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The Role of Heterogeneous Demand for Temporal and Structural Aggregation Bias*

Abstract:
Differences in estimated parameters depending on the frequency of aggregate data have been reported in several fields of economic research. Some differences are due to seasonal variations in demand, but temporal aggregation bias is reported even in seasonally adjusted models. These biases have been explained by time-nonseparable preferences and excluded dynamic components. We show that it is possible to observe temporal aggregation bias in a seasonally adjusted static model even when preferences are time-separable. This is because of changes in the distribution of exogenous factors describing the variation in seasonal demand across consumers. To show this, we develop a method for aggregation based on an Almost Ideal Demand System, where demand response varies across both consumers and time

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I. Introduction

Demand response may vary over time, e.g., over hours, seasons, and years, as well as across consumers and goods. From the theoretical literature on aggregation across individuals, we know that if consumers are heterogeneous, we may experience problems with aggregation.[1] This is because the parameters of the macro function include structural as well as behavioral micro components and will thus differ from the parameters in the micro function (Theil 1954; Stoker 1993; Blundell and Stoker 2005). The difference between the micro and macro parameters caused by these structural components creates a disaggregation bias if we use estimated micro parameters to predict aggregate demand responses as if they were macro parameters (Denton and Mountain 2001, 2004), or an aggregation bias if we use macro data to estimate behavioral parameters without correcting for structural components (Stoker 1986; Buse 1992). Based on this literature, we would expect variation in demand response over time to create structural effects on time-aggregated consumption, as well as behavioral effects.

In the empirical literature, temporal aggregation bias has been shown to exist in several fields of research, e.g., in the habit formation literature (Heien 2001), the literature on the purchasing power parity puzzle (Imbs et al. 2005), the permanent income hypothesis literature (Christiano, Eichenbaum, and Marshall 1991; Heaton, 1993), and the literature on heterogeneous panels (Pesaran and Smith 1995; Pesaran, Shin, and Smith 2002). Heien (2001) discusses the difference between habit formation and seasonal variation in demand.[2] He shows how a simple model of habit formation yields biased habit estimates if the model is estimated using annual data and there is seasonal variation in demand. He concludes that much of what has traditionally been identified as habitual behavior are, in fact, seasonal effects. Christiano, Eichenbaum, and Marshall (1991) explore the implications of temporal ag-

[1] See the work of Gorman (e.g., Blackorby and Shorrocks 1995) for a discussion.

[2] Habitual behavior is defined as a complementary relation between current and past consumption, e.g., the effect of last period's consumption on consumption in this period. On the other hand, seasonal effects are associated with effects exogenous to the consumer, such as food availability (strawberries), temperature conditions (energy use for air conditioning), and events such as holidays (turkey for Thanksgiving).

gregation bias for the test of the martingale hypothesis, and argue that the rejection of the martingale hypothesis could be due to temporal aggregation of the data. Heaton (1993) shows that whether the martingale hypothesis is rejected or not in a model adjusted for seasonal demand effects still may depend on the length of the time period. He shows that allowing for time-nonseparable preferences, due either to consumption of durables or to habit formation, can account for this difference. The studies on heterogeneous panels point out that misspecification of the aggregate demand model, ignoring heterogeneity across consumers over time, may cause biased estimates in dynamic models (see, e.g., Pesaran and Smith 1995). Bergstrom (1984) discusses the theoretical assumptions for the underlying stochastic continuous functions that are necessary for the time-aggregated function to yield its properties. He concludes that the restrictions on the underlying continuous model are quite strict, and if they are not satisfied, then temporal aggregation bias occurs.[3]

These studies point out that there is strong theoretical and empirical evidence from several fields of research that time aggregation affects estimation results. In some cases, it is assumed that the underlying microstructure is heterogeneous, but the causes for this heterogeneity are seldom modeled explicitly. In other cases, heterogeneity across consumers is ignored. Thus, all unobserved heterogeneity ends up in the error term, creating misspecification bias. None of the studies aggregates the deterministic component of the micro demand, which is the main focus in the literature on aggregation across consumers and goods. The interpretation of temporal aggregation bias is, thus, closely connected to misspecification biases when seasonal variations and dynamic components in demand are not modeled explicitly.

In the literature, the term 'aggregation bias' is used to describe two things: the bias resulting from ignoring heterogeneity in demand across consumers over time in an estimation, and the bias from ignoring structural components in aggregate demand. It is, however, important to distinguish between

[3] An early version of this discussion is in Theil (1954), where it is shown that, in most cases, the micro and macro parameters differ unless the demand functions are linear and all parameters are equal across consumers, goods, and/or time. See also Marcellino (1999) for a theoretical discussion of how temporal aggregation alters the properties existing at the disaggregated frequency in the case where there is no heterogeneity across consumers in how demand varies over time.

these two types of bias to be able to handle them properly. The bias from ignoring heterogeneity in the estimation is due to a misspecification of the econometric model (hereafter referred to as 'misspecification bias'), whereas the second is due to ignoring structural components in the aggregated parameters (hereafter referred to as 'structural aggregation bias'). The problem with structural aggregation bias is that it creates a theoretical misinterpretation of the true underlying model. Misspecification bias occurs because when we do not allow for sufficient heterogeneity across consumers and over time in the estimated demand model, the error term assumptions in the empirical model are no longer valid, and the parameter estimates are not consistent. Thus, misspecification bias is an empirical source of bias, whereas structural aggregation bias is a theoretical source of bias. When the heterogeneity in demand across consumers over time is ignored in empirical analyses, both types of biases may be experienced.

The novelty of this paper is that we merge the different traditions of describing aggregation bias across consumers and over time. This enables us to distinguish between, and identify, the different sources of bias. We extend the literature on heterogeneous panels by introducing structural components into the aggregated demand function. We also extend the discussion in Pesaran and Smith (1995) by showing that omitting variables describing heterogeneity in individual consumption over time create biased estimates not only in dynamic models but also in static models, if the omitted variables affect how consumers respond to price and income changes. Furthermore, we extend the literature on aggregation across consumers by discussing the effects of misspecification on the estimated parameters with different types of data (micro cross-section, macro time series, or panel data).

We illustrate the difference between misspecification and structural aggregation bias by developing a general framework for two-dimensional aggregation based on an Almost Ideal Demand System (AIDS), where income, prices, and the price and income derivatives are allowed to vary both across consumers and over time. The true underlying micro model is assumed to be static, that is, there is no habit formation or other dynamic component in the underlying demand. This is done to show how excluded heterogeneity in seasonal variation in demand across consumers may wrongly be interpreted as a dynamic component, even if the underlying microstructure is static.

We show how excluding heterogeneity across consumers and over time affects and biases the estimation results, depending on the specified model and the type of data available. We start by looking at the case where we have micro panel data, but we do not have information about factors creating heterogeneity in demand across consumers and over time. We use this case to isolate the source of misspecification bias and show how this may create autoregressive errors. Then, we discuss cases where we can observe both structural aggregation bias and misspecification bias, exemplified by the case of annual micro cross-sectional data and annual macro data. We compare the empirical model with the true underlying demand structure, to identify the different sources of bias. We show how structural components describing the distribution of heterogeneity across consumers and over time may create dynamic components in aggregated demand even though the underlying microstructure is static. Details on the calculations of the aggregated demand functions are given in the Appendix.

II. The Micro Model

The basis of our micro demand model is an AIDS model, where prices, total expenditure, and all parameters are allowed to vary across consumers and over time. The length of the time period t equals the shortest purchasing frequency by any of the consumers. The budget share of good i ($i = 1, \ldots, J$) for consumer h ($h = 1, \ldots, H$) at time t ($t = 1, \ldots, \ddot{T}$), w_{ht}^{i}, is given by:

$$w_{ht}^{i} = \alpha_{ht}^{i} + \sum_{j} \gamma_{ht}^{ij} \log\left(p_{ht}^{j}\right) + \beta_{ht}^{i} \log\left(\breve{x}_{ht}\right), \qquad (1)$$

where p_{ht}^{j} is the price of good j ($j = 1, \ldots, J$) for consumer h at time t and α_{ht}^{i}, γ_{ht}^{ij}, and β_{ht}^{i} are individual preference parameters, which are allowed to vary over time (t). The total real expenditure for consumer h at time t is given by $\breve{x}_{ht} = x_{ht} / P_{ht}$, where x_{ht} is total expenditure at time t. The log of the price index P_{ht} is given by $\log(P_{ht}) = \alpha^0 + \sum_i \alpha_{ht}^i \log(p_{ht}^i) + \frac{1}{2}\sum_i \sum_j \gamma_{ht}^{ji} \log(p_{ht}^i)\log(p_{ht}^j)$.

We model heterogeneity by letting all micro parameters depend on various individual and time specific variables. We assume that some variables vary both across consumers and over time. These variables are included in the vector μ_{ht} (e.g., outdoor temperature). Others vary across consumers only, included in the vector θ_h (e.g., gender), or over time only, included in the vector φ_t (e.g., interest rate). Thus, variables changing with time may be due to seasonal variation in demand or other exogenous changes such as weather conditions or policy changes. We assume that the micro parameters are linear functions of these individual- and time-specific variables, given by:

$$\alpha_{ht}^i = \alpha_0^i + \sum_m \alpha^{mi} \theta_h^m + \sum_s \alpha^{si} \mu_{ht}^s + \sum_v \alpha^{vi} \varphi_t^v \,, \tag{2a}$$

$$\beta_{ht}^i = \beta_0^i + \sum_k \beta^{ki} \theta_h^k + \sum_r \beta^{ri} \mu_{ht}^r + \sum_w \beta^{wi} \varphi_t^w \text{, and} \tag{2b}$$

$$\gamma_{ht}^{ij} = \gamma_0^{ij} + \sum_f \gamma_f^{ij} \theta_h^{jf} + \sum_d \gamma_d^{ij} \mu_{ht}^{jd} + \sum_u \gamma_u^{ij} \varphi_t^{ju} \,. \tag{2c}$$

Changes in the micro parameters ($\alpha_{ht}^i, \beta_{ht}^i$, and γ_{ht}^{ij}) are assumed to be due to changes in underlying exogenous factors included in the vectors, μ_{ht}, θ_h, and φ_t, affecting the demand response. In equations (2a)–(2c) we allow different micro parameters to be affected by different factors; e.g., the vector of μ s in β_{ht}^i may differ from the vector of μ s in γ_{ht}^{ij}. Thus, we sum over relevant variables only. In this model, the underlying behavioral parameters (α_0^i, α^{mi}, α^{si}, α^{vi}, γ_0^{ij}, γ_f^{ij}, γ_d^{ij}, γ_u^{ij}, β_0^i,

β^{ki}, β^{ri}, and β^{wi}) are constant, i.e., invariant with respect to consumer and time, even if the micro parameters, α^{i}_{ht}, γ^{ij}_{ht}, and β^{i}_{ht}, change over time and across consumers.

We assume that the properties of the standard AIDS system are fulfilled for each individual consumer *h* at each time period *t*.[4] This implies that, at a micro level, all consumers maximize a quasiconcave utility function given their budget constraint at each particular point in time. However, as we assume that heterogeneity exists across consumers and over time, the properties of the micro AIDS systems do not necessarily transfer to the aggregated demand functions (Blundell, Pashardes, and Weber 1993; Mas-Colell, Whinston, and Green 1995; Denton and Mountain 2001). This means that we assume that there exist $H\ddot{T}$ independent AIDS demand systems. As these systems cannot be exactly nonlinearly aggregated, because the parameters differ across consumers and over time, the properties of the micro functions do not hold for the aggregated functions, and we may not assume symmetry, homogeneity, etc., at an aggregate level because of the aggregation biases.[5] In this paper, we do not discuss how these restrictions will affect the various sources of bias when estimating using aggregated data when the underlying demand structure varies across consumers and/or over time. These are interesting topics for future research.

III. Micro Panel Data and Homogeneous Behavior

To illustrate the main driving forces behind misspecification bias in a static model, we look at the case where we have micro panel data for price, income, and budget shares, but no data on characteristics describing differences in demand structure across consumers and over time. We show how misspecification bias rises from correlations created by omitted variables in the error term.

Suppose that the demand structure described in Section 2, in addition to a random error term (u^{i}_{ht}) with zero expectation and constant variance, describes the true underlying micro demand struc-

[4] See Deaton and Muellbauer (1980) for the initial description of the AIDS demand system.

ture. We want to estimate demand using *micro panel data*, approximating the micro budget share function by:

$$\omega_{ht}^{i} = \alpha^{i} + \sum_{j} \gamma^{ij} \log\left(p_{ht}^{j}\right) + \beta^{i} \log\left(\breve{x}_{ht}\right) + \varepsilon_{ht}^{i}, \tag{3}$$

where ε_{ht}^{i} is an error term. This econometric specification takes into account that the budget share may vary across consumers and over time, due to variations in prices, income, and the error term. However, it ignores the heterogeneity of the true underlying demand function given by equations (1) and (2a)–(2c). That is, we assume that all parameters contributing to the heterogeneity in equations (2a)–(2c) equal zero. The question is whether estimation of equation (3) yields consistent estimates of the behavioral parameters ($\alpha_0^{i}, \gamma_0^{ij}$, and β_0^{i}). That is, does excluding the heterogeneity prevent us from obtaining good estimates of behavior?

