)}{\partial x_2^l}\right). \tag{2.2}$$

In our case the two input-output vectors will refer to two different years. That is to say, $\hat{x}^l$ will denote the growth rate of input "$l$" between these two years. $I$ is the set of inputs. $\alpha^l$ is the average output elasticity of input $l$.

Under profit maximization the marginal revenue product of an input is equal to marginal cost for a fully adjustable factor of production. Let us assume that the firm determines inputs, considering input prices to be fixed[8]. It follows that

$$\frac{x_t^l}{F(x_t)}\frac{\partial F(x_t)}{\partial x_t^l} = \frac{w_t^l x_t^l}{(1-1/e)p_t y_t} \quad t = 1,2 \tag{2.3}$$

where '$e$' is the output elasticity of demand, while $w_t^l$ and $p_t$ are the prices of input $l$ and output. Standard theory of profit maximizing behavior shows that the margin between price and marginal cost $\mu$ is given by $\mu = 1/(1-1/e)$. Combining this markup with (2.2) and (2.3), we have that the output elasticity for an adjustable input is

$$\begin{aligned} \alpha^l &= \frac{\mu}{2}\left(\frac{x_1^l w_1^l}{p_1 y_1} + \frac{x_2^l w_2^l}{p_2 y_2}\right) \\ &= \mu \bar{s}^l \end{aligned} \tag{2.4}$$

where $\bar{s}^l$ is the average cost share of input $l$ in the two years.

Various kinds of rigidities, input lags and expectational errors make it dubious to impute the marginal product of capital from observed prices on new equipment, interest rates etc. In

[7] Diewert (1976) termed this the "Quadratic approximation lemma".

[8] This is clearly the case with standard price taking behavior. It is also true with a bargaining model where the unions and the firm negotiate about the wage rate, while the firm unilaterally determines the level of employment. See Nickell et al. (1991) for a discussion.

productivity analysis of a competitive industry with constant returns to scale, this problem is dealt with by estimating the shadow price, and thereby the (shadow) elasticity of capital, as a residual. The case with imperfect competition and non-constant returns is almost as simple if we assume that we know - or can estimate - the overall scale elasticity to all factors. Denote this scale elasticity as $\eta$. Then we have that the output elasticity of capital ($\alpha^K$), obeys the following relationships

$$\begin{aligned} \alpha^K &= \eta - \sum_{l \neq K} \alpha^l \\ &= \eta - \sum_{l \neq K} \mu \bar{s}^l. \end{aligned} \tag{2.5}$$

The last equality follows from (2.4). Using the expressions in (2.4) and (2.5), equation (2.1) can be rewritten

$$\hat{y} = \mu \sum_{l \neq K} \bar{s}^l (\hat{x}^l - \hat{x}^K) + \eta \hat{x}^K. \tag{2.6}$$

## 2.1 The empirical model

In our case, the explicit regression equation corresponding to the model derived above can be stated:

$$\hat{y}_i = \gamma_0 + \mu \left[ \bar{s}_i^L (\hat{l}_i - \hat{k}_i) + \bar{s}_i^M (\hat{m}_i - \hat{k}_i) \right] + \eta \hat{k}_i + \hat{v}_i. \tag{2.7}$$

The left hand side variable is growth in output, while the right hand side variables refer to labour ($\hat{l}$), materials ($\hat{m}$) and capital ($\hat{k}$). The subscript '$i$' refers to individual plants. All growth rates correspond to changes from 1983 to 1990. We have added a constant term $\gamma_0$ which captures technological change common across plants.

The model has a very simple regression form, with only three (main) parameters to be estimated ($\gamma_0, \eta$ and $\mu$). Nevertheless, the model is entirely consistent with the widely observed fact that plants in narrow industries might differ in their productivity levels, due to differences in management, labour quality or capital vintage effects. In the absence of measurement errors, the model would provide consistent estimates of the parameters of interest even when such differences affect the plant's size and the level of its inputs. The basic insight was pointed out by Mundlak

and Hoch (1966) among others. They showed that estimating production functions in terms of growth rates, such as we do here, will eliminate the simultaneity problem encountered when estimating production relationships in levels. The error term in the levels regression will capture permanent differences in productivity, which will tend to be correlated with the regressors. Our model is consistent with differences across plants in the marginal productivity of a given factor of production. Indeed, the estimating equation (2.7) is consistent with each plant having a *separate* translog production function.

The main parametric constraint in this model is the assumption that all producers in an industry have a common scale elasticity. But notice that the model allows the plants to differ in the output elasticities of the individual inputs. In particular, the model claims that a plant with a higher (than average) output elasticity of capital will have a lower output elasticity of variable inputs, and *vice versa*. This is e.g. consistent with a situation where plants differ in their, say, capital-labour ratio, as is widely observed in micro data. Such differences could be due to temporary productivity or demand shocks and adjustment costs, or differences in the cost of capital. Take the case where a plant has a high capital labour ratio. In this case we would expect to find a high output elasticity of labour and a low output elasticity of capital. This sort of differences in the output elasticities are captured by this model.

The model can be rewritten in a manner useful for the statistical section below:

$$\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_3 = \gamma_0 \tag{2.8}$$

where $\xi_1$ is $\hat{y}_i - \hat{u}_i$. $\xi_2$ corresponds to $[\bar{s}_i^L(\hat{l}_i - \hat{k}_i) + \bar{s}_i^M(\hat{m}_i - \hat{k}_i)]$ and $\xi_3$ is $\hat{k}_i$. Also, $\gamma_2 = -\mu$ and $\gamma_3 = -\eta$.

We have a set of observables which corresponds to the $\xi$s. Reported sales is our observable proxying output. Manhours and fire insurance values are the observables for labour input and capital, while material inputs are directly reported in our data. There are a number of reasons why we expect our observables to be "error-ridden" indicators of the variables in our model. This will be discussed in more detail when we present our data set and variable construction in section 6. Measurement errors in our observable corresponding to the capital variable ($\hat{k}_i$) will give rise to an errors-in-variable problem both for $\xi_2$ and $\xi_3$ in equation (2.8). The point to notice is that the measurement errors in the capital variable will give rise to *correlated* measurement errors in two of our observables.

# 3 Bounds on the structural vector ($\tilde{\gamma}$) when the measurement errors are correlated.

The specification (2.8) is a point of departure of the present study. The structural parameters $\gamma_2$ (= – the plants' price/cost margin) and $\gamma_3$ (= – the plants' scale elasticity) are important economic entities about which we want to test specific hypotheses. However, this is not an easy problem to deal with when the structural variables or regressors $\xi_2$ and $\xi_3$ are observed with correlated measurement errors.

Although, our original concern was to solve some particular issues relating to the inference about the structural parameters $\gamma_2$ and $\gamma_3$, the ideas we are going to present have general applicability to this type of models. This will be evident in the sections to come where the results and implications are worked out. In order to show that our results, presented in the form of various lemmas and propositions, can be applied empirically, we shall steadily refer to our basic model. This is specification (2.8) supplemented by the appropriate assumptions on the stochastic variables modelled. For clarity we specify it succinctly below.

The relevant economic theory implies a linear stochastic equation between the structural variables $\xi_1$, $\xi_2$ and $\xi_3$. These variables, however, are not directly observable. What we observe are the noise-corrupted indicators of these variables. We thus have the following latent variable model:

$$\gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_3 = \gamma_0 \tag{3.1}$$

Supplemented by the measurement equations:

$$X_j = \xi_j + \varepsilon_j, \qquad j = 1,2,3,4 \tag{3.2}$$

in which the variable $X_2$ is deduced from $X_3$ and $X_4$ by the equation:

$$X_2 = s(X_4 - X_3) = s(\xi_4 - \xi_3) + s(\varepsilon_4 - \varepsilon_3), \qquad 0 < s < 1. \tag{3.3}$$

In eqs. (3.1)–(3.3) the $X_j$'s denote observable variables, the $\xi_j$'s denote the unobservable structural variables, and finally the $\varepsilon_j$'s denote the random measurement errors.

We suppose that the structural variables $(\xi_1, \xi_2, \xi_3)$ are random, but independently distributed of the random errors $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$. We observe from (3.3) that $\varepsilon_2 = s(\varepsilon_4 - \varepsilon_3)$. Thus $\varepsilon_2$

and $\varepsilon_3$ will be correlated. We suppose, however, that the random errors $\varepsilon_1$, $\varepsilon_3$, $\varepsilon_4$ are uncorrelated. Then it follows that the covariance matrix of errors $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is given by:

$$\Lambda = \begin{pmatrix} \lambda_1^2 & 0 & 0 \\ 0 & s^2(\lambda_3^2 + \lambda_4^2) & -s\lambda_3^2 \\ 0 & -s\lambda_3 & \lambda_3^2 \end{pmatrix}, \qquad \lambda_i^2 = \mathrm{var}(\varepsilon_i), \quad i = 1, 3, 4 \tag{3.4}$$

The model spesified by eqs. (3.1)–(3.4) together with our assumptions on the random structural variables and random errors stated above constitutes our model of reference.

Thus, the model we have to study is more general than the standard "Errors in variables" (EIV) models, since we cannot assume that the measurement errors are uncorrelated. In the classical treatment of EIV models it is always assumed that the covariance matrix of the random errors is diagonal (see Willassen (1987) for an up-to-date study of these models).

Let us denote by $\Omega$ the covariance matrix of the observable $X$ variables. The classical result of EIV models then says loosely speaking that if the elements of $\Omega^{-1}$ have certain patterns of signs, then the structural vector $(\tilde{\gamma})$ can be expressed as a convex combination of the columns of $\Omega^{-1}$. Hence, although the structural vector is not identifiable, with an appropriate normalization this result restricts $(\tilde{\gamma})$ to be situated in the simplex generated by the column vectors of $\Omega^{-1}$. This is a well-known result in econometrics whose origin is often connected with Frisch (1934) and Koopmans (1937). However, the first general statement and proof of this result was given by Reiersøl (1941) (Willassen, op.cit. (1987)).

This proposition is very attractive and we wish to prove a version of it in the situation that the random measurement errors are correlated. The point is that, although, the calculated simplices often will be too "wide" to be empirically useful, we shall argue that combined with relevant extraneous information these simplices prove to be useful, indeed.

