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Optimal Climate Policy under the Possibility of a Catastrophe

Abstract:
This paper concerns optimal emissions of greenhouse gases when catastrophic consequences are possible. A numerical model is presented which takes into account both continuous climate-feedback damages as well as the possibility of a catastrophic outcome. The uncertainty in the model concerns whether or not a future catastrophe will occur. However, the welfare losses imposed by such an outcome are assumed known to the decision-maker. An important result is that the possibility of a climate catastrophe is a major argument for greenhouse gas abatement even in absence of continuous damage. Special attention is given to analyses on the probability of a catastrophe and the pure rate of time preferences, and the implicit values of these parameters are calculated if the Rio stabilisation target is assumed to be optimal. Finally, the expected value of perfect information about the probability of the arrival of a catastrophe is estimated.

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1. Introduction

The natural greenhouse effect due to the concentrations of so called greenhouse gases (GHGs) in the atmosphere, was first described by the French scientist Jean Baptiste Fourier in 1824. He drew a parallel between the action of the atmosphere and the effect of glass covering a container, thus giving the name of the phenomena. Without this natural greenhouse effect, the earth's average surface temperature would have been -18°C rather than the +15°C that is observed today. In this century the greenhouse effect received little attention before the end of the 1960's. But by the middle of the 1980's, global warming was established as a problem of international concern due to an increasing awareness of environmental problems. Recently, the Intergovernmental Panel on Climate Change (see IPCC 1996a,b), has stated the possible serious impacts of the manmade greenhouse effect, and the first binding international treaty to reduce GHGs will be signed in Kyoto in December 1997.

There are at least three *uncertainty* aspects of climate change from an economist's point of view. The first is how much it will cost to reduce future CO_2 emissions, the most important greenhouse gas, which among other things depends on future emissions paths (see, e.g., Nordhaus and Yohe, 1983; Reilly *et al.*, 1987; Manne and Richels, 1994). Another aspect is the effectiveness of instruments to control CO_2 emissions (see, e.g., Peck and Teisberg, 1993; Pizer, 1997), while the most important aspect probably concerns the damage from global warming. One overview of the economic costs of global warming is given in Fankhauser (1995), while some analyses on optimal CO_2 emissions paths under uncertain climate change and uncertain climate change impacts are documented in, e.g., Nordhaus (1994a), Larson and Tobey (1994), and Eismont and Welsch (1996).

One aspect of damage uncertainty concerns *catastrophes*. The possibilities of catastrophic outcomes of global warming cannot be excluded. This may hit both regionally and globally. Examples of regional catastrophes may be disintegration of the West Antarctic ice sheet and shut down of the ocean conveyer belt, i.e., redirection of the Gulf stream. The first is regional in the sense that only coasts are affected, while the second may result in cooling in Europe. Global catastrophes may be due to a runaway greenhouse effect, i.e., accelerating positive feedback effects. Warming releases the greenhouse gas CH_4 from previously frozen soils, resulting in more warming, leading to more CH_4 etc. There is also the danger of the currently smooth climate becoming volatile, i.e., there being bigger and more frequent temperature changes. That earlier climates were like this is suggested by some results from ice-core analyses. The distinction between a regional and a global catastrophe is, however, not clear. First, a global catastrophe may not hit all regions in the same way. Second, there are links like international trade, that may lead to high negative impacts in other regions than the one that is hit. A non-climate example is an earthquake in Tokyo, or a hurricane hitting New York, that whips out large assets. As these cities are major financial centres, this will cause havoc around the globe. Something similar will probably happen if a redirection or a weakening of the Gulf stream affects Europe. Grain prices would go up with Europe failing to produce, and ultimately it may again be the poor that suffer.

Catastrophic environmental outcomes due to stock pollutants have been analysed in the theoretical economic literature. Cropper (1976) studies two types of catastrophes, one that leads to a temporary reduction in utility (catastrophic pollution), and one which is irreversible (depletion with uncertain reserves). In both cases a catastrophe occurs if an uncertain level of the stock variable (pollution or aggregated extraction) is exceeded. The probability of the catastrophe is increasing in the stock. Heal (1984) studies catastrophes related to global warming, and formulates a model similar to the first of Cropper's models. He also presents a model where the atmosphere enters as an input factor in production, either favourable or unfavourable. Initially, it is favourable, but the probability that it turns to be unfavourable increases with cumulative emissions in the atmosphere. Recently, the applications of catastrophe theory to global warming have increased. Clarke and Reed (1994) analyse irreversible catastrophic climate outcomes with catastrophic risk positively related to pollution stock levels. Both

continuous and discontinuous (catastrophes) damages are considered. By using the survivor probability as a state variable, the problem is analysed as one of deterministic control. A different approach is presented in Tsur and Zemel (1996). Here, an uncertain critical pollution level (threshold) exists which when overshot triggers a catastrophic outcome. In contrast to Cropper (1976), they assume that the probability distribution is dependent on both history and trend. Finally, Torvanger (1997) analyses an uncertain irreversible climate change in a three-generation planning model, where climate change may occur in period two and thus reduce the production available for the forthcoming generation.

Several numerical papers on integrated assessment models analyse the role of uncertainty in connection with greenhouse warming. Most of these studies attach probabilities to a set of values of important input parameters like energy efficiency, elasticities, abatement costs and technological change (see, e.g., Nordhaus and Yohe, 1983; Reilly *et al.*, 1987; Manne and Richels, 1994; Nordhaus, 1994a). Yohe (1996) applies such an approach to analyse extreme events. Sensitivity analyses are conducted with respect to two different sources. First, global climate sensitivity, which measures the temperature rise associated with a doubling of atmospheric concentration. Second, a damage sensitivity, which reflects the percentage of world GDP that would be lost annually if the global mean temperature were to rise by 3 degrees through the year 2100. Yohe works with a full range of probabilistically weighted emissions trajectories and assumes complete resolution of uncertainty by the year 2020. Another paper on optimal policy under the risk of catastrophes is Nordhaus (1994a). A calamity is assumed to take form of an extreme non-linearity in the damage function. As temperature increases up to a threshold value, marginal damages increases sharply. However, in this approach the policy makers know the threshold value with certainty. Peck and Teisberg (1994) consider catastrophic climate changes in a stochastic version of the CETA model, R-CETA, where the probability that a catastrophe will occur in a given time period is a function of temperature rise in that period. Some strong assumptions are, however, made like the probability of a loss occurring in each time period is independent of the number of losses that may have previously occurred. Furthermore, only damages which arise from catastrophes are considered.

This paper studies the optimal greenhouse policy under the possibility of an irreversible global catastrophe by presenting simulations on an integrated assessment model. A main purpose of the work is to establish a numerical model for analysing possible catastrophic outcomes arising from global warming. In contrast to, e.g., Peck and Teisberg (1994), we distinguish between two types of global warming costs - continuous deterministic costs and stochastic high consequence outcomes. Furthermore, a hazard function is applied to characterise the probability of a catastrophe induced by climate changes. The probabilities for the arrival of a catastrophe depend on temperature level, and a catastrophic event is assumed to reduce the utility of consumption (or production). The focus is on how optimal abatement policy is affected by different assumptions on the magnitude of losses due to a catastrophe, the probability for the occurrence of a catastrophe, and to changes in the pure rate of time preference and the substitution elasticity of consumption. In addition, we construct a menu for values of the rate of time preference and the perception of the likeliness of a catastrophe under the assumption of a stabilisation of global CO_2 emissions at 1990 level, which is congruent with optimality criteria. Finally, we assess the expected value of perfect information of the probability of the arrival of a catastrophe at a given point of time.

Our study is organised as follows. In Section 2, the theoretical framework is presented, and the characteristics of the optimal policy are discussed. The integrated assessment model is presented in Section 3, and Section 4 gives the simulation results. Finally, the paper ends with conclusions and policy recommendations.

2. The theory

The main purpose of this section is to illuminate the forces at play when the possibility of a catastrophe is modelled in a dynamic model of optimal greenhouse gas emissions. To do this, we

present a theoretical model which acts as a benchmark for the numerical analysis presented in the next sections. The theoretical model focuses on crucial relationships without considering all the details which inherently follows from a numerical approach. The approach chosen is to express the dynamic optimisation problem in terms of a hazard rate function in order to solve the stochastic problem by deterministic controls. This was first introduced in the economic literature by Kamien and Scwartz (1971).

The theoretical model builds upon the work of Clarke and Reed (1994) but differs in some respects. First, climatic feedback effects are not generated by the stock of carbon itself, but by the average global temperature level. This approach seems more in line with the literature on integrated assessment (see, e.g., Kverndokk, 1994; Nordhaus, 1994a). Second, the world is divided into N regions or countries, in which a social planner is deciding on optimal emissions paths for each of the regions in order to maximise global welfare. This allows for an analysis of the implications of heterogeneous characteristics across regions. Third, a more general representation of the costs that go with a catastrophic collapse is introduced.

We first consider a deterministic version of the model where catastrophic risk is not present. A social planner is to decide upon the optimal paths of greenhouse gas emissions, x_n , for each of the N regions over an finite horizon (t_1), by maximising the following time additive intertemporal welfare function (W_1);

$$(1) \qquad W_1 = \int_0^{t_1} U[\sum_n Y_n(x_n(t)), T(t)] e^{-rt} dt$$

where U[.,.] is the utility function assumed to depend positively on the sum of regional outputs[1] $[Y(t) = \sum_n Y_n(x_n(t))]$ and negatively on average global temperature level above pre-industrial level, T(t). Output for each region, Y_n, is a function of the greenhouse gas emission rate for the same region, x_n. The disutility in average global temperature level reflects various costs which are imposed by a higher global temperature due to sea level rise, adverse health effects, changes in water availability, changes in the natural growing conditions for agricultural production systems etc. (see, e.g., Fankhauser, 1995). r denotes the utility discount factor (social rate of pure time preference). Furthermore, U is continuous, twice differentiable and concave in both aggregate output (Y) and temperature level (T).

The temperature level dynamics is described by the following equation,

$$(2) \qquad \dot{T} = l[x(t), T(t)] \qquad where \quad x(t) = \sum_n x_n(t)$$

where average global temperature is a function of aggregate emissions, x(t), and the temperature level itself, T(t). The history of past emissions determines the evolution of the stock of greenhouse gases in the atmosphere which again determines average global temperature at any date. The level of temperature is introduced as an argument in l(.,.) in order to reflect depreciation processes[2].

