Discussion Papers
Statistics Norway
Research department

No. 807
April2015

Statistics Norway

Rolf Aaberge and Ugo Colombino

Labour Supply models

Statistisk sentralbyrå

Discussion Papers No. 807, April 2015
Statistics Norway, Research Department

Rolf Aaberge and Ugo Colombino

Labour Supply models

Abstract:
This paper is published as Chapter 7 of Handbook of Microsimulation Modelling edited by Cathal O'Donoghue, and issued in the series Contributions to Economic Analysis by Emerald Publishing Group. The purpose of the paper is to provide a detailed discussion in relation to the development of the field of labour supply focused microsimulation models and methodological choices.

The paper identifies three methodologies for modelling labour supply
- The Reduced Form Approach
- The Structural “Marginalist” Approach
- The Random Utility Maximisation Approach

The paper considers issues associated with the reliability of structural models relative to (ex-post) experimental or quasi-experimental analysis. Recognising however the need to undertake ex-ante analysis, it questions, whether there are alternatives to structural models and how can we evaluate structural models and how they are compared with other approaches.

The paper then describes approaches to utilising these models for policy simulation in terms of producing and interpreting simulation outcomes, outlining an extensive literature of policy analyses utilising the approach. Also labour supply is not only central to modelling behavioural response but also modelling optimal tax-benefit systems, with a focus on a computational approach, given some of the challenges of the theoretical approach. Combining labour supply results with welfare functions enables the social evaluation of policy simulations. Combining welfare functions and labour supply functions, the chapter then identifies how to model socially optimal income taxation.

Keywords: D10, D31, H21, H24, J20

JEL classification: inequality, poverty, deprivation, multidimensional well-being, capabilities and functionings

Address: Rolf Aaberge, Statistics Norway, Research Department and ESOP, Department of Economics, University of Oslo. E-mail: Rolf.Aaberge@ssb.no

Ugo Colombino, Department of Economics, University of Torino,
E-mail: ugo.colombino@unito.it

Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

ISSN 1892-753X (electronic)

Sammendrag

Artikkelen «Labour Supply Models» inngår som kapittel 7 i Handbook of Microsimulation Modelling redigert av Cathal O'Donoghue, og blir utgitt i serien Contributions to Economic Analysis av Emerald Publishing Group. Artikkelen diskuterer modellering av arbeidstilbud i mikrokonomiske simuleringsmodeller.

1. Introduction

1.1. Microsimulation meets microeconometrics

The encounter between microeconometric models of labour supply and the microsimulation approach is the result of a long process. Large microsimulation models, as originally proposed by Orcutt (1957), were meant to be behavioural, although not structural: behavioural responses were typically empirical 'reduced form' approximations, with little foundations on standard microeconomic theory (Orcutt, Greenberger, Korbel, & Rivlin, 1961). A motivation for the reduced form approach was probably a certain degree of mistrust for mainstream economic theory on the part of Orcutt and his associates. Shortly after, large microsimulation models became increasingly popular at the institutional and policy making level. For good reasons, the main research and implementation efforts were initially focussed upon the quality of data, the accuracy of the accounting relationships and representativeness of the results. For many years, the active microsimulation community has considered behavioural responses (and in particular labour supply) either unimportant or unreliable or hard to interpret. Later on, however, various motivations have progressively contributed to a more positive attitude towards the inclusion of labour supply responses into microsimulation models:

(i) The increasing policy interest in tax-benefit reforms, their effect on both distribution and efficiency and the realization that policy analysis requires structural models (a long-standing message from Marschak, 1953, possibly revived by Lucas, 1976).

(ii) The development of the first econometric approaches appropriate to deal with the complexities introduced by tax-benefit systems into the opportunity sets (e.g. Burtless & Hausman, 1978; Heckman, 1974b; Wales & Woodland, 1979).
(iii) The use of microsimulation techniques in order to compute labour supply responses, starting approximately around the early 1980s (e.g. Arrufat & Zabalza, 1986; Zabalza, 1983).[1]
(iv) The development of discrete choice labour supply models (starting with Zabalza, Pissarides, & Barton, 1980) and of models based on (various versions of) the Random Utility approach (Aaberge, Dagsvik, & Strøm, 1995; Van Soest, 1995). Compared to the models of the first structural generation (as of point (ii) above), Random Utility models are more flexible, potentially able to account for any complexity of the budget and opportunity sets and easier to be linked to large microsimulation projects.
(v) The realization of the crucial importance of heterogeneous behavioural responses in shaping the effects of policies (e.g. Aaberge, Colombino, & Strøm, 1999).

From around the second half of the 1990s a (cautious) introduction of labour supply responses into large microsimulation models begins. Klevmarken (1997) provides a report on early efforts towards that purpose. Creedy and Duncan (2002), Creedy and Kalb (2005b), Bourguignon and Spadaro (2006) and Bargain and Peichl (2013) survey more recent developments. O'Donoghue (2001) and Li and O'Donoghue (2013) provide a general discussion on microsimulation models.

1.2. A frame of reference

In this chapter we focus on microsimulation used for ex ante policy evaluation. To this end, it is required to consider a modelling framework where behavioural responses are given a *structural* representation, that is an economic model that allows separation of preferences (assumed to be invariant with respect to policy changes) and policy parameters (Hurwicz, 1962). Hereafter we sketch a simple framework within which most of what follows can be interpreted. We consider a sample of agents (households or individuals) indexed by $i = 1, \ldots, N$. Agent i faces an opportunity (or feasible) set B_i. The elements of B_i are vectors x that measure various

[1] At least up to the end of the 1970s, the typical procedure consisted of evaluating elasticities or policy effects with reference to the 'average' or in some sense 'representative' household. Even the path-breaking contributions to structural labour supply modelling (e.g. Hausman, 1985a, 1985b; Heckman, 1974a, 1974b) adopted the 'average household' approach or computed behavioural responses for different 'types' of households.

dimensions of labour supply (e.g. hours of work, sector of occupation, etc.) and consumption or investment choices (e.g. transportation, child care, training, fertility, etc.). In the simplest case $x = (c, h)$, where h = hours of work and c = net available income. The corresponding minimal representation of B_i is a budget set, e.g. $B_i = \{(c,h) : c \leq f(wh, I)\}$, where w denotes the hourly gross wage rate and $f(.,.)$ is the tax-benefit rule that transforms gross labour earnings wh and unearned income I into net available income c. More generally, there might be constraints on the feasible values of h, on the way gross earnings are generated depending on labour contracts, etc. The agents adopt decision rules D_i that, given B_i, produce the choices x_i. The crucial assumptions of structural microeconometrics are:

(i) the decision rules D_i can be identified using observations on the choice x_i and on the opportunity set B_i;
(ii) the decision rules D_i are invariant with respect to policies (Hurwicz, 1962), that is changes in B_i, so that they can be applied to a different, possibly hypothetical B_i^P in order to simulate the choices x_i^P.

Microsimulation plays therefore two roles: first (arithmetic step), it computes the new or reformed B_i^P (induced, e.g. by a reformed tax-benefit rule f^P); second (behavioural step), it produces the new choices x_i^P.

The most common representation of the decision rule is the constrained maximization of a utility function $U_i(x)$ that represents the agents' preferences: $x_i = \max_{x \in B_i} U_i(x)$. Once the preferences $U_i(\cdot)$ – and possibly some policy-invariant parameters of B_i – are estimated, the effects of a policy are simulated by $x_i^P = \max_{x \in B_i^P} U_i(x)$. The policy can be anything that can be represented as a change in B_i. A crucial contribution of microsimulation is that the agents' heterogeneity allowed for in the microeconometric model can be fully exploited when producing, interpreting and evaluating the results. Thus, these models are very useful in order to simulate changes in the tax-benefit rule: reforms of personal income taxation, of child benefits, of income support mechanisms, etc. Other reforms that can be simulated might concern changes in the admissible values of h, changes in the rules (e.g. contracts) by which gross labour earnings are generated given h, etc.

Within the general framework sketched above, one can adopt many different approaches to representing the preferences and the opportunity sets, to the assumptions concerning the agents' heterogeneity, etc. In Section 2 we discuss the main approaches to developing models of labour supply. Section 3 illustrates the use of these models in ex ante policy evaluations. The fact that microsimulation can produce highly disaggregated and multidimensional results on the one hand contribute to the richness of the policy evaluation, on the other hand calls for the development of synthetic indexes in order to guide the comparison between alternative policies: therefore, Section 4 is devoted to social evaluation of the simulation results. Section 5 focuses on recent exercises devoted to identifying optimal tax-benefit rules. Section 6 contains the conclusions and comments on future directions.

2. Modelling strategies

In this section we illustrate the main strategies that are, or have been, adopted in modelling labour supply. They are collected under three labels: the 'Reduced-form' approach, the Structural 'Marginalist' (Kuhn–Tucker) approach and the Random Utility Maximization (RUM) approach. Although the latter approach is by now the dominating paradigm, we devote some attention also to the previous two approaches since it is useful to understand the problems they encountered and why the RUM approach eventually prevailed.

We focus upon how to develop models that permit a flexible and realistic representation of complicated budget and opportunity sets and allow for a rich representation of households' and opportunities' heterogeneity. This is the most crucial problem to be faced when labour supply modelling is instrumental to policy evaluation, possibly within the context of large microsimulation projects.

In what follows we will adopt hours of work as a conventional measure of labour supply. However, most of what we write can easily be transferred to other dimensions or measures of labour supply. This observation also applies to the 'taxable income' approach (e.g. Saez, Slemrod, & Giertz, 2012), which is an important complementary perspective but does not raise new issues from the point-of-view of the modelling strategy.

Useful surveys that also cover many of the topics treated in this section are provided by Creedy and Duncan (2002), Blundell and MaCurdy (1999), Blundell (2012), Creedy and Kalb (2005b), Blundell, MaCurdy, and Meghir (2007), Meghir and Phillips (2008), Aaberge, Colombino, and Wennemo (2009), Keane (2011) and Dagsvik, Jia, Kornstad, and Thoresen (2014).

2.1. The 'Reduced Form' approach

Up to the early 1970s, empirical studies of labour supply typically adopt a 'reduced form' labour supply function

$$h = l(w, I) + e \tag{1}$$

where h denotes the observed hours of work (or some other convenient measure of labour supply), $l(w, I)$ is a function of an exogenous net wage rate w and of an exogenous net income I and e is a random variable that account for the effect of other unobserved variables. We refer to expression (1) as a reduced form since it simply embodies the hypothesis that labour supply depends on w and I, but in general it is not a correct (structural) representation of that dependence, especially when corner solutions and non-linearities of the budget constraint are present. Corner solutions (i.e. $h = 0$) are typically ignored or treated as interior solutions. Income taxes are ignored or somehow accounted for by using the average net wage rate. Examples of this type of analysis include Kosters (1966, 1969) and Bowen and Finegan (1969). Although the authors were obviously aware that h could be interpreted as the solution of a constrained utility maximization problem (with corresponding first-order Lagrange or Kuhn–Tucker conditions characterizing the solution), this theoretical background was not considered to be useful or important. In fact, if the budget constraint is linear and the solution is assumed to be interior, constrained utility maximization theory simply tells you that h will be a (linearly homogeneous) function of w and I.

Starting in the late 1960s, the importance of non-linear budget constraints – and of the theory appropriate to treat them – attracts more attention. The reason seems to be twofold. On the one hand, the newborn theory of optimal taxation suggests that important efficiency and equity effects stem from the way taxes on labour earning are designed (Mirrlees, 1971). On the other hand, at the policy level a strong interest emerges in evaluating various welfare and 'anti-poverty' programs. These policies introduce complications (non-linearities, non-convexities) into the budget sets faced by the target population, which are – in general – impossible to be adequately addressed within the reduced form approach. Various contributions tried to circumvent the difficulties with more or less ad-hoc procedures (e.g. Hall, 1973; Rosen, 1976). However, at a certain point it had to be realized that an appropriate treatment of non-linear budget constraints requires a 'structural' approach, that is a separate identification of preferences and constraints.

2.2. *The structural 'Marginalist' approach*

We denote as 'marginalist' the approach that works through the conditions for a constrained maximum of the utility function, conditions that usually involve marginal variations. Heckman's (1974b) is perhaps the first empirical paper that explicitly uses the conditions characterizing the solution of utility maximization subject to a non-linear budget constraint, with the purpose of addressing a non-standard policy evaluation problem. The policy problem addressed by Heckman (1974b) is the evaluation of a child-related welfare policy that introduces significant complications in the budget set. The author observes that in order to make such evaluation one has to estimate the preferences as separated from the constraints: '*The essence of the problem involves utility comparisons between two or more discrete alternatives. Such comparisons inherently require information about consumer preferences in a way not easily obtained from ordinary labor-supply functions*' (Heckman, 1974b, p. S136). Moreover '*… the ability to make* … (the separation between preferences and constraints) ... *is* *less important if we are willing to make the conventional assumption that wage rates are independent of hours of work ... but becomes quite important when we acknowledge the existence of progressive taxation, welfare regulations, and time and money costs of work*' (Heckman, 1974b, p. S142). The author starts from an empirical specification of the compensated labour supply function. From it, he integrates back to the indirect utility function and the uncompensated labour supply function. The estimates of the latter also identify the direct utility function. This procedure accounts for the non-linearities of the current budget constraint and permits to simulate the effects of reformed (non-linear) budget constraints. Heckman's presentation of his method was probably perceived as too specific of the addressed policy issue and to the best of our knowledge was not adopted in other contributions. However, besides the policy application, the paper in fact revives the message left by Marschak (1953) and might be considered as a 'manifesto' for a structural approach to empirical labour supply analysis with an appropriate representation of non-linear budget constraints, corner solutions and the identification of preferences as separated by constraints. The methodological importance of Heckman (1974b) is analogous to Tobin's (1958), a path-breaking contribution to linking a microeconometric analysis to the requirements of a non-standard opportunity set due to binding non-negativity constraints.[2] It is interesting to observe that, almost simultaneously, an analogous (although less constructive than Heckman's) 'manifesto' for structural analysis appears in the macroeconometrics literature (Lucas, 1976). A few years later, Burtless and Hausman (1978), Wales and Woodland (1979) and Hausman (1979, 1985b) addressed essentially the same type of problems and developed a method specifically appropriate for piecewise linear budget constraints.

