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Modeling exchange rate, inflation, and interest rate in a small open economy

A data-based approach to estimating a New Keynesian model with rational expectations

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Abstract

Building on a New Keynesian rational expectations framework, we develop a structural empirical model that jointly determines the real exchange rate, inflation, and the nominal interest rate in a small open economy. Employing a full-information system design and estimation approach that avoids imposing unduly restrictive a priori parameter constraints, we identify a forward-looking Phillips curve while addressing simultaneity bias. Our empirical findings reveal persistent exchange rate dynamics that diverge from New Keynesian rational expectations but align with prior evidence, suggesting the presence of multiple equilibria.

Keywords: New Keynesian Phillips curve, structural time series modeling, simultaneous model design and estimation.

JEL classification: C32, C51, E12, E31, F41

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Sammendrag

Valutakursen, inflasjonen og renten er sentrale størrelser i en liten, åpen økonomi som den norske. Disse variablene er nært knyttet sammen både teoretisk og empirisk, og deres samtidige bestemmelse utgjør et viktig fundament for forståelsen av økonomiens virkemåte.

I denne artikkelen presenterer vi en strukturell modell for den simultane bestemmelsen av en bilateral valutakurs, en kjerneinflasjonsindeks og en pengemarkedsrente i Norge. Modellen er basert på en ny-keynesiansk teoriramme med rasjonelle forventninger og generell likevekt, og den estimeres ved hjelp av en fullinformasjonsbasert og strengt frekventistisk metode – det vil si uten bruk av kalibrering, a priori-fordelinger eller unødvendige parameterrestriksjoner.

Modellen kombinerer teoriens langsiktige likevektsimplikasjoner med fleksibel kortsiktig dynamikk, og gir dermed rom for at data kan være styrende for modellens egenskaper. Dette står i kontrast til vanlige tilnærminger i empirisk makroøkonomisk modellering, der teorien ofte pålegges modellen gjennom kalibrering eller bayesiansk estimering. Vår tilnærming gjør det mulig å identifisere en framoverskuende Phillipskurve og en utvidet Taylorregel med renteglatting, samtidig som vi finner markant kortsiktig persistens i valutakursen. Sistnevnte funn avviker fra teorien, men er i tråd med tidligere empiriske studier.

Persistens i valutakursen kan indikere eksistensen av flere stabile løsninger, snarere enn én unik likevektsbane, og utfordrer dermed noen av de grunnleggende antakelsene i ny-keynesiansk teori. Dette kan ha betydning for hvordan man vurderer stabilitet og politikkrespons i økonomiske modeller. Samtidig viser vi at valutakurslikningen i vår modell er konsistent med den som benyttes i Finansdepartementets makroøkonomiske modell, og at vår modell omslutter resultatene fra tidligere norske studier.

På bakgrunn av disse funnene er det særlig verdt å merke seg det sentrale bidraget i artikkelen; hvordan en streng teoretisk modellramme kan forenes med en datadrevet og simultan empirisk tilnærming – uten å pålegge unødvendige og for restriktive a priori-restriksjoner. Ved å la dataene være avgjørende for modellens dynamikk, snarere enn å påtvinge teorien, viser vi at det er mulig å bevare teoretiske mekanismer i empirisk modellering – uten at det går på bekostning av statistisk validitet.

Et viktig funn i denne sammenhengen er valideringen av Phillipskurven, som knytter inflasjonsdifferansen til et aktivitetsmål, den reelle valutakursen og forventet framtidig inflasjonsdifferanse. Dette resultatet var uventet gitt tidligere funn basert på norske data, men det er robust innenfor vår modell og antyder mulige simultanitetsskjevheter i tidligere studier. Funnene understreker også den komplekse dynamikken som er involvert, og inviterer til en revurdering av etablerte empiriske rammeverk i økonomisk forskning.

1 Introduction

For a small open economy, the exchange rate plays a vital role in the functioning of the economy. Other financial variables such as interest rates and prices are also important for the working and evolution of the economy, see Woodford (2003). Joint modeling of these macro variables can thus be an important determinant of a model's properties, especially for a small open economy.

The exchange rate, inflation, and interest rate are interconnected and more or less jointly determined in economic theories. That motivates a simultaneous approach to empirical modeling of these variables. Simultaneity adds challenges in empirical modeling. Based on a theoretical New Keynesian model with rational expectations, we design and estimate a dynamic, structural, and simultaneous empirical model of a nominal exchange rate, a core consumer price index, and a money market interest rate in Norway. The theoretical model is taken to the data with a full information and strictly frequentist approach, without any a priori parameter distributions or restrictions on parameter values. We have not seen such a combination of theory and empirical methodology before in the literature. It is in contrast to the (Bayesian) approach common to empirical modeling of New Keynesian DSGE models, and also in contrast to a theory-lean (single-equation) approach often found in time series modeling.

The empirical model is based on a New Keynesian theoretical model of a two-country, two-market (domestic-foreign) economy. Within a rational expectations and general equilibrium framework, representative consumers and monopolistic producers are supposed to have equal and symmetric preferences and technology, and to maximize their utility and the discounted present value of their expected flow of future profit, as in Engel (2014). Even though the theoretical framework is rather strict, our empirical model is consistent in the long run with a New Keynesian rational expectations model. We find empirical confirmation of a forward-looking Phillips curve and an extended Taylor rule with interest rate smoothing. There is an inconsistency in the short-run dynamics, though. We find persistence in the exchange rate. Persistence implies a backward-looking exchange rate process rather than a forward-looking one, and that is not consistent with theory. On the other hand, it could be a (contributing) reason for the model to have multiple stable solutions rather than a unique, so-called saddle path solution.

The theoretical model contains a Phillips curve tailored for an open economy, where the inflation differential between two countries is modeled as a function of the real exchange rate and the expected future inflation differential.The Phillips curve dates back to the late 1950s, but received renewed interest from monetary economists with the rise of inflation targeting in central banks in the 1990s. Despite a strong theoretical foundation for a New Keynesian Philips curve, its empirical confirmation is rather weak. It is common practice in New Keynesian DSGE models to more or less impose a Phillips curve by using a mix of calibration and Bayesian estimation, e.g. Lindé et al. (2016). In the present paper, we estimate a statistically significant effect of expected future price inflation by a purely frequentist estimation method (Full Information Maximum Likelihood, abbreviated FIML), while maintaining a fully structural framework.

Using Norwegian unadjusted quarterly data, we design and estimate freely (without calibration, priors, or restrictions on the parameter values) a structural vector equilibrium correcting model (SVECM). In addition to the Phillips curve, the theory motivates long-run equilibrium conditions, which function as attractors for the short-term fluctuations of the three endogenous variables in the model. The dynamics is thus more general than in the theoretical dynamic equilibrium model per se.[1] While theory determines the underlying long-run relationships between the three modeled variables, the added short-term dynamics is crucial for the model to fit the data. Replacing restrictions with dynamics is how we make the theory data admissible.

While persistence in the exchange rate is inconsistent with the theory, the exchange rate equation is completely consistent with the exchange rate equation currently implemented in the Norwegian econometric model used by the Norwegian Ministry of Finance for forecasting purposes, cf. Boug et al. (2023). Our structural model for the simultaneous determination of prices, interest rates, and exchange rates encompasses the results in Benedictow and Hammersland (2023).

Theory with a strong standing is often more or less imposed on empirical models when the data do not support it. The novelty of our paper is that it demonstrates an empirical modeling approach that might preserve theory when data are allowed to be decisive. The approach uses simultaneous design and estimation, flexible short-term dynamics, and an empirical covariance matrix for the

[1] An unusual approach is advocated in Canova (2014), which aims at preserving the model's dynamic and systemic properties while being flexible in the design of the models' long-run structure.

vector of error terms rather than an a priori restricted diagonal covariance matrix.[2]

The rest of this paper is organized as follows. Section 2 gives an overview of so briefly presents the theoretical framework. For a detailed exposition see Engel (2014). In Section 4, we present the data used in the design and estimation of our *empirical* model. Section 5 reviews the SVECM modeling. The empirical model is evaluated in Section 6. Section 7 summarizes and concludes. Details and technicalities are relocated to several appendices.

2 Literature

There is an extensive body of literature on theoretical New Keynesian models in monetary economics, as exemplified by Gali (2018). Similarly, there is a substantial literature on empirical econometric time series modeling, with notable contributions from Hendry (1995) and a summary provided by Faust and Whitman (1997). But there is less literature on the combination of the two. One reason could be that the evaluation criteria for theoretical models and empirical models are fundamentally different. New Keynesian models focus on certain short-term properties/stylized facts, like temporal responses to shocks, typically displayed in impulse-response graphs. Empirical/quantitative properties of the models are not strong, see e.g. Chari et al. (2009), Canova (2014) and Juselius (2022). Their strength is more in policy analysis than in fitting data and forecasting. Econometric time series models try to reproduce all frequencies in the data, and avoid filtering out low-frequency information (like levels, trends, cycles). Incorporating (New Keynesian) macro theory with micro foundations is not their forte. For any kind of model, it has proved difficult to take theory to raw data without relaxing either theoretical or empirical rigor (or both). A well-known example of the former is Calvo pricing, which allows lags in the theoretical model to account for a sluggish response in the data, thereby rationalizing empirical inertia with nominal rigidities or staggered pricing; cf. Calvo (1983). Another similar example is the hybrid Phillips curve. Conversely, a common empirical practice involves relaxing the fit and statistical diagnostics to incorporate theoretical variables, such as expectations, and *a priori* imposing restrictions that maintain the model's theoretical validity. Juselius and Franchi (2007) "take a DSGE model to data" and Juselius (2022) confronts a monetary theory model for nominal exchange rate determination with German-US data 1978–1998. Both papers problematize theory models' limited ability to account for time series properties of the data and thus illustrate the methodological divide mentioned above. The present paper is an attempt to bridge the gap.

In econometric single-equation time series modeling, validating the rational expectations hypothesis remains challenging, in particular with Norwegian data. Accurately measuring the output gap and inflation expectations is difficult. It often results in biased estimates due to the failure of single-equation models to capture dynamic interactions between inflation and other endogenous variables (Rudd and Whelan, 2005). Two Norwegian studies offer valuable insights. Baardsen et al. (2004) find limited support for the traditional NKPC. Incorporating the real exchange rate as a forcing variable significantly enhances model performance. This finding is supported by Boug et al. (2017). In a preliminary VAR analysis, they identify a long-term relationship that includes the real exchange rate. Including this relationship in a conditional dynamic model of prices significantly improves the fit of their model. Lindé (2005) argues that FIML estimation can provide support for the New Keynesian Phillips Curve (NKPC) whereas single-equation (limited-information) estimation methods, e.g. GMM, fail to do so. He illustrates this by estimating a theoretical 3-equation model of inflation, output gap and interest rate on demeaned quarterly US data 1960–1997. Kleibergen and Mavroeides (2009) discuss weak instruments in GMM and reexamine evidence on the (single-equation) NKPC based on quarterly US data 1960–2007. They find that forward-looking dynamics dominate backward-looking dynamics.

Boug et al. (2017) highlight simultaneity and endogeneity issues that complicate identification of causal relationships. However, they do so without specifying a dynamic simultaneous model based on a structural theoretical formulation of the underlying mechanism. In a non-simultaneous and conditional model of price formation, they find that including both forward- and backward-looking

[2]Imposing the covariance matrix of the structural model's disturbances to be diagonal may be theoretically substantiated. However, the matter presents itself quite differently when constructing empirical models on real data since there is little to suggest that the empirical covariance matrix of an estimated structural form model should inherit the properties of its theoretical counterpart. This follows as a consequence of utilizing empirical proxies for theoretical variable constructs and from the fact that empirical models in most cases are linear approximations to nonlinear theoretical models. We may add to this the inherent problem of omitted variables and the fact that theories, after all, are revised in light of ongoing theorizing.

components improves the fit of the model on Norwegian inflation data.

Empirical support for the NKPC in an open economy — in particular incorporating the real exchange rate as a forcing variable (like in this paper) — has been explored in international studies. However, there are few studies that have investigated this for a *small* open economy like Norway, and in addition utilised a fully structural approach backed by a corresponding theoretical model. This can be seen as a shortcoming as the inclusion of the real exchange rate in the NKPC — as we show in this paper — can be theoretically motivated. It would also have significant implications for monetary policy, especially in small open economies where external shocks can substantially affect domestic inflation. Studies by Leith and Malley (2007) and Batini et al. (2005) show that real exchange rate movements significantly impact inflation dynamics. The latter study demonstrates that ignoring this factor would lead to serious model misspecification.

Batini et al. (2005) also argue that exchange rate movements have important pass-through effects on domestic prices, supporting the need for an extended NKPC in open economies. The pass-through of exchange rate changes to import prices is a crucial channel, with varying degrees of influence on domestic inflation; see e.g. Ihrig et al. (2006). Advanced econometric techniques, such as VARs and state-space models, have been employed to better capture the dynamics between inflation, the output gap, and the real changes in exchange rates; see Bussiére et al. (2014).

The present paper addresses the lack of a fully structural approach in previous analyses of the Norwegian inflation process. Our objective is to see if there is empirical support for a NKPC within a more advanced analysis. The theoretical starting point for such an analysis is a New Keynesian rational-expectations equilibrium model for a two-country, two-market economy. Since Norway is a small open economy, it is particularly relevant to include the (real) exchange rate as one of the endogenous variables in the analysis. We develop a dynamic empirical model using a simultaneous and structural model design, where all equations belonging to an underlying simultaneous structure are designed and modeled simultaneously. This is different from single-equation modeling, where the equations are modeled individually, one at a time. Our procedure incorporates both theoretical and empirical criteria, allowing theory to influence the long-term structure and empirical data to determine short-term properties.

3 Theory

We briefly present the theoretical model, summarizing Sections 1 and 2.1 of Engel (2014). The theoretical model itself is not our primary concern. It basically motivates the long-run structure of our empirical model, i.e. stable linear relationships between exchange rates, interest rates and inflation (and possibly other variables) in a small open economy and in the rest of the world.

We assume a New Keynesian model with two countries, two markets, optimizing households and monopolistic firms, competitive labor markets, mobile workers, and sticky prices. We depart from Section 2.1 in Engel (2014) by including a risk premium[3] and resorting to a monetary policy rule of flexible inflation targeting, including an output gap in the policy rule. For simplicity of exposition, we leave out the shock/error terms.

Deviation from uncovered interest rate parity in terms of a foreign exchange risk premium λ_t implies that the interest rate difference between domestic and foreign deposits can be expressed by

$$dI_t \equiv I_t - I_t^* = \mathsf{E}_t(\Delta s_{t+1}) - \lambda_t, \tag{1}$$

where the nominal interest rates on central bank deposits in the domestic and foreign currencies are denoted by I_t and I_t^*, respectively. The nominal exchange rate s_t is the logarithm of the price of the foreign currency in units of the domestic currency, and $\Delta s_{t+1} \equiv s_{t+1} - s_t$ represents the change in the exchange rate from period t to t + 1. $\mathsf{E}_t(\Delta s_{t+1}) = \mathsf{E}_t(s_{t+1}) - s_t$ is the expected change from the current period to the next conditional on all information available to the market at time period t. The real exchange rate is defined by $q_t \equiv s_t - (p_t - p_t^*)$, where p_t and p_t^* denote the logarithm of the level of domestic and foreign prices. Substituting this in (1) yields

$$dI_t = \mathsf{E}_t(\Delta q_{t+1}) + \mathsf{E}_t(d\pi_{t+1}) - \lambda_t, \tag{2}$$

where $d\pi_{t+1} \equiv \pi_{t+1} - \pi_{t+1}^*$ is the inflation difference between the two countries: $\pi_{t+1} \equiv \Delta p_{t+1} \equiv p_{t+1} - p_t$ is domestic price inflation and $\pi_{t+1}^* \equiv \Delta p_{t+1}^*$ is foreign price inflation.

Symmetry between the two countries implies that there is no "home bias" in consumption. In each country, the central bank is assumed to set its interest rate according to a Taylor-rule which reacts to the deviation of inflation from the target of the central bank, the output gap, and possibly other variables.[4]

Assuming that the inflation target, the long-term equilibrium real interest rate and potential output are the same in both countries, the interest rate difference, can then be expressed as

$$dI_t = \sigma d\pi_t + \rho dy_t + \alpha dI_{t-1}, \qquad 0 < \sigma,\ 0 < \rho,\ 0 < \alpha < 1,\ 1 < \sigma + \alpha, \tag{3}$$

where $dy_t \equiv y_t - y_t^*$ is the difference between the domestic and foreign output. It is assumed that the Taylor principle applies, so that $1 < \sigma + \alpha$.

