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The importance of entry and exit of workers

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Abstract:
Many OECD countries have experienced a slowdown in measured labour productivity from 2005 and onwards. Norway is no exception in this respect. Most countries use a simple aggregate of hours worked when measuring labour productivity. One way to improve measurement of labour services is to control for worker characteristics. A theoretical rationale for doing so is given by Diewert and Lippe (2010). We generalise previous analyses by allowing for exit and entry of workers when measuring labour services using Norwegian microdata. We find that the bias from using hours worked compared to a labour index capturing various compositional effects can be substantial and systematic over time. In the case of Norway the bias explains about a quarter of the productivity slowdown after 2005.

Keywords: Labour productivity, Index numbers, Unit value indices, Drobisch index.

JEL classification: C43, E24, J24, O47.

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Sammendrag

Måler vi produktiviteten riktig? Betydningen av tilgang og avgang av arbeidskraft for det målte fallet i produktivitetsvekst i Norge etter 2005

Produktivitetsveksten ble redusert i mange OECD land etter 2005. For Norges del inntraff reduksjonen i produktivitetsveksten omtrent samtidig med en kraftig økning i arbeidsinnvandring etter EU utvidelsen i 2004. Den negative korrelasjonen mellom innvandring og produktivitet reiser spørsmålet om den høye innvandringen kan ha resultert i en feilmåling av produktivitetsutviklingen.

Arbeidsproduktivitet blir normalt målt ved forholdet mellom produksjon (bruttoprodukt) og utførte timeverk. Utførte timeverk kan imidlertid gi et feilaktig bilde av tjenestestrømmen fra arbeidskraften. Et riktigere mål for arbeidskraftstjenester vekter arbeidstakerne på bakgrunn av deres kostnad for den enkelte bedrift, og ikke timeantallet personen jobber for bedriften, slik det er med utførte timeverk. I en periode hvor bedriftene vrir bruken av innsatsfaktorer mot arbeidskraft med lavere timelønn vil utviklingen i utførte timeverk overvurdere arbeidskraftsinnsatsen og dermed undervurdere produktivitetsutviklingen. Vi finner at ¼ av fallet i produktivitetsvekst etter 2005 kan tilskrives feilmåling av arbeidskraftsinnsatsen.

I studien dekomponerer vi også bidragene til feilmålingen. Vi deler inn arbeidskraften i tre grupper: de som kun jobber i år t, de som kun jobber i år t-1 og de som jobber i både år t-1 og år t. De to første gruppene representerer henholdsvis tilgang og avgang på arbeidskraft. Vi finner at feilmålingen av produktivitet for de som jobber i både år t-1 og år t er av samme størrelsesorden som feilmålingen for tilgang og avgang på arbeidskraft totalt.

1 Introduction

Many OECD countries have experienced a measured slowdown in labor productivity from 2005 and onwards. Norway is no exception in this respect. Using national accounts data the average productivity growth was 2.7 per cent between 2002 and 2005 and it dropped markedly after 2005, reaching -1.4 per cent in 2008, see Figure 1. This productivity slowdown occurred in tandem with a massive increase in immigration following the 2004 enlargement of the European Union. As Figure 1 illustrates, the ratio of net immigration to the total number of employees almost tripled from a level of about 0.6 before 2005 to 1.7 in 2008. Several other European countries have experienced a similar surge in immigration after 2005.

The negative correlation between net immigration and productivity growth raises a particular concern with respect to how productivity is measured. Labour productivity is defined as the ratio of the index for value added to the index for labour services. It is standard practice to use hours worked as a proxy for the labour services. However, hours worked may represent a biased proxy for labour services because a worker's contribution to labour services should be weighted by his or her cost to the firm, not the share of hours worked. For example, if there were a large number of low paid immigrants entering the labour market after 2005, and if hourly wage costs reflect marginal productivity, a measure of labour services based on hours worked would overstate the contribution to labour services and consequently understate the true development in productivity.

The bias between using hours worked and a more theoretically based index for labour services, such as Fisher or Törnqvist, is referred to as the unit value bias. In a more general context, the unit value bias has been discussed extensively in the literature, see e.g., Párniczky (1974), Timmer (1996), Balk (1998) and Silver (2010). Diewert and Lippe (2010) summarise many of these findings and analyse the unit value bias more explicitly with respect to the Laspeyres, the Paasche and the Fisher price indices.

To counteract the problems of using hours worked it is common to control for worker characteristics in a two-step procedure: the first step defines groups by worker characteristics and the second step aggregates hours worked across these groups using an index with good theoretical and axiomatic properties such as Törnqvist or Fisher, see e.g., Jorgenson et al. (1987), Jorgenson et al. (2005), Cao et al. (2009) and work based on the EU KLEMS databse such as O'Mahony and Timmer (2009), Timmer et al. (2010). Based on this framework Zoghi (2010) discusses the use of predicted wages in calculating the weighting scheme, which is the current practice at the U.S. Bureau of Labor Statistics. Using data for Norway, Hægeland (1997) calculated labour services using register data and classifications of workers according to education and sex. Nilsen et al. (2011), analysed productivity across manufacturing industries in Norway also using register data and categorised employed persons into 12 subgroups. They added to this literature by using weights

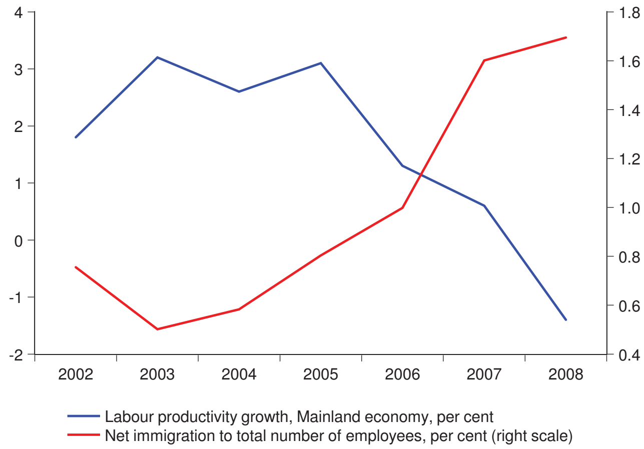

Figure 1 – Labour productivity and immigration. Labour productivity is measured as value added per hour worked in the mainland economy. Net immigration is shown relative to the total number of employees in Norway. Source: Statistics Norway.

based on predicted wages associated with individual skill attributes based on econometric wage equations.

The theoretical rationale for the two-step procedure can be found in Párniczky (1974) and Diewert and Lippe (2010). They show that the unit value bias decreases with increased disaggregation if it is compositional effects *between* the groups that contribute most to the overall bias. However, if compositional effects *within* groups are dominant, disaggregation may in fact increase the unit value bias. Note that these theoretical results follow from comparing the change in hours worked relative to indices that require underlying prices and quantities to be defined in both the base and the comparison period, such as the Fisher and Törnqvist indices. But, when applying hours worked to calculate labour services, this proxy is also calculated across those workers that were only present in *either* the comparison period or the base period. The unit value bias should consequently be defined relative to an index that allows for workers entering and exiting the labour market, a property which becomes increasingly important when net immigration surges.

In this paper, we generalise the results from Párniczky (1974) and Diewert and Lippe (2010) to allow for workers entering and exiting the labour market. To this end, we build on the theory of Feenstra (1994) who analysed the impact of new product varieties on import prices when the underlying cost function was of the constant elasticity of substitution (CES) form. This theoretical framework and some generalisations can also be found in Balk (1999). Using the case of perfect substitutes as a benchmark, we show that the contribution from entering and exiting workers on the unit value bias depends on the unit value of entering and exiting workers relative to the unit value of continuing workers. We also show theoretically that controlling for worker characteristics in the two-step procedure can exacerbate the unit value bias through entering and exiting effects.

Using Norwegian register data spanning the years 2002 to 2008, we decompose empirically the contributions from workers entering and exiting employment and those that are continuously employed. To our knowledge, this is the first study on how entry and exit effects impact aggregate wages, labour services and consequently the measure of productivity. We find that the standard practice of using hours worked overestimates labour services by approximately between 1 to 2 percentage points annually from 2002 to 2008. Correspondingly, wages and productivity are underestimated by between 1 to 2 percentage points annually. About half of the bias is attributed to a bias among continuing workers and half is attributed to the effect of workers entering and exiting employment. We also find that controlling for the level of education using the two-step procedure exacerbates the unit value bias in most years so it is within compositional effects that have been dominant in Norway between 2002 and 2008.

The backdrop of this paper is the hypothesis that mismeasurement can explain parts of the observed drop in productivity growth in Norway after 2005. On average, productivity grew 2.7 per cent annually in mainland Norway between 2002 and 2005. Between 2006 and 2008 average annual growth reduced to 0.2 per cent, down by about 2.5 percentage points. We show that the bias from using hours worked as a measure for labour services, compared to an index of labour services with desirable properties in line with index theory, increases on average with 0.7 percentage points annually after 2005. Most of this bias is due to an increasing number of entering workers with a relatively low wage rate, a development that must be seen in conjunction with the surge in immigration after 2005. Mismeasurement of productivity can thus explain about a quarter of the measured productivity slowdown after 2005.

The paper proceeds as follows: Section 2 derives theoretically the biases of using unit values and hours worked as indices for wages and labour services, respectively. Section 3 presents the data used and Section 4 outlines the empirical results. Section 5 concludes.

2 Biases of standard practice

In this section, we show theoretically the biases of using unit values as the wage index and aggregating hours worked as the quantity index. In particular, these biases will be decomposed into contributions from continuing, entering and exiting workers. In the latter part of this section we also show how the two-step procedure of splitting workers into smaller groups and then using a "proper" index to aggregate may amplify the problem caused by using hours worked as the quantity index.

