Discussion Paper

Central Bureau of Statistics, P.B. 8131 Dep, 0033 Oslo 1, Norway

No. 72 May 1992

Dynamic equilibrium adjustments to a terms of trade disturbance

by

Brita Bye and Erling Holmøy[1]

Abstract

This paper investigates how a fall in the price of imports will have dynamic effects in an open economy. We analyse the effects within an aggregated intertemporal equilibrium model with internationally mobile capital. We assume the domestic product to be an imperfect substitute for a foreign product. Hence, the model is characterized by an endogenous domestic product price and a path dependent steady state solution. Using a numerical model calibrated to the Norwegian economy we study the effects of both anticipated and unanticipated changes in the import price.

[1]The authors would like to thank Kjell Arne Brekke for useful comments on an earlier draft. We have also benefitted from discussions with Dale Jorgenson, Peter Wilcoxen and Bjørn Helge Vatne. We are also grateful to Birger Strøm for computer assistance.

Contents

1 Introduction

This paper adresses the dynamics of the macroeconomic adjustments in an open economy to an exogenous fall in import prices changing the economy's terms of trade. The effects are analysed within an intertemporal general equilibrium model where the agents are assumed to have perfect foresight. During the eighties a large litterature has emerged which especially reappraises the socalled Harberger-Meltzer-Laursen (HLM) effect within an intertemporal equilibrium framework. After the work of Laursen and Metzler (1950) and Harberger (1950) the HLM-effect has become synonymous to a positive correlation between the terms of trade and the current account balance. Relying on Keynesian savings- and investment behaviour, the cited authors provided an affirmative answer based on the following argument: A worsening of the terms of trade lowers real income, thereby savings, and given investment, leads to a deterioration of the current account balance. As is clear from this argument the effects on the current account balance are interrelated with macroeconomic adjustments in general. This becomes even more clear when the model is made more sophisticated taking intertemporal rational behaviour into account. Examples of authors working with such models in studying the relationship between the terms of trade, macroeconomic adjustments and the current account balance are Sachs (1981), Obstfeld (1982a,1982b, 1983), Dornbusch (1983), Svensson and Razin (1985), Persson and Svensson (1985), Bean (1986), Matsuyama (1987), Sen and Turnovsky (1989) and Gavin (1990). Common to all these papers is that whether or not the HLM-effect holds, depends critically upon the assumptions and specifications of the model.

What differs in our model compared to those listed above is the source of the dynamic adjustments. As in Dornbusch (1983) and Gavin (1990) the key intertemporal link is the endogeneity of the price level which enters the model as a forward looking variable through the intertemporal investment and consumption decisions. In Dornbusch (1983) and Gavin (1990) this endogeneity is due to the fact that non-traded goods are part of the composite consumption- and capital goods. Our model treats all goods as tradeables but, contrary to the papers mentioned above, we introduce endogenous prices on domestically produced goods, justified by assuming imperfect substitutability between domestic and foreign varieties. This price endogeneity gives rise to a gradual dynamic adjustment to exogenous shocks without any adjustment costs internal to the firms. Internal cost of adjustment is a fundamental source of gradual dynamics in the work of Sen and Turnovsky (1989), Matsuyama (1987), Goulder and Summers (1989) and Gavin (1990).

The other new development presented in this paper is that the model of an open economy specifying intertemporal rational behaviour and perfect foresight has been calibrated to Norwegian data and solved numerically, though at a highly aggregated level. Solving intertemporal general equilibrium models is a new stage in applied general equilibrium modelling. Pioneering works are Jorgenson and Wilcoxen (1989) and (1991) who solves a large scale model specifying 35 sectors, and Goulder and Summers (1989). These models determine the interest rate endogenously whereas the current account balance is exogenous. For a small open economy like the Norwegian a more realistic assumption is to consider the nominal interest rate as given in international financial markets. In our model both the total wealth and the composition w.r.t. financial and physical capital, are endogenous variables. We show how the economy converges towards a saddle-point stable equilibrium for all variables, including the foreign debt. As in Sen and Turnovsky (1989) and Gavin (1990) this long-run stationary equilibrium will be path dependent, i.e. the steady state solution is dependent on the initial situation and the dynamics, and can not be calculated in advance of the solution of the complete model. Hence, contrary to the model in Jorgenson and Wilcoxen (1989), our dynamic model does not have the two-point boundary structure, for which several numerical solution algorithms exist, see e.g. Wilcoxen (1990). We present a brief description of our solution procedure in appendix B.

The value of supporting theoretical models by numerical versions increases as the theoretical models yield ambiguous results due to counteracting effects whose relative strength can only be checked numerically. In addition, theoretical models have to be simplified in order to be analytically tractable. A typical example of this in intertemporal models, is the technique of linearizing the dynamic system around its steady state in order to check stability and to obtain closed formed solutions. However, if the steady state is path dependent as in our model, the linearization may be a bad approximation to the closed form solution. The accumulated foreign debt will be influenced by investments along the dynamic path. A solution for the debt-level based on linearization around steady state where no investments take place (see Sen and Turnovsky (1989) for an example of such a procedure), may result in a seriously misleading description of both the dynamics and the steady state solution of the model.

The numerical model applied in the present paper is so aggregated in many respects that the results only have illustrative interest. However, inspired by the work of Jorgenson and Wilcoxen, our intention is to develop an econometric large scale version of the model structure used in this paper, and the present analysis is a step towards this end. It should be noted that the difficulties w.r.t. the intertemporal dynamics are principally the same in the present model as in a large scale model as long as only one capital good is specified. Hence the present analysis probably provides a representative picture of the driving macroeconomic forces in a more disaggregate model derived from the same basic assumptions.

The assumption that the domestic and foreign goods are imperfect substitutes even in a small open economy like the Norwegian deserves special concern, both because it can be doubted on empirical grounds, see e.g. the discussions by Norman (1986) and Bergman (1985), and because it is a basic determinant of the speed of adjustments in the model. A vast litterature on trade and imperfect competition identify structures generating monopolistic competition and market power. And the size of the country is not necessarily relevant for this structure to be an equilibrium characteristic. Furthermore, econometric work often end up with surprisingly small trade elasticities. However, we recognize that estimates of the trade elasticities are quite uncertain. Hence we have paid special attention to how our analysis depends on them.

The paper is organized as follows. Section 2 provides an overview of the theoretical model including a discussion of the equilibrium dynamics. In section 3 the effects of a terms of trade change is discussed.

2 The Model Framework

2.1 Basic assumptions

The firms produce a combined consumption and investment commodity. No governmental sector is specified and all income in the economy is earned by the representative consumer. The supply of labour is exogenous. The income received by the consumer consists of interest on foreign net debt, sales of total production which equals labour and capital income, less the depreciation of capital.

The firms rent labour, L, but own the capital stock, K. Investment, J, is financed by retained profits[2]. The households supply a fixed amount of labour L and own the firms which entitles them to receive the cash flow. They allocate their income between consumption and savings in order to maximize an intertemporal utility function. Household savings take the form of lending to the rest of the world at a given rate of interest. The exchange rate is fixed and all values are measured in domestic currency. Firms and households demand one commodity, but this is a composite of the domestic produced commodity and two different foreign varieties, a competing and a non-competing good. For simplicity the composition is assumed to be independent of its use. The price of this macro commodity P is determined in a two stage process described by the following equations.

$$P_1 = p_1(P_1^H, P_1^I)$$

$$P = p(P_1, P_2^I)$$

The domestic price P_1 of the competing commodity is an aggregate of the domestic producer price P_1^H and the price of the competing imports P_1^I. The price of the macro commodity P is an aggregate of the price of this competing commodity P_1 and the price of non-competing imports P_2^I. Both aggregation functions are assumed to be homogenous of degree one in the prices. The domestic commodity may be exported, but competes with the foreign variant also on the world market. Due to competition among (a sufficient large number of) domestic firms, the price of the domestically produced commodity equals unit cost no matter where the commodity is sold. The production function is homogeneous of degree one in labour and capital.

2.2 Producer behavior

The decision problem of the representative firm at time 0 is to choose the time paths of the control variables labour L and gross investment J so that the present value of the cash flow, V, is maximized:

[2]In the absence of taxes, this is a harmless assumption according to the Modigliani-Miller theorem

$$V_0 = \int_{t=0}^{\infty} [P_1^H f(k)L - PJ - P^L L] e^{-rt} dt \tag{1}$$

subject to

$$\dot{K} = J - \delta K \tag{2}$$

$$K(0) = K_0 \tag{3}$$

where K_0 is predetermined. The transversality condition is $\lim_{t \to \infty} e^{-rt} PK = 0$. P^L denotes the wage rate, $k = K/L$ is the capital-labour ratio, r is the nominal world market rate of interest and δ is a constant depreciation rate. All variables are functions of time.

The necessary f.o.c. are

$$P_1^H f'(k) = (r + \delta - \frac{\dot{P}}{P})P \tag{4}$$

$$P_1^H [f(k) - kf'(k)] = P^L \tag{5}$$

Due to the assumption of constant returns to scale, the first order conditions do not determine the input levels but only the ratio k. We may interpret (4) as determining k, while (5) determines P_1^H or P^L as a result of an exit/entry mechanism ensuring that P_1^H equals unit costs.

Inserting equation (2), (4) and (5) in equation (1) and utilizing the transversality condition, it can be shown the discounted value of the firm at any given time equals the value of the capital stock.

$$V_0 = P_0 K_0$$

2.3 Consumer behavior

The household sector can be described by a representative consumer with an infinite horizon. His objective is to maximize total discounted utility with respect to total consumption C.

$$Max_{(C)} U_0 = \int_{t=0}^{\infty} u_t(C_t) e^{-\rho t} dt \tag{6}$$

subject to the budget constraint:

$$\dot{B} = rB + P_1^H f(k)L - PC - PJ \tag{7}$$

$$\lim_{t \to \infty} Be^{-rt} = 0 \tag{8}$$

$$B(0) = B_0 \tag{9}$$

B net is financial wealth and B_0 is predetermined. ρ is the subjective rate of time preference. Equation (8) is the transversality condition also called the non-Ponzi game (NPG) condition. This condition implies that the stock of net foreign claims can not grow at a rate higher than the interest rate as t goes to infinity. Hence the consumer's consumption possibilities are limited.

