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A CRIMINOMETRIC STUDY USING PANEL DATA AND LATENT VARIABLES[1]

by

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ABSTRACT

A behavioural model of crime is developed and applied on panel data on the number of crimes and clear-ups for the 53 police districts in Norway for the period 1970-78. The model consists of behavioural relations of the criminals and the police, and of measurement relations allowing for random and systematic errors in the registered crimes and clear-ups. The identification of the model, and the implied testable hypotheses on the covariance structure of the crimes and clear-ups, depend crucially on the specification of how the distribution of the police district effects of the criminal behaviour and of the police behaviour change over time. The effect of the latent clear-up probability on the crime rate is found to be significantly negative. In our preferred model, a one percent increase in the clear-up probability reduces the number of crimes by about one percent. LISREL 7 is applied.

Key words: economics of crime, deterrence, panel data, latent variables, measurement errors.
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CONTENTS

1. INTRODUCTION

Virtually all criminal legislation is pervaded by the belief that punishment has a deterrent effect on crime. This belief was strengthened by a study of Becker (1968) where, in an economic model of crime, it was assumed that crime is a risky business and that people act as rational utility maximizers. When a person considers all benefits and costs of a possible crime, the expected utility of the crime will be reduced when either the probability of being caught and punished or the severity of punishment is increased. Not surprisingly, a reduction in the expected utility of crime will lead to a reduction in the number of crimes.

In the last 20 years the hypothesis of a deterrent effect of punishment has been confirmed by several empirical studies of total crime and of various types of crime, but not by all of them. (See reviews in Blumstein, Cohen and Nagin (1978), Heineke (1978), Bleyleveld (1980), Schmidt and Witte (1984), and Cameron (1988)). Furthermore, methodological problems in the common empirical studies of crime cast doubt on a substantial part of this literature.

Most empirical studies are plagued by substantial underregistration of crime. Registration depends on the attitude of those who discover a crime, on the access to telephone, on insurance, on police routines, etc. If recording differs between police districts (in cross section studies) or over the years (in time series studies), a spurious negative correlation will appear between the crime rate and the proportion of crimes that are cleared up (see e.g. Blumstein et al., 1978). If, on the other hand, an increase in the number of policemen increases the number of crimes that are formally recorded, but not cleared up, there will be a spurious negative correlation between the number of policemen and clear-up proportion. Thus, underreporting and changes in recording will usually introduce a bias in favour of deterrence, but against the hypothesis that the police produces it (Cameron 1988). These spurious correlations impede the evaluation of criminometric studies, that most often confirm that crime increases with a decrease in the clear-up proportion, but that more police does not increase the clear-up proportion. This difficulty has inspired us to deal more explicitly with measurement errors. Especially, we introduce latent variables and employ the maximum likelihood method in estimating the structural relations of a simultaneous model.

Fisher and Nagin (1978) have discussed the serious problem of identification of models of crime. They are reluctant to accept the commonly used procedure in empirical crime studies of identifying models by excluding various socioeconomic variables from the equations. Using panel data we have succeeded in identifying our model by showing that the structural parameters are explicit functions of the theoretical 2. order moments of the crime and clear-up rates.

This paper is meant to be the first step in a more comprehensive criminometric study. When designing the model, we have emphasized simplicity in order to focus on some basic theoretical and empirical issues. In particular, we have not included sociodemographic variables explicitly. We include, however, latent police districts effects which summarize the effects of socioeconomic variables on crimes and on clear-ups, and we model the distributions of these latent variables across police districts and over time. The strength of sentences is not included as a variable, because no perceptible difference in this factor seems to exist between police districts and over time in the period studied.

The paper is organized as follows: In section 2 the criminometric model is derived by combining an equilibrium model of the latent number of crimes and clear-ups, based on behavioural relations of the offenders and the police, and measurement relations allowing for random and systematic measurement errors in the registered crimes and clear-ups. Furthermore, submodels and hypotheses are classified. Section 3 presents detailed and subtle identification results within this model class for panel data. Data and inference procedures are presented in section 4, and the empirical results in section 5. The main conclusions are summarized in section 6.

2. MODEL AND HYPOTHESES

The criminometric model is designed to describe and explain crime and clear-up rates for I (i=1,2,...I) police districts in T (t=1,2,...,T) years. Section 2.1 presents the equilibrium model of crimes and clear-ups based on behavioural relations between the true latent variables. In section 2.2 we introduce measurement relations connecting the true latent variables with the observed crimes and clear-ups. The criminometric model in final form, derived from the submodels in 2.1 and 2.2, is given in section 2.3, and in section 2.4 we define submodels and present hypotheses to be tested. Note that the equations below hold for all relevant i and t.

2.1. An equilibrium model of crimes and clear-ups

The equilibrium model consists of the following three equations:

$$P_{it} = Y_{it}/X_{it}, \tag{1a}$$

$$X_{it} = P_{it}^{b}C_{it}, \tag{1b}$$

$$Y_{it} = X_{it}^{r}U_{it}. \tag{1c}$$

X_{it} is the (true) *crime rate*, i.e. the number of crimes per 1000 inhabitants, in police district i in year t. Y_{it} is the *clear-up rate* defined as the number of clear-ups per 1000 inhabitants. P_{it} is the *clear-up proportion* defined in (1a), i.e. the number of clear-ups as a share of the number of crimes. (In the literature this concept (P_{it}) is sometimes denoted "clear-up rate", while we prefer to use this term to denote the concept symbolized by Y_{it}, treating crimes and clear-ups "symmetrically" throughout the analysis.)

The crime function (1b) says that the crime rate (X_{it}) is a simple power function of the clear-up proportion (P_{it}). It can be interpreted as a behavioural relation for an average offender with rational expectations on the probability of being caught. Furthermore, it can be derived from a utility maximizing model in the tradition of Becker (1968), keeping the severity of punishment constant. For convenience we will call the parameter b the *deterrence elasticity* and the variable C_{it} the *crime tendency* in police district i in year t. The crime tendency (C_{it}) summarizes the effect of the socioeconomic environment and other variables not explicitly modelled. The distribution of these latent crime tendencies across districts and over time will be modelled below.

The clear-up function (1c) says that the clear-up rate (Y_{it}) is a simple power function of the crime rate (X_{it}). It can be interpreted as a behavioural relation of the police. One may also interpret it as a combined relation of the behaviour of the police and the political authorities financing the police force. For convenience we will call the parameter r the *clear-up elasticity*, and the variable U_{it} the *clear-up tendency*.

We will below interpret, exploit, and/or test the following hypotheses on the deterrence elasticity (b) and the clear-up elasticity (r):

$$\text{(a) } b<0, \quad \text{(b) } r>0, \quad \text{(c) } r<1, \quad \text{(d) } d\equiv 1+b(1-r)>0. \tag{2}$$

The theory of Becker (1968) implies (2a). Relation (2b) seems reasonable because more crimes make it possible to get more cases cleared up. With more crimes, however, less police force would be available per case, thus (2c) seems plausible. Restriction (2d) secures that there will exist a meaningful and stable solution to our equilibrium model. (The significance of the sign of the "stability parameter" d is discussed below.) Assuming (2c), the restriction (2d) is equivalent to $b>-1/(1-r)$, i.e. the deterrence elasticity must not, for a fixed value of r, be too negative. Furthermore, from (2a), (2c), and (2d) it follows that

$$0<d<1. \tag{2e}$$

The system of equations (1) has three endogenous variables (P_{it}, X_{it}, Y_{it}), and two exogenous variables (C_{it}, U_{it}), with the following solution:

$$P_{it} = C_{it}^{(r-1)/d} U_{it}^{1/d}, \tag{3a}$$

$$X_{it} = C_{it}^{1/d} U_{it}^{b/d}, \tag{3b}$$

$$Y_{it} = C_{it}^{r/d} U_{it}^{(1+b)/d}. \tag{3c}$$

Assuming (2), we obtain clear-cut sign results in five out of six cases: Increased crime tendency (C_{it}) decreases the clear-up proportion (P_{it}), increases the crime rate (X_{it}) and increases the clear-up rate (Y_{it}). Increased clear-up tendency (U_{it})

increases the clear-up proportion (P_{it}), and reduces the crime rate (X_{it}), whereas the sign effect on the clear-up rate depends on the magnitude of the deterrence effect:

$$El_{U_{it}}Y_{it} = (1+b)d \gtreqless 0 \quad \text{iff } b \gtreqless -1. \tag{4}$$

Thus, if the deterrence elasticity is less than -1, an increased clear-up tendency (U_{it}) reduces the number of clear-ups (Y_{it}) due to the strong reduction in the number of crimes.

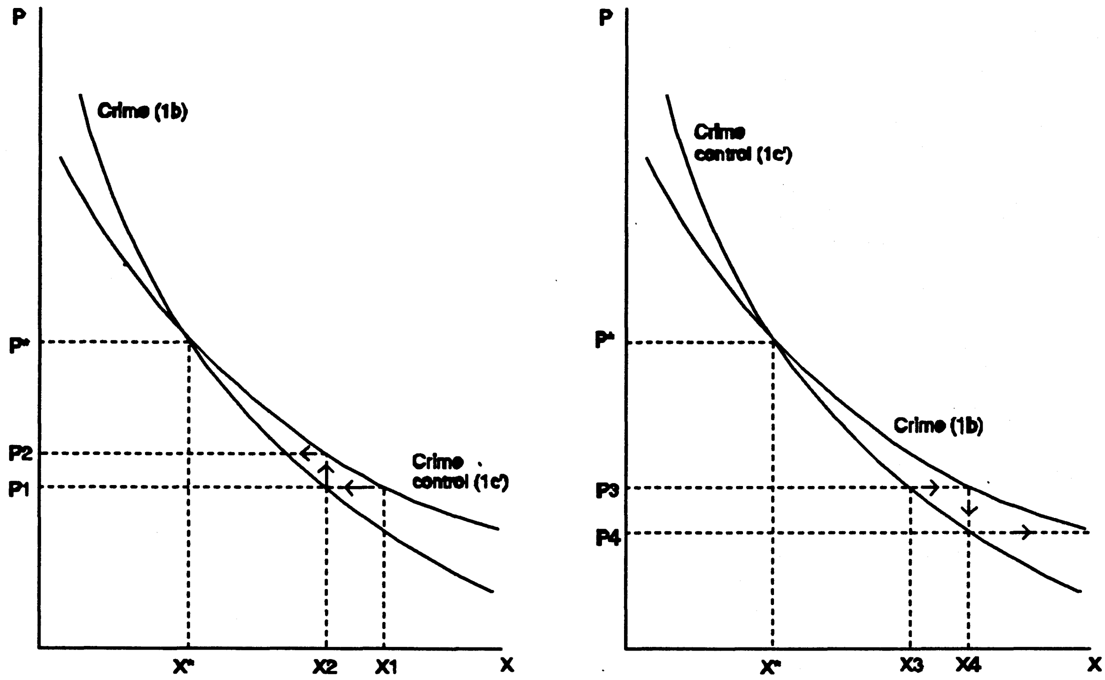
The question of stability of the equilibrium solution (3) can most easily be discussed by help of Fig. 1, where the crime rate is measured along the horizontal axis, and the clear-up proportion along the vertical one. (For convenience, the subscripts i and t are here dropped.) The crime curves illustrate relation (1b) when b<0, cf (2a). The crime control curves are obtained by eliminating the clear-up rate through substitution of (1c) into (1a):

$$P_{it} = X_{it}^{r-1}U_{it}, \quad \text{or} \tag{1c'}$$

$$X_{it} = P_{it}^{\frac{1}{r-1}} U_{it}^{\frac{1}{1-r}}. \tag{1c''}$$

Relation (1c') can be interpreted as the crime control function of the society (including the police). The clear-up activity represented by (1c) has been transformed into a function determining the clear-up probability (which again, in interaction with the crime function, determines the equilibrium values of the model).

In Fig. 1 we assume that there exist positive equilibrium values P^* and and X^* of the clear-up proportions and crime rates, respectively, and that (2a) and (2c) are satisfied. In Fig. 1 (a) the crime curve is steeper than the crime control curve, which means, cf (1b) and (1c"), that $1/(r-1) < b$, or $1+b(1-r) > 0$, which is the same as restriction (2d). Considering, according to the correspondence-principle of Samuelson (1945), our equilibrium to be the stationary solution to a corresponding dynamic model, where the society (including the police) determines the clear-up probability (cf (1c')), and the potential offenders thereafter determines the number



(a) Stable, d>0 (b) Unstable, d<0

Fig. 1 Stability of equilibrium

of crimes (cf (1b)), the following mechanism is obtained: If we start out with a hypothetical crime rate X1, the society's crime control (cf (1c')) will result in a clear-up rate P1, a rate at which crime (cf (1b)) will be reduced to X2, which again will result in a higher clear-up rate P2, etc. The crime rate and the clear-up proportion will move towards the equilibrium solution. A similar move towards equilibrium will obtain if we start from a crime rate below its equilibrium value. Thus, restriction (2d) is sufficient for a stable equilibrium under the stated conditions. If $d < 0$, we have the situation in Fig. 1 (b). Here, the society's crime control activity will produce, from a hypothetical crime rate X3, say, a clear-up proportion P3, that will result in a higher crime rate X4, which again will produce a lower clear-up proportion P4, etc. The crime rate will explode. Starting with any crime rate below X^*, the clear-up proportion will increase and the crime rate decrease. With our assumptions, we thus find that (2d) is also a necessary condition for the equilibrium solution to be stable. (If $d=0$ the two curves merge, and no single equilibrium solution is obtained.) It is straightforward to formally prove stability by analyzing an appropriate difference equation.

The model determines an equilibrium for each police district in every year. By specifying a distribution on the crime and clear-up tendencies (C_{it}, U_{it}) across police districts, and how it varies over time, we obtain a corresponding distribution of crimes and clear-ups (X_{it}, Y_{it}) through the reduced form model (3). We will give a full specification later. As for now, we introduce the following decomposition:

$$\ln C_{it} = \omega_{0t} + \omega_{1i} + t\omega_{2i}, \tag{5a}$$
$$\ln U_{it} = \lambda_{0t} + \lambda_{1i} + t\lambda_{2i}, \tag{5b}$$

where ω_{0t} and λ_{0t} are deterministic (police district invariant) time trends, and the remaining ωs and λs are time invariant latent district effects. This structure allows for a restricted evolution over time in the distribution of the crime and clear-up tendencies across police districts.

2.2. Measurement relations

Let x_{it} and y_{it} be the logs of the *registered crime and clear-up rates*, respectively. These are related to the true rates by the following equations:

$$x_{it} = \ln X_{it} + e_t + \varepsilon_{it}, \tag{6a}$$
$$y_{it} = \ln Y_{it} + f_t + \varphi_{it}. \tag{6b}$$

Here, $\exp(e_t)$ and $\exp(f_t)$ represent systematic, multiplicative measurement errors in $\exp(x_{it})$ and $\exp(y_{it})$, respectively. The terms e_t and f_t are police district invariant, but may change over time. The term e_t takes account of the problem of systematic underreporting (dark number) of crime. The variables ε_{it} and φ_{it} can be interpreted as random measurement errors.

For convenience we define the following transformed variables:

$$\chi_{it} = \ln X_{it} + e_t, \tag{7a}$$
$$\psi_{it} = \ln Y_{it} + f_t, \tag{7b}$$

$$\pi_{it} = \psi_{it} - \chi_{it} \tag{7c}$$
$$a_t = \omega_{0t} + (1+b)e_t - bf_t, \tag{7d}$$
$$k_t = \lambda_{0t} - re_t + f_t. \tag{7e}$$

In (7a) we define the log of the *latent crime rate* (χ_{it}) as the sum of the log of the true crime rate (X_{it}) and the systematic measurement error (e_t). The log of the *latent clear-up rate* (ψ_{it}), and the log of the *latent clear-up proportion* (π_{it}) are defined in (7b) and (7c). The parameters a_t and k_t are introduced in order to simplify the criminometric model below. Note that a_t and k_t are composed of the deterministic time trends of (5) and (6). We do not try to identify and estimate these components separately.

2.3. The criminometric model in final form

From (1), (5), (6), and (7) we can now derive the following criminometric model:

$$x_{it} = \chi_{it} + \varepsilon_{it}, \tag{8a}$$
$$y_{it} = \psi_{it} + \varphi_{it}, \tag{8b}$$
$$\pi_{it} = \psi_{it} - \chi_{it}, \tag{8c}$$
$$\chi_{it} = b\pi_{it} + a_t + \omega_{1i} + t\omega_{2i}, \tag{8d}$$
$$\psi_{it} = r\chi_{it} + k_t + \lambda_{1i} + t\lambda_{2i}. \tag{8e}$$

We assume the following stochastic specification:

$$E\varepsilon_{it}=E\varphi_{it}=E\omega_{1i}=E\omega_{2i}=E\lambda_{1i}=E\lambda_{2i}=0, \tag{9a}$$

$$E\varepsilon_{it}^2=\sigma_{\varepsilon\varepsilon},\quad E\varphi_{it}^2=\sigma_{\varphi\varphi},\quad E\varepsilon_{it}\phi_{it}=\sigma_{\varepsilon\varphi}, \tag{9b}$$

$$E\omega_{1i}^2=\sigma_{\omega 1\omega 1},\quad E\omega_{2i}^2=\sigma_{\omega 2\omega 2},\quad E\omega_{1i}\omega_{2i}=\sigma_{\omega 1\omega 2}, \tag{9c}$$

$$E\lambda_{1i}^2=\sigma_{\lambda 1\lambda 1},\quad E\lambda_{2i}^2=\sigma_{\lambda 2\lambda 2},\quad E\lambda_{1i}\lambda_{2i}=\sigma_{\lambda 1\lambda 2}, \tag{9d}$$

and all other covariances between the exogenous variables (ε,φ, ω, and λ) are assumed to be zero. Note that the assumptions of (9a) are innocent because of the constant terms defined in (5) and (6). Note, too, that the random measurement errors (ε_{it} and φ_{it}) are allowed to be correlated . We expect this correlation to be positive: If, in a police district, registration is particularly sloppy, some crimes that elsewhere normally would have resulted in separate files, are only informally recorded. As formal files, including eventual clear-ups, constitute the basis for the production of statistics, both the registered numbers of crimes and the registered number of clear-ups will be lower than in a similar police district with better registration procedures. This underregistration results in a positive correlation between the random measurement errors. The same will happen if some files are forgotten when statistics are produced by the end of the year.

2.4. Hypotheses and model specifications

Various specifications of the model (8)-(9) can be estimated by use of our panel data. A classification of assumptions which may be combined in various manners in order to obtain different models, is found in Table 1. Each assumption is given a label, and each model will be denoted by the corresponding combination of labels. (See Aasness, Biørn, and Skjerpen (1988) for a similar framework.) The assumptions correspond to some of the hypotheses we are interested in testing, especially hypotheses about the correlation of measurement errors, and about the distributions of latent police district effects. On the basis of the model classification of Table 1 it is possible to specify 2x4x4=32 different models defined by different assumptions in the M-, W- and L-dimensions, where these dimensions refer to correlations of measurement errors (M), correlations of police district effects on crimes (W), and correlations of police district effects on clear-ups (L). All these specific models are estimated and/or tested in the empirical analysis. We could, of course, introduce other specifications, e.g. time trends in the police district invariant terms a_t and k_t, but this is not carried out in the present paper.

3. IDENTIFICATION

Identification of most of the submodels are proven by showing that the structural parameters are explicit functions of the theoretical 2. order moments of the crime and clear-up rates, see Appendix B. The results of our investigation of identification are summarized in Table 2. Here Wi[*] (i=0,1,2,3) denotes the same assumptions as Wi in Table 1, except that all parameters assumed to be free in Table 1 now are assumed not to be zero. Lj[*] is defined similarly, and we have, for instance, that W1[*]L0 corresponds to W1L0, the difference being that $\sigma_{\omega 1 \omega 1}$ can be zero in the latter, but not in the former. Table 2 thus contains a complete set of submodels of W3L3.