In our present model of reference we know the signs of the structural parameters and a relevant instrumental variable. These two pieces of a priori information are the points of departure of our theoretical analysis.

*Extraneous information*

We suppose the availability of the following a priori information.

(i) The signs of the structural parameters, i.e. the signs of the elements of the vector $\tilde{\gamma}$, are supposed to be known.

(ii) A set of instrumental variables, although, insufficient in number to identify the structural vector ($\tilde{\gamma}$), are at our disposal.

Although, the economic theory dealing with a particular problem is incomplete and tentative, it will often be able to predict correctly the signs of the structural parameters involved. Also, research workers studying a particular application usually will have at their disposal subsidiary information in the form of one or more instrumental variables. As noted above, even if this set is insufficient to identify the structural vector, combined with the regression simplex the instrumental variables can be used to reduce the feasible region for the structural vector ($\tilde{\gamma}$) (Willassen, op.cit. p. 309).

In the analysis to come we shall use the following definitions.

DEFINITION 3.1. *A diagonal matrix $A$ is called a sign matrix if and only if its elements are* 1 *or* $-1$.

Evidently, any sign matrix $A$ satisfies $AA = I$, so that $A^{-1} = A$.

DEFINITION 3.2. *Given a class $\mathcal{B}$ of vectors, the class $C$ of all non-negative linear combinations*

$$\tilde{c} = \alpha_1\tilde{b}_1 + \alpha_2\tilde{b}_2 + \cdots + \alpha_N\tilde{b}_N, \qquad \alpha_j \geq 0, \quad j = 1, 2, \ldots, N \tag{3.5}$$

*of vectors $\tilde{b}_1, \tilde{b}_2, \ldots \tilde{b}_N$ selected from $\mathcal{B}$ is a convex cone.*

DEFINITION 3.3. *For any matrix $M$ we shall denote by* cone($M$) *the convex cone generated by the column vectors $\tilde{m}_1, \tilde{m}_2, \ldots, \tilde{m}_k$ of $M$.*

Evidently, any point $\tilde{\theta} \in$ cone($M$) can be expressed in the form:

$$\tilde{\theta} = \sum_{i=1}^{k} \alpha_i\tilde{m}_i = M\tilde{\alpha}, \qquad \alpha_i \geq 0 \quad (i = 1, 2, \ldots, k) \tag{3.6}$$

The following simple lemma will be useful.

LEMMA 3.1. *Let $M$ and $N$ be two matrices, and suppose that the number of columns of $M$ is equal to the number of rows of $N$ (say $k$). Then the* cone($MN$) *is contained in the* cone($M$) *if all elements of $N$ is non-negative. Conversely, if $P$ is another matrix and we have* cone($P$) $\subset$ cone($M$), *then there exists a matrix $N$ with non-negative elements such that $P = MN$.*

PROOF. *The assertions of the lemma is evident from the multiplication rule of matrix theory.*

■

We are then ready to work out implications of assumption (i). First, it is instructive to apply this assumption to the standard EIV model. We denote by $\tilde{X}$ the vector of observable variables, by $\tilde{\xi}$ the vector of unobservable structural variables, by $\tilde{\varepsilon}$ the vector of random measurement errors, and finally, by $\tilde{\gamma}$ the vector of structural parameters. Then, the standard EIV model is described by the following set of equations:

$$\tilde{X} = \tilde{\xi} + \tilde{\varepsilon} \tag{3.7}$$

$$(\tilde{\gamma}'\tilde{\xi}) = \sum_{i=1}^{k} \gamma_i \xi_i = \gamma_0 \tag{3.8}$$

$$(\Omega - L)\tilde{\gamma} = \tilde{O} \qquad (\tilde{O} \text{ is the zero vector of appropriate dimension}) \tag{3.9}$$

$$(\Omega - L) \quad \text{is non-negative definite.} \tag{3.10}$$

In (3.9) $\Omega$ denotes the covariance matrix of the observable variables, and $L$ is the diagonal matrix whose elements are the variances of the random errors. We suppose that $\Omega$ and $L$ are non-singular.

Setting

$$\tilde{g} = \Omega\tilde{\gamma} \tag{3.11}$$

we have because of (3.9):

$$\tilde{g} = L\tilde{\gamma} \tag{3.12}$$

Since the diagonal matrix $L$ has only positive elements, it follows from (3.12) that the column vector $\tilde{g}$ has the same sequence of signs as the structural vector $\tilde{\gamma}$.

Suppose now that we know the signs of the structural parameters $\gamma_i$. Let $A_{\tilde{\gamma}}$ denote the sign matrix whose diagonal elements correspond to the signs of the structural parameters; i.e. if $\gamma_i$ is positive the corresponding diagonal element of $A_{\tilde{\gamma}}$ is $+1$, if $\gamma_i$ is negative the diagonal element is $-1$. Post-multiplying an arbitrary matrix $B$ by the sign matrix $A_{\tilde{\gamma}}$ leaves a column vector of $B$ unchanged if the corresponding element of $A_{\tilde{\gamma}}$ is positive, and changes its sign if the corresponding element of $A_{\tilde{\gamma}}$ is negative. We shall call this procedure sign-correcting the columns of $B$ by the signs of the structural vector $\tilde{\gamma}$.

Since, $L$ is non-singular and $A_{\tilde{\gamma}}A_{\tilde{\gamma}} = I$, we attain, immediately, from (3.12):

$$\tilde{\gamma} = L^{-1}\tilde{g} = (L^{-1}A_{\tilde{\gamma}})A_{\tilde{\gamma}}\tilde{g} = (L^{-1}A_{\tilde{\gamma}})\tilde{\theta} \tag{3.13}$$

where:

$$\tilde{\theta} = A_{\tilde{\gamma}}\tilde{g} \tag{3.14}$$

Since $\tilde{g}$ has the same sequence of signs as $\tilde{\gamma}$, it follows from the definition of $A_{\tilde{\gamma}}$ that the elements of $\tilde{\theta} = A_{\tilde{\gamma}}\tilde{g}$ are non-negative. Hence, if assumption (i) is satisfied in the standard EIV model, the structural vector $\tilde{\gamma}$ is contained in the convex cone generated by the sign corrected columns of $L$. Thus, we write:

$$\tilde{\gamma} \in \text{cone}(L^{-1}A_{\tilde{\gamma}}) \tag{3.15}$$

Since $(L^{-1}A_{\tilde{\gamma}})$ is a diagonal matrix, its column vectors will be lying along the coordinate axes. These axes, the directions of which are determined by the signs of the diagonal elements of $A_{\tilde{\gamma}}$, constitute the edges of the orthant in which the structural vector $\tilde{\gamma}$ is situated.

By a similar reasoning we attain from (3.11) that:

$$\tilde{\gamma} \in \text{cone}(\Omega^{-1}A_{\tilde{\gamma}}) \tag{3.16}$$

Thus, the structural vector $\tilde{\gamma}$ is restricted to the intersection of the two convex cones generated by (3.15) and (3.16). Although, these cones viewed independently can be "large" their intersection can be "small".

We shall now relax the assumption that the covariance matrix of the random errors is diagonal, but otherwise we shall retain the specification given by eqs. (3.7)–(3.10). The covariance matrix of the random measurement errors $\tilde{\varepsilon}$ in now denoted by $\Lambda$. Thus we suppose that $\Lambda$ is an arbitrary non-negative definite, non-singular matrix. Again we shall suppose that we know the signs of the structural vector $\tilde{\gamma}$ (assumption (i)). The corresponding sign matrix is denoted by $A_{\tilde{\gamma}}$.

By applying our arguments above to this more general case, we attain the following lemma.

LEMMA 3.2. *Suppose the model is specified by eqs. (3.7)–(3.10) modified in that the general covariance matrix $\Lambda$ is substituted for the diagonal matrix $L$. Let us also suppose that the signs of the structural parameters are known, so that $A_{\tilde{\gamma}}$ is a known sign matrix. Then a necessary and sufficient condition for the structural vector $\tilde{\gamma}$ to be situated in the convex cone generated by the column vectors of $\Omega^{-1}$ sign-corrected by the signs of the vector $\tilde{\gamma}$, is that the structural vector $\tilde{\gamma}$ is contained in the convex cone generated by the column vectors of $\Lambda^{-1}$ sign-corrected by the signs of $\tilde{\gamma}$.*

PROOF. *It follows from (3.9) that the structural vector $\tilde{\gamma}$ has to satisfy the equation:*

$$A_{\tilde{\gamma}}(\Omega - \Lambda)\tilde{\gamma} = \tilde{O} \tag{3.17}$$

*Setting*

$$\tilde{\alpha} = (A_{\tilde{\gamma}}\Omega)\tilde{\gamma}$$

$$\tilde{\alpha} = (A_{\tilde{\gamma}}\Lambda)\tilde{\gamma}$$

*we have inversely since $\Omega$ and $\Lambda$ are non-singular that:*

$$\tilde{\gamma} = (\Omega^{-1}A_{\tilde{\gamma}})\tilde{\alpha} \tag{3.20}$$

$$\tilde{\gamma} = (\Lambda^{-1}A_{\tilde{\gamma}})\tilde{\alpha} \tag{3.21}$$

*The contentions of lemma 3.2 follow from eqs. (3.20) and (3.21).* ■

The structural vector $\tilde{\gamma}$ is defined in relation to the unobserved systematic variables $\xi_1, \xi_2, \ldots, \xi_k$. If the random errors $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k$ are independent of $\xi_1, \xi_2, \ldots, \xi_k$ which is the standard assumption in EIV models, the structural parameters $\gamma_1, \gamma_2, \ldots, \gamma_k$ are also independent of the elements of the covariance matrix $\Lambda$. That is, to any structural vector $\tilde{\gamma}$ there may correspond any covariance matrix $\Lambda$. Hence, the wider is the convex cone generated by the column vectors of $(\Lambda^{-1}A_{\tilde{\gamma}})$, the more likely it is that it will contain the structural vector $\tilde{\gamma}$. In the limiting case where the column vectors of $(\Lambda^{-1}A_{\tilde{\gamma}})$ are nearly proportional, the convex cone generated by these vectors will be very narrow. In this particular case it appears to be very unlikely that the structural vector $\tilde{\gamma}$ is contained in this convex cone.

