



The costs of taxation in the presence of inequality

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Abstract:

This paper provides a new and improved measure of the marginal cost of public funds (MCF). It is based on a benchmark tax which is distributionally neutral and non-distortive. This is in contrast to the MCF-measure used in the previous literature, that has used the regressive uniform lump-sum tax as the benchmark. Our proposed MCF-measure more precisely accounts for the distributional aspects of public funding (the tax scheme) and makes a clear distinction between this and the distributional aspects of the public good considered. Compared to the previous literature, we find a higher MCF both in the case of a uniform lump-sum tax and in the case of distortive taxes. Due to its regressive distributional consequences, we find that the MCF of a uniform lump-sum tax is always greater than one when not combined with distortive taxes. Moreover, we find that the MCF could be greater than one also with an optimal combination of a uniform lump-sum tax and distortive taxes.

Keywords: Marginal cost of public funds, lump-sum taxes, public goods

JEL classification: H20, H40, H50

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Sammendrag

Dette arbeidet utvikler et nytt og forbedret mål på den såkalte skattefinansieringskostnaden. En viktig forbedring er at det nye målet har som referanse en hypotetisk finansiering av offentlig konsum som er fordelingsnøytral i tillegg til at den ikke har negative effektivitetsvirkninger. Dette står i kontrast til det målet på skattefinansieringskostnaden som er brukt i den relevante forskningslitteraturen tidligere.

Den har brukt som referansemål et likt skattebeløp på hvert individ, også kalt koppskatt. En slik koppskatt er klart regressiv og kan ha svært ugunstige fordelingsegenskaper. Det er derfor uheldig å benytte denne som referanse når en skal måle de reelle fordelingseffektene av skatter. Vårt mål på skattefinansieringskostnaden vil mer presist ta hensyn til fordelingsvirkningene av skattene som brukes og gjør samtidig en klarere grenseoppgang mot hva som er fordelingseffektene av det offentlige godet som skal finansieres. Sammenlignet med tidligere litteratur finner vi at med vårt mål på skattefinansieringskostnaden så er den høyere både for en koppskatt og for alminnelige inntektsskatter. På grunn av sine regressive egenskaper finner vi at skattefinansieringskostnaden ved bruk av koppskatter er høyere enn 1 mens den tidligere har vært antatt å være 1 når den ikke kombineres med andre skatter. Vi finner altså at det koster mer enn 1 krone å finansiere offentlig konsum også med en koppskatt. Mer generelt finner vi at med vårt mål på skattefinansieringskostnaden er den høyere enn det man tidligere har antatt.

1 Introduction

Inequality is high on the political agenda, but economists still struggle to provide advice on how distributional concerns should affect public goods provision. Failure to account for the distributional aspects of both public goods and the tax schemes used to fund them, can lead to greater inequality and can make redistribution more costly. Furthermore, failure to account for distributional aspects could lead decision-makers to disregard economics as a useful source of advice on how to combat inequality.

The main contribution of this paper is to provide a new measure for the marginal cost of public funds (MCF) which more correctly accounts for the distributional aspects of taxation than the standard MCF-measure of previous literature. The paper also proposes a corresponding measure for the distributional characteristics of public goods. Together, the proposed measures distinguish clearly between the distributional characteristics of a public good on one hand, and of the taxes used to fund it on the other. The proposed measures are intuitive and should make the important insights of the literature on taxation, public goods and distribution more accessible. Hence, the paper provides a way forward for governments that want to take distribution into account in a consistent and transparent manner.

According to the original Samuelson rule, a public good shall be supplied until the aggregate marginal willingness to pay for it is equal to the marginal cost of providing it (Samuelson, 1954). If individualized lump-sum (ILS) taxes were available, there would be no deadweight loss from taxation, and redistribution could be carried out at no cost. Thus, with ILS taxes the first best could be attained. However, no government actually has unlimited access to such costless redistribution and funding. When there are costs to taxation, one must modify the Samuelson rule and pursue the second best.

The second-best solution to a government's problem of public goods provision depends on whether distribution of income and welfare across individuals matters or not. When distribution is irrelevant, the potential efficiency loss from distortive taxation is the only concern in the second best. If distortive taxation leads to inefficiently low labor supply, the second-best provision of public goods is lowered

compared to the first best (Pigou, 1947, p. 34). If the distortive tax is the only source for public funding, MCF is a number greater than one, which can be applied as a corrective factor to the marginal cost of production to find the actual cost to society of providing a public good.¹ This also applies to the MCF-concept provided in this paper, which differs from previous definitions only when distribution matters.

When distribution matters, the second best is less straightforward because the distributional aspects of both public goods and the taxes used to fund them should be taken into account. Since the poor have higher utility of money than the rich, the distributional cost of taxation will be lower if more of the tax burden is borne by the rich. However, taxes that are conditional on income are distortive and thus likely to cause efficiency losses. With regards to the public good itself, a progressive public good from which the poor derives greater utility than the rich increases welfare more than a regressive good for which it is the other way around (when the aggregate marginal willingness to pay is the same). It follows that with concerns for distribution, an additional argument is raised against the conclusion that provision of public goods should always be lower in the second best than in the first best (Pigou, 1947).

To tackle these issues in a transparent and consistent manner, both in the evaluation of public projects and in the design of tax policy, one should use methods that distinguish clearly between the distributional properties of public goods themselves and of the taxes used for funding. We show in this paper that the standard MCF measure attributes parts of the distributional effects of taxation to the public good. This leads to a significant underestimation of the funding costs and a corresponding misrepresentation of the distributional effects of public projects. The measures for the funding costs and for the distributional characteristics of public goods proposed in this paper distinguish clearly between the two sources of distributional consequences of providing a tax-funded public good, and therefore solve this problem.

To provide these measures for the funding costs and the distributional characteristics of public goods, we define a specific and hypothetical ILS tax scheme that

¹If the labour supply curve is backward bending MCF could be less than one also in representative agent models, cf. Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974).

is non-distortive and distributionally neutral. This tax scheme is used as a benchmark. The benchmark ILS tax scheme increases government revenue by reducing the utility of all individuals by the same amount. Since money has lower value to the rich than to the poor, the taxes on the rich have to be higher than the taxes on the poor, in order for utility to be reduced equally for all individuals. Because this hypothetical tax scheme is distributionally neutral and creates no efficiency loss, we set the MCF to one for this scheme.

In contrast, the previous literature implicitly uses a uniform lump-sum (ULS) tax as the benchmark for the MCF (Atkinson & Stiglitz, 1980; Sandmo, 1998; Gahvari, 2006; Kleven & Kreiner, 2006; Kreiner & Verdelin, 2012; Jacobs, 2018). Consequently, from this literature it could be concluded that the MCF of the ULS tax is one if it is the only tax and always less than one if combined with a distortive income tax to achieve the second best (Sandmo, 1998, p. 372, 376). However, a ULS tax raises government revenue by reducing income by the same amount for all individuals, which means that *utility* is reduced more for the poor than for the rich. This regressivity of the ULS tax is not taken into account in the measure of the MCF used in the previous literature. With the MCF measure proposed in this paper, the MCF of the regressive ULS tax is instead always greater than one when it is the only tax. If combined with a distortive income tax, the MCF of the ULS tax could be smaller than one also with this new measure of MCF – but it will always be greater than the MCF of the previous literature.