We start the analysis by introducing some definitions and notation. We let labour costs refer to the nominal *value* of compensation payed to employees for their work and denote labour costs for employee i at time t by V_{it}. Correspondingly, we let W_{it} and H_{it} denote the hourly wage cost and the number of hours worked. Total labour costs at time t are then given by $V_t = \sum_{i \in I_t} V_{it}$, where the set I_t holds all workers with positive working hours at time t. The index number problem is then to decompose aggregate labour costs into respective price and quantity indices, i.e.,

$$\left(\frac{V_t}{V_{t-1}}\right) = \mathcal{W} \times \mathcal{Q} \tag{2.1}$$

where $\mathcal{W}$ and $\mathcal{Q}$ represent indices for wages and labour services, respectively.

There are some workers that were employed at both time periods. We refer to those as *continuing* workers. Workers *entering* employment were employed at time t but not at time $t-1$. Workers *exiting* employment worked at time $t-1$ but not at time t. We can thus decompose aggregate growth in labour costs into contributions from continuing, entering and exiting workers. Denote the set of continuing, entering and exiting workers by C, N and X, respectively. It follows that $I_t = C \cup N$ and that $I_{t-1} = C \cup X$. Given these definitions, the above decomposition can explicitly be written as

$$\underbrace{\left(\frac{\sum_{i \in I_t} V_{it}}{\sum_{i \in I_{t-1}} V_{it-1}}\right)}_{\text{TOTAL}}$$

$$= \underbrace{\left(\frac{\sum_{i \in C} V_{it}}{\sum_{i \in C} V_{it-1}}\right)}_{\text{CONTINUING}} \times \underbrace{\left(1 + \sum_{i \in N} V_{it} \Big/ \sum_{i \in C} V_{it}\right)}_{\text{ENTERING}} \times \underbrace{\left(1 + \sum_{i \in X} V_{it-1} \Big/ \sum_{i \in N} V_{it-1}\right)^{-1}}_{\text{EXITING}}. \tag{2.2}$$

The growth contribution from entering workers is thus based on the ratio of entering to continuing workers at time t. The higher the ratio of entering to continuing workers, the higher the overall growth in aggregate labour costs. Correspondingly, the contribution from exiting workers depends on the ratio of exiting workers to continuing workers at time $t-1$. The higher the ratio of exiting to continuing workers, ceteris paribus, the lower the overall growth in aggregate labour costs.

The index number problem of decomposing aggregate labour costs into respective indices for wages and labour services can be further broken down into separate contributions from continuing, entering and exiting workers. There are thus separate index number problems for the sets of continuing, entering and exiting workers. In this paper, we will mainly focus on decomposing the change in labour services. Of course, from the product rule in Equation 2.1, any bias in the measure of labour services across either continuing, entering or exiting workers of say k per cent, is tantamount to a $(1/k)$ per cent bias in the measurement of wages. In the following we first recapitulate the standard practice of using hours worked as a measure of labour services and then compare it with our definition a "true" index with appropriate theoretical properties.

2.1 Decomposing hours worked

The standard measure of labour services is obtained by dividing the registered total hours worked at time t by the registered total hours worked at time $t-1$

$$\mathcal{Q}^{\text{Hours worked}} = \left(\frac{\sum_{i \in I_t} H_{it}}{\sum_{i \in I_{t-1}} H_{it-1}} \right), \tag{2.3}$$

where the index i runs across the sets I_t and I_{t-1} of workers with positive working hours at time t and $t-1$, respectively. As with the decomposition of total labour costs in Equation 2.2, the change in hours worked can be decomposed into contributions from continuing, entering and exiting workers by

$$\mathcal{Q}^{\text{Hours worked}} = \underbrace{\left(\frac{\sum_{i \in C} H_{it}}{\sum_{i \in C} H_{it-1}} \right)}_{\text{CONTINUING}} \times \underbrace{\left(1 + \sum_{i \in N} H_{it} \Big/ \sum_{i \in C} H_{it} \right)}_{\text{ENTERING}} \times \underbrace{\left(1 + \sum_{i \in X} H_{it-1} \Big/ \sum_{i \in C} H_{it-1} \right)^{-1}}_{\text{EXITING}}. \tag{2.4}$$

The first term after the equality sign shows the change in hours worked among continuing workers. The second term shows the contribution from entering workers. It is increasing in the ratio of hours worked of entering workers relative to hours worked of continuing workers at time t. Correspondingly, the impact from exiting workers is decreasing in the ratio of hours worked of exiting to continuing workers at time $t-1$.

2.2 Defining the "true" index

In this section we outline our concept of a "true" index both across continuing workers and those entering and exiting employment. We start by recapitulating how a wage index across continuing workers is evaluated and then outline how economic theory can guide us in deriving an index for labour services that takes into account the entry and exit of workers.

Indices calculated across continuing workers are often evaluated according to their economic and axiomatic properties. In the axiomatic approach the index should hold a number of desirable properties. For example, the *Identity* axiom states that if wages do not change between time periods neither should the overall index. The *Commensurability* axiom states that the price index should be invariant to changes in the units of measurement. The *Mean value tests* require that the overall wage index lies within the minimum and the maximum wage ratio and that the overall labour services index lies within the minimum and maximum ratio of hours worked. A thorough discussion of these and many more axioms can be found in e.g., The Consumer Price Index Manual (ILO et al. 2004b). It turns out that the Fisher index is the only index number that satisfies all of the 20 axiomatic tests that are discussed in what is labelled the first axiomatic approach. In comparison, the Törnqvist index passes 11 of these tests (ILO et al. 2004b, p. 297). In contrast, in what is labelled the second axiomatic approach, where a price index is defined by two sets of prices and two sets of values (and not quantities), it is the Törnqvist index that passes all of the axiomatic tests.

There are several practical problems with the axiomatic approaches. It is not clear what criteria to use to weight the different tests and how to decide on which of the two axiomatic approaches to use. Also, any given list of axioms can be viewed as arbitrary. Moreover, even if an index fails a particular test, it does not necessarily imply that using this index will result in a large error. Nevertheless, the Fisher and the Törnqvist indices stand out as superior to many of the other indices in the first and second axiomatic approach, respectively. Also, these indices behave similarly as they both use information about value shares in both comparison periods.

Using an economic criterion as a basis for evaluating price indices dates back to Konüs (1939). The purpose of the economic approach is to yield an index that shows the change in the wage cost between two time periods for a given level of production. Interestingly, both the Fisher and the Törnqvist index score high also when assessed by economic criteria. In a seminal article by Diewert (1976) it was shown that these indices are superlative, i.e., they are consistent with the change in wage costs when the economic framework is approximated with second-order accuracy. In particular, the Törnqvist index is exact if the cost function in the economic system is of translog form. Since both the Fisher and the Törnqvist indices score high on both axiomatic and economic test criteria they are considered by many to be superior indices. We choose the Törnqvist index as our concept of the "true" index among continuing workers since it holds desirable axiomatic and economic properties, and since it is often used to control for worker characteristics in the literature.

The impact on wages and labour services from entering and exiting workers will be analysed using the theory of new goods. We apply the results in Feenstra (1994) where the focus of analysis was the construction of a price index when the set of goods available at time t and $t-1$ differed. In the following, we first illustrate diagrammatically how entering and exiting workers impact wages. Second, we show explicitly how to construct an index for labour services which takes into account different sets of workers across time periods and this index is further decomposed into indices for continuing, entering and exiting workers.

Figure 2 illustrates the theory underlying the impact on wage costs from workers entering and exiting employment. The isocost line AA' shows the combination of hours worked between the two workers which yields the same cost for the firm. If the firm's objective is to minimise costs for a given level of production, the problem is to find a point on the isoquant where the associated isocost curve has the minimal vertical intercept. At time $t-1$, it is only worker H_2 that is available and employment is at point A. At time t, however, both workers are available for the firm. When both workers are available, the isocost curve with the minimal vertical intercept goes through point B. The entry of a new worker thus enables the firm to

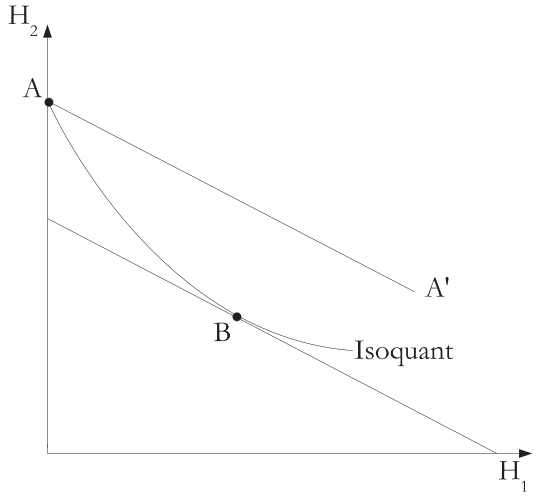

Figure 2 – Impacts on wage costs from workers entering and exiting employment.

reduce costs for a given level of production.

The size of the wage cost reduction depends on the curvature of the isoquant, or how easy it is to substitute one worker for another. When there is some sort of complementarity between workers, i.e., a worker's efficiency increases when working with others, the isoquant line will show a curvature as illustrated in Figure 2. However, if workers are perfect substitutes, the isoquant is a straight line, and there is no longer a wage cost reduction from having a new worker available for production and, consequently, there is no bias from using conventional unit value wage indices. Furthermore, the absence of a new worker bias does not require workers to be homogeneous with identical wages. As illustrated in Figure 2, worker H_2 has a higher wage than worker H_1, which reflects that they have different qualities. That workers may earn different wage rates is thus unrelated to the question of a new worker bias. A new worker bias is a result of the curvature of the isoquant, not the slope of the isocost function.

