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Selection on profitability and not productivity

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Abstract:
Foster et al. (2001) outline a framework that is commonly used to identify the contribution from firm turnover to aggregate productivity growth. The framework is not derived from economic theory and it implies that productivity levels determine the contribution from reallocation and firm turnover. In this paper, I outline an index for aggregate productivity growth based on economic theory. In contrast to common beliefs, I show that the contribution from firm turnover to aggregate productivity growth should be based on the profitability, and not the productivity, of these firms.

Keywords: Productivity, Profitability, Aggregation.

JEL classification: D24, J24, L25, O47, C43.

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Sammendrag

Måling av bidragene fra bedriftsetablering og -avvikling til aggregert produktivitet

En viktig drivkraft for den totale produktivitetsutviklingen kommer fra at etablerte og mindre lønnsomme foretak legges ned og at nye foretak etableres. Rammeverket som ofte brukes i litteraturen for å analysere bidragene fra bedriftsetablering og –avvikling har så langt ikke vært basert på økonomisk teori. Metoden til for eksempel Foster et al. (2001) baserer seg på bedriftenes produktivitetsnivå. I denne artikkelen utleder jeg en indeks for den totale produktivitetsutviklingen basert på økonomisk teori. I motsetning rammeverket som brukes i litteraturen viser jeg at bidraget fra bedriftsetablering og –avvikling skal baseres på bedriftenes lønnsomhet og ikke deres produktivitetsnivå.

1 Introduction

Foster et al. (2001) outline a framework based on a weighted average of productivity levels to identify the contribution of firm turnover, i.e., entering and exiting firms, to aggregate productivity growth. It has been used by Griliches and Regev (1995); Neil et al. (1992); Foster et al. (2006) and Foster et al. (2008), to name a few.[1]

A drawback with a method based on a weighted average of productivity levels is that it lacks theoretical rationale which again may lead to wrongful inference. For example, since the contribution from entering firms in the decomposition in Foster et al. (2001) is based on productivity levels, there is an ongoing debate about the importance of using nominal variables to measure the contribution of entering and exiting firms to aggregate productivity growth. Katayama et al. (2003) point out that productivity indices based on nominal variables may have little to do with actual productivity levels. Foster et al. (2008) find that young producers charge lower prices than incumbents and that the literature therefore understates new producer's productivity advantages and entry's contribution to aggregate productivity growth. This result hinges on the decomposition in Foster et al. (2001) being theoretically sound and, consequently, that the productivity levels of incumbent firms determine their contribution to aggregate productivity growth.

In this paper, I outline a measure based on economic theory where the contribution from entering and exiting firms is determined from nominal variables only. In contrast to common beliefs, the contribution of entering and exiting firms to aggregate productivity growth is based on the profitability, and not the productivity, of these firms. The framework in Foster et al. (2001) can therefore be biased if, say, productivity is inversely correlated with price. Also, since the value of output revenue and the value of input costs often are available, the contribution of entering and exiting firms to aggregate productivity growth can easily be identified in the proposed decomposition.

The decomposition proposed in this paper is based on the price index outlined in Feenstra (1994). He demonstrated how to incorporate new product varieties into a constant-elasticity-of-substitution (CES) aggregate of import prices. Several papers applies the Feenstra price index. For example, Broda and Weinstein (2006) use it to analyse the value to U.S. consumers from expanded import varieties. Harrigan and Barrows (2009) analyse how the end of the multifiber arrangement impacted prices and quality. Feenstra et al. (2013) consider how increased varieties affected the measurement of U.S. productivity growth. In this paper, I use the results from Feenstra (1994) to construct output and input quantity

[1]Recently, Diewert and Fox (2010) analysed a decomposition of aggregate productivity growth based on multilateral index number theory.

indices which further are used to decompose the contribution of entering and exiting firms to aggregate productivity growth.

The paper proceeds as follows. Section 2 outlines the decomposition in Foster et al. (2001). Section 3 outlines the decomposition based on economic theory. Section 4 concludes.

