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On the measurement of nonmarket hospital services in the Norwegian National Accounts

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Preface

The currently applied method at Statistics Norway for measuring somatic services rendered by central government-owned hospitals is to use the summed DRG (diagnosis related groups) points as the output volume, which is virtually not a unit cost approach for measuring nonmarket activities.

This paper proposes a method which is consistent with the tradition of applying the Laspeyres formula for volume and the Paasche formula for price indexes construction in the Norwegian national accounts. Moreover, the proposed method is the right unit cost approach, ensuring the consistency with the sound index number theory.

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Statistics Norway, 12 March 2023

Lasse Sandberg

Abstract

Somatic services rendered by central government-owned hospitals account for a significant part of nonmarket activities in Norway. The output volume of these services has been currently measured by the summed DRG (diagnosis related groups) points in the Norwegian national accounts.

This paper clarifies the important concept of unit cost in the DRG system and demonstrates that the current method is virtually not a unit cost approach for measuring nonmarket activities. Despite consistency in aggregation, the output volume and price indexes by following the current method are quite stringent.

On the contrary, the suggested method in this paper is consistent with the tradition of applying the Laspeyres formula for volume and the Paasche formula for price indexes construction in the Norwegian national accounts. More importantly, the suggested method is the right unit cost approach, ensuring the consistency with the sound index number theory.

In general, the two sets of measures, one by the current method and the other by the suggested method, will differ. The paper also illustrates that under what conditions, the estimated measures by the current method are upward- or downward biased, compared to those by the suggested method. The paper concludes that more quality work on measuring nonmarket activities is expected and therefore should be encouraged in future research at Statistics Norway.

Contents

1. Introduction

Human health and social work activities account for about ten percent of the total value-added generated in the entire economy of mainland Norway in 2017. Among the ten-percent share, nearly one-third is provided by the health enterprises and hospitals owned by the Norwegian central government. A good measurement of these nonmarket activities is therefore of significant importance.

The hospitals are partly financed through government appropriations and partly through activity-based financing that are determined according to the activity level carried out at the hospitals in Norway. The largest individual services provided by the hospitals are somatic services, psychiatry, and drug-abuse treatments.

To measure the output volume of somatic services rendered by the government-owned hospitals, the Norwegian DRG (diagnosis related groups) system is used, which classifies hospital activities into different categories based on, among other things, procedures, and diagnoses. This makes it possible to compare all types of patients and hospitals in Norway.

The application of the DRG system for compiling the output volume index of the government-owned nonmarket hospitals in the Norwegian national accounts compilation system was mentioned in e.g., Brathaug (2006) and Monsrud (2020). However, detailed information on the concrete implementation is still wanted, given that the construction and use of the DRG system varies across countries in practice (Schreyer, 2010).[1]

Research work has been continually undertaken in the world, with the view of harmonizing the construction of the DRG system across countries for the ease of international comparisons (Schreyer, 2010). In this context, countries' experiences by applying the national DRG system for various analyses can offer informed feedbacks from users' perspective and will surely strengthen the research work in this field.

The purpose of this paper is to clarify important concepts such as the unit costs in the context of the DRG system, and to make comments on the current concrete implementation method of applying the DRG system and the derived DRG points for measuring the output volume index of somatic services in the nonmarket hospitals in the Norwegian national accounts compilation system. In addition, the paper may serve as a methodology reference based on which the work quality of measuring nonmarket activities for compiling Norwegian national accounts can be better understood, and hopefully, enhanced if needed.

In the following section, a short overview is given of the commonly available approaches that can be applied for measuring nonmarket activities through the national accounts' compilation process. In Section 3, the DRG system is briefly introduced, followed by the presentation about how the Norwegian DRG cost weights and the DRG points are constructed.

Section 4 reports the current method that has been carried out in the Norwegian national accounts system of using the DRG points for measuring the output volume index of the government-owned nonmarket hospitals. In Section 5, comments on the current method are given, which demonstrates that the current method is essentially not a unit cost approach and is not consistent with the convention that has been traditionally and comprehensively applied for compiling the Norwegian national accounts. This section also illustrates that the output volume and price indexes constructed

[1] For instance, the DRG system is applied differently in Germany and Denmark as reported in Schreyer (2010).

by the current method can be biased towards either way (upwards or downwards), compared with those based on the suggested method in this paper. Section 6 concludes.

Before moving forward, some qualifications for this paper have to be put forward at this stage. First, the output concept mentioned throughout this paper refers to gross output, rather than value-added, so that we need not touch the issue about how to derive the volume index of value-added from that of gross output.[2] Second, the issues of quality-adjustment are not discussed in general, because it is believed that only if the focused issues of this paper are solved, can the quality-adjustment be further investigated.

[2] For discussions on how to construct volume index of value-added from gross output in the Norwegian national accounts system, please refer to Liu (2020).

2. Measuring nonmarket activities

The fundamental challenge for measuring nonmarket activities is that there are no or little information as regards the output of these activities, although the corresponding information of inputs is usually available. Most often than not, price information of the output is missing,[3] while quantity information is occasionally available for the nonmarket activities.