Figure 2 can also be used to illustrate the wage increase when a worker exits the labour market. When both workers are available, the isocost curve with the minimal vertical intercept goes through point B. If worker H_1 exits the labour market at time t and the firm will only employ worker H_2 (point A), the wage cost for a given amount of production increases.

Feenstra (1994) showed the intuitive results described above analytically in the case of constant elasticity of substitution (CES) production technology, based on the Sato-Vartia index (Sato 1976, Vartia 1976). Consider the CES cost function for one unit of output $\left(\sum_{i\in I_t} b_i W_{it}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ where σ is the elasticity of substitution which is assumed to exceed unity and where b_i is a quality parameter for worker i.[1] Wages

[1] If $\sigma < 1$, all workers are needed to achieve positive production, see Feenstra (1994, p. 159). The corresponding production of labour services is given by $\left(\sum_{i\in I_t} b_i^{1/\sigma} H_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$.

are assumed to reflect marginal productivity of labour services and will vary across workers if there is heterogeneity in the quality parameter. Given that the set of workers available is fixed between time periods and given by $C = I_t = I_{t-1}$, the Sato–Vartia index shows the wage index for a given unit of output. It is a geometric mean of the individual wage changes across continuing workers

$$\prod_{i \in C} (W_{it}/W_{it-1})^{x_{it}(C)}, \tag{2.5}$$

where the weights $x_i(C)$ are constructed from the expenditure shares by the relationships

$$s_{it}(C) = \frac{V_{it}}{\sum_{i \in C} V_{it}} \tag{2.6}$$

$$x_{it}(C) = \left(\frac{s_{it}(C) - s_{it-1}(C)}{\ln s_{it}(C) - \ln s_{it-1}(C)} \right) \Big/ \sum_{i \in C} \left(\frac{s_{it}(C) - s_{it-1}(C)}{\ln s_{it}(C) - \ln s_{it-1}(C)} \right). \tag{2.7}$$

The attractiveness of the Sato-Vartia wage index in Equation 2.5 is that is does not depend on the unknown quality parameters b_i, $i \in C$. However, although the Sato-Vartia index is consistent with the CES function it violates the monotonicity axiom, see Reinsdorf and Dorfman (1999). The index in Equation 2.5 requires that the same workers are working in both time periods. Feenstra (1994) generalised this result to also take into account that the set of workers (goods) might differ between time periods. More specifically, Feenstra (1994) showed that when the sets I_t and I_{t-1} differ, the total wage index is given by the product of the Sato-Vartia index in Equation 2.5 and two adjustment factors for entering and exiting workers

$$\mathcal{W} = \underbrace{\left(\prod_{i \in C} (W_{it}/W_{it-1})^{x_{it}(I)} \right)}_{\text{CONTINUING}} \times \underbrace{\left(\lambda_t^{\frac{1}{\sigma-1}} \right)}_{\text{ENTERING}} \times \underbrace{\left(\lambda_{t-1}^{-\frac{1}{\sigma-1}} \right)}_{\text{EXITING}}. \tag{2.8}$$

Henceforth, the index in Equation 2.5 will be referred to as the *Sato-Vartia-Feenstra index.* λ_r is the fraction of labour costs of the workers available at both time periods, $i \in C$, relative to labour costs aggregated across the entire set of workers $i \in I_r$ at time r, i.e.,

$$\lambda_r = \frac{\sum_{i \in C} V_{ir}}{\sum_{i \in I_r} V_{ir}}, \quad \text{for } r = t-1, t. \tag{2.9}$$

$\lambda_t^{\frac{1}{\sigma-1}}$ measures the impact from new or entering workers. For example, the higher the share of new workers, the smaller the value of λ_t and the lower the value of the Sato-Vartia-Feenstra wage index. Note that the introduction of new workers cannot lead to a higher wage index. The impact from exiting workers is opposite. If the share of workers exiting employment in $t-1$ is large, λ_{t-1} becomes small which raises the wage index. The extent to which new workers lower the wage index, and the extent to which exiting workers increase the wage index, depends on the elasticity of substitution σ. As illustrated diagrammatically in Figure 2, when workers are perfect substitutes, the elasticity of substitution goes towards infinity ($\sigma \rightarrow \infty$), the isoquant becomes linear and the impact from new workers $\lambda_t^{\frac{1}{\sigma-1}}$ goes to unity. Consequently, new workers will not reduce the wage index when workers are perfect substitutes. Correspondingly, when workers are perfect substitutes, the impact from exiting workers will not increase the wage index as $\lambda_{t-1}^{-\frac{1}{\sigma-1}}$ goes towards unity. It follows that the index in Equation 2.8 becomes the index in Equation 2.5 when workers are perfect substitutes.

We will use the case of perfect substitutes as the main alternative index from which we evaluate standard practice.[2] We will refer to this index as the "true" index. When workers are perfect substitutes it follows that any change in labour costs from entering or exiting workers is due to a change in labour services only and there is thus no impact from wage changes. To economise on notation and since it has been widely used in the literature, we will apply the Törnqvist index as an approximation of the Sato-Vartia quantity index among continuing workers. The "true" index for labour services then follows from the product rule by dividing Equation 2.2 with Equation 2.8 using the Törnqvist quantity index across continuing workers and letting $\sigma \rightarrow \infty$.

$$\mathcal{Q}^{\text{True}} = \mathcal{Q}^{\text{Törnqvist}}_{\text{CONTINUING}} \times \underbrace{\left(1 + \sum_{i \in N} V_{it} \Big/ \sum_{i \in C} V_{it}\right)}_{\text{ENTERING}} \times \underbrace{\left(1 + \sum_{i \in X} V_{it-1} \Big/ \sum_{i \in C} V_{it-1}\right)^{-1}}_{\text{EXITING}}, \tag{2.10}$$

where $\mathcal{Q}^{\text{Törnqvist}}_{\text{CONTINUING}} = \prod_{i \in C} \left(H_{it}/H_{it-1}\right)^{1/2(s_{it}+s_{it-1})}$. Note that the case of perfect substitutes is consistent with the procedure of constructing elementary aggregates when calculating indices. For example, in the consumer price index, an elementary aggregate is a group of relatively homogenous products. An elementary aggregate in the calculation of labour services would correspondingly consist of workers that are as similar as possible. Ideally, an elementary aggregate is defined by an elasticity of substitution equal to infinity within each group. Since the aggregate entering and exiting effects are a weighted average of the entering and exiting effects in each elementary aggregate, and since the Törnqvist index is approximately consistent in aggregation (Diewert 1978), the index in Equation 2.10 approximates an index based on elementary aggregates. Equation 2.10 is therefore our benchmark "true" index from which the standard practice of using hours worked will be evaluated. While the bias from using hours worked among continuing workers and an index for labour services based on an acceptable index formula is well known from e.g., the results in

[2] In the empirical section we conduct robustness checks by allowing for different elasticities of substitution.

Párniczky (1974) and Diewert and Lippe (2010), the biases from entering and exiting workers will be further analysed. To this we now proceed.

2.3 Decomposing the biases of entering and exiting workers

In the previous two sections, the impact from entering and exiting workers has been established both for the change in hours worked in Equation 2.4 and for the theoretical index Equation 2.10. In this section we compare these indices and explicitly state the bias from using hours worked.

We define the total bias by the ratio of the true index to hours worked. As with the indices for wages and labour services, the total bias can then be decomposed into the biases of continuing, entering and exiting workers

$$\text{TOTAL BIAS} = \text{CONTINUING BIAS} \times \text{ENTERING BIAS} \times \text{EXITING BIAS}.$$

The continuing bias has been extensively analysed in the literature and it measures compositional effects among continuing workers, see e.g., Diewert and Lippe (2010). The problem arises because hours worked from labour of different types are added together in Equation 2.3. For example, consider the case when the hourly wage rate of all persons is constant from one period to the next and assume further that there is a shift in demand towards lower payed labour. Since wage rates are constant, an aggregate wage index which satisfies the identity test, such as the Sato-Vartia index in Equation 2.5 or the Törnqvist index, equals unity. However, since it was assumed that there was a shift in demand towards lower paid labour, the 'average' unit value has decreased, resulting in a lower unit value wage index. The change in input mix towards lower paid labour thus causes a downward bias in the measurement of labour services: the CONTINUING BIAS is less than unity. In other words, the unit value wage index fails the identity test, i.e., if the wage of every person is identical during the two periods, then the wage index should equal unity. The unit value index also fails the axiomatic test of homogeneity (unless relative quantities do not change), i.e., if each price in the base period increases by a factor then the index should also increase by the same factor, a property regarded to be fundamental by most index number theorists, see ILO et al. (2009, Section 17.37). Also, the unit value index fails the mean value tests and is not invariant to changes in the units of measurement (ILO et al. 2009, Sections 2.22 and 2.25).

The main focus of this paper is on the entering bias and the exiting bias. Dividing the last two terms in

Equation 2.10 with the last two terms in Equation 2.4 yields

$$\text{ENTERING BIAS} = \left(\frac{1 + \sum_{i \in N} V_{it} \big/ \sum_{i \in C} V_{it}}{1 + \sum_{i \in N} H_{it} \big/ \sum_{i \in C} H_{it}} \right), \tag{2.11}$$

$$\text{EXITING BIAS} = \left(\frac{1 + \sum_{i \in X} V_{it-1} \big/ \sum_{i \in C} V_{it-1}}{1 + \sum_{i \in X} H_{it-1} \big/ \sum_{i \in C} H_{it-1}} \right)^{-1}. \tag{2.12}$$

Both of these biases relate to the relative value of labour costs to hours worked of either entering to continuing or exiting to continuing workers. In particular, if the unit value wage of entering workers are lower than the unit value wage of continuing workers, the entering bias is lower than unity, and using hours worked will overestimate the level of labour services. The reason is that the index using hours worked is based on each hour being equally important for the development of the index. However, if the unit value wage of entering workers is lower than the unit value wage of continuing workers, there are more “low productive" workers entering employment, and these hours should from theory be valued by their labour cost contribution, not the contribution from the amount of hours worked. Correspondingly, if the unit value wage of exiting workers is lower than the unit value wage of continuing workers, the exiting bias is higher than unity and using hours worked will underestimate the level of labour services.