If we know a priori which elements of $\Lambda$ are zero, eq. (3.21) can be used to derive restrictions on the structural parameters. Later on we will show how these ideas apply to our reference model.

Usually, the convex cones generated by the column vectors of the matrices $(\Omega^{-1}A_{\tilde{\gamma}})$ and $(\Lambda^{-1}A_{\tilde{\gamma}})$ will not coincide. However, the case when the cone$(\Omega^{-1}A_{\tilde{\gamma}})$ is contained in the cone$(\Lambda^{-1}A_{\tilde{\gamma}})$ deserves particular attention. We know from lemma 3.1 that if cone$(\Omega^{-1}A_{\tilde{\gamma}}) \subset$ cone$(\Lambda^{-1}A_{\tilde{\gamma}})$ then there exists a matrix $B$ with non-negative elements only, such that:

$$\Omega^{-1}A_{\tilde{\gamma}} = (\Lambda^{-1}A_{\tilde{\gamma}})B \tag{3.22}$$

The essential information is conveyed by the following proposition:

PROPOSITION 3.1. *Suppose that the covariance matrices $\Omega$ and $\Lambda$ are non-singular. Suppose that the matrix equation (3.22) is satisfied for a matrix $B$ containing only non-negative elements, and that the largest characteristic root of $|kI - A_{\tilde{\gamma}}\Lambda\Omega^{-1}A_{\tilde{\gamma}}| = 0$ is simple and equal to $k = 1$. Then the characteristic vector $\tilde{\alpha} = (A_{\tilde{\gamma}}\Omega)\tilde{\gamma}$ corresponding to the equation $(I - A_{\tilde{\gamma}}\Lambda\Omega^{-1}A_{\tilde{\gamma}})\tilde{\alpha} = \tilde{O}$ has only non-negative elements, and the structural vector $\tilde{\gamma} \subset \text{cone}(\Omega^{-1}A_{\tilde{\gamma}})$.*

PROOF. *According to (3.9) we have to solve the matrix equation:*

$$(\Omega - \Lambda)\tilde{\gamma} = \tilde{O} \tag{3.23}$$

*wrt. the structural vector $\tilde{\gamma}$. Since $A_{\tilde{\gamma}}A_{\tilde{\gamma}} = I$ it is easily verified that (3.23) is equivalent to*

$$(I - A_{\tilde{\gamma}}\Lambda\Omega^{-1}A_{\tilde{\gamma}})A_{\tilde{\gamma}}\Omega\tilde{\gamma} = \tilde{O} \tag{3.24}$$

*For this equation to have a non-zero solution wrt. $\tilde{\alpha}$ $(= A_{\tilde{\gamma}}\Omega\tilde{\gamma})$ or $\tilde{\gamma}$, it is necessary that*

$$|I - A_{\tilde{\gamma}}\Lambda\Omega^{-1}A_{\tilde{\gamma}}| = 0$$

*We observe from (3.22) that $(A_{\tilde{\gamma}}\Lambda\Omega^{-1}A_{\tilde{\gamma}}) = B$, and, by assumption $B$ is non-negative. The conclusions of the proposition then follows from the Frobenius theory of non-negative matrices.*

■

(A good review of non-negative square matrices is given by Debreu and Herstein (1953).)

A brief confrontation of prop. 3.1 with the standard EIV model is instructive. In the EIV case the covariance matrix of the random errors $\Lambda$ is diagonal. Therefore, also $\Lambda^{-1}$ is diagonal with positive entries. If we suppose that $\Omega^{-1}$ has compatible signs, there exists a sign matrix $T$ so that $(T\Omega^{-1}T)$ has only positive elements. Then we can always find a non-negative matrix $B$ so that eq. (3.22) is satisfied. This conclusion follows easily from the following facts. Since $\Lambda^{-1}$ is diagonal with positive elements and $TT = I$, (3.22) becomes:

$$(T\Omega^{-1}T) = (T\Lambda^{-1}T)B = \Lambda^{-1}B \tag{3.26}$$

Hence, we have:

$$B = \Lambda(T\Omega^{-1}T) \tag{3.27}$$

The elements of $B$ given by (3.27) are obviously positive since the elements of $\Lambda$ and $(T\Omega^{-1}T)$ are all positive. Hence, the conditions of prop. 3.1 work nicely in the classical EIV case.

Now, it is urgent to apply our theory to our model specified by eqs. (3.1)–(3.4). Our specification and a priori information imply:

$$\Lambda = \begin{pmatrix} \lambda_1^2 & 0 & 0 \\ 0 & s^2(\lambda_3^2+\lambda_4^2) & -s\lambda_4^2 \\ 0 & -s\lambda_4^2 & \lambda_3^2 \end{pmatrix} \tag{3.28}$$

$$\Lambda^{-1} = \begin{pmatrix} \frac{1}{\lambda_1^2} & 0 & 0 \\ 0 & \frac{1}{s^2\lambda_4^2} & \frac{1}{s\lambda_4^2} \\ 0 & \frac{1}{s\lambda_4^2} & \frac{(\lambda_3^2+\lambda_4^2)}{\lambda_3^2\lambda_4^2} \end{pmatrix} \tag{3.29}$$

As to the present sign matrix the relevant economic theory definitely predicts that the regressors $\xi_2$ and $\xi_3$ have a positive effect on the dependent variable $\xi_1$. Hence, writing our structural relation on the form:

$$\gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_3 = \gamma_0 \tag{3.30}$$

$\gamma_1$ compared to $(\gamma_2, \gamma_3)$ have opposite signs. We choose $\gamma_1 = 1$ and, therefore, $\gamma_2$ and $\gamma_3$ are both negative (cf. model (2.8) ).

The sign matrix $A_{\tilde{\gamma}}$ becomes:

$$A_{\tilde{\gamma}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3.31}$$

and consequently:

$$\Lambda^{-1}A_{\tilde{\gamma}} = \begin{pmatrix} \frac{1}{\lambda_1^2} & 0 & 0 \\ 0 & \frac{-1}{s^2\lambda_4^2} & \frac{-1}{s\lambda_4^2} \\ 0 & \frac{-1}{s\lambda_4^2} & \frac{-(\lambda_3^2+\lambda_4^2)}{\lambda_3^2\lambda_4^2} \end{pmatrix} \tag{3.32}$$

If $\tilde{\gamma} \in \text{cone}(\Lambda^{-1}A_{\tilde{\gamma}})$ then the equation:

$$\tilde{\gamma} = (\Lambda^{-1}A_{\tilde{\gamma}})\tilde{\alpha} \tag{3.33}$$

has a non-negative solution $\tilde{\alpha}$, i.e. $\tilde{\alpha} \geq 0$. Since the column vectors of $\Lambda^{-1}A_{\tilde{\gamma}}$ are given, a non-negative solution $\tilde{\alpha}$ will restrict the feasible region for the structural vector $\tilde{\gamma}$.

It follows from Farkas' lemma (Rockefellar (1970), sect. 22) that we can state:

LEMMA 3.3. *Either the equations:*

$$(\Lambda^{-1}A_{\tilde{\gamma}})\tilde{\alpha} = \tilde{\gamma} \tag{3.34}$$

*have a solution* $\tilde{\alpha} \geq 0$, *or the inequalities*

$$z'(\Lambda^{-1}A_{\tilde{\gamma}}) \leq \tilde{O}, \qquad z'\tilde{\gamma} > 0 \tag{3.35}$$

*have a solution, but not both.*

PROOF. *The contentions of this lemma follow from Farkas' lemma.* ■

Thus, the equations (3.34) have a non-negative solution ($\tilde{\alpha} \geq 0$), if and only if the inequalities (3.35) have no solution. Hence, we wish to deduce conditions which exclude any solution of the set of inequalities:

$$\begin{aligned}
\frac{z_1}{\lambda_1^2} &\leq 0 \\
\frac{-z_2}{s^2\lambda_4^2} - \frac{z_3}{s\lambda_4^2} &\leq 0 \\
\frac{-z_2}{s\lambda_4^2} - \frac{(\lambda_3^2+\lambda_4^2)z_3}{\lambda_3^2\lambda_4^2} &\leq 0 \\
z_1 + \gamma_2 z_2 + \gamma_3 z_3 &> 0 \quad \text{(we suppose } \gamma_1 = 1\text{)}
\end{aligned} \tag{3.36}$$

It is not difficult to verify that this system of inequalities have no solution if the condition:

$$s < \frac{\gamma_3}{\gamma_2} < s\left(1 + \frac{\lambda_4^2}{\lambda_3^2}\right), \qquad 0 < s < 1 \tag{3.37}$$

is satisfied. If the two error variances $\lambda_3$ and $\lambda_4$ are equal (3.37) becomes:

$$s \leq \frac{\gamma_3}{\gamma_2} < 2s, \qquad 0 < s < 1 \tag{3.38}$$

The bounds (3.37) or (3.38) convey useful a priori information about the ratio between the two structural parameters $\gamma_2$ and $\gamma_3$. In some econometric applications this is valuable information.

However, the general useful results are conveyed by proposition 3.1. One of the essential hypotheses assumed there is that the matrix equation (3.22) is satisfied for a non-negative matrix $B$. We wish to check this hypothesis in the present application.