Sticky (Calvo) local-currency price-setting and symmetry between the two countries imply an aggregate Phillips curve (see Appendix C for a derivation of (4) and interpretation of Θ):

$$d\pi_t = \delta q_t + \beta \mathsf{E}_t(d\pi_{t+1}), \qquad 0 < \delta = (1-\Theta)(1-\Theta\beta)/\Theta,\ 0 < \beta < 1. \tag{4}$$

The most direct and simple way to combine the equations (2)-(4) into a dynamic system of three equations for the simultaneous determination of domestic versus foreign inflation difference $d\pi$, the real exchange rate q, and the domestic versus foreign interest rate difference dI, is

$$\underbrace{\begin{bmatrix} 1 & -\delta & 0 \\ 0 & 1 & 0 \\ \sigma & 0 & \alpha \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_{t-1} \end{pmatrix}}_{\boldsymbol{z}_t} = \underbrace{\begin{bmatrix} \beta & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{B}} \underbrace{\mathsf{E}_t \begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_t \end{pmatrix}}_{\mathsf{E}_t(\boldsymbol{z}_{t+1})} + \underbrace{\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -\rho \end{bmatrix}}_{\boldsymbol{C}} \underbrace{\begin{pmatrix} \lambda_t \\ dy_t \end{pmatrix}}_{\boldsymbol{x}_t}, \tag{5}$$

[3] In the econometric model this premium will be replaced by exogenous variables intended to capture it.

[4] Denote the long-run equilibrium interest rates in the two countries, the "Wicksellian" real interest rates, by $\tilde{r}_t$ and $\tilde{r}_t^*$. Correspondingly, we denote the output and inflation gaps by $y_t^c - \bar{y}^c$ and $\pi_t^c - \bar{\pi}^c$, where the country-superscript c stands for either a blank for the domestic country or an asterisk for the foreign country. The bar denotes the potential output and the inflation target. The Taylor-rule for the domestic and the foreign country share the same coefficients and take the following forms:

$$I_t = (1-\alpha)\bar{\pi} + \tilde{r}_t + \sigma(\pi_t - \bar{\pi}) + \rho(y_t - \bar{y}) + \alpha(I_{t-1} - \tilde{r}_{t-1}) + \varepsilon_t,$$
$$I_t^* = (1-\alpha)\bar{\pi}^* + \tilde{r}_t^* + \sigma(\pi_t^* - \bar{\pi}^*) + \rho(y_t^* - \bar{y}^*) + \alpha(I_{t-1}^* - \tilde{r}_{t-1}^*) + \varepsilon_t^*.$$

The residuals ε_t and ε_t^* account for other influences on the two interest rates.

where $\boldsymbol{z}_t \equiv (d\pi_t, q_t, dI_{t-1})'$ denotes a vector of the three endogenous variables, $\boldsymbol{x}_t \equiv (\lambda_t, dy_t)'$ denotes a vector of the two exogenous variables, and $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ denote matrices of size 3×3, 3×3 and 3×2. The system (5) can then be written compactly as $\boldsymbol{A}\,\boldsymbol{z}_t = \boldsymbol{B}\,\mathsf{E}_t(\boldsymbol{z}_{t+1}) + \boldsymbol{C}\,\boldsymbol{x}_t$. In the following we drop the underbrace in the vector-matrix equations to save space.[5]

As $\boldsymbol{A}$ is invertible, (5) can be written in a forward form (where current variables depend on future expectations rather than the opposite) as $\boldsymbol{z}_t = \boldsymbol{A}^{-1}\boldsymbol{B}\,\mathsf{E}_t(\boldsymbol{z}_{t+1}) + \boldsymbol{A}^{-1}\boldsymbol{C}\,\boldsymbol{x}_t$, or explicitly as

$$\underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_{t-1} \end{pmatrix}}_{\boldsymbol{z}_t} = \underbrace{\begin{bmatrix} \beta+\delta & \delta & -\delta \\ 1 & 1 & -1 \\ \sigma(\beta-\delta)/\alpha & -\sigma\delta/\alpha & (\sigma\delta+1)/\alpha \end{bmatrix}}_{\boldsymbol{A}^{-1}\boldsymbol{B}} \underbrace{\mathsf{E}_t \begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_t \end{pmatrix}}_{\mathsf{E}_t(\boldsymbol{z}_{t+1})} + \underbrace{\begin{bmatrix} \delta & 0 \\ 1 & 0 \\ \sigma\delta/\alpha & -\rho/\alpha \end{bmatrix}}_{\boldsymbol{A}^{-1}\boldsymbol{C}} \underbrace{\begin{pmatrix} \lambda_t \\ dy_t \end{pmatrix}}_{\boldsymbol{x}_t}. \tag{6}$$

In the system (6), the two current-dated endogenous variables $d\pi_t$ and q_t depend on future expectations. They are *not* predetermined at time t and thus called free variables. The system has a unique and stable saddle-path solution if the number of (possibly complex) eigenvalues of the matrix $\boldsymbol{A}^{-1}\boldsymbol{B}$ inside the unit circle equals the number of free variables (i.e. two). If there is only one or no eigenvalue inside the unit circle, the system will exhibit multiple equilibrium solutions, commonly referred to as sunspots. Such a situation could arise if the real exchange rate followed a backward-looking process rather than a forward-looking one. The system will not have a solution if the number of eigenvalues inside the unit circle is greater than the number of free variables (i.e. three). For a more detailed explanation of these possibilities, cf. Blanchard and Kahn (1980) (or Appendix F), Woodford (2003) or Tsiaras (2017), chapter 4.

The theoretical model in this section does not lend itself to data confrontations directly, without modifications. The purely theoretical model is not data admissible. Its stylized dynamics are far too limited. Estimating a system like (6) on Norwegian data yields poor diagnostic properties and a model that cannot reproduce short- or long-term movements in the data, cf. Juselius and Franchi (2007). However, the theoretical models in this section can motivate the long-run structure of a simultaneous *empirical* model of the three variables. In the next sections, we will deviate from the theoretical equations in our design of a dynamic model, but not lose connection with the theory.

4 Data

We use quarterly data from the first quarter of year 2000 until the fourth quarter of 2019, i.e. 80 observations.The sample period begins just before the central bank of Norway began a regime of flexible inflation targeting. The sample period ends before the Corona pandemic, which was a huge shock to economies and their data-generating processes. Norwegian data are of (relatively) high quality and available to the public from Statistics Norway and Macrobond. Data for the euro zone are taken from Macrobond. Appendix D contains more details on the data and their sources.

Figure 1 shows the time series data for the variables in Section 3. Panel A shows the quarterly interbank interest rate for Norway (I_t) and the euro area (I_t^*), and their difference (dI_t). Panel B shows the logarithm of the price of euro in units of Norwegian krone (s_t), and its forward change to the next quarter (Δs_{t+1}). Panel C shows the real exchange rate (q_t), and the consumer price index for Norway (p_t) and the euro area (p_t^*). Panel D shows the difference between the quarterly inflation rates in Norway and the euro area ($d\pi_t$), and the quarterly changes in the consumer price index in Norway ($\pi_t = \Delta p_t$) and the euro area ($\pi_t^* = \Delta p_t^*$). Panel E shows the production-based output gaps in Norway ($y_t - \bar{y}_t$) and the euro area ($y_t^* - \bar{y}_t^*$), and their difference (dy_t). Panel F shows the unemployment rate in Norway (u_t) and the euro area (u_t^*), and a measure of the difference in the output gap based on the unemployment rates ($1/u_t^2 - 1/u_t^{*2}$).

The time series data discussed in this section do not unequivocally support the theoretical model presented earlier. However, by adding exogenous explanatory variables and allowing more dynamic flexibility (different lags), we shall see that the theory to a large extent can be contained within a larger dynamic econometric model that fits the data to some acceptable degree.

[5] An alternative way to combine and arrange the three equations (2)-(4) into a system, yields the expression

$$\underbrace{\mathsf{E}_t \begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_t \end{pmatrix}}_{\mathsf{E}_t(\boldsymbol{z}_{t+1})} = \underbrace{\begin{bmatrix} 1/\beta & -\delta/\beta & 0 \\ \sigma - 1/\beta & 1+\delta/\beta & \alpha \\ \sigma & 0 & \alpha \end{bmatrix}}_{\boldsymbol{D}} \underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_{t-1} \end{pmatrix}}_{\boldsymbol{z}_t} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & \rho \\ 0 & \rho \end{bmatrix}}_{\boldsymbol{G}} \underbrace{\begin{pmatrix} \lambda_t \\ dy_t \end{pmatrix}}_{\boldsymbol{x}_t}.$$

Apart from our additional exogenous term $\boldsymbol{G}\boldsymbol{x}_t$ and our exclusion of residuals for simplification of exposition, this expression is equal to equation 18 in Engel (2014).

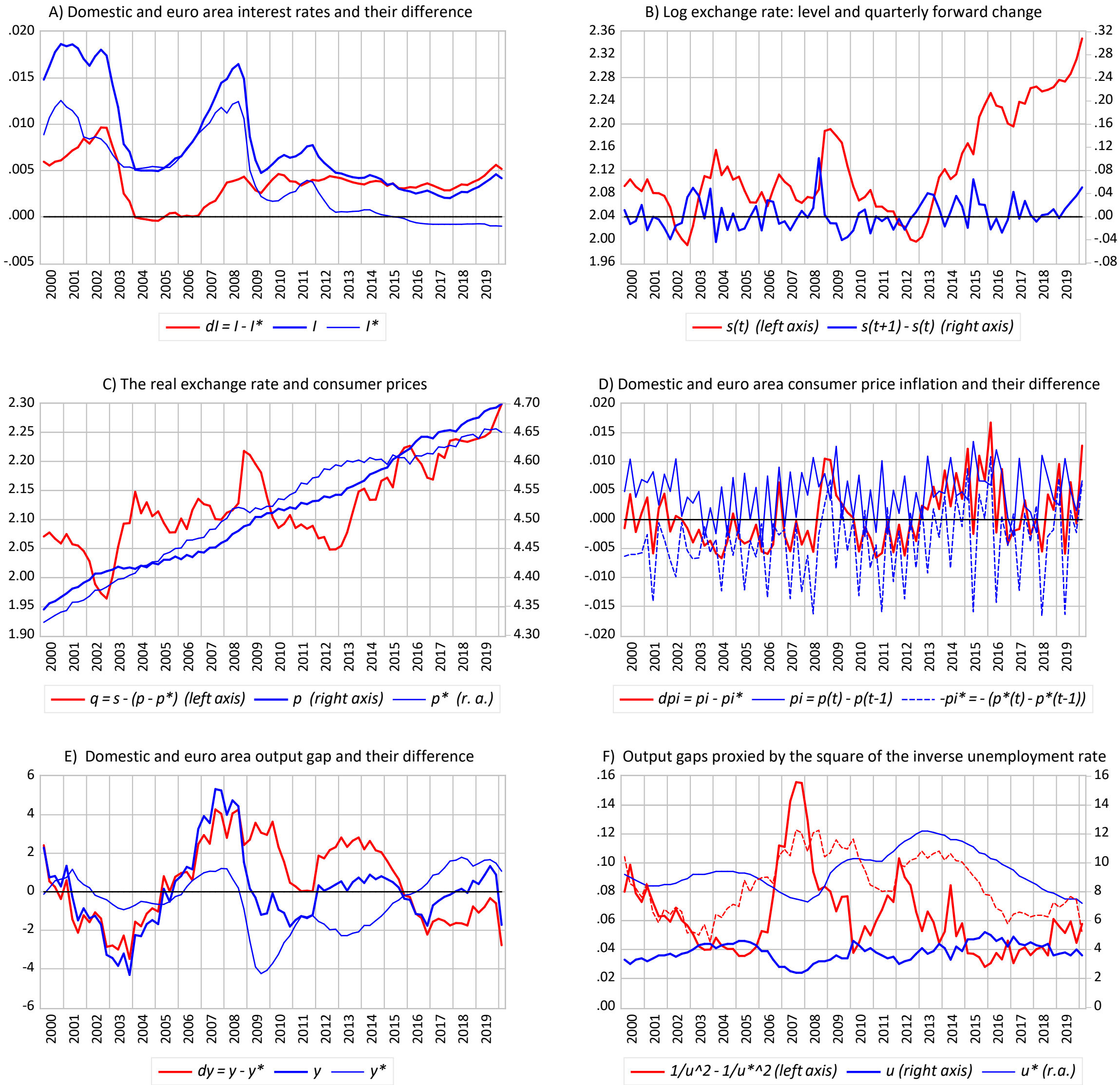

Figure 1: Time series data for variables in Section 3. In the bottom row of panels, the output gap difference dy in panel E is also graphed by a thin red line in panel F for easy comparison of the two measures of output gap difference.

Panels E and F show two different measures of the output gap difference between Norway and the euro area. The measure of output gap graphed by the red line in panel E is the difference between the actual production and an estimate of trend production. Trend production is estimated by the Hodrick-Prescott filter with default $\lambda = 1600$. The measure of output gap graphed by the red line in panel F is the inverse of the squared unemployment rate. The graph of the output gap difference in panel E is rescaled and overlayed (as a thin dashed red graph) on the output gap difference in panel F to facilitate visual comparison (the correlation is 0.56). In the empirical modeling in Section 5, both measures are used, and one will be selected and the other discarded by the data.

The time series data shown in Figure 1 cannot be said to provide unqualified support for our theoretical model without further analysis. The interest rate differential (both real and nominal), the two inflation rates, the inflation differential, and the two measures of the output gap difference all appear to be stationary time series and thus support most aspects of our theoretical framework. The relatively high degree of covariance between the two measures of the output gap also suggests that they capture more or less the same underlying economic condition. However, according to panel C in Figure 1, the level of the real exchange rate appears to be subject to a trend break in the latter part of the period (from around 2017 onward). Breaking level-stationarity by trending would make the Phillips curve relationship unbalanced. That would challenge the empirical validity of the theoretical framework.

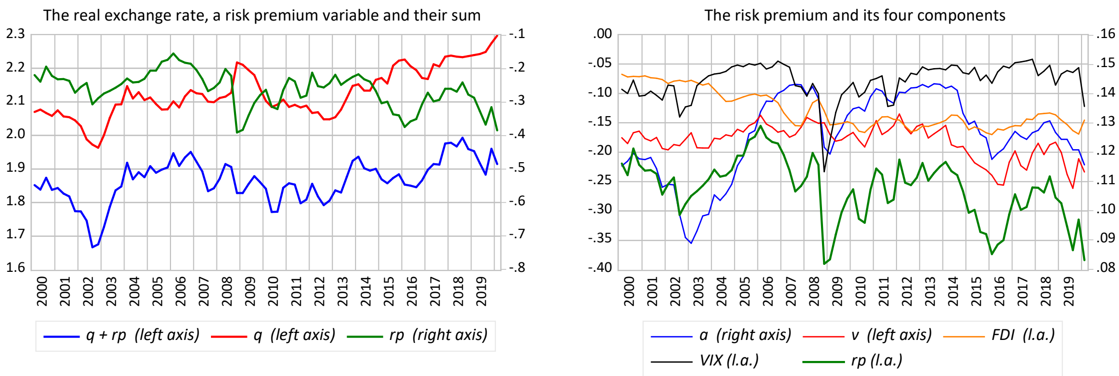

Figure 2: Time series data for the real exchange rate (q) and the risk premium (rp) (left panel), and the components of the the risk premium $rp = 0.023 * log(a) + 0.23 * log(v) - 0.004 * (FDI + VIX)$: a petroleum-related equity index (a_t), the export value of oil and gas as a share of the total value of exports (v_t), the difference between inward foreign direct investments as a percentage of GDP in the euro area and in Norway (FDI), and a volatility index related to the S&P500 stock market index in the USA (VIX).

Stationarity tests in Appendix A show that the real exchange rate q_t, if corrected for a trend, can be considered a stationary variable. The cointegration results in Benedictow and Hammersland (2023) (reproduced in Appendix B) show that this trend can be approximated by a linear combination of variables that capture a risk premium (denoted λ_t in Section 3). This is seen to some extent in Figure 2, which shows the real exchange rate and the linear combination of risk-premium variables in their analysis. The linear combination is also seen in the second to the last line in Equation (13) in Section 5.

