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EXPECTATIONS, SUBSTITUTION, AND SCRAPPING IN A PUTTY-CLAY MODEL

BY

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ABSTRACT

The paper presents a putty-clay framework for analyzing the effect of changing expectations about future prices on a firm's choice of technique, and on its anticipated scrapping of capital equipment. Particular attention is paid to the way in which the scrapping age depends on the degree of ex ante input substitution. Empirical illustrations - based on data for Norwegian manufacturing for the years 1964-1983, an ex ante technology represented by a Generalized Leontief cost function in materials, energy, labour, and capital, and an ARMA representation of the price expectation mechanism - are presented. The results indicate that the price changes in this period may have had a substantial impact on planned scrapping, and on the chosen production techniques.

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CONTENTS

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1. INTRODUCTION

The effect of the sharp increase in energy prices during the last decade on business investment, capacity utilization, capital productivity, scrapping, and related issues has received substantial attention in the recent literature [see e.g. Berndt and Wood (1984,1985)]. The fact that these price changes were to a large extent unanticipated by the market, is, in particular, a challenge to econometricians trying to quantify their effects. This raises the more general problem of formalizing and analyzing empirically how expectations about future input and output prices and unanticipated changes in these prices can affect the firms' investment and scrapping decisions. For empirical analysis of these effects, however, there is a serious problem that data on the producers' price expectations - or, more generally, information about the mechanism which links price expectations with observed prices - is almost completely lacking.

In this paper, we discuss some aspects of these problems, and focus in particular on the relationship between price expectations, choice of technique, and decisions with respect to the anticipated scrapping of capital equipment for a profit maximizing firm. We show that the degree of input substitution may crucially affect the expected service life of new capital equipment. To illustrate these theoretical conclusions, we also report some tentative empirical results for a producer with a four-factor technology based on data for Norwegian manufacturing for the years 1964-1983. Our aim is to discuss the relationship between price expectations, choice of technique, and the scrapping of capital equipment in a general setting in which not only the effects of energy price changes, but also other price changes, such as the wage rate and the price of non-energy material inputs, are brought into focus. The energy-capital substitution and the relationship between energy price shocks and capital service life is discussed in some more detail in Biørn (1986).

The technology will be represented by a vintage production model of the putty-clay type. Ex ante - i.e. before an investment is made - the firm is assumed to face a neo-classical technology with one kind of capital and one or more other (variable) inputs. Ex post - i.e. after the investment has taken its specific physical form - all inputs must be used in fixed proportions.

The putty-clay model - originally proposed by Johansen (1959,1972) - is well suited to deal with the relationship between price expectations, price shocks, and capital formation.[1] The reason for this is that it implies non-myopic decision rules, in contrast to neoclassical (putty-putty) models which assume the same degree of ex ante and ex post substitution and capital which is completely malleable. Decisions taken today will then strongly depend on expectations about the future development of prices. Further, the rigidities which exist in the adjustment of factor proportions is represented, in a logically consistent way, by the model's fundamental distinction between ex ante and ex post optimal factor proportions. Finally, since it is a vintage model it is well suited to analyzing the endogeneity of the scrapping decisions. The latter property of the model has been utilized by, inter alia, Ando et al. (1974), Malcomson (1975,1979), and Malcomson and Prior (1979). The problem of choice of technique is analyzed in Hjalmarsson (1974), and Førsund and Hjalmarsson (1986) in the context of an expanding industrial sector with increasing returns to scale, but under the assumption that each plant is infinitely long lived, thus avoiding the problem of scrapping.

The paper is organized as follows. Section 2 gives the theoretical framework in terms of a general description of the ex ante production technology. Two basic concepts involved are the terminal quasirent function and the life cycle output and input prices. We discuss the way in which the ex ante service life of the capital is related to the form of the quasirent function and the life cycle prices for each vintage; and the dependence of the planned scrapping age on the degree of ex ante input substitution. Section 3 introduces specific assumptions about the form of the capital retirement function and the price expectation functions, and gives a decomposition of the total effect of price changes on the scrapping age and choice of technique. A presentation of the data and and econometric specification, based on a four factor (materials, energy, labour, and capital) Generalized Leontief cost function, follows in section 4. In section 5, we present some simulation results which illustrate the joint determination of the scrapping age and the choice of technique.

2. THE GENERAL MODEL

Consider a producer in the process of investing in a new capital vintage. Let the ex ante technology - i.e. the set of blueprints of techniques from which he can choose - be described by the linear homogeneous production function

$$y = f(x_1, \ldots, x_m, J, t) , \tag{2.1}$$

where $(x_1, \ldots, x_m)$ is the vector of variable inputs and J is the quantity of capital invested. Technological change, represented by the time index t, is supposed to affect the ex ante technology only, i.e. all technological change is embodied in the vintage. The deterioration of the capital stock is described by the survival function $B(\tau)$, where τ denotes the age of the capital with $B(0) = 1$ and $B'(\tau) \leqq 0$. It is a technological datum which represents both the disappearance of capital goods and the decline in efficiency with age.[1] The capital input at age τ will then be $K(\tau) = B(\tau) J$.

The ex post technology is characterized by fixed factor proportions between the inputs. This implies that that the input of the i'th variable factor at age τ is equal to $x_i(\tau) = B(\tau) x_i$ and, since the technology is linear homogeneous, that output at age τ is

$$y(\tau) = B(\tau) y . \tag{2.2}$$

Let $q(t+\tau,t)$ and $p_i(t+\tau,t)$ denote the output price and the price of the i'th input, $i=1,\ldots,m$, respectively, which at time t the producer expects to prevail at the future period $t+\tau$.[2] These expectations are assumed to hold with certainty, but may be subject to revisions, as indicated by the double time subscript.[3] The ex ante quasirent from vintage t at time $t+\tau$ can then be written as

$$v(t+\tau,t) = q(t+\tau,t) y(\tau) - \sum_{i=1}^{m} p_i(t+\tau,t) x_i(\tau)$$

$$= B(\tau) \left[q(t+\tau,t) y - \sum_{i=1}^{m} p_i(t+\tau,t) x_i \right] . \tag{2.3}$$

The total profit from vintage t is equal to the discounted value of the quasirents from age 0 to the scrapping age s, less the initial investment cost,

$$V(t,s) = \int_0^s e^{-r(t)\tau} v(t+\tau,t)\, d\tau - p_J(t)\, J$$

$$= q^*(t,s)\, y - \sum_{i=1}^{m} p_i^*(t,s)\, x_i - p_J(t)\, J , \qquad (2.4)$$

where

$$q^*(t,s) = \int_0^s e^{-r(t)\tau} q(t+\tau,t)\, B(\tau)\, d\tau ,$$

$$p_i^*(t,s) = \int_0^s e^{-r(t)\tau} p_i(t+\tau,t)\, B(\tau)\, d\tau , \quad i=1,\dots,m. \qquad (2.5)$$

The latter expressions can be interpreted as the "life cycle" prices of output and inputs from age 0 to age s. The prevailing rate of discount, $r(t)$, is assumed to remain constant from time t up to the horizon.

Consider now the problem of choosing the profit maximizing technique, i.e. the input vector which for an exogenously given output y and the price expectations held at time t maximizes the ex ante life cycle profit $V(t,s)$. The maximization procedure can conveniently be divided into two stages:

(i) maximization with respect to $x_1,\dots,x_m$, and J for given s, and

(ii) maximization of the resulting function, $\Pi(t,s)$, with respect to s.

Problem (i) is formally equivalent to a neoclassical restricted profit maximization problem since the life cycle prices can be regarded as exogenous variables. Its first order conditions, subject to (2.1), are

$$p_i^*(t,s) = \lambda(t,s)\, f_i(x_1,\dots,x_m,J,t) , \quad i=1,\dots,m,$$

$$p_J(t) = \lambda(t,s)\, f_J(x_1,\dots,x_m,J,t) , \qquad (2.6)$$

where $f_i = f_i(x_1,\dots,x_m,J,t)$, $i=1,\dots,m,J$, are the partial derivatives of f with respect to the i'th input, and $\lambda(t,s)$ is the Lagrangian multiplier associated with the constraint (2.1). The solution to (2.6) is implicitly defined by the life cycle cost function dual to (2.1)

$$C(y,p_1^*,\dots,p_m^*,p_J,t) = \min_{x_1,\dots,x_m,J} \{ \sum_i p_i^* x_i + p_J J \mid y = f(x_1,\dots,x_m,J,t) \}$$

$$= y\, c(p_1^*,\ldots,p_m^*,p_J,t) \,, \tag{2.7}$$

the second equality following from the linear homogeneity of f, c being the unit cost function.

Application of Shephard's lemma to c gives the optimal input coefficients

$$a_i = \frac{x_i}{y} = \frac{\partial}{\partial p_i^*} c(p_1^*(t,s),\ldots,p_m^*(t,s),p_J(t),t) = c_i \,, \quad i=1,\ldots,m,$$

$$a_K = \frac{J}{y} = \frac{\partial}{\partial p_J} c(p_1^*(t,s),\ldots,p_m^*(t,s),p_J(t),t) = c_K \,, \tag{2.8}$$

conditional upon the service life s. The solution to problem (i) then defines the function

$$\Pi(t,s) = \max_{x_1,\ldots,x_m,J} \left[\int_0^s e^{-r(t)\tau}\, v(t+\tau,t)\, d\tau - p_J(t)\, J \right]$$

$$= y \left[q^*(t,s) - c(p_1^*(t,s),\ldots,p_m^*(t,s),p_J(t),t) \right] , \tag{2.9}$$

which represents the maximum profit attainable, given the base year scale of operation and the assumption that the equipment is to remain in service for s years.

Associated with problem (i) we also define the terminal quasirent function of vintage t

$$R(t,s) = \frac{1}{e^{-rs}\, B(s)\, y} \frac{\partial}{\partial s} \Pi(t,s)$$

$$= q(t+s,t) - \sum_{i=1}^{m} p_i(t+s,t)\, c_i(p_1^*(t,s),\ldots,p_m^*(t,s),p_J(t),t) \,, \tag{2.10}$$

which represents the current quasirent per unit of output on the equipment installed in year t and planned to be scrapped in year t+s, in the last year of its service life. The terminal quasirent function is an ex ante concept, and a change in s will result in a change in technique. This contrasts with the usual quasirent function, which is an ex post construct

struct and takes the technique as given.

This two stage argument thus permits us to start with the life cycle cost function - with life cycle prices as arguments - as a description of the ex ante technology, and then appeal to duality theory to ensure the existence of the primal production function.[4)] This is in fact the route we will follow in the empirical part of this paper.

The second stage of the optimization problem reduces to solving

$$\Pi(t) = \max_{s} \Pi(t,s) , \qquad (2.11)$$

and in the process the life cycle prices become endogenous variables. Note that both $\Pi(t,s)$ and $\Pi(t)$ are functionals, being functions of the expected price paths. The first order conditions for this problem can be written as

$$R(t,S) = 0 ,$$

which implicitly defines the scrapping age, S, as the maximizing value of the service life s. Using (2.10), this condition may also be written

$$q(t+S,t) = \sum_{i=1}^{m} p_i(t+S,t)\; c_i(p_1^*(t,S),\dots,p_m^*(t,S),p_j(t),t) , \qquad (2.12)$$

in which S is the single unknown variable. Whether this equation has a solution or not will depend on the current prices and their expected growth paths. It represents the scrapping condition, which states that vintage t will be planned to be taken out of operation when its expected average cost of the variable inputs equals the expected output price.

