Sosiale og økonomiske studier | Social and Economic Studies

Tom Kornstad

Empirical Life Cycle Models of Labour Supply and Consumption

Tom Kornstad

Empirical Life Cycle Models of Labour Supply and Consumption

Statistisk sentralbyrå • Statistics Norway
Oslo-Kongsvinger 1995

Sosiale og økonomiske studier

Serien *Sosiale og økonomiske studier* omfatter nye forskningsbidrag – monografier og redigerte arbeider – på de områder Statistisk sentralbyrå har forskningsvirksomhet. Analysemetoder og temavalg vil variere, men hovedsakelig vil arbeidene være av anvendt og kvantitativ natur med vekt på utnytting av SSBs data i analyser for samfunnsplanleggingsformål og til allmenn forståelse av sosial og økonomisk utvikling.

Social and Economic Studies

The series *Social and Economic Studies* consists of hitherto unpublished studies in economics, demography and other areas of research in Statistics Norway. Although the studies will vary in analytical methods and in subject matter, they tend to be applied studies based on quantitative analysis of the data sources of Statistics Norway. The research programmes from which the studies originate typically emphasize the development of tools for social and economic planning.

ISBN 82-537-4166-9
ISSN 0801-3845

Emnegruppe
59 Andre samfunnsøkonomiske emner
Ny 1995: 05.90 Metoder, modeller, dokumentasjon

Emneord
Arbeidstilbud
Forbruk
Konsum
Livsløpsmodell
Varige goder
Yrkesdeltaking
Økonometri

Design: Enzo Finger Design
Trykk: Falch Hurtigtrykk

Preface

This volume of Social and Economic Studies constitutes three out of four chapters of my doctoral dissertation at the Department of Economics, University of Oslo. The introductory chapter has been expanded, but apart from that the original dissertation has only been slightly modified in order to improve the exposition. The fourth chapter was originally written in Norwegian, and is therefore omitted from this volume.

I thank Steinar Strøm and John K. Dagsvik for supervising me through my Dr. Polit. dissertation research. In particular Dagsvik has been very enthusiastic and patient in teaching me the various aspects of empirical modelling. I am also grateful to Margaret Simpson for her detailed comments on chapter three. Finally I would like to thank my employer, Statistics Norway, and the Norwegian Research Council for their financial support.

Statistisk sentralbyrå, Oslo
23. juni 1995

Tom Kornstad

Abstract

Tom Kornstad

Empirical Life Cycle Models of Labour Supply and Consumption
Doctoral dissertation at the University of Oslo, Department of Economics, defended 9th April 1994

Social and Economic Studies 91 • Statistics Norway 1995

The main purpose of this doctoral thesis is to formulate and estimate a structural life cycle model of married couples' labour suppy and consumption of durables and non-durables.

According to the life cycle theory households' labour supply and consumption of durables and non-durables are determined conditional on the household's expectations about future prices and variables influencing future preferences. This fact indicates that estimation of life cycle preferences requires very much data for each household, and chapter 2 explores whether estimation of structural life cycle models is possible in the absence of some of these data. The chapter presents the various approaches, and points at the data requirements and what kind of simplifications with respect to model specifications that are invoked. These simplifications may include intertemporal as well as intratemporal separable preferences, no binding constraints in the credit market, and no income taxation. The chapter concludes that we cannot be satisfied with the state of the art.

Chapter 3 presents and estimates a structural life cycle model of married couples' labour supply and consumption of durables and non-durables. This chapter takes the problems with measuring consumption of durables and non-durables seriously, and it allows habit formation in the demand for durables. Using Hicks' composite good theorem, we can estimate a life cycle model of a particular aggregate of durables and non-durables (and leisure) without observing the consumption of non-durables, and the price and the physical stock of durables. The estimation uses data from the Income Distribution Survey 1988, 1989 and 1990, and the Standard of Living Survey 1991. The estimation results suffer from the fact that the time span of the panel data is too short.

In a fourth chapter of the thesis it is studied how the Norwegian income tax system distorts relative user prices between housing, non-durables and leisure for married and unmarried wage income earners. The analysis is based on a general life cycle model where households are assumed to have perfect certainty with respect to future prices and preferences. Empirical calculations are presented for 1985/86, 1989/90 and 1992/93, and they include average distortions and distribution tables. Even after the 1992 tax reform the income tax system still distorts some of the relative user prices considerably. This chapter of the thesis has been omitted in this volume.

Keywords: Consumption, durables, empirical, labour supply, life cycle models.

Sammendrag

Tom Kornstad

Empiriske livsløpsmodeller for arbeidstilbud og konsum
Doktoravhandling fra Sosialøkonomisk institutt, Universitetet i Oslo, forsvart 9. april 1994

Sosiale og økonomiske studier 91 • Statistisk sentralbyrå 1995

I mange problemstillinger er det viktig å ha en god forståelse av hvordan husholdenes arbeidstilbud, konsum og sparing bestemmes i et livsløpsperspektiv. Ifølge sosialøkonomisk teori kan dette analyseres med bakgrunn i livsløpsmodellen for konsum og arbeidstilbud. Denne modellen sier i grove trekk at husholdet tilpasser konsum og arbeidstilbud slik at forventet verdi av den samlete neddiskonterte nytten over livet blir maksimert, gitt at det over livet er samsvar mellom inntekter og utgifter.

Avhandlingen består av tre kapitler som kan leses separat, og en innledning. Det siste kapittel er kort omtalt nedenfor, men er ikke tatt med i denne utgaven. Formålet med avhandlingen er i første rekke å spesifisere og estimere en (strukturell) livsløpsmodell for ektefellers arbeidstilbud og konsum av varige og ikke-varige goder. Avhandlingen legger stor vekt på det metodiske aspektet ved estimering av livsløpsmodeller, og for å få en tett kobling mellom teori og empirisk modell tas det utgangspunkt i strukturelle livsløpsmodeller som estimeres fra mikrodata. I sin mest generelle form krever denne type modeller livsløpsdata for konsum og arbeidstilbud og de tilhørende prisene for hvert enkelt hushold. Siden slike datasett ikke eksisterer i noe land, har det de siste 15 årene vært lagt ned mye arbeid internasjonalt i å finne fram til spesifikasjoner som egner seg for estimering gitt de dataene man har. I kapittel 2 "Empirical approaches for analysing consumption and labour supply in a life cycle perspective" gjøres det rede for disse spesifikasjonene og hvilke forenklinger de bygger på. For eksempel blir skattesystemet ofte sett bort fra, og det gjøres vanligvis sterke a priori forutsetninger om separabilitetsegenskapene til nyttefunksjonen.

Konklusjonen på dette kapitlet er at det ennå er langt fram før vi kan være fornøyd med modellspesifikasjonene som typisk brukes i empiriske livsløpsmodeller, og de empiriske resultatene på dette feltet.

Basert på kunnskapen fra oversiktskapitlet gjøres det i kapittel 3 "An empirical life cycle model of savings, labour supply and consumption without intertemporal separability" et forsøk på å formulere og estimere en strukturell livsløpsmodell for konsum av varige og ikke-varige goder og fritid for ektepar. Det pekes på at det er store praktiske og teoretiske problemer knyttet til måling av konsum av varige og ikke-varige goder, og hva som skal menes med enhets-prisen på varige goder som for eksempel bolig. Gitt at man tar disse problemene alvorlig, står man overfor store måleproblemer ved estimering av denne type modeller.

Kapitlet peker også på at vanedannelse kan ha betydning for etterspørselen etter varige goder, og viser at dersom man tar hensyn til dette er det mulig å finne fram til en spesifikasjon som kan brukes til å estimere preferansene for fritid og et spesielt Hicks-aggregat av varige og ikke-varige goder. En viktig egenskap ved denne spesifikasjonen er at den kan estimeres uten observasjoner av konsum av ikke-varige goder og prisen på, og den fysiske beholdningen av, varige goder.

Modellen estimeres ved hjelp av data fra Inntekts- og formuesundersøkelsene 1988 til 1990 og Levekårsundersøkelsen 1991. På basis av estimeringsresultatene er det i prinsippet mulig å simulere ektefellers valg av fritid, konsum og sparing over livsløpet gitt ulike forutsetninger om skattesystem, lønnssatser og konsumpriser over livet. Estimatene på de ulike koeffisientene er imidlertid av varierende kvalitet. Årsakene til dette kan være flere, men en av grunnene skyldes at Norge har for dårlig med paneldata på dette området.

Ifølge økonomisk teori kan skattesystemet gi opphav til uheldige effektivitetsvirkninger ved at det vrir de relative prisene mellom ulike goder. Dette er emnet for den siste delen av avhandlingen, som er publisert i Kornstad (1994) under tittelen "Vridninger i lønnstakernes relative brukerpris på bolig, ikke-varige goder og fritid". Arbeidet tar utgangspunkt i en generell livsløpsmodell for konsum av ikke-varige goder, bolig og fritid, og forsøker å tallfeste hvordan omleggingen av skattesystemet fra 1986 til 1992 har endret den gjennomsnittlige vridningen i de relative prisene mellom disse godene. Beregningene er basert på data for lønnstakere hentet fra Inntekts- og formuesundersøkelsene 1985, 1986, 1989 og 1990. Ved å framskrive 1990-dataene til 1992 og 1993 er det gjort beregninger for 1992/1993. De indikerer at den gjennomsnittlige vridningen i de relative brukerprisene på bolig og ikke varige goder, og på bolig og fritid, er redusert fra 1985/86 og fram til 1992/93, både for enslige lønnstakere og for lønnstakerektepar. Størrelsen på reduksjonene avhenger av forutsetningen om prisutviklingen på boliger fra et år til det neste. Skatteomleggingen fra 1986 til 1992 har redusert den gjennomsnittlige vridningen i de relative prisene på ektefellers fritid, og på fritid og ikke-varige goder. For alle prisforholdene gjelder det at skattesystemet fortsatt gir tildels betydelige vridninger. Omleggingen av skattesystemet fra 1985/86 og fram til 1992/93 har redusert spredningen i vridningene i den relative brukerprisen mellom bolig og ikke-varige goder.

Emneord: Arbeidstilbud, konsum, livsløpsmodell, varige goder, økonometri.

Contents

1. Introduction and Overview

1.1 Introduction

The starting point of my doctoral thesis was the belief that if we were able to predict households' labour supply and financial savings decisions, microsimulation models for personal income taxation could be improved. The life cycle theory is considered since saving is an intertemporal allocation problem.

Apart from this introductory chapter, the thesis consists of three separate chapters that are all dealing with labour supply and consumption in a life cycle perspective; The first of these chapters, chapter two, surveys the approaches for estimating structural life cycle models from micro data. This chapter provides a background for assessing chapter three, where a structural life cycle model of married couples' labour supply and consumption of durables and non-durables is formulated and estimated. In chapter four I planned to use these estimation results for simulating the change in the efficiency loss due to the change in the personal income tax system from 1986 to 1992, but since the estimates became quite impresise, a less comprehensive approach was chosen. Chapter four studies the changes in the distortions in relative user prices between housing, consumption of non-durables and leisure due to the changes in the personal income tax system from 1986 to 1992. This chapter is, however, omitted from this volume since it is written in Norwegian, but below we give a summary of the chapter.

1.2 Estimation Methods

In order to simulate financial savings, demand relations for durables as well as non-durables, and labour supply for both spouses, should be developed. The decisions about all these variables are taken simultaneously, and conditional on the couple's expectations about future (after-tax) prices of these goods. These facts and the fact that the relationship between future prices and current demand may vary across life cycle, indicate that estimation requires a lot of data that must be observed for each couple. Since there is no data set that includes all these variables it becomes important to explore whether estimation is possible in the absence of some of these data. In fact, during the last decade a lot of effort has been spent trying to estimate structural[1] life cycle models of labour supply and consumption from the micro data actually available. Chapter 2, "Empirical Approaches for Analysing Consumption and Labour Supply in a Life Cycle Perspective", surveys these approaches.

The chapter is divided into two parts. In the first part households are assumed to have perfect certainty[2] about future prices and variables influencing future preferences, whereas this assumption is relaxed in the second part.

Within the perfect certainty framework it is being assumed that the household chooses labour supply and consumption of durables and non-durables in order to maximize (discounted) lifetime utility subject to a set of constraints. Three approaches are presented:

- MaCurdy's fixed effect approach, cf. MaCurdy (1981),
- Reduced form estimation, cf. Bover (1989,1991), and
- Two-stage budgeting, cf. Blundell and Walker (1986) and Blundell (1987).

Empirical life cycle models typically assume intertemporal separable preferences, and with one exception this is the case in all approaches presented in this chapter. Using duality relationships or various combinations of the first order conditions, it is then possible to find specifications that include a single conditioning variable summarising past decisions and future anticipations. Choosing a specification that includes an appropriate conditioning variable it may at least be possible to estimate life cycle consistent models from the micro data actually available. Most empirical works on the life cycle model use this approach, and MaCurdy's fixed effect approach and the use of the two-stage budgeting theory are two important examples.

MaCurdy (1981) assumes that preferences are both inter- and intraperiod separable, and uses the marginal utility of wealth constant demand functions as

[1]By a structural life cycle model I mean that the equations used for estimation are based on an explicit parameterization of the utility function. In addition there must be a "tight connection" between theory and the empirical specification used for estimation.

[2]With perfect certainty about future prices the life cycle literature really mean perfect certainty. In particular this means that households are not allowed to substitute expected prices into the wealth constraints and update these expectations each time they get new information about the future.

basis for estimation (and simulation). These equations include the marginal utility of wealth at age zero as the conditioning variable, but from an empirical point of view it is a problem that this variable is latent. MaCurdy, however, recognizes that it can be treated as a fixed effect in the estimation, and by ingenious choice of the utility function it can be eliminated by first differencing the demand functions. This transformation introduces last year's labour supply or consumption as the conditioning variable, and the parameters determining the demand responses to evolutionary price changes along a given lifetime price path can be estimated from panel data for (at least) two periods on only the quantity of the actual good and its price. The interest rate for the last period must also be observed.

In order to explain differences in labour supply across persons, MaCurdy estimates the reduced form equation for a variable related to the marginal utility of wealth. This estimation requires individual life cycle data for all exogenous variables in the model including future prices and the terminal stock of assets, and in contrast to the first stage the second stage of MaCurdy's fixed effect approach is then very data demanding. In addition, the true relationship must typically be approximated since, with only a few exceptions, it is impossible to find the reduced form equation.

MaCurdy's fixed effect approach imposes strong restrictions on within-period preferences and intertemporal substitution for the high income groups, cf. Browning, Deaton and Irish (1985) and Blundell (1987). In order to reduce restrictions, Blundell and Walker (1986) and Blundell (1987) suggest to use the two-stage budgeting theory for estimation. According to this approach, within-period demands can be written as a function of within-period full income[3] and all the current goods prices. Full income now serves as a sufficient statistic for the effects of future anticipations and past decisions on current demand, and the y-conditional demands can be estimated from cross sectional data only.

This approach typically also relies on the following results:

- If preferences are intertemporally separable, within-period allocation of full income to individual goods is completely characterized by the indirect utility function, cf. Gorman (1959).
- Flexible specifications of indirect within-period utility relax a number of the important underlying restrictions on within-period preferences in MaCurdy's fixed effect approach such as intraperiod separability.

[3] Full income is defined as the sum of the value of the household's initial time endowment, and its interest incomes and asset decumulation.

- Applying Roy's identity to indirect within-period utility, we get y-conditional demands, cf. Deaton and Muellbauer (1989).

This means that it is possible to combine explicit parameterizations of indirect utility with the use of y-conditional demands in order to make quite flexible within-period demand systems that can be estimated from cross sectional data.

The allocation of goods across time is determined by the monotononic transformation of within-period utilities. To estimate these parameters, the Euler-equation[4] for the marginal utility of wealth can be exploited, cf. Blundell (1987). This estimation requires panel data for at least two periods on full income and all goods prices, in addition to observation of the interest rate for the last periods. Compared to MaCurdy's fixed effect approach that restricts the specification of the monotonic transformation of within-period utilities, this approach allows for more flexible preferences. However, lack of panel data for many periods may reduce the theoretical advantage in empirical applications.

Bover (1989,1991) assumes that preferences are both inter- and intratemporal separable and of the so called Stone-Geary type, and that there is no income taxation. In this case it is possible to find the reduced form equation for the marginal utility of wealth, and by substituting this equation into the Frisch demand functions, it is possible to find the reduced form demand functions. This means that the household's optimization problem can be given an explicit solution, and in that respect the Stone-Geary function has favourable properties. Estimating reduced form demand equations is, however, very data demanding since it does not make use of a conditioning variable summarising future anticipations and historic decisions, and for empirical purposes this approach requires additional simplifying assumptions about future anticipations.

In the second part of this chapter it is assumed that households are uncertain about future prices. Households are now beeing assumed to choose current labour supply and consumption in order to maximize the expected value of the time-preference-discounted sum of total utility subject to a set of constraints for each period. Within this framework the following approaches are presented:

- MaCurdy's fixed effect with uncertainty, cf. MaCurdy (1985),
- Marginal rates of substitution functions, cf. MaCurdy (1983) and Altonji (1986), and
- The generalized method of moments, cf. Hansen and Singleton (1982).

[4]The Euler equation for the marginal utility of wealth is included in the first order conditions for optimum.

MaCurdy (1985) modifies MaCurdy (1981) by allowing for uncertainty about future prices and variables influencing future preferences. We only notice that the fixed effect approach cannot easily be modified to allow for uncertainty.

The use of the marginal rate of substitution functions for estimation is another example of specifications where the effects of future anticipations and historic decisions are represented by a single conditioning variable. This approach exploits the fact that if we combine the first order conditions for two goods, for instance current consumption and current labour supply, the marginal utility of wealth-variable is eliminated. Then the consumption of another good acts as the conditioning variable. This approach is then very similar to the first stage of MaCurdy's fixed effect approach, but if we only use the first order conditions for current goods, estimation only requires cross sectional data (on prices and consumption of at least two goods, in addition to taste shifter and instrument variables). It also differs from the fixed effect approach in the sense that while the fixed effect approach does not yield estimates of the parameters of preferences, but uses the estimated relationships directly for simulation, the marginal rate of substitution function approach estimates the parameters of preferences.

As it is laid out above the marginal rate of substitution functions approach cannot be used for estimation of intertemporal preferences. To do this, MaCurdy (1983) suggests to use the Euler equation for the marginal utility of wealth. This approach is quite similar to the approach described for the two-stage budgeting procedure, but estimation now requires panel data for at least two periods for all goods influencing preferences, in addition to the interest rate for the last observation period.

The basic idea underlying the generalized method of moments (GMM), is that life cycle models yield a set of stochastic Euler equations that characterise optimum[5]. One example is the equation we obtain when we substitute the first order conditions for consumption into the Euler equation for the marginal utility of wealth. These equations imply a set of population orthogonality conditions that depend on the parameters of the preferences and some observable variables. By setting the sample analogies equal zero (according to a certain metric), we can estimate the parameters of preferences.

GMM can be applied on time series, and cross section or panel data sets where the orthogonality conditions are based on averages across consumers, and is then quite flexible with respect to data specifications. According to Hansen and Singleton (1982), the estimator is consistent even in the case that the error

[5]The method for estimating intertemporal preferences in the marginal rate of substitution function approach can be viewed as a variant of this approach.

terms of the orthogonality conditions are serially correlated and the instruments are not exogenous, but only predetermined. The approach includes a method for minimizing the asymptotic covariance matrix of the estimator, and a statistic for testing model specifications.

Despite the fact that there are many approaches that can be used for estimating life cycle models, the chapter concludes that we cannot be satisfied with the current stage of development and knowledge of the empirical life cycle model of labour supply and consumption. This is due to lack of good life cycle data for separate households, that force econometricians to make unrealistic assumptions. For instance, most works are incomplete in the sense that they do not account for the simultaneity in the determination of labour supply and consumption, and in particular the consumption of durables. Most analyses also treat income taxation and the existence of bequests and inheritances superficially, and the possibilities of constraints in various markets are often disregarded. If such constraints are considered, the model is typically estimated from a subsample of unconstrained households without trying to correct for the possible estimation bias.

1.3 A Particular Empirical Specification

Based on the knowledge from the survey, in the third chapter a structural life cycle model of married couples' labour supply and consumption is formulated and estimated on Norwegian data. The chapter, titled "An Empirical Life Cycle Model of Savings, Labour Supply and Consumption without Intertemporal Separability", focuses on the following facts:

- There are important measurement errors in the of consumption of non-durables since our annual data are based on accounts of expenditures over a few (two) weeks. This problem is typical not only for Norwegian data, but is a general problem.

- Measuring consumption and stock of durables is also problematic, and this is due to lack of theoretical foundations as well as practical concerns. These measurement problems imply that it is problematic to measure the purchase price of homogeneous durable goods and the user price of durables.

- Empirical life cycle models typically ignore the consumption of durables. If it is included, the measurement problems are typically not taken seriously.

The measurement problems are important since the estimates may be biased if the standard first order conditions for non-durables and durables are estimated without taking account of these errors. Ignoring consumption of durables implies that the model cannot be used for simulation of financial saving decisions.

Most empirical analyses assume intertemporal separability, but Muellbauer (1986) claims that "Evidence from the estimation of complete systems of demand equations suggests that habits or persistence effects play an important role in consumer behaviour.". In view of this evidence we introduce habits in the demand for durables. Habits are typically taken account of by assuming that when we are evaluating preferences, consumption should be measured relative to a reference consumption bundle, cf. Houthakker and Taylor (1970) and Phlips (1972,1974). According to our hypothesis the reference consumption bundle is last year's demand for durables after deduction for physical and what we label psychological depreciation. Psychological depreciation is introduced to account for the assumption that households may want to change the stock of durables even if it is in good condition from a physical point of view. In this case the psychological deprecation parameter is positive. It may also be negative, which may indicate that durables are of sentimental value. The fact that psychological depreciated goods can be sold in the marked while physical depreciated goods cannot, makes the distinction between psychological and physical depreciation important.

An advantage of this specification is the following: If we are not particularly interested in the separate demand for durables and non-durables, we can aggregate these two goods using the Hicks composite good theorem. Rearranging the specification of this aggregate, the composite good can be expressed as the sum of the cash flow related to purchases of durables and non-durables, and the psychological depreciation parameter multiplied by the value, D, of last year's demand for durables measured in the prices of the current period. It also follows that if we observe the household's income components, interest expenditures, financial savings and D, we observe those components of the composite good that are being needed for estimation of the psychological depreciation parameter and the other parameters determining the preferences for the composite good. This means that we can estimate a life cycle model of labour supply and consumption of durables and non-durables without observing the consumption of non-durables, and the price and physical stock of durables.

The model formulation assumes that within-period preferences and the monotonic transformation of these are of Box-Cox type. Within-period preferences are being estimated by the marginal rate of substitution functions approach, and in order to take account of economic and statistical endogeneity in the estimation, a number of instrumental equations have been introduced.

All these equations are treated as a simultaneous equation system in the ML estimation.

This estimation only requires cross sectional data, and the data are obtained by linking the Income Distribution Survey 1990 with the Standard of Living Survey 1991[6]. In order to obtain data on financial savings, these data are also linked with the Income Distribution Survey 1989.The data are linked on the basis of personal identification numbers, and they are collected by Statistics Norway.

In order to estimate the parameters of the monotonic transformation of within-period preferences, the first order conditions for the composite good are substituted into the Euler equation for the marginal utility of wealth. This estimation requires panel data for two periods, and the data are obtained by linking the Income Distribution Survey 1989 and 1990, the Standard of Living Survey 1991, and an additional postal survey conducted as a part of the Income Distribution Survey 1989. In order to obtain data on financial savings, these data are also linked with the Income Distribution Survey 1988. A ML procedure is used for estimation.

Life cycle studies of labour supply and consumption typically find that the intertemporal substitution elasticities are small, and the estimates are often not significantly different from zero, cf. MaCurdy (1981), Altonji (1986) and Attanasio and Browning (1993). With some exceptions, our estimates are not very precise either. In particular, the parameter for psychological depreciation is not statistically significant at a five percent significance level. One reason may be that the data situation that does not allow modelling person specific fixed effects in the wage equations and the other instrumental equations. Another reason may be the non-linearity in the two equations based on the marginal rate of substitution functions. The fact that the samples are quite small (327 and 229 observations) is also of importance.

1.4 Price Distortions from Income Taxation

The last part of the dissertation is written in Norwegian, and is therefore omitted from this volume. The original paper is presented as essay 3 in Kornstad (1994), and a modified version of the paper is published in Kornstad (1993). In English the title would be "Distortions in relative user prices between housing, non-durables and leisure for wage earners 1985/86 to 1992/93". The paper studies how the personal income tax system changes relative prices between various goods.

[6]The Standard of Living Survey 1991 includes data on wage rates, labour supply and the market value of durables in 1990 (not 1991).

In Norway there are two types of personal income taxes called net and gross income taxes. The net income taxes correspond to the traditional taxation of incomes after deductions of interest expenditures and other expenses. In contrast, the gross income taxes for wage earners are levied on gross labour and pension incomes without any types of income deductions. In particular, interest expenditures and interest incomes are not included in the tax base of these taxes.

During the period 1987 to 1992 the taxation of net incomes relative to gross labour and pension incomes changed several times, and the analysis reviews the most important changes in the taxation of wage income earners from 1986 to 1990 and 1992. This review shows that while the marginal net income tax rates have been reduced from 1986 to 1992, the gross income tax rates have increased, in particular for the high income groups. The reductions in the highest marginal net income tax rates have been substantial, from a maximum of 66.4 percent in 1986 to 28.0 percent in 1992. And while about 55 percent of the wage earners had a rate of 30.0 percent or more in 1992, the tax rate is 28.0 percent (for all incomes in excess of 21700/43400 Nkr) in 1992.

These changes in the net income tax rates have also reduced the marginal taxation of interest incomes and deductions from 1986 to 1992, since interest incomes and expenditures are not included in the gross income taxation.

The highest marginal tax rates on wage incomes have also been reduced from 1986 to 1992, but since the marginal gross income taxation of high incomes have increased, this reduction is considerably smaller than the reduction in the highest marginal net income tax rates. The empirical calculations in the paper also show that the marginal taxation of wage incomes has not been reduced for all income groups.

The changes in the relative taxation of wage versus interest incomes imply that the distortions in the relative user prices between housing and consumption of non-durables and leisure may have changed from 1986 to 1992. The changes in the marginal taxation of high wage incomes relative to lower wage incomes may change the distortions in the relative (user) price of female and male leisure for married couples. The analysis studies these problems both theoretically and empirically. The user prices are derived from a general life cycle model of labour supply and consumption of non-durables and housing. The model specification includes income taxation of wage and interest incomes as well as stipulated income from housing, and households are assumed to have perfect certainty about future prices and preferences. Taxation of wealth is ignored.

Ideally, the analysis requires comparisons of the actual relative user prices (net of taxes), and the relative user prices in the case that there is no income taxation. To do that we need a microsimulation model that can simulate the prices in an economy with no income taxation. The model should include both demand and supply of the actual good. The life cycle model in essay two can principally be used for simulation of demand, but since most of the estimates are quite imprecise, and the simulation algorithm is not developed yet, this model is not used. Since we do not either have access to a model that can simulate supply, we approximate and assume that pre-tax prices are independent of the actual tax system. The analysis suffers from this fact, and all results should then be viewed as approximates.

The theoretical analysis discusses the signs of the partial derivatives of the relative user prices with respect to the net and gross income tax rates. The empirical calculations use data from the Income Distribution Survey 1985, 1986, 1989 and 1990. By projecting the Income Distribution Survey 1990, calculations for 1992/93 are also presented. The data are tax return data collected from taxation authorities, and we present figures for married and unmarried wage income earners. The figures include average distortions and distribution tables.

The calculations show that even after the income tax reform in 1992, income taxes still distort some of the relative user prices considerably. The average distortion in the relative user prices between married males' leisure and the couples' consumption of non-durables is about 55 percent of the relative pre-tax user prices in 1992. It is in favour of leisure, and it has been reduced from 1986 to 1992.

