## Structural Analysis of Discouraged Worker Behavior



John K. Dagsvik, Tom Kornstad and Terje Skjerpen

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#### Abstract

Discouraged workers are those who have given up search due to (perceived) low chances of obtaining work. This paper develops a stochastic structural econometric framework for analyzing discouraged worker behavior based on the theory of job search and the notion of probabilistic rationality and risky choice. Subsequently, we conduct an empirical analysis of immigrant women from non-Western countries and for women born in Norway by using micro data on labor force participation and unemployment. We find that the discouraged worker effect and modified unemployment rate are particularily high among immigrant women with low wage rates and many children.


Keywords: Discouraged workers; Modified unemployment rate; Female immigrants; Labor force participation; Random utility modeling

JEL classification: C33; C35; J21; J22; J61; J64
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## Sammendrag

Et vanlig fenomen som observeres på mange arbeidsmarkeder er at arbeidstilbudet ser ut til å avhenge av konjunktursvingninger. Individer som søker arbeid synes å bli motløse under ugunstige konjunktursituasjoner fordi de tror at deres sjanser til å finne en akseptabel jobb er så små at kostnader og stress knyttet til jobbsøking oppveier de forventede fordelene med å søke. De potensielle arbeiderne trekker seg dermed ut av arbeidsmarkedet. Innen vårt rammeverk er en kvinne som ikke jobber, definert som motløs hvis hun $\emptyset n s k e r ~ a ̊ ~ s ø k e ~ e t t e r ~ a r b e i d ~ u n d e r ~ v e l d i g ~ g o d e ~ k o n j u n k t u r f o r h o l d, ~$ men har trukket seg fra arbeidsstyrken under de aktuelle forholdene. I denne artikkelen studerer vi motløs arbeidereffekten separat for gifte/samboende kvinner født i ikke-vestlige land og i Norge med utgangspunkt i en modell basert på teorien for jobbsøking. Modellen spesifiserer hvordan sannsynligheten for å være i arbeidsstyrken avhenger av en parameter som representerer søkekostnad per tidsenhet samt sannsynligheten for å bli sysselsatt, gitt jobb søking. Denne modellen er konsistent med aktører (kvinner) som i gjennomsnitt antas å ha rasjonell atferd i sine valg om å være i arbeidsstyrken eller ikke. Modellen benyttes til å analysere motløs arbeidereffekten separat for kvinnelige innvandrere og kvinner født i Norge ved hjelp av paneldata fra Arbeidskraftsundersøkelsene (AKU) for hvert kvartal fra andre kvartal 1988 til fjerde kvartal 2010.

Vi finner at estimert søkekostnad per tidsenhet er betydelig høyere for kvinner født i Norge enn for innvandrerkvinner fra ikke-vestlige land. En innvandrerkvinne med samme sannsynlighet for å skaffe seg arbeid som en kvinne født i Norge vil dermed ha større sjanse for å søke arbeid enn en kvinne født i Norge. Andelen motløse arbeidere er imidlertid for de fleste grupper mye høyere for innvandrerkvinner enn for kvinner født i Norge. Årsaken er at estimert total (forventet) søkekostnad (søkekostnad ganger forventet søketid) i gjennomsnitt er høyere for innvandrerkvinner enn for kvinner født i Norge.

Vi foreslår også et mål for ledighet som vi kaller modifisert arbeidsledighetsrate. Denne omfatter både registrerte arbeidsledige og motløse arbeidere. Eksempelvis er arbeidsledighetsraten for gifte eller samboende kvinner fra ikke-vestlige land om lag 17 prosent i 2005, mens den modifiserte arbeidsledighetsraten er om lag 25 prosent.

## 1. Introduction

A common phenomenon observed in many labor markets is that the supply of labor appears to depend on business cycles fluctuations, see Figure 1. Workers who are searching for work become discouraged under unfavorable business cycle conditions because they believe that their chances of finding an acceptable job are so small that the cost of searching for work outweighs the expected benefits from searching. Therefore, a worker who does not work is defined as discouraged if she would like to search for work under "peak business cycle conditions" (suitably defined) but has withdrawn from the labor force under the actual conditions. This notion is consistent with the definition used by many statistical agencies. ${ }^{1}$ For economies in a boom, discouraged workers provide a hidden source of manpower since their participation rates tend to increase when the chances of receiving job offers increase. In contrast, if the economy is in a recession, potential workers withdraw from the labor market and by doing so observed unemployment is reduced.

Figure 1. The fraction of women in the labor force (right axis) and one minus the unemployment rate (left axis). Women 15-74 years


Source: https://www.ssb.no/statbank/table/03781/

[^0]These workers are not captured by standard measures of unemployment, as they are not in the labor force. So, they can be considered as hidden unemployed persons. The size of the discouraged worker effect is seen as depending on preferences, wages, the expected search cost, which itself depends on the instantaneous search cost (search cost per unit of time) as well as on the chances of getting suitable job offers. Search costs include monetary as well as psychological "costs".

In this paper we develop a stochastic structural econometric framework for analyzing discouraged worker behavior. The paper contains several novel methodological contributions. First, we use the search theoretic framework of Burdett and Mortensen (1978) to obtain a characterization of the agent's decision rule (search or not to search) in terms of job arrivaland separation rates. Specifically, we show that the decision rule can be viewed as if the agent makes a static choice under risk where the risk is represented by weights, analogous to outcome probabilities, that are functions of the job arrival, job separation and discounting rates. Second, we assume a stochastic version of the expected utility theory based on the probabilistic theory of risky choice proposed by Blavatskyy (2007) and Dagsvik (2007, 2015). Third, we use probabilistic rationality assumptions to motivate assumptions about the population distribution of the decision rule (Luce and Suppes, 1965; Luce, 1977). These assumptions imply explicit closed form expressions for the probability of being in the labor force as well as the corresponding (one-step) probabilities for transitions into and out of the labor force as functions of the primitives of the model. Subsequently, we conduct an empirical analysis of the discouraged worker phenomenon for married/cohabitating women born in non-Western countries and women born in Norway separately by using micro data on labor force participation, unemployment and selected individual characteristics. ${ }^{2}$

In several countries labor market survey questionnaires include questions intended to measure the extent of the discouraged worker phenomenon. Typical survey data do not, however, fully capture the conditions under which persons would wish to search for work. Although these data yield estimates of the fractions of workers that are discouraged they might be difficult to interpret. ${ }^{3}$ In contrast, structural analyses can identify separately the role of

[^1]preference representations, wages and search costs in explaining discouraged worker behavior. Accordingly, a structural approach may provide a deeper understanding of the differences in labor market behavior among various groups in the population (as we do in this study). Second, in addition to measuring the actual (observed) discouraged worker effect, it might also be of interest to assess the discouraged worker effect in hypothetical settings: that is, under conditions different from those that gave rise to the observed business cycles. Third, it is also of considerable interest to analyze how the discouraged worker effect varies by key determinants such as wage rates, search costs (latent), the probability of being unemployed and non-labor income and demographic covariates.

In this paper we also propose a new measure of unemployment that captures both observed and hidden (discouraged) unemployed. This measure - which we refer to as the modified unemployment rate - thus captures the total effect of barriers and latent search costs in the labor market on actual employment. The modified unemployment rate is 1 minus the number of employed women under the actual conditions divided by the (potential) number of women in the labor force under peak conditions.

The motivation for comparing immigrant women and women born in Norway is that the level of labor force participation is observed to be significantly lower among immigrant women than among women born in Norway and it is of interest to examine why. Typically, two explanations have been offered for this phenomenon. The first explanation is that the two groups of women have different preferences for work due to differences in cultural background. Many immigrant women come from societies where women often do not participate in paid work (Bredtmann and Otten, 2022). The second explanation is related to barriers to employment and discouraged worker behavior. According to our findings, the discouraged worker effect is particularly important among groups of immigrant women with low wage rates and many children, and, to a lesser extent, also among women born in Norway with low educational qualifications.

Most studies of the discouraged worker phenomenon are based on macro data (Ehrenberg and Smith, 1988). ${ }^{4}$ These studies are typically not very explicit about the precise

[^2]definition of the discouraged worker effect. ${ }^{5}$ Empirical studies based on micro data include Ham (1986), Blundell et al. (1987, 1998), Connolly (1997), Başlevent and Onaran (2003), Bloemen (2005), Hotchkiss and Robertson (2006), and Dagsvik et al. (2013). The article that is closest in spirit to our study is Blundell et al. (1998), who analyze labor force participation, unemployment, and hours of work, within a framework that can be used to analyze the discouraged worker effect. As we do not analyze the supply of hours of work relation, we are, in contrast to Blundell et al. (1998), unable to distinguish between concepts such as fixed costs of working and search costs. However, our model is derived from weaker assumptions than those postulated by Blundell et al. (1998). The present paper also differs from Dagsvik et al. (2013). First, the model of Dagsvik et al. (2013) was derived from a one state search model without discounting and estimated from time series of independent cross sections whereas the model proposed in the current paper departs from the search model of Burdett and Mortensen (1978). ${ }^{6}$ Second, our data set includes panel observations, which makes key parameter estimates more precise compared to pure cross section data and furthermore enables us to estimate probabilities for transitions into and out of the labor force.

The paper is organized as follows. In Section 2 the implication of the theory of job search is discussed. In Section 3 the search theory is extended to a probabilistic theory and the implications for the probabilities of being in the labor force and transitions between the states "in the labor force" and "out of the labor force" are derived. In Section 4 the empirical application is discussed, including estimation results, the discouraged worker effect and modified unemployment rate for the two groups of women. Section 5 concludes.

## 2. Implications from the theory of job search

In this section we establish a theoretical framework for analyzing behavior with respect to unemployment and labor force participation. For now, we suppress the indexation of time epochs in the notation. Job offers arrive randomly in continuous time according to a Poisson process. Jobs are characterized by job-specific wage rates and a vector of other job attributes (type of tasks to be perform, location, etc). No job offer arrives when the agent is out of the

[^3]labor force. Let $V_{0}$ be the lifetime value of non-participation (out of the labor force), $V_{1}$ the value of search (participation), $V_{2}$ the value of being employed, $\rho$ the preference rate of time, $\lambda$ the arrival intensity of job offers when searching for work, $\mu$ the intensity of being laid off if employed, and $1-q$ the probability of being unemployed conditional on job search. Thus, $q$ is the probability of working, conditional on being in the labor force. Lifetime utility is assumed additive separable over time. Given a job offer with wage rate $W$, and specific job attributes, $U_{2}(y)=U_{2}(W, y)$ denotes the period indirect utility of this offer where $y$ represents non-labor income. The values of future job offers are uncertain to the agent. The period utility of being out of the labor force is $U_{0}=U_{0}(y)$ and the period utility of search is denoted $U_{1}=U_{0}\left(y-c^{*}\right)$ where $c^{*}$ is the instantaneous cost of search. It is assumed that the individual must search in order to get job offers.

## Assumption 1

The individuals behave, according to the theory of intertemporal choice under uncertainty, as if future preferences and environment were stationary with infinite planning horizon. Moreover, given that the individual works, the discounted expected value of working in the next period equals the current value of working.

Assumption 1 asserts that the theory of job search, pioneered by Lippman and McCall (1976), Burdett and Mortensen (1978) and others, provides a reasonable description of labor market behavior. The stationary postulate may be motivated by the lack of information available to the agent about future business cycles. However, the woman may update her evaluation at each time epoch due to new information about the evolvement of the business cycle. Consequently, the value of searching for work may vary over time. For further references about the theory of job search, see Rogerson et al. (2005) and the references therein.

Let $d=P\left(V_{2}>V_{1} \mid \Omega\right)$ and $\bar{U}_{2}=E\left(U_{2}\left|V_{2}>V_{1}\right| \Omega\right)$ where $\Omega$ represents the information known to the decision-maker. That is, $d$ is the probability that a job offer is acceptable and $\bar{U}_{2}$ is the expected period utility of a job offer, given that the offer is acceptable. Here, the
expectation is taken with respect to future wage offers and uncertain non-pecuniary job attributes. The agent will search (enter the labor force) if $V_{1}>V_{0}$.

## Theorem 1

Assume that Assumption 1 holds. Then $\rho V_{0}=U_{0}, \rho V_{2}=U_{2}$ and

$$
\rho V_{1}=\frac{\lambda d \bar{U}_{2}+(\rho+\mu) U_{1}}{\rho+\lambda d+\mu}
$$

The proof of Theorem 1 is given in Appendix A. The result of Theorem 1 is based on a version of the search model given in Burdett and Mortensen (1978). Unfortunately, the right side of the equation for the value of search in Theorem 1 also depends on $V_{1}$ through $d$ which implies that an explicit closed form expression for $V_{1}$ cannot be obtained. Note that due to the stationary assumption, the probability of being employed given labor force participation is equal to

$$
q=\frac{\lambda d}{\mu+\lambda d}
$$

From Theorem 1 the next result follows readily:

## Corollary 1

Under the assumption of Theorem 1 the value of search can be expressed as

$$
\rho V_{1}=\psi(q, \mu) \bar{U}_{2}+(1-\psi(q, \mu)) U_{1}
$$

(1)
where

$$
\psi(q, \mu)=\frac{\lambda d}{\lambda d+\mu+\rho}=\frac{q}{1+\rho(1-q) \mu^{-1}} .
$$

Corollary 1 shows that $(q, \mu)$ is a sufficient statistic for the effect of the labor market environment on the value of search. This is in contrast to $(\lambda d, \mu)$ which depends on $d$ which does not have an empirical counterpart. The relation in (1) can be interpreted as the expected
utility of risky search as if the agent perceives outcome probabilities $\psi(q, \mu)$ and $1-\psi(q, \mu)$, respectively. ${ }^{7}$ Note that $\psi(q, \mu)$ accounts for the effect of discounting, uncertain job arrivals and uncertain job separation. When $\rho$ increases, $\psi(q, \mu)$ decreases because the agent becomes less and less concerned about the future. Accordingly, the value of being in the labor force tends towards $\bar{U}_{2}$. The case where $q=1$ (which can only happen if $\lambda=\infty$ and $\mu=0$ ) represents a reference setting where there is no uncertainty and the labor force participation decision only depends on preferences and the distribution of the wage rate. This case will hardly be attained in real labor markets, but it serves as a useful benchmark for defining the discouraged worker effect, to be discussed below in the empirical section.