Substituting (2.5) into the terminal quasirent function (2.10) and differentiating with respect to s gives

$$R_s(t,s) = \frac{\partial}{\partial s} R(t,s) \qquad (2.13)$$

$$= \frac{\partial q(t+s)}{\partial s} - \sum_{i=1}^{m} \frac{\partial p_i(t+s)}{\partial s}\, c_i - e^{-rs}\, B(s) \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij}\, p_i(t+s)\, p_j(t+s).$$

The quadratic form in this expression will be non-positive due to the concavity of c, and it measures the curvature of the factor price frontier in the direction of the price change vector $(\partial p_1^*/\partial s,\dots,\partial p_m^*/\partial s)$ induced by a change in the expected service life. Suitably normalized, it may be interpreted as a directional shadow elasticity of substitution, and (2.13) shows that R will fall more slowly as a function of the anticipated service life the greater is this substitution effect.[5] This implies that a change in the service life will have a smaller impact on the profitability of investment as it is easier to adjust the technology to the changing prices. Define

$$c_{is} = \sum_{j=1}^{m} c_{ij} \frac{\partial p_j^*(t,s)}{\partial s} = e^{-rs} B(s) \sum_{j=1}^{m} c_{ij} p_j(t+s) , \qquad i=1,\ldots m,K, \tag{2.14}$$

which measures the effect on the i'th input coefficient of a lengthening of the anticipated service life. Few general conclusions can be stated about the sign of c_{is}, $i=1,\ldots,m$. It will be negative if all inputs are substitutes ($c_{ij} > 0$ for $i,j = 1,\ldots m,K$, $j \neq i$) and $p_i^*(t)$ and $p_i(t+s)$ are roughly proportional. (This follows from the linear homogeneity of the cost function and the fact that $c_{ii} < 0$.) In this case, a lengthening of the service life will lead to the use of a technique which is less intensive in the use of the variable inputs. On the other hand, if the i'th input is complementary to capital ($c_{iK} < 0$), then there will be a tendency, depending on the behaviour of the prices, to use more of that input as the service life is increased. The c_{Ks} term will be positive if all inputs are substitutes to capital, but it could be negative for some price configurations if some input is complementary to capital.

Whether (2.12) in fact gives a maximum must be checked by computing the second derivative of Π, which is given by [see (2.10) and (2.13)]

$$\Pi_{ss}(t,s) = -\left[r - \frac{B'(s)}{B(s)}\right] e^{-rs} B(s)\, y\, R(t,s) + e^{-rs} B(s)\, y\, R_s(t,s) . \tag{2.15}$$

At a critical point, $R(t,s) = 0$: thus we have a maximum at $s = S$ if $\Pi_{ss}(t,S) < 0$, or equivalently if $R_s(t,S) < 0$, i.e. $R_s(t,S) < 0$ is a necessary condition for profit maximization at $s = S$.

3. SCRAPPING DECISIONS AND CHOICE OF TECHNIQUE

In this section, we discuss the effect of changes in prices and price expectations on the scrapping plans and on the choice of technique, and present and interpret a decomposition of these changes. For this purpose, it is necessary to parametrize the retirement function and the price expectation functions, which thus far have been unspecified. Assume now that the retirement of capital follows the exponential function

$$B(\tau) = e^{-\delta\tau}, \qquad \delta > 0, \tag{3.1}$$

and that the output and input prices are expected to grow from time t at the rates $\pi_q = \pi_q(t)$ and $\pi_i = \pi_i(t)$, i=1,...,m, respectively,

$$\begin{aligned} q(t+\tau,t) &= e^{\pi_q\tau}\, q(t), \\ p_i(t+\tau,t) &= e^{\pi_i\tau}\, p_i(t), \qquad i=1,\dots,m, \end{aligned} \tag{3.2}$$

where q(t) and $p_i(t)$ are the prices observed at time t. The life cycle output and input prices (2.5) become

$$\begin{aligned} q^*(t,s) &= q(t)\int_0^s e^{-(r+\delta-\pi_q)\tau}\, d\tau = \frac{q(t)}{r+\delta-\pi_q}\left[1 - e^{-(r+\delta-\pi_q)s}\right], \\ p_i^*(t,s) &= p_i(t)\int_0^s e^{-(r+\delta-\pi_i)\tau}\, d\tau = \frac{p_i(t)}{r+\delta-\pi_i}\left[1 - e^{-(r+\delta-\pi_i)s}\right]. \end{aligned} \tag{3.3}$$

The scrapping condition (2.12) can be written

$$e^{\pi_q S}\, q(t) = \sum_{i=1}^{m} e^{\pi_i S}\, p_i(t)\, c_i, \tag{3.4}$$

while the terminal quasirent function (2.10) takes the form

$$R(t,s) = e^{\pi_q s}\, q(t) - \sum_{i=1}^{m} e^{\pi_i s}\, p_i(t)\, c_i(p_1^*,\dots,p_m^*,p_J,t). \tag{3.5}$$

Its derivative with respect to s [see (2.13)], evaluated at s = S, becomes

$$R_s(t,S) = \pi_q q(t+S) - \sum_{i=1}^{m} \pi_i \, p_i(t+S) \, c_i$$

$$- e^{-(r+\delta)S} \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} \, p_i(t+S) \, p_j(t+S) \, . \qquad (3.6)$$

Equation (3.6) can be interpreted as the sum of a direct price and a substitution effect on the terminal quasirent due to a change in the service life. The term

$$\pi_q \, q(t+S) - \sum_{i=1}^{m} \pi_i \, p_i(t+S) \, c_i \qquad (3.7)$$

shows the change in the quasirent which would follow from a change in the service life if the technique were held fixed, while

$$- e^{-(r+\delta)S} \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} \, p_i(t+S) \, p_j(t+S) = - \sum_{i=1}^{m} c_{is} \, p_i(t+S) \, ,$$

reflects the change in technique induced by a lengthening of the service life. The fact that the latter is a negative semidefinite quadratic form implies that this substitution effect is always positive. We see that $R_s(t,S)$ will be zero if these two effects just balance each other. Whether or not such a situation can occur depends on the ex ante technology and on the values of q, π_q, p_i, π_i, p_J, and r. The negative semi-definiteness of the quadratic form and the necessary condition for profit maximization at S, $R_s(t,S) < 0$, imply that the direct price effect (3.7) must be negative.

Using the functional forms (3.1) and (3.2), solving the scrapping condition (3.4), and substituting for S in the profit function (2.9) and the factor demand equations (2.8) gives the solution to the output constrained profit maximization problem, determining profit, scrapping, and the choice of technique as functions of the level and rate of change of the output and variable input prices, the level of the investment price, the interest rate, and the level of the technology. Formally this can be written

$$\Pi(q,\pi_q,p_1,\dots,p_m,\pi_1,\dots,\pi_m,p_J,r,t) \, ,$$

$$S(q,\pi_q,p_1,\dots,p_m,\pi_1,\dots,\pi_m,p_J,r,t) \, , \qquad (3.8)$$

$$a_i(q,\pi_q,p_1,\dots,p_m,\pi_1,\dots,\pi_m,p_J,r,t) \, , \qquad i=1,\dots,m,$$

$$a_K(q,\pi_q,p_1,\ldots,p_m,\pi_1,\ldots,\pi_m,p_J,r,t) .$$

The functions S, $a_1,\ldots,a_m$, and a_K are homogeneous of degree zero in q, $p_1,\ldots,p_m$, and p_J, and their values are unaffected by equal changes in π_q, $\pi_1,\ldots,\pi_m$, and r.

It will in general be impossible to determine these functions explicitly. We will instead express their derivatives in terms of the derivatives of the cost function and the life cycle price functions. The change in the scrapping age brought about by a change in q, π_q, p_i, π_i, $i=1,\ldots,m$, p_J, r, and t, respectively, is found by totally differentiating the scrapping condition (3.4), and is given by [provided $R_s(t,S) \neq 0$]

$$\frac{dS}{dq} = - \frac{q(t+S)}{q(t)\; R_s(t,S)} ,$$

$$\frac{dS}{d\pi_q} = - \frac{S\; q(t+S)}{R_s(t,S)} ,$$

$$\frac{dS}{dp_i} = \frac{1}{p_i(t)\; R_s(t,S)} \left[p_i(t+S)\; c_i + e^{(r+\delta)S}\; c_{is}\; p_i^* \right] , \qquad i=1,\ldots,m,$$

(3.9)

$$\frac{dS}{d\pi_i} = \frac{1}{R_s(t,S)} \left[S\; p_i(t+S)\; c_i + e^{(r+\delta)S}\; c_{is}\; \frac{\partial p_i^*}{\partial \pi_i} \right] , \qquad i=1,\ldots,m,$$

$$\frac{dS}{dp_J} = \frac{1}{R_s(t,S)}\, e^{(r+\delta)S}\; c_{Ks} ,$$

$$\frac{dS}{dr} = \frac{e^{(r+\delta)S}}{R_s(t,S)} \sum_{i=1}^{m} c_{is}\, \frac{\partial p_i^*}{\partial r} ,$$

$$\frac{dS}{dt} = \frac{1}{R_s(t,S)} \sum_{i=1}^{m} c_{it}\; p_i(t+S) ,$$

where c_{is} is given by (2.14), and represents the response of the technique to a change in the service life. Note the key role played by $R_s(t,S)$ in these expressions. If $R_s(t,S)$ is large, i.e. if a change in the service life has a large effect on the terminal quasirent, then changes in the prices and the interest rate will have a small effect on the scrapping age.

Differentiating with respect to r is equivalent to differentiating with respect to δ, i.e. a change in the rate of interest and a change in the rate of deterioration have the same effect on the scrapping age. Further we see that an equal change in all expected growth rates of prices $\pi_q, \pi_1, \ldots, \pi_m$, will also have the same effect as a change in r, but with opposite sign. The derivative with respect to r may therefore, with reversed sign, be taken as a measure of the effect of a change in the inflationary expectations. The derivative with respect to t reflects the effect of the technical change only, i.e. the effect of using the technology which will prevail in year t+dt, rather than at t.

The signs of the derivatives (3.9) are in most cases ambiguous. Only the effect of an increase in the output price or in its growth rate are predictable: both will lead to an increase in the scrapping age. An increase in an input price or its growth rate will tend to decrease the scrapping age if the substitution possibilities are small. The derivatives dS/dp_i and $dS/d\pi_i$ will always be negative if $c_{is} > 0$. The effect of an increase in the price of investment goods is unambiguously negative if all variable inputs are substitutes to the capital good. Only in extreme cases of complementarity and price changes would it seem possible for an increase in the investment price to lead to a lengthening of the scrapping age. The effect of technical progress depends on its specific pattern, but if it is Hicks neutral, which implies that $c_{it} < 0$, then technical change will always lead to a lengthening of the planned scrapping age, since it reduces production costs when the output price remains unaffected.

Consider next the change in the input coefficients a_i, $i=1,\ldots,m,K$, which follows from a change in the output price and its rate of increase,

$$\frac{da_i}{dq} = c_{is}\frac{dS}{dq}, \qquad i=1,\ldots,m,K,$$

$$\frac{da_i}{d\pi_q} = c_{is}\frac{dS}{d\pi_q}, \qquad i=1,\ldots,m,K. \tag{3.10}$$

Observe that the effect on the technique is due entirely to the induced change in the scrapping age.