In this case, the error term in the model involved includes components describing how demand changes across consumers and over time, in addition to the random error term:

$$\varepsilon_{ht}^{i} = \begin{cases} u_{ht}^{i} \\ + \sum_{m} \alpha^{mi} \theta_{h}^{m} + \sum_{s} \alpha^{si} \mu_{ht}^{s} + \sum_{v} \alpha^{vi} \varphi_{t}^{v} \\ + \sum_{j} \left[\sum_{f} \gamma_{f}^{ij} \theta_{h}^{jf} + \sum_{d} \gamma_{d}^{ij} \mu_{ht}^{jd} + \sum_{u} \gamma_{u}^{ij} \varphi_{t}^{ju} \right] \log\left(p_{ht}^{j}\right) \\ + \left[\sum_{k} \beta^{ki} \theta_{h}^{k} + \sum_{r} \beta^{ri} \mu_{ht}^{r} + \sum_{w} \beta^{wi} \varphi_{t}^{w} \right] \log\left(\breve{x}_{ht}\right) \end{cases}. \tag{4}$$

[5] We do not discuss these properties further in this paper. This is a topic for future analyses.

The last three lines of the error term may create *misspecification bias* in the estimated coefficients. This misspecification biases the results in two different ways, depending on whether the excluded variables describe the heterogeneity in demand across consumers or over time in consumption level (α_{ht}^{i}) or price and income response (γ_{ht}^{ij} and β_{ht}^{i}).

Excluded cross-sectional or time heterogeneity in the intercept (second line of the error term) biases the results if the excluded regressors are correlated with prices or income (i.e., the included regressors). On the other hand, if the included regressors are strictly exogenous and the coefficients differ randomly (i.e., independently distributed across consumers and over time), there are no biases in the estimated coefficients in a *static* model (Pesaran and Smith 1995). However, it is reasonable to believe that many excluded variables describing differences in budget share levels, across consumers and over time, are correlated with income and prices. For example, income is correlated with dwelling size, type of residence, education, number of household members, etc. Thus, it is reasonable to assume that some of the heterogeneity across consumers creates misspecification bias in demand estimations.

The second source of misspecification bias is due to excluded heterogeneity across consumers and over time in the price and income responses. The omission of these variables introduces a dependency on prices and income directly into the error term as these variables are multiplied by the logarithms of prices and income. This is seen from the last two lines in equation (4), where prices and income enter directly into the error term. Thus, excluded heterogeneity in the price and income responses always biases the estimated coefficients. It is interesting to note that, since income/total expenditure is highly correlated over time, the error terms in equation (4) are also correlated over time. This is also true with respect to price correlation over time. Excluded heterogeneity in the price and income parameters may thus create autoregressive error terms.

Even though we estimate the demand model using micro panel data (not aggregated in any way), the exclusion of variables describing heterogeneity in demand across consumers and over time may create systematic misspecification bias in the estimated coefficients also in static analyses. Thus, such an estimation does not yield consistent estimates of the underlying behavioral parameters (α_0^i , α^{mi} , α^{si} , α^{vi} , γ_0^{ij} , γ_f^{ij} , γ_d^{ij} , γ_u^{ij} , β_0^i , β^{ki} , β^{ri} , and β^{wi}), but estimation of parameters corresponding to a combination of behavior and the distribution of the error and included regressors. The autocorrelation, resulting from omitting heterogeneity in price and income parameters, may easily be interpreted as excluded dynamic components in demand, even if the underlying micro demand model is static.

IV. Annual Micro Cross-section Data and Heterogeneous Behavior

Assume now that we have annual cross-sectional micro data for a good that is consumed on a more regular basis (e.g., every day or week) and that there is time variation in demand. As we do not have information about the time variation in demand, we assume that the coefficients are equal within the year in the model. However, we allow parameters to differ across consumers. We approximate the micro budget share function in year *T* by:

$$\omega_{hT}^i = \alpha_h^i + \sum_j \gamma_h^{ij} \log\left(\overline{p_{hT}^j}\right) + \beta_h^i \log\left(\overline{\breve{x}_{hT}}\right) + \varepsilon_{hT}^i \ , \tag{5}$$

where $\overline{p_{hT}^j}$ is the mean price of good *j* for consumer *h* and $\overline{\breve{x}_{hT}}$ is the mean real income for consumer *h* in year *T*. In this empirical model, the heterogeneity in the parameters across consumers is assumed to be given by:

$$\alpha_h^i = \dot{\alpha}^i + \sum_m \dot{\alpha}^{mi} \theta_h^m \ , \tag{6a}$$

$$\beta_h^i = \dot{\beta}^i + \sum_k \dot{\beta}^{ki} \theta_h^k \quad \text{, and} \tag{6b}$$

$$\gamma_h^{ij} = \dot{\gamma}^{ij} + \sum_f \dot{\gamma}_f^{ij} \theta_h^{jf} \ . \tag{6c}$$

This econometric specification allows for heterogeneity across consumers, but ignores heterogeneity in the demand function over time when the true underlying demand function is given by equations (1) and (2a)–(2c). It is a function of mean prices and real income, and ignores the distribution in prices and income over the year. The question is whether an estimation of the demand system in equations (5) and (6a)–(6c) yields consistent estimates of either the underlying behavioral parameters or the true time-aggregated parameters, described in the next section.

IV.A. Aggregation over Time

We aggregate the demand structure in Section 2 over time to find the time-aggregated demand during a year T.[6] The budget share aggregated over time period T, measured in terms of the arithmetic mean values of all variables, is defined as:[7]

$$w_{hT}^i = \frac{\overline{q_{hT}^i}\ \overline{p_{hT}^i}}{\overline{x_{hT}}} \ . \tag{7}$$

To find the explicit expression for household h's budget share in period T [(6a)–(6c)], we use that the mean consumption in year T is $\overline{q_{hT}^i} = \frac{1}{T}\sum_{t=1}^{T}\left\{\left[\alpha_{ht}^i + \sum_j \gamma_{ht}^{ij}\log\left(p_{ht}^j\right) + \beta_{ht}^i \log\left(\breve{x}_{ht}\right)\right]\frac{x_{ht}}{p_{ht}^i}\right\}$,

[6] This relates to the average sampling case discussed in Marcellino (1999).

[7] Hereafter, 'bar' denotes the arithmetic mean, e.g., $\overline{q_{hT}^i} = \frac{1}{T}\sum_{t=1}^{T} q_{ht}^i$.

given the demand structure in equations (1) and (2). Rearranging, we have the following demand function for consumer h in year T (see the Appendix, Section A.II, for more detailed calculations):

$$\overline{q^i_{hT}} = \left[\begin{array}{l} \alpha^i_0 S^0_{hT} + \sum_m \alpha^{mi} \theta^m_h S^0_{hT} + \sum_s \alpha^{si} \overline{\mu^s_{hT}} S^{\mu^s}_{hT} + \sum_v \alpha^{vi} \overline{\varphi^v_T} S^{\varphi^v}_{hT} \\ + \sum_j \left(\gamma^{ij}_0 S^{p^j}_{hT} + \sum_f \gamma^{ij}_f \theta^{jf}_h S^{p^j}_{hT} + \sum_d \gamma^{ij}_d \overline{\mu^{jd}_{hT}} S^{\mu^{jd}}_{hT} + \sum_u \gamma^{ij}_u \overline{\varphi^{ju}_T} S^{\varphi^{ju}}_{hT} \right) \log\left(\overline{p^j_{hT}}\right) \\ + \left(\beta^i_0 S^x_{hT} + \sum_k \beta^{ki} \theta^k_h S^x_{hT} + \sum_r \beta^{ri} \overline{\mu^r_{hT}} S^{\mu^r}_{hT} + \sum_w \beta^{wi} \overline{\varphi^w_T} S^{\varphi^w}_{hT} \right) \log\left(\overline{\breve{x}_{hT}}\right) \end{array} \right] \frac{\overline{x_{hT}}}{\overline{\overline{p^i_{hT}}}} . \tag{8}$$

We see from equation (8) that the demand function of household h in year T is written as a function of the behavioral parameters (α^i_0, α^{mi}, α^{si}, α^{vi}, γ^{ij}_0, γ^{ij}_f, γ^{ij}_d, γ^{ij}_u, β^i_0, β^{ki}, β^{ri}, and β^{wi}), arithmetic mean values for total expenditure ($\overline{x_{hT}}$), total real expenditure ($\overline{\breve{x}_{hT}}$), prices ($\overline{p^j_{hT}}$, $\overline{p^i_{hT}}$), and time-specific characteristics ($\overline{\varphi_T}$, $\overline{\mu_{hT}}$), as well as consumer-specific characteristics (θ_h) and aggregation factors (S_{hT}) describing the distribution over time of different variables affecting consumer h's demand. The aggregation factors measure the relative importance of changes in behavior during the period (e.g., due to seasonal variation in demand) for consumer h's annual consumption by measuring the correlation of prices, total expenditure, total real expenditure, and consumer characteristics of household h during time period T. See also the Appendix, Section A.II, for a discussion of the aggregation factors.

Inserting the expression for mean consumption in equation (8) into the expression for the budget share in year T in equation (7), we can write the budget share of consumer h in year T as a function of the time-aggregated micro parameters ($\tilde{\alpha}^i_{hT}$, $\tilde{\gamma}^{ij}_{hT}$, and $\tilde{\beta}^i_{hT}$), mean prices, and mean total real expenditure:

$$w_{hT}^{i} = \widetilde{\alpha}_{hT}^{i} + \sum_{j} \widetilde{\gamma}_{hT}^{ij} \log\left(\overline{p_{hT}^{j}}\right) + \widetilde{\beta}_{hT}^{i} \log\left(\overline{\breve{x}_{hT}}\right), \tag{9}$$

where the relationships between the behavioral and time-aggregated micro parameters in period *T* are given by:

$$\widetilde{\alpha}_{hT}^{i} = \alpha_{0}^{i} S_{hT}^{0} + \sum_{m} \alpha^{mi} \theta_{h}^{m} S_{hT}^{0} + \sum_{s} \alpha^{si} \overline{\mu_{hT}^{s}} S_{hT}^{\mu^{s}} + \sum_{v} \alpha^{vi} \overline{\varphi_{T}^{v}} S_{hT}^{\varphi^{v}}, \tag{10a}$$

$$\widetilde{\gamma}_{hT}^{ij} = \gamma_{0}^{ij} S_{hT}^{p^{j}} + \sum_{f} \gamma_{f}^{ij} \theta_{h}^{ij} S_{hT}^{p^{j}} + \sum_{d} \gamma_{d}^{ij} \overline{\mu_{hT}^{jd}} S_{hT}^{\mu^{jd}} + \sum_{u} \gamma_{u}^{ij} \overline{\varphi_{T}^{u}} S_{hT}^{\varphi^{ju}}, \text{ and} \tag{10b}$$

$$\widetilde{\beta}_{hT}^{i} = \beta_{0}^{i} S_{hT}^{x} + \sum_{k} \beta^{ki} \theta_{h}^{k} S_{hT}^{x} + \sum_{r} \beta^{ri} \overline{\mu_{hT}^{r}} S_{hT}^{\mu^{r}} + \sum_{w} \beta^{wi} \overline{\varphi_{T}^{w}} S_{hT}^{\varphi^{w}}. \tag{10c}$$

As shown in equations (10a)–(10c), the time-aggregated micro parameters ($\widetilde{\alpha}_{hT}^{i}$, $\widetilde{\gamma}_{hT}^{ij}$, and $\widetilde{\beta}_{hT}^{i}$) depend on the behavioral parameters (α_{0}^{i}, α^{mi}, α^{si}, α^{vi}, γ_{0}^{ij}, γ_{f}^{ij}, γ_{d}^{ij}, γ_{u}^{ij}, β_{0}^{i}, β^{ki}, β^{ri}, and β^{wi}), the mean values of the variables describing heterogeneity over time in year *T* for consumer *h* ($\overline{\varphi_{T}}$, $\overline{\mu_{hT}}$), the consumer-specific characteristics (θ_{h}), and the aggregation factors (S_{hT}).

There are two types of structural components in the time-aggregated parameters. First, the mean values of the variables with a seasonal pattern ($\overline{\varphi_{T}}$ and $\overline{\mu_{hT}}$) capture the structural effects on the time-aggregated micro parameters of changes in the mean of the exogenous variables during the year. Second, the aggregation factors capture the structural effects of changes in the distribution of exogenous variables describing the heterogeneity in behavior during the year.

These structural components cause the time-aggregated micro parameters to differ from the behavioral and the micro parameters described in equations (2a)–(2c), and changes in these structural components over time cause the time-aggregated micro parameters to change over time, even if the underlying behavioral parameters are stable. Note also that the time-aggregated micro parameters differ from the behavioral parameters even if there were no variations in demand over time, because both prices and income may change during period *T*. This is because the aggregation factors describing the distribution of prices and income over time may differ from unity (see Halvorsen and Larsen 2006). This is because of the nonlinearity of the AIDS-function.

IV.B. Discussion of Bias

The first question is: given the underlying time-aggregated micro demand system in equations (9) and (10), will estimation using annual cross-sectional micro data based on equations (5) and (6) give consistent estimates of either the true underlying behavioral or the time-aggregated micro parameters?