If the output of nonmarket activities is goods, it is in principle possible for them to be priced, conditional on that the equivalent goods can be found in the market with transactions. However, if the output of nonmarket activities is in the form of services, it will be more difficult for them to be priced, because services are usually of more unique characteristics than goods.

Depending on the extent to which the information about the output is available, there are several approaches for measuring the output volume index of nonmarket activities.

2.1. No information at all

For some types of nonmarket activities, information neither about price nor quantity, or volume of the output produced exist. For instance, collective services rendered by general government, such as public administration and defense, belong to this category.

As a convention, in addition to letting the value of the output of these nonmarket activities be equal to that of input, the output volume growth is set equal to the input volume growth and the corresponding output price growth is set equal to an index of input price growth (United Nations, 2009; Eurostat, 2013).

This conventional approach can be interpreted as to measure real resources that are needed or used for providing nonmarket outputs. The basic rationale behind this approach is that the output produced must be worth at least the sum of input costs, otherwise it would not have been delivered. As such, the sum of input can be considered as an acceptable indicator of the output produced by these nonmarket activities when only input information can be found.

At Statistics Norway, collective services rendered by general government are measured by this conventional approach (Sørensen, 2017), there are also other nonmarket services being measured by this approach in the Norwegian national accounts system (e.g., Monsrud, 2020).

Following this so-called '(output = input) convention' (Atkinson, 2005), the multifactor productivity level, defined as the output volume divided by the input volume, will automatically take the value of one, leading trivially to a zero growth of multifactor productivity for production unit delivering these nonmarket outputs.

To fully account input, the opportunity cost of capital is suggested to be added in capital services in government sector, which are currently measured by depreciation only (United Nations, 2009; Eurostat, 2013). On the contrary, capital services in market sector include both depreciation and the opportunity cost of capital that is tied up in holding productive assets.

[3] Some individual services provided by nonmarket activities will charge the recipients at nonmarket prices, which are either nearly zero prices or highly subsidized prices that do not cover their unit costs of production.

The omission will lead to a substantial underestimate of government sector input costs and hence underestimated economy wide GDP. Accounting for capital services in the same way regardless of the ownership of capital has been advocated by many (e.g., Atkinson, 2005; Diewert, 2017).

2.2. When quantity information is available

Without the price information of the output, the quantity information may be available for some types of nonmarket activities. There are in general three approaches for measuring the output volume under such circumstances.

Using volume indicators

Some quantity indicators can be regarded as reasonable proxies for the target output volume of these types of nonmarket outputs that are usually not directly observable. For example, the output volume can be proxied by the number of students and/or student-hours for nonmarket education services, and the number of beds for nonmarket nursing and social care in institutions.

Following this approach, the growth of quantity indicators between two time periods is considered as being equal to that of the target output volume of these nonmarket activities. Then the output price index of these nonmarket activities in concern is implicitly derived by dividing the value growth by that of volume indicators.

Examples of applying this approach in the Norwegian national accounts system can be found in Dam and Sørensen (2008) for measuring nonmarket education services, and in Øynes (2018) for measuring nonmarket nursing and social care in institutions, respectively.

Users' valuation approach

In the cases of nonmarket activities, users' valuations are usually made pertaining to the 'outcomes' rather than the 'outputs' of these activities, such as test scores of students and increased future earnings for nonmarket education services, and changes in health status between before- and after treatment for nonmarket health services (Schreyer, 2010, 2012).

Broadly speaking, outputs refer closely to activities that are within the production boundary of national accounts, e.g., the provision of education or health services, while outcomes are often beyond. Although transitions from inputs to outputs and further to outcomes are all subject to various environmental factors, arguably, more such factors will play a part in the transition from outputs to outcomes (Liu and Fraumeni, 2016).

Thus, outcomes are not yet considered to be proper measures of outputs as endorsed by national accountants. Nonetheless, information about outcomes can well provide a sensible tool for explicit quality adjustment of the outputs. Despite the existence of excellent studies measuring the output of nonmarket activities based on outcomes (e.g., Jorgenson and Fraumeni, 1992a, 1992b; O'Mahony and Stevens, 2004), more research along this line are still needed.

Unit cost approach

When producer's unit costs of production of some types of nonmarket activities are known, the information can be utilized for forming the output volume index for these nonmarket activities.

An (average) unit cost measures the costs per unit of output during a time period. For nonmarket activities, unit costs can replace output prices to value different kinds of services. The unit cost approach has long been suggested as an option for volume measurement (Scitovski, 1967). Hill (1975) noted that unit costs should be equal to selling prices for competitive market activities and advocated the general use of unit costs to value outputs for nonmarket activities as well. Schreyer

(2012) formally developed the 'price equals unit cost' methodology to value nonmarket outputs in much more detail.

In the cases of nonmarket hospital services which is the focus of this paper, unit costs are the costs per unit of hospital services. As hospital services have been defined as the number of treatments of particular diseases, unit costs are the costs per treatment of a disease. It merits to emphasize that despite the fact that 'costs' enter the picture, unit costs are defined via outputs (treatments) and not inputs (Schreyer, 2012).