These relationships can be seen more clearly by defining the unit value wage u by the aggregate labour costs relative to the number of hours worked, i.e., $u_t(Z) = \sum_{i \in Z} V_{it} \big/ \sum_{i \in Z} H_{it}$ in any given set Z. The biases above can then be approximated by[3]

$$\text{ENTERING BIAS} \approx \left(\frac{u_t(N)}{u_t(C)} \right)^{\omega_N}, \tag{2.13}$$

$$\text{EXITING BIAS} \approx \left(\frac{u_{t-1}(X)}{u_{t-1}(C)} \right)^{-\omega_X}. \tag{2.14}$$

where the weight ω is defined as the ratio of hours worked of entering or exiting workers to continuing workers, respectively, i.e., $\omega_N = \left(\frac{\sum_{i \in N} H_{it}}{\sum_{i \in C} H_{it}} \right)$ and $\omega_X = \left(\frac{\sum_{i \in X} H_{it-1}}{\sum_{i \in C} H_{it-1}} \right)$. For example, consider the case when the unit value wage of new workers is 80 per cent the unit value of continuing workers, and the hours worked by entering workers is 5 per cent the hours worked of continuing workers. Using hours worked as an index for labour services will then lead to an overvaluation of labour services by approximately 1 percentage points, i.e., ENTERING BIAS$=0.8^{0.05} = 0.99$. In contrast, if the unit value wage of exiting workers is 80 per cent the unit value of continuing workers, and the hours worked by exiting workers is 5 per cent the hours worked of continuing workers, this will lead to an undervaluation of labour services by approximately 1 percentage point, i.e., EXITING BIAS$=0.8^{-0.05} = 1.01$.

[3] See the appendix, Section 6.1.

2.4 The two-step procedure – controlling for worker characteristics

The literature that tries to control for the "quality" of labour divides the labour force into different groups defined by characteristics such as education, age, sex etc and then in a second step, applies an index, such as the Törnqvist index, to aggregate these groups. In this section we analyse the theoretical rationale for this two-step procedure and show that it may in some cases amplify the problems caused using hours worked as an index for labour services.

We divide the workforce into two complement sets consisting of those that are skilled (S) and those that are unskilled (U) so that $I_t = S_t \cup U_t$. We also maintain the notation used so far, so e.g., S_N represents the set of skilled workers entering the workforce at time t, S_X represents the set of skilled exiting the workforce at time $t-1$ and S_C represents the set of skilled workers that are continuing. It follows that the aggregate number of hours worked by for example skilled workers can be written as $\sum_{i \in S_t} H_{it}$. The Törnqvist index across skilled and unskilled labour can then be written as

$$\mathcal{Q}^{\text{Two-step}} = \left(\frac{\sum_{i \in S_t} H_{it}}{\sum_{i \in S_{t-1}} H_{it-1}} \right)^{\overline{\upsilon_t(S_t, I_t)}} \left(\frac{\sum_{i \in U_t} H_{it}}{\sum_{i \in U_{t-1}} H_{it-1}} \right)^{(1-\overline{\upsilon_t(S_t, I_t)})}, \tag{2.15}$$

where $\upsilon_t(S_t, I_t) = \sum_{i \in S_t} V_{it} / \sum_{i \in I_t} V_{it}$ is the labour cost share of high skilled and where the overline is the moving average operator between time $t-1$ and t, i.e., $\overline{\upsilon_t(S_t, I_t)} = 1/2\, [\upsilon_t(S_t, I_t) + \upsilon_{t-1}(S_{t-1}, I_{t-1})]$. This expression can be compared to the index in Equation 2.4. The total bias between these indices can also be split into three components: the bias of continuing workers, the bias of entering workers and the bias of exiting workers. The bias of continuing workers from the two-step procedure was analysed in e.g., Diewert and Lippe (2010). They found that the bias decreases with increased disaggregation if there are compositional effects between the groups that contribute most to the overall bias. The bias of entering and exiting workers between the group based index and hours worked differs from the entering and exiting bias based on the theoretical index. As an approximation, the two-step entering bias, can be written[4]

$$\text{TWO-STEP ENTERING BIAS } \approx e^{\left(\overline{\upsilon_t(S_t, I_t)} - \psi_t(S_C, C) \right) \left[\left(\frac{\sum_{i \in S_N} H_{it}}{\sum_{i \in S_C} H_{it}} \right) - \left(\frac{\sum_{i \in U_N} H_{it}}{\sum_{i \in U_C} H_{it}} \right) \right]}, \tag{2.16}$$

where the weight $\psi_t(S_C, C)$ is the share of high skilled hours worked of continuing workers evaluated at time t, i.e. $\psi_t(S_C, C) = \left(\frac{\sum_{i \in S_C} H_{it}}{\sum_{i \in C} H_{it}} \right)$. The bias is larger than unity if both brackets are positive (or if both are negative). The first bracket is positive if high skilled workers are paid more than low skilled workers and the high skilled hours worked share of continuing workers is the same as high skilled hours worked share for all

[4] See the appendix, Section 7.

workers, i.e., if $\psi_t(S_C, C) \approx \psi_t(S_t, I_t)$. The second bracket is positive if skilled entry is proportionally larger than unskilled entry. To see this, let the weight $\theta_t = \left(\frac{\sum_{i \in S_C} H_{it}}{\sum_{i \in U_C} H_{it}}\right)$ denote the ratio of skilled to unskilled man-hours across continuing workers at time t. The last bracket will then be positive if skilled entry exceeds weighted unskilled entry, i.e., $\sum_{i \in S_N} H_{it} > \theta_t \ \sum_{i \in U_N} H_{it}$. For example, if there are twice as many skilled man-hours compared with unskilled man-hours among continuing workers ($\theta_t = 2$) and there are 1 million unskilled man-hours entering the labour market, there must be more than 2 million skilled man-hours entries for the two-step entering bias to be larger than unity.

Correspondingly, the two-step exiting bias can be approximated by[4]

$$\text{TWO-STEP EXITING BIAS } \approx e^{\left(\overline{\upsilon_t(S_t,I_t)} - \psi_t(S_C,C)\right)\left[\left(\frac{\sum_{i \in U_X} H_{it}}{\sum_{i \in U_C} H_{it}}\right) - \left(\frac{\sum_{i \in S_X} H_{it}}{\sum_{i \in S_C} H_{it}}\right)\right]}. \tag{2.17}$$

The bias is larger than unity if both brackets are positive (or if both are negative). The first bracket is positive if skilled workers are paid more than unskilled workers and the skilled hours worked share of continuing workers are the same as skilled hours worked share for all workers, i.e., if $\psi_t(S_C, C) \approx \psi_t(S_t, I_t)$. The second bracket is positive if unskilled exit is proportionally larger than skilled exit. In particular, the last bracket is positive if $\theta_{t-1} \sum_{i \in U_X} H_{it} > \sum_{i \in S_X} H_{it}$, where the weight $\theta_{t-1} = \left(\frac{\sum_{i \in S_c} H_{it-1}}{\sum_{i \in U_c} H_{it-1}}\right)$ denotes the man-hour ratio of skilled to unskilled continuing workers at time t. For example, if there are twice as many skilled man-hours compared to unskilled man-hours among continuing workers ($\theta_{t-1} = 2$) and there are 1 million unskilled man-hours exiting the labour market, there must be less than 2 million skilled man-hours exiting the labour market for the bias to be greater than unity.

The purpose of splitting the workforce into groups and then aggregating using for example a Törnqvist index is to reduce the bias from using hours worked. It is thus of particular interest to analyse whether the two-step procedure actually reduces the overall bias, e.g., to analyse when the theoretical entering and exiting biases in Equation 2.13 and Equation 2.14 will be below unity and at the same time, the group biases in Equation 2.16 and Equation 2.17 will be above unity. The two-step procedure will amplify the problem caused by using hours worked if there are a large number of newly educated skilled workers entering the labour force with a relatively low wage. In this case, the relatively low wage of the newly educated workers leads to an entering bias lower than unity in Equation 2.13. In contrast, since skilled workers overall have a higher wage than unskilled, the two-step entering bias is larger than unity in Equation 2.16. The two-step procedure will also worsen the problem when there is a large number of unskilled workers exiting the labour market with a relatively high wage. In this case, the relatively high wage of the unskilled leads to the exiting bias in Equation 2.14 being lower than unity. In contrast, since unskilled workers earn less than skilled, the two-step exiting bias in Equation 2.17 will be above unity. These examples illustrate that there can be situations where the 2-step procedure yields entering and/or exiting effects that are further away from the true indices than the standard practice of using hours worked.

Importantly, the two-step procedure can also exacerbate the overall bias even though the individual entering or exiting biases are reduced. This occurs when there are asymmetric reductions in biases from exiting and entering workers. To illustrate, consider the case when the overall bias is lower than unity due to for example a large entry of workers with relatively low wages. Also, let the bias from exiting workers be larger than unity due to a lower wage among workers exiting employment. If skilled entry is proportionally equal to unskilled entry, the two-step entry bias in Equation 2.16 will be unity. But, if there is a large number of unskilled workers that exit employment with relatively low wages, both the exiting bias and the two-step exiting bias are above unity. Although the two-step procedure has reduced the exiting bias, since the exiting bias is above unity while the overall bias is below unity, a reduction of the exiting bias exacerbates the overall bias. Whether the two-step procedure actually worsens the problem of using hours worked as a quantity index, also depends on the two-step bias of continuing workers. In the empirical section, we decompose these effects separately.