Under the assumption of no seasonal variation in demand over time, which is implicitly assumed in the estimation, the relationship between the parameters in equation (6) and the true underlying time-aggregated micro parameters in equations (10a)–(10c) is given by:

$$\alpha_h^i = \dot{\alpha}^i + \sum_m \dot{\alpha}^{mi} \theta_h^m = \alpha_0^i S_{hT}^0 + \sum_m \alpha^{mi} S_{hT}^0 \theta_h^m \, , \tag{11a}$$

$$\beta_h^i = \dot{\beta}^i + \sum_k \dot{\beta}^{ki} \theta_h^k = \beta_0^i S_{hT}^x + \sum_k \beta^{ki} S_{hT}^x \theta_h^k \text{, and} \tag{11b}$$

$$\gamma_h^{ij} = \dot{\gamma}^{ij} + \sum_f \dot{\gamma}_f^{ij} \theta_h^{jf} = \gamma_0^{ij} S_{hT}^{p^j} + \sum_f \gamma_f^{ij} S_{hT}^{p^j} \theta_h^{jf} \, . \tag{11c}$$

This means that the coefficients to be estimated are the behavioral parameters multiplied by the aggregation factors describing the distribution of prices and income over time. If S_{hT}^{0}, $S_{hT}^{p^j}$, and S_{hT}^{x} differ from one, i.e., if prices and/or income vary during the year, we will experience structural aggregation bias using annual cross-section data to estimate behavioral parameters. Thus, even if we may assume that there is no variation in demand over time, we still experience structural aggregation bias because of differences in prices and income during the period. The estimated parameters thus include structural components over time, and may not be given a pure behavioral interpretation. These structural effects on the coefficients are due to the nonlinearity of the AIDS system in prices and income. Thus, we may not interpret the estimated coefficients using annual micro data as behavioral coefficients, either with or without seasonal variation in demand, as long as prices and/or income change over time in a nonlinear demand model.

The next question is whether estimation using annual micro data yields consistent estimates of the time-aggregated micro parameters, even if the parameters in equation (5) may not be interpreted as behavioral parameters. The difference between the time-aggregated micro parameters in equations (10a)–(10c) and the parameters in equations (11a)–(11c) is given by:

$$\tilde{\alpha}_{hT}^{i} - \alpha_{h}^{i} = \sum_{s} \alpha^{si} \overline{\mu_{hT}^{s}} S_{hT}^{\mu^{s}} + \sum_{v} \alpha^{vi} \overline{\varphi_{T}^{v}} S_{T}^{\varphi^{v}} \quad , \tag{12a}$$

$$\tilde{\gamma}_{hT}^{ij} - \gamma_{h}^{ij} = \sum_{d} \gamma_{d}^{ij} \overline{\mu_{hT}^{jd}} S_{hT}^{\mu^{jd}} + \sum_{u} \gamma_{u}^{ij} \overline{\varphi_{T}^{u}} S_{T}^{\varphi^{u}} \text{ , and} \tag{12b}$$

$$\tilde{\beta}_{hT}^{i} - \beta_{h}^{i} = \sum_{r} \beta^{ri} \overline{\mu_{hT}^{r}} S_{hT}^{\mu^{r}} + \sum_{w} \beta^{wi} \overline{\varphi_{T}^{w}} S_{T}^{\varphi^{w}} \ . \tag{12c}$$

The difference between the true underlying time-aggregated micro parameters and the time-aggregated parameters in equation (5) is due to ignoring behavioral changes over time. Thus, the parameters in equation (5) may not be interpreted as the true underlying time-aggregated micro parameters if there are variations in demand over time.

Finally, we may ask if the estimation of equations (5) and (6) using annual micro data yields consistent estimates of the time-aggregated micro parameters in the estimation ($\dot{\alpha}^i$, $\dot{\alpha}^{mi}$, $\dot{\beta}^i$, $\dot{\beta}^{ki}$, $\dot{\gamma}^{ij}$, and $\dot{\gamma}_f^{ij}$), or whether the excluded seasonal variation creates biased estimates of the parameters in equations (6a)–(6c) also. The error term in this empirical model is given by:

$$\varepsilon_{hT}^i = \begin{cases} \overline{u_{hT}^i} \\ + \sum_s \alpha^{si} \overline{\mu_{hT}^s} S_{hT}^{\mu^s} + \sum_v \alpha^{vi} \overline{\varphi_T^v} S_{hT}^{\varphi^v} \\ + \sum_j \left[\sum_d \gamma_d^{ij} \overline{\mu_{hT}^{jd}} S_{hT}^{\mu^{jd}} + \sum_u \gamma_u^{ij} \overline{\varphi_T^{ju}} S_{hT}^{\varphi^{ju}} \right] \log\left(p_{ht}^j\right) \\ + \left[\sum_r \beta^{ri} \overline{\mu_{hT}^r} S_{hT}^{\mu^r} + \sum_w \beta^{wi} \overline{\varphi_T^w} S_{hT}^{\varphi^w} \right] \log\left(\breve{x}_{ht}\right) \end{cases} \tag{13}$$

where $\overline{u_{hT}^i} = \frac{1}{T}\sum_{t=1}^{T} u_{ht}^i$ is the mean error term in year T. The last three lines of the error term result from excluded variables describing the heterogeneity over time in the consumption level and in price and income responses. Excluded time heterogeneity in the intercept (second line in the error term) biases the results if they are correlated with price or income. Excluded heterogeneity over time in price and income responses (the last two lines in the error term) always biases the estimated coefficients, as it introduces a dependence on prices and income directly into the error term, and causes the error term to be correlated with included regressors (prices and income). Thus, estimation using annual cross-section micro data does not yield consistent estimates of the time-aggregated parameters in the estimation of equations (5) and (6) as long as there are seasonal variations in demand.

In conclusion, estimated parameters on annual micro data may be interpreted neither as the true underlying time-aggregated micro parameters ($\tilde{\alpha}^i_{hT}$, $\tilde{\gamma}^{ij}_{hT}$, and $\tilde{\beta}^i_{hT}$) because of excluded heterogeneity over time, nor as behavioral parameters (α^i_0 , α^{mi} , α^{si} , α^{vi} , γ^{ij}_0 , γ^{ij}_f , γ^{ij}_d , γ^{ij}_u , β^i_0 , β^{ki} , β^{ri} , and β^{wi}) because of the structural components in the parameters. The time-aggregated parameters to be estimated ($\dot{\alpha}^i$, $\dot{\alpha}^{mi}$, $\dot{\beta}^i$, $\dot{\beta}^{ki}$, $\dot{\gamma}^{ij}$, and $\dot{\gamma}^{ij}_f$) include elements of both behavioral and structural changes over time, but we are not able to separate the structural from the behavioral elements without micro information about time changes in demand response. Additionally, the estimation does not yield consistent estimates of the parameters $\dot{\alpha}^i$, $\dot{\alpha}^{mi}$, $\dot{\beta}^i$, $\dot{\beta}^{ki}$, $\dot{\gamma}^{ij}$, and $\dot{\gamma}^{ij}_f$ because of the misspecification bias.

V. Annual Macro Data

Suppose that only annual macro data are available, and that we use these to estimate demand behavior. As we have information about neither consumer heterogeneity nor time variation in demand, we assume that the coefficients are constant across consumers within the year. That is, we assume that all parameters contributing to the heterogeneity in the parameters in equations (2a)–(2c) are equal to zero. The annual aggregated budget share function in year *T* is then given by:

$$\omega^i_T = \alpha^i + \sum_j \gamma^{ij} \log\left(\overline{\overline{p^j_T}}\right) + \beta^i \log\left(\overline{\overline{\breve{x}_T}}\right) + \varepsilon^i_T \,, \tag{14}$$

where $\overline{\overline{p^j_T}}$ and $\overline{\overline{\breve{x}_T}}$ are the mean price of good *j* and mean real income in year *T*. The question is, what types of biases would we expect, and will the estimates for the parameters in (14) be consistent estimates of any interpretable parameters, e.g., annual macro parameters? To answer this question, we need an expression for the true underlying time-aggregated macro parameters, given the micro demand structure defined in equations (1) and (2), which is described in the next section.

V.A. Aggregation both across Consumers and over Time

We aggregate the macro budget share function over time to find, e.g., annual consumption of a particular good for the household sector. We assume that consumption is aggregated across all consumers *H* and over time during the year *T*. The time-aggregated macro budget share over period *T* is given by:[8]

$$w_T^i = \frac{\overline{\overline{q_T^i}}\,\overline{\overline{p_T^i}}}{\overline{\overline{x_T}}} . \tag{15}$$

To find an expression for the time-aggregated macro budget share, we calculate mean consumption in year *T*, $\overline{\overline{q_T^i}} = \frac{1}{T}\sum_{t=1}^{T}\overline{q_t^i} = \frac{1}{T}\sum_{t=1}^{T}\left\{\left[\tilde{\alpha}_t^i + \sum_j \tilde{\gamma}_t^{ij}\log\left(\overline{p_t^j}\right) + \tilde{\beta}_t^i \log\left(\overline{\breve{x}_t}\right)\right]\frac{\overline{x_t}}{\overline{\overline{p_t^i}}}\right\}$. Rearranging, we have the following consumer- and time-aggregated demand function (see the Appendix, Section A.III, for more detailed derivations):

$$\overline{\overline{q_T^i}} = \left[\begin{array}{l} \alpha_0^i \overline{S_T^0}\zeta_T^0 + \sum_m \alpha^{mi}\overline{\theta^m}\,\overline{S_T^{\theta^m}}\zeta_T^{\theta^m} + \sum_s \alpha^{si}\overline{\overline{\mu_T^s}}\,\overline{S_T^{\mu^s}}\zeta_T^{\mu^s} + \sum_v \alpha^{vi}\overline{\varphi_T^v}\,\overline{S_T^0}\zeta_T^{\varphi^v} \\ + \sum_j \left[\gamma_0^{ij}\overline{S_T^{p^j}}\zeta_T^{p^j} + \sum_f \gamma_f^{ij}\overline{\theta^{jf}}\,\overline{S_T^{\theta^{jf}}}\zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij}\overline{\overline{\mu_T^{jd}}}\,\overline{S_T^{\mu^{jd}}}\zeta_T^{\mu^{jd}} + \sum_u \gamma_u^{ij}\overline{\varphi_T^{ju}}\,\overline{S_T^{p^j}}\zeta_T^{\varphi^{ju}}\right]\log\left(\overline{\overline{p_T^j}}\right) \\ + \left[\beta_0^i \overline{S_T^x}\zeta_T^x + \sum_k \beta^{ki}\overline{\theta^k}\,\overline{S_T^{\theta^k}}\zeta_T^{\theta^k} + \sum_r \beta^{ri}\overline{\overline{\mu_T^r}}\,\overline{S_T^{\mu^r}}\zeta_T^{\mu^r} + \sum_w \beta^{wi}\overline{\varphi_T^w}\,\overline{S_T^x}\zeta_T^{\varphi^w}\right]\log\left(\overline{\overline{\breve{x}_T}}\right) \end{array}\right]\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}} . \tag{16}$$

The time-aggregated macro demand function is a function of the behavioral parameters (α_0^i, α^{mi}, α^{si}, α^{vi}, γ_0^{ij}, γ_f^{ij}, γ_d^{ij}, γ_u^{ij}, β_0^i, β^{ki}, β^{ri}, and β^{wi}), arithmetic mean values across individuals and over time for total expenditure ($\overline{\overline{x_T}}$), total real expenditure ($\overline{\overline{\breve{x}_T}}$), prices ($\overline{\overline{p_T^j}}$, $\overline{\overline{p_T^i}}$), characteristics ($\overline{\varphi_T}$, $\overline{\overline{\mu_T}}$, and $\overline{\theta}$), the period mean of the aggregation factors S_t describing the distribution of income, prices, and characteristics across consumers ($\overline{S_T}$), and the aggregation factors describing the distribution of income, prices, characteristics, and the aggregation factors S_t across consumers and over time (ζ_T). See the Appendix for more information.

The aggregation factors (S_t) measure the distribution of prices, total expenditure, total real expenditure, and consumer characteristics across consumers at a particular point in time *t*. If all variables are equal for all individuals, the aggregation factors equal one.[9] The aggregation factors may be interpreted as weights measuring the relative importance for the macro consumption of consumers with different behaviors. The aggregation factors ζ_T depend on the distribution of the mean of the prices, total expenditure, total real expenditure, consumer characteristics, and the aggregation factors S_t over the period *T*. These aggregation factors may be interpreted as weights measuring the relative importance for aggregate consumption of differences in behavior at different points in time within the year *T*.

Inserting the expression for mean consumption over period *T* in equation (16) into the expression for the macro budget share in equation (15), we can write the time-aggregated macro budget share as a

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[8] Hereafter, 'double bar' denotes the global mean, i.e., the arithmetic mean over both dimensions; e.g., $\overline{\overline{q^i}} = \frac{1}{T}\sum_{t=1}^{T}\overline{q^i_{\cdot t}}$

[9] The symmetry property is due to the linearity in the specification of heterogeneity in behavior across consumers over time, described in equations (2a)–(2d).

function of the time-aggregated macro parameters, mean prices, and mean total real expenditure in period *T*:

$$w_T^i = \tilde{\tilde{\alpha}}_T^i + \sum_j \tilde{\tilde{\gamma}}_T^{ij} \log\left(\overline{\overline{p_T^j}}\right) + \tilde{\tilde{\beta}}_T^i \log\left(\overline{\overline{x_T}}\right), \tag{17}$$

where the relationship between the behavioral and time-aggregated macro parameters in period *T* is given by:

$$\tilde{\tilde{\alpha}}_T^i = \alpha_0^i \overline{S_T^0} \zeta_T^0 + \sum_m \alpha^{mi} \overline{\theta^m} \overline{S_T^{\theta^m}} \zeta_T^{\theta^m} + \sum_s \alpha^{si} \overline{\overline{\mu_T^s}} \overline{S_T^{\mu^s}} \zeta_T^{\mu^s} + \sum_v \alpha^{vi} \overline{\varphi_T^v} \overline{S_T^0} \zeta_T^{\varphi^v} , \tag{18a}$$

$$\tilde{\tilde{\gamma}}_T^{ij} = \gamma_0^{ij} \overline{S_T^{p^j}} \zeta_T^{p^j} + \sum_f \gamma_f^{ij} \overline{\theta^{jf}} \overline{S_T^{\theta^{jf}}} \zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij} \overline{\overline{\mu_T^{jd}}} \overline{S_T^{\mu^{jd}}} \zeta_T^{\mu^{jd}} + \sum_u \gamma_u^{ij} \overline{\varphi_T^{ju}} \overline{S_T^{p^j}} \zeta_T^{\varphi^{ju}} \text{, and} \tag{18b}$$

$$\tilde{\tilde{\beta}}_T^i = \beta_0^i \overline{S_T^x} \zeta_T^x + \sum_k \beta^{ki} \overline{\theta^k} \overline{S_T^{\theta^k}} \zeta_T^{\theta^k} + \sum_r \beta^{ri} \overline{\overline{\mu_T^r}} \overline{S_T^{\mu^r}} \zeta_T^{\mu^r} + \sum_w \beta^{wi} \overline{\varphi_T^w} \overline{S_T^x} \zeta_T^{\varphi^w} . \tag{18c}$$

We see from equations (18a)–(18c) that the parameters of the time-aggregated macro demand function in year *T* depend on the behavioral parameters (α_0^i, α^{mi}, α^{si}, α^{vi}, γ_0^{ij}, γ_f^{ij}, γ_d^{ij}, γ_u^{ij}, β_0^i, β^{ki}, β^{ri}, and β^{wi}) and structural components such as the mean value of variables describing heterogeneity in behavior across consumers and over time, the mean distribution of variables describing the heterogeneity in behavior across consumers in year *T* (represented by the mean *S*), and the distribution of variables describing the heterogeneity in behavior during the year (represented by ζ s). (See the Appendix, Sections A.I and A.III, for more information.)