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[1] One problem with this formulation is that distributional considerations are not taken into account. An extreme outcome of the maximisation problem can be that it is optimal that a small island state disappears as a result of sea level rise due to high abatement costs in larger economies. In this paper, however, we do not study the regional aspects.

[2] In the numerical analysis, richer and more detailed interlinkages between greenhouse gas emissions, temperature level, and carbon sequestration effects are modelled.

The model represented in (1) and (2) shares similarities with other integrated assessment models in that there is a trade-off between immediate and future welfare due to the presence of continuous climate-feedback effects. We now extend the above model in order to take into account the possibility of a future catastrophic event. The social planner is exposed to uncertainty but it is assumed that he is able to attach subjective probabilities to the occurrence of such an event. Furthermore, it is assumed that no new information is acquired so that these probabilities are not revised over time. However, the probabilities depend on global mean temperature above pre-industrial level. As a consequence, the probabilities for a catastrophic event can be denoted as endogenous. We rule out the case where the evolution in the temperature level is stochastic. The temperature level can at any time be predicted exactly from the history of greenhouse gas emissions.

Tsur and Zemel (1996) present a model in which a catastrophic collapse will occur if accumulated pollution reaches an unknown critical level. This model can be said to introduce a learning process in that the unknown threshold value always must be higher than the current pollution level as long as a catastrophe is not triggered[3]. As a consequence, it is possible for the decision-maker to avoid altogether the occurrence of risk in this model. Our approach, however, focus more on the genuine stochastic nature of the processes involved in climatic changes. Such phenomena are inadequately understood and there is an inherent complexity present at virtually every stage. The dynamic nature of the overall climate system suggests that the occurrence of catastrophes are not restricted to the average temperature level alone but depends on complex interactions between several causal factors. There are possible feedback effects which over time may strengthen or weaken the effects from any temperature level. Besides, long time lags are involved so consequences are imposed slowly and with great deal of inertia. Examples here may be the melting down of ice caps and shifts in ocean streams.

Most analysis on catastrophic events portray the occurrence of a catastrophe as causing a sudden change in welfare and/or production. As a consequence, the nature of catastrophic costs are inherently different from continuous climate-feedback effects. It is also assumed that the decision-maker is fully informed about the magnitude of this sudden change. Here we model catastrophic costs as a utility loss. This approach is along the lines of Cropper (1976), Clarke and Reed (1994), Peck and Teisberg (1994) and Tsur and Zemel (1996)[4].

Introducing this feature in our deterministic model implies that the objective function in (1) can be described as follows;

$$
(3a) \qquad W_2 = E\left\{ \int_0^{Min(t_1,\tau)} U[\sum_n Y_n(x_n(t)), T(t)]e^{-rt}dt \quad + \quad \int_{Min(t_1,\tau)}^{t_1} V[\sum_n Y_n(x_n(t)), T(t)]e^{-rt}dt \right\}
$$

The planner is to maximise the discounted present expected value of the flow of utilities over a finite horizon. The expectation operator is taken with respect to the random variable τ which is the instant of time a catastrophic event occurs. The first term in the parenthesis is the stream of discounted utilities received until a catastrophe occurs. The second term is the discounted sum of the utilities received in a post-catastrophic world.

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[3] Tsur and Zemel (1996) define this uncertainty endogenous while they denote the type of uncertainty present in Clarke and Reed (1994) as exogenous since the probabilities do not depend on history but only on the pollution level (temperature level).

[4] Cropper (1976) and Clarke and Reed (1994) define a catastrophe as an event which reduces society's utility (or consumption) to zero. Clarke and Reed also consider a case where utility is reduced to a constant level much less than zero. Tsur and Zemel (1996) model the penalty as an additive subtraction to utility at the instant of time a catastrophe occurs, while Peck and Teisberg (1994) assume that if a loss occurs in a period t, consumption is multiplied by factor less than one.

As can be seen from (3a), we allow for the utility in the post-catastrophic world, V(.,.), to be different from utility in a pre-catastrophic world, U(.,.). If we consider the situation where V(.,.) = 0, utility will drop to zero in all time periods following a catastrophe. If V(.,.) = U(.,.) there are no utility costs associated with a catastrophic event[5].

In the following, an infinite horizon is assumed, and W_2 can be written as follows,

$$(3b) \qquad W_3 = E\left\{ \int_0^\infty U[\sum_n Y_n(x_n(t)), T(t)] 1_{\{t < \tau\}} e^{-rt} dt + \int_0^\infty V[\sum_n Y_n(x_n(t)), T(t)] 1_{\{t \geq \tau\}} e^{-rt} dt \right\}$$

where $1_{\{.\}}$ denotes the indicator function that assumes the values one or zero depending on whether the argument is true or false.

We will assume that the social planner's beliefs about a catastrophic event at an arbitrarily instant of time can be represented by a hazard function, $\Psi(t)$, which is a conditional probability rate that can be defined as follows (see, e.g., Kiefer, 1988; Clarke and Reed, 1994);

$$(4) \qquad \Psi(t) = \lim_{\varepsilon \to 0} \frac{\Pr(t \leq \tau < t + \varepsilon | \tau \geq t)}{\varepsilon}$$

The hazard function is the change in the conditional probability of a catastrophe to occur in the time interval (t, t+ε) given that a catastrophe has not occurred at all earlier instants of time[6]. Furthermore, a hazard function is related to a survivor function, S, in the following way;

$$(5) \qquad S(t) = e^{-\int_0^t \psi(s)ds}$$

From (5) it is seen that the survivor function evaluated at time t is dependent of all hazard functions from the initial planning date and until t. The survivor function can be defined as the probability of experiencing no catastrophe by date t seen from the initial date. By defining the following relationship;

$$(6) \qquad y(t) = -\ln S(t)$$

we can introduce the survivor function indirectly as a state variable in our problem. From (6) the two following relationships can be derived;

$$(7) \qquad e^{-y(t)} = S(t)$$

$$(8) \qquad \dot{y} = \Psi(t)$$

From (8) we observe that the time derivative of y(t) defined in (6) equals the hazard rate function. The derived properties of the hazard function can now be utilised in the stochastic model. First, we assume that the hazard function is related to the level of temperature in the following way;

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[5] In a numerical model, we have to be careful about the specific form of the utility function and the scaling of consumption chosen. While consumption is always non-negative, the utility may be both negative or positive. One example is a logarithmic utility function, U(C) = lnC, where U < 0 for C < 1. A catastrophe defined as V = 0 may in this case be desirable if C < 1. This may be the reason why some authors define a catastrophe as if the utility is much less than prior values (see, e.g., Clarke and Reed, 1994).

[6] Most probability distributions can be expressed in terms of hazard functions.

(9) $\quad \Psi(t) = h(T(t))$

where a higher average global temperature above pre-industrial level means that the beliefs about the occurrence of a catastrophe, represented by the hazard function, will undergo a positive shift.

The objective function presented in (3b) can be rewritten, and the planners' maximisation problem becomes as follows;

(10) $\quad \underset{x_i(t) \ ...x_N(t)}{Max} \ W_4 = \int\limits_o^\infty \{U[\sum\limits_n Y_n(x_n(t)), T(t)]S(t)e^{-rt}\}dt + \int\limits_0^\infty \{V[\sum\limits_n Y_n(x_n(t)), T_t)](1-S(t))e^{-rt}\}dt$

given that $x_n(t) \geq 0$ for all n, $T(0) = T_0 > 0$ and the following two constraints

(11) $\quad \dot{T} = l(x(t), T(t))$

(12) $\quad \dot{y} = h(T(t))$

The introduction of hazard functions has enabled us to solve the stochastic maximisation problem by using deterministic control. Note that the beliefs about a catastrophic event appear both in the criteria function (10) and in one differential equation (12). The function presented in (10) can be given a straightforward explanation. It is a weighted sum of the stream of utilities given a pre-catastrophe situation, U(.,.), and a post-catastrophic situation, V(.,.), where S(t) and (1 - S(t)) are the weights at any point in time. As a consequence, the objective function is the expected discounted stream of utility over the planning horizon.

The current value Hamiltonian (H) for this problem, using (7), is as follows

(13)
$$H = U[\sum\limits_n Y_n(x_n(t)), T(t)]e^{-y(t)} + V[\sum\limits_n Y_n(x_n(t)), T(t)](1-e^{-y(t)})$$
$$+ \lambda[l(x(t), T(t))] + \mu[h(T(t)]$$

where λ and μ are the adjoint variables. The necessary conditions for the optimal path, assuming interior solutions, are (time references are omitted)

(14) $\quad \dfrac{\partial H}{\partial x_n} = \dfrac{\partial U(.,.)}{\partial Y}\dfrac{\partial Y_n(.)}{\partial x_n}e^{-y(t)} + \dfrac{\partial V(.,.)}{\partial Y}\dfrac{\partial Y_n(.)}{dx_n}(1-e^{-y(t)}) + \lambda\dfrac{\partial l(.,.)}{\partial x_n} = 0 \quad n = 1,...,N$

(15) $\quad \dot{\lambda} - r\lambda = -\dfrac{\partial H}{\partial T} = -\dfrac{\partial U(.,.)}{\partial T}e^{-y(t)} - \dfrac{\partial V(.,.)}{\partial T}(1-e^{-y(t)}) - \lambda\dfrac{\partial l(.,.)}{\partial T} - \mu\dfrac{\partial h(.)}{\partial T}$

(16) $\quad \dot{\mu} - r\mu = -\dfrac{\partial H}{\partial y} = (U[.,..] - V[.,..])e^{-y(t)}$

(17) $\quad \lim\limits_{t\to\infty} e^{-rt}\lambda(t)T(t) = 0, \qquad \lim\limits_{t\to\infty} e^{-rt}\mu(t)y(t) = 0$

Eq. (14) describes the optimal emission path for each region by balancing short-term benefits against long-term costs. Along the optimal pattern of greenhouse gas accumulation the expected marginal

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increase in benefits associated with a higher emission rate must equal the expected change in costs that goes with the same change. Since the decision-maker cannot predict if and when a catastrophe occurs, his beliefs about marginal benefits are crucial. This is captured through the presence of a survivor function in the first two terms in (14) which are the expected increase in utility at instant t if emissions are increased at the margin at the same time. The third term in (14) is the costs expected along the optimal path from higher emissions, and equals the marginal change in the temperature dynamics from higher emissions (from region n) evaluated by the shadow price of temperature (λ).