2 Decomposition in Foster et al. (2001)

The framework in Foster et al. (2001) is based on a weighted arithmetic average of productivity levels across firms. Let q_{it}^{Y} and q_{it}^{L} denote the *volume* of outputs and inputs in firm i at period t, respectively. To economise on notation, I assume that each firm produces a single output with a single input. The *level* of productivity Π_{it} in firm i at time t is thus defined as the ratio of outputs to inputs in real terms $\Pi_{it} = q_{it}^{Y}/q_{it}^{L}$. The weighted arithmetic average productivity level $(\overline{\Pi}_{t})$ across all firms present at time t can then be written

$$\overline{\Pi}_{t} = \sum_{i \in I_t} s_{it}^{Y} \Pi_{it}, \tag{1}$$

where the weights s_{it}^{Y} are output shares and I_t denotes the set of all firms present at time t. If we let V_{it}^{Y} denote the value of outputs produced by firm i, the output share can more explicitly be defined by $s_{it}^{Y} = V_{it}^{Y} / \sum_{i \in I_t} V_{it}^{Y}$. It then follows that the change in average productivity can be decomposed into contributions from continuing firms, entering firms and exiting firms by

$$\begin{aligned}\Delta\overline{\Pi}_{t} = \sum_{i \in C} s_{it-1}^{Y} \Delta\Pi_{it} + \sum_{i \in C} \left(\Pi_{it-1} - \overline{\Pi}_{t-1}\right) \Delta s_{it}^{Y} + \sum_{i \in C} \Delta s_{it}^{Y} \Delta\Pi_{it} \\ + \sum_{i \in N} s_{it}^{Y} \left(\Pi_{it} - \overline{\Pi}_{t-1}\right) - \sum_{i \in X} s_{it-1}^{Y} \left(\Pi_{it-1} - \overline{\Pi}_{t-1}\right),\end{aligned} \tag{2}$$

where the sets C, N and X holds continuing, entering and exiting firms, respectively. It is assumed that the set of continuing firms is non-empty. The first term represent a within component showing the weighted average of productivity *growth* across continuing firms. The second term represents a between component across continuing firms. The third term is a covariance terms. The last two terms represent the contribution from entering and exiting plants, respectively. All of the five terms require data on both real and nominal variables.

As pointed out by Foster et al. (2001), the second term and the two last terms involve deviations from the initial average productivity level. An increase in the output share for a continuing firm will thus only contribute positively if the initial productivity level was higher than average productivity. Also, a new firm will only contribute positively if it has higher than average productivity and an exiting firm will only contribute positively if it has lower than average productivity.

There are several other possible ways in which aggregate productivity growth could be decomposed. It is thus an open question what criteria should be applied to choose among different decompositions. Both axiomatic and economic criteria can be used. For these reasons, a decomposition based on economic theory is called for.

3 A decomposition based on economic theory

In the following I outline a decomposition that identifies the contribution of entering and exiting firms to aggregate productivity growth using firms' level of profitability. The index proposed thus depends on nominal variables. I begin by introducing some notation and definitions of profitability and aggregate productivity before I state the decomposition of aggregate productivity growth.

By profitability I refer to how the level of profits varies between firms. Let Y denote outputs, L denote inputs, and let the profit (π_{it}) of firm i at time t be defined as the difference between value of output (V_{it}^Y) and input costs (V_{it}^L), both in nominal terms, i.e., $\pi_{it} = V_{it}^Y - V_{it}^L$. Firm i is said to be more *profitable* than firm k if it generates larger profits, i.e., if $\pi_{it} > \pi_{kt}$. This definition is in line with Balk (1998, 2003) and Diewert (2014) who defined profitability as the value of outputs divided by the value of inputs.

The profitability between firms can also be expressed in terms of output and input shares. Let s_{it}^j denote the nominal value share of firm i, $s_{it}^j = V_{it}^j / \sum_{i \in I_t} V_{it}^j$, for both outputs and inputs ($j = Y, L$). It then follows that the profit of firm i is greater than the profit of firm k if the difference between output and input shares is larger for firm i: $s_{it}^Y - s_{it}^L > s_{kt}^Y - s_{kt}^L$.[2] Correspondingly, the profitability of entering and exiting firms can be compared with the profitability of continuing firms. Let s_{Nt}^Y and s_{Nt}^L denote the nominal output and input shares of entering firms at time t, and let s_{Ct}^Y and s_{Ct}^L denote the nominal output and input shares of continuing firms at time t, i.e., $s_{Nt}^j = \sum_{i \in N} V_{it}^j / \sum_{i \in I_t} V_{it}^L$ and $s_{Ct}^j = \sum_{i \in C} V_{it}^j / \sum_{i \in I_t} V_{it}^L$ for $j = Y, L$. Since the input and output shares of continuing and entering firms at time t sum to unity, $s_{Ct}^j + s_{Nt}^j = 1$, the profitability of entering firms is said to be higher than the profitability of continuing firms if $s_{Nt}^Y - s_{Nt}^L > 0$. Correspondingly, the profitability