The results obtained above will enable us to obtain further characterizations of individual labor market behavior, and in particular when a woman is discouraged from search. This topic will be discussed further in the empirical application below.

## 3. Participation and transition probabilities

The search theoretic discussion above is relevant for a single agent but is silent about the distributional aspects of behavior in a population characterized by observed as well as unobserved heterogeneity. In the previous section we showed that the theory of search with discounting can be rationalized as if the agent (decision maker) behaves according to the static (one shot) expected utility theory for binary choice under risk with outcome probabilities $\psi(q, \mu)$ and $1-\psi(q, \mu) .{ }^{8}$ The developments in this section relax the theory above by assuming specific probabilistic axioms that are analogous of the axioms of expected utility theory for risky choice where the risk is represented by $\psi(q, \mu)$ and $1-\psi(q, \mu)$. Such axioms where proposed independently by Blavatskyy (2007) and Dagsvik (2007, 2015). The weakest set of axioms was proposed by Dagsvik (2007) and is, in addition to regularity conditions, probabilistic (aggregate) versions of the Archimedean and Independence axioms of expected utility

[^4]theory (Karni and Schmeidler, 1991). This means that in a population the Archimedean and Independence axioms are only assumed to hold on average. These axioms imply that
\[

$$
\begin{equation*}
\rho V_{1}=h\left[\psi(q, \mu) u_{2}+u_{1}(1-\psi(q, \mu))\right]+\tilde{\varepsilon}_{1} \quad \text { and } \quad \rho V_{0}=h\left(u_{0}\right)+\tilde{\varepsilon}_{0} \tag{2}
\end{equation*}
$$

\]

where $h$ is a strictly increasing transformation, $u_{2}$ is a systematic term representing the mean utility (in the population) of acceptable jobs, $u_{1}$ is the mean utility of search, $u_{0}$ is the mean utility of being out of the labor force and ( $\tilde{\varepsilon}_{0}, \tilde{\varepsilon}_{1}$ ) are zero mean random variables that are independent of $u_{j}, j=1,2$, and $\psi(q, \mu)$. By postulating a stronger version of the Independence axiom (Strong probabilistic independence) if follows that $h(x)=x$ (Dagsvik, 2007). The latter result has also been obtained by Blavatskyy (2007) based on similar axioms. Thus, the approaches of Blavatskyy and Dagsvik imply that the decision rule can be expressed as expected utility (possibly transformed) plus noise, where the noise enters in an independent and additive separable way.

In the following let $t$ index time (year). The discussion above provides a motivation for the next assumption.

## Assumption 2

The value functions have the structure

$$
\rho V_{1}(t)=\psi(q(t), \mu(t)) u_{2}(t)+(1-\psi(q(t), \mu(t)))\left(u_{0}(t)-c\right)+\tilde{\varepsilon}_{1}(t)
$$

and

$$
\rho V_{0}(t)=u_{0}(t)+\tilde{\varepsilon}_{0}(t)
$$

where $c=: u_{0}(t)-u_{1}(t)$ and $\left\{\tilde{\varepsilon}_{j}(t)\right\}$ are zero mean random variables that are independent of $u_{1}(t), u_{0}(t)$ and $\psi(q(t), \mu(t))$. The mean disutility of search across individuals (period-specific) $c=u_{0}(t)-u_{1}(t)$ is time invariant.

## Assumption 3

The random terms $\tilde{\varepsilon}_{j}(t), j=0,1$, have the structure

$$
\tilde{\varepsilon}_{j}(t)=Z_{j}+\varepsilon_{j}(t)
$$

where $\varepsilon_{0}(t)$ and $\varepsilon_{1}(t)$ are zero mean random variables that are mutually independent and independent across time. The random variables $Z_{0}$ and $Z_{1}$ are independent, time invariant and independent of $\left\{\varepsilon_{j}(t)\right\}$.

The role of the random error terms $\left\{\varepsilon_{1}(t), \varepsilon_{0}(t)\right\}$ is to capture the effect of unobservables that vary randomly over time whereas the random effects $\left\{Z_{1}, Z_{0}\right\}$ account for time invariant unobservables. The terms $\left\{\varepsilon_{1}(t), \varepsilon_{0}(t)\right\}$ may also capture the effect of aspects that are uncertain to the individual herself, i.e., the difficulty the individual may have for assessing a definite value of the alternatives once and for all. Assumption 3 implies that the autocorrelation function of the error terms $\left\{\tilde{\varepsilon}_{j}(t)\right\}$ is constant.

In order to obtain a model that can be confronted with the data it is necessary to specify distributions of the stochastic error terms and the random effects.

## Assumption 4

The random effects have the property that $\varepsilon_{j}(t)+Z_{j}$ has the same distribution as $\varepsilon_{j}(t) / \alpha, j=0$, l, where $\alpha \leq 1$ is a positive scalar. Furthermore, $\left\{\varepsilon_{j}(t)\right\}$ have Gumbel cdf $\exp (-\exp (-x / \theta))$ for real $x$ where $\theta>0$ is a constant.

Assumption 4 can be motivated by specific probabilistic rationality assumptions based on the so-called product rule (Luce and Suppes, 1965, p. 350), see Appendix A for details.

## Theorem 2

Assume that Assumptions 2, 3 and 4 hold. Then the random variables $\exp \left(\theta Z_{j}\right), j=0$, 1, have stable cdf $S_{\alpha}\left((\cos (\alpha \pi / 2))^{1 / \alpha}, 1,0\right)$ where $0<\alpha \leq 1 .{ }^{9}$ Furthermore,

$$
\begin{equation*}
P\left(\rho V_{1}(t)>\rho V_{0}(t) \mid Z_{1}-Z_{0}\right)=\frac{1}{1+\exp \left(-v(t)-\theta\left(Z_{1}-Z_{0}\right)\right)} \tag{3}
\end{equation*}
$$

[^5]and
\[

$$
\begin{equation*}
P(t)=: P\left(\rho V_{1}(t)>U_{0}(t)\right)=E P\left(\rho V_{1}(t)>U_{0}(t) \mid Z_{1}-Z_{0}\right)=\frac{1}{1+\exp (-\alpha v(t))} \tag{4}
\end{equation*}
$$

\]

where

$$
v(t)=: \theta v_{1}(t)-\theta v_{0}(t)=\theta\left(u_{2}(t)-u_{0}(t)\right) \psi(q(t), \mu(t))-\theta c(1-\psi(q(t), \mu(t))) .
$$

The parameter $\alpha$ has the interpretations

$$
\begin{equation*}
\operatorname{Var}\left(\theta\left(Z_{1}-Z_{0}\right)\right)=\left(\alpha^{-2}-1\right) \frac{\pi^{2}}{3} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\operatorname{Corr}\left(Z_{j}+\varepsilon_{j}(t), Z_{j}+\varepsilon_{j}(r)\right)=1-\alpha^{2} \tag{6}
\end{equation*}
$$

for $j=0,1$, and $t \neq r$.

The proof of Theorem 2 is given in Appendix A. In Appendix B (Theorem B1) we prove that if the aggregate probability of being in the labor force has the binary logit functional form, the structure given in (4) is the only possible one. ${ }^{10}$ To the best of our knowledge, the property of the distribution of the random effects is novel. This distribution has the convenient property that the choice probability remains invariant under aggregation of the random effect apart from a scale transformation of the structural part $v(t)$. The square of the scale parameter $\theta$ is inversely proportionate to the variance of $\varepsilon_{j}(t)$ and $\theta$ can therefore be interpreted as a precision parameter. ${ }^{11}$

To estimate the model efficiently from panel data with observations points in time we need the corresponding joint probabilities of being "out of the labor force" or "in the labor force" at two consecutive points in time. Let $Y(t)=1$ if the agent is in the labor force in year $t$ and 0 otherwise, and define the transition probabilities for flows into and out of the labor force by

$$
Q_{01}(t-1, t)=P(Y(t)=1 \mid Y(t-1)=0), Q_{10}(t-1, t)=P(Y(t)=0 \mid Y(t-1)=1),
$$

$Q_{00}(t-1, t)=1-Q_{01}(t-1, t)$ and $Q_{11}(t-1, t)=1-Q_{10}(t-1, t)$. We have the following result:

[^6]
## Theorem 3

Under Assumptions 2, 3 and 4 we have that

$$
Q_{01}(t-1, t)=g_{\alpha}(v(t-1)-v(t)) P(t)
$$

and

$$
Q_{10}(t-1, t)=g_{\alpha}(v(t)-v(t-1))(1-P(t))
$$

where

$$
g_{\alpha}(x)=\frac{\exp (\alpha x)-1}{\exp (x)-1}
$$

for $x \neq 0,0<\alpha \leq 1$. Moreover, $g_{\alpha}(x)$ is continuously differentiable and decreasing in $x$, $0<g_{\alpha}(x) \leq 1$ and $g_{\alpha}(0)=\alpha, g_{\alpha}^{\prime}(0)=-0.5 \alpha(1-\alpha)$ for $x=0$.

The proof of Theorem 3 is given in Appendix A. To the best of our knowledge, the result of this theorem is novel. The function $g_{\alpha}(\theta v(t)-\theta v(r))$ for $r<t$, has the interpretation

$$
g_{\alpha}(\theta v(t)-\theta v(r))=\frac{\operatorname{Cov}(Y(r), Y(t))}{P(r)(1-P(t))}
$$

which shows that $g_{\alpha}(\theta v(t)-\theta v(r))$ is a measure of the serial correlation in the stochastic error terms of the preference representation. In the stationary case where $v(t)=v(r), g_{\alpha}(0)=\alpha$ coincides with the usual definition of the correlation coefficient. Note that for $x \neq 0, g_{1}(x)=1$ whereas $g_{\alpha}(x) \rightarrow 0$ when $\alpha \rightarrow 0$. The case where $\alpha=1$ corresponds to serial independence whereas the case when $\alpha$ approaches zero corresponds to the degenerate model with random effect with infinitely large variance.

When $v(t)$ is approximately time invariant the model becomes stationary (approximately) and the transition probabilities become $Q_{01}(t-1, t) \cong \alpha P$ and $Q_{10}(t-1, t) \cong \alpha(1-P)$. Once the model given by Theorems 2 and 3 has been specified empirically and estimated it allows the researcher to predict the size of the labor force and the flows into and out of the labor force under counterfactual changes in the unemployment rate, the wage rate and other characteristics. A further advantage is that the participation probability and the likelihood function can be expressed in closed form by simple formulas.

## 4. An empirical application

This section specifies the empirical model and reports results from an empirical application on discouraged worker behavior, with special reference to the difference between immigrant women from non-Western countries and women born in Norway.

### 4.1. Specification of the empirical model

We shall now specify further details of the empirical model and discuss some aspects of the estimation procedure. All the parameters of the model are estimated jointly, including the wage equation and the probability $q_{i}(t) .{ }^{12}$ Unfortunately, our sample is too small to produce reliable estimates of $\mu(t)$ conditional on the selected covariates and we are therefore unable to identify and estimate $\rho$. Since

$$
1-\psi(q(t), \mu(t))=\frac{\rho+\mu(t)}{\rho+\mu(t)+\lambda(t) d(t)}=\frac{1}{1+\lambda(t) d(t) /(\rho+\mu(t))},
$$

$1-\psi(q(t), \mu(t))$ will increase when $1-q(t)$ increases, and vice versa, because

$$
\frac{\lambda(t) d(t)}{\rho+\mu(t)}
$$

decreases when the economy heads towards recessions, and vice versa. Thus, the theoretical counterpart of the unemployment rate, $1-q(t)$, evidently is a proxy for $1-\psi(q(t), \mu(t))$. Note furthermore that a first order Taylor expansion of $1-\psi(q(t), \mu(t))$ around the means $(\bar{q}, \bar{\mu})$, where $\bar{q}$ and $\bar{\mu}$ are the respective means across individuals and across time, yields

$$
c(1-\psi(q(t), \mu(t))) \cong c(1-q(t)) \cdot \frac{\bar{\mu}+\rho}{\bar{\mu}+\rho(1-\bar{q})}=\tilde{c}(1-q(t))
$$

and

$$
\begin{equation*}
\tilde{c}=c \cdot \frac{\bar{\mu}+\rho}{\bar{\mu}+\rho(1-\bar{q})} \geq c . \tag{7}
\end{equation*}
$$

Note that the factor $(\bar{\mu}+\rho) /(\bar{\mu}+\rho(1-\bar{q}))$ is increasing as a function of $\rho$. Eq. (7) implies that when $\psi(q(t), \mu(t))$ is replaced by $q(t)$ in the empirical model, the estimate of $\tilde{c}$ will be

[^7]an upward biased estimate of $c$. The bias will differ among the two groups of women we are studying due to differences in unemployment and lay-off rates.

In the following it is convenient to introduce indexation of the individuals, subscript $i$.