Eqs. (15) and (16) determine the adjustments in the shadow values λ and μ along the optimal paths, respectively. μ(t) will be denoted the shadow value of a catastrophe and is the price associated with changes in the hazard rate function. Using (15) the shadow value of a catastrophic event can be expressed as follows:

$$(18) \qquad \mu(t) = -e^{rt}\int_t^\infty \{U[.,.] - V[.,.]\}S(v)e^{-rv}dv = -E\left\{\int_t^\tau [U(.,.) - V(.,.)]e^{-r(v-t)}dv\right\}$$

Thus, it depends on a stream of utility differentials across the possible outcomes until a catastrophic collapse. Or to put it another way, at any future date the difference between utility in a pre-catastrophic situation and utility in a post-catastrophic situation is multiplied with the survivor function evaluated at the same future dates. The shadow price of a catastrophic collapse is therefore the current discounted expected net gain in absolute utilities, measured in negative value. A higher emission rate in period t will increase the average global temperature level in all future periods, as a consequence the probability distribution for a catastrophic event will undergo a shift in the same time interval. Higher future probabilities for a catastrophe make utility losses more likely and in this way represents costs for the planner.

From (16) the shadow value of temperature can be expressed as follows,

$$(19) \qquad \lambda(t) = e^{(rt+\int_0^t \frac{\partial l(.,.)}{\partial T}ds)} \int_t^\infty \left[\frac{\partial U[.,.]}{\partial T}S(v) + \frac{\partial V[.,.]}{\partial T}(1-S(v)) + \mu\frac{\partial h(.)}{\partial T}\right]e^{-(rv+\int_0^v \frac{\partial l(.,.)}{\partial T}ds)}dv$$

$$= e^{(rt+\int_0^t \frac{\partial l(.,.)}{\partial T}ds)}\left[E\{\int_t^\tau \frac{\partial U[.,.]}{dT}e^{-(rv+\int_0^v \frac{\partial l(.,.)}{\partial T}ds)}dv\} + E\{\int_\tau^\infty \frac{\partial V[.,.]}{dT}e^{-(rv+\int_0^v \frac{\partial l(.,.)}{\partial T}ds)}dv\} + \mu\frac{\partial h(.)}{dT}e^{-(rv+\int_0^v \frac{\partial l(.,.)}{\partial T}ds)}dv\right]$$

Several effects are important for the determination of the shadow price of temperature. First, we observe that the shadow price is not only adjusted for discounting but also for depreciation. The depreciation effect pulls in the same way as a higher time preference rate, in that less weight is given to future consequences relatively to immediate ones. The first two terms within the parenthesis of (19) are costs associated with continuous climate-feedback effects. Higher emissions increase the average global temperature at all future dates, thus imposing a utility penalty in all future time periods independent of whether a catastrophe occurs or not. The first term is the expected decrease in utility from period t and until a catastrophe occurs, while the second term is the expected decrease in post-catastrophic utility. Together the two terms are the current value of the excepted marginal loss in utility following from a higher temperature level from period t and till the end of the planning horizon. The third term within the parenthesis is costs associated with a catastrophic collapse; the change in the hazard rate function from a marginal change in the temperature level, which captures shifts in the beliefs about future occurrence of a catastrophe, is evaluated using the shadow value of a catastrophe.

A full presentation of the optimal path of greenhouse gas emissions for region n can now be derived by inserting (18) and (19) into (14).

Let us look at the situation in which V(.,.) = U(.,.). It follows from (18) that the shadow price of a catastrophe becomes zero under this assumption. Furthermore, the hazard functions (or the survivor functions) can now be cancelled out in (14) and (19). As a consequence, we have arrived at a situation where the social planner when deciding on optimal emissions paths, no longer pays attention to the possibility of a catastrophic collapse. This conclusion makes sense since there are no costs associated with a catastrophic event. The social planner will as a consequence optimise in a deterministic world and only pay attention to continuous climate-feedback effects. If V(.,.) = 0 as in Cropper (1976) and Clarke and Reed (1994), a quite large penalty associated with a catastrophe is imposed[7]. Another possibility is that the utility in a post-catastrophic world is reduced to some low level for the rest of the planning horizon; $V(.,.) = \bar{V}$. The utility function is now independent of both its arguments after a catastrophe[8]. Thus, two marginal effects presented in (14) and (19) disappear. First, a higher emission rate (more output) will not contribute to increases in utility in a post-catastrophic world. Furthermore, a higher temperature has no negative effect on utility via continuous climate-feedback effects for the time periods following a catastrophic event[9].

If we consider the case where the beliefs about a catastrophic event is not influenced by the average global temperature level (exogenous uncertainty) it follows from (19) that the shadow price of a catastrophe does not play a role for optimal behaviour since $\partial h / \partial T = 0$. This conclusion share similarities with the one arrived at when V(.,.) = U(.,.). However, being exposed to the possibility of a catastrophic collapse has consequences for the optimal emission path under this assumption in contrast to a situation where V(.,.) = U(.,.). When human made actions have no effects on the likelihood of a bad outcome it is self-evident that the planner cannot be better off with changing emissions (ignoring continuous climate-feedback effects). However, the planner takes into consideration the fact that a catastrophic collapse will inflict a penalty upon him. This follows from the presence of survivor functions and expectation operators in (14) and (19). A constant hazard rate appears as a (constant) risk premium which is added to the time preference rate in the model, and we know that a higher time preference implies that less weight is given to future generations. Thus, emissions are increased compared to the case when V(.,.) = U(.,.).

3. Overview of the model

This section provides an overview of the model. Our simulations are based upon an extended version of the integrated assessment model presented in Kverndokk (1994). The features described in

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[7] However, it is possible to impose even worse penalties by allowing for negative utilities.

[8] If V ≠ U and if V is dependent on its argument, we get a problem with time consistency. The planner commits to the emissions path at time 0, and the occurrence of a catastrophe will make it optimal to change the path. When a catastrophe occurs, the argument to reduce emissions because of the possibility of a catastrophe disappears, and the optimal post-catastrophic emissions will jump upwards. An expected utility approach as the one we use, smoothens the paths, and the pre-catastrophic emissions will be higher and the post-catastrophic emissions will be lower than the time consistent emission paths. A catastrophe may occur at every point of time, and it is therefore difficult to say if the time inconsistent emissions are higher or lower than the optimal path. Thus, for V ≠ U and for V dependent on its argument, we consider average paths. If more than one catastrophe can happen, but only one at the time, optimal emissions will smooth and closely follow the average path. However, all the scenarios used in the numerical simulations below are time consistent.

[9] One option could also be to let the utility function experience a shift in a post-catastrophic situation compared to a pre-catastrophic situation, i.e., V = βU, β < 1. An interpretation of this is that the planner's evaluation of a given production - temperature level mix is reduced after a catastrophic collapse. However, if β ≠ 0, we get a problem with time consistency as described in footnote 8.

Paragraphs 3.1 through 3.3 are the same as in the original model, while the features described in Paragraph 3.4 are unique to the model of this paper.

The world is divided into seven blocks of countries: (1) USA, (2) EU, (3) the rest of OECD (ROECD), (4) the former Soviet Union (EX-USSR), (5) China, (6) India, and (7) the rest of the world (REST). Although this paper is based upon the seven-region model, the results are presented at the global rather than regional level. A long planning horizon is justified by the long-term impacts of global warming. Impacts are only felt after some 30 to 50 years, but may then persist for as much as two or three centuries. Thus, the model is simulated for the period 1990 to 2230, giving a planning horizon of 240 years.

3.1. The Production Module

The gross domestic product (GDP) in a country, P, is the product of two functions. First, the income function, Y, which is production in the absence of the climate-feedback effect. Second, D, which reflects continuous climate feedback effects.

(20) $\qquad P_n(t) = D_n[T(t)] \cdot Y_n[x_n(t); t]$

The *income function* in equation (20) is taken from Kverndokk (1993),

$$Y_n[x_n(t); t] = \hat{Y}_n(t) - G_n[x_n(t); t]$$

(21)

$$G_n[x_n(t); t] = \frac{q_n(t)\hat{x}_n(t)}{b_n(t)} \left(\frac{\hat{x}_n(t) - x_n(t)}{\hat{x}_n(t)} \right)^{b_n(t)}, \quad x_n(t) \leq \hat{x}_n(t)$$

where (the subscript is left out for simplicity):
b = the constant elasticity of abatement costs with respect to abatement ($\hat{x}$ - x),
q = the shadow price of CO_2 when x = 0, i.e., the tax on CO_2 emissions which leads to a total substitution away from fossil fuels to non-fossil backstop technologies (the switch price of CO_2),
Y = GDP in the absence of climate change,
$\hat{Y}$ = GDP in the absence of climate change and emissions constraints (business as usual (BAU) GDP without climate change),
$\hat{x}$ = CO_2 emissions from energy use in the absence of any emissions constraints (BAU CO_2),
G = abatement costs for abatement equal to $\hat{x}$ - x.

The income function is concave in CO_2 emissions for b > 1 and q > 0. The condition $b \geq q\hat{x}/\hat{Y}$ ensures that GDP is non-negative. The data are mainly taken from the OECD's GREEN model (see Burniaux *et al.*, 1992a,b).

The exogenous BAU scenario with no climate feedback ($\hat{x}$ and $\hat{Y}$) is defined so that the emissions maximise the production in the absence of global warming, i.e., $G_n[\cdot] = 0$. The *q-values* are based on Burniaux *et al.* (1992a,b), and they are assumed to decline due to an expected increase in fossil fuel prices, as well as falling costs of backstop technologies. However, the switch prices are constant from 2090 onwards. The *b-values* are calibrated such that the carbon taxes in the GREEN model (the Toronto scenario) equal the respective shadow prices of carbon in our model in 2015 at the specific emission levels. The exogenous paths of the b-parameters follow the paths for the autonomous energy-efficiency parameter (AEEI) in the GREEN model, which is a growth of 1 per cent p.a. in all regions from 1990 to 2050. We assume a uniform growth of 0.5 per cent p.a. until 2100 and zero growth in the parameter thereafter.