3 Data

Our dataset holds information about hours worked and labour costs for all employed persons in Norway between 2002 and 2008. It is based on information from the Register of Employers and Employees and the Pay Statements Register. Labour costs are measured by wage income and include wages and other remunerations.[5] Wage costs per hour are constructed as annual labour costs divided by contractual annual working hours. We have trimmed the data by removing workers with a registered hourly wage above NOK 4 000 and below NOK 40. Workers registered with more than 4 000 working hours a year are also removed. As a robustness check, we change cut-off points to workers earning more than NOK 5 000 and less than NOK 30, and workers with more than 5 000 working hours. This way of treating the data is also compared with not trimming the data. In total, our benchmark trimmed dataset holds 2.9 million annual observations, which amounts to 98 per cent of the total number of observations. The results presented below are not sensitive to the level of trimming. In the appendix, Section 7.1, we compare our measure of contractual hours with the measure of actual hours worked in the National Accounts. Further details about the register-based micro data and how they compare to data from e.g., the Labour Force Survey can be found in Villund (2009) and Aukrust et al. (2010). Information about worker's level of education is taken from the Population's

[5] It also includes income earned at sea and company benefits such as a car or a phone.

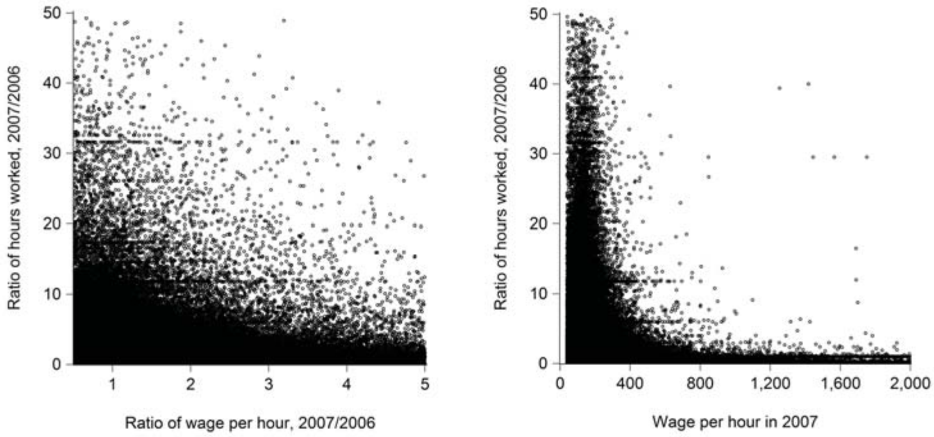

Figure 3 – Wages and hours worked. The left panel shows the ratio of hours worked and the ratio of wage per hour between 2007 and 2006. The right panel shows the ratio of hours worked between 2007 and 2006 and wage per hour in 2007. Source: Statistics Norway, authors' calculations.

level of education statistics.[6] There are ten educational levels based on the revised Norwegian Standard Classification of Education (NUS2000): 0–No education and preschool education, 1–Primary education, 2–Lower secondary education, 3–Upper secondary (basic), 4–Upper secondary (final year), 5–Post-secondary non-tertiary education, 6–First stage of tertiary education (undergraduate level), 7–First stage of tertiary education (graduate level), 8–Second stage of tertiary education, graduate level, 9–Unspecified. We define high skilled as workers with a NUS2000 level from 4–8, i.e., from Upper secondary final year to the second stage of tertiary education. Low skilled is thus defined as workers with no education to basic upper secondary education, and it also covers workers with an unspecified level of education. About 1.5 per cent of the workers are registered with an unspecified level of education. Most of these are immigrants.

Figure 3 shows descriptive evidence for wages and hours worked. The left panel shows the ratio of hours worked and the ratio of wage per costs hour between 2007 and 2006. There is a clear negative correlation between growth in hours worked and growth in wages. The right panel shows the ratio of hours worked between 2007 and 2006 and wage per hour in 2007. There is also a clear negative correlation between growth in hours worked and the wage level. From theory we know that a negative correlation between wage changes and changes in hours worked yields a positive bias between the Laspeyres index and the Paasche index, see e.g., ILO et al. (2004a, p. 285). We also know that a negative correlation between the wage level and changes in hours worked yields a positive bias between the overall change hours worked and for example the Törnqvist or Fisher index, see e.g., Diewert and Lippe (2010). The extent to which the negative correlations shown in Figure 3 impact the indices for labour services will be further analysed in the empirical section.

[6]See https://www.ssb.no/en/utdanning/statistikker/utniv

4 Empirical results

We now turn to our empirical findings based on the theoretical framework in Section 2. We start by decomposing labour costs into wage changes and labour services changes using the standard practice. Second, we take a closer look at how indices such as Laspeyres, Paasche, Törnqvist and Fisher compare with the standard practice when calculated across continuing workers. Third, based on the theory of entering and exiting workers, we compare the standard practice with our preferred "true" index and decompose the contributions from continuing, entering and exiting workers. Forth, we analyse how controlling for the level of education using the two-step procedure performs empirically. Fifth, we conduct a robustness analysis allowing for different elasticities of substitution in our definition of the "true" index. Finally, we show how mismeasurement of labour services using the standard practice has impacted the development of productivity in Norway.

4.1 Decomposing labour costs using the standard practice

In Table 1 we show the results of decomposing the logarithmic change in labour costs into its respective price and quantity components applying the unit value index. Labour services are thus measured by hours worked, see Equation 2.4. Labour costs growth was temporarily reduced to 3.29 per cent in 2003 before increasing to 10.15 per cent in 2008. The contribution from continuing, entering and exiting workers changed during this time period. From 2002 to 2005 the contribution from continuing workers was higher than the total figure, indicating that the impact from exiting workers outweighed the impact from entering workers. In tandem with a booming economy and an increase in immigration after 2004, the impact from entering workers increased and the reduction in wage costs from exiting workers was reduced. The large increase in total labour costs was due to an increase in labour costs among continuing workers. Growth in labour costs is decomposed into wage growth and growth in labour services. From 2002 to 2005 most of the labour cost growth was attributed to wage growth. After 2005, a larger portion of the total labour cost growth is due to labour services growth, measured by hours worked. Workers entering the workforce reduced total wages by between 1.3 and 1.8 percentage points. In contrast, exiting workers contributed to an increase in total wages between 0.8 and 1.0 percentage points. As a result, the impact from entering workers is larger than from exiting workers. The increased negative impact on unit values from entering workers after 2005 may reflect the relatively low wages paid to immigrants in conjunction with the large increase in immigration.

Table 1 – Decomposing labour costs using unit values

	Labour costs[a]				Wage costs per hour[b]				Labour services (Hours worked)[c]			
	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting
2002	5.70	6.30	5.33	−5.92	4.52	5.11	−1.60	1.01	1.18	1.19	6.93	−6.93
2003	3.29	4.40	5.20	−6.31	3.68	4.25	−1.43	0.86	−0.39	0.15	6.63	−7.17
2004	3.92	4.09	4.78	−4.95	2.87	3.20	−1.33	1.00	1.05	0.89	6.11	−5.95
2005	4.68	5.24	4.32	−4.88	4.64	5.09	−1.49	1.03	0.04	0.14	5.81	−5.91
2006	8.20	7.53	5.42	−4.76	4.85	5.81	−1.78	0.81	3.35	1.72	7.20	−5.57
2007	9.34	8.91	5.22	−4.79	5.85	6.84	−1.79	0.80	3.49	2.07	7.01	−5.59
2008	10.15	8.91	5.66	−4.43	5.56	6.51	−1.76	0.81	4.59	2.40	7.43	−5.23

The product rule of indices (Value index = Price index × Quantity index) is decomposed using unit values as the price index into contributions from continuing, entering and exiting workers. Measured as the logarithmic difference in per cent.

[a] Decomposed using Equation 2.2.

[b] Decmoposed as the ratio of Equation 2.2 and Equation 2.4.

[c] Decomposed using Equation 2.4.

Source: Statistics Norway, authors' calculations.

Table 2 – Labour services across continuing workers. Growth rates.

	Hours worked[a]	Törnqvist I[b]	Törnqvist II[c]	Fisher	Laspeyres	Paasche
2002	1.19	0.65	0.57	0.29	4.46	−3.87
2003	0.15	−0.32	−0.31	−0.75	3.10	−4.59
2004	0.89	0.24	0.20	−0.13	3.50	−3.75
2005	0.14	−0.50	−0.57	−0.96	2.43	−4.35
2006	1.72	1.10	0.91	0.77	4.19	−2.65
2007	2.07	1.00	0.81	0.56	3.97	−2.84
2008	2.40	1.62	1.47	1.29	4.75	−2.18

[a] From Table 1: Hours worked across continuing workers.
[b] Measured directly using the Törnqvist quantity index: $\prod (H_{it}/H_{it-1})^{1/2(s_{it}+s_{it-1})}$.
[c] Measured indirectly from the Törnqvist price (wage) index by applying the product rule: $\left(\sum V_{it}/\sum V_{it-1}\right)/\prod (W_{it}/W_{it-1})^{1/2(s_{it}+s_{it-1})}$.
Measured as the logarithmic difference in per cent. Source: Statistics Norway, authors' calculations.