Table 1
Classification of hypotheses and models[a]

Assumptions with respect to correlations of measurement errors

Label	Parameter restriction $\sigma_{\varepsilon\varphi}$	Interpretation
M0	0	No correlation of measurement errors
M1	free	Measurement errors correlated

Assumptions with respect to correlations of police district effects on crimes

Label	Parameter restriction $\sigma_{\omega 1 \omega 1}$	$\sigma_{\omega 2 \omega 2}$	$\sigma_{\omega 1 \omega 2}$	Interpretation
W0	0	0	0	No district effect in crime
W1	free	0	0	Time invariant district effect in crime
W2	free	free	0	Trend in distribution of district effect in crime
W3	free	free	free	Time invariant and trend effects correlated

Assumptions with respect to correlations of police district effects on clear-ups

Label	Parameter restriction $\sigma_{\lambda 1 \lambda 1}$	$\sigma_{\lambda 2 \lambda 2}$	$\sigma_{\lambda 1 \lambda 2}$	Interpretaion
L0	0	0	0	No district effect in clear-ups
L1	free	0	0	Time invariant district effect in clear-up
L2	free	free	0	Trend in distribution of district effect in clear-up
L3	free	free	free	Time invariant and trend effects correlated

[a] A model is specified by a combination of 3 labels: e.g. model M0W1L1 is a model where there is no correlation of measurement errors, and no trends in the police district effects on crimes and clear-ups.

A particular problem arises in models $W3^*L3^*$ and $W2^*L2^*$. Here identification of r (or b) requires the solution of a second order equation in this parameter, and we will in general have two different roots, corresponding to two observationally equivalent structures. The model can nevertheless be identified if only one of the two solutions satisfy a priori restrictions on the set of parameter values. The simplest case is to assume (2e), i.e. $0<d<1$, since we have shown (Appendix B.10) that only one of the two solutions can satisfy this restriction.

If one is not willing to use (2e) as a maintained assumption, for example because one is interested in testing this hypothesis, or the hypothesis of $b<0$, there are still possibilities for discriminating between the two observationally equivalent structures, combining a priori and empirical information. We will give an example of this. Consider the following set of restrictions, cf (2b) and (2d):

Table 2

Identification of submodels of W3L3[ab]

	$W3^*$	$W2^*$	$W1^*$	$W0^*$
$L3^*$	Identified if assuming (2e) or #A=1	Identified	Identified	Not identified[c]
$L2^*$	Identified	Identified if assuming (2e) or #A=1	Identified	Not identified[c]
$L1^*$	Identified	Identified	Not identified[e]	Not identified[c]
$L0^*$	Not identified[d]	Not identified[d]	Not identified[d]	Not identified

[a] See section 2.4 and Table 1 for definitions of models. The results hold for both M0 and M1.

[b] $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\varphi}$ are identified for W3L3 (and for all submodels).

[c] b is identified.

[d] r is identified.

[e] If one of the 4 non-identified parameters is given a fixed value, the remaining ones are identified.

$\Theta=\{\theta\in R^n \,|\, \Sigma_{\omega\omega}, \Sigma_{\lambda\lambda}, \text{ and } \Sigma_{\varepsilon\varphi} \text{ are all positive semidefinit, } r>0,\ d>0\},$ (10)

where θ are the unknown parameters of our model, and

$$\Sigma_{\omega\omega} = \begin{bmatrix} \sigma_{\omega 1\omega 1} & \sigma_{\omega 1\omega 2} \\ \sigma_{\omega 1\omega 2} & \sigma_{\omega 2\omega 2} \end{bmatrix}, \qquad \Sigma_{\lambda\lambda} = \begin{bmatrix} \sigma_{\lambda 1\lambda 1} & \sigma_{\lambda 1\lambda 2} \\ \sigma_{\lambda 1\lambda 2} & \sigma_{\lambda 2\lambda 2} \end{bmatrix}, \qquad \Sigma_{\varepsilon\varphi} = \begin{bmatrix} \sigma_{\varepsilon\varepsilon} & \sigma_{\varepsilon\varphi} \\ \sigma_{\varepsilon\varphi} & \sigma_{\varphi\varphi} \end{bmatrix}.$$

Let $\Sigma(\theta)$ denote the theoretical covariance matrix of the observed variables as a function of the unknown parameters θ of our model. Let

$$A = \{\theta\in R^n \,|\, \Sigma(\theta)=\Sigma\} \cap \Theta$$

for an arbitrary value of the covariance matrix Σ. If, for a given model, the number of elements in A is equal to one (#A=1), we consider the corresponding solution the only one that can be accepted. The number of elements in A can depend on Σ, and the question of identification of $W3^*L3^*$ and $W2^*L2^*$ thus involves empirical issues. In the empirical analysis below we argue that only one of the two solutions of $W3^*L3^*$ is relevant in our case.

It is demonstrated in Appendix B (Section B.9) that, Wi^*Lj^* is observationally equivalent to Wj^*Li^* for i≠j and i,j=0,1,2,3. Restriction (10) will in some of these cases determine which of two "symmetric" models is relevant or acceptable. Furthermore, it is shown that assuming (2e) for one such model, the symmetric one is unstable, i.e. d<0. That is, within the set of two symmetric models $\{Wi^*Lj^*, Wj^*Li^*\}$ (i≠j, i,j=1,2,3), we can identify the correct model under assumption (2e).

The parameters $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\varphi}$ are identified for W3L3 as a whole. Six of the submodels are completely identified. Identification of b is further obtained in the three first models of the last column, whereas identification of the remaining parameters here requires one supplementary piece of information (e.g. fixing the value of one of them). Similarly, r is identified in the three first models of the last line, and here too one more piece of information is necessary in order to identify the remaining parameters.

4. DATA AND ESTIMATION

The model is estimated by use of data on the number of crimes and clear-ups for 53 police districts in Norway for the period 1970-78, (cf Central Bureau of Statistics, annual). Our main reasons for choosing this period is the absence of substantial changes in legal rules or registration practices, and the wish to make comparisons with Eide (1987). These data are transformed into crime rates and clear-up rates (Tables A3 and A5) and further into logs of these rates (Tables A7 and A8). Finally, the logs are used to calculate a covariance matrix of the log numbers of crime and clear-up rates for the nine years. This covariance matrix (see Appendix A) is all the data we use in our econometric analysis.

Let S be this sample covariance matrix of our observed variables, and

$$F = \ln|\Sigma(\theta)| + tr(S\Sigma(\theta)^{-1}) - \ln|S| - 2T, \qquad (11)$$

where "tr" is the trace operator, i.e. the sum of the diagonal elements of the matrix.

Minimization of F w.r.t. θ is equivalent to maximization of the likelihood function when assuming that all the observed variables (i.e. the lnx's and lny's) are multinormally distributed. (All the first order moments are used to estimate the constant terms a_t and k_t.) We have used the computer program LISREL 7 by Jöreskog and Sörbom (1988) to perform the numerical analysis.

A standard measure of the goodness of fit of the entire model in LISREL is $GFI = 1 - tr[(\Sigma^{-1}S - I)^2]/tr[(\Sigma^{-1}S)^2]$, where I is the identity matrix; GFI = 1 indicates perfect fit. Standard asymptotic t-values and χ^2- statistics are utilized. We use a significance level of 0.01 as a standard in our test, but report also significance probabilities.

We will test a specific model 0 (the null hypothesis) against a more general model 1 (the maintained hypothesis) by a likelihood ratio test. Let F_0 and F_1 be the minimum of F under model 0 and model 1, respectively, and let s be the difference in the number of parameters of the two models. It can be shown that minus twice the logarithm of the likelihood ratio is equal to $I(F_0 - F_1)$, where I is the number of police districts. According to standard theory this statistic is approximately χ^2 distributed with s degrees of freedom. The χ^2 value for each model, given in Table 3, is defined as IF_0, which can be interpreted as the test statistic above when the alternative hypothesis is an exactly identified model, giving a perfect fit to the sample covariance matrix and accordingly $F_1=0$. The test statistic $I(F_0 - F_1)$ for an arbitrary pair of models may thus be computed by simply subtracting the corresponding pair of χ^2 values. The significance probability corresponding to the value of a test statistic, i.e. the probability of getting a χ^2 value greater than the value actually obtained given that the null hypothesis is true, is reported in Table 4.

LISREL 7 minimizes the function F without imposing any constraints on the admissible values of the parameter vector θ. Thus the LISREL estimate of a parameter which we interpret as a variance, may well turn out to be negative. This may be considered as a drawback of this computer program. However, if our model and its interpretation is correct, the LISREL estimates should turn out to have the expected signs, apart from sampling errors. Thus, if for a given model all the estimated variances are positive, and all the estimates of the covariance matrices $\Sigma_{\omega\omega}$, $\Sigma_{\lambda\lambda}$, and $\Sigma_{\varepsilon\varphi}$ are positive semidefinite, we will take this as a confirmation that the model has passed an important test. This in fact happened in our empirical analysis.

If one is unwilling to assume normality of the observed variables, the estimators derived from minimizing F above can be labelled quasi maximum likelihood estimators. These estimators will be consistent, but their efficiency and the properties of the test procedures are not so obvious. A large literature on the robustness of these types of estimators and test procedures for departure from normality prevails, see e.g. Jøreskog and Sørbom (1988) for an extensive list of references, with quite different results depending on the assumptions and methods used. A recent and growing literature shows, however, that the estimators and test statistics derived under normality assumptions within LISREL type of models retain their asymptotic properties for wide departures from normality, exploiting assumptions on independently distributed nonnormal latent variables, see e.g. Anderson and Amemiya (1988), Amemiya and Anderson (1990), Browne (1987), and Browne and Shapiro (1988).

The assumption of normality can be tested by use of the (moment coefficient of) skewness $m_3/\sqrt{m_2}^3$ and the (moment coefficient of) kurtosis m_4/m_2^2. In a normal distribution the skewness is equal to zero, and the kurtosis is equal to three. Given that the distribution is normal, the observed skewness and kurtosis are asymptotically independent, and can thus be used for two asymptotically independent tests of normality. Skewness and kurtosis for our samples have been calculated (by SPSS) for the crime and clear-up rates, and for their logs, and are included in Tables A3-A8. In 98% of all samples of size 50 from a normal population we have that the absolute value of skewness is less than 0.787, and the value of kurtosis is within the interval [1.95, 4.88][2]. We find that normality is rejected for the crime rate (Table A3) by the skewness test for all years, and by the kurtosis test for two years. As for the clear-up rate (Table A5), normality is rejected by both tests for all years. The log of crime rates (Table A7) passes the skewness test for all years, but the kurtosis test for none, whereas the log of clear-up rates (Table A8) passes the skewness test in three years, and the kurtosis test also in three years. Obviously, a logarithmic specification of our model is to be preferred to a linear one. The values of the observed kurtosis are low, indicating platykurtic or "flat" distributions. This departure from normality is considered in the χ^2 tests below.

As the distribution of the logs of the crime and clear-up rates are found to be platykurtic this χ^2 test may not be satisfactory. However, the test can easily be corrected by dividing the difference of the corresponding pairs of χ^2 values by the relative multivatiate kurtosis when performing the significance test. As the relative multivariate kurtosis for our covariance matrix is very close to one (1.06), the results of the likelihood ratio tests are not changed by this adjustment, and we find it unneccessary to present details.

5. EMPIRICAL RESULTS

5.1. Likelihood ratio tests

All 32 models classified in Table 1 have been fitted. Table 3 contains for all models

[2]The critical values of skewness and kurtosis can be found in Pearson (1965). A discussion of the present tests of normality is found in White and MacDonald (1980).

Table 3
Overview of fitted models

M1-models[a]

District effects on clear-ups		District effects on crime W3	W2	W1	W0
L3	df	160	161	162	163[c]
	χ^2	283.12	283.71	296.02	496.79
	GFI	0.647	0.645	0.638	0.397
L2	df	161	162	163	164[c]
	χ^2	283.71	296.96	300.83	506.16
	GFI	0.645	0.636	0.634	0.391
L1	df	162	163	164	165[c]
	χ^2	296.02	300.83	405.46	605.96
	GFI	0.638	0.634	0.513	0.333
L0	df	163[b]	164[b]	165[b]	166[bc]
	χ^2	496.79	506.16	605.96	1469.3
	GFI	0.397	0.391	0.333	0.186

M0-models[a]

District effects on clear-ups		District effects on crime W3	W2	W1	W0
L3	df	161	162	163	164[c]
	χ^2	585.37	589.11	589.19	688.05
	GFI	0.471	0.464	0.464	0.431
L2	df	162	163	164	165[c]
	χ^2	589.11	607.51	607.53	701.24
	GFI	0.464	0.462	0.461	0.427
L1	df	163	164	165	166[c]
	χ^2	589.19	607.53	727.16	798.44
	GFI	0.464	0.461	0.401	0.389
L0	df	164[b]	165[b]	166[b]	167[bc]
	χ^2	688.05	701.24	798.44	2037
	GFI	0.431	0.427	0.389	0.090

[a] See section 3 regarding the symmetry between WiLj and WjLi (i≠j; i=0,1,2,3).
[b] The model is estimated for a fixed value of b, any b would give the same χ^2.
[c] The model is estimated for a fixed value of r, any r would give the same χ^2.

the degrees of freedom (df), the goodness of fit (GFI), and the likelihood ratio χ^2 test statistic for each model against a model with no restriction on the covariance matrix.

First, we have studied the presence of correlation of measurement errors by testing M0 against M1. For all (16) possible combinations of maintained assumptions in the W- and L-dimensions M0 is rejected, even at a level of significance of 10^{-6}.

Table 4.1 presents significance probabilities for tests of each of the hypotheses in the W-dimension against a more general hypothesis of the same dimension. These tests are performed for each of the alternative maintained assumptions in the L-dimension. Table 4.2 contains similar tests of the L-dimension. From Tables 4.1 and 4.2 we conclude that the hypotheses of W0, L0, W1, and L1 are rejected. We have further found (not included in Table 4) that W0L0 is rejected against W1L1, W1L1 against W2L2, and W2L2 against W3L3. This leaves us with the general model M1W3L3 and the two non-rejected models M1W3L2 and M1W2L3. The choice between them can be made on the basis of parsimony, and of the acceptability of the estimated parameters. It will be argued below that M1W3L2 is the model to be preferred.

5.2. Evaluation of models not rejected by likelihood ratio tests

As identification of certain parameters in some of our models depends on the solution of a second order equation, there will in general exist two observationally equivalent structures, and correspondingly two global minima to the fit function in (11). Depending on the starting values, LISREL will find one or the other of these two solutions. The second one, which has the same F-value as the first, can be located by choosing appropriate starting values. This is done for the model M1W3L3, where we obtain the solutions I and II, the parameter estimates of which are given in Table 5. The two solutions are further characterized in Fig. 2, where the minimum value of F is plotted for various given values of r. The two global minima of F are obtained for those values of r that correspond to the solutions I and II. As a check of our conclusions, the minimum value of F has been calculated for a series of values of r in the interval [-200, 200]. F is decreasing for values of r to the left of the lower solution. For values of r higher than 1.7, F is decreasing, but very slowly, and does not reach lower than 2.752 in the interval studied.

Table 4

Significance probabilities in likelihood ratio tests[a]

1. Tests of district effects on crimes

Maintained assumptions	Null and alternative hypotheses			
	W0 against W1	W1 against W2	W2 against W3	W1 against W3
M1L3	0.000000	0.000451	0.442419	0.001581
M1L2	0.000000	0.049156	0.000273	0.000192
M1L1	0.000000	0.000000	0.028295	0.000000
M1L0	0.000000	0.000000	0.002206	0.000000

2. Tests of district effects on clear-ups

Maintained assumptions	Null and alternative hypotheses			
	L0 against L1	L1 against L2	L2 against L3	L1 against L3
M1W3	0.000000	0.000451	0.442419	0.001581
M1W2	0.000000	0.049156	0.000273	0.000192
M1W1	0.000000	0.000000	0.028295	0.000000
M1W0	0.000000	0.000000	0.002206	0.000000

[a] The equality of the significance probabilities between Tables 4.1 and 4.2 is due to the symmetry between the models WiLj and WjLi, cf Table 3.

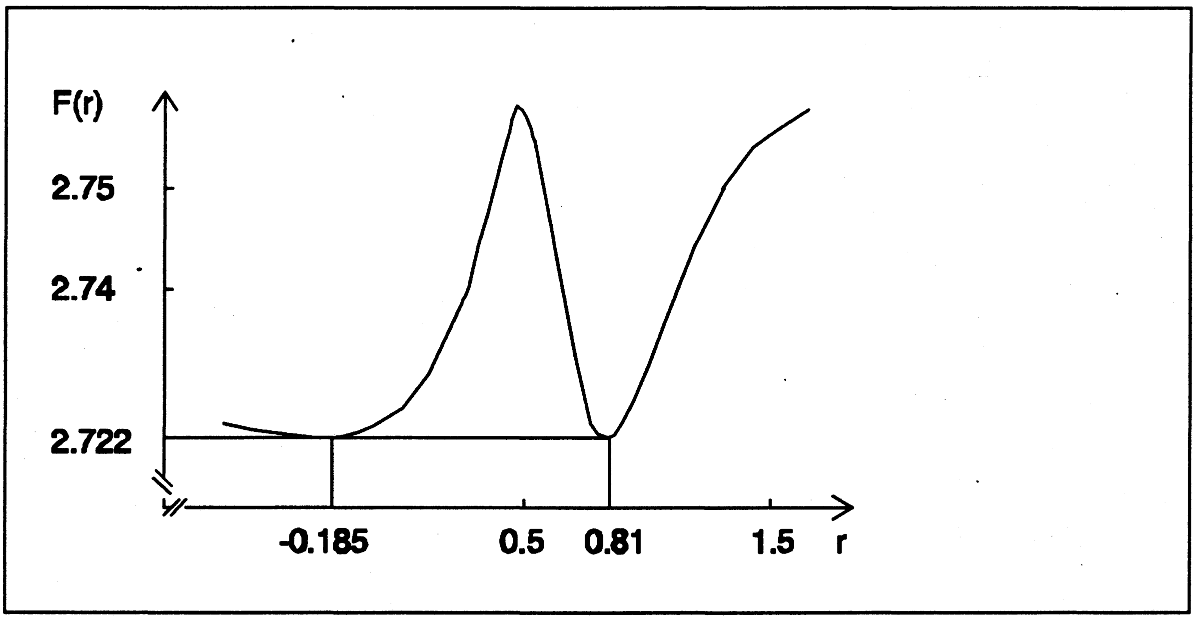


Fig. 2. F-values of M1W3L3 with two solutions

Solution II violates restrictions (2b) and (2d), cf (10), whereas all the estimates in solution I seem sensible. Thus, we prefer solution I.

We observe that the estimates of M1W3L3^{I} and M1W3L3^{II} are almost identical with those of M1W3L2 and M1W2L3, respectively. Furthermore, from the estimates of b and r we calculate the value of the stability parameter d to be 0.83 in M1W3L2 and -4.99 in M1W2L3. Thus we prefer the former model to the latter, cf section 3. The final choice is then between M1W3L3^{I} and M1W3L2. The latter being more parsimonious, we consider this model to be the preferred one. We focus on this model in sections 5.3 to 5.5, and discuss robustness of results across models in section 5.6.

5.3. The deterrence and clear-up elasticities

The estimate of the deterrence elasticity (b) is significantly negative in our preferred model, and close to -1. The estimate of the clear-up elasticity (r) is about 0.8 in our preferred model, and the confidence interval is clearly within the boundaries argued a priori, cf (2). These estimates of b and r imply that the estimate of the stability parameter d is 0.8, and the corresponding confidence interval is clearly within the boundaries (0,1), in agreement with our hypothesis (2e).