Integrating the budget constraint and using the NPG condition gives that the value of total discounted consumption is equal to the initial level of financial and real capital in addition to human capital $H(0)$.

$$\int_{t=0}^{\infty} P_t C_t e^{-rt} dt = B_0 + P_0 K_0 + H(0)$$

where

$$H(0) = \int_{t=0}^{\infty} P_t^L L_t e^{-rt} dt$$

From the utility maximization we have the following first order condition:

$$\frac{u'_C}{P} = \mu \tag{10}$$

where μ is the costate variable associated with the financial wealth accumulation equation (7). The costate variable evolves according to

$$\dot{\mu} = \frac{u'_C}{P}(r - \rho) \tag{11}$$

The real value of the marginal utility per money unit is constant as long as $r = \rho$. This is a necessary assumption to obtain a stationary path.[3] We assume $r = \rho$ in the rest of the paper.

We choose the following well known specification of the utility function:

[3] This is only necessary in the "last" part of the simulation period.

$$u(c_t) = \frac{\sigma^c}{\sigma^c - 1} c^{(\frac{\sigma^c - 1}{\sigma^c})} \qquad , \qquad when \quad \sigma^c \neq 1$$

$$u(c_t) = \ln c_t \qquad , \qquad when \quad \sigma^c = 1$$

σ^c is the intertemporal elasticity of substitution.

From the first order conditions of intertemporal utility maximization we get the following simple relationship between consumption and the costate variable μ:

$$c = \frac{1}{(\mu P)^{\sigma^c}} \tag{12}$$

2.4 Macroeconomic equilibrium

The dynamic equilibrium is described by the following equations:

$$P = p(P_1, P_2^I) \tag{13.a}$$

$$P_1 = p_1(P_1^H, P_1^I) \tag{13.b}$$

$$P_1^H f'(k) = (r + \delta - \frac{\dot{P}}{P})P \tag{13.c}$$

$$P_1^H(f(k) - kf'(k)) = P^L \tag{13.d}$$

$$f(k) = p_1'(P_1, P_2^I)p_{1H}'(P_1^H, P_1^I)(c + \dot{k} + \delta k) + a \tag{13.e}$$

$$a = a(\frac{P_1^H}{P_1^I}) \tag{13.f}$$

$$\dot{b} = rb + P_1^H f(k) - P(c + \dot{k} + \delta k) \tag{13.g}$$

$$c = \frac{1}{(\mu P)^{\sigma^c}} \tag{13.h}$$

$$\lim_{t \to \infty} be^{-rt} = 0, b(0) = b_0, k(0) = k_0 \tag{13.i}$$

Small letters denote per unit of labour variables. Equations (13.a) and (13.b) is the price functions. The partial derivatives of the price function are defined as $\frac{\partial p}{\partial P_1} = p_1'$ and $\frac{\partial p_1}{\partial P_1^H} = p_{1H}'$ a.s.o. By Shepards lemma the partial derivatives equal the commodity shares. Equations (13.c) and (13.d) follow from optimal producer behaviour. (13.e) is the equilibrium condition in the product market. (13.f) determines the export demand as a decreasing function of the domestic price relative to the world market price. (13.g) gives the change in the stock of net foreign debt/assets. (13.h) follows from the optimal consumer behaviour. (13.i) is the NPG condition. The stock variables are predetermined at the beginning of the planning horizon, equation (13.i).

2.5 Equilibrium dynamics

We eliminate P from the system using (13.a) and (13.b). We also assume, for simplicity, that all exogenous variables are constant through time. The costate variable μ is constant and equal to its solution in the stationary (steady state) equilibrium due to our assumption of $r = \rho$. $\dot{P}_1^I = \dot{P}_2^I = 0$ and hence we can define $g(P_1^H) \equiv p(p_1(P_1^H, P_1^I), P_2^I)$. The dynamic system derived from (13.c), (13.e) and (13.g) then takes the following form:

$$\dot{P}_1^H = h_1(k, P_1^H) = \frac{1}{p_1' p_{1H}'}((r+\delta)g(P_1^H) - P_1^H f'(k)) \tag{14}$$

$$\dot{k} = h_2(k, P_1^H; \mu) = \frac{1}{p_1' p_{1H}'}(f(k) - a) - \frac{1}{(\mu g(P_1^H))^{\sigma c}} - \delta k \tag{15}$$

$$\dot{b} = rb + P_1^H f(k) - g(P_1^H)(\frac{1}{(\mu g(P_1^H)^{\sigma c}} + \dot{k} + \delta k) \tag{16}$$

For any given μ, the three variables P_1^H, b and k can be solved from equations (14), (15) and (16). However the resulting accumulation of b will in general be inconsistent with the NPG condition imposed by (13.i) and will eventually explode. The complete equilibrium therefore requires a solution for μ that satisfies (13.i). The model can be separated into an "inner" system, consisting of equations (14) and (15), and an "outer" system constituted by the intertemporal budget constraints (13.g) and (13.i). We have utilized this separable structure extensively in the solution of the model.

In order to analyse the stability properties of the model we will assume that a stationary solution really exist. We investigate the local behavior of the "inner" system by linearizing it around the steady state solution $(\bar{k}, \bar{P}_1^H)$. Defining $h_{1k} = \frac{\partial h_1(\bar{k}, \bar{P}_1^H)}{\partial k}$, $h_{1P} = \frac{\partial h_1(\bar{k}, \bar{P}_1^H)}{\partial P_1^H}$ a.s.o. for the other elements of the Jacobi matrix, the linear system takes the following form:

$$\begin{pmatrix} \dot{P}_1^H \\ \\ \dot{k} \end{pmatrix} = \begin{pmatrix} h_{1P} & h_{1k} \\ \\ h_{2P} & h_{2k} \end{pmatrix} \times \begin{pmatrix} P_1^H - \bar{P}_1^H \\ \\ k - \bar{k} \end{pmatrix} \tag{17}$$

The elements in the Jacobian are given in appendix A. The corresponding characteristic equation becomes:

$$\lambda^2 - (h_{1P} + h_{2k})\lambda + D = 0 \tag{18}$$

where D is the determinant of the linearized system, defined by

$$D = h_{1P}h_{2k} - h_{1k}h_{2P} < 0 \tag{19}$$

The characteristic equation has one positive and one negative root for the eigenvalue λ since $D < 0$, see appendix A. This implies in turn that the stationary solution at least locally is saddle point stable. However it is more difficult to find the steady state solution for the whole model. One approximate method is to rely on the linearized version of the dynamic system. This is the method used by Sen and Turnovsky (1989). Linearization makes it possible to obtain closed form solutions to the differential equations, and thereby a static model to determine the steady state solution. Applying this method to our model, involves the following steps:

Let λ_1 denote the negative eigenvalue corresponding to the stable path. This eigenvalue cannot be calculated without knowing the stationary solution. Since $k(0) = k_0$ the stable solution of $k(t)$ has the following form:

$$k(t) = \bar{k} + (k_0 - \bar{k})e^{\lambda_1 t} \tag{20}$$

Inserting this into the linearized form of (15) we obtain:

$$P_1^H(t) = \bar{P}_1^H + \frac{\lambda_1 - h_{2k}}{h_{2P}}(k_0 - \bar{k})e^{\lambda_1 t} \tag{21}$$

From equation (16) we get

$$\begin{aligned} \dot{b} &= r(b-\bar{b}) + (P_1^H f'(\bar{k}) - \delta P)(k_0 - \bar{k}) \\ &+ (f(\bar{k}) - (\delta\bar{k} + (1-\sigma^c)\frac{1}{(\mu P)^{\sigma c}})p_1' p_{1H}')(P_1^H - \bar{P}_1^H) \end{aligned} \tag{22}$$

Substituting the solution of P_1^H from (21) and k from (20) we obtain the following differential equation in b:

$$\dot{b} - rb = \alpha(k_0 - \bar{k})e^{\lambda_1 t} - r\bar{b}$$

where

$$\alpha = rP + \frac{\lambda_1 - h_{2k}}{h_{2P}}(f(\bar{k}) - (\delta\bar{k} + (1-\sigma^c)\frac{1}{(\mu P)^{\sigma c}})p_1' p_{1H}')$$

As $b(0) = b_0$ we have the solution:

$$b = \bar{b} + \frac{\alpha(k_0 - \bar{k})e^{\lambda_1 t}}{\lambda_1 - r} + [(b_0 - \bar{b}) - \frac{\alpha(k_0 - \bar{k})}{\lambda_1 - r}]e^{rt}$$

But according to the transversality condition $\lim_{t\to\infty} be^{-rt} = 0$, the last term must vanish implying that:

$$b = \bar{b} + \frac{\alpha(k_0 - \bar{k})e^{\lambda_1 t}}{\lambda_1 - r} \tag{23}$$

which is the solution consistent with long run solvency. Equation (23) represents the budget constraint regarded from the initial point of time.

Combining (23) with (14), (15) and (16) and setting $\dot{P}_1^H = \dot{k} = \dot{b} = 0$ gives us a static system to determine the steady state values of $\bar{P}_1^H$, $\bar{k}$, $\bar{b}$ and μ. (Remember that λ_1 and α are functions of the steady state values). Having determined the complete system of P_1^H, k and b, it is trivial to find the time paths for the remaining variables in the model. Due to the simplifying assumption of constant exogenous variables and hence an autonomous system, combined with the linearization, all information about the paths is comprised into two parameters only, the initial point and the growth rate given by the negative eigenvalue.

