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Labor Supply as a Choice among Latent Job Opportunities
A Practical Empirical Approach

Abstract:
In this paper, we discuss aspects of a particular framework for modeling labor supply and the application of this approach in practical policy simulation experiments. This modeling framework differs from the standard models of labor supply in that the notion of job choice is fundamental. Specifically, the worker is assumed to have preferences over a latent worker-specific choice set of jobs from which he or she chooses his or her preferred job. A job is characterized with fixed (job-specific) working hours and other non-pecuniary attributes. As a result, observed hours of work are interpreted as the job-specific (fixed) hours of work that is associated with the chosen job.

We then show that our framework is practical with respect to applications in empirical analysis and simulation experiments, and is able to produce satisfactory out-of-sample predictions by estimating the model on Norwegian microdata from 1997 and predicting the corresponding microdata from 2003.

Keywords: Labor supply, non-pecuniary job attributes, non-convex budget sets, latent choice sets, random utility models.

JEL classification: J22, C51

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1. Introduction

It has recently become more common for policy-makers to employ micro-based behavioral labor supply modeling tools in their ongoing preparations of national budgets, to assess the revenue and distributional effects of prospective changes in the tax and transfer system. Meeting this rising demand for reliable behavioral micro-simulation models has proven to be a rather difficult task and there is no generally accepted best approach to achieving this goal. In this paper, we present a modeling framework that has been developed and established to serve Norwegian policy-makers. We discuss how our modeling framework meets important criteria, such as having a sound theoretical basis and being practical in empirical analysis and simulation experiments, and being able to produce satisfactory out-of-sample predictions.

In the traditional approach (the standard approach), individual labor supply is viewed as a choice among feasible leisure and disposable income combinations. This approach has been criticized for ignoring an important behavioral aspect, namely that an agent in the labor market typically has preferences over job types and may face restrictions on his or her choice among job opportunities and hours of work. What complicates the matter further is that these restrictions are typically latent, since the researcher is usually ignorant about which agents face restrictions. Although there have been several attempts to take into account restrictions on hours of work, see for example Ilmakunnas and Pudney (1990), Kapteyn, Kooreman and van Soest (1990), Dickens and Lundberg (1993), these approaches are nevertheless centered on the standard approach. Recently, the discrete choice approach to labor supply modeling has gained widespread popularity, mainly because it is much more practical than the conventional continuous approach based on marginal calculus, see for example van Soest (1995) and Ilmakunnas and Pudney (1990). For example, with the discrete choice approach, it is easy to deal with nonlinear and nonconvex economic budget constraints, unlike in the Hausman model, cf. Hausman (1985) and Hausman and Ruud (1984). However, from a theoretical perspective, the conventional discrete choice approach represents no essential departure from the standard approach. This is because the only new assumption postulated is that the set of feasible hours of work is finite and that the random components of the utility function have particular distributional properties. Unfortunately, in situations with latent rationing of hours of work choices, the conventional discrete choice approach does not seem to be particularly practical, unless the set of potential alternatives is small (see, for example, Ilmakunnas and Pudney, 1990).

In this paper, we propose an alternative approach, based on Dagsvik (1994) and Dagsvik and Strøm (2006). See also Aaberge, Dagsvik and Strøm (1995) and Aaberge, Colombino and Strøm (1999). In this alternative approach, labor supply behavior is viewed as an outcome of agents choosing from a set of job ‘packages’, each of which is characterized by an offered wage rate, offered hours of work and nonpecuniary (qualitative) attributes describing the nature of the job-specific tasks to be performed. Thus, the hours of work of a given job are assumed fixed. In a modeling context where job-type is allowed to be a decision variable, workers may face additional constraints because the set of available jobs may be constrained. The individual-specific sets of feasible jobs are endogenous in the sense that they are determined by market equilibrium conditions and/or by negotiations between unions and employers. However, to the individual agent, the set of job opportunities may be viewed as given, although it is latent to the researcher. Similarly to the models of van Soest (1995) and van Soest, Das and Gong (2002), ours is formulated within a discrete choice framework. Theoretically, however, our alternative approach differs fundamentally from this and other previous approaches, since it accommodates the concept of ‘job’, and accounts for (latent) restrictions on hours of work and job opportunities. In the most general case, the distribution of hours of work can, however, be continuous, as demonstrated by Dagsvik (1994) and Dagsvik and Strøm (2006). The alternative point of departure we propose represents a powerful modeling strategy because it leads to an empirical framework that is flexible and practical to apply, and which we suggest is consistent with crucial features of the "true" choice setting. In particular, unlike in standard models, within our approach it is easy to account for latent choice restrictions.

To illustrate the potential of the framework, we conduct an empirical application. This application involves formulating and estimating models for the joint labor supply of married couples (within a unitary modeling framework), as well as models for single individuals, based on the alternative approach mentioned above. Subsequently, we discuss how the modeling framework can be applied to undertaking practical simulation experiments to, for example, determine the effect of alternative labor market and tax policies. For this purpose, it is important to link the labor supply models to a micro-population that is representative of the Norwegian population with respect to a set of variables that are taken as exogenously given in the labor supply models. Although our empirical application is similar to the ones discussed in Aaberge, Dagsvik and Strøm (1995), Aaberge, Colombino and Strøm (1999) and Dagsvik and Strøm (2006), the discussion in this paper puts more emphasis on the integration of the behavioral models into the established system of routines for tax policy simulations. A suitable representative micro-population (LOTTE population) has already been established by Statistics Norway and has been applied extensively to nonbehavioral tax policy analyses, cf. Statistics Norway (2006). Recall that the labor supply models are estimated conditionally on given household types and characteristics and given nonlabor income components. By aggregating the labor supply models over the micro-population for the different household types, one can obtain the (unconditional) distributions of hours of work, tax revenue and disposable income. As a special case, we obtain the respective moments of these variables. In addition, we discuss how the labor supply model can readily be extended to a joint model for labor supply and consumer demand. We also develop the appropriate analytic formulas for calculating effects from policy reforms (such as wage elasticities) conditional on the (chosen) level of income. See also Dagsvik, Locatelli and Strøm (2006) for an analogous discussion and analysis based on a labor supply model that allows for choice of sector (private and public). They also analyze the effect of specific tax reforms.

An important part of any behavioral model assessment is an examination of within-sample, and in particular, out-of-sample predictive performance. This is because these models are intended for predicting behavior under alternative budget constraints that typically differ from those observed in the data. In our empirical application, we show that not only are the models able to reproduce the within-sample data well, they also produce excellent seven-year-ahead out-of-sample predictions of the distribution of disposable income and labor supply (hours of work) in 2003 under appropriate updating of the wage rate equations and price levels (inflation). Despite small changes in the budget restrictions between 1997 and 2003, the excellent prediction properties of the models indicate that they represent structural relations. However, since the changes in the tax system and the wage rates from 1997 to 2003 are small this prediction exercise does not provide a very serious test of the behavioral properties of the models.

In this paper, we also make a theoretical contribution by adopting a novel approach to dealing with unobserved heterogeneity in individuals' latent choice sets of feasible alternatives. Up to now, unobserved heterogeneity in preferences and choice sets has been modeled by applying a particular multidimensional Poisson process representation (see Dagsvik, 1994, and Dagsvik and Strøm, 2006). The formalism of the Poisson process is somewhat abstract and may appear less intuitive than the formulation used in this paper. In contrast, we apply a more conventional formulation in which the parameters representing the choice set are specified as random effects. Nevertheless, the two types of representation are equivalent in the sense that they yield empirical models of the same structure.

The paper is organized as follows. In Section 2, we discuss the model. In Section 3, we present the empirical specification. In Section 4, we describe the data set. In Section 5, we report the estimation results and in Section 6, we discuss the implied wage elasticities.

2. The modeling framework

In this section, we present the basic structure of the modeling approach. The models considered in this paper differ somewhat from previous models estimated by Aaberge, Dagsvik and Strøm (1995), Aaberge, Colombino and Strøm (1999) and Dagsvik and Strøm (1997). In these papers, wage rates are assumed to be job specific and distributed across jobs according to a distribution function that varies across individuals by observable individual characteristics. In contrast, we assume that each individual faces only one individual-specific wage rate with a distribution that varies across individuals only by observable individual characteristics, similarly to the specification used by Dagsvik and Strøm (2004, 2006). However, unlike in this paper, Dagsvik and Strøm (2004, 2006) explicitly model the sectoral choice (between public and private sector) for married women, given the husbands labor market choice, but do not account for the simultaneous labor supply behavior of the spouses.

2.1. Single-individual households

Let $U(C,h,z)$ be the (ordinal) utility function of the household, where *C* denotes household (real) disposable income, *z* indexes market and nonmarket opportunities, or job-types, and *h* is hours of work. Let the positive indices, $z = 1, 2, \ldots,$ refer to market opportunities (jobs) and let $z = 0$ refer to the nonmarket alternative. For a market opportunity (job) *z*, there are associated hours of work, $H(z)$, and unobservable nonpecuniary attributes, such as the nature of the job-specific tasks to be performed, and location of the workplace. How these are determined is discussed later. For given hours and wage rates, *h* and *w*, the economic budget constraint is represented by

$$C = f(hw, I), \tag{2.1}$$

where *I* is nonlabor income, which includes the income of the husband and $f(\cdot)$ is the function that transforms gross income into after-tax household income. The income of the husband is treated as given. The function $f(\cdot)$ captures all details of the tax and benefit system. Our first assumption concerns the structure of preferences. The utility function is assumed to have the structure

$$U(C,h,z) = v(C,h)\varepsilon(z), \tag{2.2}$$

for $z = 0, 1, 2, \ldots,$ where $v(\cdot)$ is a positive deterministic function and $\varepsilon(z)$ is a positive random taste shifter. The random taste shifter is assumed to account for the unobservable individual characteristics and nonpecuniary job-type attributes that affect utility, and is allowed to vary both across households and opportunities. Thus, this formulation implies that the agent may have preferences over nonpecuniary job attributes. For simplicity, we shall use the notation

$$\psi(h,w,I) \equiv v\big(f(hw,I),h\big). \tag{2.3}$$

The term $\psi(h,w,I)$ is the representative utility of jobs with hours of work *h*, a given wage rate *w* and nonlabor income *I*. In addition to (2.1), there are restrictions on the set of feasible market opportunities faced by a specific worker. This is because there are job types for which the worker is not qualified and there may be no jobs available for which he or she is qualified.

Consider first the case in which there is no unobserved heterogeneity in the choice sets. Moreover, assume that the wage rates only depend on individual characteristics and do not vary across jobs. Although it would be of interest also to allow wage rates to vary across jobs this raises serious identification problems, which we currently are unable to deal with in a satisfactory way. Let $B(h,w)$ denote the agent's set of available jobs with hours of work; that is, this set contains those jobs z for which $H(z)=h$. Let $m(h,w)$ be the number of jobs in $B(h,w)$, which may depend on the wage rate. For the nonmarket alternative, one can normalize such that $m(0,w)=1$. The choice sets $\{B(h,w)\}$ are unobserved to the researcher. Let D be the set of feasible hours of work. This set is equal for all households. Prior to job search, the individual-specific choice set of jobs may even be unknown to the agent and may be revealed through the search process in which the agent learns gradually about his or her (equilibrium) choice set. See Dagsvik (2000) for details of the interpretation of choice sets unknown to the agent. The random error terms $\{\varepsilon(z)\}$ are assumed to be independent of offered hours and wages and are independent and identically distributed (i.i.d.) across jobs and individuals with type I extreme value distribution; that is, the cumulative distribution function is equal to $\exp(-1/x)$, defined for positive values of x. This particular distribution function is consistent with the property that the choice of jobs satisfies the assumption of independence from irrelevant alternatives (IIA). Recall that the basic underlying intuition of the IIA assumption is that the agent's ranking of job opportunities from a subset, B (say), within the choice set of feasible jobs with given job-specific hours of work and wage rate, does not change if the choice set of feasible jobs is altered.