Table 5
Estimates of non-rejected models[ab]

Parameter	M1W3L3^{I}	M1W3L2	M1W2L3	M1W3L3^{II}
b	-.844	-0.864	-5.175	-5.260
	(0.357)	(0.311)	(2.237)	(2.646)
r	0.810	0.807	-0.158	-0.185
	(0.096)	(0.084)	(0.417)	(0.501)
$\sigma_{\omega1\omega1}$	0.269	0.267	1.065	1.202
	(0.069)	(0.065)	(1.179)	(1.534)
$\sigma_{\omega2\omega2}$	0.0010	0.0009	0.0093	0.0112
	(0.0004)	(0.0003)	(0.0095)	(0.0133)
$\sigma_{\omega1\omega2}$	-0.0094	-0.0094	0[c]	-0.0198
	(0.0036)	(0.0035)		(0.0385)
$\sigma_{\lambda1\lambda1}$	0.043	0.040	0.357	0.377
	(0.015)	(0.013)	(0.310)	(0.381)
$\sigma_{\lambda2\lambda2}$	0.0004	0.0003	0.0013	0.0013
	(0.0002)	(0.0001)	(0.0012)	(0.0015)
$\sigma_{\lambda1\lambda2}$	-0.0007	0[c]	-0.0126	-0.0132
	(0.0010)		(0.0110)	(0.0136)
$\sigma_{\varepsilon\varepsilon}$	0.028	0.028	0.028	0.028
	(0.002)	(0.002)	(0.002)	(0.002)
$\sigma_{\varphi\varphi}$	0.067	0.067	0.067	0.067
	(0.005)	(0.005)	(0.005)	(0.005)
$\sigma_{\varepsilon\varphi}$	0.032	0.032	0.032	0.032
	(0.003)	(0.003)	(0.003)	(0.003)
d	0.839	0.833	-4.992	-5.233
	(0.045)	(0.045)	(1.631)	(1.764)

[a] See Table 1 for definitions of models. Solutions I and II correspond to the two solutions of a second order equation obtained in identifying the model.
[b] Standard errors in parentheses.
[c] A priori restriction.

5.4. Distribution of crime and clear-up tendencies

The estimates of the distribution parameters of the district effects on crime are also given in Table 5. Straightforward calculation shows that for our preferred model the variance of the crime tendency, var $\ln C_{it} = \sigma_{\omega1\omega1} + t^2\sigma_{\omega2\omega2} + 2t\sigma_{\omega1\omega2}$, is estimated to be positive for all years, i.e. for t=1,2,...,9. As this estimate is not restricted to positive values by LISREL, we take the result as a confirmation that our model, and our interpretation of it, has passed an interesting test.

We note that $\sigma_{\omega1\omega2}$ is significantly negative. Furthermore, the estimates indicate a decrease in the variance of the district effects over time. Denoting the first difference operator by Δ, we see in fact that Δvar $\ln C_{it} = 2(t-1)\sigma_{\omega2\omega2} + 2\sigma_{\omega1\omega2}$ is negative for the whole period. The estimate of var $\ln C_{it}$ is, in this period, reduced from 0.250 to 0.186. The estimate of the variance of the crime tendency is thus substantially reduced during the period.

The estimates of the variances of the district effects on clear-ups ($\sigma_{\lambda1\lambda1}$ and $\sigma_{\lambda2\lambda2}$) are positive, as expected, and significantly different from zero in our preferred model. The change over time in the variance of the clear-up tendency is negligible.

5.5. Measurement errors

The estimates of the variances and the covariance of the errors of measurement are positive and highly significant. This confirms our hypothesis in section 2.4 of a positive $\sigma_{\varepsilon\varphi}$.

5.6. Robustness of results

Table 6 show the estimates of all models with two global maxima (solutions I and II). We observe that for all four solutions II the estimates of both r and d are negative. These models are thus rejected according to (10).

Tables 7 and 8 contain the estimates of all identified M1- and M0-models, respectively (solutions II not included). The M0-models are strongly rejected against the corresponding M1-models, and we may thus expect that at least some of the estimators of the structural parameters are biased in the M0-models. But no clear-cut results emerge by comparing Tables 7 and 8. For example, the estimates of b and r are rather close to each other for M1W3L3 and M0W3L3. For the latter, however, the estimate of the variance of λ_2 is negative, thus modelling measurement errors correctly can be important for estimating the other parameters.

Just like in our preferred model, the estimate of b is found to be negative in all but 2 of the 16 estimated models in Tables 7 and 8. The two models in question, M1W2L1 and M1W3L1 have not significant estimates of b. They are strongly rejected by the likelihood ratio tests, and have some quite nonsensical estimates. Thus, we do not give them weight as evidence on b. We conclude that the estimated sign of b is robust across models, although the value varies substantially. This result suggests that misspecification in modelling may not hinder the sign of the deterrence elasticity to be correctly determined, but that a reliable estimate of its value requires thorough empirical analysis.

The estimate of r is, as expected, and just as in our preferred model, located in the interval [0,1] for all 11 models where the estimate is significant. The estimate is positive in 3 of the remaining models, and negative in two, i.e. in M0W1L2 and M1W2L3. Of these, model M0W1L2 is firmly rejected. According to Table 4, M1W2L3 is not rejected against M1W3L3. We nevertheless disregard the former model, because the estimated value of d is significantly negative, and because its symmetric counterpart M1W3L2 is perfectly acceptable. Thus, none of the more interesting models have estimates of r that are outside the assumed interval.

For all models the variance of the crime tendency (var $\ln C_{it}$) is found to be positive in all years. We note that $\sigma_{\omega 1 \omega 2}$ is significantly negative for the fitted models where this parameter is not zero a priori (i.e. for the W3-models). Furthermore, the estimates indicate a decrease in the variance of the district effects over time for most models.

The estimates of the variances of the district effects on clear-ups ($\sigma_{\lambda 1 \lambda 1}$ and $\sigma_{\lambda 2 \lambda 2}$ are positive in all models, but one. Here, in M0W3L3, the estimate of $\sigma_{\lambda 1 \lambda 1}$ is negative, but not significant.

The estimates of the variances and the covariance of the errors of measurement are very robust with respect to model specifications.

Table 6
Estimates of models with two global maxima[ab]

Parameter	M1W3L3^I	M1W3L3^{II}	M1W2L2^I	M1W2L2^{II}	M0W3L3^I	M0W3L3^{II}	M0W2L2^I	M0W2L2^{II}
b	-0.844	-5.260	-0.916	-4.526	-0.919	-4.889	-0.127	-3.488
	(0.357)	(2.646)	(0.554)	(2.548)	(0.298)	(1.602)	(1.000)	(1.020)
r	0.810	-0.185	0.779	-0.092	0.795	-0.088	0.713	-6.887
	(0.096)	(0.501)	(0.124)	(0.661)	(0.067)	(0.353)	(0.084)	(62.18)
$\sigma_{\omega1\omega1}$	0.269	1.202	0.218	0.763	0.271	0.457	0.377	0.432
	(0.069)	(1.534)	(0.078)	(1.126)	(0.063)	(0.565)	(0.236)	(0.291)
$\sigma_{\omega2\omega2}$	0.0010	0.0112	0.0006	0.0074	0.0014	-.0067	0.0017	-0.00031
	(0.0004)	(0.0133)	(0.0004)	(0.0093)	(0.0003)	(0.0047)	(0.0011)	(0.0020)
$\sigma_{\omega1\omega2}$	-0.0094	-0.0198	0[c]	0[c]	-0.0114	0.0647	0[c]	0[c]
	(0.0036)	(0.0385)			(0.0034)	(0.0423)		
$\sigma_{\lambda1\lambda1}$	0.043	0.377	0.037	0.260	0.0191	0.320	0.0355	23.43
	(0.015)	(0.381)	(0.015)	(0.394)	(0.0138)	(0.251)	(0.0096)	(383.4)
$\sigma_{\lambda2\lambda2}$	0.0004	0.0013	0.0004	0.0007	-0.00028	0.0016	-0.0000	0.104
	(0.0002)	(0.0015)	(0.0001)	(0.0049)	(0.00017	(0.0011)	(0.0002)	(1.703)
$\sigma_{\lambda1\lambda2}$	-0.0007	-0.0132	0[c]	0[c]	0.0027	-0.0134	0[c]	0[c]
	(0.0010)	(0.0136)			(0.0011)	(0.0095)		
$\sigma_{\varepsilon\varepsilon}$	0.028	0.028	0.028	0.028	0.028	0.028	0.027	0.027
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$\sigma_{\varphi\varphi}$	0.067	0.067	0.068	0.068	0.065	0.065	0.065	0.065
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
$\sigma_{\varepsilon\varphi}$	0.032	0.032	0.033	0.033	0[c]	0[c]	0[c]	0[c]
	(0.003)	(0.003)	(0.003)	(0.003)				
d	0.839	-5.232	0.798	-3.943	0.812	-4.318	0.964	-26.510
	(0.045)	(1.764)	(0.048)	(1.177)	(0.053)	(1.493)	(0.277)	(209.9)

[a] See Table 1 for definitions of models. Solutions I and II correspond to the two solutions of a second order equation obtained in identifying the model.
[b] Standard errors in parentheses.
[c] A priori restriction.

Table 7
Estimates of M1-models[ab]

Parameter	M1W3L3	M1W3L2	M1W3L1	M1W2L3	M1W2L2	M1W2L1	M1W1L3	M1W1L2
b	-.844	-0.864	1.405	-5.175	-0.916	86.198	-2.193	-1.692
	(0.357)	(0.311)	(2.119)	(2.237)	(0.554)	(1281)	(0.476)	(0.278)
r	0.810	0.807	0.544	-0.158	0.779	0.409	1.712	1.012
	(0.096)	(0.084)	(0.099)	(0.417)	(0.124)	(0.097)	(1.073)	(0.172)
$\sigma_{\omega 1\omega 1}$	0.269	0.267	0.953	1.065	0.218	604.5	0.207	0.171
	(0.069)	(0.065)	(1.146)	(1.179)	(0.078)		(0.071)	(0.036)
$\sigma_{\omega 2\omega 2}$	0.0010	0.0009	0.0046	0.0093	0.0006	3.832	0[c]	0[c]
	(0.0004)	(0.0003)	(0.0058)	(0.0095)	(0.0004)	(112.4)		
$\sigma_{\omega 1\omega 2}$	-0.0094	-0.0094	-0.026	0[c]	0[c]	0[c]	0[c]	0[c]
	(0.0036)	(0.0035)	(0.0034)					
$\sigma_{\lambda 1\lambda 1}$	0.043	0.040	0.043	0.357	0.037	0.060	0.483	0.081
	(0.015)	(0.013)	(0.012)	(0.310)	(0.015)	(0.023)	(0.888)	(0.051)
$\sigma_{\lambda 2\lambda 2}$	0.0004	0.0003	0[c]	0.0013	0.0004	0[c]	0.0023	0.0005
	(0.0002)	(0.0001)		(0.0012)	(0.0001)		(0.0041)	(0.0002)
$\sigma_{\lambda 1\lambda 2}$	-0.0007	0[c]	0[c]	-0.0126	0[c]	0[c]	-0.013	0[c]
	(0.0010)			(0.0110)			(0.024)	
$\sigma_{\varepsilon\varepsilon}$	0.028	0.028	0.028	0.028	0.028	0.029	0.028	0.029
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$\sigma_{\varphi\varphi}$	0.067	0.067	0.071	0.067	0.068	0.072	0.071	0.072
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
$\sigma_{\varepsilon\varphi}$	0.032	0.032	0.034	0.032	0.033	0.035	0.034	0.035
	(0.003)	(0.003)	(0.0029	(0.003)	(0.0029)	(0.0029)	(0.0029)	(0.0029)
d	0.839	0.833	1.641	-4.992	0.798	51.950	2.561	1.020
	(0.045)	(0.045)	(1.089)	(1.631)	(0.048)	(764.2)	(2.653)	(0.294)

[a] See Table 1 for definitions of models. Only solutions I are included; see Table 6 for solutions II.
[b] Standard errors in parentheses.
[c] A priori restriction.

Table 8
Estimates of M0-models[ab]

Parameter	M0W3L3	M0W3L2	M0W3L1	M0W2L3	M0W2L2	M0W2L1	M0W1L3	M0W1L2
b	-.919	-1.442	-1.429	-16.097	-0.127	-0.085	-14.278	-3.449
	(0.298)	(0.407)	(0.395)	(34.83)	(1.000)	(1.226)	(25.11)	(1.169)
r	0.795	0.938	0.930	0.307	0.713	0.710	0.300	-10.71
	(0.067)	(0.134)	(0.123)	(0.196)	(0.084)	(0.098)	(0.193)	(168.1)
$\sigma_{\omega1\omega1}$	0.271	0.248	0.249	14.678	0.377	0.386	11.537	0.413
	(0.063)	(0.056)	(0.055)	(69.75)	(0.236)	(0.291)	(45.17)	(0.304)
$\sigma_{\omega2\omega2}$	0.0014	0.0014	0.0014	0.009	0.0017	0.0017	0[c]	0[c]
	(0.0003)	(0.0005)	(0.0005)	(0.061)	(0.0011)	(0.0014)		
$\sigma_{\omega1\omega2}$	-0.0114	-0.0116	-0.0121	0[c]	0[c]	0[c]	0[c]	0[c]
	(0.0034)	(0.0044)	(0.0043)					
$\sigma_{\lambda1\lambda1}$	0.0191	0.0567	0.0566	0.119	0.0355	0.0347	0.122	52.93
	(0.0138)	(0.0273)	(0.0257)	(0.072)	(0.0096)	(0.0085)	(0.073)	(1558)
$\sigma_{\lambda2\lambda2}$	-0.00028	0.00004	0[c]	·0.0006	-0.0000	0[c]	0.0007	0.235
	(0.00017	(0.00013)		(0.0003)	(0.0002)		(0.0003)	(6.930)
$\sigma_{\lambda1\lambda2}$	0.0027	0[c]	0[c]	-0.006	0[c]	0[c]	-0.006	0[c]
	(0.0011)			(0.003)			(0.003)	
$\sigma_{\varepsilon\varepsilon}$	0.028	0.027	0.027	0.027	0.027	0.027	0.027	0.027
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$\sigma_{\varphi\varphi}$	0.065	0.064	0.064	0.064	0.065	0.066	0.064	0.065
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)
$\sigma_{\varepsilon\varphi}$	0[c]	0[c]	0[c]	0[c]	0[c]	0[c]	0[c]	0[c]

[a] See Table 1 for definitions of models. Only solutions I are included; see Table 6 for solutions II.
[b] Standard errors in parentheses.
[c] A priori restriction.

5.7. Estimate of M0W1L1

As shown in Table 2, model W1L1 cannot be identified without further restrictions. In Table 9 we present the results when a constant value of r is introduced as such a restriction. The table illustrates the effect of choosing various values of this parameter. We see that an increase in r causes a strong decrease in the estimate

Table 9[a]

Estimate of M0W1L1 with r constant

r	0.5	0.6	0.7	0.82	0.85	0.90
b	4.284	1.017	-0.241	-1.004	-1.135	-1.319
	(3.924)	(0.967)	(0.439)	(0.285)	(0.271)	(0.258)
$\sigma_{\omega1\omega1}$	2.502	0.623	0.284	0.188	0.174	0.165
	(3.077)	(0.351)	(0.098)	(0.046)	(0.042)	(0.038)
$\sigma_{\lambda1\lambda1}$	0.043	0.034	0.033	0.039	0.043	0.049
	(0.010)	(0.009)	(0.009)	(0.010)	(0.011)	(0.012)
$\sigma_{\varepsilon\varepsilon}$	0.041	0.041	0.041	0.041	0.041	0.041
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
$\sigma_{\varphi\varphi}$	0.074	0.074	0.074	0.074	0.074	0.074
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
χ^2	727.16	727.16	727.16	727.16	727.16	727.16
GFI	0.401	0.401	0.401	0.401	0.401	0.401

[a] Standard errors in parentheses.

of b. The estimates of the measurement errors do not depend on the value of r. Observe that the χ^2-value, as expected, is independent of r.

6. CONCLUSIONS

A new criminometric model is derived from a theory of criminal and police behaviour, and measurement relations with random and systematic registration errors. The effects of the socioeconomic environment are summarized by the latent district effects in the crime and clear-up functions, and the distribution of these latent variables across police districts and over time is modelled.

The model is not identified if the latent district effects are constant over time. If the latent district effects vary over time, many submodels are identified. In the general model there will be two observationally equivalent structures, and correspondingly two global maxima in the likelihood function, due to the two solutions of a 2. order equation. However, by reasonable a priori restrictions in the parameter space, only one of the two solutions come out as empirically relevant.

The model was applied successfully on panel data on the number of crimes and clear-ups for the 53 police districts in Norway for 1970-78, confirming the hypothesis that our approach is fruitful. The model and the theoretical and empirical analysis can be extended in various ways. The present paper may thus be used as a starting point for further research in criminometrics.

The deterrence elasticity was found to be significantly negative and close to -1. The estimate of this elasticity varies considerably between submodels, and illustrates the importance of our systematic approach to classifying, estimating, testing, and evaluating submodels.

The variance of the latent district effects in the crime function decreased during the period. The estimates of the variances and the covariance of the measurement errors were significantly positive and very robust with respect to model specifications.

REFERENCES

Aasness, J., E. Biørn and T. Skjerpen (1988): Engel functions, panel data and latent variables. *Discussion Paper No 41, Central Bureau of Statistics*, Oslo

Amemiya, Y. and T. W. Anderson (1990): Asymptotic Chi-square tests for a large class of factor analysis models, *The Annals of Statistics*, 18, 1453-63.

Anderson, T.W. and Y. Amemiya (1988): The asymptotic normal distribution of estimators in factor analysis under general conditions, *The Annals of Statistics,* 14, 759-71.

Becker, G.S. (1968): Crime and punishment: an economic approach. *J. Pol. Econ.*, 76, 2 (March/April), 169-217.

Bleyleveld, D. (1980): *A bibliography on general deterrence research*. Westmead, Saxon House.

Blumstein, A., J. Cohen and D. Nagin, eds. (1978): *Deterrence and incapacitation: estimating the effects of criminal sanctions on crime rates*. National Academy of Sciences, Washington, D.C.

Browne, M. W. (1987): Robustness of statistical inference in factor analysis and related models, *Biometrica,* 74, 375-84.

Browne, M. W. and A. Shapiro (1988): Robustness of normal theory methods in the analysis of linear latent models, *British Journal of Mathematical and Statistical Psychology*, 41, 193-208.

Cameron, S. (1988): The economics of crime deterrence: A survey of theory and evidence. *Kyklos*, 41, 2, 301-23.

Central Bureau of Statistics (annual): Criminal Statistics. *Norges Offisielle Statistikk,* Oslo.

Eide, E. (1987): Punishment works - but do not waste money on the police. Paper presented to the Econometric Society European Meeting, Copenhagen, Aug. 1987.

Fisher, F. M. and D. Nagin (1978); On the feasibility of identifying the crime function in a simultaneous model of crime rates and sanction levels. In: *Blumstein et al.* (1978).

Heineke, J. M. (1978): *Economic models of criminal behavior*. Amsterdam, North-Holland.

Jöreskog, K. G. and D. Sörbom (1988): *LISREL 7: A guide to the program and applications*, SPSS Inc., Chicago.

Pearson, E. S. (1965): Tables of percentage points of $\sqrt{b_1}$ and b_2 in normal sampels; a rounding off. *Biometrika*, 52, 282-85.

Samuelson, P. A. (1945): *Foundations of economic analysis*. Cambridge, Mass.

Schmidt, P. and A. D. Witte (1984): *An economic analysis of crime and justice: Theory, methods and applications*. Orlando, Academic Press.

White, H. and G. M. MacDonald (1980): Some large-sample tests for nonnormality in the linear regression model, *J. of the American Statistical Association*, 75, 16-28.

APPENDIX A: DATA

Data on the number of crimes in Table 2 is found in Central Bureau of Statistics (annual): *Crime Statistics*. Data on the number of clear-ups are unpublished statistics from the Central Bureau of Statistics.

A crime is registered by the police for statistical purposes in the year when the investigation is closed. Accordingly, the statistics give information on investigated cases and not about crimes committed in the statistical year. The average period of investigation has for the country as a whole and for all crimes taken together increased from 4 months in 1970-1972 to 4.9 months in the period 1980-1982. Cases shelved for observation are considered closed, but if such cases are subsequently cleared up, a new statistical report is submitted. If this happens the same year, the first report is not included in the statistics, but if it happens in a subsequent year, the crime is registered as a separate crime and clear-up that year.

The only noticeable legal change of some importance to the number of crimes during the studied period, was the decriminalizing of "naskeri" (petty larceny of less than about 50 kr.) by the end of 1972. Until then this crime was included in the group of petty larceny, which accounted for about 30 per cent of total crimes. The number of crimes in this group did not decline from 1972 to 1973.