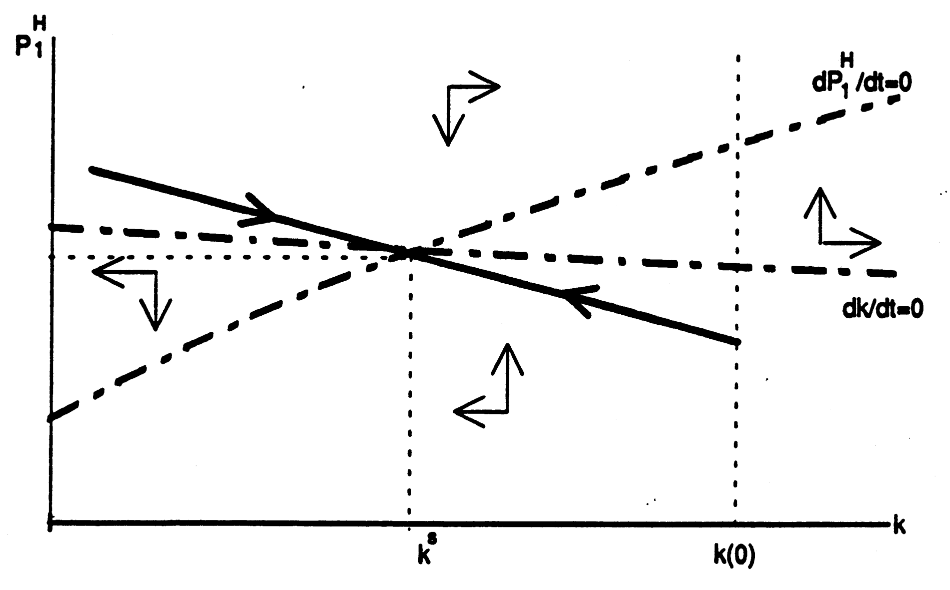
However, we suspect it to be too optimistic in general to accept the solution of the linearized model as a sufficiently good approximation to the true solution. The reason is that linearization around steady state will ignore the effects of investment expenditure on the net exports. In particular adjustments of the capital stock may play a crucial role when determining the dynamics. This is in fact the case in Sen and Turnovsky (1989) and also in our simulations that are presented in section 3.3. Any error in the solution of b is transmitted to the other variables through the constant co-state variable μ.

We have simulated a numerical version of the model given by (13.a) - (13.i). The numerical results are described in section 3.3 and the solution procedure is briefly presented in appendix B.

2.6 Phase-space analysis

The properties of the dynamic system given by (14) and (15) can be illustrated in the phase diagram in figure 2.6.

Figure 2.6: Phase diagram



As noted, this phase diagram is conditional on the stationary solution of μ. We denote the loci generated by setting $\dot{P}_1^H = 0$ in (14) and $\dot{k} = 0$ in (15) as locus 1 and 2 respectively. They are both approximated by straight lines in the figure. We find the slope of these loci by logarithmic differentiation of the stationary solution. ($\check{P}_1^H$, $\check{k}$, a.s.o. represents the logarithmic derivative.)[4]

$$\check{P}_1^H = \frac{1}{\theta^I}\frac{\theta^L}{\sigma^k}\check{k} \tag{24}$$

$$(\theta_k - \frac{\delta^H}{x}k)\check{k} = -s_{1H}\check{P}_1^H - s_{12}\theta_1^H\check{P}_1^H - \theta^I\check{P}_1^H - \frac{c^H}{x}\sigma^c\theta_1\theta_1^H\check{P}_1^H \tag{25}$$

where

$$s_{1H} \equiv (1 - \frac{a}{x})\theta_1^I(\sigma^{P_1} - 1) + \frac{a}{x}\alpha^a$$

$$s_{12} \equiv (1 - \frac{a}{x})\theta_2(\sigma^P - 1)$$

and

$$\theta^I \equiv (1 - \theta_1^H\theta_1)$$

$$c^H \equiv \theta_1^H\theta_1 c, \qquad \delta^H \equiv \theta_1^H\theta_1\delta$$

θ_1^j, $j = H, I$, is the budget share of the competing commodity for the domestic and imported competing goods respectively. θ_i, $i = 1, 2$, is the budget share for the competing commodity and non-competing imports respectively. θ^I can naturally be interpreted as the total import share. σ^{P_1} is the substitution elasticity between domestic production and competing imports, while

[4]Total logarithmic differentiation of the stationary solution gives:

$$\check{P}_1^H = \frac{1}{\theta^I}(\theta_1\theta_1^I\check{P}_1^I + \theta_2\check{P}_2^I + \frac{\theta^L}{\sigma^k}\check{k})$$

$$(\theta_k - \frac{\delta^H}{x}k)\check{k} = s_{1H}(\check{P}_1^I - \check{P}_1^H) + s_{12}(\check{P}_2^I - \theta_1^H\check{P}_1^H - \theta_1^I\check{P}_1^I) + \check{P}^I - \theta^I\check{P}_1^H - \frac{c^H}{x}\sigma^c(\check{\mu} + \theta_1\theta_1^H\check{P}_1^H + \check{P}^I)$$

where

$$\check{P}^I = \theta^I\theta_1^I\check{P}_1^I + \theta_2\check{P}_2^I$$

σ^P is the substitution elasticity between the competing commodity and non-competing imports. σ^k is the substitution elasticity between labour and capital in the production function. s_{1H} denotes the substitution effects between the domestic product and the competing world market product. It is determined by the trade elasticities and the importance of imports and exports indicated by the export share a/x and θ^I, i.e. the degree of "openness" of the economy in the initial stationary equilibrium. s_{1H} is positive as long as the substitution elasticity is larger than 1. s_{12} denotes the substitution effects between the composite competing commodity and non-competing imports.

Equation (24) is the first order condition for capital on elasticity form given constant exogenous variables. Locus 1 combines those values of k and P_1^H that are consistent with a rate of return to capital that requires no capital gains for satisfying the arbitrage equilibrium for the two assets in our model. Given import prices, locus 1 gives P_1^H as an increasing function of k. A larger stock of capital which gives a reduction in the rate of return on capital must be accompanied by an increase in the product price relatively to the user cost of capital. The elasticity of P_1^H w.r.t. k increases with the ratio between the cost share of labour and the elasticity of factor substitution. This ratio expresses the relative decrease in the marginal productivity of capital. The larger is this decrease, the higher is the increase in the output price necessary for having the first order condition fulfilled without capital gains. However, an increase in P_1^H will be carried over to the price on the capital good but the increase in P is smaller the larger is the total import share. Hence, the larger is the total import share, the smaller is the required increase in P_1^H as k increases along locus 1.

Equation (25) is the relationship between changes in P_1^H, k and μ that are consistent with equilibrium in the commodity market when there is no net investment. We can interpret the equation in the following way: a small increase in k yields a relative increase in output equal to the marginal elasticity of capital times the relative increase in k. In producer optimum the marginal elasticity is equal to the cost share of capital in the stationary equilibrium. Adjusting for increased depreciation yields the left hand side of the equation. The increase in the supply has to be offset by an equal relative increase in demand for the domestic good, and this increase has to be brought about by a change in P_1^H. A change in P_1^H affects the demand for the domestic good through four channels identified on the right hand side of the equation. First, we have a positive substitution effect that is crucially dependent on the trade elasticities. If $\sigma^{P_1} > 1$ a decrease in P_1^H, cet.par., brings about a substitution in favour of the domestic variety on both the domestic and the world market.

The second channel through which P_1^H affects the demand for the domestic good, is the substitution effect between good 1 and 2. We assume that $\sigma^P > 1$. A decrease in P_1^H is carried over to a decrease in P_1, and this effect is stronger the larger is the budget share of the domestic variety in the composite commodity 1. A relative decrease in P_1 increases the demand for commodity 1 both for consumption and replacement purposes. This implies an increase in the demand for both the domestic and the imported variety of commodity 1. The strength of this positive effect is of course dependent on the substitution possibilities between commodity 1 and 2, and the relative importance of domestic demand in the total demand for the domestic good.

The third channel through which P_1^H affects the demand for the domestic variety is a terms of trade effect since a fall in P_1^H for a given level of consumption expenditure, increases consumption when measured in units of the domestic product.

The fourth channel is the intertemporal substitution effect; a fall in the domestic price level implies an increase in the consumption of the domestic variety for a given value of μ.

All effects in equation (25) contribute to a negative relationship between P_1^H and k; a reduction in P_1^H is necessary to meet the increased supply. The slope of locus 2 is steeper the larger is the marginal elasticity of capital measured by the cost share of capital. The locus will be closer to horisontal the higher is the value of s_{1H}, i.e. the larger are the trade elasticities and the degree of openness. But, as noted, these considerations are somewhat partial because so far we have ignored the intertemporal relations that determine μ.

3 The effects of a change in terms of trade

We want to study the effects of a fall in the import prices. With reference to the effects of the European internal market or trade liberalization between Eastern and Western Europe it is not too difficult to motivate such an applied exercise in order to illustrate the properties of the intertemporal model.

3.1 Phase-space analysis

As pointed out in the previous section our dynamic system does not have the so-called two-point boundary property in which case we could have calculated the stationary solution independent of the initial situation and the dynamic transition path of the system. This makes it in general impossible to derive explicit analytical results even for the long run impacts of an exogenous change. The phase diagram presented in Figure 2.6.1 can provide some insight to how shocks will affect the stationary long-run equilibrium and the dynamics, but the properties of the phase-diagram are contingent on the real value of the marginal utility, μ, which has to be determined simultaneously with the whole model.