We also propose a new measure for the distributional characteristics of public goods. We define a public good as distributionally neutral if an additional unit increases the utility of all individuals equally, regardless of their income. Correspondingly, public goods that increase the utility of the rich more than the poor are defined as regressive, and vice versa. Based on these definitions, we introduce a corrective factor which can be applied to the aggregate marginal willingness to pay for the public good to account for the distributional aspects. For distributionally neutral public goods, the corrective factor is equal to one, while for regressive and progressive goods it is less than one and greater than one, respectively.

Our measure for the distributional characteristics of the public good also differs from the previous literature. The corresponding corrective factor of the previous literature is set equal to one when the *marginal willingness to pay* for a public good

is equal for all individuals, regardless of their income, see for example Atkinson and Stiglitz (1980), Sandmo (1998) or Wilson (1991). However, for the *marginal willingness to pay* for a public good to be the same for all individuals, the *marginal utility* of the good must be higher for the poor than for the rich. Within the new framework proposed in this paper, such a public good is defined as progressive and its corrective factor is greater than one.

Atkinson and Stiglitz (1980) and Sandmo (1998) are seminal contributions to the literature on inequality and provision of public goods. Atkinson and Stiglitz (1980, p. 494) emphasizes that "the conditions for the optimum supply of public goods are influenced by distributional considerations". Correspondingly, Sandmo (1998) argues that tax schemes are designed to take into account governments' preferences for income distribution. Therefore, when measuring the costs of taxation, one should include distributional effects.

Essentially, as pointed out by Slemrod and Yitzhaki (2001, p. 192), the choice of benchmark for the MCF is about normalization. If the cost and benefits of public goods are to be expressed in monetary terms or by the use of a numeraire good, marginal utility has to be translated through normalization. The choice of Atkinson and Stiglitz (1980) and Sandmo (1998) was to divide the shadow cost of the public budget constraint by the *average* marginal utility of money. This approach has been adopted by a number of later contributions to this literature, see for example Gahvari (2006); Kleven and Kreiner (2006); Kreiner and Verdellin (2012); Jacobs (2018). This normalization is usually considered the standard approach for measuring the MCF.^{2,3}

Ballard and Fullerton (1992) divide the early contributions to the literature on the MCF into two categories. The first category includes Harberger (1964) and Browning (1976, 1987), and was labeled the Pigou-Harberger-Browning (PHB) approach. This approach was essentially based on measuring the deadweight loss

²In the case where the government maximizes a social welfare function of the type $W = W(v_1, \dots, v_n)$, the variable used for normalization is $\bar{\beta} = \frac{1}{n} \sum_i \beta_i$ where $\beta_i = W_i \lambda_i$ represents the marginal social value of money of consumer i .

³Because the MCF of lump-sum taxes could be different from one when they are combined with distortionary taxes, Jacobs (2018) proposes a normalization method previously used by Håkonsen (1998), based on Diamond (1975). This normalization implies that the MCF of a lump-sum tax is always one. Holtmark (2019) discusses Jacobs' proposal.

when a distortionary tax is used to collect revenue, while the entire revenue is paid back to individuals as a lump-sum transfer. In this way the income effect of the tax was cancelled out. Using this approach, the MCF of a distortionary tax is always greater than one, since it only captures the substitution effect. The conclusions from this strand of the literature are in line with Pigou's conjecture that with distortionary taxes the Samuelson rule will lead to oversupply of public goods (Pigou, 1947, p. 34).

Ballard and Fullerton (1992) labeled the second category of this early literature the Stiglitz-Dasgupta-Atkinson-Stern (SDAS) approach after Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974). These contributions show that because of the income effect on labour supply, funding through distortionary taxes does not always imply that the Samuelson rule leads to overprovision of public goods. If labor supply is inefficiently low, for example due to other distortionary taxes that are already in place, the income effect of additional taxation normally leads to an increase in labor supply, representing an efficiency gain. Thus, if the income effect is stronger than the substitution effect for the specific tax scheme in question, a marginal increase in the revenue raised by taxation could improve efficiency. If that is the case, the resulting MCF would be less than one.⁴

As most of the more recent literature, we investigate distributional aspects within an SDAS-approach.

The paper is organized as follows. The model, definitions and main results are presented in Section 2, while Section 3 presents some numerical examples to illustrate the main findings of the paper. Section 4 concludes. An appendix includes a proof related to the main results and a detailed description of the model used in the numerical examples.

⁴It should here be noted that also Wildasin (1984, p. 229) pointed to the difference between what he called the Pigou-Browning approach and the Atkinson-Stern-approach, respectively.

2 The model

Let there be a total of n individuals, with subscript $i \in \{1, \dots, n\}$ denoting individual i . Individual i 's utility is given by:

$$u_i = u(c_i, l_i, G), \quad (1)$$

where c_i is consumption of a (composite) private good, l_i is leisure and G is consumption of a pure public good. We make the assumptions that $u_{ic}, u_{il}, u_{iG} > 0$, that $u_{icc} < 0$, and that $u_{ic} \rightarrow \infty$ as $c_i \rightarrow 0 \forall i$. Individuals provide labour supply h_i within the time endowment T . Hence, $h_i + l_i = T$. Following Mirrlees (1971), the labour productivity of individual i is given by w_i , and varies across individuals. This is the source of inequality in this model.

Labour is used in the production of both the private consumption good and the public good. Let q be the unit cost of producing the public good in terms of the private good, such that:

$$\sum_i c_i + qG = \sum_i w_i h_i.$$

For our notation to be in line with the literature in the previous literature, we let $\lambda_i \equiv u_{ic}$ denote the marginal utility of income (net of taxes) for individual i and $\bar{\lambda}$ be its average. Furthermore, for notational simplicity, we define

$$m_i = \frac{u_{iG}}{\lambda_i},$$

individual i 's marginal willingness to pay for the public good.