Table A.1
Covariance matrix of logs of crime rates

Crimes	Crimes								
	1970	1971	1972	1973	1974	1975	1976	1977	1978
1970	.4019								
1971	.3614	.3695							
1972	.3693	.3673	.3947						
1973	.3714	.3718	.3912	.4187					
1974	.3411	.3367	.3476	.3645	.3660				
1975	.3390	.3402	.3524	.3689	.3505	.3836			
1976	.3107	.3149	.3209	.3390	.3311	.3551	.3475		
1977	.3001	.3070	.3143	.3324	.3151	.3351	.3240	.3331	
1978	.2942	.2894	.2990	.3074	.2973	.3162	.2956	.3017	.3449

Table A.2
Covariance matrix of logs of crime and clear-up rates

Clear-ups	Crimes								
	1970	1971	1972	1973	1974	1975	1976	1977	1978
1970	.2931	.2590	.2524	.2444	.2302	.2217	.2093	.1957	.1859
1971	.2525	.2622	.2430	.2441	.2144	.2143	.2010	.1918	.1727
1972	.2577	.2670	.2834	.2684	.2386	.2283	.2123	.2053	.1897
1973	.2506	.2713	.2762	.2962	.2458	.2526	.2344	.2242	.1931
1974	.2204	.2334	.2388	.2445	.2533	.2334	.2231	.1993	.1734
1975	.2117	.2205	.2313	.2374	.2222	.2539	.2321	.2056	.1827
1976	.2147	.2286	.2376	.2428	.2366	.2586	.2658	.2250	.1788
1977	.1882	.2062	.2135	.2231	.1911	.2075	.2085	.2231	.1758
1978	.1980	.1998	.2069	.1978	.1883	.1879	.1775	.1803	.2110

Table A.3
Covariance matrix of logs of clear-up rates

Clear-ups	Clear-ups								
	1970	1971	1972	1973	1974	1975	1976	1977	1978
1970	.2746								
1971	.2279	.2522							
1972	.2165	.2219	.2834						
1973	.1996	.2181	.2350	.2864					
1974	.1796	.1850	.2117	.2210	.2512				
1975	.1642	.1634	.1819	.1997	.1920	.2380			
1976	.1766	.1706	.1969	.2074	.2031	.2060	.2837		
1977	.1499	.1591	.1712	.1903	.1566	.1638	.1830	.2284	
1978	.1672	.1554	.1872	.1605	.1440	.1387	.1582	.1529	.2244

Table A2
Number of crimes, police districts, 1970-78

Police district	No	1970	1971	1972	1973	1974	1975	1976	1977	1978	Mean	St.dev.
Oslo	1	21395	23550	23559	26966	28648	31616	26411	29299	32836	27142.22	1210.51
Halden	2	378	521	512	503	551	632	528	546	625	532.89	23.38
Sarpsborg	3	1066	1143	1291	1386	1879	1760	1658	1516	1901	1511.11	97.41
Fredrikstad	4	1090	1513	1836	2036	1234	2335	2238	1818	1580	1742.22	134.28
Moss	5	514	620	636	667	808	966	1123	1288	1535	906.33	109.28
Follo	6	904	1129	1245	1233	1230	1382	1236	1327	1293	1219.89	43.36
Romerike	7	1933	2177	2333	3017	3687	3420	3523	3758	4015	3095.89	241.01
Kongsvinger	8	346	480	413	343	484	383	598	682	599	480.89	38.41
Hamar	9	304	686	660	938	811	1077	1063	1071	1009	846.56	81.75
Østerdal	10	412	510	565	571	723	489	563	569	537	548.78	26.21
Gudbrandsd.	11	369	392	549	566	468	629	691	622	617	544.78	35.18
Vest-Oppl.	12	768	1240	1073	1244	1607	1729	1680	1162	1365	1318.67	98.39
Ringerike	13	748	1144	1294	1544	1951	1783	1798	1564	1583	1489.89	117.53
Asker & B.	14	3271	2688	4315	5609	4066	4725	3581	3440	3715	3934.44	272.22
Drammen	15	1821	2019	2215	2719	2652	3154	2879	3285	3628	2708.00	189.71
Kongsberg	16	541	559	588	659	636	806	674	639	711	645.89	25.55
N.-Jarlsb.	17	650	692	770	902	1230	849	929	1058	1243	924.78	67.78
Tønsberg	18	962	1329	1759	1920	1506	2220	1918	1876	2516	1778.44	146.19
Sandefjord	19	522	710	869	815	985	1081	1151	952	1286	930.11	73.17
Larvik	20	432	400	531	576	801	1183	1175	860	988	771.78	95.39
Skien	21	606	1048	1149	1081	1533	1407	1448	1596	1547	1268.33	102.03
Telemark	22	628	1172	1073	1088	1170	1986	2157	1863	2100	1470.78	175.01
Notodden	23	240	335	373	376	473	426	490	550	400	407.00	28.82
Rjukan	24	88	100	114	111	102	142	133	159	114	118.11	7.06
Kragerø	25	154	211	199	297	224	194	158	217	226	208.89	13.28
Arendal	26	775	682	1024	890	1069	1261	1203	1131	1729	1084.89	97.01
Kristians.	27	2293	2814	2511	2689	2652	3420	2969	2486	3964	2866.44	164.87
Vest-Agder	28	190	259	184	217	338	356	395	435	426	311.11	31.57
Rogaland	29	812	954	891	1249	1101	1070	1251	1217	1397	1104.67	60.40
Stavanger	30	2023	2429	2370	2327	2426	2522	2529	2182	2703	2390.11	62.84
Haugesund	31	813	851	872	743	769	1137	1262	1590	2042	1119.89	139.88
Hardanger	32	357	374	420	227	317	354	272	178	211	301.11	26.11
Hordaland	33	1421	1575	1037	903	1189	1334	1301	1278	1523	1284.56	68.37
Bergen	34	4721	4462	6668	6986	7921	7065	6684	6263	7371	6460.11	364.40
Sogn	35	78	69	67	86	180	166	168	130	254	133.11	20.14
Fjordane	36	170	196	183	161	125	168	144	194	645	220.67	50.52
Sunnmøre	37	807	941	957	1235	1156	825	1077	876	731	956.11	53.18
Romsdal	38	265	456	441	425	483	486	490	497	481	447.11	22.78
Nordmøre	39	471	479	678	665	473	476	431	566	391	514.44	31.55
Ut-Trønd.	40	494	596	749	670	778	943	958	1108	1062	817.56	66.79
Trondheim	41	3862	4093	4205	3707	3898	3349	3260	3235	4097	3745.11	119.01
Inn-Trønd.	42	371	703	704	690	764	881	1032	1086	976	800.78	69.32
Namdal	43	178	234	237	220	228	280	270	230	235	234.67	9.16
Helgeland	44	871	822	910	737	777	758	848	912	920	839.44	21.98
Bodø	45	815	932	1140	1005	1231	1141	878	917	1077	1015.11	44.08
Narvik	46	451	463	486	532	512	524	462	476	460	485.11	9.52
Lof. & Ves.	47	325	323	488	503	547	491	527	546	603	483.67	30.52
Senja	48	418	399	486	516	445	480	505	479	683	490.11	25.79
Troms	49	1088	1311	1608	1574	1785	1411	1808	1189	2368	1571.33	122.27
Vest-Finnm.	50	396	357	258	372	396	598	694	652	478	466.78	46.87
Vardø	51	151	191	163	231	166	187	194	140	108	170.11	11.22
Vadsø	52	89	102	100	109	140	187	218	139	94	130.89	14.17
Sør-Var.	53	233	163	152	156	258	234	292	237	253	219.78	15.85
Mean		1228	1389	1508	1641	1728	1858	1734	1737	1986	1645.44	78.45
St.dev		2971	3238	3307	3777	3995	4361	3649	4018	4527	3760.33	174.17
Kurtosis		42.68	44.20	39.63	40.55	41.47	43.69	42.04	44.63	43.43	42.48	.56
Skewness		6.28	6.42	6.00	6.09	6.18	6.37	6.21	6.46	6.35	6.26	.05

Table A3
Crime rates, police districts, 1970-78

Police district	No	1970	1971	1972	1973	1974	1975	1976	1977	1978	Mean	St.dev.
Oslo	1	43.90	48.90	49.54	57.06	61.15	67.94	57.04	63.35	71.32	57.80	2.87
Halden	2	11.02	15.21	14.77	14.48	15.71	18.03	15.08	15.59	17.85	15.30	.64
Sarpsborg	3	14.77	15.56	17.35	18.51	24.83	23.10	21.76	19.79	24.57	20.03	1.18
Fredrikstad	4	17.35	23.84	28.67	31.42	19.06	36.05	34.64	28.23	24.62	27.10	2.04
Moss	5	10.44	12.34	12.49	13.04	15.72	18.59	21.32	24.11	28.41	17.38	1.93
Follo	6	15.75	18.50	19.77	19.12	18.59	20.50	17.81	18.71	17.86	18.51	.42
Romerike	7	12.86	14.04	14.74	18.61	22.33	20.42	20.75	21.94	23.21	18.77	1.23
Kongsvinger	8	6.53	9.07	7.74	6.38	8.95	7.04	10.88	12.34	10.77	8.86	.67
Hamar	9	4.01	8.91	8.51	12.04	10.36	13.73	13.48	13.57	12.73	10.82	1.02
Østerdal	10	8.27	10.35	11.50	11.62	14.68	9.88	11.35	11.38	10.63	11.07	.54
Gudbrandsd.	11	5.50	5.86	8.15	8.31	6.83	9.16	10.02	8.99	8.89	7.97	.49
Vest-Oppl.	12	7.33	11.74	10.06	11.57	14.83	15.88	15.38	10.62	12.44	12.21	.88
Ringerike	13	11.72	17.83	19.98	23.67	29.72	26.95	27.04	23.36	23.54	22.65	1.72
Asker & B.	14	31.30	24.82	38.73	49.35	35.45	40.88	30.92	29.74	32.07	34.81	2.29
Drammen	15	19.25	20.95	22.68	27.54	26.48	30.92	27.96	31.67	34.89	26.93	1.63
Kongsberg	16	14.28	14.60	15.11	16.80	16.11	20.20	16.77	15.72	17.40	16.33	.56
N.-Jarlsb.	17	14.06	14.72	16.04	18.42	24.81	16.97	18.49	20.92	24.46	18.77	1.23
Tønsberg	18	17.25	23.64	31.04	33.74	26.45	38.89	33.45	32.45	43.37	31.14	2.47
Sandefjord	19	14.79	19.86	23.99	22.24	26.60	29.14	30.88	25.39	33.98	25.21	1.84
Larvik	20	11.97	10.99	14.48	15.67	21.63	31.67	31.31	22.93	26.22	20.76	2.49
Skien	21	11.19	19.28	20.93	19.67	27.93	25.33	25.83	28.33	27.31	22.87	1.77
Telemark	22	11.68	21.82	19.84	20.12	21.67	36.81	39.78	33.87	37.73	27.04	3.16
Notodden	23	9.26	13.03	14.55	14.64	18.44	16.53	18.95	21.15	15.31	15.76	1.11
Rjukan	24	10.31	11.96	14.06	13.99	13.05	18.23	17.33	20.81	14.85	14.95	1.03
Kragerø	25	10.56	14.39	13.48	20.03	15.02	12.93	10.52	14.36	14.92	14.02	.89
Arendal	26	10.61	9.25	13.72	11.77	13.97	16.21	15.27	14.21	21.48	14.05	1.12
Kristians.	27	28.37	34.31	30.07	31.83	31.00	39.64	34.05	28.19	44.51	33.55	1.70
Vest-Agder	28	3.85	5.25	3.69	4.32	6.65	6.93	7.62	8.34	8.08	6.08	.58
Rogaland	29	10.27	11.78	10.77	14.80	12.85	12.23	14.07	13.47	15.19	12.83	.54
Stavanger	30	18.11	21.65	20.90	20.27	20.85	21.37	21.12	18.03	22.06	20.48	.46
Haugesund	31	10.77	11.27	11.42	9.67	9.92	14.54	15.96	19.95	25.37	14.32	1.68
Hardanger	32	12.95	13.69	15.39	8.39	11.70	13.14	10.07	6.61	7.86	11.09	.94
Hordaland	33	6.24	6.75	7.52	6.50	8.43	9.26	8.91	8.61	10.11	8.04	.42
Bergen	34	40.79	39.36	31.46	32.69	36.91	33.01	31.29	29.44	34.79	34.42	1.22
Sogn	35	1.94	1.71	1.65	2.14	4.51	4.18	4.24	3.28	6.41	3.34	.51
Fjordane	36	2.79	3.24	2.99	2.61	2.01	2.67	2.27	3.03	10.02	3.51	.78
Sunnmøre	37	7.29	8.48	8.52	10.86	10.09	7.14	9.29	7.52	6.24	8.38	.47
Romsdal	38	5.29	9.00	8.61	8.21	9.26	9.21	9.19	9.26	8.93	8.55	.40
Nordmøre	39	7.53	7.72	10.90	10.69	7.55	7.59	6.87	8.96	6.23	8.23	.51
Ut-Trønd.	40	4.76	5.74	7.22	6.47	7.52	9.12	9.26	10.67	10.22	7.89	.64
Trondheim	41	30.11	31.44	31.90	27.60	28.66	24.47	23.66	23.34	29.62	27.87	1.04
Inn-Trønd.	42	4.54	8.58	8.49	8.25	8.98	10.22	11.88	12.45	11.12	9.39	.75
Namdal	43	4.89	6.49	6.57	6.10	6.32	7.76	7.50	6.38	6.49	6.50	.26
Helgeland	44	10.75	10.24	11.38	9.19	9.70	9.47	10.56	11.37	11.50	10.46	.27
Bodø	45	12.57	14.35	17.48	15.29	18.56	17.08	13.05	13.59	15.91	15.32	.66
Narvik	46	12.58	13.09	13.71	15.07	14.48	14.88	13.18	13.68	13.29	13.77	.27
Lof. & Ves.	47	5.29	5.35	8.11	8.30	9.00	8.06	8.70	9.06	10.02	7.99	.51
Senja	48	8.89	8.55	10.37	10.94	9.36	10.05	10.57	10.03	14.31	10.34	.53
Tromsø	49	12.15	14.55	17.66	16.94	18.92	14.78	18.77	12.32	24.42	16.72	1.21
Vest-Finnm.	50	9.73	8.76	6.30	8.89	9.32	13.90	16.12	15.16	11.13	11.03	1.04
Vardø	51	14.68	18.32	15.62	22.37	15.80	17.97	18.93	13.59	10.54	16.42	1.08
Vadsø	52	6.01	6.96	6.77	7.30	9.30	12.35	14.37	9.21	6.25	8.72	.92
Sør-Var.	53	22.05	15.55	14.29	14.64	23.90	21.55	26.81	21.70	23.50	20.44	1.42
Mean		12.66	14.60	15.50	16.42	17.21	18.77	18.35	17.56	19.53	16.73	.73
St. dev.		8.67	8.97	9.30	10.68	10.30	11.91	10.36	10.20	12.36	10.31	.42
Kurtosis		4.17	3.88	2.71	4.09	5.05	4.34	2.41	6.45	4.69	4.20	.40
Skewness		1.91	1.71	1.45	1.74	1.69	1.69	1.28	1.86	1.74	1.67	.07

Table A4
Number of clear-ups, police districts, 1970-78

Police district	No	1970	1971	1972	1973	1974	1975	1976	1977	1978	Mean	St.dev.
Oslo	1	4870	5085	4942	5627	5088	5131	4540	4455	5597	5037.22	126.59
Halden	2	233	264	231	244	267	253	209	214	211	236.22	7.00
Sarpsborg	3	367	382	477	447	837	538	608	543	759	550.89	50.58
Fredrikstad	4	466	620	728	882	459	1142	895	746	607	727.22	69.44
Moss	5	238	257	228	250	273	255	368	409	400	297.56	22.93
Follo	6	247	344	266	313	265	276	325	366	297	299.89	12.60
Romerike	7	836	757	602	892	1274	862	969	977	877	894.00	57.27
Kongsvinger	8	267	310	225	122	179	101	266	284	292	227.33	24.01
Hamar	9	212	370	312	422	374	451	545	480	464	403.33	31.26
Østerdal	10	213	165	198	167	198	148	206	238	196	192.11	8.70
Gudbrandsd.	11	102	88	144	127	122	166	271	199	130	149.89	17.64
Vest-Oppl.	12	321	655	451	454	689	721	698	341	375	522.78	52.19
Ringerike	13	304	356	453	575	705	619	682	421	451	507.33	45.06
Asker & B.	14	603	619	555	791	636	957	684	662	689	688.44	37.85
Drammen	15	754	682	680	766	626	669	777	831	764	727.67	20.71
Kongsberg	16	166	193	169	153	144	222	238	159	205	183.22	10.31
N.-Jarlsb.	17	192	276	354	251	344	223	301	276	343	284.44	17.80
Tønsberg	18	307	331	455	456	263	442	435	430	424	393.67	22.89
Sandefjord	19	239	305	341	318	357	336	487	425	452	362.22	24.63
Larvik	20	158	117	202	145	213	362	327	146	160	203.33	26.91
Skien	21	196	259	358	476	497	371	443	368	322	365.56	30.95
Telemark	22	189	336	240	446	301	317	545	409	373	350.67	33.88
Notodden	23	72	119	144	104	198	138	97	210	93	130.56	14.87
Rjukan	24	38	37	26	40	44	80	35	92	17	45.44	7.71
Kragerø	25	58	86	68	124	64	46	32	79	57	68.22	8.31
Arendal	26	354	352	454	328	447	455	533	504	641	452.00	31.43
Kristians.	27	784	1000	883	887	879	959	884	959	1055	921.11	25.24
Vest-Agder	28	103	124	93	93	91	212	110	162	174	129.11	13.65
Rogaland	29	489	590	550	681	506	593	603	535	625	574.67	19.07
Stavanger	30	804	1107	1006	789	911	809	978	787	856	894.11	35.92
Haugesund	31	417	468	470	446	304	309	333	426	395	396.44	20.64
Hardanger	32	248	192	225	111	165	166	144	63	129	160.33	17.94
Hordaland	33	629	487	369	378	422	498	624	583	708	522.00	37.86
Bergen	34	1591	1233	2185	1834	2196	1609	1594	1322	1715	1697.67	104.92
Sogn	35	48	46	33	42	88	62	45	32	48	49.33	5.34
Fjordane	36	90	114	80	91	62	95	46	103	243	102.67	17.76
Sunnmøre	37	438	462	523	696	585	360	574	439	382	495.44	34.06
Romsdal	38	127	262	257	216	246	233	211	227	180	217.67	13.30
Nordmøre	39	260	302	425	380	232	205	233	353	218	289.78	24.93
Ut-Trønd.	40	232	307	343	220	294	362	423	533	531	360.56	36.27
Trondheim	41	1384	1577	1120	793	840	899	721	628	694	961.78	103.65
Inn-Trønd.	42	233	363	441	447	505	536	612	664	511	479.11	40.57
Namdal	43	102	159	122	124	117	132	132	117	116	124.56	4.97
Helgeland	44	489	393	477	336	328	316	424	328	401	388.00	20.70
Bodø	45	472	504	592	516	673	607	307	433	505	512.11	33.58
Narvik	46	221	239	190	215	203	183	163	168	173	195.00	8.23
Lof. & Ves.	47	177	164	239	257	257	249	263	262	280	238.67	12.67
Senja	48	194	176	215	213	180	217	225	254	299	219.22	12.02
Troms	49	502	564	587	519	546	513	598	441	631	544.56	18.15
Vest-Finnm.	50	247	278	97	125	114	182	240	272	135	187.78	22.75
Vardø	51	65	102	71	133	92	61	80	66	38	78.67	8.64
Vadsø	52	50	54	37	78	85	95	151	56	30	70.67	11.66
Sør-Var.	53	138	70	54	65	71	82	107	101	114	89.11	8.63
Mean		425	466	471	483	488	487	497	464	498	475.44	7.55
St. dev.		694	717	717	788	741	723	643	619	778	713.33	18.58
Kurtosis		33.49	34.29	30.19	36.25	29.60	33.77	30.91	34.30	36.89	33.30	.86
Skewness		5.38	5.43	5.09	5.63	5.02	5.36	5.03	5.38	5.68	5.33	.08