We restrict ourselves to study a permanent decrease in the import price P_1^I. Let us see why this implies a downward shift in locus 1 and possibly locus 2. We repeat the logarithmically differentiated equations from section 2 when $\check{P}_1^I < 0$.

$$\check{P}_1^H = \frac{1}{\theta^I}(\theta_1\theta_1^I\check{P}_1^I + \frac{\theta^L}{\sigma^k}\check{k})$$

$$(\theta_k - \frac{\delta^H}{x}k)\check{k} = s_{1H}(\check{P}_1^I - \check{P}_1^H) - s_{12}(\theta_1^H\check{P}_1^H + \theta_1^I\check{P}_1^I) + \check{P}^I - \theta^I\check{P}_1^H - \frac{c^H}{x}\sigma^c(\check{\mu} + \theta_1\theta_1^H\check{P}_1^H + \check{P}^I)$$

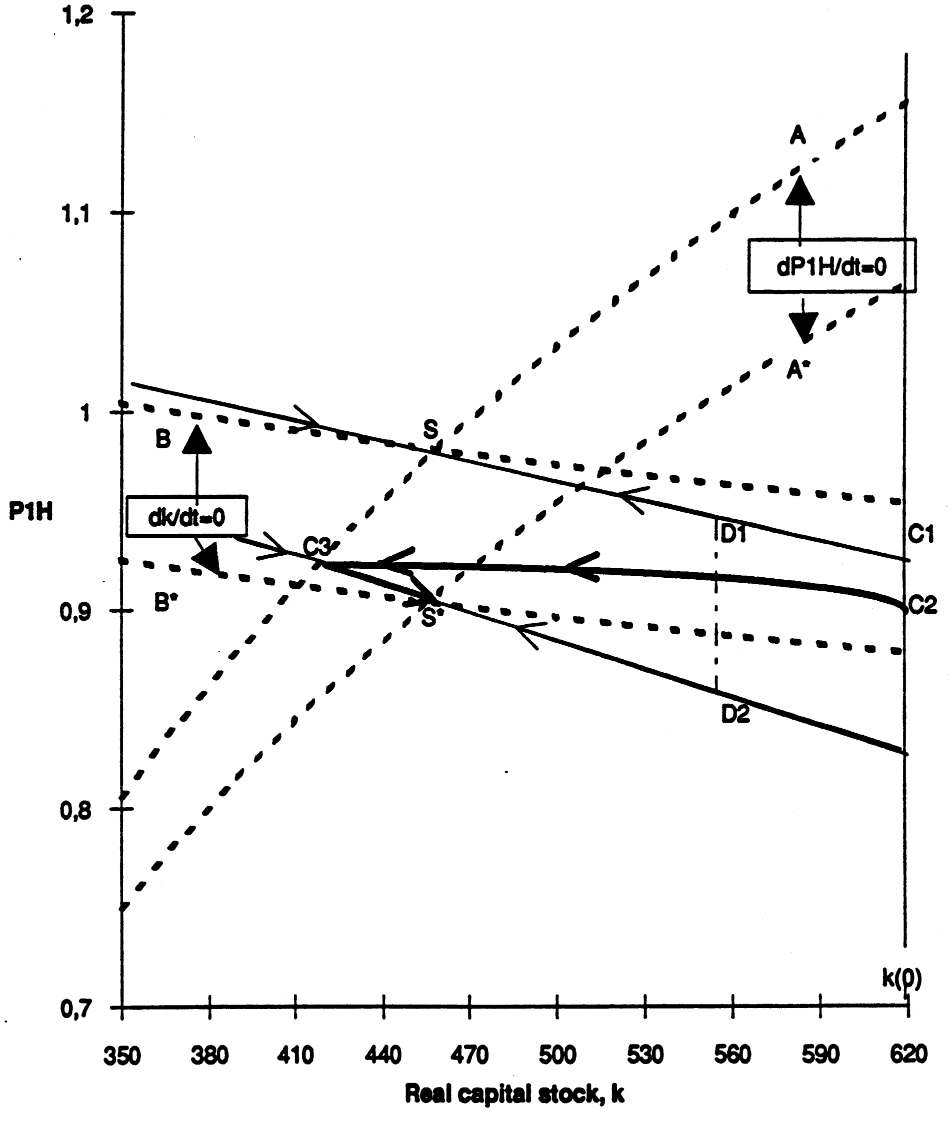
$$\text{where} \quad \check{P}^I = \theta^I\theta_1^I\check{P}_1^I$$

A fall in P_1^I makes locus 1 shift downward. Consider the case where the share of commodity 2 is negligible. Then the shift in this locus measured by the relative change in P_1^H at a given level of k is exactly equal to the relative decrease in P_1^I independent of any elasticities. The reason is that if the fixed value of k is to prevail being optimal, P_1^H and P has to change by the same rate to keep the nominal return to capital equal to the capital costs. Since the price function of commodity 1 is homogeneous of degree one, this is the only consistent solution to the shift effect. If the share of commodity 2 is not negligible and the price of this commodity is kept constant, then P will decrease less than proportionally to P_1^I even if P_1^H is reduced by the same rate as P_1^I. In order to keep the fixed k optimal, the relative decrease in P_1^H therefore has to be less than the decrease in P_1^I. Taking account of second order effects will reinforce the negative shift.

Locus 2 may shift in both directions. Large substitution elasticities between the tradeable commodity 1 and non-competing imports, contribute to a positive shift. In the case where s_{1H} is negligible, but not s_{12}, the decrease in P_1^I will increase the demand for the domestic product through an increase in the demand for good 1. P_1^H must increase in order to neutralize the demand for the domestic product. On the other hand large trade elasticities combined with a high degree of openness (i.e. large s_{1H}) contribute to a negative shift. The term $(\check{P}^I - \theta^I \check{P}_1^H)$ represents a revaluation effect on consumption measured in units of the domestic product. This effect contributes to a negative shift. The last term in the equation denoting locus 2 expresses the intertemporal substitution effect of price changes, reinforcing the positive shift.

In Figure 3.1 the shift in the two loci are negative. We have then made implicit assumptions about the intertemporal income effect working through μ, so that the negative effects upon locus 2 dominate. Hence, the steady state level of P_1^H decreases. The effect on the stock of capital is more ambiguous depending on the elasticity of substitution between the domestically and the imported good respectively.

Figure 3.1: Dynamic effects of a fall in the import price P1I.



Although the steady state solution depends on the change in μ which in turn is path dependent, the dynamics of the inner system in k and P_1^H is qualitatively independent of μ. Presupposing that the steady state changes as depicted in figure 3.1, we can use the phase diagram to show the dynamics of k and P_1^H caused by a reduction in P_1^I. The dotted curves A and B are the same loci as those drawn in figure 2.6. A^* and B^* are the same pair of loci after a reduction in P_1^I. These loci are drawn according to the numerical simulations reported in section 3.3. Assume that the fall in P_1^I is announced at time t_0 to take place at time t_1 in the future. At time t_0 the economy is on its way from C_1 towards S along the saddle path associated with S. The new steady state is at S^* and at time t_1 the economy must be on the new saddle path associated with S^*; otherwise, the economy will not converge to the new equilibrium. Prior to t_1, dynamics are determined by the equations of motions associated with S. At time t_0, $k = k(0)$ is given. Finally, the adjustment path must be such that there is no expected discrete change in P_1^H during the adjustment process. If such a change were expected, the price of capital would be expected to change by a discrete amount, generating the possibility of arbitrarily large capital gains per unit of time. Accordingly the adjustment path is one that follows the dynamics dictated by the arrows around S until P_1^I actually changes; precisely at that moment the adjustment path has to hit the saddle path leading to S^*. Such a path is shown by $C_1C_2C_3S^*$ in figure 3.1. Note that some kind of overshooting occurs before the economy hits the new saddle path.

Suppose that the fall in P_1^I at time t_1 is not announced. The economy then moves along the saddle path associated with S until it is surprised at time t_1 by this unanticipated fall in the import price. At time t_1 the capital stock is given so there is an unexpected downwards jump in P_1^H from point D_1 to D_2 (in figure 3.1) on the new saddle path associated with S^*.

We will support this examination of the dynamics caused by a fall in P_1^I with some more intuitive interpretations in section 3.3 where we present numerical illustrations.

3.2 Results in the perfect substitutability case

So far we have only discussed effects on the steady state solution for k and P_1^H, ignoring the general intertemporal income effect through μ. In order to better identify the main intertemporal equilibrium mechanisms, it seems useful to accompany the phase diagram with some sort of reference case where the effects are not too different from those actually operating in our model but sufficiently transparent to be explained analytically. As our reference case we will asume that the two varieties of commodity 1 are perfect substitutes with a common exogenous price P_1. As will be shown in the following section the dynamics in this case degenerates to immediate jumps and have a recursive structure.

We stress that this model is not very interesting on its own account; its only purpose is to facilitate the interpretation of the more general case where the varieties are imperfect substitutes. The model will in this case where P_1^H is equal to P_1^I, have a recursive structure. P, k and P^L are determined successively by the following equations:

$$P = P(P_1, P_2^I) \tag{26.a}$$

$$P_1 f'(k) = (r + \delta - \frac{\dot{P}}{P})P \tag{26.b}$$

$$P_1(f(k) - kf'(k)) = P^L \tag{26.c}$$

If all exogenous variables are constant the paths for all the endogenous variables become stationary except for a possible jump if k_0 is different from the stationary level. For simplicity, we disregard this possibility. We have also simplified the analysis by assuming that $\sigma^c = 1$. The consumption expenditure y is constant, and the intertemporal budget constraint implies that y is determined by

$$y = rb_0 + rP_0k_0 + P^L$$

The following equation determine net exports z as the difference between domestic output and demand for the commodity.

$$f(k) = c + \dot{k} + \delta k + z \tag{26.d}$$

What happens in this model if the agents experience an anticipated permanent decrease in P_1 in period t^*?[5] For the variables P, k and P^L the answers

[5]We consider the model in discrete time to simplify the interpretations of the model's dynamic properties.

are straightforward according to equations (26.a)-(26.c). P is reduced to a new level from period t^*. For the periods $[0, t^* - 2]$ k and P^L will be unchanged. In period $t^* - 1$ the agents correctly anticipate a negative capital gain increasing the user cost of capital and reducing the capital stock and the wage rate. In the remaining periods $[t^*, \infty >$ k and P^L find their new stationary values. Since P_1/P is reduced, k and P^L are lower compared to the reference path. The new constant level of consumption expenditure, $\tilde{y}$, declines due to the reduction in the wage rate from period $t^* - 1$ and onwards. The variables with a tilde represents the calculations with lower P_1. Hence, we have:

$$\tilde{y} - y = y + (\frac{1}{1+r})^{t^*-1}(\tilde{P}^L_{t^*-1} - P^L_{t^*-1}) + (\tilde{P}^L - P^L)\sum_{t=t^*}^{\infty}(\frac{1}{1+r})^t$$

The two other components determining $\tilde{y}$, the return from the initial value of financial and physical capital, is unchanged. Hence, the relative reduction in y is larger the higher is the share of wage income in the household's total income. The consumption volume, $c = y/P$, decreases in the periods up to t^*. The new stationary level may however change in both directions. Somewhat loosely, we can say that it will increase if wages is a relatively unimportant source of income and the budget share of good 1 is "large". For marginal changes in P_1 we have

$$\frac{\tilde{c} - c}{c} = \frac{\tilde{y} - y}{y} - \theta_1(\frac{\tilde{P}_1 - P_1}{P_1})$$

The value of net exports will increase relative to the reference path in the periods $[0, t^* - 2]$ since y falls while the output value and investment expenditure is constant. In period $t^* - 1$ the increase in net exports is stronger due to negative net investment which dominates the negative effect of a reduced production capacity. In period t^* the change in the trade surplus compared to the reference path is ambiguous; both the output value and the consumption expenditure is lower and investment is likely to be higher than in the reference path since the decline in the capital stock is likely to be stronger in period $t^* - 1$ than in period t^*.