Let $\varphi(h\,|\,w,I)$ denote the probability that the agent chooses a particular job with offered hours h, given wage rate w and nonlabor income I (and given individual characteristics) and let D be the set of feasible hours. From standard results in discrete choice theory (McFadden, 1984), it follows that the probability that a *specific* job, z (say), within $B(h,w)$ is chosen is given by

$$P\Big(\psi\big(h,w,I\big)\varepsilon(z)=\max_{x\in D,}\max_{k\in B(x,w)}\big(\psi\big(x,w,I\big)\varepsilon(k)\big)\Big)=\frac{\psi(h,w,I)}{\sum_{x\in D}\sum_{k\in B(x,w)}\psi(x,w,I)}=\frac{\psi(h,w,I)}{\sum_{x\in D}\psi(x,w,I)m(x,w)}.$$

The probability of choosing *any* job within $B(h,w)$ is thus obtained by summing the choice probabilities above over all jobs in $B(h,w)$, which yields

$$\varphi(h\,|\,w,I) = \sum_{z\in B(h,w)} P\Big(\psi(h,w,I)\varepsilon(z) = \max_{x\in D}\max_{k\in B(x,w)}\big(\psi(x,w,I)\varepsilon(k)\big)\Big)$$

(2.4)

$$= \sum_{z\in B(h,w)} \frac{\psi(h,w,I)}{\sum_{x\in D}\sum_{z\in B(x,w)}\psi(x,w,I)} = \frac{\psi(h,w,I)m(h,w)}{\psi(0,w,I)+\sum_{x>0,x\in D}\psi(x,w,I)m(x,w)}$$

for $h>0$, and similarly when $h=0$. The resulting expression is a choice model that is analogous to a multinomial logit model with representative utility terms $\{\psi(h,w,I)\}$ weighted by the frequencies of feasible jobs, $\{m(h,w)\}$. Unfortunately, the frequencies $\{m(h,w)\}$ are not directly observable, but under specific assumptions, one can identify $m(h,w)$ and $\psi(h,w,I)$ and estimate their parameters. We return to this issue below. Above we have suppressed the fact that the systematic part of the utililty function and the terms $\{m(h,w)\}$ depend on individual characteristics such as schooling and demographic variables. The specification of the functional form and how household characteristics enter the model will be considered in Section 3.

2.2. Married couples

Taking the unitary model as a point of departure[1], the model of joint labor supply for married couples is similar to the model for single individuals. Let $U(C,h_F,h_M,z)$ denote the utility function of the household, where h_F and h_M are hours of work for the female and the male and $z=(z_F,z_M)$ indexes the combination of jobs for the female and male in the household, respectively. Similarly to single-individual households, we assume that $U(C,h_F,h_M,z)=v(C,h_F,h_M)\varepsilon(z)$, which is interpreted analogously to the single-individual case above. The budget constraint in this case can be written as

(2.5) $$C = f\left(h_F w_F, h_M w_M, I\right),$$

where w_F and w_M are the respective wage rates for the female and male and $f(\cdot)$ is the function that transforms gross income to disposable income for the household. Let $\varphi(h_F,h_M\,|\,w_F,w_M,I)$ be the joint density of hours of work for the female and male in the household, given wage rates and nonlabor income. The empirical counterpart of this density is the fraction of couples in which the

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[1] Despite the expanding literature on household decision-making, no consensus has been reached as to what approach is best to describe the interaction between husband and wife (see Bergstrom, 1997, Blundell and MaCurdy, 1999, and Jia, 2005, for discussion of household behavior models). We follow the traditional 'common preference model' and assume the couple maximize a joint utility function subject to pooled budget constraints. However, the model can also be interpreted as a special case of the collective labor supply model (see Chiappori, 1988, 1992) when bargaining power is not affected by labor market decisions.

husband works h_F hours and the wife works h_M hours, within the subpopulation of couples with wage rates and nonlabor income equal to (w_F, w_M, I). We assume further that the offered hours, H_F, to the female and H_M to the male, are independent. Define

(2.6) $$\psi(h_F, h_M, w_F, w_M, I) \equiv v\big(f(h_F w_F, h_M w_M, I), h_F, h_M\big).$$

Then, under assumptions similar to those made for single-individual households, it follows that the conditional density of (h_F, h_M), given that $h_M > 0$, is given by

(2.7) $$\varphi(h_F, h_M \mid w_F, w_M, I) = \frac{\psi(h_F, h_M, 0, w_M, I)\, m_F(h_F, w_F) m_M(h_M, w_M)}{M(w_F, w_M, I)},$$

for $h_F > 0, h_M > 0$. In addition

(2.8) $$\varphi(0, h_M \mid w_F, w_M, I) = \frac{\psi(0, h_M, w_F, w_M, I)\, m_M(h_M, w_M)}{M(w_F, w_M, I)}$$

for $h_F = 0$ and $h_M > 0$, where $m_F(h_F, w_F)$ and $m_M(h_M, w_M)$ are the number of feasible jobs with offered hours h_F for the female and h_M for the male, w_F and w_M are the respective wage rates for the female and the male, and

(2.9)

$$M(w_F, w_M, I) = \sum_{y>0, y\in D} \psi(0, y, 0, w_M, I) m_M(y, w_M) + \sum_{y\in D, y>0,} \sum_{x\in D, x>0} \psi(x, y, w_F, w_M, I)\, m_F(x, w_F) m_F(y, w_M).$$

Note that the expressions in (2.6) to (2.9) are analogous to those for single-individual households.

2.3. Unobserved heterogeneity in individual choice sets

In the preceding analysis, we treated the parameters that represent the sizes of the choice sets as constant within observationally identical households. This means that unobserved heterogeneity is ruled out. Dagsvik (1994) discusses a general framework for dealing with stochastic choice sets that accommodates unobserved heterogeneity in the choice sets. This framework is based on a particular formal nonhomogeneous multidimensional Poisson process representation. This means that the attributes and taste-shifters associated with the respective alternatives are viewed as independently scattered realizations according to a location-dependent intensity measure. The reason why the locations of the points of the process are random is that the researcher does not observe which attributes are feasible. An additional explanation is that the agent is viewed as boundedly rational, as assumed by Thurstone (1927), and only makes his or her choice from a subset of his or her ‘objective’ choice set. Dagsvik and Strøm (2006) discuss this framework in the context of labor supply modeling. In this paper we discuss an alternative approach. This alternative approach has the advantage of being analogous the more traditional random effect type of approach. See also Dagsvik, Strøm and Jia (2006) for a similar approach.

For simplicity, we consider only single-individual households. Assume that the random error terms $\{\varepsilon(z)\}$ introduced in (2.2) are replaced by $\{\tilde{\varepsilon}(z)\}$, which are defined by

$$\tilde{\varepsilon}(z) = \varepsilon(z)\kappa(z) . \tag{2.10}$$

The terms $\varepsilon(z), z = 0,1,2,...,$ are i.i.d. with type-I extreme value distribution, as in Subsection 2.1. We interpret these terms as random to the agent himself in the sense that it is difficult for him to assess utility precisely once and for all. Thus, in replications of identical choice settings, the individual may vary his or her tastes in a manner that is not predictable by him or her. In contrast, the term $\kappa(z)$ is interpreted as representing the value of the unobservable attributes of job *z* that are perceived as perfectly predictable to the agent. However, $\{\kappa(z)\}$ is not observed by the researcher, and is represented as a random variable. We assume that $\{\kappa(z)\}$ and $\{\varepsilon(z)\}$ are independent. Let $\varphi\left(h \mid w, I, \{\tilde{m}(h', w), h' \in D\}\right)$ denote the conditional probability of supplying *h* hours of work given the wage rate, nonlabor income and $\{\tilde{m}(h,w), h \in D\}$. Similarly to (2.4), it follows immediately that the conditional density of supplied hours of work, given $\{\kappa(z)\}$, has the structure

$$\begin{aligned} &\sum_{z\in B(h,w)} P\left(\psi(h,w,I)\tilde{\varepsilon}(z) = \max_x \max_{k\in B(x,w)} \left(\psi(x,w,I)\tilde{\varepsilon}(z)\right) \mid \{\kappa(z)\}\right) \\ &= \varphi\left(h \mid w, I, \{\tilde{m}(h',w), h' \in D\}\right) = \frac{\psi(h,w,I)\tilde{m}(h,w)}{\sum_{x\in D}\psi(x,w,I)\tilde{m}(x,w)}, \end{aligned} \tag{2.11}$$

where

$$\tilde{m}(h,w) = \sum_{z\in B(h,w)} \kappa(z) .$$

It follows from (2.11) that the set $\{\tilde{m}(h,w), h \in D\}$ represents a sufficient set of random variables for the latent choice sets $\{B(h,w), h \in D\}$. Note that when $\kappa(z) = 1$, (2.11) reduces to (2.4). It follows immediately that the unconditional choice probability of working hours *h* is given by

$$\varphi\left(h \middle| w, I\right) = E\varphi\left(h \middle| w, I, \left\{\tilde{m}(h', w), h' \in D\right\}\right), \tag{2.12}$$

where the last expectation is taken with respect to $\left\{\tilde{m}(h, w), h \in D\right\}$.

A challenging issue is how to characterize the distribution of the terms $\left\{\tilde{m}(h, w), h \in D\right\}$. Our approach to this challenge is to postulate a reasonable invariance property, which is discussed in Appendix B. It can be demonstrated that the postulated invariance assumption, together with the requirement that $\tilde{m}(h)$ is positive, imply that the distribution of $\tilde{m}(h)$ is strictly stable and totally skew to the right. Recall that the class of Stable distributions represents a generalization of normal distributions. In particular, a general version of the central limit theorem yields the class of Stable distributions (see, for example, Embrechts, Klüppelberg and Mikosch, 1997). We refer to Appendix B for a detailed description of the family of Stable distributions. Thus, given that $\tilde{m}(h, w)$, $h \in D$, are independent and distributed according to a strictly Stable distribution that is totally skew to the right, it is shown in Appendix B that

$$\varphi(h | w, I) = E\left(\frac{\psi(h, w, I)\tilde{m}(h, w)}{\sum_{x \in D}\psi(x, w, I)\tilde{m}(x, w)}\right) = \frac{\psi^{\alpha}(h, w, I)m(h, w)}{\psi^{\alpha}(0, w, I) + \sum_{x \in D, x > 0}\psi^{\alpha}(x, w, I)m(x, w)}, \tag{2.13}$$

for $h > 0$, and similarly for $h = 0$, where $0 < \alpha < 1$ is a parameter of the Stable distribution and $\log m(h, w)$ is equal to $\alpha E \log \tilde{m}(h, w)$, apart from an additive constant, and with the normalization $m(0, w) = 1$. Thus, we have obtained the remarkable result that the structure of the choice probabilities is invariant under aggregation across unobserved choice sets (with suitable reinterpretation of the opportunity distribution), except for a power transformation of the systematic part of the utilities. In other words, we have demonstrated that the structure of the labor supply choice probabilities given in Section 2.1 is consistent with the stochastic choice sets of feasible jobs provided that the systematic part of the utility function *v* has a functional form that is invariant under arbitrary (increasing) power transformations. An analogous argument applies to the model for married couples. As will be clear from the empirical specification below the parameter α cannot be identified and can therefore be normalized to one. Note furthermore, that $m(h, w)$ can no longer be interpreted as the number of feasible jobs with hours of work h.

2.4. Equilibrium and identification issues

We have not yet discussed the structure of the opportunity measures $\left\{m(h, w)\right\}$ in equilibrium. Although a thorough analysis of this issue is beyond the scope of this paper, we nevertheless provide some clarification of the issue in this section. In what follows, we assume that $m(h,w)$ is multiplicatively separable in h and w; that is, $m(h,w) = g(h)\theta(w)$ for $h > 0$. See Dagsvik and Strøm (2004) for a justification of this assumption. Without loss of generality, we can normalize so that $g(h)$ is a probability density. Within the setting discussed in Sections 2.1 and 2.2 the term $\theta(w)$ is interpreted as the number of jobs that are feasible to the individual, and $g(h)$ is interpreted as the fraction of feasible jobs that have offered hours, $H(z)$, equal to h. With this notation, we have

$$\varphi(h \mid w, I) = \frac{\theta(w)\psi(h,w,I)g(h)}{\psi(0,0,I) + \theta(w)\sum_{x \in D, x>0} \psi(x,w,I)g(x)}, \tag{2.14}$$

for $h > 0$, and

$$\varphi(0 \mid w, I) = \frac{\psi(0,0,I)}{\psi(0,0,I) + \theta(w)\sum_{x \in D, x>0} \psi(x,w,I)g(x)}, \tag{2.15}$$

where we have used the fact that $\psi(0,w,I) = \psi(0,0,I)$. We call $\theta(w)g(h)$ *the opportunity density* (individual specific). However, within the extended setting discussed in Section 2.3 the interpretation of $g(h)$ and $\theta(w)$ is no longer so simple since these terms now depend on preferences through $\{\kappa(z)\}$. We therefore could call $\theta(w)g(h)$ the "quality adjusted" opportunity density, but for simplicity we shall still continue to use the terminology "opportunity density" in this case. As mentioned in Section 2.1 the choice probabilities in (2.14) and (2.15) also depend on socio-demographic variables that affect the systematic term of the utility function, $v(C,h)$.