[2] $\pi_i > \pi_k \Leftrightarrow V_{it}^Y - V_{it}^L > V_{kt}^Y - V_{kt}^L \Leftrightarrow V_{it}^Y / V_{it}^L > V_{kt}^Y / V_{kt}^L \Leftrightarrow s_{it}^Y / s_{it}^L > s_{kt}^Y / s_{kt}^L \Leftrightarrow s_{it}^Y - s_{it}^L > s_{kt}^Y - s_{kt}^L$.

of exiting firms is said to be higher than the profitability of continuing firms if $s^Y_{Xt-1} - s^L_{Xt-1} > 0$, where s^Y_{Xt-1} and s^L_{Xt-1} denote the nominal output and input shares of exiting firms evaluated at $t-1$: $s^j_{Xt-1} = \sum_{i \in X} V^j_{it-1} / \sum_{i \in I_{t-1}} V^L_{it-1}$ for $j = Y, L$.

Aggregate productivity is defined as the ratio of an aggregate output index relative to an aggregate input index, i.e., Q^Y/Q^L, where Q^Y and Q^L are the output and input quantity indices, respectively. This definition is standard in the index number literature and is applied in e.g. Diewert and Nakamura (2003) and OECD (2001).

The decomposition proposed is based on economic theory. In particular, it is assumed that both outputs and inputs are aggregated in a CES framework (U_t)

$$U_t = \left(\sum_{i \in I_t} a^j_i (q^j_{it})^{\frac{\sigma^j - 1}{\sigma^j}} \right)^{\frac{\sigma^j}{\sigma^j - 1}} \quad \text{for } j = Y, L, \tag{3}$$

where σ^j denotes the elasticity of substitution which is assumed to exceed unity, I_t is the set of either outputs or inputs varieties in period t and a^j_i is a quality parameter for variety i. The set of input and output varieties can vary between time periods. It is only in the special case where all varieties are identical ($\sigma^j \to \infty$ and $a^j_i = 1$) that aggregation can be undertaken using a summation of quantities. If buyers minimise costs and if all varieties are equal one would expect unit prices to be equal. However, if unit prices differ across products, and if price variation reflects heterogenous varieties, an index based on summation of quantities will be biased, see e.g., Diewert and Lippe (2010). In contrast to a simple summation of quantities, the above framework allows for different qualities of both outputs and inputs and, thus, also price variation across both outputs and inputs.[3]

It is assumed that buyers of both inputs and outputs minimise costs. Optimal expenditure shares are then given by:[4]

$$s^j_{it} = \frac{(a^j)^{\sigma^j} p^{1-\sigma^j}_{it}}{\sum_{i \in I_t} b^j_i p^{1-\sigma^j}_{it}} \quad \text{for } j = Y, L, \text{ and } i \in I_t, \tag{4}$$

where p^j_{it} is the unit price of input or output variety i at time t (all prices are assumed positive).

[3]Price variation does not always reflect corresponding differences in qualities of the goods or services sold. Price variation can also be caused by lack of information, price discrimination or the existence of parallel markets. It is pointed out in the System of National Accounts 2008 that: "If there is doubt as to whether the price differences constitute price discrimination, it seems preferable to assume that they reflect quality differences, as they have always been assumed to do so in the past" (European Commision et al., 2009, 15.75). The framework in this article attributes price variation to quality differences and not price differences.

[4]The minimum cost of obtaining one unit of services from either outputs (Y) or inputs (L) are then given by $\left(\sum_{i \in I_t} (a^j)^{\sigma^j} (p^j_{it})^{1-\sigma^j} \right)^{\frac{1}{1-\sigma^j}}$ for $j = Y, L$.