The signs of the estimated elements of the symmetric matrix $\Omega^{-1} = (\Omega_{ij})$ are in all our samples given by:

$$\begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{pmatrix} = \begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix} \tag{3.39}$$

Sign-correcting the columns of $\Omega^{-1}$ by the sign matrix (3.31), implies

$$(\Omega^{-1}A_{\tilde{\gamma}}) = \begin{pmatrix} \Omega_{11} & -\Omega_{12} & -\Omega_{13} \\ \Omega_{21} & -\Omega_{22} & -\Omega_{23} \\ \Omega_{31} & -\Omega_{32} & -\Omega_{33} \end{pmatrix} = \begin{pmatrix} + & + & + \\ - & - & - \\ - & - & - \end{pmatrix} \tag{3.40}$$

Similarly we attain:

$$(\Lambda^{-1}A_{\tilde{\gamma}}) = \begin{pmatrix} \frac{1}{\lambda_1^2} & 0 & 0 \\ 0 & \frac{-1}{s^2\lambda_4^2} & \frac{-1}{s\lambda_4^2} \\ 0 & \frac{-1}{s\lambda_4^2} & \frac{-(\lambda_3^2+\lambda_4^2)}{\lambda_3^2\lambda_4^2} \end{pmatrix} \tag{3.41}$$

By comparing elements it is evident that we can find a non-negative matrix $B = (b_{ij})$ so that the matrix equation (3.22) is satisfied. In the next section we elaborate on a formal statistical analysis of this condition.

The hypothesis that the largest root of the characteristic equation:

$$|kI - A_{\tilde{\gamma}}\Lambda\Omega^{-1}A_{\tilde{\gamma}}| = 0 \tag{3.42}$$

is simple and equal to $k = 1$ is much more tricky to verify. It can be done along the lines followed by Willassen ((1987), sect. 2). But a complete analysis of this question will lead us away from our main line of reasoning. We thus assume that this hypothesis is fulfilled.

It follows from prop. (3.1) that:

$$\tilde{\gamma} = (\Omega^{-1}A_{\tilde{\gamma}})\tilde{\alpha}, \qquad \text{where } \tilde{\alpha} \geq \tilde{O} \tag{3.43}$$

That is, the structural vector $\tilde{\gamma}$ is a non-negative linear combination of the column vectors of $(\Omega^{-1}A_{\tilde{\gamma}})$. Above we have assumed that $\gamma_1$ is positive. If the structural variable $\xi_1$ denotes our dependent variable, it is reasonable to normalize the structural vector $\tilde{\gamma}$ by requiring $\gamma_1 = 1$.

Hence, we can write our structural equation:

$$\xi_1 = \gamma_0 + g_2\xi_2 + g_3\xi_3, \qquad \text{where } g_2 = -\gamma_2,\ g_3 = -\gamma_3 \tag{3.44}$$

If $\tilde{g}' := (1, g_2, g_3)$ we observe that $\tilde{g} = A_{\tilde{\gamma}}\tilde{\gamma}$. Using (3.43) we attain:

$$\tilde{g} = (A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}})\tilde{\alpha}, \qquad \tilde{\alpha} \geq \tilde{O} \tag{3.45}$$

It follows from eq. (3.22)

$$A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}} = (A_{\tilde{\gamma}}\Lambda^{-1}A_{\tilde{\gamma}})B \tag{3.46}$$

for a non-negative matrix $B$. We verify directly by inspection that $(A_{\tilde{\gamma}}\Lambda^{-1}A_{\tilde{\gamma}})$ has only non-negative elements. It then follows that the elements of $(A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}})$ are all positive.

For brevity we define:

$$H^{-1} := (A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}}) \tag{3.47}$$

$$\tilde{e}' := \{\text{a row vector whose elements are all 1}\} \tag{3.48}$$

$$D := \{\text{a diagonal matrix with elements consisting of the first row of } H^{-1}\} \tag{3.49}$$

$$\begin{pmatrix} \tilde{e}' \\ P \end{pmatrix} := (H^{-1}D^{-1}) \tag{3.50}$$

In the present application the matrix $P$ consists of 2 rows and 3 column vectors: $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$. We observe from (3.50)

$$P = \{\tilde{P}_1, \tilde{P}_2, \tilde{P}_3\} = \begin{pmatrix} \frac{-\Omega_{12}}{\Omega_{11}} & \frac{-\Omega_{22}}{\Omega_{12}} & \frac{-\Omega_{23}}{\Omega_{13}} \\ \frac{-\Omega_{13}}{\Omega_{11}} & \frac{-\Omega_{23}}{\Omega_{12}} & \frac{-\Omega_{33}}{\Omega_{13}} \end{pmatrix} \tag{3.51}$$

Then we can prove:

PROPOSITION 3.2. *Let all the hypotheses of proposition 3.1 be satisfied. Then the structural vector $\tilde{g}$ is situated in the convex hull generated by the column vectors of the matrix $(H^{-1}D^{-1})$.*

PROOF. *According to (3.45) we have:*

$$\tilde{g} = \begin{pmatrix} 1 \\ g_2 \\ g_3 \end{pmatrix} = H^{-1}\tilde{\alpha} = (H^{-1}D^{-1})D\tilde{\alpha} \tag{3.52}$$

*Setting $\tilde{w} = D\tilde{\alpha}$ it follows that $\tilde{w} \geq \tilde{O}$. From our normalization $g_1 := 1$ it also follows that the components of $\tilde{w}$ add to 1. Hence, in the $(g_2, g_3)$-plane we have the representation:*

$$\begin{pmatrix} g_2 \\ g_3 \end{pmatrix} = w_1\tilde{P}_1 + w_2\tilde{P}_2 + w_3\tilde{P}_3 \tag{3.52}$$

*where* $\sum_{j=1}^{3} w_j = 1$. ■

If we define

$$\mathcal{P} := \{\text{the convex hull generated by the vectors } \tilde{P}_1,\ \tilde{P}_2,\ \tilde{P}_3\}, \tag{3.54}$$

we note that in the present application $\mathcal{P}$ is a triangle in $\mathcal{R}_+^2$.

REMARK: The analysis carried out above rests on the assumption that we know a priori the signs of the structural parameters. The signs of the elements of $\Omega^{-1}$ then implied that the first row of the sign-corrected matrix $(\Omega^{-1}A_{\tilde{\gamma}})$ has only positive elements. This will always be the case when we suppose that the sequence of signs of the structural parameters is the same as the sequence of signs of the elements of the first row (or column) of $\Omega^{-1}$ ($\Omega^{-1}$ is, of course, symmetric). From least square regression theory we know that by regressing $X_1$ on the remaining random variables, the vector of parameters determined is a positive multiple of the first row (or column) of $(\Omega^{-1})$. Hence, the a priori information on the signs of the structural parameters used in the present applications simply says that the signs of the structural parameters $\gamma_2$ and $\gamma_3$ are the same as the signs of the parameters attained by regressing $X_1$ on $X_2$ and $X_3$. That is, as the signs of the components of the vector $(-\Omega_{21}/\Omega_{11}, -\Omega_{31}/\Omega_{11})$. Hence, the signs predicted by the relevant economic theory agree with the signs of the parameter estimates attained by least square regression. In a way this is reassuring. Although the least square estimators are biased in the presence of measurement errors, they will usually have the correct signs.

# 4 Restricting the convex hull $\mathcal{P}$ by using instrumental variables

In the present section we shall work out implications of our a priori assumption (ii). In order to be spesific we shall study the trivariate case implied by our model of reference. Hence, we wish to combine the simplex generated by the vectors $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$ of (3.51) with the information conveyed by an instrumental variable $Z$.

In our application we observe a variable $Z$ which certainly is correlated with the observable variables $X_1$, $X_2$, $X_3$ and which is reasonably supposed to be independent of the random measurement errors $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$. This means that $Z$ will serve as a proper instrumental variable in the present specification. However, in order to identify the structural parameters $(g_2, g_3)$ defined in eq. (3.44), we need two instrumental variables at least. Hence, with only one such variable the

parameters $(g_2, g_3)$ are still not identifiable. But we hope that a proper use of the instrument $Z$ will make it possible to dwindle considerably the feasible region for the structural parameters $(g_2, g_3)$.

Before we investigate this idea we have to clarify by a statistical test whether the matrix equation (3.22) is satisfied for a non-negative matrix $B$ of full rank. Equivalently, if

$$(A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}}) = (A_{\tilde{\gamma}}\Lambda^{-1}A_{\tilde{\gamma}})B \tag{4.1}$$

where $\Lambda^{-1}$ is given by (3.29) and the sign matrix by (3.31).

From (3.29) and (3.31) it is readily verified that the matrix $(A_{\tilde{\gamma}}\Lambda^{-1}A_{\tilde{\gamma}})$ is non-negative by assumption. Thus, we shall consider eq. (3.22) or (4.1) to be verified if it can be justified by statistical testing that the elements of $(A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}})$ are all positive. In effect, we wish to test if the matrix $\Omega^{-1}$ has compatible signs (see definition 3.2). In this respect the following lemma is useful.

LEMMA 4.1. *A matrix $M = (m_{ij})$ has compatible signs if and only if the product $(m_{ik}m_{il}m_{jk}m_{jl})$ is positive for all indeces $i$, $j$, $k$, $l$.*

PROOF. *Evident.* ■

In applying this lemma to the matrix $\Omega^{-1}$ we note the following facts. Since $\Omega$ is symmetric and positive definite the same is true for $\Omega^{-1}$. This implies that the diagonal elements of $\Omega^{-1}$ are all positive with probability 1. Using this fact together with lemma 4.1 we conclude that $\Omega^{-1}$ has compatible signs if and only if the product:

$$\theta = \Omega_{12}\Omega_{13}\Omega_{23} > 0 \tag{4.2}$$

Thus, we wish to test the hypothesis:

$$\mathrm{H}_0 : \theta \leq 0 \qquad \text{against} \qquad \mathrm{H}_1 : \theta_1 > 0 \tag{4.3}$$

In order to test this hypothesis we need the sample counterparts $S$, $S^{-1}$ and $S_{ij}$ of $\Omega$, $\Omega^{-1}$ and $\Omega_{ij}$. We start with the consistent estimator $\hat{\theta}$ of $\theta$ given by:

$$\hat{\theta} = S_{12}S_{13}S_{23} \tag{4.4}$$

It is evident that the finite sample distribution of $\hat{\theta}$ (4.4) is very difficult to attain. Therefore, in order to test $\mathrm{H}_0$ of 4.3 we have to resort to a large-sample test. From standard asymptotic

theory (Serfling (1980)) we know that under quite general conditions the statistic $\sqrt{N}(\hat{\theta} - \theta)$ is asymptotically normal with mean zero and variance $\sigma^2_{\hat{\theta}}$. Estimates of the variance $\sigma^2_{\hat{\theta}}$ are difficult to attain, but recent resampling techniques are tailored to handle situations like this. Thus we calculated an estimate of $\sigma^2_{\hat{\theta}}$ by the Bootstrap method (Efron (1982)).