2.1 The first-best allocation

We define the first-best allocation as the maximum of a utilitarian welfare function $W = \sum_i u_i$.⁵ Given this welfare function, the first-best allocation in this economy

⁵It is common in this literature on the MCF to let the government maximize a more general welfare function. The use of the less general, additive welfare function applied here does not influence the main results but make the introduction to our approach significantly more readable.

is defined by the following $2n$ first-order conditions:

$$\begin{aligned}\sum_j u_{jG} &= q\lambda_i \quad \forall i \\ \sum_j u_{jG} &= qu_{il}\omega_i \quad \forall i.\end{aligned}$$

The original Samuelson rule – defining the first-best provision of the public good – follows from these equations:

$$\sum_i m_i = q. \tag{2}$$

2.2 The MCF and the second-best allocation

To provide the public good, the government must raise public revenue. First, consider the case where the government can tax wage income, but can also use both a uniform and an individual lump-sum tax. Let $t > 0$ represent a linear income tax while b_i and b represent individual and uniform lump-sum taxes, respectively. The lump-sum taxes could be positive, zero, or negative, while we only consider non-negative income taxes. Individual i then maximizes u_i with respect to c_i and l_i , taking G as given, subject to the budget constraint:

$$c_i = (1 - t)w_i h_i - b_i - b,$$

The following first-order conditions solves individual i 's problem:

$$\frac{u_{il}}{\lambda_i} = (1 - t)\omega_i$$

defining $c_i(t, b, b_i, \omega_i, G)$, $h_i(t, b, b_i, \omega_i, G)$ and $l_i(t, b, b_i, \omega_i, G)$. Let $v_i = v(t, b, b_i, w_i, G)$ be the indirect utility of individual i . Using the envelope theorem gives:

$$\begin{aligned}\frac{\partial v_i}{\partial b_i} &= \frac{\partial v_i}{\partial b} = -\lambda_i, \\ \frac{\partial v_i}{\partial t} &= -\lambda_i w_i h_i.\end{aligned} \tag{3}$$

The government maximizes the utilitarian welfare function, which is now given by $W = \sum_i v_i$, subject to the public budget constraint

$$R = t \sum_i w_i h_i + \sum_i b_i + nb = qG. \quad (5)$$

The corresponding Lagrange-function is:

$$L_g = \sum_i v(t, b, b_i, w_i, G) + \mu \left(t \sum_i w_i h_i + \sum_i b_i + nb - qG \right).$$

When individual lump-sum taxes are available, neither a uniform lump-sum tax nor a linear income tax will be applied by a welfare maximizing government. Consider therefore the case with individual lump-sum taxes as the only source for public funding, i.e. $t = b = 0$. The first order conditions for the government's maximization problem are then:

$$\begin{aligned} \lambda_i &= \mu, \quad \forall i = 1, \dots, n, \\ \sum_i u_{iG} &= \mu q. \end{aligned}$$

This set of $n + 1$ first order conditions gives equation (2). Hence, we have the well known result that public spending, G , and a set of ILS taxes could be set such that first best, the Samuelson rule in (2), is achieved. It follows that with ILS taxes there are no additional costs related to tax funding of public goods, in terms of distortions or in terms of distributional effects.

Next, consider the more realistic case where the government can use only a uniform lump-sum tax combined with a positive linear income tax. Now, $b_i = 0$ and public revenue is $R = t \sum_i w_i h_i + nb$. We rule out the case where $b \geq w_j T$ if $w_j = \min(w_1, \dots, w_n)$ in which the model has no solution.

The first-order conditions for the government's maximization problem with respect to the uniform lump-sum tax, b , the income tax, t , and the size of the

public sector, G , are:

$$\sum_i \lambda_i = \mu \left(t \sum_i w_i \frac{\partial h_i}{\partial b} + n \right), \quad (6)$$

$$\sum_i \lambda_i w_i h_i = \mu \left(\sum_i w_i h_i + t \sum_i w_i \frac{\partial h_i}{\partial t} \right), \quad (7)$$

$$\sum_i u_{iG} = \mu \left(q - t \sum_i w_i \frac{\partial h_i}{\partial G} \right). \quad (8)$$

Together with the public budget constraint, these three first order conditions give second-best levels of the shadow-cost of the public budget constraint, μ , the ULS-tax b , the linear income tax t , and public consumption G in second-best.

2.3 Benchmarks, definitions and interpretation of the first-order conditions

Below we rewrite the first-order conditions (6)-(8) into two equations that provide expressions for the size of the MCF. As mentioned in the introduction, the shadow cost of the public budget constraint, μ , represents MCF. However, to have an operative corrective factor that could be used in cost-benefit analyses, this shadow cost has to be normalized in some way or another. It has been common to use the average marginal utility of money, $\bar{\lambda}$, for this normalization, see for example Atkinson and Stiglitz (1980), Sandmo (1998) and several later contributions. However, $\bar{\lambda}$ represents the aggregate welfare loss to individuals of collection of one dollar with a uniform lump-sum tax, which means that each consumer pays the same tax, in this case $1/n$ dollar. This is a regressive funding scheme. Hence, normalizing μ with $\bar{\lambda}$ means that we evaluate the distributional properties of taxes using a regressive tax as the benchmark for evaluation. We find it more reasonable to evaluate the distributional effects of taxes using a distribution-neutral tax scheme as the benchmark. We therefore make the following definition:

Definition 1 (Distribution-neutrality). *Additional revenue collection is distribution-neutral if designed such that all individuals experience the same loss of utility. A*

public good is distribution-neutral on the margin if $u_{1G} = \dots = u_{nG}$.

This means that we need to define a factor for normalization which represents the welfare costs of a distribution-neutral funding scheme. This benchmark scheme should also be non-distortionary, as the ULS tax scheme of the previous literature. From (3) we have that the increase db_i in the ILS tax paid by individual i when the tax is increased so that the utility of individual i is reduced by $-du_i$, is given by

$$db_i = -\frac{1}{\lambda_i} du_i, \quad (9)$$

where $1/\lambda_i$ represents the money needed to increase the utility of individual i by one unit. It follows that $\sum_i (1/\lambda_i)$ represents the aggregate lump-sum transfer needed to increase the utility of all individuals by one unit, i.e. in a distribution-neutral manner. Along these lines, consider a case where a set of individual lump-sum taxes are designed to give an aggregate additional tax revenue dR in a distribution-neutral manner, i.e. such that all individuals experience the same utility loss $-du$. From (9) we have that

$$dR = -\left(\sum_i \frac{1}{\lambda_i}\right) du.$$

Define

$$\theta = \frac{1}{\frac{1}{n} \sum_i \frac{1}{\lambda_i}}. \quad (10)$$

Then we have that:

$$-n \cdot du = \theta \cdot dR. \quad (11)$$

It follows that θ represents the welfare cost of a tax change which is distribution-neutral and does not influence efficiency. A less precise interpretation is that θ represents the welfare gain of one additional dollar distributed to the individuals such that all have the same utility gain. Put differently, θ represents the welfare

gain of a dollar that is shared by the individuals in a distribution-neutral manner.

Before we proceed, we define the MCF concept of this paper:

Definition 2 (The marginal cost of public funds). *The marginal cost of public funds (MCF) is the marginal welfare cost of revenue collection relative to the marginal welfare cost of a distribution-neutral revenue collection that does not alter efficiency.*

Definition 2 means that θ represents a benchmark for measuring the MCF.