## Assumption 5

The systematic term of utility of the job offers is given by

$$
\begin{equation*}
u_{i 2}(t)=E\left(\log W_{i}(t) \mid U_{i 2}(t)>\rho V_{i 1}(t)\right) \tag{8}
\end{equation*}
$$

The wage rate is represented by

$$
\begin{align*}
& \log W_{i}(t)=\beta_{0 t}+X_{i 2}(t) \beta+\delta_{i}(t)  \tag{9}\\
& E\left(\delta_{i}(t) \mid U_{i 2}(t)>\rho V_{i 1}(t), \varepsilon_{i 1}(t)-\varepsilon_{i 0}(t)+Z_{i 1}-Z_{i 0}\right) \cong \kappa\left(\varepsilon_{i 1}(t)-\varepsilon_{i 0}(t)+Z_{i 1}-Z_{i 0}\right) \tag{10}
\end{align*}
$$

where $\kappa$ is a constant, $\delta_{i}(t)$ is a zero mean random term that is independent of $X_{2 i}(t)$, the intercept $\beta_{0 t}$ depends on time, $X_{i 2}(t)$ is a vector of covariates (given in the text column of Table D1 in Appendix D in the online supplementary section), and $\delta_{i}(t)$ is independent of $\varepsilon_{i 2}(t)$ and $Z_{i 2}$.

The wage equation in (9) is used to predict wage rates for all women in the empirical model. The relation in (10) allows for self-selection effects because wages are not observed for those who do not work. In addition, the motivation for introducing the wage equation is to accommodate for measurement error (see Appendix C in the online supplementary section). Note that the wage equation above is a conditional wage equation given employment. ${ }^{13}$

It follows from (8) and (9) that

$$
\begin{equation*}
u_{i 2}(t)=\beta_{0 t}+X_{i 2}(t) \beta . \tag{12}
\end{equation*}
$$

Since the number of observations in the respective subgroups of individuals in the labor force surveys is rather small, a (reduced form) logit model for $q_{i}(t)$ as a function of selected covariates is applied in the model for labor force participation instead of the corresponding observed

[^8]fractions for the respective population groups (see Appendix C in the online supplementary section).

## Assumption 6

The systematic part of the utility of being out of the labor force is given by

$$
\begin{equation*}
u_{i 0}(t)=X_{i 0}(t) \gamma \tag{11}
\end{equation*}
$$

where $X_{i 0}(t)$ is a vector of covariates (specified in Table 1).

From (2), (8), (11) and (12) and Assumption 6 it follows that

$$
\begin{equation*}
v_{i}(t) \cong \theta\left(\beta_{0 t}+X_{i 2}(t) \beta-X_{i 0}(t) \gamma\right) q_{i}(t)-\theta \tilde{c}\left(1-q_{i}(t)\right) \tag{13}
\end{equation*}
$$

which together with (4) gives the probability of participation. Thus, the probability of labor force participation is expressed in terms of characteristics of the individual through the utility of working, the utility of being out of the labor force, modified instantaneous search cost (latent) and the probability of receiving an acceptable job offer. With $\left\{v_{i}(t)\right\}$ given by (13) it follows from Theorem 3 that the joint choice probabilities $Q_{i j k}(t-1, t)$ can be readily computed.

## Theorem 4

Assume that Assumptions 3 and 5 hold. Then

$$
\begin{equation*}
E\left(\delta_{i}(t) \mid U_{i 2}>\rho V_{i 1}(t)>U_{i 0}(t)\right) \cong-\alpha \kappa \theta^{-1}\left[\log P_{i}(t)+\left(P_{i}(t)^{-1}-1\right) \log \left(1-P_{i}(t)\right)\right] . \tag{14}
\end{equation*}
$$

The proof of this theorem is given in Appendix A.
When forming the likelihood function it is assumed that the conditional error term in the wage equation for those who work is normally distributed and serially correlated. ${ }^{14} \mathrm{By}$

[^9]using (9) and (10) we can form the likelihood of the wage observations conditional on the subsample of those who work. Hence, we can obtain the joint likelihood function by multiplying the conditional likelihood of the wage rate observations by the likelihood of the observations that identify which labor market states the individuals occupy at each moment in time (including the employment and unemployment state).

Identification is discussed in Appendix B. The likelihood function is found in the same appendix.

### 4.2. Data and estimation results

The data used in the estimation of the model are obtained by linking information from the Norwegian Labor Force Surveys (LFS) 1988-2010 with information from the Norwegian Educational Database, registries with income information from the tax authorities (1988-2010) and the population registries with information about family composition in different years, as well as country of birth for immigrants and their first year of residence in Norway. Information about whether the person lives in a densely populated area is also obtained from the population registries. All registers and survey data are linked using a personal identification key. In the selec tion of the sample we include only married/cohabiting women aged 25-60 years. The lower age limit excludes most women enrolled in higher education, while the upper age limit excludes women that might withdraw from the labor force due to early retirement. In addition to the selection based on age and marital status, we also exclude women who are disabled or claim that they are unable to work according to information in the LFS. Self-employed women and women hired in firms run by family members are also excluded as well as immigrants with missing information about education. To ensure consistency over time non-labor income is constrained to include only the wage income of the partner. If the partner is self-employed, a stipulated measure of income from work as used by the tax authorities is applied. Hourly wages and non-labor income are measured in constant 2010 NOK prices by using the CPI.

Table 1. Estimation results for the probability of labor force participation. Women born in non-Western countries and women born in Norway

|  | Women born in non- <br> Western countries |  | Women born in Norway |  |
| :--- | :---: | :---: | :---: | :---: |
| Variable/parameter | Estimate | $t$-value | Estimate | $t$-value |
| Precision, constant, $\theta_{0}$ | 5.856 | 4.338 | -7.577 | -2.594 |
| Precision, age effect, $\theta_{1}$ |  |  | 7.968 | 5.125 |
| Precision, age squared effect, $\theta_{2}$ |  |  | -0.707 | -3.686 |
| Serial correlation, $\alpha$ | 0.454 | 17.623 | 0.339 | 67.928 |
| Search cost (modified) per unit of | 1.971 | 2.618 | 4.366 | 5.918 |
| time, $\tilde{c}$ |  |  |  |  |
| Constant, $\gamma_{1}$ | 4.534 | 8.008 | 3.041 | 14.187 |
| Age, $\gamma_{2}$ | -0.063 | -1.893 | 0.024 | 3.301 |
| (Age/10) ${ }^{2}, \gamma_{3}$ | 0.102 | 2.354 | -0.0001 | -0.0198 |
| (Real non-labor income) $\times 10^{-5}, \gamma_{4}$ | 0.043 | 2.800 | 0.030 | 13.983 |
| No. of children 0-3, $\gamma_{5}$ | 0.376 | 4.099 | 0.251 | 18.671 |
| No. of children 4-6, $\gamma_{6}$ | 0.108 | 1.714 | 0.166 | 17.012 |
| No. of children 7-18, $\gamma_{7}$ | 0.157 | 3.740 | 0.076 | 17.941 |
|  |  |  |  |  |
| No. of observations | 3,448 |  | 104,202 |  |
| No. of observation units | 1,724 |  | 52,101 |  |
| Log-likelihood | $-1,218.7$ |  | $35,440.5$ |  |
| McFadden's $\rho^{2}$ | 0.41 |  | 0.65 |  |

Note: McFadden's __ is based on the loglikelihood for participation and transitions into and out of the labor force, cf. (B3) in the online supplementary section.

In addition to the dataset used in the estimation of the model, we have also constructed a dataset for simulations. This dataset is based on registry data covering the whole Norwegian population as of 2005. In the construction of this dataset we have tried to use the same selection rules as for the main dataset. The reason for introducing a specific dataset for simulations is that we want simulations to be representative for smaller groups of women, in particular among immigrants. More information about the data is given in Appendix C in the online supplementary section.

Recall that all the parameters of the model are estimated simultaneously. Table D1 and Table D2 in Appendix D in the online supplementary section contain the estimation results for the wage equation and the job offer probability, respectively. In Table 1 we report parameter
estimates of the main model, i.e., the probability of labor force participation for women born in non-Western countries and women born in Norway.

From the table we notice that the estimate of $\alpha$, which represents the variance of the random effects, is significantly higher for the women born in non-Western countries than it is for those born in Norway. This result indicates that the effect of latent permanent factors (perhaps cultural background variables) is more important for immigrant women than for women born in Norway as regards labor force participation. Recall that $\theta$ is inversely proportionate to the standard deviation of $\varepsilon_{0 i}(t)-\varepsilon_{1 i}(t)$, which captures the effects of serially independent unobservables on preferences. In the empirical model $\theta$ is assumed to be a constant for immigrant women but allowed to depend on age and age squared for women born in Norway. ${ }^{15}$ From Table 1 we note that the estimate of the parameter that represents the (modified) search costs (disutility) per unit of time ( $\tilde{c}$ ) is more than twice as large for women born in Norway compared to women born in non-Western countries. Recall that due to (7) one reason why the estimate of $\tilde{c}$ for immigrant women is lower than the estimate of $\tilde{c}$ for women born in Norway may be due to the fact that the factor $(\bar{\mu}+\rho) /(\bar{\mu}+\rho(1-\bar{q}))$ is lower for immigrant women than for women born in Norway provided $\rho$ is the same for both groups. Specifically, the average job separation rates are of order of magnitudes equal to 0.04 and 0.008 , respectively, in our sample. Since the corresponding average unemployment rates are 0.10 and 0.02 , respectively, it follows that if $\rho \cong 0.017$ then the bias corrected estimates of search costs become the same for immigrant women and women born in Norway (given the approximation related to Eq. (7)).

[^10]Table 2. Observed and predicted labor force participation rates

|  | Women born in non-Western <br> countries |  |  |  | Women born in Norway |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Period | Observed | Std. dev. | Predicted | Observed | Std. dev. | Predicted |
| $1988-1990$ | 0.714 | 0.032 | 0.726 | 0.829 | 0.003 | 0.836 |
| $1991-1993$ | 0.665 | 0.024 | 0.638 | 0.856 | 0.002 | 0.860 |
| $1994-1996$ | 0.648 | 0.020 | 0.645 | 0.880 | 0.002 | 0.877 |
| $1997-1999$ | 0.725 | 0.023 | 0.729 | 0.907 | 0.003 | 0.906 |
| $2000-2002$ | 0.749 | 0.022 | 0.750 | 0.927 | 0.003 | 0.926 |
| $2003-2005$ | 0.776 | 0.018 | 0.785 | 0.937 | 0.002 | 0.935 |
| $2006-2008$ | 0.845 | 0.014 | 0.841 | 0.953 | 0.002 | 0.956 |
| $2009-2010$ | 0.865 | 0.019 | 0.895 | 0.967 | 0.003 | 0.970 |
| $1988-2010$ | 0.752 | 0.007 | 0.754 | 0.891 | 0.001 | 0.893 |

The estimate of the parameter relating to real non-labor income is not significantly different between the two groups and indicates a negative relationship between labor market participation and non-labor income for both groups. With respect to the variables capturing the number of children in different age intervals, the estimates indicate that an increase in the number of children reduces labor market participation, and the effect is larger for younger than for older children. The effects are fairly similar for both population groups.

To assess the fit of the model we have calculated McFadden's $\rho^{2}$. The values of $\rho^{2}$ are 0.41 for immigrant women and 0.65 for women born in Norway (Table 1), which indicate that the fit is fairly good for both groups. In addition, we have computed the predicted participation probabilities obtained from the model. Table 2 displays aggregate predicted participation rates based on the estimated model together with observed participation rates in the sample. The predicted figures are average predicted participation probabilities across all women in the actual group, and we use the same samples as used in the estimations. We notice that the estimated models fit the data quite well, and that the predictions capture the increase in labor market participation over time for both groups of women.

### 4.3. The discouraged worker effect and barriers to employment

In this section we report the results of using the estimated model to make predictions of the discouraged worker effects and modified unemployment for subgroups of the two groups of females we are studying. To emphasize that the value functions and the participation
probability depend on $q_{i}$ we now write $P_{i}=P_{i}\left(q_{i}\right)$ and $V_{1 i}=V_{1 i}(q)$ where the time index is suppressed. Recall that we define the peak business cycle condition of the labor market as the case where $q=1$. This is a reference case where there is no uncertainty and labor force participation is based only on preferences and the distribution of the wage rate. The highest predicted value of $q$ in the sample is in fact close to one.

From Section 2 (last paragraph) it follows that the probability of being discouraged is

$$
P\left(\rho V_{1 i}(1)>U_{0 i}>\rho V_{1 i}\left(q_{i}\right)\right)=P_{i}(1)-P_{i}\left(q_{i}\right) .
$$

Similarly, we obtain that the modified unemployment rate is given by $\left(P_{i}(1)-q_{i} P_{i}\left(q_{i}\right)\right) / P_{i}(1)$. The numerator in the last formula is the probability of being in the labor force in the reference case minus the probability of working. In other words, the numerator is the probability of being discouraged or unemployed. The denominator is the probability of being in the labor force in the reference case. Table 3 displays measures of the discouraged worker effect and the total effect of barriers to employment for different population groups, based on registry data for the entire population as of 2005. Here, "barriers" are represented by $\left\{q_{i}\right\}$. Specifically, we have divided the immigrant women in the registry data into 12 specific subgroups depending on their duration of residence in Norway, their actual education and their age. We have also provided results for women born in Norway (see the four last rows). The last column of the table shows the number of observations in each group for the entire population in 2005.

