Taxation of fuel and vehicles when emissions are constrained

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Abstract:
A tax on fuel combined with tax exemptions or subsidies for fuel-efficient vehicles is implemented in many countries to fulfill the Paris agreement and to curb mileage-related externalities from road traffic. The present study shows that a tax on fuel should be combined with heavier taxation of low- and zero emission vehicles to curb mileage-related externalities and to fulfill emission targets within the transport sector. The emission target is fulfilled by adjusting the CO_2-tax component on fuel. The road user charge on fuel is designed to curb mileage-related externalities. The heavier tax on low- and zero emission vehicles prevent motorists from avoiding the road user charge on fuel by purchasing low- and zero emission vehicles.

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Sammendrag

Mange land kombinerer avgifter på drivstoff med avgiftsfritak og subsidier til elbiler og kjøretøy med et lavt drivstofforbruk for å innfri nasjonale CO_2-utslippsmål. I Norge har vi både en CO_2-avgift og en veibruksavgift på drivstoff. Veibruksavgiftens formål er å sørge for at bilistene betaler for eksterne kostnader som køer, ulykker, støy, veislitasje og lokal forurensing når de bruker veiene. Denne studien analyserer hvordan avgifter på drivstoff og kjøretøy bør kombineres for å dempe slike veibruksrelaterte kostnader når et CO_2-utslippsmål samtidig skal innfris.

Studien finner at det fortsatt bør være en CO_2-avgift samt en veibruksavgift på forbruk av drivstoff. Nivået på CO_2-avgiften bør imidlertid tilpasses slik at avgiften reflekterer marginalkostnaden ved å innfri utslippsmålet. Det vil medføre at husholdningene tar hensyn til utslippsmålet når de tilpasser omfanget av kjøring, samt når de velger kjøretøy. Veibruksavgiften bør settes lik gjennomsnittet av de veibruksrelaterte kostnadene per liter drivstoff. Det vil bety at kjøretøy med lavere drivstofforbruk enn gjennomsnittet påføres en veibruksavgift på drivstoff som er lavere enn veibrukskostnadene som påføres samfunnet, og vice versa.

En effektiv beskatning av kjøretøy må kompensere for avvik mellom veibruksrelaterte kostnader og veibruksrelaterte avgifter på drivstoff for de forskjellige kjøretøyene. Elbiler, og mer generelt, biler som bruker lite drivstoff, bør derfor pålegges høyere kjøpsavgifter enn biler som bruker mye drivstoff. En ekstra kjøpsavgift på lavutslippskjøretøy kombinert med avgiftene på drivstoff innebærer at rasjonelle husholdninger tar hensyn til veibruksrelaterte kostnader samt at utslippsmålet skal innfris når de velger kjøretøy. Ettersom brukere av elbiler ikke betaler veibruksavgifter bør også kjøp av elbiler avgiftsbelegges kraftigere enn kjøp av bensin- og dieselbiler. Politikkanbefalingene som kommer ut av denne studien forutsetter bl.a. at den teknologiske utviklingen av elbiler er upåvirket av politikken, samt at subsidier til nye ladestasjoner tilpasses for å høste eventuelle gevinster forbundet med utbygging av ladestasjoner. Den teoretiske analysen tar ikke hensyn til andre sosiale gevinster og tap. Andre transportpolitiske virkemidler samt andre former for transportatferd er også utelatt.

1. Introduction

Several studies investigate how taxes on fuels and vehicles should be designed to curb traffic related externalities in the form of CO_2-emissions, local air pollution, accidents, congestion and noise, see e.g. Innes (1996), Fullerton and West (2002, 2010), Parry and Small (2005) and Bjertnæs (2019a). However, countries participating in the Paris agreement have adopted targets with respect to greenhouse gas emissions. A number of countries faced with such targets have introduced emission targets for their transport sector. Several European countries have introduced bonus-malus schemes with tax exemptions and subsidies for purchase of low- and zero emission vehicles to lower greenhouse gas emissions, see Klier and Linn (2015). A CO_2 emission standard for passenger cars, Regulation (EU) 2019/631, applies in EU countries from 2020 as part of a strategy to fulfill emission targets for new passenger cars. Several EU countries have also adopted domestic targets even though the Effort sharing regulation 2021-2030 incorporates flexibility for participating countries. Efficient taxation of road transport in the presence of emission targets are however an underexplored topic in the literature.

The present study contributes to the literature by analyzing efficient combinations of taxes on fuels and vehicles when emissions from road transport are restricted by an emission target. The study finds that a tax on fuel should be combined with heavier taxation of low- and zero emission vehicles to fulfill the emission target and to curb mileage-related externalities. Furthermore, the study finds that emission targets should be fulfilled by adjusting the CO_2 component of the fuel tax. The road user charge on fuel should be designed to curb mileage-related externalities. However, the households' choice of vehicle is distorted by the tax on fuel, as the road user charge on fuel deviates from the mileage-related externality. The heavier tax on low- and zero emission vehicles is designed to neutralizes this distortion. Hence, the tax on fuel combined with the heavier tax on low- and zero emission vehicles implements the socially desirable allocation of vehicles.

The rest of the paper is divided into three sections; Section 2 provides a literature review, Section 3 presents the model and results, and Section 4 concludes.

2. Literature review

Parry and Small (2005) show that the optimal uniform tax rate on gasoline consists of an adjusted Pigouvian tax component, which includes damage from carbon emissions and other driving-related externalities, a Ramsey tax component designed to raise tax revenue, and a congestion feedback component, which captures welfare gains as labor supply increases with lower congestion. The

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component of the tax related to externalities due to congestion and accidents as well as the Ramsey tax component are dominant, while the Pigouvian elements related to global warming and congestion feedback are modest. Policy instruments in Parry and Small (2005) are restricted, however, as a perfect tax on driving-related externalities are excluded. The tax-induced gain in terms of reduced externalities per liter of fuel is consequently diminished as households avoid the mileage-related tax component by purchasing more fuel-efficient vehicles. Parry and Small's estimated optimal tax rates on gasoline are reduced accordingly. A range of other studies have adopted their method to calculate optimal tax rates on fuel; see e.g. Anton-Sarabia and Hernandez-Trillo (2014), Lin and Zeng (2014), and Anderson and Auffhammer (2014). Differentiated taxes on purchase of vehicles are not considered in these studies, even though Innes (1996), Fullerton and West (2002, 2010) and De Borger (2001) show that restrictions on taxes on the use of vehicles imply that taxes on the purchase of vehicles are desirable[1]. Indeed, Bjertnæs (2019a) shows that such avoidance should be neutralized by heavier taxation of fuel-efficient vehicles, and hence, that the gasoline tax rate should not be reduced due to such avoidance.