Table A5
Clear-up rates, police districts, 1970-78

Police district	No	1970	1971	1972	1973	1974	1975	1976	1977	1978	Mean
Oslo	1	9.99	10.56	10.39	11.91	10.86	11.03	9.81	9.63	12.16	10.70
Halden	2	6.79	7.71	6.66	7.02	7.61	7.22	5.97	6.11	6.03	6.79
Sarpsborg	3	5.08	5.20	6.41	5.97	11.06	7.06	7.98	7.09	9.81	7.30
Fredrikstad	4	7.42	9.77	11.37	13.61	7.09	17.63	13.85	11.58	9.46	11.31
Moss	5	4.83	5.12	4.48	4.89	5.31	4.91	6.99	7.66	7.40	5.73
Follo	6	4.30	5.64	4.22	4.85	4.01	4.09	4.68	5.16	4.10	4.56
Romerike	7	5.56	4.88	3.80	5.50	7.72	5.15	5.71	5.70	5.07	5.45
Kongsvinger	8	5.04	5.86	4.22	2.27	3.31	1.86	4.84	5.14	5.25	4.20
Hamar	9	2.80	4.81	4.02	5.42	4.78	5.75	6.91	6.08	5.85	5.16
Østerdal	10	4.28	3.35	4.03	3.40	4.02	2.99	4.15	4.76	3.88	3.87
Gudbrandsd.	11	1.52	1.32	2.14	1.86	1.78	2.42	3.93	2.88	1.87	2.19
Vest-Oppl.	12	3.06	6.20	4.23	4.22	6.36	6.62	6.39	3.12	3.42	4.85
Ringerike	13	4.76	5.55	6.99	8.81	10.74	9.36	10.26	6.29	6.71	7.72
Asker & B.	14	5.77	5.72	4.98	6.96	5.55	8.28	5.91	5.72	5.95	6.09
Drammen	15	7.97	7.08	6.96	7.76	6.25	6.56	7.55	8.01	7.35	7.28
Kongsberg	16	4.38	5.04	4.34	3.90	3.65	5.56	5.92	3.91	5.02	4.64
N.-Jarlsb.	17	4.15	5.87	7.37	5.13	6.94	4.46	5.99	5.46	6.75	5.79
Tønsberg	18	5.50	5.89	8.03	8.01	4.62	7.74	7.59	7.44	7.31	6.90
Sandefjord	19	6.77	8.53	9.41	8.68	9.64	9.06	13.07	11.33	11.94	9.83
Larvik	20	4.38	3.21	5.51	3.94	5.75	9.69	8.71	3.89	4.25	5.48
Skien	21	3.62	4.76	6.52	8.66	9.05	6.68	7.90	6.53	5.68	6.60
Telemark	22	3.52	6.26	4.44	8.25	5.57	5.88	10.05	7.44	6.70	6.45
Notodden	23	2.78	4.63	5.62	4.05	7.72	5.35	3.75	8.08	3.56	5.06
Rjukan	24	4.45	4.43	3.21	5.04	5.63	10.27	4.56	12.04	2.21	5.76
Kragerø	25	3.98	5.87	4.61	8.36	4.29	3.07	2.13	5.23	3.76	4.59
Arendal	26	4.85	4.77	6.08	4.34	5.84	5.85	6.77	6.33	7.96	5.87
Kristians.	27	9.70	12.19	10.57	10.50	10.27	11.12	10.14	10.87	11.85	10.80
Vest-Agder	28	2.09	2.51	1.87	1.85	1.79	4.13	2.12	3.11	3.30	2.53
Rogaland	29	6.18	7.29	6.65	8.07	5.91	6.78	6.78	5.92	6.80	6.71
Stavanger	30	7.20	9.87	8.87	6.87	7.83	6.86	8.17	6.50	6.99	7.68
Haugesund	31	5.52	6.20	6.16	5.80	3.92	3.95	4.21	5.35	4.91	5.11
Hardanger	32	9.00	7.03	8.24	4.10	6.09	6.16	5.33	2.34	4.81	5.90
Hordaland	33	2.76	2.09	2.68	2.72	2.99	3.46	4.27	3.93	4.70	3.29
Bergen	34	3.75	10.88	10.31	8.58	10.23	7.52	7.46	6.21	8.09	9.23
Sogn	35	1.19	1.14	.81	1.05	2.20	1.56	1.14	.81	1.21	1.23
Fjordane	36	1.48	1.88	1.31	1.48	1.00	1.51	.73	1.61	3.77	1.64
Sunnmøre	37	3.96	4.16	4.66	6.12	5.11	3.12	4.95	3.77	3.26	4.34
Romsdal	38	2.54	5.17	5.02	4.17	4.72	4.42	3.96	4.23	3.34	4.17
Nordmøre	39	4.16	4.87	6.83	6.11	3.70	3.27	3.71	5.59	3.47	4.63
Ut-Trønd.	40	2.24	2.96	3.31	2.12	2.84	3.50	4.09	5.13	5.11	3.48
Trondheim	41	10.79	12.11	8.50	5.90	6.18	6.57	5.23	4.53	5.02	7.20
Inn-Trønd.	42	2.85	4.43	5.32	5.34	5.94	6.22	7.05	7.61	5.82	5.62
Namdal	43	2.80	4.41	3.38	3.44	3.24	3.66	3.67	3.25	3.20	3.45
Helgeland	44	6.04	4.90	5.97	4.19	4.09	3.95	5.28	4.09	5.01	4.83
Bodø	45	7.28	7.76	9.08	7.85	10.15	9.09	4.56	6.42	7.46	7.74
Narvik	46	6.16	6.76	5.36	6.09	5.74	5.20	4.65	4.83	5.00	5.53
Lof. & Ves.	47	2.88	2.72	3.97	4.24	4.23	4.09	4.34	4.35	4.65	3.94
Senja	48	4.13	3.77	4.59	4.52	3.79	4.54	4.71	5.32	6.26	4.62
Troms	49	5.61	6.26	6.45	5.59	5.79	5.37	6.21	4.57	6.51	5.82
Vest-Finnm.	50	6.07	6.82	2.37	2.99	2.68	4.23	5.57	6.32	3.14	4.47
Vardø	51	6.32	9.78	6.80	12.88	8.76	5.86	7.81	6.41	3.71	7.59
Vadsø	52	3.38	3.68	2.50	5.22	5.65	6.27	9.95	3.71	1.99	4.71
Sør-Var.	53	13.06	6.68	5.08	6.10	6.58	7.55	9.82	9.25	10.59	8.30
Mean		5.22	5.78	5.60	5.79	5.78	5.93	6.17	5.82	5.65	5.75
St.dev.		2.71	2.55	2.45	2.76	2.53	2.85	2.69	2.40	2.53	2.61
Kurtosis		1.79	.41	- .21	.75	- .37	4.27	.73	.73	.62	1.10
Skewness		1.91	1.71	1.41	1.74	1.69	1.69	1.28	1.86	1.74	1.67

Table A6
Clear-up proportions, police districts, 1970-78

Police district	No	1970	1971	1972	1973	1974	1975	1976	1977	1978	Mean	St. dev.
Oslo	1	.23	.22	.21	.21	.18	.16	.17	.15	.17	.19	.01
Halden	2	.62	.51	.45	.49	.48	.40	.40	.39	.34	.45	.03
Sarpsborg	3	.34	.33	.37	.32	.45	.31	.37	.36	.40	.36	.01
Fredrikstad	4	.43	.41	.40	.43	.37	.49	.40	.41	.38	.41	.01
Moss	5	.46	.41	.36	.37	.34	.26	.33	.32	.26	.35	.02
Follo	6	.27	.30	.21	.25	.22	.20	.26	.28	.23	.25	.01
Romerike	7	.43	.35	.26	.30	.35	.25	.28	.26	.22	.30	.02
Kongsvinger	8	.77	.65	.54	.36	.37	.26	.44	.42	.49	.48	.05
Hamar	9	.70	.54	.47	.45	.46	.42	.51	.45	.46	.50	.03
Østerdal	10	.52	.32	.35	.29	.27	.30	.37	.42	.36	.36	.02
Gudbrandsd.	11	.28	.22	.26	.22	.26	.26	.39	.32	.21	.27	.02
Vest-Oppl.	12	.42	.53	.42	.36	.43	.42	.42	.29	.27	.40	.02
Ringerike	13	.41	.31	.35	.37	.36	.35	.38	.27	.28	.34	.01
Asker & B.	14	.18	.23	.13	.14	.16	.20	.19	.19	.19	.18	.01
Drammen	15	.41	.34	.31	.28	.24	.21	.27	.25	.21	.28	.02
Kongsberg	16	.31	.35	.29	.23	.23	.28	.35	.25	.29	.28	.01
N.-Jarlsb.	17	.30	.40	.46	.28	.28	.26	.32	.26	.28	.32	.02
Tønsberg	18	.32	.25	.26	.24	.17	.20	.23	.23	.17	.23	.01
Sandefjord	19	.46	.43	.39	.39	.36	.31	.42	.45	.35	.40	.02
Larvik	20	.37	.29	.38	.25	.27	.31	.28	.17	.16	.27	.02
Skien	21	.32	.25	.31	.44	.32	.26	.31	.23	.21	.30	.02
Telemark	22	.30	.29	.22	.41	.26	.16	.25	.22	.18	.25	.02
Notodden	23	.30	.36	.39	.28	.42	.32	.20	.38	.23	.32	.02
Rjukan	24	.43	.37	.23	.36	.43	.56	.26	.58	.15	.38	.05
Kragerø	25	.38	.41	.34	.42	.29	.24	.20	.36	.25	.32	.02
Arendal	26	.46	.52	.44	.37	.42	.36	.44	.45	.37	.42	.02
Kristians.	27	.34	.36	.35	.33	.33	.28	.30	.39	.27	.33	.01
Vest-Agder	28	.54	.48	.51	.43	.27	.60	.28	.37	.41	.43	.04
Rogaland	29	.60	.62	.62	.55	.46	.55	.48	.44	.45	.53	.02
Stavanger	30	.40	.46	.42	.34	.38	.32	.39	.36	.32	.38	.01
Haugesund	31	.51	.55	.54	.60	.40	.27	.26	.27	.19	.40	.05
Hardanger	32	.69	.51	.54	.49	.52	.47	.53	.35	.61	.52	.03
Hordaland	33	.44	.31	.36	.42	.35	.37	.48	.46	.46	.41	.02
Bergen	34	.34	.28	.33	.26	.28	.23	.24	.21	.23	.27	.01
Sogn	35	.62	.67	.49	.49	.49	.37	.27	.25	.19	.43	.05
Fjordane	36	.53	.58	.44	.57	.50	.57	.32	.53	.38	.49	.03
Sunnmøre	37	.54	.49	.55	.56	.51	.44	.53	.50	.52	.52	.01
Romsdal	38	.48	.57	.58	.51	.51	.48	.43	.46	.37	.49	.02
Nordmøre	39	.55	.63	.63	.57	.49	.43	.54	.62	.56	.56	.02
Ut-Trønd.	40	.47	.52	.46	.33	.38	.38	.44	.48	.50	.44	.02
Trondheim	41	.36	.39	.27	.21	.22	.27	.22	.19	.17	.25	.02
Inn-Trønd.	42	.63	.52	.63	.65	.66	.61	.59	.61	.52	.60	.02
Namdal	43	.57	.68	.51	.56	.51	.47	.49	.51	.49	.53	.02
Helgeland	44	.56	.48	.52	.46	.42	.42	.50	.36	.44	.46	.02
Bodø	45	.58	.54	.52	.51	.55	.53	.35	.47	.47	.50	.02
Narvik	46	.49	.52	.39	.40	.40	.35	.35	.35	.38	.40	.02
Lof. & Ves.	47	.54	.51	.49	.51	.47	.51	.50	.48	.46	.50	.01
Senja	48	.46	.44	.44	.41	.40	.45	.45	.53	.44	.45	.01
Tromsø	49	.46	.43	.37	.33	.31	.36	.33	.37	.27	.36	.02
Vest-Finnm.	50	.62	.78	.38	.34	.29	.30	.35	.42	.28	.42	.05
Vardø	51	.43	.53	.44	.58	.55	.33	.41	.47	.35	.45	.03
Vadsø	52	.56	.53	.37	.72	.61	.51	.69	.40	.32	.52	.04
Sør-Var.	53	.59	.43	.36	.42	.28	.35	.37	.43	.45	.41	.03
Mean		.46	.44	.40	.40	.38	.36	.37	.37	.33	.39	.01
St.dev.		.13	.13	.12	.13	.12	.12	.11	.12	.12	.12	.00
Kurtosis		-.41	-.43	-.41	-.37	-.52	-.70	-.03	-.63	-.97	-.50	.09
Skewness		1.91	1.71	1.45	1.74	1.69	1.69	1.28	1.86	1.74	1.67	.07

Table A7
Log of crime rates, police districts, 1970-78

Police district	No	X70	X71	X72	X73	X74	X75	X76	X77	X78	Mean	St.dev
Oslo	1	3.78	3.89	3.90	4.04	4.11	4.22	4.04	4.15	4.27	4.05	.05
Halden	2	2.40	2.72	2.69	2.67	2.75	2.89	2.71	2.75	2.88	2.72	.04
Sarpsborg	3	2.69	2.74	2.85	2.92	3.21	3.14	3.08	2.99	3.20	2.98	.06
Fredrikstad	4	2.85	3.17	3.36	3.45	2.95	3.58	3.55	3.34	3.20	3.27	.08
Moss	5	2.35	2.51	2.52	2.57	2.75	2.92	3.06	3.18	3.35	2.80	.11
Follo	6	2.76	2.92	2.98	2.95	2.92	3.02	2.88	2.93	2.88	2.92	.02
Romerike	7	2.55	2.64	2.69	2.92	3.11	3.02	3.03	3.09	3.14	2.91	.07
Kongsvinger	8	1.88	2.20	2.05	1.85	2.19	1.95	2.39	2.51	2.38	2.16	.08
Hamar	9	1.39	2.19	2.14	2.49	2.34	2.62	2.60	2.61	2.54	2.32	.12
Østerdal	10	2.11	2.34	2.44	2.45	2.69	2.29	2.43	2.43	2.36	2.39	.05
Gudbrandsd.	11	1.70	1.77	2.10	2.12	1.92	2.21	2.30	2.20	2.18	2.06	.07
Vest-Oppl.	12	1.99	2.46	2.31	2.45	2.70	2.77	2.73	2.36	2.52	2.48	.08
Ringerike	13	2.46	2.88	2.99	3.16	3.39	3.29	3.30	3.15	3.16	3.09	.09
Asker & B.	14	3.44	3.21	3.66	3.90	3.57	3.71	3.43	3.39	3.47	3.53	.06
Drammen	15	2.96	3.04	3.12	3.32	3.28	3.43	3.33	3.46	3.55	3.28	.06
Kongsberg	16	2.66	2.68	2.72	2.82	2.78	3.01	2.82	2.75	2.86	2.79	.03
N.-Jarlsb.	17	2.64	2.69	2.78	2.91	3.21	2.83	2.92	3.04	3.20	2.91	.06
Tønsberg	18	2.85	3.16	3.44	3.52	3.28	3.66	3.51	3.48	3.77	3.41	.09
Sandefjord	19	2.69	2.99	3.18	3.10	3.28	3.37	3.43	3.23	3.53	3.20	.08
Larvik	20	2.48	2.40	2.67	2.75	3.07	3.46	3.44	3.13	3.27	2.96	.13
Skien	21	2.42	2.96	3.04	2.98	3.33	3.23	3.25	3.34	3.31	3.10	.09
Telemark	22	2.46	3.08	2.99	3.00	3.08	3.61	3.68	3.52	3.63	3.23	.13
Notodden	23	2.23	2.57	2.68	2.68	2.91	2.81	2.94	3.05	2.73	2.73	.08
Rjukan	24	2.33	2.48	2.64	2.64	2.57	2.90	2.85	3.04	2.70	2.68	.07
Kragerø	25	2.36	2.67	2.60	3.00	2.71	2.56	2.35	2.66	2.70	2.62	.06
Arendal	26	2.36	2.22	2.62	2.47	2.64	2.79	2.73	2.65	3.07	2.62	.08
Kristians.	27	3.35	3.54	3.40	3.46	3.43	3.68	3.53	3.34	3.80	3.50	.05
Vest-Agder	28	1.35	1.66	1.31	1.46	1.89	1.94	2.03	2.12	2.09	1.76	.10
Rogaland	29	2.33	2.47	2.38	2.69	2.55	2.50	2.64	2.60	2.72	2.54	.04
Stavanger	30	2.90	3.08	3.04	3.01	3.04	3.06	3.05	2.89	3.09	3.02	.02
Haugesund	31	2.38	2.42	2.44	2.27	2.29	2.68	2.77	2.99	3.23	2.61	.11
Hardanger	32	2.56	2.62	2.73	2.13	2.46	2.58	2.31	1.89	2.06	2.37	.09
Hordaland	33	1.83	1.91	2.02	1.87	2.13	2.23	2.19	2.15	2.31	2.07	.05
Bergen	34	3.71	3.67	3.45	3.49	3.61	3.50	3.44	3.38	3.55	3.53	.03
Sogn	35	.66	.54	.50	.76	1.51	1.43	1.44	1.19	1.86	1.10	.16
Fjordane	36	1.03	1.18	1.10	.96	.70	.98	.82	1.11	2.30	1.13	.15
Sunnmøre	37	1.99	2.14	2.14	2.39	2.31	1.97	2.23	2.02	1.83	2.11	.06
Romsdal	38	1.67	2.20	2.15	2.11	2.23	2.22	2.22	2.23	2.19	2.13	.06
Nordmøre	39	2.02	2.04	2.39	2.37	2.02	2.03	1.93	2.19	1.83	2.09	.06
Ut-Trønd.	40	1.56	1.75	1.98	1.87	2.02	2.21	2.23	2.37	2.32	2.03	.09
Trondheim	41	3.40	3.45	3.46	3.32	3.36	3.20	3.16	3.15	3.39	3.32	.04
Inn-Trønd.	42	1.51	2.15	2.14	2.11	2.19	2.32	2.47	2.52	2.41	2.20	.09
Namdal	43	1.59	1.87	1.88	1.81	1.84	2.05	2.01	1.85	1.87	1.86	.04
Helgeland	44	2.37	2.33	2.43	2.22	2.27	2.25	2.36	2.43	2.44	2.34	.03
Bodø	45	2.53	2.66	2.86	2.73	2.92	2.84	2.57	2.61	2.77	2.72	.04
Narvik	46	2.53	2.57	2.62	2.71	2.67	2.70	2.58	2.62	2.59	2.62	.02
Lof. & Ves.	47	1.67	1.68	2.09	2.12	2.20	2.09	2.16	2.20	2.30	2.06	.07
Senja	48	2.18	2.15	2.34	2.39	2.24	2.31	2.36	2.31	2.66	2.33	.05
Troms	49	2.50	2.68	2.87	2.83	2.94	2.69	2.93	2.51	3.20	2.79	.07
Vest-Finnm.	50	2.28	2.17	1.84	2.18	2.23	2.63	2.78	2.72	2.41	2.36	.10
Vardø	51	2.69	2.91	2.75	3.11	2.76	2.89	2.94	2.61	2.36	2.78	.07
Vadsø	52	1.79	1.94	1.91	1.99	2.23	2.51	2.67	2.22	1.83	2.12	.10
Sør-Var.	53	3.09	2.74	2.66	2.68	3.17	3.07	3.29	3.08	3.16	2.99	.07
Mean		2.34	2.51	2.57	2.61	2.68	2.75	2.75	2.71	2.80	2.64	.05
St. dev.		.63	.61	.63	.65	.60	.62	.59	.58	.59	.61	.01
Kurtosis		.44	1.40	1.58	.85	1.11	.40	1.19	.79	- .64	.93	.14
Skewness		- .11	- .47	- .67	- .42	- .48	- .23	- .55	- .41	.17	.39	.06