Although we have assumed that the agent's taste-shifters are (stochastically) independent of offered hours and wage rates, the distribution of wage rates and the opportunity density will depend on the *distribution* of the preferences due to equilibrium conditions. In other words, the market forces that regulate the balance between supply and demand, be it a market-clearing regime or not, are assumed to operate solely at the aggregate level. Consequently, the opportunity density depends on the production technologies of firms as well as on the contracts and wage-setting policies of unions and firms. It is beyond the scope of this paper to discuss fully how the opportunity density, $\theta(w)g(h)$, through market equilibrium processes, depends on the systematic part of the utility function, $\psi(\cdot)$. Consequently, the estimated model can only be applied to simulate behavior conditional on the opportunity density. In our empirical application below, we assume that the density function $g(h)$ is exogenously given in the short run. In the Norwegian economy, normal working hours are typically determined once or twice every decade. In contrast, the mean level of offered wages for different groups are, in the unionized part of the economy, set annually. The parameter $\theta(w)$, which represents the size of the choice set of feasible jobs, will vary over the business cycle. Dagsvik (2000) considers equilibrium conditions in a setting in which the labor market is viewed as a matching game where workers and firms search and compete in order to obtain the best possible match with a potential partner. He shows that the choice model has the same structure as does the model given in (2.14) and (2.15), where $\theta(w)g(h) = b(w)g(h)V$. Whereas $g(h)$ is determined by preferences and institutional regulations determined infrequently, typically once or twice every decade, $b(w)$ is the systematic term in the conditional profit function of the firm and V is the total number of vacancies in the economy. That is, $b(w)$ is a function that measures the representative profit from hiring the agent at wage rate w. The decomposition, $\theta(w) = b(w)V$, implies that when V is observed, one can express the model in terms of V, a representative utility function and a representative conditional profit function. No additional equilibrium conditions need be imposed because the vacancy variable V is a sufficient statistic for the equilibrium relations. If cross-sections for several periods over which there are business cycle variations are available, one can use V as an instrument to control for the restrictions that arise because of the equilibrium conditions. In addition to changes in prices and wages over the business cycle, variations in V capture effects that operate through the equilibrium relationships, including destruction and creation of jobs. However, it is difficult to identify the structure of $b(w)$ because it depends on both the wage rate and those variables that represent worker qualifications, such as the length of schooling and experience. Specifically, $b(w)$ will be decreasing in w, whereas the (mean) effect of length of schooling may be ambiguous because, for some jobs, the worker may be overqualified. Therefore, in the empirical analysis that follows, we use a reduced form specification of $\theta(w)$.

2.5. Comparison with the standard approach

In the standard approach to labor supply modeling, the researcher typically chooses a specification of an individual labor supply function (hours of work function) that is consistent with the maximization of a quasi-concave utility function in disposable income and leisure, subject to the economic budget constraint. Except for the Hausman approach, the budget constraint is usually approximated by a suitable smooth version that implies a convex budget set. Suppose for example that the labor supply function has the structure

(2.16) $$\tilde{h} = \alpha + \beta\tilde{w}(h) + X\gamma + \delta\tilde{I}(h) + \varepsilon,$$

when

(2.17) $$\alpha + \beta\tilde{w}(0) + X\gamma + \delta\tilde{I}(0) + \varepsilon > 0,$$

and $h = 0$ otherwise, where $\tilde{w}(h)$ is the marginal wage rate, $\tilde{I}(h)$ is so-called virtual nonlabor income, X is a vector of individual characteristics that affects preferences, ε is a random error term with a normal distribution $N(0,\sigma)$ and α, β, γ, σ and δ are unknown parameters. The inequality in (2.17) is a condition for working. In general, when the tax system is nonlinear, the marginal wage rate and virtual income depend on hours of work and hence, they are endogenous. As a result, one cannot estimate the model by using OLS. Additional complications follow from the condition given by (2.17) and from the fact that the wage rate is not observed for those who do not work. Now, suppose that the parameters of this labor supply function have been estimated. Then, to derive hours of work when (2.17) holds, given the wage rate and nonlabor income, one needs to solve for h in the nonlinear equation given in (2.16). Let us denote by $\tilde{h}_i = F\left(w_i, I_i, X_i, \varepsilon_i\right)$ the resulting labor supply function of worker i; this is the solution of (2.16) for hours of work, where w_i is the wage rate and I_i is nonlabor income for worker i. Then, one can simulate the conditional distribution of labor supply (hours of work) by drawing T i.i.d. error terms $\{\varepsilon_i\}$ from the normal distribution $N(0,\sigma)$ and one can compute the simulated conditional distribution as

$$(2.18) \qquad P\left(\tilde{h}_i \le y \mid w_i, I_i, X_i\right) = \sum_{\left\{k;F\left(w_i,I_i,X_i,\varepsilon_k\right)\le y\right\}} F\left(w_i, I_i, X_i, \varepsilon_k\right)\frac{1}{T}.$$

The summation on the right-hand side of (2.18) is taken over all k such that supplied hours are less than or equal to y. The empirical counterpart of (2.18) is the fraction of agents with characteristics $\left(w_i, I_i, X_i\right)$ that supply hours of work of less than or equal to y. The corresponding unconditional labor supply distribution can be obtained by computing

$$(2.19) \qquad P\left(\tilde{h}_i \le y\right) = \frac{1}{N}\sum_{i\in\Omega} P\left(\tilde{h}_i \le y \mid w_i, I_i, X_i\right),$$

where Ω denotes a representative micro-population of size N. In principle, this can be done with general utility specifications and the corresponding labor supply functions, but, as mentioned above, this will in most cases be rather cumbersome in practice. The reason for this is that the class of utility functions that imply explicit and tractable labor supply functions, such as the one in (2.16), is rather limited and, hence, in more general cases, one is forced to work with nonlinear specifications. Thus, even when the budget constraint is simplified to ensure a convex budget set, the estimation and simulation of labor supply responses is not straightforward.

In contrast, the modeling framework applied in this paper differs in several aspects from the standard approach. First, it allows for preferences to depend on nonpecuniary job attributes and, second, it accounts for possible constraints on the set of feasible job offers. In addition to these theoretical aspects, this framework has the advantage that it does not require an explicit derivation of an individual labor supply function. Instead, the distribution (probability density) of labor supply is modeled directly and expressed explicitly as a function of the systematic term of the utility function. As a result, one need not simplify the budget constraint (2.1). In addition, the systematic term of the utility function, $v(C,h)$, can be quite general because this approach does not depend on the derivations of, and solutions to, first-order conditions. Furthermore, the simulation of distributional effects is straightforward because, as mentioned above, the model is represented explicitly in terms of probability densities, cf. (2.14) and (2.15).

The conventional discrete choice approach also shares many of the practical features discussed above because no marginal calculations are needed, cf. van Soest (1995). See also the review by Creedy and Kalb (2005). Specifically, it enables the researcher to straightforwardly apply quite general specifications of the utility function and the budget constraint. However, as mentioned in the introduction, it does not accommodate the feature that preferences typically depend on nonpecuniary job attributes, and because it is basically a version of the standard approach, it cannot deal with latent restrictions on choice opportunities.

3. Empirical specification

As discussed in Dagsvik and Strøm (2004, 2006), we can in general only identify the product $v(C,h_F,h_M)g_F(h_F)g_M(h_M)$ nonparametrically. To disentangle $v(C,h_F,h_M)$ from $g_F(h_F)$ and $g_M(h_M)$, we assume that the clustering of hours of work at part-time and full-time work is due to technological organizational constraints and/or regulation of hours introduced by unions and/or the government. The terms $g_F(h_F)$ and $g_M(h_M)$ are meant to capture this aspect of the labor market in the highly unionized Norwegian economy. Thus, through parametric identification, our model implies that the observed concentration of hours of work around part-time and full-time work arises because there are institutional constraints in the labor market rather than because individuals have strong preferences for full-time and part-time work. Note that there is no need to identify the terms $v(C,h_F,h_M)$, $g_F(h_F)$ and $g_M(h_M)$ separately if one is only interested in simulating the effects of changes in wage rates, the budget constraint and demographic variables affecting preferences. This identification is, however, crucial if one also wishes to simulate the effect on labor supply of changes in institutional hours of work restrictions.

We consider only models that are conditional on the male working. This is because it is likely that many males who do not work do so either because they cannot obtain suitable work or because they have health problems.

In the empirical specifications to be estimated, we assume that the density of offered hours, $g_k(h),\ k = F, M$, is uniform except for peaks at full-time and part-time hours. Because we established above that the opportunity densities may depend on preferences, we allow them to vary across household types; that is, those for single males, single females and married couples. Uniformly distributed offered hours are consistent with the assumption of a perfectly competitive economy. The full-time peak in the hours distribution captures institutional restrictions and technological constraints and hence market imperfections in the economy. We specify seven intervals for hours of work. The medians of the intervals are 315, 780, 1,040, 1,560, 1,976, 2,340 and 2,600. Thus, the set *D* consists of these points. The full-time peak occurs in the fifth interval, in which the median is 1,976 annual hours. The part-time peak is related to the third interval, which has a median of 1,040 annual hours. These intervals correspond to the most common agreements of what constitutes full- and half-time annual hours of work. To deal with the problem of wage rates being unobserved for those who do not work and of wage rates possibly being correlated with the taste-shifters in the utility function, we estimate instrumental wage rate equations. For $k = F, M$, we assume that

$$W_k = \bar{w}_k \eta_k \quad , \tag{3.1}$$

for $j = 1, 2$, where $\{\eta_{ik}\}$ are random terms that are lognormally distributed; that is, $\log \eta_k, k = F, M$, are independent and normally distributed, $N(0, \sigma_k)$. We assume that $\log \bar{w}_k$ is a linear function of length of schooling, experience and experience squared. When the wage rate equations are inserted into the model for married couples and when the error terms in these equations are integrated out, we obtain the following empirical model for the joint labor supply density for married couples

$$\bar{\varphi}_j\left(h_F, h_m \mid \bar{w}_F, \bar{w}_M, I\right) = E\left[\frac{\psi\left(h_F, h_M; \bar{w}_F \eta_F, \bar{w}_M \eta_M, I\right) g_F\left(h_F\right) g_M\left(h_M\right) \theta_F}{M\left(\bar{w}_F \eta_F, \bar{w}_M \eta_M, I\right)}\right], \tag{3.2}$$

for $h_F > 0, h_M > 0$, and similarly for $h_F = 0, h_M > 0$. The expectation in (3.2) is taken with respect to the error terms in the wage rate equations. In practice, we compute the expectation by simulation when estimating the model. The term θ_F is assumed to depend on the wage rates solely through the length of schooling. In this context, we assume that:

$$\log \theta_F = f_{F1} + f_{F2} S \ , \tag{3.3}$$

where S is the length of schooling.