Innes (1996) and Fullerton and West (2002, 2010) study the optimal design of taxes on both fuels and vehicles. Innes (1996) shows that optimal vehicle taxes, or their regulatory equivalents, approximately equal the social cost of a vehicle's predicted emissions less the portion of costs that is internalized by a uniform gasoline tax. Fullerton and West (2002) extend his analysis and explore tax combinations that implement the social planner choices of mileage, engine size, pollution control equipment, and fuel type. They find that vehicles with bigger engines should be subsidized (taxed) if the tax rate on fuel, which equals the marginal damage per gallon of fuel, more (less) than completely internalizes the impact of engine size. According to their study, empirical investigations are required to determine whether to tax or subsidize vehicles with large engines. Fullerton and West (2010) extend the analysis in Fullerton and West (2002) with vehicle age and simulate different scenarios. They find that the three-part instrument involving a gas tax, an engine-size subsidy, and a new-car subsidy maximize welfare. The engine-size subsidy does not increase welfare significantly, however. Bjertnæs (2019a) develops theories in Innes (1996) and Fullerton and West (2002) into operational tax formulas that are comparable with current taxation of fuel and vehicles. Scenarios with myopic behavior and electric vehicles (EVs) are included. Bjertnæs (2019a) shows that the tax on fuel-efficient vehicles should exceed the tax on fuel-intensive vehicles, and that the efficient tax on fuel equals the average marginal damage per liter fuel consumed. Hence, avoidance of road user charges on fuel by purchasing more

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[1] Subsidizing substitutes for polluting goods might be desirable when governments are unable to tax emissions directly, see Sandmo (1976).

fuel-efficient vehicles is neutralized by the heavier tax on low- and zero emission vehicles in this case. The Ramsey tax component on fuels is excluded in Bjertnæs (2019a) because Jacobs and de Mooij (2015) show that a Pigouvian tax on polluting goods without a Ramsey tax component is part of a welfare-maximizing tax system within a Mirrlees-economy framework[2]. The Pigouvian solution in Jacobs and de Mooij (2015) is not attainable, however, when policy instruments are restricted to a uniform tax on fuel and differentiated taxes on vehicles.

As mentioned, countries participating in the Paris agreement have adopted emission targets. Within some countries, such emission targets have given rise to ambitious emission targets for the transport sector. Many countries have implemented taxes on fuel combined with tax exemptions or subsidies for fuel-efficient vehicles to fulfill the Paris agreement and to curb mileage-related externalities. Efficient taxation of road transport in the presence of emission targets are however an underexplored topic in the literature. The present study contributes to the literature by analyzing efficient combinations of taxes on fuel and vehicles when emissions is restricted by an emission target. The cost per emission unit within the model framework in Bjertnæs (2019a) is replaced with an emission target for road transport. The study shows that optimal tax formulas in Bjertnæs (2019a) are unchanged when this emission target is implemented[3]. Hence, the emission target is fulfilled by adjusting the CO_2-tax component on fuel. The CO_2-tax on fuel adjusts households driving and choice of vehicles so that the target is satisfied. The road user charge on fuel is designed to curb mileage-related externalities. The choice of vehicle is distorted by the tax on fuel, however, as the road user charge on fuel deviates from the mileage-related externality. The heavier tax on low- and zero emission vehicles is designed to neutralizes this distortion. Implementation of a road user charge based on driving might be an alternative, see Bjertnæs (2019a) and Bjertnæs (2019b).

3. The model framework

The model framework in Bjertnæs (2019a) is extended with an emission target for road transport. Other aspects of the model framework are identical. This section therefore draws heavily on the presentation of the model framework in Bjertnæs (2019a).

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[2] A general set of assumptions excludes the Ramsey tax component from a welfare-maximizing tax system according to Atkinson and Stiglitz (1976). However, results in the literature differ on the issue of whether environmental taxes should deviate from the Pigouvian rate due to tax revenue requirements. The optimal tax rate in Parry and Small (2005) is lower due to tax revenue requirements. Jaeger (2011), however, finds that the need for tax revenue contributes to increasing the optimal environmental tax wedge to higher than the Pigouvian tax rate. The optimal CO_2 tax also exceeds the quota price when the government purchase quotas and the marginal cost of public funds exceed one, according to Bjertnæs et. al. (2013).

[3] The mathematical contributions within the present paper is consequently marginal. Mathematics within economics is however mostly limited to applications of mathematical theorems. The mathematical contribution is consequently marginal within most economic research.

3.1 Households

All households have the same income. The income is spent on a vehicle, on fuel, and on a non-polluting good. There are two types of vehicles; fuel-efficient and fuel-intensive. Households' preferences are identical except that they consider the advantages and disadvantages of fuel-intensive cars differently. Each household chooses one car, which is either fuel efficient or fuel intensive. Household $i's$ utility, u_i, excluding externalities, is given by the quasilinear utility function

$$(1) \qquad u_i = u(m_i) + b_i + c_i,$$

when a fuel-intensive vehicle is chosen. The utility, u_i, is determined by driving distance measured in kilometers, m_i, consumption of a non-polluting consumer good, c_i, and the utility associated with owning a fuel-intensive vehicle, b_i. Household $i's$ utility when choosing a fuel-efficient vehicle equals the utility function in equation (1), but with b_i removed from the equation. The marginal utility of additional driving distance is positive, $u' > 0$, but declines as the driving distance increases, $u'' < 0$. This feature of the utility function captures that some trips are more important/ necessary to households than other trips. The vehicle-specific utility parameter, b_i, differs across households as transportation needs and requirements differ across households. The parameter is high for households which prefer high engine power due to e.g. heavy loads and frequent use of trailer. Range anxiety associated with EVs might be another reason when fossil fuel vehicles are compared with EVs. Note that some households may dislike the fuel-intensive vehicle, i.e., their utility parameter, b_i, is negative. Such vehicle specific preferences are implemented to study the allocation of vehicles. The specification of utility is chosen to be able to study the tradeoff faced by the government when taxes on fuel and vehicles are designed to satisfy a constraint on emissions, and to arrive at optimal tax formulas for fuel and vehicles in this setting. Transportation-policy aspects which are excluded from the model framework is discussed in later sections. Household $i's$ budget constraint is given by the equation

$$(2) \qquad c_i = y + k - \left(p_f + t_f\right)f_j m_i - t_{car,j} - p_{car,j},$$

where $j = high, low$ indicates fuel-intensive and fuel-efficient vehicle, respectively. Consumption of the non-polluting good, c_i, equals a fixed income, y, plus government transfers, k, minus costs of fuel, $(p_f + t_f)f_j m_i$, which is given by the price per liter of fuel, p_f, the tax per liter of fuel, t_f, and the fuel economy measured in liters per kilometer, f_j, minus the tax on the chosen vehicle, $t_{car,j}$, minus the price of the chosen vehicle, $p_{car,j}$. Utility maximization with respect to m_i implies that