As a test statistic of the null hypothesis $H_0$ of (4.3) we shall use:

$$T = \frac{\hat{\theta}}{\sigma^2_{\hat{\theta}}} \tag{4.5}$$

We reject $H_0$ if $T \geq t_\varepsilon$ where $P_{\theta=0}\{T \geq t_\varepsilon\} = \varepsilon$. (Here $P_{\theta=0}\{\cdot\}$ denotes the standard normal distribution, and $\varepsilon$ is the chosen significance level).

If we reject $H_0$ of (4.3) we conclude that there exists a sign matrix $A = (a_{ii})$ so that by a proper selection of the diagonal elements $a_{ii}$ $(= \pm 1)$, the sign-corrected matrix $(A\Omega^{-1}A)$ has only positive elements. In the present application the distribution of signs of the estimated elements of $\Omega^{-1}$ are shown in (3.39). We observe that the estimated values of $\Omega_{12}$ and $\Omega_{13}$ are negative while the estimated value of $\Omega_{23}$ is positive. Thus, it is obvious that the sign matrix $A$ which makes $(A\Omega^{-1}A)$ positive in this application is the sign matrix $A_{\tilde{\gamma}}$ given by eq. (3.31).

Having verified that $(A_{\tilde{\gamma}}\Omega^{-1}A_{\tilde{\gamma}})$ is positive, and since $(A_{\tilde{\gamma}}\Lambda^{-1}A_{\tilde{\gamma}})$ is non-negative by the specification of $\Lambda$ (3.4), we conclude that eq. (4.1) is satisfied for a non-negative matrix $B$. Thus, we have shown that an important hypothesis of prop. 3.1 is satisfied in our application. Having clarified these details we are ready to introduce our instrumental variable.

Under the hypotheses of prop. 3.1 the structural parameters $(g_2, g_3)$ are shown to be situated in the simplex $\mathcal{P}$ generated by the points (vectors) $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$. In empirical applications this simplex is often too wide to be of any practical use. Except in rare cases, an appropriate use of our instrumental variable $Z$ will restrict considerably the feasible region for the structural parameters.

From the definitions (3.47)–(3.51) it follows that in the present trivariate case, $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$ are given by:

$$\tilde{P}_1 = \begin{pmatrix} \dfrac{-\Omega_{12}}{\Omega_{11}} \\ \dfrac{-\Omega_{13}}{\Omega_{11}} \end{pmatrix}, \qquad \tilde{P}_2 = \begin{pmatrix} \dfrac{-\Omega_{22}}{\Omega_{12}} \\ \dfrac{-\Omega_{23}}{\Omega_{12}} \end{pmatrix}, \qquad \tilde{P}_3 = \begin{pmatrix} \dfrac{-\Omega_{23}}{\Omega_{13}} \\ \dfrac{-\Omega_{33}}{\Omega_{13}} \end{pmatrix} \tag{4.6}$$

where $\Omega_{ij}$ is the cofactor of the element $\omega_{ij}$ of $\Omega$.

The edges of the simplex $\mathcal{P}$ are generated by the three straight lines:

$$\begin{pmatrix} g_2 \\ g_3 \end{pmatrix} = t\tilde{P}_1 + (1-t)\tilde{P}_2, \qquad 0 \leq t \leq 1 \tag{4.7}$$

$$\begin{pmatrix} g_2 \\ g_3 \end{pmatrix} = t\tilde{P}_1 + (1-t)\tilde{P}_3, \qquad 0 \leq t \leq 1 \tag{4.8}$$

$$\begin{pmatrix} g_2 \\ g_3 \end{pmatrix} = t\tilde{P}_2 + (1-t)\tilde{P}_3, \qquad 0 \leq t \leq 1 \tag{4.9}$$

Since $\Omega$ is a symmetric matrix the cofactors $\Omega_{ij} = \Omega_{ji}$. By eliminating the parameter $t$ in (4.7)–(4.9) we attain 3 linear equations in $(g_2, g_3)$. By using the symmetry of $\Omega$ together with an appropriate use of identity (11.7.3) in Cramér (1946 p. 111) these calculations simplify greatly. Corresponding to (4.7)–(4.9) we attain the 3 equations:

$$\omega_{13} = g_2\omega_{23} + g_3\omega_{33} \tag{4.10}$$

$$\omega_{12} = g_2\omega_{22} + g_3\omega_{23} \tag{4.11}$$

$$\omega_{11} = g_2\omega_{12} - g_3\omega_{13} \tag{4.12}$$

We note that (4.10) and (4.11) correspond to the normal equations we attain by least squares regression of $X_1$ on $X_2$ and $X_3$.

Then we combine our structural eq. (3.44) i.e.

$$\xi_1 = \gamma_0 + g_2\xi_2 + g_3\xi_3 \tag{4.13}$$

with our instrumental variable $Z$. Multiplying (4.13) by $(z - E(Z))$ and then taking expectations on both sides of the resultant equation, we attain:

$$\omega_{1z} = g_2\omega_{2z} + g_3\omega_{3z} \quad (\mathcal{I}) \tag{4.14}$$

where $\omega_{1z} = \text{cov}(X_i, Z)$, $i = 1, 2, 3$.

In deriving (4.14) we have used the fact that the instrumental variable $Z$ is independent of all the random measurement errors $(\varepsilon_i)$. Since $Z$ is an instrumental variable this assumption follows by definition.

The following requirement on the instrumental variable $Z$ is reasonable and intuitive:

> *The instrumental variable $Z$ is sensible if and only if the straight line given by (4.14) intersects the simplex $\mathcal{P}$ whose edges are given by (4.10)–(4.12)* (4.15)

Hence, if $Z$ is a sensible instrument the feasible set for the structural parameters $(g_2, g_3)$ reduces to the intersection between the triangle $\mathcal{P}$ and the straight line $\mathcal{I}$. If $\mathcal{F}$ denotes this intersection we have:

$$\mathcal{F} = \{(g_2, g_3) \in (\mathcal{P} \cap \mathcal{I}) \,)\} \tag{4.16}$$

The straight line $\mathcal{I}$ (4.14) will, except in rare case, intersect only two of the edges of $\mathcal{P}$. Which two will be determined by the $\omega$'s. In the present application the line $\mathcal{I}$ intersects the two edges given by (4.10) and (4.11). For clarity we state the necessary details in the following proposition:

PROPOSITION 4.1. *Let us suppose that the structural vectors $(g_2, g_3)$ are situated in the simplex $\mathcal{P}$ whose edges are given by (4.10), (4.11) and (4.12). Let us also suppose that the instrumental variable $Z$ is sensible in that (4.15) is satisfied. Then the structural parameters $(g_2, g_3)$ are restricted to the intersection $\mathcal{F}$ of (4.16), and, therefore, contained in the rectangle $(G_2 \times G_3)$ given by:*

$$G_2 = \left[\frac{(\omega_{12}\omega_{3z} - \omega_{23}\omega_{1z})}{(\omega_{22}\omega_{3z} - \omega_{23}\omega_{2z})}; \frac{(\omega_{13}\omega_{3z} - \omega_{1z}\omega_{33})}{(\omega_{23}\omega_{3z} - \omega_{2z}\omega_{33})}\right] \tag{4.17}$$

$$G_3 = \left[\frac{(\omega_{23}\omega_{1z} - \omega_{2z}\omega_{13})}{(\omega_{23}\omega_{3z} - \omega_{2z}\omega_{33})}; \frac{(\omega_{22}\omega_{12} - \omega_{12}\omega_{2z})}{(\omega_{22}\omega_{3z} - \omega_{23}\omega_{2z})}\right] \tag{4.18}$$

PROOF. *The intervals (4.17) and (4.18) are attained by solving eqs. (4.10) and (4.14), and then (4.11) and (4.14).* ■

The following figure is instructive.

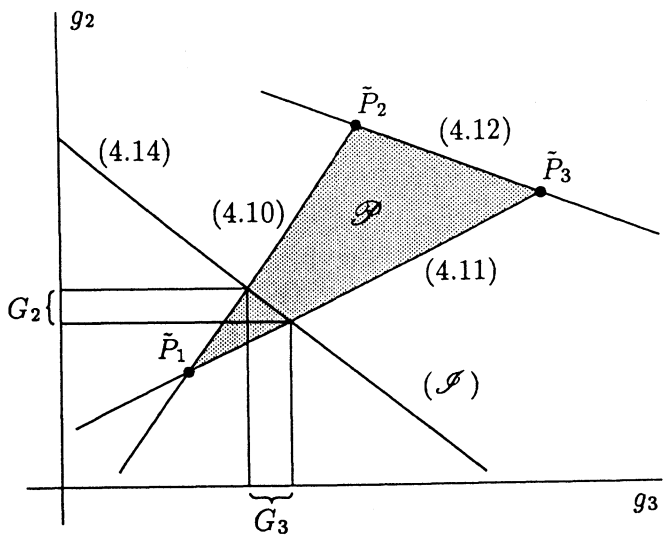


Figure 1.

# 5 Testing hypotheses on the unidentifiable structural parameters $(g_2, g_3)$

The object of testing hypotheses on the unidentifiable structural parameters may seem strange, but in cases where it is possible to identify bounds for their positions, it can be done. But the standard approaches cannot be used directly. Usually they have to be adapted in some way or the other.

Let us suppose we wish to test the hypothesis

$$\mathrm{H}_0 : g_2 = g_2^* \qquad \text{against} \qquad \mathrm{H}_1 : g_2 \neq g_2^* \tag{5.1}$$

Above we have shown that, under the stated conditions, both of $g_2$ and $g_3$ are positive. Hence, we assume that $g_2^*$ is some positive constant.