However, considering the periods $t^* - 1$ and t^* as a unity, there will be negative net investment and consequently an increase in the value of net exports. In period $t^* + 1$ the net export value reaches its new stationary level which has to equal the interest paid on the foreign debt. The value of foreign debt at the beginning of period t^* will be reduced relative to the reference path due to the trade surpluses in the periods $[0, t^*]$. Hence, the reduction in the stationary net export value will be stronger the longer into the future t^* is. The dynamics of the the trade surplus reflects the rational intertemporal consumer behaviour. The clairvoyant consumer foresees a fall in future income and starts saving in order to find a new constant level of consumption expenditure that is consistent with long-run solvency. This increase in savings is absorbed by changes in financial wealth because investment per capita in the alternative asset, real capital, is determined by relative prices only.

3.3 Results in the imperfect substitutability case

As already pointed out no general results can be derived in this case. However numerical simulations can shed light over the mechanisms at work and which effects that are the most important determinants of the characteristics of the solution. The figures presented in this section are based on a numerically implemented version of the theoretical model presented in section 2 calibrated to Norwegian data in 1989[6]. We have used the model to simulate effects of a permanent reduction in the price of competing imports, P_1^I. This is done by first simulating a reference path where all exogenous variables are kept constant. We have then compared this solution with the results from the simulations of a partial anticipated and unanticipated permanent reduction in P_1^I respectively. In both cases the reduction is 10% and is implemented 4 periods after the initial period. The most important parameters in the model are given in table 3.3.1. In the simulations the intertemporal elasticity of substitution is equal to unity. Hence the consumption expenditure level is endogenous but constant over time. (This choice was made in order to simplify the interpretation of the results; we have no problems with obtaining numerical solutions with other values.)

[6]The only difference is that the offshore sector is treated separately and exogenously. Hence the production sector corresponds to mainland Norway.

Table 3.3.1: Some important parameters in the numerical model

Elasticities		Budget shares	
Substitution between the domestic and the imported product, σ^{P_1}	4	Share of competing imports in good 1, θ_1^I	0.44
Substitution between good 1 and non-competing imports, σ^P	1.5	Share of non-competing imports in total demand, θ_2	0.13
Export price elasticity, α^a	5	Export/output ratio,sector 1	0.26
Substitution between labour and capital, σ^k	2	Cost share of labour, sector 1, θ^l	0.6
Intertemporal elasticity of substitution, σ^c	1		

Anticipated reduction:

Even though the imperfect substitutability (IPS) case is much more complicated than the case of perfect substitutability (PS), the basic mechanisms discussed in the previous section are still at work but more or less modified through the endogeneity of P_1^H. In the following we will therefore concentrate on the effects brought about by this endogeneity. We pay special attention to the way endogenous price adjustments generate more gradual dynamics than in the PS-case.

Compared to the PS-case, negatively sloped export demand functions imply some degree of market power on the export market. In order to obtain a given improvement in the value of the export surplus, the necessary reduction in P_1^H is smaller the more imperfect is the foreign product as a substitute for the domestic one. As shown in figure 3.3.1, P_1^H falls less than 10% even in the steady state. This represent an improvement in terms of trade compared to the PS-case. In addition we have second order effects through the adjustment of the capital stock that increase the present value of total income further. Through the intertemporal budget constraint the consumption expenditure, y, is positively related to changes in total wealth. Compared to the PS-case it is much harder to calculate the net effect on y because there are many more transitory adjustments that have to be accounted for. However, it seems intuitively reasonable that the terms of trade gain makes it necessary to reduce y by a smaller amount than in the PS-case. Cet.par. a decrease in y implies a negative shift in the demand function for the domestic product and a lower value of P_1^H in order to clear supply and demand in this market. In the phase diagram drawn in the figures 2.6 and 3.1, the general equilibrium effect through y should be represented by a negative shift in locus 2, but this shift is smaller the more imperfect substitutes the two products become.

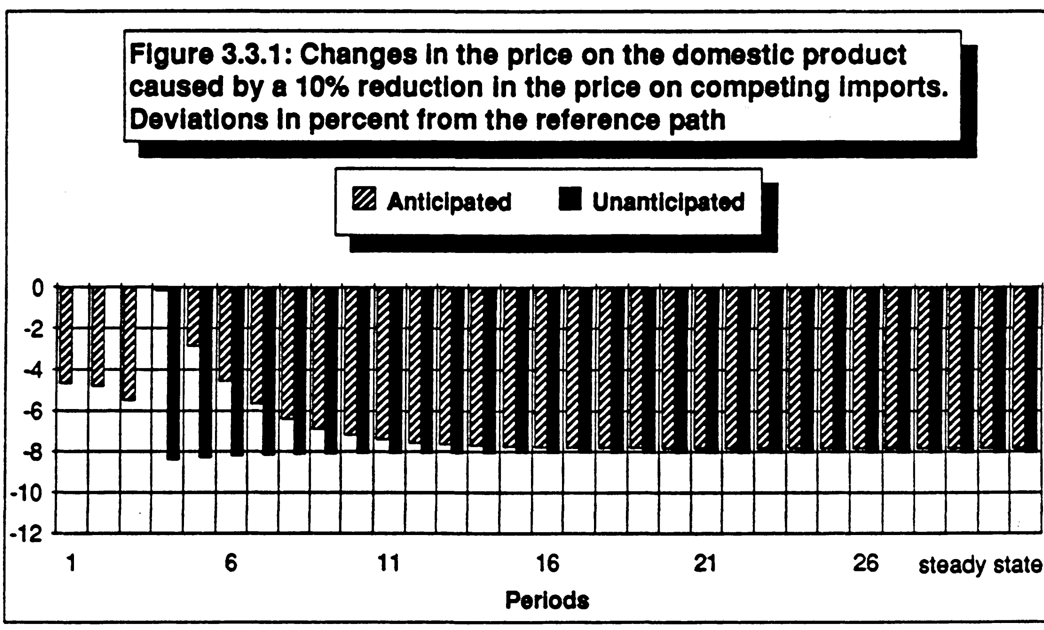


Figure 3.3.1: Changes in the price on the domestic product caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path

Turning to the dynamics, they fit very well with the dynamics depicted in the phase diagram in figure 3.1. In this section we present a more intuitive interpretation of the results. As to the changes in P_1^H, two characteristics call for an explanation. First, why do prices change before the exogenous price reduction is implemented in period 4. Second, why is the reduction in P_1^H exceptionally small in the first period where the economy experiences a negative price impulse? The basic element in the explanation of the first characteristic is the forward looking investment behaviour captured by the capital gains term in the user cost expression. Investors know in the initial period that the import price will fall 4 periods ahead, but it is not rational for them as individual price takers to take notice of any changes in prices until the period before they change. However, the equilibrium effect of the collective adjustments in period 3, induce endogenous price changes. Therefore announced price changes in future periods are spread backwards in time.

Let us try to make the interpretation of the results more concrete. Consider a representative investor who takes the price of the domestic product as fixed in a first iteration towards a plan for rational adjustment. In period 3 he knows that the import price will fall in the next period and that this will be carried over to a reduction in the macro price index representing the price on capital goods. Cet.par. this negative capital gain reduces the demand for capital and, relatively to the reference path, investment falls. The rational investor will now as a second step in his iterative reasoning towards a rational adjustment, take the effects of endogenous changes in the P_1^H into account. Consider P_1^H as the price equilibrating supply and demand in the market for the domestic product. The reduction in investment will lead to excess supply in the domestic product market as long as P_1^H is constant[7]. Thus the attempt to get rid of parts of the capital stock in period 3, requires a reduction in P_1^H in this period in order to keep the domestic product market in equilibrium because a fall in P_1^H increases demand through higher exports and declining import shares. In addition real consumption increases. On the other hand the investment respons requires some restrictions on the elasticities in the model, especially the elasticity of factor substitution, for a reduction in P_1^H to be an equilibrium response. A fall in P_1^H will reduce

[7]Though lower investment has a negative capacity effect on output even in the same period, the relative decline in output is only a fraction of the decline in demand. For small changes this fraction equals the marginal elasticity or the cost share of capital.

the price of output relative to the price of capital, thereby reinforcing the negative investment response. The same line of reasoning can be repeated for all periods back to the initial period where the predetermined capital stock determines the initial investments. Thus, the initial predetermined capital stock is a determinant of the equilibrium price adjustments.

As the endogenous prices change in several periods, the effects on the optimal capital stock become twofold. First, a reduction in P_1^H will reduce the output price relative to the price on capital goods as long as import prices are constant, see figures 3.3.1 and 3.3.2. Second, the capital gains term is affected. However, the figures 3.3.3 and 3.3.4 indicate that both effects contribute to a lower capital stock relative to the reference path. However, it is important to note that a reduction of the capital stock does not imply a reduction of investment in all periods. This is easily seen from figure 3.3.4. The changes in the capital stock implies corresponding changes in the capital/labour ratio. The changes in the wage rate consistent with the changes in the prices and capital stock are shown in figure 3.3.5.