Table A8
Log of clear-up rates, police districts, 1970-78

Police district	No	Y70	Y71	Y72	Y73	Y74	Y75	Y76	Y77	Y78	Mean	St.dev.
Oslo	1	2.30	2.36	2.34	2.48	2.39	2.40	2.28	2.27	2.50	2.37	.03
Halden	2	1.92	2.04	1.90	1.95	2.03	1.98	1.79	1.81	1.80	1.91	.03
Sarpsborg	3	1.63	1.65	1.86	1.79	2.40	1.95	2.08	1.96	2.28	1.96	.08
Fredrikstad	4	2.00	2.28	2.43	2.61	1.96	2.87	2.63	2.45	2.25	2.39	.09
Moss	5	1.58	1.63	1.50	1.59	1.67	1.59	1.94	2.04	2.00	1.73	.07
Follo	6	1.46	1.73	1.44	1.58	1.39	1.41	1.54	1.64	1.41	1.51	.04
Romerike	7	1.72	1.59	1.34	1.71	2.04	1.64	1.74	1.74	1.62	1.68	.06
Kongsvinger	8	1.62	1.77	1.44	.82	1.20	.62	1.58	1.64	1.66	1.37	.13
Hamar	9	1.03	1.57	1.39	1.69	1.56	1.75	1.93	1.81	1.77	1.61	.08
Østerdal	10	1.45	1.21	1.39	1.22	1.39	1.10	1.42	1.56	1.36	1.35	.05
Gudbrandsd.	11	.42	.27	.76	.62	.58	.88	1.37	1.06	.63	.73	.10
Vest-Oppl.	12	1.12	1.82	1.44	1.44	1.85	1.89	1.85	1.14	1.23	1.53	.10
Ringerike	13	1.56	1.71	1.95	2.18	2.37	2.24	2.33	1.84	1.90	2.01	.09
Asker & B.	14	1.75	1.74	1.61	1.94	1.71	2.11	1.78	1.74	1.78	1.80	.05
Drammen	15	2.08	1.96	1.94	2.05	1.83	1.88	2.02	2.08	1.99	1.98	.03
Kongsberg	16	1.48	1.62	1.47	1.36	1.29	1.72	1.78	1.36	1.61	1.52	.05
N.-Jarlsb.	17	1.42	1.77	2.00	1.63	1.94	1.49	1.79	1.70	1.91	1.74	.06
Tønsberg	18	1.71	1.77	2.08	2.08	1.53	2.05	2.03	2.01	1.99	1.92	.06
Sandefjord	19	1.91	2.14	2.24	2.16	2.27	2.20	2.57	2.43	2.48	2.27	.06
Larvik	20	1.48	1.17	1.71	1.37	1.75	2.27	2.16	1.36	1.45	1.63	.12
Skien	21	1.29	1.56	1.88	2.16	2.20	1.90	2.07	1.88	1.74	1.85	.09
Telemark	22	1.26	1.83	1.49	2.11	1.72	1.77	2.31	2.01	1.90	1.82	.10
Notodden	23	1.02	1.53	1.73	1.40	2.04	1.68	1.32	2.09	1.27	1.56	.11
Rjukan	24	1.49	1.49	1.17	1.62	1.73	2.33	1.52	2.49	.80	1.62	.16
Kragerø	25	1.38	1.77	1.53	2.12	1.46	1.12	.76	1.65	1.33	1.46	.12
Arendal	26	1.58	1.56	1.81	1.47	1.76	1.77	1.91	1.85	2.07	1.75	.06
Kristians.	27	2.27	2.50	2.36	2.35	2.33	2.41	2.32	2.39	2.47	2.38	.02
Vest-Agder	28	.74	.92	.62	.62	.58	1.42	.75	1.13	1.19	.89	.09
Rogaland	29	1.82	1.99	1.89	2.09	1.78	1.91	1.91	1.78	1.92	1.90	.03
Stavanger	30	1.97	2.29	2.18	1.93	2.06	1.92	2.10	1.87	1.94	2.03	.04
Haugesund	31	1.71	1.82	1.82	1.76	1.37	1.37	1.44	1.68	1.59	1.62	.06
Hardanger	32	2.20	1.95	2.11	1.41	1.81	1.82	1.67	.85	1.57	1.71	.13
Hordaland	33	1.02	.74	.98	1.00	1.10	1.24	1.45	1.37	1.55	1.16	.08
Bergen	34	2.62	2.39	2.33	2.15	2.33	2.02	2.01	1.83	2.09	2.20	.08
Sogn	35	.18	.13	-.21	.04	.79	.45	.13	-.21	.19	.17	.10
Fjordane	36	.39	.63	.27	.39	-.00	.41	-.32	.48	1.33	.40	.14
Sunnmøre	37	1.38	1.43	1.54	1.81	1.63	1.14	1.60	1.33	1.18	1.45	.07
Romsdal	38	.93	1.64	1.61	1.43	1.55	1.49	1.38	1.44	1.21	1.41	.07
Nordmøre	39	1.42	1.58	1.92	1.81	1.31	1.18	1.31	1.72	1.25	1.50	.08
Ut-Trønd.	40	.80	1.08	1.20	.75	1.04	1.25	1.41	1.64	1.63	1.20	.10
Trondheim	41	2.38	2.49	2.14	1.78	1.82	1.88	1.65	1.51	1.61	1.92	.11
Inn-Trønd.	42	1.05	1.49	1.67	1.68	1.78	1.83	1.95	2.03	1.76	1.69	.09
Namdal	43	1.03	1.48	1.22	1.23	1.18	1.30	1.30	1.18	1.16	1.23	.04
Helgeland	44	1.80	1.59	1.79	1.43	1.41	1.37	1.66	1.41	1.61	1.56	.05
Bodø	45	1.99	2.05	2.21	2.06	2.32	2.21	1.52	1.86	2.01	2.02	.07
Narvik	46	1.82	1.91	1.68	1.81	1.75	1.65	1.54	1.57	1.61	1.70	.04
Lof. & Ves.	47	1.06	1.00	1.38	1.44	1.44	1.41	1.47	1.47	1.54	1.36	.06
Senja	48	1.42	1.33	1.52	1.51	1.33	1.51	1.55	1.67	1.83	1.52	.05
Troms	49	1.72	1.83	1.86	1.72	1.76	1.68	1.83	1.52	1.87	1.76	.04
Vest-Finnm.	50	1.80	1.92	.86	1.09	.99	1.44	1.72	1.84	1.15	1.42	.13
Vardø	51	1.84	2.28	1.92	2.56	2.17	1.77	2.05	1.86	1.31	1.97	.11
Vadsø	52	1.22	1.30	.92	1.65	1.73	1.84	2.30	1.31	.69	1.44	.15
Sør-Var.	53	2.57	1.90	1.62	1.81	1.88	2.02	2.28	2.22	2.36	2.08	.10
Mean		1.52	1.65	1.61	1.63	1.65	1.67	1.71	1.67	1.63	1.64	.02
St. dev.		.63	.61	.63	.65	.60	.62	.59	.58	.59	.61	.01
Kurtosis		.25	1.39	2.05	.82	1.29	.70	4.10	3.93	.79	1.70	.47
Skewness		- .30	- .91	-1.12	- .76	- .88	- .44	-1.49	-1.30	- .53	-.01	.33

APPENDIX B: IDENTIFICATION

B.1. Introduction

Identification is proved by showing that the structural parameters are explicit functions of the theoretical 2. order moments of the crime and clear-up rates. It turns out to be suitable to first deduce differences of the 2. order moments, and general 2. order differences of the 2. order moments of the crime and clear-up rates. Identification of $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\varphi}$ is proved for our most general model under investigation (M1W3L3), and for all its submodels. Then, identification of r, b and the remaining variances and covariances is considered for various submodels. For two of these there exist two observationally equivalent structures. They may still be identified if one is willing to make restrictions in the parameter space as discussed in section 3, and empirically analyzed in section 5. It is further proved that full identification is not obtainable for some of the submodels. The 1. order moments are used to identify a_t and k_t, whereas our assumption of normality excludes the possibility of obtaining supplementary information from moments of higher order than two.

B.2. Derivation of 2. order moments

Omitting subscript i for all variables we have for all t

(B1) $x_t = \chi_t + \varepsilon_t$,

(B2) $y_t = \psi_t + \phi_t$,

(B3) $\chi_t = a_t + b\pi_t + \omega_1 + t\omega_2$,

(B4) $\pi_t = \psi_t - \chi_t$,

(B5) $\psi_t = k_t + r\chi_t + \lambda_1 + t\lambda_2$.

Defining $\tilde{\chi}_t = \chi_t - E\chi_t$, etc., we obtain

(B1') $\tilde{x}_t = \tilde{\chi}_t + \varepsilon_t$,

(B2') $\tilde{y}_t = \tilde{\psi}_t + \phi_t$, (B3') $\tilde{\chi}_t = b\tilde{\pi}_t + \omega_1 + t\omega_2$,

(B4') $\tilde{\pi}_t = \tilde{\psi}_t - \tilde{\chi}_t$,

(B5') $\tilde{\psi}_t = r\tilde{\chi}_t + \lambda_1 + t\lambda_2$.

Substitution of (B4') and (B5') into (B3') gives

(B3*) $d\tilde{\chi}_t = \omega_1 + t\omega_2 + b(\lambda_1 + t\lambda_2)$,

where d=1+b(1-r) as defined in (2d).

Assuming $d \neq 0$, we have

(B3'') $\tilde{\chi}_t = \frac{1}{d}(\omega_1 + t\omega_2) + \frac{b}{d}(\lambda_1 + t\lambda_2)$,

which substituted into (B5') gives

(B5'') $\tilde{\psi}_t = \frac{r}{d}(\omega_1 + t\omega_2) + \frac{1+b}{d}(\lambda_1 + t\lambda_2)$.

Substitution of (B3'') into (B1'), and (B5'') into (B2') gives

(B1'') $\tilde{x}_t = \frac{1}{d}(\omega_1 + t\omega_2) + \frac{b}{d}(\lambda_1 + t\lambda_2) + \varepsilon_t$,

and

(B2'') $\tilde{y}_t = \frac{r}{d}(\omega_1 + t\omega_2) + \frac{1+b}{d}(\lambda_1 + t\lambda_2) + \phi_t$.

For convenience we define

$$K = \left(\frac{1}{d}\right)^2.$$

Note that it follows from (2d) that K>0.

From (B1'') and (B2'') we obtain, with the stochastic specifications assumed in 2.3, the following second order moments for t,s=1,2,...,T:

(B6) $\text{cov}(x_t,x_s) = K[\sigma_{\omega_1\omega_1} + (t+s)\sigma_{\omega_1\omega_2} + ts\sigma_{\omega_2\omega_2} + b^2\sigma_{\lambda_1\lambda_1} + b^2(t+s)\sigma_{\lambda_1\lambda_2} + b^2 ts\sigma_{\lambda_2\lambda_2}] + \delta_{ts}\sigma_{\varepsilon\varepsilon}$,

(B7) $\text{cov}(y_t,y_s) = K[r^2\sigma_{\omega_1\omega_1} + r^2(t+s)\sigma_{\omega_1\omega_2} + r^2 ts\sigma_{\omega_2\omega_2}$

$+(1+b)^2\sigma_{\lambda_1\lambda_1} + (1+b)^2(t+s)\sigma_{\lambda_1\lambda_2} + (1+b)^2 ts\sigma_{\lambda_2\lambda_2}] + \delta_{ts}\sigma_{\phi\phi}$, (B8) $\mathrm{cov}(x_t,y_s) = K[r\sigma_{\omega_1\omega_1}+r(t+s)\sigma_{\omega_1\omega_2}+rts\sigma_{\omega_2\omega_2}$

$+ (1+b)b\sigma_{\lambda_1\lambda_1}+(1+b)b(t+s)\sigma_{\lambda_1\lambda_2}+(1+b)bts\sigma_{\lambda_2\lambda_2}] + \delta_{ts}\sigma_{\varepsilon\phi}$,

where δ_{ts} are Kronecker deltas.

B.3. Differences of 2. order moments

We define, for all possible combinations of i, j, t, and s, the differences of 2. order moments

(B9) $\Delta_{ij}\mathrm{cov}(x_t,x_s) = \mathrm{cov}(x_t,x_s)-\mathrm{cov}(x_{t-i},x_{s-j})$,

and similarly for $\mathrm{cov}(y_t,y_s)$ and $\mathrm{cov}(x_t,y_s)$. With these definitions we obtain from (B6)-(B8):

(B10) $\Delta_{ij}\mathrm{cov}(x_t,x_s) = K[(i+j)\sigma_{\omega_1\omega_2}+(is+jt-ij)\sigma_{\omega_2\omega_2}+b^2[(i+j)\sigma_{\lambda_1\lambda_2}+(is+jt-ij)\sigma_{\lambda_2\lambda_2}]]$

$+ (\delta_{ts}-\delta_{t-i,s-j})\sigma_{\varepsilon\varepsilon}$,

(B11) $\Delta_{ij}\mathrm{cov}(y_t,y_s) = K[r^2[(i+j)\sigma_{\omega_1\omega_2}+(is+jt-ij)\sigma_{\omega_2\omega_2}]+(1+b)^2[(i+j)\sigma_{\lambda_1\lambda_2}+(is+jt-ij)\sigma_{\lambda_2\lambda_2}]]$

$+ (\delta_{ts}-\delta_{t-i,s-j})\sigma_{\varphi\varphi}$,

(B12) $\Delta_{ij}\mathrm{cov}(x_t,y_s) = K[r[(i+j)\sigma_{\omega_1\omega_2}+(is+jt-ij)\sigma_{\omega_2\omega_2}]+(1+b)b[(i+j)\sigma_{\lambda_1\lambda_2}+(is+jt-ij)\sigma_{\lambda_2\lambda_2}]]$

$+ (\delta_{ts}-\delta_{t-i,s-j})\sigma_{\varepsilon\varphi}$.

For notational convenience we define, by the first equation in each of the following expressions

(B13) $D_{xx} = \Delta_{1,-1}\mathrm{cov}(x_t,x_t) = K[\sigma_{\omega_2\omega_2}+b^2\sigma_{\lambda_2\lambda_2}] + \sigma_{\varepsilon\varepsilon}$,

(B14) $D_{yy} = \Delta_{1,-1}\mathrm{cov}(y_t,y_t) = K[r^2\sigma_{\omega_2\omega_2}+(1+b)^2\sigma_{\lambda_2\lambda_2}] + \sigma_{\varphi\varphi}$,

(B15) $D_{xy} = \Delta_{1,-1}\mathrm{cov}(x_t,y_t) = K[r\sigma_{\omega_2\omega_2}+(1+b)b\sigma_{\lambda_2\lambda_2}] + \sigma_{\varepsilon\varphi}$,

(B16) $E_{xx} = \Delta_{11}\mathrm{cov}(x_t,x_s) = K[2\sigma_{\omega_1\omega_2}+(s+t-1)\sigma_{\omega_2\omega_2}+b^2[2\sigma_{\lambda_1\lambda_2}+(s+t-1)\sigma_{\lambda_2\lambda_2}]]$,

(B17) $E_{yy} = \Delta_{11}\mathrm{cov}(y_t,y_s) = K[r^2[2\sigma_{\omega_1\omega_2}+(s+t-1)\sigma_{\omega_2\omega_2}]+(1+b)^2[2\sigma_{\lambda_1\lambda_2}+(s+t-1)\sigma_{\lambda_2\lambda_2}]]$, (B18) $E_{xy} = \Delta_{11}cov(x_t,y_s) = K[r[2\sigma_{\omega_1\omega_2}+(s+t-1)\sigma_{\omega_2\omega_2}]+(1+b)b[2\sigma_{\lambda_1\lambda_2}+(s+t-1)\sigma_{\lambda_2\lambda_2}]]$.

B.4. General 2. order differences of 2. order moments

We define, for all possible combinations of k, l, m, n, i, j, t, and s, the following general 2. order differences of 2. order moments:

(B19) $\Delta_{klmn}\Delta_{ij}cov(x_t,x_s) = cov(x_t,x_s)-cov(x_{t-i},x_{s-j}) - [cov(x_{t-k},x_{s-l})-cov(x_{t-k-m},x_{s-l-n})]$,

and similarly for $cov(y_t,y_s)$ and $cov(x_t,y_s)$. From (B10)-(B12) we then obtain

(B20) $\Delta_{klmn}\Delta_{ij}cov(x_t,x_s) = K[K_1(\sigma_{\omega_1\omega_2}+b^2\sigma_{\lambda_1\lambda_2})+K_2(\sigma_{\omega_2\omega_2}+b^2\sigma_{\lambda_2\lambda_2})]+M\sigma_{\varepsilon\varepsilon}$,

(B21) $\Delta_{klmn}\Delta_{ij}cov(y_t,y_s) = K[K_1(r^2\sigma_{\omega_1\omega_2}+(1+b)^2\sigma_{\lambda_1\lambda_2})+K_2(r^2\sigma_{\omega_2\omega_2}+(1+b)^2\sigma_{\lambda_2\lambda_2})]+M\sigma_{\varphi\varphi}$,

(B22) $\Delta_{klmn}\Delta_{ij}cov(x_t,y_s) = K[K_1(r\sigma_{\omega_1\omega_2}+(1+b)b\sigma_{\lambda_1\lambda_2})+K_2(r\sigma_{\omega_2\omega_2}+(1+b)b\sigma_{\lambda_2\lambda_2})]+M\sigma_{\varepsilon\varphi}$,

where

$K_1 = i+j-m-n$,

$K_2 = (i-m)s+(j-n)t-ij+ml+kn+mn$,

$M = \delta_{t,s}-\delta_{t-i,s-j}-\delta_{t-k,s-l}+\delta_{t-k-m,s-l-n}$.

Note that M can take the following values: -2, -1, 0, 1, 2. For the purpose of generating some of the identification results we define

(B23) $\mathcal{F} = \{(t,s,i,j,k,l,m,n) \mid K_1=0, K_2\neq 0\}$,

(B24) $\mathcal{G} = \{(t,s,i,j,k,l,m,n) \mid K_1\neq 0, K_2=0\}$.

These sets are non-empty: all elements satisfying i=m, j=n, and il+kj≠0 constitute a subset of $\mathcal{F}$; an example of an element in $\mathcal{G}$ is t=4, s=3, i=2, j=-1, k=l=m=n=1. From (B20)-(B23) it then follows:

(B25) $F_{xx} = \dfrac{\Delta_{klmn}\Delta_{ij}cov(x_t,x_s)}{K_2} = K(\sigma_{\omega_2\omega_2}+b^2\sigma_{\lambda_2\lambda_2})$,

(B26) $F_{yy} = \dfrac{\Delta_{klmn}\Delta_{ij}cov(y_t,y_s)}{K_2} = K[r^2\sigma_{\omega_2\omega_2}+(1+b)^2\sigma_{\lambda_2\lambda_2}]$,

(B27) $F_{xy} = \dfrac{\Delta_{klmn}\Delta_{ij}cov(x_t,y_s)}{K_2} = K[r\sigma_{\omega_2\omega_2}+(1+b)b\sigma_{\lambda_2\lambda_2}]$,

where the Fs are defined by the first equation in these formulas.
Similarly, it follows from (B20)-(B22) and (B24):

$$\text{(B28)}\quad G_{xx} = \frac{\Delta_{klmn}\Delta_{ij}\mathrm{cov}(x_t,x_s)}{K_1} = K[\sigma_{\omega_1\omega_2}+b^2\sigma_{\lambda_1\lambda_2}]\ ,$$

$$\text{(B29)}\quad G_{yy} = \frac{\Delta_{klmn}\Delta_{ij}\mathrm{cov}(y_t,y_s)}{K_1} = K[r^2\sigma_{\omega_1\omega_2}+(1+b)^2\sigma_{\lambda_1\lambda_2}]\ ,$$

$$\text{(B30)}\quad G_{xy} = \frac{\Delta_{klmn}\Delta_{ij}\mathrm{cov}(x_t,y_s)}{K_1} = K[r\sigma_{\omega_1\omega_2}+(1+b)b\sigma_{\lambda_1\lambda_2}]\ .$$

We note that the Ds, Es, Fs, and Gs are observables. Viewed as sample statistics these expressions will depend on k, l, m, n, i, j, t, and s, (the Ds and Es only on t and s), but viewed as population parameters they will, according to our model, be independent of these values. (In fact, any weighted average of any possible set of $\Delta_{klmn}\Delta_{ij}\mathrm{cov}(x_t,x_s)/K_2$ will be a consistent estimate of the population parameter F_{xx}, and similarly for F_{yy} and F_{xy}, and for all the Ds, Es, and Gs.) The equations defining the Ds, Es, Fs, and Gs can be used for various tests. In this appendix they are used only to study identification.