5 Empirical modeling

In the natural sciences, a model is predominantly perceived as a simplified representation of a complex nature. In economics, a model can often be conceived more as a tool. The theoretical model in Section 3 aims to aid our thinking and understanding of the partial economy. Thus, it can be seen as a tool for logical economic thinking. Other types of models are better suited for applied economic analysis or for economic forecasting. The dynamic econometric model we propose in this section has several purposes. Being an empirical model, it tries to explain and mimic levels and movements of data. We incorporate theory in the form of equilibrium relationships between levels of variables. These relationships more or less explain the levels and persistent movements of time series data. Changes and temporary movements of data are mimicked by adding differenced variables at future, current and lagged datings to the relationships. The model blends long-run level-relationships and short-term changes to fit the data to certain statistical degrees. Our simultaneous multivariate model tries to connect and explain three variables of interest. Using econometric methodology, the model brings theory to data *without* imposing an overly number of overidentifying theoretical restrictions.

Based on the theoretical framework in Section 2, we design and estimate an empirical Structural Vector Equilibrium Correction Model (SVECM) for the simultaneous determination of three variables: the logarithm of the real exchange rate, $q = s - dp$, where $dp = p - p^*$ is the difference between the logarithms of the domestic and foreign price levels; the 3-months money-market interest rate, I; and the quarterly change in the logarithm of the fraction of the domestic to foreign consumer price index adjusted for taxes and energy prices, $d\pi = \pi - \pi^* = \Delta dp$. The starting point for our empirical modeling is a General Unrestricted Model (GUM) that encompasses (includes, encloses) both the data and the theoretical model outlined in Section 2 and specified in system (5). Our modeling approach seeks to preserve the long-run properties of the theoretical model while being flexible with the short-term dynamics of the econometric model.

Incorporating more dynamic terms than in the theoretical model increases the empirical model's ability to fit ('explain') data ex post and predict data ex ante, but has a potential cost of jeopardizing the theoretical model's systemic properties. A theoretical model expresses economic logic in a simplified and stylized form. Hence, it provides predominantly qualitative explanations of economic behavior. An empirical model tries to make sense of the data in light of economic theory and to some extent quantify the theory. In dynamic modeling with time series data, it is often necessary to make changes and extensions to the theoretical model. Changes in the dating of some variables compared to the theoretical model is hardly an objection when theory is mainly about the long run or equilibrium and has little to say about the short run and dating of variables. However, time is an important dimension in the New Keynesian models with rational expectations, but mainly logically and qualitatively, i.e. forward-looking vs. *not* forward-looking. Adding lagged and differenced variables to improve the fit of the model to data generally changes some properties of the empirical model relative to the theoretical model. Theoretical models with forward-looking agents (only) typically respond quickly to changes and struggle to explain the sluggish behavior observed in the data. Adding lagged variables improves on this, like in the Calvo-pricing theory.

We define a vector of model variables $\boldsymbol{v}_t' = (s_t, p_t, p_t^*, \lambda_t, I_t, I_t^*, y_t, y_t^*)$, where the topscript $*$ denotes the euro area. The risk premium λ_t is a function of variables introduced in Section 3 and explained below. Deterministic terms such as centered seasonals, step and impulse dummies[6] are collected into a vector $\boldsymbol{d}_t$. The GUM corresponding to the SVECM can be written as

$$\Delta dp_t = c_\pi + \delta q_t + \beta_\pi \mathsf{E}_t \Delta dp_{t+1} + \gamma_\pi \Delta dp_{t-1} + \boldsymbol{\theta}_\pi{}' \boldsymbol{d}_t + \epsilon_t^{dp}, \tag{7}$$

$$\Delta q_t = c_q - \phi_q\{q_{t-1} + \lambda_{t-1} + \alpha\, dI_{t-1} - \omega_q d\pi_t\} + \beta_q \mathsf{E}_t q_{t+1} + \sum\nolimits_{i=0}^{k} \boldsymbol{\mu}_i{}'\, \Delta \boldsymbol{v}_{t-i} + \boldsymbol{\theta}_q{}' \boldsymbol{d}_t + \epsilon_t^q, \tag{8}$$

$$\Delta I_t = c_I - \phi_I\{dI_{t-1} - \omega_I d\pi_t - \rho\, dy_t\} + \sum\nolimits_{i=0}^{k} \boldsymbol{\eta}_i{}'\, \Delta \boldsymbol{v}_{t-i} + \boldsymbol{\theta}_I{}' \boldsymbol{d}_t + \epsilon_t^I, \tag{9}$$

where lower normal case letters denote logarithmically transformed variables and lower bold case letters denote column vectors. The endogenous variable ΔI_t on the left side of (9) differs from the one in the theoretical model in Section 3. The dynamic effect of the foreign interest rate ΔI^* is relegated to the exogenous and dynamic part ($\boldsymbol{v}$) of the system. For practical use of the model, the timing of the interest rate variable is period t instead of $t-1$ (as in the theoretical model). The first part on the right side of each equation, set in normal case letters, is an equilibrium condition that reflects the long-run outcome of the theoretical model. It exerts a pull towards an equilibrium level of the dependent variable on the left side of the equation. Note that the Phillips curve in (7) models dp as an equilibrium-correction mechanism: $dp - s - c = -q - c$ is dp's deviation from its equilibrium solution $s + c$, where c represents the part of the unrestricted constant c_π that belongs to the equilibrium solution. In (7), we also allow for the possibility of a hybrid Phillips curve specification, in that the lagged inflation differential enters as a potential explanatory variable. In (8) and (9), the lagged variables in $\boldsymbol{v}_{t-i}$ and the deterministic terms in $\boldsymbol{d}_t$ capture much of the short-term movements of the data that are not explained by theory. There is a residual ϵ_t in each equation.

In the system (7)-(9), the loading parameters δ, ϕ_q and ϕ_I determine the speed or strength of equilibrium correction of the three variables dp, q and I. The parameters α, ω_q, ω_I, and ρ define the long-run cointegration structure in (8) for the real exchange rate q and in (9) for the interest rate I. Vectors $\boldsymbol{\mu}_i$ and $\boldsymbol{\eta}_i$ contain parameters for dynamic effects on the exchange rate and the interest rate.

We define the vector $\boldsymbol{z} = (d\pi, q, dI)'$ and write the system (7)-(9) in matrix notation as:

$$\underbrace{\begin{pmatrix} \Delta dp_t \\ \Delta q_t \\ \Delta I_t \end{pmatrix}}_{\Delta \boldsymbol{z}_t^\dagger} = \underbrace{\begin{pmatrix} c_\pi \\ c_q \\ c_I \end{pmatrix}}_{\boldsymbol{c}} + \underbrace{\begin{bmatrix} \gamma_\pi & 0 & 0 \\ 0 & -\phi_q & -\phi_q\alpha \\ 0 & 0 & -\phi_I \end{bmatrix}}_{\boldsymbol{F}} \underbrace{\begin{pmatrix} d\pi_{t-1} \\ q_{t-1} \\ dI_{t-1} \end{pmatrix}}_{\boldsymbol{z}_{t-1}} + \underbrace{\begin{bmatrix} 0 & \delta & 0 \\ \phi_q\omega_q & 0 & 0 \\ \phi_I\omega_I & 0 & 0 \end{bmatrix}}_{\boldsymbol{H}} \underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_t \end{pmatrix}}_{\boldsymbol{z}_t}$$

$$+ \underbrace{\begin{bmatrix} \beta_\pi & 0 & 0 \\ 0 & \beta_q & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{J}} \underbrace{\mathsf{E}_t \begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_{t+1} \end{pmatrix}}_{\mathsf{E}_t \boldsymbol{z}_{t+1}} + \underbrace{\begin{bmatrix} 0 & 0 \\ -\phi_q & 0 \\ 0 & \phi_I\rho \end{bmatrix}}_{\boldsymbol{K}} \underbrace{\begin{pmatrix} \lambda_{t-1} \\ dy_t \end{pmatrix}}_{\boldsymbol{x}_t}$$

$$+ \sum\nolimits_{i=0}^{k} \boldsymbol{\Pi}_i\, \Delta \boldsymbol{v}_{t-i} + \boldsymbol{\Theta} \boldsymbol{d}_t + \underbrace{\begin{pmatrix} \epsilon_t^{dp} \\ \epsilon_t^q \\ \epsilon_t^I \end{pmatrix}}_{\boldsymbol{\epsilon}_t}, \tag{10}$$

[6] In addition to a dummy variable for the 2008 financial crisis and a pair of structural dummies intended to capture a structural break during the period 2016-2018, these dummies account for seasonal variations and a few unexplained outliers.

where $\Delta dp = d\pi$. This equation system can be written compactly as

$$\Delta \boldsymbol{z}_t^{\dagger} = [\boldsymbol{c} + \boldsymbol{F}\boldsymbol{z}_{t-1} + \boldsymbol{H}\boldsymbol{z}_t + \boldsymbol{J}\,\mathsf{E}_t(\boldsymbol{z}_{t+1}) + \boldsymbol{K}\boldsymbol{x}_t] + \sum_{i=0}^{k} \boldsymbol{\Pi}\Delta \boldsymbol{v}_{t-i} + \boldsymbol{\Theta}\boldsymbol{d}_t + \boldsymbol{\epsilon}_t. \tag{11}$$

The vector $\boldsymbol{z}_t^{\dagger}$ of endogenous variables on the left side of the equation differs from the one in the theoretical model in Section 3 and from $\boldsymbol{z}_t$ (hence the topscript †). The interest rate in $\boldsymbol{z}_t^{\dagger}$ refers to the domestic rate and not the difference in interest rates with foreign countries: $dI = I - I^*$. Changes in the foreign interest rate ΔI_t^* are thereby not assumed to spill over to the domestic interest rate one-to-one in the short term. That is contrary to what applies in the long run, when such a restriction is imposed. The terms in brackets in (11) define the equilibrium relationships, while the latter three terms in (11) are the impulses and shocks that cause the endogenous variables to fluctuate around their equilibrium values. The two terms before the residual vector in (11) contain many lagged variables and dummies. Their number will be reduced, and the remaining ones will be made explicit after a reduction process from the GUM has produced the final model.[7]

In (8), the foreign exchange premium λ_t is not modeled as a parameterized function of variables intended to capture the risk premium. To address this shortcoming, we will follow Benedictow and Hammersland (2023) and allow the risk premium to be captured by the following variables: v_t = the export value of oil and gas as a share of the total value of exports, FDI = the difference between inward foreign direct investments as a percentage of GDP in the euro area and in Norway, a_t = a petroleum-related equity index, and VIX = a volatility index related to the S&P500 stock market index in the USA. These variables have been inspired by theory and claims to insights made by market participants. As discussed in Benedictow and Hammersland (2023), some of them can even be related to the behavior of investors that envision a transition of the Norwegian economy towards fading petroleum revenues and a green shift.

In the interest of unconstrained estimation, we do not impose any theoretical parameter constraints on the system at the outset nor through the reduction process — not even the Taylor principle ($\sigma + \alpha > 1$).

In the process of 'pruning' the GUM (11) to a smaller and more interpretable model at little cost in terms of fit to the data, we first substitute all theory variables by their empirical equivalents. For practical econometric reasons, we add definitional equations (identities) to the system. They make the level of the real exchange rate an endogenous variable and enhance the model's ability to simulate and forecast the levels of the variables.[8] After an iterative reduction process of simultaneous system estimation by FIML, testing, reformulation, estimation, testing etc., we arrive at the following parsimonious representation, where $SD171_t = \{1\ if\ t >= 2017Q1\ else\ 0\}$ is a step dummy capturing a trend break, and the variables D_π, D_q and D_I are shorthand for deterministic dummies that are linear expressions of variables and estimated parameters:

$$\Delta dp_t = -\underset{(7.11)}{27.17} + \underset{(0.034)}{0.13}\,q_t + \underset{(0.07)}{0.17}\,\Delta dp_{t+1} + D_{\pi,t} + \tilde{\epsilon}_t^{dp}, \tag{12}$$

$$\begin{aligned}\Delta q_t = {} & \underset{(0.06)}{0.22} + \underset{(0.098)}{0.11}\,\Delta q_{t-1} + \underset{(0.085)}{0.30}\,\Delta q_{t-2} - \underset{(0.016)}{0.097}\,\Delta p_t^{oil} + \underset{(0.015)}{0.055}\,\Delta p_{t-1}^{oil} - \underset{(0.009)}{0.046}\,\Delta dI_t + \underset{(0.008)}{0.031}\,\Delta dI_{t-1} \\ & + \underset{(0.0013)}{0.004}\,\Delta(FDI_{t-1}\cdot SD171_{t-1}) + \underset{(0.052)}{0.138}\,\Delta(a_{t-1}\cdot SD171_{t-1}) + \underset{(0.0003)}{0.0006}\,\Delta VIX_t \\ & - \underset{(0.03)}{0.11}\,(q_{t-1} + 0.023\,a_{t-1} - 0.004\,FDI_{t-1} + 0.23\,v_{t-1} - 0.004\,VIX_{t-1}) \\ & - \underset{(0.001)}{0.004}\,(dI_{t-1} - \Delta dp_t) + D_{q,t} + \epsilon_t^{q},\end{aligned} \tag{13}$$

$$\begin{aligned}\Delta I_t = {} & \underset{(0.06)}{0.007} + \underset{(0.06)}{0.25}\,\Delta I_{t-1} + \underset{(0.06)}{0.90}\,\Delta I_t^* - \underset{(0.08)}{0.18}\,\Delta I_{t-1}^* \\ & - \underset{(0.014)}{0.034}\,(dI_{t-1} - (dp_t - dp_{t-4})) + \underset{(0.006)}{0.015}\,dy_t + D_{I,t} + \epsilon_t^{I}.\end{aligned} \tag{14}$$

Vector SEM-AR 1-1 test: $F(9,126) = 1.1703$ [0.3178]

Vector ARCH 1-1 test: $F(9,100) = 0.88047$ [0.5438]

Vector normality test: $\chi^2(6) = 5.6863$ [0.4592]

[7] In the reduction process, the General Unrestricted Model (GUM) in (11) should be viewed as a conceptual point of departure rather than a literal specification of a starting point for a reduction process. The point of departure is constrained by the available degrees of freedom. Consequently, (11) should be understood as a necessary delineation of the possibility space from which an initial specification is drawn for a series of reductions, guided by statistical congruence and theoretical admissibility.

[8] For an explicit account of these identities, we refer the reader to Appendix I.