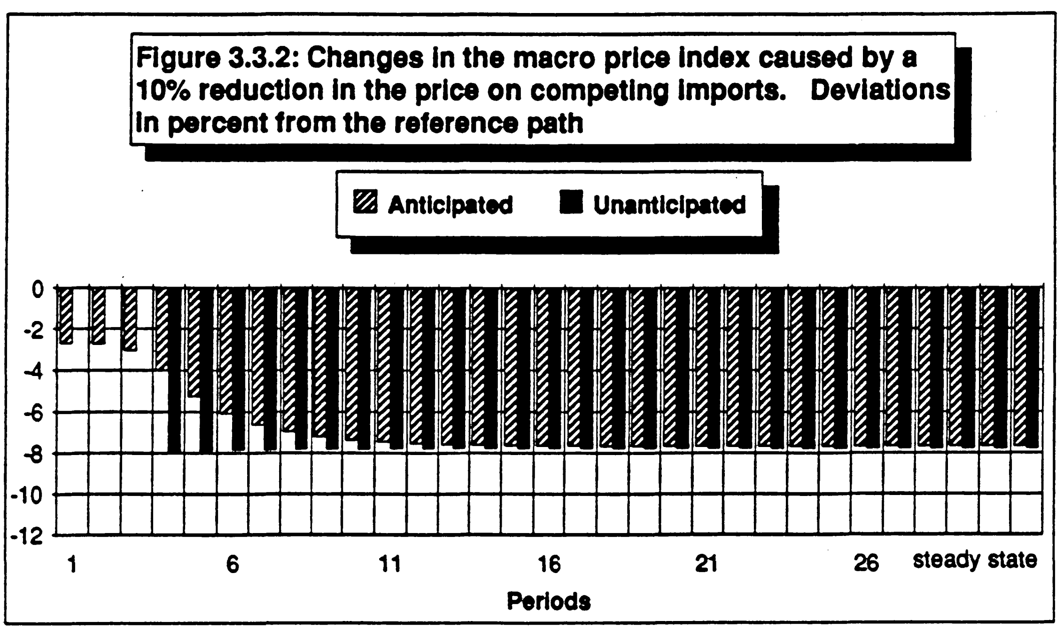


Figure 3.3.2: Changes in the macro price index caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path

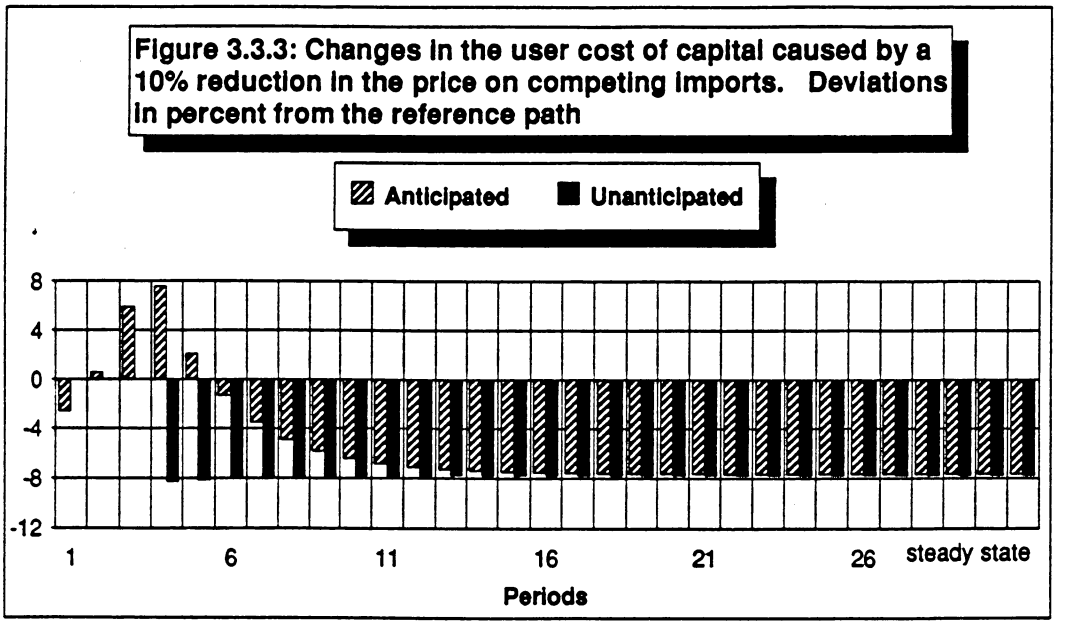


Figure 3.3.3: Changes in the user cost of capital caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path

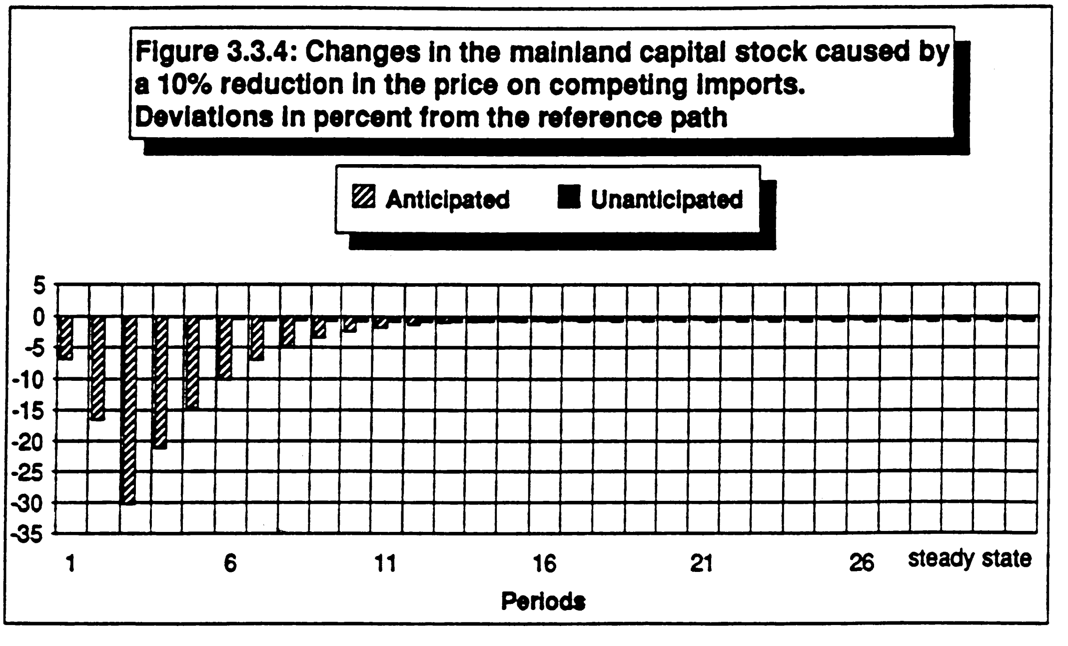


Figure 3.3.4: Changes in the mainland capital stock caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path

Figure 3.3.5: Changes in the wage rate caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path

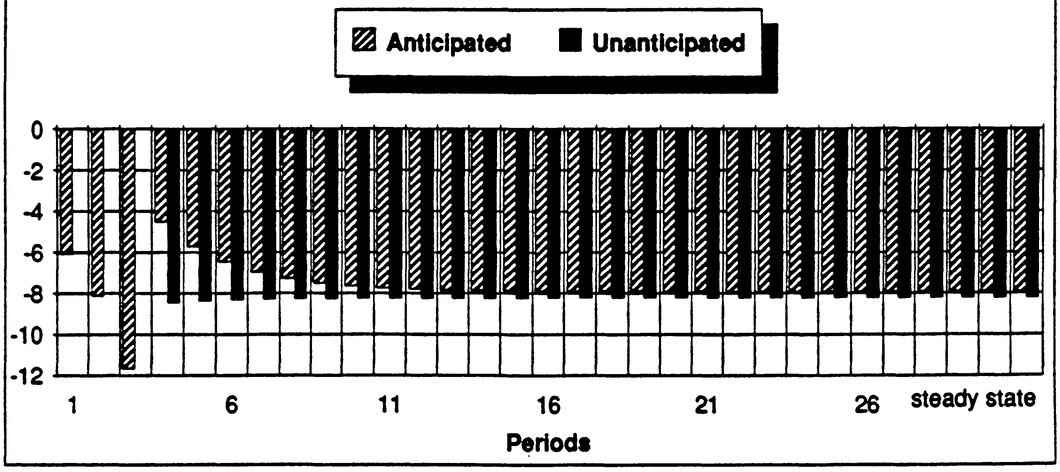
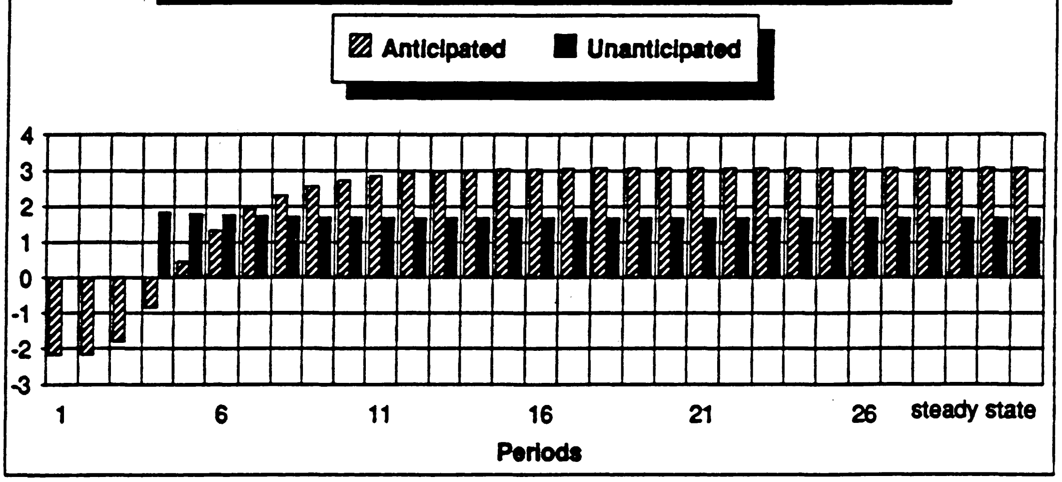


Figure 3.3.6: Changes in consumption caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path



We are now ready to explain what may look paradoxical: *Why is the fall in P_1^H exceptionally small in period 4, the same period as the reduction in P_1^I is implemented?* In this period the consumers benefit from the reduced price on imports through a fall in P which is their cost of living index. The consumption expenditure is constant so that real consumption increases by the same proportion as P. Also, the fall in P reduces, cet.par., the user cost of capital giving rise to an increase in the capital stock. Both effects contribute to a positive shift in the demand function for the domestic good from this period on. However, the total net effect is ambiguous due to loss of international competitiveness reducing exports and increasing the import share of competing imports. In the PS-case this effect will of course always dominate and forces P_1^H to exactly follow P_1^I. In our numerical model it turns out that the positive effects are strong enough to keep P_1^H almost unchanged compared to the reference path, in order to keep the product market balanced. This is particularly due to the strength of the investment impulse because real consumption actually declines relative to the reference path. Thus investment in this period is financed both by a trade deficit and by postponing consumption which increases from period 5 and thereafter.