We choose $v(\cdot)$ to be of the form

(3.4)

$$\log v\left(C,h_F,h_M\right)$$
$$=\alpha_2\left(\frac{\left[10^{-4}\left(C-C_0\right)\right]^{\alpha_1}-1}{\alpha_1}\right)+\left(\frac{\left(L_F-L_0\right)^{\alpha_3}-1}{\alpha_3}\right)\left(\alpha_5+\alpha_6\log A_F+\alpha_7\left(\log A_F\right)^2+\alpha_8 CU6+\alpha_9 CO6\right)$$
$$+\left(\alpha_{10}+\alpha_{11}\log A_M+\alpha_{12}(\log A_M)^2+\alpha_{13}CU6+\alpha_{14}CO6\right)\left(\frac{\left(L_M-L_0\right)^{\alpha_4}-1}{\alpha_4}\right)$$
$$+\alpha_{15}\left(\frac{\left(L_M-L_0\right)^{\alpha_4}-1}{\alpha_4}\right)\left(\frac{\left(L_F-L_0\right)^{\alpha_3}-1}{\alpha_3}\right),$$

where A_k, $k=F,M$, is age for gender k, $CU6$ and $CO6$ are the number of children below or equal to and above the age of six, respectively, C is given by (2.7), L_k, $k=F,M$, is leisure for gender k, with $L_k-L_0=1-h_k/3{,}650$, and α_j, $j=1,2,...,15$, are unknown parameters. Observe that we have subtracted from total annual hours a 'subsistence' level, amounting to 5,110 hours, which allows for sleep and rest. This corresponds to about 14 hours per day reserved for sleep and rest. The term C_0 is an income subsistence level. We have chosen C_0 to be approximately NOK 40,000$\sqrt{\mathrm{N}}$, where N is the number of persons in the household. Disposable income, C, is measured as the sum of the annual wage incomes of the woman and her husband after tax, household capital income after tax and child allowances. The tax functions and the child allowance rule are described in Appendix F of Dagsvik and Strøm (2004). If $\alpha_1<1$, $\alpha_3<1$, $\alpha_4<1$, $\alpha_2>0$, and the term in front of leisure is positive, and α_9 is sufficiently large, then $\log v\left(C,h\right)$ is increasing in C, decreasing in (h) for fixed C and strictly concave in $\left(C,h\right)$. Dagsvik and Strøm (2004) provide a theoretical justification of the functional form in (3.4).

To control for selection bias when estimating the wage equations, we apply the estimation procedure proposed by Dagsvik and Strøm (2004). Conditional on the estimated parameters of the wage equations, the remaining parameters of the model are estimated in a second stage by using the maximum likelihood procedure. It would also have been desirable to allow η_1 and η_2 to be correlated. However, because this complicates the computations considerably, we have chosen to leave this challenge for another occasion. This two-stage procedure has the added advantage that it reduces the measurement error caused by a negative correlation between hours of work and wage rates.

4. Data

The data are obtained by merging the Labor Force Survey of 1997 with two different register data sets that contain additional information about incomes, family composition, children and education. The concepts applied in the Labor Force Survey are consistent with the official statistics from Statistics Norway and the recommendations of the International Labor Organization (ILO). Note that persons were asked about their attachment to the labor market during a particular week in the first quarter of 1997.

Information about actual and formal working time in main and secondary jobs and information on background variables, such as demographic characteristics and occupation, was also obtained from the Labor Force Survey by using personal identification numbers. Conditional on labor market participation, respondents are also asked whether they consider themselves to be self-employed or employees, and based on this information, we have excluded self-employed persons from the sample used for estimation. Working time is measured as formal hours of work in both the main and second job. If this information is missing and the respondent is participating in the labor market, information about actual working time is used.

Information on education is obtained from the National Education Database, which is a register database that can be linked to the Labor Force Survey by using personal identification numbers. Whereas the Labor Force Survey yields detailed information about employment and hours of work, it does not provide information about annual labor incomes that can be used in the calculations of (average) gross wage rates and nonlabor income. To obtain this information, we utilized the Tax Return Register (which includes more detailed information about, for example, employee income, self-employment income, taxable pensions). These data can be linked to the Labor Force Survey by using the personal identification numbers. Nominal hourly wage rates are measured as labor incomes (for main and second jobs) divided by (normal) total annual hours of work (for main and second jobs). The sample includes persons aged between 26 and 62. The motivation for this is that for women below 26 years of age, education is an important activity, and many of those older than 62 years of age have retired. The number of children includes all children aged less than 19. A person is defined as working if he or she works at least one hour per week. Households in which one of the adults has income from self-employment that exceeds NOK 80,000 are excluded. Also excluded are households in which one of the adults works more than 80 hours, or receives a wage rate of less than NOK 50 or more than

NOK 400. In Table 1, we report the summary statistics for the sample used to estimate the labor supply model.

Table 1. Summary statistics for individuals in the sample, 1997

	Couples				Single Male / Single Female			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Male age	44.9	8.5	26	62	37.1	9.2	26	62
Male education	12.6	2.7	6	20	12.3	2.6	6	20
Male experience	25.3	9.1	1	46	17.7	9.8	1	46
Male nonlabor income	6,776	12,503	0	79,518	10,723	15,661	0	79,197
Male wage rate	155.3	54.6	50.4	400	137.7	47.3	50.1	387.4
Male weekly hours of work	38.5	5.4	2	80	37.9	6.7	2	75
Female age	42.6	8.5	26	62	38.9	9.8	26	62
Female education	12.1	2.6	0	20	12.3	2.7	0	20
Female experience	23.6	9.5	2	51	19.6	10.7	1	49
Female nonlabor income	18,671	17,415	0	79,752	16,578	22,287	0	79,627
Female wage rate	120.2	37.8	50	385.4	121.7	40.2	50.2	373.5
Female weekly hours of work	27.3	12.5	0	60	43.2	14.4	0	77.7
No. of children 0–7	0.48	0.78	0	4				
No. of children 8–18	0.78	0.93	0	4				
Number of observations	2,511				Male: 2,095 Female: 1,907			

5. Estimation results

5.1. Estimates of the wage rate equations

In this section, we report estimates of the wage rate equations. The wage equation is specified in a conventional way; that is, the logarithm of observed wage rates, $\log W_k, k = F, M$, is assumed to depend linearly on experience, experience squared and the education level. Experience is defined as age minus years of schooling minus seven. As shown in Table 2, the selection bias in the wage equations is negligible. Since the selection bias is negligible, we have not reported the corresponding bias for males. The estimates of the variances of the error terms in the wage equations are large. Thus, it seems important to account for the error terms in the wage equations when estimating the structural model.

Table 2. Estimates of wage equations, females and males, 1997

Variables	Males		Females		Females (selection corrected)	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
Constant	4.0897	0.030	4.1082	0.031	4.1145	0.038
Experience in years/10	0.2234	0.018	0.1429	0.017	0.1409	0.018
(Experience in years/10)^2	–0.0382	0.004	–0.0225	0.003	–0.0221	0.004
Education in years	0.0440	0.002	0.0388	0.017	0.0386	0.002
Married	0.0548	0.009	–0.0223	0.008	–0.0213	0.009
Log(P)					0.0132	0.045
Variance	0.3029		0.2755		0.2755	
No. observations	5,448		5,074		5,074	
R^2	0.15		0.10		0.10	

5.2. Estimates of labor supply probabilities

Estimates of the parameters of the structural choice model are reported in Table 3. For married couples, all exponents (α_1, α_3 and α_4) are significantly less than unity and, thus, the estimates imply that the deterministic part of the utility function is quasi-concave. We note that the parameter associated with the interaction term between male and female leisure is not significantly different from zero. Hence, we cannot reject the hypothesis that the deterministic part of the utility function is additively separable in the leisure of the female and that of the male. The marginal utilities of consumption and leisure (for all relevant ages) are positive. The marginal utilities of female and male leisure are convex and increasing functions of age, and imply that the marginal utility of leisure for females increases up to the age of 33 and then decreases, and that the marginal utility of leisure for males is increasing up to age 36 and thereafter decreases. The marginal utility of female leisure depends on the number of children in each age group, unlike the marginal utility of male leisure, which does not depend significantly on the number of children. The implication of the exponent α_1 being significantly different from zero is that agents care not only about relative consumption levels (beyond subsistence), absolute levels also matter. Note that the measure of the number of available jobs for females, m_F, depends positively on the length of schooling (S) (both for married and single females). Note also that the full-time and part time peaks in the opportunity density of hours for males are substantially higher than the corresponding peaks for females. This is probably because of differences in preferences between females and males, which is possible according to the interpretation of the opportunity density given in Section 2.3. The results for single males and females are similar to those for couples, except that the coefficients associated with leisure are not significantly different from zero. Figures 1 and 2 display the observed and (aggregate) predicted values of participation and hours of work for married couples; note that the model predicts these aggregates quite well. The corresponding figures for single females and single males are given in Appendix D. McFadden's ρ^2 given in Table 3 also confirms the good fit of the models.

Table 3. Parameter estimates of the labor supply probabilities

	Parameter	Married Couples		Single Males		Single Females	
		Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
Preferences:							
Consumption							
Exponent	α_1	0.6643	0.054	0.7919	0.206	0.5656	0.109
Scale 10^{-4}	α_2	1.8411	0.352	0.3509	0.126	0.3424	0.075
Female leisure							
Exponent	α_3	–0.8334	0.182			–4.2964	0.776
Constant	α_5	11.8387	1.888			0.4491	0.355
	α_6	–12.5285	1.945			–0.5867	0.469
Log(age/10) squared	α_7	5.2456	0.733			0.2181	0.174
No. children below or equal 7 years	α_8	0.9682	0.168				
No. children above 7 years	α_9	0.5075	0.094				
Male leisure							
Exponent	α_4	–1.8043	0.430	0			
Constant	α_{10}	3.8929	1.112	8.2806	4.110		
Log(age/10)	α_{11}	–4.3054	1.142	–11.2454	6.509		
Log(age/10) squared	α_{12}	1.6682	0.444	4.3352	2.454		
No. children below 6 years	α_{13}	0.0547	0.051				
No. children above 6 years	α_{14}	0.0083	0.029				
Leisure interaction	α_{15}	0.2047	0.147				
The parameters θ_F; $\log\theta_F = f_{F1} + f_{F2}S$							
Constant	f_{F1}	–3.5041	0.435			–5.3010	0.995
Education	f_{F2}	1.2389	0.366			2.8963	0.925
Opportunity density of offered hours							
Male full-time peak		2.3769	0.086	2.5580	0.082		
Female full-time peak		1.4380	0.296			1.7042	0.076
Male part-time peak		1.0960	0.063	–0.1767	0.206		
Female part-time peak		0.5622	0.067			0.3832	0.097
Number of Observations		2,511		2,095		1,907	
Log likelihood		–5,706.5		–1,841.3		–2,272.9	
McFadden's ρ^2		0.44		0.55		0.43	

5.3. Comparison with the standard discrete choice labor supply model

In this section, we compare our model with a version of the standard discrete choice modeling approach (van Soest, 1995). To this end it is assumed that the utility function for married couples has the structure

$$U(C,h_F,h_M) = v(C,h_F,h_M)\varepsilon(C,h_F,h_M), \tag{5.1}$$

where v is the systematic term and $U(C,h_F,h_M) = v(C,h_F,h_M)\varepsilon(C,h_F,h_M)$, and $\varepsilon(C,h_F,h_M)$ is a positive random error term. As above, hours are discrete, with $h_F, h_M \in D$. In addition (2.1) holds. This means that after the budget constraint has been taken into account, the error term will, for a given household, only depend on C through the couple's hours of work. The systematic term v is assumed to be a quadratic polynomial, given by

$$\begin{aligned} \log v(C,h_F,h_M) = {} & \alpha_1(C-\gamma_L-\gamma_F) + \beta_{M1}h_M + \beta_{F1}h_F + \alpha_2(C-\gamma_L-\gamma_F)^2 + \beta_{FM}h_F h_M \\ & + \beta_{M2}h_M^2 + \beta_{F2}h_F^2 + \alpha_3(C-\gamma_L-\gamma_F)h_M + \alpha_4(C-\gamma_L-\gamma_F)h_F. \end{aligned} \tag{5.2}$$

To account for observable differences in preferences between households, β_{M1} and β_{F1} are typically specified as functions of personal and household characteristics in the same way as in (3.4). The wage equations used are the same as those estimated in Section 5.1. The estimation procedure for this model is the same as that discussed in Section 3 and the estimates are given in Table C.1 of Appendix C.