The structural marginalist approach can be represented by any of the following four alternative ways:

(1) Specify a direct utility $u(c,h)$ function and solve

$$\begin{aligned}&\max_{c,h}\ u(c,h)\\&\text{s.t.}\\&c = wh + I\end{aligned} \tag{2}$$

to get a labour supply function $h(w,I)$, where

$c =$ consumption (=income)

$h =$ hours of work

[2] In his comment to Heckman's paper, Rosen (1974) writes: '... *Heckman clearly has opened a lot of new territory in the economics of labour supply .... Future studies in this area are bound to be affected for many years to come by Heckman's paper*.'

w = wage rate

I = exogenous income.

Wales and Woodland (1979), Zabalza (1983) and Arrufat and Zabalza (1986) adopt this approach starting from a CES utility function.

(2) Specify the indirect utility function $u^*(w,I)$ and obtain the labour supply function through Roy's Theorem: $h(w,I) = \frac{\partial u^*(w,I)}{\partial w} / \frac{\partial u^*(w,I)}{\partial I}$. An example is provided by Wales and Woodland (1976), who start from an Indirect Translog utility function.

(3) Specify the uncompensated labour supply function $h(w,I)$ – for example a linear function – remembering that if we want it to be consistent with the model of constrained utility maximization, it will have to satisfy Slutsky's conditions. In fact, this is the approach followed for example by Burtless and Hausman (1978) and Hausman (1980, 1985a), where linear or log-linear labour supply functions are specified. The indirect utility function can be retrieved by 'integrating' the Roy's identity (treated as a differential equation). The dual relationship between the indirect and the direct utility function allows, if needed, to recover the latter from the former.

(4) Specify the compensated labour supply function $\tilde{h}(w,u)$ and use Shephard's Lemma $-\frac{\partial e(w,u)}{\partial w} = \tilde{h}(w,u)$ to recover (by integration) the expenditure function, that is the minimum unearned income needed to attain the utility level u, given the wage rate w. Then, by setting $e(w,u)=I$ we obtain (by inversion) the indirect utility function $u = u^*(w,I)$. Roy's Theorem can then be applied to obtain the uncompensated labour supply function $h(w,I)$. The method used by Heckman (1974b) is in fact a variant of this fourth way of proceeding (Rosen, 1974).

All the above four routes guarantee a strict consistency between the labour supply functions and the preferences. Note that procedure (1) directly applies the Lagrange or Kuhn–Tucker conditions charactering the maximization of a well-behaved utility function subject to the budget constraint, whereas procedures (2), (3) and (4) work through an indirect application of the Kuhn–Tucker conditions. The advantage of the structural 'marginalist' modelling approach – independent of whether procedure (1), (2), (3) or (4) is used – is that it allows identification of consumption-leisure preferences. Thus, given the estimates of the labour supply function one can also identify the preferences. It is important to remember that what can be directly estimated with observed data is just the uncompensated labour supply $h(w,I)$. Preferences can be recovered either because $h(w,I)$ is obtained by the maximization of a direct utility function $u(h,c)$ – as in procedure (1) – or because we exploit Roy's Theorem or Shephard's Lemma as in procedures (2), (3) and (4). Given the preferences, one can simulate new choices under a reformed budget constraint. In choosing the (direct or indirect) specification of the preferences or of the labour supply function, typically a trade-off between flexibility and analytical tractability has to be faced: in this respect, a very useful reference is Stern (1986).

2.2.1. Dealing with corner solutions

If the analysis includes the possibility of corner solutions, the optimal labour supply h^* solves

$$\begin{aligned} &\max_{c,h} \quad u(c,h) \\ &\text{s.t.} \\ &c = wh + I \\ &h \geq 0 \end{aligned} \tag{3}$$

If $h(w,I)$ solves $\max\limits_{c,h} u(c,h)$ s.t. $c = wh + I$ then it is easily verified that

$$h^* = \begin{cases} h(w,I) \text{ if } h(w,I) > 0 \\ 0 \qquad\quad h(w,I) \leq 0 \end{cases} \tag{4}$$

In view of the empirical analysis, usually we must account for unobserved (by the analyst) heterogeneity of preferences and/or for measurement/ optimization errors.

Heterogeneity of preferences can be introduced into the labour supply function by assuming that one or more of its parameters depend on observed and/or unobserved variables. For example, $h(w,I)$ might be specified as:

$$h(w,I) = \eta(w,I) + \varepsilon_1 \tag{5}$$

where $\eta(w,I)$ is a function of observed variables and ε_1 is a random variable that accounts for unobserved heterogeneity of preferences. If ε_1 is the only source of randomness from the analyst's point-of-view, then the analyst is assuming that the observed value h is uniquely generated by the solution of problem (3). During the pre-Heckman eve, it was common to simply use OLS on the sample with positive values of h.

Heckman (1974a) adopts a Tobit-like approach: assuming $\varepsilon_1 \sim N(0,\sigma^2)$ the contribution to the likelihood is $\phi \ \frac{h-\eta\ (w,I)}{\sigma}$ if the household works h hours and $\Phi \ \frac{-\eta(w,I)}{\sigma}$ if the household does not work. Alternatively, one might use Selection-Corrected Least Squares as elaborated by Heckman (1979).

2.2.2. Dealing with unobserved wage rates

Parallel to the corner solutions, we face the problem of unobserved wage rates for those who do not work. One could use a two-equation 'Tobit' as in Heckman (1974a) or adopt a multi-step selection-correction approach (Heckman, 1979), namely: estimate a wage equation (corrected for non-random sample selection) on the working sub-sample; compute the systematic part of the wage equation and impute it to everyone. The random error of the wage equation can be accounted for in different ways depending on the functional form of $\eta(w,I)$ and on the assumptions made on correlation between the wage equation and the hours equation.

2.2.3. *Optimization errors, involuntary unemployment, quantity constraints*

Measurement or optimization errors can also be accounted for. For example, we might assume that what we observe is

$$h = h^* + \varepsilon_2 \tag{6}$$

where h^* is the desired choice and ε_2 is a random variable that accounts for the analyst's inability to accurately measure the choice made and/or for the decision maker's inability to implement the optimal choice. The measurement/optimization error can be specified in different ways depending on what we assume about the process that generates it. For example, a common assumption is that if desired hours are positive then it can happen that actual hours are different and possibly even equal to 0; however, if desired hours are 0, then also actual hours are 0. According to these assumptions, we then have:

$$h = \begin{cases} \eta(w,I) + \varepsilon_1 + \varepsilon_2 & \text{if } \eta(w,I) + \varepsilon_1 > 0 \text{ and } \eta(w,I) + \varepsilon_1 + \varepsilon_2 > 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Specification (7) also accounts for involuntary unemployment, that is the event: $\eta(w,I)+\varepsilon_1 > 0$ and $\eta(w,I)+\varepsilon_1 + \varepsilon_2 < 0$. Alternatively, one could specify a complementary equation that generates involuntary unemployment, for example as in Blundell, Ham, and Meghir (1987).

A different perspective to look at the possible divergence between optimal and observed hours is to think of workers as 'captive' to certain choices, as in Harris and Duncan (2002). Some authors, within the 'marginalist' approach, have exploited datasets containing explicit information on quantity constraints in the opportunity sets, for example Ham (1982), Colombino and Zabalza (1982), Colombino (1985), Altonji and Paxson (1988) and Ilmakunnas and Pudney (1990).

2.2.4. *Non-linear budget constraints*

Let us consider the following modification of problem (2):

$$\begin{aligned} &\max_{c,h}\ u(c,h) \\ &\text{s.t.} \\ &c = \\ &f\ (w\ \ h\ \ , I) \end{aligned} \tag{8}$$

Here the function $f(.,.)$ represents the tax-benefit rule, that is the rule according to which the gross earnings wh and the exogenous gross income I are turned into net available income c (=consumption). If $u(.,.)$ and $f(.,.)$ are differentiable, $u(.,.)$ is quasi-concave and $f(.,.)$ is concave in h (i.e. the budget set is convex), then the following condition (together with the budget constraint) is necessary and sufficient for a (interior) solution of problem (8):

$$-\frac{\partial u}{\partial h} \Big/ \frac{\partial u}{\partial c} = \frac{\partial f}{\partial h} \tag{9}$$

The condition is not sufficient anymore if $u(.,.)$ is not quasi-concave and/or the budget set is not concave. In these cases, the sufficient conditions for identifying a solution might become very cumbersome and unpractical to use in applied research. Of course, also the non-differentiability of $f(.,.)$ creates problems. However, most actual or reformed tax-benefit rules belong to the piecewise linear family, that is they can be represented as a combination of linear segments. Starting with Burtless and Hausman (1978), a procedure has been designed for identifying the solution on convex budged sets defined by piecewise linear constraints. Let us suppose that as long as the consumer's earnings do not exceed a certain amount E, she is not required to pay taxes on her earnings. However, for every Euro of earnings above E she has to pay taxes according to a marginal tax rate τ. The first segment has slope w, the second segment has slope equal to $w(1-\tau)$. It is useful to define

$H = (E/w)$=hours of work corresponding to the 'kink',

and $I + E - w(1-\tau)H = I + E\tau$ = 'virtual' exogenous income associated to the second segment (i.e. the intercept of the line that lies on the second segment). Note that the exogenous income associated to the first segment is instead I, which is assumed to be tax-free.

Then the problem is:

$$\begin{aligned} &\max_{c,h} u(c,h) \\ &\text{s.t.} \\ &c \le I + wh \\ &c \le I + E\tau + w(1-\tau)h \\ &h \ge 0 \end{aligned} \tag{10}$$

Now define $h(n,q)$ as the 'virtual' labour supply given a wage rate n and an exogenous income q, that is the value of h that solves the problem

$$\begin{aligned} &\max_{c,h} u(c,h) \\ &\text{s.t.} \\ &c = q + nh \end{aligned} \tag{11}$$

The solution to problem (10) is then characterized as follows:

$$h^* = \begin{cases} 0 \text{ if } h(w, I) \le 0 \\ h(w, I) \text{ if } 0 < h(w, I) < H \\ H \text{ if } h(w, I) \ge H \text{ and } h(w(1-\tau), I + E\tau) \le H \\ h\ (w(1-\tau), I + E\tau) \text{ if } h\ (w(1-\tau), I + E\tau) > H \end{cases} \tag{12}$$

The same procedure can be used to characterize the solution when the problem involves more than two segments and can be extended (with due modifications) to cases with non-convex budget sets. The method originally proposed by Heckman (1974b) also adopts a very similar logic.

The structural 'marginalist' approach can be extended in many directions. Instead of representing the budget constraint with a combination of linear segments (which in most cases in fact correspond to the real system), one could use a smooth non-linear approximation (e.g. Flood & MaCurdy, 1992). Random components capturing preference heterogeneity and/or measurement/optimization errors can be specified in a way similar to what illustrated in the linear budget constraint case. In principle it can also be extended to cover simultaneous household decisions, although most of the applications treat unconditional husband's decisions or wife's decisions conditional on husband's ones. Useful presentations are provided by Hausman (1979, 1985a), Moffitt (1986), Heckman and MaCurdy (1986) and Blundell and MaCurdy (1999). Duncan and Stark (2000) have developed an algorithm for generating piecewise linear budget constraints for estimation or simulation purposes. Applications to different countries and different tax-benefit rules and reforms include Burtless and Hausman (1978), Hausman (1979, 1980, 1985a, 1985b), Blomquist (1983), Zabalza (1983), Arrufat and Zabalza (1986), Blomquist and Hansson-Brusewitz (1990), Bourguignon and Magnac (1990), Colombino and Del Boca (1990), MaCurdy, Green, and Paarsch (1993), Triest (1990), Van Soest, Woittiez, and Kapteyn (1990) and Bloemen and Kapteyn (2008).[3] More general surveys, also covering contributions that belong to the structural 'marginalist' approach, include Blundell and MaCurdy (1999), Blundell et al. (2007), Meghir and Phillips (2008) and Keane (2011).