The average distortion in the relative user price between housing and non-durables, and between housing and leisure, has been reduced from 1985/86 to 1992/92, for both unmarried and married wage income earners. The order of the reduction depends largely on the assumption about how the tax system influences purchase prices of housing. If prices are independent, the calculations show that the change in the tax system from 1985/86 to 1992/93 has increased the average distortion in relative user prices between housing and leisure for married females and males respectively.

According to the calculations, the change in the taxation of wage incomes from 1986 to 1992 has reduced the average distortion in the relative price between female and male leisure slightly. The average distortion was about 17 percent of relative pre-tax prices in 1992, and the distortion subsidizes male leisure compared to female leisure.

If pre-tax prices are independent of the tax system, the calculations also show that the distribution of the distortions in the relative user price between housing and non-durables, and between married couples' leisure, has been more narrow from 1985/86 to 1992/93. The reason is that the progression in the marginal taxation of both net and gross income has been reduced. In contrast, the distortions in the relative user price between housing and leisure for married couples still vary much across couples. This is due to the fact that the reduction in the marginal taxation of interest incomes typically has been larger than the reduction in the marginal taxation of wage incomes.

2. Empirical Approaches for Analysing Consumption and Labour Supply in a Life Cycle Perspective

2.1 Introduction

The life cycle model introduced by Modigliani and Brumberg in 1955 has received a great deal of attention both for its micro and macro economic implications, and during the last decade many have tried to estimate structural life cycle models from micro data, cf. King (1985). A major problem in that respect is that, ideally, estimation of the life cycle model for consumption and labour supply requires complete, individual lifetime data for a great variety of variables such as household labour supply, the consumption of durables and non-durables, and their expected prices; including interest and income tax rates. Today there is no single data set containing all these variables, and the challenge has been to find specifications that can be used for estimation and identification of the parameters of interest given the data actually available. This chapter surveys the *approaches* for estimating *structural* life cycle models of labour supply and consumption demand from *micro* data. Today there exists no comprehensive survey within this field. The papers of Browning, Deaton and Irish (1985), and Blundell (1987), may be regarded as survey papers, but they omit important approaches such as the one based on the marginal rate of substitution functions, cf. MaCurdy (1983), and the general method of moments, cf. Hansen and Singleton (1982). They also lack a discussion of the problems related to MaCurdy's fixed effect approach when households are uncertain about future prices and preferences, and they don't consider personal income taxation.

Apart from this introductory section and the concluding section, this chapter is divided into three main parts. In Section 2.2 we discuss the theoretical framework for the econometric approaches presented in the two subsequent sections. We present (Section 2.2.1) the life cycle model, cf. Ghez and Becker (1975) and King (1985), including the specification of the wealth constraints and the specification of possible constraints in the labour and credit markets.

Then we discuss (Section 2.2.2) the first order conditions in the case that the household has perfect knowledge of future prices, and point out that, in a life cycle context, the relevant demand functions are the Frisch demands, cf. MaCurdy (1981), Heckman (1974,1976), Browning, Deaton and Irish (1985) and Blundell (1987). From an econometric point of view, the usefulness of these functions depends on the separability properties of preferences and wealth constraints, cf. Blomquist (1985), and Section 2.2.3 focuses on this fact. This section also focuses on the fact that the practical usefulness of the intertemporal separability assumption may depend on whether there are binding constraints in the credit market in current as well as historic periods.

From Section 2.3 we consider in more details the various approaches for estimation. First we look at methods that assume that households have perfect knowledge of future prices. We start out (Section 2.3.1) with MaCurdy's fixed effect approach, cf. MaCurdy (1981). This approach uses a (first) differenced marginal utility of wealth constant function for estimation of the parameters determining the responses of labour supply to evolutionary wage changes along a given life cycle wage path. In order to explain differences in labour supply across persons, the reduced form equation for a variable related to the marginal utility of wealth, must be estimated. This estimation requires individual life cycle data, and the true relationship must typically be approximated. Section 2.3.1.1 presents the model specifications, and in Section 2.3.1.2 we view the estimation procedure. The restrictions on intra- and intertemporal preferences are discussed in Section 2.3.1.3, and in Section 2.3.1.4 we discuss this approach in the case that we allow for income taxation.

Section 2.3.2 reviews how Heckman and MaCurdy (1980) modify MaCurdy's fixed effect approach by taking the decision of working or not into consideration in the estimation. By estimating a bivariate fixed effect Tobit model, they eliminate the possible selection bias from using a subsample of households that are unconstrained in the labour markets.

In Section 2.3.3 we discuss some works that utilize the fact that if preferences can be described by a Stone-Geary function, it is possible to find an explicit solution for the marginal utility of wealth and the reduced form equations for the household's decision variables, cf. Bover (1989,1991) and Biørn (1981). A weakness of this approach is that estimation essentially requires complete life cycle data, and the approach typically requires rather arbitrary assumptions about lifetime prices; including interest and income tax rates.

We end (Section 2.3.4) the review of approaches that assume perfect certainty, by discussing how the interpretation of the life cycle theory as a two-stage budgeting process can be used for estimation of within-period preferences from cross section data only, cf. Blundell (1987). Provided that panel data are available for all goods, this approach can also be combined with a particular use of the Euler equation for the marginal utility of wealth, for estimation of the remaining parameters of lifetime utility.

In Section 2.4 we turn to methods that assume that households do not have perfect knowledge of future prices and variables influencing future preferences. These methods assume that households maximize expected utility. First we comment on the changes in the optimization problem and the first order conditions compared with the perfect certainty case (Section 2.4.1). Section 2.4.2 discusses the problems related to MaCurdy's fixed effect approach in the uncertainty case, cf. MaCurdy (1985). This approach turns out to be unsuccessful, and in (1983), MaCurdy suggests to use the marginal rate of substitution functions for estimation (Section 2.4.3). This method allows for more flexible functional forms than the fixed effect approach, but it requires that we observe prices and consumption for at least two goods for the current period.

The marginal rate of substitution functions cannot be used for estimation of the parameters of the transformation of within-period utilities, but MaCurdy shows that this estimation is possible through a particular use of the Euler equation for marginal utility of wealth. This method can be viewed as a special case of the generalized method of moments, cf. Hansen and Singleton (1982). The generalized method of moments exploits the fact that the Euler equation for the marginal utility of wealth, and the other first order conditions, imply a set of population orthogonality conditions that can be used for estimation of the parameters of the utility function. In Section 2.4.4 we discuss this approach, and comment on some of its limitations (Section 2.4.4.1). Section 2.5 summarizes the chapter.

The chapter uses subscript i for person or household, and subscript t for period.

2.2 Theoretical Framework

This section presents and discusses the life cycle model of labour supply and consumption including durable goods. The aim of the discussion is to clarify some aspects that are relevant for estimation.

2.2.1 Consumption and Labour Supply Behaviour

Assume the household consists of one adult whose lifetime preferences can be described by the utility function

$$\sum_{t=0}^{T}(1+\rho)^{-t}U_t\left(C_t,\, K_t,\, H_t\right), \tag{1}$$

where T is the planning horizon, ρ is the time preference rate, subscript t

denotes period, $U_t\,(C_t,\,K_t,\,H_t)$ is within-period utility, C_t and K_t are Hicks composite goods grouped as consumption of non-durables and durables, and H_t is labour supply. Within-period utilities are assumed to be strictly concave in consumption of goods and leisure.

The separability properties of preferences are an important issue in this section, and in order to clarify the discussion, we now look at the definition. According to Deaton and Muellbauer (1989), within-period preferences are weakly separable if $U_t(C_t, K_t, H_t)$ can be written as $\Phi_t[U_{ct}(C_t), U_{kt}(K_t), U_{ht}(H_t)]$, where U_{ct}, U_{kt} and U_{ht} are the functional forms of subutilities, and Φ is some function that is increasing in all its arguments. That is, the conditional orderings on C_t, K_t and H_t respectively are independent of the consumption levels of the other goods. If $U_t(C_t, K_t, H_t)$ can be written as

$$\Phi_t[U_{ct}(C_t) + U_{kt}(K_t) + U_{ht}(H_t)], \tag{2}$$

within-period preferences are additively (strongly) separable. Intraperiod preferences are non-separable if none of these two cases are relevant.

The separability properties of intertemporal preferences are defined analogously.

While standard economic theory allows for non-separable preferences both within and between periods, empirical specifications typically assume at least intertemporal additive separability. This assumption requires cardinal utility and that the demand for a particular good is not influenced by habits. Recently several authors have emphasized the importance of allowing for non-separable intertemporal utility, cf. Bover (1991), Hotz, Kydland and Sedlacek (1988), Johnson and Pencavel (1984) and Muellbauer (1986), but with one exception in section 2.3.3 we shall not discuss such specifications. Hence, with this exception, throughout the chapter we assume intertemporal additive separability and that the monotonic transformation of lifetime preferences is the identity transformation, cf. equation (1).

The household faces the within-period wealth constraints,

$$\begin{aligned} A_t - A_{t-1} &= w_t H_t + r_t A_{t-1} - S_t(r_t A_{t-1}, w_t H_t) - p_t C_t \\ &- q_t\,[K_t - (1-\delta_f)K_{t-1}], \end{aligned} \tag{3}$$

where $t = 0, 1, 2, \ldots, T$, and δ_f is the depreciation rate for durables. The prices p, q and w are nominal prices of non-durables, durables and leisure, and r is the nominal pre-tax interest rate. Since households are assumed to have perfect knowledge of future prices and variables influencing future preferences, expected prices coincide with realized prices. The variable A_t is nominal net value of interest-bearing assets at the end of period t, and if debt exceeds interest-bearing claims, it is negative. The possibility of capital gains on durables is taken account of through changes in q_t from one period to another, and the assumption that durables do not depreciate completely during one period, that is, $0 \leq \delta_f < 1$.

The specification of the tax function captures that income taxes $S_t(r_t A_{t-1}, w_t H_t)$ are assessed on wage and interest incomes. If the household's net interest income is negative, it allows for deduction of interests on debt by the tax assessment. We also assume that the tax function is continuous and convex, and that it has continuous first- and second order partial derivatives. Wealth taxes are ignored.

The specification of the wealth constraint implies that we disregard that households can place their wealth into various kinds of securities, such as bonds, stocks and shares, pension funds, arts and antiques. With the exception of the fact that inheritance at the beginning of the planning period can be (exogenously) included into the value of A_0, we do not either consider the possibilities of inheritance and bequest. In particular, this means that we treat A_T as exogenous.

The period-specific wealth constraints can often be more conveniently represented by amalgamating them into a lifetime wealth constraint

$$\begin{aligned} d_T A_T - A_0 &= \sum_{t=0}^{T} d_t \left\{ w_t H_t - S_t(r_t A_{t-1},\, w_t H_t) \right\} \\ &- \sum_{t=0}^{T} d_t \left\{ p_t C_t + q_t \left[K_t - (1-\delta_f) K_{t-1} \right] \right\}, \end{aligned} \tag{4}$$

where $d_t \equiv 1/[(1+r_0)(1+r_1)\cdots(1+r_t)]$, for $t = 0, 1, \ldots, T$, is the discount rate that converts income in period t into its period 0 equivalent, and where $r_0 \equiv 0$.

In order to get an idea of the problems involved when we consider credit market constraints, we follow Mariger (1987) and assume that households must borrow against mortgage in property. That is, net debt, $-A_t$, cannot exceed a fraction κ of the market value of the property, $q_t K_t$,

$$-A_t \leq \kappa q_t K_t, \quad t = 0, 1, \ldots, T. \tag{5}$$

Here κ is the fraction between maximum debt and the market value of durables, and it is assumed that it is equal to all persons. In particular, this means that it is independent of income and changes in institutionally determined constraints.

The household may also be constrained in the labour market, for example due to institutional constraints, but this will not be discussed here. Only the non-negative restriction will be accounted for, i.e.,

$$H_t \geq 0, \quad t = 0, 1, \ldots, T. \tag{6}$$

Maximization of the lifetime utility (1) subject to the within-period wealth constraints (3), the borrowing constraints (5), the non-negative constraints (6), and given values of initial stock of assets and durables, and terminal stock of assets, yields a set of first order conditions. They include the within-period wealth constraints, the borrowing constraints, the non-negative constraints, and

$$\frac{\partial U_t}{\partial C_t} = \lambda_t p_t, \quad t = 0, 1, \ldots, T, \tag{7}$$

$$\frac{\partial U_t}{\partial K_t} = \lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1} q_{t+1} - \gamma_t \kappa q_t, \quad t = 0, 1, \ldots, T, \tag{8}$$

$$-\frac{\partial U_t}{\partial H_t} = \lambda_t m_t + \alpha_t, \quad t = 0, 1, \ldots, T, \tag{9}$$

and the Euler equation[7]

$$\lambda_t = \frac{1}{1+\rho}(1 + R_{t+1})\lambda_{t+1} + \gamma_t, \quad t = 0, 1, \ldots, T, \tag{10}$$

where

$$m_t = w_t \left[1 - \frac{\partial S_t(r_t A_{t-1}, w_t H_t)}{\partial (w_t H_t)}\right] \tag{11}$$

and

$$R_{t+1} \equiv r_{t+1} \left[1 - \frac{\partial S_{t+1}(r_{t+1} A_t, w_{t+1} H_{t+1})}{\partial (r_{t+1} A_t)}\right] \tag{12}$$

are the after-tax marginal wage and interest rate, that are both endogenous for the household. The Lagrange multipliers $\lambda_t \equiv (1+\rho)^t \lambda_t^*$, $\gamma_t \equiv (1+\rho)^t \gamma_t^*$ and $\alpha_t \equiv (1+\rho)^t \alpha_t^*$ can be associated with the wealth constraint, the borrowing constraint and the labour supply constraint respectively. Superscript $*$ means that the values of the multipliers are discounted to period 0, and multipliers without superscript are current values.

According to the Envelope Theorem, the Lagrange multipliers signify the marginal rate of change of the maximum value of the utility with respect to a change in the constraint, cf. Takayama (1986). Thus they express the shadow price of the actual resource. In particular, λ_t^* can be interpreted as the marginal utility of wealth in period t discounted into the first planning period 0, while λ_t is measured in current values.

All the Lagrange multipliers are non-negative, and if a constraint is not binding, the multiplier is zero. Hence, $\gamma > 0$ if and only if the borrowing constraint (5) is

[7] This equation is obtained by differentiating the objective function (i.e. the Lagrangian) with respect to A_t.

binding. Similarly, $\alpha > 0$ if and only if the labour supply constraint (6) is binding. Since the wealth constraint is always binding, $\lambda_t > 0$ for all t.

2.2.2 Interpretation of the First Order Conditions

The first order conditions constitute a simultaneous equation system that implicitly defines the lifetime demand path for all goods, and the Lagrange parameters, as functions of initial and terminal value of assets, initial stock of durables, lifetime prices; including the interest rate and all the formal income tax rates in the tax tables, and the preferences. In most cases it is, however, impossible to obtain a closed form solution for these functions.

It turns out that in a life cycle perspective, the relevant demand functions are the so called Frisch demands, cf. Frisch (1932). These functions are characterized by demand being conditioned on the marginal utility of wealth, and if preferences are additively separable within periods (in addition to additively separable between periods) and the monotonic transformation function Φ_t is the identity transformation, each of the first order conditions (7) to (9) implicitly define the Frisch demands. In what follows we make that assumption. This means that when we now turn to the discussion of some properties of the first order conditions that are relevant for estimation, we also discuss the properties of the Frisch demand functions.

The first order condition for non-durables, cf. equation (7), and the Euler equation (10) imply

$$\frac{\partial U_{t+1}}{\partial C_{t+1}} = \frac{1+\rho}{1+R_{t+1}} \left[\frac{\partial U_t}{\partial C_t} - \gamma_t \right], \tag{13}$$

when $p_t = 1$ for all $t = 0, 1, \ldots, T$. That is, if preferences are both inter- and intraperiod additively separable, if the monotonic transformation function Φ_t is the identity transformation, if there are no binding borrowing constraints, and if R and p as well as preferences are constant over time, then consumption (or its marginal utility) at age t is the only relevant variable to predict consumption (or its marginal utility) at age $t+1$. Hall (1978) also assumes that $(1+\rho)/(1+R_{t+1})$ is constant[8] (at least over a decade or two), and tests this implication on macro time series data.

If the tax function is convex, the first order condition for labour supply and leisure demand, cf. equation (9), can be rewritten as

$$\begin{cases} \tilde{H}_t = 0 & \text{if} \quad -\frac{1}{\lambda_t}\frac{\partial U_t}{\partial H_t}\Big|_{H=0} \geq m_t|_{H=0} \\ \tilde{H}_t > 0 & \text{otherwise, and determined by} \quad -\frac{1}{\lambda_t}\frac{\partial \tilde{U}_t}{\partial H_t} = \tilde{m}_t, \end{cases} \tag{14}$$

[8]The constancy of $(1+\rho)/(1+R_{t+1})$ is of cause a strong simplification even if we disregard interest income taxation. While ρ is a constant, even the pre-tax interest rate varies across time and households.

where tilde denotes that the variable is evaluated at optimum. That is, the decision of working or not is determined by comparing the marginal utility of leisure at zero hours work (measured in money), $-\frac{1}{\lambda_t}\frac{\partial U_t}{\partial H_t}\Big|_{H=0}$, with the price of leisure, that is, the after-tax marginal wage rate at zero hours labour supply, $m_t|_{H=0}$. If the marginal utility of leisure at zero hours work exceeds the after-tax marginal wage rate at zero hours, the person chooses not to work. Contrary, if the marginal utility of leisure at zero hours work is smaller than the wage rate, he chooses to work, and decreases leisure until the marginal utility of leisure equals the marginal wage rate. Labour supply then has two different dimensions, annual participation in the work force, and annual hours of work. Heckman and MaCurdy (1980) utilize this property and estimate a bivariate Tobit life cycle model of female labour supply, cf. section 2.3.2.

Condition (14) is often referred to as a local criterion as opposed to the global criterion, where the utility levels of various combinations of labour supply and consumption must be compared. The global criterion should be used if the budget sets are non-convex, but we do not go further into this problem.

Notice also that the marginal tax rate on labour income at zero hours is endogenous, and can well be positive, since taxation of labour income depends on the level of interest income. It also depends on the income of the other spouse if the household consists of two adults that are treated as one taxpayer by the tax assessment.

The specification of the credit market constraint (5) implies that whether the credit market constraint is binding or not depends on the demand for durables. However, in order to simplify the discussion of the first order condition for durables, we now argue as if this on/off decision is independent of consumer behaviour. The demand for durables, cf. equation (8), can then be written as

$$(15)\quad \begin{cases} \tilde{K}_t = -\frac{A_t}{\kappa q_t} \text{ if } \frac{\partial U_t}{\partial K_t}\Big|_{k=-\frac{A}{\kappa q}} < \lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1}q_{t+1} \\ \tilde{K}_t > -\frac{A_t}{\kappa q_t} \text{ otherwise, and determined by } \frac{\partial \tilde{U}_t}{\partial K_t} = \lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1}q_{t+1}, \end{cases}$$

where we recall that tildes denote that the variable is evaluated at the optimum. The marginal cost, $\lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1}q_{t+1}$, is now measured in terms of utility. If the household is constrained in the credit market, the demand for durables is determined by the constraint, and the marginal utility of durables, evaluated at the constraint, is less than the marginal cost. Hence, the credit market constraint (5) increases the demand for durables relative to the demand in the case of no binding constraints. Contrary, if the household is unconstrained in the credit market, the household adjusts the demand for durables until the marginal utility equals the marginal cost.

Economic theory gives few guidelines with respect to how to measure demand for durables. It should be measured in physical units or in real values, but apart from that the theory is of little help. An example of the problems involved, is the measuring of housing consumption with all its dimensions such as location, number of rooms and quality. Problems with this definition also lead to problems with the definition of purchase and user price of durables, cf. chapter 3.

Notice that our treatment of durables assumes that there are no fixed costs in the demand for durables, and that durables can be treated as continuous variables. These approximations may be reasonable for white goods, but not for other kinds of durables such as housing.

While the first order conditions (7) to (9) determine allocation of resources within a particular period, the Euler equation (10) determines allocation of resources over time. It can be rewritten as

$$\lambda_t = \frac{1}{1+\rho}(1+R_{t+1})\lambda_{t+1} \quad \text{if} - A_t < \kappa q_t K_t \tag{16}$$

and

$$\lambda_t \geq \frac{1}{1+\rho}(1+R_{t+1})\lambda_{t+1} \quad \text{if} - A_t = \kappa q_t K_t. \tag{17}$$

If the household is unconstrained in the credit market, saving should be adjusted until the marginal utility of wealth in period t equals the (discounted) marginal utility of using the same resources next year added after-tax interest incomes. If the household is constrained, the marginal utility in period t exceeds the marginal utility of postponing consumption one period.

In the present case of perfect certainty, all the λ's can be calculated once and for ever, and the updating mechanism is given by the Euler equation (10). This means that if the household is unconstrained in all markets, if it knows $(1+R_{t+1})/(1+\rho)$ for all periods, and if it is able to calculate the marginal utility of wealth for a particular period, cf. equation (31), it can use the Euler equation to calculate λ for all t, and plug these values into the first order conditions to find consumption and labour supply. However, if the marginal tax rate on interest incomes depends on labour and interest incomes, R_{t+1} is endogenous and cannot be treated as an exogenous variable with respect to the first order conditions.

As will be evident from the following analysis, equation (16) and (17) have been utilized in various ways to obtain equations that are suitable for estimation. Since some households can be constrained, the possibility of borrowing constraints complicates estimation. One solution is to estimate the model from a subgroup of unconstrained households, but then we should correct for the possible selection bias, cf. Heckman (1979). Another possibility is to use the complete sample and estimate a switching regimes model, cf. Hajivassiliou and Ioannides (1991), but most works disregard the possibility of this constraint since one usually cannot identify what households are constrained, cf. Hall (1978), MaCurdy (1981,1985) and Blundell (1987).

2.2.3 Separability, Constraints and Frisch Demands

The functional form of the Frisch demands depends only on the specification of within-period utility and, as we have seen, on whether corner solutions are present. Their usefulness, however, depends on the separability properties of preferences, the possibility of binding constraints in the labour and credit markets, and the specification of lifetime wage rates and the income tax system, and this section focuses on these topics.

Apart from the difficulties related to α_t and γ_t being latent and the endogeneity of R_{t+1} and m_t, the fact that λ_t is latent and varying across age complicates the estimation of the Frisch demand functions. The problem related to λ_t being age-specific can be circumvented by expressing these equations as functions of λ_0 (or λ_0^*). Substituting the Euler equations successively into each other, leads to

$$\lambda_t = (1+\rho)^t \lambda_0 D_t - \overline{\gamma}_{t-1}, \tag{18}$$

where

$$D_t = \frac{1}{(1+R_0)(1+R_1)\cdots(1+R_t)} \tag{19}$$

is the after-tax marginal discount rate that transforms income in period t into its period 0 equivalent, and

$$\overline{\gamma}_{t-1} = \frac{1+\rho}{1+R_t}\gamma_{t-1} + \frac{(1+\rho)^2}{(1+R_t)(1+R_{t-1})}\gamma_{t-2} + \cdots + D_t(1+\rho)^t\gamma_0.$$

Hence, the marginal utility of wealth at age t can be expressed as a difference consisting of a function of the marginal utility of wealth at age 0, and a weighted sum of the multipliers of the borrowing constraints.

Substituting equation (18) into the first order conditions (7) to (9), implies

$$\frac{\partial U_t}{\partial C_t} = \left[(1+\rho)^t \lambda_0 D_t - \overline{\gamma}_{t-1}\right] p_t, \tag{20}$$

$$\frac{\partial U_t}{\partial K_t} = \left[(1+\rho)^t \lambda_0 D_t - \overline{\gamma}_{t-1}\right] q_{ut} - \left[\kappa q_t - \frac{1-\delta_f}{1+R_{t+1}} q_{t+1}\right] \gamma_t \tag{21}$$

and

$$-\frac{\partial U_t}{\partial H_t} = \left[(1+\rho)^t \lambda_0 D_t - \overline{\gamma}_{t-1}\right] m_t + \alpha_t, \tag{22}$$

where

$$q_{ut} \equiv q_t - \frac{1-\delta_f}{1+R_{t+1}} q_{t+1}$$

is the user price of durables. Since R_{t+1} depends on the marginal taxation of interest incomes, the user price is endogenous.

At this moment we notice that if preferences are non-separable both between and within periods, all the marginal utilities depend on the consumption of all goods in all periods, and estimation of a particular first order condition is very data demanding even if we could observe λ. If preferences are additively separable between, but not within periods (and the monotonic transformation of lifetime preferences is the identity transformation), the marginal utilities for a particular period all depend on the consumption of all goods in that period, and estimation of a particular first order condition requires less data. If, however, preferences are both intra- and intertemporal additively separable, and the monotonic transformations of both intra- and intertemporal preferences are the identity transformation, the marginal utilities only depend on the consumption of the actual good, and estimation is considerably simplified.

In the case of no binding credit market constraints in any historic period as well as no binding constraints in the labour and credit market in the current period, γ_t, $\overline{\gamma}_{t-1}$ and α_t equal zero. If we also assume additive within-period utility and that the monotonic transformation Φ_t of within period preferences is the identity transformation, the Frisch demand functions become

(23) $$C_t = C_t\left[(1+\rho)^t \lambda_0 D_t p_t\right], \quad t = 0, 1, \ldots, T,$$

(24) $$K_t = K_t\left[(1+\rho)^t \lambda_0 D_t q_{ut}\right], \quad t = 0, 1, \ldots, T,$$

and

(25) $$H_t = H_t\left[(1+\rho)^t \lambda_0 D_t m_t\right], \quad t = 0, 1, \ldots, T,$$

where the C_t-, K_t- and H_t-functions are the inverse of the functions for the subutilities of C, K and H respectively.

This specification illustrates the advantage of what MaCurdy (1981) labels the λ-constant demand functions. In the case of no binding constraints in any historic period in the credit market, no current binding constraints in the credit and labour markets, and inter- and intraperiod additive separable preferences with Φ_t equal to the identity transformation, the arguments of the demand functions, apart from the discounting rate, are reduced to prices observable within the *current* period and the (latent) *life cycle component* λ_0. This means that with the exception of the information that is included in D_t, m_t and q_{ut}, λ_0

summarizes all historic and future information relevant to the household's current decisions.

If there is no income taxation, D_t, m_t and q_{ut} are exogenous, and λ_0 summarizes all historic and future information relevant to the household's current decisions. The marginal utility of wealth at age zero can then be thought of as a statistic[9] representing the household's (perfect) expectations about future (pre-tax) wage and interest rates, the purchase prices of durables and non-durables, and realized values of these variables earlier in life. Initial and terminal (net) wealth also influence consumption and labour supply through the marginal utility of wealth.

Since λ_0 is independent of time, it can be treated as a fixed effect during estimation, and if the Frisch demand functions are additive in λ_0 or its logarithm, the problem related to λ_0 being latent can be overcome by differencing the Frisch demands, cf. MaCurdy (1981) and section 2.3.1. This approach also reduces the problem related to the fact that estimation of the λ-constant demand functions is quite data demanding even if we could observe λ_0, since the after-tax discount rate D_t is endogenous and depends on $R_0, R_1, \ldots, R_t$, cf. equation (43).

While empirical analyses typically assume that life cycle wage rates are independent of labour supply, they can also be thought of as being affected by work experience, cf. Heckman (1981) and Nakamura and Nakamura (1985)[10]. This assumption introduces a kind of non-separability through the wealth constraint, since current labour supply decisions can be viewed as an investment to increase future wages. The marginal wage rate then depends on future hours, and both future wage rates and hours enter into the first order conditions. The data requirement thus increases.