Figure 1. Predicted and observed distributions of hours of work for married males, 1997

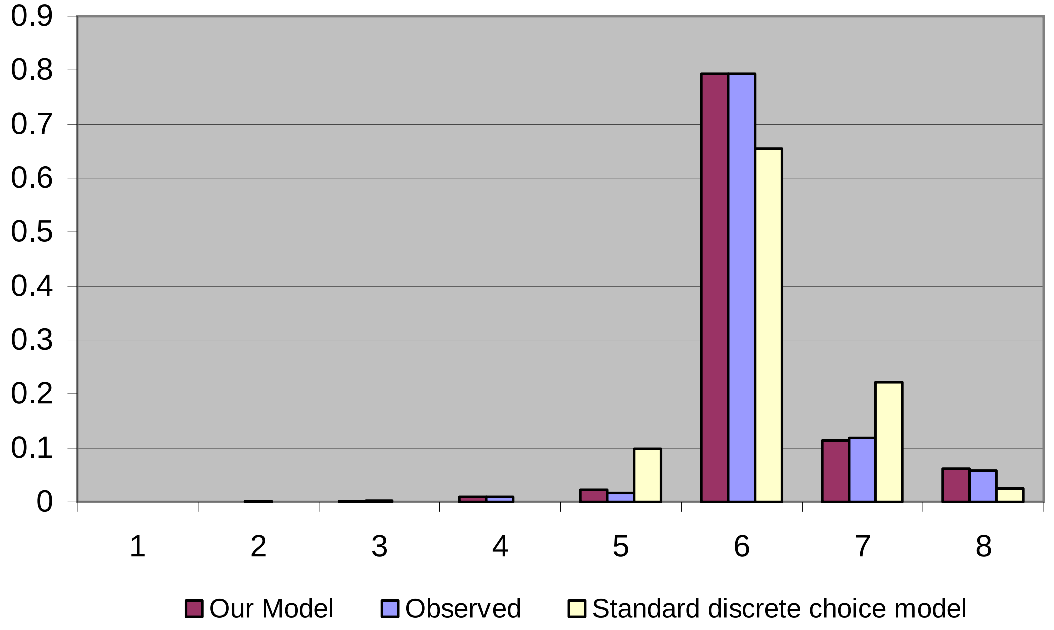
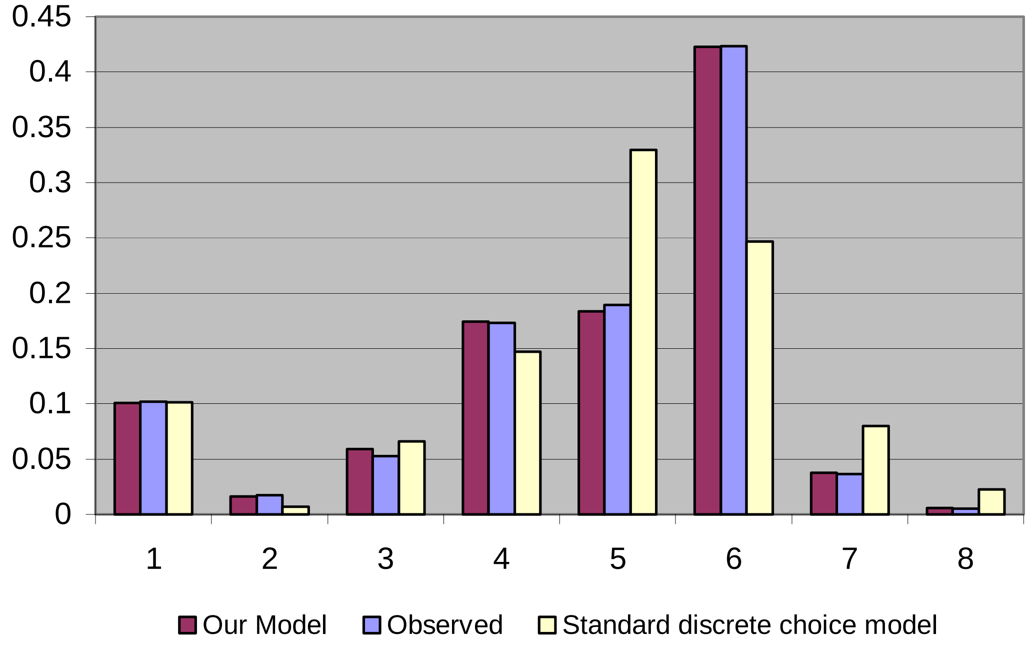


Figure 2. Predicted and observed distributions of hours of work for married females, 1997



Figures 1 and 2 show that our model predicts the labor supply probabilities much better than does the standard discrete choice model with the quadratic polynomial utility specification. The main problem with the standard discrete model is its inability to account for the concentration of part-time and full-time hours of work. For a further discussion and comparison of our type of approach with the standard discrete model, see Dagsvik and Strøm (2004, 2006).

6. Model simulations

6.1. Practical simulation of effects of policy reforms

A major motivation for developing behavioral models is the need for assessing the impact of policy reforms such as for example changes in direct and indirect taxes, the distributions of length of schooling and wage rates. The modeling approach discussed in this paper represents a convenient framework for simulating measures of behavioral responses to such reforms. Particular measures of interest are, (i) wage elasticities, (ii) income distribution, (iii) hours of work distribution, (iv) indexes of inequality in income (Gini coefficient), (v) compensating variation (CV), (vi) labor force participation, and (vii) transitions between labor market states. By means of the models estimated above these measures, apart from (vii), can be calculated and used to compare alternative policy regimes. As regards (vii) one cannot simulate transitions, say, from one year to the next, because the model does not include transition probabilities. To establish structural transition probabilities to this end will be a task for research in the future.

The practical simulations with our model differ from simulations based on the conventional approach. In the conventional approach the hours of work equation (including draws from the distribution of the error term) is used to simulate the respective (stochastic) realizations for each individual. Similarly, one can simulate disposable income for each household by combining the simulation from the wage equation and the labor supply equation. In contrast, simulations based on the modeling approach presented in this paper do not produce individual simulations. Instead the respective probability distributions under alternative policy regimes are calculated, from which one can calculate relevant measures, such as for example (i) to (vi). Although it is possible to simulate individual realizations as in the conventional approach, this is less convenient and also less precise because it implies additional error due to simulation uncertainty, unless one uses a very large number of draws from the distribution of the error term. The reason why it is unnecessary to simulate stochastic individual responses is because measures such as the theoretical income distribution, hours of work distribution, the Gini coefficient, mean wage elasticities, mean wage elasticities conditional on a given income decile, and the distribution of CV, is because these measures can be derived directly from the model. To realize how this can be done we have carried out selected simulation exercises below, namely the calculation of different type of wage elasticities, and within-sample and out-of-sample prediction of the hours of work- and income distribution. As an example, consider the calculation of the Gini coefficient, G (say), for disposable income. By well known results the Gini coefficient can be expressed as

$$(6.1) \qquad G = \frac{\int_0^\infty F(y)\left(1-F(y)\right)dy}{\int_0^\infty \left(1-F(y)\right)dy}$$

where $F(y)$ is the cumulative distribution of disposable income. In Appendix A it is demonstrated how one can simulate $F(y)$. Subsequently, one can use (6.1) to simulate G.

As regards policy experiments and the calculation of CV we refer to Dagsvik, Locatelli and Strøm (2006) who have carried out simulations of the effect of selected policy reforms. In particular, they demonstrate how one can apply the method of Dagsvik and Karlström (2005) to calculate the mean and the distribution of CV that follows from these reforms.

6.2. Aggregate wage elasticities

In this subsection, we report selected wage elasticities. We have chosen to calculate elasticities that take into account both the systematic terms and the unobservables in the model. This means that we account for how the mean of the *distribution* of labor supply is affected by changes in, say, wage levels. We refer to these elasticities as *aggregate* elasticities because they take into account unobserved and observed heterogeneity in the population. In Tables 4 and 5, we report what we term aggregate uncompensated elasticities. They are calculated as follows. For each household, we simulate the change in the choice probabilities of working and the expected hours of work for females and males following a 10 per cent increase in wage rates. Subsequently, we aggregate over the sample to obtain the corresponding change in the mean probability of working and mean expected hours of work. To obtain elasticities, we multiply these figures by 10 and divide by the respective mean probability of working and the mean expected hours of work.

In general, the tables show that the uncompensated wage elasticities are moderate for married females but small for males and single females. For married females, the own wage elasticity of the probability of working is equal to 0.33, which means that if the wage rates of married females were to increase by 5 per cent (say), then the aggregate fraction of married female working would increase by 1.5 per cent. If both male and female wage rates were increased, then the corresponding elasticity of the probability of working is equal to 0.22. This means that the fraction of married females working would increase by one per cent in this case.

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Table 4. Uncompensated wage elasticities for married couples

		Female base value	Male base value	Female own wage elasticity	Female cross wage elasticity	Male own wage elasticity	Male cross wage elasticity	Female elasticity with respect to both wage rates	Male elasticity with respect to both wage rates
Probability of working	Whole sample	0.89		0.33	–0.14			0.22	
	Lowest decile	0.87		0.42	–0.18			0.28	
	2nd to 8th decile	0.90		0.33	–0.14			0.22	
	Highest decile	0.92		0.25	–0.09			0.17	
Mean hours of work conditional on working	Whole sample	1,601	2,015	0.28	–0.09	0.08	–0.02	0.20	0.06
	Lowest decile	1,581	2,002	0.29	–0.09	0.07	–0.02	0.21	0.05
	2nd to 8th decile	1,602	2,015	0.28	–0.09	0.08	–0.02	0.20	0.06
	Highest decile	1,618	2,030	0.27	–0.08	0.09	–0.01	0.19	0.08
Unconditional mean hours of work	Whole sample	1,444		0.61	–0.23			0.42	
	Lowest decile	1,383		0.71	–0.26			0.48	
	2nd to 8th decile	1,445		0.61	–0.22			0.42	
	Highest decile	1,500		0.52	–0.18			0.37	

Conditional on working, the wage elasticity of mean hours of work is 0.28 for married females. Note also that the elasticities conditional on income groups decrease slightly by income for females but increase slightly for males. However, the elasticities with respect to change in both wage rates remain practically constant over income groups. The corresponding unconditional elasticities for the females measure the effect on total mean hours of work of a change in wages. Table 4 shows that the unconditional elasticities for married females range from 0.71 in the lowest decile to 0.52 in the highest decile of disposable income. The figure for the whole population is 0.61. This means that a 5 per cent increase in the wage rate of married females increases total mean annual hours of work by 44 hours.

Table 5. Uncompensated wage elasticities for single individuals

	Male base value	Male wage elasticity	Female base value	Female wage elasticity
Probability of working			0.97	0.023
Mean hours of work conditional on working	1,982	0.03	1,766	0.002
Unconditional mean hours of work			1,720	0.004

6.3. Out-of-sample prediction

As shown in Section 5, our model fits the data well. A more interesting test of the performance of the model is to check the extent to which the model is able to predict out-of-sample labor supply behavior. In policy simulation experiments, for predicting the effect of tax reforms for example, we are also interested in assessing how the distribution of disposable income, taxes paid, and inequality measures are affected by the reform. Detailed instructions on how to construct the distribution of disposable income based on the predicted choice probabilities can be found in Appendix A. In this section, we report results from out-of-sample simulation experiments.

To this end, we use two different samples from Norwegian populations to perform out-of-sample prediction exercises. In the first exercise, we use the most recent available data from the same source as our sample for estimation, namely the Labor Force Survey of 2003, merged with the Tax return register for the same year. The advantage of using this sample is that we can construct all variables in the same way as we did for the estimating sample and apply the same sample-selection rules.

For the second prediction exercise, we use different data, which were obtained by selecting data from the income and property statistics for households (LOTTE population), which is a representative sample survey of households, in which information from various registers is combined with household composition data from interviews. The only sample selection criterion for this simulation is the requirement that the individuals should be wage earners between 26 and 62 years of age. In this data set, we have detailed income data but no information about hours of work. Thus, we only compare the actual and predicted distributions of different income variables.

Two parameters are important when using the model estimated for one year (the base year) to predict labor market behavior in another year (the simulation year): These are the wage growth rate and the inflation rate, both measured from base year to simulation year. We use the observed wage growth rate and the wage regression for the base year to generate the wage rate in the simulation year. It is also necessary to adjust incomes in the simulation year by using the inflation rate to compute real income in the base year for undertaking the model simulations.

Although the out-of-sample predictions for labor supply behavior are not as good as the within-sample predictions, our model predicts the proportions in each category well. As expected, the standard discrete choice model predicts poorly in this case. Figures 3 and 4 illustrate the observed and predicted distributions of labor supply for married couples based on the first sample, which is described above.