Let us denote the two bounds of the closed interval $G_2$ of (4.17) by $\alpha_1$ and $\alpha_2$. Since the interval bounded by $\alpha_1$ and $\alpha_2$ contains the structural parameter $g_2$, the hypothesis $\mathrm{H}_0$ of (5.1) will imply the hypothesis $\mathrm{F}_0$ given by

$$\mathrm{F}_0 : \{0 < \alpha_1 \leq g_2^* \leq \alpha_2\} \tag{5.2}$$

The inequalities of (5.2) represent a hypothesis on $\alpha_1$ and $\alpha_2$ which are parameters in the joint distribution of the observable variables $\{X_1, X_2, X_3, Z\}$.

Since the hypothesis $F_0$ is implied by the hypothesis $H_0$, we should reject $H_0$ if we reject $F_0$. On the other hand if we accept $F_0$ then $g_2^*$ is a possible value of $g_2$, and, therefore, we should also abstain from rejecting $H_0$ of (5.1).

Several test procedures can be designed to test the hypothesis $F_0$ or $H_0$. Perhaps the more accessible approach to test $H_0$ of (5.1) is the following based upon constructing a confidence interval for the closed interval:

$$[\alpha_1, \alpha_2] \tag{5.3}$$

We know from (4.17) that $g_2$ is contained in this interval. Since $\alpha_1 \leq g_2 \leq \alpha_2$ we observe that a confidence interval for (5.3) will also be a confidence interval for the structural coefficient $g_2$. Hence, we shall ask for a confidence interval for the interval (5.3).

A confidence interval will cover the interval (5.3) if and only if it covers its end-points. Thus we start on constructing confidence intervals for the two end-points $\alpha_1$ and $\alpha_2$. From (4.17) we observe that consistent estimators of $\alpha_1$ and $\alpha_2$ are given by:

$$\hat{\alpha}_1(N) = \frac{(s_{12}s_{3z} - s_{23}s_{1z})}{(s_{22}s_{3z} - s_{23}s_{2z})} \tag{5.4}$$

$$\hat{\alpha}_2(N) = \frac{(s_{13}s_{3z} - s_{1z}s_{33})}{(s_{23}s_{3z} - s_{2z}s_{33})} \tag{5.5}$$

where $s_{ij}$ denotes elements of the sample covariance matrix $S$, and $s_{iz}$ denotes the sample covariances between $X_i$ and the random instrument $Z$. Both $\hat{\alpha}_1$ and $\hat{\alpha}_2$ depend on $(N)$ via the sample moments.

In the following we will need some results from multivariate statistical analysis to be found in Anderson (1984). In particular we shall use his theorems (4.2.2) and (4.2.3). For the sake of clarity we reproduce theorem (4.2.3).

THEOREM (4.2.3) (Anderson (op.cit.)). *Let $\{U(N)\}$ be a sequence of m-component random vectors and $b$ a fixed vector such that $\sqrt{N}(U(N) - b)$ has a limiting distribution $N(\tilde{O}, T)$ as $N \to \infty$. Let $f(u)$ be a vector-valued function of $u$ such that each component $f_i(u)$ has a non-zero differential at $u = b$, and let $(\partial f_j/\partial u_i)_{u=b}$ be the $i,j$th component of $\Phi_b$. Then $\sqrt{N}(f(u(N)) - f(b))$ has the limiting distribution $\mathcal{N}(\tilde{O}, \Phi_b' T \Phi_b)$.*

This theorem also covers the case when $f(u)$ is a real-valued function satisfying the appropriate conditions. If $f(u)$ is a scalar function, $\Phi_b$ will be a vector. Hence, we can apply these theorems to the functions of $(s_{ij}, s_{iz})$ given by (5.4) and (5.5). We conclude that $\sqrt{N}(\hat{\alpha}_1(N) - \alpha_1)$ and $\sqrt{N}(\hat{\alpha}_2(N) - \alpha_2)$ have limiting normal distributions given by $\mathcal{N}(0, \sigma^2_{\hat{\alpha}_1})$ and $\mathcal{N}(0, \sigma^2_{\hat{\alpha}_2})$. The variances $\sigma^2_{\hat{\alpha}_1}$ and $\sigma^2_{\hat{\alpha}_2}$ are attained by calculating the relevant quadratic forms $(\tilde{\Phi}'_b T \tilde{\Phi}_b)$, or they can be calculated by the bootstrap method.

Let $V_1$ and $V_2$ be two random variables having the standard normal distribution $N(0,1)$. Then we can find a positive number $t$ such that:

$$\Pr\{V_1 \geq t\} = \Pr\{V_2 \leq -t\} = \frac{\varepsilon}{2} \tag{5.6}$$

As a confidence interval for the interval (5.3) we choose the interval:

$$[\hat{\alpha}_1 - t\sigma_{\hat{\alpha}_1}; \hat{\alpha}_2 + t\sigma_{\hat{\alpha}_2}] \tag{5.7}$$

The probability that the confidence interval (5.7) covers the interval (5.3) is equal to the probability of the set $\{V_1 \leq t; V_2 \geq -t\}$. The probability of this set is exactly $(1 - (\varepsilon/2))^2$ if $V_1$ and $V_2$ are independent. Generally, $V_1$ and $V_2$ are correlated so that the confidence coefficient of the interval (5.7) is approximately $(1 - (\varepsilon/2))^2$ which is slightly greater than $(1 - \varepsilon)$.

Since $\alpha_1 \leq g_2 \leq \alpha_2$ the probability that the interval (5.7) covers the structural coefficient $g_2$ is greater than the probability that it covers the interval (5.3). Thus, the interval (5.7) considered as a confidence interval for $g_2$ has a confidence coefficient greater than $(1 - \varepsilon)$. The probability that the confidence interval (5.7) covers $g_2$ depends on the position of $g_2$ in the interval $[\alpha_1, \alpha_2]$. In order to find this probability exactly for various values of $g_2$, we would need the joint distribution of $\hat{\alpha}_1$ and $\hat{\alpha}_2$. However, we observe directly that if $g_2$ is equal to one of the end-pints of the interval, that is $g_2 = \alpha_1$ or $g_2 = \alpha_2$, the probability that the interval (5.7) covers $g_2$ will be only slightly less than $(1 - (\varepsilon/2))$. For intermediate values of $g_2$, i.e. $\alpha_1 < g_2 < \alpha_2$, this probability can be considerably closer to 1.

These considerations should be borne in mind when we wish to apply the confidence interval (5.7) to test the null hypothesis $H_0$ of (5.1). Having chosen a confidence coefficient of $(1 - \varepsilon)$ and then calculated the interval (5.7), we recommend the following approach. If the interval (5.7) does not cover the specified value $g_2 = g_2^*$, then $H_0$ should be rejected straight away. If the value $g_2^*$ is just inside this interval indicating that $\alpha_1 \approx g_2^*$ or $\alpha_2 \approx g_2^*$ the decision of

accepting/rejecting $H_0$ should be taken only after we have considered the appropriate confidence coefficient which is approximately $(1-(\varepsilon/2))$ in these two cases.

Finally we note that the length of the confidence interval (5.7) will not tend to zero as the number of observations $(n)$ tends to infinity. We note that irrespective of what method we have used to construct a confidence interval for the interval $[\alpha_1, \alpha_2]$, the estimated interval should tend to the interval $[\alpha_1, \alpha_2]$ as $N \to \infty$.

Testing hypotheses on unidentifiable structural parameters is not entirely new. However, we know of only two previous studies, namely Moran (1956) and Willassen (1984).

# 6 The data and estimation procedures

## 6.1 Measurement errors and the data

The empirical analysis has examined 9 different industry groups: Textiles (ISIC 32), Wood products (33), Paper products (34), Chemicals (35), Mineral products (36), Metal products (381), Machinery (382), Electrical equipments (383) and Transport equipment (384). The unit of observation is a plant. The samples cover most of Norwegian manufacturing in 1983 and 1990. See the appendix for some further details on sample extraction.

As mentioned above, reported sales is our observable proxying output. Manhours and fire insurance values are the observables for labour input and capital, while materials are directly reported in our data. The number of employees is used as an instrument in the last set of regressions presented below. Notice that we have used the number of manhours in our construct of share weighted measure of variable inputs per unit of capital. The number of manhours and employees are reported as separate items in our data. Deflators are taken from the Norwegian National Accounts. Some details on variable construction are presented in the appendix.

There are several reasons why we would expect there to be an "errors-in-variables" problem in our data set[9]. Measurement errors in our regressors are created by well-known sources of noise such as differences in dating of the inputs and the output of production, misunderstanding of the questionnaire and typing errors. Furthermore, there are discrepancies between the variables needed in our model, and those available in our data set. Our production relationship requires observations on growth in the real, quality adjusted inputs. This is not available. E.g., we

[9]See Griliches (1986) for a general discussion of data problems and measurement errors in panel data .

use growth in manhours as our labour input variable, while growth in quality adjusted hours (considering differences in schooling, on-the-job training etc.) would be more correct.

For capital we have used growth in the fire insurance values, deflated by the capital price index from the National Accounts. This is clearly only an imperfect measure of the growth of capital services. This is even more so, as we know that this variable have not been subject to less scrutiny by the data collectors, as compared to most of the other variables.

An additional source of random errors is due to our assumption that the cost shares measures the output elasticities of the variable factors up to a markup coefficient. Random departures from this assumption, as well as unobservable factors (for us as outside observers) which enter the profit maximization problem will add noise to the relationship between the cost share and the output elasticity of a factor (cf. (2.4) ).

## 6.2 The bootstrap

We have used a bootstrap procedure to test whether the covariance matrix of our observable variables have compatible sign, as required for the identification of the parameter bounds. As shown in section 4, this amounts to testing whether $\theta = \Omega_{12}\Omega_{13}\Omega_{23}$ is positive, where $\Omega_{ij}$ is an element of the inverse of the covariance matrix of the $\xi$'s in our structural model. There are two reasons why it is desirable to estimate the distribution of $\hat{\theta}$ by means of a bootstrap procedure. It is simpler than deriving the analytical expression for the asymptotic variance of of $\hat{\theta}$, as $\hat{\theta}$ is a complicated function of the sample moments. The bootstrap is also claimed to a provide better estimate of the finite sample distribution of a test statistic such as $\hat{\theta}$[10]. In particular, a bootstrap test is not confined to situations where the test statistic has a normal distribution, where the first two moments are sufficient to identify the confidence interval.