(3) $\qquad u'_{m_i}(m_i) = (p_f + t_f)f_j,$

which implicitly defines the following function:

(4) $\qquad m_i = m_j(t_f).$

Equation (3) shows that the marginal gain in utility of one additional kilometer, u'_{m_i}, equals the private cost of driving one additional kilometer, $(p_f + t_f)f_j$. Hence, driving is restricted to trips where the benefit exceeds the costs[4]. Equation (3) also implies that total driving distance is longer for households with a fuel-efficient vehicle compared to households with a fuel-intensive vehicle. This is one of the challenges connected with the transition towards fuel-efficient vehicles, and hence, a novel feature of the model framework.

As mentioned each household chooses one car, which is either fuel efficient or fuel intensive. The impact of a tax on purchase of fuel-intensive vehicles on the choice of vehicles is identical with the impact of a subsidy on purchase of fuel-efficient vehicles. The tax on purchase of fuel-intensive vehicles is also equivalent with a subsidy on fuel-efficient vehicles within the government optimization problem. The tax on purchase of fuel-intensive vehicles, $t_{car,high}$, is therefore labeled t_{car}, and the tax on purchase of fuel-efficient vehicles is set equal to zero. The indirect utility function net of externalities for each household, i, for each type of vehicle, is found by inserting equation (2) into equation (1), and then implementing equation (4).

(5) $\quad v_{i,high} = u\left(m_{high}(t_f)\right) + b_i + y + k - (p_f + t_f)f_{high}m_{high}(t_f) - t_{car} - p_{car,high}$, and

$\qquad v_{i,low} = u\left(m_{low}(t_f)\right) + y + k - (p_f + t_f)f_{low}m_{low}(t_f) - p_{car,low},$

Assume that households are ranked from high to low according to their utility parameter, b_i, and that the first N households have chosen the fuel-intensive vehicle. Assume that the accumulated utility from their b_i-utility parameter, BA, is given by the expression

(6) $\qquad BA = b_{max}N - {}^a/_2 N^2,$

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[4] Vehicle maintenance and capital depreciation are excluded from the operating costs of vehicles to simplify the model framework. However, a tax designed to correct for negative externalities is not influenced by these operating costs when externalities are not influenced by them. Maintenance could be preserved by maintenance control, for example.

where parameter $a > 0$ and no restrictions are imposed on parameter b_{max}. Households choose the type of vehicle that maximizes utility. Households therefore choose the fuel-intensive vehicle up to the point where household number N is indifferent between types of vehicles. This equilibrium condition is given by the expression

(7)
$$u\left(m_{high}(t_f)\right) + b_{max} - aN + y + k - (p_f + t_f)f_{high}m_{high}(t_f) - t_{car} - p_{car,high}$$
$$= u\left(m_{low}(t_f)\right) + y + k - (p_f + t_f)f_{low}m_{low}(t_f) - p_{car,low}.$$

Households that derive higher utility from owning a fuel-intensive vehicle will choose a fuel-intensive vehicle. Households that derive lower utility from owning a fuel-intensive vehicle will choose a fuel-efficient vehicle. Equation (7) determines the number of households which choose the fuel-intensive vehicle, as a function of fuel taxes, vehicle taxes, exogenous parameters and prices. Taxation of both fuel and vehicles is crucial for choice of vehicles, see Sallee et al. (2016) and Busse et al. (2013). This feature is crucial for taxation designed to facilitate the transition towards fuel-efficient vehicles, and hence, is a novel feature of the model framework. Equation (7) is written as equation (8) to simplify notations.

(8)
$$N = N(t_f, t_{car})$$

The total number of households is $\bar{N}$. Hence, the number of households that choose the fuel-efficient vehicle amounts to

(9)
$$N_{low} = \bar{N} - N.$$

3.2 The emission target

Consumption of each liter of fuel generates a fixed amount of CO_2 emission. Hence, the CO_2 emission target translates into a fuel consumption target, S_{CO2}.

(10)
$$S_{CO2} = Nf_{high}m_{high}(t_f) + (\bar{N} - N)f_{low}m_{low}(t_f).$$

The fuel consumption target, S_{CO2}, equals the number of liters of fuel consumed by households with fuel-intensive vehicles, $Nf_{high}m_{high}(t_f)$, plus the number of liters of fuel consumed by households with fuel-efficient vehicles, $(\bar{N} - N)f_{low}m_{low}(t_f)$. A share of the current lifetime emissions from vehicles originates from production of vehicles and energy; see Hawkins et al. (2012). However, CO_2 emissions from production of energy and vehicles are excluded from the model framework. This assumption is appropriate when all polluters pay for their own emissions. The assumption is also

relevant when these emissions are included in an emission trading system like that of the EU, and thus are neutralized by adjustments in other emission sources.

3.3 Social costs

The cost of mileage-related damage, S_d, is given by the expression

(11) $\qquad S_d = p_d N m_{high}(t_f) + p_d(\bar{N} - N)m_{low}(t_f).$

S_d equals the damage per kilometer, p_d, multiplied by the number of kilometers driven by households with fuel-intensive vehicles, $N m_{high}(t_f)$, plus the damage per kilometer, p_d, multiplied by the number of kilometers driven by households with fuel-efficient vehicles, $(\bar{N} - N)m_{low}(t_f)$. The costs of traffic congestion and damage due to accidents dominates, while the costs of local pollution are more modest. These costs are influenced by a range of factors like drinking and driving, reckless driving and speeding. It is assumed that the present level of drinking and driving, reckless driving and speeding is preserved by current traffic laws and regulations.

3.4 Taxation of fuel and vehicles

Tax revenue collected is transferred to households. Each household receives a lump-sum transfer, k. The transfer is chosen to conform to the constraint of a balanced government budget. The government budget constraint is given by the following equation

(12) $\qquad \bar{N}k = N t_f f_{high} m_{high}(t_f) + N t_{car} + (\bar{N} - N)t_f f_{low} m_{low}(t_f).$

Total transfers, $\bar{N}k$, equal tax revenue from taxation of fuel for fuel-intensive vehicles, $N t_f f_{high} m_{high}(t_f)$, plus tax revenue from taxation of fuel-intensive vehicles, $N t_{car}$, plus tax revenue from taxation of fuel for fuel-efficient vehicles, $(\bar{N} - N)t_f f_{low} m_{low}(t_f)$.