The tests refer to, respectively, a test for the absence of serial correlation in the residuals (AR), a test for the absence of autoregressive conditional heteroscedasticity (ARCH) effects, and a test for whether the residuals follow a multivariate normal distribution, all pertaining to the system level, with p-values reported in brackets. By using the observed value Δdp_{t+1} instead of its unobserved expectation $\mathsf{E}_t(\Delta dp_{t+1})$ in Equation (12), we have implicitly relegated the expectation term to the residual, which becomes $\tilde{\epsilon}^{\pi}_t = \epsilon^{\pi}_t + \beta(\mathsf{E}_t(\Delta dp_{t+1}) - \Delta dp_{t+1})$. If agents had perfect foresight, the expected values of the regressor would be equal to the observed values of the regressor: $\mathsf{E}_t(\Delta dp_{t+1}) = \Delta dp_{t+1}$ for all t, and $\tilde{\epsilon}^{\pi}_t = \epsilon^{\pi}_t$. With rational expectations instead of omniscient perfect forsight, there will be an expectation error: $\varepsilon^{\mathsf{E}dp}_{t+1} = \mathsf{E}_t(\Delta dp_{t+1}) - \Delta dp_{t+1}$.[9] We re-estimate the system with an instrumental variable $\widehat{\Delta dp}_{t+1}$ replacing the observed value Δdp_{t+1} in Equation (12). In Appendix E we discuss instrumentation and show estimation results with different instruments for the future inflation term. Using a linear combination of exogenous and predetermined endogenous variables as instrument, the re-estimated system is

$$\Delta dp_t = \underset{(6.19)}{-25.91} + \underset{(0.029)}{0.12}\, q_t + \underset{(0.18)}{0.95}\, \widehat{\Delta dp}_{t+1} + D_{\pi,t} + \tilde{\epsilon}^{\pi}_t, \tag{15}$$

$$\begin{aligned}\Delta q_t = {} & \underset{(0.06)}{0.20} + \underset{(0.10)}{0.11}\,\Delta q_{t-1} + \underset{(0.087)}{0.35}\,\Delta q_{t-2} - \underset{(0.017)}{0.082}\,\Delta p^{oil}_t + \underset{(0.016)}{0.075}\,\Delta p^{oil}_{t-1} - \underset{(0.009)}{0.045}\,\Delta dI_t + \underset{(0.008)}{0.03}\,\Delta dI_{t-1} \\ & + \underset{(0.0015)}{0.004}\,\Delta(FDI_{t-1}\cdot SD171_{t-1}) + \underset{(0.05)}{0.13}\,\Delta(a_{t-1}\cdot SD171_{t-1}) + \underset{(0.0003)}{0.0004}\,\Delta VIX_t \\ & - \underset{(0.03)}{0.10}\,(q_{t-1} + 0.023\,a_{t-1} - 0.004\,FDI_{t-1} + 0.23\,v_{t-1} - 0.004\,VIX_{t-1}) \\ & - \underset{(0.001)}{0.004}\,(dI_{t-1} - d\pi_t) + D_{q,t} + \tilde{\epsilon}^{q}_t,\end{aligned} \tag{16}$$

$$\begin{aligned}\Delta I_t = {} & \underset{(0.058)}{0.012} + \underset{(0.059)}{0.25}\,\Delta I_{t-1} + \underset{(0.06)}{0.90}\,\Delta I^*_t - \underset{(0.08)}{0.19}\,\Delta I^*_{t-1} \\ & - \underset{(0.014)}{0.036}\,(dI_{t-1} - (dp_t - dp_{t-4})) + \underset{(0.006)}{0.015}\,dy_t + D_{I,t} + \epsilon^{I}_t.\end{aligned} \tag{17}$$

Vector SEM-AR 1-1 test: $F(9, 126) = 1.4271$ [0.1810]
Vector ARCH 1-1 test: $F(9, 100) = 0.63414$ [0.7669]
Vector normality test: $\chi^2(6) = 3.2762$ [0.7735]

We see that both the FIML estimate of β and its statistical significance increase a lot, while the other coefficients in the system (15)-(17) are not much affected relative to their values in (12)-(14). The results of this re-estimation clearly indicate that the significance of the forward-looking inflation term in the system (12)-(14) is not due to a simultaneity bias. Quite the opposite, since the importance of the future inflation term actually increases when correcting for a potential simultaneity bias using a valid instrument in (12). The empirical correlation between the observed regressor Δdp_{t+1} and the residual $\tilde{\epsilon}^{\pi}_t$ in (12) is small, but still it seems to cause a large downward bias in the estimate of β relative to using the instrument in (15). Under the axiom of correct specification, this is because the instrument is less correlated with the unobserved expectation error than the observed series. A larger $\hat{\beta}$ compensates for that (cf. Table 2 in Appendix E).

The estimations with observed values or instrument values in place of the unobserved expectations confirm the pure forward-looking Phillips curve in the model. In the literature, models with a pure forward-looking Phillips curve have calibrated and/or estimated values of β close to our high estimate 0.95; e.g. Syed Kanwar Abbas and Sgro (2011) report estimates between 0.7 and 1. More common are models with the hybrid Phillips curve, where a weight of β and $1-\beta$ is placed on the forward expectation and the lagged inflation term, respectively. In a single-equation model, Kleibergen and Mavroeides (2009) discuss (weak) identification and explain why it may be statistically difficult to differentiate between a forward-looking, a backward-looking and a hybrid

[9] The residual in (12) is a sum of the original residual and the expectation error: $\tilde{\epsilon}^{\pi}_t = \epsilon^{\pi}_t + \beta\varepsilon^{\mathsf{E}dp}_{t+1}$. If this combined residual is correlated with the regressor Δdp_{t+1}, it causes a bias in the estimator of β:

$$\begin{aligned} cov(\tilde{\epsilon}^{\pi}_t, \Delta dp_{t+1}) &= cov(\epsilon^{\pi}_t + \beta\varepsilon^{\mathsf{E}dp}_{t+1}, \mathsf{E}_t(\Delta dp_{t+1}) - \varepsilon^{\mathsf{E}dp}_{t+1}) \\ &= cov(\epsilon^{\pi}_t, \mathsf{E}_t(\Delta dp_{t+1})) + \beta\, cov(\varepsilon^{\mathsf{E}dp}_{t+1}, \mathsf{E}_t(\Delta dp_{t+1})) - cov(\epsilon^{\pi}_t, \varepsilon^{\mathsf{E}dp}_{t+1}) - \beta\, var(\varepsilon^{\mathsf{E}dp}_{t+1}) \end{aligned}$$

The in-sample correlation (normalized covariance) is $\widehat{corr}(\tilde{\epsilon}^{\pi}_t, \Delta dp_{t+1}) = -0.061$. This is a small number.

Phillips curve. If we add a lagged term Δdp_{t-1} as a regressor in (15) and re-estimate the system (15)-(17) with no added restrictions, we obtain

$$\Delta dp_t = \underset{(6.80)}{-26.83} + \underset{(0.032)}{0.13}\, q_t + \underset{(0.19)}{0.96}\, \widehat{\Delta dp}_{t+1} + \underset{(0.08)}{0.025} \Delta dp_{t-1} + D_{\pi,t} + \tilde{\epsilon}_t^{\pi}. \tag{18}$$

The lagged inflation term does not enter significantly when added to Equation 15 in the system (15)-(17). Hence, (18) does not support a hybrid Phillips curve. During the reduction process from the GUM to the system (15)-(17), lagged inflation was also found to be insignificant and thus excluded from the model. There is no simultaneity issues with the real exchange rate q_t since it is an endogenous variable within the system due to the set of included identities.[10]

When deflating the nominal interest differential dI_{t-1} in (16), we use the (annualized) country-difference in quarterly price changes one period ahead (relative to the lagged dating of the equilibrium terms): $\Delta dp_t \equiv \Delta p_t - \Delta p_t^* = dp_t - dp_{t-1}$, thus turning the real expression into an ex ante real interest rate differential. In (17), where the inflation difference is derived from the inflation gaps of the two underlying country-specific Taylor rules — and its effect thus represents responses of interest rates to deviations from the two countries' inflation targets — we use the country-difference in 4-quarter price changes: $\Delta_4 dp_t \equiv dp_t - dp_{t-4} = \Delta_4 p_t - \Delta_4 p_t^*$. The latter difference is less volatile and preferred by econometric evaluation criteria.

The cointegrating structure of the estimated dynamic models in (12)-(14) and (15)-(17) defines the long-run or equilibrium part of the models. It serves as an attractor on the dynamic evolution of the three endogenous variables. To find the equilibrium part of the model, we disregard the dummy terms, and equal the level terms on the right-hand side of the equations to zero. Letting the endogenous variable in these level expressions refer to time period t (instead of $t-1$ as in the case of the variables included in the two error correction terms that apply to the real exchange rate and the interest rate in the system), we then get:

$$s_t = p_t - p_t^* \equiv dp_t \tag{19}$$

$$q_t = -0.023\, a_t + 0.004\, FDI_t - 0.23\, v_t + 0.004\, VIX_t - 0.036\, (dI_t - \Delta dp_{t+1}) \tag{20}$$

$$dI_t = (dp_{t+1} - dp_{t-3}) + 0.44\, dy_{t+1} \tag{21}$$

Equation 19 follows directly from the definition $q_t \equiv s_t - (p_t - p_t^*)$. The long-run part (20) and (21) of the estimated econometric model (13)-(14) is theoretically permissible in the sense of being in line with the New-Keynesian rational expectation model of a two-country, two-market economy in Section 3.

In the system (19)-(21), we have taken the model's long-run cointegrating structure out of its dynamic context. The first equation is in accordance with the purchasing power parity (PPP) theory and implies that the real exchange rate is a (trend-)stationary variable in itself (cf. Footnote 10). The other two equations are linear combinations of variables which, in themselves, could be stationary and/or non-stationary. (20) is a risk-adjusted uncovered interest parity (UIP) equation in line with the theoretical framework in Section 3. (21) reflects the policy rule of two countries with a flexible inflation target. The interest rate differential between the two countries will then be a function of the inflation gap difference and the output gap difference between the two countries. In (21), the weight given to the output gap compared to the inflation gap is given by the ratio 0.015/0.034=0.44. Appendix A shows stationarity tests. Appendix B provides technical information on some of the most prominent cointegrating relationships, particularly the one pertaining to the real exchange rate.

6 Model evaluation

We have subjected the model to diagnostic tests, as reported below the system (12)-(14) and the system (15)-(17), and in appendices A and B. To see how the model predicts the data in-sample, we carry out three dynamic simulations of the systems. Each simulation differs from the other simulations by the series used for the unobserved expectation $\mathsf{E}(\Delta dp_{t+1})$, cf. Appendix E for details:

[10] To render the empirical version of the Phillips curve balanced we specify a break in 2017, making the real exchange rate a trend stationary variable. We may get rid of this break by substituting the real exchange rate with its risk premium adjusted equivalent, $(q_{t-1} + 0.023\, a_{t-1} - 0.004\, FDI_{t-1} + 0.23\, v_{t-1} - 0.004\, VIX_{t-1})$. The estimated effect of the real exchange rate then falls from 0.13 to 0.077, but the effect is still significant and the effect of the forward-looking inflation term becomes stronger and more significant. The other parts of the system are otherwise little affected by this modification.

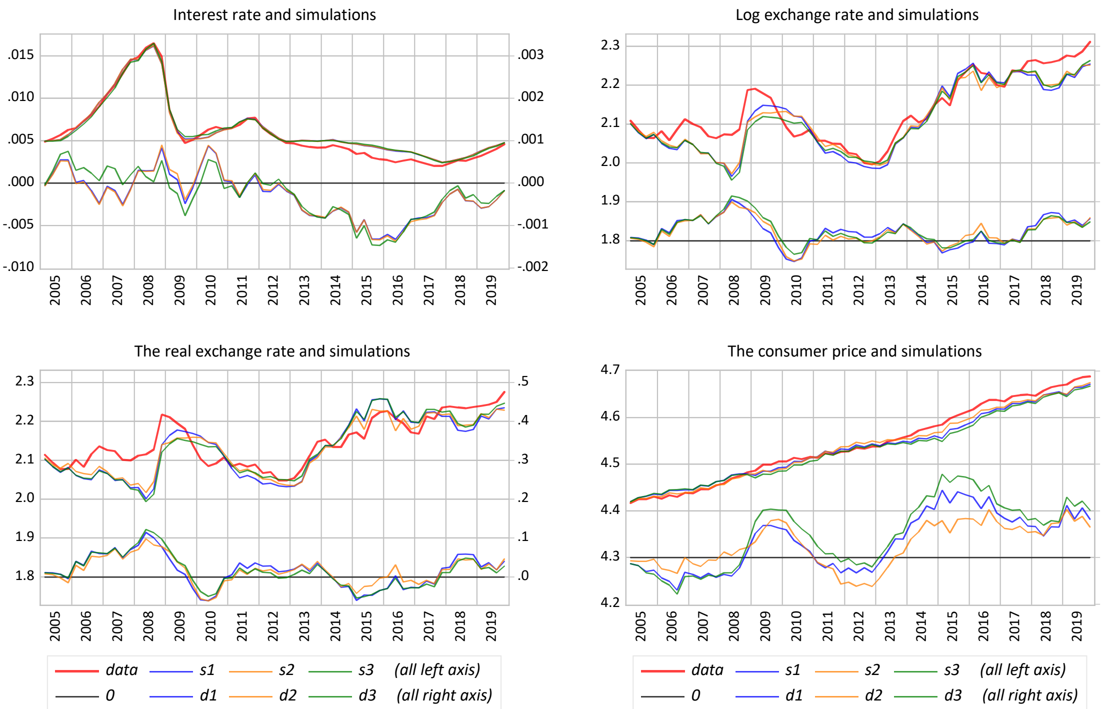

Figure 3: Time series data and dynamic system simulations with different instruments for $\mathsf{E}(\Delta dp_{t+1})$, The upper bundle of graphs in each panel show the data (red graph) and the simulations s1-s3. The lower bundles show the corresponding differences d1-d3 between the data series and the simulations.

(i) data Δdp_{t+1} (perfect foresight),

(ii) data smoothed by moving average: $\overline{\Delta dp}_{t+1}$ defined by (33) in Appendix E, and

(iii) the instrument $\widehat{\Delta dp}_{t+1}$ defined by (32) in Appendix E.

We simulate the system (12)-(14) substituting (i) or (ii) for the expectations term in the Phillips curve equation. We simulate the system (15)-(17) with the instrument (iii). The two systems differ mainly by the estimated value of the coefficient β_π, that is the weight placed on the expectation term. The upper bundle of graphs in each panel in Figure 3 shows the data series (red graph) and its three dynamic simulations over 60 periods, starting in the first quarter of 2005. The series are shown in (log) levels, measured on the left axis. Simulated values are used for lagged endogenous variables in all dynamic simulations. The legends s1, s2, and s3 denote the simulations with series (i), (ii) and (iii) approximating $\mathsf{E}(\Delta dp_{t+1})$. The lower bundle of graphs in each panel contains the corresponding differences d1, d2, and d3 between the data series and the simulations. They are measured on the right axis.

The graphs of the dynamic system simulations show that the model tracks the historical developments of the data quite well and shows no signs of instability in the sample. The model maps the endogenous variables $d\pi$, q and I from past periods and the upcoming period to the current period. The eigenvalues of the dynamic mapping determine whether the developments of the variables over time are stable or not. A technical eigenvalue analysis that is relegated to Appendix G supports stability. Although the model fits the data fairly well, there are longer periods when the model deviates from the data series for the interest rate and the consumer price.

The simulations are not highly sensitive to the treatment of the expectation term. The series simulated with smoothed observations $\overline{\Delta dp}_{t+1}$ for the expectation (s2, orange graphs) is marginally closer to the data series (red graphs) than the other two simulated series. The series simulated with the instrument $\widehat{\Delta dp}_{t+1}$ (s3, green graphs) are barely closer to the data series than the simulations with perfect foresight (s1, blue graphs). This may be due to the instrument (iii) being an endogenous variable. It is defined by (32) in Appendix E, and is incorporated into the model as an equation. During dynamic simulation of the system (15)-(17), the current value of the instrument depends on the previously simulated lagged values dI_{t-1} and Δdp_{t-2}.

Figure 4 shows forecasting properties of the model (15)-(17) estimated on data up to and

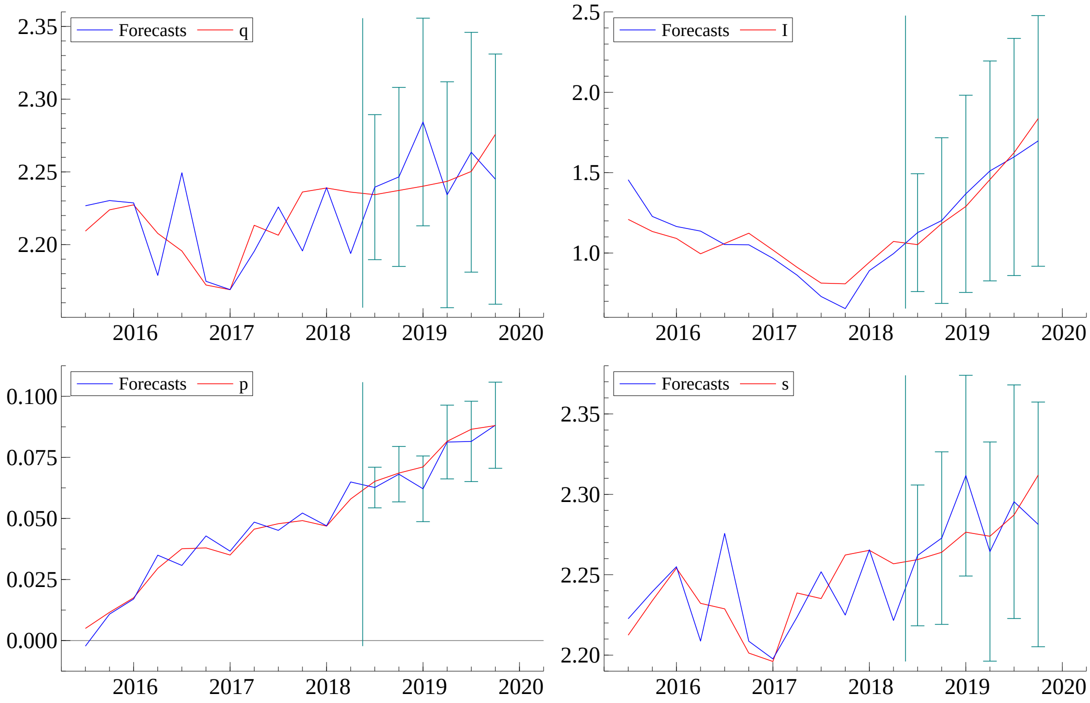

Figure 4: Dynamic forecasts from 2018Q3, using exogenous data and simulated lagged values.

including the 2nd quarter of 2018. The shorter sample causes only minor changes to some estimates. The dynamic simulations start in 2010 from observed data values and using simulated values of lagged endogenous variables and observed values of exogenous variables. The model is simulated dynamically six periods beyond the estimation period (ex post). The simulations support the model's good fit to the data.