Figure 3. Predicted and observed distributions of hours of work for married males, 2003

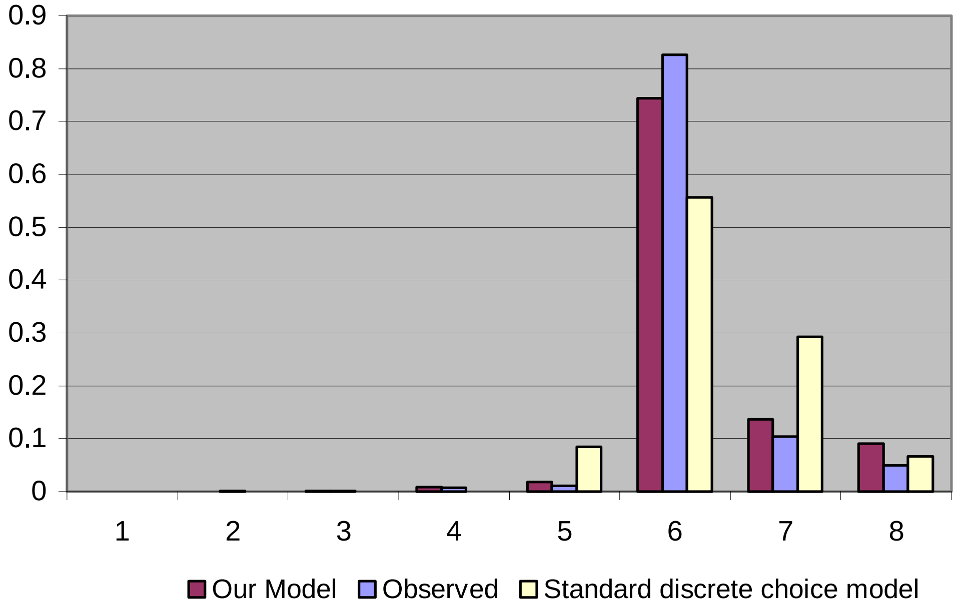


Figure 4. Predicted and observed distributions of hours of work for married females, 2003

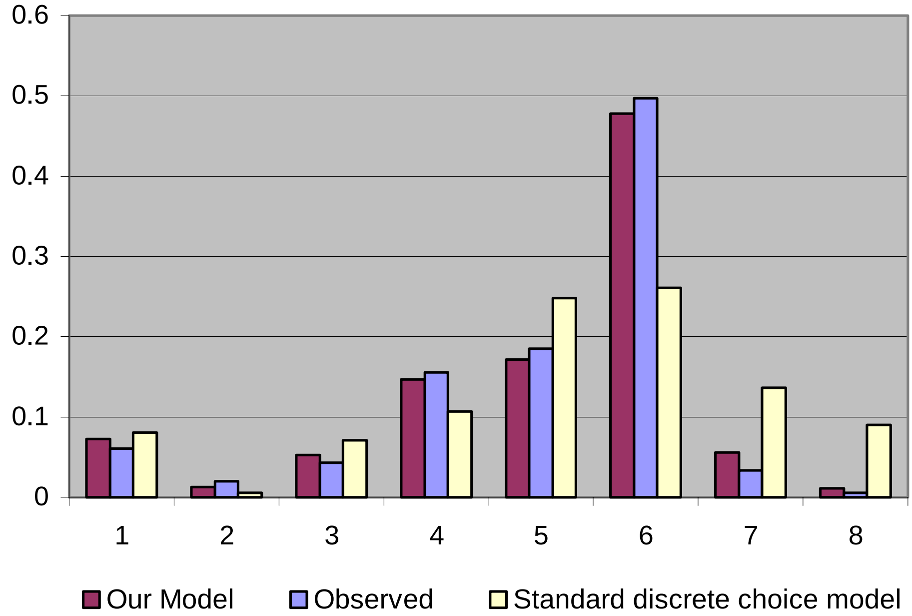
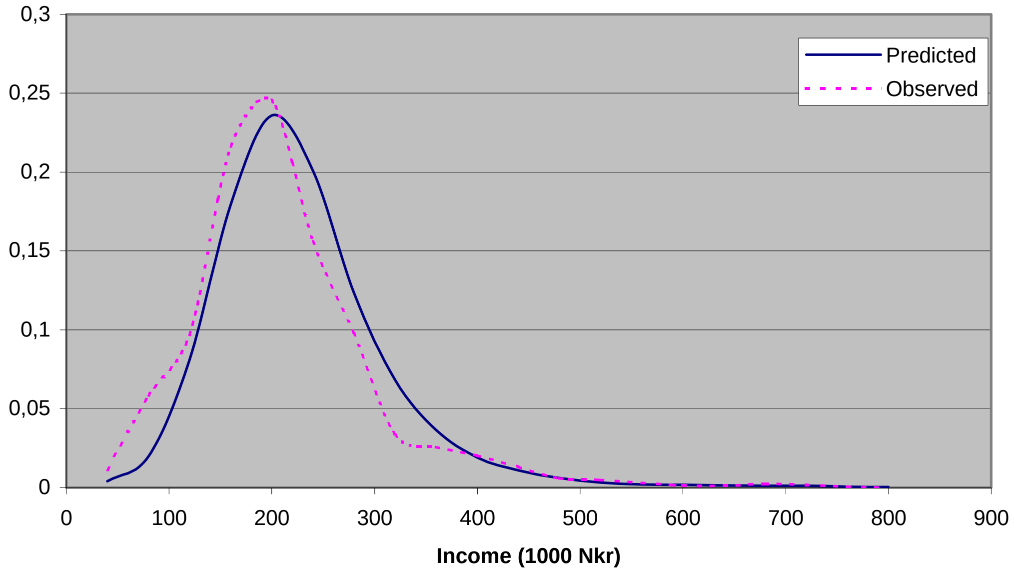


Figure 5 shows the observed and predicted disposable income distribution based on the LOTTE data. In this case, our model predicts well, which is an indication of its good performance. However, we need to be careful when interpreting this result. The distribution of disposable income depends mainly on the wage rate equations and the labor supply model (conditional on wage rates).

We have found that the shape of the distribution of disposable income appears to be quite robust with respect to moderate changes in the distribution of hours of work. Thus, it seems that the distribution of the error term in the wage rate equation is of crucial importance in this context. For example, the standard discrete choice model yields a similar distribution of disposable income to the one obtained from our model. Thus, a poor fit of the distribution of disposable income is not necessarily a sign of a poorly fitting underlying behavioral model, but could result from a misspecified distribution of the error terms in the wage rate equations. In fact, a closer look at the wage rate equations reveals that the assumption of normally distributed error terms in the wage rate equations seems restrictive. In simulations not reported, it is found that the wage rate equations are not capable of reproducing the right tails of the distribution of observed wages in the 1997- sample. This is not surprising because it is well known that the right tail of the lognormal distribution is not heavy enough to capture the right tails of most income distributions. In fact, a closer look at Figure 5 reveals that the right tail of the empirical density seems fatter than the tail of the corresponding simulation.

Figure 5. Observed and predicted density of disposable income for married couples, LOTTE 2003



6.4. Consistency with consumer demand relations

So far, we have not discussed explicitly the relationship between the labor supply model and the demand for consumption goods. In this section, we discuss how the labor supply model developed above can be made consistent with consumer demand relations and can thus be used to simulate the joint effect of changes in direct and indirect taxes as well as of changes in wage rates and commodity prices. First, note that the utility function $U(C, h, z)$ can be interpreted as conditional indirect utility given (C, h, z). This means that one can view the agent's choice behavior as a two-stage process: In the first stage, the agent chooses the preferred job, from which disposable income is earned. In the second stage, the agent allocates disposable income to consumption of different commodities. In the first-stage choice, the agent takes into account that the second stage allocation will be optimal (according to her or his preferences) given the prices of the commodities.

Recall that the estimated empirical specification of $v(C, h)$ has the structure

(6.2) $$\log v(C,h) = a_1 (C - C_0)^{\alpha_1} + a_2 (L - L_0)^{\alpha_2},$$

where a_2 depends on demographic household characteristics. We assume that α_1, a_2 and α_2 are independent of the vector of commodity prices, p. Consequently, only the term $a_1 (C - C_0)^{\alpha_1}$ matters for the second-stage allocation. The terms a_1 and C_0 are functions of the commodity prices. Hence, our utility function $U(C, L, z)$ can be interpreted as a conditional indirect utility function, and can be written as

(6.3) $$\log U(C,h,z) = a_1^* \left(\frac{C - C_0(p)}{P(p)} \right)^{\alpha_1} + a_2 (L - L_0)^{\alpha_2} + \log \varepsilon(z),$$

where $a_1^* / P(p)^{\alpha_1} = a_1$, and $P(p)$ and $C_0(p)$ are linear-homogeneous, concave, decreasing functions of the commodity prices p. In the context of the consumer demand relations, (6.3) is equivalent to an indirect utility function of the Gorman Polar form, cf. Gorman (1953). The corresponding demand relations follow from Roy's identity.

Thus, when the functions $C_0(p)$ and $P(p)$ are determined (and suitably calibrated to be consistent with the subsistence level and the estimate of a_1 in the period for which the model was estimated) one can carry out policy simulations to assess the effect of changes in goods prices, taxes, indirect taxes and wage rates. (Note that indirect taxes enter the model through transformations of prices.) In fact, an extensive consumer demand system for Norway that is consistent with the above formulation has been developed by Statistics Norway. A brief descriptive summary of this demand system is given in Aasness, Bye and Mysen (1996), see Statistics Norway (2006) for more detailed information.

7. Conclusion

In this paper, we have discussed the application of a particular modeling framework for empirical analysis of labor supply behavior with a view towards model assessment and practical use in policy simulation experiments. An essential feature of the modeling framework is that it allows for latent job opportunities and restrictions on the latent set of feasible jobs. Furthermore, we have developed a novel approach to accommodating unobserved heterogeneity in the latent choice sets of job opportunities.

To demonstrate the applicability of our approach, we estimated models for single females, single males and married couples. The estimated models reproduce the data within the sample well. The (uncompensated) wage elasticities implied by the models are small or moderate in magnitude. In addition, the wage elasticities computed conditional on deciles of disposable income are found to decline gradually by decile. To test the performance of the model, we have used it to predict out-of-sample behavior in 2003 on two different data sets. The results show that our model is able to predict the actual distribution of hours of work and disposable income quite well.

For purposes of comparison, we also have estimated a standard discrete choice model with a quadratic polynomial specification of the structural term of the utility function for married couples. The fit of this estimated model is poor, as is its out-of-sample predictive performance. This is mainly because this model cannot explain the observed peaks at full- and part-time hours of work.

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Simulation of the distribution of disposable income for single households

Recall that disposable income for single households is given by:

(A.1) $$C = f\left(hW, I\right),$$

where the agent's wage rate *W* is predicted by the wage equation

(A.2) $$\log W = X\beta + \eta,$$

where η is assumed normally distributed with zero mean and variance σ^2. Let us extend the notation introduced in Section 2.1 by letting $\varphi(h \mid W, I, Z)$ be the probability density of realized hours of work (that is, hours of work of the chosen job), conditional on the wage equation, nonlabor income *I* and the characteristics *Z* that affect preferences. Furthermore, let $K\left(hW, I, y\right) = 1$ if $f\left(hW, I\right) \le y$ and zero otherwise. Then

(A.3)
$$\begin{aligned}
&P\left(C \le y\right) = P\left(f\left(hW, I\right) \le y\right) = EP\left(f\left(hW, I\right) \le y \middle| W, I, Z\right) = EK\left(h\exp\left(X\beta + \eta\right), I, y\right) \\
&= \sum_h E\left(K\left(h\exp\left(X\beta + \eta\right), I, y\right)\varphi\left(h \middle| W, I, Z\right)\right) \\
&\cong \frac{1}{NT}\sum_{i=1}^{N}\sum_{r=1}^{T}\sum_h K\left(h\exp\left(X_i\beta + \eta_r\right), I_i, y\right)\varphi\left(h \middle| \exp\left(X_i\beta + \eta_r\right), I_i, Z_i\right)
\end{aligned}$$

where *N* is the size of the micro-simulation population and *T* is the number of i.i.d. draws from the normal distribution with variance σ^2. The simulation of the conditional distribution given the characteristics of a specified population group is the same. In addition, the simulation of disposable income for couples is analogous to the case considered above.