The welfare function is given by the indirect utility function minus driving related social costs. The sum of indirect utility functions net of externalities, equation (5), is found by accumulating over the number of individuals choosing fuel-efficient and fuel-intensive vehicles. The accumulated utility associated with owning a fuel-intensive vehicle is given by equation (6). The driving related social costs is given by equations (11). The government budget constraint, equation (12), and the condition determining the allocation of vehicles, equation (8), are incorporated in the welfare function. The

government chooses the uniform tax rate on fuel, t_f, and the tax on purchase of fuel-intensive vehicles, t_{car}, to maximize welfare given the emission target, equation (10). The problem is

$$
(13) \qquad \underset{t_f, t_{car}}{\text{Max}} \; \bar{N}y + N(t_f, t_{car})u\left(m_{high}(t_f)\right) + b_{max}N(t_f, t_{car}) - \tfrac{1}{2}aN(t_f, t_{car})^2
$$

$$
+ \left(\bar{N} - N(t_f, t_{car})\right)u\left(m_{low}(t_f)\right) - N(t_f, t_{car})[p_{car,high} + p_f f_{high}m_{high}(t_f)]
$$

$$
- \left(\bar{N} - N(t_f, t_{car})\right)[p_{car,low} + p_f f_{low}m_{low}(t_f)] - p_d N(t_f, t_{car})m_{high}(t_f)
$$

$$
p_d(\bar{N} - N(t_f, t_{car}))m_{low}(t_f),
$$

subject to the emission target

$$
S_{CO2} = N f_{high}m_{high}(t_f) + (\bar{N} - N)f_{low}m_{low}(t_f).
$$

The Lagrangian of the government's maximization problem is

$$
(14) \qquad L = \bar{N}y + N(t_f, t_{car})u\left(m_{high}(t_f)\right) + b_{max}N(t_f, t_{car}) - \tfrac{1}{2}aN(t_f, t_{car})^2
$$

$$
+ \left(\bar{N} - N(t_f, t_{car})\right)u\left(m_{low}(t_f)\right) - N(t_f, t_{car})[p_{car,high} + p_f f_{high}m_{high}(t_f)]
$$

$$
- \left(\bar{N} - N(t_f, t_{car})\right)[p_{car,low} + p_f f_{low}m_{low}(t_f)] - p_d N(t_f, t_{car})m_{high}(t_f)
$$

$$
- p_d(\bar{N} - N(t_f, t_{car}))m_{low}(t_f)
$$

$$
- p_{CO2}\left[N(t_f, t_{car})f_{high}m_{high}(t_f) + \left(\bar{N} - N(t_f, t_{car})\right)f_{low}m_{low}(t_f) - S_{CO2}\right],
$$

where p_{CO2} equals the shadow price of the fuel consumption target. The tax on fuel affects the number of fuel-intensive vehicles, $N(t_f, t_{car})$, the driving distance of fuel-intensive vehicles, $m_{high}(t_f)$, and the driving distance of fuel-efficient vehicles, $m_{low}(t_f)$. The tax on purchase of fuel-intensive vehicles affects the number of fuel-intensive vehicles, $N(t_f, t_{car})$. Note that choice of transfers, k, is excluded from the optimization problem as the government budget constraint is incorporated in the welfare function. First order conditions and tax formulas become identical with first order conditions and tax formulas in Bjertnæs (2019a). Hence, interpretation of results is therefore closely related to interpretations in Bjertnæs (2019a). The first order conditions imply that

(15)
$$u\left(m_{high}(t_f)\right) + b_{max} - aN - p_{car,high} - p_f f_{high} m_{high}(t_f)$$
$$-p_{CO2} f_{high} m_{high}(t_f) - p_d m_{high}(t_f)$$
$$= u\left(m_{low}(t_f)\right) - p_{car,low} - p_f f_{low} m_{low}(t_f) - p_{CO2} f_{low} m_{low}(t_f) - p_d m_{low}(t_f).$$

See appendix A. Second order conditions are presented in Appendix B. Equation (15) shows that benefits minus the private and social costs of one additional fuel-intensive vehicle equal the benefits minus private and social costs of one additional fuel-efficient vehicle[5].

Tax theory is unable to produce a unique optimal tax rate on polluting goods due to the choice of normalization, see Fullerton (1997). The explanation is that the allocation of resources is unchanged when a uniform tax increase on consumer goods is combined with a proportional, revenue-neutral reduction in taxation of income. The optimal tax rate on fuel is therefore labeled the optimal additional tax rate on fuel. This tax rate equals

(16)
$$t_f^* = p_{CO2} + \frac{\left(N m_{high}`t_f + (\bar{N} - N) m_{low}`t_f\right) p_d}{N m_{high}`t_f f_{high} + (\bar{N} - N) m_{low}`t_f f_{low}}.$$

The optimal additional tax rate on fuel, t_f^*, equals the shadow price per liter of fuel, p_{CO2}, plus the road user charge on fuel, labeled t_d, given by the second term on the right-hand side of equation (16). This road user charge equals the reduction in mileage-related damage due to a marginal tax increase on fuel (the numerator), divided by the reduction in fuel consumption due to a marginal tax increase on fuel (the denominator). Thus, the road user charge on fuel equals the reduction in mileage-related damage per liter of reduced fuel consumption due to a marginal tax increase on fuel. The road user charge on fuel exceeds mileage-related externalities, p_d, for fuel-intensive vehicles. The road user charge on fuel is lower than mileage-related externalities, p_d, for fuel-efficient vehicles[6]. The welfare-maximizing driving distance for fuel-intensive (fuel-efficient) vehicles is lower (higher) than the social planner

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[5] A detailed inspection of equation (15) shows that the driving-related utility of one additional fuel-intensive vehicle, $u\left(m_{high}(t_f)\right)$, plus the additional utility of owing a fuel-intensive vehicle, $b_{max} - aN$, minus the producer price of a fuel-intensive vehicle, $p_{car,high}$, minus the production cost of fuel for one additional fuel-intensive vehicle, $p_f f_{high} m_{high}(t_f)$, minus shadow costs related to CO_2 emissions of one additional fuel-intensive vehicle, $p_{CO2} f_{high} m_{high}(t_f)$, minus mileage-related damage attributable to one additional fuel-intensive vehicle, $p_d m_{high}(t_f)$, equal the driving-related utility of one additional fuel-efficient vehicle, $u\left(m_{low}(t_f)\right)$, minus the producer price of a fuel-efficient vehicle, $p_{car,low}$, minus the production cost of fuel for one additional fuel-efficient vehicle, $p_f f_{low} m_{low}(t_f)$, minus shadow costs related to CO_2 emissions from one additional fuel-efficient vehicle, $p_{CO2} f_{low} m_{low}(t_f)$, minus mileage-related damage related to one additional fuel-efficient vehicle, $p_d m_{low}(t_f)$.