A unit cost index is therefore a weighted average of unit costs of particular disease/treatment, where the cost share of each category of treatment constitutes the weight. Such a unit cost index mimics a price index and can be used for deflation when production is on a nonmarket basis.

Applying a unit cost index to an index of total costs is tantamount to constructing a direct volume index. Specifically, if a unit cost index is constructed by means of the Paasche index formula, then the corresponding volume index is the Laspeyres volume index, on the other hand, if a unit cost index is constructed by means of the Laspeyres index formula, then the corresponding volume index is the Paasche volume index.

Note that both the Laspeyres and Paasche indexes are of some asymmetric characteristics in term of using only one time period cost share as weight,[4] and neither of them is superlative index. [5] Certainly, a unit cost index can be constructed by using superlative indexes as well, such as Fisher index and Törnqvist index.

Fisher volume (or unit cost) index is a geometric mean of the Laspeyres and Paasche volume (or unit cost) indexes. Therefore, if a unit cost index is constructed by means of the Fisher index formula, then the corresponding volume index is also of the form of Fisher index formula.

Törnqvist index is the weighted geometric mean of the unit costs or quantity relatives using arithmetic averages of the cost shares in the two periods as weights. Thus, if a unit cost index is constructed by means of the Törnqvist index formula, then the corresponding volume index is called the implicit Törnqvist volume index, which is slightly different from the Törnqvist volume index.[6]

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[4] For more general discussions on index number theory, please refer to e.g., Diewert (2012).
[5] 'Superlative' index numbers are those that can be directly derived from functional forms that provide a second-order approximation to an arbitrary, twice differentiable linear homogenous function, covering a wide range of utility, production, distance, cost or revenue functions. A 'superlative' index is called 'exact' if it can be directly derived from a particular functional form (Diewert, 1978). For example, Törnqvist index is exact for the translog flexible functional form, and Fisher index is exact for a quadratic functional form.
[6] See footnote 4.

3. The DRG system and points

3.1. The DRG system

Diagnosis Related Groups (DRG) is a system for describing the patient case-mix in hospital care. It was originally developed in the US in 1970s (Fetter *et al*. 1976) and introduced in Norway in 1984/85. The Norwegian version of the DRG was based on a Nordic joint system called NordDRG [7], and was employed for activity-based financing [8] unveiled in Norway in 1997.

The objective of the DRG system is to create relatively cost homogeneous categories to compare hospital performance, focusing on the total hospital spell as the final product, measured as discharges defined according to the inpatient's diagnosis and treatment. For example, the variables used for category grouping in the Norwegian DRG system include diagnosis, procedures, sex, age, and status at discharge. [9]

The DRG system is particularly useful for measuring the output volume of hospital services because it provides information on unit costs per category of treatments and on the number of treatments that have been carried out. By construction, each category of treatments stands for a relatively homogenous services and thus, in principle, construction of a unit cost or of a volume index from the most detailed level of categories is feasible.

3.2. The DRG points

Assume the DRG system in time period *t* (e.g. a year *t*) has defined *N* categories of treatments, for each category *i* (*i* = 1, 2, …, *N*), there are Q_{it} treatments being carried out, which cost V_{it} in nominal monetary values in time period t (e.g. millions NOK in current prices), thus the (average) unit cost of treatment category *i*, P_{it}, can be defined as:

(1) $$P_{it} = \frac{V_{it}}{Q_{it}}.$$

In the Norwegian DRG system, a DRG (cost) weight D_{it} is defined for each category *i*, reflecting the medical complexity of treatment and resource consumption of one specific patient category in relation to the average for all patient categories.

The primary rationale behind this weighting system is that a simple operation or treatment will cost less and therefore should be assigned a relatively low DRG weight, to the contrary, a large and complicated operation or treatment will be more resource-intensive, and thus, ought to be highly weighted.

The cost for an average patient for all categories of treatments in the DRG system in time period *t*, $\overline{D_t}$, can be calculated as:

(2) $$\overline{D_t} = \frac{\sum_i V_{it}}{\sum_i Q_{it}}.$$

Then the DRG weight D_{it} is defined for each category *i* as:

[7] See https://www.nordcase.org/

[8] *Innsatsstyrt Finansiering (ISF)* in *Norwegian*: https://www.helsedirektoratet.no/tema/finansiering/innsatsstyrt-finansiering-og-drg-systemet/innsatsstyrt-finansiering-isf

[9] For more information about the Norwegian DRG system, please refer to: https://www.helsedirektoratet.no/tema/finansiering/innsatsstyrt-finansiering-og-drg-systemet/drg-systemet

(3) $$D_{it} = \frac{P_{it}}{D_t}.$$

Note that the (cost) weight as given by (3) is a relative weighting structure that is calibrated so that the average treatment has a value of one.

More important to know is that the term of '(cost) weight' is not used in the sense of a set of shares that sum to unity with which national accountants are familiar, but rather it is used in the sense of an adjustment coefficient. Therefore, the DRG cost weight is possible to be larger than one for some categories defined in the DRG system.