The introduction of income taxes, $S_t(r_t A_{t-1}, w_t H_t)$, also leads to a kind of non-separability that may reduce the usefulness of the marginal utility of wealth constant functions (23) to (25), and the methods based on these functions. If one does not observe current interest incomes, the calculation of the after-tax

[9]Friedman (1957) argues that the consumer's aggregate consumption is related to "permanent" and "transitory" income. Mincer (1962) transfers this theory to the labour supply market, and assumes that labour supply is related to transitory and permanent wage rates and incomes. Comparing these theories with our marginal utility of wealth constant functions, we find that permanent income and wage rate play the same role as the marginal utility of wealth, while transitory incomes and wage rate play the role of the current prices.

[10]Using estimates from participation equations only, according to these authors it is impossible to separate this kind of dependence from the dependence due to intertemporal non-separable preferences. Hotz, Kydland and Sedlacek (1988) show that the separability properties of lifetime preferences can be studied by expressing the Euler equation in terms of consumption since consumption does not depend on the process generating the life cycle wages.

marginal wage and interest rate requires observations on A_{t-1}, and we need panel data for (at least) two years. The calculation of the after-tax discount rate D_t, cf. equation (19), requires panel data for even more periods, and the after-tax wage and interest rate, and the user price of durables, are now endogenous. In order to simplify estimation, income taxes are often omitted. Blomquist (1985) shows that the lifetime wealth constraint (4) then becomes separable in goods and prices across periods, and the only way a price change can influence demand in other periods, is through the wealth effect (λ_0). The responses to price changes in other periods are then of a very special form, since there is no effect via the discounted prices corresponding to $D_t p_t$, $D_t q_{ut}$ and $D_t m_t$, cf. Blomquist.

The possibility of constraints in the credit market also complicates estimation of the Frisch demands considerably. Comparing equations (7) to (9) with (20) to (22), we notice that the substitution of the Euler equation (18) into the first order conditions introduces the borrowing constraint multipliers for all earlier periods into these conditions. Since these multipliers are latent, this substitution probably introduces more problems that it solves, and the specifications (7) to (9) seem to be more attractive.

In order to use the Frisch demands to explain differences in demand across households, the relation between λ_0 and all the exogenous variables must be determined. Substituting the λ-constant demand functions (23) to (25) into the lifetime wealth constraint in the case of no binding constraints in any market in any period, leads to

$$\begin{aligned} & d_T A_T - A_0 \\ = & \sum_{t=0}^{T} d_t \Big\{ w_t H_t \left[(1+\rho)^t \lambda_0 D_t m_t\right] - S_{\lambda t} - p_t C_t \left[(1+\rho)^t \lambda_0 D_t p_t\right] \\ (26) \quad - & q_t \left[K_t \left[(1+\rho)^t \lambda_0 D_t q_{ut}\right] - (1-\delta_f) K_{t-1} \left[(1+\rho)^{t-1} \lambda_0 D_{t-1} q_{ut-1}\right]\right]\Big\}, \end{aligned}$$

where $S_{\lambda t}$ denotes that the λ-constant demand functions are also substituted into the tax function. This function implicitly defines λ_0 as a function of the household's initial value of assets and durables, terminal value of assets, all prices for all periods, and the time preference rate and the other parameters determining lifetime preferences. If there are binding constraints in some markets, the shadow prices of these constraints are also included.

In most cases this equation cannot can be given a closed form solution with respect to λ_0. Even if we could, cf. section 2.3.3, the fact that existing panel data do not contain *complete lifetime* price paths, including the interest rate and the income tax rate paths, and seldom initial and terminal assets and equities, complicates estimation of the reduced form equation for λ_0. The fact that λ_0 is latent complicates estimation further.

2.3 Estimation in the Perfect Certainty Case

While the aim of section 2.2 was to present some theoretical considerations that can be used for estimation of the life cycle model, we are now going to take a closer look at some important contributions and methods within this field. In this section we limit ourselves to methods particularly suited for the perfect certainty case.

2.3.1 MaCurdy's Fixed Effect Approach

The use of the marginal utility of wealth constant functions for estimation of the life cycle model can be associated with Thomas E. MaCurdy (1981)[11], and we will now study how he utilizes these functions to estimate an intertemporal model of labour supply.

2.3.1.1 Model specifications

Compared with our general model presented in section 2.2, MaCurdy disregards consumption of durables, personal income taxation and the possibility of credit market constraints. The lifetime wealth constraint is then

$$A_{i0} + \sum_{t=0}^{T} d_t w_{it} H_{it} = \sum_{t=0}^{T} d_t C_{it}, \tag{27}$$

where subscript i denotes person and terminal wealth, A_T, is assumed to be zero. The wage and interest rate, and the stock of assets, are now measured in real terms.

MaCurdy assumes that preferences can be described according to

$$\sum_{t=0}^{T} (1+\rho)^{-t} \left(\Gamma_{cit} C_{it}^{\omega_c} - \Gamma_{hit} H_{it}^{\omega_h} \right), \tag{28}$$

where ω_c and ω_h are time- and person-invariant modifiers of preferences, and Γ_{cit} and Γ_{hit} are person- and time-specific modifiers. The modifier of preferences for leisure, Γ_{hit}, is assumed to be randomly distributed across the population according to

$$\Gamma_{hit} \equiv \exp(\sigma_i - \xi_t^*), \tag{29}$$

where σ_i is a non-stochastic parameter and ξ_t^* is a stochastic parameter. Since the consumer is assumed to have perfect knowledge of future prices and

[11] The original paper was part of his Ph.D. thesis, "Two Essays on the Life Cycle", University of Chicago, 1978.

preferences, ξ_t^* is introduced in order to take account of the fact that the econometrician cannot observe all factors that influence preferences for work.

The taste modifier for goods, Γ_{cit}, is not specified further since MaCurdy focuses on labour supply and does not use the first order condition for consumption.

Assuming the real rate of interest is constant over the life cycle and equal to r with the exception of for period zero where it is equal to ρ, the marginal utility of wealth constant labour supply function (25) now becomes

$$\text{(30)} \qquad \ln H_{it} = F_{i0} + bt + a \ln w_{it} + \xi_{it},$$

where $a \equiv 1/(\omega_h - 1)$, $\xi_{it} \equiv a\xi_{it}^*$ and $F_{i0} \equiv [1/(\omega_h - 1)](\ln \lambda_{i0} - \sigma_i - \ln \omega_h)$. This specification also requires that $\ln(1+r)$ and $\ln(1+\rho)$ can be approximated by r and ρ.

The assumptions about the interest rate imply that $(1+\rho)^t D_{it}$ in equation (25) is being reduced to bt, where $b \equiv a(\rho - r)$, and the interest rate becomes part of the b-parameter.

Since the specification of preferences means that it is impossible to find a reduced form solution for λ_0, cf. equation (26), MaCurdy assumes that "$\ln \lambda_{i0}$ can be approximated as a linear function of measured characteristics, Z_i, net natural log of wages at each age, initial wealth and an unobserved random variable, ζ_i, representing unmeasured characteristics". Hence

$$\text{(31)} \qquad F_{i0} = Z_i\phi + \sum_{t=0}^{T^*} \beta_t \ln w_{it} + A_{i0}\theta + \zeta_i,$$

where T^* denotes the age of retiring, β_t and θ are scalars, and ϕ is a vector of constants that are assumed to be constant across consumers.

Estimation of this equation requires that the lifetime wage path and initial assets can be observed for each consumer. To observe the wage rates outside the sample period, and for the unemployed workers, MaCurdy assumes that wages follow a quadratic function in age,

$$\text{(32)} \qquad \ln w_{it} = \pi_{0i} + t\pi_{1i} + t^2\pi_{2i} + \varsigma_{1it},$$

where ς_{1it} is a random variable, and where the coefficients π_{0i}, π_{1i} and π_{2i} are linear functions of some age-invariant characteristics of the person, M_i,

$$\text{(33)} \qquad \pi_{ji} = M_i g_j, \quad j = 0, 1, 2,$$

where g_0, g_1 and g_2 are vectors of parameters. The variables included in M, are education and variables related to family background.

Most data set do not include even extensive measures of the consumers' current wealth, and to predict initial assets[12], MaCurdy assumes that the optimal lifetime income stream from buying assets, rA_{it}, can be approximated by a quadratic function in age,

$$rA_{it} = \pi_{3i} + t\pi_{4i} + t^2\pi_{5i} + \varsigma_{2it}. \tag{34}$$

Here ς_{2it} is a random variable, and the parameters π_{3i}, π_{4i} and π_{5i} are linear functions of a vector of exogenous and time invariant determinants, J_i, of property incomes,

$$\pi_{ji} = J_i h_j, \quad j = 3, 4, 5, \tag{35}$$

where h_j is the vector of coefficients.

2.3.1.2 Estimation

MaCurdy does not estimate the parameters of the utility function, but applies equations (30) and (31) for prediction of labour supply.

The estimation of equation (30) takes into account that λ_{i0} is latent and correlated with the exogenous variables. Since F_{i0} is a linear transformation of λ_{i0} and some time invariant parameters, F_{i0} can be treated as a fixed effect. MaCurdy then estimates a and ρ, determined through $b \equiv a(\rho - r)$, by working on a first differenced version of the Frisch supply function (30),

$$\Delta \ln H_{ij} = b + a\Delta \ln w_{ij} + \Delta\xi_{ij}, \qquad i = 1, 2, \ldots, n, \quad j = 2, 3, \ldots, j^*, \tag{36}$$

where Δ is the difference operator, i.e. $\Delta X_t \equiv X_t - X_{t-1}$. Subscript j denotes sample period, j^* is the number of sample periods, and n is the number of workers in the sample. The variables H_{ij} and w_{ij} then denote person i's labour supply and wage rate in sample period j.

Since there may be omitted variables that are correlated with both the wage rate growth and the error term $\Delta\xi_{ij}$, the wage rate is treated as endogenous in the estimation. The parameters of equation (36) can then be (consistently) estimated by standard 2SLS[13].

[12] Notice that MaCurdy's specification of the property income path seems to overdetermine the property income path. When consumption and labour supply are determined through the first order conditions, the period-specific wealth constraints can be utilized to derive saving in all periods. Given initial assets and a lifetime path for the interest rate we can then find assets and property incomes in all periods. Hence the a priori assumption of a particular lifetime property income path can lead to inconsistency problems.

[13] MaCurdy also considers another specification of equation (36) where the interest rate is allowed to vary over time.

An advantage of this approach is that the parameters determining the responses to current price changes along a given life cycle price path can be estimated from panel data for (at least) two periods on only the actual good in question (including the price), in addition to the relevant instrument variables. In MaCurdy's case this means that he can estimate the parameters related to labour supply without having data on total consumption. Since many data sets do not include observations of both labour supply and consumption including the consumption of durables, this fact is important.

A problem related to first differencing in particular, but also higher order differencing, is that it tends to accentuate measuring noise relative to the observed changes in the differenced variables, and the precision of the estimated parameters is reduced.

To estimate the reduced form equation for F_{i0}, MaCurdy uses the wage and property income path equations and obtains

$$F_{i0} = Z_i\phi + \pi_{0i}\overline{\beta}_0 + \pi_{1i}\overline{\beta}_1 + \pi_{2i}\overline{\beta}_2 + \pi_{3i}\overline{\theta} + \varrho_i, \tag{37}$$

where

$$\overline{\theta} = \theta/r, \qquad \overline{\beta}_j = \sum_{t=0}^{T*} t^j\beta_t, \quad j = 0,1,2,$$

and ϱ is the error term.

The equations (33), (35) and (37) constitute a simultaneous equation system, that can be estimated if we have data on F_0, Z, and π_0 to π_3 for all consumers. The consumer characteristics, Z_i, are observed directly, and MaCurdy shows that unbiased estimates for all the π_i's can be obtained by making some particular transformations of equations (32) and (34). Estimates for all the F_{i0}'s are obtained from equation (30) inserted the estimates for b and a from the estimation of equation (36). Using these results, MaCurdy estimates the simultaneous equation system (33), (35) and (37) by ordinary 2SLS. The substitutions imply that the error term and the right hand side variables of the F_0-equation become correlated, but according to MaCurdy, the estimates are consistent for a "sufficiently large" number of observations per consumer.

The estimation of the reduced form equation for F_{i0} is very data demanding since it requires life cycle data for each person. This means that if we want to estimate all aspects of demand, the fixed effect approach necessarily involves some kind of arbitrary assumptions about the lifetime price paths. In most cases estimation also requires approximations to the true relationship. This reduces the tight connection between theory and estimated regression function.

2.3.1.3 Restrictions on intra- and intertemporal preference

MaCurdy's fixed effect approach requires that the marginal utility of wealth constant demand functions are linear functions of λ_0 or $\ln \lambda_0$. From the first order conditions (7) to (9) it should be evident that within-period preferences must be additive if the λ-constant demand functions are to be linear in λ_0, cf. also Blundell and Walker (1986). Browning, Deaton and Irish (1985) recognize that this (unfortunate) result may be due to the fact that we start out from the direct (within period) utility function, and that if we start out from another specification, this restriction may be relaxed. In order to reduce restrictions as far as possible, they suggest to start out from the profit function. This approach views the household's decision problem as a profit maximization problem, where the household produces utility, U, from consuming the goods C, K and H. To ease notation, the subscript *it* is now suppressed. Lifetime profit (and utility) is the sum of the profits (and utilities) for all periods, since we still assume intertemporal additivity. The within period profit function is defined as

$$\pi(p^u, \underline{p}) = \max_u \left\{ p^u U - c(U, \underline{p}) \right\}, \tag{38}$$

where $\underline{p}$ denotes (to ease notation) the price[14] vector corresponding to the input vector (C, K, H), $p^u \equiv 1/\lambda$ is the marginal cost/price of utility, and $c(U, \underline{p})$ is the (standard) cost/expenditure function corresponding to the preferences $U(C, K, H)$. All prices are pre-tax prices since Browning et al. ignore income taxation.

This function defines the household's maximum profit attainable, $\pi(p^u, \underline{p})$, from selling utility U to itself at a price p^u, and at a production cost $c(U, \underline{p})$, where the cost function represents the technology of utility production. Hence, it represents household preferences as a function of the price of utility and the prices of the goods, just like the cost function represents preferences in terms of utility and prices of goods. The link between periods is the reciprocals of the marginal utility of wealth, p^u.

A favourable property of this approach is that by differentiating (minus) the profit function with respect to the price of a particular good, we get the Frisch demand function for the good. This fact follows directly from specification (38). Browning et al. use this result and find the most general profit function that satisfies the condition that the partial derivatives of the Frisch demands with respect to all prices, are independent of the marginal utility of wealth. Based on the specification of this general function they conclude that "the treatment of p^u as additive in the hours and quantities demanded implies intraperiod quasi-homotheticity." In other words, viewing the households' decision problem

[14] The relevant price of K is the user price.

as a profit maximization problem, the requirement that the Frisch demands are linear in $p^u \equiv 1/\lambda$ implies that demand is linearly related to within-period full income. Full income at age t, Y_t, is defined as the sum of the value of the household's initial time endowment $\overline{L}$, and its interest incomes and asset decumulation,

$$Y_t = w_t\overline{L} + r_t A_{t-1} - (A_t - A_{t-1}). \tag{39}$$

The profit function approach, however, allows for non-separable within period preferences.

Blundell, Fry and Meghir (1985) show an analogous result within the same framework, that is, relaxing intraperiod additive separability leads to unitary within-period full income elasticities if the loglinear Frisch demands are to be loglinear in λ_0. The use of Frisch demand functions that are derived either from a specification of direct utility or from the profit function, then introduces quite strong restrictions on intratemporal preferences.

Intuitively we would consider that the use of highly aggregated Hicks composite goods reduces the restrictiveness of assuming intraperiod additive separability. Intraperiod additive separability can then turn out to be a reasonable approximation for life cycle models of highly aggregated Hicks composite goods.

The assumption of intertemporal additive separability can also be questioned. According to Blundell (1987), the indirect within-period utility at age t corresponding to the profit function of Browning et al., is of the Gorman Polar form

$$G_t = I_t\left\{\frac{Y_t - a_t(\underline{p}_t)}{b_t(\underline{p}_t)}\right\}, \tag{40}$$

where I_t is some concave monotonic transformation that determines the intertemporal allocation of the goods, and $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$ are particular concave linear homogeneous functions of the vector $\underline{p}_t$ of prices discounted to period zero.

Blundell uses this specification to discuss the restrictions on intertemporal substitution implied by the profit function of Browning, Deaton and Irish. He follows Browning (1985) and assumes that the intertemporal substitution possibilities best can be measured by the intertemporal elasticity of substitution, $\phi = G_y/YG_{yy}$, where $G_y = \partial G/\partial Y$ and $G_{yy} = \partial G_y/\partial Y$. The utility index (40) implies

$$\phi_t = \frac{I_t'\{[Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)\}b_t(\underline{p}_t)}{I_t''\{[Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)\}Y_t}, \tag{41}$$

where I_t' denotes partial derivative. Since $Y_t - a_t(\underline{p}_t)$ is dominated by Y_t, this means that $b_t(\underline{p}_t)$ represents the substitution possibilities for rich people. This result and the fact that the specification of the profit function implies that $b_t(\underline{p}_t)$ is a linear function of all the prices of $\underline{p}_t$, means that the use of linear Frisch demands constraints intertemporal substitution for the high income groups.

It can be argued that this matter is not very important since the substitution possibilities are also determined by the derivatives of the monotonic transformation I_t. Many works, however, let I_t be the identity transformation, and Blundell shows that the substitution elasticity for the loglinear and the exponential transformations approaches -1 and 0 as income increases. Linear or loglinear Frisch demands then also seem to restrict intertemporal substitution possibilities considerably.

2.3.1.4 Income taxation and MaCurdy's fixed effect approach

As mentioned MaCurdy disregarded income taxation and assumed that the interest rate was constant across periods and equal for all persons. In this case the Frisch demands only include current variables. Without these simplifications the Frisch labour supply function corresponding to equation (30) becomes

$$\ln H_{it} = F_{i0} + a \ln \left[D_{it}(1+\rho)^t m_{it} \right] + \xi_{it}, \tag{42}$$

where we recall that D_{it} is the after-tax marginal discount rate defined in equation (19), and m_{it} is the after-tax marginal wage rate. Since the discount rate D_{it} is endogenous, we can hardly assume it is constant during the life cycle, and in no way we can reasonably assume it is equal for all persons. Taking also into consideration that D_{it} depends on $R_{i0}, R_{i1}, \ldots, R_{it}$, the introduction of income taxes immediately seems to complicate estimation considerably. However, first differencing the Frisch demands yields

$$\Delta \ln H_{it} = a\rho + a[\Delta \ln m_{it} - \ln(1 + R_{it})] + \Delta\xi_{it}, \tag{43}$$

which illustrates that the method of differencing is attractive even in this case. MaCurdy's fixed effect approach can then very well be used even in the case of personal income taxation, but we must take account of that the introduction of income taxes makes m_{it} and R_{it} endogenous to the consumer and dependent on A_{t-1}.

2.3.2 A Fixed Effect Tobit Model

Heckman and MaCurdy (1980) extend MaCurdy's fixed effect approach by implementing a bivariate fixed effect Tobit model for married females' labour supply. This work uses the utility function

$$\sum_{t=0}^{T} (1+\rho)^{-t} \left[\Gamma_{ct} C_t^{\omega_c} + \Gamma_{lt} (\overline{L} - H_t)^{\omega_l} \right], \tag{44}$$

where $\overline{L}$ is total time available in the period, and ω_l and ω_c are unknown coefficients. The age- and person-specific modifier of tastes for leisure, Γ_{lt}, is assumed to be related to a set of observed consumer characteristics, N_t, and a stochastic component, ϵ_{1t}, according to $\Gamma_{lt} \equiv \exp(N_t\psi + \epsilon_{1t})$, where ψ is a vector of coefficients. The taste modifier for consumption, Γ_{ct}, is not specified further.

The wage rate equations (32) and (33) are now modified to

$$\text{(45)} \qquad \ln w_t = M_t\beta_m + \epsilon_{2t},$$

where M is a vector of observed consumer characteristics, β_m is the corresponding vector of coefficients, and ϵ_{2t} is a stochastic component. Identification of ω_l, ψ and $(\rho - r)$ requires that M includes at least one variable that is not included in N, cf. equation (45) and (49).

It is also assumed that the disturbance vector $(\epsilon_{1t}, \epsilon_{2t})$ follows a components of variance scheme,

$$\text{(46)} \qquad \epsilon_{jt} = \eta_j + \mu_{jt}, \quad j = 1, 2,$$

where the vector (μ_{1t}, μ_{2t}) is allowed to be correlated within-periods, but is assumed to be serially uncorrelated and generated by a bivariate normal distribution. The components η_1 and η_2 are left freely correlated. Thus, the econometric model has the structure of a bivariate Tobit model, cf. also Chamberlain (1984).

Assuming $\ln[(1+\rho)/(1+r)]$ can be approximated by $\rho - r$, the labour supply function corresponding to equation (14) now becomes (cf. appendix A)

$$\text{(47)} \qquad \ln(\overline{L} - H_t) = \begin{cases} \ln \overline{L} & \text{if } \; V_{1t} \geq -f - \mathcal{N}_t + \ln \overline{L}, \\ f + \mathcal{N}_t + V_{1t} & \text{if } \; V_{1t} < -f - \mathcal{N}_t + \ln \overline{L}, \end{cases}$$

where

$$\text{(48)} \qquad f = \frac{1}{\omega_l - 1}(\ln \lambda_0 - \ln \omega_l - \eta_1 + \eta_2), \quad V_{1t} = \frac{1}{\omega_l - 1}(\mu_{2t} - \mu_{1t})$$

and

$$\text{(49)} \qquad \mathcal{N}_t = \frac{\rho - r}{\omega_l - 1}t - N_t\frac{\psi}{\omega_l - 1} + M_t\frac{\beta_m}{\omega_l - 1}.$$

This approach then takes into consideration that labour supply has two different dimensions, cf. the discussion of equation (14), and the simultaneous likelihood function for the model consisting of the labour supply function (47) and the wage rate equation can be found. Since f includes the marginal utility of wealth, f is correlated with all the exogenous variables of the model, and to overcome this problem, it is treated as a fixed effect to be estimated.

Again there is no reduced form equation for λ_0 or f. Heckman and MaCurdy approximate, and regress estimated fixed effects, f, (obtained from the ML-estimation) on education, average household income, future fertility plans, premarital work experience and realized fertility measures. The effects of lifetime prices and initial wealth as determinants of demand, are ignored.

Compared with the estimation strategy found in MaCurdy (1981), this estimation method reduces the possible selection bias from using a subsample of unconstrained persons. Regarding the estimation of the responses to current price changes along a given life cycle price path, both methods have in common that they only require observations of variables related to the actual good in question. Both methods typically require approximations to the true relationship for the reduced form equation for the marginal utility of wealth. The way they estimate the fixed effects (F_0 and f) is different. While MaCurdy's approach involves a quite cumbersome procedure, f is estimated simultaneously with all the other parameters in the approach of Heckman and MaCurdy. Both methods require panel data (Heckman and MaCurdy use panel data for eight years) for estimation of these effects. Depending on the parameter specifications, Heckman and MaCurdy can estimate the parameters determining the responses to current price changes along a given life cycle price path, from cross section data. In contrast, MaCurdy's fixed effect approach requires panel data for at least two periods.

2.3.3 Reduced Form Estimation

In this section we review some works where it is possible to find the explicit solution for the household's optimization problem.

Bover (1989) recognizes that when preferences can be described by a Stone-Geary function, it is possible to find the reduced form equation for λ_0. This equation can then be substituted into the Frisch demands.

Assuming the Stone-Geary utility function

$$\sum_{t=0}^{T}(1+\rho)^{-t}[B_{hit}\ln(b_h - H_{it}) + B_{cit}\ln(C_{it} - b_c)], \tag{50}$$

no income taxation and the lifetime wealth constraint (27), the reduced form for λ_0 is

$$\lambda_{i0} = \frac{\sum_{t=0}^{T}(1+\rho)^{-t}(B_{hit} + B_{cit})}{A_{i0} + \sum_{t=0}^{T}(1+r)^{-t}(b_h w_{it} - b_c)}. \tag{51}$$

Here b_h, b_c and ρ are unknown coefficients, B_{hit} and B_{cit} are taste modifiers, and A_{i0} is the initial stock of assets measured in real terms.

As we would expect from the discussion of equation (26), λ_0 is a function of preferences, initial wealth, and the lifetime price paths, here represented by the (real) interest, r, and wage rate, w_{it}. Since no data sets include all these variables, estimation requires further assumptions. Bover assumes that $B_{cit} = 1 - B_{hit}$, that the time horizon is infinite, and that $\rho = r$, where r is not observed. She also specifies the wage equation

$$w_{it} = l_{0i} + l_{1i}t + \nu_{it}, \tag{52}$$

where l_{0i} and l_{1i} are "unspecified functions of time invariant determinants of wages", and ν_{it} is an unobserved random variable.

The earnings function now becomes

$$w_{it}H_{it} = b_h w_{it} - B_{hit}\left[b_h\hat{l}_{0i} + \frac{b_h}{r}\hat{l}_{1i} + \frac{r}{1+r}A_{i0} - b_c\right], \tag{53}$$

where $\hat{l}_{0i}$ and $\hat{l}_{1i}$ are unbiased estimates of l_{0i} and l_{1i} obtained from a separate estimation of the wage equation. Assuming further that B_{hit} can be related to observable variables and a random error, Bover uses this specification as basis for estimating a linear earnings equation on w_{it}, $\hat{l}_{0i}$, $\hat{l}_{1i}$, A_{i0}, individual characteristics, and interactions of characteristics with $\hat{l}_{0i}$, $\hat{l}_{1i}$ and A_{i0}, by minimizing a generalized instrumental variables criterion function. From this estimation she obtains estimates of all the parameters of the utility function, including the time preference rate $\rho = r$.

Regarding the properties of this approach, we notice that it is based on the relevance of the Stone-Geary utility index. To find the reduced form expression for λ_0, one has to assume that there are no binding constraints in any market in any period. Estimation requires observations of variables related to the actual good, initial assets and demographic variables, but since most data sets do not include observations of initial assets, estimation can be more complicated than it may look like at first glance. Bover also disregards income taxation, and this decision simplifies estimation and reduces the data requirement.

An explicit solution of the optimization problem is also obtained in Biørn (1981), where Biørn estimates a Stone-Geary utility index with the household's aggregate consumption as the only argument in the utility index. By splitting income at the beginning of the planning horizon into permanent and transitory income, he relates consumption to permanent and transitory incomes.

In Bover (1991), Bover modifies her Stone-Geary specification described above, to allow for habits;

$$\sum_{k=t}^{T}(1+\rho)^{t-k}\left[B_{hik}\ln(b_h + \delta H_{ik-1} - H_{ik}) + B_{cik}\ln(C_{ik} - b_c)\right]. \tag{54}$$

This specification implies that the utility from supplying H_k hours of labour depends on H_{k-1}, and the δ-parameter reflects to what extent the household considers last year's labour supply in its determination of current labour supply.

Since we do not observe $(H_k - \delta H_{k-1})$ (or δH_{k-1}), it seems difficult to estimate this specification, but rewriting the wealth constraint in terms of $H_k^+ \equiv H_k - \delta H_{k-1}$, the Frisch labour supply and consumption demand functions become

$$H_{it}^+ = b_h - \frac{B_{hit}}{w_{it}^+}\lambda_{0i}^{-1} \quad \text{and} \quad C_{it} = b_c + B_{cit}\lambda_{0i}^{-1}, \tag{55}$$

where

$$w_{ik}^+ = \sum_{j=0}^{T-k}\left(\frac{\delta}{1+r}\right)^j w_{ik+j}. \tag{56}$$

Bower now allows for wage replanning[15]. The specification of the wage equation is similar to equation (52), but it is now assumed that l_0 and l_1 are both time- and person-specific. It is still assumed that $B_{cik} = 1 - B_{hik}$, and that the taste modifier B_{hik} can be related to observable consumer characteristics, N_{ik}, and a random error. Using a plausible approximation to the reduced form for λ_t, the earnings function corresponding to equation (53) becomes

$$\begin{aligned} w_{it}H_{it} &= b_h w_{it} + \delta w_{it}H_{it-1} + a_1 N_{it} + a_2 N_{it}\hat{w}_{it} + a_3 N_{it}\hat{l}_{1it} \\ &+ a_4 N_{it}A_{it} + a_5 H_{it-1}N_{it}\hat{w}_{it} + a_6 H_{it-1}N_{it}\hat{l}_{1it}, \end{aligned} \tag{57}$$

where a_j, $j = 1, 2, \ldots, 6$, are nonlinear functions of the structural parameters, and $\hat{w}_t$ is the predicted wage rate from the estimated wage equation. Notice that the habit parameter δ from the utility function, is the coefficient on the variable $w_{it}H_{it-1}$.