[6] This result is consistent with the result in Diamond (1973).

solution as policy tools are restricted. This outcome shows that the approach in Fullerton and West (2002), where the tax system is designed to implement the social planner solution, is inconsistent with the optimal tax solutions in the present study.

The welfare-maximizing tax on fuel-intensive vehicles equals

$$(17) \qquad t^*_{car} = \frac{\frac{\bar{N}-N}{\bar{N}}(f_{low}-f_{high})}{\frac{N}{\bar{N}}\frac{m_{high}`t_f}{m_{low}`t_f}f_{high}+\frac{\bar{N}-N}{\bar{N}}f_{low}}p_d m_{high}(t^*_f) + \frac{\frac{N}{\bar{N}}\frac{m_{high}`t_f}{m_{low}`t_f}(f_{low}-f_{high})}{\frac{N}{\bar{N}}\frac{m_{high}`t_f}{m_{low}`t_f}f_{high}+\frac{\bar{N}-N}{\bar{N}}f_{low}}p_d m_{low}(t^*_f).$$

Both terms on the right side are negative. Hence, there should be heavier taxes on fuel-efficient vehicles than on fuel-intensive vehicles. Inserting the expression for the road user charge on fuel, t_d, from equation (16) into equation (17) implies that

$$(18) \qquad t^*_{car} = (p_d - t_d f_{high})m_{high}(t_f) - (p_d - t_d f_{low})m_{low}(t_f).$$

Equation (18) shows that the optimal tax on fuel-intensive vehicles, t^*_{car}, equals mileage-related damage minus road user charges for fuel-intensive vehicles, $(p_d - t_d f_{high})m_{high}(t_f)$, minus the difference between mileage-related damage and road user charges for fuel-efficient vehicles, $(p_d - t_d f_{low})m_{low}(t_f)$. Future taxes on fuel are fully accounted for by households with rational expectations. Therefore, the CO$_2$ tax on fuel provides a correct incentive for the choice of vehicle in this case. The choice of vehicle is distorted, however, as the mileage-related tax on fuel deviates from the mileage-related externality. The heavier tax on fuel-efficient vehicles neutralizes this distortion. Hence, household's choice of vehicles implements the socially desirable allocation of vehicles given by equation (15).

The model framework is unable to distinguish between a tax on fuel-efficient vehicles and a subsidy on fuel-intensive vehicles. However, a welfare maximizing tax system consists of a Pigouvian tax on polluting goods designed to correct for externalities according to Jacobs and de Mooij (2015). Adopting this insight implies that tax formulas within the present study should be interpreted as environmental taxes designed to correct for externalities. Hence, purchase of fuel-intensive vehicles should be subsidized with an amount which equals the difference between road user charges on fuel and the mileage-related damage associated with each fuel-intensive vehicle, i.e. the first expression on the right-hand side of equation (18). Purchase of fuel-efficient vehicles should be taxed with the

difference between mileage-related damage and road user charges on fuel associated with each fuel-efficient vehicle, i.e. the second expression on the right-hand side of equation (18). Fullerton (1997) shows that the optimal commodity tax on clean and polluting goods is uniform when combined with an optimal environmental tax on polluting goods. Hence, tax formulas within the present study should be combined with a uniform commodity tax on fuel, both types of vehicles, and on the non-polluting good according to this insight.

Some limitations should be considered when results are interpreted. The simple one-period model framework adopted, with specific externalities and preferences with respect to driving and type of vehicle, suggests that results are limited to specific settings. A share of the mileage-related damage might e.g. be related to the weight of vehicles, and hence, to the fuel consumption of vehicles, see Anderson and Auffhammer (2014). Hence, a mileage-related tax on fuel might be desirable to correct for this share of the mileage-related externalities. The model framework excludes choices, such as economical driving (Bjertnæs 2019b), and other externalities, like the race for status. Other policy tools designed to reduce traffic-related externalities, like parking fees, toll roads and CAFE standards, are omitted from the model framework. Heterogeneity along dimensions like demand for driving, income and environmental awareness are also excluded. The simple model framework is, however, able to arrive at optimal tax formulas that are mainly determined by the damage fuel and vehicles inflict upon society. Such damage is determined by empirical estimates, so tax formulas are mainly determined by these estimates.

3.5 Electric Vehicles

A user charge on EVs is desirable to correct for mileage-related externalities. However, this section analyzes optimal taxation of fuel and purchases of EVs when the use of EVs is not taxed. The problem is analyzed within the present model framework by replacing low-emission vehicles with EVs, and by assuming that the private cost of using an EV is zero. CO_2 emissions from production of electricity and EVs are excluded. Thus, the driving distance for EVs is determined by the condition, $u'_{m_{low}} = 0$. Private operating costs of EVs is excluded from the model framework in this case. However, a tax designed to correct for negative externalities is not influenced by such operating costs when externalities are not influenced by such operating cost. Driving distance, and hence, mileage-related externalities are magnified when private operating costs equals zero. This problem is however solved by implementing appropriate driving distance for EVs within optimal tax formulas.

The maximization problem of the government is found by inserting $f_{low} = 0$, and by assuming that $m_{low}(t_f)$ is fixed in problem (13). First order conditions imply that

$$(19) \qquad u'_{m_{high}} = p_f f_{high} + p_{CO2} f_{high} + p_d.$$

Inserting equation (19) into equation (3) gives

$$(20) \qquad t_f^{**} = p_{CO2} + \frac{p_d}{f_{high}}.$$

Thus, the optimal tax difference between fuel and non-polluting consumer goods equals the shadow price of CO_2 emissions plus the mileage-related marginal damage of road transport. The first order condition with respect to t_{car} combined with equations (20) and (7) implies that

$$(21) \qquad t_{car}^{**} = -p_d m_{low}.$$

Equation (21) shows that the optimal additional tax on purchase of EVs equals mileage-related damage associated with EVs. The shadow price of CO_2 emissions and mileage-related damage due to fossil fuel vehicles with an average fuel efficiency is incorporated into the price of fuel. The cost of mileage-related damage associated with EVs is incorporated into the price of the vehicle. Thus, rational households face costs attributable to externalities when choosing between a fossil fuel vehicle with average fuel consumption and an EV. Note that greater damage from CO_2 emissions, preferences for vehicles due to factors such as range anxiety, and price differences between vehicles do not alter the optimal additional tax on EVs expressed by equation (21).