Simulation of conditional aggregate elasticities given that disposable income is restricted

We now consider the derivation of the joint probability of hours of work and disposable income (consumption). This simultaneous choice probability is needed for computing conditional elasticities such as the wage elasticity of labor supply, given the level of consumption. We only consider the case of single-individual households. Let *H* denote the chosen hours of work, that is, hours of work in the chosen job, and let

$$\tilde{\varphi}\left(h, y \mid I\right) = P\left(H = h, C \le y \middle| I\right).$$

Implicit in the above definition is that we only condition on the explanatory variables in the wage equation. The error terms in the wage equations will be integrated out. From (A.1), it follows that

(A.4)
$$\begin{aligned}\tilde{\varphi}(h,y\mid I) &= P\left(f(hW,I)\le y,\, H=h \middle| I\right)\\ &= E_W\left\{P\left(f(hW,I)\le y \middle| H=h,W \middle| I\right)\varphi(h\mid W,I)\right\},\end{aligned}$$

where E_W denotes the expectation with respect to W, where W is the wage rate. Note that, when (H,W) are given, then $f(hW,I)$ is nonstochastic and the probability

$$P\left(f(hW,I)\le y \middle| H=h,W,I\right)$$

is equal to zero or one. Hence, we can express (A.4) as

(A.5)
$$\tilde{\varphi}(h,y\mid I) = E_W\left\{K(hW,I,y)\varphi(h\mid W,I)\right\}.$$

Similarly

(A.6)
$$\tilde{\varphi}(0,y\mid I) = K(0,I,y)E_W\varphi(0\mid W,I).$$

In practice, the probability in (A.5) is computed by stochastic simulation as follows. Let W^r be given by the wage equation

(A.7)
$$\log W^r = X\beta + \sigma\eta^r,$$

where $\eta_j^r, r=1,2,...,M,$ are independent draws from $N(0,1)$. If M is large, then

(A.8)
$$\tilde{\varphi}(h,y\mid I) \cong \frac{1}{M}\sum_{r=1}^{M} K\left(hW^r,I,y\right)\varphi\left(h\mid W^r,I\right).$$

Once we have obtained $\tilde{\varphi}(h,y\mid I)$, it follows immediately that, for example

(A.9)
$$P\left(H=h,\middle| y_1 < C \le y_2, I\right) = \frac{\tilde{\varphi}(h,y_2\mid I)-\tilde{\varphi}(h,y_1\mid I)}{\sum_{x>0,x\in D}\left(\tilde{\varphi}(x,y_2\mid I)-\tilde{\varphi}(x,y_1\mid I)\right)+\tilde{\varphi}_0(0,y_2\mid I)-\tilde{\varphi}_0(0,y_1\mid I)}.$$

Equation (A.9) represents the conditional density of chosen hours given that disposable income lies within the interval $\left(y_1, y_2\right)$. From this expression, we can compute the corresponding conditional mean hours of work and different types of conditional elasticities, such as the conditional elasticity of mean hours given that disposable income lies within some interval. We can also use (A.8) to simulate the marginal distribution of disposable income because this distribution is given by

(A.10) $$\tilde{\varphi}(y \mid I) \equiv \sum_{x>0,\, x\in D} \tilde{\varphi}\left(x, y \mid I\right) + \varphi\left(0, y \mid I\right).$$

Appendix B

Properties of the family of Stable distributions

A Stable distribution with parameters α, σ, β and μ, is often denoted by $S_\alpha(\sigma,\beta,\mu)$. The parameter α is restricted such that $0<\alpha\le 2$, and is an index that characterizes the heaviness of the tails, whereas σ is a positive scale parameter that is similar to the standard deviation. The parameter β is restricted to the closed interval $[-1,1]$ and it characterizes the skewness of the distribution; $\beta=0$ implies symmetry whereas $\beta=1$ $(\beta=-1)$ implies that the distribution is totally skewed to the right (left). The parameter μ is a location parameter that coincides with the expectation when $\alpha>1$, whereas the expectation is not defined when $\alpha\le 1$. When $\alpha=2$, the distribution reduces to the normal distribution, in which case, β vanishes. However, when $\alpha<2$, the variance is infinite. It also follows from condition (ii) that $\mu=0$. If we impose (i), it follows in addition that $\alpha<1$ and $\beta=1$, cf. Samorodnitsky and Taqqu (1994). When μ = 0 we say that the distribution is strictly stable.

Assumptions that yield positive strictly stable random variables

These are the following: (i) $\tilde{m}(h,w)>0$; (ii) any positive value of $\tilde{m}(h,w)$ is possible, (iii) for any hours of work, h_1 and h_2, and nonnegative constants, b_1 and b_2, then $b_1\tilde{m}(h_1,w)+b_2\tilde{m}(h_2,w)$ should have the same distribution as $c\tilde{m}(h_1,w)$, provided that $\tilde{m}(h_1,w)$ and $\tilde{m}(h_2,w)$ are independent, where c is a positive constant that may depend on h_1, h_2, b_1 and b_2. The motivation for (i) is obvious: unless this condition is satisfied, for some hours of work, the conditional choice probabilities would be zero or negative. Condition (ii) also seems highly plausible. Condition (iii) implies that for any $h_1,h_2,...,h_r$ within D, the distribution of the conditional aggregate choice probabilities, given by

$$\sum_{k=1}^{r}\varphi\left(h_k\mid w,I,\{\kappa(z)\}\right),$$

which are random variables because they depend on $\{\kappa(z)\}$ through $\{\tilde{m}(h,w),h\in D\}$, across unobservable choice sets belongs to the same family of distributions as the conditional choice probabilities, $\varphi\left(h_k\,|w,I,\{\kappa(z),z=0,1,..\}\right)$. In other words, requirement (iii) implies that the distribution of the conditional choice probabilities is invariant under the aggregation of alternatives (hours of work). Since the aggregation level within the total set of feasible hours is somewhat arbitrary, it seems reasonable that the distributional properties of the model do not depend critically on the partition of the set of feasible hours into alternatives. It follows that from the theory of Stable distributions that Assumption (iii) implies that $\tilde{m}(h,w)$ is stably distributed, see for example Samorodnitsky and Taqqu (1994). Furthermore, Assumption (i) implies that $\alpha < 1$ and $\beta = 1$. Finally, Assumption (iii) implies that $\mu = 0$.

Proof of equation (2.15)

For expositional simplicity, we simplify notation in proving the result. Consider the choice among M discrete alternatives. Alternative j has utility $U_j = v_j \tilde{m}_j \varepsilon_j$, where ε_j, $j = 1,2,...,M$, are i.i.d. positive random variables with c.d.f. $\exp(-1/x)$, for $x > 0$, where $\{v_j\}$ are positive deterministic terms and $\tilde{m}_j, 1,2,...,M$, are independent with m_j and distributed according to $S_\alpha(\sigma_j, 1, 0)$, where σ_j is a positive parameter. Moreover, $\{\tilde{m}_j\}$ and $\{\varepsilon_j\}$ are independent.

Consider the c.d.f. of $\tilde{m}_j \varepsilon_j$. Because $\tilde{m}_j$ and ε_j are independent, it follows from Proposition 1.2.12 in Samorodnitsky and Taqqu (1994, p. 15) that

(B.1) $$P\left(\tilde{m}_j \varepsilon_j \le x\right) = EP\left(\varepsilon_j \le \frac{x}{\tilde{m}_j} \middle| \tilde{m}_j\right) = E\exp\left(-\tilde{m}_j x^{-1}\right) = \exp\left(-k\sigma_j^\alpha x^{-\alpha}\right)$$

where $k = 1/\cos(\alpha\pi/2)$. However, this implies that $\tilde{m}_j \varepsilon_j$ has the same distribution as $k^{1/\alpha} \sigma_j \varepsilon_j^{1/\alpha}$. Furthermore, this implies that the utility function U_j is equivalent to the utility function $\tilde{U}_j = v_j^\alpha \sigma_j^\alpha \varepsilon_j$ because $k^{-1} U_j^\alpha$ has the same distribution as $\tilde{U}_j$. According to results that are well known, it follows that

(B.2) $$P\left(U_j = \max_k U_k \middle| \{\tilde{m}_k\}\right) = \frac{v_j \tilde{m}_j}{\sum_k v_k \tilde{m}_k},$$

and that

(B.3) $$P\left(\tilde{U}_j = \max_k \tilde{U}_k\right) = \frac{v_j^\alpha \sigma_j^\alpha}{\sum_k v_k^\alpha \sigma_k^\alpha}.$$

Consequently, it follows that

$$E\left(\frac{v_j\tilde{m}_j}{\sum_k v_k\tilde{m}_k}\right) = EP\left(U_j = \max_k U_k \middle| \{\tilde{m}_k\}\right)$$

(B.4)

$$= P\left(U_j = \max_k U_k\right) = P\left(\tilde{U}_j = \max_k \tilde{U}_k\right) = \frac{v_j^\alpha \sigma_j^\alpha}{\sum_k v_k^\alpha \sigma_k^\alpha}.$$

The result in (2.15) follows immediately from (B.4). It only remains to prove that $E\log\tilde{m}_j = \log\sigma_j$. Because $\tilde{m}_j\varepsilon_j$ has the same distribution as $k^{1/\alpha}\sigma_j\varepsilon_j^{1/\alpha}$, if follows that $E\log\tilde{m}_j + E\log\varepsilon_j = \log\sigma_j + \log k/\alpha + E\log\varepsilon_j/\alpha$. Moreover, $\log\varepsilon_j$ has the distribution $\exp(-\exp(-x))$, which has a mean equal to Euler's constant, 0.5772. Hence, $E\log\tilde{m}_j = \log\sigma_j + c = c + \log m_j/\alpha$, where c is a constant. Hence, the proof is complete.

Q.E.D.

Appendix C

Estimation results from the traditional model

Table C.1. Parameter estimates for the traditional discrete choice model

	Parameter	Estimate	S.E.
Preferences:			
Consumption	α_1	0.6865	0.2072
Consumption squared	α_2	0.0019	0.0017
Female leisure β_{F1}			
Constant	β_{F11}	28.3200	2.1910
(age/10)	β_{F12}	–5.8966	3.0280
(age/10) squared	β_{F13}	0.9364	0.3421
No. of children below 6 years	β_{F14}	3.1722	0.5000
No. of children above 6 years	β_{F15}	1.4945	0.2893
Female leisure squared	β_{F2}	–22.4610	1.7702
Male leisure β_{M1}			
Constant	β_{M11}	97.2747	1.9108
(age/10)	β_{M12}	–4.0438	1.8206
(age/10) squared	β_{M13}	0.6137	0.2501
No. of children below 6 years	β_{M14}	–0.7105	0.6104
No. of children above 6 years	β_{M15}	–1.0290	0.4812
Male leisure squared	β_{M2}	–115.1640	0.6064
Leisure interaction	β_{MF}	48.5725	1.8799
Consumption and female leisure interaction	α_4	0.6034	0.0810
Consumption and male leisure interaction	α_3	0.9158	0.1260
Fixed cost of working for female	γ_F	1.4744	0.1655
Fixed cost of working for male	γ_M	NA	
Number of observations		2,511	
Log likelihood		–6,484.1	
McFadden's ρ^2		0.368	

More results for in-sample and out-of-sample predictions

Goodness of fit of the model

Figure D.1. Predicted and observed distributions of hours of work for single females, 1997