Table 3. Labor force participation, employment and discouraged worker effect by age, education and duration of residence. 2005

| Group | Duration of res. <br> (D) | Length of education (E) | Age <br> (A) | Participation rate ${ }^{\mathrm{a}}$ | Employ- <br> ment <br> rate ${ }^{\text {b }}$ | Un-em-ployment rate ${ }^{\mathrm{c}}$ | Disc. worker effect ${ }^{\text {d }}$ | Modified unem-ployment rate ${ }^{\mathrm{e}}$ | No. of obs. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Women born in non-Western countries |  |  |  |  |  |  |  |  |  |
| 1 | $\mathrm{D} \leq 5$ | $\mathrm{E} \leq 13$ | A<35 | 0.559 | 0.450 | 0.227 | 0.270 | 0.479 | 4,144 |
| 2 | $\mathrm{D} \leq 5$ | $\mathrm{E} \leq 13$ | $\mathrm{A} \geq 35$ | 0.595 | 0.485 | 0.210 | 0.243 | 0.442 | 3,253 |
| 3 | $\mathrm{D} \leq 5$ | $\mathrm{E}>13$ | A<35 | 0.794 | 0.679 | 0.140 | 0.126 | 0.260 | 2,319 |
| 4 | $\mathrm{D} \leq 5$ | $\mathrm{E}>13$ | $\mathrm{A} \geq 35$ | 0.800 | 0.696 | 0.135 | 0.117 | 0.245 | 1,601 |
| 5 | $5<\mathrm{D} \leq 10$ | $\mathrm{E} \leq 13$ | A<35 | 0.670 | 0.580 | 0.147 | 0.168 | 0.322 | 2,902 |
| 6 | $5<\mathrm{D} \leq 10$ | $\mathrm{E} \leq 13$ | $\mathrm{A} \geq 35$ | 0.723 | 0.639 | 0.126 | 0.132 | 0.267 | 3,088 |
| 7 | $5<\mathrm{D} \leq 10$ | $\mathrm{E}>13$ | A<35 | 0.833 | 0.759 | 0.092 | 0.074 | 0.169 | 1,149 |
| 8 | $5<\mathrm{D} \leq 10$ | $\mathrm{E}>13$ | $\mathrm{A} \geq 35$ | 0.854 | 0.786 | 0.082 | 0.062 | 0.145 | 1,729 |
| 9 | D> 10 | $\mathrm{E} \leq 13$ | A $<40$ | 0.806 | 0.759 | 0.062 | 0.058 | 0.129 | 6,127 |
| 10 | D $>10$ | $\mathrm{E} \leq 13$ | $\mathrm{A} \geq 40$ | 0.826 | 0.786 | 0.051 | 0.046 | 0.103 | 8,479 |
| 11 | D $>10$ | $\mathrm{E}>13$ | A $<40$ | 0.898 | 0.868 | 0.035 | 0.022 | 0.058 | 2,491 |
| 12 | D>10 | $\mathrm{E}>13$ | $\mathrm{A} \geq 40$ | 0.910 | 0.884 | 0.029 | 0.016 | 0.046 | 4,117 |
| Women born in Norway |  |  |  |  |  |  |  |  |  |
| 13 |  | $\mathrm{E} \leq 13$ | A<35 | 0.867 | 0.832 | 0.042 | 0.065 | 0.110 | 61,910 |
| 14 |  | $\mathrm{E} \leq 13$ | $\mathrm{A} \geq 35$ | 0.924 | 0.905 | 0.020 | 0.028 | 0.050 | 279,315 |
| 15 |  | $\mathrm{E}>13$ | A<35 | 0.942 | 0.927 | 0.016 | 0.014 | 0.030 | 60,557 |
| 16 |  | E>13 | $\mathrm{A} \geq 35$ | 0.969 | 0.962 | 0.007 | 0.005 | 0.012 | 153,427 |

${ }^{\text {a }}$ Participation rate: $P(q) .{ }^{\mathrm{b}}$ Employment rate: $P(q) q .{ }^{\mathrm{c}}$ Unemployment rate: $1-q \cdot{ }^{\mathrm{d}}$ Discouraged worker effect:
$P(1)-P(q) .{ }^{\mathrm{e}}$ Modified unemployment rate: $(P(1)-P(q) q) / P(1)$.
As regard barriers to employment, we note that the (predicted) unemployment rate for young immigrant women with a low level of schooling and a short time since arrival (group 1 ) is high, about 23 percent, decreasing to about 13 per cent for older women with time 5-10 years since arrival and low level of education (group 6). The lowest unemployment rate among immigrant women is for group 12 , which contains women with high education, aged $45+$, who have been in Norway for more than 10 years. In contrast, the (predicted) unemployment rate for women born in Norway is much lower for all levels of education and age. For this group the unemployment rate varies between 1 and 4 percent.

As mentioned above, we use the modified unemployment rate to measure the total effect of barriers to employment. This rate is highest among immigrant women with short duration of residence in Norway and low education (groups 1 and 2). For these two groups, which differ with respect to age, the modified unemployment rate is over 40 percent. At the other end of the scale, we find well-educated immigrant women who have lived for a long period of time in Norway (groups 11 and 12). For these two groups, the modified unemployment rates are about 6 and 5 percent respectively, mirroring a low level of discouraged workers. Among
immigrant women with short duration of residence in Norway and low education, barriers are thus substantial. However, as duration of residence increases, immigrant women seem to integrate into the Norwegian labor market. Barriers then decrease and employment among immigrant women increases. For Norwegian-born women the barriers are typically much smaller, but not always. By comparing the figures for the four groups in Table 3 (groups 13-16), we note that the modified unemployment rate is highest for young women born in Norway with a low level of schooling (group 13). For this group the modified unemployment rate is 11 percent. For the other three groups, barriers are much smaller, and the low rates are due to both a low unemployment rate and a low discouraged worker effect.

Table 4. Predicted fraction of labor force participation and discouraged worker effect by wage rate, age, number of children and unemployment rate ${ }^{\text {a }}$



Immigrant women

| 1 | 180 | 30 | 0 | 0 | 0 | 0.25 | 0.838 | 0.143 | 0.359 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 180 | 30 | 0 | 0 | 0 | 0.15 | 0.928 | 0.053 | 0.196 |
| 3 | 180 | 30 | 1 | 1 | 1 | 0.25 | 0.591 | 0.313 | 0.510 |
| 4 | 180 | 30 | 1 | 1 | 1 | 0.15 | 0.753 | 0.151 | 0.292 |
| 5 | 100 | 30 | 1 | 0 | 0 | 0.25 | 0.431 | 0.368 | 0.595 |
| 6 | 100 | 30 | 1 | 0 | 0 | 0.15 | 0.595 | 0.204 | 0.367 |
| 7 | 100 | 30 | 1 | 1 | 1 | 0.25 | 0.309 | 0.354 | 0.650 |
| 8 | 350 | 30 | 0 | 0 | 0 | 0.05 | 0.994 | 0.002 | 0.052 |
| 9 | 350 | 30 | 1 | 1 | 1 | 0.15 | 0.932 | 0.050 | 0.193 |
| 10 | 180 | 45 | 0 | 1 | 1 | 0.15 | 0.820 | 0.118 | 0.257 |
| 11 | 180 | 45 | 0 | 1 | 4 | 0.15 | 0.612 | 0.200 | 0.359 |
| 12 | 100 | 45 | 0 | 1 | 1 | 0.25 | 0.389 | 0.370 | 0.615 |
| 13 | 100 | 45 | 0 | 1 | 4 | 0.25 | 0.200 | 0.275 | 0.684 |
| 14 | 100 | 45 | 0 | 1 | 4 | 0.15 | 0.294 | 0.180 | 0.472 |

Women born in Norway

| 15 | 205 | 30 | 0 | 0 | 0 | 0.025 | 0.991 | 0.003 | 0.028 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 16 | 205 | 30 | 1 | 1 | 1 | 0.025 | 0.956 | 0.016 | 0.041 |
| 17 | 140 | 30 | 0 | 0 | 0 | 0.05 | 0.952 | 0.029 | 0.078 |
| 18 | 140 | 30 | 1 | 1 | 1 | 0.05 | 0.802 | 0.103 | 0.158 |
| 19 | 305 | 30 | 0 | 0 | 0 | 0.025 | 0.998 | 0.001 | 0.026 |
| 20 | 305 | 30 | 1 | 0 | 0 | 0.025 | 0.994 | 0.002 | 0.027 |
| 21 | 305 | 30 | 1 | 1 | 1 | 0.025 | 0.988 | 0.005 | 0.030 |
| 22 | 140 | 45 | 0 | 1 | 1 | 0.05 | 0.811 | 0.121 | 0.173 |
| 23 | 140 | 45 | 0 | 1 | 4 | 0.05 | 0.608 | 0.217 | 0.300 |
| 24 | 205 | 45 | 0 | 1 | 1 | 0.05 | 0.960 | 0.028 | 0.077 |
| 25 | 205 | 45 | 0 | 1 | 4 | 0.05 | 0.896 | 0.070 | 0.119 |

Women born in Norway with immigrants' characteristics

| 26 | 180 | 30 | 0 | 0 | 0 | 0.25 | 0.473 | 0.518 | 0.642 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 27 | 180 | 30 | 0 | 0 | 0 | 0.15 | 0.864 | 0.128 | 0.260 |
| 28 | 180 | 30 | 0 | 0 | 0 | 0.05 | 0.978 | 0.014 | 0.063 |
| 29 | 180 | 30 | 1 | 0 | 0 | 0.25 | 0.322 | 0.659 | 0.754 |
| 30 | 180 | 30 | 1 | 0 | 0 | 0.15 | 0.755 | 0.226 | 0.346 |
| 31 | 180 | 30 | 1 | 0 | 0 | 0.05 | 0.952 | 0.028 | 0.078 |
| 32 | 180 | 30 | 1 | 1 | 1 | 0.25 | 0.205 | 0.753 | 0.840 |
| 33 | 180 | 30 | 1 | 1 | 1 | 0.15 | 0.606 | 0.351 | 0.462 |

${ }^{\text {a }}$ We also assume that non-labor income, measured in 2010 prices, is 380,000 NOK for immigrant women (cases $1-14,26-$ 33) and 490,000 NOK for women born in Norway (cases $15-25$ ). ${ }^{\mathrm{b}}$ In constant 2010 NOK prices. ${ }^{\mathrm{c}}$ Discouraged worker effect: $P(1)-P(q) .{ }^{\mathrm{d}}$ Modified unemployment rate: $(P(1)-P(q) q) / P(1)$.

Above we have discussed the distribution of the discouraged worker effect in the labor market for the target population. By dividing the population into subgroups, we found that there is considerable heterogeneity in the barriers due to the composition of the different groups.

We next wish to take a closer look at the difference in behavior for given population groups facing hypothetical levels of real wage rates and probabilities of getting acceptable job offers (Table 4). Specifically, we simulate the labor market behavior of the respective groups of women with a given wage rate, age, real non-labor income, number of children in the three different age groups and a given unemployment rate. The upper part of Table 4 contains simulations for 14 different types of immigrant women, while the lower part contains similar results for 19 groups of women born in Norway. For the latter group we present simulations for women with characteristics that are not only representative for women born in Norway (cases 15-25), but also similar to the ones used for immigrant women (cases 26-33). Since the two groups of women vary systematically with respect to wage rate and the probability of obtaining a job, the assumptions being made about the level of these variables differ across the two groups. The lowest values of the mean wage rates correspond (approximately) to the first decile in the distribution of that variable in our sample for 2010, whereas the highest value is the ninth decile, and the value in the middle is the median value.

Looking at the results given in Table 4 for women born in non-Western countries, we see that the discouraged worker effect varies from 0.002 to 0.370 . The lowest rate is found for a childless woman aged 30 years with a predicted wage equal to 350 NOK and a probability of obtaining an acceptable job equal to 0.95 , which corresponds to an unemployment rate of 5 percent. The highest rate is for a woman aged 45 with a wage equal to 100 NOK, with two children, one in the oldest age group and one in the next oldest age group, and a probability of obtaining an acceptable job equal to 0.75 . As expected, the discouraged worker effect and the modified unemployment rate increase when there is an increase in the unemployment rate.

By comparing the simulation results for immigrant women with different wage rates, we also notice that the discouraged worker effect depends on the wage rate. For women with high wage rates, the effect is small (cases 8 and 9), even when the unemployment rate is moderate, and the woman has many children (case 9). For women with low wage rates, the discouraged worker effect is considerably higher, even when the unemployment rate is moderate (cases 6 and 14). Note that the modified unemployment rate is particularly high for the groups

12, 13, among immigrant woman and 29 and 32 among women born in Norway. The relationship between the number of children and the discouraged worker effect is more complicated, since a change in the number of children yields shifts in preferences. For young women with medium wage rates (cases 1-4), we note that more children increase the discouraged worker effect. However, by comparing the discouraged worker effect for cases 12 and 13 we note that there is a decrease for women with several children. Both these groups face low mean wage rates and belong to the oldest age group. Thus, we would expect them to have low participation rates. The participation rates are 0.370 for women with only two children and 0.275 for women with five children, according to the results in Table 4. Compared to immigrant women, there is much less variation in the discouraged worker effect among women born in Norway. For these women (cases 15-25), the discouraged worker effect lies between 0.001 and 0.217 , and it is only for middle-aged women with a low wage rate, a low $q$ and five children (case 23) that the discouraged worker effect is higher than 0.2.

In the final part of Table 4 we present some simulations for women born in Norway, but with mean wage rates and unemployment rates that are more representative for immigrant women. Cases 26-28 relate to young women born in Norway with a mean wage rate equal to the median wage rate for immigrant women with different hypothetical unemployment rates. While these women are childless, others have one young child (cases 29-31) or a child in each of the three age groups (cases 32 and 33 ). By comparing women with similar characteristics - cases 26, 27, 32 and 33 for women born in Norway and cases 1-4 for immigrant women - we find that the discouraged worker effect is lower among immigrant women than among women born in Norway. The reason is that the estimate of the search cost per unit of time, $c$, is lower among immigrant women than women born in Norway.

Looking at the figures for the modified unemployment rate, we find that this rate is particularly high for the groups $7,12,13$, among immigrant woman and 29 and 32 among women born in Norway. Common for all these groups of women is that the unemployment rate is high. The immigrant women have low wages, and the women in groups 7 and 13 have several children. The immigrant women in group 12 are older women compared to the women in the other groups.