In the Norwegian DRG system, a DRG point DP_{it} is defined as the product of the DRG weight D_{it} and the corresponding number of treatments Q_{it} for category *i*:

(4) $$DP_{it} = D_{it} * Q_{it}.$$

Therefore, the total number of DRG points across all categories in the DRG system in time period t, DP_t, becomes:

(5) $$DP_t = \sum_i DP_{it}.$$

Commonly, the total DRG points can be regarded as the total activity levels where hospital treatments are adjusted for differences in patient compositions. Defined as such, one DRG point can essentially be considered as the cost for an average patient.[10]

[10] For instance, the reimbursement for one DRG point is set at NOK 44,654 in 2019, which is the cost for an average patient.

4. The current method

Using the above definitions (1) to (4), equation (5) can be written as:

(6) $$DP_t = \sum_i DP_{it} = \sum_i (D_{it} * Q_{it}) = \frac{\sum_i (P_{it} * Q_{it})}{\overline{D_t}} = \frac{\sum_i V_{it}}{\overline{D_t}} = \sum_i Q_{it}.$$

As shown in (6), the total number of DRG points are equal to the total number of treatments across all categories in the DRG system in time period *t*, although the DRG points for a specific category of treatments is not equal to the number of treatments in this category, as indicated by (4).

In recognition with this observation, the output volume Y_t of somatic services of the government-owned hospitals in time period *t* is currently measured as the total number of DRG points in the Norwegian national accounts system, i.e.,

(7) $$Y_t = DP_t,$$

and the corresponding volume growth (index) between time period *t-1* and *t*, $\frac{Y_t}{Y_{t-1}}$, as:

(8) $$\frac{Y_t}{Y_{t-1}} = \frac{DP_t}{DP_{t-1}} = \frac{\sum_i Q_{it}}{\sum_i Q_{it-1}}.$$

The implicit price growth (index) between time period *t-1* and *t*, $\frac{P_t}{P_{t-1}}$, is then calculated as:

(9) $$\frac{P_t}{P_{t-1}} = \left(\frac{\sum_i V_{it}}{\sum_i V_{it-1}}\right) / \left(\frac{Y_t}{Y_{t-1}}\right) = \frac{\overline{D_t}}{\overline{D_{t-1}}} = \left(\frac{\sum_i (P_{it} * Q_{it})}{\sum_i Q_{it}}\right) / \left(\frac{\sum_i (P_{it-1} * Q_{it-1})}{\sum_i Q_{it-1}}\right).$$

It is worth mentioning that not all of the output generated by the nonmarket hospitals owned by the central government are estimated by using the DRG points in the current Norwegian national accounts compilation system.

For example, hospital stays are used as weights for measuring psychiatry services, the number of day care and outpatient consultations is used for other various services. For drug-abuse treatments, the days of stay are used as a volume indicator, while the number of assignments is used for measuring ambulance services. Furthermore, for some production of the nonmarket hospitals where it is not possible to measure the volume directly, the use is made of cost price indices (Monsrud, 2020).

However, the focus of this paper is placed exclusively on those somatic services provided by the government-owned nonmarket hospitals that utilize the DRG system and the derived DRG points for measuring the output volume index of these nonmarket health services.

5. Comments on the current method

5.1. Consistency

There are two shortcomings by following the current method to measure the output volume index of somatic services rendered by the government-owned hospitals as reported in Section 4. The first is that the current method is essentially not a unit cost approach, and thus measures by using the current method is not easily aggregated in a consistent way, compared to those by following the suggested method which will be proposed and illustrated later in this paper.

The second shortcoming is that the current method is not in accordance with the long-time tradition in the Norwegian national accounts compilation system where the Laspeyres index formula has been almost universally applied for volume index construction, and the Paasche index formula for the corresponding price index construction.

With this tradition being respected, and given the definitions as outlined in subsection 3.2, the output volume index of somatic services provided by the government-owned hospitals, $\frac{Y_t^L}{Y_{t-1}^L}$, is suggested to be defined as the following Laspeyres volume index:

(10) $$\frac{Y_t^L}{Y_{t-1}^L} = \frac{\sum_i (P_{it-1} * Q_{it})}{\sum_i (P_{it-1} * Q_{it-1})} = \frac{\sum_i (D_{it-1} * Q_{it})}{\sum_i (D_{it-1} * Q_{it-1})}.$$

As shown in (10), the usual price of output (which is not observable in the cases of nonmarket activities) is replaced by the unit cost of the output, P_{it-1}, which serves as the weighting mechanism. Note that the use is made of (3) for the second equality in (10).

With the output volume index being defined as in (10), the corresponding output price index can be derived by applying the product test according to the index number theory (Frisch, 1930), i.e. it can be derived by dividing the cost value ratio by the output volume index as given in (10).