The effect of a change in an input price and its expected growth rate is given by

$$\frac{da_i}{dp_k} = c_{ik}\frac{p_k^*}{p_k} + c_{is}\frac{dS}{dp_k}, \qquad i=1,\ldots,m,K, \quad k=1,\ldots,m,$$

$$\frac{da_i}{d\pi_k} = c_{ik}\frac{\partial p_k^*}{\partial \pi_k} + c_{is}\frac{dS}{d\pi_k}, \qquad i=1,\ldots,m,K, \quad k=1,\ldots,m. \tag{3.11}$$

The first term of these expressions represents the direct substitution effect of an increase in the price p_k, or its growth rate π_k, on the input coefficient a_i, with the the scrapping age held constant. The second term represents the indirect effects which come from the induced change in the scrapping age. The effects of a change in the investment price, the interest rate, and the technology are given by

$$\frac{da_i}{dp_J} = c_{iK} + c_{is}\frac{dS}{dp_J}, \qquad i=1,\ldots,m,K,$$

$$\frac{da_i}{dr} = \sum_{j=1}^{m} c_{ij}\frac{\partial p_j^*}{\partial r} + c_{is}\frac{dS}{dr}, \qquad i=1,\ldots,m,K, \tag{3.12}$$

$$\frac{da_i}{dt} = c_{it} + c_{is}\frac{dS}{dt}, \qquad i=1,\ldots,m,K.$$

All sets of derivatives (3.11) and (3.12) have two components: a direct substitution effect brought about by the price change, the interest change, or technical change with the scrapping age kept constant, and an indirect effect brought about by the induced effect on the scrapping age. These direct effects are all, except for the own derivatives c_{ii}, uncertain as to sign, and even c_{it} can have either sign as long as the the nature of the technical change is unspecified. The signs of the indirect effects are also indeterminate since c_{is} may have either sign.

Most analyses of the choice of technique in putty-clay models take the service life as exogenously given [cf. e.g. Fuss (1977,1978) and Berndt and Wood (1984,1985)]. These studies thus ignore the induced effects in (3.10) and in the last terms of (3.11) and (3.12). Particularly extreme is the neglect of the effect in (3.10), i.e. the claim that a change in the output price, or in its expected growth rate, will not affect the choice of technique.

The two stage optimization discussed in section 2 led naturally to a decomposition of the change in the input coefficients into two effects, one representing the effect when the service life is held constant, the other reflecting induced changes via changes in the service life. We now consider a similar decomposition of the effect on the service life, holding first the technique constant and then allowing it to respond to changes in the exogenous variables. Since the derivative of the terminal quasirent function (3.6) when no substitution is possible (i.e. when all $c_{ij} = 0$), reduces to

$$\bar{R}_s = \bar{R}_s(t,S) = \pi_q\, q(t+S) - \sum_{i=1}^{m} \pi_i\, p_i(t+S)\, a_i , \tag{3.13}$$

the expressions for the change in the scrapping age (3.9), can, by using (3.10)-(3.13), be decomposed as follows

$$\frac{dS}{dq} = - \frac{q(t+S)}{q(t)\,\bar{R}_s} + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{dq} ,$$

$$\frac{dS}{d\pi_q} = - \frac{S\, q(t+S)}{\bar{R}_s} + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{d\pi_q} ,$$

$$\frac{dS}{dp_i} = \frac{p_i(t+S)\, a_i}{p_i(t)\,\bar{R}_s} + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{dp_i} , \qquad i=1,\dots,m, \tag{3.14}$$

$$\frac{dS}{d\pi_i} = \frac{S\, p_i(t+S)\, a_i}{\bar{R}_s} + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{d\pi_i} , \qquad i=1,\dots,m,$$

The first terms in (3.14) give the effect of output and input price changes on the scrapping age if the technique were kept constant. These terms, which are positive for the output price and negative for the input prices, represent the effects which would be realized under a fixed coefficient ex ante technology (clay-clay), i.e. under such a technology an increase in an input price, or in its growth rate, would always lead to a reduction in the scrapping age. The second term in (3.14) represents the additional effect of allowing the technique to respond to the price changes. Further

$$\frac{dS}{dp_J} = 0 + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{dp_J} ,$$

$$\frac{dS}{dr} = 0 + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{dr} , \tag{3.15}$$

$$\frac{dS}{dt} = 0 + \frac{1}{\bar{R}_s} \sum_{j=1}^{m} p_j(t+S) \frac{da_j}{dt} .$$

An increase in the investment price, in the rate of interest, or in the technology has no effect on the scrapping age when the technique is held constant: this is indicated by the zeros in (3.15). The second term again represents the additional effect of allowing the technique to respond to price changes, and this effect may be of either sign.

In section 5, we numerically illustrate these decompositions.

4. DATA AND ECONOMETRIC SPECIFICATION

The data are taken from the Norwegian annual national accounts. For convenience, we use data for an aggregate sector, total manufacturing, although the putty-clay concept is commonly interpreted as genuinely microeconomic, essentially related to the individual firm, or even to a single production plant within a firm (confer Johansen (1972, section 2.2)). Our empirical calculations below then serve to illustrate the effect of input and output price changes on the scrapping behaviour of a "typical" Norwegian manufacturing firm.

In empirical work, the putty-clay framework seems to be used more frequently for energy-intensive manufacturing sectors than for others [see for instance Førsund and Hjalmarsson (1983)]. To some extent, this may be due to the fact that such sectors often have a relatively simple input and output structure, which makes appropriate data rather easily accessible. A priori, however, the assumption of ex post fixity of factor proportions may be just as valid for labour as it is for energy, and the majority of manufacturing sectors in Norway are relatively labour intensive. Of course, energy price shocks (for instance the two OPEC induced shocks in the 1970's) will have their largest impact on energy intensive sectors, whereas labour intensive sectors will be most strongly affected by labour price shocks (an example may be the wage inflation in Norway at the mid-1970's). It is the total effect of such factor price changes - whether they come through the energy price, the labour price, or the price of other materials, or more likely, a combination - that we intend to illustrate. Our parametrization of the technology implies that all factors are subject to a putty-clay technology. We do not, as Biørn (1986), allow for the possibility that it may be neoclassical for some factors, clay-clay (i.e. fixed proportions both ex ante and ex post) for others, and putty-clay for the rest (although the clay-clay specification is a special case of the chosen functional form).

We use a technology with four inputs (i.e. m = 3): materials (M), energy (E), labour (L), and capital (K). A description of the basic data is given in appendix A. As is evident from the above description of the putty-clay model, it places heavy demands on data. Its emphasis on vintage specific output and inputs, and expectations about future prices is, in particular, problematic for the econometric implementation, since neither of these variables can easily be observed - if they are observable at all. In this section, we describe the procedure chosen for constructing the expected price paths for the output and the variable inputs (section 4.1), and present the functional form used for the ex ante technology and the estimates of its parameters (section 4.2).

4.1. The price expectation process

Let p_t denote an arbitrary input price or the output price in year t and define

$\pi_t = p_t/p_{t-1} - 1$ = rate of price increase observed in year t, annual rate,

π_t^* = rate of price increase for the future time period as expected in year t, annual rate.

We assume that the producers form their price expectations by smoothing the observed rates of price increase by means of the process

$$\pi_t^* = (1 - \gamma)\, \pi_{t-1}^* + \gamma\, \{\mu\, \pi_t + (1 - \mu)\, \pi_{t-1}\}\ , \qquad (4.1)$$

where γ and μ are constants between zero and one. Formally, (4.1) defines π_t^* as an ARMA (1,1) process on π_t. The lower the values of γ and μ, the more sluggish is the response of the expected future rate of price increase formed in year t to the rate of price increase actually observed in this year. Since the lag polynomial connecting π_t and π_t^* is given by

$$\pi_t^* = \frac{\gamma\ (\mu + (1-\mu)\ L)}{1 - (1-\gamma)\ L}\ \pi_t = B(L)\ \pi_t\ , \qquad (4.2)$$

L being the lag operator, with coefficients adding to unity (i.e. B(1) = 1), the mean lag between the actual and expected rate of price increase can be expressed as [see e.g. Dhrymes (1971, p. 8)]

$$m = B'(1) = \frac{1 - \gamma\,\mu}{\gamma}\ , \qquad \gamma \neq 0\ . \qquad (4.3)$$

This smoothing may be one (although imperfect) way of taking account of uncertainty with respect to future prices.

This ARMA specification includes as special cases:

$\gamma = 0$:	Static price expectations, i.e. no revision of future rates of increase.
$\mu = 1,\ 0 < \gamma < 1$:	Adaptive expectations based on current rate of increase.
$\mu = 0,\ 0 < \gamma < 1$:	Adaptive expectations based on rate of increase in previous year.
$\gamma = 1,\ 0 < \mu < 1$:	Moving average of rates of increase in current and previous year.
$\gamma = 1,\ \mu = 0$:	Instantaneous adjustment of price expectations to rate of increase in previous year.
$\gamma = \mu = 1$:	Instantaneous adjustment of price expectations.

The expected rate of price increase and the interest rate are converted to rates expressed in continuous time by means of the transformations $\pi(t) = \log(1 + \pi_t^*)$ and $r(t) = \log(1 + r_t)$. Note that we assume that expectations about the interest rates are adjusted instantaneously.

Since no data on price expectations are available, by means of which we could estimate γ and/or μ econometrically, values must be assigned to these parameters a priori. We have selected the following four alternative processes:

$$\begin{aligned} &\gamma = 0.1,\ \mu = 1.0, \quad \text{i.e. } m = 9, \\ &\gamma = 0.2,\ \mu = 1.0, \quad \text{i.e. } m = 4, \\ &\gamma = 0.5,\ \mu = 0.5, \quad \text{i.e. } m = 1.5, \\ &\gamma = 1.0,\ \mu = 1.0, \quad \text{i.e. } m = 0, \end{aligned} \tag{4.4}$$

of which we regard the second as our base specification. This adaptive expectation process is, to some extent, in line with the results of an econometric investigation of genuine expectations data from British manufacturing industries [see Pesaran (1985, table 2 A)]. His results are, however, sensitive to his choice of model specification and measure of price expectations, and there are indications that the lag between the actual and expected inflation rates, as perceived by the producers, may be shorter (or the expectation process more complicated) than our base specification implies.

4.2. Ex ante technology: Generalized Leontief

We assume that the ex ante technology is linear homogeneous and can be represented by the Generalized Leontief (GL) unit cost function [see Diewert (1971)]

$$c(p,t) = e^{-\varepsilon t}\left[\sum_{i=1}^{m}\sum_{j=1}^{m} b_{ij}\,(p_i^* \, p_j^*)^{1/2} + 2\sum_{j=1}^{m} b_{jK}\,(p_j^* \, p_J)^{1/2} + b_{KK}\, p_J\right], \tag{4.5}$$

where $B = [b_{ij}]$ is a matrix of coefficients. Technical change is assumed to be Hicks neutral at the constant rate ε.

To this parametrization of the technology correspond the input coefficient equations [see (2.8)]

$$a_i(t) = e^{-\varepsilon t}\left[\sum_{j=1}^{m} b_{ij}\left[\frac{p_j^*}{p_i^*}\right]^{1/2} + b_{iK}\left[\frac{p_J}{p_i^*}\right]^{1/2}\right], \quad i=1,\dots,m,$$

$$a_K(t) = e^{-\varepsilon t}\left[\sum_{j=1}^{m} b_{Kj}\left[\frac{p_j^*}{p_J}\right]^{1/2} + b_{KK}\right]. \tag{4.6}$$

Substituting (4.6) into the scrapping condition (3.4) gives

$$q(t)\, e^{\pi_q S} = e^{-\varepsilon t}\sum_{i=1}^{m} p_i(t)\, e^{\pi_i S}\left[\sum_{j=1}^{m} b_{ij}\left[\frac{p_j^*(t)}{p_i^*(t)}\right]^{1/2} + b_{iK}\left[\frac{p_J(t)}{p_i^*(t)}\right]^{1/2}\right]. \tag{4.7}$$

This equation together with (3.3) gives a set of m+1 equations in the unknown variables $S, p_1^*, \dots, p_m^*$. In actually determining and understanding the solution, we found the terminal quasirent function (3.5) very useful.