In the second half of the 1980s the structural 'marginalist' approach was thought to be a dominating paradigm and a special number of the Journal of Human Resources (1990) was dedicated to applications of this method to various countries. The same issue of the JHR, however, also collects most of the critiques that eventually led to adopting alternative approaches. The method proposed by Heckman as well as the method

[3] Hausman and Wise (1980), although applied to the demand for housing and not to labour supply, is a very clear illustration of how the structural marginalist approach can be applied to non-convex budget sets.

proposed by Hausman and co-authors in practice turn out to be not so easily applicable to problems that are more complicated than those for which they were originally exemplified. First, the application is general and straightforward with convex budget sets (e.g. those generated by progressive taxation) and a two-good case (e.g. leisure and consumption in the individual labour supply model). Instead, it is more case-specific and tends to become computationally cumbersome when the decision makers face non-convex budget sets and/or when more than two goods are choice variables (e.g. in the case of a many-person household). Second, in view of the computational problems, the above approach essentially forces the researcher to choose relatively simple specifications for the utility function or the labour supply functions. Third, computational and statistical consistency of ML estimation of the model requires imposing a priori the quasi-concavity of the utility function (e.g. Kapteyn, Kooreman, & van Soest, 1990; MaCurdy et al., 1993).[4]

2.3. *The random utility maximization approach*

As a response to the problems mentioned above, since the early 1990s researchers have made use of another innovative research effort which matured in the first half of the 1970s, that is the random utility maximization (RUM) model – or some variations of it – developed by McFadden (1974, 1984). The crucial advantage of this approach is that the solution of the utility maximization problem is represented in terms of comparisons of absolute values of utility rather than in terms of marginal variations of utility, and it is not affected by the specification of the utility function nor of the tax-benefit rule. This approach is very convenient when compared to the previous ones, since it does not require going through complicated Kuhn–Tucker conditions involving derivatives of the utility function and of the budget constraints. Therefore, it is not affected by the complexity of the rule that defines the budget set or by how many goods are contained in the utility function. Equally important, the deterministic part of the utility function can be specified in a very flexible way without worrying about the computational problems. The most popular version adopts the Extreme Value distribution for the stochastic component, which leads to an easy and intuitive expression for the probability that any particular alternative is chosen (i.e. the Multinomial or Conditional Logit model).

---

[4] The simultaneous household decision model Hausman and Ruud (1984) has essentially remained an isolated contribution. On the difficulties of applying the 'marginalist' approach outside the simplest scenarios, see also Bloemen and Kapteyn (2008).

2.3.1. *The discrete choice model*

This approach essentially consists in representing the budget set with a set of discrete alternatives or jobs. Early and path-breaking contributions include Zabalza et al. (1980), where labour supply is represented in terms of probabilities of choosing alternative hours of work or alternative jobs. This contribution, however, is essentially an ordinal probit analysis. Especially in view of modelling simultaneous household decisions, the Conditional Multinomial Logit model is much more convenient. This is the line chosen by Van Soest (1995). Although this very influential contribution can be classified as belonging to the RUM family, we denote it more specifically as a Discrete Choice (DC) model. First, the discreteness of the opportunity set is a distinctive feature of it (this is not the case in general for RUM models). Second, the random term that generates the probabilistic choices is given an eclectic interpretation that includes both the RUM-McFadden (1974, 1984) interpretation and the optimization error interpretation (the latter leading to a non-random utility model). Besides Van Soest (1995), many contributions have adopted the DC model during the last two decades. Among others: Duncan and Giles (1996), Bingley and Walker (1997), Blundell, Duncan, McCrae, and Meghir (2000), Van Soest, Das, and Gong (2002), Creedy, Kalb, and Scutella (2006), Haan and Steiner (2005), Brewer, Duncan, Shephard, and Suarez (2006), Labeaga, Oliver, and Spadaro (2008), Fuest, Peichl, and Schaefer (2008), Haan and Wrohlich (2011), Blundell and Shephard (2012), Bargain, Decoster, et al. (2013), and Bargain, Orsini, and Peichl (2014).

The DC model typically treats (also) couples with simultaneous decisions of the two partners, but in order to keep the illustration simple, we will discuss the singles case below: the extension to couples is straightforward. The household chooses among $H+1$ alternatives or $h=0, 1, \ldots, T$. The utility derived from alternative h is first defined as non-stochastic, $v(f(wh,I),h)$, where w is the fixed (individual-specific) gross wage rate, I is the exogenous income and $f(.,.)$ is the tax-transfer rule that transforms gross incomes into net available income. In order to model the observed hours of work as the result of a probabilistic process, a random variable ε is added to the previously defined utility function: $v(f(wh,I),h)+\varepsilon$. As mentioned above, the random term is typically given two different interpretations (e.g. Van Soest, 1995): (i) the utility contribution of unobserved characteristics of the alternative choices; (ii) a measurement/optimization error. Interpretation (i) is compatible with the classic RUM interpretation and implies that the household are observed as choosing exactly what they prefer, and what they prefer is decided on the basis of $v(f(wh,I),h)+\varepsilon$. Interpretation (ii) instead implies that the household's preference are measured by $v(f(wh,I),h)$ but the alternative to which they are matched does not maximize $v(f(wh,I),h)$ but rather $v(f(wh,I),h)+\varepsilon$: this might happen because they make errors or because some other unexpected process displaces them from the preferred choices. The two interpretations, in principle, have also different implications in view of the simulation and of the welfare evaluation. The contributions adopting the DC approach stress the importance of a very flexible specification of $v(f(wh,I),h)$ and of checking for its quasi-concavity (e.g. Van Soest, 1995; Van Soest et al., 2002). This focus of attention suggests that this approach tends to consider $v(f(wh,I),h)$ as the true utility function and ε as a measurement/optimization error.[5] Consistently, preference heterogeneity is preferably introduced through random preference parameters.

By assuming that ε is i.i.d. Type I Extreme Value, one gets the Multinomial Logit or Conditional Logit expression for the probability that the household is observed working h hours:[6]

$$P(h) = \frac{\exp\{v(f(wh,I),h)\}}{\sum_{y=0}^{T}\exp\{v(f(wy,I),y)\}} \tag{13}$$

Model (13) usually does not fit labour supply data very well. For example van Soest (1995) notes that the model over-predicts the number of people working part-time. More generally, certain types of jobs might dif- fer according to a number of systematic factors that are not accounted for by the observed variables contained in v: (a) availability or density of job-types; (b) fixed costs; (c) search costs; (d) systematic utility components. In order to account for these factors the following 'dummies refinement' can be adopted. Let us define subsets $S_0, \ldots, S_L$ of the set $(0, 1, \ldots, H)$. Clearly, the definition of the subsets should reflect some hypothesis upon the differences between the values of h with respect to the factors (a) and (b) mentioned above. Now we specify the choice probability as follows:

$$P(h) = \frac{\exp\{v(f(wh,I),h) + \sum_{\ell}\gamma_{\ell}1(h\in S_{\ell})\}}{\sum_{y=0}^{T}\exp\{v(f(wy,I),y) + \sum_{\ell}\gamma_{\ell}1(y\in S_{\ell})\}} \tag{14}$$

[5] A motivation for interpreting ε as a measurement/optimization error in DC models is the relatively small number of values of h that are typically allowed to belong to the opportunity set, in many cases just three (non-participation, part-time and full-time). Since the observed distribution of hours worked is much more dispersed, it makes sense to allow for a measurement/optimization error.

[6] The derivation of the Conditional Logit expression for utility maximization under the assumption that the utility random components are i.i.d. Type I extreme value distributed is due to McFadden (1974). It is conventional to call Conditional Logit a Multinomial Logit model with generic attributes (i.e. attributes – like hours or income – whose values vary across alternatives).

where $\mathbf{1}(e)=1$ iff e is true. Many papers have adopted this refinement, for example Van Soest (1995), Callan and Van Soest (1996) and Kalb (2000) among others. Aaberge et al. (1995, 1999), Dagsvik and Strøm (2006), Colombino, Locatelli, Narazani, and O'Donoghue (2010) and Colombino (2013) also implement a similar procedure, which however is based on a specific structural interpretation of the dummies and of their coefficients (see expressions (21) and (22)). An alternative adjustment consists of imputing a monetary cost (or benefit) to some ranges of work hours:

$$P(h) = \frac{\exp\ v(f(wh, I) + \sum_{\ell} c_{\ell} 1(h \in S_{\ell}), h)}{\sum_{y=0}^{T} \exp\ v(f(wy, I) + \sum_{\ell} c_{\ell} 1(y \in S_{\ell}), y)} \tag{15}$$

A popular specification of the (15)-type is interpreted as accounting for fixed costs of working c (e.g. Duncan & Harris, 2002; see also the survey by Blundell et al., 2007).

2.3.2. *The random utility—Random opportunities model*

The Random Utility—Random Opportunities (RURO) model is an extension of McFadden's RUM model. The utility is assumed to be of the following form:

$$U(f(wh, I), h, j) = v(f(wh, I), h) + \varepsilon(w, h, j) \tag{16}$$

where h is hours of work, w is the wage rate, I is the exogenous income, f is a tax-transfer function that transforms gross incomes into net income, j is a variable that captures other job and/or individual characteristics and ε is a random variable that varies across market and non-market alternatives.

A first difference with respect to the DC model is that the utility function is directly specified as stochastic. The random component is interpreted as in McFadden's (1974) presentations of the Conditional Logit model: besides the observed characteristics, there are other characteristics j of the job or of the household-job match that are observed by the household but not by the econometrician. Commuting time or required skill (when not observed by the analyst) are possible examples of the characteristics captured by j. Their effect upon utility is captured by $\varepsilon(w,h,j)$.

Second, the households maximize their utility by choosing not simply hours but rather opportunities ('jobs') defined by hours of work h, wage rates w (which can change across jobs for the same household) and other unobserved (by the analyst) attributes j. In the DC model, the households' choices (how many hours of work) are analogous to the choices of a consumer deciding how many units of a consumption good (like meat, milk or gasoline) to buy every week. In the RURO model, the household is closer to the McFadden's commuter choosing among car, train or the

BART shuttle when travelling along the San Francisco Bay (Domencich & McFadden, 1975) or to the McFadden's household choosing among different apartment in different locations (McFadden, 1978).

Third, besides not observing the other job characteristics j, the analyst does not know exactly which and how many jobs are contained in the household opportunity set; therefore the opportunity set can be seen as random from the analyst's viewpoint. The opportunity set will in general contain more than one job of the same (w,h) type. These jobs will differ depending on the value of other unobserved (by the analyst) attributes. This implies that the number (or the density) of jobs belonging to the different types will plays a crucial role in the model.

In Aaberge et al. (1995) the range of values of (w,h) is assumed to be continuous. Let B be the set of admissible values of (w,h) and $p(x,y)$ the density of jobs of type (x,y). The household chooses h and j so as to maximize $v(f(wh,I),h)+\varepsilon(j)$. Then it turns out that we get the (continuous) conditional logit expression for the probability density function of a (w,h) choice:

$$\varphi(w,h) = \frac{\exp\left\{v(f(wh,I),h)\right\}p(w,h)}{\int_{(x,y)\in B} \exp\left\{v(f(xy,I),y)\right\}\, p(x,y)\,\mathrm{d}x\,\mathrm{d}y} \tag{17}$$

Expression (17) is based on Dagsvik (1994). The model is close to the continuous spatial model developed by Ben-Akiva and Watanatada (1981). It can also be seen as an extension of the McFadden's Conditional Logit model where the systematic utility of a job type (w,h) is 'weighted' by the number of jobs of that type available in the opportunity set. On the foundations and various applications of RURO models, see also Dagsvik (2000) and Dagsvik et al. (2014). Aaberge et al. (1999) formally derive a discrete version of model (17):

$$\varphi(w,h) = \frac{\exp\left\{v(f(wh,I),h)\right\}\, p(w,h)}{\sum_{(x,y)\in B} \exp\left\{v(f(xy,I),y)\right\}p(x,y)} \tag{19}$$

The discrete version can be interpreted either as a more realistic representation or as computational simplification of the continuous version.[7]

[7] Tummers and Woittiez (1991) and Dickens and Lundberg (1993) develop labour supply models not based on the same stochastic assumptions as RURO's where different hours of work have a different probability of being available and thus have some similarity with model (18). An alternative way to account for quantity constraints in the opportunity set is developed by Harris and Duncan (2002).

So far, in all the applications of the RURO the opportunity density $p(w,h)$ is first factorized as

$$p(w,h) = \begin{cases} p_1 g_1(h) g_2(w) \text{ if } h > 0 \\ 1 - p_1 \text{ if } h = 0 \end{cases} \tag{19}$$

where p_1 denotes the density of alternatives with $h > 0$, that is market jobs, and $g_1(h)$ and $g_2(w)$ are the densities of w and h conditional on $h > 0$. The conditional density of hours is specified as uniform-with-peaks (to be estimated) corresponding to part-time and full-time. The conditional density of wage rates is assumed to be log-normal. Details can be found in the work by Aaberge et al. (1995, 1999, 2013). All the densities p_1, $g_1(h)$, $g_2(w)$ and the density of w can depend on household or job characteristics.

From expressions (13) and (18), we can see that the solution of the utility maximization problem is expressed in terms of comparisons of absolute values of utility rather than in terms of marginal variations of utility and it is not affected by the specification of $v(.,.)$ or $f(.,.)$. One can choose relatively general and complicated specifications for v and/or accounting for complex tax-transfer rules f without affecting the characterization of behaviour and without significantly affect the computational burden involved by the estimation or simulation of the model. This holds for both the DC model and the RURO model (whether in continuous or discrete version). It is not often realized in the literature that the advantages of the RUM approach are due more to the representation of choice as the maximization of a random utility, rather than to the discreteness of the choice set.