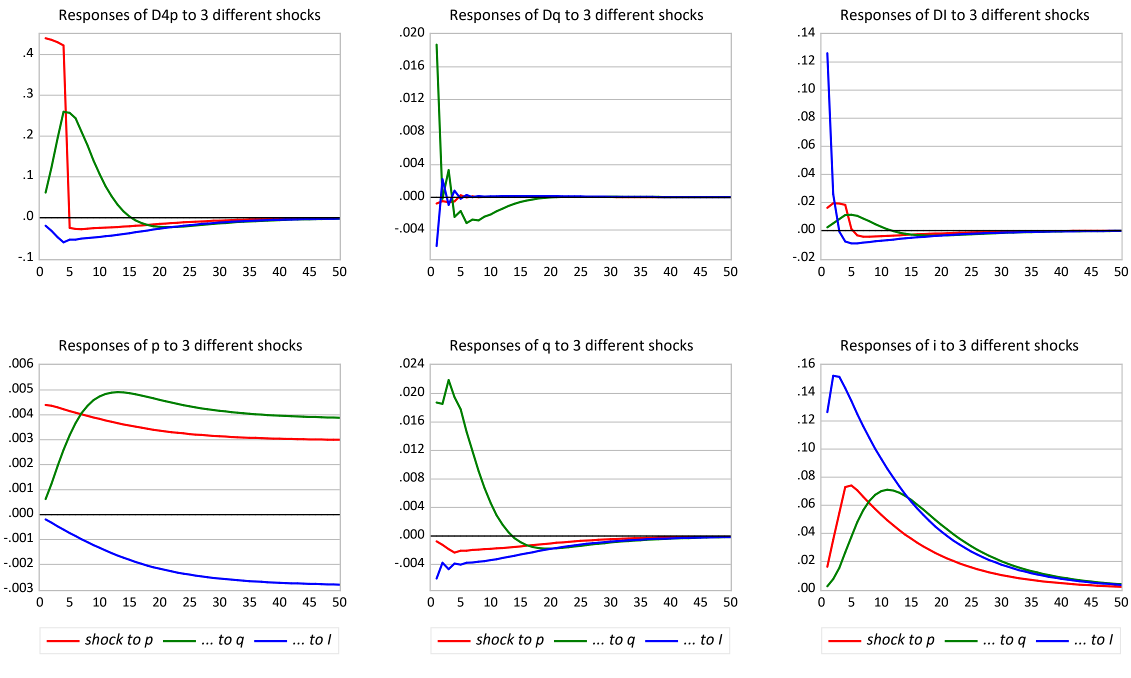

Figure 5: Impulse responses over 50 quarters to a single-period shock to an individual equation. The upper panels show D4p $\equiv \Delta_4 p_t = p_t - p_{t-4}$, Dq $\equiv \Delta q$, and DI $\equiv \Delta I$.

It is common in the literature on DSGE and New Keynesian models to show impulse-response graphs. It is less common in the time series literature. However, we graph the responses of each endogenous variable to 3 shocks. Each shock is to the error term in one of the structural equations (15)-(17), and it lasts only a single period. The size of each shock is equal to the standard deviation of the empirical residual in each equation. Each panel in Figure 5 shows the response of one endogenous variable to a shock to the residual in each of the 3 structural equations (15)-(17). The panels in the upper row show the changes in each endogenous variable in response to the impulse shock. The lower row of panels shows the accumulated effects of the impulse shock to the level of each endogenous variable. The left panels show the inflation and the price level. The center panels show the change in and the level of the real exchange rate, while the change in and level of the interest rate are displayed in the right panels. A red graph is the response to a (supply) shock to the inflation (π or Δp). A green graph is the response to a (terms-of-trade) shock to the real exchange rate (Δq). A blue graph is the response to a (policy-implementation-error) shock to the interest rate (ΔI).

The upper row of panels shows that each impulse shock causes a relatively large immediate change in the variable to which it belongs, but the change does not last. The lower row of panels shows that the change accumulates in level and lasts longer. A more detailed account of the impulse responses related to the three types of structural shocks that we consider here – a supply shock, an exogenous terms-of-trade shock and a relative monetary policy shock – is relegated to Appendix H. It should be noted that the effect of a monetary policy shock, as shown by the blue graphs in Figure 5, is consistent with the inflation response in Christiano et al. (2005), since the effect on inflation of an interest rate shock reaches its maximum after about 4-5 quarters. As shown by the accumulated impulse responses in the lower panels, it takes a long time for the effect of the shock to be fully neutralized, with the price level not returning to its pre-shock level even in 50 periods.

7 Conclusion

Theoretical economic models are logical and highly simplified in order to be analytically tractable. By construction, they are not likely to 'fit' the data. Taking a theoretical model to data generally requires adaptations, of either the data, the model or the empirical method. The information content in the data can be reduced by filtering (demeaning, seasonally adjustment) and/or down-weighted by a priori information (Bayesian approach). That makes it less demanding for the model to "explain" the data. The model can be made more flexible and thus data admissible by adding variables and lags (the hybrid NKPC, structural time series models). Finally, the method of data confrontation can favor economic criteria over statistical criteria. The balance between theoretical and empirical qualities of the quantified model may depend on the purpose of the model.

We use a data-driven method to take a theoretical New-Keynesian rational expectations model to empirical data. Our primary objective is to explore the feasibility of validating this kind of models without preimposing their structural integrity and therefore their presumed validity. We embed the theoretical structure in a larger dynamic and simultaneous equation system that offers flexibility. The system is then statistically reduced to a smaller data admissible system, where the theoretical model is part of the long-run 'attractor' for the short-term dynamics.

Traditionally, estimation methods for New Keynesian models rely heavily on a combination of calibration and Bayesian inference procedures. Such approaches typically require imposing various a priori constraints on the structural parameters of the models. Our procedure deliberately avoids such constraints. This not only facilitates a more malleable validation process, but also ensures that both the model structure and its quantification emerge from testable interaction with the data.

Employing this innovative method, we demonstrate that it can be possible to substantiate a New Keynesian rational expectation model without reliance on *a priori* information or restrictions. The long-term structure of the empirical model developed in this paper closely reflects important aspects of our theoretical framework. Specifically, the real exchange rate equation aligns with deviations from uncovered interest rate parity in the form of a risk premium, while the interest rate is set according to a rule based on flexible inflation targeting. This affirms the robustness and relevance of our approach.

To elaborate, this paper provides empirical support for a theoretical framework that addresses the simultaneous determination of relative interest rates, prices, and exchange rates. This empirical structure largely aligns with predictions of a New Keynesian model within a two-country, two-market economy. The only notable deviation is observed in the short-term dynamics of the exchange rate. Contrary to the forward-looking exchange rate process posited by the New Keynesian theory model, our estimated model suggests a backward-looking exchange rate process. This feature also implies that the number of free variables exceeds the number of stable forward-looking eigenvalues in the model. Consequently, the model has multiple equilibria, meaning that its stability does not depend on being positioned at a uniquely defined saddle-path.

The behavior of the exchange rate equation in our model concurs with the findings reported by Benedictow and Hammersland (2023). Although their study is based on a simultaneous analysis for identifying the model's underlying long-run cointegration structure, a simultaneous approach is not apply when designing the model's dynamic specification. Thus, our analysis not only reinforces the structural foundation laid out in Benedictow and Hammersland (2023) but also addresses previously identified gaps in the literature. Additionally, our results corroborate the theoretical framework anticipated by a two-country two-market model where the central banks of both countries engage in symmetric and flexible inflation targeting.

A significant finding of our study is the validation of the Phillips curve, which connects the inflation differential with an activity indicator, the real exchange rate, and the expected future inflation differential. This finding was unexpected in light of previous studies using Norwegian data. However, it is robust within our model and suggests potential simultaneity biases in earlier research methodologies and estimations. It also underscores the complex dynamics at play and invites a reevaluation of established empirical frameworks within economic research.

The paper raises several questions that merit further investigation. One immediate extension would be to examine further whether the real exchange rate should be modeled empirically as a backward- or forward-looking process. This distinction has direct implications for the model's solution properties—specifically, whether it follows a saddle-path or allows for multiple equilibria. A more detailed empirical analysis of this aspect, possibly using data from other countries, could help assess the robustness and generalizability of our findings.

A Statistical testing

We test for stationarity of the data series, using the augmented Dickey-Fuller test (ADF). The null hypothesis is that a unit root is present in a time series. The results are shown below. The starred t-values reject the hypothesis, and the series are assumed to be level-stationary. The twelve first series are judged to have a stochastic trend.

Table 1: Augmented Dickey Fuller tests (ADF)

Variable	Lagged level parameter	t-value	Conclusion
s_t	-0.043(2)	-1.088	I(1)
p_t	0.006(2)	1.06	I(1)
p_t^*	-0.016(2)	-2.02	I(1)
v_t	-0.09(2)	-1.31	I(1)
p_t^{oil}	-0.075(2)	-2.07	I(1)
$dp_t = p - p_t^*$	-11.85(2)	1.56	I(1)
$s_t + p_t^* - p_t$	-0.07(1)	-1.6	I(1)
a_t	-0.055(2)	-1.74	I(1)
FDI_t	-0.049(2)	-1.12	I(1)
VIX_t	-0.25(2)	-2.68	I(1)
I_t	-0.067(2)	-2.83	I(1)
I_t^*	-0.036(2)	-1.70	I(1)
dI_t	-0.099(2)	-3.36**	I(0)
$d\pi_t$	-0.43(2)	-3.18**	I(0)
$dI_t - d\pi_{t+1}$	-0.39(2)	-3.05**	I(0)
$dp_t - dp_{t-4}$	-0.12(2)	-3.00**	I(0)
$dI_t - (dp_{t+1} - dp_{t-3})$	-0.14(2)	-3.12**	I(0)
$\tilde{q}_t$	-0.195(2)	-3.15**	I(0)
y_t	-0.18(3)	-2.9**	I(0)
$\left\{ \begin{array}{c} q_t + 0.023a_t + 0.227v_t \\ -0.004(FDI + VIX)_t \end{array} \right\}$	-0.12(1)	$-2.17^{\dagger}$	I(0)

Note: The 5 and 1 percent critical values of the ADF-test are taken from MacKinnon (2010). They are -2.90 and -3.52 when a constant has been included in the specification of the model and the number of observations is 76. The asterisks ** and *** denote statistical significance to a level of 5 and 1 percent, respectively. Numbers in parentheses after the lagged parameter indicate the number of lags in differences utilized when performing the individual ADF tests. The series for $\tilde{q}_t$ is the real exchange rate adjusted for a trend break in the first quarter of 2017.

† Regarding the stationarity test of the cointegrating relationship, we refer to the cointegration analysis presented in Benedictow and Hammersland (2023). A summary of this analysis is reproduced in Appendix B. It provides strong evidence for this relationship, indicating that it enters as a distinct cointegrating relation within a five-dimensional simultaneous cointegration structure.

B The results of the cointegration analysis in Benedictow and Hammersland (2023)

The cointegration analysis in Benedictow and Hammersland (2023) uses a five-dimensional conditional Vector Error Correction Model (VECM) of order 2. Their model simultaneously determines the real exchange rate $q = s - (p - p^*)$, the real interest rate differential between Norway and the euro area $dr = r - r^* = dI - d\pi$, a Norwegian petroleum-related equity index a, the ratio of the value of oil and gas exports to the total value of Norwegian exports v, and the difference between foreign direct investment as a percentage of GDP in the euro area and in Norway FDI. In addition to a constant and dummies for structural breaks, their model includes the Brent blend crude oil price measured in US dollars, p^{oil}, and the S&P volatility index VIX as exogenous variables, both serving as sources of a common trend. These variables are the same as in our analysis.

The identified system of 5 cointegrating linear combinations, the loading matrix and a test of overidentifying restrictions are as follows.

The identified long-run structure is

$$\hat{\beta}' \begin{pmatrix} Y_t \\ X_t \end{pmatrix} = \begin{pmatrix} q_t + \underset{(0.013)}{0.023}\, a_t + \underset{(0.063)}{0.227}\, v_t - 0.004(FDI + VIX)_t \\ (dI - d\pi)_t \\ (v - p^{oil})_t \\ a_t + \underset{(0.014)}{0.043}\, VIX_t - p^{oil}_t \\ FDI_t + \underset{(2.47)}{81.65}\, p^{oil}_t \end{pmatrix} \tag{22}$$

The equilibrium correction coefficient matrix is

$$\begin{pmatrix} \Delta q \\ \Delta dr \\ \Delta v \\ \Delta a \\ \Delta FDI \end{pmatrix} : \begin{pmatrix} \hat{\alpha}_{11} & \hat{\alpha}_{12} & \hat{\alpha}_{13} & \hat{\alpha}_{14} & \hat{\alpha}_{15} \\ \hat{\alpha}_{21} & \hat{\alpha}_{22} & \hat{\alpha}_{23} & \hat{\alpha}_{24} & \hat{\alpha}_{25} \\ \hat{\alpha}_{31} & \hat{\alpha}_{32} & \hat{\alpha}_{33} & \hat{\alpha}_{34} & \hat{\alpha}_{35} \\ \hat{\alpha}_{41} & \hat{\alpha}_{42} & \hat{\alpha}_{43} & \hat{\alpha}_{44} & \hat{\alpha}_{45} \\ \hat{\alpha}_{51} & \hat{\alpha}_{52} & \hat{\alpha}_{53} & \hat{\alpha}_{54} & \hat{\alpha}_{55} \end{pmatrix} = \begin{pmatrix} \underset{(0.06)}{-0.15} & \underset{(0.001)}{-0.002} & \underset{(0.03)}{0.05} & \underset{(0.007)}{-0.18} & \underset{(0.0004)}{0.0008} \\ \underset{(7.52)}{-22.49} & \underset{(0.16)}{-0.96} & \underset{(3.38)}{3.92} & \underset{(0.85)}{1.17} & \underset{(0.04)}{0.05} \\ \underset{(0.30)}{0.51} & \underset{(0.006)}{0.000005} & \underset{(0.136)}{0.08} & \underset{(0.034)}{-0.07} & \underset{(0.0017)}{0.0006} \\ \underset{(7.03)}{-1.89} & \underset{(0.15)}{-0.05} & \underset{(3.16)}{-5.86} & \underset{(0.8)}{-0.59} & \underset{(0.04)}{-0.06} \\ \underset{(0.20)}{-0.23} & \underset{(0.004)}{0.007} & \underset{(0.09)}{-0.41} & \underset{(0.023)}{0.0015} & \underset{(0.001)}{-0.005} \end{pmatrix}$$

LR test of overidentifying restrictions: $\chi^2(11) = 17.898\,[0.0840]$.

Based on this model, the reduced form analysis in Benedictow and Hammersland (2023) provides support for no less than five cointegrating vectors. The final overidentified structure is displayed in equation (22) above.[11]

In the cointegration structure above, the first two cointegrating relationships pertain to the real krone-euro exchange rate and the real interest rate differential. The latter reflects a stationary relationship in itself. The remaining three relationships illustrate the degree of oil dependence in the Norwegian economy, measured by two distinct types of investor behavior and the relative contribution of oil and gas exports to the total value of Norwegian exports. They have no implications for the results in the present paper.