Our numerical examples are based on estimates of the b_{ij} coefficients from a neo-classical model with a homothetic technology with Hicks neutral technical change, derived from national accounts data, i.e. data aggregated across vintages, for the years 1962 - 1981 [see Bye and Frenger (1985)]. The associated estimate for the technical change parameter was $\varepsilon = 0.0112$. The estimated cost function was concave for all years in the sample period. Table 4.1 presents the 1981 values of the shadow elasticities of substitution (SES) and the Hicks-Samuelson elasticities of substitution (HSES),[1)] computed on the basis of the estimated parameters.

TABLE 4.1. Shadow (SES) and Hicks-Samuelson (HSES) elasticities of substitution implied by estimated GL technology

	SES			HSES
	E	L	K	
M	0.3600	0.9036	0.5474	0.7297
E		0.4205	0.2789	0.3463
L			0.7580	0.9494
K				0.5733

We then assume that our ex ante model has the same second order properties, as described by the shadow elasticities of substitution, in 1980 as the model estimated in Bye and Frenger (1985). We determine the GL function which implies these shadow elasticities, given the life cycle prices and the input coefficient ratios in the base year. The coefficients of this function are presented in table 4.2. The negative value of b_{EK} shows that there is a complementary relationship between energy and capital.

TABLE 4.2. GL coefficients b_{ij} estimated from national account data, neo-classical specification

	M	E	L	K
M	0.2908	0.0269	0.2416	0.1305
E		0.0118	0.0081	-0.0304
L			-0.0893	0.1455
K				-0.0119

Since data on vintages are unavailable, a problem arises in the determination of the level of the cost structure of new investment. We have decided to impose an exogenously given profit rate by writing

$$q^*(t_0) = \beta\ c(p_1^*(t_0),\dots,p_m^*(t_0),p_J(t_0),t_0)\ , \tag{4.8}$$

where t_0 denotes a given base year. Then the life cycle profit for the capital invested in this year will be $(\beta-1)$ times its life cycle cost. In particular, we assume that the capital invested in 1980 - which is the base year of our national accounts data - is expected to earn a zero profit over its anticipated service life, i.e. $t_0 = 1980$ and $\beta = 1$, which implies that the life cycle output value of the "base vintage" (1980) exactly exhausts its life cycle cost (inclusive of the investment cost). The role of this assumption is that it permits us to determine the "efficiency parameter" of the ex ante cost function, and thus the scaling of the input coefficients. For the other vintages, a non-zero profit (positive or negative) will, of course, normally occur.

It is reasonable to believe that the above estimates, based on average data, seriously underestimate the elasticities of substitution of the ex ante technology. To compensate for this, we therefore also consider specifications with higher values for these elasticities. Technically, these have been computed by magnifying all second order derivatives of the cost function at the base point by an exogenously given scaling factor α, while holding the first derivatives, i.e. the input coefficients, constant. Since the second order derivatives of the cost function (4.5) are given by

$$
\begin{aligned}
c_{ij} &= \frac{1}{2}\,\frac{b_{ij}}{(p_i^*p_j^*)^{1/2}}\ , && i,j=1,\dots,m,K,\ i\neq j,\\
c_{ii} &= -\frac{1}{2\,p_i^*}\left[\sum_{\substack{j=1\\ j\neq i}}^{m} b_{ij}\left[\frac{p_j^*}{p_i^*}\right]^{\frac{1}{2}} + b_{iK}\left[\frac{p_J}{p_i^*}\right]^{\frac{1}{2}}\right]\ , && i=1,\dots,m,\\
c_{KK} &= -\frac{1}{2\,p_J}\sum_{j=1}^{m} b_{Kj}\left[\frac{p_j^*}{p_J}\right]^{\frac{1}{2}}\ ,
\end{aligned}
\tag{4.9}
$$

rescaling these derivatives is equivalent to rescaling the off-diagonal coefficients by the factor α

$$b_{ij}(\alpha) = \alpha\, b_{ij}, \qquad i,j=1,\ldots,m,K, \quad j\neq i, \qquad (4.10)$$

and then defining the diagonal coefficients residually,

$$b_{ii}(\alpha) = a_i(t_0) - \alpha \sum_{\substack{j=1 \\ j\neq i}}^{m} b_{ij}\left[\frac{p_j^*(t_0)}{p_i^*(t_0)}\right]^{\frac{1}{2}} - \alpha\, b_{iK}\left[\frac{p_J(t_0)}{p_i^*(t_0)}\right]^{\frac{1}{2}}, \qquad i=1,\ldots,m,K. \qquad (4.11)$$

This insures that the input coefficients remain unchanged. Since the definitions of the elasticities of substitution are linear in the second derivatives, this will change these elasticities by the same factor α.

An exogenous deterioration of production capacity at a (continuous) rate of 10 per cent ($\delta = 0.10$) is assumed over the capital's life cycle. Since only the sum of the retirement rate δ and the interest rate r occurs in the model [cf. (3.3) and (3.6)], a non-zero value of δ may be interpreted as including a risk premium claimed by the firm (or its owners) for undertaking an uncertain investment project. Or more precisely, $r + \delta$ can be reinterpreted as the sum of the market interest rate (on approximate risk-free assets), the rate of retirement (decline in efficiency), and the risk premium. The value $\delta = 0.10$ can thus, for instance, represent a retirement rate of 6 per cent and a risk premium of 4 per cent. From this it follows that the actual ex ante life cycle profit for the base vintage may be positive even if the profit based on the interest rate inclusive of risk premium is restricted to zero.

5. EMPIRICAL ILLUSTRATIONS

In this section, we attempt, by combining the theoretical results in sections 2 and 3 with the parametric ex ante technology and the data described in section 4, to assess empirically the effect of the price changes on the scrapping plans and choice of technique in the period 1964-1983.

5.1. Properties of base year technology

Let us first consider the behaviour of the model in the base year 1980, when the scaling factor α is unity, using the base specification of the price expectations. The current prices and their expected rate of change are reported in the first two columns of table 5.1, the life cycle prices are presented in the third column, while the prices expected to prevail at the time of scrapping are given in the last column. The anticipated scrapping age of new equipment installed in 1980 was 14.6 years.

Table 5.1. Basic price data; 1980.
Base alternative for price expectations.
$\alpha = 1$, $\delta = 0.10$, $r = 0.1071$, $S = 14.65$

	Initial price $q(t), p_i(t)$	Rate of increase $\pi_q(t), \pi_i(t)$	Life cycle price $q^*(t), p_i^*(t)$	Terminal price $q(t+S), p_i(t+S)$
y	1.0000	0.0825	6.7326	3.3472
M	1.0000	0.0840	6.7852	3.4211
E	1.0000	0.1407	9.3683	7.8534
L	1.0000	0.1059	7.6407	4.7199
K	1.0000	-	1.0000	-

The base year input coefficients are given in the first column of table 5.2. The second column presents the elasticity of the technology with respect to the service life. An increase in the scrapping age S will, as remarked in section 3, change the life cycle prices, which induces a change in the optimal technique. The directional shadow elasticity of substitution in the direction of the induced change is 0.52. This brings about a substantial substitution of capital for labour, while the input coefficients for materials and energy change relatively little. Note, however, the signs of the elasticities for materials and energy: we get larger use of materials and a smaller use of energy, despite the complementarity between energy and capital, and despite the tendency to use less of the variable factors as S increases, because of the substantially lower growth rate expected for the price of materials than for the prices of energy and labour.

Table 5.2. Input coefficients and technique elasticities
Base alternative for price expectations. α = 1.

	Input coef. a_i	Technique elast.[a)]
M	0.6289	0.0081
E	0.0321	-0.0103
L	0.2000	-0.0995
K	0.6371	0.1893

a) Elasticity of input coefficients with respect to service life. Cf. (2.14).

Table 5.3 presents, for α = 1, 3, and 5, the basic elasticities of the principal endogenous variables of the model, S, a_M, a_E, a_L, and a_K, with respect to the base year prices, their expected rate of growth, the rate of interest, and technical change. Let us concentrate on the results for α = 1. The consequences of a 1% increase in the base year output price, its expected growth rate remaining unchanged, can be read off the top line of the table. It will increase the expected scrapping age of new capital by 6.6%, i.e. by almost one year. It will lead to a greater use of materials, the input coefficient a_M increasing by 0.05%, while energy and labour decrease by 0.07% and 0.66%, respectively. And the new technique will be more capital intensive, a_K increasing by 1.25%. And the increased use of materials is a consequence of the lower growth rate of the price of this input.

An increase in the investment price by 1% will reduce the scrapping age by 0.33%, and lead to a technique which is more intensive in the use of materials and labour, and less intensive in the use of capital and its complementary input, energy.

The planned service life is rather sensitive to changes in the expected growth rate of prices. An increase in the growth rate of the output price by one percentage point will lead to almost a doubling of the scrapping age (97% increase). The sign structure of the effects of a change in the growth rate of the output price on the choice of technique is the same as the effect we observe for a change in the level of the output price. The sign structure is also the same for all input price changes and the change in their growth rates, with one exception: An increase in the level of the base year labour price will lead to a more capital intensive technique, while an increase in its growth rate will reduce the capital intensity.

An increase in the rate of interest (or equivalently a uniform decrease in the expected rate of inflation) by one percentage point will lead to a reduction in the scrapping age by about 2%, and to the use of a 3% less capital intensive technique. This illustrates the non-neutral nature of the

Table 5.3. 1980 elasticities.[a] Base alternative for price expectations.