Note that expression (13) can be seen as a special case of expression (18) when the wage rate w is treated as a fixed characteristic of the household (invariant with respect to the alternatives) and $p(x,y)$=constant for all (x,y).

It is also useful to observe that the opportunity density $p(x,y)$ can be specified in such a way that expression (18) reduces to a DC model with dummies refinement. For example, Colombino (2013) starts by considering a model with fixed individual-specific wage rates:

$$\varphi(h) = \frac{\exp\left\{v(f(wh,I),y)\right\}p(h)}{\sum_{y \in B} \exp\left\{v(f(wy,I),y)\right\}p(y)} \tag{20}$$

By specifying the opportunity density $p(y)$ as uniform-with-peaks, we get the following expression:

$$\varphi(h) = \frac{\exp\left\{v(f(wh,I),h) + \right\}\gamma_0 \mathbf{1}(h>0) + \sum_{\ell=1}^{L} \gamma_\ell \mathbf{1}(h \in S_\ell)}{\sum_{y \in B} \exp\left\{v(f(wy,I),x) + \gamma_0 \mathbf{1}(y>0) + \sum_{\ell=1}^{L} \gamma_\ell \mathbf{1}(y \in S_\ell)\right\}} \tag{21}$$

with

$$\gamma_0 = \ln J + A_0,\ \gamma_\ell = \ln\left(\frac{J_\ell}{J}\right) + A_\ell \tag{22}$$

$J =$ number of alternatives with $h > 0$,
$J_l =$ number of alternatives with $h \in S_l$ (e.g. S_l might be the set of hours values classified as 'part-time'),
A_0 and A_l are constants.

Expression (21) is formally equivalent to the DC model with the 'dummies refinement': however, here the coefficients γ have a specific structural interpretation, which – as we will see in the section dedicated to policy simulation – can be used to develop an equilibrium simulation procedure.

2.3.3. *The representation of the opportunity set*

In the continuous version of the RURO model, the opportunity set in principle can contain the whole positive quadrant, that is all the positive values of (w,h). If instead one adopts a discrete representation of the choice set (as in the DC model or as in the (18)-version of the RURO model) then one has to decide which alternatives are to be included in the opportunity set (besides the chosen alternative). DC models typically assume the opportunity set is fixed and imputed to every household. For example, one might divide the hours interval $(0,T)$ into equal sub-intervals and pick one value in each sub-interval (e.g. the midpoint, or a randomly chosen point). The wage rate is also fixed and household-specific: therefore, for every value h, the corresponding gross earnings are equal to wh. In the RURO models, the opportunity set is unknown since the opportunity density $p(w,h)$ must be estimated. The opportunity set used in the estimation (and in the simulations) can then be interpreted as a sample drawn from an unknown population. Therefore, the sampling method emerges as a relevant issue. Aaberge et al. (1995, 1999), Aaberge, Colombino, and Strøm (2004), Aaberge et al. (2013) sample alternative (w,h) values from a pre-estimated density $q(w,h)$ and, following McFadden (1978) and Ben-Akiva and Lerman (1985), and use a re-weighted version of expression (18):

$$\varphi(w,h) = \frac{\exp\{v(f(wh,I),h)\}p(w,h)/q(w,h)}{\sum_{(x,y)\in\hat{B}} \exp\ v(f(xy,I),y)\ p(x,y)/q(x,y)} \tag{23}$$

where $\hat{B}$ is the set of sampled alternatives. Expression (23) can also be interpreted as a computational approximation to expression (17). The same method is explained in detail and applied by Train, McFadden, and Ben-Akiva (1987). Aaberge et al. (2009) discuss and evaluate different methods of representing the opportunity set and find that they might have an important impact on the results of the policy simulation.

2.3.4. *Unobserved wage rates*

As in the 'marginalist' approach, also in the RUM approach the problem of unobserved wage rates for those who are not working can be solved either with a simultaneous procedure or with a two-step procedure. When adopting a simultaneous estimation with a DC model, one should also treat the wage rate w as an endogenous outcome and account for the fact that w is not observed for the non-workers in the sample. For that purpose we must specify a probability density function $m(w)$. Starting from expression (13), the likelihood of an observation with non-zero hours h and wage rate w would then be:

$$P(w,h) = \frac{\exp\{v(f(wh,I),h)\}}{\sum_{k=0}^{T} \exp\{v(f(wk,I),k)\}} m(w) \tag{24}$$

The likelihood of an observation with $h=0$ and unobserved wage rate would instead be:

$$P(h=0) = \int \frac{\exp\{v(f(0,I),0)\}}{\sum_{k=0}^{T} \exp \; v(f(wk,I),k)} m(w)\,\mathrm{d}w \tag{25}$$

In RURO models, the wage rate is endogenous from the very start. Therefore (in the continuous version), the likelihood of a choice (w,h) is given by (18) or (23). For example, by inserting (19) into (18) we get

$$\varphi(w,h) = \begin{cases} \dfrac{\exp\{v(f(wh,I),h)\}p_1 g_1(h) g_2(w)}{\exp\{v(f(0,I),0)\}(1-p_1) + \sum_{(x,y)\neq 0} \exp\{v(f(xy,I),y)\}p_1 g_1(y) g_2(x)\,\mathrm{d}x\,\mathrm{d}y} & \text{if } h>0 \\ \dfrac{\exp\{v(f(0,I),0)\}(1-p_1)}{\exp\{v(f(0,I),0)\}(1-p_1) + \sum_{(x,y)\neq 0} \exp\{v(f(xy,I),y)\}p_1 g_1(y) g_2(x)\,\mathrm{d}x\,\mathrm{d}y} & \text{if } h=0 \end{cases} \tag{26}$$

Alternatively, one could use a two-step procedure for imputing unobserved wages. In the first step, the wage equation is estimated. In the second step, the predicted wage rate replaces the missing values (or, alternatively, both the missing and the observed values). The random term of the wage equation is added to the systematic part and integrated (or 'averaged') out with a simulation procedure (e.g. Van Soest, 1995). Löffler, Peichl, and Siegloch (2013) illustrate that the estimated labour supply elasticities can be very sensitive to the way unobserved wage rates are treated.

Both the simultaneous and the two-steps procedures illustrated above assume that the random term of the wage equation is uncorrelated with the random term of the utility function. However, one might want to allow for a correlation of the wage rate random component with one or more random parameters of $v(f(wh,I),h)$ – due, for example, to a dependence of the wage rate on previous decisions – (e.g. Blundell & Shephard, 2012; Breunig, Cobb-Clark, & Gong, 2008; Gong & Van Soest, 2002; Löffler et al., 2013).

2.3.5. *Involuntary unemployment*

Apparently, RUM-type models do not leave much space to the possibility of involuntary unemployment, since also $h=0$ is an optimal choice (non-participation). If, however, ε is interpreted as an optimization error rather than as part of the utility, then some of the individuals with $h=0$ might be interpreted as involuntary unemployed. Maybe they could be identified as those with $h=0$ and systematic utility sufficiently close (in some sense) to the systematic utility of those with $h>0$. To the best of our knowledge, this line of research has never been pursued. Instead, some contributions have taken involuntary unemployment into account by complementing the basic DC model with an exogenous latent index equation (Blundell et al., 2000). Euwals and van Soest (1999) have used subjective evaluations together with observed outcome to model the differences between actual and desired labour supply. In RURO models, ε is strictly interpreted as part of the utility function and therefore $h=0$ is an optimal choice. However, there is a sense in which these models also account for involuntary unemployment: the opportunity density $p(w,h)$ allows for a different availability of different opportunities to different households, therefore it can happen that some households have no (or very few) available opportunities with $h>0$.

2.3.6. *Generalizations and developments*

Both the DC and the RURO model can be easily generalized to include several dimensions of choice. Besides simultaneous decisions on the part of partners in a couple, one might include other decisions such as: labour supply of other members of the household, consumption of goods and services, fertility, choice of child-care mode, sector of employment, other dimensions of labour supply (occupational choice, educational choices, job search activities, etc.) and so on. For example, Aaberge, Colombino, Strøm, and Wennemo (2007), Aaberge and Colombino (2006, 2013) and Dagsvik and Strøm (2006), and Dagsvik, Locatelli, and Strøm (2009) include the choice between private sector and public sector employment; Kornstad and Thoresen (2007) model the simultaneous choice of labour supply and child-care; Haan and Wrohlich (2011) analyse fertility and employment, Flood, Hansen, and Wahlberg (2004), Hoynes (1996) and Aaberge and Flood (2013) analyse labour supply and welfare participation.

A potential limitation of the RUM models based on the independent and identical extreme value distribution for the random component ε is the Independence-of-Irrelevant-Alternatives assumption, which in turn implies restrictions on the behavioural responses (e.g. Ben-Akiva & Lerman, 1985). Some contributions have opted for alternative distributional assumptions (e.g. Keane & Moffitt, 1998). However, advances with simulation-based methods (Train, 2009), have made it feasible to overcome this limitation by assuming GEV distributions (e.g. Nested Logit models) or random parameters, while preserving the main convenient analytical advantages of the extreme value distributions. By assuming that one or more preference parameters are random, one gets the so-called Mixed Logit model (McFadden & Train, 2000). When it comes to RURO models, expressions (17) and (18) are also close to a Mixed Logit model since the wage rate w is random. See also the survey by Keane and Wasi (2013).

Due to space limitations, we can only mention two important developments in labour supply analysis which also, recently, tend to adopt the RUM modelling strategy:

(i) Stochastic dynamic programming (SDP) models, for example Miller and Sanders (1997), Wolpin (1996), Grogger (2003), Swann (2005), Todd and Wolpin (2006), Keane and Wolpin (2002a, 2002b), Keane (2011), Keane, Todd, and Wolpin (2011). There are various motivations for using SDP models. First, many choices – notably human capital decisions, occupational choices, fertility, etc. – have important intertemporal implications: namely, the effects of decisions taken today have important effects in the future (e.g. Miller & Sanders, 1997). Second, many policies have an intrinsic intertemporal dimension, for example there might be time limits, or it might be that the amount of services I decide to get today affects the amount of services I can get tomorrow (Swann, 2005). Third, an important source of uncertainty in current decisions is the expectation of future changes in policies, for example expectations on whether a certain policy is temporary or permanent (Keane & Wolpin, 2002a, 2002b).

(ii) Non-unitary models of household behaviour, where the household is not represented as a fictitious individual but rather as a set of individuals who – somehow – arrive at a collective decision. A major aim is developing models that can analyse intra-household allocation of resources (e.g. among genders) and the effects of policies upon different member of the households. As to the way of modelling the process that leads to the collective decision, there are two main lines of research: (i) The 'sharing rule' approach, for example Chiappori (1988, 1992), Donni (2003, 2007), Vermeulen (2005), Vermeulen et al. (2006), Bloemen (2010). Here, the intra-household allocation process is given a 'reduced form' representation: this way of proceeding requires minimal a priori assumptions (namely, the household attains, somehow, a Pareto-efficient allocation), but in principle makes the model not applicable to ex-ante policy evaluation, unless one is prepared to assume that the 'sharing rule' is policy-invariant; (ii) The explicit structural representation of intra-household allocation process. For example, McElroy and Horney (1981) have proposed Nash bargaining. Other types of solution are of course possible. So far, this second approach has been much less popular than the 'sharing rule' one, although its structural character makes it more promising in view of policy simulation (e.g. Bargain & Moreau, 2013; Del Boca & Flinn, 2012; Hernæs, Jia, & Strøm, 2001).

2.4. How reliable are structural models?

Many authors have raised doubts upon the reliability of structural models as compared with the (supposed) robustness of evidence produced by (ex-post) experimental or quasi-experimental analysis (e.g. Bargain & Doorley, 2013; Blundell, Duncan, & Meghir, 1998; Brewer et al., 2006). Provided we want ex-ante policy evaluation, the issue is twofold:

(i) Are there alternatives to structural models?
(ii) How do we evaluate structural models and how do they compare with other approaches?

When answering question (i) one has to carefully distinguish between type of data and type of models (or parameters) to be estimated. Often we observe a tendency to associate structural models with observational data and ex-post programme evaluation with experimental or quasi-experimental data. Although this is what goes on in most cases, in principle nothing prevents the use of experimental or quasi-experimental data for the estimation of structural models. Another possible source of confusion comes from erroneously associating structural modelling with the use of convenient parametric functional forms: although this might be a common practice, most of the research done on non-parametric estimation addressed to policy evaluation is definitely structural (e.g. Blomquist & Newey, 2002; Manski, 2012; Matzkin, 2013; Todd & Wolpin, 2008; Varian, 2012). What counts in view of ex-ante evaluation is that a set of relevant parameters (or primitives) be identified as policy independent (Hurwicz, 1962). Depending on the class of policies we are interested in, different sets or combinations of parameters might be sufficient for the purpose (Marschak, 1953). Of course, the point is that in general experimental or quasi-experimental data, *by themselves*, are not sufficient to identify policy-invariant parameters. For that purpose they must be analysed by a model, either in explicit form (e.g. Bargain & Doorley, 2013; Card & Hyslop, 2005; Todd & Wolpin, 2006), or in an implicit form – as for example with 'statistical extrapolation' (e.g. Chetty, 2009). The availability of experimental or quasi-experimental evidence promises to improve the internal validity (or the identification conditions) of the model, but does not overcome the need for a structural approach. Therefore, the answer to question (i) is negative: ex-ante evaluation requires a structural model, whether parametric or non-parametric, explicit or implicit, estimated on observational or (quasi-) experimental data, etc. It is fair to say, however, that more effort would be desirable on developing models or analysis that somehow go beyond the mainstream of a parametric model estimated on observational data. Let us turn to question (ii). The structural econometric community has abandoned the ideal of the correct specification. Models are approximations. Ordinary statistical testing is informative on the precision of the parameter estimates of the model but less so on how useful the estimated model is. This pragmatic approach would seem to entail a shift of focus from the issue of identification to the issues of external validation and out-of-sample prediction performance (Keane, 2010; Wolpin, 2007), although this conclusion is debatable (e.g. Blundell, 2010; Imbens, 2010). The amount of out-of-sample testing so far is limited (e.g. Aaberge & Colombino, 2006, 2013; Aaberge et al., 2009; Aaberge & Flood, 2013; Keane & Moffitt, 1998; Keane & Wolpin, 2002a, 2002b, 2007) but reassuring. A supplementary evidence provided by out-of-sample prediction exercises suggests that flexible theoretical models – as compared with structural models – tend to perform better in-sample but worse out-of-sample.