Some additional aspects should be considered, however. First, one may argue that driving distance is likely to differ among households with an EV, and hence, that a tax on EVs consequently deviates from mileage-related damage for some households. The present study is unable to illuminate on this issue. Diamond (1973) however argue that a uniform price which corrects for externalities which differ among households should be set equal to a weighted average of externalities[7]. Second, several empirical studies find that households have rational expectations when purchasing vehicles; see Sallee et al. (2016) and Busse et al. (2013). The analyzes above have adopted this assumption. Some studies

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[7] Differences in mileage-related costs across geographic regions call for geographic tax differentiation across regions. Implementation of geographic tax differentiation favors an annual vehicle tax, as differentiated taxes on purchases are more likely to be subject to evasion.

find support for myopic behavior, however; see Grigolon et al. (2014), Allcott and Wozny (2014) and Gillingham et al. (2021). Bjertnæs (2019a) shows that the optimal additional tax on zero emission vehicle is positive in the case with myopic behavior. Hence, the additional tax on zero emission vehicles designed to neutralize distortions due to a mileage tax on fuel exceeds tax rebates designed to correct for myopic behavior. Third, several car manufacturers have recently been caught manipulating tests to classify their vehicles as fuel efficient. Taxes are avoided and customers are cheated. Customers may however benefit as prices are reduced, see Reynaert and Sallee (2021). The heavier tax on low- and zero-emission vehicles lowers incentives for such avoidance, and hence contributes to solving this problem. Improved testing is of course an alternative solution to this problem. Fourth, countries have implemented tax exemptions and subsidies for EVs to promote the development of clean-transport technology, and possibly to prepare their car industry for an electric future. The present study shows that the optimal additional tax on EVs equals the value of their mileage-related externalities when the use of EVs is untaxed and other market imperfections are absent. This optimal additional tax is reduced if sales of new EVs boost technological development. The optimal tax is also reduced if rebates are designed to protect the domestic car industry. It is challenging to quantify such externalities, but additional adverse impacts, such as increased car use and less public transport, should be expected; see Holtsmark and Skonhoft (2014) and Aasness and Odeck (2015). Fifth, externalities associated with a network of charging stations could also justify tax exemptions for the purchase of EVs; see Greaker and Midttømme (2016). Shanjun et al. (2017) find, however, that direct subsidies for investing in charging stations are more efficient than subsidies for EVs.

4. Conclusion

Several European countries redesigned their vehicle tax system in the mid-2000s and implemented bonus-malus schemes that favored fuel-efficient vehicles. Some countries imposed a CO_2-based tax on purchase of vehicles, while other countries imposed annual CO_2-based registration taxes; see Klier and Linn (2015). According to their study, CO_2-based tax on purchase of vehicles leads to larger reductions in the average emission rates of new vehicles. The emission reduction of such taxation is eroded as sales of new vehicles expand, however (Alberini and Bareit, 2017), and as the retirement of high-emitting vehicles is postponed (Alberini et al., 2018). The annual CO_2-based registration tax, levied on both new and existing vehicles, is not burdened by these undesirable impacts according to Alberini et al. (2018). The impact of these annual taxes on the average emission rates of new vehicles is modest, however, (Klier and Linn, 2015), and the cost per ton of reduced CO_2 emissions is substantial (Alberini and Bareit, 2017). Additional tax exemptions and subsidies for purchase of low- and zero emission vehicles were later introduced in many countries to fulfill emission targets.

Efficient taxation of road transport in the presence of emission targets are an underexplored topic in the literature. The present study contributes to the literature by analyzing efficient combination of taxes on fuel and vehicles when emissions from road transport is restricted by an emission target. The study finds that a tax on fuel should be combined with heavier taxation of low- and zero emission vehicles to fulfill the emission target and to curb mileage-related externalities. The emission target is fulfilled by adjusting the CO_2-tax component on fuel. The road user charge on fuel is designed to curb mileage-related externalities. The choice of vehicle is distorted by the tax on fuel, however, as the road-user charge on fuel deviates from the mileage-related externality. The heavier tax on low- and zero emission vehicles is designed to neutralize this distortion.

The expansion of EVs create a need for road user charges that are not based on fuel. A few countries have introduced GPS-based road user charges on heavy duty vehicles, but systems for light-duty passenger vehicles are lagging. Bjertnæs (2019a) shows that the optimal tax on EVs equals the tax on fossil fuel vehicles when the road user charge is based on GPS tracking, the tax on fuel equals the marginal damage of CO_2 emissions, and other market imperfections are absent[8]. This solution leads to a more efficient allocation of vehicles and driving than the solution with a uniform tax on fuel combined with heavier taxation of fuel-efficient vehicles; see also Ashley et al. (2017) and Montag (2015). However, a GPS-based system is more costly to administer and is likely to impose information-processing costs and undesirable surveillance; see Parry et al. (2007). One may argue that a road-user charge based on odometer readings or pay-as-you-drive insurance combined with congestion charges and toll roads resembles GPS-based road user charges. However, such charges are also costly to administer, are susceptible to evasion, and leads to undesirable traffic planning designed to avoid toll stations; see Parry (2002).

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[8] Myopic behavior calls for tax differentiation according to Jansen and Denis (1999).