The implicitly derived and also suggested output price index is the Paasche index:

(11) $$\frac{P_t^P}{P_{t-1}^P} = \left(\frac{\sum_i V_{it}}{\sum_i V_{it-1}}\right) / \left(\frac{Y_t^L}{Y_{t-1}^L}\right) = \frac{\sum_i (P_{it} * Q_{it})}{\sum_i (P_{it-1} * Q_{it})}.$$

If one defines the cost value share of category *i* in time period *t-1* as:

(12) $$s_{it-1}^V = \frac{P_{it-1} * Q_{it-1}}{\sum_i (P_{it-1} * Q_{it-1})} = \frac{D_{it-1} * Q_{it-1}}{\sum_i (D_{it-1} * Q_{it-1})},$$

where the use is also made of (3) for the second equality in (12), which indicates that the cost value share of category *i* can also be interpreted as its corresponding DRG points share (see equation (4)).

Inserting (12) into (10) yields:

(13) $$\frac{Y_t^L}{Y_{t-1}^L} = \sum_i \left(s_{it-1}^V * \frac{Q_{it}}{Q_{it-1}}\right),$$

Comparing (8) with (13) will immediately confirm that the current estimate of the output volume index for nonmarket hospitals in the Norwegian national accounts system is the growth of the total number of treatments between time period *t-1* and *t*, without any weighting for each category of treatments in the DRG system (see (8)).

On the contrary, the output volume index by using the suggested Laspeyres formula as shown in (13) is a (time period *t-1*) cost share weighted, or equivalently, the DRG points weighted, average of the growth of treatments in individual category between time period *t-1* and *t*, where the unit cost as well as the DRG points play an important role through weighting (see (12)).

If one defines a quantity or treatment share of category *i* in time period *t* as:

(14) $$s_{it}^{Q} = \frac{Q_{it}}{\sum_i Q_{it}},$$

Then the output price index by following the current method and given in (9) can be expressed as:

(15) $$\frac{P_t}{P_{t-1}} = \left(\frac{\sum_i (P_{it} * Q_{it})}{\sum_i Q_{it}}\right) / \left(\frac{\sum_i (P_{it-1} * Q_{it-1})}{\sum_i Q_{it-1}}\right) = \frac{\sum_i \left(s_{it}^{Q} * P_{it}\right)}{\sum_i \left(s_{it-1}^{Q} * P_{it-1}\right)}.$$

Recall that (11) gives rise to the implicitly derived output price index (the Paasche price index) by following the suggested method in this paper, it can also be rewritten as:

(16) $$\frac{P_t^P}{P_{t-1}^P} = \frac{\sum_i (P_{it} * Q_{it})}{\sum_i (P_{it-1} * Q_{it})} = \left(\sum_i \left(s_{it}^{V} * \left(\frac{P_{it}}{P_{it-1}}\right)^{-1}\right)\right)^{-1},$$

where s_{it}^{V} is the cost share of category *i* in time period *t* which is defined as:

(17) $$s_{it}^{V} = \frac{P_{it} * Q_{it}}{\sum_i (P_{it} * Q_{it})} = \frac{D_{it} * Q_{it}}{\sum_i (D_{it} * Q_{it})}.$$

Note that the second identity indicates that the cost share of category *i* can also be interpreted as its corresponding DRG points share (see equation (4)).

As shown by (16) and (17), the implicit and suggested Paasche output price index is a (time period *t*) cost share weighted, and also the DRG points weighted, harmonic average of the ratios of the unit cost in individual category between time period *t-1* and *t*, where, again, the unit cost and the DRG points play an important role through weighting.

Clearly, the suggested output price index as defined by (16) is the right unit cost index, which is a weighted average of unit costs of individual category of treatments, with the cost share, and the corresponding DRG points of each category of treatments constituting the weight. It has been proved that both the Laspeyres and Paasche indexes have the property of consistency in aggregation, which means that the value of an index calculated in two stages coincides necessarily with the value of the index as calculated in a single stage (Diewert, 1978).

The output price index as given by (9) by following the current method is, unfortunately, not a unit cost in essence. [11] It is basically a volume indicator approach as discussed in subsection 2.2. Although the output volume and price index generated by the current method are of the property of consistency in aggregation, they are quite stringent.

Note that the discussions so far have focused implicitly on the total economy with all categories of treatment *i* (*i* = 1, 2, ..., *N*) included. Now consider a below-total economy level, say, a group of hospitals[12] with only a *s* part of the total categories of treatment being concerned, i.e., $i \in s$ and $s \subset (i = 1, 2, ..., N)$.

[11] See the definition of unit cost index in subsection 2.2. across levels

[12] The group of hospitals can contain only one hospital, or several hospitals combined together.