[11] The F-test for the number of overidentifying restrictions shows that the final and overidentified system, consisting of five cointegrating relations, constitutes a valid constraint on a correspondingly accurately identified long-run structure. For those wanting more information about the identification process itself, we refer to Benedictow and Hammersland (2023).

C Derivation of the aggregate Phillips curve relationship

It is assumed that domestic and foreign consumers have identical preferences. If faced with the same price, they would demand the same basket of goods and services, and purchasing parity would hold all the time. We follow Engel (2014) and assume pricing to market that arises from a particular type of nominal price stickiness, denoted local currency pricing, or LCP, in the literature. Firms are assumed to set different prices for their goods sold in each country, and each of those prices is sticky in the currency in which it is set. Hence, the model is based on the so-called Calvo price setting, where each firm at every point of time has a given and constant probability of changing its two prices.

The log of the aggregate price of the goods produced at home for sale in the domestic market (denoted by a subscript H for home) is given by

$$p_{H,t} = (1 - \Theta)\tilde{p}_{H,t} + \Theta p_{H,t-1}, \tag{23}$$

where $\tilde{p}_{H,t}$ is the log of the price of the fraction of firms $1 - \Theta$ that get the possibility of changing their price in period t and Θ denotes the constant probability of not changing the price.

Firms produces output using only labor. Hence, $w_t - a_t$ is the log of the unit cost, where w_t is the the log of the nominal wage, and a_t the log of labor productivity. With a discount factor equal to β and a probability Θ that the price will not change, the firms that get the opportunity to reset their prices at time t set them to maximise the expected present discounted value of profits. The optimal price thus satisfies the following recursion:

$$\tilde{p}_{Ht} = (1 - \Theta\beta)(w_t - a_t) + \Theta\beta \mathsf{E}_t \tilde{p}_{Ht+1}. \tag{24}$$

With some manipulation[12], (23) and (24) can be combined to arrive at

$$\pi_{H,t} = \delta(w_t - a_t - p_{H,t}) + \beta \mathsf{E}_t \pi_{H,t+1}, \tag{25}$$

where $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\delta = (1 - \Theta)(1 - \Theta\beta)/\Theta$. Note that the greater the probability is that a business will be able to change its prices (lower Θ), the greater is δ.

Similar derivation as for (25) and using that the cost per unit in foreign currency of the home good is $w_t - s_t - a_t$, we derive an equation for the evolution of prices of home goods sold in the foreign country in terms of the foreign currency $p^*_{H,t}$:

$$\pi^*_{H,t} = \delta(w_t - a_t - s_t - p^*_{H,t}) + \beta \mathsf{E}_t \pi^*_{H,t+1}, \tag{26}$$

where $\pi^*_{Ht} = p^*_{Ht} - p^*_{Ht-1}$. Combining (25) and (26) gives:

$$\pi_{Ht} - \pi^*_{Ht} = \delta(s_t + p^*_{Ht} - p_{Ht}) + \beta E_t(\pi_{Ht+1} - \pi^*_{Ht+1}) \tag{27}$$

So far we have only looked at the price setting of goods produced at home. Under the assumption of symmetry, the deviations from the law of one price should be equal for home and foreign produced goods. Denoting the price of foreign goods at home and abroad by $p_{F,t}$ and $p^*_{F,t}$, the latter in terms of foreign currency, we have that:

$$p_{H,t} - s_t - p^*_{H,t} = p_{F,t} - s_t - p^*_{F,t} \Leftrightarrow p_{H,t} - p^*_{H,t} = p_{F,t} - p^*_{F,t} \tag{28}$$

The assumption of identical preferences implies that the price indices at home and abroad are given by $p_t = (p_{H,t} + p_{F,t})/2$ and $p^*_t = (p^*_{H,t} + p^*_{F,t})/2$. Due to (28), these expressions imply that $q_t = s_t + p^*_t - p_t = s_t + p^*_{H,t} - p_{H,t} = s_t + p^*_{F,t} - p_{F,t}$ and $\pi_{H,T} - \pi^*_{H,T} = \pi_{F,t} - \pi^*_{F,t} = \pi_t - \pi^*_t$.[13] Based on these identities, (27) can be expressed in aggregate terms, implying that

$$\pi_t - \pi^*_t = \delta q_t + \beta \mathsf{E}_t(\pi_{t+1} - \pi^*_{t+1}) \tag{29}$$

which is the aggregate Phillips curve for our two-countries, two-markets economy.

[12] Inserting the expression for $\tilde{p}_{Ht}$ in (24) into (23) $\Rightarrow p_{Ht} = (1-\Theta)(1-\Theta\beta)(w_t - a_t) + \Theta\beta(1-\Theta)E_t\tilde{p}_{Ht+1} + \Theta p_{Ht-1}$. Solving (23) for $(1-\Theta)E_t\tilde{p}_{Ht+1}$ and substituting the resulting expression for the similar term in the aforementioned equation and adding and substituting $(1-\Theta)(1-\Theta\beta)p_{Ht}$, should then lead us almost directly to (25).

[13] $q_t = (s_t + p^*_t) - p_t = (s_t + (p^*_{H,t} + p^*_{F,t})/2) - (p_{H,t} + p_{F,t})/2 = (s_t + p^*_{H,t}) - p_{H,t} = (s_t + p^*_{F,t}) - p_{F,t}$, and $\pi_t - \pi^*_t = p_t - p_{t-1} - (p^*_t - p^*_{t-1}) = (p_{H,t} + p_{F,t} - (p_{H,t-1} + p_{F,t-1}))/2 - (p^*_{H,t} + p^*_{F,t} - (p^*_{H,t-1} + p^*_{F,t-1}))/2 = (p_{H,t} - p_{H,t-1} - (p^*_{H,t} - p^*_{H,t-1}))/2 + (p_{F,t} - p_{F,t-1} - (p^*_{F,t} - p^*_{F,t-1}))/2 = (\pi_{H,t} - \pi^*_{H,t})/2 + (\pi_{F,t} - \pi^*_{F,t})/2 = \pi_{H,t} - \pi^*_{H,t} = \pi_{F,t} - \pi^*_{F,t}$.

D Variable sources and definitions

Variables:

S = krone per euro exchange rate. Source: Macrobond

I = nominal interest rate Norway. Source: Statistics Norway

I^* = nominal interest rate euro area. Source: Statistics Norway

P = consumer price index Norway. Source: Statistics Norway

P^* = consumer price index euro area. Source: Statistics Norway

VIX = S&P500 volatility index. This is a measure of the volatility on the S&P500 equity index, which is made up of 500 of the largest companies traded on US stock markets. Source: Chicago Board Options Exchange

P^{oil} = oil price (Brent blend) in USD per barrel. Source: IMF

A = Norwegian petroleum-related equity index. Source: Macrobond

FDI = the difference between foreign direct investment as a percentage of GDP in the euro area and in Norway: Sources Macrobond, Oxford Economics and Swedbank

V = ratio of the value of Norwegian oil and gas exports to the value of total Norwegian exports. Source: Statistics Norway

u = Rate of unemployment (ILO), Norway

u^* = Rate of unemployment (ILO), euro area

Definitions:

s = logarithm of the nominal exchange rate (NOK)

p = logarithm of the core consumer price index in Norway

p^* = logarithm of the core consumer price index in the euro area

$dp = p - p^*$ = logarithm of relative core consumer prices between Norway and the euro area

$dI = I - I^*$

$q = s - (p - p^*) = s - dp$ = logarithm of the real exchange rate

$\pi = 400 * (p_t - p_{t-1})$ = Core consumer price inflation in Norway, annualized, percentage points

$\pi^* = 400 * (p_t^* - p_{t-1}^*)$ = Core consumer price inflation in euro area, annualized, percentage points

$d\pi = \pi - \pi^*$ = inflation difference between Norway and the euro area, annualised and in percentage points

$r_t = I_t - \pi_{t+1}$ = real ex ante domestic interest rate in period t

$I_t - (p_{t+1} - p_{t-3})$ = real ex ante domestic interest rate utilizing 4 quarter changes in the core consumer price index, percentage points

$r_t^* = I_t^* - (\pi_{t+1}^*)$ = real ex ante foreign interest rate in period t

$I_t^* - (p_{t+1}^* - p_{t-3}^*)$ = real ex ante foreign interest rate utilizing 4 quarter changes in the core consumer price index, percentage points

$dr_t = r_t - r_t^* = dI_t - d\pi_{t+1}$ = real ex ante interest rate differential

$dI_t - (dp_{t+1} - dp_{t-3})$ = real ex ante interest rate differential utilizing 4 quarter core CPI differences

v = logarithm of the ratio of the value of oil and gas to the value of total Norwegian exports

a = logarithm of the petroleum-related equity index

p^{oil} = logarithm of the oil price in USD

dy = output gap difference based on unemployment rates $(1/u_t^2 - 1/u_t^{*2})$.

Note: lower case letters indicate logarithms with the exception of the interest rate r and r^*. An asterisk indicates the euro area.

E IV estimation of the open economy Phillips curve

Under the assumptions of a correctly specified model and perfect foresight, the FIML estimator $\hat{\beta}_\pi$ in the Phillips curve equation (7) as part of the 3-equation system is asymptotically unbiased when using observed values Δdp_{t+1} for the unobserved expectations $\mathsf{E}_t\Delta dp_{t+1}$. In our empirical model, the estimator could be biased due to correlation between the regressor Δdp_{t+1} and the residual $\widehat{\tilde{\epsilon}_t^\pi}$. The correlation could be caused by misspecification, omitted variables, the finite data sample and/or (rational) expectation errors. In the FIML estimated system (12)-(14), the empirical correlation $\widehat{corr}(\Delta dp_{t+1}, \tilde{\epsilon}_t^\pi) = -0.061$. This is a fairly small number "by construction" that does not indicate an unbiased estimate of β_π.

First, we estimate the Phillips curve as a single equation (7) outside the system, using the instrumental variable (IV) method. The set of instruments for the endogenous variable q_t and the latent variable $\mathsf{E}_t\Delta dp_{t+1}$ contains non-modeled exogenous variables and lagged endogenous variables in the system (7)-(9):

$$\{const, D_{\pi,t}, \Delta p_t^{oil}, \Delta p_{t-1}^{oil}, \Delta y_t^*, \Delta y_{t-1}^*, \Delta I_t^*, \Delta I_{t-1}^*, dI_{t-1}, \Delta dp_{t-2}\}. \tag{30}$$

The IV-estimator in OxMetrics (Doornik (2018)) then yields

$$\Delta dp_t = \underset{(9.20)}{-27.38} + \underset{(0.044)}{0.13}\,\hat{q}_t + \underset{(0.202)}{0.997}\widehat{\Delta dp}_{t+1} + D_{\pi,t} + \epsilon_t^\pi, \tag{31}$$

AR 1-5 test: $F(5,69) = 1.8872$ [0.1078]	ARCH 1-4 test: $F(4,72) = 0.99259$ [0.4172]
Normality test: $\chi^2(2) = 2.5899$ [0.2739]	Hetero test: $F(6,72) = 1.2598$ [0.2868]
Hetero-X test: $F(7,71) = 1.5023$ [0.1805]	RESET23 test: $F(2,72) = 0.82029$ [0.4444]

Sargent's specification test: $\chi^2(6) = 11.599$ [0.072]

The diagnostic tests indicate that the estimated Phillips curve equation (31) satisfies most white noise requirements, including no autocorrelation (AR) and absence of different forms of heteroscedasticity, such as time-varying volatility (ARCH), general heteroscedasticity (Breusch-Pagan or White tests), and heteroscedasticity related to specific explanatory variables (Hetero-X test). The Normality test suggests that the residuals are normally distributed, and the RESET test confirms the absence of non-linear misspecification. More importantly, the test by Sargan (1964) does not reject the null hypothesis that the instruments are uncorrelated with the error term. P-values are reported in brackets. We conclude that the instruments in Equation (30), used to estimate the Phillips curve with a single-equation IV procedure, are valid.

Beyond the estimates themselves, our motivation for the single equation IV estimation (31) is to demonstrate the validity of our instruments using a formally accepted test, specifically the Sargan test. We first regress Δdp_{t+1} on the instruments (30):

$$\begin{aligned}\widehat{\Delta dp}_{t+1} = & -\underset{(1.85)}{3.64}\Delta p_t^{oil} - \underset{(1.92)}{3.99}\Delta p_{t-1}^{oil} - \underset{(620)}{768}\Delta y_t^* - \underset{(710)}{297}\Delta y_{t-1}^* + \underset{(1.18)}{1.58}\Delta I_t^* + \underset{(1.034)}{0.68}\Delta I_{t-1}^* \\ & + \underset{(0.27)}{0.12}(I - I^*)_{t-1} + \underset{(0.13)}{0.13}\Delta dp_{t-2} + \tilde{D}_{\pi,t},\end{aligned} \tag{32}$$

where $\tilde{D}_{\pi,t}$ is shorthand for the aggregate effect of a structural break in 2017, seasonal variation, and an estimated constant. We then use the predicted (fitted) values from this regression as an instrument for the endogenous latent variable $\mathsf{E}_t\Delta dp_{t+1}$ in the FIML estimation of the simultaneous system, yielding (15)-(17). This two-step procedure ensures that the instruments used in the second step, the FIML estimation of the system, are appropriately derived from an IV analysis. Note that no statistical method exists to test for the validity of an instrument in an equation that is part of a simultaneous system of equations like (15)-(17). Consequently, we tested the validity of the instruments in a separate and preliminary IV estimation of the Phillips curve.

In addition to using the observed values Δdp_{t+1} in the system estimation (12)-(14) and the instrument (32) in the system estimation (15)-(17), we also use a centered moving average over 5 periods:

$$\overline{\Delta dp}_{t+1} = MA(\Delta dp_{t+1}, 5) \tag{33}$$

as an approximation to the latent variable $\mathsf{E}_t\Delta dp_{t+1}$. The smoothed series $\overline{\Delta dp}$ is ad hoc, with high correlation with the observations (used in place of the unobserved expectations): $\widehat{corr}(\overline{\Delta dp}, \Delta dp) = 0.75$. The IV $\widehat{\Delta dp}$ is less correlated with the observations: $\widehat{corr}(\widehat{\Delta dp}, \Delta dp) = 0.45$. The two left

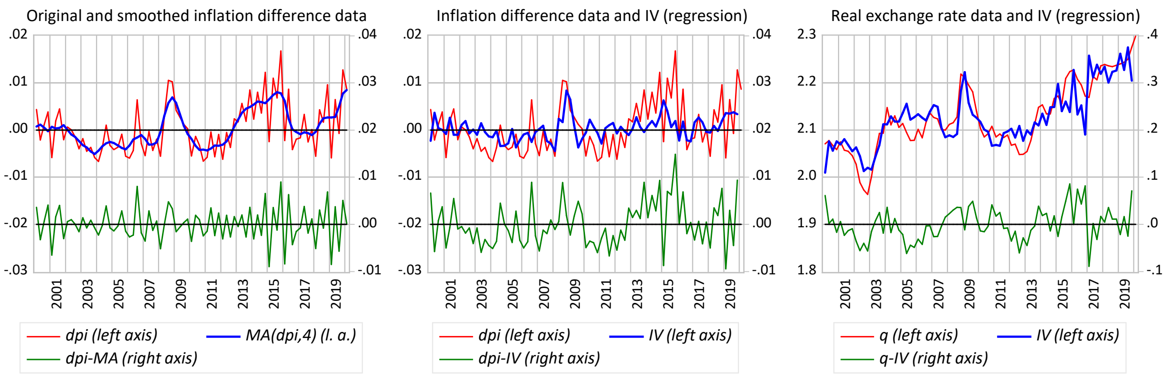

Figure 6: Time series data for Δdp_{t+1} and q_t (red graphs), and their instruments $\widetilde{\Delta dp}_{t+1}$, $\widehat{\Delta dp}_{t+1}$, and $\widehat{q}_t$ (blue graphs). The lower graphs (green) are the differences between data and instruments.

panels in Figure 6 show the data series Δdp in red, and the two instruments $\overline{\Delta dp}$ and $\widehat{\Delta dp}$ in blue. The lower green graphs show the differences between the same data series and the instrument series: $\Delta dp - \overline{\Delta dp}$ and $\Delta dp - \widehat{\Delta dp}$. The right panel shows the instrument $\hat{q}_t$ used in (31).