The annual *growth rates for BAU GDP* in the absence of climate change from 1990 to 2050 are based on Burniaux *et al.* (1992a). From 2051 onward, they build on the long-run growth rates in Manne and Richels (1992) and Cline (1992), leading to an almost 21-fold multiple of gross world product (GWP) by 2230 if there are no feedbacks from global warming. The *BAU CO_2 growth rates* from fossil fuel use are based on GREEN to the middle of the next century and build on the long-run growth rates from Manne and Richels (1992) and Cline (1992) thereafter. This gives an average annual growth rate of 0.4 per cent for the period 2100-2230. Under these assumptions, BAU CO_2 emissions from energy use increase to almost 60 billion ton carbon by 2230.

The *damage function* in equation (20) is specified in the following way, involving no damage in 1990:

$$(22) \qquad D_n[T(t)] \; = \; 1 \; - \; k_n \cdot \left(\frac{T(t) \; - \; T(0)}{\Lambda \; - \; T(0)} \right)^{\gamma}, \; \; \gamma \geq 1$$

Thus, k_n is the relative GDP loss in region n due to the climate-feedback effect at temperature increase Λ, where Λ is set to 2.5°C, i.e., $1 - D_n[\cdot] = k_n$ when $T(t) = \Lambda$. For an additive specification of the damages, see Fankhauser and Kverndokk (1996).
Several studies, see, e.g., Nordhaus (1991), Cline (1992), and Fankhauser (1992), estimate that a temperature increase of 2.5°C since preindustrial time will cause a global damage in the interval of 1-2 per cent of GWP. In this study we assume that the total damage at 2.5°C actual global temperature increase is 2 per cent of GWP.

For parameter calibration of the *k-values*, we use the damage distribution pattern in Fankhauser (1992). *T(0)*, the global temperature increase from preindustrial time to 1990, is set equal to 0.6°C, based on IPCC (1996b), while γ is set equal to 1.3, which is the estimate of Cline (1992).

3.2. The Climate Module

This model represents the 17 major GHGs, where only CO_2 emissions from energy use are specified endogenously. Thus, we assume that an international agreement only concerns these emissions and that other emissions, including CO_2 emissions due to changes in land-use patterns, are not affected.
The climate module of the model consists of a stock equation, (23), and several temperature equations, (24) and (25).

The *stock equation* gives the atmospheric concentration of CO_2 at different emission levels. Emissions of GHGs increase the corresponding concentrations in the atmosphere. However, the observed concentrations are less than they would have been if all emissions since pre-industrial time had been added to a constant pre-industrial stock. For CO_2, this is due to the removal of carbon from the atmosphere into the oceans and the terrestrial biosphere. One way of modelling this is to assume an annual depreciation of the stock of GHGs based on the lifetime of the actual GHG. We have chosen this model for the accumulation of CO_2 in the atmosphere, where the preindustrial stock is assumed to be the equilibrium stock. This means that if all anthropogenic emissions were eliminated, the atmospheric concentration would approach the preindustrial level. However, we do not apply a constant depreciation rate, due to saturation of the carbon-sink capacity of the oceans (see IPCC 1996b). This is shown in equation (23), where Q(t) is the atmospheric concentration of CO_2 at time t, Q^p is the preindustrial concentration, Z(t) is the difference between concentration at time t and preindustrial time, L(t) is the lifetime of CO_2, and X(t) is the global emissions in PPM at time t. The parameters are set according to IPCC (1992, 1996b) and Peck and Teisberg (1992).

$$Q(t) = Q^p + Z(t),$$

$$Z(t) = Z(t - 1) \cdot (1 - \frac{1}{L(t)}) + X(t), \quad t > 0$$

The *temperature equations* describe the reaction of temperature to changes in atmospheric GHG concentrations or radiative forcing, i.e., heating in watt per square metre (W m^{-2}), and consist of functions for potential and actual temperature increase. Potential temperature increase is the increase in the geophysical equilibrium. Before fully warming the earth's surface, the greenhouse effect must first heat up the oceans (ocean terminal lag). This may take as long as three decades or more, and the potential temperature increase is therefore the increase in temperature after full adjustment. Potential temperature increase due to CO_2 emissions, T_{CO2}^P, is specified according to Peck and Teisberg (1992). The A and B parameters in equation (24) are fit so that the increase is zero at the preindustrial concentration and 2.5°C for twice the preindustrial CO_2 concentration.

(24)
$$T_{CO2}^P(t) = A \ln Q(t) - B,$$

$$T_{OGHG}^P(t) = \theta R(t),$$

$$T^P(t) = T_{CO2}^P(t) + T_{OGHG}^P(t)$$

The potential temperature increase due to emissions of the 16 other GHGs, T_{OGHG}^P, is the product of the total warming coefficient, θ (measured in °C/W m^{-2}), and the radiative forcing above preindustrial level, $R(t)$ (measured in W m^{-2}). The time series for radiative forcing, where CFCs are phased out by 2020, are taken from the calculations of Hoel and Isaksen (1993).

Based on Nordhaus (1991), the increase in actual temperature (T) in each period is governed by a lagged adjustment process due to the thermal inertia of oceans as mentioned above, where the speed of adjustment is determined by the delay parameter α, which is set equal to 0.025.

(25) $\quad T(t) = \alpha T^P(t) + (1 - \alpha)T(t - 1)$

3.3. The Consumption Module

Assuming that there is a constant ratio between consumption and production, we can as well specify the utility as a function of production. We have chosen an isoelastic *utility function* where $1/\sigma$ is the elasticity of substitution between consumption at two points in time.

(26)
$$U[P(t)] = \frac{1}{1 - \sigma}[P(t)^{1-\sigma} - 1], \quad \sigma \neq 1, \sigma \geq 0$$

$$U[P(t)] = \ln P(t), \quad \sigma = 1$$

Nordhaus (1991, 1993) uses a σ value equal to 1, while Cline (1992) proposes a value of 1.5 for USA. In this model σ is set equal to 1.

3.4. The Stochastic Module

The hazard rate function is assumed to be a convex function in temperature, where the probability of a catastrophe to occur is zero for the temperature level in 1990. A Weibull distribution (see, e.g., Kiefer 1988) is chosen which gives the following hazard function:

$$(27) \qquad \Psi[T(t)] = \varphi\eta \cdot [T(t) - T(0)]^{\eta-1}$$

η is set to 2.5 as we want a convex function, while φ is calibrated so that there is a 4.8 per cent probability of a catastrophe in 2090 if we follow the BAU emissions path. This assumption is based on Nordhaus (1994b) who asks an expert panel of their subjective probabilities for a catastrophe to occur at a temperature increase of 3°C in 2090[10]. A catastrophe in the Nordhaus study is defined as a loss of more than 25 per cent of world GDP (economic equivalent to the Great Depression) [11].

4. Simulation results[12]

4.1. Scenarios

Below we describe briefly the different scenarios to which the model is applied. Six different scenarios are considered of which four include the possibility of a catastrophic occurrence.

i) The BAU scenario represents the uncontrolled emissions, i.e., no future policies are taken to slow global warming. It is based upon exogenous estimates on future CO_2 emissions for each of the 7 regions modelled. The exogenous production paths are adjusted for feedback effects from global warming (see Section 3 and Kverndokk, 1994, for further details).

ii) NOCAT [V(.,.) = U(.,.)]. The social planner is to decide on an optimal emission path for each region under the assumption of a catastrophe not being possible. In contrast to BAU, the emissions under this scenario follow from optimising behaviour.

iii) CAT1 [V(.,.) = 0, calibrated hazard]. The scenario represents a situation where utility drops to zero if a catastrophic event occurs. The hazard rate function is calibrated as outlined in Paragraph 3.4.

iv) CAT2 [V(.,.) = $\bar{U}$ = U(Y1990, T1990), calibrated hazard]. In this scenario we have assumed that utility, if a catastrophe occurs, is reduced to the utility level of 1990. The calibrated hazard function is similar to the one applied in CAT1.

v) ONLYCAT [V(.,.) = $\hat{U}$ = U(Y$_{1990}$), calibrated hazard]. This scenario equals CAT2 with the exception of T no longer being an argument in the utility function. As a consequence we are in a situation where climatic costs are only associated with a catastrophe, i.e., continuous climate-feedback effects are absent.

vi) CAT3 [V(.,.) = 0, φ = 0.01 and η = 1, see Eq.27]. In this run, a constant hazard is assumed together with a payoff under a catastrophe as in CAT1. As a consequence, the social planner is aware of the

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[10] The chosen estimate is the mean probability for a catastrophic event among the survey participants.

[11] Another paper that studies the probability of a climate catastrophe is Bentley (1997). Bentley concludes that a rapid sea-level rise from a collapse of the West Antarctic ice sheet is unlikely to happen over the next century or two. The probability of such an event in this time period is in the order of 0.1%.

[12] The simulations were carried out for the time period 1990 to 2230 using the GAMS/MINOS system (Brooke et al., 1992). Results are not reported for the last 50 years to avoid extreme end effects.

possibility of a catastrophic occurrence, but the emissions and temperature level trajectories do not have any effect on the probabilities of a loss. The hazard rate is assumed equal to 1 per cent.

4.2. Emission paths and the development in global mean temperature

Figures 4.1, 4.2 and 4.3 show the paths of global carbon emissions and global mean temperature under the six scenarios. In all scenarios, the time preference rate is 3 percent[13].