3. Policy simulation

3.1. Producing simulation outcomes

We start by asking, when is information on behavioural responses needed? Non-behavioural simulations may be sufficiently informative provided the policy changes or the reforms can be represented as marginal changes in net wages and/or in unearned income. Let $u^*(w,I)$ be the indirect utility function, where w is the net wage rate and I is the unearned income. Let us suppose that the reform can be represented as a marginal change $(\mathrm{d}w,\mathrm{d}I)$. Then we have: $\mathrm{d}u^* = (\partial u^*/\partial w)\,\mathrm{d}w + \mu \mathrm{d}I$, where $\mu \equiv (\partial u^*/\partial I)$ is the marginal utility of income. By applying Roy's Theorem, we get: $(\mathrm{d}u^*/\mu) = h\,\mathrm{d}w + \mathrm{d}I$. The right-hand side is the change in the budget, conditional on the pre-reform labour supply h. The left-hand side is the monetary equivalent of the change in utility. Therefore, the result tells us that the change in the budget (i.e. the basic result produced by a non-behavioural simulation) is a money-metric measure of the change in utility. Similar arguments can be generalized so that a non-behavioural simulation can be complemented by point-estimates of elasticities or other local measures of behavioural responses (e.g. Chetty, 2009).

When the reforms involve non-marginal changes in the budget constraint, we typically want a prediction of the new choices, in particular of the new value of h or some function of it. Within the 'reduced form' and the 'marginalist' approaches (as defined in Section 2) we usually estimate a labour supply function and (directly or indirectly) a utility function. With non-linear budget constraints and corner solutions (the case commonly faced by analyses adopting the 'marginalist' approach), it is in general possible to identify the distribution of the random component capturing unobserved heterogeneity of preferences and/or the distribution of the measurement/optimization error (whichever is present in the model). Non-convex budget sets in general require recovering also a direct or indirect representation of the utility function in order to be able to simulate the optimal decision. Given the estimates of the (non-random parameters of) the labour supply function and/or of utility function, those random components are simulated so that their values are compatible with the observed values of h. Arrufat and Zabalza (1986) provide a clear and exhaustive explanation of this procedure. With DP or RURO models, we can choose between two alternative procedures:

Compute the expected chosen value of the variable of interest, based upon the estimated choice probabilities, for example Colombino et al. (2010) and Colombino (2013).

Simulate the value of the systematic utility and of the random component corresponding to each alternative in the opportunity set. Identify the alternative with the highest utility and compute the corresponding value of the variable of interest. Typically, the random components are kept fixed across the different policy regimes that one might want to simulate and compare. As to the current policy regime, simulation might be used as well: its results will not be identical to the observations but reasonable close at least in large samples. Alternatively, one might adopt the procedure suggested by Creedy and Kalb (2005b), that is generating a vector of random components that, given the estimated parameters of the utility function, are consistent with the observed choices under the current policy regime.

When simulating sample aggregates, such as the total hours worked or total gross income, the two procedures (a) and (b) should be asymptotically equivalent, however they might diverge on small samples or subsamples. Overall, we can observe that, as far as labour supply models are concerned, so far we lack a rigorous investigation of the statistical properties of different methods of producing microsimulation outcomes.[8]

3.1.1. Interpretation of the policy simulation results: short-run, long-run, comparative statics

There appears to be a consensus that the results of non-behavioural policy microsimulation should be interpreted as 'the day after' predictions,

[8] The systematic analysis of the statistical properties of alternative methods for producing predictions is more advanced in other areas where RUM models are used, for example Watanatada and Ben-Akiva (1979) and Ben-Akiva and Lerman (1985).

that is predictions of the very short term, when agents and market interactions did not have time yet to adjust to the new policy. As argued above, even in the long-run, non-behavioural results might be considered a sufficient statistic provided the reforms can be represented as marginal changes in the budget constraint. The interpretation of behavioural microsimulation results raises more complicated and controversial issues. The typical policy simulation exercise computes the labour supply effects while leaving the wage rates unchanged. Some authors (e.g. Creedy & Duncan, 2005) interpret this scenario as the 'month after' prediction, with households making new choices but the market mechanisms still late in the process of adjusting wage rates, labour demand etc. An alternative interpretation might view the typical simulation exercise as a 'very long-run' prediction, with a perfectly elastic labour demand defined by the current wage rates. In any case, comparative statics is the appropriate perspective with behavioural microsimulation models based on a static representation of agents' choices, that is we want to compare two different equilibria induced by two different policies. With the notion of equilibrium, we refer in general to a scenario in which the economic agents make optimal choices (i.e. they choose the best alternative among those available in the opportunity set) and their choices are mutually consistent or feasible. The comparative statics perspective is relevant both when the new equilibrium is reached in a short time and is maybe temporary (as might be the case with an intervention explicitly designed to have an immediate effect) and when instead we evaluate reforms of institutions or policies with long-run (and possibly long-standing) effects. In order to produce simulation results that respect the comparative statics perspective, Creedy and Duncan (2005) and Peichl and Siegloch (2012) have proposed procedures where DC labour supply models (as defined in Section 2) are complemented by a function of labour demand and the wage rates are adjusted so that an appropriate or feasible equilibrium criterion is satisfied. These procedures, however, in general would not be consistent with RURO models, which already include a representation of the density of market jobs of different types at the time of observation. In general, the notion of equilibrium will imply some relationship between the opportunity density and the size and composition of labour supply: since a reform will induce a change in labour supply, it follows that in equilibrium also the opportunity density will have to change. This observation carries over to DC models with dummies refinement, to the extent that the alternative-specific constants reflect also the demand side (e.g. the availability of jobs): a new equilibrium induced by a reform should entail a change of the alternative-specific constants. Colombino (2013) proposes and exemplifies an iterative simulation procedure that exploits the structural interpretation of the coefficients of the alternative-specific constants given in expression (22) of Section 2.

3.2. Examples of simulations addressing specific policies or issues

As explained in Section 1, the microeconometric models of labour supply can be, and have been, used to evaluate by simulation a very large variety of policies and reforms. Most applications concern tax-benefit and welfare policies. While it would be clearly impossible to present an exhaustive list, in Table 1 we summarize a selection of notable examples.

Table 1. Tax and benefit analyses based on structural microeconometric models

Type of policy analysis	Country	Contributions
Flat tax	Australia	Creedy, Kalb, and Kew (2003), Creedy and Kalb (2005a), Scutella (2004)
	Germany	Beninger et al. (2006), Fuest el al. (2008)
	Italy	Aaberge, Colombino, and Strøm (2000), Aaberge, Colombino, Strøm, and Wennemo, 2000), Aaberge et al. (2004)
	Norway	Aaberge et al. (1995), Aaberge, Colombino, Strøm, and Wennemo (2000)
	Spain	Labeaga et al. (2008)
	Sweden	Blomquist (1983), Blomquist and Hansson-Brusewitz (1990), Aaberge, Colombino, Strøm, and Wennemo (2000)
Unconditional transfers and Basic Income	Australia	Scutella (2004)
	Canada	Clavet, Duclos, and Lacroix (2013)
	Germany	Horstschräer, Clauss, and Schnabel (2010)
	Italy	Colombino (2013, 2014), Colombino et al. (2010), and Colombino and Narazani (2013, 2014)
Mean-tested transfers, Negative Income Tax and Work Fare	France	Bargain and Doorley (2013), Gurgand and Margolis (2008)
	Canada	Fortin, Truchon, and Beausejour (1993)
	Italy	Aaberge, Colombino, Strøm, (2000), Aaberge et al. (2004), Colombino et al. (2010), and Colombino and Narazani (2013, 2014)
	US	Burtless and Hausman (1978)
In-work benefits, Tax credits and Wage subsidies	Australia	Creedy (2005)
	Belgium	Decoster and Vanleenhove (2012)
	Canada	Clavet et al. (2013)
	France	Bargain and Orsini (2006)
	Germany	Haan and Myck (2007), Bargain and Orsini (2006)
	Italy	Colombino (2014), Colombino and Narazani (2013, 2014), Colonna and Marcassa (2012), De Luca, Rossetti, and Vuri (2012), Figari (2011), Haan and Steiner (2008), Pacifico (2013)
	Sweden	Flood, Wahlberg, and Pylkkänen (2007)
	Sweden	Aaberge and Flood (2013)

Table 1. (*Continued*)

Type of policy analysis	Country	Contributions
	UK	Duncan and Giles (1996), Bingley and Walker (1997), Blundell (2006), Blundell and Hoynes (2004), Blundell et al. (2000), Blundell, Brewer, Haan, and Shephard (2009), Blundell and Shepard (2012), Brewer (2001, 2009), Brewer et al. (2006), Brewer, Francesconi, Gregg, and Grogger (2009)
	US	Aaberge and Flood (2013), Keane (1995), Kean and Moffitt (1998), Meyer and Rosenbaum (2001), Meyer and Holtz-Eakin (2002), Blank (2002), Meyer and Holtz-Eakin (2002), Hotz and Scholz (2003), Fang and Keane (2004), Eissa and Hoynes (2004, 2006, 2011), Grogger (2003), Grogger and Karoly (2005), Moffitt (2006), and Eissa, Kleven, and Kreiner (2008)
Welfare participation and labour supply	Sweden	Flood et al. (2004), Aaberge and Flood (2013)
	US	Moffitt (1983), Fraker and Moffitt (1988), Hoynes (1996), Keane and Moffitt (1998)
Child care and labour supply	Australia	Kalb (2009)
	Belgium	Van Klaveren and Ghysels (2012)
	Germany	Wrohlich (2008), Haan and Wrohlich (2011)
	Italy	Del Boca (2002), Del Boca and Vuri (2007)
	Norway	Kornstad and Thoresen (2006, 2007)
	Russia	Lokshin (2004)
	Sweden	Gustafsson and Stafford (1992)
	US	Heckman (1974a, 1974b), Rosen (1976), Blau and Robbins (1988), Ribar (1995)
Fertility and labour supply	Italy	Del Boca (2002)
	Germany	Haan and Wrohlich (2011)
	US	Hotz and Miller (1988)
Optimal taxation	Australia	Creedy and Hérault (2012)
	Germany	Blundell et al. (2009), Bach, Corneo, and Steiner (2012)
	Italy	Aaberge and Colombino (2012)
	Norway	Aaberge and Colombino (2006, 2013),
	Sweden	Ericson and Flood (2012)
	UK	Blundell et al. (2009), Blundell and Shepard (2012)

3.3. *Identifying optimal systems*

3.3.1. *Empirical applications of theoretical optimal taxation model*

Labour supply is central not only in the design and evaluation of specific tax-transfer reforms, but also in the identification of optimal tax-transfer systems. To see this, it is useful to review briefly the basic framework adopted by theoretical optimal taxation (Mirrlees, 1971). Agents – households – differ by their market productivity = wage rate w. They solve

$$\begin{aligned}
&\max_{c,h} u(c,h)\\
&\text{s.t.}\\
&c = wh - T(wh)
\end{aligned} \tag{27}$$

where
$u(c,h) = u(c) - h$ = utility function, assumed – for simplicity of illustration – to be separable in c and h and with no income effects;
c = net available income;
h = hours of work;
$T(z)$=tax to be paid by an agent with earnings $z = wh$.

The Social Planner solves

$$\begin{aligned}
&\max_{c,h} \int_0^\infty W(u^w)f(w)\,\mathrm{d}w\\
&\text{s.t.}\\
&\int_0^\infty T(z^w)f(w)\,\mathrm{d}w = R\\
&u^w = \max_{c,h}\ u(c,h)\ \text{c.v.}\ c = wh - T(wh)
\end{aligned} \tag{28}$$

where
$W(\cdot)$ = Social Welfare function
R = total tax revenue to be collected (exogenously given)
$z^w \equiv wh^w$
$h^w = \arg\max_h u(c,h)$ c.v. $c = wh - T(wh)$
$f(w) = F_w(w)$ = probability density function of w.

The solution to problem (28) can be expressed as follows: T

$$\frac{{}_z(z^w)}{1 - T_z(z^w)} = \left[1 + \frac{1}{\xi(w)}\right] \times \left[\frac{(1 - F(w))}{wf(w)}\right] \times \left[\frac{\int_w^\infty (1 - \Omega^m)f(m)\mathrm{d}m}{1 - F(w)}\right] \tag{29}$$

where $T_z(z^w)$=marginal tax rate for a household with productivity w (and therefore earnings z^w);
$\xi(w)$=labour supply elasticity of a household with productivity w;
Ω^m = marginal social weight given to the consumption of a household with productivity w.