In the FIML estimated system (15)-(17) with an instrument for the unobserved expectation $\mathsf{E}_t(\Delta dp_{t+1})$ in (15), almost all estimates of the coefficients in the system change relative to the uninstrumented FIML estimation (12)-(14). However, the changes in the equations for Δq_t and ΔI_t are small and economically insignificant. More interesting and important are the estimates of the coefficients in the Phillips curve equation, which are displayed in Table 2. The table presents the estimation results for the Phillips curve equation using three different methods: single-equation OLS, single-equation IV estimation considering the endogenous nature of the variables q_t and Δdp_{t+1}, and simultaneous FIML estimation of all three behavioral equations in the system, with the identities in Appendix I included. The first column lists the various regressors, with the last row indicating the two variables treated as endogenous in the single-equation IV estimation. In the fifth column of the last row, the correlations of the residuals with the regressor q_t (top) and Δdp_{t+1} (bottom) are shown in braces, based on this method.

Using instruments increases the estimated effect ($\hat{\beta}$) of the inflation expectation term in all estimations. The smoothed-data $\overline{\Delta dp}_{t+1}$ produces smaller and imprecise estimates of the constant c and the effect δ of the real exchange rate q_t. The system/FIML estimates are more precise than the single-equation/OLS estimates. The single-equation estimates are not shown just for comparisons. They are close to the system estimates, and thus add confidence in the system 'identification' of the foreward-looking Phillips curve.

$\Delta dp_t = c + \delta\, q_t + \beta\, \mathsf{E}_t(\Delta dp_{t+1}) + D_{\pi,t} + \epsilon_t^{\pi}$								
Instruments	Single equation estimation				System estimation			
	$\hat{c}$	$\hat{\delta}$	$\hat{\beta}$	$\widehat{corr}(IV, \epsilon_t^{\pi})$	$\hat{c}$	$\hat{\delta}$	$\hat{\beta}$	$\widehat{corr}(IV, \epsilon_t^{\pi})$
Δdp_{t+1}	-25.34 (8.25)	0.12 (0.039)	0.15 (0.10)	$-52\cdot 10^{-17}$	-27.17 (7.11)	0.13 (0.034)	0.17 (0.07)	-0.061
$\overline{\Delta dp}_{t+1}$	-7.70 (7.57)	0.039 (0.036)	0.75 (0.13)	$-51\cdot 10^{-17}$	-13.18 (6.43)	0.063 (0.031)	0.64 (0.11)	-0.030
$\widehat{\Delta dp}_{t+1}$	—	—	—	—	-25.91 (6.19)	0.124 (0.029)	0.95 (0.18)	-0.030
$q_t, \Delta dp_{t+1}$	-27.38 (9.20)	0.13 (0.044)	0.997 (0.202)	$\left\{ \begin{matrix} 41 \\ -29 \end{matrix} \right\} \cdot 10^{-17}$	—	—	—	—

Table 2: Estimates of the Phillips curve equation with observations and different instruments for $\mathsf{E}_t(\Delta dp_{t+1})$ in single-equation OLS estimation and in system FIML estimation.

F Method of Blanchard and Kahn

We are using the method of Blanchard and Kahn (1980) to solve the system in (6). The composite 3×3 matrix $\boldsymbol{A}^{-1}\boldsymbol{B}$ and 3×2 matrix $\boldsymbol{A}^{-1}\boldsymbol{C}$ on the right side of (6) are in this Appendix denoted by $\boldsymbol{A}$ and $\boldsymbol{B}$. A 3×3 matrix pertaining to the parameters associated with a suppressed error term, here denoted by $\boldsymbol{C}$, are also included. We partition these matrices according to the partitioning of the vector of endogenous variables $\boldsymbol{z}_t = (\pi_t - \pi_t^*, q_t, I_{t-1} - I_{t-1}^*)'$ into a vector of two free variables $\boldsymbol{z}_t^1 = (\pi_t - \pi_t^*, q_t)'$ and the predetermined variable $z_t^2 = (I_{t-1} - I_{t-1}^*)$ (in our theoretical model only one). The model (6) can be written as

$$\underbrace{\begin{pmatrix} \boldsymbol{z}_t^1 \\ z_t^2 \end{pmatrix}}_{\boldsymbol{z}_t} = \underbrace{\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21}' & a_{22} \end{bmatrix}}_{\boldsymbol{A}} \mathsf{E}_t \underbrace{\begin{pmatrix} \boldsymbol{z}_{t+1}^1 \\ z_{t+1}^2 \end{pmatrix}}_{\boldsymbol{z}_{t+1}} + \underbrace{\begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{b}_2' \end{bmatrix}}_{\boldsymbol{B}} \boldsymbol{f}_t + \underbrace{\begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{c}_{12} \\ \boldsymbol{c}_{21}' & c_{22} \end{bmatrix}}_{\boldsymbol{C}} \begin{pmatrix} \boldsymbol{\omega}_{t+1} \\ \boldsymbol{\upsilon}_{t+1} \end{pmatrix} \tag{34}$$

where the matrix elements $\boldsymbol{A}_{11}$, $\boldsymbol{B}_1$ and $\boldsymbol{C}_{11}$ are 2×2 matrices, $\boldsymbol{a}_{12}$, $\boldsymbol{a}_{21}$, $\boldsymbol{c}_{21}$, and $\boldsymbol{b}_2$ are column vectors with 2 elements, and a_{22} is a scalar. The vector $\boldsymbol{f}_t = (\lambda_t, dy_t)'$ holds the exogenous forcing variables in (6). The expectation error terms are $\boldsymbol{\omega}_{t+1} = (\omega_{\pi,t+1}, \omega_{q,t+1})' = (\mathsf{E}_t(\pi_{t+1}) - \pi_{t+1}, \mathsf{E}_t(q_{t+1}) - q_{t+1})'$, while $\boldsymbol{\upsilon}_{t+1} = \boldsymbol{\varepsilon}_t - \boldsymbol{\varepsilon}_t^*$ represents a structural shock originating from the two country specific monetary policy rules (see Footnote 4 for an explanation). Hence, $\boldsymbol{c}_{12}$ is a 2×1 vector and c_{22} is a scalar.

The Blanchard-Kahn method begins with a Jordan matrix decomposition $\boldsymbol{A} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{-1}$. The elements of the diagonal $\boldsymbol{\Lambda}$ are the eigenvalues of $\boldsymbol{A}$ ordered by increasing absolute value down the diagonal. The corresponding eigenvectors form the columns of the matrix $\boldsymbol{Q}$. The diagonal matrix $\boldsymbol{\Lambda}$ can be written as

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \lambda_{22} \end{bmatrix}. \tag{35}$$

For a unique solution, the two eigenvalues on the diagonal of $\boldsymbol{\Lambda}_{11}$ must be within the unit circle, and λ_{22} must be outside the unit circle. The eigenvalues in $\boldsymbol{\Lambda}_{11}$ will then be forward stable since $\boldsymbol{\Lambda}_{11}^t \to 0$ as $t \to \infty$. The eigenvalue λ_{22} will be unstable or explosive, since $|\lambda_{22}^t|$ diverges as t increases. The matrix $\boldsymbol{Q}$ is partitioned conformably as

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{q}_{12} \\ \boldsymbol{q}_{21}' & q_{22} \end{bmatrix}. \tag{36}$$

When the number of forward stable eigenvalues is equal to the number of free variables, which is normally assumed in the literature and in the paragraph above, the system is said to be saddle path stable and a unique solution to the model exists. Otherwise, the model is either undetermined, in the sense of generating an infinite number of solutions, or unstable.

Proceeding in the case of saddle-path stability, we insert (35) and (36) into the Jordan-decomposed version of (34) and multiply the system by $\boldsymbol{Q}^{-1}$. We denote the transformed variable vectors by $\widetilde{\boldsymbol{z}}_t^1$ and $\widetilde{z}_t^2$, such that

$$\begin{pmatrix} \widetilde{\boldsymbol{z}}_t^1 \\ \widetilde{z}_t^2 \end{pmatrix} = \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{q}_{12} \\ \boldsymbol{q}_{21}' & q_{22} \end{bmatrix}^{-1} \begin{pmatrix} \boldsymbol{z}_t^1 \\ z_t^2 \end{pmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{Q}}_{11} & \widetilde{\boldsymbol{q}}_{12} \\ \widetilde{\boldsymbol{q}}_{21}' & \widetilde{q}_{22} \end{bmatrix} \begin{pmatrix} \boldsymbol{z}_t^1 \\ z_t^2 \end{pmatrix}. \tag{37}$$

Then (34) becomes

$$\begin{pmatrix} \widetilde{\boldsymbol{z}}_t^1 \\ \widetilde{z}_t^2 \end{pmatrix} = \begin{bmatrix} \boldsymbol{\Lambda}_{11} & 0 \\ 0 & \lambda_{22} \end{bmatrix} \begin{pmatrix} \widetilde{\boldsymbol{z}}_{t+1}^1 \\ \widetilde{z}_{t+1}^2 \end{pmatrix} + \begin{bmatrix} \widetilde{\boldsymbol{B}}_1 \\ \widetilde{\boldsymbol{b}}_2' \end{bmatrix} \boldsymbol{f}_t + \begin{bmatrix} \widetilde{\boldsymbol{C}}_{11} & \widetilde{\boldsymbol{c}}_{12} \\ \widetilde{\boldsymbol{c}}_{21}' & \widetilde{c}_{22} \end{bmatrix} \begin{pmatrix} \boldsymbol{\omega}_{t+1} \\ \upsilon_{t+1} \end{pmatrix}, \tag{38}$$

where

$$\begin{bmatrix} \widetilde{\boldsymbol{B}}_1 \\ \widetilde{\boldsymbol{b}}_2' \end{bmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{Q}}_{11} & \widetilde{\boldsymbol{q}}_{12} \\ \widetilde{\boldsymbol{q}}_{21}' & \widetilde{q}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{b}_2' \end{bmatrix} \tag{39}$$

and

$$\begin{bmatrix} \widetilde{\boldsymbol{C}}_{11} & \widetilde{\boldsymbol{c}}_{12} \\ \widetilde{\boldsymbol{c}}_{21}' & \widetilde{c}_{22} \end{bmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{Q}}_{11} & \widetilde{\boldsymbol{q}}_{12} \\ \widetilde{\boldsymbol{q}}_{21}' & \widetilde{q}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{c}_{12} \\ \boldsymbol{c}_{21}' & c_{22} \end{bmatrix}. \tag{40}$$

This transformation effectively decouples the system, so that the non-predetermined variables $\boldsymbol{z}_t^1 = (\pi_t - \pi_t^*, q_t)'$ depend only upon the forward-stable eigenvalues $\boldsymbol{\Lambda}_{11}$ of the matrix $\boldsymbol{A}$ as expressed in the upper row in (38).

Having decoupled the system, we derive a solution for the non-predetermined variables by taking the expectation on both sides of (38) (remember $\mathsf{E}_t(\boldsymbol{\omega}_{t+1}) = \mathbf{0}$ and $\mathsf{E}_t(\upsilon_{t+1}) = 0$) and performing a forward iteration on the upper portion of the equation system. This is accomplished as follows. First, re-express, the upper portion of (38) as

$$\widetilde{\boldsymbol{z}}_t^1 = \boldsymbol{\Lambda}_{11}\mathsf{E}_t(\widetilde{\boldsymbol{z}}_{t+1}^1) + \widetilde{\boldsymbol{B}}_1\boldsymbol{f}_t. \tag{41}$$

This implies that $\widetilde{\boldsymbol{z}}_{t+1}^1 = \boldsymbol{\Lambda}_{11}\mathsf{E}_t(\widetilde{\boldsymbol{z}}_{t+2}^1) + \widetilde{\boldsymbol{B}}_1\mathsf{E}_t(\boldsymbol{f}_{t+1})$ which can be substituted into (41) to obtain

$$\widetilde{\boldsymbol{z}}_t^1 = \boldsymbol{\Lambda}_{11}^2\mathsf{E}_t(\widetilde{\boldsymbol{z}}_{t+2}^1) + \boldsymbol{\Lambda}_{11}\widetilde{\boldsymbol{B}}_1\mathsf{E}_t(\boldsymbol{f}_{t+1}) + \widetilde{\boldsymbol{B}}_1\boldsymbol{f}_t \tag{42}$$

Since $\boldsymbol{\Lambda}_{11}$ contains eigenvalues with magnitude less than 1, $\boldsymbol{\Lambda}_{11}^t$ disappears as t approaches infinity. Continuation of this process of iteration can thus be written as

$$\widetilde{\boldsymbol{z}}_t^1 = \sum_{i=0}^{\infty}\boldsymbol{\Lambda}_{11}^i\widetilde{\boldsymbol{B}}_1\mathsf{E}_t(\boldsymbol{f}_{t+i}). \tag{43}$$

By using (37), we can map this expression back into an expression for $\boldsymbol{z}_t^1$. We then get

$$\widetilde{\boldsymbol{z}}_t^1 = \widetilde{\boldsymbol{Q}}_{11}\boldsymbol{z}_t^1 + \widetilde{\boldsymbol{q}}_{12}z_t^2 \Rightarrow \boldsymbol{z}_t^1 = \widetilde{\boldsymbol{Q}}_{11}^{-1}\widetilde{\boldsymbol{z}}_t^1 - \widetilde{\boldsymbol{Q}}_{11}^{-1}\widetilde{\boldsymbol{q}}_{12}z_t^2.$$

Inserting the expression (43) for $\widetilde{\boldsymbol{z}}_t^1$ in the above expression then gives

$$\boldsymbol{z}_t^1 = \widetilde{\boldsymbol{Q}}_{11}^{-1}\sum_{i=0}^{\infty}\boldsymbol{\Lambda}_1^i\boldsymbol{B}_1\mathsf{E}_t(\boldsymbol{f}_{t+i}) - \widetilde{\boldsymbol{Q}}_{11}^{-1}\widetilde{\boldsymbol{q}}_{12}z_t^2. \tag{44}$$

Finally, to solve the nonexplosive portion of the system, we can start by multiplying (34)by $\boldsymbol{A}^{-1}$ and setting the disturbance term equal to its zero-expected value, giving

$$\begin{pmatrix} \boldsymbol{z}_{t+1}^1 \\ z_{t+1}^2 \end{pmatrix} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{\boldsymbol{a}}_{12} \\ \widetilde{\boldsymbol{a}}_{21}' & \widetilde{a}_{22} \end{bmatrix} \begin{pmatrix} z_t^1 \\ z_t^2 \end{pmatrix} + \begin{bmatrix} \widetilde{A}_{11} & \widetilde{\boldsymbol{a}}_{12} \\ \widetilde{\boldsymbol{a}}_{21} & \widetilde{a}_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ \boldsymbol{b}_2' \end{bmatrix} f_t, \tag{45}$$

where the $\widetilde{A}_{ij}$ matrices represent the partitions of A^{-1} conformable with z_t^1 and z_t^2. By expanding the lower part of (45) we then get that

$$z_{t+1}^2 = \widetilde{\boldsymbol{a}}_{21}'z_t^1 + \widetilde{a}_{22}z_t^2 + (\widetilde{\boldsymbol{a}}_{21}'B_1 + \widetilde{a}_{22}\boldsymbol{b}_2')f_t. \tag{46}$$

Substituting for z_t^1 using (44) then yields a solution for z_t^2 depending on the exogenous forcing variables and the past.

$$z_{t+1}^2 = \widetilde{\boldsymbol{a}}_{21}'\widetilde{\boldsymbol{Q}}_{11}^{-1}\sum_{i=0}^{\infty}\boldsymbol{\Lambda}_1^i\boldsymbol{B}_1\mathsf{E}_t(\boldsymbol{f}_{t+i}) + (\widetilde{a}_{22} - \widetilde{\boldsymbol{a}}_{21}'\widetilde{\boldsymbol{Q}}_{11}^{-1}\widetilde{\boldsymbol{q}}_{12})z_t^2 + (\widetilde{\boldsymbol{a}}_{21}'B_1 + \widetilde{a}_{22}\boldsymbol{b}_2')f_t. \tag{47}$$

G Eigenvalues of the model's dynamic mapping

The dynamic properties of our model decide whether it has a unique, so-called saddle-path solution, multiple solutions, or no stable solution. We investigate the solution alternatives by approximating the eigenvalues of the model. Since the model contains endogenous variables with a lead and several lags, it is not straightforward to do so. First, we write the model as a certain vector-matrix equation. Next, the matrices and vectors are 'stacked' in a companion form, which allows us to calculate its eigenvalues.