$\alpha = 1,\ \varepsilon = 0.01$

	S	aM	aE	aL	aK
pQ	6.6011	0.0534	-0.0683	-0.6565	1.2494
pM	-4.3394	-0.3039	0.4018	1.0008	-0.5546
pE	-0.4879	0.0212	-0.3111	0.0709	-0.1653
pL	-1.4367	0.1922	0.1289	-0.5803	0.0436
pJ	-0.3371	0.0371	-0.1513	0.1651	-0.5731
πQ	96.6980	0.7816	-1.0008	-9.6168	18.3017
πM	-62.6596	-1.9133	2.5166	9.2110	-10.4632
πE	-7.2211	0.0963	-1.8716	0.8561	-1.8158
πL	-24.8986	0.9360	0.8936	-1.5586	-2.9520
r	-1.9187	0.0995	-0.5379	1.1083	-3.0706
t	0.0660	-0.0095	-0.0107	-0.0166	0.0025

$\alpha = 3,\ \varepsilon = 0.01$

	S	aM	aE	aL	aK
pQ	9.5935	0.2326	-0.2978	-2.8623	5.4471
pM	-6.5868	-0.9661	1.2753	3.6729	-2.9397
pE	-0.6838	0.0588	-0.9273	0.2713	-0.6072
pL	-0.8533	0.5908	0.3685	-1.9151	0.4622
pJ	-1.4696	0.0838	-0.4186	0.8332	-2.3624
πQ	140.5330	3.4076	-4.3631	-41.9286	79.7939
πM	-92.5307	-6.4641	8.4770	36.5452	-48.3503
πE	-10.3392	0.2133	-5.5178	3.4986	-7.2179
πL	-29.2976	2.7012	2.8174	-3.3634	-11.3537
r	-8.3653	0.1420	-1.4135	5.2483	-12.8722
t	0.0959	-0.0077	-0.0130	-0.0386	0.0445

$\alpha = 5,\ \varepsilon = 0.01$

	S	aM	aE	aL	aK
pQ	17.5494	0.7092	-0.9080	-8.7266	16.6075
pM	-12.5618	-1.8516	2.4345	9.0927	-10.5539
pE	-1.2048	0.0769	-1.5185	0.7111	-1.5049
pL	0.6976	1.0474	0.5338	-3.9630	2.2379
pJ	-4.4805	0.0181	-0.5419	2.8858	-6.7867
πQ	257.0780	10.3891	-13.3018	-127.8340	243.2790
πM	-171.9510	-13.9831	18.2374	100.4010	-155.7410
πE	-18.6296	0.0205	-8.7673	9.9534	-19.8752
πL	-40.9940	4.0293	5.3007	0.2104	-29.9912
r	-25.5046	-0.4559	-1.4690	17.2698	-37.6729
t	0.1755	-0.0029	-0.0191	-0.0973	0.1561

a) Elasticities for pQ, pM, pE, pL, and pJ, derivatives of logarithms for πQ, πM, πE, πL, r, and t.

inflationary expectations in the model. (Equal changes in the interest rate and the inflation rate - as is predicted by the simple version of "Fisher's law" - will, however, have no effect on the ex ante decisions.) Assuming a 1% rate of Hicks neutral technical change results in a lengthening of the scrapping age by 0.07%. This induces a non-neutral and capital using effect on the chosen technique: the demand for capital will actually increase, while its complementary input, energy, will decrease by less than 1%.

Let us now apply the decomposition presented in section 3 to get a better understanding of the changes in the input structure and the scrapping implied by the model. Table 5.4 presents a decomposition of the change in technique, based on (3.10) - (3.12). Column A shows the effect which would obtain if the scrapping age were held constant, column B gives the adjustments induced by the change in the scrapping age. The total effect (which, of course, coincides with the result in table 5.3) is given in the third column. The sign pattern of the price effects in column A (the primary effects) are the same as those that would be observed in a corresponding neoclassical model: all own price effects are negative, while the cross price effects are positive for substitutes and negative for complements (i.e. energy and capital). A change in the output price has no effect on the technique since it does not affect the relative life cycle input prices when the service life is held constant [cf. (3.10)]. Increasing the rate of interest will lead to a less capital intensive technique, while technical change will affect all input coefficients proportionately. The sign pattern in column B (the secondary effects) depends on the sign of the technique elasticities [see table 5.2] and on the dS/dp_i and $dS/d\pi_i$ terms [see (3.10)-(3.12)]. An increase in the output price, or in its rate of growth, leads to a more material and capital intensive technique and a lower energy and labour intensity. This is due to the secondary effect via the scrapping age, the primary effect being zero, and the sign of the effect is determined by the technique elasticities of table 5.2.

Table 5.5 presents a similar decomposition of the effect on the scrapping age [see (3.14) and (3.15)]. In column A, the technique (i.e. the input coefficients) is held constant, column B gives the changes in the scrapping age which are induced by changes in technique, and the last column shows the total effect. Column A thus gives the effect which would have been obtained if the technology had been Leontief (clay-clay), with coefficients equal to those observed in the base year, and shows that in this case increasing the output price will increase the scrapping age, while increases in the input prices will reduce the scrapping age. The secondary effects reported in table 5.5 reinforce the primary effects with one exception: the change in technique induced by an increase in the wage

Table 5.4. 1980 elasticities[a)] of input coefficients. Decomposition. Base alternative for price expectations. $\alpha = 1$, $\varepsilon = 0.01$

A. Elasticity with no change in service life
B. Correction due to change in service life

Elasticity of	A	B	Total
a_M w.r.t.			
pQ	0.0000	0.0534	0.0534
pM	-0.2688	-0.0351	-0.3039
pE	0.0251	-0.0039	0.0212
pL	0.2038	-0.0116	0.1922
pJ	0.0398	-0.0027	0.0371
πQ	0.0000	0.7816	0.7816
πM	-1.4068	-0.5064	-1.9133
πE	0.1547	-0.0584	0.0963
πL	1.1372	-0.2012	0.9360
r	0.1150	-0.0155	0.0995
t	-0.0100	0.0005	-0.0095
a_E w.r.t.			
pQ	0.0000	-0.0683	-0.0683
pM	0.3569	0.0449	0.4018
pE	-0.3162	0.0050	-0.3111
pL	0.1140	0.0149	0.1289
pJ	-0.1547	0.0035	-0.1513
πQ	0.0000	-1.0008	-1.0008
πM	1.8681	0.6485	2.5166
πE	-1.9463	0.0747	-1.8716
πL	0.6359	0.2577	0.8936
r	-0.5577	0.0199	-0.5379
t	-0.0100	-0.0007	-0.0107
a_L w.r.t.			
pQ	0.0000	-0.6565	-0.6565
pM	0.5692	0.4316	1.0008
pE	0.0224	0.0485	0.0709
pL	-0.7232	0.1429	-0.5803
pJ	0.1316	0.0335	0.1651
πQ	0.0000	-9.6168	-9.6168
πM	2.9794	6.2316	9.2110
πE	0.1379	0.7182	0.8561
πL	-4.0348	2.4762	-1.5586
r	0.9175	0.1908	1.1083
t	-0.0100	-0.0066	-0.0166
a_K w.r.t.			
pQ	0.0000	1.2494	1.2494
pM	0.2667	-0.8213	-0.5546
pE	-0.0730	-0.0923	-0.1653
pL	0.3156	-0.2719	0.0436
pJ	-0.5093	-0.0638	-0.5731
πQ	0.0000	18.3017	18.3017
πM	1.3961	-11.8594	-10.4632
πE	-0.4491	-1.3667	-1.8158
πL	1.7605	-4.7125	-2.9520
r	-2.7075	-0.3631	-3.0706
t	-0.0100	0.0125	0.0025

a) Elasticities for pQ, pM, pE, pL, and pJ, derivatives of logarithms for πQ, πM, πE, πL, r, and t.

rate leads to a positive secondary effect on the scrapping age.

Table 5.5. 1980 elasticities[a)] of service life. Decomposition. Base alternative for price expectations. $\varepsilon = 0.01$

A. Elasticity with no change in input coefficients
B. Correction due to change in input coefficients

Elasticity of S w.r.t	A	B	Total
pQ	5.7104	0.8907	6.6011
pM	-3.6705	-0.6689	-4.3394
pE	-0.4296	-0.0583	-0.4879
pL	-1.6103	0.1736	-1.4367
pJ	0.0000	-0.3371	-0.3371
πQ	83.6508	13.0473	96.6980
πM	-53.7686	-8.8910	-62.6596
πE	-6.2931	-0.9281	-7.2211
πL	-23.5891	-1.3095	-24.8986
r	0.0000	-1.9187	-1.9187
t	0.0000	0.0660	0.0660

a) Elasticities for pQ, pM, pE, pL, and pJ, derivatives of logarithms for πQ, πM, πE, πL, r, and t.

Consider, as an example, an increase in the expected rate of growth of the price of materials, π_M, by one percentage point. This will bring about a more labour intensive technique (3.0%) when holding S constant [table 5.4]. Allowing for the induced reduction in the scrapping age by 63%, or 9.2 years [table 5.5], which leads to increased labour demand [$c_{LS} < 0$] [table 5.2], we get a secondary effect on the labour coefficient of 6.2%, so that the total effect is an increase in the labour coefficient by 9.2% [table 5.4]. The secondary effect thus exceeds by far the primary effect in this case. We observe a different pattern when we consider the effect of the increase in π_M on the capital stock. Holding S constant leads to an increased demand for capital (primary effect equal to 1.4%), but when we allow for the induced reduction in S, this effect is reversed (secondary effect equal to -11.9%), and we end up with a more than 10 per cent less capital intensive technique. The end result is thus markedly different from what would have been predicted by a neoclassical model.

Let us now return to table 5.3 and consider what happens when we increase the ex ante elasticities of substitution by increasing the value of the scaling parameter α. Then the terminal quasirent function becomes flatter, i.e. $R_s(t,S)$ decreases in absolute value. This leads to a change in the elasticities, although they do not increase pari-passu with α. In fact, the sign of some of the effects for $\alpha = 5$ differs from that when $\alpha = 1$ or 3. A notable example occurs for the labour price when α increases from 3 to 5: the signs of dS/dp_L and $da_L/d\pi_L$ are both reversed, from negative to positive. The high degree of substitition possibilities makes it profitable to substitute the fixed factor for labour to such an extent that variable costs at scrapping actually fall. This leads to an increase in the scrapping age, and to the use of a technique which is much less intensive in the use labour, and substantially more intensive in the use of capital. The use of the other two inputs also increases. An increase in the growth rate of the price of labour will increase the variable costs at scrapping and lead to a reduction in the scrapping age. The secondary effect of a reduction in the scrapping age is sufficient to counter the neoclassical effect, and we end up with a more labour intensive, and much less capital intensive, technique.

In the base year, we obtain a critical value for $\bar{\alpha}$ = 7.41. The derivative of the terminal quasirent function $R_s(t,s)$ [see (2.13)] would become positive if α were larger than this critical value and our base point would cease to be a profit maximizing point. All values of α considered in table 5.3 are less than this critical value.

5.2. Simulations for the years 1964 -1983

Let us now consider the behaviour of the model when simulated over the entire observation period, 1964 - 1983. In table 5.6 are given simulated values of the ex ante service life, with the base specification of the price expectation process (γ = 0.2, μ = 1), for three alternative values of the scaling factor, α = 1, α = 2, and α = 3, and two values of the rate of technical change, ε = 0 and ε = 1%. For the base vintage, 1980, the service life, S = 14.6 years, is independent of the value of α. This follows from the way in which the GL cost function parameters have been constructed [cf. section 4.2]. The service life shows substantial cyclical variations. On the whole, the year-to-year changes tend to be larger, the larger is the scaling factor, i.e. the higher is the overall degree of ex ante substitution between the inputs. This is consistent with the pattern of the elasticities in table 5.3. The service life is quite sensitive to variations in the rate of technological change; the more efficient is a vintage, the longer is its profitable service period. A 1% rate of technical change from 1980 to 1983 will, for instance, increase the ex ante service life of the 1983 vintage from 14.2 years to 17.0 years as compared with a situation with no technical change. (Compare columns 1 and 4 of table 5.6, part A.) Part B of the table shows, not surprisingly, that the year-to-year fluctuations are smaller the smoother is the price expectation process.

Particularly interesting is the behaviour of the service life in the years 1973-1975 and 1978-1980. These periods included the two OPEC induced energy price shocks (1973/74, 1979/80), the sharp rise in the international raw material prices (1973/75), years with a substantial rise in the Norwe-

TABLE 5.6. Ex ante service life in years.