## 5. Concluding remarks

In this paper we have analyzed labor force participation and the discouraged worker phenomenon for married/cohabiting women born in non-Western countries and Norway, respectively. We have applied an empirical modelling framework based on the theory of job search and the notion of probabilistic rationality. The model is estimated separately for immigrant women from non-Western countries and women born in Norway. According to our estimation results, the two groups differ with respect to the estimate of the (modified) search costs per unit of time. Women born in Norway have higher probabilities of getting acceptable job offers, but they also have considerably higher estimate of $\tilde{c}$ (modified) search costs per unit of time compared to immigrant women. In total, however, the proportion of discouraged workers is significantly higher for some groups of immigrant women than for women born in Norway. The reason is that due to the lower probabilities of getting an acceptable job offer immigrant women will, on average, need to search for a longer time than women born in Norway to get a job.

Among immigrant women, duration of residence has a similar effect on barriers to employment as educational qualifications, as it is positively correlated with the wage rate and the probability of getting an acceptable job offer.

Although the analysis has focused on data from Norway, we believe that our results also are of interest to other countries. As mentioned in the introduction, several countries (such as the US) collect data on discouraged workers. Such data are evidently useful for descriptive purposes. Still, we believe it is of interest to conduct a structural analysis that intends to explain how labor force participation varies over the cycle and depends on key socio-economic variables (expected cost of search, wage rates, etc.) which can be used to analyze counterfactuals. Similar to conventional labor supply models, the model developed in this paper can be used to carry out policy simulations of the effect of changing wage rates, or length of schooling which affect the wage rates, on the modified unemployment rate, as well as on the probability of being in the labor force, conditional on the unemployment rates.

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## Appendix A

## Proofs

## Proof of Theorem 1:

In the following it will be useful to introduce indexation of the periods relevant to the individual agent. Let $\tilde{V}_{s j}, j=0,1,2$, denote the value function at time $s$ (continuous) of being in the respective states and let $y_{s}$ be exogenous non-labor income. Let $U_{s 2}=U_{2}\left(W_{s}, y_{s}\right)$ be the indirect utility dual to the direct utility of disposable income and leisure. There is no cost of working. Let $U_{s 1}=U\left(y_{s}-c^{*}\right)$ be the utility of zero hours of work subject to the budget constraint with monetary search costs, $c^{*}$. Similarly, let $U_{s 0}=U\left(y_{s}\right)$ be the direct utility of not working (zero hours of work) subject to the budget constraint. Let $E_{s}$ denote the expectation operator given the information available to the agent at time $s$. Let $\tilde{s}=s+\Delta s$. As in Burdett and Mortensen (1978) we have that

$$
\begin{align*}
& \quad(1+\rho \Delta s) \tilde{V}_{s 0}=U_{s 0} \Delta s+\Delta s E_{s} \max \left(\tilde{V}_{\tilde{s} 0}, \tilde{V}_{\tilde{s} 1}\right)+o(\Delta s),  \tag{A.1}\\
& (1+\rho \Delta s) \tilde{V}_{s 1}=U_{s 1} \Delta s+\lambda \Delta s E_{s} \max \left(\tilde{V}_{\tilde{s} 0}, \tilde{V}_{\tilde{s} 1}, \tilde{V}_{\tilde{s} 2}\right)+(1-\lambda \Delta s) E_{s} \max \left(\tilde{V}_{\tilde{\tilde{s}} 0}, \tilde{V}_{\tilde{s} 1}\right)+o(\Delta s) \tag{A.2}
\end{align*}
$$

and

$$
\begin{equation*}
(1+\rho \Delta s) \tilde{V}_{s 2}=U_{s 2} \Delta s+(1-\mu \Delta s) E_{s} \max \left(\tilde{V}_{\tilde{s} 0}, \tilde{V}_{\tilde{s} 1}, \tilde{V}_{\tilde{z_{2}}}\right)+\mu \Delta s E_{s} \max \left(\tilde{V}_{\tilde{s} 0}, \tilde{V}_{\tilde{s} 1}\right) \tag{A.3}
\end{equation*}
$$

The first equation above states that since there is no job offer when out of the labor force the value of being out of the labor force, adjusted for discounting, equals the period-specific utility of being out of the labor force plus the maximum of the value of being out of the labor force and the value of searching in the next period. The second equation accommodates that two things may happen when searching. A job offer may arrive with probability $\lambda$ or no job offer arrives, which has probability $1-\lambda$. The corresponding expected value therefore equals

$$
\lambda \Delta s E_{s} \max \left(\tilde{V}_{\tilde{s} 0}, \tilde{V}_{\tilde{s} 1}, \tilde{V}_{\tilde{s}, 2}\right)+(1-\lambda \Delta s) E_{s} \max \left(\tilde{V}_{\tilde{s} 0}, \tilde{V}_{\tilde{s}, 1}\right)+o(\Delta s)
$$

The interpretation of (A.3) is analogous.
Consider next the implications of Assumption 1. Stationary environment in this context means that the perceived subjective distribution of wage rates and job-specific attributes associated with future job offers is time invariant. Stationarity and infinite planning horizon imply
that the individual's predictions of non-labor incomes for $s$ greater than the current time epoch, are time invariant and $y_{s}=y$ is also time invariant. Accordingly, since the future period utility function is time invariant (by assumption) it follows that (A.1) to (A.3) reduce to

$$
\begin{align*}
& (1+\rho \Delta s) \tilde{V}_{0}=U_{0} \Delta s+\bar{V}_{0},  \tag{A.4}\\
& (1+\rho \Delta s) \tilde{V}_{1}=U_{1} \Delta s+\lambda \Delta s E_{s} \max \left(\tilde{V}_{1}, \tilde{V}_{s+1,2}\right)+(1-\lambda \Delta s) \tilde{V}_{1}+o(\Delta s) \tag{A.5}
\end{align*}
$$

and

$$
\begin{equation*}
(1+\rho \Delta s) \tilde{V}_{s 2}=U_{s 2} \Delta s+(1-\mu \Delta s) \tilde{V}_{s 2}+\mu \Delta s \tilde{V}_{1} \tag{A.6}
\end{equation*}
$$

Let $\bar{U}_{2}=E_{s}\left(U_{s+1,2} \mid \tilde{V}_{s+1,2}>\tilde{V}_{1}\right), \bar{V}_{2}=E_{s}\left(\tilde{V}_{s+1,2} \mid \tilde{V}_{s+1,2}>\tilde{V}_{1}\right)$ and $d=E_{s} 1\left\{\tilde{V}_{s+1,2}>\tilde{V}_{1}\right\}$. Note that

$$
\begin{equation*}
E_{s} \max \left(\tilde{V}_{1}, \tilde{V}_{s+1,2}\right)=d E_{s}\left(\tilde{V}_{s+1,2} \mid \tilde{V}_{s+1,2}>\tilde{V}_{1}\right)+(1-d) \tilde{V}_{1} . \tag{A.7}
\end{equation*}
$$

It follows from (A.5) and (A.7) that

$$
\begin{equation*}
(\rho+\lambda) \tilde{V}_{1}=U_{1}+d \lambda \bar{V}_{2} . \tag{A.8}
\end{equation*}
$$

From (A.6) it follows similarly that

$$
\begin{equation*}
(\rho+\mu) \bar{V}_{2} d=\bar{U}_{2} d+\mu \tilde{V}_{1} . \tag{A.9}
\end{equation*}
$$

When solving for $\bar{V}_{2}$ in (A.8) and inserting the solution in (A.9), we obtain that

$$
\rho \tilde{V}_{1}=\frac{(\rho+\mu) U_{1}+\bar{U}_{2} \lambda d}{\rho+\lambda d+\mu}
$$

From (A.6) it follows that

$$
(\rho+\mu)\left(\tilde{V}_{2}-\tilde{V}_{1}\right)=U_{2}-\rho \tilde{V}_{1} .
$$

Accordingly, we realize that $\tilde{V}_{2}>\tilde{V}_{1}$ and $\tilde{V}_{1}>\tilde{V}_{0}$ are equivalent to

$$
U_{2}>\frac{(\rho+\mu) U_{1}+d \lambda \bar{U}_{2}}{\rho+\lambda d+\mu} \quad \text { and } \quad \frac{(\rho+\mu) U_{1}+d \lambda \bar{U}_{2}}{\rho+\lambda d+\mu}>U_{0}
$$

respectively. Consequently, it will be convenient to redefine the value of search as

$$
\rho V_{1}=\frac{(\rho+\mu) U_{1}+d \lambda \bar{U}_{2}}{\rho+\lambda d+\mu}
$$

and the values of being employed and out of the labor force as $U_{2}$ and $U_{0}$, respectively.
Q. E. D.

## Lemma A1:

Assume that $Y=Z_{1}-Z_{0}+\varepsilon$, where $\varepsilon, Z_{0}$ and $Z_{1}$ are independent, $\varepsilon$ has a standard logistic cdf $(1+\exp (-x))^{-1}$ for real $x$ and $\exp \left(Z_{j}\right), j=0,1$, are independent and have stable distribution $S_{\alpha}\left((\cos (\alpha \pi / 2))^{1 / \alpha}, 1,0\right)$ where $0<\alpha \leq 1$. Then $\alpha^{-1} Y$ has a standard logistic cdf.

## Proof of Lemma A1:

Let $\varepsilon_{j}, j=0,1$, be independent random variable with standard Gumbel cdf $\exp (-\exp (-x))$ for real $x$. Then it follows readily that $\varepsilon_{1}-\varepsilon_{0}$ has standard logistic cdf. Moreover

$$
\begin{align*}
& P\left(\alpha Z_{j}+\alpha \varepsilon_{j} \leq y\right)=E P\left(\varepsilon_{j} \leq y \alpha^{-1}-Z_{j} \mid Z_{j}\right)  \tag{A.10}\\
& =E \exp (-\exp (Z-y / \alpha))=\exp (-\exp (-y)) .
\end{align*}
$$

The last equality in (A.10) follows from Proposition 1.2.12 in Samorodnitsky and Taqqu (1994, p. 15). Note that the expression on the right side of (A.10) is a standard Gumbel cdf. Hence, it follows that $\alpha \varepsilon_{1}-\alpha \varepsilon_{0}+\alpha Z_{1}-\alpha Z_{0}$ has the same distribution as $\varepsilon_{1}-\varepsilon_{0}$, namely the standard logistic cdf.
Q. E. D.

## Proof of Theorem 2:

From Luce and Suppes (1965, p. 350) it follows that the product rule is equivalent to binary choice probabilities having the logit functional form. Furthermore, the logit model is equivalent to the corresponding utility differences having a logistic cdf as in (3). By Lemma A1 it follows that the unconditional cdf also becomes a logistic cdf. Hence, (4) follows. To prove (5) we use the fact that $\operatorname{Var}\left(\varepsilon_{1}-\varepsilon_{0}\right)=2 \operatorname{Var}_{1}=\pi^{2} / 3$. Since (A.10) yields that $\varepsilon_{j}+Z_{j}$ has the same distribution as $\varepsilon_{j} / \alpha$ it follows that

$$
\operatorname{Var}\left(\alpha \varepsilon_{1}-\alpha \varepsilon_{0}+\alpha Z_{1}-\alpha Z_{0}\right)=\alpha^{2} \operatorname{Var}\left(\varepsilon_{1}-\varepsilon_{0}\right)+\alpha^{2} \operatorname{Var}\left(Z_{1}-Z_{0}\right)=\operatorname{Var}\left(\varepsilon_{1}-\varepsilon_{0}\right)
$$

from which (5) follows. Moreover, (6) follows from (5).
Q.E.D.

## Lemma A2

Let

$$
g_{\alpha}(x)=\frac{e^{\alpha x}-1}{e^{x}-1} \text { for real } x \neq 0 \text { and } g_{\alpha}(0)=\alpha
$$

defined for $0<\alpha \leq 1$. The function $g_{\alpha}(x)$ is continuous differentiable and non-increasing.

## Proof of Lemma A2:

It is straightforward to verify that $g_{\alpha}(x)$ is continuously differentiable for all $x$. Moreover,

$$
\frac{g_{\alpha}^{\prime}(x)}{g_{\alpha}(x)}=\frac{f_{\alpha}(x)}{\left(1-e^{-\alpha x}\right)\left(1-e^{-x}\right)}
$$

where

$$
f_{\alpha}(x)=\alpha\left(1-e^{-x}\right)-1+e^{-\alpha x} .
$$

By the mean value theorem we get

$$
\begin{equation*}
f_{\alpha}(x)-f_{\alpha}(0)=f_{\alpha}(x)=f_{\alpha}^{\prime}(\tilde{x}) x=\alpha\left(e^{-\tilde{x}}-e^{-\alpha \tilde{x}}\right) x \tag{A.11}
\end{equation*}
$$

where $\tilde{x}$ is a number within the interval $(0, x)$. Since $f_{\alpha}^{\prime}(x)<0$ for $x>0$ and $f_{\alpha}^{\prime}(x)>0$ for $x<0$ it follows from (A.11) that $f_{\alpha}(x) \leq 0$.
Q. E. D.

## Proof of Theorem 3:

Let $Z=Z_{1}-Z_{0}$ and recall that

$$
P(t)=\frac{1}{1+\exp (-\alpha v(t))} \quad \text { and } \quad g_{\alpha}(x)=\frac{\exp (\alpha x)-1}{\exp (x)-1}
$$

Note that

$$
\begin{aligned}
& \frac{1}{1+\exp (-v(t-1)-\theta Z)}\left(1-\frac{1}{1+\exp (-v(t)-\theta Z)}\right) \\
& =\frac{1}{1-\exp (v(t)-v(t-1))}\left(\frac{1}{1+\exp (-v(t-1)-\theta Z)}-\frac{1}{1+\exp (-v(t)-\theta Z)}\right) .
\end{aligned}
$$

By using the result of Lemma A1 we obtain that

$$
\begin{aligned}
& P(t-1) Q_{10}(t-1, t)=E\left(\frac{1}{1+\exp (-v(t-1)-\theta Z)}\left(1-\frac{1}{1+\exp (-v(t)-\theta Z)}\right)\right) \\
& =\frac{1}{1-\exp (v(t)-v(t-1))}\left(E\left(\frac{1}{1+\exp (-v(t-1)-\theta Z)}\right)-E\left(\frac{1}{1+\exp (-v(t)-\theta Z)}\right)\right) \\
& =\frac{P(t-1)-P(t)}{1-\exp (v(t)-v(t-1))}=g_{\alpha}(v(t-1)-v(t)) P(t)(1-P(t-1)) .
\end{aligned}
$$

The formulas for $Q_{j k}(t-1, t)$ for other combinations of $j$ and $k$ are proved in a similar way. The properties of $g_{\alpha}(x)$ follow from Lemma A2.
Q.E.D.