B.5. Identification of $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\varphi}$ for all models

From (B25)-(B27) and (B13)-(B15) we obtain

$$\text{(B31)}\quad \sigma_{\varepsilon\varepsilon} = D_{xx} - F_{xx}\ ,$$

$$\text{(B32)}\quad \sigma_{\varphi\varphi} = D_{yy} - F_{yy}\ ,$$

$$\text{(B33)}\quad \sigma_{\varepsilon\varphi} = D_{xy} - F_{xy}\ ,$$

which identify the variances and covariances of the errors of measurement for M1W3L3 and for all its submodels. Note that the parameter $\sigma_{\varepsilon\varphi}$ is not involved in the identification of $\sigma_{\varepsilon\varepsilon}$ and $\sigma_{\varphi\varphi}$, and neither in the identification of the other parameters below. Our identification results are thus equally valid for M0W3L3, where $\sigma_{\varepsilon\varphi}$=0, and for M1W3L3, where this covariance is not restricted.

B.6. Identification of r and b

Relationship between b and r

From (B7) and (B8) we obtain, for $t \neq s$

(B37) $\quad cov(y_t,y_s)-rcov(x_t,y_s) = K(1+b)d[\sigma_{\lambda_1\lambda_1}+(t+s)\sigma_{\lambda_1\lambda_2}+ts\sigma_{\lambda_2\lambda_2}].$

From (B8) and (B6) we obtain, also for $t \neq s$

(B38) $\quad cov(x_t,y_s)-rcov(x_t,x_s) = Kbd[\sigma_{\lambda_1\lambda_1}+(t+s)\sigma_{\lambda_1\lambda_2}+ts\sigma_{\lambda_2\lambda_2}].$

Assuming $\sigma_{\lambda1\lambda1}\neq 0$, (which for $t\neq s$ secures that the square bracket in (B38) is not zero), we divide (B37) by (B38) to obtain

(B39) $$b = \frac{cov(x_t,y_s)-rcov(x_t,x_s)}{cov(y_t,y_s)-cov(x_t,y_s)+r(cov(x_t,x_s)-cov(x_t,y_s))},$$

which identifies b in all models where r already has been identified, with the exception of the four submodels in W3L0 (when $\sigma_{\lambda1\lambda1}=0$, and the denominator in (B39) also is zero). In section B.7 below it is demonstrated that b is not identifiable in W3L0.

Solving (B39) with respect to r gives, if the denominator is different from zero,

(B40) $$r = \frac{(1+b)cov(x_t,y_s)-bcov(y_t,y_s)}{(1+b)cov(x_t,x_s)-bcov(x_t,y_s)},$$

which identifies r if b already has been identified. In model W0L3 the denominator (and also the numerator) is zero, and (B40) cannot be used to identify r in this case (including four submodels). It can be demonstrated, analogously to the proof in section B.7 of the non-identifiabilify of b in W3L0, that r cannot be identified in W0L3.

We will now first identify r in those models where this is easily done by help of Fs and Gs. Equation (B39) can in these cases be used to identify b. Similarly, we will identify b in some of the remaining models, and use (B40) to identify r. Finally, identification of these two parameters in two of the models will be obtained by a more comprehensive analysis. The number of the equations used to identify each parameter in the various models are given in Table B1. The first four rows of this table show the results for the four submodels of $W3^*L3$, the next four

Table B1

Overview of equations used to identify the parameters in the different models[ac]

Model	Parameter							
	r	b	$\sigma_{\omega1\omega2}$	$\sigma_{\omega2\omega2}$	$\sigma_{\omega1\omega1}$	$\sigma_{\lambda1\lambda2}$	$\sigma_{\lambda2\lambda2}$	$\sigma_{\lambda1\lambda1}$
W3*L3*[b]	B56	B39	B28/B29	B25/B26	B6/B8	B28/B29	B25/B26	B6/B8
W3*L2*	B41	B39	B28/B29	B25/B26	B6/B8	0	B25/B26	B6/B8
W3*L1*	B42	B39	B28/B29	B25/B26	B6/B8	0	0	B6/B8
W3*L0*	B42	NI	NI[d]	NI[d]	NI[d]	0	0	0
W2*L3*	B40	B47	0	B25/B26	B6/B8	B28/B29	B25/B26	B6/B8
W2*L2*[b]	B61	B39	0	B25/B26	B6/B8	0	B25/B26	B6/B8
W2*L1*	B42	B39	0	B25/B26	B6/B8	0	0	B6/B8
W2*L0*	B42	NI	0	NI[d]	NI[d]	0	0	0
W1*L3*	B40	B48	0	0	B6/B8	B28/B29	B25/B26	B6/B8
W1*L2*	B40	B48	0	0	B6/B8	0	B25/B26	B6/B8
W1*L1*	NI[d]	NI[e]	0	0	NI[f]	0	0	NI[f]
W1*L0*	B46	NI	0	0	NI[d]	0	0	0
W0*L3*	NI	B48	0	0	0	NI[e]	NI[e]	NI[e]
W0*L2*	NI	B48	0	0	0	0	NI[e]	NI[e]
W0*L1*	NI	B49	0	0	0	0	0	NI[e]
W0*L0*	NI	NI	0	0	0	0	0	0

[a] The numbers refer to the equations used for identification. NI=not identifiable.

[b] Identified assuming 0<d<1 or #A=1, cf section 3.

[c] The parameters $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\varphi}$ are identified by (B31), (B32), and (B33), respectively.

[d] Identified if b is given a fixed value.

[e] Identified if r is given a fixed value.

[f] Identified if either b or r is given a fixed value.

those of $W2^*L3$, etc.

*W3*L2**

For model $W3^*L2^*$ identification of r is obtained by dividing (B30) by (B29), which gives

$$\text{(B41)} \quad r = \frac{G_{xy}}{G_{xx}}.$$

*W3*L1*, W3*L0*, W2*L1*, and W2*L0**

For these models identification of r is obtained by dividing (B27) by (B25), which gives

$$\text{(B42)} \quad r = \frac{F_{xy}}{F_{xx}}.$$

*W1*L1*, W1*L0*, W1*L1*, and W1*L0**

For model W1L1 (including the four above mentioned submodels) the differences of the 2. order moments, (B10)-(B12), and the differences of these differences, (B20)-(B22) are zero, and one is left with the simple covariances for identification. (B6)-(B8) are now simplified to

$$\text{(B43)} \quad cov(x_t,x_s) = K[\sigma_{\omega_1\omega_1} + b^2\sigma_{\lambda_1\lambda_1}] + \delta_{ts}\sigma_{\varepsilon\varepsilon} ,$$

$$\text{(B44)} \quad cov(y_t,y_s) = K[r^2\sigma_{\omega_1\omega_1} + (1+b)^2\sigma_{\lambda_1\lambda_1}] + \delta_{ts}\sigma_{\phi\phi} ,$$

$$\text{(B45)} \quad cov(x_t,y_s) = K[r\sigma_{\omega_1\omega_1} + (1+b)b\sigma_{\lambda_1\lambda_1}] + \delta_{ts}\sigma_{\varepsilon\phi} .$$

For submodel $W1^*L1^*$, for which $\sigma_{\omega1\omega1} \neq 0$ and $\sigma_{\lambda1\lambda1} \neq 0$, we thus, for $t \neq s$, have only three equations, independent of time, in the four parameters b, r, $\sigma_{\omega1\omega1}$, and $\sigma_{\lambda1\lambda1}$. They cannot all be identified without further information. If, for instance, r is given a fixed value, the remaining parameters are easily identified, cf section 5.7.

For submodel $W1^*L0^*$ identification is obtained by dividing, for $t \neq s$, (B45) by (B43), which in this case, when $\sigma_{\lambda1\lambda1}=0$, gives

(B46) $r = \dfrac{cov(x_t, y_s)}{cov(x_t, x_s)}$.

For submodel W0*L0*, r cannot be identified even if b is given a fixed value, and vice versa.

*W2*L3**

Identification of b in model W2*L3* is obtained by dividing (B28) by (B30) which gives

(B47) $b = \dfrac{G_{xx}}{G_{xy} - G_{xx}}$.

*W1*L3*, W1*L2*, W0*L3*, and W0*L2**

Identification of b in these four models is obtained by dividing (B25) by (B27), which gives, after some ordering

(B48) $b = \dfrac{F_{xx}}{F_{xy} - F_{xx}}$.

*W0*L1**

Identification of b in W0*L1* is obtained by dividing (B43) by (B45), which gives

(B49) $b = \dfrac{cov(x_t, x_s)}{cov(x_t, y_s) - cov(x_t, x_s)}$.

*W3*L3**

Multiplying (B25) by r, and subtracting (B27) from the resulting equation gives

(B50) $F_{xx}r - F_{xy} = K[rb^2 - (1+b)b]\sigma_{\lambda_2\lambda_2} = -Kbd\sigma_{\lambda_2\lambda_2}$.

Multiplying (B27) by r, and subtracting the resulting equation from (B26) gives

(B51) $F_{yy}-F_{xy}r = K[(1+b)^2-r(1+b)b]\sigma_{\lambda_2\lambda_2} = K(1+b)d\sigma_{\lambda_2\lambda_2}$.

Assuming $\sigma_{\lambda2\lambda2} \neq 0$ and $b \neq -1$ (identification when $b = -1$ is demonstrated below), we divide (B50) by (B51) to obtain

(B52) $$\frac{F_{xx}r-F_{xy}}{F_{yy}-F_{xy}r} = -\frac{b}{(1+b)}.$$

Multiplying, for i=j=1, (B16) by r and subtracting (B18) from the resulting equation gives:

(B53) $rE_{xx}-E_{xy} = -\ Kbd[2\sigma_{\lambda_1\lambda_2}+(t+s-1)\sigma_{\lambda_2\lambda_2}]$.

Multiplying, also here for i=j=1, (B16) by r, and subtracting the resulting equation from (B17) gives

(B54) $E_{yy}-rE_{xy} = K(1+b)d[2\sigma_{\lambda_1\lambda_2}+(t+s-1)\sigma_{\lambda_2\lambda_2}]$.

Observing that, for at least some t and s, $2\sigma_{\lambda1\lambda2}+(t+s-1)\sigma_{\lambda2\lambda2} \neq 0$, we divide (B53) by (B54) to obtain

(B55) $$\frac{rE_{xx}-E_{xy}}{E_{yy}-rE_{xy}} = -\frac{b}{1+b}.$$

From equations (B52) and (B55) then follows

(B56) $r^2[F_{xy}E_{xx}-F_{xx}E_{xy}] + r[F_{xx}E_{yy}-F_{yy}E_{xx}] + [F_{yy}E_{xy}-F_{xy}E_{yy}] = 0$.

This equation cannot be used to identify r if the first two square brackets are zero. Inserting from equations (B16)-(B18) and (B25)-(B27) into (B56) gives

(B57) $r^2[2K^2bd\sigma_{\lambda_2\lambda_2}\sigma_{\omega_1\omega_2}]+r[2K^2(r^2b^2-(1+b)^2)\sigma_{\lambda_2\lambda_2}\sigma_{\omega_1\omega_2}]+[2K^2r(1+b)d\sigma_{\lambda_2\lambda_2}\sigma_{\omega_1\omega_2}] = 0,$

where the three square brackets are equal to those of (B56), respectively. Equation (B57) shows that all three square brackets in (B56) are equal to zero except for W3*L2* and W3*L3*. Identification of b and r in the former model is already proved. For the latter model, (B56) is a second order equation in r that in general has two different solutions. This problem of two roots is discussed in Sections 3 and B.10, where additional assumptions are introduced in order to choose between the two. If r is identified by this procedure, identification of b follows from (B39).

*W2*L2**

Observing that we in model $W2^*L2^*$ have $\sigma_{\omega1\omega2} = \sigma_{\lambda1\lambda2} = 0$, multiplication of (B6) by r and subtraction of (B8) from the resulting equation gives, for $t \neq s$

$$\text{(B58)} \quad r\text{cov}(x_t,x_s) - \text{cov}(x_t,y_s) = -Kbd(\sigma_{\lambda_1\lambda_1} + ts\sigma_{\lambda_2\lambda_2}).$$

Multiplying (B8) by r and subtracting the resulting equation from (B7) gives

$$\text{(B59)} \quad \text{cov}(y_t,y_s) - r\text{cov}(x_t,y_s) = K(1+b)d(\sigma_{\lambda_1\lambda_1} + ts\sigma_{\lambda_2\lambda_2}).$$

Dividing the former by the latter gives

$$\text{(B60)} \quad \frac{r\text{cov}(x_t,x_s) - \text{cov}(x_t,y_s)}{\text{cov}(y_t,y_s) - r\text{cov}(x_t,y_s)} = -\frac{b}{1+b}.$$

From (B52) (which is valid also for the present model) and (B60) follows

$$\text{(B61)} \quad r^2[F_{xy}\text{cov}(x_t,x_s) - F_{xx}\text{cov}(x_t,y_s)]$$
$$+ r[F_{xx}\text{cov}(y_t,y_s) - F_{yy}\text{cov}((x_t,x_s)]$$
$$+ [F_{yy}\text{cov}(x_t,y_s) - F_{xy}\text{cov}(y_t,y_s)] = 0.$$

Here, too, identification of r is not possible if the square brackets are equal to zero. Inserting (B6)-(B8) (for $t \neq s$, and $\sigma_{\omega1\omega2} = \sigma_{\lambda1\lambda2} = 0$) and (B25)-(B27) into (B61) gives

$$\text{(B62)} \quad r^2[-K^2bd(\sigma_{\lambda_1\lambda_1}\sigma_{\omega_2\omega_2} - \sigma_{\lambda_2\lambda_2}\sigma_{\omega_1\omega_1})]$$
$$+ r[K^2(b^2r^2 - (1+b)^2)(\sigma_{\lambda_1\lambda_1}\sigma_{\omega_2\omega_2} - \sigma_{\lambda_2\lambda_2}\sigma_{\omega_1\omega_1})]$$
$$+ [K^2r(1+b)d(\sigma_{\lambda_1\lambda_1}\sigma_{\omega_2\omega_2} - \sigma_{\omega_1\omega_1}\sigma_{\lambda_2\lambda_2})] = 0.$$

The three brackets in (B62), corresponding to those of (B61), are in general not zero, and equation (B61) can be used to identify r and b by the same procedure as for $W3^*L3^*$.

Identification of r when b = -1

Discussing identification of $W3^*L3^*$ and $W2^*L2^*$ we used the assumption $b \neq -1$. When $b = -1$, it follows from (B40) that

$$\text{(B63)} \quad r = \frac{\text{cov}(y_t,y_s)}{\text{cov}(x_t,y_s)},$$

which identifies r. Identification of the remaining parameters does not depend on the value of b, cf section B.8.

B.7. Proof of non-identifiability of b in W3L0

If $\sigma_{\lambda 1\lambda 1} = 0$, which then gives model W3L0, full identification of all parameters are not possible. Additional information, for instance a fixed value of one of the parameters, is needed to obtain complete identification.

Rewriting (B6)-(B8)

(B69) $\mathrm{cov}(x_t,x_s) - \delta_{ts}\sigma_{\varepsilon\varepsilon} = K[\sigma_{\omega_1\omega_1}+(t+s)\sigma_{\omega_1\omega_2}+ts\sigma_{\omega_2\omega_2}],$

(B70) $\mathrm{cov}(y_t,y_s) - \delta_{ts}\sigma_{\phi\phi} = Kr^2[\sigma_{\omega_1\omega_1}+(t+s)\sigma_{\omega_1\omega_2}+ts\sigma_{\omega_2\omega_2}],$

(B71) $\mathrm{cov}(x_t,y_s) - \delta_{ts}\sigma_{\varepsilon\phi} = Kr[\sigma_{\omega_1\omega_1}+(t+s)\sigma_{\omega_1\omega_2}+ts\sigma_{\omega_2\omega_2}],$

we remark that $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, $\sigma_{\varepsilon\varphi}$, and r are already identified in $W3^*L0^*$, $W2^*L0^*$, and $W1^*L0^*$, cf Table B1. (In submodel $W0^*L0^*$ r cannot be identified, and the proof below is irrelevant.)

Assume that $\hat{b}$, $\hat{\sigma}_{\omega1\omega1}$, $\hat{\sigma}_{\omega2\omega2}$, and $\hat{\sigma}_{\omega1\omega2}$ satisfy (B69)-(B71) for given values of $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, $\sigma_{\varepsilon\varphi}$, and r. Then it can be shown that the same must be true for

$$b^* = \frac{\sqrt{h}(1+\hat{b}-\hat{b}r)-1}{1-r}, \quad \sigma^*_{\omega_1\omega_1} = h\hat{\sigma}_{\omega_1\omega_1}, \quad \sigma^*_{\omega_1\omega_2} = h\hat{\sigma}_{\omega_1\omega_2}, \quad \sigma^*_{\omega_2\omega_2} = h\hat{\sigma}_{\omega_2\omega_2},$$

where h is a positive scalar. For the above assertion to hold we must have $hK(b^*,r) = K(\hat{b},r)$, where

$$K(b^*,r) = \left[\frac{1}{1+b^*-rb^*}\right]^2, \quad \text{and} \quad K(\hat{b},r) = \left[\frac{1}{1+\hat{b}-r\hat{b}}\right]^2$$

Inserting for b^* we obtain

$$hK(b^*,r) = h\left[\frac{1}{1+b^*-rb^*}\right]^2 = \left[\frac{\sqrt{h}}{1+\frac{\sqrt{h}(1+\hat{b}-\hat{b}r)-1}{1-r} - r\left(\frac{\sqrt{h}(1+\hat{b}-\hat{b}r)-1}{1-r}\right)}\right]^2 =$$

$$\left[\frac{\sqrt{h}}{1+\frac{(1-r)[\sqrt{h}(1+\hat{b}-\hat{b}r)-1]}{1-r}}\right]^2 = \left[\frac{1}{1+\hat{b}-\hat{b}r}\right]^2 = K(\hat{b},r)$$

This means that b, $\sigma_{\omega1\omega1}$, $\sigma_{\omega2\omega2}$, and $\sigma_{\omega1\omega2}$ cannot be identified. If, for instance, b is given a fixed value, the three remaining parameters can be obtained from (B69)-(B71).

A similar proof, not given in this paper, will show that r is not identifiable in W0L3.

B.8. Identification of $\sigma_{\omega1\omega1}$, $\sigma_{\lambda1\lambda1}$, $\sigma_{\omega2\omega2}$, $\sigma_{\lambda2\lambda2}$, $\sigma_{\omega1\omega2}$, and $\sigma_{\lambda1\lambda2}$

For the submodels where b and r have been identified, the parameters $\sigma_{\omega2\omega2}$ and $\sigma_{\lambda2\lambda2}$ are identified by (B25) and (B26), which are two linear equations in these parameters. Thereafter we identify the parameters $\sigma_{\omega1\omega2}$ and $\sigma_{\lambda1\lambda2}$ by (B28) and (B29), and finally we determine $\sigma_{\omega1\omega1}$ and $\sigma_{\lambda1\lambda1}$ from (B6) and (B8).

B.9. Observational equivalence between Wi*Lj* and Wj*Li*

The demonstration of identification above needs a qualification. The models Wi*Lj* and Wj*Li* for $i \neq j$ and i,j=0,1,2,3 can be shown to be pairwise observationally equivalent. Here we restrict ourselves to prove this proposition for the most general pair of models, i.e. W2*L3* and W3*L2*. The proof for other pairs is analogous, but somewhat simpler.