Define the total DRG points of the group of hospitals as DP_{st} at time period *t*, then the volume growth (index) of the group of hospitals between time period *t-1* and *t*, $\frac{Y_{st}}{Y_{st-1}}$, can be calculated by following the current method as:

(18) $$\frac{Y_{st}}{Y_{st-1}} = \frac{DP_{st}}{DP_{st-1}} = \frac{\sum_{i\in s} DP_{it}}{\sum_{i\in s} DP_{it-1}} = \frac{\sum_{i\in s}(D_{it} * Q_{it})}{\sum_{i\in s}(D_{it-1} * Q_{it-1})} = \frac{\overline{D_{t-1}} * \sum_{i\in s}(P_{it} * Q_{it})}{\overline{D_t} * \sum_{i\in s}(P_{it-1} * Q_{it-1})}.$$

Then the implicit price growth (index) of the group of hospitals between time period *t-1* and *t*, $\frac{P_{st}}{P_{st-1}}$, can be calculated as:

(19) $$\frac{P_{st}}{P_{st-1}} = \left(\frac{\sum_{i\in s} V_{it}}{\sum_{i\in s} V_{it-1}}\right) / \left(\frac{Y_{st}}{Y_{st-1}}\right) = \left(\frac{\sum_{i\in s}(P_{it} * Q_{it})}{\sum_{i\in s}(P_{it-1} * Q_{it-1})}\right) / \left(\frac{Y_{st}}{Y_{st-1}}\right) = \frac{\overline{D_t}}{\overline{D_{t-1}}}.$$

As shown by (18) and (19), following the current method, the output volume index of the group of hospitals is the ratio of the sum of the DRG points between time period *t-1* and *t*, while the output price index of the group of hospitals is uniformly equal to the ratio of the average patient cost across all categories of treatments in the DRG system between time period *t-1* and *t*, regardless of the size of the group of hospitals (see also the second equality in (9)).

If one think of the uniform price index $\frac{\overline{D_t}}{\overline{D_{t-1}}}$ as a general price index (see (19)), such as the headline CPI, then the volume index across all levels ($\frac{\overline{D_{t-1}} * \sum_{i\in s}(P_{it} * Q_{it})}{\overline{D_t} * \sum_{i\in s}(P_{it-1} * Q_{it-1})}$.) are derived by merely adjusting the value (expenditure/cost) ratio by the general and same price change such as a headline CPI at all levels, without concerning various possible relative price changes among different categories of treatment, and thus implicitly attributing all the change effects to the volume change. This does not make sense.

To sum up, compared with the current method applied at Statistics Norway, the suggested method in this paper is considered to be an improvement. In fact, the same method as suggested in this paper has been applied at the Office of National Statistics (ONS) for non-market output measurement for the National Accounts in the United Kingdom, which is presented in a recent note by ONS (2021).

In the next subsection, we shall explore under what conditions, the two sets of measures, one constructed by the current method (defined by (8) and (9)), and the other by the suggested method in this paper (defined by (10) and (11)), will be equal to each other.

5.2. Conditions for two sets of measures being equal

Sufficient condition

First, let us check some sufficient conditions for the two sets of measures being equal. Based on the definitions and discussions so far, the following two lemmas can be given:

Lemma 1: *If* $P_{it-1} = P_{ct-1}$, *then* $\frac{Y_t}{Y_{t-1}} = \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} = \frac{P_t^P}{P_{t-1}^P}$.

Proof: Inserting $P_{it-1} = P_{ct-1}$ respectively into (9) and (10) and using (11) and (8) yields:

(20) $$\frac{P_t}{P_{t-1}} = \left(\frac{\sum_i(P_{it} * Q_{it})}{\sum_i Q_{it}}\right) / \left(\frac{\sum_i(P_{it-1} * Q_{it-1})}{\sum_i Q_{it-1}}\right) = \frac{\sum_i(P_{it} * Q_{it})}{\sum_i(P_{ct-1} * Q_{it})} = \frac{P_t^P}{P_{t-1}^P},$$

(21) $$\frac{Y_t^L}{Y_{t-1}^L} = \frac{\sum_i (P_{it-1} * Q_{it})}{\sum_i (P_{it-1} * Q_{it-1})} = \frac{\sum_i (P_{ct-1} * Q_{it})}{\sum_i (P_{ct-1} * Q_{it-1})} = \frac{\sum_i Q_{it}}{\sum_i Q_{it-1}} = \frac{Y_t}{Y_{t-1}}.$$

Q.E.D.

Lemma 1 indicates that if the unit cost for each category of treatments in the DRG system is the same in time period *t-1*, then the output volume and price indexes measured by the current method are the same as those by following the suggested Laspeyres volume and Paasche price index formulas, respectively.

Lemma 2: *If* $Q_{it} = Q_{ct}$ *and* $Q_{it-1} = Q_{ct-1}$ *then* $\frac{Y_t}{Y_{t-1}} = \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} = \frac{P_t^P}{P_{t-1}^P}$.