## Lemma A3

Let $V_{j}=v_{j}+\varepsilon_{j}, j=0,1$, where $v_{1}$ and $v_{0}$ are deterministic terms and $\varepsilon_{1}-\varepsilon_{0}$ has cdf $\{1+\exp (-x)\}^{-1}$ for real $x$ and let $P_{1}=P\left(V_{1}>V_{0}\right)$. Then $E\left(\varepsilon_{1}-\varepsilon_{0} \mid V_{1}>V_{0}\right)=-\log P_{1}-\left(P_{1}^{-1}-1\right) \log \left(1-P_{1}\right)$.

## Proof of Lemma A3:

We have that

$$
\begin{aligned}
& E\left(\varepsilon_{1}-\varepsilon_{0}\right) 1\left\{V_{1}>V_{0}\right\}=E\left(V_{1}-V_{0}\right) 1\left\{V_{1}>V_{0}\right\}+\left(v_{0}-v_{1}\right) P\left(V_{1}>V_{0}\right) \\
& =E\left(V_{1}-V_{0}\right) 1\left\{V_{1}>V_{0}\right\}+P_{1} \log \left(\frac{1-P_{1}}{P_{1}}\right) .
\end{aligned}
$$

Moreover, using integration by parts it follows that

$$
\begin{aligned}
& E\left(V_{1}-V_{0}\right) 1\left\{V_{1}>V_{0}\right\}=\int_{0}^{\infty} \frac{x \exp \left(v_{1}-v_{0}-x\right) d x}{\left(1+\exp \left(v_{1}-v_{0}-x\right)\right)^{2}}=-\left.\right|_{0} ^{\infty} x\left(1-\frac{1}{1+\exp \left(v_{1}-v_{0}-x\right)}\right) \\
& +\int_{0}^{\infty}\left(1-\frac{1}{1+\exp \left(v_{1}-v_{0}-x\right)}\right) d x=\int_{0}^{\infty} \frac{\exp \left(v_{1}-v_{0}-x\right)}{1+\exp \left(v_{1}-v_{0}-x\right)} d x \\
& =-\left.\right|_{0} ^{\infty} \log \left(1+\exp \left(v_{1}-v_{0}-x\right)\right)=\log \left(1+\exp \left(v_{1}-v_{0}\right)\right)=-\log \left(1-P_{1}\right) .
\end{aligned}
$$

Hence,

$$
E\left(\varepsilon_{1}-\varepsilon_{0} \mid V_{1}>V_{0}\right)=-\log P_{1}-\left(P_{1}^{-1}-1\right) \log \left(1-P_{1}\right)
$$

> Q.E.D.

## Proof of Theorem 4:

Recall that by Lemma A1 that $\theta Z_{i 1}-\theta Z_{i 0}+\theta \varepsilon_{i 1}(t)-\theta \varepsilon_{i 0}(t)$ has the same distribution as $\alpha^{-1}\left(\theta \varepsilon_{i 1}(t)-\theta \varepsilon_{i 0}(t)\right)$. By Assumption 5 and Lemma A3

$$
\begin{aligned}
& E\left(\delta_{i}(t) \mid V_{i 2}(t)>V_{i 1}(t)>V_{i 0}(t)\right)=\kappa \alpha \theta^{-1} E\left(\theta Z_{i 1}-\theta Z_{i 0}+\theta \varepsilon_{i 1}(t)-\theta \varepsilon_{i 0}(t) \mid V_{i 1}(t)>V_{i 0}(t)\right) \\
& =\tau \alpha^{-1} \theta^{-1} E\left(\theta \varepsilon_{i 1}(t)-\theta \varepsilon_{i 0}(t) \mid V_{i 1}(t)>V_{i 0}(t)\right)=-\tau \alpha^{-1} \theta^{-1}\left[\log P_{i}(t)+\left(P_{i}(t)^{-1}-1\right) \log \left(1-P_{i}(t)\right)\right]
\end{aligned}
$$

Q.E.D.

## Appendix B

## Identification

If the wage rates for those who work are known $u_{2}(t)$ can be recovered. Assume now that $P(t), q(t), \mu(t), u_{2}(t)$ and $\left\{Q_{j k}(t-1, t)\right\}$ are known and $u_{0}(t)$ is specified as in (11).

Then it follows from Theorem 2 that

$$
\log \left(\frac{P(t)}{1-P(t)}\right)=\alpha v(t)=\left(u_{2}(t)-u_{0}(t)\right) \alpha \theta \psi_{\rho}(q(t), \mu(t))-\alpha \theta c\left(1-\psi_{\rho}(q(t), \mu(t))\right.
$$

which shows that $\alpha \theta v(t)$ is known. Furthermore, it follows from Theorem 3 that

$$
\frac{(1-P(t-1)) Q_{01}(t-1, t)}{(1-P(t-1)) P(t)}=g_{\alpha}(v(t-1)-v(t))=\frac{\exp (\alpha v(t-1)-\alpha v(t))-1}{\exp ((\alpha v(t-1)-\alpha v(t)) / \alpha)-1}
$$

which implies that $\alpha$ is identified. Let $z(t)=(1-q(t)) / \mu(t)$. We get that

$$
\begin{equation*}
\frac{\partial(\alpha \theta v(t))}{\partial q(t)}=\alpha \theta\left(u_{2}(t)-u_{0}(t)+c\right) \frac{1}{1+\rho z(t)} \tag{B.1}
\end{equation*}
$$

and

$$
\frac{\partial(\alpha \theta v(t))}{\partial z(t)}=-\alpha \theta\left(u_{2}(t)-u_{0}(t)+c\right) \frac{\rho}{(1+\rho z(t))^{2}}
$$

which implies that

$$
z(t)-\frac{\partial(\alpha \theta v(t)) / \partial q(t)}{\partial(\alpha \theta v(t)) / \partial z(t)}=\frac{1}{\rho} .
$$

The last relation shows that $\rho$ is identified and thus the function $\psi_{\rho}$ can be recovered. Moreover,

$$
\frac{\partial \theta v(t)}{\partial u_{2}(t)}=\theta \psi_{\rho}(q(t), \mu(t)) \quad \text { and } \quad \frac{\partial\left(\theta v(t) \psi_{\rho}(t)^{-1}\right)}{\partial\left(\psi_{\rho}(t)^{-1}\right)}=\theta c
$$

which shows that $\theta$ and $c$ are identified. From (B.1) it follows that $u_{0}(t)$ is identified. In our case where $\psi(q(t), \mu(t))$ is replaced by $q(t)$ only $\tilde{c}$ is identified.

## Probabilistic rationality and the product rule

Let $a_{0}, a_{1}$ and $a_{2}$ denote the alternatives "out of the labor force", "search" and "employed", respectively, and let $\succ$ denote "preferred to". Luce (1959) and Luce and Suppes (1965) proposed the product rule, as a characterization of probabilistic rationality in the following sense:

Consider a choice experiment where an individual is asked to make binary choices from the set $\left\{a_{0}, a_{1}, a_{2}\right\}$. Consider 3 binary choices from the respective choice sets $\left\{a_{j}, a_{k}\right\},\left\{a_{k}, a_{q}\right\}$ and $\left\{a_{q}, a_{j}\right\}, j, k, q \in\{0,1,2\}$. The product rule is given by

$$
P\left(a_{j} \succ a_{k}\right) P\left(a_{k} \succ a_{q}\right) P\left(a_{q} \succ a_{j}\right)=P\left(a_{j} \succ a_{q}\right) P\left(a_{q} \succ a_{k}\right) P\left(a_{k} \succ a_{j}\right) .
$$

When the choices are independent the left side of the equation above is the probability that the intransitive chain $a_{j} \succ a_{k} \succ a_{q} \succ a_{j}$ is realized and the right side is the probability that the intransitive chain $a_{j} \succ a_{q} \succ a_{k} \succ a_{j}$ is realized. Thus, the product rule asserts that there are no reasons for an intransitive chain in one direction to be more probable than an intransitive chain in the opposite direction. In other words, the product rule captures the notion that departure from rationality is - on average - not systematic.

Let $\xi_{j k}=Z_{j}-Z_{k}$ and $V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)=v_{j}+Z_{j}+\varepsilon_{j}$ for $j=0,1,2$. The following assumption is a statement of the conditional product rule given $\left\{Z_{j}\right\}$.

## Assumption B1

Let $\left\{\varepsilon_{j}, \varepsilon_{j}^{\prime}, \varepsilon_{j}^{\prime \prime}, Z_{j}\right\}$ be independent random variables. Then
$P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{k}, Z_{k}, \varepsilon_{k}\right) \mid \xi_{j k}\right) P\left(V\left(v_{k}, Z_{k}, \varepsilon_{k}^{\prime}\right)>V\left(v_{q}, Z_{q}, \varepsilon_{q}^{\prime}\right) \mid \xi_{k q}\right) P\left(V\left(v_{q}, Z_{q}, \varepsilon_{q}^{\prime \prime}\right)>V\left(v_{j}, Z_{j}, \varepsilon_{j}^{\prime \prime}\right) \mid \xi_{q j}\right)$ $=P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{q}, Z_{q}, \varepsilon_{q}\right) \mid \xi_{j q}\right) P\left(V\left(v_{q}, Z_{q}, \varepsilon_{q}^{\prime}\right)>V\left(v_{k}, Z_{k}, \varepsilon_{k}^{\prime}\right) \mid \xi_{k q}\right) P\left(V\left(v_{k}, Z_{k}, \varepsilon_{k}^{\prime \prime}\right)>V\left(v_{j}, Z_{j}, \varepsilon_{j}^{\prime \prime}\right) \mid \xi_{j k}\right)$.

We realize that Assumption B1 is a version of the product rule. The empirical counterpart of Assumption B1 is the following: Consider a series of choice experiments conducted on a homogeneous population where each individual is presented with 3 binary choices at different points in time, namely the choice from $\left\{a_{j}, a_{k}\right\},\left\{a_{k}, a_{q}\right\}$ or $\left\{a_{j}, a_{q}\right\}, j, k, q=0,1,2$. By homogeneous population we mean a population where the individuals have i. i. d. preferences. Luce and Suppes (1965, p. 350) demonstrated that Assumption B1 implies that

$$
P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{k}, Z_{k}, \varepsilon_{k}\right) \mid \xi_{j k}\right)=\frac{1}{1+\exp \left(-\tilde{v}_{j}+\tilde{v}_{k}\right)}
$$

where $\tilde{v}_{j}$ is a function of $v_{j}+Z_{j}$. To this end, the next lemma is useful.

## Lemma B1

Assume that $\left\{\eta_{j}\right\}$ are random variables and that

$$
P\left(\eta_{j}+v_{j}>\eta_{k}+v_{k}\right)=\frac{e^{\tilde{\nu}_{j}}}{e^{\tilde{v}_{j}}+e^{\tilde{v}_{k}}}
$$

where $\left\{v_{j}\right\}$ are arbitrary deterministic scales on the real line and $\left\{\tilde{v}_{j}\right\}$ are functions of $\left\{v_{j}\right\}$. Then

$$
\tilde{v}_{j}=\tau v_{j}
$$

where $\tau>0$ is a constant.

## Proof of Lemma B1:

Let $G_{j k}(x)=P\left(\eta_{k}-\eta_{j} \leq x\right)$. From the assumption it follows that

$$
\begin{equation*}
G_{j k}\left(v_{j}-v_{k}\right)=L\left(\tilde{v}_{j}-\tilde{v}_{k}\right) \tag{B.2}
\end{equation*}
$$

for all $j$ and $k$, where $L(x)=1 /\left(1+e^{-x}\right)$. Since $\tilde{v}_{j}$ only depends on $j$ it cannot depend on $v_{k}$. Hence, $\tilde{v}_{j}$ is a function (unknown) of $v_{j}$, say, $\tilde{v}_{j}=f_{j}\left(v_{j}\right)$. By differentiation of (B.2) with respect to $v_{j}$ and $v_{k}$, respectively, we get

$$
G_{j k}^{\prime}\left(v_{j}-v_{k}\right)=L^{\prime}\left(f_{j}\left(v_{j}\right)-f_{k}\left(v_{k}\right)\right) f_{j}^{\prime}\left(v_{j}\right) \quad \text { and } \quad-G_{j k}^{\prime}\left(v_{j}-v_{k}\right)=-L^{\prime}\left(f_{j}\left(v_{j}\right)-f_{k}\left(v_{k}\right)\right) f_{k}^{\prime}\left(v_{k}\right)
$$

The equations above imply that $f_{j}^{\prime}\left(v_{j}\right)=f_{k}^{\prime}\left(v_{k}\right)$ which implies that $f_{j}^{\prime}\left(v_{j}\right)$ is a constant, say $\alpha$.

## Q. E. D.

The result of Lemma B1 is similar to the result of Theorem 2 in Strauss (1979). By using the result of Lemma B1 we obtain that

$$
P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{k}, Z_{k}, \varepsilon_{k}\right) \mid \xi_{j k}\right)=\frac{1}{1+\exp \left(-\tilde{v}_{j}+\tilde{v}_{k}\right)}=\frac{1}{1+\exp \left(-\tau\left(v_{j}-v_{k}+Z_{j}-Z_{k}\right)\right)}
$$

With no loss of generality we can normalize by letting $\tau=1$.
The next assumption is a statement of an unconditional (aggregate) version of the product rule in a homogeneous population.