4.2 Comparing standard practice with other classical indices

Table 2 shows the logarithmic change of different indices for labour services across continuing workers only. There are large differences across the measures of labour services. *Törnqvist I* is based on a Törnqvist quantity index. Overall, the annual growth is lower than for hours worked, approximately 0.5 – 1 percentage points. Correspondingly, wage growth is overvalued by 0.5 – 1 percentage points. Note that the increase in labour services from 2005 to 2008 is about at the same level as for hours worked. Interestingly, for some years there is a significant difference between the Törnqvist quantity index and the quantity index measured indirectly using a Törnqvist wage index and the product rule, referred to as *Törnqvist II*. This discrepancy is increasing somewhat in the years after 2005. Also, the Fisher index shows lower growth than the other indices. As illustrated by Dumagan (2002), this may reflect large variations in wage shares and hours worked across time. These patterns should also be seen in conjunction with the discrepancy between the Laspeyres and the Paasche indices in Table 2. A positive bias between the Laspeyres index and the Paasche index occurs when there is a negative correlation between price *changes* and volume changes, see e.g., ILO et al. (2004a, p. 285). There has thus been a large shift towards using labour that has become cheaper. In contrast, the bias between hours worked (unit value index) and for example the Fisher index is driven by a correlation between the wage *level* and volume changes. Consequently, Table 2 also shows that there has been a large increase in employment for workers with low wage levels.

4.3 Comparing standard practice with the true index

Table 3 compares the standard practice using hours worked as a measure for labour services with the true index calculated both across continuing, entering and exiting workers, as defined in Equation 2.4 and Equation 2.10. As mentioned, the Törnqvist quantity index is chosen as the preferred alternative index

Table 3 – Comparing standard practice with the true index

	Hours worked[a]				True index[b]				Bias			
	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting
2002	1.18	1.19	6.93	−6.93	0.05	0.65	5.33	−5.92	−1.13	−0.54	−1.60	1.01
2003	−0.39	0.15	6.63	−7.17	−1.43	−0.32	5.20	−6.31	−1.04	−0.47	−1.43	0.86
2004	1.05	0.89	6.11	−5.95	0.07	0.24	4.78	−4.95	−0.98	−0.65	−1.33	1.00
2005	0.04	0.14	5.81	−5.91	−1.07	−0.50	4.32	−4.88	−1.10	−0.65	−1.49	1.03
2006	3.35	1.72	7.20	−5.57	1.77	1.10	5.42	−4.76	−1.58	−0.62	−1.78	0.81
2007	3.49	2.07	7.01	−5.59	1.42	1.00	5.22	−4.79	−2.07	−1.07	−1.79	0.80
2008	4.59	2.40	7.43	−5.23	2.86	1.62	5.66	−4.43	−1.73	−0.78	−1.76	0.81

[a] Hours worked is exactly decomposed into continuing, entering and exiting workers using the identity Equation 2.4.
[b] The true index is the sum of the contributions from continuing workers (Törnqvist) and entering and exiting workers when workers are assumed to be perfect substitutes, see Equation 2.10.
Measured as the logarithmic difference in per cent. Source: Statistics Norway, authors' calculations.

since this index is often used in the literature to control for compositional effects and it holds desirable axiomatic and theoretical properties. The table shows the biases between the standard practice and the true index, in particular the entering and exiting biases as defined in Equation 2.11 and Equation 2.12. The total bias in hours worked ranges from -1 to -2 percentage points annually. The bias of the wage index will thus correspondingly range from 1 to 2 percentage points annually. About half of this bias is attributed to compositional effects among continuing workers and half of this bias is attributed to the impact from entering and exiting workers. Note that the bias from entering workers ranges from -1.3 to -1.8 percentage points, but is somewhat offset by the bias from exiting workers which ranges from 0.8 to 1.0 percentage points. Interestingly, the overall bias increases (in absolute value) after 2005, which is the period when productivity growth in Norway decreased.

4.4 The two-step procedure – controlling for the level of education

Table 4 compares the standard practice for hours worked with the Törnqvist index across hours worked in two educational groups, as the approximations in Equation 2.16 and Equation 2.17 illustrates. In contrast to the negative bias between the "true" index and hours worked, the total bias when aggregating across two educational groups is positive from 2002 to 2006. About a third of this bias is due to continuing workers. From the results in (Diewert and Lippe 2010, p. 704), we know that this is caused by compositional effects within groups being dominant. Interestingly, most of the total bias is a result of exiting workers. The reason is, however, not that the bias for exiting workers has the "wrong" sign with respect to the true index. It is rather caused by the biases from entering and exiting workers having opposite signs and it is mostly the exiting effect which is controlled for in the two-step procedure. As the overall bias is negative and the exiting bias in Table 3 is positive, the two-step procedure exacerbates the overall bias since it is the exiting bias that is mostly reduced. Towards the end of the sample period, the total bias changes from being positive to being negative, a change mainly caused by an increased negative bias among entering workers. From Equation 2.16 this is caused by the weighted number of unskilled entries into the labour market exceeding the number of skilled entries, which should be viewed in light of the large increase in immigration during this time period.

4.5 Robustness with respect to the elasticity of substitution

Our definition of the "true" index is based on the assumption of workers being perfect substitutes, i.e., the elasticity of substitution σ is set to infinity. Although the assumption allows for heterogenous workers earning different wages, it may be restrictive, and hence influence the overall index. In this section, we analyse how sensitive the aggregate index is to different assumptions about the elasticity of substitution. To this end, we

Table 4 – Labour services, controlling for the level of education: two-step aggregation

	Hours worked[a]				Two-step index (two educational groups)[b]				Bias			
	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting	Total	Continuing	Entering	Exiting
2002	1.18	1.19	6.93	−6.93	1.35	1.25	6.93	−6.83	0.16	0.06	0.00	0.10
2003	−0.39	0.15	6.63	−7.17	−0.21	0.21	6.63	−7.05	0.18	0.06	0.00	0.12
2004	1.05	0.89	6.11	−5.95	1.20	0.95	6.09	−5.84	0.15	0.07	−0.02	0.11
2005	0.04	0.14	5.81	−5.91	0.14	0.19	5.76	−5.81	0.11	0.05	−0.04	0.10
2006	3.35	1.72	7.20	−5.57	3.42	1.76	7.12	−5.46	0.06	0.03	−0.08	0.11
2007	3.49	2.07	7.01	−5.59	3.44	2.07	6.84	−5.48	−0.05	0.01	−0.17	0.11
2008	4.59	2.40	7.43	−5.23	4.43	2.38	7.19	−5.13	−0.16	−0.02	−0.24	0.11

[a] Hours worked is exactly decomposed into continuing, entering and exiting workers using the identity Equation 2.4.

[b] The two-step procedure is based on two educational groups, where high skilled are defined as workers with a NUS2000 level from 4–8, i.e., from upper secondary final year to the second stage of tertiary education. The aggregate index is calculated as a Törnqvist index of the two groups, see Equation 2.15.

Measured as the logarithmic difference in per cent. Source: Statistics Norway, authors' calculations.

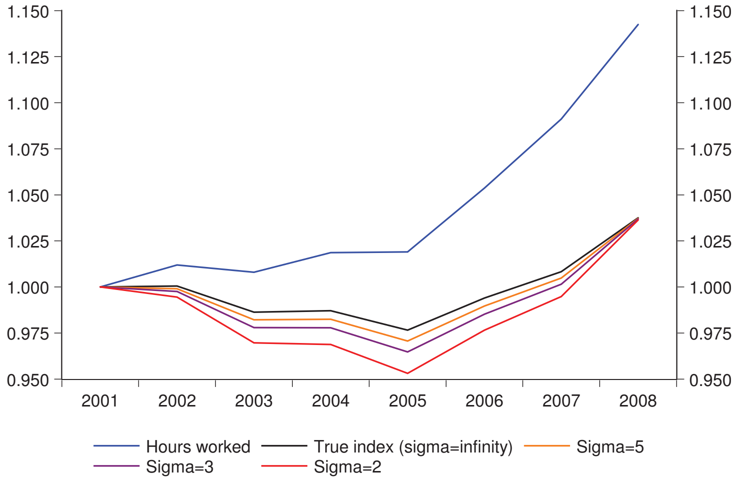

Figure 4 – Labour services. Index: 2001=1. Compares the index for hours worked with theoretical indices based on different values of the elasticity of substitution σ. Our definition of a "true" index is based on workers being perfect substitutes, which is obtained when the elasticity of substitution is set to infinity.

consider the theoretical index ($\mathcal{Q}_\sigma$) for any value of the elasticity of substitution:[7]

$$\mathcal{Q}_\sigma = \mathcal{Q}^{\text{Törnqvist}}_{\text{CONTINUING}} \times \underbrace{\left(1 + \sum_{i \in N} V_{it} \Big/ \sum_{i \in C} V_{it}\right)\left(\lambda_t^{-\frac{1}{\sigma-1}}\right)}_{\text{ENTERING}} \times \underbrace{\left(1 + \sum_{i \in X} V_{it-1} \Big/ \sum_{i \in C} V_{it-1}\right)^{-1}\left(\lambda_{t-1}^{\frac{1}{\sigma-1}}\right)}_{\text{EXITING}},$$

where λ_r for $r = t, t-1$, defined in Equation 2.9, is the fraction of expenditure on the workers available at both time periods, $i \in I$, relative to the expenditure on the entire set of workers $i \in I_r$ at time r. Since λ_r is less than unity, it follows that a lower value of the elasticity of substitution increases the contribution from entering workers and decreases the contribution from exiting workers. In Figure 4, we compare the index for hours worked with the theoretical index based on different values of the elasticity of substitution. Interestingly, the "true" index as we define it, with an elasticity of substitution equal to infinity, represents a conservative measure throughout the sample period relative to indices based on lower values of σ. This is driven by the large effect from exiting workers between 2001 and 2005. In tandem with increased immigration after 2005, the contribution from entering workers yields a larger increase in indices with a lower σ. In 2008, all of the theoretical indices are approximately 3.6 per cent higher than the value in 2001. In contrast, the index for hours worked is 14.2 per cent higher in 2008 than the value in 2001. The overvaluation from using hours worked as an index for labour services is thus robust to different values of the elasticity of substitution.

[7] The theoretical index follows from the product rule by dividing Equation 2.2 with Equation 2.8 using the Törnqvist quantity index.