Let b', r', d', $\sigma_{\omega1\omega1}'$, $\sigma_{\omega2\omega2}'$, $\sigma_{\lambda1\lambda1}'$, $\sigma_{\lambda2\lambda2}'$, $\sigma_{\varepsilon\varepsilon}'$, $\sigma_{\varphi\varphi}'$, and $\sigma_{\varepsilon\varphi}'$ be the values of b, r, d, $\sigma_{\omega1\omega1}$, $\sigma_{\omega2\omega2}$, $\sigma_{\lambda1\lambda1}$, $\sigma_{\lambda2\lambda2}$, $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\varphi}$, respectively, in W3L2, and b", r", d", $\sigma_{\omega1\omega1}''$, $\sigma_{\omega2\omega2}''$, $\sigma_{\lambda1\lambda1}''$, $\sigma_{\lambda2\lambda2}''$, $\sigma_{\varepsilon\varepsilon}''$, $\sigma_{\varphi\varphi}''$, and $\sigma_{\varepsilon\varphi}''$ the values of the same parameters in W2L3. For W3L2 to be observationally equivalent to W2L3 they have to imply the same theoretical moments of the observable variables. This means that the following identities must hold for all possible combinations of t and s (cf (B6)-(B8)):

(B72) $$\Big[K(b',r')[\sigma'_{\omega_1\omega_1}+b'^2\sigma'_{\lambda_1\lambda_1}] - K(b'',r'')[\sigma''_{\omega_1\omega_1}+b''^2\sigma''_{\lambda_1\lambda_1}]\Big]$$
$$+ \Big[K(b',r')\sigma'_{\omega_1\omega_2} - K(b'',r'')b''^2\sigma''_{\lambda_1\lambda_2}\Big](t+s)$$
$$+ \Big[K(b',r')[\sigma'_{\omega_2\omega_2}+b'^2\sigma'_{\lambda_2\lambda_2} - K(b'',r'')[\sigma''_{\omega_2\omega_2}+b''^2\sigma''_{\lambda_2\lambda_2}]\Big]ts + \delta_{ts}[\sigma'_{\varepsilon\varepsilon}-\sigma''_{\varepsilon\varepsilon}] = 0,$$

(B73) $$\Big[K(b',r')[r'^2\sigma'_{\omega_1\omega_1}+(1+b')^2\sigma'_{\lambda_1\lambda_1}] - K(b'',r'')[r''^2\sigma''_{\omega_1\omega_1}+(1+b'')^2\sigma''_{\lambda_1\lambda_1}]\Big]$$
$$+ \Big[K(b',r')r'^2\sigma'_{\omega_1\omega_2} - K(b'',r'')(1+b'')^2\sigma''_{\lambda_1\lambda_2}\Big](t+s)$$
$$+ \Big[K(b',r')[r'^2\sigma'_{\omega_2\omega_2}+(1+b')^2\sigma'_{\lambda_2\lambda_2}] - K(b'',r'')[r''^2\sigma''_{\omega_2\omega_2}+(1+b'')^2\sigma''_{\lambda_2\lambda_2}]\Big]ts$$
$$+ \delta_{ts}[\sigma'_{\varphi\varphi}-\sigma''_{\varphi\varphi}] = 0\ ,$$

(B74) $$\Big[K(b',r')[r'\sigma'_{\omega_1\omega_1}+(1+b')b'\sigma'_{\lambda_1\lambda_1}] - K(b'',r'')[r''\sigma''_{\omega_1\omega_1}+(1+b'')b''\sigma''_{\lambda_1\lambda_1}]\Big]$$
$$+ \Big[K(b',r')r'\sigma'_{\omega_1\omega_2} - K(b'',r'')(1+b'')b''\sigma''_{\lambda_1\lambda_2}\Big](t+s)$$
$$+ \Big[K(b',r')[r'\sigma'_{\omega_2\omega_2}+(1+b')b'\sigma'_{\lambda_2\lambda_2}] - K(b'',r'')[r''\sigma''_{\omega_2\omega_2}+(1+b'')b''\sigma''_{\lambda_2\lambda_2}]\Big]ts$$
$$+ \delta_{ts}[\sigma'_{\varepsilon\varphi}-\sigma''_{\varepsilon\varphi}] = 0.$$

For (B72)-(B74) to hold as identities the following relations must apply:

(B75) $$K(b',r')[\sigma'_{\omega_1\omega_1}+b'^2\sigma'_{\lambda_1\lambda_1}] = K(b'',r'')[\sigma''_{\omega_1\omega_1}+b''^2\sigma''_{\lambda_1\lambda_1}]\ ,$$

(B76) $$K(b',r')\sigma'_{\omega_1\omega_2} = K(b'',r'')b''^2\sigma''_{\lambda_1\lambda_2}\ .$$

(B77) $$K(b',r')[\sigma'_{\omega_2\omega_2}+b'^2\sigma'_{\lambda_2\lambda_2}] = K(b'',r'')[\sigma''_{\omega_2\omega_2}+b''^2\sigma''_{\lambda_2\lambda_2}]\ ,$$

(B78) $$\sigma'_{\varepsilon\varepsilon} = \sigma''_{\varepsilon\varepsilon}\ ,$$

(B79) $$K(b',r')[r'^2\sigma'_{\omega_1\omega_1}+(1+b')^2\sigma'_{\lambda_1\lambda_1}] = K(b'',r'')[r''^2\sigma''_{\omega_1\omega_1}+(1+b'')^2\sigma''_{\lambda_1\lambda_1}]\ ,$$

(B80) $$K(b',r')r'^2\sigma'_{\omega_1\omega_2} = K(b'',r'')(1+b'')^2\sigma''_{\lambda_1\lambda_2}\ ,$$

(B81) $$K(b',r')[r'^2\sigma'_{\omega_2\omega_2}+(1+b')^2\sigma'_{\lambda_2\lambda_2}] = K(b'',r'')[r''^2\sigma''_{\omega_2\omega_2}+(1+b'')^2\sigma''_{\lambda_2\lambda_2}]\ ,$$

(B82) $\sigma'_{\varphi\varphi} = \sigma''_{\varphi\varphi}$,

(B83) $K(b',r')[r'\sigma'_{\omega_1\omega_1}+(1+b')b'\sigma'_{\lambda_1\lambda_1}] = K(b'',r'')[r''\sigma''_{\omega_1\omega_1}+(1+b'')b''\sigma''_{\lambda_1\lambda_1}]$,

(B84) $K(b'r')r'\sigma'_{\omega_1\omega_2} = K(b'',r'')(1+b'')b''\sigma''_{\lambda_1\lambda_2}$.

(B85) $K(b',r')[r'\sigma'_{\omega_2\omega_2}+(1+b')b'\sigma'_{\lambda_2\lambda_2}] = K(b'',r'')[r''\sigma''_{\omega_2\omega_2}+(1+b'')b''\sigma''_{\lambda_2\lambda_2}]$,

(B86) $\sigma'_{\varepsilon\varphi} = \sigma''_{\varepsilon\varphi}$.

From (B75)-(B86) it is now possible to solve for

$\sigma''_{\varepsilon\varepsilon}$, $\sigma''_{\varphi\varphi}$, $\sigma''_{\varepsilon\varphi}$, b'' , r'' , $\sigma''_{\omega_1\omega_1}$, $\sigma''_{\omega_2\omega_2}$, $\sigma''_{\lambda_1\lambda_1}$, $\sigma''_{\lambda_1\lambda_2}$, and $\sigma''_{\lambda_2\lambda_2}$ as functions of $\sigma'_{\varepsilon\varepsilon}$, $\sigma'_{\varphi\varphi}$, $\sigma'_{\varepsilon\varphi}$, b' , r' , $\sigma'_{\omega_1\omega_1}$, $\sigma'_{\omega_1\omega_2}$, $\sigma'_{\omega_2\omega_2}$, $\sigma'_{\lambda_1\lambda_1}$, and $\sigma'_{\lambda_2\lambda_2}$.

From (B78), (B82), and (B86) we have

(I) $\sigma''_{\varepsilon\varepsilon} = \sigma'_{\varepsilon\varepsilon}$, $\quad \sigma''_{\varphi\varphi} = \sigma'_{\varphi\varphi}$, $\quad \sigma''_{\varepsilon\varphi} = \sigma'_{\varepsilon\varphi}$.

Dividing (B84) by (B76) we obtain

(II) $b'' = \dfrac{1}{r'-1}$.

Multiplying (B75) by r', and subtracting the resulting equation from (B83) we have

(B87) $K'(b',r')[(1+b')b'-r'b'^2]\sigma'_{\lambda_1\lambda_1} = K(b'',r'')[r''-\dfrac{1+b''}{b''}]\sigma''_{\omega_1\omega_1}$.

Multiplying (B83) by r', and subtracting the resulting equation from (B79) we have

(B88) $K'(b',r')[(1+b')^2-r'(1+b')b']\sigma'_{\lambda_1\lambda_1} = K(b'',r'')[r''^2-r''\dfrac{1+b''}{b''}]\sigma''_{\omega_1\omega_1}$.

Dividing (B88) by (B87), assuming $\sigma_{\omega1\omega1}$ and $\sigma_{\lambda1\lambda1} \neq 0$, and thus excluding the pair of models (W1*L0*, W0*L1*) from the proof, we obtain

(III) $r'' = \dfrac{(1+b')^2-r'(1+b')b'}{(1+b')b'-r'b'^2} = \dfrac{1+b'}{b'}$.

From (B76) we obtain

$$\text{(IV)}\quad \sigma''_{\lambda_1\lambda_2} = \frac{K(b',r')}{K[b''(r'),r''(b')]b''(r')^2}\sigma'_{\omega_1\omega_2} = \frac{1}{b'^2}\sigma'_{\omega_1\omega_2}\ .$$

From (B87) we obtain

$$\text{(V)}\quad \sigma''_{\omega_1\omega_1} = \frac{K(b',r')}{K[b''(r'),r''(b')]}\ \frac{(1+b')b'-r'b'^2}{r''(b')-\dfrac{1+b''(r')}{b''(r')}}\sigma'_{\lambda_1\lambda_1} = \frac{1}{(1-r')^2}\sigma'_{\lambda_1\lambda_1}\ .$$

From (B85) and (V) we obtain

$$\text{(VI)}\quad \sigma''_{\lambda_1\lambda_1} = \frac{K(b',r')}{K[b''(r'),r''(b')](b''(r'))^2}[\sigma'_{\omega_1\omega_1}+b'^2\sigma'_{\lambda_1\lambda_1}] - \frac{\sigma''_{\omega_1\omega_1}(b',r',\sigma'_{\lambda_1\lambda_1})}{(b''(r'))^2} = \frac{1}{b'^2}\sigma'_{\omega_1\omega_1}.$$

Multiplication of (B77) by r', and subtraction of the resulting equation from (B85) gives

$$\text{(B89)}\quad K'(b',r')[(1+b')b'-r'b'^2]\sigma'_{\lambda_2\lambda_2} = K(b'',r'')[r''-\frac{1+b''}{b''}]\sigma''_{\omega_2\omega_2}\ ,$$

from which we obtain

$$\text{(VII)}\quad \sigma''_{\omega_2\omega_2} = \frac{K(b',r')}{K[b''(r'),r''(b')]}\ \frac{(1+b')b'-r'b'^2}{r''(b')-\dfrac{1+b''(r')}{b''(r')}}\sigma'_{\lambda_2\lambda_2} = \frac{1}{(1-r')^2}\sigma'_{\lambda_2\lambda_2}\ .$$

From (B77) and (VII) we, finally, obtain

$$\text{(VIII)}\quad \sigma''_{\lambda_2\lambda_2} = \frac{K(b',r')}{K[b''(r'),r''(b')]}\ \frac{1}{(b''(r'))^2}[\sigma'_{\omega_2\omega_2}+b'^2\sigma'_{\lambda_2\lambda_2}] - \frac{\sigma''_{\omega_2\omega_2}(b',r',\sigma'_{\lambda_1\lambda_1})}{(b''(r'))^2}$$

$$= \frac{1}{b'^2}\sigma'_{\omega_2\omega_2}\ .$$

Thus we have proved that given an arbitrary set of parameters in $W2^{*}L3^{*}$ we can find a corresponding set of parameters in $W3^{*}L2^{*}$ which makes the models observationally equivalent.

From II and III we obtain the following relation between the stability parameters d' and d" (cf (2d)) in W3L2 and W2L3, respectively:

(IX) $d'' = 1+b''(1-r'') = 1+\frac{1}{(r'-1)b'} = \frac{d'}{d'-1}$, or

(IX') $d' = \frac{d''}{d''-1}$.

Note that (IX) implies that if 0<d"<1, then d'<0, and vice versa. Thus assuming (2e), i.e. 0<d<1, only one of the symmetric models is relevant. That is, within the set of two symmetric models $\{Wi^*Lj^*, Wj^*Li^*\}$ we can identify the correct model under assumption (2e).

B.10. Observational equivalence between $WiLi^{I}$ and $WiLi^{II}$, i=2,3.

A pairwise observational equivalence analogous to that between Wi^*Lj^* and Wj^*Li^* exists between the two solutions of model WiLi, i=2,3, i.e. between $WiLi^{I}$ and $WiLi^{II}$. Here we restrict ourselves to demonstrate that the parameters of $W3L3^{II}$ are functions of those of $W3L3^{I}$. The proof for the two solutions of W2L2 is analogous, but somewhat simpler. Observational equivalence between the two solutions of W3L3 requires that identities analogous to (B72)-(B74) hold. Let b^I, r^I, etc. denote the values of the parameters in $W3L3^{I}$, and b^{II}, r^{II} those in $W3L3^{II}$. For all possible combinations of t and s we must now have

(B92)
$$\left[K(b^{I},r^{I})[\sigma^{I}_{\omega_1\omega_1}+b^{I2}\sigma^{I}_{\lambda_1\lambda_1}] - K(b^{II},r^{II})[\sigma^{II}_{\omega_1\omega_1}+b^{II2}\sigma^{II}_{\lambda_1\lambda_1}]\right]$$
$$+\left[K(b^{I},r^{I})[\sigma^{I}_{\omega_1\omega_2}+b^{I2}\sigma^{I}_{\lambda_1\lambda_2}] - K(b^{II},r^{II})[\sigma^{II}_{\omega_1\omega_2}+b^{II2}\sigma^{II}_{\lambda_1\lambda_2}]\right](t+s)$$
$$+\left[K(b^{I},r^{I})[\sigma^{I}_{\omega_2\omega_2}+b^{I2}\sigma^{I}_{\lambda_2\lambda_2}] - K(b^{II},r^{II})[\sigma^{II}_{\omega_2\omega_2}+b^{II2}\sigma^{II}_{\lambda_2\lambda_2}]\right]ts + \delta_{ts}[\sigma^{I}_{\varepsilon\varepsilon}-\sigma^{II}_{\varepsilon\varepsilon}] = 0,$$

(B93)
$$\left[K(b^{I},r^{I})[r^{I2}\sigma^{I}_{\omega_1\omega_1}+(1+b^{I})^2\sigma^{I}_{\lambda_1\lambda_1}] - K(b^{II},r^{II})[r^{II2}\sigma^{II}_{\omega_1\omega_1}+(1+b^{II})^2\sigma^{II}_{\lambda_1\lambda_1}]\right]$$
$$+\left[K(b^{I},r^{I})[r^{I2}\sigma^{I}_{\omega_1\omega_2}+(1+b^{I})^2\sigma^{I}_{\lambda_1\lambda_2}] - K(b^{II},r^{II})[r^{II2}\sigma^{II}_{\omega_1\omega_2}+(1+b^{II})^2\sigma^{II}_{\lambda_1\lambda_2}]\right](t+s)$$
$$+\left[K(b^{I},r^{I})[r^{I2}\sigma^{I}_{\omega_2\omega_2}+(1+b^{I})^2\sigma^{I}_{\lambda_2\lambda_2}] - K(b^{II},r^{II})[r^{II2}\sigma^{II}_{\omega_2\omega_2}+(1+b^{II})^2\sigma^{II}_{\lambda_2\lambda_2}]\right]ts$$
$$+\ \delta_{ts}[\sigma^{I}_{\varphi\varphi}-\sigma^{II}_{\varphi\varphi}] = 0\ ,$$

(B94) $\Big[K(b^I,r^I)[r^I\sigma^I_{\omega_1\omega_1}+(1+b^I)b^I\sigma^I_{\lambda_1\lambda_1}] - K(b^{II},r^{II})[r^{II}\sigma^{II}_{\omega_1\omega_1}+(1+b^{II})b^{II}\sigma^{II}_{\lambda_1\lambda_1}\ ]\Big]$

$+ \Big[K(b^I,r^I)[r^I\sigma^I_{\omega_1\omega_2}+(1+b^I)b^I\sigma^I_{\lambda_1\lambda_2}] - K(b^{II},r^{II})[r^{II}\sigma^{II}_{\omega_1\omega_2}+(1+b^{II})b^{II}\sigma^{II}_{\lambda_1\lambda_2}]\Big](t+s)$

$+ \Big[K(b^I,r^I)[r^I\sigma^I_{\omega_2\omega_2}+(1+b^I)b^I\sigma^I_{\lambda_2\lambda_2}] - K(b^{II},r^{II})[r^{II}\sigma^{II}_{\omega_2\omega_2}+(1+b^{II})b^{II}\sigma^{II}_{\lambda_2\lambda_2}]\Big]ts$

$+ \delta_{tu}[\sigma^I_{\varepsilon\varphi}-\sigma^{II}_{\varepsilon\varphi}] = 0.$

For (B92)-(B94) to hold as identities relations analogous to (B75)-(B86) must apply. In three of these relations, those corresponding to (B76), (B80), and (B84), some additional terms have to be included. The three enlarged relations are:

(B95) $K(b^I,r^I)[\sigma^I_{\omega_1\omega_2}+b^{I2}\sigma^I_{\lambda_1\lambda_2}] = K(b^{II},r^{II})[\sigma^{II}_{\omega_1\omega_2}+b^{II2}\sigma^{II}_{\lambda_1\lambda_2}]$,

(B96) $K(b^I,r^I)[r^{I2}\sigma^I_{\omega_1\omega_2}+(1+b^I)^2\sigma^I_{\lambda_1\lambda_2}] = K(b^{II},r^{II})[r^{II2}\sigma^{II}_{\omega_1\omega_2}+(1+b^{II})^2\sigma^{II}_{\lambda_1\lambda_2}]$,

(B97) $K(b^I,r^I)[r^I\sigma^I_{\omega_1\omega_2}+(1+b^I)b^I\sigma^I_{\lambda_1\lambda_2}] = K(b^{II},r^{II})[r^{II}\sigma^{II}_{\omega_1\omega_2}+(1+b^{II})b^{II}\sigma^{II}_{\lambda_1\lambda_2}]$.

In the remaining equations it suffices to substitute the model notations ^{I} and ^{II} for the apostrophes used to designate the two models in (B75)-(B86). It can now easily be shown that when substituting solutions analogous to (I)-(VII), i.e.

(Ia) $\sigma^{II}_{\varepsilon\varepsilon} = \sigma^I_{\varepsilon\varepsilon}$, $\sigma^{II}_{\varphi\varphi} = \sigma^I_{\varphi\varphi}$, $\sigma^{II}_{\varepsilon\varphi} = \sigma^I_{\varepsilon\varphi}$.

(IIa) $b^{II} = \dfrac{1}{r^I-1}$.

(IIIa) $r^{II} = \dfrac{1+b^I}{b^I}$.

(IVa) $\sigma^{II}_{\lambda_1\lambda_2} = \dfrac{1}{b^{I2}}\sigma^I_{\omega_1\omega_2}$.

(Va) $\sigma^{II}_{\omega_1\omega_1} = \dfrac{1}{(1-r^I)^2}\sigma^I_{\lambda_1\lambda_1}$.

(VIa) $\sigma^{II}_{\lambda_1\lambda_1} = \dfrac{1}{b^{I2}}\sigma^I_{\omega_1\omega_1}$.

(VIIa) $\sigma^{II}_{\omega_2\omega_2} = \dfrac{1}{(1-r^I)^2}\sigma^I_{\lambda_2\lambda_2}$.

(VIIIa) $\sigma^{II}_{\lambda_2\lambda_2} = \frac{1}{b^{I2}}\sigma^{I}_{\omega_2\omega_2}$.

and in addition

(Xa) $\sigma^{II}_{\omega_1\omega_2} = \frac{1}{(1-r^{I})^2}\sigma^{I}_{\lambda_1\lambda_2}$,

into these relations, in each of them the left hand side turns out to be equal to the right hand side. The two solutions are thus observationally equivalent.

From (IIa) and (IIIa) we obtain, analogous to (IX) and (IX')

(IXa) $d^{II} = \frac{d^{I}}{d^{I}-1}$

(IXa') $d^{I} = \frac{d^{II}}{d^{II}-1}$.

Note that (IXa) implies that if $0<d^{II}<1$, then $d^{I}<0$, and vice versa. Thus assuming (2e), i.e. $0<d<1$, only one of the two solutions is relevant, which means that the model is identified. This is empirically illustrated in Table 6.

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