Define the following variables:

$$y_i = w_i h_i, \quad (12)$$

$$Y = \sum_i y_i, \quad (13)$$

$$\delta_G = \frac{\text{cov}\left(\frac{1}{\lambda_i}, u_{iG}\right)}{\left(\frac{1}{n} \sum_i \frac{1}{\lambda_i}\right) \left(\frac{1}{n} \sum_i u_{iG}\right)}, \quad (14)$$

$$\delta_t = \frac{\text{cov}(\lambda_i, y_i)}{\bar{\lambda} \bar{y}}, \quad (15)$$

where $\bar{y} = (1/n) \sum_i y_i$. Dividing both sides of the first order conditions (6), (7), and (8) with θ and using the formula for a covariance:

$$\sum_i \frac{u_{iG}}{\lambda_i} = n \cdot \text{cov}\left(\frac{1}{\lambda_i}, u_{iG}\right) + \frac{1}{n} \sum_i \frac{1}{\lambda_i} \sum_i u_{iG},$$

gives then the two following first order conditions for optimal taxation and size of the public sector:

$$\frac{1}{1 + \delta_G} \sum_i m_i = \frac{\bar{\lambda}}{\theta} \frac{1}{1 + \frac{t}{n} \frac{\partial Y}{\partial b}} \left[q - \frac{\partial R}{\partial G} \right], \quad (16)$$

$$\frac{1}{1 + \delta_G} \sum_i m_i = \frac{\bar{\lambda}(1 + \delta_t)}{\theta} \frac{1}{1 + \frac{t}{Y} \frac{\partial Y}{\partial t}} \left[q - \frac{\partial R}{\partial G} \right]. \quad (17)$$

Now compare the equations (16) and (17) with the original Samuelson rule which says that $\sum_i m_i = q$ in the first best. The left hand sides of (16) and (17) include in addition to $\sum_i m_i$ a fraction which corrects the benefit side of the equation with regard to the distributional properties of the public good. If the public good is distribution-neutral according to Definition 1, then $\delta_G = 0$ and there should be no correction of the Samuelson rule related to distributional properties of the public good.

If there is a pattern where the poor have higher marginal benefits from the public project than the rich, then $cov(1/\lambda_i, u_{iG}) < 0$, and we are dealing with a progressive public good. In that case $\delta_G < 0$ and (16) and (17) show that such distributional characteristics of the public good draw in the direction of more supply than given by the Samuelson rule, and the other way around when the rich have higher marginal benefits from the public good compared to the poor.

The case when all individuals have the same willingness to pay for the public good, i.e. $m_1 = \dots = m_n$, represents a special case of a progressive good. We will return to that case below.

With regard to the right hand sides of (16) and (17), the square brackets represent the net costs of the public project when it is taken into account that public projects might influence labour supply and thereby also public revenue. The two fractions on the right hand sides together represent the MCF of the ULS tax and the linear income tax, respectively. The second of these two fractions in each first-order condition represents the revenue effects of altered labour supply caused by the a marginal increase in the ULS tax and the linear income tax, respectively. These fractions are standard in the literature within the SDAS-approach, irrespective of whether a single individual is considered as in Mayshar

(1990, 1991) or there is a set of heterogenous individuals as in Dahlby (1998); Sandmo (1998); Mayshar and Yitzhaki (1995); Slemrod and Yitzhaki (2001).

The first fractions on the right hand sides of equations (16) and (17) are different from previous literature. They represent factors for correction of the production cost with regards to the distributional aspects of each tax, when a distribution-neutral tax-funding scheme is used as the benchmark. To investigate closer these these expressions, we simplify and move to the case where a ULS scheme is the only source for public funding.

2.4 Distributional effects when a public good is funded with a uniform lump-sum tax

When considering the case with a uniform lump-sum tax as the only source for public revenue, the first-order condition (16) can be written as follows:

$$\frac{1}{1 + \delta_G} \sum_i m_i = \frac{\bar{\lambda}}{\theta} q. \quad (18)$$

Again, the corrective factor related to distributional characteristics of the public good is on the left hand side, while distributional effects related to funding are accounted for on the right hand side. Because only lump-sum taxes are in use, there are no terms related to effects on efficiency.

First, consider the right hand side of (18). $\bar{\lambda}$ represents the welfare loss to individuals if one additional USD is collected with ULS. In comparison, θ , defined in (10), measures the aggregate welfare loss if one additional USD is collected in a distribution-neutral ILS-scheme. Because there are no efficiency losses from ULS-funding, the MCF of an ULS-funded tax is related to its effect on distribution only. It follows that $\bar{\lambda}/\theta$ represents MCF using ULS, cf. Definition 2. We have the following result:

Proposition 1 (MCF of ULS). *When a uniform lump-sum tax is the only tax implemented, and at least two of the n individuals have different productivities, then*

- $MCF > 1$

- $MCF \rightarrow \infty$ as $b \rightarrow w_j T$ when $w_j \leq w_i \forall j \neq i$.

Proof. With regard to the first part of Proposition 1, it was shown above that $\bar{\lambda}/\theta$ represents MCF when a uniform lump-sum tax is the only tax in use. It is shown in Appendix A that $\bar{\lambda}/\theta > 1$ if there are variation in individuals' productivities. With regard to the second part, it follows from the properties of the utility function that $\lambda_j \rightarrow \infty$ as $b \rightarrow w_j T$ from below. Hence, $\bar{\lambda}/\theta \rightarrow \infty$ as $b \rightarrow w_j T$. ■

The first part of Proposition 1 reflects that ULS is a regressive tax. If individuals' productivities vary, ULS funding will always give higher funding costs than a distribution-neutral ILS tax. This is in contrast to what follows from the standard definition of MCF, see for example Sandmo (1998, p. 372) who states that when a uniform lump-sum tax is the only source of government revenue, then $MCF = 1$.⁶

The second part of Proposition 1 says that if there are no individuals with lower productivity than individual j , then MCF of a lump-sum tax becomes large when the tax converges from below toward individual j 's maximum possible income, i.e. $b \rightarrow w_j T$. This reflects that distributional concerns have been included in MCF of the uniform lump-sum tax. In contrast, the standard MCF-measure of the uniform lump-sum tax is still 1 in this situation even though the poorest individual has close to zero after-tax income. These properties of MCF with a lump-sum tax is illustrated by Figure 1 (p. 24).

Before we proceed, a result comes out of this discussion:

Proposition 2 (Optimal provision of a distribution neutral public good).

When the public good on the margin is distribution-neutral and a uniform lump-sum tax is the only funding source, the level of public consumption should be lower than prescribed by the Samuelson rule.

Proof. When the public good is distribution-neutral on the margin, then the fraction on the left hand side of equation (18) is equal to 1. The fraction on the right hand side, $\bar{\lambda}/\theta$, is greater than one when the funding source is a lump-sum tax.

⁶Sandmo (1998) stated that "It may seem surprising that the MCF in this case does not reflect distributional concerns at all; after all, this is a regressive form of tax finance. [...] But it should be stressed that this does not imply absence of distributional concerns from the cost-benefit calculations; it simply means that these concerns are reflected in the evaluation of benefits." We argue that, partly, these distributional concerns should have been included in the costs.