We put the endogenous variables in the System (12)-(14) into a vector $\boldsymbol{z}_t = (d\pi_t, q_{t-1}, I_{t-1})'$. All other terms — exogenous variables including I^* and π^*, and dummies — are collected in a vector $\boldsymbol{f}$. The System (12)-(14) can then be written as:

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0.004 & 1 & 0.073 \\ 0.034 & 0 & 1.25 \end{pmatrix}}_{\boldsymbol{A}_1} \underbrace{\begin{pmatrix} d\pi_t \\ q_{t-1} \\ I_{t-1} \end{pmatrix}}_{\boldsymbol{z}_t} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.19 & -0.031 \\ 0.034 & 0 & -0.25 \end{pmatrix}}_{\boldsymbol{A}_2} \underbrace{\begin{pmatrix} d\pi_{t-1} \\ q_{t-2} \\ I_{t-2} \end{pmatrix}}_{\boldsymbol{z}_{t-1}} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0.034 & 0 & 0 \end{pmatrix}}_{\boldsymbol{A}_3} \underbrace{\begin{pmatrix} d\pi_{t-2} \\ q_{t-3} \\ I_{t-3} \end{pmatrix}}_{\boldsymbol{z}_{t-2}}$$

$$+ \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.034 & 0 & 0 \end{pmatrix}}_{\boldsymbol{A}_4} \underbrace{\begin{pmatrix} d\pi_{t-3} \\ q_{t-4} \\ I_{t-4} \end{pmatrix}}_{\boldsymbol{z}_{t-3}} = \underbrace{\begin{pmatrix} 0.17 & 12.94 & 0 \\ 0 & 1 & 0.046 \\ 0 & 0 & 1 \end{pmatrix}}_{\boldsymbol{A}_0} \underbrace{\begin{pmatrix} d\pi_{t+1} \\ q_t \\ I_t \end{pmatrix}}_{\boldsymbol{z}_{t+1}} + \boldsymbol{G}\,\boldsymbol{f},$$

where $\mathbf{G}$ is a matrix of zeros and the estimated coefficient of the terms in $\mathbf{f}$. In (14), there is a term $dp_t - dp_{t-4}$, which is equal to $\Delta dp_t + \Delta dp_{t-1} + \Delta dp_{t-2} + \Delta dp_{t-3}$. Since $d\pi_t \equiv \Delta dp_t$, the term $dp_t - dp_{t-4}$ is written as $d\pi_t + d\pi_{t-1} + d\pi_{t-2} + d\pi_{t-3}$ in the expression above.

To facilitate the calculation of the roots of the System (12)-(14), we truncate the number of lags in the expression above to two. This should not change the dynamic properties of the system to any significant degree as it only involves removing a third lag in q_t and slightly changing the operational definition of the inflation variable $d\pi$ used in the interest rate equation. The advantage of doing so is that the eigenvalues can be easily calculated. Doing this and, as a consequence, approximating the last inflation term in (14) by the difference over two periods, gives the following companion form representation of the endogenous part of our system:

$$\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_t \\ \boldsymbol{z}_{t-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{t+1} \\ \boldsymbol{z}_t \end{bmatrix}$$

$$\Updownarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.004 & 1 & 0.073 & 0 & 0.19 & -0.031 \\ 0.034 & 0 & 1.26 & 0.034 & 0 & -0.25 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\pi_t \\ q_{t-1} \\ I_{t-1} \\ d\pi_{t-1} \\ q_{t-2} \\ I_{t-2} \end{bmatrix} = \begin{bmatrix} 0.17 & 12.94 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.046 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\pi_{t+1} \\ q_t \\ I_t \\ d\pi_t \\ q_{t-1} \\ I_{t-1} \end{bmatrix}$$

Since the matrix on the left is nearly singular, we opt to calculate the system's eigenvalues using the inverse of what corresponds to the $\boldsymbol{A}$ matrix in System (34) in Appendix F. This approach reverses the conditions required for the system to produce a stable saddle path or multiple equilibria. Instead of needing the number of free variables to match the number of forward-stable eigenvalues, the condition for a stable saddle path now requires that the number of free variables equals the number of eigenvalues outside the unit circle. Alternatively, for the system to generate multiple equilibria, the number of eigenvalues outside the unit circle must be fewer than the number of free variables.

By using the two matrices in the expression above to compute the inverse of the $\boldsymbol{A}$ matrix from system (34) and determine its eigenvalues, we obtain the following eigenvalues in descending order:

$$\mathbf{5.54}, \quad 0.40, \quad 0.04, \quad 0.02, \quad 7.1{\cdot}10^{-5}, \quad -3.0{\cdot}10^{-17}.$$

Only one eigenvalue exceeds one (and thus lies outside the unit circle), while all the others are well within the unit circle. Given that the number of free variables is two, this suggests the presence of multiple equilibria.

H Impulse responses

We apply an impulse shock to the error term in each structural equation of the simultaneous model and simulate the response of the endogenous variable over the next 50 quarters. The shock is a one standard deviation increase that lasts for one period only. The shock is applied to each equation one at a time.

The shock to the inflation equation (π or Δp) can be interpreted as a supply shock. The shock to the real exchange rate equation (q) can be interpreted as an exogenous terms-of-trade shock, and the shock to the interest rate equation (I) as a policy-implementation-error shock (Lubik and Schorfheide, 2004). The standard errors of these shocks are, respectively, 1.77, 0.019, and 0.127.

Figure 7 below, shows responses of the endogenous variables –in differences and in levels – to a positive terms-of-trade shock to the real exchange equation (for Δq). The shock leads to an immediate real depreciation of the krone exchange rate and an increase in inflation, peaking after approximately 5 quarters. This inflation increase triggers an almost immediate policy response, with the interest rate peaking after about 10 quarters. The interest rate response helps to curb inflation, which then falls sharply over the next few years. However, as indicated by the accumulated impulse responses (level variable responses) in the bottom row of panels, the price level never returns to its pre-shock level. In contrast, the real exchange rate and interest rate eventually return to their pre-shock levels after about 40-50 quarters.

Figure 8 on the top of the next page, shows responses of the endogenous variables – in differences and in levels – to a contractionary exogenous monetary policy shock to the interest rate equation (for ΔI). The shock immediately leads to an appreciation of the krone and lower inflation, while the nominal interest rate rises. The main effect on inflation occurs after about 4-5 quarters, consistent with Christiano et al. (2005). The interest rate quickly falls below the steady state after the disturbance, but as shown by the accumulated impulse responses, it takes a long time for the shock's effect to be fully neutralized, with the price level not returning to its pre-shock level even after 50 quarters.

Figure 9 at the bottom of the next page, shows responses of the endogenous variables – in differences and in levels – to a positive exogenous supply shock to the inflation equation (for ΔP).

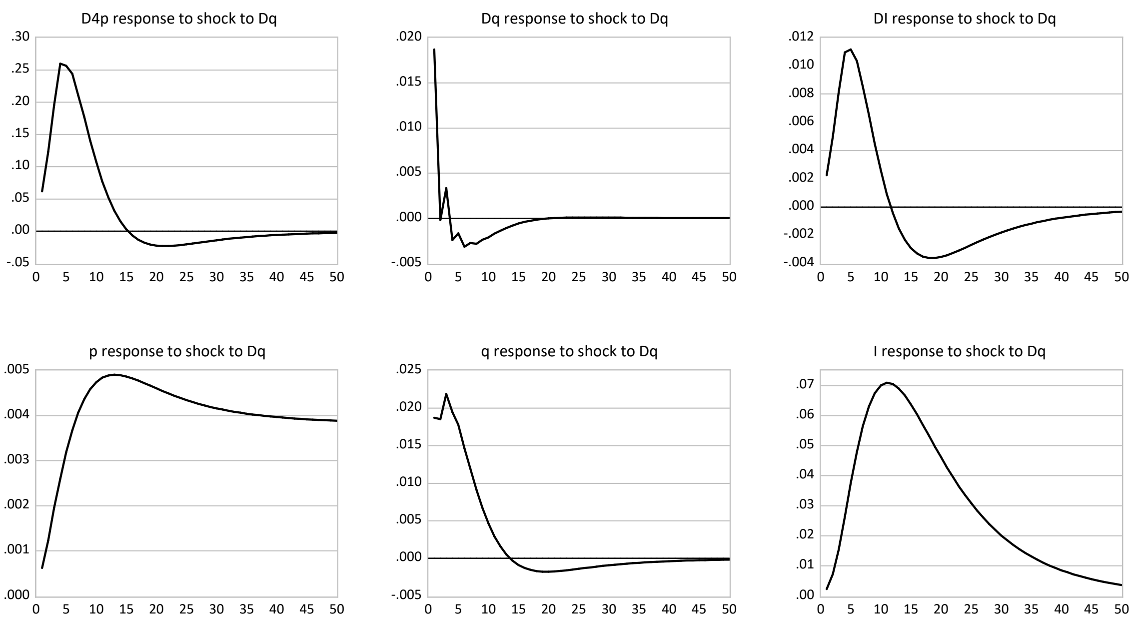

Figure 7: Impulse responses over 50 quarters to a positive 1 standard deviation, 1 period, (terms-of-trade) shock to the real exchange rate equation (Dq=Δq). The panels in the upper row show the impulse responses of the changes in the endogenous variables. In the headings, D4p denotes $\Delta_4 p_t = p_t - p_{t-4}$, Dq denotes Δq_t, and DI denotes ΔI_t. The lower panels show the accumulated responses of the level of the variables. The left panels show the impulse responses of inflation and the price level. The center panels show the corresponding responses of the real exchange rate, while the results for the interest rate are displayed in the right panels.

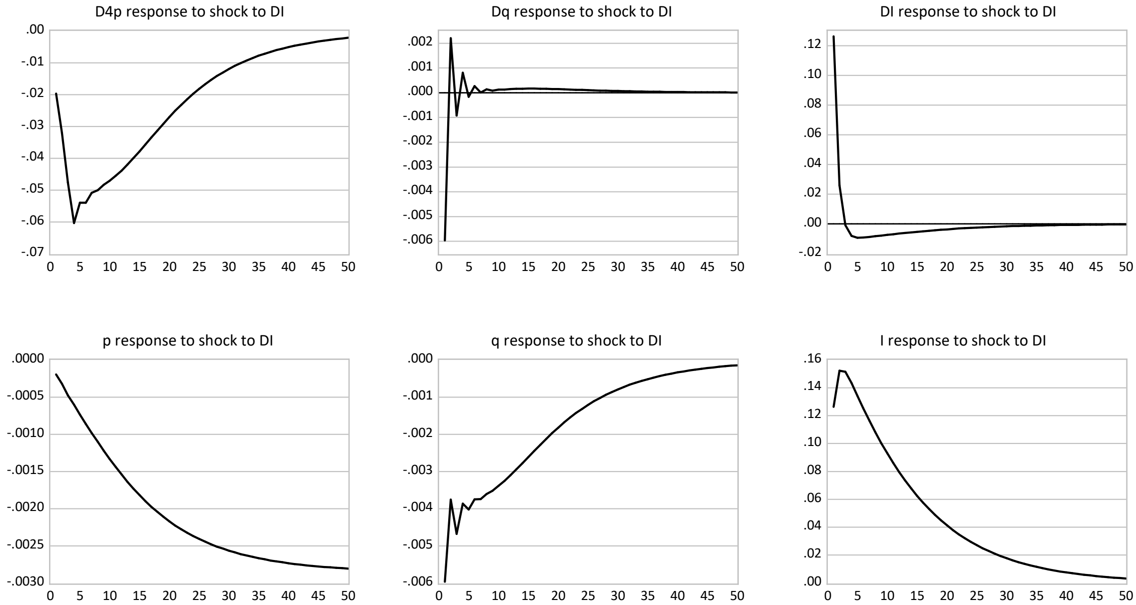

Figure 8: Impulse responses over 50 quarters to a positive 1 standard deviation, 1 period, (policy-implementation-error) shock to the interest rate equation (DI=ΔI). See Figure 7 for explanations of the panels.

The shock immediately raises inflation and interest rates, leading to an appreciation of the krone exchange rate. While the shock increases inflation by about 0.4 percent, it quickly subsides after 5 quarters as the shock drops out of the four-quarter change in prices and the interest rate change affects the trajectory. Unlike the other shocks, the effect of the supply shock appears to be fully neutralized after 50 quarters, with all variables returning to their pre-shock levels.

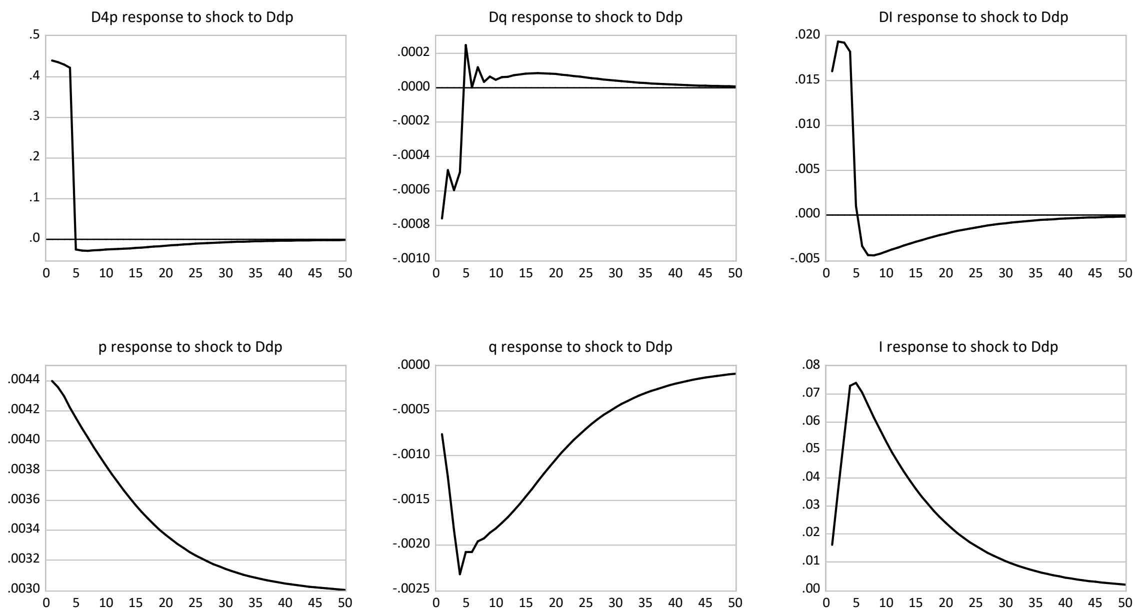

Figure 9: Impulse responses over 50 quarters to a positive 1 standard deviation, 1 period, (supply) shock to the inflation equation (Dp=Δp). See Figure 7 for explanations of the panels.

I System identities

In the reduction process from the GUM to the final model, definitional equations (identities) have been added to the system (7)-(9) or (10), primarily to preserve information about the level of variables. Equations that calculate levels are

$$Ci_t = Ci_{t-1} + \Delta Ci_t, \; i = 1, 2, ..., 6,$$

where

$$\begin{aligned}
C1_t &\equiv q_t + 0.023a_t + 0.23v_t - 0.004(FDI + VIX)_t \\
C2_t &\equiv dI_t - d\pi_{t+1} \\
C3_t &\equiv dI_t - (dp_{t+1} - dp_{t-3}) \\
C4_t &\equiv q_t = s_t - dp_t = s_t - (p_t - p_t^*) \\
C5_t &\equiv p_t \\
C6_t &\equiv I_t
\end{aligned}$$

The three first identities for the equilibrium correction terms capture dynamic feedback effects during dynamic simulations and forecasts. Additionally, a fourth identity for the real exchange rate q_t eliminates the need to instrument this variable in (7), thereby mitigating potential issues with simultaneity bias. The last two identities calculate the levels of the domestic price and the domestic interest rate.

This approach has been used to improve the model's ability to simulate and predict levels of variables that can be directly inferred from the variables being modeled, like the level of the domestic price index (p_t) above. In addition to the dynamic equations above, two identities are added for the level of the domestic interest rate (I_t) and the nominal exchange rate:

$$\begin{aligned}
I_t &= dI_t + I_t^* \\
s_t &= q_t + p_t - p_t^* = q_t + dp_t
\end{aligned}$$

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