During the rest of the dynamic path P_1^H converges monotonically to the steady state equilibrium being close to 8% below the steady state level in the reference path. This development reflects that the capital stock gradually converges to its new steady state level, reducing the investment demand. Both the capital stock and the financial wealth are almost unaffected if we compare the two steady state solutions. Consequently the accumulated transitory changes in both real investment and financial investment through the trade surplus are zero in the long run.

Figure 3.3.7: Changes in the trade balance caused by a 10% reduction in the price on competing imports. Deviations in NOK per man hour from the reference path

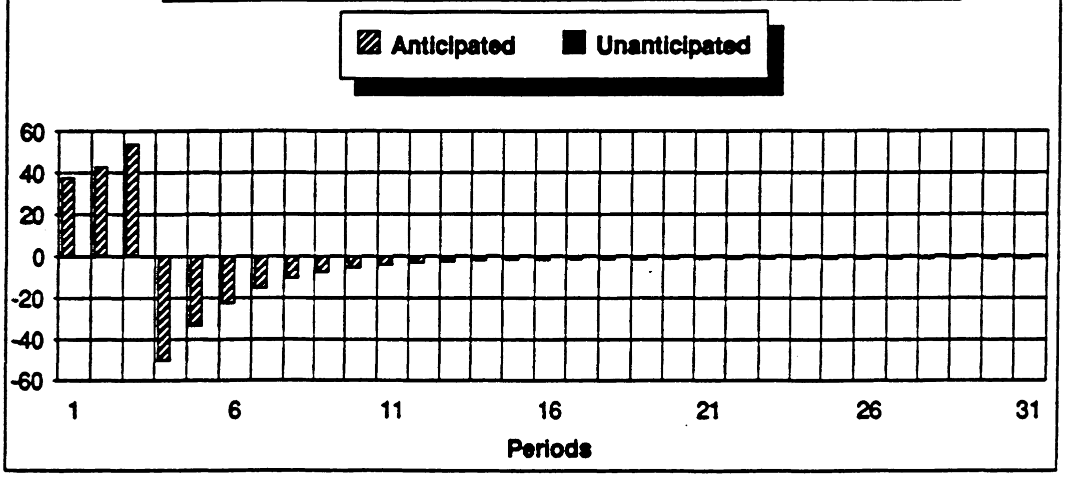
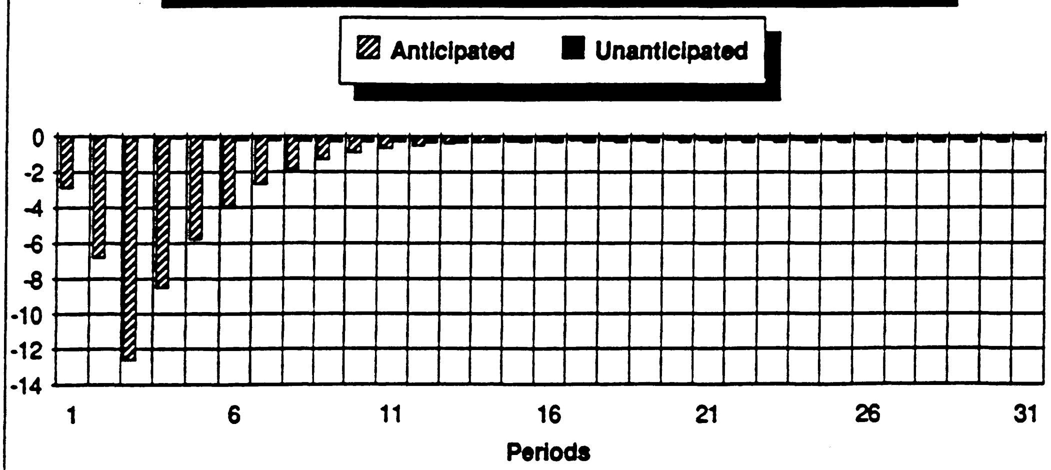


Figure 3.3.8: Changes in mainland output caused by a 10% reduction in the price on competing imports. Deviations in percent from the reference path



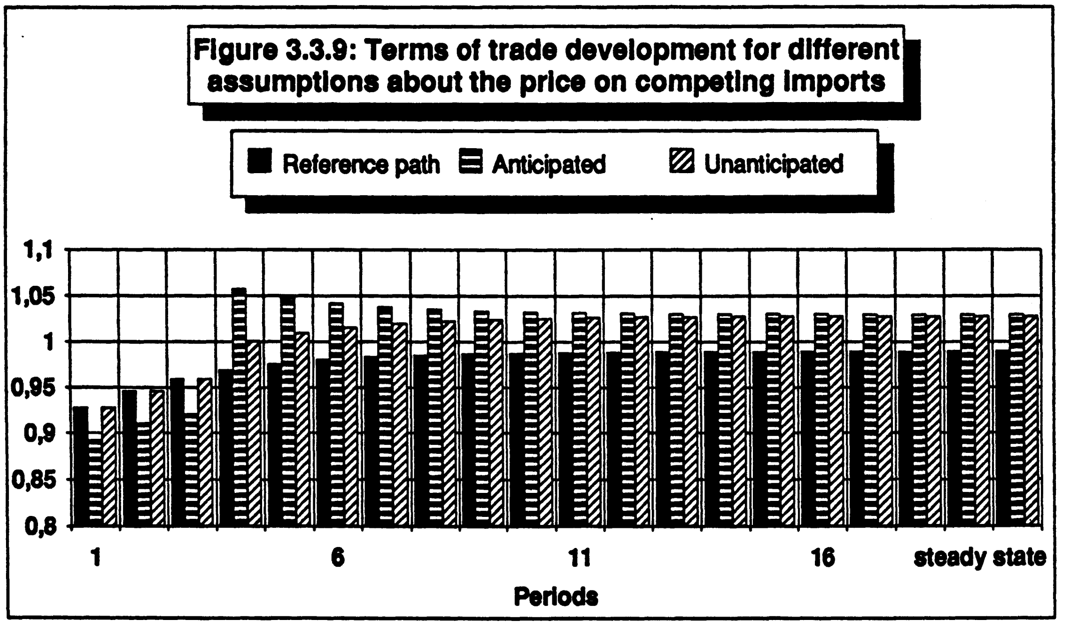


Figure 3.3.9: Terms of trade development for different assumptions about the price on competing imports
Reference path
Anticipated
Unanticipated
1,1
1,05
1
0,95
0,9
0,85
0,8
1
6
11
16
steady state
Periods

Whether or not the effects of this import price fall are in accordance with the Harberger-Laursen-Meltzer (HLM) effect depends on the total effect on the terms of trade. Figure 3.3.9 shows the terms of trade development along the three paths. The terms of trade is calculated as the ratio between the price index on total exports (good 1 and petroleum) and the price index on total imports of good 1 and 2. Although the export price of mainland output is endogenously reduced, the implementation of a lower P_1^I has a positive net effect upon the term of trade. The opposite is true in the periods between the announcement and the implementations, where only P_1^H is reduced. In all periods the HLM-effect is rejected; an improvement of the terms of trade is negatively correlated to an increase in the current account balance, see figure 3.3.7. In our model, as in the one discussed by Sen and Turnovsky (1989), the financing of real investment is a major determinant of these current account dynamics. The HLM-effect was stated in a model relying on Keynesian savings- and investment behaviour which turns out to be quite different from the rational intertemporal behaviour described in our model.

Unanticipated reduction

In this case the economy evolves along the reference path until period 4 where P_1^I is reduced by 10%. Compared to the previous full information case where the price reduction was anticipated, the capital stock is not optimally adjusted in the first three periods. Especially, investors have wrong expectations in period 3, and as discussed above, the general equilibrium effects implies wrong expectations in the earlier periods as well. Intuitively this suboptimality reduces the intertemporal consumption budget and welfare, and the reduction in the consumption expenditure (from period 4 on) is larger than in the case of an anticipated price reduction. Compared to the case of an anticipated price change, the consumers find that they have been consuming too much relative to what they would have done if they had known better in the first three periods. Roughly speaking, this "too high" level of consumption has to be paid for by lower stationary real consumption, see figure 3.3.6. Due to the solution of the intertemporal price system it turns out that savings through an increase in the capital stock are not optimal, but channelled to the other asset; the value of the trade surplus is slightly increasing in the short as well as in the long run and this is brought about by a stronger reduction in the price of the domestic product than in the case where the fall in the import price was anticipated. However, this effect is not sufficiently strong in the numerical model to cause significant differences between the two steady state solutions. Any difference between the steady state solutions corresponding to the anticipated and the unanticipated exogenous change is due to the path dependence of the steady state solution.

Without any path dependence, the two paths would converge to exactly the same steady state solution. In our simulations the path dependence has minor effects upon the steady state solutions, but this is of course not generally true. It is the length of the period from the announcement till the implementation of the exogenous change in which the economy is suboptimally adjusted that matters. The longer is this period, the larger is the value of consumption possibilities foregone, and the larger are also the differences between the two dynamic equilibrium paths in the long run.

The short run responses may be different depending on whether or not the exogenous change is anticipated. However, this is because the short run effects in the anticipated case are very different from the long run effects, whereas the short- and long-run effects are rather similar in the case where the economy is surprised. To understand this, recall that in the anticipated case it is the adjustment of the capital stock in the period before the implementation of the exogenous change, that is the driving force behind the dynamics. In the present case the changes in relative prices do not give such strong incentives for changes in the portfolio composition between real and financial assets.