V.B. Discussion of Bias

As in the case of annual micro data, the aggregations across consumers and over time imply structural components in the time-aggregated macro parameters. Thus, the parameters in equation (14) may not be interpreted as behavioral parameters. The next question is whether the parameters in equation (14) may be interpreted as behavioral parameters under the assumption that there is no variation in demand across consumers over time. Under these assumptions, the relationship between the estimated demand structure in equation (14) and the true underlying demand structure is given by:

$$\alpha^i = \alpha_0^i \overline{S_T^0} \zeta_T^0 \, , \tag{19a}$$

$$\beta^i = \gamma_0^{ij} \overline{S_T^{p^j}} \, \zeta_T^{p^j} \text{ , and} \tag{19b}$$

$$\gamma^{ij} = \beta_0^i \overline{S_T^x} \zeta_T^x \, . \tag{19c}$$

This means that, given these assumptions, the coefficients are a product of behavioral parameters, and the aggregation factors describing the distribution of prices and income over time and across consumers. If the aggregation factors differ from one, we experience structural aggregation bias using annual macro data to estimate behavioral parameters. Thus, even if the assumption that there are no variations in demand across consumers and over time were true, we would experience structural aggregation bias because of differences in prices and income across consumers and over time.[10] Thus, we may not interpret the estimated coefficients using annual macro data as estimating behavioral coefficients, even if consumers are homogeneous and/or there is no seasonal variation in demand.

The next question is whether estimating the parameters in equation (14) gives consistent estimates of the time-aggregated macro parameters, even if it may not be given a pure behavioral interpre-

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[10] These structural effects on the coefficients are due to the nonlinearity of the AIDS system. If the demand function was linear, and there is no heterogeneity in demand across consumers and over time, we are in the case of perfect linear aggregation (Forni and Brighi 1991).

tation. The difference between the time-aggregated macro parameters and the parameters in equations (19a)–(19c) is given by:

$$\tilde{\tilde{\alpha}}_T^i - \alpha^i = \sum_m \alpha^{mi} \overline{\theta^m} \, \overline{S_T^{\theta^m}} \zeta_T^{\theta^m} + \sum_s \alpha^{si} \overline{\overline{\mu_T^s}} \, \overline{S_T^{\mu^s}} \zeta_T^{\mu^s} + \sum_v \alpha^{vi} \overline{\varphi_T^v} \, \overline{S_T^0} \zeta_T^{\varphi^v} \quad , \tag{20a}$$

$$\tilde{\tilde{\gamma}}_T^{ij} - \gamma^{ij} = \sum_f \gamma_f^{ij} \overline{\theta^{jf}} \, \overline{S_T^{\theta^{jf}}} \zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij} \overline{\overline{\mu_T^{jd}}} \, \overline{S_T^{\mu^{jd}}} \zeta_T^{\mu^{jd}} + \sum_u \gamma_u^{ij} \overline{\varphi_T^{ju}} \, \overline{S_T^{p^j}} \zeta_T^{\varphi^{ju}} \quad \text{, and} \tag{20b}$$

$$\tilde{\tilde{\beta}}_T^i - \beta^i = \sum_k \beta^{ki} \overline{\theta^k} \, \overline{S_T^{\theta^k}} \zeta_T^{\theta^k} + \sum_r \beta^{ri} \overline{\overline{\mu_T^r}} \, \overline{S_T^{\mu^r}} \zeta_T^{\mu^r} + \sum_w \beta^{wi} \overline{\varphi_T^w} \, \overline{S_T^x} \zeta_T^{\varphi^w} \quad . \tag{20c}$$

The difference between the true underlying time-aggregated macro parameters and the parameters in equation (14) is due to ignoring behavioral differences across consumers and over time. Thus, estimations using annual macro data may not be interpreted as the true underlying time-aggregated macro parameters.

The final question is whether the estimation yields consistent estimates of the parameters in equation (14), even if we are not able to separate the structural and the behavioral components of these parameters. The answer is no, because we may experience misspecification bias from excluded variables describing the heterogeneity in demand across consumers and over time. These elements end up in the stochastic error term (ε_T^i) and makes it correlated with prices and income. The error term in the estimation using annual macro data is given by:

$$\varepsilon_T^i = \left(\begin{array}{l} \overline{\overline{u_T^i}} \\ + \sum_m \alpha^{mi} \overline{\theta^m} \, \overline{S_T^{\theta^m}} \, \zeta_T^{\theta^m} + \sum_s \alpha^{si} \overline{\overline{\mu_T^s}} \, \overline{S_T^{\mu^s}} \, \zeta_T^{\mu^s} + \sum_v \alpha^{vi} \overline{\varphi_T^v} \, \overline{S_T^0} \zeta_T^{\varphi^v} \\ + \sum_j \left[+ \sum_f \gamma_f^{ij} \overline{\theta^{jf}} \, \overline{S_T^{\theta^{jf}}} \, \zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij} \overline{\overline{\mu_T^{jd}}} \, \overline{S_T^{\mu^{jd}}} \, \zeta_T^{\mu^{jd}} + \sum_u \gamma_u^{ij} \overline{\varphi_T^{ju}} \, \overline{S_T^{p^j}} \, \zeta_T^{\varphi^{ju}} \right] \log\left(\overline{\overline{p_T^j}} \right) \\ + \left[\sum_k \beta^{ki} \overline{\theta^k} \, \overline{S_T^{\theta^k}} \, \zeta_T^{\theta^k} + \sum_r \beta^{ri} \overline{\overline{\mu_T^r}} \, \overline{S_T^{\mu^r}} \, \zeta_T^{\mu^r} + \sum_w \beta^{wi} \overline{\varphi_T^w} \, \overline{S_T^x} \zeta_T^{\varphi^w} \right] \log\left(\overline{\overline{\breve{x}_T}} \right) \end{array} \right., \qquad (21)$$

where $\overline{\overline{u_T^i}} = \frac{1}{H} \sum_{h=1}^{H} \overline{u_{hT}^i}$ is the global mean error term during period *T*.

As for the case of annual micro data, results from an estimation using annual macro data may not be interpreted as neither behavioral nor the true underlying time-aggregated macro parameters because of the structural biases. Neither will they be consistent estimates of the parameters in the empirical model because of the misspecification bias.

Note that both the misspecification and the structural biases may create autoregressive errors in an estimation using annual macro data. This may happen because income and prices in one period are likely to be correlated with income and prices in the next period, or if the aggregation factors and/or mean consumer characteristics change over time. Thus, in this model, correcting the error term for autocorrelation picks up effects of excluded seasonal variations or changes in the distribution of prices, income, and other variables over time. This implies that autoregressive error terms do not necessarily mean that the true underlying demand structure is dynamic, but may just as well be a result of structural components and misspecification biases in the estimation.

Note also that changes in the aggregation factors over time, caused, e.g., by changes in the price and/or income distribution, may create dynamic components in the time-aggregated *consumption* even in a static micro model. For example, assume that the distribution of electricity prices during a year changes over time, increasing the difference between low summer and high winter prices, but leaving the annual mean price unaffected. This shifts the consumption pattern during the year, reducing winter consumption and increasing summer consumption. This change in the annual consumption pattern is likely to affect annual consumption even if the annual mean price does not change, because the electricity consumption for space heating is higher during the winter. Looking at annual data, consumption may change from one year to another even if annual prices and income are unchanged. The problem is that even if annual prices are stationary over time, weekly or daily prices may not be. This has implications for the estimation of time-aggregated parameters based on annual macro data, as the annual means dilute the true underlying correlation between price and consumption over time.

VI. Conclusions

In this paper, we have discussed different types of bias resulting from ignoring heterogeneity in demand across consumers and over time: a misspecification bias resulting from a correlation between the deterministic component and the error term, and an aggregation bias due to structural components in the aggregated parameters. Depending on the type of data available (annual cross-section micro data, annual macro data, micro panel data, etc.), and how the estimated model accounts for heterogeneity in behavior across consumers and over time, we may experience both structural aggregation bias and misspecification bias in estimated coefficients.

We show that ignoring heterogeneity in demand response to price and income change, either across consumers or over time, creates *misspecification bias* in the estimated coefficients, also in static models. This is because neglected heterogeneity leads to there being price and income effects in the error term. The more heterogeneity that is unaccounted for, the more are sources of misspecification bias introduced into the estimation (*ceteris paribus*). Thus, consistent estimates of the key parameters in the model are not obtained. This misspecification bias occurs at all levels of aggregation in the data, and is due to the omission of relevant variables. This exclusion of heterogeneity in demand may also create autoregressive error terms.

We also see that estimations using aggregated data cannot be given a pure behavioral interpretation, as it creates structural aggregation bias. The structural aggregation bias increases with the level of aggregation. Furthermore, omitting variables describing the heterogeneity in demand, either across consumers or over time, in the estimation using aggregate data makes the parameters to be estimated different from the true underlying aggregated parameters. Finally, changes in the structural components describing the distributions of price, income, and other relevant variables over time may create dynamic components in aggregated data even if there are no dynamic elements in the micro functions. This is because changes in the distributions of relevant variables, e.g., prices and income, may change aggregate demand even if the mean prices and income do not change. Likewise, if the distribution of consumers with different characteristics changes over time, aggregate demand changes even if the behavior of consumers within each group does not change. If these structural components are excluded from estimations using aggregate data, the error terms become autoregressive.

In a series of papers, Pesaran and his colleagues have discussed the misspecification bias that occurs in dynamic models if we exclude dynamic components in estimations using heterogeneous panels (see Pesaran and Smith 1995). This paper shows that misspecification bias is just as problematic in static models. It is not only the exclusion of dynamic components that creates this bias, but also excluded variables describing heterogeneity across consumers and over time in the response to price and income changes. This implies that excluded dynamic components are not the only cause of autoregressive residuals in estimations. This is important to bear in mind when interpreting results from estimations corrected for autocorrelation. It may be difficult, when only macro data are available, to distinguish between excluded heterogeneity and excluded dynamic components in the micro function. A good example of this is in the analyses of Heien (2001), who shows that much of what previously has been interpreted as habit may be time variation in demand.[11]

[11] Habit introduces dynamic components in the demand function, whereas time variation in demand is purely static.

In this paper, we apply explicit functional forms to illustrate the different biases that occur when estimating using aggregate data when the underlying demand structure changes across consumers and over time. This makes it easier to interpret the different sources of bias. We use an AIDS model as it is sufficiently complicated to illustrate different potential sources of bias and it is often used in the literature on aggregation across consumers. However, as shown in the theoretical literature, problems with aggregation of heterogeneous demand functions, both across consumers and/or over time, are general for all types of functional forms, unless the underlying demand structure allows for exact linear aggregation.[12] That is, unless all consumers actually react identically to all changes in prices and income, face the same prices, and have the same underlying *linear* demand structure without any (seasonal) variation in demand over time, different combinations of these biases occur, depending on the heterogeneity in the underlying demand structure and the type of data available.

This has implications for the interpretation and use of results from empirical analyses, especially using aggregated data (including annual cross-sectional micro data), but also on micro panel data. First, one should always try to obtain as much information as possible about variables that may create heterogeneity in consumption across consumers and over time, to reduce the sources of misspecification bias due to omission of relevant variables. Second, it is important to be careful with the interpretation of estimated parameters based on aggregate data, as they will always be a mixture of behavioral and structural components. Thus, if the structural changes in demand are expected to be large, because of differences in demand across consumers and/or over time, the use of such parameters to predict changes in demand, e.g., due to policy changes, may be problematic. This implies that the less aggregated data and the more information about explanatory variables that is available, the easier it is to interpret the results. However, when micro estimates are used to predict the aggregate effect on demand due to a policy instrument, we need the predictions at a micro level (micro simulations) and then

[12] Some biases are avoided in the case of perfect nonlinear aggregation, but not all, because we normally allow income to vary across consumers even if prices are not allowed to vary. This implies that, as long as the income distribution does not change over time, aggregation biases are avoided (Deaton and Muellbauer 1980; Forni and Brighi 1991). However, perfect nonlinear aggregation does not exclude problems with disaggregation biases (see Halvorsen 2006).

to sum up. The reason for this disaggregation bias is that the micro and the aggregated parameters differ if demand differs across consumers and/or over time. In most cases, we need to live with these problems, failing to have better data. It is, however, important to be aware of the potential problems to avoid misinterpretations and misspecification bias.