Proof: Inserting $Q_{it} = Q_{ct}$ *and* $Q_{it-1} = Q_{ct-1}$ into (8) and (9) yields:

(22) $$\frac{Y_t}{Y_{t-1}} = \frac{\sum_i Q_{it}}{\sum_i Q_{it-1}} = \frac{Q_{ct}}{Q_{ct-1}}.$$

(23) $$\frac{P_t}{P_{t-1}} = \left(\frac{\sum_i (P_{it} * Q_{it})}{\sum_i Q_{it}}\right) / \left(\frac{\sum_i (P_{it-1} * Q_{it-1})}{\sum_i Q_{it-1}}\right) = \frac{\sum_i P_{it}}{\sum_i P_{it-1}}.$$

Then, inserting $Q_{it} = Q_{ct}$ *and* $Q_{it-1} = Q_{ct-1}$ into (10) and (11) gives:

(24) $$\frac{Y_t^L}{Y_{t-1}^L} = \frac{\sum_i (P_{it-1} * Q_{it})}{\sum_i (P_{it-1} * Q_{it-1})} = \frac{Q_{ct}}{Q_{ct-1}}.$$

(25) $$\frac{P_t^P}{P_{t-1}^P} = \frac{\sum_i (P_{it} * Q_{it})}{\sum_i (P_{it-1} * Q_{it})} = \frac{\sum_i P_{it}}{\sum_i P_{it-1}}.$$

Apparently, (22) and (24) are equal, and so are (23) and (25).

Q.E.D.

Lemma 2 states that if the number of treatments in time period *t* is the same across all categories in the DRG system, and that in time period *t-1* is the same across all categories as well,[13] then the output volume and price indexes measured by the current method are the same as those by following the suggested Laspeyres volume and Paasche price index formulas, respectively.

It is not common that the sufficient conditions for the two sets of measures being equal as stated in Lemma 1 and Lemma 2 hold in practice, which seemly implies that the two sets of measures are seldom equal to each other in practice. However, such a statement can only be made with fallacy.

It would be wrong if one draws such a quick conclusion that the two sets of measures are seldom equal in practice simply based on these two Lemmas. The reason is that Lemma 1 and Lemma 2 offer only two sets of sufficient conditions for the two sets of measures being equal. Recall that if a sufficient condition does not hold, it does not necessarily mean that the two sets of measures will not be equal, and there may exist many different (from Lemma 1 and Lemma 2) sufficient conditions.

In order to draw firm conclusions, the necessary conditions for the two sets of measures being equal should be identified. Only if such necessary conditions break, can one be certain that the two sets of measures must not be equal.

[13] Note that the number of treatments for category *i* in time period *t* is not necessarily required to be equal to that for the same category *i* in period *t-1*.

Necessary condition

Now let us have a look at the necessary condition for the two sets of measures being equal. Based on the definitions and discussions, the lemma giving the necessary condition can be written as:

Lemma 3: *If* $\frac{Y_t}{Y_{t-1}} = \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} = \frac{P_t^P}{P_{t-1}^P}$, *then* $\frac{\sum_i Q_{it}}{\sum_i Q_{it-1}} = \frac{\sum_i (P_{it-1} * Q_{it})}{\sum_i (P_{it-1} * Q_{it-1})}$,

equivalently,

If $\frac{Y_t}{Y_{t-1}} = \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} = \frac{P_t^P}{P_{t-1}^P}$, *then* $\sum_i \left(s_{it}^Q - s_{it-1}^Q\right) * P_{it-1} = 0$.

Proof: Equalizing either (8) with (10) or (9) with (11) gives rise to the same necessary condition as:

(26) $\frac{\sum_i Q_{it}}{\sum_i Q_{it-1}} = \frac{\sum_i (P_{it-1} * Q_{it})}{\sum_i (P_{it-1} * Q_{it-1})}$.

By inserting (14) into (26) and rearranging, the necessary condition becomes:

(27) $\sum_i \left(s_{it}^Q - s_{it-1}^Q\right) * P_{it-1} = 0$.

Q.E.D.

The first part of Lemma 3 (also see (26)) says that the necessary condition for the two sets of measures being equal is that the growth of the number of total treatments between time period *t-1* and *t* is equal to the growth of the cost share weighted number of individual category of treatments between time period *t-1* and *t* (see (13)).

The second part of Lemma 3 (also see (27)) indicates that the necessary condition for the two sets of measures being equal is that the sum of time period *t* treatment share weighted average of unit cost levels in time period *t-1* should be equal to the sum of time period *t-1* treatment share weighted average of the same unit cost levels in time period *t-1*.

5.3. Direction of possible biasedness

Most likely, the distribution of treatment share among categories changes between time period *t-1* and *t* in the DRG system in practice. The necessary condition as stated in Lemma 3 and given in (26) and (27) implies that the effect of the treatment share increases for some categories will be exactly offset by the effect of the treatment share decreases for other categories, taking into account that the unit costs vary across these categories. Therefore, the necessary condition is quite stringent, and will seldom hold in general.

As mentioned, if the necessary condition breaks, then one can be certain that the two sets of measures, one by the current method and the other by the suggested method, will be different.

Using the measures by following the suggested method in this paper as benchmark, the direction of possible biasedness of the measures by the current method relative to the benchmark can be investigated. Based on the necessary condition as given in (27), the following lemma regarding the direction of possible biasedness can be given:

Lemma 4: *If* $\sum_i \left(s_{it}^Q - s_{it-1}^Q\right) * P_{it-1} > 0$, *then* $\frac{Y_t}{Y_{t-1}} < \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} > \frac{P_t^P}{P_{t-1}^P}$,

and

If $\sum_i \left(s_{it}^Q - s_{it-1}^Q\right) * P_{it-1} < 0$, *then* $\frac{Y_t}{Y_{t-1}} > \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} < \frac{P_t^P}{P_{t-1}^P}$.