A. Variation with scaling factor α for γ = 0.2, μ = 1.0.

Vintage	Scaling factor/Rate of technical change			
	α=1, ε=0	α=2, ε=0	α=3, ε=0	α=1, ε=0.01
1964	22.50	19.58	17.30	11.89
1965	20.82	18.40	16.41	10.73
1966	18.37	16.15	14.18	8.89
1967	15.27	13.18	10.87	6.77
1968	15.30	13.59	11.28	7.80
1969	15.44	13.85	11.60	8.27
1970	17.82	16.68	15.19	10.73
1971	14.93	14.14	12.71	8.43
1972	16.74	18.30	24.20	11.38
1973	14.15	15.56	22.36	9.07
1974	11.83	11.12	9.79	6.90
1975	14.03	15.92	23.95	10.00
1976	11.37	12.68	*	7.97
1977	10.25	10.65	13.59	7.52
1978	10.00	9.62	*	8.10
1979	19.76	22.33	26.44	18.75
1980	14.65	14.65	14.65	14.65
1981	14.28	14.41	14.59	15.23
1982	14.11	13.94	13.68	16.08
1983	14.22	15.03	16.42	17.03

B. Variation with price expectation process for α = 1

Vintage	Parameters in ARMA process for price expectations			
	γ = 0.1 μ = 1.0 mean lag: 9 years	γ = 0.2 μ = 1.0 mean lag: 4 years	γ = 0.5 μ = 0.5 mean lag: 1.5 years	γ = 1.0 μ = 1.0 mean lag: 0 years
1964	21.47	22.50	21.97	50.08
1965	20.50	20.82	22.26	21.33
1966	18.92	18.37	16.79	17.03
1967	16.73	15.27	12.54	12.96
1968	16.37	15.30	12.12	20.41
1969	16.03	15.44	13.78	25.24
1970	17.02	17.82	18.00	49.86
1971	15.25	14.93	16.35	13.30
1972	16.00	16.74	15.89	27.57
1973	14.43	14.15	14.57	12.61
1974	13.20	11.83	8.05	6.89
1975	13.88	14.03	9.38	*
1976	12.01	11.37	10.43	12.36
1977	11.02	10.25	7.13	15.14
1978	10.38	10.00	7.00	22.51
1979	15.06	19.76	23.85	*
1980	13.45	14.65	22.33	8.74
1981	13.43	14.28	10.65	18.82
1982	13.07	14.11	10.99	25.81
1983	13.06	14.22	12.38	21.00

*) No (positive and finite) profit maximizing service life exists.

gian labour cost (1974/75), and years in which a wage and prize freeze was in effect in Norway (1978/79) (cf. tables A.1 and A.2 in the data appendix). From 1973 to 1974, the estimated ex ante service life is reduced from 12.6 to 6.9 years if the producers are assumed to react with no lag in their price expectations ($\gamma = \mu = 1$), even with the scaling factor α set as low as unity (last column of table 5.6, part B). It is reduced from 14.2 to 11.8 years when the more sluggish process with a mean lag of m = 4 years is assumed (i.e. the base alternative) and from 14.4 to 13.2 years when the process is even smoother (mean lag m = 9 years). In the latter two cases, the service life is reduced further, to about 10 years, over the following four years, after a temporary increase in 1975. With a higher degree of ex ante substitution, represented by the more realistic value $\alpha = 3$ (the "correct" value may be still higher), we find that the planned service life may drop by more than 50 per cent from 1973 to 1974 and increase again in the following year. [Compare the first and third column of table 5.6, part A.] The behaviour of the planned service life from 1973 to 1975 can also, to a large extent, be ascribed to the difference between the rates of increase of the output and material prices in these years. This follows from the fact that the elasticities of S with respect to $\dot{\pi}_Q$ and $\dot{\pi}_M$ are fairly large, even with $\alpha = 1$ [confer table 5.3]. A similar effect of the price changes occurred in the years 1978/1980.

The changes in technique which are induced by these price changes are reported in tables B.1 - B.4 in appendix B. From 1973 to 1974, assuming the base specification of price expectations, we find that the input coefficient of capital (a_K) declined from 0.630 to 0.618 for $\alpha = 1$, from 0.655 to 0.575 for $\alpha = 2$, and from 0.821 to 0.469, i.e. by more than 40 per cent, for $\alpha = 3$ (table B.4). At the same time, the material intensity (a_M) was somewhat reduced (table B.1), the energy coefficient (a_E) was substantially reduced - for $\alpha = 3$ for instance by more than 30 per cent (table B.2) - while the input coefficient of labour (a_L) was increased - for $\alpha = 3$ for instance by more than 180 per cent (from 0.098 to 0.278) (table B.3). During the next 1 - 2 years, these changes are, however, to some extent reversed. This conclusion is confirmed by the bottom parts of these tables, which show the sensitivity of the results to the assumed price expectation process. A similar change in the input structure occurred from 1978 to 1980, simultaneously with the change in the service life.

Changes in technology of this order of magnitude, which at a first glance may seem surprisingly large, are quite reasonable when we recall that they reflect the changes in the relative life cycle prices which are induced by the price changes through the changes in the ex ante service lives and real interest rates for a marginal vintage. Ex ante life cycle prices for the base specification of price expectations with $\alpha = 1$ are reported in table 5.7. Table 5.8 shows that dramatic changes in the relative life cycle prices occurred from 1973 to 1975 and from 1978 to 1980 (the life cycle capital price of a vintage being by assumption its investment price). In 1974, capital was, in comparison with labour, a substantially more expensive input in the ex ante life cycle sense than in 1973, in particular for $\alpha = 3$, and therefore the producers would have found it profitable to operate the former vintage with a more labour intensive technique than the latter. From 1973 to 1974, for instance, the energy/capital life cycle price ratio showed a substantially larger increase than the corresponding annual prices. This is the net effect of the dramatic rise in both the level and the rate of increase of the energy price, and the reduction in the service life. The year-to-year changes in technique and relative life cycle prices are not so dramatic when a more sluggish price response is assumed (confer the bottom part of tables B.1 - B.4), but in the perspective of 3 - 5 years it is still substantial.

Our assumption of a zero ex ante life cycle profit for the 1980 vintage is reflected in the simulation results reported above. A positive shift in the constant term of the ex ante production function (or equivalently, a negative shift in the constant term of the dual unit cost function) will, in general, lead to a positive shift in both the ex ante profitability and the ex ante service life of each vintage. Table B.5 in appendix B shows that the profitability of the investment, defined as the ratio between the maximal life cycle profit and the life cycle cost, tends to decline over time when no technical change is assumed but shows an increase when a one per cent rate of (embodied) technical change is assumed.

TABLE 5.7. Ex ante life cycle prices.[a)]
Base alternative for price expectations. $\alpha = 1$, $\varepsilon = 0$

Vintage	Materials	Energy	Labour	Capital
1964	2.5018	1.4043	1.6889	0.3912
1965	2.6415	1.3549	1.8536	0.4089
1966	2.6144	1.4618	1.9337	0.4234
1967	2.4271	1.6248	2.0675	0.4305
1968	2.3446	1.5030	2.2852	0.4339
1969	2.4335	1.5118	2.4379	0.4558
1970	2.9567	1.5371	2.9499	0.5069
1971	3.0323	1.9132	3.2636	0.5357
1972	3.0437	2.0043	3.8964	0.5621
1973	3.2648	2.0741	4.1010	0.5742
1974	4.3630	4.1739	4.3226	0.6632
1975	5.0377	4.9798	6.3085	0.7318
1976	4.7789	4.4535	6.0945	0.7952
1977	4.7974	4.7905	6.1051	0.8604
1978	4.5088	4.4810	5.9410	0.9207
1979	6.2021	7.0630	8.0697	0.9163
1980	6.7852	9.3683	7.6407	1.0000
1981	7.3148	11.2381	8.1602	1.0508
1982	7.3514	11.2815	8.3992	1.1385
1983	7.3202	12.5374	9.2799	1.1868

a) The life cycle capital price of vintage t is, by definition, the investment price in year t.

TABLE 5.8. Annual rate of increase of life cycle prices, per cent. $\varepsilon = 0$
A. Base specification of price expectations

	1973/74	1974/75	1978/79	1979/80
Materials, $\alpha = 1$	33.6	15.5	37.6	9.4
$\alpha = 2$	25.1	26.2	43.4	6.9
$\alpha = 3$	6.3	55.1	*	4.5
Energy, $\alpha = 1$	101.2	19.3	57.6	32.6
$\alpha = 2$	85.9	35.4	67.9	27.5
$\alpha = 3$	53.8	86.6	*	21.9
Labour, $\alpha = 1$	5.4	45.9	35.8	-5.3
$\alpha = 2$	-4.8	66.5	44.7	-8.9
$\alpha = 3$	-28.6	134.9	*	-12.7
Capital	15.5	10.3	9.1	5.1

B. Instantaneous adjustment of price expectations, 1973/74

	Life cycle price	Annual price
Materials, $\alpha = 1$	30.4	23.9
Energy, $\alpha = 1$	245.4	48.8
Labour, $\alpha = 1$	-24.1	16.4
Capital	15.5	15.5

APPENDIX A. DATA

The data for this analysis are aggregates for total manufacturing taken from the Norwegian annual national accounts. In this appendix, we give a brief description of these data.

Basic data

Our basic data, all of which refer to total manufacturing, are the following:

- y : Gross production at constant (1980) prices, mill. Nkr.
- M : Input of other materials, valued at constant (1980) prices, mill. Nkr.
- E_1: Energy input, electricity, valued at constant (1980) prices mill. Nkr.
- E_2: Energy input, fuel etc., valued at constant (1980) prices, mill. Nkr.
- L: Labour input, mill. hours worked.
- J : Gross investment in fixed capital (aggregate of buildings, machinery and transport equipment), valued at constant (1980) prices, mill. Nkr.
- q : Price index gross production, 1980 = 1.
- p_M: Price index, input of other materials, 1980 = 1.
- p_{E1}: Price index, energy input, electricity, 1980 = 1.
- p_{E2}: Price index, energy input, fuel, 1980 = 1.
- p_L: Hourly wage rate (wages paid), Nkr.
- p_J: Price index, gross investment, 1980 = 1.
- r : Interest rate (pro anno) on loans from commercial banks to corporations.

The data on y, M, J, q, p_M, and p_J are taken from the Norwegian annual national accounts, L and p_L are taken from labour market statistics, and E_1, E_2, p_{E1}, and p_{E2} from energy statistics published by the Central Bureau of Statistics of Norway. The labour market data and energy data are integrated into the national accounts. The series for the interest rate r is calculated and published by the Bank of Norway.

Energy aggregates

From the data on the two energy commodities electricity and fuel, we have constructed

E : Aggregate energy input, valued at constant (1980) prices, mill. Nkr., and

p_E: Price index, aggregate energy input, 1980 = 1,

as CES aggregates:

$$E = \{\beta_1(E_1/\beta_1)^{-\eta} + \beta_2(E_2/\beta_2)^{-\eta}\}^{-1/\eta} , \quad (A.1)$$

$$p_E = \{\beta_1 p_{E1}^{1-\lambda} + \beta_2 p_{E2}^{1-\lambda}\}^{1/(1-\lambda)} , \quad (A.2)$$

where β_1 is the electricity share and β_2 is the fuel share in total energy cost in 1980 (β_1 = 0.4906, β_2 = 0.5094), λ is the elasticity of substitution between electricity and fuel, and $\eta = (1-\lambda)/\lambda$. Eq. (A.1) may be interpreted as the "production function" for the aggregate energy input and (A.2) as its dual unit cost function. We assume that these aggregation functions have the same CES form ex ante and ex post, which implies that electricity can be substituted for fuel to the same degree after the capital has been installed and the technology chosen, as it could ex ante. The elasticity of substitution is set to $\lambda = 0.4$, which concurs with the results of an investigation of the electricity-fuel substitution in Norwegian manufacturing by Bye (1984, table 3).