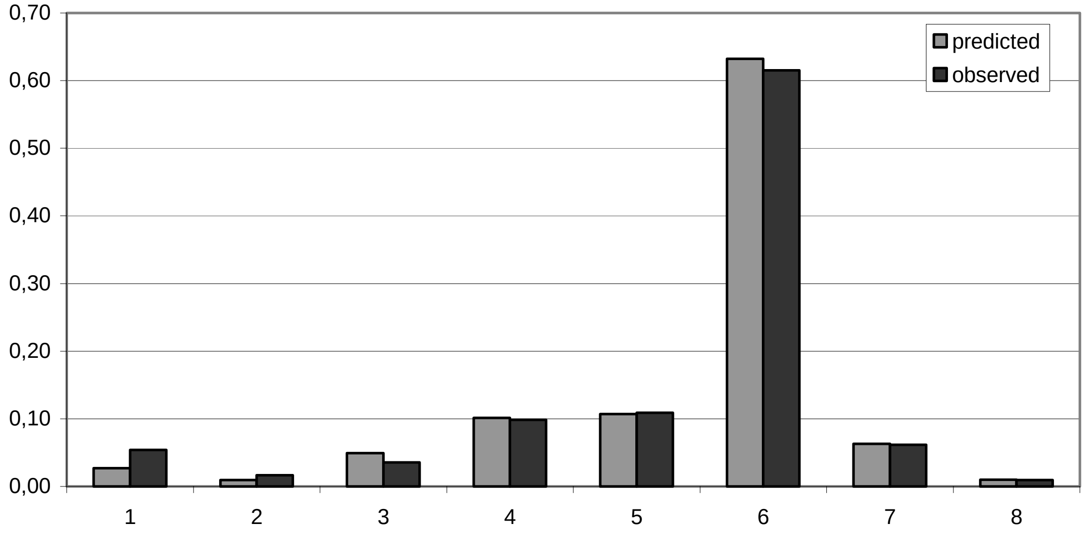
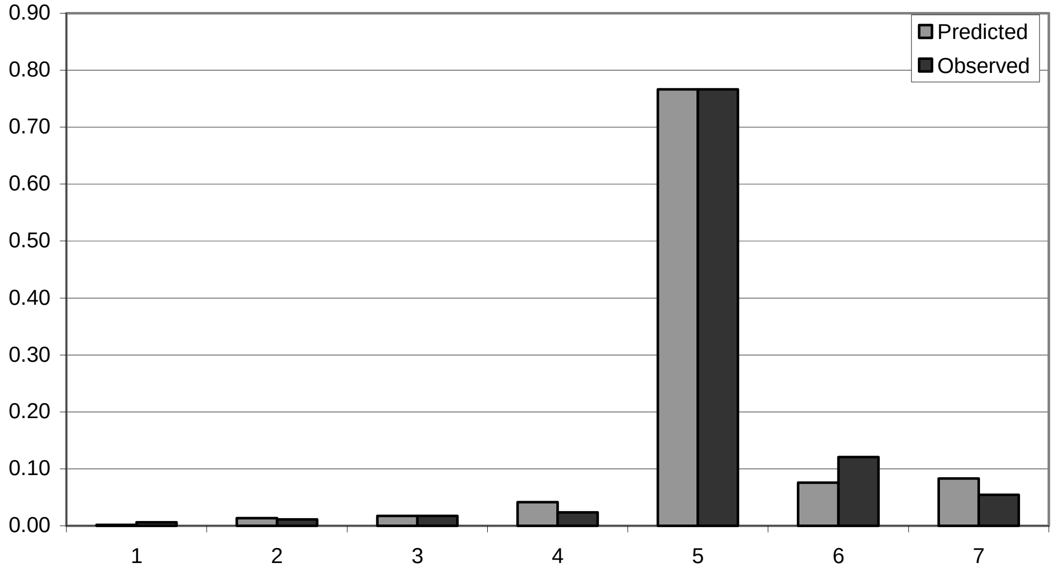


Figure D.2. Predicted and observed distributions of hours of work for single males, 1997



Out-of-sample predictions for 2003

Figure D.3. Predicted and observed distributions of hours of work for single males, 2003

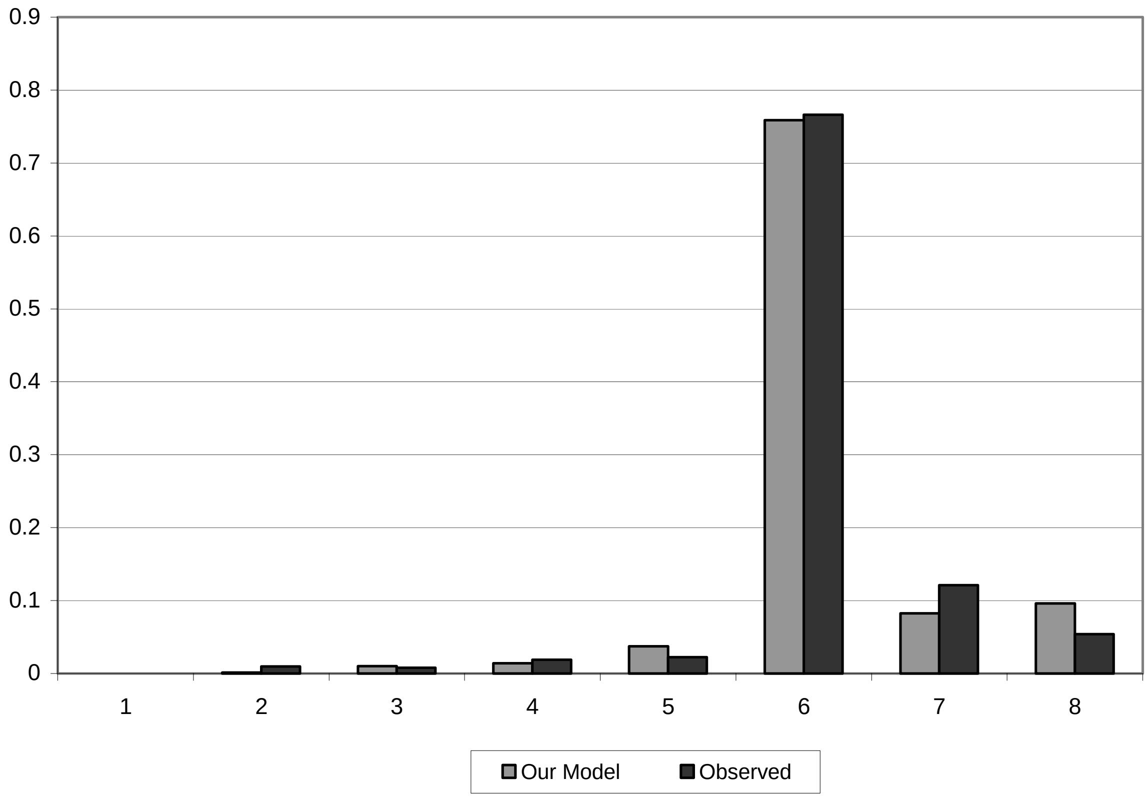


Figure D.4. Predicted and observed distributions of hours of work for single females, 2003

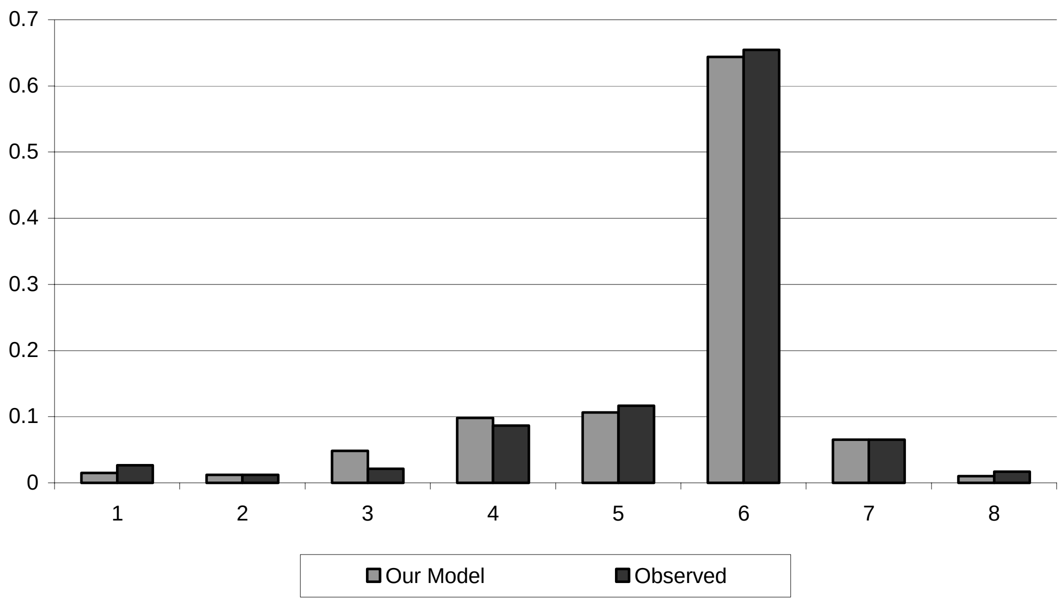


Figure D.5. Predicted and observed density of disposable income for single males, 2003, LOTTE

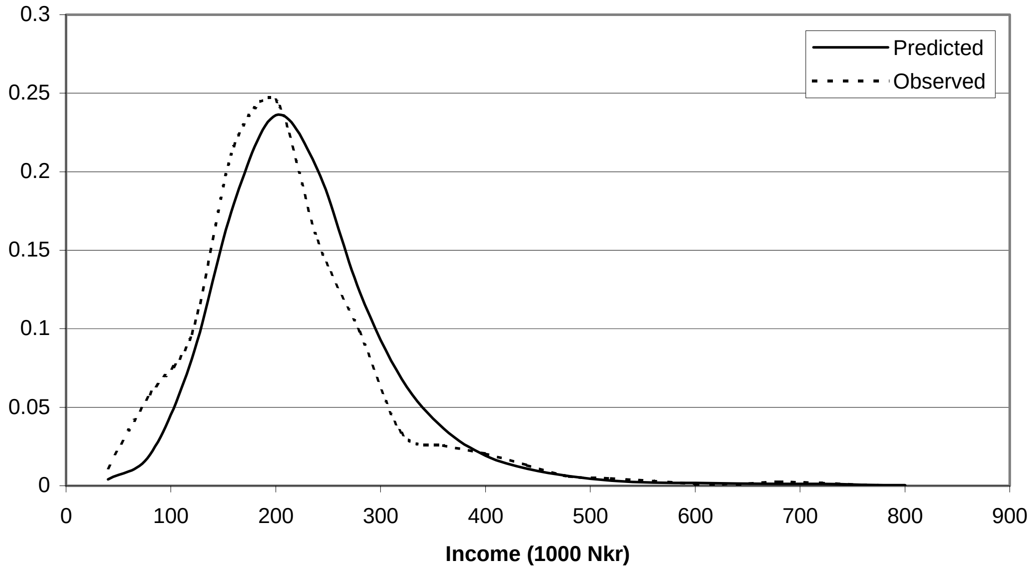
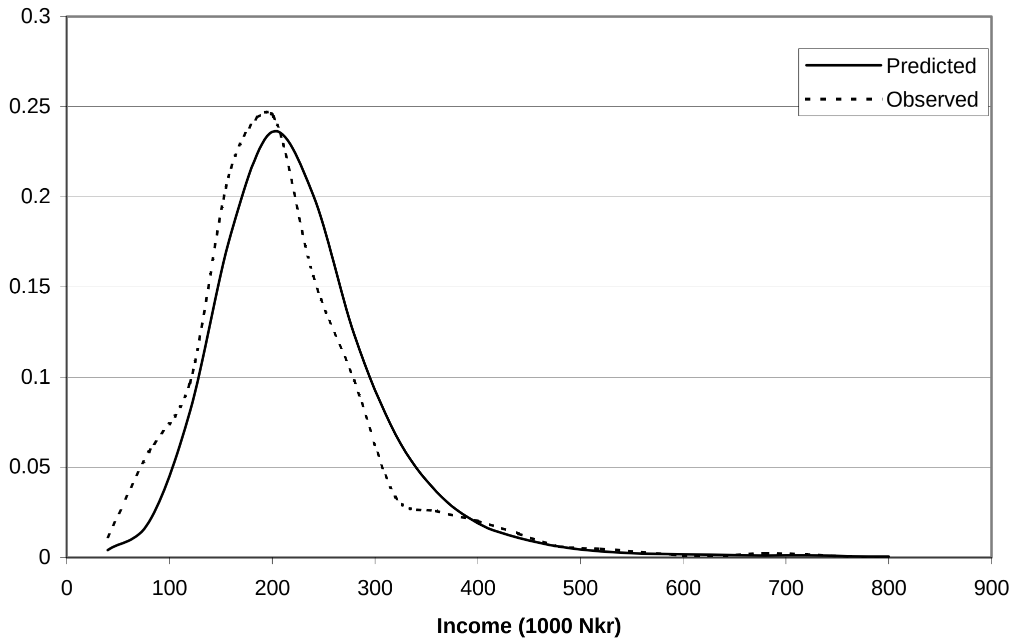


Figure D.6. Predicted and observed density of disposable income for single females, 2003, LOTTE



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