Appendix: Aggregation

In this Appendix, we show how to aggregate the true underlying micro demand structure described in Section 2 across consumers, over time, and both across consumers and over time.

A.I. Aggregation across Consumers

We start by aggregating across consumers to find the macro budget share at a given point in time t. The macro budget share at time t, measured in terms of the arithmetic mean values of all variables, is defined as:[13]

$$w_t^i = \frac{\overline{q_t^i}\,\overline{p_t^i}}{\overline{x_t}}. \tag{A1}$$

The mean consumption at time t, $\overline{q_t^i} = \frac{1}{H}\sum_h \left\{ \left[\alpha_{ht}^i + \sum_j \gamma_{ht}^{ij} \log\left(p_{ht}^j\right) + \beta_{ht}^i \log\left(\breve{x}_{ht}\right) \right] \frac{x_{ht}}{p_{ht}^i} \right\}$,

given the demand structure in equations (1) and (2), is used to find an expression for aggregate demand. To simplify the calculation, we calculate the effect on the intercept, the price effects, and the income effects separately. First, we aggregate the intercept in the micro function, resulting in the term forming the macro intercept parameter in period t ($\tilde{\alpha}_t^i$):

$$\frac{1}{H}\sum_h \alpha_{ht}^i \frac{x_{ht}}{p_{ht}^i} = \frac{1}{H}\sum_h \left[\left(\alpha_0^i + \sum_m \alpha^{mi}\theta_h^m + \sum_s \alpha^{si}\mu_{ht}^s + \sum_v \alpha^{vi}\varphi_t^v \right) \frac{x_{ht}}{p_{ht}^i} \right]$$

[13] Hereafter, 'bar' denotes the arithmetic mean, e.g., $\overline{q_t^i} = \frac{1}{H}\sum_h q_{ht}^i$.

$$
= \left(\begin{array}{l} \alpha_0^i \dfrac{1}{H} \sum_h \dfrac{x_{ht}}{p_{ht}^i} + \dfrac{1}{H} \sum_h \sum_m \alpha^{mi} \theta_h^m \dfrac{x_{ht}}{p_{ht}^i} \dfrac{\overline{\theta^m}}{\overline{\theta^m}} + \dfrac{1}{H} \sum_h \sum_s \alpha^{si} \mu_{ht}^s \dfrac{x_{ht}}{p_{ht}^i} \dfrac{\overline{\mu_t^s}}{\overline{\mu_t^s}} \\ + \sum_v \alpha^{vi} \varphi_t^v \dfrac{1}{H} \sum_h \dfrac{x_{ht}}{p_{ht}^i} \end{array} \right) \frac{\overline{p_t^i}}{\overline{x_t}} \frac{\overline{x_t}}{\overline{p_t^i}}
$$

$$
= \left[\begin{array}{l} \alpha_0^i \dfrac{1}{H} \sum_h \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} + \sum_m \alpha^{mi} \overline{\theta^m} \dfrac{1}{H} \sum_h \dfrac{\theta_h^m}{\overline{\theta^m}} \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} + \sum_s \alpha^{si} \overline{\mu_t^s} \dfrac{1}{H} \sum_h \dfrac{\mu_{ht}^s}{\overline{\mu_t^s}} \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} \\ + \sum_v \alpha^{vi} \varphi_t^v \dfrac{1}{H} \sum_h \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} \end{array} \right] \frac{\overline{x_t}}{\overline{p_t^i}}
$$

$$
= \left[\alpha_i^0 S_t^0 + \sum_m \alpha^{mi} \overline{\theta^m} S_t^{\theta^m} + \sum_s \alpha^{si} \overline{\mu_t^s} S_t^{\mu^s} + \sum_v \alpha^{vi} \varphi_t^v S_t^0 \right] \frac{\overline{x_t}}{\overline{p_t^i}} = \widetilde{\alpha}_t^i \frac{\overline{x_t}}{\overline{p_t^i}} .
$$

Second, we aggregate the price effects, resulting in the term including the macro price parameter in period t ($\widetilde{\gamma}_t^{ij}$):

$$
\frac{1}{H} \sum_h \sum_j \gamma_{ht}^{ij} \log\left(p_{ht}^j\right) \frac{x_{ht}}{p_{ht}^i} = \frac{1}{H} \sum_h \sum_j \left(\gamma_0^{ij} + \sum_f \gamma_f^{ij} \theta_h^{jf} + \sum_d \gamma_d^{ij} \mu_{ht}^{jd} + \sum_u \gamma_u^{ij} \varphi_t^{ju} \right) \log\left(p_{ht}^j\right) \frac{x_{ht}}{p_{ht}^i}
$$

$$
= \left\{ \sum_j \left[\begin{array}{l} \gamma_0^{ij} \dfrac{1}{H} \sum_h \log\left(p_{ht}^j\right) \dfrac{x_{ht}}{p_{ht}^i} + \sum_f \gamma_f^{ij} \dfrac{1}{H} \sum_h \theta_h^{jf} \log\left(p_{ht}^j\right) \dfrac{x_{ht}}{p_{ht}^i} \dfrac{\overline{\theta^{jf}}}{\overline{\theta^{jf}}} \\ + \sum_d \gamma_d^{ij} \dfrac{1}{H} \sum_{h=1} \mu_{ht}^{jd} \log\left(p_{ht}^j\right) \dfrac{x_{ht}}{p_{ht}^i} \dfrac{\overline{\mu_t^{jd}}}{\overline{\mu_t^{jd}}} + \sum_u \gamma_u^{ij} \varphi_t^{ju} \dfrac{1}{H} \sum_h \log\left(p_{ht}^j\right) \dfrac{x_{ht}}{p_{ht}^i} \end{array} \right] \frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{p_t^j}\right)} \right\} \frac{\overline{x_t}}{\overline{p_t^i}} \frac{\overline{p_t^i}}{\overline{x_t}}
$$

$$
= \left\{ \sum_j \left[\begin{array}{l} \gamma_0^{ij} \dfrac{1}{H} \sum_h \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} \dfrac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_t^j}\right)} + \sum_{f=1} \gamma_f^{ij} \overline{\theta^{jf}} \dfrac{1}{H} \sum_h \dfrac{\theta_h^{jf}}{\overline{\theta^{jf}}} \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} \dfrac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_t^j}\right)} \\ + \sum_d \gamma_d^{ij} \overline{\mu_t^{jd}} \dfrac{1}{H} \sum_h \dfrac{\mu_{ht}^{jd}}{\overline{\mu_t^{jd}}} \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} \dfrac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_t^j}\right)} + \sum_u \gamma_u^{ij} \varphi_t^{ju} \dfrac{1}{H} \sum_h \dfrac{x_{ht}}{\overline{x_t}} \dfrac{\overline{p_t^i}}{p_{ht}^i} \dfrac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_t^j}\right)} \end{array} \right] \log\left(\overline{p_t^j}\right) \right\} \frac{\overline{x_t}}{\overline{p_t^i}}
$$

$$
=\left\{\sum_j\left[\gamma_{ij}^0\, S_t^{p^j} + \sum_f \gamma_f^{ij}\,\overline{\theta^{ij}}\, S_t^{\theta^{ijf}} + \sum_d \gamma_d^{ij}\,\overline{\mu_t^{jd}}\, S_t^{\mu^{jd}} + \sum_u \gamma_u^{ij}\varphi_t^{ju} S_t^{p^j}\right]\log\left(\overline{p_t^j}\right)\right\}\frac{\overline{x_t}}{\overline{p_t^i}} = \left\{\sum_j \widetilde{\gamma}_t^{ij}\,\log\left(\overline{p_t^j}\right)\right\}\frac{\overline{x_t}}{\overline{p_t^i}}\,.
$$

Finally, we aggregate the total real expenditure effects, resulting in the term forming the macro total real expenditure parameter in period t ($\widetilde{\beta}_t^i$):

$$
\frac{1}{H}\sum_h \beta_{ht}^i \log\left(\breve{x}_{ht}\right)\frac{x_{ht}}{p_{ht}^i} \;=\; \frac{1}{H}\sum_h\left(\beta_0^i + \sum_k \beta^{ki}\theta_h^k + \sum_r \beta^{ri}\mu_{ht}^r + \sum_w \beta^{wi}\varphi_t^w\right)\log\left(\breve{x}_{ht}\right)\frac{x_{ht}}{p_{ht}^i}
$$

$$
=\left[\begin{array}{l}
\beta_0^i\,\dfrac{1}{H}\displaystyle\sum_h \log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p_{ht}^i} + \displaystyle\sum_k \beta^{ki}\,\dfrac{1}{H}\displaystyle\sum_h \theta_h^k \log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p_{ht}^i}\,\dfrac{\overline{\theta^k}}{\overline{\theta^k}} \\
+\displaystyle\sum_r \beta^{ri}\,\dfrac{1}{H}\displaystyle\sum_h \mu_{ht}^r \log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p_{ht}^i}\,\dfrac{\overline{\mu_t^r}}{\overline{\mu_t^r}} + \displaystyle\sum_w \beta^{wi}\varphi_t^w\,\dfrac{1}{H}\displaystyle\sum_h \log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p_{ht}^i}
\end{array}\right]\frac{\overline{x_t}}{\overline{p_t^i}}\,\frac{\overline{p_t^i}}{\overline{x_t}}\,\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\breve{x}_t}\right)}
$$

$$
=\left[\begin{array}{l}
\beta_0^i\,\dfrac{1}{H}\displaystyle\sum_h \dfrac{x_{ht}}{\overline{x_t}}\,\dfrac{\overline{p_t^i}}{p_{ht}^i}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_t}\right)} + \displaystyle\sum_k \beta^{ki}\,\overline{\theta^k}\,\dfrac{1}{H}\displaystyle\sum_h \dfrac{\theta_h^k}{\overline{\theta^k}}\,\dfrac{x_{ht}}{\overline{x_t}}\,\dfrac{\overline{p_t^i}}{p_{ht}^i}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_t}\right)} \\
+\displaystyle\sum_r \beta^{ri}\,\overline{\mu_t^r}\,\dfrac{1}{H}\displaystyle\sum_h \dfrac{\mu_{ht}^r}{\overline{\mu_t^r}}\,\dfrac{x_{ht}}{\overline{x_t}}\,\dfrac{\overline{p_t^i}}{p_{ht}^i}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_t}\right)} + \displaystyle\sum_w \beta^{wi}\varphi_t^w\,\dfrac{1}{H}\displaystyle\sum_h \dfrac{x_{ht}}{\overline{x_t}}\,\dfrac{\overline{p_t^i}}{p_{ht}^i}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_t}\right)}
\end{array}\right]\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right)
$$

$$
=\left[\beta_0^i S_t^x + \sum_k \beta^{ki}\,\overline{\theta^k}\, S_t^{\theta^k} + \sum_r \beta^{ri}\,\overline{\mu_t^r}\, S_t^{\mu^r} + \sum_w \beta^{wi}\varphi_t^w S_t^x\right]\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right) = \widetilde{\beta}_t^i\,\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right).
$$

Rearranging, we have the following macro demand function at time t:

$$\overline{q^i_{\cdot t}} = \left[\begin{array}{l} \alpha^i_0 S^0_t + \sum_m \alpha^{mi} \overline{\theta^m} S^{\theta^m}_t + \sum_s \alpha^{si} \overline{\mu^s_t} S^{\mu^s}_t + \sum_v \alpha^{vi} \varphi^v_t S^0_t \\ + \sum_j \left(\gamma^{ij}_0 S^{p^j}_t + \sum_f \gamma^{ij}_f \overline{\theta^{jf}} S^{\theta^{jf}}_t + \sum_d \gamma^{ij}_d \overline{\mu^{jd}_t} S^{\mu^{jd}}_t + \sum_u \gamma^{ij}_u \varphi^u_t S^{p^j}_t \right) \log\left(\overline{p^j_t}\right) \\ + \left(\beta^i_0 S^x_t + \sum_k \beta^{ki} \overline{\theta^k} S^{\theta^k}_t + \sum_r \beta^{ri} \overline{\mu^r_t} S^{\mu^r}_t + \sum_w \beta^{wi} \varphi^w_t S^x_t \right) \log\left(\breve{\overline{x}}_t\right) \end{array} \right] \frac{\overline{x_t}}{\overline{p^i_t}}, \tag{A2}$$

where the aggregation factors at time t (S_t) are defined by:

$$S^0_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} , \tag{A3a}$$

$$S^{\theta^m}_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\theta^m_h}{\theta^m} , \tag{A3b}$$

$$S^{\mu^s}_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\mu^s_{ht}}{\mu^s_t} , \tag{A3c}$$

$$S^{p^j}_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\log\left(p^j_{ht}\right)}{\log\left(\overline{p^j_t}\right)} , \tag{A3d}$$

$$S^{\theta^{jf}}_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\log\left(p^j_{ht}\right)}{\log\left(\overline{p^j_t}\right)} \frac{\theta^{jf}_h}{\theta^{jf}} , \tag{A3e}$$

$$S^{\mu^{jd}}_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\log\left(p^j_{ht}\right)}{\log\left(\overline{p^j_t}\right)} \frac{\mu^{jd}_{ht}}{\mu^{jd}_t} , \tag{A3f}$$

$$S^x_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\log\left(\breve{x}_{ht}\right)}{\log\left(\breve{\overline{x}}_t\right)} , \tag{A3g}$$

$$S^{\theta^k}_t = \frac{1}{H} \sum_{h=1}^{H} \frac{x_{ht}}{x_t} \frac{\overline{p^i_t}}{p^i_{ht}} \frac{\log\left(\breve{x}_{ht}\right)}{\log\left(\breve{\overline{x}}_t\right)} \frac{\theta^k_h}{\theta^k} , \text{ and} \tag{A3h}$$

$$S_t^{\mu^r} = \frac{1}{H}\sum_{h=1}^{H}\frac{x_{ht}}{\bar{x}_t}\frac{\overline{p_t^i}}{p_{ht}^i}\frac{\log(\breve{x}_{ht})}{\log(\bar{\breve{x}}_t)}\frac{\mu_{ht}^r}{\overline{\mu_t^r}}. \tag{A3i}$$