Hence, there should be a correction on the cost side of the Samuelson equation but not on the benefit side.■

It could at this point be useful to go back to two important previous studies. In the case with a ULS tax and when $t = 0$, both Atkinson and Stiglitz (1980, p. 496) and Sandmo (1998, p. 372) formulated the f.o.c. as follows:

$$\sum_i m_i(1 + \delta_z) = q,$$

where they defined

$$\delta_z = \frac{\text{cov}(\lambda_i, m_i)}{\lambda \bar{m}}.$$

In both these contributions δ_z was labeled the *distributional characteristics of the public good*. We argue, however, that their variable δ_z represents not only distributional properties of the public good, but also distributional properties of the ULS tax. To see this, use the formula for the covariance to find that:

$$1 + \delta_z = \frac{\sum_i u_{iG}}{\frac{1}{n} \sum_i \lambda_i \sum_i \frac{u_{iG}}{\lambda_i}}. \quad (19)$$

Recall the case discussed above, where individual j has lowest productivity and that $\lambda_j \rightarrow \infty$ as $b \rightarrow w_j T$. It follows from (19) that the corrective factor $(1 + \delta_z) \rightarrow 0$ as $b \rightarrow w_j T$. This does not reflect that distributional effects of the public good makes additional production unacceptable. It reflects the prohibitive distributional costs of increasing the lump-sum tax further when a level is reached where individual j is left with close to zero net income despite his labour supply is close to T .

Consider again the case where $m_1 = \dots = m_n$, i.e. all individuals have the same willingness to pay for the public good, which means that the distributional characteristics, δ_z , as defined in Atkinson and Stiglitz (1980, p. 496) and Sandmo (1998, p. 372), is zero. In good match, Wilson (1991, p. 159) defined a public good as distribution-neutral when individuals have the same willingness to pay

for the public good.⁷ We have found it more reasonable to consider public goods with such distributional properties as *progressive*, because it means that even the poorest individual is willing to pay as much for a unit of the good as the richest one. In accordance with this, $\delta_G < 0$, and the corrective factor $1/(1 + \delta_G) > 1$ when $m_1 = \dots = m_n$. Hence, the progressivity of the public good draws in the direction of higher supply of the public good than given by the Samuelson rule. At the same time, the poor pay the same tax as the rich. This regressive restriction from the tax system increases the costs of funding because the poor have a higher marginal utility of money compared to the rich, thus $\bar{\lambda}/\theta > 1$. When $m_1 = \dots = m_n$, the benefits related to the progressivity of the public project happens to be exactly as big as the distributional cost related to a ULS tax, i.e. $1/(1 + \delta_G) = \bar{\lambda}/\theta$. Hence, correction of the Samuelson rule with regard to distributional effects of the public good exactly offsets the corresponding appropriate correction related to the funding costs.⁸

2.5 Distributional and efficiency aspects of a linear income tax

This section considers the case with an income tax. The starting point is then the first order condition (17). Because the left hand side of (17) has already been discussed, the focus is in the following on the right hand side. The second fraction represents what Mayshar and Yitzhaki (1995) labeled the marginal efficiency costs of funding (MECF). Studies with single individual models, as for example Mayshar (1991) and Wildasin (1984), see equation (1) in both, include corresponding expressions. If the labour supply elasticity with respect to the effective wage rate is positive (negative), i.e. the substitution effect is stronger (smaller) than the income effect, then $MECF > 1$ (< 1).

The focus in this paper is on the first fraction on the r.h.s. which corrects for

⁷Wilson (1991) defined a public good as *distributionally-neutral* when the covariance between the individuals willingness to pay for the public good, m_i , and the "social marginal utility of income", as defined by Diamond (1975), is zero. When there are no distortionary taxes, Diamond's social marginal utility of money is identical to the private utility of money, λ_i .

⁸This has relevance to the discussion in Kaplow (1996, 2004). He considered several cases where distributional effects of the public good were offset by distributional effects of a tax adjustment, see also Christiansen (1981, 2007).

the distributional effect of the income tax. The factor $(1 + \delta_t)$, which Sandmo (1998, p. 373) labeled the distributional characteristic of the marginal tax rate, is also found in Atkinson and Stiglitz (1980, p. 494). As already mentioned, both used the regressive ULS-tax as a benchmark for measuring the MCF. Because we have found that a distribution-neutral funding scheme is a better choice as benchmark, we multiply with $(\bar{\lambda}/\theta)$, the distributional costs of ULS relative to a distribution-neutral funding scheme, see equation (16).

With regard to the size of δ_t , individual incomes y_i are assumed to increase with ability w_i (agent monotonicity). Thus, $cov(\lambda_i, y_i)$ is negative, which means that $\delta_t < 0$. Using the formula for the covariance we find that:

$$\delta_t = \frac{\frac{1}{n} \sum_i \lambda_i w_i h_i}{\bar{\lambda} \bar{y}} - 1. \quad (20)$$

Because the fraction is positive, we have that $0 < 1 + \delta_t < 1$. Thus, this factor reduces the MCF of a linear income tax compared to the MCF of a ULS tax, which reflects that a linear income tax always has better distributional properties than a ULS tax. At the same time, from Proposition 1 we know that $\bar{\lambda}/\theta > 1$. This means that we cannot on a general basis conclude whether $\bar{\lambda}(1 + \delta_t)/\theta$ is greater or smaller than one. We have the following result:

Proposition 3 (MCF of a linear income tax). *When a linear income tax is the only tax implemented, and at least two of the n individuals have different productivities, then the MCF with a distribution neutral benchmark could be both smaller than, equal to, or greater than 1, but will always be greater than the MCF measure based on the standard definition.*

Proof. Both the first and the second fraction on the right hand side of equation (17) could both be both greater, equal to, or smaller than one. The difference between an MCF with a distribution-neutral benchmark and the standard MCF measure is that the first represents the shadow cost of the public constraint divided with θ while the latter with $\bar{\lambda}$. It is shown in the appendix that $\bar{\lambda} > \theta$. ■

2.6 Optimal taxation

In the optimal tax system, a trade-off is made between the distributional cost of the ULS tax and the efficiency loss caused by the income tax. When both taxes are set optimally, the MCF must be the same for both sources of funds (Wilson, 1991, p. 159). Since the MCF for the ULS tax and for the linear income tax are equal to the terms preceding the net production cost, $[q - \partial R/\partial G]$, in the respective first order conditions (16) and (17), these terms must be equal to each other and both represent the MCF in the optimal tax system. Thus, in an optimal tax system we have that:

$$MCF = \frac{\bar{\lambda}}{\theta} \frac{1}{1 + \frac{t}{n} \frac{\partial Y}{\partial b}} = \frac{\bar{\lambda}(1 + \delta_t)}{\theta} \frac{1}{1 + \frac{t}{Y} \frac{\partial Y}{\partial t}}. \quad (21)$$

It can be seen from this expression that at the optimum the difference between the distributional cost of the income tax compared to the lump-sum tax, given by the expression $(1 + \delta_t)$, is equal to the difference between the efficiency terms, represented by the second fractions on the right hand sides of equations (16) and (17).