Let Q^Y and Q^L denote the overall output and input volume *indices* from the CES aggregate in equation (3), so that $Q^j = U_t^j / U_{t-1}^j$ for $j = Y, L$, and let Q_C^Y and Q_C^L be the corresponding quantity indices across continuing firms.[5] Also, let s_{Cit}^j denote the nominal value share of a continuing firm i, evaluated relative to the nominal value across all continuing firms: $s_{Cit}^j = V_{it}^j / \sum_{i \in C} V_{it}^j$. The contributions from within firm productivity growth, reallocation of inputs between existing firms and the contributions from entering and exiting firms to aggregate productivity growth are then shown by the following result:

Proposition 1 (Aggregate productivity growth)**.** *Let the output and input quantity indices Q^j be based on the CES aggregation in equation (3) when $\sigma^j > 1$ and the optimal shares in equation (4). Assume that the quantity index across continuing firms are approximately equal to a geometric Laspeyres quantity index, i.e., $Q_C^j \approx \sum_{i \in C} s_{Cit-1}^j \Delta \ln q_{it}^j$ for $j = Y, L$. Aggregate productivity growth can then approximately be decomposed by*

$$\begin{aligned} \ln\left(Q^Y/Q^L\right) \approx & \sum_{i \in C} s_{Cit-1}^Y \Delta \ln(q_{it}^Y / q_{it}^L) + \sum_{i \in C} \left(s_{Cit-1}^Y - s_{Cit-1}^L\right) \Delta \ln q_{it}^L \\ & + \left(\frac{\sigma^Y}{1-\sigma^Y}\right) \ln(1 - s_{Nt}^Y) - \left(\frac{\sigma^L}{1-\sigma^L}\right) \ln(1 - s_{Nt}^L) \\ & - \left(\left(\frac{\sigma^Y}{1-\sigma^Y}\right) \ln(1 - s_{Xt-1}^Y) - \left(\frac{\sigma^L}{1-\sigma^L}\right) \ln(1 - s_{Xt-1}^L)\right). \quad (5) \end{aligned}$$

Proof. See the Appendix. □

The first term is a weighted average of productivity growth measured in log points. Note that the shares in the first two terms are evaluated at $t-1$. This is due to the choice of the geometric Laspeyres index as an aggregator formula. Another weighting scheme could have been used. For example, if the Törnqvist index is applied, the weights should be the average of the shares between the two consecutive time periods. The second term is often referred to as the reallocation term. It shows the effect on aggregate productivity from inputs moving between continuing firms. Note that the impact from reallocation depends on the value shares s_{Cit-1}^Y and s_{Cit-1}^L. As defined above, the difference between the output and the input shares $\left(s_{Cit-1}^Y - s_{Cit-1}^L\right)$ represents a measure of *profitability*. The reallocation term is positive if inputs are reallocated towards more profitable firms and it is negative if inputs are moved towards less profitable firms. The first two terms, which are summed across continuing firms, have been used in the literature to decompose aggregate productivity growth into between and within effects

[5] Specifically, $Q_C^j = U_{Ct}/U_{Ct-1}$, where $U_{Ct} = \left(\sum_{i \in C} a_i^j (q_{it}^j)^{\frac{\sigma^j - 1}{\sigma^j}}\right)^{\frac{\sigma^j}{\sigma^j - 1}}$.

across continuing firms (or industries) and it is outlined in e.g., OECD (2001, p. 145). The framework above is thus a generalisation that also takes into account the effect from entering and exiting firms, represented by the last two terms. They depend on the elasticity of substitution between both outputs and inputs and the value shares of entering and exiting outputs and inputs. Everything else equal, the impact from entering firms increases in the output elasticity of substitution and decreases in the input elasticity of substitution. The intuition behind this result is that a higher elasticity of substitution means that there is less to be gained from a new variety in terms of reduced costs. For a given value share, a higher elasticity of substitution of for example outputs represents an increase in the quantity index and thus an increase in productivity.

It is of particular interest to analyse the decomposition when these elasticities are large since the literature has taken as benchmark the case when firms produce a homogenous good using a homogenous input. Also, from a practical point of view, the output elasticity can safely be assumed to be large when analysing firms at a highly disaggregated industry level. In the following corollary, aggregate productivity growth is decomposed in the case of inputs and outputs being homogenous across firms and when the entry and exit shares are relatively small:

Corollary 1 (Homogenous outputs and homogenous inputs)**.** *Consider Proposition 1 when outputs produced by firms are homogenous ($\sigma^Y \to \infty$), inputs used by firms are homogenous ($\sigma^L \to \infty$) and when the entry and exit shares $s^Y_{Nt}, s^L_{Nt}, s^Y_{Xt-1}$ and s^L_{Xt-1} are relatively small. Aggregate productivity growth can then approximately be decomposed by*

$$\ln\left(Q^Y/Q^L\right) \approx \sum_{i\in C} s^Y_{Cit-1}\Delta\ln\Pi_{it} + \sum_{i\in C}\left(s^Y_{Cit-1} - s^L_{Cit-1}\right)\Delta\ln L_{it} + \left(s^Y_{Nt} - s^L_{Nt}\right) - \left(s^Y_{Xt-1} - s^L_{Xt-1}\right). \quad (6)$$

Proof. Follows since $\ln(1+z) \approx z$ when $z \approx 0$. □

The importance of profitability for the contribution from firm turnover to aggregate productivity growth is explicitly shown in equation (6). An entering firm will contribute positively to aggregate productivity growth if its profitability is higher than the average profitability of continuing firms, i.e., if $s^Y_{Nt} > s^L_{Nt}$. Correspondingly, an exiting firm will contribute positively to aggregate productivity growth if its profitability is lower than the average profitability of continuing firms, i.e., if $s^Y_{Xt-1} > s^L_{Xt-1}$. Equation (6) has been derived under the assumption that the shares $s^Y_{Nt}, s^L_{Nt}, s^Y_{Xt-1}$ and s^L_{Xt-1} are relatively small. If they are large, the entry and exit terms should be replaced by $\ln\left(\frac{1-s^L_{Nt}}{1-s^Y_{Nt}}\right)$ and $-\ln\left(\frac{1-s^L_{Xt-1}}{1-s^Y_{Xt-1}}\right)$, respectively. Nevertheless, there is a clear correspondence between the entry and exit terms and the reallocation term since they are all related to the level of profitability and not the level of productivity. The aggregate productivity growth obtained by using equation (6) will therefore differ from the aggregate productivity growth obtained by equation (2). Since the contributions to aggregate productivity growth from reallocation and firm turnover depend on productivity levels in equation (2) and on profitability levels in equation (6), there will be a larger discrepancy between the two measures if productivity is inversely correlated with price, as has been found in the literature. Also, in contrast to the decomposition in equation (2), there is no need for variables measured in real terms to calculate the impact from entering and exiting firms on aggregate productivity growth. This is empirically important as detailed price and quantity information may not be available at the firm level. Since the value of output revenue and the value of input costs often are available, the contribution of entering and exiting firms to aggregate productivity growth can easily be identified by the proposed method of decomposition in this paper.

4 Conclusion

Foster et al. (2001) outline a framework that is commonly used to identify the contribution of entering and exiting firms to aggregate productivity growth. The framework is not derived from economic theory and it implies that productivity levels determine the contribution from reallocation and firm turnover. In this paper, I have outlined an index for aggregate productivity growth based on economic theory. In contrast to common beliefs, the contribution of entering and exiting firms to aggregate productivity growth is based on the profitability, and not the productivity, of these firms. Therefore, the standard framework used in the literature to measure aggregate productivity growth may be biased if, for example, productivity is inversely correlated with price.

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5 Appendix: Proof of Proposition 1

I start by outlining how the basic index number problem of splitting a value ratio into price and quantity components can be further decomposed into the contributions from continuous, entering and exiting varieties. The product rule states that the ratio between two time periods of the sum of values equals the product of a price and a quantity index. Explicitly, this can be written as

$$\left(\frac{\sum_{i\in I_t} V_{it}}{\sum_{i\in I_{t-1}} V_{it-1}}\right) = \mathcal{P} \times \mathcal{Q}, \tag{7}$$

where where V_{it} is the nominal expenditure on variety i at time t, $\mathcal{P}$ denotes the price index, $\mathcal{Q}$ denotes the quantity index and I_t and I_{t-1} denote the sets of varieties available at t and $t-1$, respectively. Note that the left hand side of equation (7) can be decomposed into contributions from continuous, entering and exiting varieties by

$$\underbrace{\left(\frac{\sum_{i\in I_t} V_{it}}{\sum_{i\in I_{t-1}} V_{it-1}}\right)}_{\text{TOTAL}} = \underbrace{\left(\frac{\sum_{i\in C} V_{it}}{\sum_{i\in C} V_{it-1}}\right)}_{\text{CONTINUING}} \times \underbrace{\left(1+\sum_{i\in N} V_{it}/\sum_{i\in C} V_{it}\right)}_{\text{ENTERING}} \times \underbrace{\left(1+\sum_{i\in X} V_{it-1}/\sum_{i\in C} V_{it-1}\right)^{-1}}_{\text{EXITING}} \tag{8}$$