Figure 5 – Labour productivity. Mainland Norway. Measured as the index for value added relative to the index for labour services, using Equation 2.10 and Equation 2.4. 2001=1.

4.6 Implications for productivity measurement

Figure 5 shows how mismeasured labour services have impacted the measured level of productivity in Mainland Norway. The series "Hours worked" represents the official index for labour productivity in Mainland Norway from Statistics Norway, normalised to unity in 2001. The series "True index" represents an adjusted series where the adjustment is the bias defined by the difference between Equation 2.10 and Equation 2.4. In contrast to what official figures shows, productivity did not drop in 2008 but increased modestly by 0.3 percent. Compared with the period between 2001 and 2005, average labour productivity growth as measured by official statistics was reduced by 2.5 percentage points in the period after 2005. Figure 5 shows that the bias increases in the same time period. The average total bias between 2002 and 2005 was 1.1 percentage points and it changed to 1.8 percentage points on average from 2006 to 2008, up by 0.7 percentage points. Mismeasurement can therefore explain about a quarter of the measured drop in productivity growth after 2005.

5 Conclusions

The purpose of this paper has been to analyse the measured slowdown in productivity growth in Norway after 2005. To that end, we have computed indices for labour services with good theoretical properties that take into account the effects from workers entering and exiting the workforce. We have shown theoretically the poor properties of the standard practice of using hours worked as and index for labour services. Central in our framework is the assumption that wages reflect marginal productivity. Also, we have analysed theoretically the two-step procedure often used in the literature to counteract the weaknesses with unit values and hours worked. In particular, we have shown that the two-step procedure can exacerbate the unit value bias through entering and exiting effects if there is asymmetry in the reduction of entering and exiting biases.

The theoretical results have been empirically illustrated in the case of Norway between 2002 and 2008 using register data. We found that using hours worked overestimated growth in labour services by approximately between 1 to 2 percentage points annually. Wages have been correspondingly underestimated. About half of this was attributed to a bias among continuing workers and half was attributed to the effect of workers entering and exiting employment. In addition, we found that the two-step procedure exacerbated the unit value bias in the first half of the sample when controlling for workers' level of education. Importantly, our findings show that an increasing overestimation of labour services can account for about a quarter of the measured productivity slowdown after 2005.

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6 Appendix

6.1 Approximation of unit value biases

The entering and exiting bias can be written as the ratio of the last two terms in Equation 2.2 and Equation 2.4

$$\left(\frac{1+\sum_{i\in N} V_{it}\big/\sum_{i\in C} V_{it}}{1+\sum_{i\in N} H_{it}\big/\sum_{i\in C} H_{it}}\right)\left(\frac{1+\sum_{i\in X} V_{it-1}\big/\sum_{i\in C} V_{it-1}}{1+\sum_{i\in X} H_{it-1}\big/\sum_{i\in C} H_{it-1}}\right)^{-1}. \tag{6.1}$$

Given that the number of workers entering and exiting is small relative to the number of workers available in both time periods, and by applying the approximation $\ln(1+z)\approx z$ when z is small, the logarithm of the entering bias (the first parenthesis) can be written as

$$\ln\left(\frac{1+\sum_{i\in N} V_{it}\big/\sum_{i\in C} V_{it}}{1+\sum_{i\in N} H_{it}\big/\sum_{i\in C} H_{it}}\right) \approx \sum_{i\in N} V_{it}\Big/\sum_{i\in C} V_{it} - \sum_{i\in N} H_{it}\Big/\sum_{i\in C} H_{it} \tag{6.2}$$

$$= \frac{\sum_{i\in N} H_{it}}{\sum_{i\in C} H_{it}}\left(\frac{\sum_{i\in N} V_{it}\big/\sum_{i\in N} H_{it}}{\sum_{i\in C} V_{it}\big/\sum_{i\in C} H_{it}} - 1\right) \tag{6.3}$$

$$\approx \left(\frac{\sum_{i\in N} H_{it}}{\sum_{i\in C} H_{it}}\right)\ln\left(\frac{u_t(N)}{u_t(C)}\right), \tag{6.4}$$

where the last equality follows from the definition of unit values as the aggregate labour costs relative to the number of hours worked, i.e., $u_t(Z)=\sum_{i\in Z} V_{it}\big/\sum_{i\in Z} H_{it}$ in any given set Z. Taking the exponential on the right hand side of Equation 6.4, and by applying the corresponding approximation for the set of exiting workers, yields the approximate expression for the aggregate entering and exiting unit value bias in Equation 6.1

$$\left(\frac{u_t(N)}{u_t(C)}\right)^{\left(\frac{\sum_{i\in N} H_{it}}{\sum_{i\in C} H_{it}}\right)}\left(\frac{u_{t-1}(X)}{u_{t-1}(C)}\right)^{-\left(\frac{\sum_{i\in X} H_{it-1}}{\sum_{i\in C} H_{it-1}}\right)}. \tag{6.5}$$

The two terms are the biases of entering and exiting workers, as given in Equation 2.13 and Equation 2.14, respectively.

7 Two-step biases

In this section we derive the TWO-STEP ENTERING BIAS in Equation 2.16 and the TWO-STEP EXITING BIAS in Equation 2.17. Since the workforce can be split into two complement sets consisting of those that are skilled (S) and those that are unskilled (U) so that $I_t = S_t \cup U_t$, the contribution from workers entering the workforce in Equation 2.4 can be approximately decomposed by[8]

$$
\begin{aligned}
\ln\left(1+\sum_{i\in N} H_{it}/\sum_{i\in C} H_{it}\right) &= \ln\left(1+\frac{\sum_{i\in S_N} H_{it}+\sum_{i\in U_N} H_{it}}{\sum_{i\in S_C} H_{it}+\sum_{i\in U_C} H_{it}}\right)\\
&\approx \psi_t(S_C,C)\left(\frac{\sum_{i\in S_N} H_{it}}{\sum_{i\in S_C} H_{it}}\right)+(1-\psi_t(S_C,C))\left(\frac{\sum_{i\in U_N} H_{it}}{\sum_{i\in U_C} H_{it}}\right) \qquad (7.1)
\end{aligned}
$$

where the weight $\psi_t(S,I)$ is the share of man-hours carried out by skilled among continuing workers evaluated at time t, i.e. $\psi_t(S_C,C) = \left(\frac{\sum_{i\in S_C} H_{it}}{\sum_{i\in C} H_{it}}\right)$. Correspondingly, the contribution from workers exiting the workforce in Equation 2.4 can then be approximately decomposed[9]

$$
\begin{aligned}
&\ln\left(1+\sum_{i\in X} H_{it-1}/\sum_{i\in C} H_{it-1}\right)^{-1}\\
&\qquad\approx -\psi_{t-1}(S_C,C)\left(\frac{\sum_{i\in S_X} H_{it-1}}{\sum_{i\in S_C} H_{it-1}}\right)-(1-\psi_{t-1}(S_C,C))\left(\frac{\sum_{i\in U_X} H_{it-1}}{\sum_{i\in U_C} H_{it-1}}\right). \qquad (7.2)
\end{aligned}
$$

These entering and exiting terms will be compared with the entering and exiting terms in the two-step procedure. The Törnqvist index across skilled and unskilled labour in Equation 2.15 can be written in logs as

$$
\overline{\upsilon_t(S_t,I_t)}\ln\left(\frac{\sum_{i\in S_t} H_{it}}{\sum_{i\in S_{t-1}} H_{it-1}}\right)+\left(1-\overline{\upsilon_t(S_t,I_t)}\right)\ln\left(\frac{\sum_{i\in U_t} H_{it}}{\sum_{i\in U_{t-1}} H_{it-1}}\right).
$$

By applying Equation 2.4 for both skilled and unskilled, the Törnqvist index can *approximately* be decomposed into contributions from continuing, entering and exiting workers[10]

$$
\begin{aligned}
&\overline{\upsilon_t(S_t,I_t)}\ln\left(\frac{\sum_{i\in S_C} H_{it}}{\sum_{i\in S_C} H_{it-1}}\right)+\left(1-\overline{\upsilon_t(S_t,I_t)}\right)\ln\left(\frac{\sum_{i\in U_C} H_{it}}{\sum_{i\in U_C} H_{it-1}}\right)\\
&\quad+\overline{\upsilon_t(S_t,I_t)}\left(\frac{\sum_{i\in S_N} H_{it}}{\sum_{i\in S_C} H_{it}}\right)+\left(1-\overline{\upsilon_t(S_t,I_t)}\right)\left(\frac{\sum_{i\in U_N} H_{it}}{\sum_{i\in U_C} H_{it}}\right)\\
&\quad-\overline{\upsilon_t(S_t,I_t)}\left(\frac{\sum_{i\in S_X} H_{it-1}}{\sum_{i\in S_C} H_{it-1}}\right)-\left(1-\overline{\upsilon_t(S_t,I_t)}\right)\left(\frac{\sum_{i\in U_X} H_{it-1}}{\sum_{i\in U_C} H_{it-1}}\right). \qquad (7.3)
\end{aligned}
$$

It follows that the (log of the) entering bias from using the two-step procedure can be approximated by the difference between the entering and exiting terms in the expression above with Equation 7.1 and

[8] Since $\ln(1+z)\approx z$ for $z\approx 0$.
[9] Since $\ln(1+z)^{-1}\approx -z$ for $z\approx 0$.
[10] For example, $\ln\left(1+\frac{\sum_{i\in S_N} H_{it}}{\sum_{i\in S_C} H_{it}}\right)\approx\left(\frac{\sum_{i\in S_N} H_{it}}{\sum_{i\in S_C} H_{it}}\right)$ when the share of entering skilled workers are relatively small.