Figure 4.1. Global carbon emissions (billion tons), comparing different scenarios
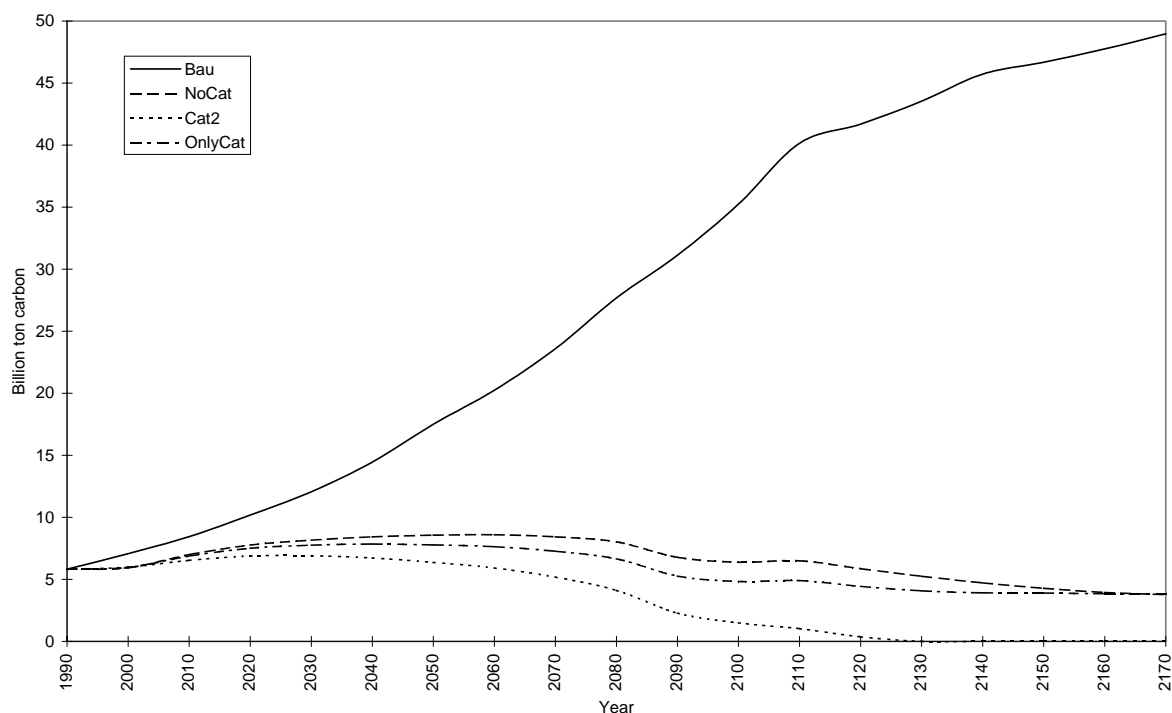
In Figure 4.1, the emissions paths for four out of the six scenarios are presented. The uncontrolled path (BAU) shows annual emissions of 7 billion tons in year 2000, rising to 35 billion tons by year 2100, and 47 billion tons by 2170. When climate-feedback effects are taken into account (NOCAT), the optimal paths of carbon emissions are dramatically reduced compared to the BAU scenario, i.e., to a level equal to 6 billion tons both in year 2000 and 2100, and to 4 billion tons in 2170. This gives emission reduction rates compared to BAU of about 10 per cent in 2000, rising to 80 per cent in 2090. Compared to other integrated assessment models, the optimal emissions path is in line with the emissions in the power damage function case in the CETA model (Peck and Teisberg, 1992), however, it is less than the emissions path in the linear damage function case, especially for the period 2050-2150. Compared to DICE, see, Nordhaus (1993,1994a), the optimal reduction rate starts at almost the same level, but while the reduction rate increases steadily in our model, the optimal reduction rate in DICE rises to about 15 per cent late in the next century. The difference is mainly due to lower abatement costs and higher damage costs in our model[14]

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[13] Many numerical studies apply a similar value, see, e.g., Manne and Richels (1992), Peck and Teisberg (1992), and Nordhaus (1994a).

[14] Fankhauser and Kverndokk (1996) do some sensitivity analysis on the income function of our model. They conclude that using an income function based on abatement cost data from GREEN, gives much higher optimal abatement than for instance if abatement cost data from Global 2100 (Manne and Richels, 1992) were used.

While NOCAT only considers continuous climate-feedback effects, the ONLYCAT scenario only considers discontinuous climate-feedback, i.e., the only problem with global warming is the possibility of a climate catastrophe. As seen from Figure 1, the risk of a catastrophic outcome alone, induces the planner to reduce emissions even more than if only continuous damage was considered. Thus, even if we do not believe in the climate costs included in the continuous damage function, the possibility of a catastrophe may itself be a good argument for substantial emissions reductions.

The CAT2 scenario includes both types of climate change damages. However, the emissions are not reduced much compared to NOCAT and ONLYCAT before the end of the next century. The reason is that a very high abatement is undertaken even if only one of the damage components are considered. Higher abatement means very high abatement costs as the marginal abatement costs are increasing.

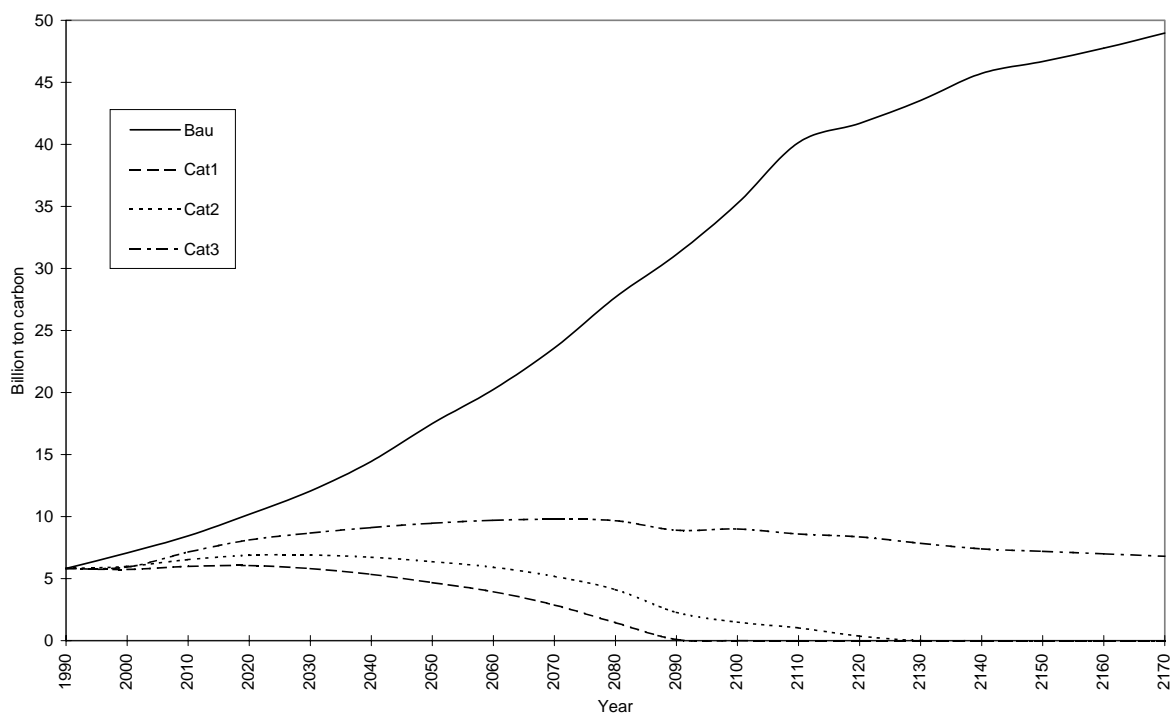**Figure. 4.2. Global carbon emissions (billion tons), both continuous and discontinuous damage**

Figure 4.2 compares the optimal emissions paths for the scenarios which take account on both continuous and discontinuous damage. The penalty from a catastrophe differs between CAT1 and CAT2 with the strongest penalty in CAT1. As seen from the figure, the magnitude of the penalty is important, especially from the end of the next century. For CAT1 and CAT2, optimal emission are slowed down to 1 billion ton in year 2085 and 2115, respectively.

Scenario CAT3 differs from the other catastrophe scenarios since the hazard rate function is constant and the probability of a catastrophic event becomes very high over time. This is seen from Table 4.1 where the probability of a catastrophe by 2090 along the optimal emission paths for different scenarios are shown.

Table 4.1. Probabilities of the arrival of a catastrophe by year 2090 and year 2170

	CAT1	CAT2	ONLYCAT	CAT3
2090	4%	4%	4%	63%
2170	12%	13%	15%	84%

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The incentives for emission control are weaker in CAT3 compared to the scenario where only continuous feedback effects are considered (NOCAT). This result may at first sight appear surprising but coincides with our expectations of Section 2 and with the conclusions arrived at in Clarke and Reed (1994) and Torvanger (1997). In a situation where future utility may become unavailable due to exogenous events, it becomes rational to redistribute utility from the far to the near future. As mentioned in Section 2, a constant hazard rate appears as a constant risk premium added to the pure rate of time preference. In fact, CAT3 coincides with NOCAT given a 4 percent utility discount factor. When the probability of a catastrophe depends on the level of temperature, the implications are different. The prospect of affecting the likeliness of an adverse outcome act as an incentive to slow down emissions.

To get more information on when a catastrophe is likely to happen in the model, we used the distribution of the stochastic variable τ (the time the catastrophe occurs) that follows from scenario CAT2. Drawing 10 different numbers from this distribution, three outcomes were within our planning horizon (2080, 2180 and 2210). The rest of the outcomes were after 2230.