The time series for q, p_M, p_E, p_L, p_J, and r for the years 1963-1984 are given in table A.1. The corresponding series for the smoothed rates of price increase, in continuous time, $\pi(t)$ (cf. section 4.1) are given in table A.2.

Input coefficients

These data define the following input coefficients for materials, energy, and labour

$$a_M = M/y, \qquad a_E = E/y, \qquad a_L = L/y.$$

The capital coefficient a_K is constructed from national accounts data, and defined as the ratio between the volume of the (gross) capital stock and gross production.

TABLE A.1. Basic data for prices and interest rate

Year	q 1980=1	p_M 1980=1	p_E 1980=1	p_L NKr	p_J 1980=1	r per cent p.a.
1963	0.3373	0.3469	0.2322	9.47	0.3819	5.37
1964	0.3515	0.3621	0.2193	10.11	0.3912	5.34
1965	0.3642	0.3767	0.2171	11.06	0.4089	5.41
1966	0.3715	0.3846	0.2279	12.04	0.4234	5.55
1967	0.3756	0.3844	0.2467	13.49	0.4305	5.65
1968	0.3793	0.3817	0.2408	14.80	0.4339	5.77
1969	0.3961	0.3977	0.2474	16.26	0.4558	6.54
1970	0.4310	0.4340	0.2482	18.04	0.5069	6.67
1971	0.4560	0.4576	0.2837	20.65	0.5357	6.68
1972	0.4715	0.4602	0.2916	22.98	0.5621	6.76
1973	0.5044	0.4973	0.3112	26.06	0.5742	6.90
1974	0.6047	0.6160	0.4632	30.34	0.6632	7.54
1975	0.6737	0.6672	0.5148	36.80	0.7318	8.04
1976	0.7233	0.7186	0.5613	42.18	0.7952	8.49
1977	0.7798	0.7763	0.6451	47.34	0.8604	8.94
1978	0.8192	0.8107	0.6891	52.00	0.9207	10.73
1979	0.8971	0.8799	0.7787	54.76	0.9163	10.95
1980	1.0000	1.0000	1.0000	60.29	1.0000	11.30
1981	1.1073	1.1088	1.2050	67.41	1.0508	12.15
1982	1.1843	1.1847	1.3385	73.74	1.1385	12.95
1983	1.2512	1.2320	1.5162	81.68	1.1868	13.04

TABLE A.2. Expected rates of increase of prices of output and variable inputs in continuous time, $\pi(t)$.

Parameters in price expectation process:

a : $\gamma = \mu = 1.0$; mean lag = 0 years.
b : $\gamma = \mu = 0.5$; mean lag = 1.5 years.
c : $\gamma = 0.2$, $\mu = 1.0$; mean lag = 4.0 years.
d : $\gamma = 0.1$, $\mu = 1.0$; mean lag = 9.0 years.

A. Output price, $\pi_Q(t)$

Year	a	b	c	d
1964	0.04134	0.01749	0.01588	0.01265
1965	0.03545	0.02800	0.01982	0.01495
1966	0.01985	0.02784	0.01983	0.01544
1967	0.01100	0.02166	0.01807	0.01500
1968	0.00990	0.01607	0.01644	0.01449
1969	0.04328	0.02141	0.02186	0.01741
1970	0.08440	0.04296	0.03469	0.02431
1971	0.05644	0.05683	0.03908	0.02757
1972	0.03343	0.05093	0.03795	0.02816
1973	0.06746	0.05076	0.04392	0.03216
1974	0.18136	0.08910	0.07296	0.04812
1975	0.10794	0.11761	0.08005	0.05426
1976	0.07115	0.10376	0.07828	0.05597
1977	0.07522	0.08859	0.07767	0.05791
1978	0.04918	0.07552	0.07203	0.05704
1979	0.09090	0.07289	0.07584	0.06048
1980	0.10858	0.08643	0.08247	0.06539
1981	0.10190	0.09588	0.08639	0.06910
1982	0.06723	0.09031	0.08258	0.06892
1983	0.05495	0.07582	0.07712	0.06753

B. Material cost, $\pi_M(t)$

Year	a	b	c	d
1964	0.04306	0.01558	0.01372	0.00999
1965	0.03935	0.02848	0.01890	0.01297
1966	0.02087	0.02932	0.01930	0.01376
1967	-0.00049	0.01983	0.01537	0.01235
1968	-0.00696	0.00812	0.01094	0.01043
1969	0.04085	0.01269	0.01700	0.01352
1970	0.08744	0.03889	0.03149	0.02116
1971	0.05288	0.05472	0.03580	0.02438
1972	0.00572	0.04223	0.02986	0.02253
1973	0.07758	0.04226	0.03959	0.02817
1974	0.21409	0.09661	0.07701	0.04840
1975	0.07984	0.12326	0.07757	0.05159
1976	0.07412	0.10039	0.07688	0.05386
1977	0.07721	0.08811	0.07695	0.05622
1978	0.04345	0.07439	0.07034	0.05495
1979	0.08181	0.06862	0.07265	0.05767
1980	0.12800	0.08706	0.08396	0.06493
1981	0.10323	0.10148	0.08785	0.06883
1982	0.06622	0.09322	0.08356	0.06857
1983	0.03920	0.07321	0.07484	0.06567

TABLE A.2. (cont.)

C. Energy cost, $\pi_E(t)$

Year	a	b	c	d
1964	-0.05720	-0.01134	0.00112	0.01370
1965	-0.00972	-0.02220	-0.00104	0.01138
1966	0.04820	-0.00105	0.00901	0.01512
1967	0.07954	0.03200	0.02352	0.02176
1968	-0.02431	0.03048	0.01413	0.01724
1969	0.02699	0.01618	0.01672	0.01822
1970	0.00361	0.01577	0.01411	0.01677
1971	0.13339	0.04356	0.03913	0.02906
1972	0.02755	0.06290	0.03682	0.02891
1973	0.06507	0.05473	0.04254	0.03259
1974	0.39761	0.15444	0.12435	0.07570
1975	0.10556	0.20977	0.12062	0.07872
1976	0.08645	0.15453	0.11388	0.07950
1977	0.13923	0.13407	0.11900	0.08564
1978	0.06604	0.11880	0.10863	0.08369
1979	0.12220	0.10673	0.11136	0.08761
1980	0.25010	0.14829	0.14069	0.10510
1981	0.18646	0.18416	0.15001	0.11354
1982	0.10512	0.16556	0.14119	0.11270
1983	0.12461	0.14056	0.13790	0.11390

D. Labour cost, $\pi_L(t)$

Year	a	b	c	d
1964	0.06535	0.06749	0.06764	0.06792
1965	0.09049	0.07276	0.07225	0.07020
1966	0.08507	0.08030	0.07483	0.07170
1967	0.11307	0.08978	0.08259	0.07591
1968	0.09301	0.09646	0.08468	0.07764
1969	0.09378	0.09493	0.08651	0.07926
1970	0.10392	0.09690	0.09002	0.08175
1971	0.13522	0.10836	0.09922	0.08723
1972	0.10681	0.11476	0.10074	0.08921
1973	0.12598	0.11560	0.10584	0.09295
1974	0.15219	0.12746	0.11529	0.09903
1975	0.19296	0.15038	0.13131	0.10883
1976	0.13635	0.15774	0.13232	0.11162
1977	0.11554	0.14200	0.12899	0.11201
1978	0.09384	0.12355	0.12206	0.11021
1979	0.05174	0.09860	0.10838	0.10451
1980	0.09607	0.08645	0.10593	0.10367
1981	0.11175	0.09523	0.10710	0.10448
1982	0.08974	0.09803	0.10365	0.10302
1983	0.10216	0.09700	0.10335	0.10293

APPENDIX B. INPUT COEFFICIENTS AND PROFIT RATES

TABLE B.1. Vintage specific material input coefficients, a_M.

A. Variation with scaling factor α for $\gamma = 0.2$, $\mu = 1.0$.

Vintage	Scaling factor/Rate of technical change			
	$\alpha=1, \varepsilon=0$	$\alpha=2, \varepsilon=0$	$\alpha=3, \varepsilon=0$	$\alpha=1, \varepsilon=0.01$
1964	0.5611	0.4892	0.4144	0.6498
1965	0.5638	0.4950	0.4235	0.6469
1966	0.5712	0.5096	0.4449	0.6494
1967	0.5908	0.5468	0.4970	0.6632
1968	0.6070	0.5793	0.5431	0.6719
1969	0.6103	0.5868	0.5556	0.6704
1970	0.6055	0.5795	0.5494	0.6604
1971	0.6177	0.6039	0.5848	0.6664
1972	0.6421	0.6613	0.7084	0.6841
1973	0.6377	0.6521	0.7014	0.6745
1974	0.6085	0.5871	0.5645	0.6457
1975	0.6376	0.6534	0.7108	0.6635
1976	0.6428	0.6613	*	0.6645
1977	0.6455	0.6632	0.6942	0.6626
1978	0.6539	0.6781	*	0.6654
1979	0.6452	0.6652	0.6907	0.6510
1980	0.6289	0.6289	0.6289	0.6289
1981	0.6288	0.6288	0.6288	0.6288
1982	0.6337	0.6385	0.6431	0.6217
1983	0.6506	0.6732	0.6979	0.6330

B. Variation with price expectation process for $\alpha = 1$, $\varepsilon = 0$

Vintage	Parameters in ARMA process for price expectations			
	$\gamma = 0.1$ $\mu = 1.0$	$\gamma = 0.2$ $\mu = 1.0$	$\gamma = 0.5$ $\mu = 0.5$	$\gamma = 1.0$ $\mu = 1.0$
1964	0.5457	0.5611	0.5938	0.5360
1965	0.5490	0.5638	0.5910	0.5616
1966	0.5556	0.5712	0.6008	0.5803
1967	0.5714	0.5908	0.6250	0.6229
1968	0.5852	0.6070	0.6484	0.6354
1969	0.5899	0.6103	0.6534	0.6047
1970	0.5884	0.6055	0.6410	0.5644
1971	0.5992	0.6177	0.6468	0.6326
1972	0.6179	0.6421	0.6802	0.6909
1973	0.6184	0.6377	0.6821	0.6164
1974	0.5989	0.6085	0.6404	0.5728
1975	0.6202	0.6376	0.6530	*
1976	0.6264	0.6428	0.6804	0.6441
1977	0.6299	0.6455	0.6810	0.6401
1978	0.6375	0.6539	0.6888	0.6617
1979	0.6308	0.6452	0.6799	*
1980	0.6200	0.6289	0.6513	0.6077
1981	0.6192	0.6288	0.6539	0.6234
1982	0.6226	0.6337	0.6641	0.6371
1983	0.6348	0.6506	0.6863	0.6848

*) No (positive and finite) profit maximizing service life exists.

TABLE 8.2. Vintage specific energy input coefficients, a_E.