Inserting the expression for mean consumption into the expression for the macro budget share in (A1), we can write the macro budget share at time *t* as a function of the macro parameters ($\tilde{\alpha}_t^i$, $\tilde{\gamma}_t^{ij}$, and $\tilde{\beta}_t^i$), mean prices, and mean total real expenditure:

$$w_t^i = \tilde{\alpha}_t^i + \sum_j \tilde{\gamma}_t^{ij}\log\left(\overline{p_t^j}\right) + \tilde{\beta}_t^i \log\left(\bar{\breve{x}}_t\right), \tag{A4}$$

where the relationships between the micro and macro parameters at time *t* are given by:

$$\tilde{\alpha}_t^i = \alpha_0^i S^0 + \sum_m \alpha^{mi}\overline{\theta^m}S_t^{\theta^m} + \sum_s \alpha^{si}\overline{\mu_t^s}S_t^{\mu^s} + \sum_v \alpha^{vi}\varphi_t^v S_t^0, \tag{A5a}$$

$$\tilde{\gamma}_t^{ij} = \gamma_0^{ij} S_t^{p^j} + \sum_f \gamma_f^{ij}\overline{\theta^{jf}}S_t^{\theta^{jf}} + \sum_d \gamma_d^{ij}\overline{\mu_t^{jd}}\,S_t^{\mu^{jd}} + \sum_u \gamma_u^{ij}\varphi_t^u S_t^{p^j}\text{ , and} \tag{A5b}$$

$$\tilde{\beta}_t^i = \beta_0^i S_t^x + \sum_k \beta^{ki}\overline{\theta^k}\,S_t^{\theta^k} + \sum_r \beta^{ri}\overline{\mu_t^r}\,S_t^{\mu^r} + \sum_w \beta^{wi}\varphi_t^w S_t^x\,. \tag{A5c}$$

A.II. Aggregation over Time

Assume now that we aggregate over time to find the consumers' demand during a period (e.g., annual demand). We assume that consumption is aggregated over all *t* during the year *T*, but not necessarily over all time periods ($t = 1, \ldots, \ddot{T}$), e.g., if we have observations for ten years, the time period *T* may be a year, a season, or a month. This means that the time period *T* is a subset of time periods in the available time frame $\ddot{T}$. In our estimation using annual cross-sectional data, we assume that we only have information for one year ($T = \ddot{T}$).

The budget share aggregated over the time period *T*, measured in terms of the arithmetic mean values of all variables, is defined as:[14]

$$w_{hT}^{i} = \frac{\overline{q_{hT}^{i}}\ \overline{p_{hT}^{i}}}{\overline{x_{hT}}}. \tag{A6}$$

The mean consumption during period *T*, $\overline{q_{hT}^{i}} = \frac{1}{T}\sum_{t}\left\{\left[\alpha_{ht}^{i} + \sum_{j}\gamma_{ht}^{ij}\log\left(p_{ht}^{j}\right) + \beta_{ht}^{i}\log\left(\breve{x}_{ht}\right)\right]\frac{x_{ht}}{p_{ht}^{i}}\right\}$, given the demand structure in equations (1) and (2), is used to find an expression for time-aggregated demand. To simplify the calculation, we calculate the effect on the intercept, the price effects, and the income effect separately. First, we aggregate the intercept term in the micro function over time, resulting in the term forming the intercept parameter during period *T* ($\tilde{\alpha}_{hT}^{i}$):

$$\frac{1}{T}\sum_{t}\alpha_{ht}^{i}\frac{x_{ht}}{p_{ht}^{i}} = \frac{1}{T}\sum_{t}\left[\left(\alpha_{0}^{i} + \sum_{m}\alpha^{mi}\theta_{h}^{m} + \sum_{s}\alpha^{si}\mu_{ht}^{s} + \sum_{v}\alpha^{vi}\varphi_{t}^{v}\right)\frac{x_{ht}}{p_{ht}^{i}}\right]$$

$$= \begin{pmatrix} \alpha_{0}^{i}\frac{1}{T}\sum_{t}\frac{x_{ht}}{p_{ht}^{i}} + \sum_{m}\alpha^{mi}\theta_{h}^{m}\frac{1}{T}\sum_{t}\frac{x_{ht}}{p_{ht}^{i}} + \frac{1}{T}\sum_{t}\sum_{s}\alpha^{si}\mu_{ht}^{s}\frac{x_{ht}}{p_{ht}^{i}}\frac{\overline{\mu_{hT}^{s}}}{\overline{\mu_{hT}^{s}}} \\ + \sum_{v}\alpha^{vi}\frac{1}{T}\sum_{t}\varphi_{t}^{v}\frac{x_{ht}}{p_{ht}^{i}}\frac{\overline{\varphi_{T}^{v}}}{\overline{\varphi_{T}^{v}}} \end{pmatrix}\frac{\overline{p_{hT}^{i}}}{\overline{p_{hT}^{i}}}\frac{\overline{x_{hT}}}{\overline{x_{hT}}}$$

---

[14] 'Bar' denotes the arithmetic mean, e.g., $\overline{q_{hT}^{i}} = \frac{1}{T}\sum_{t=1}^{T}q_{ht}^{i}$.

$$
= \left[\begin{array}{l} \alpha_0^i \frac{1}{T}\sum_t \frac{x_{ht}}{x_{hT}} \frac{\overline{p_{hT}^i}}{p_{ht}^i} + \sum_m \alpha^{mi}\theta_h^m \frac{1}{T}\sum_t \frac{x_{ht}}{x_{hT}} \frac{\overline{p_{hT}^i}}{p_{ht}^i} + \sum_{s=1} \alpha^{si}\overline{\mu_{hT}^s} \frac{1}{T}\sum_t \frac{\mu_{ht}^s}{\mu_{hT}^s}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^i}}{p_{ht}^i} \\ + \sum_v \alpha^{vi}\overline{\varphi_T^v}\frac{1}{N}\sum_t \frac{\varphi_t^v}{\varphi_T^v}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^i}}{p_{ht}^i} \end{array} \right] \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}}
$$

$$
= \left[\alpha_i^0 S_{hT}^0 + \sum_m \alpha^{mi}\theta_h^m S_{hT}^0 + \sum_s \alpha^{si}\overline{\mu_{hT}^s} S_{hT}^{\mu^s} + \sum_v \alpha^{vi}\overline{\varphi_T^v} S_{hT}^{\varphi^v} \right] \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}} = \tilde{\alpha}_{hT}^i \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}} .
$$

Second, we aggregate the price effects, resulting in the term forming the macro price parameter during period T ($\tilde{\gamma}_{hT}^{ij}$):

$$
\frac{1}{T}\sum_t\sum_j \gamma_{ht}^{ij}\log\left(p_{ht}^j\right)\frac{x_{ht}}{p_{ht}^i} = \frac{1}{T}\sum_t\sum_j \left(\gamma_0^{ij} + \sum_f \gamma_f^{ij}\theta_h^{jf} + \sum_d \gamma_d^{ij}\mu_{ht}^{jd} + \sum_u \gamma_u^{ij}\varphi_t^{ju} \right)\log\left(p_{ht}^j\right)\frac{x_{ht}}{p_{ht}^i}
$$

$$
= \left\{ \sum_j \left[\begin{array}{l} \gamma_0^{ij}\frac{1}{T}\sum_t \log\left(p_{ht}^j\right)\frac{x_{ht}}{p_{ht}^i} + \sum_f \gamma_f^{ij}\theta_h^{jf}\frac{1}{T}\sum_t \log\left(p_{ht}^j\right)\frac{x_{ht}}{p_{ht}^i} \\ + \sum_d \gamma_d^{ij}\frac{1}{T}\sum_t \mu_{ht}^{jd}\log\left(p_{ht}^j\right)\frac{x_{ht}}{p_{ht}^i}\frac{\overline{\mu_{hT}^{jd}}}{\overline{\mu_{hT}^{jd}}} + \sum_u \gamma_u^{ij}\frac{1}{T}\sum_t \varphi_t^{ju}\log\left(p_{ht}^j\right)\frac{x_{ht}}{p_{ht}^i}\frac{\overline{\varphi_T^{ju}}}{\overline{\varphi_T^{ju}}} \end{array} \right] \frac{\log\left(\overline{p_{hT}^j}\right)}{\log\left(\overline{p_{hT}^j}\right)} \right\} \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}}\frac{\overline{p_{hT}^i}}{\overline{x_{hT}}}
$$

$$
= \left\{ \sum_j \left[\begin{array}{l} \gamma_0^{ij}\frac{1}{T}\sum_t \frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^i}}{p_{ht}^i}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{hT}^j}\right)} + \sum_{f=1}\gamma_f^{ij}\theta_h^{jf}\frac{1}{T}\sum_t \frac{x_{ht}}{x_{\cdot T}}\frac{\overline{p_{\cdot T}^i}}{p_{ht}^i}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{\cdot T}^j}\right)} \\ + \sum_{d=1}\gamma_d^{ij}\overline{\mu_{hT}^{jd}}\frac{1}{T}\sum_t \frac{\mu_{ht}^{jd}}{\mu_{hT}^{jd}}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^i}}{p_{ht}^i}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{hT}^j}\right)} + \sum_u \gamma_u^{ij}\overline{\varphi_T^{ju}}\frac{1}{T}\sum_t \frac{\varphi_t^{ju}}{\varphi_T^{ju}}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^i}}{p_{ht}^i}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{hT}^j}\right)} \end{array} \right] \log\left(\overline{p_{hT}^j}\right) \right\} \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}}
$$

$$
= \left\{ \sum_j \left[\gamma_0^{ij} S_{hT}^{p^j} + \sum_f \gamma_f^{ij}\theta_h^{jf} S_{hT}^{p^j} + \sum_d \gamma_d^{ij}\overline{\mu_{hT}^{jd}} S_{hT}^{\mu^{jd}} + \sum_u \gamma_u^{ij}\overline{\varphi_T^{ju}} S_{hT}^{\varphi^{ju}} \right] \log\left(\overline{p_{hT}^j}\right) \right\} \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}} = \left\{ \sum_j \tilde{\gamma}_{hT}^{ij}\log\left(\overline{p_{hT}^j}\right) \right\} \frac{\overline{x_{hT}}}{\overline{p_{hT}^i}}
$$

Finally, we aggregate the total real expenditure effects, forming the term including the macro total real expenditure parameter ($\tilde{\beta}_{hT}^i$):

$$\frac{1}{T}\sum_t \beta^i_{ht}\log\left(\breve{x}_{ht}\right)\frac{x_{ht}}{p^i_{ht}} \;=\; \frac{1}{T}\sum_t\left(\beta^i_0+\sum_k\beta^{ki}\theta^k_h+\sum_r\beta^{ri}\mu^r_{ht}+\sum_w\beta^{wi}\varphi^w_t\right)\log\left(\breve{x}_{ht}\right)\frac{x_{ht}}{p^i_{ht}}$$

$$=\left[\begin{array}{l}\beta^i_0\dfrac{1}{T}\displaystyle\sum_t\log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p^i_{ht}}+\sum_k\beta^{ki}\theta^k_h\dfrac{1}{T}\sum_t\log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p^i_{ht}}\\ +\displaystyle\sum_r\beta^{ri}\dfrac{1}{T}\sum_t\mu^r_{ht}\log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p^i_{ht}}\dfrac{\overline{\mu^r_{hT}}}{\overline{\mu^r_{hT}}}+\sum_w\beta^{wi}\dfrac{1}{T}\sum_t\varphi^w_t\log\left(\breve{x}_{ht}\right)\dfrac{x_{ht}}{p^i_{ht}}\dfrac{\overline{\varphi^w_T}}{\overline{\varphi^w_T}}\end{array}\right]\frac{\overline{x_{hT}}}{\overline{x_{hT}}}\,\frac{\overline{p^i_{hT}}}{\overline{p^i_{hT}}}\,\frac{\log\left(\overline{\breve{x}_{hT}}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}$$

$$=\left[\begin{array}{l}\beta^i_0\dfrac{1}{T}\displaystyle\sum_t\dfrac{x_{ht}}{\overline{x_{\cdot t}}}\,\dfrac{\overline{p^i_{\cdot t}}}{p^i_{ht}}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{\cdot t}}\right)}+\sum_k\beta^{ki}\theta^k_h\dfrac{1}{T}\sum_t\dfrac{x_{ht}}{\overline{x_{hT}}}\,\dfrac{\overline{p^i_{hT}}}{p^i_{ht}}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}\\ +\displaystyle\sum_r\beta^{ri}\,\overline{\mu^r_{hT}}\,\dfrac{1}{T}\sum_t\dfrac{\mu^r_{ht}}{\overline{\mu^r_{hT}}}\,\dfrac{x_{ht}}{\overline{x_{hT}}}\,\dfrac{\overline{p^i_{hT}}}{p^i_{ht}}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}+\sum_w\beta^{wi}\,\overline{\varphi^w_T}\,\dfrac{1}{T}\sum_t\dfrac{\varphi^w_t}{\overline{\varphi^w_T}}\,\dfrac{x_{ht}}{\overline{x_{hT}}}\,\dfrac{\overline{p^i_{hT}}}{p^i_{ht}}\,\dfrac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}\end{array}\right]\frac{\overline{x_{hT}}}{\overline{p^i_{hT}}}\log\left(\overline{\breve{x}_{hT}}\right)$$

$$=\left[\beta^i_0 S^x_{hT}+\sum_k\beta^{ki}\theta^k_h S^x_{hT}+\sum_r\beta^{ri}\,\overline{\mu^r_{hT}}\,S^{\mu^r}_{hT}+\sum_w\beta^{wi}\,\overline{\varphi^w_T}\,S^{\varphi^w}_{hT}\right]\frac{\overline{x_{hT}}}{\overline{p^i_{hT}}}\log\left(\overline{\breve{x}_{hT}}\right)=\widetilde{\beta}^i_{hT}\,\frac{\overline{x_{hT}}}{\overline{p^i_{hT}}}\log\left(\overline{\breve{x}_{hT}}\right).$$