## Assumption B2

Let $\left\{\varepsilon_{j}, \varepsilon_{r}^{\prime}, \varepsilon_{k}^{\prime \prime}, Z_{j}, Z_{r}^{\prime}, Z_{k}^{\prime \prime}\right\}$ be independent random variables. Then
$P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{k}, Z_{k}, \varepsilon_{k}\right)\right) P\left(V\left(v_{k}, Z_{k}^{\prime}, \varepsilon_{k}^{\prime}\right)>V\left(v_{q}, Z_{q}^{\prime}, \varepsilon_{q}^{\prime}\right)\right) P\left(V\left(v_{q}, Z_{q}^{\prime \prime}, \varepsilon_{q}^{\prime \prime}\right)>V\left(v_{j}, Z_{j}^{\prime \prime}, \varepsilon_{j}^{\prime \prime}\right)\right)$
$=P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{q}, Z_{q}, \varepsilon_{q}\right)\right) P\left(V\left(v_{q}, Z_{q}^{\prime}, \varepsilon_{q}^{\prime}\right)>V\left(v_{k}, Z_{k}^{\prime}, \varepsilon_{k}^{\prime}\right)\right) P\left(V\left(v_{k}, Z_{k}^{\prime \prime}, \varepsilon_{k}^{\prime \prime}\right)>V\left(v_{j}, Z_{j}^{\prime \prime}, \varepsilon_{j}^{\prime \prime}\right)\right)$.

Assumption B2 is analogous to Assumption B1. An empirical counterpart of Assumption B2 is as follows: Consider a series of choice experiments over time conducted on a population where no individual is presented with more than one binary choice, either the choice from $\left\{a_{j}, a_{k}\right\},\left\{a_{k}, a_{q}\right\}$ or $\left\{a_{j}, a_{q}\right\}$. Under the assumptions above and when the sample is large the respective binary choice fractions that satisfy the product rule will be approximately the same as if all the individuals were presented with all 3 binary choice experiments. Average intransitive behavior is therefore not systematic. Assumption B2 together with Lemma B1 imply that unconditional choice probabilities are given by

$$
P\left(V\left(v_{j}, Z_{j}, \varepsilon_{j}\right)>V\left(v_{k}, Z_{k}, \varepsilon_{k}\right)\right)=\frac{1}{1+\exp \left(-\alpha\left(v_{j}-v_{k}\right)\right)}
$$

where $\alpha>0$ is a constant.

## Supplementary Material

## Appendix C

## The likelihood function

Let $S$ denote the sample of individuals who are observed and let

$$
\theta v_{i}(t)=\left(\beta_{0 t}+X_{2 i}(t) \beta-X_{0 i}(t) \gamma\right) \theta q_{i}(t)-\theta c\left(1-q_{i}(t)\right)
$$

and recall that

$$
q_{i}(t)=\frac{1}{1+\exp \left(-B_{i}(t) b\right)} \text { and } P_{i}(t)=\frac{1}{1+\exp \left(-\alpha \theta v_{i}(t)\right)}
$$

where $B_{i}(t)$ is a vector of individual characteristics that is given in Table D 2 . Recall also that by Theorem 2 we have that

$$
g_{\alpha}(x)=\frac{\exp (\alpha x)-1}{\exp (x)-1}
$$

for $x \neq 0, g_{\alpha}(0)=\alpha$ and $g_{\alpha}^{\prime}(0)=-0.5 \alpha(1-\alpha)$. The function $g_{\alpha}(x)$ is well defined on the whole real line and is non-increasing and continuously differentiable. Moreover, $0<g_{\alpha}(x) \leq 1$, and by Theorem 3 we have that

$$
\begin{aligned}
& \quad R_{i 01}(t-1, t)=:\left(1-P_{i}(t-1)\right) Q_{i 01}(t-1, t) \\
& =g\left(\theta v_{i}(t-1)-\theta v_{i}(t)\right) \cdot \frac{1}{\left[1+\exp \left(\alpha \theta v_{i}(t-1)\right)\right]\left[1+\exp \left(-\alpha \theta v_{i}(t)\right)\right]}, \\
& R_{i 10}(t-1, t)=: P_{i}(t-1) Q_{i 10}(t-1, t)=g\left(\theta v_{i}(t)-\theta v_{i}(t-1)\right) \cdot \frac{1}{\left[1+\exp \left(-\alpha \theta v_{i}(t-1)\right)\right]\left[1+\exp \left(\alpha \theta v_{i}(t)\right)\right]}, \\
& \\
& R_{i 11}(t-1, t)=: P_{i}(t-1) Q_{i 11}(t-1, t)=\frac{1}{1+\exp \left(-\alpha \theta v_{i}(t)\right)} \\
& -g\left(\theta v_{i}(t-1)-\theta v_{i}(t)\right) \cdot \frac{1}{\left[1+\exp \left(\alpha \theta v_{i}(t-1)\right)\right]\left[1+\exp \left(-\alpha \theta v_{i}(t)\right)\right]}, \\
& \\
& R_{i 00}(t-1, t)=:\left(1-P_{i}(t-1)\right) Q_{i 00}(t-1, t)=\frac{1}{1+\exp \left(\alpha \theta v_{i}(t-1)\right)} \\
& \\
& -g\left(\theta v_{i}(t-1)-\theta v_{i}(t)\right) \cdot \frac{1}{\left[1+\exp \left(\alpha \theta v_{i}(t-1)\right)\right]\left[1+\exp \left(-\alpha \theta v_{i}(t)\right)\right]} .
\end{aligned}
$$

For $t \geq 2$, let $Y_{i j k}(t-1, t)=1$ if individual $i$ occupies state $j$ in year $t-1$ and state $k$ in year $t$, and zero otherwise, $j, k=0,1$, and let $t(i)$ be the last year individual $i$ is observed. For $t=1$, $Y_{i j k}(0,1)=: 0$. Moreover, let $R_{i j k}(t-1, t)$ be the joint probability that individual $i$ is in state $j$ in year $t-1$ and in state $k$ in year $t$ where $j, k=0,1$. Then the $\log$-likelihood $\log L_{1}$, associated with the panel observations of being in state 0 or state 1 at two consecutive years may be expressed as follows

$$
\begin{equation*}
\log L_{1}=\sum_{i \in S} \sum_{k=0,1} \sum_{j=0,1} Y_{i j k}(t(i)-1, t(i)) \log R_{i j k}(t(i)-1, t(i)) . \tag{C.1}
\end{equation*}
$$

Consider next the pseudo-likelihood of working versus being unemployed. Let $A_{i 1}(t)=1$ if individual $i$ is in the labor force in year $t$ and zero otherwise, and $A_{i 2}(t)=1$ if individual $i$ works in year $t$ and zero otherwise. Furthermore, $A_{i 1}(0)=: A_{i 2}(0)=: 0$. Note that $E\left(A_{i 2}(t) \mid A_{i 1}(t)=1\right)=q_{i}(t)$. However, it may be the case that $E\left(A_{i 2}(t) A_{i 2}(t-1) \mid A_{i 1}(t)=A_{i 1}(t-1)=1\right) \neq q_{i}(t) q_{i}(t-1)$. A log-pseudo-likelihood function associated with the individuals who are working or being unemployed is given by

$$
\begin{align*}
& \log L_{2}=\sum_{i \in S_{3}} A_{i 1}(t(i))\left\{A_{i 2}(t(i)) \log q_{i}(t(i))+\left(1-A_{i 2}(t(i))\right) \log \left(1-q_{i}(t(i))\right)\right\}  \tag{C.2}\\
& +\sum_{i \in S_{3}} A_{i 1}(t(i)-1)\left\{A_{i 2}(t(i)-1) \log q_{i}(t(i)-1)+\left(1-A_{i 2}(t(i)-1)\right) \log \left(1-q_{i}(t(i)-1)\right)\right\} .
\end{align*}
$$

According to Gourieroux and Montfort (1995, Ch. 8.4.2) pseudo maximum likelihood estimation based on (C.2) yields consistent estimation because $E\left(A_{i 2}(t) \mid A_{i 1}(t)=1\right)=q_{i}(t)$.

Consider next the pseudo likelihood of the wage observations. Let $W_{i}(t)$ denote the wage rate of individual $i$ in year $t, r=\alpha \kappa \theta^{-1}$ and

$$
\Lambda_{i}(t)=\log P_{i}(t)+\left(P_{i}(t)^{-1}-1\right) \log \left(1-P_{i}(t)\right) .
$$

The wage rate is only observed for those who work. Recall that the corresponding wage equation that accommodates potential selection bias is given by

$$
\log W_{i}(t)=\beta_{0 t}+X_{2 i}(t) \beta-r \Lambda_{i}(t)+\delta_{i}^{*}(t)
$$

where

$$
\begin{equation*}
\delta_{i}^{*}(t)=\tau \delta_{i}^{*}(t-1)+\zeta_{i}(t) \tag{C.3}
\end{equation*}
$$

$|\tau|<1$ and $\left\{\zeta_{i}(t)\right\}$ is white noise, $\sigma^{2}=\operatorname{Var} \delta_{i}^{*}(t), \operatorname{Var} \zeta_{i}(t)=\left(1-\tau^{2}\right) \sigma^{2}$. By using (C.3) we obtain that

$$
\begin{align*}
\log W_{i}(t) & =\beta_{0 t}-\tau \beta_{0, t-1}+\tau \log W_{i}(t-1)+X_{2 i}(t) \beta-X_{2 i}(t-1) \beta \tau  \tag{C.4}\\
& -r \Lambda_{i}(t)+\tau r \Lambda_{i}(t-1)+\zeta_{i}(t)
\end{align*}
$$

For $t \geq 2$, let $G_{i 1}(t)=1$ if individual $i$ works in year $t$ but does not work in year $t-1$ and zero otherwise. For $t=1, G_{i 1}(1)=1$ if individual $i$ works in year 1 and zero otherwise. For $t \geq 2$, let $G_{i 2}(t)=1$ if individual $i$ works in year $t-1$ and year $t$ and 0 otherwise. For $t=1, G_{i 2}(1)=0$. Let

$$
\begin{gathered}
\log L_{3}=-\sum_{i \in S} G_{i 1}(t(i))\left(\frac{\left(\log W_{i}(t(i))-\beta_{0 t(i)}-X_{2 i}(t(i)) \beta+r \Lambda_{i}(t(i))\right)^{2}}{2 \sigma^{2}}+\log \sigma\right) \\
-\sum_{i \in S} G_{i 2}(t(i))\left\{\left(\frac{\log W_{i}(t(i))-\beta_{0 t(i)}+\tau \beta_{0, t(i)-1}-\tau \log W_{i}(t(i)-1)-X_{2 i}(t(i)) \beta+X_{2 i}(t(i)-1) \beta \tau}{2\left(1-\tau^{2}\right) \sigma^{2}}\right.\right. \\
\left.\left.+\frac{\left[r \Lambda_{i}(t(i))-\tau r \Lambda_{i}(t(i)-1)\right]}{2\left(1-\tau^{2}\right) \sigma^{2}}\right)^{2}+\log \sigma+0.5 \log \left(1-\tau^{2}\right)\right\} .
\end{gathered}
$$

The expression in (C.5) is a log-pseudo-likelihood function because it is based on the assumption that $\left\{\zeta_{i}(t)\right\}$ are normally distributed whereas the true distribution may be different.

Hence, from (C.1), (C.2) and (C.5) we obtain that the total log-pseudo-likelihood function is given by

$$
\log L=\log L_{1}+\log L_{2}+\log L_{3}
$$

## Appendix D

## Estimation of the wage equations and the job offer probabilities

## The wage equations with measurement errors

For simplicity we suppress the time and individual indexation here. The data on wage rates contain measurement errors. Let $H$ denote the true amount of hours of work and let $\tilde{H}$ be the corresponding observed hours of work where $\tilde{H}=H \xi$ so that $\xi$ represents the measurement error. Let $m$ represent gross wage income. In our sample the wage rate $\tilde{W}$ is computed as $\tilde{W}=m / \tilde{H}=m / H \xi=W / \xi$ where $W$ is the true wage rate. Hence, it follows that $\log W=\log \tilde{W}+\log \xi$ so that the wage equation in (C.4) extended to accommodate measurement errors (with time and individual indexation) becomes

$$
\begin{gathered}
\tilde{W}_{i}(t)=\beta_{0 t}-\tau \beta_{0, t-1}+\tau \log W_{i}(t-1)+X_{2 i}(t) \beta-X_{2 i}(t-1) \beta \tau \\
-r \Lambda_{i}(t)+\tau r \Lambda_{i}(t-1)+\zeta_{i}(t)+\log \xi_{i}(t) .
\end{gathered}
$$

Table D1 displays the estimation results for the wage equations. We note that most of the variables enter the real wage equations in a significant manner. The estimated return on education is somewhat larger for women born in Norway than for women born in non-Western countries. In both equations experience has a positive effect on the real wage, but it is not easy to compare the results for the two groups since for immigrant women experience and duration of residence to some extent pick up similar features. The dummy for urbanity enters, as expected, with a positive and significant value. For both groups the estimated time effects are all positive and increase over time, accounting for business cycle variations and general growth of real wages over time which is not due to changes in the other explanatory variables. The estimate of the selection effect, $r$, is quite imprecise for both groups. We have allowed the disturbances to be autocorrelated according to an $\operatorname{AR}(1)$ process for both groups. The estimate of the autocorrelation parameter, $\tau$, is 0.57 and 0.54 for immigrant women and women born in Norway, respectively.