A. Variation with scaling factor α for γ = 0.2, μ = 1.0.

Vintage	Scaling factor/Rate of technical change			
	α=1,ε=0	α=2,ε=0	α=3,ε=0	α=1,ε=0.01
1964	0.0405	0.0483	0.0552	0.0449
1965	0.0421	0.0513	0.0596	0.0455
1966	0.0407	0.0485	0.0551	0.0430
1967	0.0382	0.0433	0.0466	0.0392
1968	0.0390	0.0451	0.0494	0.0405
1969	0.0395	0.0462	0.0510	0.0409
1970	0.0429	0.0531	0.0623	0.0445
1971	0.0401	0.0478	0.0543	0.0409
1972	0.0401	0.0488	0.0595	0.0418
1973	0.0409	0.0506	0.0636	0.0415
1974	0.0354	0.0386	0.0410	0.0360
1975	0.0363	0.0408	0.0456	0.0374
1976	0.0363	0.0410	*	0.0365
1977	0.0350	0.0381	0.0425	0.0350
1978	0.0343	0.0364	*	0.0343
1979	0.0347	0.0373	0.0399	0.0350
1980	0.0321	0.0321	0.0321	0.0321
1981	0.0311	0.0301	0.0291	0.0307
1982	0.0308	0.0296	0.0284	0.0302
1983	0.0300	0.0278	0.0255	0.0290

B. Variation with price expectation process for α = 1, ε = 0

Vintage	Parameters in ARMA process for price expectations			
	γ = 0.1 μ = 1.0	γ = 0.2 μ = 1.0	γ = 0.5 μ = 0.5	γ = 1.0 μ = 1.0
1964	0.0367	0.0483	0.0457	0.0569
1965	0.0380	0.0513	0.0497	0.0501
1966	0.0374	0.0485	0.0463	0.0410
1967	0.0361	0.0433	0.0407	0.0360
1968	0.0369	0.0451	0.0402	0.0435
1969	0.0371	0.0462	0.0419	0.0452
1970	0.0392	0.0531	0.0471	0.0626
1971	0.0378	0.0478	0.0454	0.0369
1972	0.0379	0.0488	0.0420	0.0407
1973	0.0387	0.0506	0.0432	0.0446
1974	0.0348	0.0386	0.0371	0.0321
1975	0.0353	0.0408	0.0361	*
1976	0.0351	0.0410	0.0380	0.0394
1977	0.0340	0.0381	0.0362	0.0350
1978	0.0332	0.0364	0.0351	0.0368
1979	0.0337	0.0373	0.0367	*
1980	0.0316	0.0321	0.0332	0.0310
1981	0.0307	0.0301	0.0324	0.0305
1982	0.0302	0.0296	0.0323	0.0329
1983	0.0295	0.0278	0.0317	0.0293

*) No (positive and finite) profit maximizing service life exists.

TABLE B.3. Vintage specific labour input coefficients, a_L.

A. Variation with scaling factor α for γ = 0.2, μ = 1.0.

Vintage	Scaling factor/Rate of technical change			
	α=1, ε=0	α=2, ε=0	α=3, ε=0	α=1, ε=0.01
1964	0.2822	0.3761	0.4813	0.3672
1965	0.2744	0.3598	0.4561	0.3571
1966	0.2667	0.3457	0.4380	0.3499
1967	0.2460	0.3080	0.3942	0.3327
1968	0.2254	0.2635	0.3261	0.2966
1969	0.2214	0.2534	0.3073	0.2832
1970	0.2187	0.2433	0.2778	0.2681
1971	0.2087	0.2228	0.2500	0.2599
1972	0.1853	0.1623	0.1071	0.2214
1973	0.1865	0.1640	0.0977	0.2231
1974	0.2184	0.2416	0.2776	0.2551
1975	0.1835	0.1553	0.0791	0.2088
1976	0.1841	0.1587	*	0.2080
1977	0.1867	0.1702	0.1256	0.2055
1978	0.1855	0.1738	*	0.1973
1979	0.1791	0.1532	0.1207	0.1821
1980	0.2000	0.2000	0.2000	0.2000
1981	0.2012	0.2020	0.2024	0.1981
1982	0.1997	0.1998	0.2007	0.1937
1983	0.1867	0.1714	0.1526	0.1781

B. Variation with price expectation process for α = 1, ε = 0

Vintage	Parameters in ARMA process for price expectations			
	γ = 0.1 μ = 1.0	γ = 0.2 μ = 1.0	γ = 0.5 μ = 0.5	γ = 1.0 μ = 1.0
1964	0.2851	0.2822	0.2784	0.2509
1965	0.2774	0.2744	0.2731	0.2192
1966	0.2694	0.2667	0.2657	0.2131
1967	0.2510	0.2460	0.2462	0.1758
1968	0.2333	0.2254	0.2199	0.1510
1969	0.2290	0.2214	0.2102	0.1685
1970	0.2268	0.2187	0.2093	0.1839
1971	0.2162	0.2087	0.2008	0.1544
1972	0.1980	0.1853	0.1752	0.1087
1973	0.1944	0.1865	0.1713	0.1577
1974	0.2144	0.2184	0.2283	0.2347
1975	0.1916	0.1835	0.2049	*
1976	0.1892	0.1841	0.1751	0.1382
1977	0.1894	0.1867	0.1921	0.1440
1978	0.1891	0.1855	0.1925	0.1317
1979	0.1847	0.1791	0.1746	*
1980	0.1971	0.2000	0.2071	0.1932
1981	0.1981	0.2012	0.2141	0.1620
1982	0.1981	0.1997	0.2051	0.1518
1983	0.1887	0.1867	0.1847	0.1241

*) No (positive and finite) profit maximizing service life exists.

TABLE B.4. Vintage specific capital input coefficients, a_K.

A. Variation with scaling factor α for γ = 0.2, μ = 1.0.

Vintage	Scaling factor/Rate of technical change			
	α=1, ε=0	α=2, ε=0	α=3, ε=0	α=1, ε=0.01
1964	0.5628	0.4668	0.3481	0.5852
1965	0.5742	0.4884	0.3784	0.5780
1966	0.5667	0.4692	0.3395	0.5451
1967	0.5577	0.4406	0.2621	0.5012
1968	0.5687	0.4696	0.3077	0.5310
1969	0.5707	0.4772	0.3254	0.5389
1970	0.6012	0.5496	0.4698	0.5872
1971	0.6002	0.5470	0.4525	0.5549
1972	0.6174	0.6215	0.7115	0.6054
1973	0.6303	0.6546	0.8207	0.5909
1974	0.6180	0.5749	0.4692	0.5433
1975	0.6783	0.7705	1.0595	0.6384
1976	0.6388	0.6858	*	0.5866
1977	0.6120	0.6024	0.7288	0.5658
1978	0.5794	0.5085	*	0.5536
1979	0.6749	0.7337	0.8167	0.6766
1980	0.6371	0.6371	0.6371	0.6371
1981	0.6384	0.6417	0.6482	0.6391
1982	0.6191	0.5987	0.5735	0.6193
1983	0.6202	0.6140	0.6256	0.6174

B. Variation with price expectation process for α = 1, ε = 0

Vintage	Parameters in ARMA process for price expectations			
	γ = 0.1, μ = 1.0	γ = 0.2, μ = 1.0	γ = 0.5, μ = 0.5	γ = 1.0, μ = 1.0
1964	0.5647	0.5628	0.5761	0.7001
1965	0.5734	0.5742	0.6061	0.6786
1966	0.5707	0.5667	0.5850	0.6166
1967	0.5695	0.5577	0.5500	0.5887
1968	0.5784	0.5687	0.5536	0.6414
1969	0.5745	0.5707	0.5720	0.6996
1970	0.5872	0.6012	0.6298	0.8620
1971	0.5922	0.6002	0.6540	0.6609
1972	0.6022	0.6174	0.6460	0.6936
1973	0.6207	0.6303	0.6596	0.7015
1974	0.6177	0.6180	0.5664	0.6256
1975	0.6450	0.6783	0.6351	*
1976	0.6274	0.6388	0.6666	0.7022
1977	0.6122	0.6120	0.5583	0.7072
1978	0.5803	0.5794	0.5267	0.6667
1979	0.6436	0.6749	0.6830	*
1980	0.6281	0.6371	0.6598	0.6157
1981	0.6333	0.6384	0.6016	0.7283
1982	0.6166	0.6191	0.5958	0.6826
1983	0.6208	0.6202	0.6072	0.6612

*) No (positive and finite) profit maximizing service life exists.

TABLE 8.5. Ex ante life cycle profit rate.[a)b)]

Base alternative for price expectations

Vintage	Scaling factor/Rate of technical change			
	$\alpha=1, \varepsilon=0$	$\alpha=2, \varepsilon=0$	$\alpha=3, \varepsilon=0$	$\alpha=1, \varepsilon=0.01$
1964	0.1409	0.1726	0.2106	-0.0226
1965	0.1218	0.1500	0.1835	-0.0303
1966	0.0972	0.1211	0.1508	-0.0452
1967	0.0713	0.0871	0.1112	-0.0656
1968	0.0677	0.0773	0.0929	-0.0561
1969	0.0597	0.0679	0.0811	-0.0536
1970	0.0656	0.0729	0.0820	-0.0353
1971	0.0413	0.0457	0.0520	-0.0522
1972	0.0507	0.0537	0.0560	-0.0297
1973	0.0279	0.0308	0.0332	-0.0444
1974	0.0106	0.0132	0.0172	-0.0569
1975	0.0216	0.0228	0.0245	-0.0293
1976	-0.0046	-0.0022	*	-0.0494
1977	-0.0195	-0.0173	-0.0133	-0.0553
1978	-0.0320	-0.0291	*	-0.0558
1979	0.0216	0.0235	0.0260	0.0117
1980	0.0000	0.0000	0.0000	0.0000
1981	-0.0014	-0.0013	-0.0012	0.0086
1982	-0.0101	-0.0100	-0.0098	0.0099
1983	-0.0120	-0.0103	-0.0086	0.0181

a) The profit rate is defined as the ratio between the life cycle profit and the total life cycle cost.

b) An asterisk (*) indicates that no (positive and finite) profit maximizing service life exists.

FOOTNOTES

Section 1:

1) Johansen here, mainly due to his emphasis on the planning context, did not elaborate these aspects of the model either in his presentation of the model or in the applications he made of it, though he mentioned the role played by expectational variables in rather general terms: cf. Johansen (1972, pp. 33, 201, and 225). See also Johansen (1967).

Section 2:

1) Confer e.g. Biørn (1983) for an interpretation of this function in a neo-classical context.

2) We are making the simplifying assumption that producers form independent expectations about output and input prices. Endogenous output prices are discussed in Koizumi (1969), Malcomson (1975), and Frenger (1985b).

3) Some consequences of (stochastically specified) price uncertainty are discussed, within the framework of a simple putty-clay model, by Moene (1985).

4) This approach is also used in Fuss (1977,1978) in describing his putty-semiputty technology. He also uses life cycle prices, but needs cross products of the expected price paths due to the flexibility of the ex post technology. Our GL model is in fact his putty-clay model, but he assumes that the planning horizon (service life) is exogenously given and constant.

5) See Frenger (1985a,1986) for a more detailed exposition of this argument.

Section 4:

1) The SES was defined by McFadden (1963), and measures the elasticity of the factor ratio a_i/a_j w.r.t. the price ratio p_i/p_j, holding total cost, output, and all other prices constant. The HSES was introduced by Hicks (1963, pp. 339, 379) and by Samuelson (1968, p. 468), and measures the response of the demand for a factor to a change in its price, when all other prices change proportionately so as to leave total cost constant. It represents a renormalization of the own Allen-Uzawa elasticity of substitution. Both the SES and the HSES are special cases of the directional shadow elasticity of substitution, and its minimum and maximum values in 1981 were 0.2746 and 0.9733, respectively [see Frenger (1985a) for definitions of these elasticities].

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