Figure. 4.3. Development in global mean temperature above preindustrial level
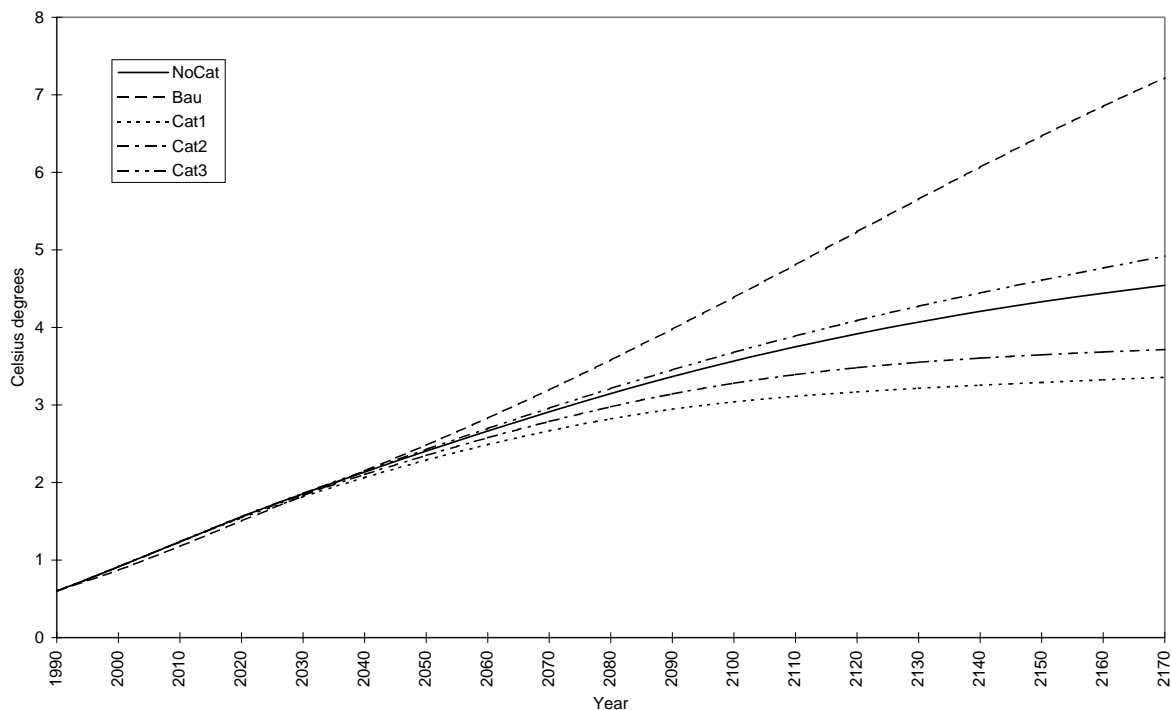
The development in global mean temperature under the different scenarios is presented in Figure 4.3. A striking feature is the modest differences up to year 2050. This is due to two aspects. First, emissions do not diverge much the first decades following 1990. Second, the lag in temperature because of the thermal inertia of oceans, prevents large deviations initially. The uncontrolled path (BAU) yields a temperature which is more than 4 degrees higher by year 2100 and 7 degrees higher by year 2170, compared to the pre-industrial level. If continuous global warming costs are considered (NOCAT) the rise in temperature at the same two dates amounts to about 3 and 4 degrees, respectively. For the two scenarios with endogenous hazard, the temperature level experiences a further decline. Compared to preindustrial, level the temperature is now 3.3 and 3.7 degrees higher in year 2100 and year 2170 for CAT2, and 3.0 and 3.4 degrees higher at the same years for CAT1.

We also compared the per capita utility (U/N, where N is population) for the different scenarios, using a population scenario mainly based on Burniaux *et al.* (1992a,b). While the utilities under the BAU

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and NOCAT scenarios are deterministic, the utilities under the catastrophe scenarios are expected utilities. First, comparing BAU and NOCAT, the global per capita utility is higher for BAU up to 2080, but from then on it reverses. This is because greenhouse policy gives immediate costs but long run gains (compare also with the annual gains from CO_2 abatement in Kverndokk, 1994). Under the catastrophe scenarios, the danger of further damages threatens the world. Thus, the expected per capita utilities under CAT1 and CAT2 are lower than the corresponding deterministic utilities. Further, expected utility is lower in CAT1 than CAT2 due to higher penalty at a catastrophic outcome.

4.3. Sensitivity analyses; pure time preference rate, the substitution elasticity of consumption and the hazard rate function

Every integrated assessment model of climate change contains a range of uncertain parameters, and sensitivity analyses are crucial. In this paper, we have chosen to concentrate on two important ethical parameters; the rate of pure time preference (utility discounting) and the elasticity of the marginal utility of consumption (consumption discounting). In addition, we also take a further look at the hazard rate function and conduct sensitivity analyses on the probability of a catastrophe. All sensitivity analyses are performed on scenario CAT2. For further sensitivity analyses on the deterministic version of the model, see Kverndokk (1994).

Figure 4.4. Sensitivity analyses on the pure rate of time preference (CAT2). Global carbon emissions (billion tons)
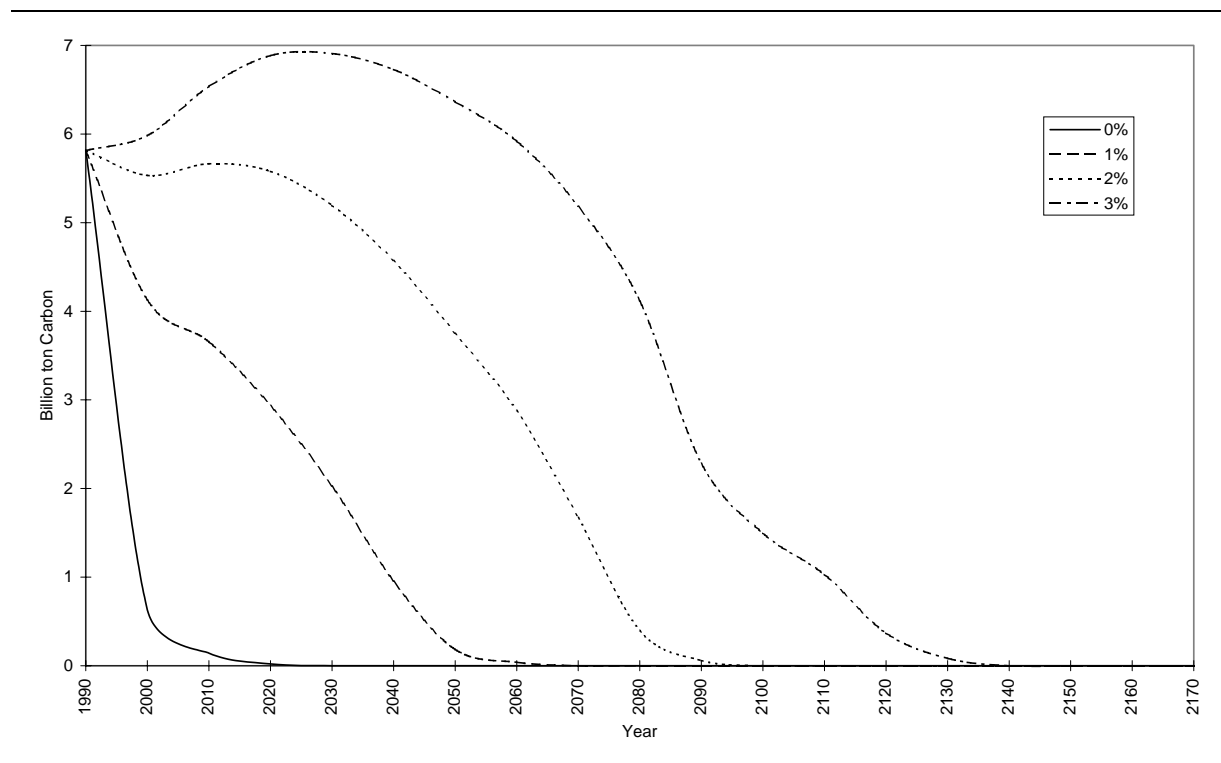
In the literature there is an ongoing debate on the size of the rate of pure time preference. Utilitarians like Ramsey and Harrod have been advocates against a positive utility discounting (see, e.g., Dasgupta and Heal, 1979, for a discussion of such issues). In the context of global warming, similar issues are discussed in Cline (1993) and Birdsall and Steer (1993). In most studies the estimate of the pure time preference rate lies within the range of 0 to 3 per cent. Nordhaus (1994a) and Peck and Teisberg (1992) for example apply a rate equal to 3. In Figure 4.4 we have relaxed our earlier assumption of a time preference rate equal to 3 percent and compare global carbon emission paths for rates of time preference equal to 2, 1 and 0.

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Changes in the time preference rate affect the emissions trajectory in the expected direction. An increasing rate speeds up emissions. Furthermore, we observe that the optimal emission path is very sensitive to the degree of utility discounting. A higher utility discount factor increases annual emissions or postpones the date it becomes optimal to reach a specific low target level. From Figure 4.4 we observe that global carbon emissions are reduced from 6 billion tons to 1 billion ton by year 2110, 2075, 2040, and 2000 for global time preference rates equal to 3, 2, 1, and 0 respectively. Another observation is that the variation in annual emissions appear very early in time. A rate of time preference equal to zero implies a rapid and sharp reduction in global carbon emissions. Furthermore, the variation observed across emission trajectories are at least as sensitive to discount rate changes as to various assumptions on the magnitude of losses associated with catastrophic events. In fact, a very high time preference rate will imply that any assumption of magnitude of catastrophic losses have insignificant effects. This result emphasises the importance of this parameter.

In Table 4.2 the probabilities for the arrival of a catastrophic outcome by two different dates for the four values of the pure time preference rate are presented. We notice that the difference in percentage points across the various scenarios increase with the time horizon.

Table 4.2. Probabilities of the arrival of a catastrophe by year 2090 and 2170 for different values of the pure rate of time preference (CAT2)

|	0%	1%	2%	3%
2090	2%	3%	4%	4%
2170	6%	8%	11%	13%

Figure 4.5. Sensitivity analysis on the elasticity of marginal consumption (NOCAT). Carbon emissions (billion tons)
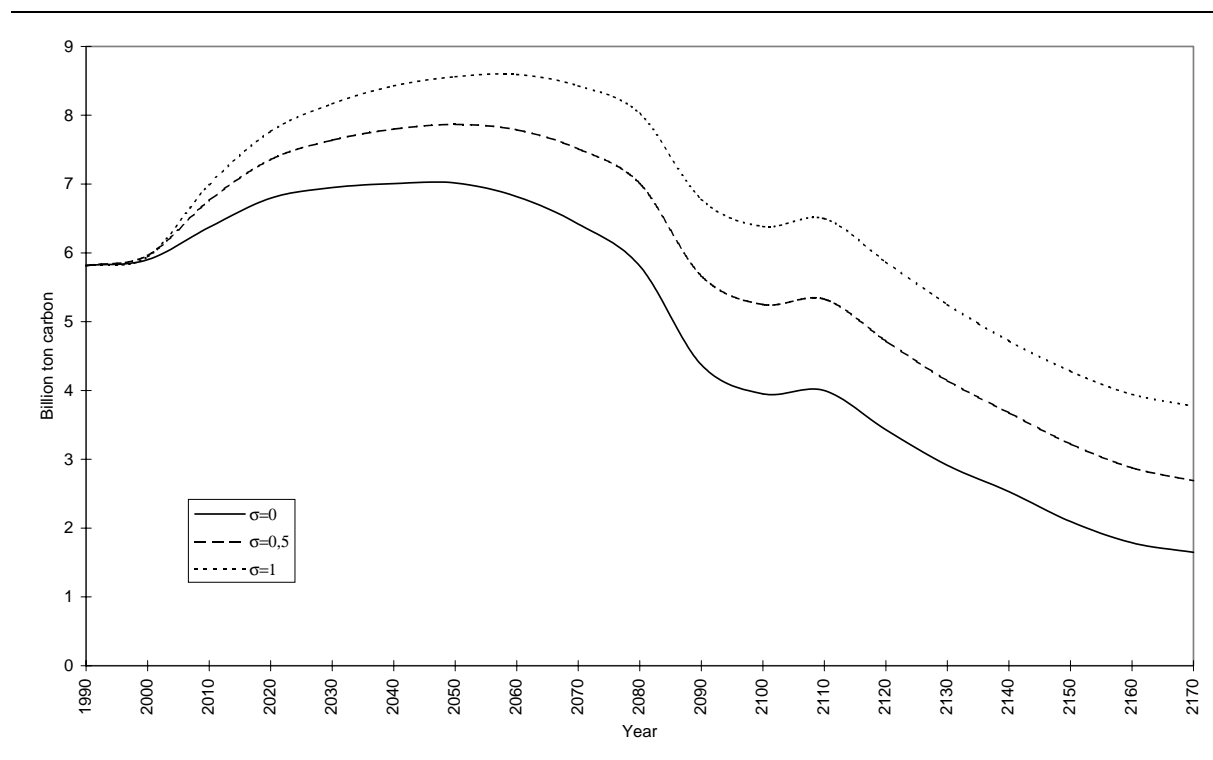
Figures 4.5 and 4.6 consider the implications of changing the intertemporal substitution elasticity $1/\sigma$. The value of σ determines the shape of the utility function in that the larger σ, the more concave is the utility function. Dasgupta and Heal (1979) find that a higher value of σ reflects a situation with a less