Estimation of this equation allows for identification of all the structural parameters. Despite the fact that this equation does only include current variables and variables lagged one period, estimation requires panel data for more periods in order to estimate the wage equation. (Bover uses 10 years of panel data from the Michigan Panel of Income Dynamics.) Notice also that in contrast to specification (53), that includes initial assets, this specification includes current assets, a great advantage from an econometric point of view. With the exception that this specification considers habits, the approach is quite similar to Bover's approach discussed above, and the properties of these two approaches are quite similar.

[15]This assumption introduces a particular kind of uncertainty, but according to Bover there is no obvious extension to allow for general uncertainty.

2.3.4 Two-Stage Budgeting

From an econometric point of view estimation of the Frisch demand functions is complicated by the fact that the marginal utility of wealth cannot be observed. In order to find a specification of the demand functions where it is possible to observe all variables, Blundell (1987) and Blundell and Walker (1986) suggest to use the two-stage budgeting theory for estimation. This approach has also been used in Blundell, Browning and Meghir (1989,1994), in Blundell, Meghir and Neves (1993), and in Attanasio and Browning (1993). It is particularly relevant for estimation of the parameters of within-period preferences, but at the end of this section we show how it can be combined with a particular use of the Euler equation in order to estimate the parameters of the monotonic transformation of within-period preferences.

For simplicity we assume that there are no binding constraints in any market and no durable goods. Assuming intertemporal additive separability, the life cycle model presented in section 2.2.1 can be viewed as a two-stage budgeting process. In the first step the household determines the lifetime assets path $(A_1, A_2, \ldots, A_{T-1})$ according to the Euler equation (10). This allocation depends on lifetime prices; including the interest rates and the formal income tax rates in the tax tables, initial and terminal value of assets, and the preferences, cf. the first order conditions (3), (7) and (9) to (12) where $K_t = q_t = \delta_f = 0$. Using the definition of full income, cf. equation (39), we find that the allocation of full income across periods is determined by the same variables plus the initial time endowment $\overline{L}$. Estimation of this relationship is then very data demanding, and it is typically not estimated.

Notice that we don't need to change the definition of full income when we introduce personal income taxation, and that the allocation of full income accross periods is consistent with the optimizing values of C_t and H_t for $t = 0, 1, \ldots, T$.

Conditional on full income, the household in the next step determines the allocation of full income to all goods. At age t this allocation is determined by maximizing within-period utility

$$U_t\,(C_t, H_t) \tag{58}$$

with respect to C_t and H_t, subject to the within-period wealth constraint

$$w_t(\overline{L} - H_t) + S_t(r_t A_{t-1},\, w_t H_t) + p_t C_t = Y_t. \tag{59}$$

This wealth constraint corresponds to equation (3) when we ignore durables and use the time budget constraint $L_t + H_t = \overline{L}$.

The first order conditions corresponding to this optimization problem implicitly define the demand for goods and leisure as functions of w_t, p_t, Y_t and $r_t A_{t-1}$, and the parameters of the tax function and the preferences. In order to simplify the exposition further, ignore income taxation and assume that these demand functions can be given closed form solutions. If $\underline{C}_t = (C_t, L_t)$ is the vector of consumption goods with corresponding price vector $\underline{p}_t = (w_t, p_t)$, the within-period demands can be written as

$$\underline{C}_t = \underline{f}_t\left(\underline{p}_t, Y_t\right), \tag{60}$$

where $\underline{f}_t$ is the vector of demand functions.

These demand functions are called y-conditional demands since consumption is chosen conditional on full income. They are homogenous of degree zero in prices, $\underline{p}_t$, and full income, Y_t, and the functional form varies across goods. According to Blundell and Walker (1986), the demand functions for all goods generally change if there are binding constraints in any market in period t. In contrast binding constraints in the future only change the value of Y. The assumption of intertemporal additive separability then allows for decentralization over time.

Although we are assuming perfect certainty, we notice that the introduction of uncertainty about future prices and preferences leaves the two-stage budgeting model almost unchanged, since all uncertainty is captured through the distribution of full income across the life cycle, cf. Blundell and Walker (1986).

From the discussion of the determination of full income it is evident that the full income variable in the y-conditional demands summarises the effects of past decisions and future prices and preferences, on current decisions. In that respect full income serves the same purpose in the y-conditions demands as the marginal utility of wealth does in the Frisch demand functions.

Compared with the Frisch demands, the y-conditional demands, however, only include variables that can be observed, and they can be observed within the current period. Estimation of the demand function for a particular good requires observations of the demand for the good, the price vector $\underline{p}_t$ and full income. If we take account of income taxation, $r_t A_{t-1}$ must also be observed in addition to the parameters of the tax system. Since many surveys are either income or expenditure surveys, it can be hard to find data sets that include all these variables.

Blundell (1987) points at that empirical specifications based on parametrization of direct utility, can often be shown to be unnecessarily restrictive compared to dual representations. In that respect the following results are important:

1. If preferences are intertemporally separable, within-period allocation of

full income to individual goods is completely characterized by the indirect utility function, and the allocation is invariant to monotonic transformations of the utility function, cf. Gorman (1959). Such transformations only influence intertemporal allocation.

2. Applying Roy's identity[16] to indirect within-period utility, yields the y-conditional demands.

Hence, it is possible to combine explicit parametrizations of indirect utility with the use of y-conditional demand functions.

In order to get more insight in this approach, assume within-period utility is of the general Gorman Polar form (40). While the fixed effect approach requires a particular specification of the price indices $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$, the y-conditional approach does only require that $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$ are concave linear homogenous functions of $\underline{p}_t$, cf. Blundell (1987). Blundell and Walker (1986) discuss particular forms of these functions, with emphasis on their flexibility and how they can be related to demographic variables, but we do not go further into these items here.

Using Roy's identity to equation (40), the within-period demand for non-durables is given by

$$C_t = a_t'(\underline{p}_t) + \frac{b_t'(\underline{p}_t)\left[Y_t - a_t(\underline{p}_t)\right]}{b_t(\underline{p}_t)}, \tag{61}$$

where $a_t'(\underline{p}_t)$ and $b_t'(\underline{p}_t)$ are the derivatives of $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$ with respect to the price of the good, p_t.

While this specification is linear in Y_t, estimation of the y-conditional demands does not require that. Notice also that despite the fact that the two-stage procedure implies that Y_t is predetermined, Blundell claims there is no reason to assume that Y_t is exogenous from an econometric point of view. Since we are assuming perfect certainty, the reason must be that there are variables that are known for the consumer, but unobserved for the econometrician. If these variables are autocorrelated, Y_t will be correlated with the error term of the y-conditional demands, and estimation requires an instrument variable method. Blundell suggests a method for estimation, and testing the exogeneity assumption, proposed by Hausman (1978).

An important advantage of the y-conditional demands is that these equations can be estimated without direct knowledge of the form of the equation determining the allocation of Y_t accross periods, since full income is observable.

[16]See Deaton and Muellbauer (1989).

This fact is related to the fact that in contrast to the Frisch demands, which include the partial derivative of the monotonic transformation of the within-period utilities, the specification of the y-conditional demands is independent of the choice of this transformation, that is, the I_t-function in equation (40). Hence, for general choices of the monotonic transformation, it is impossible to derive estimates of the parameters of the I_t-function from estimating y-conditional demands.

However, in a life cycle perspective it is important to study intertemporal allocations. Blundell then shows that if there are no binding constraints in the credit market, the parameters of the monotonic transformation of within-period utility can be estimated by utilizing the Euler equation (10), cf. also Hall (1978). To clarify this method, assume indirect within-period utility is given by equation (40). The marginal utility of wealth is now $\lambda_t^* = I_t'/b_t(\underline{p}_t)$, where I_t' denotes partial derivatives with respect to $[Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)$. Substituting this result into the Euler equation for the marginal utility of wealth, leads to

$$I_t' = (1+\rho)\frac{b_t(\underline{p}_t)}{b_{t-1}(\underline{p}_{t-1})}\frac{1}{1+R_t}I_{t-1}'. \tag{62}$$

Using the parameter estimates of b_t and b_{t-1} from the estimation of equation (61), and panel data on Y and $\underline{p}$, this equation can be used as a basis for estimating ρ and the parameters of the I_t-functions. In section 2.4.3 we show how this approach modifies in the uncertainty case.

Blundell uses this approach for a particular specification of the Gorman Polar form (40), that is

$$G_t^* = \frac{\tau_t(Y_t^*)^o}{o}, \tag{63}$$

where $Y_t^* = [Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)$ is "real supernumary" outlay, and τ_t and o are coefficients.

Equation (62) now becomes

$$\Delta \ln Y_t^* \equiv o^*\Delta \ln \tau_t - o^*\Delta \ln b_t(\underline{p}_t) + o^* \ln\left(\frac{1+R_t}{1+\rho}\right), \tag{64}$$

where $o^* \equiv 1/(1-o)$. Assuming τ_t is normalized such that $\tau_0 = 1$, and that the time preference rate ρ is known, we can estimate o and τ_t, for $t = 1, 2, \ldots, T$, if we have panel data for full income, the price vector $\underline{p}_t$ and R_t, for all the periods $t = 0, 1, 2, \ldots, T$.

While the fixed effect approach restricts the specification of the monotonic transformation of within-period utilities, cf. section 2.3.1.3, there is no such a priori restrictions in the present method. It therefore seems to allow for more flexible preferences, cf. Blundell (1987). However, estimation of a rich specification of the I-function tends to require panel data for many years, cf. specification (64), and if we do not have access to all these data, the theoretical advantage can be reduced in empirical applications.

2.4 Estimation in the Uncertainty Case

This section presents the main approaches for estimating life cycle models in the case that the household is uncertain about future prices and variables influencing future preferences. To get a basic understanding of the various estimation strategies, we first study the modifications of the optimization problem, and the interpretation of the first order conditions, compared to the perfect certainty case.

2.4.1 Consumption and Labour Supply Behaviour

We assume that the household is uncertain about future prices and variables influencing future preferences. In contrast, current exogenous (pre-tax) prices as well as variables influencing current preferences, are realized at the beginning of the period, and they are then known. At each age t the household[17] maximizes the expected value of the time-preference-discounted sum of total utility,

$$(65) \qquad U_t\left(C_t,\, K_t,\, H_t\right) + \frac{1}{1+\rho} E_t \left\{ \sum_{k=t+1}^{T} (1+\rho)^{t+1-k} U_k\left(C_k,\, K_k,\, H_k\right) \right\},$$

with respect to $C_{t'}$, $K_{t'}$ and $H_{t'}$, for $t' = t, t+1, \ldots, T$, subject to the within-period wealth constraints, the borrowing constraints and the labour supply constraints presented in section 2.2.1, and given values of A_{t-1}, $(1-\delta_f)K_{t-1}$ and A_T. Here p_t, q_t, w_t, r_t and all the parameters of the tax function S_t as well as the value of current taste modifier variables are known in period t, while $p_{t'}$, $q_{t'}$, $w_{t'}$, $r_{t'}$ and all the parameters of the tax function $S_{t'}$, for $t' > t$, as well as the value of future taste modifier variables, are not known. The expectation operator E_t indicates that the household accounts for all information available in period t, in the calculation of expected values. Since households are assumed to have rational[18] expectations, E_t denotes both the household's subjective expectation and the mathematical conditional expectation as of period t.

We also assume that the law of the motion for these exogenous state variables is independent of the household's decisions.

---

[17]For simplicity we assume a single person household.

[18]Begg (1982) defines rational expectations as "The hypothesis of Rational Expectations asserts that the unobservable subjective expectations of individuals are exactly the true mathematical conditional expectations implied by the model itself".

To study this optimization problem, we can use a dynamic programming formulation, cf. Rust (1992) and MaCurdy (1983,1985). Let

$$V_{t+1}[A_t,(1-\delta_f)K_t] = \max\left\{E_{t+1}\left[\sum_{k=t+1}^{T}(1+\rho)^{t+1-k}U_k\left(C_k,\,K_k,\,H_k\right)\right]\right\} \tag{66}$$

be the value function at age $t+1$, where the maximization is carried out with respect to $C_{t'}$, $K_{t'}$ and $H_{t'}$, for $t'=t+1,t+2,\ldots,T$, subject to the wealth, the labour supply and the borrowing constraints for all periods, and given values for A_t, $(1-\delta_f)K_t$ and A_T. This function shows the household's *maximum* expected lifetime utility at age $t+1$ when the wealth, the labour supply and the borrowing constraints for period $t+1,t+2,\ldots,T$ are satisfied, the household is endowed with initial wealth equal to A_t and $(1-\delta_f)K_t$, and the value of A_T is given. Since $V_{t+1}[A_t,(1-\delta_f)K_t]$ is a function of all the parameters generating future preferences and prices[19], and the household is uncertain about their realized values as of age t, $V_{t+1}[A_t,(1-\delta_f)K_t]$ is uncertain at age t.

The Lagrange function corresponding to the household's optimization problem (65) now becomes

$$\begin{aligned} L_t &= U_t(C_t,\,K_t,\,H_t)+\frac{1}{1+\rho}E_tV_{t+1}[A_t,(1-\delta_f)K_t] \\ &- \lambda_t\left[A_t-(1+r_t)A_{t-1}-w_tH_t+S_t(r_tA_{t-1},w_tH_t)+p_tC_t\right. \\ &+ \left.q_t[K_t-(1-\delta_f)K_{t-1}]\right]+\gamma_t(A_t+\kappa q_tK_t)+\alpha_tH_t. \end{aligned} \tag{67}$$

The first order conditions for the decision variables C_t, K_t, H_t and A_t include all the constraints and

$$\frac{\partial U_t}{\partial C_t}=\lambda_t p_t, \tag{68}$$

$$\frac{\partial U_t}{\partial K_t}=\lambda_t q_t-\frac{1}{1+\rho}E_t\left[\frac{\partial V_{t+1}[A_t,(1-\delta_f)K_t]}{\partial[(1-\delta_f)K_t]}(1-\delta_f)\right]-\gamma_t\kappa q_t, \tag{69}$$

$$-\frac{\partial U_t}{\partial H_t}=\lambda_t m_t+\alpha_t \tag{70}$$

and

$$\frac{1}{1+\rho}E_t\frac{\partial V_{t+1}[A_t,(1-\delta_f)K_t]}{\partial A_t}-\lambda_t+\gamma_t=0. \tag{71}$$

The Envelope Theorem implies

$$\frac{\partial V_{t+1}[A_t,(1-\delta_f)K_t]}{\partial[(1-\delta_f)K_t]}(1-\delta_f)=(1-\delta_f)\lambda_{t+1}q_{t+1}$$

[19]We suppress these arguments for simplicity.

and

$$\frac{\partial V_{t+1}[A_t,(1-\delta_f)K_t]}{\partial A_t} = (1+R_{t+1})\lambda_{t+1}.$$

Thus, the first order condition for durables, and the updating equation for the marginal utility of wealth, can be rewritten as

$$\text{(72)} \qquad \frac{\partial U_t}{\partial K_t} = \lambda_t q_t - \frac{1-\delta_f}{1+\rho} E_t[\lambda_{t+1} q_{t+1}] - \gamma_t \kappa q_t$$

and

$$\text{(73)} \qquad \lambda_t = \frac{1}{1+\rho} E_t[(1+R_{t+1})\lambda_{t+1}] + \gamma_t.$$

The first order conditions are quite similar to the conditions in the perfect certainty case, cf. section 2.2. The main difference is that in an environment of uncertainty, the marginal utility of wealth, λ_{t+1}, and the after-tax interest rate, R_{t+1}, become stochastic. In the case of no current binding constraint in the credit market, the Euler equation (73) says that saving should be adjusted until the marginal utility of wealth in the current period, equals *expected* marginal utility of using the same resources next year added after-tax interest incomes. If the household is constrained in the credit market, the marginal utility is greater than the expected marginal utility.

If there are no binding constraints in any market, demand is determined by substituting the value of λ_t into the demand functions. If the good is a durable good, the value of $E_t[\lambda_{t+1} q_{t+1}]$ must also be substituted into the demand function. To complete this view of behaviour, we also assume that the household at the beginning of the planning period calculates λ_t according to the Euler equation (73), and $E_t[\lambda_{t+1} q_{t+1}]$, and that it uses all available information about future prices and variables influencing future preferences in this calculation. As time passes by, the household continuously acquires new, unanticipated information about these variables. When the forecasting errors are realized, the household (continuously) revises the value of $E_t[(1+R_{t+1})\lambda_{t+1}]$ and $E_t[\lambda_{t+1} q_{t+1}]$, and simultaneously adjusts demand according to the unanticipated elements. The effects of unanticipated price changes are then taken account of only through the changes in $\lambda_t = \frac{1}{1+\rho} E_t[(1+R_{t+1})\lambda_{t+1}]$ and $E_t[\lambda_{t+1} q_{t+1}]$, while the responses to anticipated price changes can be measured by the coefficients on the (current) price variables, cf. equation (80). The importance of distinguishing between these two aspects has been recognized since the works of Heckman (1974,1976), and Ghez and Becker (1975).

To improve the knowledge about the stochastic process generating the marginal utility of wealth, notice that we can always express the actual value of $\ln \lambda_t$ as the sum of its expected value viewed from period $t-1$, and a one period forecast error ϵ_t that represents unanticipated realizations of the stochastic variables in period t. It can be shown, cf. MaCurdy (1985), that this decomposition implies

$$\ln \lambda_t = \rho_t^* + \ln \lambda_{t-1} + \epsilon_t, \tag{74}$$

where

$$\rho_t^* \equiv \ln \left[\frac{1+\rho}{1+R_t}\right] - \ln \left[E_{t-1}(e^{\epsilon_t})\right]. \tag{75}$$

Notice that ρ_t^* is known at time t (but not at time $t-1$) since R_t is known at time t.

Repeated substitution gives

$$\ln \lambda_t = \sum_{j=0}^{t} \rho_j^* + \ln \lambda_0 + \sum_{j=1}^{t} \epsilon_j, \tag{76}$$

and in order to adjust for the mean value of $\ln(1+R_t)$ across periods, we define $\rho_t^{**} = \rho_t^* - \overline{\rho^*}$, where $\overline{\rho^*} = (1/t)\sum_{j=0}^{t} \rho_j^*$. We then find

$$\ln \lambda_t = \ln \lambda_0 + \overline{\rho^*} t + \sum_{j=0}^{t} \rho_j^{**} + \sum_{j=1}^{t} \epsilon_j, \tag{77}$$

which indicates that in an environment where the household continuously revises its predictions of the marginal utility of wealth to account for new information, $\ln \lambda_t$ follows a stochastic process resembling a random walk with drift. Substituting this result into the first order conditions, implies that consumption and labour supply follow a nonstationary stochastic process over life cycle.

We end this section by noticing that the introduction of uncertainty seems to complicate the estimation of preferences for durables. In the perfect certainty case with no binding constraints in the credit market, we can use the Euler equation to express the first order condition for durables in terms of λ_t only, and not both λ_t and λ_{t+1}, cf. equation (24). In the uncertainty case this is more problematic, since the Euler equation includes $E_t[(1+R_{t+1})\lambda_{t+1}]$, while the first order condition for durables includes $E_t[\lambda_{t+1} q_{t+1}]$, cf. equations (73) and (72). Since the expectation of a product in general cannot be rewritten as the product of the expectation of the two components, we must assume that both the after-tax interest rate, and the purchase price of durables, are non-stochastic if we want to follow the strategy from the perfect certainty case.

2.4.2 MaCurdy's Fixed Effect Approach with Uncertainty

We are now going to study how MaCurdy (1985)[20] suggests to utilize the Frisch demand functions for estimation of a life cycle model of labour supply in the case of uncertainty. This paper generalizes MaCurdy (1981) by allowing for uncertainty, and since the perfect certainty case is treated thoroughly in section 2.3.1, we focus only on the estimation problems particular to the uncertainty case.

Assume preferences can be described by

$$\sum_{t=0}^{T}(1+\rho)^{-t}\left(\Gamma_{cit}C_{it}^{\omega_c}-\Gamma_{hit}H_{it}^{\omega_h}\right), \tag{78}$$

where ω_c and ω_h are unknown parameters. In contrast to specification (29) the taste modifier for labour is now related to a vector N_{it} of measured household characteristics, and some unmeasured (random) characteristics ϵ_{1it} according to

$$\Gamma_{hit}=\exp\left(-N_{it}\psi-\epsilon_{1it}\right), \tag{79}$$

where ψ is a vector of parameters. MaCurdy disregards income taxation and the possibility of binding constraints in the credit and labour markets, and the Frisch labour supply function now becomes

$$\ln H_{it}=F_{it}+a\ln w_{it}+N_{it}\psi^*+\epsilon_{1it}^*, \tag{80}$$

where $a\equiv 1/(\omega_h-1)$, $F_{it}\equiv a[\ln\lambda_{it}-\ln\omega_l]$, $\psi^*\equiv a\psi$ and $\epsilon_{1it}^*\equiv a\epsilon_{1it}$. We recall that w and A (equation (82)) are measured in real terms.

As argued in section 2.2.3 λ_{i0} is latent and correlated with all the exogenous variables in the model. Using the definition of F_{it} and the Euler equation for the marginal utility of wealth, this implies that F_{it} is latent and correlated with all the exogenous variables in the model. In (1981) MaCurdy solved this estimation problem by expressing the Frisch demands in terms of F_{i0} before he applied first differencing to eliminate F_{i0}. Using equation (76), the labour supply equation (80) can be expressed in terms of F_{i0},

$$\ln H_{it}=F_{i0}+a\sum_{j=0}^{t}\rho_{ij}^*+a\ln w_{it}+N_{it}\psi^*+\eta_{it}, \tag{81}$$

where $\eta_{it}\equiv\epsilon_{1it}^*+a\sum_{j=0}^{t}\epsilon_{ij}$ is the disturbance term. The disturbance term represents the effects from omitted variables and the accumulation of all past forecast errors.

[20]He did not estimate the model in this paper.

Differencing this equation eliminates F_{i0}, but we are left with the unobservable variable ρ^*_{it}, that is both household- and time-specific. Since ρ^*_{it} includes the after-tax interest rate, it is correlated with all the exogenous variables in this equation, and the error term η_{it}. If the distribution generating the forecast errors is such that $E_{t-1}(e^{\epsilon_{it}})$ varies with changes in household characteristics, ρ^*_{it} is also correlated with N_{it} and η_{it} even in the absence of income taxation. This means that without further assumptions, the first differencing approach cannot be used for estimation of this equation.

If we can assume that ρ^*_{ij} is constant across both households and periods, the method of first differencing can be used. The introduction of uncertainty gives an additional argument for treating the wage rate (and the wage rate growth) as well as R_t as endogenous in the estimation. Since the wage rate and the after-tax interest rate include unanticipated components, $\ln w_{it}$ and R_t are correlated with current and past values of ϵ_{it}. A similar argument can be applied on N_{it}, that is, for N_{it} being exogenous in the empirical analysis, it must both be known prior to period one and uncorrelated with the unobserved taste factors ϵ^*_{1it}. This fact is also of significance for the choice of instrument variables, in that, for a variable to be a valid instrument, it must be uncorrelated with all unanticipated components.

We then conclude that if we can assume that ρ^*_{ij} is constant across both consumers and time, the parameters determining the responses to anticipated price changes can be estimated from panel data for at least two periods for the actual good in question. In most cases, however, this assumption seems quite unreasonable.

Regarding the estimation of the reduced form equation for the marginal utility of wealth, MaCurdy suggests to approximate the relationship corresponding to equation (31), by

$$F_{it} = Z_{it}\phi_t + \sum_{j=t}^{T} \beta_{tj} E_t \ln w_{ij} + A_{it}\theta_t + \zeta^*_{it}, \tag{82}$$

where subscript t on all the parameters and the variables indicates that they are observed or anticipated at age t. MaCurdy interprets A_{it} as the stock of assets at the beginning of period t, and it is then exogenous for the household as of period t.

Since the household is assumed to continuously replan as it acquires new information, there is one (different) equation for each year. Subscript t is introduced on all parameters to take account of that the parameter values may change systematically with age. The reason is that, as the household updates expectations about the distribution of future preferences and prices, this may well change the mathematical form of the reduced form expression for the marginal utility of wealth. Hence, the (mathematical) form of this equation may change across periods, but in order to simplify, MaCurdy assumes that only the parameter values are changed.

Apart from the huge problems related to obtaining information about the households' expectations about future wages and other exogenous variables, the fact that F_{it} varies across both households and periods, makes it hard to think of how we can estimate these equations without further assumptions. Recall also that even for the most simple models, it is impossible to find the reduced form expressions for λ_{it}. We then conclude that MaCurdy's fixed effect approach cannot be used for estimation of life cycle models under uncertainty.

2.4.3 Marginal Rate of Substitution Functions

This section focuses on how the marginal rate of substitution functions[21] can be utilized for estimation of within- period preferences, cf. Altonji (1986) and MaCurdy (1983). We also show how this approach can be combined with a particular use of the Euler equation to estimate the parameters of the monotonic transformation of within-period preferences, cf. also the perfect certainty case in section 2.3.4.

When we wish to estimate the first order equations (68), (70) and (72), it is a problem that λ_t is latent[22]. These equations can, however, be viewed as (three) separate expressions that can be used for elimination of λ_t. Using the first order conditions for consumption and labour supply, we find

$$\frac{-\alpha_t - \partial U_t/\partial H_t}{\partial U_t/\partial C_t} = \frac{m_t}{p_t}. \tag{83}$$

Here $-(\partial U_t/\partial H_t)/(\partial U_t/\partial C_t)$ is the marginal rate of substitution between leisure and non-durables, and if the consumer is not constrained in the labour market, the equation implies that, in equilibrium the person sets the marginal rate of substitution between leisure and non-durables equal to the real marginal wage rate.

Choosing a particular specification of the utility function, equation (83) does only include variables that can be observed, the unknown parameters of the within-period preferences, and the Lagrange multipliers of binding constraints. Using a subsample of unconstrained households, the marginal rate of

[21] According to Rust (1992) this method can be thought of as a special case of the method of moments, cf. the next section, but with a different orthogonality condition.

[22] In the estimation of equation (72) it is also a problem that $E_t[\lambda_{t+1}q_{t+1}]$ is latent.

substitution functions can then be used for estimation of the parameters of within-period preferences[23].

If preferences are additively separable within periods (in addition to intertemporal additively separable) and the transformation function Φ_t is the identity transformation, estimation requires cross section observations on prices and consumption of at least two goods, and the taste shifter variables. We also need a set of instrument variables since the marginal rate of substitution functions generally are not reduced form equations. If within-period preferences are non-separable, we must observe the consumption of all goods, but we can still use cross section data.

Equation (83) cannot be used for estimation of the monotonic transformation of within-period preferences. Assume that the within-period utility can be written as

$$U_t(C_t K_t, H_t) = \Phi_t\left[U_t^*(C_t, K_t, H_t)\right], \tag{84}$$

where Φ_t is a monotonically increasing function and U_t^* is a function that is additively separable in C_t, K_t and H_t. The equilibrium condition corresponding to equation (83) is now identical with equation (83), and it is impossible to identify the parameters of the Φ-transformation.