We have carried out two alternative procedures to test whether $\theta$ is positive. Both procedures are based on a draw of 100 bootstrap samples of full sample size. Efron and Tibshirani (1986) claims that between 50 and 200 bootstrap samples are sufficient in most situations. From each bootstrap sample we have estimated a $\theta$. The two test procedures use the resulting sample of $\theta$-estimates differently. The first procedure assumes the distribution of $\hat{\theta}$ to be normal. It follows that a test can be carried out on the basis of the test statistic $\mu_{\hat{\theta}}/\sigma_{\hat{\theta}}$, where the numerator and the denominator are the mean and the standard deviation of the estimated sample of $\hat{\theta}$s (cf.

[10]See Efron (1982) and Efron and Tibshirani (1986).

section 4). Clearly, this test-statistic should exceed the desired percentile in the standard normal distribution to conclude that we can reject the hypothesis that $\theta$ is negative.

The alternative procedure, termed "the percentile method" (Efron, 1982, ch. 10.5), identifies the confidence region directly from the estimated distribution of $\hat{\theta}$s. In our case this is very easy: By identifying the smallest $\hat{\theta}$ estimate, we automatically get an estimate of the one percent lower bound on $\theta$. Efron and Tibshirani (1986) discuss more elaborate procedures to identify confidence intervals based on the bootstrap method, which are more robust if the $\hat{\theta}$ has a non-normal distribution. We have not considered such methods.

## 6.3 Estimating the variances of the parameter bounds

The variances of the parameter bounds have been estimated by two alternative methods: a bootstrap procedure and analytically by asymptotic theory. The bootstrap procedure is carried out in the same manner as explained above. Using 100 bootstrap samples we have obtained a sample of estimates for both the upper and lower parameter bounds for each of the two parameters of interest ($\gamma_2$ and $\gamma_3$). Using these samples we have estimated the variance of each of the parameter bounds. These variances are reported below.

An alternative procedure is to analytically identify the variances using asymptotic theory as discussed in section 5. This is somewhat more cumbersome, as the following derivation illustrates. Take the bound for the $g_2$ parameter identified in (5.4), which we termed $\hat{\alpha}_1(N)$ in section 5. The variance of $\sqrt{N}\ \hat{\alpha}_1(N)$, denoted $\sigma_{\hat{\alpha}_1}$, is given by

$$\sigma_{\hat{\alpha}_1} = \left(\frac{\partial \hat{\alpha}_1}{\partial s}\right)' V(s) \left(\frac{\partial \hat{\alpha}_1}{\partial s}\right). \tag{6.9}$$

The first term on the right hand side is the transposed of the gradient of the $\hat{\alpha}_1$-bound with respect to each of the covariances it depends on. We have stacked this set of covariances in a vector; $s = (s_{1z}, s_{2z}, s_{3z}, s_{12}, s_{22}, s_{23})$. The second term is the covariance matrix of this vector of covariances.

Using the expression in (4.17), we have that

$$\left(\frac{\partial \hat{\alpha}_1}{\partial s}\right)' = \frac{1}{D}\left[-s_{23},\ \alpha_1 s_{23},\ \psi s_{23},\ s_{3z},\ -\alpha_1 s_{3z},\ -\psi s_{3z}\right]. \tag{6.10}$$

Here we have defined $D \equiv s_{22}s_{3z} - s_{23}s_{2z}$, and $\psi \equiv (s_{22}s_{1z} - s_{12}s_{2z})/D$.

The $s$-vector consists of sample covariances. The asymptotic covariance matrix ( $V(s)$ ) of such a vector has a normal distribution given our assumption of independent observations (across units). This follows from the multivariate central limit theorem presented in Anderson (1984, Theorem 3.4.3). The explicit formula for the elements of the covariance matrix $V(s)$ can be derived by using results in Anderson (1984, chs. 2 and 3). This derivation require the assumption of a multivariate normal distribution of the variables. Consider an element in the $s$-vector: $s_{ij}$, where $i$ and $j$ are a suitable pair of the indecies $1, 2, 3, z$. Then we have that $(N-1)s_{ij} = \sum_{n=1}^{N}(X_i^n - \bar{X}_i)(X_j^n - \bar{X}_j) = \sum_{n=1}^{N} Y_{ij}^n$. Each element $(Y_{ij}^n)$ in this sum is independent with mean $\omega_{ij}$. The covariance between two such elements ($Y_{ij}^n$ and $Y_{lk}^n$) follows from the second, third and fourth order moments of the distribution. In the case of normality, we have that

$$\begin{aligned} \text{cov}(Y_{ij}^n, Y_{lk}^n) &= E\{[(X_i^n - \mu_i)(X_j^n - \mu_j) - \omega_{ij}][(X_l^n - \mu_l)(X_k^n - \mu_k) - \omega_{lk}]\} \\ &= \omega_{ik}\omega_{jl} + \omega_{il}\omega_{jk}, \end{aligned} \tag{6.11}$$

where we have replaced sample means by population means; the $\mu$'s as we focus on asymptotic results. The final equality follows from the properties of the second, third and fourth order moments of the normal distribution (see Anderson, 1984, ch. 2.6.2):

$$\begin{aligned} E[(X_i^n - \mu_i)(X_j^n - \mu_j)] &= \omega_{ij}, \\ E[(X_i^n - \mu_i)(X_j^n - \mu_j)(X_l^n - \mu_l)] &= 0, \\ E[(X_i^n - \mu_i)(X_j^n - \mu_j)(X_l^n - \mu_l)(X_k^n - \mu_k)] &= \omega_{ij}\omega_{kl} + \omega_{ik}\omega_{jl} + \omega_{il}\omega_{jk} \end{aligned} \tag{6.12}$$

From (6.11) and theorem 3.4.3 in Anderson (1984), it follows that

$$\text{cov}(s_{ij}, s_{lk}) = (\omega_{ik}\omega_{jl} + \omega_{il}\omega_{jk})/N, \tag{6.13}$$

and in particular

$$\text{var}(s_{ij}) = (\omega_{ij}^2 + \omega_{ii}\omega_{jj})/N. \tag{6.14}$$

With these expressions at hand, we have computed the asymptotic standard error for the $\alpha_1$, replacing the covariances in (6.13) and (6.14) by the corresponding empirical covariances. In

the same way we have derived the analytical expression for the asymptotic standard errors for the other bounds, and estimated their values. These are reported below.

# 7 Empirical results

## 7.1 Ordinary regression results

Table 1 presents results from ordinary regressions using the model in equation (2.7). In most of the industries, the markups (cf. $\mu$ in (2.7)) are small. In none of the industries can price taking behavior be rejected. Furthermore, the results reveal no presence of increasing returns in any of the industries. Most industries reveal significant and substantial decreasing returns to scale.

While the results in table 1 are not entirely implausible, several researchers have raised the question of whether estimates of production parameters similar to ours, tend to be downward biased due to measurement errors in the regressors[11]. The statistical framework presented above is suited to address this issue. Our next step is to use this framework to provide parameter bounds, considering the possibility of measurement errors in our regressors.

## 7.2 Estimating the parameter bounds

The standard model for bracketing parameter estimates in the presence of (uncorrelated) measurement errors in the regressors is to identify the bounds by the set of direct and reversed regression. As shown in section 3, this procedure is also valid in the more general case with correlated measurement errors, as long as the condition (4.2) is fulfilled. We have tested this condition by means of a bootstrap test, as suggested in the previous section. Table 2 presents the results for each industry separately. The first row reports the test statistic based on the assumption of an approximate normal distribution for $\hat{\theta}$. The next row presents the one percent lower bound on $\theta$ identified by the "percentile method" discussed above. Both test statistics suggest that the $\theta$s are non-negative for all our industries (though the evidence is a bit weak according to the first test-statistic for the industries 37, 382, 383 and 384). We conclude that proposition 4.2 seems applicable in our case.

Table 3 presents the corners of the simplex $\mathcal{P}$ defined in (3.51). These corners have been identified by running the set of direct and reverse regressions. The parameter bounds in table 3

[11] See e.g. Mairesse (1990).

are too wide to be of much value. Notice however that even the upper bounds for the price cost margins are lower than the estimates found by Hall (1988, 1990) and Domowitz et al. (1988).

As explained in section 5, parameter bounds for our model can be obtained by identifying the intersection between the simplex in the $(\mu, \eta)$-space and the line traced out by our (single) instrument (cf. (4.17) and (4.18) ).

The results are presented in table 4 and 5. Table 4 presents the direct estimates of the bounds, with the asymptotic standard errors in parentheses. The results presented in table 5 are based on the first two moments of the set of bound-estimates obtained using the bootstrap samples. The results in table 4 and 5 are very similar, the main difference being that the standard errors based on the bootstrap are somewhat higher.

Comparing the bounds in table 3 and table 4 (or 5), the usefulness of a single instrument clearly comes out. Even without complete identification of the parameters, the single instrument narrows down the parameter bounds substantially. The standard errors are quite small. The estimated lower bounds in table 4 for the price-cost margins are on average 3.2 percent higher than the OLS estimates (cf. table 1). The corresponding ratio for the scale elasticity is 3.7 percent. If we replace the lower bounds by the average of the upper and the lower bounds, the ratios are 4.6 percent and 4.1 percent. The major conclusion which emerges from this empirical analysis is that the ordinary regression results are not entirely misleading. In particular, even considering the impact of errors in variables, our estimates have lead us to conclude there is not a large margin between price and marginal costs in the industries we have considered. Our estimates are e.g. considerably lower than those identified by Hall (1988, 1990) and Domowitz et al. (1988). With respect to scale economies, we do not find (significant) increasing returns to scale to be present in any of our industries. In most industries we can not reject constant returns to scale (cf. ISICs 32, 35, 36, 382, 383 and 384 in table 4), in the sense that either the (estimated) upper or the lower bound of the scale parameter has a 95 percent confidence interval covering the value one. See the discussion in section 5. Eight out of nine industries have a scale elasticity at least 0.95 or higher, in the sense that the ninety five percent confidence interval (cf. (5.7)) for the lower bound cover the parameter value 0.95. There is one industry characterized by very low parameter estimates (ISIC 34).