Proposition 4 (MCF with optimal taxation). *With an optimal combination of a linear income tax t with a lump-sum tax b , then*

- $MCF \stackrel{\leq}{\equiv} 1$
- $MCF \rightarrow \infty$ as $tw_jT + b \rightarrow w_jT$ when $w_j \leq w_j \forall j \neq i$.

Proof. The first statement of Proposition 4 can be proven either by a similar argument as in the proof of Proposition 3, or by the fact that the first fraction of the right hand side of equation (16) is greater than one by Proposition 1 while the second fraction of the right hand side of equation (17) is less than one because leisure is a normal good so that the income effect on labour is positive.

With regard to the second statement of Proposition 4, we know from the proof of Proposition 1 that $\bar{\lambda}/\theta \rightarrow \infty$ as $b \rightarrow w_jT$. From (20) we have that $(1 + \delta_t)$ does not converge toward zero when $b \rightarrow w_jT$. Neither are there reasons why

the denominators of the second fractions on the right hand sides of (16) and (17) should converge toward infinity when $b \rightarrow w_j T$. ■

The conclusion that MCF could be both less than, equal to or greater than one in the optimal tax system, differs from the previous literature. When the regressive ULS tax is used as the benchmark, the MCF in the optimal tax system is always less than one at the optimal tax system, see for example Sandmo (1998, p. 376). However, as discussed in this paper, for this MCF measure to be used, the benefit side of the Samuelson rule, for all but the most progressive goods must be adjusted downwards.

3 Numerical illustrations

To illustrate some of the results of the present paper, simulations of a numerical model with 10 individuals were carried out. For detailed description of the applied, model, see Appendix A. The utility functions given in equation (1) were specified as CES-functions that include private consumption, leisure, and public consumption. The elasticity of substitution between leisure and private consumption was assumed to be 1.25, which means that the labour supply curve is upward sloping. The public budget constraint applies which means that in all simulations the entire net public revenue is used for production of the public good or transfers (negative lump-sum taxes).

Figure 1 shows numerical examples where public consumption is funded with either a ULS tax or a linear income tax. The horizontal axis measures public consumption. The vertical axis measures MCF.

The green solid double-line shows MCF of a uniform lump-sum tax when the ULS-tax is the only source of public funding and MCF is defined according to Definition 2, where distribution neutrality is given by Definition 1. In accordance with Proposition 1, and because a ULS tax is regressive, MCF with ULS is found to be greater 1 at all tax levels and it converges toward infinity when the lump-sum tax becomes close to 24 because the individual with lowest productivity has a time endowment $T = 24$ and a productivity $w_1 = 1$, cf. the second bullet point of Proposition 1. For comparison, MCF of a uniform lump-sum tax using the standard definition is given by the green, broken double-line. Because we here

considered a case where the lump-sum tax is the only tax, and because the standard definition of MCF uses the distributional properties of a uniform lump-sum tax as the benchmark, this MCF measure is always equal to 1, see for example Sandmo (1998, p. 372).

The black solid line shows MCF of a linear income tax when the this tax is the only source of public funding and MCF is defined according to Definition 2. For tax rates below 0.34, MCF of the income tax is less than 1. For comparison, the black broken line shows MCF of a linear income tax with what has been the standard approach. In accordance with Proposition x, this line is below the solid line, which represents MCF of the linear income tax with our definition.

Figure 2 shows a numerical example where different levels of public consumption are funded with an optimal combination of a ULS-tax and a linear income tax. For zero and low levels of public consumption the lump-sum tax is negative. In accordance with Proposition y, the lower diagram shows that MCF with our definition and with optimal taxation could be both below, equal to, and higher than 1. As in the case where ULS-tax is the only source of public funding, MCF converges toward infinity due to the unfortunate effects of income distribution when the total tax bill becomes close to the maximum possible income level of the poorest consumer. However, because the lump-sum tax now is combined with an income tax, this happens at a higher level of public consumption.

The broken line of the lower diagram (Figure 2) shows that MCF with the standard definition and optimal taxation is always below 1, in accordance with findings in several papers, see for example Sandmo (1998, p. 376).

4 Conclusion

Decisions about the provision of public goods should be based on a clear understanding of both the benefits of public goods and the costs of funding them with taxes. If there is inequality and the benefits of public goods are measured in monetary terms, attention must be paid to the fact that money has greater value to the poor than to the rich. When redistribution is costly, a corrective factor has to be applied to the willingness to pay for a public good in order to provide more of the goods that benefit the poor and less of the goods that benefit the rich. For