Expressions like (29) or more general versions of it have been used by many authors (e.g. Tuomala, 1990) to perform illustrative simulation exercises where optimal taxes are computed given imputed or calibrated measures of $\xi(w)$ and $F(w)$. A typical criticism moved to these exercises is that they do not account properly for the heterogeneity of the preferences and productivity across the population (e.g. Tuomala, 2010). Revesz (1989), Diamond (1998) and Saez (2001, 2002), among others, present reformulations of Mirrlees's model that are more directly interpretable in terms of empirically observable variables and make it more convenient to

account for agents' heterogeneity. These reformulations have been used in conjunction with microeconometric models. In particular, Saez (2002) develops a discrete model that assigns a crucial role to the relative magnitude of the labour supply elasticities at the extensive and at the intensive margin. There are $J+1$ types of job, each paying (in increasing order) z_0, z_1, ..., z_J. Job '0' denotes non-working conditions (non-participation or unemployment). Net available income on job j is $c_j = z_j - T_j$ where T_j is the tax paid at income level z_j. Each agent is characterized by one of the potential incomes z_0, z_1, ..., z_J and if she decides to work she is allocated to the corresponding job. The agent of type j decides to work if $c_j \geq c_0$. The extensive margin (or participation) elasticity is defined as $\eta_j = \frac{c_j - c_0}{\pi_j} \frac{\partial \pi_j}{\partial (c_j - c_0)}$, where π_j is the proportion of agents on job of type j. Working agents can also move to a different job if income opportunities change, but the movements (for reasons implicit in the assumptions of the model) are limited to adjacent jobs (i.e. from job j to job $j-1$ or job $j+1$). The intensive margin elasticity is defined as:
$\xi_j = \frac{c_j - c_{j-1}}{\pi_j} \frac{\partial \pi_j}{\partial (c_j - c_{j-1})}$. Then it turns out that the optimal taxes satisfy:

$$\frac{T_j - T_{j-1}}{c_j - c_{j-1}} = \frac{1}{\xi_j} \frac{\sum_{k=j}^{J} \pi_k \left[1 - \Omega_k - \eta_k \frac{T_k - T_0}{c_k - c_0}\right]}{\pi_j} \tag{30}$$

where Ω_k is the marginal social value of income at job k. The model is attractive in view of empirical applications because it seems to fit well to the DC framework. Recent applications include: Blundell et al. (2009) (optimal taxation of single mothers in Germany and the United Kingdom); Haan and Wrohlich (2010) (optimal design of children benefits in Germany); Immervoll, Kleven, Kreiner, and Saez (2007) (evaluation of income maintenance policies in European countries).

The studies coupling theoretical optimal taxation results with microsimulation proceed as follows. The researcher looks for an analytical solution to the optimal taxation problem, that is a 'formula' that allows to compute the optimal taxes or marginal tax rates as function of exogenous variables and parameters. Next, the numerical simulations consist in calculating the analytical solution with exogenous variables and parameters assigned numerical values produced by microeconometric estimates. There are two main problems with this procedure. First, in order to get an analytical solution we must adopt many simplifying and restrictive assumptions. Second, when we 'feed' the formulas with empirical measures, we are very likely to face an inconsistency between the theoretical results and the empirical evidence, since the latter was typically generated under assumptions that are very different for those that made it possible obtaining the former. For example, Saez (2002) assumes there are no income effects and specifies a very special and limited representation of choices at the intensive margin. None of these assumptions are shared by the typical microeconometric models used to simulate the elasticities.

3.3.2. *Identification of an optimal rule by searching the policy space*

A number of studies have used labour supply microsimulation models to explore the policy space defined by certain types of tax-transfer systems. Fortin et al. (1993) calibrate (on the basis of previous estimates or 'reasonable' imputation) a collective 'marginalist' model of household labour supply and run it in order to identify the best income support mechanism within a set including many versions of the Negative Income Tax and of a Workfare system. The contributions mentioned hereafter adopt a RURO approach. Aaberge et al. (2004) evaluate rules such as the Flat Tax, the Negative Income Tax and the Workfare as hypothetical reforms in Italy. Colombino et al. (2010), Colombino and Narazani (2013) and Colombino (2013) focus on Basic Income policies. Ericson and Flood (2012) look for welfare improving changes in the Swedish tax-benefit system.

3.3.3. *Optimal taxation by simulation*

In order to overcome the drawbacks of the simulation exercises coupled with theoretical optimal taxation (see Section 3.3.1), recent contributions have proposed a computational approach (Aaberge & Colombino, 2006, 2012, 2013; Blundell & Shephard, 2012). Modern microeconometric models of labour supply are based on very general and flexible assumptions. They can accommodate many realistic features such as general structures of heterogeneous preferences, simultaneous decisions of household members, non-unitary mechanisms of household decisions, complicated (non-convex, non-continuous, non-differentiable, etc.) constraints and opportunity sets, multidimensional heterogeneity of both households and jobs, quantitative constraints, etc. It is simply not feasible (at least so far) to obtain analytical solutions for the optimal taxation problem in such environments. Yet those features are very relevant and important especially in view of evaluating or designing reforms (Colombino, 2009). An alternative (or maybe complementary) procedure consists of using a microeconometric model to obtain a computational solution of the optimal taxation problem. The microeconometric model, which primarily simulates the agents' choices by utility maximization, is embedded into a global maximization algorithm that solves the social planner's problem, that is the maximization of a social welfare function subject to the public budget constraint.

The method (as presented in Aaberge & Colombino, 2013) is formulated as follows:

$$
\begin{aligned}
&\max_{\vartheta}\ W(U_1(c_1,h_1,j_1),U_2(c_2,h_2,j_2),\ldots,U_N(c_N,h_N,j_N))\\
&\text{s.t.}\\
&(c_n,h_n,j_n)=\underset{(w,h,j)\in B_n}{\arg\max}\ U_n(c,h,j)\ \text{ s.t. }\ c=f(wh,I_n;\vartheta),\ \ \forall n\\
&\sum_{n=1}^{N}(w_nh_n+I_n-f(w_nh_n,I_n;\vartheta))\geq R
\end{aligned}
\tag{31}
$$

Agent n can choose a 'job' within an opportunity set B_n. Each job is defined by a wage rate w, hours of work h and other characteristics j (unobserved by the analyst). Given gross earnings wh and gross unearned income I, net available income is determined by a tax-transfer function $c=f(wh,I;\vartheta)$ defined up to a vector of parameters ϑ. For any given tax-transfer rule (i.e. any given value of ϑ) the choices by the agents are simulated by a microeconometric model that allows for a very flexible representation of heterogeneous preferences and opportunity sets, it covers both singles and couples, accounts for quantity constraints and is able to treat any tax-transfer rule however complex. Note that it would be hopeless to look for analytical solutions of an optimal taxation problem in such an environment. The choices made by the N agents result in N positions (c_1,h_1,j_1), (c_2,h_2,j_2), …, (c_N,h_N,j_N), which are then evaluated by the social planner according to a Social Welfare function W. The Social Planner's problem therefore consists of searching for the value of the parameters ϑ that maximizes W subject to the following constraints: (i) the various positions (c_1,h_1,j_1), …, (c_N,h_N,j_N) result from utility-maximizing choices on the part of the agents (incentive-compatibility constraints); (ii) the total net tax revenue must attain a given amount R (public budget constraint).

The optimal taxation problem is solved computationally by iteratively simulating the household choices for different values of ϑ until W is maximized.

Any exercise involving a comparison between the utility levels attained by heterogeneous households requires developing comparable measures of utility or individual welfare. If, moreover, we adopt social welfare as the criterion for comparing alternative policies we must specify a Social Welfare function. The next section is devoted to these two issues.

4. *Social evaluation of policy simulations*

4.1. *Individual welfare functions*

As explained in Sections 2 and 3, empirical microeconomic models of labour supply are helpful tools for simulating the effects on households' labour supply and income from changes in tax and benefit systems or from changes in distributions of wage rates and hours of work offered by the demand side of the labour market. However, to complete the economic evaluation of policy reforms a framework for analysing the outcomes from the simulation exercises is required. It is straightforward to provide a summary of changes in employments rates and distributions of hours of work and income. However, a social planner needs information that makes it possible to compare individuals' level of welfare before and after a policy change and thus who is gaining and who is losing on the policy change. It is, however, not obvious how one should make a social evaluation of the policy effects when each individual's welfare is considered to be a function of income and leisure. The estimated utility functions (or their systematic parts) might emerge as a useful basis for making social evaluations of welfare. However, since the behaviour of an individual is invariant with respect to monotonic transformations of the utility function we face two problems. The first one concerns the construction of specific cardinal utility functions to represent the consumption/leisure preferences of individuals/households, and the second concerns the lack of convincing justification for comparing arbitrarily chosen individual cardinal utility functions and use them as arguments in a social welfare function (see e.g. the thorough discussion provided by Hammond, 1991). The origin of the problem is as stated by Hume that one cannot derive an 'ought' from an 'is'. To circumvent these problems Deaton and Muellbauer (1980), King (1983) and Hammond (1991) proposed to use a common utility function as a tool for making interpersonal comparisons of welfare, since it by definition contains within it interpersonal comparability of both welfare levels and welfare differences. The common utility function is supposed to capture the preferences of the social planner, whereas the individual/household-specific utility functions solely are assumed to capture the consumption/leisure preferences of individuals/households. The latter can be used to simulate the behaviour of individuals/households under alternative tax/benefit systems, whereas the former is designed to be used for evaluating the outcomes of simulation exercises. However, even though there was agreement about the requirement of a common utility function the problem of how to construct it would remain. As argued by Aaberge and Colombino (2013) a plausible approach is to assume that the social planner exploits the information provided by the consumption/leisure choices of the individuals/households (and moreover accounts for large heterogeneity in the availability of different jobs in the market) by estimating the common utility function. Alternatively, a specific utility function (e.g. the utility function of the poorest, the richest or the median) can be used as the common utility function. Examples of the latter approach can be found in King (1983) for housing choices and in Aaberge et al. (2004) for labour supply choices.

As opposed to the common utility approach the practice of basing social evaluations on distributions of individual-specific money-metric measures of utility differences like equivalent and compensation variation disregards the interpersonal comparability problem, which makes it difficult to judge the ethical significance of this approach.[9] An alternative and more promising approach aiming at respecting individual (consumption/leisure) preferences in welfare analyses has been proposed by Fleurbaey (2003, 2008) and Fleurbaey and Maniquet (2006) and applied by Bargain, Decoster, et al. (2013) and Decoster and Haan (2014) in analyses of labour supply. However, as acknowledged by Decoster and Haan (2014), the choice of a specific preference respecting welfare metric might have a significant impact on the result of the welfare evaluation, and moreover it is shown to depend on the degree of emphasis the welfare metric places on willingness-to-work. Thus, depending on the chosen metric a work averse or work loving individual will be more or less favoured, which means that the social planner faces the problem of giving more or less weight to people with preferences that exhibit low or high willingness-to-work.

Below we will provide an explanation of the specific version of the common utility approach employed by Aaberge and Colombino (2013) for designing optimal taxes based on a microeconomic model of labour supply. Since households differ with regard to size and composition it is required to construct a common utility function that justifies comparison of individual welfare for individuals. The common utility function (individual welfare function) V is to be interpreted just as the input of a social welfare function and thus differs from the role played by the actual utility function U for households. The individual welfare function (V) is assumed to have a functional form that is identical to the basic functional form of the systematic part of the positive utility function U, which means that the heterogeneity of the parameters of U has been removed. Thus, V is defined by

$$V(y,h) = \gamma_2\left(\frac{y^{\gamma_1}-1}{\gamma_1}\right) + \gamma_4\left(\frac{L^{\gamma_3}-1}{\gamma_3}\right) \tag{32}$$

where L is leisure, defined as $L = 1-(h/8736)$, and y is the individual's income after tax defined by

$$y = \begin{cases} c = f(wh, I) & \text{for singles} \\ \dfrac{c}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} f(w_F h_F, w_M h_M, I) & \text{for married/cohab. individuals.} \end{cases} \tag{33}$$

Thus, couples incomes are transformed into comparable individual-specific incomes by dividing the couple incomes by the square root of 2. The next problem is to assess the value of the four parameters of the

[9] See, for example Aaberge et al. (1995, 2000) and Creedy and Hérault (2012).

common utility function for individuals on the basis of the observed leisure and income data where individual incomes are defined by Eq. (33). Since the observed chosen combinations of leisure and income depend on the availability of various job opportunities, we use expression (26), where the systematic part of the utility function v is replaced by the individual welfare function (V) defined by Eq. (32), as a basis for estimating the parameters of V. Table 2 displays the parameter estimates.

A different way to circumvent the interpersonal comparability problem consists in avoiding interpersonal comparisons altogether and basing the social evaluation exclusively on intrapersonal comparisons of utility levels, which of course is less informative. A proper application of the ordinal criterion would require defining the optimal tax in a different way, for example the rule that maximizes the number of winners. However, since the winners might be the individuals with the highest pre-reform welfare levels the ordinal criterion does obviously not account for distributional effects and may, for that reason, be considered as an inappropriate social evaluation approach.