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Appendix A

For the elements of the Jacobian we have in steady state:

$$h_{1P} = -(r+\delta)(\frac{1}{\theta_1\theta_1^H} - 1) < 0 \tag{A.1}$$

P_1^H decreases when P_1^H is increased partially from its stationary level. The interpretation is that an increase in P_1^H increases the return from real capital relative to financial assets so that less (negative in steady state) capital gains are needed in order to preserve arbitrage equilibrium.

$$h_{1k} = -\frac{P_1^H}{p_1' p_{1H}'} f''(k) > 0 \tag{A.2}$$

When k increases the return from real capital decreases. In order to preserve arbitrage equilibrium capital gains are needed.

$$h_{2P} = \frac{(-\alpha^a)\frac{a}{P}}{\theta_1\theta_1^H} + \frac{\sigma^c c}{P_1^H}\theta_1\theta_1^H + \theta_1^H(\frac{c+\delta k}{P_1^H})(\sigma^P\theta_2 + \sigma^{P_1}\frac{\theta_1}{\theta_1^H}) > 0 \tag{A.3}$$

since all terms are positive. The reason is that an increase in P_1^H implies reduced export demand, increased import share and lower real consumption. All these effects contribute to increase the amount of the fixed domestic production that has to be demanded for investment if the product market shall stay in equilibrium.

$$h_{2k} = \frac{r}{\theta_1\theta_1^H} + \delta(\frac{1}{\theta^1\theta_1^H} - 1) > 0 \tag{A.4}$$

A partial increase in k increases output according to the marginal productivity and depreciation. In the stationary equilibrium the net effect is positive and product market equilibrium can only be preserved by increased capital formation when consumption, exports, the import share and prices are constant.

The determinant of the linearized system can be written:

$$D = h_{1P}h_{2k} - h_{1k}h_{2P} < 0 \tag{A.5}$$

The two solutions to the eigenvalue of the characteristic equation is:

$$\lambda_2 = \frac{h_{1P} + h_{2k}}{2} + [\frac{(-h_{1P} - h_{2k})^2}{4} - D]^{1/2} \quad (A.6)$$

which corresponds to the unstable solution. The negative eigenvalue is:

$$\lambda_1 = \frac{h_{1P} + h_{2k}}{2} - [\frac{(-h_{1P} - h_{2k})^2}{4} - D]^{1/2} \quad (A.7)$$

Appendix B

We have shown that the reduced dynamic system in P_1^H and k can be solved given a value of μ. Hence the dynamic model (13.a) - (13.i) can be divided in an "inner" and "outer" system. This property is utilized in the solution procedure of the model. The model is solved in the computer programme TROLL (Hollinger (1988)) and the solution procedure consists of three steps illustrated in figure B.1. The steps are briefly described as follows:

Step 1

We start the simulations by making an initial guess on μ and the path for the lead variable P.

Step 2

The Forward Looking simulator in TROLL uses three different kinds of iteration processes to solve the optimal path. Type 1 solves the intratemporal model (given figures on the solution of the intertemporal problem, i.e.) given a path for the lead variable P. The model given by the equation system (13.a) - (13.h) then becomes a static system in each period. The solutions for the different periods are however linked through the lag-variables k and b. The Type 1 iteration applies an algorithm based on Newton's method to solve this static system. The Type 1 iteration yields a solution for the lead variable P in all periods. Formally we have a vector valued function A that generates actual values from a vector of expectations of P, $A_i = A(P_i)$ where i denotes the number of iterations.

TROLL applies the Fair-Taylor algorithm when solving the dynamic system that results from the assumption of perfect foresight. See Fair and Taylor (1983) and Wilcoxen (1989) for a more comprehensive exposition. A perfect foresight equilibrium requires a solution to the equation:

$$A(P_i) = P_i \tag{B.1}$$

If A_i is not sufficiently close to P_i, a new vector is determined by

$$P_{i+1} = \gamma A_i + (1 - \gamma) P_i \tag{B.2}$$

γ is a constant between 0 and 1 which can be adjusted to improve the rate of convergence.

This problem is then solved in two stages. In the first stage called Type 2 iteration, the horizon is fixed to h implying that the guess vector P has h elements. The iterative procedure indicated by (B.2) goes on until convergence is obtained by satisfying (B.1) over the horizon h. In the second stage, called Type 3 iteration, the horizon is extended to $h + 1$. The Type 2 iteration process is then repeated. The solution for the h first element of the new P-vector based on the horizon equal to $h + 1$, is then compared to the solution of the P-vector based on the horizon equal to h. The horizon is extended until the difference is sufficiently small, i.e. the iteration process has converged when an extension of the horizon does not affect the dynamic solution. For every Type 2 and Type 3 iteration the static model is solved by Type 1 iteration.

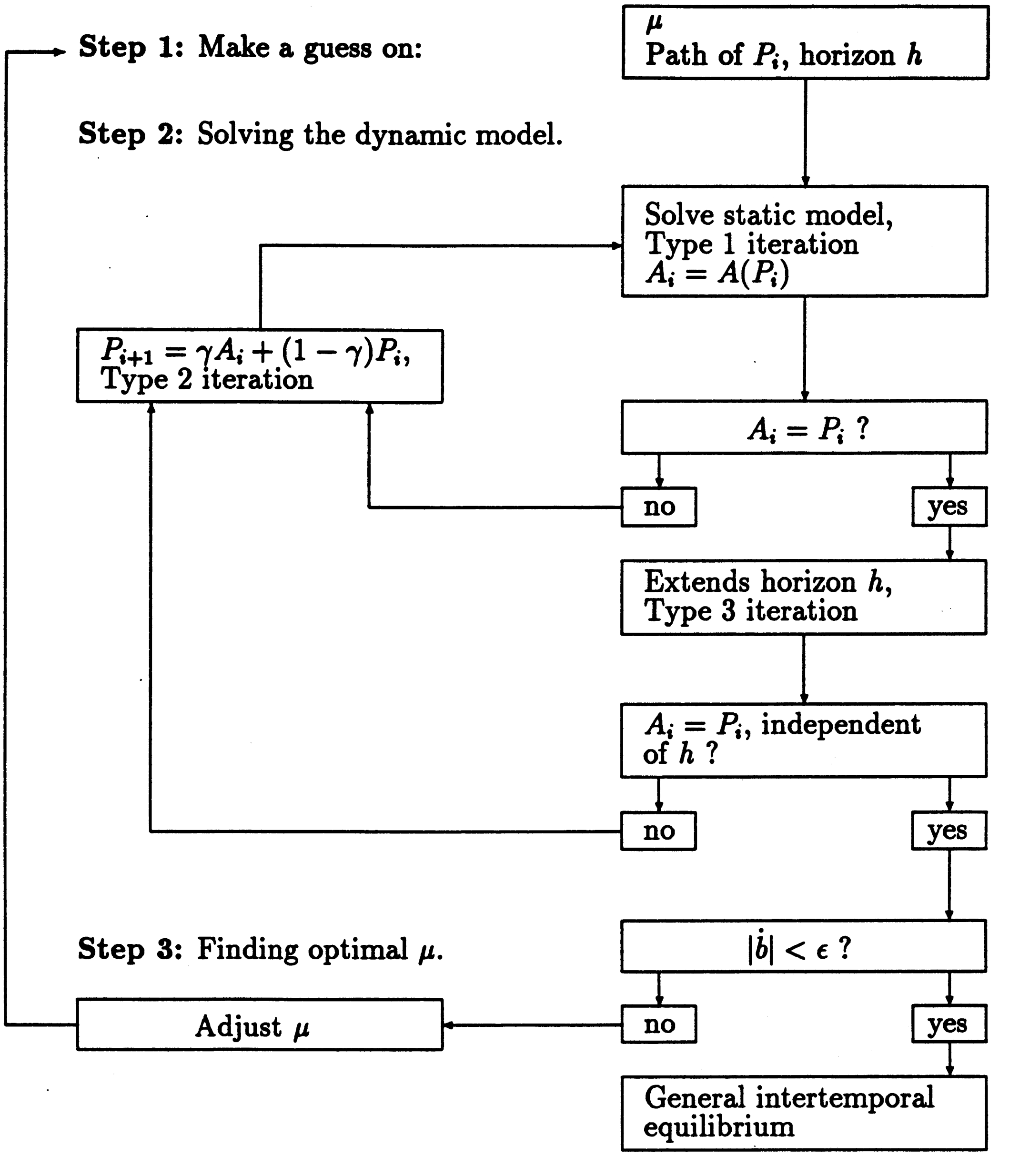
Step 3

The last step in the solution procedure is to find the equilibrium value for μ, which so far has been set by an "educated" first guess. This is equivalent to finding a path that satisfies the NPG condition, i.e. the intertemporal budget constraint. We utilize that there is a monotoneous positive relationship between μ and the change in net financial wealth $\dot{b}$. An increase in marginal utility means less consumption, thereby decreasing demand and increasing the trade surplus and the rate of wealth accumulation. According to the NPG condition the present value of b must tend to 0 as time tends to infinity. The steady state solution implies that the trade surplus/deficit will be constant. A sufficient condition for the NPG condition to be satisfied is that the current account is approximately 0 such that the trade surplus/deficit and the interest payments are approximately equal, but with opposite signs. Hence, if $\dot{b} \simeq 0$ for a given number of periods, the NPG condition will be satisfied.

We have designed an algorithm which tests for these restrictions in a particular period t'. This period has to be sufficiently far away from the initial period so that we are sure a steady state solution has been reached for the variables other than b. If $|\dot{b}| > \epsilon$ where ϵ is the convergence criterion and $\dot{b} > 0$, a smaller value of μ is chosen, and conversely if $\dot{b} < 0$.[8] The optimal value of μ must satisfy the condition $|\dot{b}| < \epsilon$ and in addition b and the value of the trade balance must have opposite signs for the rest of the simulation period to ensure that the current account is equal to zero.

[8] A complete description of this algorithm is available from the authors.

Figure B.1: Illustration of the solution procedure



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