Since the Euler equation determines the household's relative preferences for consumption and leisure in the various periods, it seems natural to use this equation for estimation of the Φ-transformation. Notice then that given equation (73) we can also write

$$\lambda_{t+1} = \frac{1+\rho}{1+R_{t+1}}\left[\lambda_t(1+e_{t+1}) - \gamma_t\right], \tag{85}$$

where e_{t+1} is the one period forecast error with respect to $\left[\frac{(1+R_{t+1})\lambda_{t+1}}{(1+\rho)\lambda_t} + \frac{\gamma_t}{\lambda_t} - 1\right]$, cf. appendix B.

Substituting for λ_t and λ_{t+1} from the first order condition for consumption, we obtain

$$\frac{\Phi'_{t+1}\partial U^*_{t+1}/\partial C_{t+1}}{p_{t+1}} = \frac{1+\rho}{1+R_{t+1}}\left[\frac{\Phi'_t\partial U^*_t/\partial C_t}{p_t}(1+e_{t+1}) - \gamma_t\right], \tag{86}$$

where Φ'_t is the partial derivative of Φ_t with respect to U_t^*.

If the household has rational expectations, e_{t+1} is uncorrelated with

$$\frac{1+\rho}{1+R_{t+1}}\left[\frac{\Phi'_t}{p_t}\frac{\partial U_t^*}{\partial C_t}\right],$$

[23] Since the various parameters of the marginal rate of substitution equations may be functions of two or more parameters from the utility function, we may, however, be confronted with an identification problem.

and specification (86) can be used for estimation of the parameters of the Φ-function and the time preference rate ρ from a sample of households that are unconstrained in the credit market, cf. the discussion of equation (62). Notice that even though within-period preferences are additive, this specification requires panel data (for at least two periods) for *all* goods influencing preferences. The reason is that even though $\partial U_t^*/\partial C_t$ does only depend on C_t, the partial derivative of Φ depends on all goods. Under particular circumstances, among them that the parameters of the Φ-function is independent of household, estimation may also be possible with time series data.

If Φ_t includes a stochastic component, the right hand side variables of equation (86) may be correlated with the error term. Assume

$$\Phi_t\left[U_t^*(C_t, K_t, H_t)\right] = \Omega_t \frac{[U_t^*(C_t, K_t, H_t) + \delta_1]^{\delta_2} - 1}{\delta_2}, \tag{87}$$

where δ_1 and δ_2 are unknown parameters, and Ω_t is related to household characteristics, N_t, and a stochastic component, φ_t, according to $\Omega_t = \exp\{N_t\phi_n + \varphi_t\}$.

The equation corresponding to equation (86) in the case of no binding constraints in the credit market, now becomes

$$\begin{aligned} \ln \frac{\partial U_{t+1}^*}{\partial C_{t+1}} - \ln \frac{\partial U_t^*}{\partial C_t} &= \ln(1+\rho) + \ln \frac{p_{t+1}}{p_t} + (1-\delta_2) \ln \frac{U_{t+1}^* + \delta_1}{U_t^* + \delta_1} \\ &- (N_{t+1} - N_t)\phi_n + [\varphi_t - \varphi_{t+1} + \ln(1 + e_{t+1})]. \end{aligned} \tag{88}$$

Since the error term $[\varphi_t - \varphi_{t+1} + \ln(1 + e_{t+1})]$ includes φ_t, the right hand side variables can be correlated with the error term, and they must be treated as endogenous in the empirical analysis. If, however, Φ_t is deterministic, the error term only includes the prediction error, and the right hand side variables can be considered as predetermined in the estimation.

According to MaCurdy, this estimation method is sensitive to measurement errors. If the error term of equation (83) includes measurement errors, the predictions of $\ln(\partial U_t^*/\partial C_t)$, $\ln(\partial U_{t+1}^*/\partial C_{t+1})$, and $\ln \frac{U_{t+1}^*+\delta_1}{U_t^*+\delta_1}$ for known δ_1, are biased. Since the prediction errors do not satisfy $X = X^* + \vartheta_x$, where X^* is the true prediction and ϑ_x is an error term that is randomly distributed and independent of all instrument variables, estimation of equation (88) then yields inconsistent parameter estimates.

The approach discussed above allows for a rich class of utility functions, since we do not need to find the closed form solution for the demand functions. We do not either need making any assumptions regarding the households' anticipations about future prices including the interest and the tax rates, and the variables influencing future preferences. As it is presented here, it is also assumed that we can ignore the possible selection problem from using a subsample of unconstrained households in the estimation. In empirical applications of this approach it may be a problem that it is difficult to find good instruments for the right-hand-side-variables of the marginal rate of substitution functions. Estimation of the transformation function Φ is also data demanding.

2.4.4 The Generalized Method of Moments

In this section we review the generalized method of moments (GMM). This approach can be associated with Hansen (1982) and Hansen and Singleton (1982). A recursive application of this approach has been used in Hotz, Kydland and Sedlacek (1988), that allows a particular kind of intertemporal non-separability in an analysis of labour supply. The estimation method in MaCurdy (1983) can also be viewed as a variant of this approach, and the approach is also described in Rust (1992) and in Hall (1993).

The basic idea underlying the generalized method of moments, is that economic models typically yield a set of stochastic Euler equations that characterize optimum. These equations imply a set of population orthogonality conditions that depend on the parameters of the objective/utility function and some observable variables. By setting the sample analogies equal zero (according to a certain metric), we can estimate the parameters of the utility function (and subsets of the parameters determining the law of motion of the state variables).

In order to clarify this approach, notice that in the case of no binding restrictions in any market, the Euler equation (73) and the Frisch labour supply function (70) imply

$$E_t\left[\frac{1}{m_t}\frac{\partial U_t}{\partial H_t} - \frac{1+R_{t+1}}{1+\rho}\frac{1}{m_{t+1}}\frac{\partial U_{t+1}}{\partial H_{t+1}}\right] = 0. \tag{89}$$

That is, in optimum the conditional expectation of the change in the utility (evaluated at the true parameter values) from reallocating labour supply corresponding to one pound labour income from period $t+1$ to period t, is equal to zero.

Depending on what variables are observed and for what periods, various combinations of the first order conditions can be substituted into the Euler equation. The resulting orthogonality conditions do only include variables that can be observed, and the parameters of the utility function. To simplify notation, write the set of orthogonality conditions that are to be estimated as

$$E_t g(x_t, x_{t+1}, \theta^*) = 0, \tag{90}$$

where x_t and x_{t+1} include the household's decision variables, the exogenous state variables such as pre-tax wage and interest rates and exogenous variables influencing preferences, and the endogenous state variables such as the marginal tax rate on wage and interest incomes, in period t and $t+1$ respectively. According to our assumptions all variables subscripted t are known in period t, while variables with subscript $t+1$ are random variables with respect to the household's information set in period t. The vector θ^* represents the true parameters of the utility function.

Notice that $g(x_t, x_{t+1}, \theta^*)$ can be interpreted as the disturbance vector, u_t, in the estimation, and that the orthogonality conditions must be chosen such that the matrix $u_t u_t'$ has full rank.

Using the law of iterated expectations, equation (90) implies

$$(91) \qquad Eg(x_t, x_{t+1}, \theta^*) = E\left[E_t g(x_t, x_{t+1}, \theta^*)\right] = 0,$$

that is, the unconditional expectations also equal zero.

The next step of the analysis rests heavily on the assumption that $\{x_t\}$ is (strictly) stationary, and before we continue the analysis, we define this term. According to Ogaki (1993), a time series $\{x_t\}$ is stationary if the joint distribution of $\{x_t, \ldots, x_{t+k}\}$ is identical to those of $\{x_{t+s}, \ldots, x_{t+s+k}\}$ for any t, k and s. This requirement is quite restrictive, and it implies that when they exist, the unconditional moments Ex_t and $E(x_t x_{t+k}')$ cannot depend on t for any k. Hence, deterministic trends, autoregressive unit roots, and unconditional heteroskedasticity are ruled out (but we may have conditional heteroskedasticity). Since macro-economic variables often exhibits nonstationarity, it also means that the econometrician must be particularly careful if he is using macro data. (Ogaki presents some examples of how aggregated variables can be transformed in order to remove nonstationarity.)

Returning to the original analysis, we notice that if the unknown parameters are independent of time, and if $\{x_t\}$ is a strictly stationary and ergodic[24] process, the time averages of functions of x_t converge to their unconditional expectations with probability one,

$$(92) \qquad \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} g(x_t, x_{t+1}, \theta^*) = E\left[g(x_t, x_{t+1}, \theta^*)\right] = 0,$$

cf. Dudley (1989).

By analogy we estimate θ^* by that value of $\hat{\theta}$ that makes the sample average $\frac{1}{T}\sum_{t=1}^{T} g(x_t, x_{t+1}, \hat{\theta})$ close to zero.

[24] See Cox and Miller (1988) page 92-93 for a definition of an ergodic process.

If the number of unknown parameters is greater than the number of orthogonality conditions, we cannot identify the parameters. Hansen and Singleton then propose to use a vector of instrument variables, v_t, that can be any function of the household's information at age t, provided it is observed by the econometrician. Apart from this, the only requirement is that v_t is predetermined as of time t, and current and lagged values of x_t can then be used. In this case the Kronecker product $E_t g(x_t, x_{t+1}, \theta^*) \otimes v_t \equiv g^*(x_t, x_{t+1}, v_t, \theta^*)$ equals zero, since v_t can be treated as a constant in the calculation of the expectations. By analogous reasoning that lead to equation (92), we find that

$$G_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} g^*(x_t, x_{t+1}, v_t, \theta), \tag{93}$$

evaluated at the true value of $\theta = \theta^*$, should approximate zero for large values of T. Assuming $\{x_t, v_t\}$ is a strictly stationary, ergodic process, Hansen and Singleton define the GMM estimator $\hat{\theta}$ as that value of θ that minimizes

$$\Upsilon_T(\theta) = G_T(\theta)' W_T G_T(\theta), \tag{94}$$

where W_T is a symmetric, positive definite weighting matrix. This estimator is consistent and asymptotic normally distributed under mild regularity assumptions, cf. Hansen (1982). According to Hansen and Singleton, it is also consistent even in the case that the error terms of the orthogonality conditions (90) are serially correlated and the instruments are not exogenous, but only predetermined.

Hansen and Singleton explain how to choose W_T optimally, in order to minimize the asymptotic covariance matrix of the estimator. The estimation method then becomes a two-stage procedure: In the *first* stage, estimate θ by equation (94) using an arbitrary weighting matrix. Calculate the optimal weighting matrix W_T^* according to

$$W_T^* = \left[\hat{R}_0 + \sum_{j=1}^{m}\left(1 - \frac{j}{m+1}\right)\left(\hat{R}_j + \hat{R}_j'\right)\right]^{-1}, \tag{95}$$

where

$$\hat{R}_j = \frac{1}{T}\sum_{t=j+1}^{T} g^*(x_t, x_{t+1}, v_t, \hat{\theta}) g^*(x_{t-j}, x_{t+1-j}, v_{t-j}, \hat{\theta})', \tag{96}$$

and where m equals the number of non-zero autocorrelations in $g^*(x_t, x_{t+1}, v_t, \hat{\theta})$. Then, in the *second* stage, estimate $\hat{\theta}$ by equation (94) using the optimal weighting matrix.

Hansen (1982) shows that T times the minimized value of equation (94) evaluated at the optimal weighting matrix,

$$TG_T(\hat{\theta})'W_T^*G_T(\hat{\theta}), \tag{97}$$

is Chi-square distributed with degrees of freedom equal the number of orthogonality conditions, O, minus the number of parameters, P. This result can be used for testing model specifications: In the estimation, P of the orthogonality conditions are set close to zero in order to estimate the P unknown parameters. If the number of orthogonality conditions exceeds the number of unknown parameters, $O - P$ of the orthogonality conditions are not used in the estimation, and the model is said to be overidentified. However, if the model specifications are true, these orthogonality conditions should also be close to zero. This means that if the test statistics (97) is "large", there is reason to believe that there is something wrong about our model specifications including the choice of the instrument variables. Thus, the general method of moments also provides a test for the validity of the model specifications.

2.4.4.1 Advantages and limitations of Euler equation methods

The main advantages of this approach are:

- Only minimal assumptions about the distribution of the unobservable exogenous state variables are required.
- Conditioned that the data satisfy the requirements of the method, GMM can be applied on time series, and cross section or panel data sets where the orthogonality conditions are based on averages for the consumers. In this latter case consistency of the estimated parameters requires that the state and control variables of the Euler equation are uncorrelated across consumers. GMM is then quite flexible with regard to data set specifications.
- The instrument variables need only be predetermined and not exogenous, and the approach gives considerable latitude in selecting instruments.
- We do not need to explicitly solve for the value function or the optimal decision rule. Instead we can use the Euler equation and the first order conditions.
- Standard gradient algorithms can be used for estimation.
- The approach yields a useful diagnostic test statistic as a by-product.

According to Rust (1992), GMM is not applicable for the following model specifications:

- Specifications with binding constraints in the goods and/or the credit markets, cf. equations (5) and (6), and the possibility of institutional constraints in the labour market. In this case the orthogonality conditions will be inequalities and not close to zero.

- Most specifications where the unobserved state variables enter the Euler equation directly, that is, not implicitly as functions of other observed state and control variables. For instance, unobserved components of the exogenous variables cannot influence preferences directly. These variables are typically correlated with the forecast errors that arise from unanticipated realizations of prices and variables influencing preferences, and they are then correlated with the price variables and the taste modifier variables in the Euler equation. Hence they cannot be treated as a part of the error term.

- Specifications where the Euler equation contains state or control variables that reflect macroeconomic shocks that are correlated across consumers. In this case the data might not satisfy the ergodicity conditions, and the result (92) may not be valid.

- Specifications where the transition probabilities for the endogenous and exogenous state variables depend on the household's decisions. In this case the first order conditions include the value function V, cf. equation (66), and it seems to be no way to substitute it out of the problem. To find the orthogonality conditions corresponding to equation (90), we then must make explicit parametric specifications of the law of motion for all the state variables.

Regarding the point of binding constraints, in some cases we can also think of specifications where the econometrician makes use of that he knows what households will be unconstrained in the future. However, as Pakes (1991) points out, "If we simply select out those observations for which it ends up being true, we will be selecting the sample on the basis of behaviour determined by information not available at date t, and any selection procedure based on such information will generate an inconsistency in the estimation procedure.".

Rust also comments on a problem with the use of the test statistic (97), that is,

- The unconditional expectation of the Euler equation can be close to zero even though the conditional expectation may be non-zero for some histories.

The point here is that while the conditional expectation of the Euler equation is zero for every history given that our model specifications are true, the criterion function (93) attempts to set the average of expectations, that is, the unconditional expectation, to zero. This means that if our model is misspecified such that the conditional expectations are non-zero, but on average equal zero, the test statistic (97) fails to reject the model specifications. See also West (1986) for an interesting illustration of this problem.

2.5 Summary

From this chapter it should be apparent that a great deal of effort has been spent trying to estimate structural life cycle models on micro data. Despite this effort most empirical analyses make use of strong simplifications. Most works are incomplete in the sense that they do not account for the simultaneity in the determination of labour supply and consumption, and in particular the consumption of durables. In addition most works disregard habits.

While these simplifications concern the specification of preferences, the specification of the wealth constraint is also simplified. Many analyses treat income taxes as well as the existence of bequests and inheritances superficially. Most analyses also disregard the many possibilities of constraints in the various markets, and even if they are taken into consideration, the model is usually estimated from a subsample of unconstrained households without trying to correct for the possible estimation bias.

There are many reasons for these simplifications, the most important being that, ideally, estimation of the full life cycle model requires complete life cycle data for variables such as female and male labour supply, consumption of non-durables and durables, the stock of assets and all the matching prices; including the interest and income tax rates. Since households adjust according to anticipated prices, and the expectations are continuously being updated as the household acquires new information, ideally, these expectations should be observed in each period. To allow for heterogeneous households, we also need observations of taste shifter variables, and the estimation of the wage equation requires variables such as female and male education and work experience. Depending on the econometric specification, we may also need further instrument variables, and the estimation of a complete life cycle model is then very data demanding.

Estimation of the preferences for durables involves a particular problem. Economic theory is weak with respect to how to measure the stock or consumption of durables, and without a precise definition, the price is also undefined. This means that the standard first order conditions are not particularly suitable for estimation of the preferences for durables.

Estimation is considerably simplified if one is willing to assume intertemporal additive separability. Within-period preferences can then be estimated using only variables currently observed, cf. the equations based on the marginal rate of substitution functions or the two-stage budgeting theory.

If we can also assume intraperiod additive separability and that the monotonic transformation Φ_t is the identity transformation, it is possible to estimate the parameters determining the responses to current price changes along a given life cycle price path, using panel data for only the actual good in question, cf. MaCurdy's fixed effect approach. Since many cross section data do not include observations of both labour supply and consumption, this property is of importance.

In order to analyse the effects of shifts in the price paths, MaCurdy suggests to estimate the reduced form equation for the marginal utility of wealth. This estimation typically requires approximations to the true relationship, and a priori assumptions about lifetime price paths. A considerable weakness of this approach is that it cannot be used if the consumers are uncertain about future prices and variables influencing future preferences. It also restricts preferences more than the approaches based on the marginal rate of substitution functions and the two-stage budgeting theory.

While the equations based on the marginal rate of substitution functions and the y-conditional demands can be used for estimation of within-period preferences, they cannot be used for estimation of the monotonic transformation of within-period preferences. It is then suggested to combine these approaches with a particular use of the Euler equation. In the two-stage budgeting theory case this method requires panel data for full income and all prices, in addition to observations of the after-tax interest rate for at least the current period. On the contrary, the estimation based on the substitution functions requires panel data for all goods and one price in addition to observations of the after-tax interest rate, and this result is independent of the separability property of within-period utility. Since most data sets do no include all these variables, and since the number of years covered by panel data is typically quite short, most analyses do not estimate this transformation.

We end this chapter noticing that common to all the surveyed estimation strategies is that they use the first order conditions as basis for deriving the specifications used for estimation. Compared to the real world, the first order conditions are restrictive in the sense that the household is to compare marginal utility with the marginal cost of buying the good. Concerning labour market decisions for instance, it might be the case that the job searchers only can choose between a limited number of packages that consist of wage rate, hours of work, fringe benefits and other working conditions, cf. Dagsvik and Strøm

(1992). The job searchers must then make utility comparisons between these packages, and the first order conditions do not hold. This argument is also relevant for some durables, and it implies that the predictions based on estimations of the first order conditions can be biased no matter how sophisticated the estimation procedure really is. Considering also all the simplifications pointed out at the beginning of this section, we conclude that we cannot be satisfied with the current stage of development and knowledge of the empirical life cycle model of labour supply and consumption.

3. An Empirical Life Cycle Model of Savings, Labour Supply and Consumption without Intertemporal Separability

3.1 Introduction

This chapter formulates and estimates an empirical life cycle model for two-person households that choose labour supply and consumption including consumption of durables in an environment of uncertainty about future prices. Ideally, estimation of this model requires complete life cycle data for all these goods and households' expectations about the corresponding prices, including the interest and the income tax rates. Since there exist no such data sets, and economic theory is rather weak on how to measure the demand for durables, empirical work treats at least some of the goods in a rather summary way. We will also argue that measurement error in the observations of the consumption of non-durables may be important. The point of departure of this work has then been to modify the theory so that a complete life cycle model of labour supply and consumption of durables and non-durables can be estimated in the absence of reliable data for consumption of non-durables, and the purchase price and the physical stock of durables. In contrast to the standard, and presumably unrealistic, assumption of intertemporal separability, we allow for a particular kind of non-separability in the demand for durables.

The chapter is organized as follows. Section 3.2 including its subsections presents the standard life cycle model of consumption of durables and non-durables, and discusses some of the problems related to estimation of preferences over these goods. We discuss the serious measurement problems, and raise the important point that the first order condition for durables does not reflect the existence of fixed costs in the demand for durables. The standard assumption of intertemporal separability is also discussed.

In Section 3.3 we present the general framework of the complete model to be estimated. We introduce a particular form of habit persistence in the demand for durables. The specification is analogous to the one that is found in Bover (1991), Hotz, Kydland and Sedlacek (1988), Johnson and Pencavel (1984) and

Muellbauer (1986). However, in contrast to these works, which analyse labour supply and consumption of non-durables, we relate this specification to the stock of durables. We argue that our specification is consistent with an interpretation that says that the household gains utility from an increase in the stock of durables relative to what "remains" from the last period. In the determination of what "remains" from the last period, we allow for physical as well as psychological depreciation. Psychological depreciation is introduced to take into account that after some time the household may tire of the present stock of durables. This means the physically existing capital stock does not yield as much utility as before, but since it can be resold in the market, it must be treated differently from the stock that is physically depreciated.

If the stock of durables is totally depreciated (in the utility context), our specification of the utility of durables implies that utility depends on the stock of durables. Hence, the standard specification of the utility of durables is a special case of our specification.

It appears that an advantage of our interpretation is that it leads us towards a specification that can be estimated in the absence of reliable observations of the consumption of non-durables, and the purchase price and the physical stock of durables. This advantage is demonstrated, before we end Section 3.3 by commenting on the estimation strategy. Section 3.4 presents a particular empirical specification which is estimated, and in Section 3.5 we look at the details of the estimation procedure. Section 3.6 gives an account of the data, and Section 3.7 presents the estimation results. Finally, some concluding remarks are made in Section 3.8.

3.2 Confronting the Standard Life Cycle Theory with Data

3.2.1 The standard framework

We now look at some problems related to the estimation of preferences over durables and non-durables. In order to focus on the main problems, assume that lifetime preferences are an inter- and intratemporal separable function of durables and ordinary consumer goods only. Thus

$$U_0 = \sum_{t=0}^{T}(1+\rho)^{-t}\left[U_{ct}(C_t) + U_{kt}(K_t)\right], \tag{98}$$

where ρ is the time preference rate, K denotes stock of durables, C is consumption of non-durables, and subscript 0 and T denote the beginning and the end of the planning horizon. We also assume perfect certainty about future prices and preferences, that labour supply, H, is exogenous, and that the subutility functions U_{ct} and U_{kt} are strictly concave and twice differentiable.

In the absence of personal income taxation, the within period wealth constraints are

(99) $w_t H_t + (1+r_t)A_{t-1} = p_t C_t + q_t\left[K_t - (1-\delta_f)K_{t-1}\right] + A_t, \quad t = 0, 1, \cdots, T,$

where A is assets measured in nominal terms, r is the nominal interest rate, w, p and q are the nominal prices of leisure, non-durables and durables, and δ_f is the physical depreciation rate for durables. If $\delta_f < 1$ (and ≥ 0), the specification of the wealth constraint implies that durables can be sold in a second hand market.

Despite the fact that households may face different[25] prices, we assume that all (pre-tax) prices are equal for all households. Except for the fact that bequests at the beginning of the planning period can be exogenously included into the value of A_0, we do not take explicit account of the possibility of bequests. Neither do we consider that families can place their wealth into other kinds of assets, such as bonds, shares, pension funds, arts or antiques.

We assume that durables are bought at the end of the period. Since this stock is continuously being depreciated during the subsequent period, $q_t(1-\delta_f)K_{t-1}$ is the market value of the stock of durables that was demanded in the last period and that is left at the end of period t.

In what follows we assume that there are no binding credit market restrictions. Maximization of the preference function (98) subject to the wealth constraint (99), given values of initial stocks of assets and durables, and the terminal stock of assets, yields the following first-order conditions,

(100) $$\frac{\partial U_{ct}}{\partial C_t} = \lambda_t p_t, \quad t = 0, 1, \cdots, T,$$

(101) $$\frac{\partial U_{kt}}{\partial K_t} = \lambda_t q_t - \frac{1}{1+\rho}\lambda_{t+1}(1-\delta_f)q_{t+1}, \quad t = 0, 1, \cdots, T,$$

and

(102) $$\lambda_t = \frac{1}{1+\rho}(1+r_{t+1})\lambda_{t+1}, \quad t = 0, 1, \cdots, T-1,$$

where λ_t is the Lagrange multiplier, i.e. the marginal utility of wealth.

3.2.2 Some data and measurement problems

Equations (100) to (102) and the wealth constraints (99) constitute a simultaneous equation system that determines C_t, K_t, A_t and λ_t, for $t = 0, 1, \cdots, T$, as functions of all the exogenous variables of the model,

[25] The households may face different prices, for example, because they live in different regions of the country or because banks offer different terms to different customers.

including the parameters of the lifetime utility index. In most cases it is, however, impossible to find these reduced form equations. If λ can be observed, however, it appears that the relevant demand functions in the life cycle context are the Frisch demands, cf. MaCurdy (1981).

Concerning the estimation of these functions, it is a problem that λ_t is latent and depends on all the exogenous variables of model. In chapter two we discussed various solutions to this problem. These solutions typically involve a two stage estimation procedure, where the first stage involves estimation of the parameters related to intraperiod allocations, while the second stage involves estimation of the parameters related to interperiod allocations. However, even if one estimates only within period preferences (stage one), which typically requires less data, it appears that the data requirement is quite large. For instance, in the estimation of the differenced marginal utility of wealth constant demand functions, MaCurdy's fixed effect approach, cf. MaCurdy (1981, 1985), demands panel data for consumption and (after-tax) prices for all goods (at least one), for at least two periods. Similarly, estimation of the y-conditional demands in the two stage budgeting theory discussed in Blundell (1987) and Blundell and Walker (1986), demands cross section data for the consumption of all goods, for a variable labelled full income[26], and for the simultaneous distribution of all current (after-tax) prices. Altonji (1986) and MaCurdy (1983) propose using the marginal rate of substitution functions, and this approach requires cross section data for all goods (at least two) and their (after-tax) prices. In addition, these approaches may need instrument and taste-modifier variables, and in order to account for eventual cohort effects, estimation of the instrument equations may require panel data. Since the decisions with respect to labour supply and consumption of non-durables and durables are taken simultaneously, ideally, our data should include observations for all these goods. Taking also into consideration the fact that estimation of the parameters determining interperiod allocations typically requires panel data, estimation of complete life cycle models of labour supply and consumption is very data demanding. This extensive data requirement is important to have in mind as we look at some problems that are particular to the estimation of the preferences over durables and non-durables.

A particular problem in the estimation of equation (100), is the measurement of consumption of non-durables. Non-durables include a variety of goods, and the measurement of the consumption of all these goods requires detailed household accounts. To reduce costs, many goods are bought infrequently, but in large quantities each time. This fact, and the fact that the consumption of many

[26]Blundell (1987) defines full income as the sum of the household's interest incomes, the value of its initial time endowment, and the asset accumulation.

goods is season specific, implies that consumption patterns should be observed over longer time periods. Hence, without having access to detailed household accounts for longer time periods, it is hard to obtain reliable observations of the consumption of non-durables.

However, if we observe the cash flow related to the purchases of durables and all income components as well as income and wealth taxes, we may use the period specific wealth constraints and the consumer price index to calculate the consumption of non-durables. But since we, in most cases, do not observe the purchase of durables in addition to all the other variables we need, this method is rarely useful.

Concerning the demand for durables, notice that if there are no binding constraints in the credit market, the first order condition (101) can be written

$$\frac{\partial U_{ct}}{\partial K_t} = \lambda_t \left[q_t - (1-\delta_f)\frac{q_{t+1}}{1+r_{t+1}} \right], \tag{103}$$

where $q_t - [(1-\delta_f)q_{t+1}/(1+r_{t+1})]$ is the user price of durables (including capital gains).

Again, measurement problems are serious. According to economic theory, consumption should be measured in physical units or in real terms, and not as an expenditure evaluated in current prices. It is also evident that consumption is a stream which should be measured per time unit, but apart from that economic theory gives very few guidelines. Since durables yield a flow of services for many periods, it is difficult to quantify the consumption of durables for a particular year.

Assuming there is a fixed relationship between the consumption and the stock of durables, the stock can be used as an argument in the utility index, cf. equation (98). The measurement problems, however, are still serious. An important reason is that durables typically have many characteristics that influence utility. For example, the utility of housing depends on characteristics such as location, the number of different kinds of rooms and "quality". The multiplicity of characteristics mean that a division of housing into homogenous subgroups will inevitably result in a huge number of groups. Even if it was possible to define these groups, it is still a problem that no existing data set includes all the information required to determine the distribution of all housing into the various groups. This fact becomes even more accentuated when we consider that estimation typically requires observations of other variables as well.