Rearranging, we have the following demand function for consumer h during period T:

$$\overline{q^i_{hT}}=\left[\begin{array}{l}\alpha^i_0\,S^0_{hT}+\displaystyle\sum_m\alpha^{mi}\theta^m_h S^0_{hT}+\sum_s\alpha^{si}\,\overline{\mu^s_{hT}}S^{\mu^s}_{hT}+\sum_v\alpha^{vi}\,\overline{\varphi^v_T}S^{\varphi^v}_{hT}\\ +\displaystyle\sum_j\left(\gamma^{ij}_0\,S^{p^j}_{hT}+\sum_f\gamma^{ij}_f\theta^{jf}_h S^{p^j}_{hT}+\sum_d\gamma^{ij}_d\,\overline{\mu^{jd}_{hT}}\,S^{\mu^{jd}}_{hT}+\sum_u\gamma^{ij}_u\,\overline{\varphi^{ju}_T}\,S^{\varphi^{ju}}_{hT}\right)\log\left(\overline{p^j_{hT}}\right)\\ +\displaystyle\left(\beta^i_0 S^x_{hT}+\sum_k\beta^{ki}\theta^k_h S^x_{hT}+\sum_r\beta^{ri}\,\overline{\mu^r_{hT}}\,S^{\mu^r}_{hT}+\sum_w\beta^{wi}\,\overline{\varphi^w_T}\,S^{\varphi^w}_{hT}\right)\log\left(\overline{\breve{x}_{hT}}\right)\end{array}\right]\frac{\overline{x_{hT}}}{\overline{p^i_{hT}}}\,, \qquad \text{(A7)}$$

where the aggregation factors during period T (S_{hT}) are defined by:

$$S_{hT}^{0} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}} , \tag{A8a}$$

$$S_{hT}^{\varphi^v} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\varphi_t^v}{\varphi_T^v}, \tag{A8b}$$

$$S_{hT}^{\mu^s} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\mu_{ht}^s}{\mu_{hT}^s}, \tag{A8c}$$

$$S_{hT}^{p^j} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{hT}^j}\right)}, \tag{A8d}$$

$$S_{hT}^{\varphi^{ju}} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{hT}^j}\right)}\frac{\varphi_t^{ju}}{\varphi_T^{ju}}, \tag{A8e}$$

$$S_{hT}^{\mu^{jd}} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\log\left(p_{ht}^j\right)}{\log\left(\overline{p_{hT}^j}\right)}\frac{\mu_{ht}^{jd}}{\mu_{hT}^{jd}}, \tag{A8f}$$

$$S_{hT}^{x} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}, \tag{A8g}$$

$$S_{hT}^{\varphi^w} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}\frac{\varphi_t^w}{\varphi_T^w}, \text{ and} \tag{A8h}$$

$$S_{hT}^{\mu^r} = \frac{1}{T}\sum_{t=1}^{T}\frac{x_{ht}}{x_{hT}}\frac{\overline{p_{hT}^{i}}}{p_{ht}^{i}}\frac{\log\left(\breve{x}_{ht}\right)}{\log\left(\overline{\breve{x}_{hT}}\right)}\frac{\mu_{ht}^r}{\mu_{hT}^r}. \tag{A8i}$$

Inserting the expression for mean consumption into the expression for the budget share during period T, we can write the budget share of consumer h in period T as a function of the time-aggregated parameters ($\tilde{\alpha}_{hT}^i$, $\tilde{\gamma}_{hT}^{ij}$, and $\tilde{\beta}_{hT}^i$), mean prices, and mean total real expenditure:

$$w_{hT}^i = \tilde{\alpha}_{hT}^i + \sum_j \tilde{\gamma}_{hT}^{ij}\log\left(\overline{p_{hT}^j}\right) + \tilde{\beta}_{hT}^i\log\left(\overline{\breve{x}_{hT}}\right), \tag{A9}$$

where the relationships between the micro and time-aggregated parameters in period T are given by:

$$\widetilde{\alpha}_{hT}^{i} = \alpha_0^i \, S_{hT}^{0} + \sum_m \alpha^{mi} \theta_h^m S_{hT}^{0} + \sum_s \alpha^{si} \, \overline{\mu_{hT}^{s}} S_{hT}^{\mu^s} + \sum_v \alpha^{vi} \, \overline{\varphi_T^{v}} S_T^{\varphi^v} \,, \tag{A10a}$$

$$\widetilde{\gamma}_{hT}^{ij} = \gamma_0^{ij} S_{hT}^{p^j} + \sum_f \gamma_f^{ij} \theta_h^{jf} S_{hT}^{p^j} + \sum_d \gamma_d^{ij} \, \overline{\mu_{hT}^{jd}} \, S_{hT}^{\mu^{jd}} + \sum_u \gamma_u^{ij} \, \overline{\varphi_T^{ju}} S_T^{\varphi^{ju}} \text{ , and} \tag{A10b}$$

$$\widetilde{\beta}_{hT}^{i} = \beta_0^i S_{hT}^{x} + \sum_k \beta^{ki} \theta_h^k \, S_{hT}^{x} + \sum_r \beta^{ri} \, \overline{\mu_{hT}^{r}} \, S_{hT}^{\mu^r} + \sum_w \beta^{wi} \, \overline{\varphi_T^{w}} S_T^{\varphi^w} \,. \tag{A10c}$$

A.III. Aggregation over Time and across Consumers

Now, we turn to the case where we aggregate the macro budget share function over time, i.e., to find annual consumption of a particular good for the household sector. We assume that consumption is aggregated over all t during the year T, but not necessarily over all time periods ($t = 1, \ldots, \ddot{T}$), e.g., if we have observations for ten years, the time period T may be a year, a season, or a month. This means that the time period T is a subset of time periods t in the available time frame $\ddot{T}$.

The time-aggregated macro budget share over a period T is given by:

$$w_T^i = \frac{\overline{\overline{q_T^i}} \; \overline{\overline{p_T^i}}}{\overline{\overline{x_T}}} \,. \tag{A11}$$

The mean consumption in period T, $\overline{\overline{q_T^i}} = \frac{1}{T}\sum_{t=1}^{T} \overline{q_t^i} = \frac{1}{T}\sum_{t=1}^{T} \left\{ \left[\widetilde{\alpha}_t^i + \sum_j \widetilde{\gamma}_t^{ij} \log\left(\overline{p_t^j}\right) + \widetilde{\beta}_t^i \log\left(\overline{\breve{x}_{\cdot t}}\right) \right] \frac{\overline{x_t}}{\overline{\overline{p_t^i}}} \right\}$,

given the macro parameters and the aggregation factors, is used to find an expression for time-aggregated macro demand. To simplify the calculation, we calculate the effect on the intercept term, the price effects, and the income effects separately. First, we aggregate the intercept in the micro function, resulting in forming the macro intercept parameter in period T ($\tilde{\tilde{\alpha}}_T^i$):

$$\frac{1}{T}\sum_t \tilde{\alpha}_t^i \frac{\overline{x_t}}{p_t^i} = \frac{1}{T}\sum_t \left[\alpha_i^0 S_t^0 + \sum_m \alpha^{mi} \overline{\theta^m} S_t^{\theta^m} + \sum_s \alpha^{si} \overline{\mu_t^s} S_t^{\mu^s} + \sum_v \alpha^{vi} \varphi_t^v S_t^0 \right] \frac{\overline{x_t}}{p_t^i}$$

$$= \begin{bmatrix} \alpha_i^0 \frac{1}{T}\sum_t S_t^0 \frac{\overline{x_t}}{p_t^i} \frac{\overline{S_T^0}}{S_T^0} + \sum_m \alpha^{mi} \overline{\theta^m} \frac{1}{T}\sum_{t=1} S_t^{\theta^m} \frac{\overline{x_t}}{p_t^i} \frac{\overline{S_T^{\theta^m}}}{S_T^{\theta^m}} \\ + \sum_s \alpha^{si} \frac{1}{T}\sum_t \overline{\mu_t^s} S_t^{\mu^s} \frac{\overline{x_t}}{p_t^i} \frac{\overline{S_T^{\mu^s}}}{S_T^{\mu^s}} \frac{\overline{\overline{\mu_T^s}}}{\mu_T^s} + \sum_v \alpha^{vi} \frac{1}{T}\sum_t \varphi_t^v S_t^0 \frac{\overline{x_t}}{p_t^i} \frac{\overline{\varphi_T^v}}{\varphi_T^v} \frac{\overline{S_T^0}}{S_T^0} \end{bmatrix} \frac{\overline{\overline{x_T}}}{p_T^i} \frac{\overline{\overline{p_T^i}}}{x_T}$$

$$= \begin{bmatrix} \alpha_i^0 \overline{S_T^0} \frac{1}{T}\sum_t \frac{S_t^0}{S_T^0} \frac{\overline{x_t}}{x_T} \frac{\overline{\overline{p_T^i}}}{p_t^i} + \sum_m \alpha^{mi} \overline{\theta^m}\, \overline{S_T^{\theta^m}} \frac{1}{T}\sum_t \frac{\overline{x_{.t}}}{x_T} \frac{\overline{\overline{p_T^i}}}{p_t^i} \frac{S_t^{\theta^m}}{S_T^{\theta^m}} \\ + \sum_s \alpha^{si} \overline{\overline{\mu_T^s}}\, \overline{S_T^{\mu^s}} \frac{1}{T}\sum_t \frac{\overline{x_t}}{x_T} \frac{\overline{\overline{p_T^i}}}{p_t^i} \frac{S_t^{\mu^s}}{S_T^{\mu^s}} \frac{\overline{\mu_t^s}}{\mu_T^s} + \sum_v \alpha^{vi} \overline{\varphi_T^v}\, \overline{S_T^0} \frac{1}{T}\sum_t \frac{\overline{x_t}}{x_T} \frac{\overline{\overline{p_T^i}}}{p_t^i} \frac{\varphi_t^v}{\varphi_T^v} \frac{S_t^0}{S_T^0} \end{bmatrix} \frac{\overline{\overline{x_T}}}{p_T^i}$$

$$= \left[\alpha_i^0 \overline{S_T^0} \zeta_T^0 + \sum_m \alpha^{mi} \overline{\theta^m}\, \overline{S_T^{\theta^m}} \zeta_T^{\theta^m} + \sum_s \alpha^{si} \overline{\overline{\mu_T^s}}\, \overline{S_T^{\mu^s}} \zeta_T^{\mu^s} + \sum_v \alpha^{vi} \overline{\varphi_T^v}\, \overline{S_T^0} \zeta_T^{\varphi^v} \right] \frac{\overline{\overline{x_T}}}{p_T^i} = \tilde{\tilde{\alpha}}_T^i \frac{\overline{\overline{x_T}}}{p_T^i} .$$

Second, we aggregate the price effects, resulting in the macro parameter for the price in period T ($\tilde{\gamma}_T^{ij}$):

$$\frac{1}{T}\sum_t \tilde{\gamma}_t^{ij} \frac{\overline{x_t}}{p_t^i} \log\left(\overline{p_t^j}\right) = \frac{1}{T}\sum_t \sum_j \left[\gamma_{ij}^0 S_t^{p^j} + \sum_f \gamma_f^{ij} \overline{\theta^{jf}} S_t^{\theta^{jf}} + \sum_d \gamma_d^{ij} \overline{\mu_t^{jd}} S_t^{\mu^{jd}} + \sum_u \gamma_u^{ij} \varphi_t^{ju} S_t^{p^j} \right] \frac{\overline{x_t}}{p_t^i} \log\left(\overline{p_t^j}\right)$$

$$
= \left\{ \sum_j \left[\begin{array}{l} \gamma_0^{ij} \frac{1}{T}\sum_t S_t^{p^j} \frac{\overline{x_t}}{\overline{p_t^i}} \log\left(\overline{p_t^j}\right) \frac{\overline{S_T^{p^j}}}{\overline{S_T^{p^j}}} + \sum_f \gamma_f^{ij} \overline{\theta^{jf}} \frac{1}{T}\sum_t S_t^{\theta^{jf}} \frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{p_t^j}\right) \frac{\overline{S_T^{\theta^{jf}}}}{\overline{S_T^{\theta^{jf}}}} \\ + \sum_d \gamma_d^{ij} \frac{1}{T}\sum_t \overline{\mu_t^{jd}} S_t^{\mu^{jd}} \frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{p_t^j}\right) \frac{\overline{\overline{\mu_T^{jd}}}}{\overline{\overline{\mu_T^{jd}}}} \frac{\overline{S_T^{\mu^{jd}}}}{\overline{S_T^{\mu^{jd}}}} + \sum_u \gamma_u^{ij} \frac{1}{T}\sum_t \varphi_t^{ju} S_t^{p^j} \frac{\overline{x_{\cdot t}}}{\overline{p_{\cdot t}^i}}\log\left(\overline{p_t^j}\right) \frac{\overline{\varphi_T^{ju}}}{\overline{\varphi_T^{ju}}}\frac{\overline{S_T^{p^j}}}{\overline{S_T^{p^j}}} \end{array}\right] \frac{\log\left(\overline{\overline{p_T^j}}\right)}{\log\left(\overline{\overline{p_T^j}}\right)} \right\} \frac{\overline{\overline{x_T}}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{\overline{p_T^i}}}
$$