4.2. Social welfare functions – the primal and dual approach

The informational structure of the individual welfare functions (defined by the common utility function (32) or Fleurbaey's preference respecting welfare metrics) allows comparison of welfare levels as well as gains and losses of different individuals due to a policy change. Comparison of distributions of individual welfare, formed for example by alternative hypothetical tax reforms, might be made in terms of dominance criteria of first- and second degree. However, since distribution functions normally intersect even second-degree dominance may not provide an unambiguous ranking of the distributions in question. Dominance criteria of higher degree can as demonstrated by Aaberge et al. (2013) provide a complete ranking, but it would in any case be helpful to quantify social welfare. To this end, let social preferences be represented by the ordering $\succcurlyeq$ defined on the family $\boldsymbol{F}$ of distributions of individual welfare. The preference ordering is assumed to be continuous, transitive and complete and to satisfy

Table 2. Estimates of the parameters of the welfare function for individuals 20 to 62 years old, Norway 1994

Variable	Parameter	Estimate	Std. dev.
Income after tax (y)			
	γ_1	−0.649	0.086
	γ_2	3.026	0.138
Leisure (L)			
	γ_3	−12.262	0.556
	γ_4	0.045	0.011

first-degree stochastic dominance as well as the following independence axiom,

Axiom (Independence). Let F_1, F_2 and F_3 be members of **F**(V) and let $\alpha \in [0,1]$. Then $F_1 \succcurlyeq F_2$ implies $\alpha F_1 + (1-\alpha)F_3 \succcurlyeq \alpha F_2 + (1-\alpha)F_3$.

This axiom focuses attention on the proportion of people $F(y)$ for a given level V of individual welfare and imposes an invariance condition on the proportions $F_1(V)$ and $F_2(V)$ being compared. Instead, we might focus on the income level $F^{-1}(t)$ that is associated with a given proportion of people t, that is, the rank in the distribution F, and impose an invariance condition on the individual welfare levels $F_1^{-1}(t)$ and $F_2^{-1}(t)$ being compared. This corresponds to an alternative version of the independence axiom, which is called the dual independence axiom in the literatures on uncertainty and inequality,

Axiom (Dual Independence). Let F_1, F_2 and F_3 be members of **F** and let $\alpha \in [0,1]$. Then $F_1 \succcurlyeq F_2$ implies $\left(\alpha F_1^{-1} + (1-\alpha)F_3^{-1}\right)^{-1} \left(\alpha F_2^{-1} + (1-\alpha)F_3^{-1}\right)^{-1}$.

The axioms require that the ordering is invariant with respect to certain changes in the distributions being compared. It is these axioms that give social preferences an empirical content. If F_1 is weakly preferred to F_2, then the Independence Axiom (similar to the expected utility theory) states that any mixture on F_1 is weakly preferred to the corresponding mixture on F_2. The intuition is that identical mixing interventions on the distributions do not affect their ranking; the ranking depends solely on how the differences between the mixed distributions are judged. Thus, the axiom requires the ordering relation $\succcurlyeq$ to be invariant with respect to aggregation of sub-populations across individual welfare. The Dual Independence axiom postulates a similar invariance property on the inverse distributions. It says that, if we consider a decomposition by sources of individual welfare, then dominance with regard to one set of sources implies, other things equal, overall dominance. The essential difference between the two axioms is that the Independence Axiom deals with the relationship between a given level of individual welfare and weighted averages of corresponding population proportions, while the Dual Independence Axiom deals with the relationship between given population proportions and weighted averages of corresponding levels of individual welfare.

The choice between the two independence axioms determines whether the associated family of welfare functions can be considered as a primal or dual family of social welfare functions.[10] The 'primal approach' is

---

[10] See Aaberge (2001) and Aaberge and Atkinson (2013) for similar discussions of how to summarize the information content of Lorenz curves and headcount curves.

analogue to the inequality framework developed by Atkinson (1970), while the 'dual approach' is analogue to the rank-dependent measurement of social welfare introduced by Weymark (1981) and Yaari (1988). As is well known the Independence Axiom justifies the following family of social welfare functions,

$$W(F) = \int_0^\infty u(x)\,\mathrm{d}F(x) \tag{34}$$

where F is a distribution with mean μ of the individual welfare V, and u is a non-decreasing concave evaluation function of individual welfare levels that reflects the preferences of a social planner who support the Independence Axiom. As demonstrated by Atkinson (1970) W can be represented by the equally distributed equivalent welfare level defined by:

$$\xi(F) = u^{-1}(W(F)) \tag{35}$$

Thus, $\xi(F)$ is the equally distributed individual welfare level that would yield the same level of social welfare as the actual distribution F. Since $\xi(F) \leq \mu$ Atkinson (1970) used $\xi(F)$ as a basis for defining the following family of inequality measures,

$$I(F) = 1 - \frac{\xi(F)}{\mu} \tag{36}$$

The following specific family of social welfare functions and associated inequality measures were introduced by Atkinson (1970),

$$\xi(F) = \left(\int_0^\infty x^{1-\theta}\ \mathrm{d}F(x)\right)^{\frac{1}{1-\theta}} \tag{37}$$

where $\theta \geq 0$ defines the degree of inequality aversion of the social welfare function. The simplest welfare function is the one that adds up the individual welfare levels, which is obtained by inserting $u(x) = x$ in Eq. (34) or $\theta = 0$ in Eq. (37). The objection to the linear additive welfare function is that the individuals are given equal welfare weights, independent of whether they are poor or rich. Concern for distributive justice requires, however, that poor individuals are assigned larger welfare weights than rich individuals. This is consistent with inserting a strictly concave u-function in Eq. (34). A similar structure is captured by the family of rank-dependent welfare functions,[11]

[11] Several other authors have discussed rationales for rank-dependent measures of inequality and social welfare, see for example Sen (1974), Hey and Lambert (1980), Donaldson and Weymark (1980, 1983), Weymark (1981), Ben Porath and Gilboa (1994), Aaberge (2007) and Aaberge et al. (2013).

$$W\ \ (F) = \int_0^1 p(t)F^{-1}(t)\,\mathrm{d}t \tag{38}$$

where F^{-1} is the left inverse of the cumulative distribution function of the individual welfare levels V with mean μ, and $p(t)$ is a positive concave weight-function defined on the unit interval.[12] The social welfare functions (38) can be given a similar normative justification as for the family (34). Given suitable continuity and dominance assumptions for the preference ordering $\succcurlyeq$ defined on the family of income distributions $\boldsymbol{F}$, Yaari (1987, 1988) demonstrated that the Dual Independence Axiom characterizes the family of rank-dependent measures of social welfare functions (38) where represents the preferences of the social planner. Aaberge (2007) proposed to use the following specification of $p(t)$,

$$p_i(t) = \begin{cases} -\log t, & i = 1 \\ \dfrac{i}{i-1}\left(1 - t^{i-1}\right), & i = 2, 3, \ldots \end{cases} \tag{39}$$

Note that the inequality aversion exhibited by the social welfare function W_i^* (associated with $p_i(t)$) decreases with increasing i. As $i \to \infty$, W_i^* approaches inequality neutrality and coincides with the linear additive welfare function defined by

$$W_\infty = \int_0^1 F^{-1}(t)\,\mathrm{d}t = \mu \tag{40}$$

It follows by straightforward calculations that $W_i \leq \mu$ for all i and that W_i is equal to the mean μ for finite i if and only if F is the egalitarian distribution. Thus, W_i can be interpreted as the equally distributed individual welfare level. As recognized by Yaari (1988) this property suggests that C_i, defined by

$$C_i = 1 - \frac{W_i}{\mu}, \quad i = 1, 2, \ldots \tag{41}$$

can be used as a summary measure of inequality and moreover can be proved to be a member of the 'illfare-ranked single-series Ginis' class introduced by Donaldson and Weymark (1980).[13] Thus, as was recognized

[12] Note that Eqs. (32)−(34) and (32), (33) and (40) can be considered as two-stage approaches for measuring social welfare where the first stage consists of using the common utility function to aggregate the two goods (consumption and leisure) for each individual into a measure of well-being and the second stage to aggregate the well-being across individuals into a measure of social welfare. As demonstrated by Bosmans, Decancq, and Ooghe (2013) the two-stage approach can be given an axiomatic normative justification.

[13] Aaberge (2007) provides an axiomatic justification for using the C_k − measures as criteria for ranking Lorenz curves.

by Ebert (1987) the justification of the social welfare function $W_i = \mu(1 - C_i)$ can also be made in terms of a value judgement of the trade-off between the mean and (in)equality in the distribution of welfare.

To ease the interpretation of the inequality aversion profiles exhibited by W_1, W_2, W_3 and W_∞. Table 3 provides ratios of the corresponding weights – as defined by (39) – of the median individual and respectively the 5 per cent poorest, the 30 per cent poorest and the 5 per cent richest individual for different social welfare criteria. As can be observed from the weight profiles provided by Table 3 W_1 will be particularly sensitive to changes in policies that affect the welfare of the poor, whereas the inequality aversion profile of W_3 is rather moderate and W_∞ exhibits neutrality with respect to inequality.

5. *Socially optimal income taxes*

A number of recent contributions identify optimal tax-benefit rules by employing a microeconometric labour supply model together with microsimulation and (some version of) the social evaluation framework presented above. Aaberge and Colombino (2006, 2013) identify the optimal income tax in Norway within the class of piecewise linear systems. Aaberge and Colombino (2012) perform a similar exercise for Italy, where however the Social Welfare criterion adopted is based on a version of the Roemer's (1998) Equality-of-Opportunity criterion. Blundell and Shephard (2012) look for an optimal tax-benefit rule for low-income families with children in the United Kingdom. Bach et al. (2012) consider optimal taxation with household income splitting. Creedy and Hérault (2012) explore welfare improving directions for tax-benefit reforms in Australia.

Instead of relying on a priori theoretical results as in previous empirical applications of optimal taxation theory, the microeconometric-simulation approach allows for a much more flexible representation of households' heterogeneous characteristics and behaviour and permits the analysis of more complicated tax-benefit rules. This has significant implications upon the results. For example, Aaberge and Colombino (2013), for each of the social welfare functions referred to in Table 3, identify the tax system that maximizes social welfare within a class of 10 parameter tax rules. The

Table 3. Distributional weight profiles of four different social welfare functions

	W_1 (Bonferroni)	W_2 (Gini)	W_3	W_∞ (Utilitarian)
$p(0.01)/p(0.5)$	6.64	1.98	1.33	1
$p(0.05)/p(0.5)$	4.32	1.90	1.33	1
$p(0.30)/p(0.5)$	1.74	1.40	1.21	1
$p(0.95)/p(0.5)$	0.07	0.10	0.13	1

results show that the marginal tax rates of each of the optimal tax systems turned out to be monotonically increasing with income and that more egalitarian social welfare functions tended to imply more progressive tax rules. Moreover, the optimal bottom marginal tax rate is negative, suggesting a mechanism close to policies like the Working Families Tax Credit in the United Kingdom, the Earned Income Tax Credit in the United States and the In-Work Tax Credit in Sweden. The overall picture emerging is in sharp contrast with most of the results obtained by the numerical exercises based on Mirrlees's optimal tax type of formulas. The typical outcome of those exercises envisages a positive lump-sum transfer which is progressively taxed away by very high marginal tax rates on lower incomes (i.e. a negative income tax mechanism), in combination (close to) flat (or even decreasing) marginal tax rates for higher incomes. The results obtained with the microsimulation approach seems to support what suggested by Tuomala (2010): the theory-based results might be enforced by the restrictive assumptions made on the preferences, the elasticities and the distribution of productivities (or wage rates), which in turn might be in conflict with the empirical evidence provided by microeconomic labour supply studies.

6. *Conclusions and future perspectives*

The original concept of microsimulation envisaged large models of the entire economic (or even socio-economic) system – as an alternative to the then dominating large macroeconometric models – including behavioural responses. The events took a different route. On the one hand, the first successful implementations of microsimulation models at the policy level were non-behavioural. On the other hand, the researchers working on microeconometric models of labour supply started using microsimulation tools for policy design and evaluation. In this chapter, we have documented the evolution of different labour supply modelling strategies, together with their notable policy applications that use microsimulation methods. Further developments, both on the microsimulation algorithms side and on the microeconometric side, might or might not favour a re-encounter between large microsimulation algorithms and behavioural labour supply analysis. While further developments on the side of microsimulation technology is documented in other chapters, on the side of microeconometric labour supply models, four research directions are likely to attract more and more attention: (i) intertemporal decisions and decisions under uncertainty; (ii) other dimensions of labour supply (educational and occupational choices, effort, etc.); (iii) modelling intra-household allocation, in particular the structural approach; (iv) development of standardized procedures for improving external (i.e. out-of-sample) validation and internal validation (e.g. non-parametric identification and estimation with experimental or quasi-experimental data) of structural models. The general problem is that there is a trade-off between the increasing theoretical sophistication of labour supply models (e.g. stochastic dynamic programming models, intra-household allocation or collective model, etc.) and their flexibility in interacting with other models representing different segments the economic system. There seem to be three – not mutually exclusive – main directions, in various degree dependent on the quality of the available data and on how sophisticated and flexible both the microeconometric methodology and the microsimulation algorithms will become. First, very specific (both methodologically and policy-wise) labour supply 'modules' can be more or less 'mechanically' linked to system-wide models, the latter being in turn micro- or macro-analytic or a combination of the two. This is close to the current most common practice on micro-macro models. Second, it might be the case that empirical research on labour supply – whether based on observational, experimental, or quasi-experimental data – at a certain point reaches a degree of robustness and generality comparable to an accounting relationship, and can therefore be permanently incorporated into a system-wide microsimulation model. Third, it might be that microeconometric results on labour supply attain a level that allows both specificity and flexibility and permits a structural (micro-founded rather than mechanical) linkage with other micro-analytic behavioural modules and system-wide algorithms.