According to equation (103), the user price of durables is the relevant price for the demand for durables. Another problem related to the estimation of preferences over durables then is how to measure the user price of durables.

This price is only rarely realized in the market, which means that for practical estimation purposes it cannot be observed directly. An alternative is to calculate it indirectly using observations of the purchase prices, the (after-tax) interest rate and the physical depreciation rate. Notice then that the practical problems related to the measurement of durables also make it problematic to quantify the purchase price of durables. Since the user price includes the purchase price from two different periods, and the depreciation rate is rarely known, this means that the indirect method can not be used either.

The difficulty of observing the purchase price of durables suggests an approximation in which we use the (real) market value of durables[27] as an argument of the utility index. A problem with this approach is that, since most second-hand durables are only rarely resold, it is difficult to obtain reliable predictions of the market values for all relevant goods. Although this problem may be serious, it is not obvious that it is more serious that the problems related to the measurement of expenditures on non-durables. It may be easier to ask for information about the most important durables than about a large number of non-durables.

The fact that many data sets do not include the market value of durables, in addition to all the other variables we need for estimation, is a serious problem. Since we typically do not observe $q_t(1-\delta_f)K_{t-1}$ and the other components of the wealth constraint, indirect observation through the wealth constraint is also difficult.

We therefore conclude that it is difficult to obtain reliable observations on the market value of durables. Due to missing theoretical foundations, it is even more difficult to determine and observe their consumption and prices. Reliable observations of the consumption of non-durables are also difficult to obtain, but this is due to practical problems and cost considerations, rather than theoretical problems.

In what follows we assume that the only consumption data we observe are $q_t K_{t-1}$ and the total cash flow related to the purchases of durables and non-durables. The cash flow may be calculated indirectly through the wealth constraints.

3.2.3 Some additional arguments in favour of an alternative specification for durables

If δ_f equals one, either because there is no second-hand market or because physical depreciation is high, the user price coincides with the purchase price.

[27] In order to get a kind of a real value the market value can eventually be divided with the consumer price index.

Since durables and non-durables are treated in the same way in the utility index, this means that, apart from the fact that some durables can be resold, standard economic theory treats durables analogously to non-durables. As will now be argued, it is not obvious that this should be the case.

Standard life cycle theory assumes that consumption of durables can be treated as a continuous variable, and that no transactions costs are incurred in purchasing these goods. According to the first order condition (101), consumption should be adjusted until the marginal utility of durables equals its (net) marginal cost. This means that the demand for durables should continuously be adjusted in accordance with changes in the user price. In the case of housing, these adjustments involve important transactions costs of both the pecuniary and non-pecuniary kinds. If we divide housing into homogeneous subgroups with respect to location, number of rooms, and quality, we can also question whether it is reasonable to assume that it is possible to make marginal changes in its consumption. (This fact is also relevant for labour supply.) Both these facts raise the question of whether the marginal, and quite fine, comparisons that follow from the first order conditions, are good approximations to the comparisons households actually make. Maybe the household adjusts more roughly, in the sense that it can only choose between a limited number of baskets of goods. In the case of the demand for housing we can think of this basket as consisting of the purchase price of the housing, location, number of rooms, and a measure of quality. Households then choose the basket that maximizes utility.

Another argument in favour of a specification for durables other than the one presented in equation (98) is the following. Economic theory allows $\partial(\partial U_t/\partial X_t)/\partial X^*_{t'} \neq 0$, where X_t and $X^*_t = C_t, K_t$, for $t, t' = 0, 1, 2, \cdots T$. That is, the marginal utilities may depend on the consumption of all goods in all periods. Estimation of specifications that are based on the first order conditions then requires life cycle data for the consumption of all goods (in addition to the other variables that are needed).

Given this fact, most empirical analyses realize that some kind of separability assumption must be invoked. Most empirical analyses assume that preferences can be reasonably approximated by an intertemporal separable function, and it is also frequently assumed that even the within-period utilities are separable, cf. equation (98).

Intertemporal separability means that $\partial(\partial U_t/\partial X_t)/\partial X^*_{t'} = 0$, where X_t and $X^*_t = C_t$, K_t, $t \neq t'$ for $t, t' = 0, 1, 2, \cdots T$. That is, the marginal utilities are functions of current consumption only, and it is possible to estimate within-period preferences from cross sectional data only, cf. Altonji (1986), Blundell (1987) and MaCurdy (1983).

Intertemporal separability does, however, imply that habits play no role in the demand for the various goods. Duesenberry's (1949) relative income hypothesis focuses on habits in aggregate demand, and Muellbauer (1986) claims that "Evidence from the estimation of complete systems of demand equations suggests that habits or persistence effects play an important role in consumer behaviour.". Bover (1991) concludes that her estimation of a life cycle model "strongly support the importance of past hours in determining current hours decisions.". We therefore conclude that the assumption of intertemporal separability seems to be rather unrealistic.

3.3 A Model with Habits or Psychological Depreciation

This section considers modifying the life cycle model outlined in the previous section to account for habits in the demand for durables. The modification relies on a story in which habits inference the demand for durables, but not the demand for non-durables. An advantage of this modification is that it can be estimated for the case in which we observe neither the consumption of non-durables nor the price and the physical stock of durables.

The model is extended to include female and male labour supply, personal income taxation and the possibility of credit market constraints, and constitutes the general framework of the empirical analysis.

3.3.1 The model formulation

Assume lifetime preferences

$$U_t = \sum_{k=t}^{T} \frac{1}{(1+\rho)^{k-t}} G\left[U_{zk}(Z_k) + \sum_{j=f,m} U_{jk}(L_{jk})\right], \tag{104}$$

where L_f and L_m are female and male leisure, and Z is a composite good defined as

$$Z_k \equiv C_k + \frac{q_k}{p_k}\left[K_k - \beta K_{k-1}\right], \tag{105}$$

where β is a constant. The good Z then includes both non-durables and durables. In the next section we present a theory that leads to this specification and the interpretation of β, but for the moment we merely postulate that Z is the relevant consumption good related to the consumption of durables and non-durables.

The G-function is a monotonic transformation of within period utility that determines the intertemporal allocation of goods. For simplicity it is assumed independent of time. Notice also that the introduction of the G-function does not reflect the fact that the conditional orderings on female labour supply may depend on male labour supply and vica versa.

The period specific wealth constraints corresponding to the utility index (104) now become

$$\begin{aligned} \sum_{j=f,m} w_{jk}H_{jk} + r_kA_{k-1} &= I_k\left(w_{fk}H_{fk}, w_{mk}H_{mk}, r_kA_{k-1}\right) \\ &+ p_kZ_k - \delta_p q_k K_{k-1} + \Delta A_k, \quad k = t, \cdots, T, \end{aligned} \tag{106}$$

where the introduction of the term $-\delta_p q_k K_{k-1}$ is due to the definition of Z_k.

In order to account for personal income taxation, we have introduced the tax function $I_k\left(w_{fk}H_{fk}, w_{mk}H_{mk}, r_kA_{k-1}\right)$. Thus, income taxes are levied on wage and interest incomes, and we ignore wealth taxes. If A is negative, the household is in a net debt position, and the specification of the tax function allows deductions of interest expenses. The index k on the tax function indicates that the marginal tax rates vary across periods.

Since purchase of durables are often financed by loan, the possibility of credit market constraints should be considered. We follow Mariger (1987), and assume that the families can borrow only against mortgage in property. That is, net debt raised at the end of the period, $-A_k$, cannot exceed a fraction κ of the market value of the stock of durables at the end of that period, q_kK_k,

$$-A_k \leq \kappa q_k K_k, \quad k = t, t+1, \cdots, T. \tag{107}$$

It is assumed that κ is invariant of both time and characteristics of the household.

As opposed to Mariger, we do not observe whether or not a household is constrained since we do not know κ and do not observe q_kK_k. (In the empirical analysis we assume κ is known and approximate q_kK_k by $q_{k+1}K_k$.)

Despite the importance of labour market constraints we only account for the non-negativity constraint,

$$H_{jk} \geq 0, \quad j = f, m, \quad k = t, t+1, \cdots, T. \tag{108}$$

Leisure and labour supply are also constrained through the time budget, that is

$$L_{jk} = \overline{L} - H_{jk}, \quad k = t, t+1, \cdots, T. \tag{109}$$

We also allow for uncertainty about future prices, including the marginal tax and interest rates. The household maximizes the expected value of the time-preference-discounted sum of lifetime preferences with respect to $Z_{t'}$, $H_{ft'}$ and $H_{mt'}$, for $t' = t, t+1, \cdots, T$, subject to the constraints stated above and given values of A_{t-1}, K_{t-1} and A_T.

Necessary conditions for an optimum are the satisfaction of all the constraints and

$$\tilde{G}' \frac{\partial \tilde{U}_{zt}}{\partial Z_t} = \lambda_t p_t, \tag{110}$$

$$\tilde{G}' \frac{\partial \tilde{U}_{jt}}{\partial L_{jt}} = \lambda_t \tilde{m}_{jt} + \alpha_{jt}, \quad \mathrm{j = f, m} \tag{111}$$

and

$$\lambda_t = \frac{1}{1+\rho} E_t \left[(1 + \tilde{R}_{t+1}) \lambda_{t+1} \right] + \psi_t, \tag{112}$$

where "tilde" denotes that the variable is evaluated at the optimum, G' is the partial derivative of G with respect to $[U_{zt}(Z_t) + \sum U_{jt}(L_{jt})]$, and

$$m_{jt} \equiv w_{jt} \left[1 - \frac{\partial I_t}{\partial (w_{jt} H_{jt})} \right] \tag{113}$$

and

$$R_{t+1} \equiv r_{t+1} \left[1 - \frac{\partial I_{t+1}}{\partial (r_{t+1} A_t)} \right] \tag{114}$$

are the after-tax marginal wage and interest rates. The Lagrange multipliers λ_t, ψ_t and α_{jt}, $j = f$, m, which are associated with the wealth constraint, the borrowing constraint and the labour supply constraints, are all household- or person-specific.

3.3.2 The specification of the composite good

To focus on the essential properties of the composite good, neglect the monotonic transformation function G of the preference function (104), and assume that the preferences over consumption of non-durables and durables can be formulated as

$$\begin{aligned} V_0 &= V_0(K_0, K_1, \cdots, K_T, C_0, C_1, \cdots, C_T) = \\ &\sum_{t=0}^{T} \frac{1}{(1+\rho)^t} \left\{ U_{kt} \left[K_t - \beta K_{t-1} \right] + U_{ct}(C_t) \right\}. \end{aligned} \tag{115}$$

If $\beta = 0$, preferences are given by

$$V_0 = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} \left\{ U_{kt}(K_t) + U_{ct}(C_t) \right\}. \tag{116}$$

That is, if $\beta = 0$, our treatment of durables coincides with the standard specification (98). In order to introduce more flexibility in the demand for durables, we assume that the preference structure underlying the demand behaviour can be defined with respect to a reference bundle, K_{t-1}, cf. Spinnewyn (1981), Phlips (1972,1974), Houthakker and Taylor (1970), and Stadt, Kapteyn and Geer (1985). The parameter β then measures the extent to which the reference bundle influences current decisions.

The choice of reference bundle varies across analyses. Houthakker and Taylor (1970) assume that when households hold physical inventory of the actual good, the demand for the good depends on its stock. The reference bundle is now the stock of the good. For other goods such as tobacco, beer and spirits households do not hold inventories of any significance. Many researchers and other people, however, believe that these stimulus are habit-forming, and we can say metaphorically that households have built up a psychological stock of smoking and drinking habits, cf. Houthakker and Taylor. The reference bundle is now this psychological stock of habits, and according to Phlips (1974), this "built-in" mechanism is typical for other kinds of non-durables as well.

According to the habit-interpretation, we would expect that β (in equation (115)) satisfies the condition $0 < \beta \leq 1$, and that β measures the extent to which habits influence current decisions[28]. Bover (1991), Hotz, Kydland and Sedlacek (1988), Johnson and Pencavel (1984) and Muellbauer (1986) use a similar specification for preferences over leisure and non-durables.

Notice also that a more general means of accounting for habits, would be to introduce $\sum_{j=1}^{J} \beta_j K_{t-j}$, where we may reasonably assume $\beta_{t-m} > \beta_{t-n}$, for $n > m$.

While the examples above are related to habit formation, Alessie and Kapteyn (1991) are concerned with the fact that household preferences are influenced by the behaviour of other groups of households. The reference bundle is then the consumption of these groups, but the reason for introducing the reference bundle is not habit formation. Phlips (1974) classifies this type of taste changes as taste changes that result from better outside information due to external influences on the household. For instance, by talking with its neighbours, or reading advertising, the household may obtain new knowledge about goods that it has not yet purchased. Another example is that the consumption of a particular good may reveal that the good is harmful to the health of the consumer.

[28] Muellbauer (1986), using quarterly US consumption data, claims that β can be negative if K_t is purchases of durable goods, but this argument does not fit our specification since K_t is stock of durables.

The reference bundle can also represent minimal requirements of different goods from a physical point of view, cf. the linear expenditure system.

These examples indicate that the reference bundle may well change over time, and that it may depend on past experience. Specification (115) implies that households have rational, in contrast to myopic, habits, cf. Muellbauer (1986) and Spinnewyn (1981). This means that households recognise that current consumption decisions influence their future marginal rates of substitution. A disadvantage of assuming rational habits from an econometric point of view is that, if households maximize utility with respect to K, the two-stage budgeting property no longer holds since the intertemporal utility function is no longer separable. In what follows we will assume that the relevant good related to the consumption of durables can be measured by $(K_t - \beta K_{t-1})$, and in this case intertemporal utility is separable (in $(K_t - \beta K_{t-1})$).

In order to present an interpretation of β which is particularly relevant for durables, split β into two components according to

(117) $$\beta \equiv 1 - \delta_f - \delta_p,$$

where we recall that δ_f is the parameter for physical depreciation, and where δ_p is the parameter we label as psychological depreciation.

It is assumed that habits are determined by the stock of durables that was demanded in the last period after deductions for physical as well as psychological depreciation. Physical depreciation equals $\delta_f K_{t-1}$. The reason for introducing psychological depreciation is that, according to our theory, the household gets tired of the current stock of durables and wants a change. An example may be the household that moves into a new flat and after some time starts looking for a new flat. Another example may be the husband who, for some months, is very fascinated with his new car, but after some time loses interest. In a utility context this depreciation should also be deducted. But since the psychologically depreciated good $\delta_p K_{t-1}$ can be sold in the market, we must distinguish between physical and psychological depreciation. At the end of period t the utility relevant stock of durables is then $(1 - \delta_f - \delta_p)K_{t-1}$.

According to this story it seems reasonable to assume that $\delta_p \geq 0$. On the other hand, if durables are not worthless with respect to utility, but for some reason or another are of sentimental value, δ_p can well be negative. It is then reasonable to assume that the size of δ_p depends on the actual good, and that it may be household- and age-specific. Despite these facts, in the empirical analysis we approximate and assume that δ_p is constant, that is, independent of all these variables.

The upper limit of β implies that $\delta_f + \delta_p \geq 0$. If $\delta_f + \delta_p = 0$, equation (115) reduces to

$$V_0 = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} \{U_{kt}[K_t - K_{t-1}] + U_{ct}(C_t)\}. \tag{118}$$

This means that for the special case that $\beta = 1$, equation (115) implies that the household is concerned about the increase in the stock of durables .

It remains to explain the relationship between equation (104) and (115) in their specification of preferences over non-durables and durables. By substituting $(K_k - \beta K_{k-1})$ into the wealth constraint (106), the wealth constraint can be written

$$\sum_{j=f,m} w_{jk}H_{jk} + r_k A_{k-1} - I_k(w_{fk}H_{fk}, w_{mk}H_{mk}, r_k A_{k-1}) =$$
$$p_k C_k + q_k[K_k - \beta K_{k-1}] - \delta_p q_k K_{k-1} + \Delta A_k. \tag{119}$$

If $(K_k - \beta K_{k-1})$ is the relevant measure for consumption of non-durables, the specification of the wealth constraint and the associated preference function imply that $(K_k - \beta K_{k-1})$ can be viewed just like any other good. (In this respect the term $-\delta_p q_k K_{k-1}$ can be interpreted as negative income.) This means that we can use the Hicks aggregation theorem for aggregation of $(K_k - \beta K_{k-1})$ and C_k into a Hicks composite good.

The application of the Hicks composite good theorem, cf. Deaton and Muellbauer (1989), requires that the price ratio q/p is constant over time. Due to substitution possibilities in consumption and production, and the fact that the production of durables and non-durables utilizes many of the same inputs, this assumption may seem reasonable in the long run[29], and there are also arguments that justify this assumption as a reasonable approximation during the sample period, cf. section 3.7. In what follows we assume that the composite good theorem can be used.

3.3.3 The measurement of the composite good

For estimation purposes we need observations of Z. Let

$$Y_k \equiv C_k + \frac{q_k}{p_k}[K_k - (1-\delta_f)K_{k-1}], \tag{120}$$

which is the cash flow from purchasing non-durables and durables measured in units of the price of non-durables. The composite good can now be expressed as

$$Z_k \equiv Y_k + \delta_p D_{k-1}, \tag{121}$$

[29] As argued in section 3.2.2 it is very difficult to determine and observe price changes on housing and other kinds of durables across time due to missing theoretical foundations.

where

$$D_{k-1} \equiv \frac{q_k K_{k-1}}{p_k}$$

is the (real) market value of the stock of durables that was demanded at the end of the previous period, measured in the current prices. If $\delta_p = 0$, then $Z_k \equiv Y_k$, and specification (104) implies that households have preferences over the cash flow related to purchase of durables and non-durables.

In what follows we assume that we can observe D_{k-1} for at least one period. It remains to give an account of how we can observe Y_t. Since our data do not include reliable observations of this variable, we use the indirect approach. We have

$$(122)\, Y_k = \frac{1}{p_k}\left[\sum_{j=f,m} w_{jk}H_{jk} + r_k A_{k-1} - I_k\left(w_{fk}H_{fk}, w_{mk}H_{mk}, r_k A_{k-1}\right) - \Delta A_k\right].$$

Apart from the unknown habit persistence parameter δ_p, we observe all components of the composite good. We are left with the problem of obtaining reliable observations of the market value on durables, but we do not need to observe the consumption of non-durables, and the price and the physical stock of durables. It is also reasonable to assume that δ_p is small compared with Y, and measurement errors in D may then be less serious than in the standard framework presented in section 3.2. Specification (105)/(104), and the first order condition (110), then circumvent the problems related to obtaining reliable observations of the consumption of non-durables. Since durables are only one of the components of Z, we also believe that it reduces the problems related to the fixed costs in the demand for durables. In section 3.4 and 3.5 we show that δ_p can be treated as a parameter to be estimated.

3.3.4 Estimation strategy

Since the composite good, Z, with the exception of the unknown parameter, δ_p, can be treated as an ordinary good, the estimation approaches used for the standard life cycle model with intertemporal separability are relevant for the model outlined in the previous section. These approaches are surveyed in chapter two, and we will now focus only on the use of the marginal rate of substitution functions, cf. MaCurdy (1983) and Altonji (1986).

According to economic theory, households adjust demand until the marginal rate of substitution between two arbitrary goods equals the relative price of the goods. For instance, if there are no binding constraints in the labour and the credit markets, we have

$$(123) \qquad \frac{\partial U_{jt}/\partial L_{jt}}{\partial U_{zt}/\partial Z_t} = \frac{m_{jt}}{p_t}, \quad j = f, m.$$

That is, in equilibrium the marginal rate of substitution between female/male leisure and the composite good is equal to the real after-tax marginal wage rate.

By choosing a particular class of lifetime preferences, the marginal utilities can be expressed in terms of the unknown parameters of the preferences and the (realized) values of L_j and Z, and we obtain two specifications that can be used for estimation of the within-period utilities.

The possibility of bias in the estimates of labour supply functions from a subsample of workers that are unconstrained in the labour market is well known, cf. Heckman and MaCurdy (1980). According to Heckman (1979), the bias can be viewed as an "omitted variables" bias.

Heckman assumes that it is possible to find a simple expression for the reduced form of the relationship to be estimated. This assumption simplifies the calculation of the omitted variable considerably, but the approach can not be easily modified to include structural equations. Except in very special cases it is impossible to find the reduced form equations of the lifetime paths for L_j and Z, so we ignore the possible selection problem.

Also notice that the marginal rate of substitution functions cannot be used to estimate the transformation function G. Using the Frisch demands for Z and the Euler equation (112) in the case of no binding constraints in the credit market, we find that

$$(124) \qquad \frac{G'\partial U_{zt}/\partial Z_t}{p_t} = \frac{1}{1+\rho} E_t \left[(1+R_{t+1}) \frac{G'\partial U_{zt+1}/\partial Z_{t+1}}{p_{t+1}} \right],$$

where G' is the partial derivative of G. Assuming e_{t+1} is the one-period forecast error with respect to $\left[\frac{(1+R_{t+1})p_t G'\partial U_{zt+1}/\partial Z_{t+1}}{(1+\rho)p_{t+1}G'\partial U_{zt}/\partial Z_t} - 1\right]$, this expression implies

$$(125) \qquad \frac{G'\partial U_{zt+1}/\partial Z_{t+1}}{p_{t+1}} = \frac{1+\rho}{1+R_{t+1}} \left[\frac{G'\partial U_{zt}/\partial Z_t}{p_t} \right] (1+e_{t+1}),$$

cf. appendix C.

If expectations are rational, e_{t+1} is uncorrelated with

$$(126) \qquad \frac{1+\rho}{1+R_{it+1}} \left[\frac{G'\partial U_{zt}/\partial Z_t}{p_t} \right],$$

and we have found an equation that can be used as a basis for estimation of the parameters of the G-function and the time preference rate ρ. In this situation, as with the estimation of equations based on the marginal rate of substitution functions, we face a possible selection problem if the relationship is estimated from a subsample of households that are unconstrained in the credit market.

Section 3.5.1 discusses the estimation procedure in more details.

3.4 A Particular Empirical Specification

In what follows we assume that the monotonic transformation function G is given by

$$G = \frac{U_{ik}^{\theta} - 1}{\theta}, \tag{127}$$

and that (within-period) preferences are of the Box-Cox type

$$U_{ik} = \frac{(Z_{ik} + z_0)^{\sigma} - 1}{\sigma} + \Gamma_{ik} \frac{L_{fik}^{\gamma} - 1}{\gamma} + \Omega_{ik} \frac{L_{mik}^{\omega} - 1}{\omega}, \tag{128}$$

where subscript i denotes household and θ, σ, γ and ω are unknown coefficients. If $\theta = 1$, the coefficients γ and ω determine the intertemporal substitution elasticities $1/(\gamma - 1)$ and $1/(\omega - 1)$, which measure the percentage change in the consumption of leisure in any two periods in response to a percentage change in the relative wage rate for those periods, cf. Heckman and MaCurdy (1980). The interpretation of σ is less straightforward, since $\beta \neq 0$, implying that Z is not an ordinary physical good.

If the within-period utilities are to be strictly concave, σ, γ and ω must all be less than one. Since the cash flow related to the purchases of durables and non-durables, as well as the parameter δ_p, are allowed to be negative, Z can be negative. If z_0 is zero and Z is negative, the (Box-Cox) utility of Z is undefined; in order to avoid this problem, we introduce the parameter z_0. Notice also that since δ_p is unknown, the smallest value of $Y_t + \delta_p D_{t-1}$ can not be observed, and we (must) treat z_0 as a parameter to be estimated.

It is also assumed that the person- and age-specific modifiers of tastes, Γ_{ik} and Ω_{ik}, can be related to a vector of exogenous and observable consumer characteristics, X_{ik} and B_{ik}, and unmeasured characteristics, ε_{ik} and η_{ik}, according to $\Gamma_{ik} = \exp(X_{ik}\phi_x + \varepsilon_{ik})$ and $\Omega_{ik} = \exp(B_{ik}\phi_b + \eta_{ik})$. The error terms are needed since the econometrician cannot observe all components that influence preferences.

The first order conditions for consumption and female and male leisure become

$$U_{it}^{\theta-1}(Z_{it} + z_0)^{\sigma-1} = \lambda_{it} p_t, \tag{129}$$

$$U_{it}^{\theta-1}\Gamma_{it} L_{fit}^{\gamma-1} = \lambda_{it} m_{fit} + \alpha_{fit} \tag{130}$$

and

$$U_{it}^{\theta-1}\Omega_{it} L_{mit}^{\omega-1} = \lambda_{it} m_{mit} + \alpha_{mit}. \tag{131}$$

Assuming no binding constraints, either in the labour markets or in the credit market, the relationship between female leisure and the consumption good Z corresponding to equation (123), can be written

$$(132) \qquad \ln L_{fit} = X_{it}a_1 + a_2 \ln \frac{m_{fit}}{p_t} + a_3 \ln(Y_{it} + z_0 + \delta_p D_{it-1}) + \nu_{it},$$

where $a_1 = -\phi_x/(\gamma - 1)$, $a_2 = 1/(\gamma - 1)$, $a_3 = (\sigma - 1)/(\gamma - 1)$ and $\nu_{it} = -\varepsilon_{it}/(\gamma - 1)$.

The similar relationship between male leisure and Z can be written

$$(133) \qquad \ln L_{mit} = B_{it}b_1 + b_2 \ln \frac{m_{mit}}{p_t} + b_3 \ln(Y_{it} + z_0 + \delta_p D_{it-1}) + \mu_{it},$$

where $b_1 = -\phi_b/(\omega - 1)$, $b_2 = 1/(\omega - 1)$, $b_3 = (\sigma - 1)/(\omega - 1)$ and $\mu_{it} = -\eta_{it}/(\omega - 1)$.

It also follows that the Euler equation corresponding to equation (125) can be written

$$(134) \quad \ln\left[(1 + R_{it+1})\frac{p_t}{p_{t+1}}\left(\frac{Z_{it+1} + z_0}{Z_{it} + z_0}\right)^{\sigma-1}\right] = \ln(1+\rho) + (1-\theta)\ln\frac{U_{it+1}}{U_{it}} + \xi_{it+1},$$

where $\xi_{it+1} = \ln(1 + e_{it+1})$.

3.5 Estimating the Subutilities

Ideally, equation (132), (133) and (134) should be estimated as a simultaneous equation system. We simplify, and use a two stage estimation procedure where we in the first stage estimate the subutilities, cf. equation (132) and (133), before we in the second stage estimate the monotonic transformation G, and the time preference rate ρ, cf. equation (134).

Comparing equation (132) and (133), we notice that the definitions of the parameters a_2, a_3, b_2 and b_3 yield four equations for the determination of the three coefficients γ, σ and ω. The model specification then implies a particular kind of parameter restriction across the two equations, which should be accounted for in the estimation. Moreover, notice that these equations are non-linear in δ_p, and that there are endogenous right-hand-side variables. Estimation then requires a non-linear simultaneous equations procedure; and we apply a full-information maximum likelihood procedure.

First, however, we must choose the taste modifier variables for female and male leisure. We assume that preferences depend on age, and for the female, also on the size of the household. The taste modifier variable X then includes the number of children with age less than 21 years and the age of the female, while

B is the age of the male. Even though the decision to have children depends on the behaviour of the household, we assume that these taste modifiers can be reasonably viewed as exogenous in the econometric specification, cf. also the discussion of the wage rate equations (135).

Estimation also requires a decision about what right-hand-side variables should be treated as endogenous from an econometric point of view, and a specification of the instrument equations for these variables. Notice that the logarithm of the after-tax wage rate can be decomposed into two components according to $\ln m_j = \ln(1 - \partial I/\partial(w_j H_j)) + \ln w_j$.