$$= \underbrace{\left(\frac{\sum_{i\in C} V_{it}}{\sum_{i\in C} V_{it-1}}\right)}_{\text{CONTINUING}} \times (1-s_{Nt})^{-1} \times (1-s_{Xt-1}), \tag{9}$$

where C, N and X represent the sets of continuing, entering and exiting varieties, respectively. Assume that the price and quantity indices on the right hand side of equation (7) also can be decomposed into a product of contributions from continuous, entering and exiting varieties, i.e.,

$$\mathcal{P} \times \mathcal{Q} = \mathcal{P}_C \times \mathcal{P}_N \times \mathcal{P}_X \times \mathcal{Q}_C \times \mathcal{Q}_N \times \mathcal{Q}_X, \tag{10}$$

where $\mathcal{P}_C$ and $\mathcal{Q}_C$ are the price and quantity indices of continuous varieties, $\mathcal{P}_N$ and $\mathcal{Q}_N$ are the price and quantity indices of entering varieties and $\mathcal{P}_X$ and $\mathcal{Q}_X$ are the price and quantity indices of exiting varieties.

Feenstra (1994) derived the explicit expressions for the price indices $\mathcal{P}_C, \mathcal{P}_N$ and $\mathcal{P}_X$ based on equation (3) and the optimal shares in equation (4). Sato (1976) and Vartia (1976) showed how the price index develop when the same varieties are present in both time periods, i.e., they provided an explicit expression for the price index $\mathcal{P}_C$. Feenstra (1994) showed that the price indices for entering and exiting varieties can be written as

$$\mathcal{P}_N = (1-s_{Nt})^{\frac{1}{\sigma-1}}, \quad \mathcal{P}_X = (1-s_{Xt-1})^{\frac{-1}{\sigma-1}}. \tag{11}$$

Notice that when σ approaches infinity, and the varieties are perfect substitutes, both price indices in equation (11) goes towards unity, i.e., if $\sigma \to \infty$ then $\mathcal{P}_N = \mathcal{P}_X = 1$. It then follows that aggregate price index equals the price index calculated across continuing varieties, i.e., if $\sigma \to \infty$ then $\mathcal{P} = \mathcal{P}_C$.

Given the above price indices, the quantity indices $\mathcal{Q}_C, \mathcal{Q}_N$ and $\mathcal{Q}_X$ can be backed out, using the product rule in equations (7) and (10) in combination with the decomposition in equation (8), as

$$\mathcal{Q} = \mathcal{Q}_C \times \mathcal{Q}_N \times \mathcal{Q}_X = \mathcal{Q}_C \times (1-s_{Nt})^{\frac{\sigma}{1-\sigma}} \times (1-s_{Xt-1})^{\frac{-\sigma}{1-\sigma}}. \tag{12}$$

The first term after the second equality ($\mathcal{Q}_C$) is the quantity index for continuing varieties. It is calculated indirectly using the price index across continuing varieties, which in the CES case is based on the Sato-Vartia index. In Proposition 1, the geometric Laspeyres quantity index was used to approximate this term: $\ln \mathcal{Q}_C \approx \sum_{i \in C} s_{Cit-1} \ln(q_{it}/q_{it-1})$. The choice of the geometric Laspeyres quantity index is due to notational convenience. The Fisher and Törnqvist index are good alternatives. Taking logs of the above quantity index and inserting the geometric Laspeyres quantity index yields

$$\ln \mathcal{Q} \approx \sum_{i \in C} s_{Cit-1} \Delta \ln q_{it} + \left(\frac{\sigma}{1-\sigma}\right) \ln(1 - s_{Nt}) - \left(\frac{\sigma}{1-\sigma}\right) \ln(1 - s_{Xt-1}). \tag{13}$$

Productivity is defined by a ratio of outputs to inputs in real terms, i.e., $\mathcal{Q}^Y/\mathcal{Q}^L$, where $\mathcal{Q}^Y$ is the quantity index of outputs and $\mathcal{Q}^L$ is the quantity index of inputs. Taking logs of this ratio and using equation (13) both for the output and the input index yields equation (5).

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