egalitarian distribution of consumption across generations since a higher σ implies that the curvature of the utility function is changing in favour of immediate utility compared to distant[15]. In Figure 4.5 the NOCAT scenario is considered for σ taking values between zero and one (while r is set equal to 3 per cent). When only continuous damage costs are considered, a higher σ also yield an upward shift in the carbon emission path. In spite of the difficulties in comparing changes in r with those in σ, the main impression is that emission paths are less sensitive to σ than to r.

Figure 4.6. Sensitivity analyses on the elasticity of marginal consumption (CAT2). Carbon emissions (billion tons)
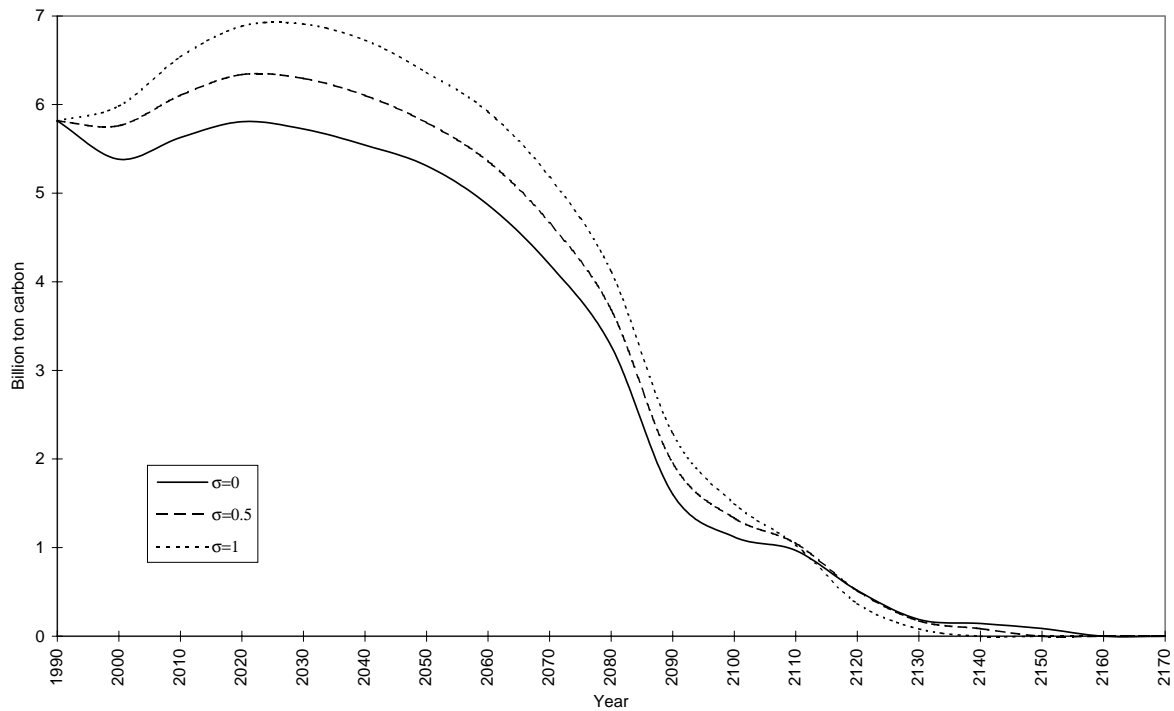
If the possibility of a catastrophic collapse is included (CAT2), the results from a higher σ changes to some extent (see Figure 4.6). For the first 120 years a similar pattern is observed in both scenarios. However, at later dates the picture changes. Beyond year 2110, annual carbon emissions become lower for higher values of σ. As a consequence, including the possibility of the arrival of a catastrophic outcome in our numerical model changes the magnitude and partly the direction of the effects arising from changing the intertemporal substitution elasticity compared to a situation where only continuous costs are introduced. The result may be due to two contradictory effects. First, a higher σ gives a higher discount rate as above. Second, the more concave the utility function is, the less is the increase in utility for higher consumption. In addition, higher emissions increases the probability of a catastrophe. Thus, for high values of consumption, the expected gains from rising emissions when σ increases may be negative. This finding is consistent with the results arrived at in Torvanger (1993), where low probability - high consequence outcomes are considered in an three generation planning model on climate change.

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[15] A general problem when analysing consumption discounting in dynamic models is that changing the parameter reflecting the intertemporal elasticity of substitution have consequences for the structure of risk preferences, and in some studies σ is denoted the degree of risk aversion. Here, we think of σ as the elasticity of marginal consumption, as there are no stochastic variables in the utility function.

Figure. 4.7. Sensitivity analyses on the probability of a catastrophe (CAT2). Carbon emissions (billion tons)
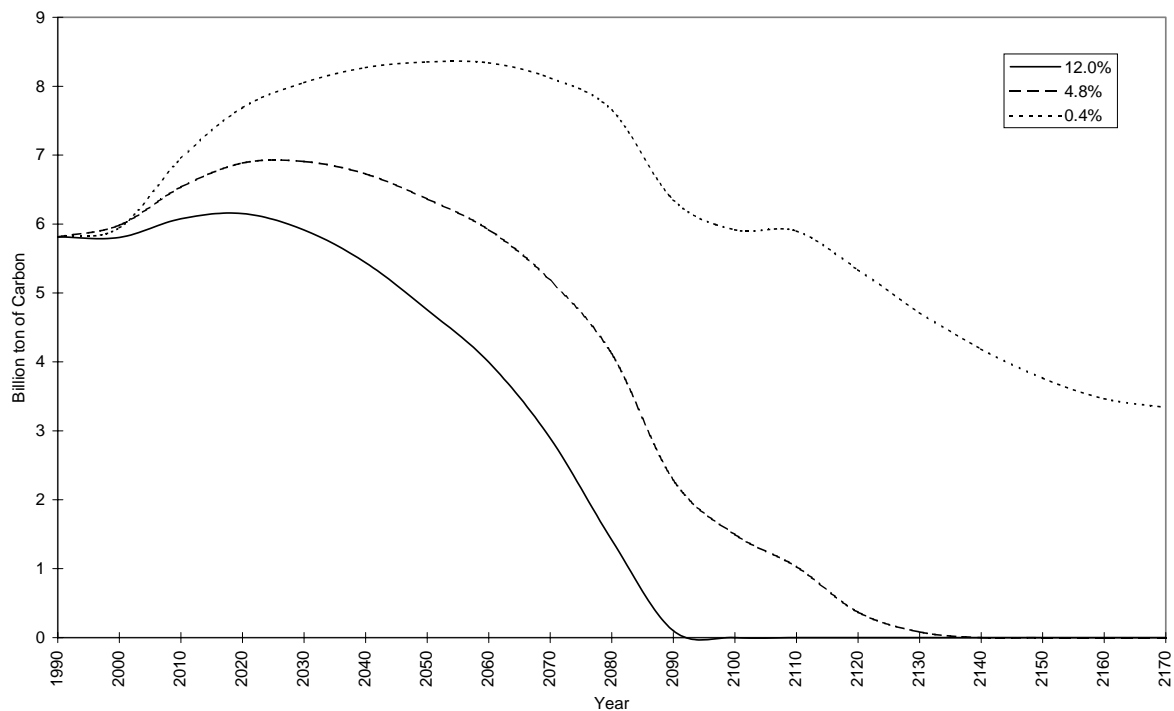

Figure 4.7 shows the results from the sensitivity analyses on the probability of a catastrophe. The expert judgements presented in Nordhaus (1994b) stress the uncertainty attached to the likeliness of future catastrophes in that the results show strong dependency on the scientific background of those surveyed. So far we have considered a hazard function which was calibrated on the basis of the mean of survey participants estimates (the probability of the arrival of a catastrophic event by year 2090 is 4.8 per cent). The average opinion of non-environmental economists was 0.4 per cent, while the same perception of natural scientists amounted to 12 per cent. Calibrating the hazard function in accordance with these numbers provides quite different optimal emissions paths under CAT2. Thus, optimal emissions are very sensitive to the perception of the likeliness of a catastrophe. We follow up this problem in Paragraph 4.5 below.

## 4.4. Optimal policy and the Rio conference on global warming

There have been efforts in the literature to reveal preferences of policy makers on the basis of policy recommendations and policy choices by implicit optimisation techniques. An early study on such techniques is Christiansen and Jansen (1978), while Eyckmans *et al.* (1993) present an application in the context of global warming issues. Below we try a related approach on the basis of the recommendation made by the Rio Conference on the stabilisation of global CO_2 emissions at 1990 level.