Since we do not allow for learning-by-doing, the household is price-taker in the labour market; this assumption does not necessarily mean that the econometrician should look at the (pre-tax) wage rates as exogenous, cf. Heckman (1974a). If there are unobserved variables that influence the person's preferences for leisure, and that are correlated with the wage rate, the wage rate should be treated as endogenous in the empirical analysis. An example may be the comparison between a person who is very motivated for doing a good job at work, and one who is lazy and does not like to work. In this case we expect that the motivated person obtains a higher wage rate than the lazy one, even if we adjust for differences in education and work experience. It is also reasonable to believe that the motivated person has different preferences for work and leisure than the lazy one, and we have a situation where the unobserved variable influences both preferences and the wage rate. We may label this kind of endogeneity "statistical endogeneity".

Many empirical analyses account for this endogeneity of the wage rate by introducing a separate wage equation. This equation typically includes the number of years of education and a measure of work experience as explanatory variables. Important explanatory variables such as motivation and ability are rarely included, and the specification suffers from the omission of these unobservables.

Another problem related to the specification of the wage rate equation is that estimation ideally requires individual life cycle data. If the wage rate equation is estimated from cross sectional data, the estimates also reflect differences in wage rates across cohorts, and the estimated wage equation will not be well-suited for predicting the wage rate for a particular person.

One may include person-specific fixed effects in order to control for unobserved variables. MaCurdy (1981) assumes that wages follow a quadratic function in age with intercept and slope coefficients that depend on age-invariant characteristics of the consumer. This specification is then estimated using panel data for 10 years.

We do not have access to panel data that can be used for estimation of wage equations with person-specific fixed effects. Despite this fact, we find the argument for treating the pre-tax wage rate as endogenous quite convincing. Most empirical analyses also seem to adopt this assumption. Based on Heckman (1974), and the findings in Dagsvik and Strøm (1992), we assume that the logarithm of the wage rate is a linear function of work experience, experience squared and education, that is,

$$(135) \qquad \ln w_{jit} = M_{jit} g_{aj} + v_{jit}, \quad j = f, m,$$

where M represents the consumer characteristics that are assumed to be exogenous from an econometric point of view, g_a is the corresponding vector of coefficients, and v is an error term. Education is measured in years, and work experience is measured as age minus the years of education minus the age at the beginning of education.

The discussion above and the fact that the wage rate equation is estimated from cross sectional data imply that one must take the greatest care in the interpretation of the regression coefficients.

Since the marginal tax rate depends on the wage and interest incomes, and these depend on the behaviour of the household, the marginal tax rate is endogenous even for the household. To find the instrument for the logarithm of the after-tax marginal wage rate, it remains to determine the instrument for $\ln(1 - \partial I/\partial(w_j H_j))$. We assume that the relevant explanatory variables are the same as for the wage equations, but in addition we include the number of children with age less than 21 years. This variable is included because both labour supply and the demand for durables and non-durables may be influenced by the size of the household. Wage incomes, financial savings and interest incomes and deductions are then influenced by the size of the household, and when the size of these variables is determined, the marginal tax rate is also determined. The instrument equations for the marginal tax rates are then

$$(136) \qquad \ln\left(1 - \frac{\partial I}{\partial(w_j H_j)}\right) = B_{jit} g_{bj} + \epsilon_{jit}, \quad j = f, m,$$

where B is the vector of consumer characteristics, that is, experience, experience squared, education and the number of children with age less than 21 years, g_b is the corresponding vector of coefficients, and ϵ is the error term. Based on the above assumption, the vector B can be treated as exogenous in the estimation.

The cash flow related to purchases of durables and non-durables is endogenous for the household. We use education and education squared for the male as instrument variables for Y and add work experience for the female. Education is assumed to reflect the income potential of the household, but since the education of married females and males is (positively) correlated, we include only the education of the male in order to reduce the number of unknown coefficients. Using education squared, acknowledges that the consumption potential increases less than pre-tax wage incomes, since the tax system is progressive. Work experience is introduced as a substitute for age, since cash flow related to purchases of non-durables and, particularly durables, depends on age relative to the age when one finished education.

The instrument equation for the cash flow related to the purchases of durables and non-durables is then

$$Y_{it} = S_{it} g_s + \varsigma_{it}, \tag{137}$$

where S represents the consumer characteristics that are assumed to be exogenous from an econometric point of view, g_s is the corresponding vector of coefficients, and ς is an error term.

An alternative instrumental variable for Y would be gross or net income lagged one period, but since we find it reasonable to assume that there are omitted variables that are correlated with both lagged income and the error term in equation (132) and (133), we do not prefer this alternative.

The purchase price of non-durables is assumed equal to one in the estimation of the equations based on the marginal rate of substitution functions. It is also assumed that D_{t-1} is endogenous from an econometric point of view, although it is predetermined from the perspective of the household at age t. This decision is based on the assumption that there are omitted variables that are serially correlated and that influence preferences as well as D_{t-1}. The instruments used are female and male education, and an index of the number of inhabitants in the area where the household lives. Education is assumed to reflect income and consumption possibilities, while the number of inhabitants is introduced to try to account for the increase of the average price of housing as population density increases.

The instrument equation for the stock of durables is

$$D_{it-1} = J_{it} g_j + \psi_{it}, \tag{138}$$

where J is the vector of household characteristics that are assumed to be exogenous from an econometric point of view, g_j is the corresponding vector of coefficients, and ψ is an error term.

The complete model to be estimated consists of the structural equations (132) and (133) and the instrument equations (135) to (138). The properties of the error terms are as follows. Let $\mathbf{u}_{it} = (\nu_{it}, \mu_{it}, v_{fit}, v_{mit}, \epsilon_{fit}, \epsilon_{mit}, \varsigma_{it}, \psi_{it})'$ be the vector of error terms for the simultaneous equation system. The subsequent analysis assumes that

(139) $$E\mathbf{u}_{it} = \mathbf{0}, \quad \forall i, t,$$

and that

(140) $$E(\mathbf{u}_{it}\mathbf{u}_{it}') = \sum, \quad \forall i, t.$$

It is also assumed that $\mathbf{u}_{it}$ is serially correlated, but uncorrelated between households. Assuming, in addition, that the components of $\mathbf{u}_{it}$ follow a multivariate normal distribution, we can estimate the system by a full-information maximum likelihood procedure. All equations are identifiable, cf. Stewart and Wallis (1982), and from the parameters of these equations it is possible to identify the parameters of preferences.

3.5.1 Estimating the monotonic transformation

Using the specification of within-period preferences U, the parameter estimates for σ, θ, ω, the taste modifier parameters ϕ_x and ϕ_b, and panel data observations on Y, $q_t K_{t-1}$, L_f, L_m as well as the taste modifier variables, it is possible to estimate $\ln(U_{t+1}/U_t)$ and $((Z_{t+1} + z_0)/(Z_t + z_0))^{\sigma-1}$ for use in the estimation of the Euler equation (134).

Estimating the Euler equation requires that we decide which households are unconstrained in the credit market, cf. (107). We do not have access to that kind of information, and for simplicity[30] we assume that $\kappa = 0.9$. Since we cannot observe $q_t K_t$, but only $q_t K_{t-1}$, we also assume that households are unconstrained in the credit market if their net debt is less than 90 per cent of the market value of their durables measured in the prices of the next period, that is, if

(141) $$-A_t < 0.9 q_{t+1} K_t.$$

The selection problem that may be involved by estimating the Euler equation from a subsample of unconstrained households, will be ignored.

The properties of the error term $\xi_{it+1} = \ln(1 + e_{it+1})$ also require comment. Since ξ_{it+1} is a non-linear function of e_{it+1}, the assumption that $E_t e_{it+1} = 0$ does not imply that $E_t \xi_{it+1} = 0$. In what follows we assume that the (conditional) variance of e_{it+1} is constant across the sample, and that it is small. By approximating $\ln(1 + e_{it+1})$ with a Taylor expansion, we then find that $E_t \xi_{it+1}$ is small, and that the (conditional) variance of ξ_{it+1} is constant.

[30] One reason may be that households cannot finance the purchase of durables without having any own capital. By choosing $\kappa = 0.9$, we approximate.

The Euler equation can then be estimated by a maximum likelihood procedure that treats the estimates for $\ln(U_{t+1}/U_t)$ as exogenous.

3.6 Data

The data are obtained from the panel data sections of the Income Distribution Survey 1988, 1989 and 1990, the Standard of Living Survey 1991, and an additional postal survey conducted as part of the Income Distribution Survey 1989. All data sets were collected and prepared by Statistics Norway, and I have linked them on the basis of personal identification numbers.

The Income Distribution Surveys consist mainly of tax return data collected from taxation authorities. They yield quite detailed information about wage incomes, dividends, interest incomes and expenditures, financial wealth and debt, and income and property taxes. Transfer payments received, including child benefit, are also reported, and all these variables are used in the calculation of the cash flow related to the purchases of durables and non-durables. By linking two successive surveys, this survey also yields information about financial savings.

The Income Distribution Surveys also contain information about education, and the number and age of children. Given the tables of formal marginal tax rates, they also provide enough information for calculation of the marginal tax rates on wage and interest incomes. I have calculated the marginal tax rates in accordance with the set of regulations, and the possibility of separate taxation of the wage incomes of spouses is considered.

The ordinary Income Distribution Surveys also include the value of the stock of durables measured in the prices relevant for taxation, but these data underestimate stocks considerably. Instead I have used the additional postal survey conducted as part of the Income Distribution Survey 1989, and the Standard of Living Survey 1991. These two surveys include questions about the market value of housing, cottages, cars and private motor boats in 1989 and 1990 (not 1991) respectively, and the variable $q_t K_{t-1}$ includes all these durables.

Information about working time in 1989 and 1990 is also obtained from these two surveys. The spouses are asked about their average working time per week and the number of weeks worked. Multiplying these two variables, yields an estimate for labour supply. Leisure is defined as 8736 hours (= total number of hours in a year) minus hours of labour supply.

The (marginal) wage rate is measured as wage income divided by hours of labour supplied. If the worker gets overtime pay, and has much overtime, this wage rate will exceed his normal wage rate. In contrast, if the worker does not get overtime pay, and works much overtime, the observed wage rate underestimates the standard wage rate.

The data set used for estimation of equation (132) and (133) and the instrument equations (135) to (138) is obtained in the following way. First we have linked the Income Distribution Survey 1989 and 1990 with the Standard of Living Survey 1991. This linkage yields observations of L_{fit}, L_{mit}, w_{fit}, w_{mit}, Y_{it}, D_{it-1}, the taste modifier variables X_{it} and B_{it}, and the exogeneous variables M_{fit}, M_{mit}, B_{fit}, B_{mit}, S_{it} and J_{it} for the year 1990. (The Income Distribution Survey 1989 is included only to get data for financial savings through 1990.) By linking the Income Distribution Survey 1988 and 1989 and the additional survey to the Income Distribution Survey 1989 we get the same information for 1989. (The Income Distribution Survey 1988 is included only to get data for financial savings through 1989.) Finally, we combine these two subsamples into one sample, but in order to eliminate the estimation problems related to serial correlation of the error terms of our regression equations, all persons that are included in both subsamples are excluded.

The selection rules are as follows. Only married couples for which both spouses are between 30 and 55 years in 1990 are included. Those couples for which at least one of the spouses has entrepreneurial income in excess of wage income are excluded. Couples for which one or both spouses receive disablement benefit and who have wage and entrepreneurial incomes less than 53 440 Norwegian kroner, are also excluded. Both spouses must work between 200 and 3500 hours[31] a year, and the household must own their housing (not be tenants), to be included.

For some reason, the measurement method for the wage rate seems to bias the distribution of the wage rate, at least for females, cf. appendix D and table 4. By comparing the upper tail of the wage rate distribution for females and males (see table 4), we find that the average female wage rate is considerably higher than the average male wage rate, even if we controle for differences in age and education. Since we don't find this result reasonable, all couples for which the wage rate of the female exceeds 230 kroner or the wage rate of the male exceeds 270 kroner, are omitted from the analysis. The sample then consists of 327 observations. For further information about the sample, see summary statistics in table 1.

The pre-tax nominal interest rate is obtained by averaging over the quarterly observations of the private banks lending rents inclusive fees as they are presented in the statistics from the Norwegian Bank. This simplification may

[31]The method for the calculation of the wage rates implies that we observe the wage rate for all persons satisfying this labour supply constraint.

Table 1: Summary statistics for the 327 married couples used for estimation of the equations based on the marginal rate of substitution functions.

	Average	Standard deviation	Minimum value	Maximum value
Female leisure per year	7270	553	5616	8528
Male leisure per year	6717	353	5304	8466
Female gross wage rate, NOK per hour (among those who work)	94.7	30.2	43.1	219.8
Male gross wage rate, NOK per hour (among those who work)	117.7	33.8	67.3	266.7
Female marginal tax rate on wage incomes	0.407	0.090	0.231	0.62
Male marginal tax rate on wage incomes	0.523	0.095	0.266	0.62
Marginal tax rate on interest incomes/expenditures	0.394	0.064	0.260	0.456
Cash flow related to purchase of durables and non-durables	320361	227438	-41572	1498183
Stock of durables	1602300	879527	56000	5400000
Female education in years	10.9	2.2	8.0	17.5
Male education in years	11.7	2.8	8.0	19.0
Female age	41.5	6.3	30.0	55.0
Male age	43.9	6.2	30.0	55.0
Female experience in years	24.6	7.0	9.5	40.0
Male experience in years	26.2	7.1	8.5	41.0
Number of children 0-20 years	1.39	1.03	0.0	4.0
Index for inhabitants	3.42	1.30	1.0	5.0

bias the estimation results since the (pre-tax nominal) interest rate do vary across households. The average consumer price index published by Statistics Norway, is used to measure p_t/p_{t+1}. The average is based on monthly observations.

Estimating the Euler equation (134) requires panel data for L_f, L_m, Y and D for two successive periods. The data are obtained by linking the Income Distribution Survey 1989 and 1990 with the Standard of Living Survey 1991, and the Income Distribution Survey 1988 and 1989 with the postal survey to the Income Distribution Survey 1989. These two subsamples are then linked, and the final sample consists of 229 households. In contrast to the procedure for the sample used in the estimation of the equations based on the marginal rate of substitution functions, only those households that are observed in both subsamples, are included.

3.6.1 Some arguments in favour of Hicks aggregation

Complete household expenditure data for Norway are collected by Statistics Norway. The data for non-durables are based on accounts of expenditures over two weeks, and expenditures for the whole year are found by multiplying these data by 26. The measurement errors introduced by this procedure can be huge, and we prefer to obtain the data from another data source.

This chapter suggests aggregating C_k and $(K_k - \beta K_{k-1})$ using the Hicks composite good theorem. In light of price reductions for housing over the last years, the use of this theorem may be questioned. There are, however, some arguments that favour this assumption. The first is that the consumer price index, which we assume can be used to measure the price of non-durables[32], includes expenditures related to housing as well. According to Statistics Norway (1991), the budget share of expenditures related to lighting and heating is about 5 per cent. The second point to notice is that durables include many variants, such as cars, private motorboats, furniture and kitchen and leisure equipment, and price changes for these goods have been less dramatic than for housing. The third point is that, as we have argued in section 3.3.2, it seems reasonable to assume that the price path for durables is more or less similar to the price path for non-durables in the long run. So even if it can be questioned whether this approach is appropriate for the years included in this analysis, the approach may well be more defendable in other periods.

3.6.2 Measurement error

The possibility of measurement errors in some of the endogenous variables in the model makes the consequences of measurement errors of interest. The estimates of the equations based on the marginal rate of substitution functions are consistent only if the measurement error is of the classical errors in the variables type. This means that the observed endogenous variable F in the regression function is related to the true variable F_0 according to $F = F_0 + \varpi$, where ϖ is a randomly distributed error term that is distributed independently of all instrument variables. Given the results presented in appendix D, the measurement problems related to durables, and the fact that the cash flow related to the purchases of durables and non-durables is observed indirectly through the wealth constraint, this is a very strong assumption.

The same issues of measurement errors arise in the discussion of the consequences of measurement errors in the estimation of the Euler equation (134). If there are measurement errors in equation (132) and (133), the error terms of these equations include measurement errors. This means that the

[32]Unfortunately this index also includes prices of durables, and the observed price of non-durables may then be biased.

estimate for U is biased, and when we use this estimate in the Euler equation, the measurement errors will be non-linear. In this case, we do not have a classical errors in the variables scheme, and the estimates will no longer be consistent.

3.7 Empirical Results

This section reports the estimates of the parameters of the subutility and monotonic transformation functions. Since we are not particularly interested in the results from the estimation of the instrument equations, these results are presented in appendix E. That section also presents the estimates of the parameters of equation (132) and (133).

As it will become evident from this section, the estimation results are, with some exceptions, not very satisfying. One reason may be that we cannot model person-specific fixed effects in the wage equations as well as the other instrument equations, since we do not have access to panel data of sufficient length. Another reason may be the non-linearity in the two equations based on the marginal rate of substitution functions. We may also reasonably assume that treating Y_t and D_{t-1} as endogenous variables in the estimation enlarges the problems related to non-linearity. The fact that the samples include only 327 and 229 observations is also relevant. No matter the reason, the estimation results should be viewed tentatively, and as part of on-going research.

Since many of the estimates are quite imprecise in the sense that they are not statistically significant, it may be appropriate to clarify the implications of this result. The fact that a parameter is not statistically significant according to the standard T-test does not necessarily mean that we believe it is zero. If we have prior grounds for believing that it is different from zero (For instance, we may believe that females' preferences for leisure increase with an increase in the number of children.), we may continue to believe that even if the T-test does not support our original view. According to Wonnacott and Wonnacott (1985), this conclusion is particularly relevant if the test is based on a small sample. The estimation results presented in this section, are based on a small sample.

Table 2 reports the estimates of the parameters of the subutilities. A priori we believe that the parameters of the taste modifiers are different from zero. The estimates of the intercepts of the taste modifiers are quite large, but none of them are statistically significant. We also notice that the estimate of the coefficient on the age variable is approximately the same for females as for males. The sign of these coefficients indicates that an increase in age decreases the preferences for leisure. This means that the marginal rate of substitution between female/male leisure and the composite good decreases, and female/male labour supply increases. These results should, however, be viewed

Table 2: FIML estimates† for the subutility.

ϕ_x				ϕ_b		
Constant	Age	Children‡	γ	Constant	Age	ω
32.237	-0.0051	0.0565	-2.228	20.179	-0.0048	-0.8791
(26.496)	(0.0053)	(0.0408)	(2.999)	(11.950)	(0.0033)	(1.3955)

z_0	δ_p	σ
30241.9	0.0333	0.9558
(31029.4)	(0.0662)	(0.0359)

† Standard errors in parentheses.
‡ The number of children with age less than 21 years.

in light of the fact that none of these estimates are determined precisely.

The sign of the estimate of the coefficient on children indicates that females' preferences for leisure increase with an increase in the number of children. The marginal rate of substitution between female leisure and the composite good then increases, and female labour supply decreases. The estimate is within about one standard error of the estimate of Heckman and MaCurdy (1980), but it is imprecise.

The estimates for γ and ω are not significantly different from zero. We do not have knowledge of similar studies using Norwegian data, and cannot use this sort of information for claiming that these parameters are different from zero. Both estimates are, however, consistent with the results in Aaberge, Dagsvik and Strøm (1990), although they use a static and somewhat different model specification. The estimate for γ is also consistent with the results in Heckman and MaCurdy (1982). The implications of the finding that γ and ω are not significantly different from zero, are then ambiguous, in particular when we consider that the sample is small.

Both estimates are consistent with the assumption of a strict concave utility function. Since the absolute value of γ exceeds that of ω, the estimates also imply that male labour supply is more responsive to wage changes than female labour supply, cf. the first order conditions (130) and (131). This result seems to be in contrast with most empirical findings, which indicate that female labour supply is most responsive to wage changes.

If we look at the estimates of the parameters related to the preferences over the composite good, we notice that the standard errors of both z_0 and δ_p exceed the absolute value of the parameter estimates. These results do not support our theory that there is psychological depreciation in the consumption of durables, but again we should recall that the sample is small. The reason for introducing z_0 implies that z_0 should be greater than zero.

In contrast to all the other estimates, the estimate for σ is very precise. It is consistent with the assumption that preferences are strictly concave. Since there are no other studies that use our specification of the composite good Z, this result must be compared with the result of studies[33] that use the standard consumption measure, cf. section 3.2.1. MaCurdy (1983) includes imputed service flow for durables in the consumption measure, and finds that the parameter corresponding to σ is 0.34, with standard error equal to 0.21.

It remains to present the estimates from the estimation of the Euler equation (134), cf. table 3. Notice that the standard errors in this table do not account for the dependence of the variable $\ln(U_{t+1}/U_t)$ on estimated quantities.

Table 3: ML estimation† of the monotonic transformation.

ρ	θ
0.0663	(0.8881)
(0.0026)	(0.0682)

† The asymptotic standard errors in parenthesis are not adjusted for the use of the instruments.

In this case, the estimates for ρ and θ are both statistically significant. The estimate for θ implies that $\theta - 1$ is negative (= - 0.112), and this result is consistent with risk averted households.

3.8 Summary

This chapter formulates and estimates a life cycle model of labour supply and consumption of durables and non-durables for married couples in an environment of uncertainty and personal income taxes. While standard economic theory assumes that preferences are exogenously determined, we respond to the fact that many empirical studies have presented evidence in favour of utility being a relative concept, by specifying preferences subject to a reference standard. According to Boyer (1983), the authors of these studies generally argue that although the assumption of given preferences can be reasonable in the short run, in a life cycle perspective it should be challenged. The lack of a well-established economic theory of habit formation, however,

[33] The estimate for σ is very similar to the result in the (static) model of Aaberge, Dagsvik and Strøm (1990).

raises the question of how habit formation should be modelled. We introduce habit formation only in the demand for durables, and propose to use last year's demand for durables after deductions for physical and what we label psychological depreciation as the reference standard.

Since we do not have access to data for the consumption of non-durables, we aggregate non-durables and the good $(K_t - \beta K_{t-1})$ into a Hicks composite good. We then find a specification of the life cycle model of labour supply and consumption of durables and non-durables that can be estimated in the absence of data on the consumption of non-durables, and the price and the physical stock of durables. We argue that if the psychological depreciation parameter δ_p is small, this specification reduces the bias in the estimate of σ if there are measurement errors in observations of market value of the stock of durables.

In order to allow for more flexibility in lifetime utility, we decompose the period-specific subutilities into one component that determines within-period allocations, and a monotonic transformation that influences the allocations between periods.

In an empirical application of this specification, we assume that preferences are of the Box-Cox type, and suggest estimating the parameters of the subutilities by using the marginal rate of substitution functions between female/male leisure and the composite good. It is also possible to estimate the psychological depreciation parameter from these functions.

The monotonic transformation of the within-period utilities cannot be identified from these functions, but can be estimated by applying the Euler equation.

The data are obtained by linking the Income Distribution Surveys 1988, 1989 and 1990, the additional postal survey of the Income Distribution Survey 1989, and the Standard of Living Survey 1991. All data are linked on the basis of personal identification numbers. The estimates of θ and ω indicate that males' demand for leisure is more responsive to wage changes than females', but the estimation results should be viewed as tentative. None of these estimates are significantly different from zero. In contrast, the estimate of σ, which influences the preferences for consumption of durables and non-durables, is precise. All estimates are consistent with the assumption of strictly concave preferences.

The estimate of the psychological depreciation parameter is not statistically significant. The analysis then does not support our hypothesis that we should consider physical as well as psychological depreciation in the determination of the reference standard.

Finally, we estimate the Euler equation, but as we have argued, for this estimation to give consistent estimates, there is no room for measurement errors. It is then reasonable to assume that the estimates for θ and the time preference rate are biased.

APPENDIX A: The Derivation of the Labour Supply Equation (47)

The household is assumed to maximize lifetime utility

$$(142)\qquad \sum_{t=0}^{T}(1+\rho)^{-t}\left[\Gamma_{ct}C_t^{\omega_c}+\Gamma_{lt}(\overline{L}-H_t)^{\omega_l}\right]$$

subject to the wealth constraint

$$(143)\qquad A_{i0}+\sum_{t=0}^{T}\frac{1}{(1+r)^t}w_{it}H_{it}=\sum_{t=0}^{T}\frac{1}{(1+r)^t}C_{it},$$

the non-negativity constraint

$$(144)\qquad H_t\geq 0,$$

and a terminal condition for A_{i0}. (All variables are defined in the text.)

The first order condition for leisure corresponding to equation (9), is

$$(145)\qquad \frac{\omega_l}{(1+\rho)^t}\Gamma_{lt}L_t^{\omega_l-1}=\lambda_0\frac{w_t}{(1+r)^t}+\alpha_t.$$

Similarly to equation (14), this equation implies

$$(146)\qquad \begin{cases} L_t=\overline{L} & \text{if } \frac{\omega_l}{\lambda_0(1+\rho)^t}\Gamma_{lt}\overline{L}_t^{\omega_l-1}\geq\frac{w_t}{(1+r)^t} \\ L_t<\overline{L} & \text{otherwise, and determined by } \frac{\omega_l}{\lambda_0(1+\rho)^t}\Gamma_{lt}L_t^{\omega_l-1}=\frac{w_t}{(1+r)^t}, \end{cases}$$

where we have suppressed tilde to simplify notation.

If we take the logarithm and assume that $\ln\left(\frac{1+\rho}{1+r}\right)\approx\rho-r$, the condition

$$(147)\qquad \frac{\omega_l}{\lambda_0(1+\rho)^t}\Gamma_{lt}\overline{L}_t^{\omega_l-1}\geq\frac{w_t}{(1+r)^t}$$

can be rewritten as

(148) $$\ln \omega_l - \ln \lambda_0 - (\rho - r)t + \ln \Gamma_{lt} + (\omega_l - 1)\ln \overline{L} - \ln w_t \geq 0.$$

Using the wage equation

(149) $$\ln w_t = M_t\beta_m + \epsilon_{2t},$$

the taste modifier equation

(150) $$\Gamma_{lt} \equiv \exp(N_t\psi + \epsilon_{1t}),$$

and the components of variance scheme

(151) $$\epsilon_{jt} = \eta_j + \mu_{jt}, \quad j = 1, 2,$$

for the disturbance vector $(\epsilon_{1t}, \epsilon_{2t})$, this equation implies

(152) $$\begin{aligned} \tfrac{\mu_{2t}-\mu_{1t}}{\omega_l - 1} &\geq \tfrac{1}{\omega_l - 1}[-\ln \lambda_0 + \ln \omega_l + \eta_1 - \eta_2 + N_t\psi - M_t\beta_m - (\rho - r)t] \\ &+ \ln \overline{L}, \end{aligned}$$

since $(\omega_l - 1) < 0$. Using the definition of f, V_{1t} and $\mathcal{N}_t$ from equation (48) and (49), this equation can be written

(153) $$V_{1t} \geq -f - \mathcal{N}_t + \ln \overline{L}.$$

The first line of equation (146) can then be written

(154) $$\ln L_t = \ln \overline{L} \text{ if } V_{1t} \geq -f - \mathcal{N}_t + \ln \overline{L},$$

and this expression is equal to the first line of equation (47).

The second line of equation (146) says that the demand for leisure is determined by the equation

(155) $$\frac{\omega_l}{\lambda_0(1+\rho)^t}\Gamma_{lt}L_t^{\omega_l - 1} = \frac{w_t}{(1+r)^t}.$$

Solving this equation with respect to $\ln L_t$, and using the approximation $\ln\left(\frac{1+\rho}{1+r}\right) \approx \rho - r$, yields

(156) $$\ln L_t = \frac{1}{\omega_l - 1}\left[\ln w_t + \ln \lambda_0 + (\rho - r)t - \ln \omega_l - \ln \Gamma_{lt}\right].$$

Using the wage equation (149), the taste modifier equation (150), the components of variance scheme (151), and the definitions of f, V_{1t} and $\mathcal{N}_t$, the second line of (47) can easily be found.