$$
= \left\{\sum_j\left[\begin{array}{l} \gamma_0^{ij}\overline{S_T^{p^j}}\frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{S_t^{p^j}}{\overline{S_T^{p^j}}} + \sum_f \gamma_f^{ij}\overline{\theta^{jf}}\,\overline{S_T^{\theta^{jf}}}\frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{S_t^{\theta^{jf}}}{\overline{S_T^{\theta^{jf}}}} \\ +\sum_d\gamma_d^{ij}\overline{\overline{\mu_T^{jd}}}\,\overline{S_T^{\mu^{jd}}}\frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{\overline{\mu_t^{jd}}}{\overline{\overline{\mu_T^{jd}}}}\frac{S_t^{\mu^{jd}}}{\overline{S_T^{\mu^{jd}}}} + \sum_u \gamma_u^{ij}\overline{\varphi_T^{ju}}\,\overline{S_T^{p^j}}\frac{1}{T}\sum_t\frac{\overline{x_{\cdot t}}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{\varphi_t^{ju}}{\overline{\varphi_T^{ju}}}\frac{S_t^{p^j}}{\overline{S_T^{p^j}}}\end{array}\right]\log\left(\overline{\overline{p_T^j}}\right)\right\}\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}}
$$

$$
= \left\{\sum_j\left[\gamma_0^{ij}\overline{S_T^{p^j}}\,\zeta_T^{p^j} + \sum_f\gamma_f^{ij}\overline{\theta^{jf}}\,\overline{S_T^{\theta^{jf}}}\,\zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij}\overline{\overline{\mu_T^{jd}}}\,\overline{S_T^{\mu^{jd}}}\,\zeta_T^{\mu^{jd}} + \sum_u\gamma_u^{ij}\overline{\varphi_T^{ju}}\,\overline{S_T^{p^j}}\,\zeta_T^{\varphi^{ju}}\right]\log\left(\overline{\overline{p_T^j}}\right)\right\}\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}} = \left\{\sum_j\tilde{\tilde{\gamma}}_T^{ij}\log\left(\overline{\overline{p_T^j}}\right)\right\}\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}}
$$

Finally, we aggregate the total real expenditure effects, resulting in the macro parameter for the total real expenditure in period T ($\tilde{\beta}_T^i$):

$$
\frac{1}{T}\sum_t\tilde{\beta}_t^i\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right) = \frac{1}{T}\sum_t\left[\beta_i^0 S_t^x + \sum_k\beta^{ki}\overline{\theta^k}\,S_t^{\theta^k} + \sum_r\beta^{ri}\overline{\mu_t^r}\,S_t^{\mu^r} + \sum_w\beta^{wi}\varphi_t^w S_t^x\right]\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right)
$$

$$
= \left[\begin{array}{l} \beta_i^0\frac{1}{T}\sum_t S_t^x\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right)\frac{\overline{S_T^x}}{\overline{S_T^x}} + \sum_k\beta^{ki}\overline{\theta^k}\frac{1}{T}\sum_t S_t^{\theta^k}\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right)\frac{\overline{S_T^{\theta^k}}}{\overline{S_T^{\theta^k}}} \\ +\sum_r\beta^{ri}\frac{1}{T}\sum_t\overline{\mu_t^r}\,S_t^{\mu^r}\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right)\frac{\overline{\overline{\mu_T^r}}}{\overline{\overline{\mu_T^r}}}\frac{\overline{S_T^{\mu^r}}}{\overline{S_T^{\mu^r}}} + \sum_w\beta^{wi}\frac{1}{T}\sum_t\varphi_t^w S_t^x\frac{\overline{x_t}}{\overline{p_t^i}}\log\left(\overline{\breve{x}_t}\right)\frac{\overline{\varphi_T^w}}{\overline{\varphi_T^w}}\frac{\overline{S_T^x}}{\overline{S_T^x}}\end{array}\right]\frac{\overline{\overline{x_T}}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{\overline{p_T^i}}}\frac{\log\left(\overline{\overline{\breve{x}_T}}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}
$$

$$
= \left[\begin{array}{l} \beta_i^0\overline{S_T^x}\frac{1}{T}\sum_t\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{S_t^x}{\overline{S_T^x}} + \sum_k\beta^{ki}\overline{\theta^k}\,\overline{S_T^{\theta^k}}\frac{1}{T}\sum_t\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{S_t^{\theta^k}}{\overline{S_T^{\theta^k}}} \\ +\sum_r\beta^{ri}\overline{\overline{\mu_T^r}}\,\overline{S_T^{\mu^r}}\frac{1}{T}\sum_t\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{\overline{\mu_t^r}}{\overline{\overline{\mu_T^r}}}\frac{S_t^{\mu^r}}{\overline{S_T^{\mu^r}}} + \sum_w\beta^{wi}\overline{\varphi_T^w}\,\overline{S_T^x}\frac{1}{T}\sum_t\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{p_t^i}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{\varphi_t^w}{\overline{\varphi_T^w}}\frac{S_t^x}{\overline{S_T^x}}\end{array}\right]\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}}\log\left(\overline{\overline{\breve{x}_T}}\right)
$$

$$=\left[\beta_i^0 \overline{S_T^x}\zeta_T^x + \sum_k \beta^{ki}\overline{\theta^k}\,\overline{S_T^{\theta^k}}\zeta_T^{\theta^k} + \sum_r \beta^{ri}\overline{\overline{\mu_T^r}}\,\overline{S_T^{\mu^r}}\zeta_T^{\mu^r} + \sum_w \beta^{wi}\overline{\varphi_T^w}\,\overline{S_T^x}\zeta_T^{\varphi^w}\right]\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}}\log\left(\overline{\overline{\breve{x}_T}}\right) = \tilde{\tilde{\beta}}_T^i \frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}}\log\left(\overline{\overline{\breve{x}_T}}\right)$$

. Rearranging, we have the following consumer- and time-aggregated demand function:

$$\overline{\overline{q_T^i}} = \left[\begin{array}{l} \alpha_0^i \overline{S_T^0}\zeta_T^0 + \sum_m \alpha^{mi}\overline{\theta^m}\,\overline{S_T^{\theta^m}}\zeta_T^{\theta^m} + \sum_s \alpha^{si}\overline{\overline{\mu_T^s}}\,\overline{S_T^{\mu^s}}\zeta_T^{\mu^s} + \sum_v \alpha^{vi}\overline{\varphi_T^v}\,\overline{S_T^0}\zeta_T^{\varphi^v} \\ + \sum_j \left[\gamma_0^{ij}\overline{S_T^{p^j}}\zeta_T^{p^j} + \sum_f \gamma_f^{ij}\overline{\theta^{jf}}\,\overline{S_T^{\theta^{jf}}}\zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij}\overline{\overline{\mu_T^{jd}}}\,\overline{S_T^{\mu^{jd}}}\zeta_T^{\mu^{jd}} + \sum_u \gamma_u^{ij}\overline{\varphi_T^{ju}}\,\overline{S_T^{p^j}}\zeta_T^{\varphi^{ju}}\right]\log\left(\overline{\overline{p_T^j}}\right) \\ + \left[\beta_0^i \overline{S_T^x}\zeta_T^x + \sum_k \beta^{ki}\overline{\theta^k}\,\overline{S_T^{\theta^k}}\zeta_T^{\theta^k} + \sum_r \beta^{ri}\overline{\overline{\mu_T^r}}\,\overline{S_T^{\mu^r}}\zeta_T^{\mu^r} + \sum_w \beta^{wi}\overline{\varphi_T^w}\,\overline{S_T^x}\zeta_T^{\varphi^w}\right]\log\left(\overline{\overline{\breve{x}_T}}\right) \end{array}\right]\frac{\overline{\overline{x_T}}}{\overline{\overline{p_T^i}}}, \quad \text{(A12)}$$

where the aggregation factors are defined by:

$$\zeta_T^0 = \frac{1}{T}\sum_{t=1}^{T}\frac{S_t^0}{\overline{\overline{S_T^0}}}\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T}^i}}{\overline{p_t^i}}, \tag{A13a}$$

$$\zeta_T^{\theta^m} = \frac{1}{T}\sum_{t=1}^{T}\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{S_t^{\theta^m}}{\overline{\overline{S_T^{\theta^m}}}}, \tag{A13b}$$

$$\zeta_T^{\mu^s} = \frac{1}{T}\sum_{t=1}^{T}\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{S_t^{\mu^s}}{\overline{\overline{S_T^{\mu^s}}}}\frac{\overline{\mu_t^s}}{\overline{\overline{\mu_T^s}}}, \tag{A13c}$$

$$\zeta_T^{\varphi^v} = \frac{1}{T}\sum_{t=1}^{T}\frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\varphi_t^v}{\overline{\overline{\varphi_T^v}}}\frac{S_t^0}{\overline{\overline{S_T^0}}}, \tag{A13d}$$

$$\zeta_T^{p^j} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{S_t^{p^j}}{\overline{\overline{S_T^{p^j}}}}, \tag{A13e}$$

$$\zeta_T^{\theta^{jf}} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T}^i}}{\overline{p_t^i}}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{S_t^{\theta^{jf}}}{\overline{\overline{S_T^{\theta^{jf}}}}}, \tag{A13f}$$

$$\zeta_T^{\mu^{jd}} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{\overline{\mu_t^{jd}}}{\overline{\overline{\mu_T^{jd}}}}\frac{S_t^{\mu^{jd}}}{S_T^{\mu^{jd}}}, \tag{A13g}$$

$$\zeta_T^{\varphi^{ju}} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{p_t^j}\right)}{\log\left(\overline{\overline{p_T^j}}\right)}\frac{\varphi_t^{ju}}{\varphi_T^{ju}}\frac{S_t^{p^j}}{S_T^{p^j}}, \tag{A13h}$$

$$\zeta_T^{x} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{S_t^{x}}{S_T^{x}}, \tag{A13i}$$

$$\zeta_T^{\theta^k} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{S_t^{\theta^k}}{S_T^{\theta^k}}, \tag{A13j}$$

$$\zeta_T^{\mu^r} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{\overline{\mu_t^r}}{\overline{\overline{\mu_T^r}}}\frac{S_t^{\mu^r}}{S_T^{\mu^r}} \text{ , and} \tag{A13k}$$

$$\zeta_T^{\varphi^w} = \frac{1}{T}\sum_t \frac{\overline{x_t}}{\overline{\overline{x_T}}}\frac{\overline{\overline{p_T^i}}}{\overline{p_t^i}}\frac{\log\left(\overline{\breve{x}_t}\right)}{\log\left(\overline{\overline{\breve{x}_T}}\right)}\frac{\varphi_t^w}{\varphi_T^w}\frac{S_t^{x}}{S_T^{x}}. \tag{A13l}$$

Inserting the expression for mean consumption over period T into the expression for the macro budget share, we can write the time-aggregated macro budget share as a function of the macro parameters, mean prices, and mean total real expenditure in the period T:

$$w_T^i = \tilde{\tilde{\alpha}}_T^i + \sum_j \tilde{\tilde{\gamma}}_T^{ij}\log\left(\overline{\overline{p_T^j}}\right) + \tilde{\tilde{\beta}}_T^i \log\left(\overline{\overline{\breve{x}_T}}\right), \tag{A14}$$

where the relationships between the micro and macro parameters in the period T are given by:

$$\tilde{\tilde{\alpha}}_T^i = \alpha_0^i\,\overline{S_T^0}\,\zeta_T^0 + \sum_m \alpha^{mi}\,\overline{\theta^m}\,\overline{S_T^{\theta^m}}\,\zeta_T^{\theta^m} + \sum_s \alpha^{si}\,\overline{\overline{\mu_T^s}}\,\overline{S_T^{\mu^s}}\,\zeta_T^{\mu^s} + \sum_v \alpha^{vi}\,\overline{\varphi_T^v}\,\overline{S_T^0}\,\zeta_T^{\varphi^v}, \tag{A15a}$$

$$\widetilde{\widetilde{\gamma}}_T^{ij} = \gamma_0^{ij}\overline{S_T^{p^j}}\,\zeta_T^{p^j} + \sum_f \gamma_f^{ij}\overline{\theta^{jf}}\,\overline{S_T^{\theta^{jf}}}\,\zeta_T^{\theta^{jf}} + \sum_d \gamma_d^{ij}\overline{\overline{\mu_T^{jd}}}\,\overline{S_T^{\mu^{jd}}}\,\zeta_T^{\mu^{jd}} + \sum_u \gamma_u^{ij}\overline{\varphi_T^{ju}}\,\overline{S_T^{p^j}}\,\zeta_T^{\varphi^{ju}} \text{, and} \tag{A15b}$$

$$\widetilde{\widetilde{\beta}}_T^{i} = \beta_0^{i}\overline{S_T^{x}}\zeta_T^{x} + \sum_k \beta^{ki}\overline{\theta^{k}}\,\overline{S_T^{\theta^{k}}}\,\zeta_T^{\theta^{k}} + \sum_r \beta^{ri}\overline{\overline{\mu_T^{r}}}\,\overline{S_T^{\mu^{r}}}\,\zeta_T^{\mu^{r}} + \sum_w \beta^{wi}\overline{\varphi_T^{w}}\,\overline{S_T^{x}}\zeta_T^{\varphi^{w}}\,. \tag{A15c}$$

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