A natural question to ask is what considerations acted as the benchmark for the actual recommendation made. If the policy advice of the Rio conference were made in order to achieve efficiency, their perceptions about technology, abatement costs, ethical parameters, and damage costs would all be of importance. Here we focus on the role of two influential factors for the policy recommendation to be fulfilled; the pure rate of time preference and the probability of a catastrophic collapse to occur in the absence of no policy action. This can only be done under the assumptions that the Rio recommendations were based upon the same technology, abatements costs and estimates on continuous climate-feedback as those that matters in our model, and that losses associated with a

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possible catastrophic outcome coincide with those of CAT2. The BAU scenario also acts as a benchmark for how the Rio conference perceived future development in carbon emissions in the absence of climate policy.

For different values of the pure rate of time preference and the coefficient φ in our specification of the hazard function (see Eq. 27), it is now possible to identify combinations that ensures the optimal path of carbon emissions never to overshoot global carbon emissions in 1990. The results from this procedure appear in Figure 4.8. We have arrived at a menu, or a mix of the minimum probabilities for the arrival of a catastrophe by 2090 along BAU and values of the rate of pure time preference, which secures that the stabilisation goal suggested by the Rio conference is never overshot throughout the planning horizon. Not surprisingly we observe that the higher the pure rate of time preference the higher is the perception of the likeliness of the arrival of catastrophe by 2090 along BAU.

Figure 4.8. Menu of combinations of probabilities of losses and pure rates of time preference which fulfils the Rio recommendation (CAT2)
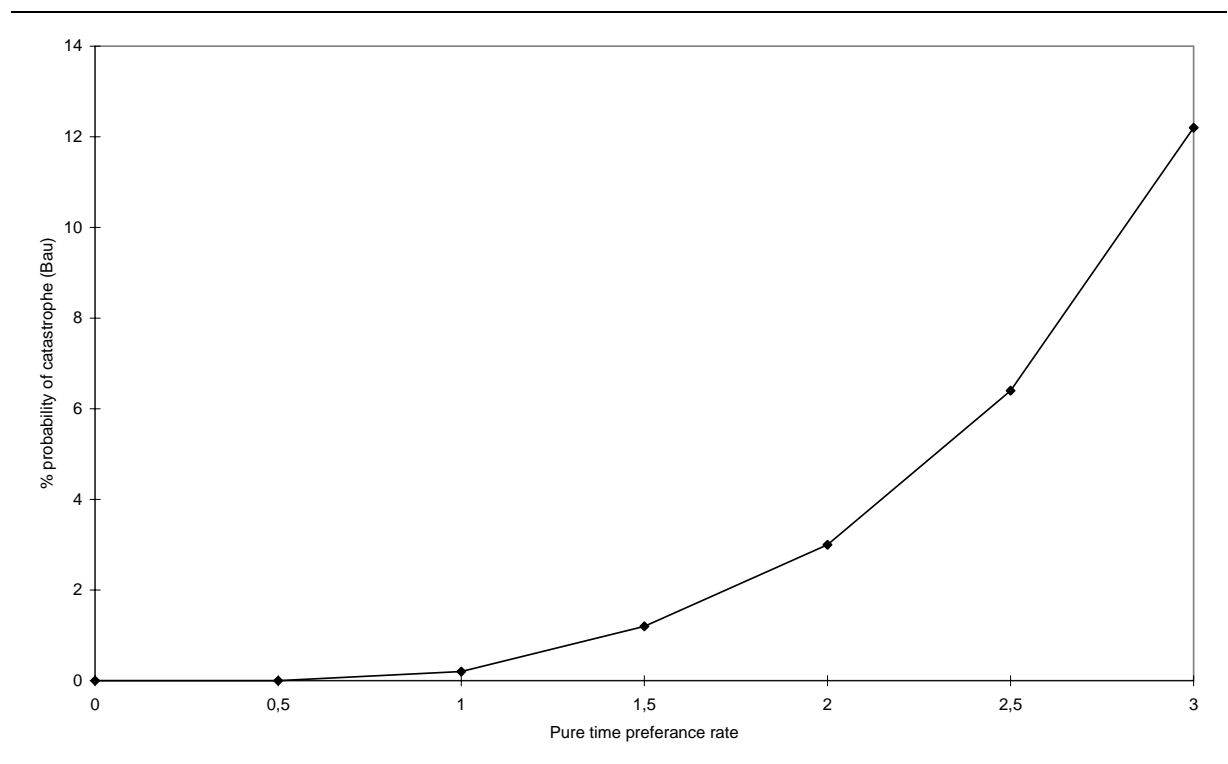
Under our assumptions it follows that if the Rio conference did not discriminate among generations (zero utility discounting), its recommendation could be made independent of the perception of the probability of the arrival of a catastrophe by year 2090 along the BAU path; the minimum probability is equal to zero. However, if future utility is discounted, the suggested policy recommendations are compatible with positive probabilities of the arrival of a catastrophic event. Given a time preference rate equal to 3 per cent, the policy recommendation is optimal under the assumption that the probability of the arrival of a catastrophe by year 2090 along the uncontrolled path is no lower than 13 per cent. Starting with beliefs about the probability of a catastrophe, a 4.8 per cent probability as in Nordhaus (1994b), implies a pure rate of time preference of about 2.3.

The above menu is very tentative in nature but stresses the importance of assumptions made on utility discounting (ethical considerations) and whether a catastrophe is perceived as likely or not for the optimal design of global control policies. The position of the menu in the diagram will of course depend on assumptions made on all other relevant parameters, however the shape will still be valid.

4.5. The expected value of perfect information about the probability of a catastrophe

So far the simulation results are based upon specific knowledge about the probability distribution for a catastrophe given any temperature level. However, there is considerable uncertainty attached to the probability distributions. More accurate information about such relationships represents a potential gain for a decision maker, and below an attempt is made to determine the magnitude of such gains.

As mentioned above, the mean probability of the arrival of a catastrophe by year 2090 from the expert judgements presented in Nordhaus (1994b) is 4.8 per cent. The average opinion of non-environmental economists was 0.4 per cent, while the same perception of natural scientists amounted to 12 per cent. Assuming that the true probability is one out of the three estimates means that there are now two sources of uncertainty in the model; i) the time of the occurrence of the catastrophe, τ, and ii) the probability of the catastrophe. The first source may be thought of as uncertainty due to Nature, while the second source is uncertainty due to lack of knowledge. The probability distribution of τ is endogenous, and we focus on the probability of a catastrophe under BAU in 2090.

Peck and Teisberg (1993) define the expected value of perfect information (EVPI) as the difference between two terms. First, the expected value of the states, where each state is obtained as if the world is known before a policy is adopted. Second, the expected value obtained if one and only one policy is adopted across all possible states of the world. Thus, we have the following definition of EVPI

$$(28) \qquad EVPI = \sum_i p_i \cdot [\max_{k_i} V(k_i, W_i)] - \max_K [\sum_i p_i \cdot V(K, W_i)]$$

where
V = value function for policy and state of the world
W_i = the state of the world i (the probability of a catastrophic outcome)
p_i = probability of state i
K = common policy for all states of the world
k_i = policy for state i

Assuming the three possible states, we can apply Eq. (28) to arrive at an estimate for EVPI. Each state, W_i, is attached to a particular estimate of the probability of the arrival of catastrophic outcome by year 2090 where all states are assumed equally likely, see Table 4.3.

Table 4.3. The different states of the world with corresponding probabilities

W_i	0.4%	4.8%	12.0%
p_i	1/3	1/3	1/3

EVPI can now be calculated by calibrating our specification of the hazard function for each state of the world. This is done under the assumption of a identity utility function ($\sigma = 0$) and a pure rate of time preference rate equal to 3 percent. The procedure yields an estimate of EVPI equal to 589 billion dollars (1985), which is the discounted gain in utility measured in consumption units for obtaining precise information on the relevant probability distribution for the arrival of catastrophe. The estimate arrived at represents 4.0 per cent of world GDP in 1990. Thus, there are high potential gains associated with acquiring more knowledge about the likeliness of catastrophic events. There are several studies in the literature presenting estimates on the value of information of climate parameters (see, e.g., Peck and Teisberg , 1993b; Manne and Richels, 1992), but no other studies on the probability of a catastrophe. Yohe (1996), however, presents estimates of EVPI for catastrophic losses. In spite of differences both in methodology and underlying assumptions, the EVPI of the probability of a catastrophe arrived at in this study, seems to be a bit higher than the estimates for other climate parameters in the greenhouse literature, stressing the importance of this parameter.

5. Conclusions

This study analyses the consequences for optimal CO_2 emissions when explicit attention is paid to the possibility of a catastrophe. The uncertainty in our model concerns the question of whether a catastrophe will occur or not. All other potential uncertain variables are assumed known to the social planner. The source of uncertainty is introduced by applying a hazard rate function. Such an approach is already applied in theoretical studies, but to our knowledge this is the first attempt on a numerical integrated assessment model on global warming.

The simulation results show that even if we do not believe in climate catastrophes, a high abatement is called for due to continuous climate-feedback effects. On the other hand, this study supports Schelling (1992) who suggests that the probability of high-consequence outcomes is a major argument for cutting current GHG emissions; if we only consider catastrophic impacts, the emissions reductions required are actually higher than the corresponding reductions due to continuous damage. Taking account of both types of damages requires even higher abatement. The results may have important policy implications, as high reductions of GHGs are called for even if we trust only parts of the story. The debate on GHG emissions reductions under the Climate Convention will continue even after the Kyoto conference, and it is to hope that policymakers will pay more attention to the possibility of a catastrophic outcome and the impacts on future generations in the negotiations to come.

The reported results also stress the fact that optimal abatement is sensitive to crucial parameters, especially the probability of a catastrophe and the pure rate of time preferences. As the probability of a catastrophe is important for the magnitude of optimal abatement, and the value of information of this probability seems to be high, a call for more research on how climate catastrophes are linked to GHG emissions seems necessary. Also, the value of the pure rate of time preference, an ethical parameter, is at least as important for optimal carbon abatement as the perception of the likeliness of a catastrophe and the magnitude of losses associated with such an outcome. Thus, ethical considerations matter.

Finally, this study ignores the distributional effects of a catastrophe. As mentioned earlier, an extreme outcome of the maximisation problem can be that it is optimal that a small island state disappears as a result of sea level rise due to high abatement costs in larger economies. An other interesting distributional problem is how a regional catastrophe may have impacts on other regions via international trade and migration. Distributional issues of catastrophes are important subjects of future research.

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