APPENDIX B: The Derivation of Equation (85)

The Euler condition (73) for the marginal utility of wealth can be rewritten as

$$E_t \left[\frac{(1+R_{t+1})\lambda_{t+1}}{(1+\rho)\lambda_t} + \frac{\gamma_t}{\lambda_t} - 1 \right] = 0. \tag{157}$$

Given this equation we also have

$$\frac{(1+R_{t+1})\lambda_{t+1}}{(1+\rho)\lambda_t} + \frac{\gamma_t}{\lambda_t} - 1 = E_t \left[\frac{1+R_{t+1}\lambda_{t+1}}{(1+\rho)\lambda_t} + \frac{\gamma_t}{\lambda_t} - 1 \right] + e_{t+1}, \tag{158}$$

where e_{t+1} is the one period forecast error with respect to $\left[\frac{(1+R_{t+1})\lambda_{t+1}}{(1+\rho)\lambda_t} + \frac{\gamma_t}{\lambda_t} - 1 \right]$.

Since the left hand side of equation (157) can be rewritten as

$$\frac{1}{(1+\rho)\lambda_t} E_t \left[(1+R_{t+1})\lambda_{t+1} \right] + \frac{\gamma_t}{\lambda_t} - 1, \tag{159}$$

equation (158) implies

$$E_t \left[(1+R_{t+1})\lambda_{t+1} \right] = (1+R_{t+1})\lambda_{t+1} - (1+\rho)\lambda_t e_{t+1}. \tag{160}$$

Substituting this expression into the Euler equation (73) for the marginal utility of wealth, yields

$$\lambda_t = \frac{1}{1+\rho} \left[(1+R_{t+1})\lambda_{t+1} - (1+\rho)\lambda_t e_{t+1} \right] + \gamma_t. \tag{161}$$

Rearranging this equation gives equation (85).

APPENDIX C: Derivation of Equation (125)

Equation (124) can be rewritten as

$$(162) \qquad E_t\left[\frac{(1+R_{t+1})G'\frac{\partial U_{zt+1}}{\partial Z_{t+1}}p_t}{(1+\rho)G'\frac{\partial U_{zt}}{\partial Z_t}p_{t+1}}-1\right]=0.$$

The one-period forecast error is defined according to

$$(163) \quad e_{t+1}=\left[\frac{(1+R_{t+1})G'\frac{\partial U_{zt+1}}{\partial Z_{t+1}}p_t}{(1+\rho)G'\frac{\partial U_{zt}}{\partial Z_t}p_{t+1}}-1\right]-E_t\left[\frac{(1+R_{t+1})G'\frac{\partial U_{zt+1}}{\partial Z_{t+1}}p_t}{(1+\rho)G'\frac{\partial U_{zt}}{\partial Z_t}p_{t+1}}-1\right].$$

This equation can be rewritten as

$$(164) \quad E_t\left[(1+R_{t+1})\frac{G'\frac{\partial U_{zt+1}}{\partial Z_{t+1}}}{p_{t+1}}\right]=(1+R_{t+1})\frac{G'\frac{\partial U_{zt+1}}{\partial Z_{t+1}}}{p_{t+1}}-(1+\rho)\frac{G'\frac{\partial U_{zt}}{\partial Z_t}}{p_t}e_{t+1}.$$

Substituting this result into equation (124), yields

$$(165) \qquad \frac{G'\frac{\partial U_{zt}}{\partial Z_t}}{p_t}=\frac{1}{1+\rho}\left[(1+R_{t+1})\frac{G'\frac{\partial U_{zt+1}}{\partial Z_{t+1}}}{p_{t+1}}-(1+\rho)\frac{G'\frac{\partial U_{zt}}{\partial Z_t}}{p_t}e_{t+1}\right],$$

and solving this equation with respect to $\frac{G'\partial U_{zt+1}/\partial Z_{t+1}}{p_{t+1}}$, we find equation (125).

APPENDIX D: Measurement Errors in the Wage Rate

The wage rate is measured by dividing wage incomes with labour supply, where labour supply is measured by multiplying average labour supply per week by the number of weeks worked, cf. section 3.6. For some reason this procedure seems to bias the distribution of the wage rate, at least for females, cf. table 4. This

Table 4: The distribution of female and male wage rate for employed persons with wage rate in excess of 200 NOK and less than 800 NOK.

	Female			Male		
Variable	Number	Mean	Variance	Number	Mean	Variance
Wage rate	51	343.8	19197.0	88	281.7	10738.4
Hours†	51	403.7	60186.3	88	1642.3	676742.7
Age	51	40.0	46.6	88	43.6	48.2
Education	50	10.6	4.2	87	13.4	7.5

† Labour supply measured in hours.

table shows the number of married females and males with wage rate in excess of 200 Norwegian kr and less than 800 kr, their average wage rate and average labour supply, for persons who are classified as employees, cf. section 3.6. The upper limit of the wage rate is chosen such that very extreme observations are omitted, and the data are obtained by linking the Income Distribution Survey 1990 with the Standard of Living Survey 1991. In order to increase the number of observations, it is now not conditioned that the families also are included in the Income Distribution Survey 1989, as is the case with the sample used in the empirical analysis, and the total sample consists of about 950 observations.

We notice that the average female wage rate is about 22 per cent higher than the average wage rate for males, and that the variance is much higher for females than for males. We also notice that average female labour supply is only about 400 hours, or about one fourth of average male labour supply. Even though the income effect of wage changes may reduce labour supply, these results seem unreasonable.

Taking also into consideration that the females in average are younger than the males and that they have shorter education, the wage rate for at least the females must be biased. As a consequence of these and some other results, all females with wage rate less than kr 40 and greater than kr 230 are omitted from the empirical analysis. For the males the corresponding limits are 50 and 270 kr.

APPENDIX E: Some Further Estimation Results

The aim of this appendix is to give some further information about the estimates of the parameters of the subutilities by presenting the estimates for the parameters of the two marginal rate of substitution functions (132) and (133), and the instrument equations (135) to (138). (All the equations are estimated as a simultaneous equations system by FIML.)

Table 5 shows the estimates of the parameters of the marginal rate of substitution functions. With the exception of the intercepts, none of the parameters have a t-statistic in excess of two.

Table 5: FIML estimates† for the coefficients of the marginal rate of substitution functions.

a_1						
Constant	Age	Children‡	a_2	a_3	z_0	δ_p
9.987	-0.0016	0.0175	-0.3098	0.0137	30241.9	0.0333
(1.083)	(0.0021)	(0.0103)	(0.2878)	(0.0142)	(31029.4)	(0.0662)

b_1		
Constant	Age	b_2
10.738	-0.0025	-0.5322
(1.627)	(0.0026)	(0.3952)

† Standard errors in parentheses.
‡ The number of children with age less than 21 years.

We do not present the multiple correlation coefficients for the following reason. In the reduced form case, the multiple correlation coefficient, R^2, measures the proportion of the original variance in the dependent variable that is explained by the regression as a whole. The marginal rate of substitution function, however, includes endogenous right-hand-side variables, and the meaning of R^2 becomes unclear. Further, the total sum of squares cannot be partitioned into explained and unexplained sums of squares for simultaneous equation estimates, since the residuals are not orthogonal to all of the explanatory variables. The residual sum of squares can then exceed the total sum of squares, and in this case R^2 will be negative if it is calculated via the residual sum of squares.

Table 6 presents the results for the wage rate equations. All estimates are

Table 6: FIML estimates† for the coefficients of the wage rate equation for females and males.

	Constant	Experience	Experience squared	Education
Female	3.920	0.0179	-0.00040	0.0373
	(0.237)	(0.0122)	(0.00023)	(0.0112)
Male	4.166	0.0188	-0.00037	0.0298
	(0.188)	(0.0105)	(0.00019)	(0.0067)

† Standard errors in parentheses.

consistent with the results of Dagsvik and Strøm (1992). The parameters of the female wage rate equation are estimated less precisely than the parameters of the male equation.

Table 7: FIML estimates† for the coefficients of the instrument equation for $\ln(1 - \partial I/\partial(wH))$ for females and males.

	Constant	Experience	Exp. squared	Education	Children‡
Female	0.0032	-0.0238	0.00044	-0.0245	0.0207
	(0.1286)	(0.0082)	(0.00017)	(0.0046)	(0.0106)
Male	-0.1222	-0.0216	0.00033	-0.0263	-0.00072
	(0.1634)	(0.0101)	(0.00019)	(0.0059)	(0.0039)

† Standard errors in parentheses.
‡ The number of children with age less than 21 years.

Regarding the parameters of the instrument equation for $\ln(1 - \partial I/\partial(wH))$, cf. table 7, we notice that while the parameter related to the number of children with age less than 21 years is statistically significant for females, it is determined rather imprecisely for males. The estimates for the parameters related to experience, experience squared and education for the females are within the estimates for the males plus or minus one standard error.

Table 8: FIML estimates† for the coefficients of the instrument equation for cash flow related to purchase of durables and non-durables.

Constant	Male education	Education squared	Female experience
-124399	63799.2	-1967.59	-693.798
(275512)	(40069.2)	(1479.99)	(2285.59)

† Standard errors in parentheses.

The lack of variables in our data that can reasonably be assumed exogenous from an econometric point of view, and the fact that the few we have access to vary little across the sample, make it hard to find instruments that can reproduce the large variation in cash flow related to the purchases of durables in the sample. From table 8 we also notice that the parameters related to male education and education squared, and to the age of the female relative to the age she completed her education, are all imprecisely determined.

Table 9: FIML estimates† for the coefficients of the instrument equation for the stock of durables.

	Education		
Constant	Female	Male	Inhabitants
135258	66332.4	49135.4	48865.6
(306063)	(24659.0)	(22763.2)	(43467.2)

† Standard errors in parentheses.

Table 9 shows that the parameters of the instrument equation for the stock of durables are more precisely determined. The parameters related to female and male education are both statistically significant.

References

Alessie, R. and A. Kapteyn (1991): Habit Formation, Interdependent Preferences and Demographic Effects in the Almost Ideal Demand System, *The Economic Journal* **101**, 404-419.

Altonji, J.G. (1986): Intertemporal Substitution in Labour Supply: Evidence from Micro Data, *Journal of Political Economy* **94**, 176-215.

Attanasio, O.P. and M. Browning (1993): Consumption Over the Life Cycle and Over the Business Cycle, Working Paper 4453, National Bureau of Economic Research, Cambridge.

Begg, D.K.H. (1982): *The Rational Expectations Revolution in Macroeconomics*, Philip Allan, Oxford.

Biørn, E. (1981): The Consumption Function and the Life Cycle Hypothesis: An analysis of Norwegian Household Data, *The Scandinavian Journal of Economics* **82**, 464-480.

Blomquist, N.S. (1985): Labour Supply in a Two-Period Model: The Effect of a Nonlinear Progressive Income Tax, *Review of Economic Studies* **52**, 515-524.

Blundell, R., V. Fry and C. Meghir (1985): Preference Restrictions in Empirical Models of Life-Cycle Behaviour under Uncertainty, Mimeo, Department of Economics UCL, July 1986.

Blundell, R. and I. Walker (1986): A Life-Cycle Consistent Empirical Model of Family Labour Supply Using Cross-Section Data, *Review of Economic Studies* **53**, 539-558.

Blundell, R. (1987): Econometric Approaches to the Specification of Life-Cycle Labour Supply and Commodity Demand Behaviour, *Econometric Reviews* **5** (No. 1), 103-165.

Blundell, R., M. Browning and C. Meghir (1989): A Microeconometric Model of Intertemporal Substitution and Consumer Demand, Warwick Economic Research Papers 324, University of Warwick, Coventry.

Blundell, R., C. Meghir and P. Neves (1993): Labour Supply and Intertemporal Substitution, *Journal of Econometrics* **59**, 137-160.

Blundell, R., M. Browning and C. Meghir (1994): Consumer Demand and the Life-Cycle Allocation of Household Expenditures, *Review of Economic Studies* **61**, 57-80.

Bover, O. (1989): Estimating Intertemporal Labour Supply Elasticities Using Structural Model, *The Economic Journal* **99** (No. 398), 1026-1039.

Bover, O. (1991): Relaxing Intertemporal Separability: A Rational Habits Mode of Labor Supply Estimated from Panel Data, *Journal of Labor Economics* **9** (No. 1), 85-100.

Boyer, O. (1983): Rational Demand and Expenditure Patterns under Habit Formation, *Journal of Economic Theory* **31**, 27-53.

Browning, M.J. (1985): Which Demand Elasticities Do we Know and Which Do we Need to Know for Policy Analysis, McMaster Economics Discussion Paper 85-13.

Browning, M.J., A.S. Deaton and M. Irish (1985): A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle, *Econometrica* **53**, 503-544.

Browning, M.J. (1991): A Simple Nonadditive Preference Structure for Models of Household Behaviour over Time, *Journal of Political Economy* **99** (No. 3), 607-637.

Brumberg, A. and F. Modigliani (1955): "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data" in K.K. Kurihara (ed.): Post-Keynesian Economics, London: Allen and Unwin.

Chamberlain, G. (1984): "Panel Data" in Z. Griliches and M.D. Intriligator (eds.): *Handbook of Econometrics,* Vol. II, Elsevier Science Publishers BV.

Cox, D.R. and H.D. Miller (1970): *The Theory of Stochastic Processes,* Methuen & Co Ltd.

Dagsvik, J.K. and S. Strøm (1992): Labor Supply with Non-Convex Budget Sets, Hours Restrictions and Non-Pecuniary Job-Attributes, Discussion Papers 76, Statistics Norway.

Deaton, A. and J. Muellbauer (1989): *Economics and Consumer Behavior*, Cambridge University Press.

Dudley, R.M. (1989): *Real Analysis and Probability*, Wadsworth & Brooks/Cole, Pacific Groves.

Frisch, R. (1932): *New Methods of Measuring Marginal Utility*, Tubigen: J.C.B. Mohr.

Ghez, G. and G.S. Becker (1975): *The Allocation of Time and Goods over the Life-Cycle*, New York: Columbia University Press.

Gorman, W.M. (1959): Separable Utility and Aggregation, *Econometrica* **27**, 469-481.

Hajivassiliou, V.A. and Y.M. Ioannides (1991): Dynamic Switching Regression Models of the Euler Equation: Consumption and Liquidity Constraints, unpublished working paper revised April 1991.

Hall, E. (1978): Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence, *Journal of Political Economy* **86** (No. 6), 971-988.

Hall, A. (1993): "Some Aspects of Generalized Method of Moments Estimation" in G.S. Maddala, C. R. Rao and H.D. Vinod (eds.): *Handbook of Statistics*, Vol. 11, Elsevier Science Publishers BV.

Hansen, L.P. (1982): Large Sample Properties of Generalized Method of Movements Estimators, *Econometrica* **50**, 1029-1054.

Hansen, L.P. and K. Singleton (1982): Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica* **50**, 1269-1286.

Hausman, J.A. (1978): Specification Tests in Econometrics, *Econometrica* **48**, 697-720.

Heckman, J.J. (1974a): Shadow Prices, Market Wages, and Labour Supply, *Econometrica* **62** (No. 4), 679-694.

Heckman, J.J. (1974b): Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle, *The American Economic Review* **64** (No. 1), 188-194.

Heckman, J.J. (1976): A Life-Cycle Model of Earnings, Learning and Consumption, *Journal of Political Economy* **84** (No. 4), 11-44.

Heckman, J.J. (1979): Sample Selection Bias as a Specification Error, *Econometrica* **47**, 153-161.

Heckman, J.J. and T.E. MaCurdy (1980): A Life Cycle Model of Female Labour Supply, *Review of Economic Studies* **47**, 47-74.

Heckman, J.J. (1981): "Heterogeneity and State Dependence" in S. Rosen (ed.) *Studies in Labor Markets*, Chicago: University of Chicago Press.

Heckman, J.J. and T.E. MaCurdy (1982): Corrigendum on A Life Cycle Model of Female Labour Supply, *Review of Economic Studies* **69**, 659-660.

Hotz, V.J., F.E. Kydland and G.L. Sedlacek (1988): Intertemporal Preferences and Labor Supply, *Econometrica* **56** (No. 2), 335-360.

Houthakker, H. and L. Taylor (1970): *Consumer Demand in the United States*, 2nd. ed., Cambridge: Harvard University Press.

Johnson, T.R. and J.H. Pencavel (1984): Dynamic Hours of Work Functions for Husbands, Wives and Single Females, *Econometrica* **52** (No. 2), 363-389.

Kapteyn, A. (1980): A Theory of Preference Formation, MRG Working Paper 7933, University of Southern California.

King, M. (1985): "The Economics of Saving. A Survey of Recent Contributions", in K.J. Arrow and S. Honkapohja (eds.): *Frontiers of Economics*, Chapter 4, Basil Blackwell Ltd. 227-295.

Kornstad, T. (1993): Vridninger i Lønnstakernes Relative Brukerpris på Bolig, Ikke-Varige Goder og Fritid, *Norsk Økonomisk Tidsskrift* **107**, 1-32.

Kornstad, T. (1994): Empirical Life Cycle Models of Labour Supply and Consumption, Økonomiske doktoravhandlinger nr. 20, Universitetet i Oslo, Sosialøkonomisk Institutt.

MaCurdy, T.E. (1980): An Intertemporal Analysis of Taxation and Work Disincentives, Working Papers in Economics E-80-4, The Hoover Institution, Stanford University.

MaCurdy, T.E. (1981): An Empirical Model of Labor Supply in a Life-Cycle Setting, *Journal of Political Economy* **89** (No. 6), 1059-1084.

MaCurdy, T.E. (1983): A Simple Scheme for Estimating an Intertemporal Model Labor Supply and Consumption in the Presence of Taxes and Uncertainty, *International Economic Review* **24** (No. 2), 265-289.

MaCurdy, T.E. (1985): "Interpreting Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty" in J.J. Heckman and B. Singer (eds): *Longitudinal Analysis of Labor Market Data*, Chapter 3, London: Cambridge University Press, 111-155.

Mariger, R.P. (1987): A Life-Cycle Consumption Model with Liquidity Constraints: Theory and Empirical Results, *Econometrica* **55** (No. 3), 533-557.

Muellbauer, J. (1986): Habits, Rationality and Myopia in the Life-Cycle Consumption Function, Discussion Paper 112, Centre for Economic Policy Research, London.

Nakamura, A. and M. Nakamura (1985): *The Second Paycheck: A Socioeconomic Analysis of Earnings*, New York: Academic Press.

Ogaki, M. (1993): "Generalized Method of Moments: Econometric Applications" in G.S. Maddala, C. R. Rao and H.D. Vinod (eds.): *Handbook of Statistics*, Vol. 11, Elsevier Science Publishers BV.

Pakes, A. (1991): "Estimation of Dynamic Structural Models: Problems and Prospects Part II: Mixed Continuous-Discrete Models and Market Interactions", forthcoming in C. Sims and J.J. Laffont (eds.): *Proceedings of the 6th World Congress of Econometric Society*, Barcelona, Cambridge University Press.

Phlips, L. (1972): A Dynamic Version of the Linear Expenditure Model, *Review of Economics and Statistics* **54**, 450-458.

Phlips, L. (1974): "Applied Consumption Analysis", in C.J. Bliss and M.D. Intrilligator (eds.): *Advanced Textbooks in Economics*, Vol. 5, North-Holland/American Elsevier.

Rust, J. (1992): "Stochastic Decision Processes", preliminary draft of Chapter 16 in R. Eagle and D. McFadden (eds.): *Handbook of Econometrics*, Vol. 4.

Spinnewyn, F. (1981): Rational Habit Formation, *European Economic Review* **15**, 91-109.

Stadt, H.van de, A. Kapteyn, S. van de Geer (1985): The Relativity of Utility: Evidence from Panel Data, *The Review of Economics and Statistics* **67** (No. 2), 179-187.

Statistics Norway (1991): *Konsumprisindeksen* (The Consumer Price Index), Reports 91/8.

Stewart, M.B. and K.F. Wallis (1982): *Introductory Econometrics*, Oxford: Basil Blackwell.

Takayama, A. (1986): *Mathematical Economics*, 2nd edition, Cambridge University Press.

West, K.D. (1986): A Variance Bounds Test of the Linear Quadratic Inventory Model, *Journal of Political Economy* **94** (No. 2), 374-401.

Wonnacott, R.J. and T.H. Wonnacott (1985): *Introductory Statistics*, 4th edition, USA: John Wiley and Sons.

Aaberge, R., J.K. Dagsvik and S. Strøm (1990): Labor Supply, Income Distribution and Excess Burden of Personal Income Taxation in Norway, Discussion Papers 54, Statistics Norway.

Utkommet i serien Sosiale og økonomiske studier (SØS)

Issued in the series Social and Economic Studies (SES)

ISSN 0085-4344 (t.o.m. nr. 63)
ISSN 0801-3845

59 Økonomi, befolkningsspørsmål og statistikk. Utvalgte arbeider av Petter Jakob Bjerve. *Economy, Population Issues and Statistics Selected works by Petter Jakob Bjerve.* 1985-431s. 50 kr. ISBN 82-537-2236-2

60 Erik Hernæs: Framskriving av befolkningens utdanning. Revidert modell. *Projections of the Educational Characteristics of the Population. A Revised Model.* 1985-95s. 25 kr. ISBN 82-537-2296-6

61 Tiril Vogt: Vannkvalitet og helse. Analyse av en mulig sammenheng mellom aluminium i drikkevann og aldersdemens. *Water Quality and Health. Study of a Possible Relation between Aluminium in Drinking Water and Dementia.* 1986-77s. 30 kr. ISBN 82-537-2370-9

62 Lars B. Kristofersen: Dødelighet blant yrkesaktive. Sosiale ulikheter i 1970-årene. *Mortality by Occupation. Social Differences in the 1970s.* 1986-54s. 40 kr. ISBN 82-537-2398-9

63 Liv Anne Støren: Levekår blant utenlandske statsborgere 1983. *Living Conditions among Foreign Citizens.* 1987-299s. 55 kr. ISBN 82-537-2432-2

64 Marit Wårum: Tidsbruk og aktivitet i nærmiljø. *Neighbourhood Activity and the Use of Time.* 1987-91s. 45 kr. ISBN 82-537-2534-5

65 Knut H. Alfsen, Torstein Bye, Lorents Lorentsen: Natural Resource Accounting and Analysis. The Norwegian Experience 1978 - 1986. *Naturressursregnskap og analyser. Norske erfaringer.* 1987-71s. 40 kr. ISBN 82-537-2560-4

66 Liv Grøtvedt: Støy og helse. Analyse av støyopplevelser i Norge. *Noise and Health. Study on Noise Annoyance in Norway.* 1988-71s. 45 kr. ISBN 82-537-2574-4

67 Øystein Olsen, Kjell Roland: Modeling Demand for Natural Gas. A Review of Various Approaches. *Etterspørsel etter naturgass. En oversikt over ulike modellopplegg.* 1988-81s. 40 kr. ISBN 82-537-2665-1

68 Miljøstatistikk 1988. Naturressurser og miljø. *Environmental Statistics. Natural Resources and the Environment.* 1988-291s. 70 kr. ISBN 82-537-2664-3

69 Bernt Aardal, Henry Valen: Velgere, partier og politisk avstand. 1989-329s. 125 kr. ISBN 82-537-2762-3

70 Sosialt utsyn 1989. *Social Survey.* 1989-230s. 125 kr. ISBN 82-537-2776-3

71 Anne Lise Ellingsæter: Normalisering av deltidsarbeidet. En analyse av endring i kvinners yrkesaktivitet og arbeidstid i 80-årene. *Normalization of Part-Time Work. A Study of*

Women's Employment and Working Time Patterns in the 1980s. 1989-127s. 75 kr. ISBN 82-537-2779-8

72 Dag Album: Individ, arbeid og inntekt. En fordelingsanalyse. *Individuals, Jobs and Earnings. A study of Distribution*. 1989-198s. 85 kr. ISBN 82-537-2850-6

73 Kjell Arne Brekke, Asbjørn Torvanger (red.): Vitskapsfilosofi og økonomisk teori. *Philosophy of Science and Economic Theory*. 1990-315s. 115 kr. ISBN 82-537-2857-3

74 Henry Valen, Bernt Aardal, Gunnar Vogt: Endring og kontinuitet Stortingsvalget 1989. 1990-172s. 100 kr. ISBN 82-537- 2963-4

75 Odd Aukrust: Økonomisk forskning og debatt. *Economic research and debate*. Utvalgte artikler 1942-1989. 1990-383s. 125 kr. ISBN 82-537-2984-7

76 Gustav Haraldsen, Hege Kitterød: Døgnet rundt. Tidsbruk og tidsorganisering 1970-90. Tidsnyttingsundersøkelsene. 1992-185s. 189 kr. ISBN 82-537-3639-8

77 Jan-Erik Lystad: Norsk hotellnæring 1950-1990. 1992-174s. 115 kr. ISBN 82-537-3677-0

78 Olav Ljones, Bjørg Moen, Lars Østby (red.): Mennesker og modeller Livsløp og kryssløp. 1992-336s. 165 kr. ISBN 82-537-3699-1

79 Inger Gabrielsen: Det norske skattesystemet 1992. *The Norwegian Tax System*. 1992-175s. 115 kr. ISBN 82-537- 3728-9

80 Einar Bowitz: Offentlige stønader til husholdninger En økonometrisk undersøkelse og modellanalyse. 1992-119s. 100 kr. ISBN 82-537-3785-8

81 Svein Blom, Turid Noack og Lars Østby: Giftermål og barn – bedre sent enn aldri? 1993-167s. 115 kr. ISBN 82-537- 3808-0

82 Rolf Aaberge, Tom Wennemo: Inntektsulikhet og inntektsmobilitet i Norge 1986-1990. 1993-46s. 90 kr. ISBN 82-537-3911-7

83 Ingvild Svendsen: Empirical Tests of the Formation of Expectations – A Survey of Methods and Results. 1993-52s. 75 kr. ISBN 82-537-3948-6

84 Bjørn E. Naug: En økonometrisk analyse av utviklingen i importandelene for industrivarer 1968-1990. *An Econometric Analysis of the Development of Manufacturing Import Shares 1968-1990*. 1994-78s. 95 kr. ISBN 82-537-3955-9

85 Einar Bowitz og Ådne Cappelen: Prisdannelse og faktoretterspørsel i norske næringer. *Price Formation and Factor Demand in Norwegian Industries*. 1994-177s. 125 kr. ISBN 82-537-4024-7.

86 Klaus Mohn: Modelling Regional Producer Behaviour – A Survey. *Modellering av regional produsentatferd – En litteraturoversikt*. 1994-71s. 95 kr. ISBN 82-537-4042-5.

87 Knut A. Magnussen: Old-Age Pensions, Retirement Behaviour and Personal Saving. A Discussion of the Literature. *Alderspensjon, pensjoneringsatferd og privat sparing. En disku-*

sjon av litteraturen. 1994-69s. 95 kr. ISBN 82-537-4050-6.

88 Klaus Mohn, Lasse S. Stambøl og Knut Ø. Sørensen: Regional analyse av arbeidsmarked og demografi - Drivkrefter og utviklingstrekk belyst ved modellsystemet REGARD. *Regional Analysis of Labour Market and Demography with the Model REGARD.* 1994-165s. 125 kr. ISBN 82-537-4082-4.

89 Nils Martin Stølen: Wage Formation and the Macroeconomic Functioning of the Norwegian Labour Market. *Lønnsdannelse og den makroøkonomiske funksjonsmåten til det norske arbeidsmarkedet.* 1995-306s. 180 kr. ISBN 82-537-4141-3.

90 Øystein Kravdal: Sociodemographic Studies of Fertility and Divorce in Norway with Emphasis on the Importance of Economic Factors. *Sosiodemografiske studier av fruktbarhet og skilsmisse i Norge med vekt på betydningen av økonomiske faktorer.* 1994-267s. 155 kr. ISBN 82-537-4088-3.

91 Tom Kornstad: Empirical Life Cycle Models of Labour Supply and Consumption. *Empiriske livsløpsmodeller for arbeidstilbud og konsum.* 1995-115s. 110 kr. ISBN 82-537-4166-9.

C

Statistisk sentralbyrå
Statistics Norway