# 8 Conclusions

We have presented a simple regression model applicable to a panel data set on plants, firms or industries. In our application, the framework is consistent with a very heterogeneous production structure across plants. Each plant can have its separate translog production function. The framework imposes the assumption that plants within an industry have a common degree of scale economies and the same margin between price and marginal costs.

The model has been estimated for a number of manufacturing industries using plant level data. The results suggest small margins between price and marginal costs, and scale economies below unity. Other researchers have obtained similar results using a simpler production model. There is a widespread view that these results are largely artifacts due to measurement errors. Our analysis has led us to conclude to the contrary. Correcting for measurement errors we find parameter estimates which are only slightly higher (less than 5 percent) than the OLS-estimates.

To reach this conclusion we have introduced some new statistical results which extend a classical result on regression models with "errors-in-variables" due to Reiersøl (1941). Our new results show how to identify parameter bounds when measurement errors are correlated across regressors. We also identify the advantage of having a few instrumental variables, even if they are not sufficiently numerous to fully identify the parameters. These new results should be useful to applied researchers beyond the empirical model presented here. They provide an alternative to instrumental variable regressions when a sufficient number of instruments are not available or one is worried about the assumptions required to justify the chosen instruments.

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## Appendix: Details on the construction of the data set

The applied sample covers ten out of thirteen 2.5-digit (ISIC) manufacturing industries. We have left out the sector "Food, beverages and tobacco" (ISIC 31) because it is heavily regulated, questioning the validity of the behavioral model applied above. "Basic metals" (ISIC 37), "Scientific instruments" (ISIC 385) and "Other manufacturing industries" (ISIC 39) are too small to provide reliable results based on asymptotic statistical theory.

The sample was constructed from the "Panel data"-files for Norwegian manufacturing plants[12]. These files are constructed on the basis of the census carried out by Statistics Norway[13].

In the current study, only operating plants with at least five employees have been included. All observations which did not report the variables required have been eliminated. Also we removed observations with extreme value added per unit of labour input or extreme value added per unit of capital. Extreme values were defined as outside a 300 percent interval of the median values for each year and each 5-digit industry.

[12]See Halvorsen et al. (1991) for documentation.
[13]NOS (several years) reports a variety of summary statistics.

**Table 1: OLS-estimates of markups (price-cost margins) and scale economies.[1] Long differences. 1983-90.**

| Industry (ISIC-code) | Textiles (32) | Wood prod. (33) | Paper prod. (34) | Chemi-cals (35) | Mineral prod. (36) | Metal prod. (381) | Machi-nery (382) | El. equip. (383) | Transp. equip. (384) |
|---|---|---|---|---|---|---|---|---|---|
| Markup | 0.977 | 1.010 | 0.852 | 1.053 | 1.002 | 0.979 | 0.971 | 1.038 | 1.021 |
| | (.035) | (.020) | (.034) | (.031) | (.034) | (.032) | (.045) | (.033) | (.022) |
| Scale coefficient | 0.934 | 0.932 | 0.799 | 0.955 | 0.925 | 0.907 | 0.928 | 1.002 | 0.989 |
| | (.035) | (.018) | (.036) | (.029) | (.030) | (.025) | (.040) | (.038) | (.031) |
| Intercept | 0.218 | 0.144 | 0.246 | 0.158 | 0.117 | 0.230 | 0.223 | 0.154 | 0.170 |
| | (.019) | (.011) | (.020) | (.023) | (.020) | (.016) | (.025) | (.030) | (.020) |
| RMSE | 0.203 | 0.177 | 0.217 | 0.179 | 0.185 | 0.222 | 0.255 | 0.226 | 0.184 |
| $R^2$ | 0.87 | 0.90 | 0.78 | 0.88 | 0.88 | 0.84 | 0.81 | 0.88 | 0.94 |
| N | 228 | 658 | 632 | 257 | 206 | 424 | 265 | 138 | 293 |

Footnote: 1) Asymptotic standard errors robust to general cross-sectional heteroskedasticity are presented in parentheses.

**Table 2: Test for compatible signs of $\Omega^{-1}$. The test statistics are based on bootstrap estimation of the distibution of the $\theta$-parameter in (4.4) and (4.5).**

| Industry (ISIC-code) | Textiles (32) | Wood prod. (33) | Paper prod. (34) | Chemi-cals (35) | Mineral prod. (36) | Metal prod. (381) | Machi-nery (382) | El. equip. (383) | Transp. equip. (384) |
|---|---|---|---|---|---|---|---|---|---|
| $t_\theta$ [1)] | 2.17** | 2.79*** | 2.66*** | 2.34*** | 2.80*** | 2.60*** | 1.34* | 1.46* | 1.89* |
| $\hat{p}^{\theta}_{0.01}$ [2)] | 5068 | 15086 | 2232 | 11833 | 12618 | 3949 | 718 | 1897 | 10367 |
| Size of bootstrap sample | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Footnotes: [1)] This is the ratio of the mean and the standard deviation of the sample of bootstrap estimates of $\theta$. Cf. (4.4) and (4.5).
[2)] This is the 1 % percentile in the distribution of the sample of bootstrap estimates of $\theta$.
*: Significantly greater than zero at 10 percent level.
**: Significantly greater than zero at 5 percent level.
***: Significantly greater than zero at 1 percent level.

**Table 3: Estimates of the bounds for the price-cost margins (μ) and scale coefficients (η) from the set of direct and reverse regressions.**

| Industry (ISIC-code) | Textiles (32) | Wood prod. (33) | Paper prod. (34) | Chemi-cals (35) | Mineral prod. (36) | Metal prod. (381) | Machi-nery (382) | El. equip. (383) | Transp. equip. (384) |
|---|---|---|---|---|---|---|---|---|---|
| Direct: | | | | | | | | | |
| $\mu_D$ | 0.977 | 1.010 | 0.852 | 1.053 | 1.003 | 0.979 | 0.971 | 1.038 | 1.021 |
| $\eta_D$ | 0.934 | 0.932 | 0.799 | 0.955 | 0.925 | 0.907 | 0.928 | 1.002 | 0.989 |
| Reverse 1: | | | | | | | | | |
| $\mu_{R1}$ | 1.154 | 1.139 | 1.142 | 1.262 | 1.179 | 1.208 | 1.208 | 1.225 | 1.109 |
| $\eta_{R1}$ | 1.024 | 1.013 | 0.991 | 1.057 | 1.029 | 1.048 | 1.048 | 1.101 | 1.040 |
| Reverse 2: | | | | | | | | | |
| $\mu_{R2}$ | 1.072 | 1.097 | 1.057 | 1.165 | 1.115 | 1.131 | 1.131 | 1.142 | 1.073 |
| $\eta_{R2}$ | 1.071 | 1.059 | 1.039 | 1.097 | 1.064 | 1.119 | 1.119 | 1.172 | 1.063 |
| N | 228 | 658 | 632 | 257 | 206 | 424 | 265 | 138 | 293 |

**Table 4: Parameter bounds for markups (price-cost margins) and scale economies. Long differences 1983-90. Asymptotic standard errors in parentheses.**

| Industry (ISIC-code) | Textiles (32) | Wood prod. (33) | Paper prod. (34) | Chemi-cals (35) | Mineral prod. (36) | Metal prod. (381) | Machi-nery (382) | El. equip. (383) | Transp. equip. (384) |
|---|---|---|---|---|---|---|---|---|---|
| Markup Up. bound | 1.095 (.032) | 1.039 (.017) | 0.937 (.025) | 1.081 (.034) | 1.038 (.037) | 1.024 (.027) | 1.062 (.038) | 1.066 (.040) | 1.086 (.019) |
| Markup Low. bound | 1.032 (.045) | 1.027 (.020) | 0.906 (.031) | 1.067 (.046) | 1.024 (.047) | 1.007 (.031) | 1.017 (.048) | 1.051 (.047) | 1.051 (.024) |
| Scale Up. bound | 1.013 (.030) | 0.957 (.016) | 0.862 (.024) | 0.972 (.029) | 0.951 (.033) | 0.947 (.025) | 0.979 (.035) | 1.023 (.039) | 1.032 (.019) |
| Scale Low. bound | 0.994 (.033) | 0.950 (.019) | 0.855 (.025) | 0.969 (.032) | 0.946 (.037) | 0.935 (.028) | 0.983 (.034) | 1.017 (.041) | 1.027 (.019) |
| N | 228 | 658 | 632 | 257 | 206 | 424 | 265 | 138 | 293 |

**Table 5: Parameter bounds for markups (price-cost margins) and scale economies. Long differences 1983-90. Bootstrap estimates of the standard errors are presented in parentheses.**

| Industry (ISIC-code) | Textiles (32) | Wood prod. (33) | Paper prod. (34) | Chemi-cals (35) | Mineral prod. (36) | Metal prod. (381) | Machi-nery (382) | El. equip. (383) | Transp. equip. (384) |
|---|---|---|---|---|---|---|---|---|---|
| Markup Up. bound | 1.100 (.051) | 1.042 (.022) | 0.943 (.041) | 1.079 (.047) | 1.038 (.051) | 1.034 (.043) | 1.063 (.048) | 1.073 (.056) | 1.084 (.029) |
| Markup Low. bound | 1.037 (.036) | 1.029 (.019) | 0.912 (.034) | 1.069 (.034) | 1.025 (.041) | 1.015 (.038) | 1.015 (.041) | 1.054 (.041) | 1.051 (.023) |
| Scale Up. bound | 1.010 (.039) | 0.957 (.020) | 0.869 (.040) | 0.970 (.037) | 0.949 (.042) | 0.955 (.035) | 0.987 (.035) | 1.023 (.047) | 1.027 (.030) |
| Scale Low. bound | 0.992 (.037) | 0.950 (.018) | 0.862 (.039) | 0.968 (.034) | 0.943 (.038) | 0.943 (.030) | 0.985 (.035) | 1.017 (.044) | 1.020 (.034) |
| N | 228 | 658 | 632 | 257 | 206 | 424 | 265 | 138 | 293 |

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