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Appendix A

First order equations w.r.t.

t_{car}: $-\frac{1}{a}u\left(m_{high}(t_f)\right) - \frac{1}{a}b_{max} + N + \frac{1}{a}u\left(m_{low}(t_f)\right) + \frac{1}{a}(p_{car,high} + p_f f_{high}m_{high}(t_f))$

$-\frac{1}{a}\left(p_{car,low} + p_f f_{low}m_{low}(t_f)\right) + \frac{1}{a}p_{CO2}f_{high}m_{high}(t_f) - \frac{1}{a}p_{CO2}f_{low}m_{low}(t_f) + \frac{1}{a}p_d m_{high}(t_f)$

$-\frac{1}{a}p_d m_{low}(t_f) = 0$

Note that $\frac{\partial N}{\partial t_{car}} = \frac{1}{-a}$ according to equation (7). If we multiply by $-a$, then

$u\left(m_{high}(t_f)\right) + b_{max} - aN - p_{car,high} - p_f f_{high}m_{high}(t_f) - p_{CO2}f_{high}m_{high}(t_f) - p_d m_{high}(t_f)$

$= u\left(m_{low}(t_f)\right) - p_{car,low} - p_f f_{low}m_{low}(t_f) - p_{CO2}f_{low}m_{low}(t_f) - p_d m_{low}(t_f).$

First order equations w.r.t. t_f:

$\frac{f_{low}m_{low}(t_f) - f_{high}m_{high}(t_f)}{a}[u\left(m_{high}(t_f)\right) + b_{max} - aN(.) - u\left(m_{low}(t_f)\right) - p_{car,high}$

$-p_f f_{high}m_{high}(t_f) + p_{car,low} + p_f f_{low}m_{low}(t_f) - p_{CO2}f_{high}m_{high}(t_f) + p_{CO2}f_{low}m_{low}(t_f)$

$-p_d m_{high}(t_f) + p_d m_{low}(t_f)] + N(.)u'_m m_{high}'_{t_f} - N(.)p_f f_{high}m_{high}'_{t_f}$

$-N(.)p_{CO2}f_{high}m_{high}'_{t_f} - N(.)p_d m_{high}'_{t_f} + (\bar{N} - N(.))u'_m m_{low}'_{t_f} - (\bar{N} -$

$N(.))p_f f_{low}m_{low}'_{t_f}$

$-(\bar{N} - N(.))p_{CO2}f_{low}m_{low}'_{t_f} \quad -(\bar{N} - N(.))p_d m_{low}'_{t_f} = 0$.

Note that equation (7) implies that $\frac{\partial N}{\partial t_f} = \frac{f_{low}m_{low}(t_f) - f_{high}m_{high}(t_f)}{a}$. The first order equation w.r.t t_{car}

implies that the parameters in the first bracket equal zero. Hence, these conditions imply that

$\frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}u'(m_{high}) + \frac{\bar{N} - N}{\bar{N}}u'(m_{low}) = \frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}p_f f_{high} + \frac{\bar{N} - N}{\bar{N}}p_f f_{low}$

$+\frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}p_{CO2}f_{high} + \frac{\bar{N} - N}{\bar{N}}p_{CO2}f_{low} + \frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}p_d + \frac{\bar{N} - N}{\bar{N}}p_d$

Multiplying equation (3) by $\frac{N}{\bar{N}}$ and $\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}$ gives

$\frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}u'(m_{high}) = \frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}p_f f_{high} + \frac{N}{\bar{N}}\frac{m_{high}`_{t_f}}{m_{low}`_{t_f}}t_f f_{high}$

Multiplying equation (3) by $\frac{\bar{N} - N}{\bar{N}}$ gives

$$\frac{\bar{N}-N}{\bar{N}}u'(m_{low}) = \frac{\bar{N}-N}{\bar{N}}p_f f_{low} + \frac{\bar{N}-N}{\bar{N}}t_f f_{low}$$

Summing these equations:

$$\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}u'(m_{high}) + \frac{\bar{N}-N}{\bar{N}}u'(m_{low}) = \frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}p_f f_{high} + \frac{\bar{N}-N}{\bar{N}}p_f f_{low}$$

$$+ \frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}t_f f_{high} + \frac{\bar{N}-N}{\bar{N}}t_f f_{low}$$

The first order conditions w.r.t. t_f and t_{car}, and this equation imply that

$$\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}t_f f_{high} + \frac{\bar{N}-N}{\bar{N}}t_f f_{low} = \frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}p_{CO2} f_{high} + \frac{\bar{N}-N}{\bar{N}}p_{CO2} f_{low}$$

$$+ \frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}p_d + \frac{\bar{N}-N}{\bar{N}}p_d$$

Hence,

$$t_f^* = p_{CO2} + \frac{\left(Nm_{high}\grave{}_{t_f} + (\bar{N}-N)m_{low}\grave{}_{t_f}\right)p_d}{Nm_{high}\grave{}_{t_f}f_{high} + (\bar{N}-N)m_{low}\grave{}_{t_f}f_{low}}$$

Substituting t_f^* in equation (7) gives

$$u\left(m_{high}(t_f^*)\right) + b_{max} - aN - p_{car,high} - t_{car} - p_f f_{high} m_{high}(t_f^*)$$

$$-p_{CO2}f_{high}m_{high}(t_f^*) - \frac{\left(\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}+\frac{\bar{N}-N}{\bar{N}}\right)p_d f_{high}}{\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}f_{high}+\frac{\bar{N}-N}{\bar{N}}f_{low}}m_{high}(t_f^*)$$

$$= u\left(m_{low}(t_f^*)\right) - p_{car,low} - p_f f_{low} m_{low}(t_f^*)$$

$$-p_{CO2}f_{low}m_{low}(t_f^*) - \frac{\left(\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}+\frac{\bar{N}-N}{\bar{N}}\right)p_d f_{low}}{\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}f_{high}+\frac{\bar{N}-N}{\bar{N}}f_{low}}m_{low}(t_f^*)$$

Hence,

$$u\left(m_{high}(t_f^*)\right) + b_{max} - aN - p_{car,high} - t_{car} - p_f f_{high} m_{high}(t_f^*)$$

$$-p_{CO2}f_{high}m_{high}(t_f^*) - p_d m_{high}(t_f^*) - \frac{\left(\frac{\bar{N}-N}{\bar{N}}\right)[f_{high}-f_{low}]}{\frac{N}{\bar{N}}\frac{m_{high}\grave{}_{t_f}}{m_{low}\grave{}_{t_f}}f_{high}+\frac{\bar{N}-N}{\bar{N}}f_{low}}p_d m_{high}(t_f^*)$$

$$= u\left(m_{low}(t_f^*)\right) - p_{car,low} - p_f f_{low} m_{low}(t_f^*)$$

$$-p_{CO2}f_{low}m_{low}(t_f^*) - p_d m_{low}(t_f^*) - \frac{\left(\frac{N}{\overline{\overline{N}}}\right)\frac{m_{high}`t_f}{m_{low}`t_f}[f_{low} - f_{high}]}{\frac{N}{\overline{\overline{N}}}\frac{m_{high}`t_f}{m_{low}`t_f}f_{high} + \frac{\overline{N} - N}{\overline{N}}f_{low}} p_d m_{low}(t_f^*)$$

Implementing first order conditions v.r.t. t_{car} gives

$$t_{car}^* = \frac{\frac{\overline{N} - N}{\overline{N}}(f_{low} - f_{high})}{\frac{N}{\overline{N}}\frac{m_{high}`t_f}{m_{low}`t_f}f_{high} + \frac{\overline{N} - N}{\overline{N}}f_{low}} p_d m_{high}(t_f^*) + \frac{\frac{N}{\overline{\overline{N}}}\frac{m_{high}`t_f}{m_{low}`t_f}(f_{low} - f_{high})}{\frac{N}{\overline{N}}\frac{m_{high}`t_f}{m_{low}`t_f}f_{high} + \frac{\overline{N} - N}{\overline{N}}f_{low}} p_d m_{low}(t_f^*)$$

Both expressions on the right-hand side are negative. This proves that t_{car}^* is negative.