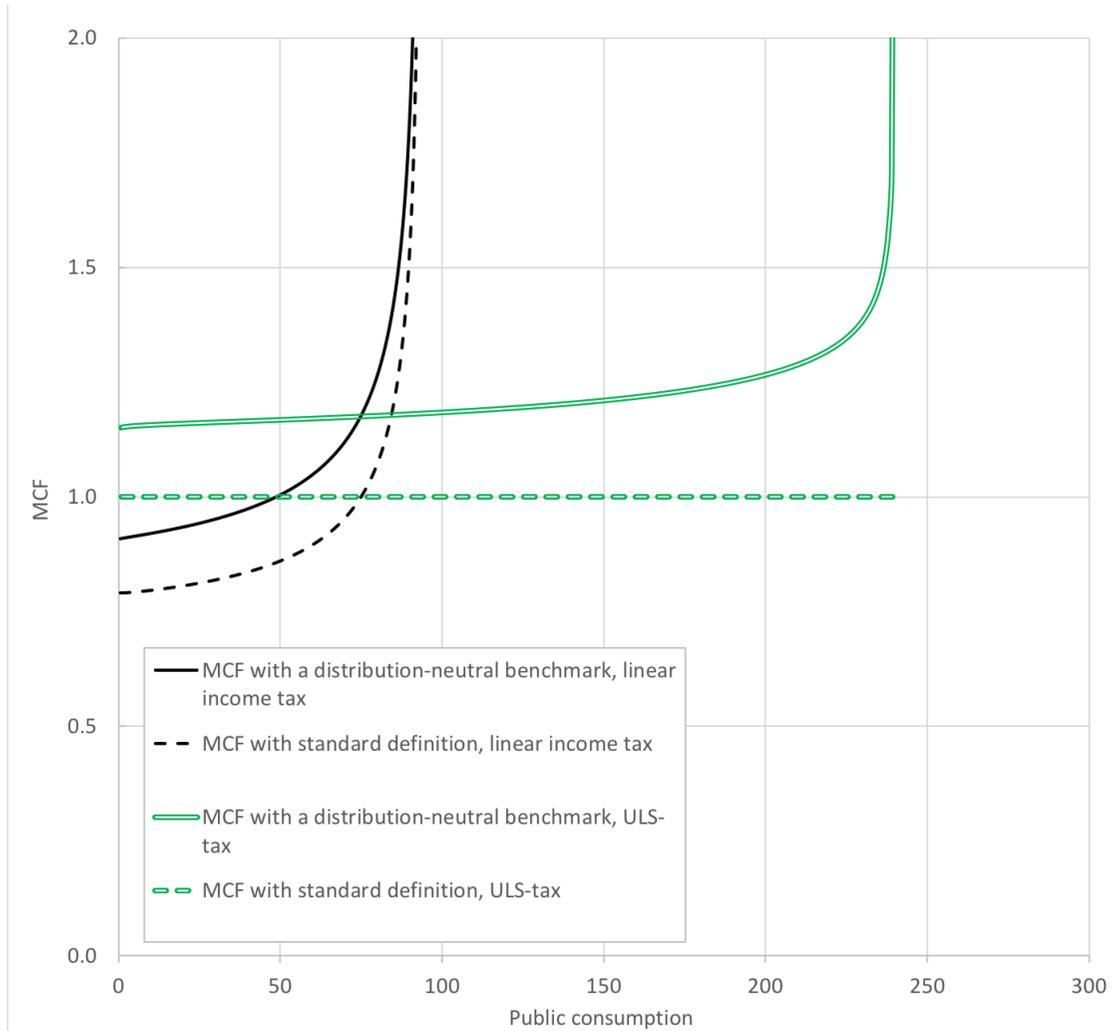


Figure 1: Two numerical examples with different levels of public consumption funded with a lump-sum tax or a linear income tax, respectively. The solid double-line shows MCF of a uniform lump-sum tax when we have defined MCF as in this paper (with a distribution-neutral benchmark). The broken double-line shows MCF of a uniform lump-sum tax with the standard definition. The black solid line shows MCF as defined in the present paper with a linear income tax. The broken black line shows MCF with the standard definition and a linear income tax.

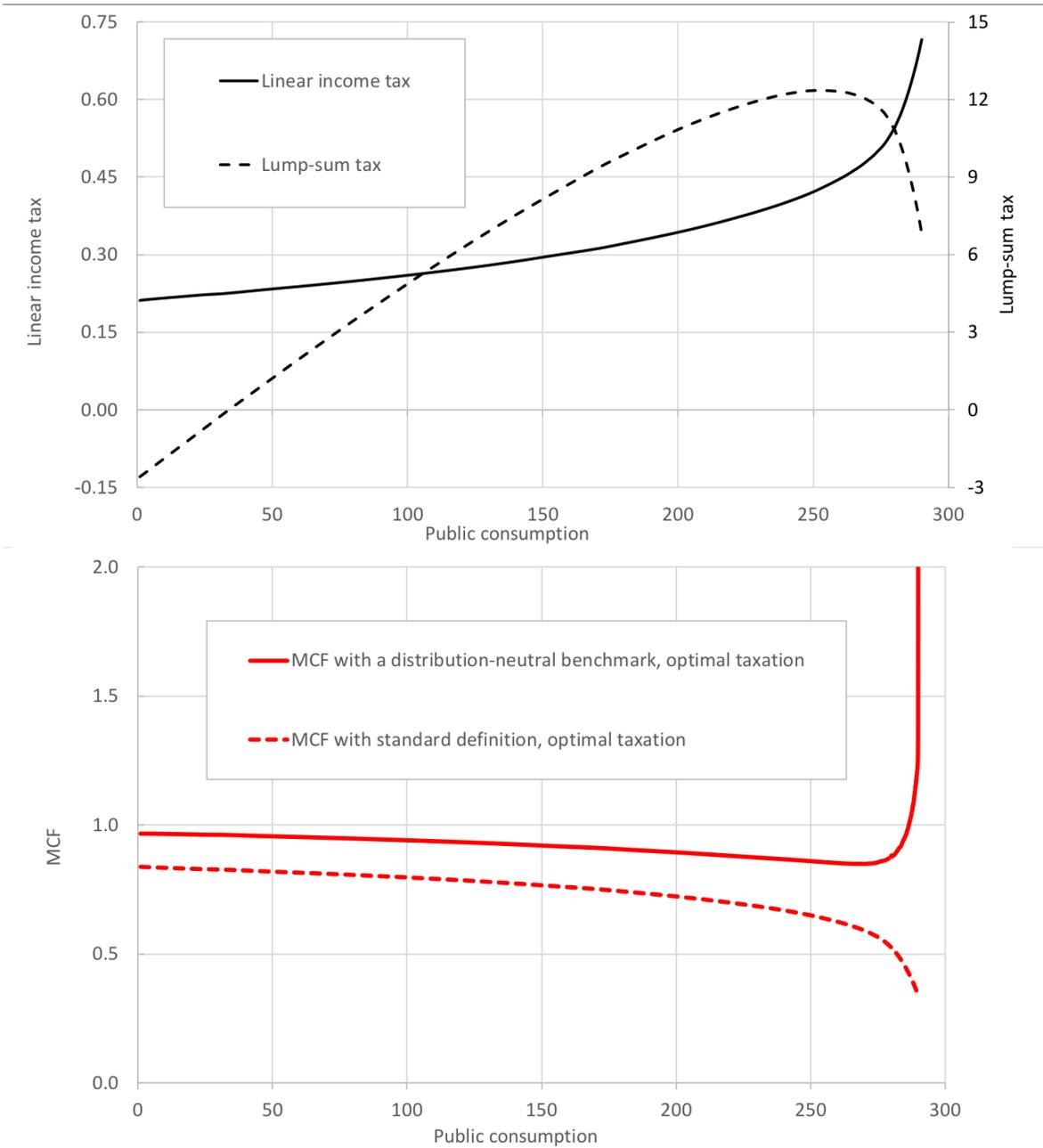


Figure 2: A numerical example with different levels of public consumption funded with an optimal combination of a uniform lump-sum tax and a linear income tax. The upper diagram shows the tax levels and the lower the MCF-levels.

the funding, both the efficiency loss from distortive taxation and the distributional properties of taxes have to be taken into account. The MCF should be a corrective factor that captures both these aspects of taxation, to be applied to the direct cost of providing a public good to arrive at the actual cost to society of providing it.

The main contribution of this paper is to establish a measure for the MCF that more correctly accounts for the distributional aspects of taxation. Because this MCF measure is based on a non-distortive and distributionally neutral benchmark tax, it accurately captures all the distributional effects of a specific tax or the marginal source of public funds. Correspondingly, the paper also proposes a corrective factor for the willingness to pay for public goods that accurately captures the distributional characteristics of a public good.

This paper shows that the MCF measure of previous literature attributes parts of the distributional effects of taxation to the measure for the distributional characteristics of the public good. The reason for this failure to account precisely for the distributional effects of taxation is that the literature uses the regressive ULS tax as the benchmark against which the distributional effects of other taxes are measured.