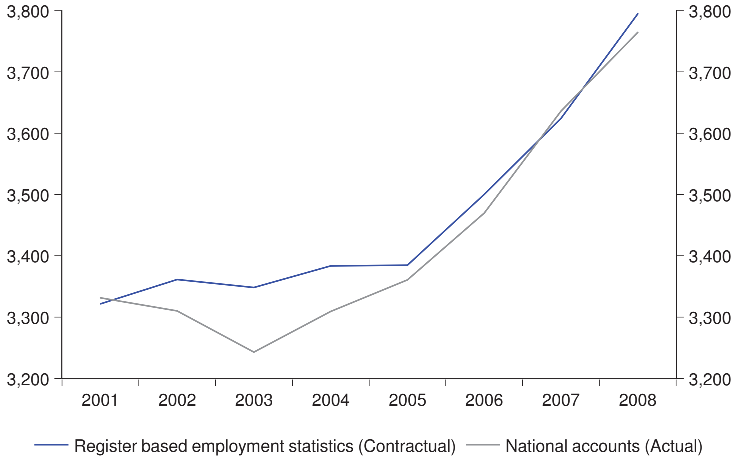

Figure 6 – Hours worked (Millions). Compares register based employment statistics for contractual hours worked with national accounts data on actual hours worked.

Equation 7.2, respectively

$$\overline{\upsilon_t(S_t, I_t)} \left(\frac{\sum_{i \in S_N} H_{it}}{\sum_{i \in S_C} H_{it}} \right) + \left(1 - \overline{\upsilon_t(S_t, I_t)}\right) \left(\frac{\sum_{i \in U_N} H_{it}}{\sum_{i \in U_C} H_{it}} \right) - \left(\psi_t(S, I) \left(\frac{\sum_{i \in S_N} H_{it}}{\sum_{i \in S_C} H_{it}} \right) + (1 - \psi_t(S, I)) \left(\frac{\sum_{i \in U_N} H_{it}}{\sum_{i \in U_C} H_{it}} \right) \right).$$

Taking the exponential of this expression yields the TWO-STEP ENTERING BIAS in Equation 2.16. Correspondingly, taking the exponential of the difference between the exiting terms in Equation 7.3 and Equation 2.16 yields the TWO-STEP EXITING BIAS in Equation 2.17.

7.1 Contractual hours as a proxy for actual hours worked

In our analysis of measuring labour services contractual hours represents a proxy for actual hours worked. The purpose of this section is to evaluate the discrepancy between contractual hours and actual hours worked and to analyse if this discrepancy increased after 2005 when productivity growth slowed.[11] To this end, we will compare data on actual and contractual hours from register based statistics, the National Accounts and the Labour Force Survey (LFS). We will also take a closer look at aggregate rates for overtime and sickness absence.

Figure 6 compares register-based employment statistics for contractual hours worked with national accounts data on actual hours worked. Both series start at about 3 300 million hours worked in 2001 and end at about 3 800 million hours worked. However, in the first couple of years, there is some discrepancy

[11] See ILO (2008) for the different concepts of hours worked.

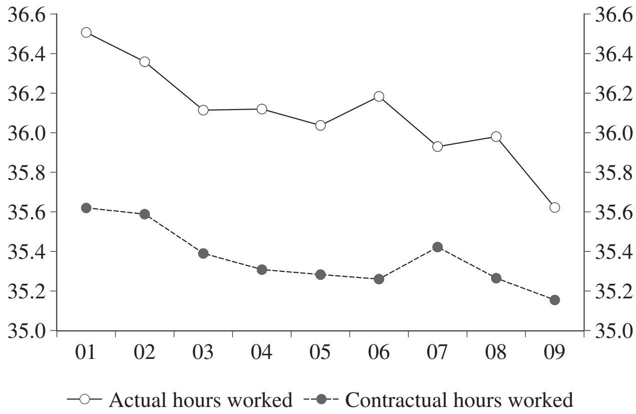

Figure 7 – Weekly actual and contractual hours worked. Mean across 3rd quarter observations from the Labour Force Survey. Workers with either actual or contractual weekly hours below 5 or higher than 90 hours are excluded. Source: Statistics Norway.

between the two series. While there is a modest increase in contractual hours worked from 2001 to 2005, the number of actual hours worked as measured by the National Accounts drops from 3 331 million hours worked in 2001 to 3 242 in 2003, before increasing to 3 360 in 2005. Although there are some discrepancies between these series in the short term, over the entire sample they show broadly the same increase in hours worked.

Figure 7 shows actual and contractual working hours. The data represents the mean across 3rd quarter observations from the LFS.[12] Workers with either actual or contractual weekly hours below 5 or higher than 90 are excluded. There has been a reduction in both weekly contractual and actual working hours from 2000 to 2009, from 35.6 to 35.2 and from 36.7 to 35.6, respectively. Although there was some variation between 2005 and 2008, neither contractual nor actual working hours changed much between these years: the mean contractual working hour was 35.3 in both 2005 and 2008 and the mean actual working hour was 36.0 in both 2005 and 2008. According to these data, the discrepancy between the two different measures of labour services in Table 3 between 2005 and 2008 can thus not be explained by differences between contractual and actual working hours.

A different way to analyse the wedge between contractual and actual working hours is to look at overtime and sickness absence rates. The overtime rate is defined as the ratio of overtime hours to contractual hours and the sickness absence rate is defined as the ratio of sickness absence hours to contractual hours. In contrast to actual hours worked, contractual hours worked excludes irregular overtime and includes absence from work such as sickness absence. To illustrate, we let actual hours worked be defined as the sum of contractual hours worked and irregular overtime hours but excluding sickness absence, as a crude approximation. Moreover,

[12]Thanks to Tom Kornstad for providing these series.

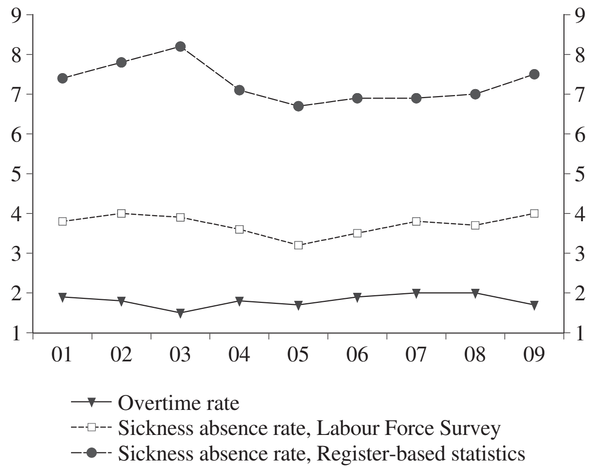

Figure 8 – Overtime and sickness absence rates. Per cent. Overtime rate is measured as the ratio of overtime hours to contractual hours. The target figure for sickness absence rate used in the Labour force survey is the number of employees who have been absent during the whole registration week in per cent of employees in total while register based statistics measure man-days lost due to own sickness as a percentage of contractual man-days. Source: Statistics Norway.

let the overtime rate be defined by the ratio of overtime hours to contractual hours and let the sickness absence rate be defined by the ratio of sickness absence hours to contractual hours. It then follows that[13]

$$\begin{aligned}&\text{Percentage change in hours worked} - \text{Percentage change in contractual hours}\\ &\qquad\approx \text{Change in overtime rate} - \text{Change in sickness absence rate.}\end{aligned}$$

The wedge between the percentage change in hours worked and contractual hours is thus the difference between the change in overtime and sickness absence rate, measured in percentage points.

Figure 8 shows overtime and sickness absence rates measured in per cent. The overtime rate has been fairly constant ranging from 1.5 per cent in 2003 to 2.0 per cent in 2007 and 2008. In 2005 the overtime rate was 1.7 per cent. The change in overtime rate from 2005 to 2008 was thus 0.3 percentage points during these four years. Two measures of sickness absence rates are shown in Figure 8. The measure used in the LFS is the number of employees who have been absent during the whole registration week in per cent of employees in total while register-based statistics measure man-days lost due to own sickness as a percentage of contractual man-days. Differences in levels between these series are due to how register-based statistics include persons on partial sick leave and also cases of sickness absence shorter than one week, which are not included in the

[13]Let H_t denote actual hours worked at time t, C_t contractual hours worked, O_t irregular overtime hours and S_t sickness absence hours, where $H_t = C_t + O_t - S_t$, and let the overtime rate (r_t^O) be defined by $r_t^O = O_t/C_t$ and the sickness absence rate (r_t^S) by $r_t^O = S_t/C_t$. It follows that $H_t = C_t(1+r_t^O - r_t^S)$. When the overtime and sickness absence rates are close to zero $H_t = C_t(1+r_t^O - r_t^S)$ can be approximated as $\ln H_t - \ln C_t \approx r_t^O - r_t^S$, since $\ln(1+z) \approx z$ for $z \approx 0$, and the first difference is given by $\Delta \ln H_t - \Delta \ln C_t \approx \Delta r_t^O - \Delta r_t^S$, which is the expression on this page.

absence rates from the LFS. In addition, the register-based statistics also take into account both the working time and the duration of the sickness absence when the portion of sickness absence is calculated. Although the levels between the two series differ, the changes in the series show broadly the same development. From 2005 to 2008 the sickness absence rate increased 0.6 percentage points from 3.2 to 3.8 per cent according to the LFS. According to register based statistics the increase was 0.3 percentage points from 6.7 to 7.0 per cent. Since increases in overtime and sickness absence rates have been small and since they have offsetting effects on the wedge between contractual and actual hours worked, the total impact from these changes is negligible.

The purpose of this section has been to evaluate contractual hours as a proxy for actual hours worked. Overall, both series show the same development from 2001 to 2008. Also, since the change in overtime rates and absence sickness rates have been very small, we conclude that it is not the wedge between these measures that explains the measured drop in productivity after 2005.

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