Proof: Inserting (14) into $\sum_i \left(s_{it}^Q - s_{it-1}^Q\right) * P_{it-1} > 0$ and rearranging yields:

(28) $$\sum_i \left(\frac{Q_{it}}{\sum_i Q_{it}} * P_{it-1}\right) > \sum_i \left(\frac{Q_{it-1}}{\sum_i Q_{it-1}} * P_{it-1}\right).$$

Because the assumptions of $\sum_i Q_{it} > 0$ and $\sum_i (P_{it-1} * Q_{it-1}) > 0$ will generally hold in practice, rearranging (28) gives:

(29) $$\frac{Y_t}{Y_{t-1}} = \frac{\sum_i Q_{it}}{\sum_i Q_{it-1}} < \frac{\sum_i (P_{it-1} * Q_{it})}{\sum_i (P_{it-1} * Q_{it-1})} = \frac{Y_t^L}{Y_{t-1}^L}.$$

Because the assumptions of $\frac{Y_t}{Y_{t-1}} > 0$ and $\frac{Y_t^L}{Y_{t-1}^L} > 0$ will generally hold in practice, using (9), (11), and (29) leads to:

(30) $$\frac{P_t}{P_{t-1}} = \left(\frac{\sum_i V_{it}}{\sum_i V_{it-1}}\right) / \left(\frac{Y_t}{Y_{t-1}}\right) > \left(\frac{\sum_i V_{it}}{\sum_i V_{it-1}}\right) / \left(\frac{Y_t^L}{Y_{t-1}^L}\right) = \frac{P_t^P}{P_{t-1}^P}.$$

By following the same fashion, it can be easily proved that if $\sum_i \left(s_{it}^Q - s_{it-1}^Q\right) * P_{it-1} < 0$, then $\frac{Y_t}{Y_{t-1}} > \frac{Y_t^L}{Y_{t-1}^L}$, *and* $\frac{P_t}{P_{t-1}} < \frac{P_t^P}{P_{t-1}^P}$.

Q.E.D.

Lemma 4 implies that if the average unit cost of those categories of treatments with increased treatment shares is larger than that of those categories of treatments with decreased treatment shares between time *t-1* and *t*, then the estimated output volume index by the current method will be downward biased, and the estimated output price index upward biased, compared to the corresponding volume and price indexes estimated by following the suggested method in this paper.

On the other hand, if the average unit cost of those categories of treatments with increased treatment shares is lower than that of those categories of treatments with decreased treatment shares between time *t-1* and *t*, then the estimated output volume index by the current method will be upward biased, and the estimated output price index downward biased, compared to the corresponding volume and price indexes estimated by following the suggested method in this paper.

In general, biased output volume index will result in biased measure of productivity growth along the same direction, i.e., other things being unchanged, upward (or downward) biased output volume index will lead to an upward (or downward) biased measure of productivity growth for the nonmarket activities discussed in this paper.

6. Conclusions

Somatic services rendered by the central government-owned hospitals account for a significant part of nonmarket activities in the Norwegian economy. The output volume index of these services has been measured by using the DRG system and the derived DRG points in the Norwegian national accounts.

This paper clarifies the important concepts such as the unit cost in the DRG system and demonstrates that the current method applied is essentially not a unit cost approach for measuring nonmarket activities. Despite consistency in aggregation, the output volume and price indexes by following the current method are quite stringent.

On the contrary, the suggested method in this paper is consistent with the tradition of applying the Laspeyres formula for volume index and the Paasche formula for price index construction in the Norwegian national accounts system. More important, the suggested method is the right unit cost approach, ensuring the consistency with the sound index number theory.

In general, the two sets of measures, one by the current method and the other by the suggested method, will differ. The paper also illustrates that under what conditions, the estimated measures by the current method are upward- or downward biased, compared to those by the suggested method. Clearly, other things unchanged, a biased measure of the output volume index will lead to a biased measure of productivity growth along the same direction for these important nonmarket activities in the Norwegian economy.

So far, this paper has implicitly assumed that the total categories of treatments are the same between time period *t-1* and *t*. However, there has been continuous reclassification of categories in the DRG system. To tackle this issue, one practically possible method is to group the detailed lower-level categories into larger groups, as implemented by Destatis (Statistisches Bundesamt, 2008).

However, another critical issue still exists even after the regrouping, i.e., how to deal with the appearance of new category of treatments, such as the treatment of Covid-19 disease. This is fundamentally a 'new good' problem, and is closely related, but not the same as, the quality adjustment issue.

As stated at the beginning of this paper, quality adjustment, whether implicit or explicit, is not discussed in this paper, the reason is that only if the issues discussed in this paper are solved, can the quality adjustment issue be investigated. To conclude, more quality work on measuring nonmarket activities is expected and therefore should be encouraged in future research.

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