First order equations w.r.t. p_{CO2}:

$$S_{CO2} = Nf_{high}m_{high}(t_f) + (\overline{N} - N)f_{low}m_{low}(t_f)$$

Appendix B

Second order conditions for the government maximization problem, equation (13). First order conditions solve the maximization problem if the Lagrangian is concave. Hence,

$$\frac{\partial^2 L}{\partial t_f \partial t_f} = \frac{f_{low} m_{low}(t_f) - f_{high} m_{high}(t_f)}{a}\left[-a\left[\frac{f_{low} m_{low}(t_f) - f_{high} m_{high}(t_f)}{a}\right]\right.$$

$$+ u'_m m_{high}{'}_{t_f} - p_f f_{high} m_{high}{'}_{t_f} - p_{CO2} f_{high} m_{high}{'}_{t_f} - p_d m_{high}{'}_{t_f}$$

$$\left. - u'_m m_{low}{'}_{t_f} + p_f f_{low} m_{low}{'}_{t_f} + p_{CO2} f_{low} m_{low}{'}_{t_f} + p_d m_{low}{'}_{t_f}\right]$$

$$+ \frac{f_{low} m_{low}(t_f) - f_{high} m_{high}(t_f)}{a}\left[u'_m m_{high}{'}_{t_f} - p_f f_{high} m_{high}{'}_{t_f} - p_{CO2} f_{high} m_{high}{'}_{t_f}\right.$$

$$\left. - p_d m_{high}{'}_{t_f} - u'_m m_{low}{'}_{t_f} + p_f f_{low} m_{low}{'}_{t_f} + p_{CO2} f_{low} m_{low}{'}_{t_f} + p_d m_{low}{'}_{t_f}\right]$$

$$+ N(t_f, t_{car})\left[u'_{m'm} m_{high}{'}_{t_f} m_{high}{'}_{t_f} + u'_m m_{high}{'}_{t_f}{'}_{t_f} - p_f f_{high} m_{high}{'}_{t_f}{'}_{t_f}\right.$$

$$- p_{CO2} f_{high} m_{high}{'}_{t_f}{'}_{t_f}$$

$$- p_d m_{high}{'}_{t_f}{'}_{t_f} - u'_{m'm} m_{low}{'}_{t_f} m_{low}{'}_{t_f} - u'_m m_{low}{'}_{t_f}{'}_{t_f} + p_f f_{low} m_{low}{'}_{t_f}{'}_{t_f} + p_{CO2} f_{low} m_{low}{'}_{t_f}{'}_{t_f}$$

$$\left. + p_d m_{low}{'}_{t_f}{'}_{t_f}\right] + \bar{N}\left[u'_{m'm} m_{low}{'}_{t_f} m_{low}{'}_{t_f} + u'_m m_{low}{'}_{t_f}{'}_{t_f} - p_f f_{low} m_{low}{'}_{t_f}{'}_{t_f}\right.$$

$$- p_{CO2} f_{low} m_{low}{'}_{t_f}{'}_{t_f}$$

$$- p_d m_{low}{'}_{t_f}{'}_{t_f}\right] < 0$$

$$\frac{\partial^2 W}{\partial t_{car} \partial t_{car}} = \frac{1}{-a} < 0$$

$$\frac{\partial^2 L}{\partial t_f \partial t_{car}} = \frac{f_{low} m_{low}(t_f) - f_{high} m_{high}(t_f)}{a}$$

$$- \frac{1}{a}\left[u'_m m_{high}{'}_{t_f} - p_f f_{high} m_{high}{'}_{t_f} - p_{CO2} f_{high} m_{high}{'}_{t_f}\right.$$

$$\left. - p_d m_{high}{'}_{t_f} - u'_m m_{low}{'}_{t_f} + p_f f_{low} m_{low}{'}_{t_f} + p_{CO2} f_{low} m_{low}{'}_{t_f} + p_d m_{low}{'}_{t_f}\right]$$

The second order condition is satisfied if

$$\frac{\partial^2 L}{\partial t_{car} \partial t_{car}} < 0 \text{ and } \frac{\partial^2 L}{\partial t_{car} \partial t_{car}} \frac{\partial^2 L}{\partial t_f \partial t_f} - \left(\frac{\partial^2 L}{\partial t_f \partial t_{car}}\right)^2 > 0$$

The first inequality condition is satisfied if $a > 0$.

The second inequality condition is satisfied when

$$-\frac{1}{a} N(t_f, t_{car})\left[u'_{m'm} m_{high}{'}_{t_f} m_{high}{'}_{t_f} + u'_m m_{high}{'}_{t_f}{'}_{t_f} - p_f f_{high} m_{high}{'}_{t_f}{'}_{t_f}\right.$$

$$- p_{CO2} f_{high} m_{high}{'}_{t_f}{'}_{t_f}$$

$$- p_d m_{high}{'}_{t_f}{'}_{t_f} - u'_{m'm} m_{low}{'}_{t_f} m_{low}{'}_{t_f} - u'_m m_{low}{'}_{t_f}{'}_{t_f} + p_f f_{low} m_{low}{'}_{t_f}{'}_{t_f}$$

$$+p_{CO2}f_{low}m_{low}{'}_{t_f}{'}_{t_f} +p_d m_{low}{'}_{t_f}{'}_{t_f}\Big] - \frac{1}{a^2}\Big[u{'}_m m_{high}{'}_{t_f} - p_f f_{high}m_{high}{'}_{t_f} - p_{CO2}f_{high}m_{high}{'}_{t_f}$$

$$-p_d m_{high}{'}_{t_f} - u{'}_m m_{low}{'}_{t_f} + p_f f_{low}m_{low}{'}_{t_f} + p_{CO2}f_{low}m_{low}{'}_{t_f} +p_d m_{low}{'}_{t_f}\Big]^2 > 0$$

Parameter values and functional forms are restricted to those that satisfy this condition.