We believe that our proposed measure for the MCF and for the distributional characteristics of public goods are more intuitive and more in line with the general understanding of inequality and redistribution. A clear distinction between the distributional effects of the taxes and those of public goods should make the important insights from the literature on distributional concerns more accessible. The result could be that distributional effects are more properly accounted for in cost-benefit analyses of public projects.

With the measures of this paper, the case where all individuals on the margin get the same utility from the public good is particularly straightforward. With such public goods, which we have defined as distributionally neutral, there is no need to correct the left hand (benefit) side of the Samuelson rule (the total willingness to pay for the public good) because the corrective factor of this paper is one in this case. For comparison, the case with the MCF measure of the previous literature, the case that does not require correction of the benefit side is when the willingness to pay is the same for all individuals. For all individuals to have the same willingness to pay, and because the poor have higher margin utility of income,

they must also get greater utility from that public good. With our definitions \hat{A} – and to most people, we believe \hat{A} – such a public good is progressive. Hence, our setup is also a response to the timely concern of Christiansen (2007) about the difficulties on collecting this type of information empirically.

For all public goods but the most progressive, the willingness to pay has to be adjusted downwards when the MCF measure of the previous literature is used. The reason for this downwards adjustment is that the regressive ULS tax used as the benchmark for measuring the distributional effects of public spending. If the MCF measure of the previous literature is applied without the corresponding adjustment on the benefit side, the net benefits to society will be misrepresented and this may lead to public goods provision which does not lead to the second best. Because the MCF measure of the previous literature is lower than the real costs of taxation, neutral, regressive and even some progressive public goods will be overprovided. On the other hand, sufficiently progressive public goods will still be underprovided, compared to the second best.

Appendices

A Proof of Proposition 1

Proof. For the first part of first part of Proposition 1, it is necessary to show that $\bar{\lambda}/\theta > 1$. An underlying assumption here is that marginal utility of money is positive for all individuals.

We have that from the definition of θ in (10) that

$$\bar{\lambda}/\theta = \frac{1}{n^2} \sum_i \lambda_i \sum_j \frac{1}{\lambda_j}.$$

Further manipulation gives that:

$$\bar{\lambda}/\theta = \frac{1}{n} + \frac{1}{n^2} \sum_i \sum_{j \neq i} \frac{\lambda_i}{\lambda_j}. \quad (\text{A.1})$$

Let $j \neq i$, then the last term on the right hand side of (A.1) could be manipulated to:

$$\begin{aligned} \frac{1}{n^2} \sum_{i < n} \sum_{j > i} \left(\frac{\lambda_i}{\lambda_j} + \frac{\lambda_j}{\lambda_i} \right) &= \frac{1}{n^2} \sum_{i < n} \sum_{j > i} \left(\frac{\lambda_i^2}{\lambda_j \lambda_i} + \frac{\lambda_j^2}{\lambda_i \lambda_j} \right) \\ &= \frac{1}{n^2} \sum_{i < n} \sum_{j > i} \left(\frac{\lambda_i^2 + \lambda_j^2}{\lambda_j \lambda_i} \right) \\ &= \frac{2}{n^2} \sum_{i < n} \sum_{j > i} \left(\frac{(\lambda_i - \lambda_j)^2 + 2\lambda_i \lambda_j}{2\lambda_j \lambda_i} \right) \end{aligned}$$

Hence, we have that

$$\bar{\lambda}/\theta = \frac{1}{n} + \frac{2}{n^2} \sum_{i < n} \sum_{j > i} \left[\frac{(\lambda_i - \lambda_j)^2 + 2\lambda_i \lambda_j}{2\lambda_j \lambda_i} \right]. \quad (\text{A.2})$$

The expression within the square brackets cannot be less than one. There are $n(n-1)/2$ of these. Hence, the second term on the right hand side cannot be

smaller than $1 - 1/n$. Thus $\bar{\lambda}/\theta \geq 1$. ■

B Description of the model used in the numerical example

In the numerical model used for illustrative purposes in section 4, there are $n = 10$ individuals with the following utility function:

$$u_i = x \left(\alpha^{1-\rho} c_i^\rho + \beta^{1-\rho} l_i^\rho + \gamma^{1-\rho} G^\rho \right)^{\frac{1}{\rho}}$$

where x , α , β , γ , and ρ are parameters. Define

$$s = \frac{1}{1 - \rho}.$$

s is the elasticity of substitution between consumption and leisure and was set to 1.25. The parameters α and β were calibrated such that $\alpha^{1-\rho} = 0.3$ and $\beta^{1-\rho} = 0.7$.

The wage rates were determined such that:

$$\begin{aligned} w_1 &= 1 \\ w_i &= (w_{i-1})^g \quad \text{if } i > 1, \end{aligned}$$

where g is a parameter which was calibrated to a value that gave a Gini-index of gross income distribution of 27.5 in second best. Both the individual wage rates and individual before and after tax incomes in second best are shown in Figure 2.

(B.0)

The complete list of applied parameter values follows:

Parameter	Value	Parameter	Value
x	0.25	γ	0.10
α	0.22	β	0.64
ρ	0.20	s	1.25
T	24.00	g	1.03

(B.1)

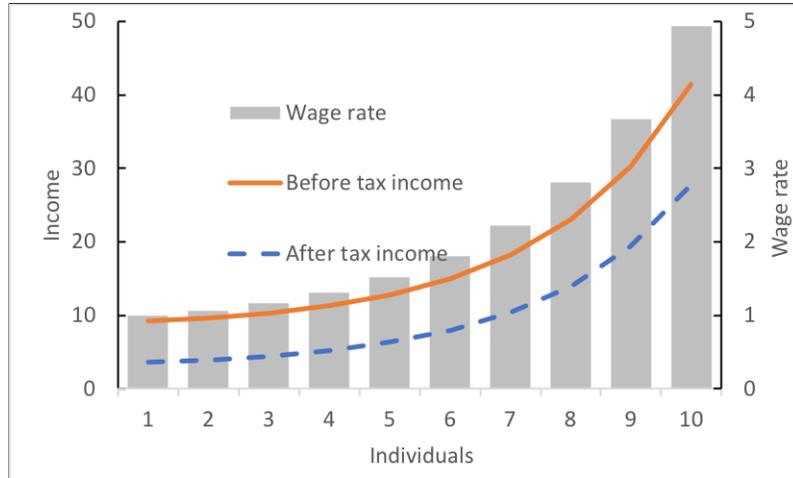


Figure 3: Wage rates w_i and gross and net incomes in second best, for $i = 1, \dots, 10$.

Second best was achieved with an effective tax rate $t^* = 0.25$ combined with an effective lump-sum transfer $a^* = -3.30$. individuals spent between 8.4 and 9.2 time units working of total time endowments of $T = 24$ time units. An effective level $t^* = 0.41$ gave welfare maximum if the lump-sum transfer was set to zero.

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