Table D1. Estimates of the wage equations ${ }^{\text {a }}$

|  | Women born in non-Western |  | Women born in Norway |  |
| :--- | :--- | :--- | :--- | :--- |
| Variables | countries |  |  |  |
|  | Estimate | $t$-value | Estimate | $t$-value |
| Constant | 3.999 | 38.765 | 4.128 | 388.947 |
| Education | 0.037 | 9.672 | 0.042 | 85.964 |
| Experience | 0.013 | 2.452 | 0.018 | 33.464 |
| Experience squared/100 | -0.016 | -1.335 | -0.028 | -26.068 |
| Dummy for urbanity |  |  | 0.032 | 14.867 |
| Duration of residence/10 | 0.125 | 2.743 |  |  |
| Duration of residence sq./100 | -0.017 | -1.306 |  |  |
| D91T93b | 0.072 | 1.602 | 0.056 | 18.464 |
| D94T96 | 0.106 | 2.232 | 0.081 | 24.439 |
| D97T99 | 0.153 | 3.104 | 0.112 | 31.107 |
| D00T02 | 0.197 | 3.956 | 0.159 | 42.075 |
| D03T05 | 0.197 | 4.193 | 0.210 | 50.575 |
| D06T08 | 0.310 | 6.681 | 0.279 | 64.297 |
| D09T10 | 0.351 | 6.866 | 0.351 | 60.515 |
| Selection effect, $r=\alpha \kappa \theta^{-1}$ | 0.052 | 1.376 | -0.008 | -2.108 |
| Autocorrelation parameter, $\tau$ | 0.572 | 24.623 | 0.542 | 151.078 |
| Std. deviation, $\sigma$ | 57.732 | 0.295 | 403.879 |  |
| No. of observations |  |  | 104,202 |  |

${ }^{\text {a }} \log W$ is the left-hand side variable. ${ }^{\text {b }}$ The variable D9193 is a dummy for the years 1991-1993, with a similar notation for the other time dummies. The dummy D8890 is excluded since a constant term is present.

## The probability of obtaining a job conditional on search

Table D 2 shows the results from the estimation of the $q$-relations (probability of facing a job offer, given participation). The model specifications are somewhat different for the two groups with respect to which explanatory variables are included. For both groups, educational attainment and non-labor income (log-transformed) are used as regressors. All the four estimates have a positive sign. The estimate of the parameter attached to the education variable is larger for women born in Norway than for women born in non-Western countries. For the estimated parameters attached to non-labor income it is the other way around. Whereas the job probability of women born in Norway is assumed to depend on a second order polynomial in experience, the job probability of women born in non-Western countries depends on a second order polynomial in duration of residence. As seen from Table D2, both the estimated functions have the inverted u-shaped form. The job probability increases with experience and duration of residence. For women born in non-Western countries the number of children in the two youngest age-groups enters as explanatory variables and with a negative sign. The largest
estimated effect, in absolute value, is related to the variable for the middle group, i.e., children aged 3-6 years. For women born in Norway only the number of children in the middle age group plays a role. At the bottom of Table D2 the estimates of the coefficients representing the time effects are displayed. We note that these estimates reflect the business cycle fluctuations.

Table D2. Estimates of the probability of being employed, $q$, conditional on being in the labor force

| Variables | Women born in non-Western <br> countries |  | Women born in Norway |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Estimate | $t$-value | Estimate | $t$-value |
| Education | 0.064 | 4.049 | 0.258 | 24.786 |
| Work experience |  |  | 0.073 | 7.781 |
| Work experience squared/100 | 0.137 | 5.423 | -0.049 | -2.626 |
| Duration of residence | -0.176 | -2.038 |  |  |
| Duration of residence squared/100 | 4.070 | 0.214 | 7.142 |  |
| Log (real non-labor income) | 0.303 | -2.507 |  |  |
| No. of children aged 0-3 | -0.252 | -3.265 | -0.151 | -3.842 |
| No. of children aged 4-6 | -0.348 | -2.792 | -3.117 | -7.938 |
| D88T90 | -2.600 | -3.616 | -3.241 | -8.154 |
| D91T93 | -3.430 | -3.738 | -3.205 | -7.987 |
| D94T96 | -3.525 | -3.467 | -2.983 | -7.500 |
| D97T99 | -3.306 | -3.572 | -3.040 | -7.422 |
| D00T02 | -3.460 | -3.264 | -3.329 | -7.901 |
| D03T05 | -3.122 | -3.155 | -3.009 | -7.214 |
| D06T08 | -3.071 | -2.505 | -2.885 | -6.325 |
| D09T10 | -2.453 |  | 104,202 |  |
| No. of observations | 3,448 |  |  |  |

[^11]
## Appendix E

## Detailed information about the data

The Norwegian LFS follow the international recommendations for labor force surveys when it comes to classification of persons as being employed, unemployed or outside the labor force, etc. Working time is measured as contractual hours of work on an annual basis in both the main and any possible second jobs. If this information is missing and the respondent is active in the labor market, information about actual working time is used. Nominal hourly wages are measured as annual labor income divided by annual working time. To ensure time consistency, we have chosen to use a measure of non-labor income that includes salary of the husband as well as stipulated labor income for self-employed husbands. The nominal hourly wage and non-labor income variables are deflated by the official Norwegian consumer price index, with 2010 as the reference year. Based on the information in the registries, we also calculate the number of children in each household aged 0-3 years, 4-6 years and 7-18 years separately for each year. Education is measured in years of achieved level of schooling and work experience is defined as age minus length of schooling minus 7. Duration of residence in Norway in a specific year is calculated as the number of years from the first year of residence. Urbanity is represented by a dummy variable which is equal to one if the person lives in a densely populated area, and zero otherwise. In our data a collection of houses is registered as a densely populated area if there are at least 200 people living there and the distances

The Norwegian LFS are quarterly and the samples are rotating. In the estimation of the model we make use of the fact that it is possible to observe a person in the same quarter in two consecutive years. Thus, each woman in the sample is observed twice, and by observing women in the same quarter in both years, we avoid problems related to seasonal fluctuations. Note, however, that the sample includes observations from all four quarters during a year.

Table E1. Summary statistics for women born in non-Western countries and women born in Norway (1999)

|  | Women born in non-Western countries |  |  | Women born in Norway |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Variable | Mean | Std. dev. | Min. | Max. | Mean | Std. dev. | Min. | Max. |
| Age | 37.1 | 7.3 | 25 | 60 | 42.0 | 9.5 | 25 | 60 |
| Education (years) | 12.4 | 3.3 | 6 | 20 | 12.6 | 2.8 | 9 | 20 |
| Experience (years) | 18.6 | 7.5 | 5 | 41 | 23.4 | 10.4 | 2 | 41 |
| \# children 0-3 years | 0.4 | 0.6 | 0 | 2 | 0.3 | 0.5 | 0 | 2 |
| \# children 4-6 years | 0.2 | 0.5 | 0 | 2 | 0.2 | 0.5 | 0 | 2 |
| \# children 7-18 years | 0.9 | 1.1 | 0 | 4 | 0.7 | 1.0 | 0 | 4 |
| Non-labor income $^{\mathrm{a}}$ | 343,531 | 185,439 | 58,401 | 185,439 | 385,643 | 185,243 | 58,401 | $1,325,439$ |
| Wage rate $^{\mathrm{b}}$ | 136.1 | 20.7 | 100.6 | 214.9 | 152.7 | 18.4 | 114.1 | 227.1 |
| Participation rate $^{13}$ | 0.75 | 0.44 | 0 | 1 | 0.91 | 0.29 | 0 | 1 |
| Unemployment rate $^{\mathrm{c}}$ | 0.099 | 0.299 | 0 | 1 | 0.023 | 0.149 | 0 | 1 |


Number of observations are 1,262 for immigrant woman and 46,214 for women born in Norway.
The reason we exclude women who are observed only once is that the behavior of this group of women in the labor market seems to be fundamentally different from that of other women. ${ }^{16}$

The empirical analysis is done separately for women born in Norway and for female non-Western immigrants. Non-Western immigrants include immigrants born in Eastern Europe, Africa, Asia, South and Central America and countries in Oceania except Australia and New Zealand. We have excluded immigrants born in Western Europe, Australia, New Zealand and North America because we want to focus on immigrants with a substantially different cultural background compared to those born in Norway. In total the sample consists of 52,101 women born in Norway and 1,724 immigrant women. Table E1 gives summary statistics for the women for 1999, which is in the middle of the period of analysis.

So far, we have discussed the data used in the estimations. It is, however, of interest to apply the model for prediction of participation, unemployment and discouraged workers for the whole Norwegian population. As the sample used in the estimations of the model is not representative for our target population, especially with respect to immigrant women, we have prepared another data set for prediction purposes. This data set is based on the Norwegian Income Registry 2005, representing the total Norwegian population. ${ }^{17}$ For all women (of interest) it contains information about all the individual explanatory variables of the model: that is, non-

[^12]labor income, length of schooling, (potential) work experience, duration of residence, urbanity dummy, age and the number of children in the specific age groups.

The selection rules used for the micro population are as follows: we have removed women with public and private pensions in excess of 125,000 NOK (in nominal terms). This income limit is consistent with the maximum pension income in the data used in the estimation of the participation model, and by imposing this restriction we omit women who are unable to participate in the labor market. To capture the fact that we are modeling the decision to participate in paid work and not self-employment, women with more income from self-employment than wage income are excluded. In addition, about 20 per cent of the immigrant women are excluded due to missing information about their educational attainment. As in the estimations, we have carried out the simulations separately for married/cohabitating women of age $25-60$ born in non-Western countries (41,339 obs.) and Norway (555,209 obs.).


[^0]:    ${ }^{1}$ For example, the US Bureau of Labor Statistics defines discouraged workers as "persons not in the labor force who want and are available for a job and who have looked for work sometime in the past 12 months, but who are not currently looking because they believe there are no jobs available or there are none for which they would qualify."

[^1]:    ${ }^{2}$ Occasionally, we use the term "immigrant women" as a synonym for "women born in non-Western countries".
    ${ }^{3}$ The Norwegian Labor Force Surveys also collect direct information on the discouraged worker effect, but the figures from the surveys are not presented in the official statistics. One reason might be that the number of individuals providing this information is rather small as only individuals participating in the survey for the first or eighth time are asked the question.

[^2]:    ${ }^{4}$ Some recent studies that have analyzed the discouraged worker issue using macro time series data are Benati (2001), Darby et al. (2001), Vendrik and Cörvers (2009), Österholm (2010), Emerson et al. (2011), Fuchs and Weber (2017) and Provenzano (2017). All these studies find significant discouraged worker effects, at least for subgroups in their samples.

[^3]:    ${ }^{5}$ To make the concept of discouraged worker precise one needs to define under which conditions (wage rate and non-labor income) a woman would like to obtain employment.
    ${ }^{6}$ Another relevant theoretical analysis is Pissarides (1976).

[^4]:    ${ }^{7}$ Note that when $\rho \rightarrow 0, \psi(q, \mu)$ reduces to $q$. In this case $\bar{U}_{2}$ can be interpreted as the lump sum value over the infinite horizon (Flinn and Heckman, 1982) and the right side of (1) reduces to $E \max \left(U_{2}, U_{1}\right)$ which is the expected value of being in the labor force in the one shot case.
    ${ }^{8}$ The importance of theoretically justified stochastic choice models for risky outcomes has been emphasized by Harless and Camerer (1994) and Hey and Orme (1994). Hey and Orme (1994) write: "... we are tempted to conclude by saying that our study indicates that behavior can reasonably well be modeled (to what may be termed as "a reasonable approximation) as expected utility plus noise" (pp. 1321-1322).

[^5]:    ${ }^{9}$ The notation $S_{\alpha}(\sigma, \beta, \mu)$ means a stable distribution with index $0<\alpha \leq 2$ and scale parameter $\sigma>0$, skewness parameter $\beta$ and location parameter $\mu$. See Samorodnitsky and Taqqu (1994) for more information about Stable distributions.

[^6]:    ${ }^{10}$ The tails of the logistic distribution are fatter than the tails of the normal distribution. Since the participation probabilities for some groups of women are close to 1 the tail properties matter.
    ${ }^{11}$ The variance of the standard Gumbel distribution is equal to $\pi^{2} / 6$.

[^7]:    ${ }^{12}$ The hours of work variable is plagued with measurement errors. Since the wage rate is obtained by dividing labor income by hours of work the measurement errors in hours of work are thus transmitted to the data on wage rates (see Appendix C in the online supplementary section for a discussion of this issue).

[^8]:    ${ }^{13}$ Unfortunately, we are not able to identify the (unconditional) wage equation that corresponds to the unconditional wage distribution.

[^9]:    ${ }^{14}$ The estimation procedure can be interpreted as a pseudo maximum likelihood procedure, based on the assumption that the probabilities $P_{i}(t)$ and $q_{i}(t)$ are correctly specified, the true mean of $\delta_{i}^{*}(t)$ is zero and the variance of $\delta_{i}^{*}(t)$ does not depend on explanatory variables. The reason why pseudo maximum likelihood estimation works in this case is that the model belongs to the quadratic exponential family with first- and second-order moments of the dependent variables equal to the corresponding true moments. See Gourieroux and Monfort (1995, Ch. 8.4.2) for a discussion of the conditions of pseudo maximum likelihood estimation.

[^10]:    ${ }^{15}$ The reason age was omitted in the specification for immigrants was that due to the relatively small sample size of immigrant women we were not able to estimate significant age effects for this group.

[^11]:    ${ }^{\text {a }}$ The variable D9193 is a dummy for the years 1991-1993, with a similar notation for the other time dummies.

[^12]:    ${ }^{16}$ One of the most frequent reasons for non-response in the LFS is the difficulty of getting in contact with the interview objects.
    ${ }^{17}$ Data for the Income Registry cannot be used for estimation of the model as it does not include sufficient information about labor market participation and unemployment.