References

Aaberge, R. (2007). Gini's nuclear family. *Journal of Economic Inequality*, *5*(3), 305–322.

Aaberge, R., & Atkinson, A. B. (2013). *The median as watershed*. Discussion Paper No. 749. Research Department, Statistics Norway.

Aaberge, R., & Colombino, U. (2006). *Designing optimal taxes with a microeconometric model of labour supply*. IZA Discussion Paper No. 2468.

Aaberge, R., & Colombino, U. (2012). Accounting for family background when designing optimal income taxes: A microeconometric simulation analysis. *Journal of Population Economics*, *25*(2), 741–761.

Aaberge, R., & Colombino, U. (2013). Using a microeconometric model of household labour supply to design optimal income taxes. *Scandinavian Journal of Economics*, *115*(2), 449–475.

Aaberge, R., Colombino, U., Holmøy, E., Strøm, B., & Wennemo, T. (2007). Population ageing and fiscal sustainability: An integrated micro-macro analysis of required tax changes. In A. Harding & A. Gupta (Eds.), *Population ageing, social security and taxation: Modelling our future*. Bingley, UK: Emerald Group Publishing Limited.

Aaberge, R., Colombino, U., & Strøm, S. (1999). Labour supply in Italy: An empirical analysis of joint household decisions, with taxes and quantity constraints. *Journal of Applied Econometrics*, *14*(4), 403–422.

Aaberge, R., Colombino, U., & Strøm, S. (2000). Labor supply responses and welfare effects from replacing current tax rules by a flat tax: Empirical evidence from Italy, Norway and Sweden. *Journal of Population Economics*, *13*(4), 595–621.

Aaberge, R., Colombino, U., & Strøm, S. (2004). Do more equal slices shrink the cake? An Empirical investigation of tax-transfer reform proposals in Italy. *Journal of Population Economics*, *17*(4), 767–785.

Aaberge, R., Colombino, U., Strøm, S., & Wennemo, T. (2000). Joint labour supply of married couples: Efficiency and distributional effects of tax reforms. In H. Sutherland, L. Mitton, & M. Weeks (Eds.), *Microsimulation in the new millenium: Challenges and innovations, department of applied economics*. Cambridge: Cambridge University.

Aaberge, R., Colombino, U., & Wennemo, T. (2009). Evaluating alternative representations of the choice set in models of labor supply. *Journal of Economic Surveys*, *23*(3), 586–612.

Aaberge, R., Dagsvik, J. K., & Strøm, S. (1995). Labor supply responses and welfare effects of tax reforms. *Scandinavian Journal of Economics*, *97*(4), 635–659.

Aaberge, R., & Flood, L. (2013). *U.S. versus Sweden: The effect of alternative in-work tax credit policies on labour supply of single mothers*. IZA Discussion Paper No. 7706.

Aaberge, R., Havnes, T., & Mogstad, M. (2013). *A theory for ranking distribution functions*. Discussion Paper No. 763. Research Department, Statistics Norway.

Altonji, J. G., & Paxson, C. H. (1988). Labor supply preferences, hours constraints, and hours-wage trade-offs. *Journal of Labor Economics*, *6*(2), 254–276.

Arrufat, J. L., & Zabalza, A. (1986). Female labor supply with taxation, random preferences, and optimization errors. *Econometrica*, *54*(1), 47–63.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory*, *2*(3), 244–263.

Bach, S., Corneo, G., & Steiner, V. (2012). Optimal top marginal tax rates under income splitting for couples. *European Economic Review*, *56*(6), 1055–1069.

Bargain, O., Decoster, A., Dolls, M., Neumann, D., Peichl, A., & Siegloch, S. (2013). Welfare, labor supply and heterogeneous preferences: Evidence for Europe and the US. *Social Choice and Welfare*, *41*(4), 789–817.

Bargain, O., & Doorley, K. (2013). *Putting structure on the RD design: Social transfers and youth inactivity in France*. IZA Discussion Paper No. 7508.

Bargain, O., & Orsini, K. (2006). In-work policies in Europe: Killing two birds with one stone? *Labour Economics*, *13*(6), 667–697.

Bargain, O., Orsini, K., & Peichl, A. (2014). Comparing labor supply elasticities in Europe and the US: New results. *Journal of Human Resources*, *49*, 723–738.

Bargain, O., & Peichl, A. (2013). *Steady-state labor supply elasticities: A survey*. ZEW Discussion Paper No. 13-084.

Ben-Akiva, M., & Lerman, S. (1985). *Discrete choice analysis: Theory and application to travel demand*. Cambridge, MA: MIT Press.

Ben-Akiva, M., & Watanatada, T. (1981). Application of a continuous spatial choice logit model. In C. F. Manski & D. McFadden (Eds.), *Structural analysis of discrete data with econometric applications*. MIT Press.

Ben Porath, E., & Gilboa, I. (1994). Linear measures, the Gini index, and the income-equality trade-off. *Journal of Economic Theory*, *64*, 443–467.

Beninger, D., Bargain, O., Beblo, M., Blundell, R., Carrasco, R., Chiuri, M.-C., … Vermeulen, F. (2006). Evaluating the move to a linear tax system in Germany and other European countries. *Review of Economics of the Household*, *4*(2), 159–180.

Bingley, P., & Walker, I. (1997). The labour supply, unemployment and participation of lone mothers in in-work transfer programmes. *The Economic Journal*, *107*(444), 1375–1390.

Blank, R. M. (2002). Evaluating welfare reform in the United States. *Journal of Economic Literature, American Economic Association*, *40*(4), 1105–1166.

Blau, D. M., & Robbins, P. K. (1988). Child care costs and family labor supply. *Review of Economics and Statistics*, *70*, 374–381.

Bloemen, H. (2010). An empirical model of collective household labour supply with non-participation. *Economic Journal*, *120*(543), 183–214.

Bloemen, H. G., & Kapteyn, A. (2008). The estimation of utility-consistent labor supply models by means of simulated scores. *Journal of Applied Econometrics*, *23*(4), 395–422.

Blomquist, N. S. (1983). The effect of income taxation on the labor supply of married men in Sweden. *Journal of Public Economics*, *22*(2), 169–197.

Blomquist, N. S., & Hansson-Brusewitz, U. (1990). The effect of taxes on male and female labor supply in Sweden. *Journal of Human Resources*, *25*(3), 317–357.

Blomquist, S., & Newey, W. (2002). Nonparametric estimation with nonlinear budget sets. *Econometrica*, *70*(6), 2455–2480.

Blundell, R. (2006). Earned income tax credit policies: Impact and optimality: The Adam Smith lecture, 2005. *Labour Economics*, *13*(4), 423–443.

Blundell, R. (2010). Comments on: Michael P. Keane 'structural vs. atheoretic approaches to econometrics. *Journal of Econometrics*, *156*(1), 25–26.

Blundell, R. (2012). Tax policy reform: The role of empirical evidence. *Journal of the European Economic Association*, *10*(1), 43–77.

Blundell, R., & Hoynes, H. W. (2004). Has 'in-work' benefit reform helped the labor market? (pp. 411–460). National Bureau of Economic Research, Inc. Retrieved from http://ideas.repec.org/h/nbr/nberch/6753.html

Blundell, R., Brewer, M., Haan, P., & Shephard, A. (2009). Optimal income taxation of lone mothers: An empirical comparison of the UK and Germany. *Economic Journal*, *119*(535), 101–121.

Blundell, R., Duncan, A., McCrae, J., & Meghir, C. (2000). The labour market impact of the working families' tax credit. *Fiscal Studies*, *21*(1), 75–103.

Blundell, R., Duncan, A., & Meghir, C. (1998). Estimating labor supply responses using tax reforms. *Econometrica*, *66*(4), 827–861.

Blundell, R., Ham, J., & Meghir, C. (1987). Unemployment and female labour supply. *The Economic Journal*, *97*, 44–64.

Blundell, R., & MaCurdy, T. (1999). Labor supply: A review of alternative approaches. In O. Ashenfelter & D. Card (Eds.), *Handbook of labor economics* (1st ed., Vol. 3, pp. 1559–1695). Amsterdam: Elsevier.

Blundell, R., MaCurdy, T., & Meghir, C. (2007). Labor supply models: Unobserved heterogeneity, nonparticipation and dynamics. In J. Heckman & E. Leamer (Eds.), *Handbook of econometrics* (1st ed., Vol. 6). Amsterdam: Elsevier.

Blundell, R., & Shephard, A. (2012). Employment, hours of work and the optimal taxation of low-income families. *Review of Economic Studies*, *79*(2), 481–510.

Bosmans, K., Decancq, K., & Ooghe, E. (2013). *What do normative indices of multidimensional inequality really measure?* Université catholique de Louvain, Center for Operations Research and Econometrics (CORE). Retrieved from http://ideas.repec.org/p/cor/louvco/2013035.html

Bourguignon, F., & Magnac, T. (1990). Labor supply and taxation in France. *The Journal of Human Resources*, *25*(3), 358–389.

Bourguignon, F., & Spadaro, A. (2006). Microsimulation as a tool for evaluating redistribution policies. *Journal of Economic Inequality*, *4*(1), 77–106.

Bowen, W. G., & Finegan, T. A. (1969). *The economics of labor force participation*. Princeton, NJ: Princeton University Press.

Breunig, R., Cobb-Clark, D. A., & Gong, X. (2008). Improving the modelling of couples' labour supply. *The Economic Record*, *84*(267), 466–485.
Brewer, M. (2001). Comparing in-work benefits and the reward to work for families with children in the US and the UK. *Fiscal Studies*, *22*(1), 41–77.
Brewer, M. (2009). *How do income-support systems in the UK affect labour force participation?* IFAU – Institute for Evaluation of Labour Market and Education Policy. Working Paper No. 2009–027. Retrieved from http://ideas.repec.org/p/hhs/ifauwp/2009_027.html
Brewer, M., Duncan, A., Shephard, A., & Suarez, M. J. (2006). Did working families' tax credit work? The impact of in-work support on labour supply in Great Britain. *Labour Economics*, *13*(6), 699–720.
Brewer, M., Francesconi, M., Gregg, P., & Grogger, J. (2009). Feature: In-work benefit reform in a cross-national perspective: Introduction. *Economic Journal*, *119*(535), F1–F14.
Burtless, G., & Hausman, J. A. (1978). The effect of taxation on labor supply: Evaluating the Gary negative income tax experiment. *Journal of Political Economy*, *86*(6), 1103–1130.
Callan, T., & van Soest, A. H. O. (1996). *Family labour supply and taxes in Ireland*. Tilburg University, Center for Economic Research. Retrieved from http://ideas.repec.org/p/dgr/kubcen/199426.html
Card, D., & Hyslop, D. (2005). Estimating the effects of a time-limited-earnings subsidy for welfare-leavers. *Econometrica*, *73*(6), 1723–1770.
Chetty, R. (2009). Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annual Review of Economics, Annual Reviews*, *1*(1), 451–488.
Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica*, *56*(1), 63–90. doi:10.2307/1911842
Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy*, *100*(3), 437–467.
Clavet, N.-J., Duclos, J.-Y., & Lacroix, G. (2013). Fighting poverty: Assessing the effect of guaranteed minimum income proposals in Quebec. *Canadian Public Policy*, *39*(4), 491–516.
Colombino, U. (1985). A model of married women's labour supply with systematic and random disequilibrium components. *Ricerche Economiche*, *39*(2), 165–179.
Colombino, U. (2009). Optimal income taxation: Recent empirical applications. *Rivista Italiana degli Economisti*, *14*(1), 47–70.
Colombino, U. (2013). A new equilibrium simulation procedure with discrete choice models. *International Journal of Microsimulation*, *6*(3), 25–49.
Colombino, U. (2014). *Five crossroads on the way to basic income: An Italian tour*. IZA Discussion Paper No. 8087.

Colombino, U., & Del Boca, D. (1990). The effect of taxes on labor supply in Italy. *Journal of Human Resources*, *25*(3), 390–414.

Colombino, U., Locatelli, M., Narazani, E., & O'Donoghue, C. (2010). Alternative basic income mechanisms: An evaluation exercise with a microeconometric model. *Basic Income Studies*, *5*(1), 1–31.

Colombino, U., & Narazani, E. (2013). Designing a universal income support mechanism for Italy: An exploratory tour. *Basic Income Studies*, *8*(1), 1–17.

Colombino, U., & Narazani, E. (2014). *Closing the gender gap: Gender based taxation, wage subsidies or basic income?* WP 12-2014. Torino: Dipartimento di Economia e Statistica.

Colombino, U., & Zabalza, A. (1982). *Labour supply and quantity constraints. Results on female participation and hours in Italy*. Discussion Paper No. 125. Centre for Labour Economics, London School of Economics.

Colonna, F., & Marcassa, S. (2012). *Taxation and labor force participation: The case of Italy*. CEPREMAP Working Papers (Docweb) 1203.

Creedy, J. (2005). An in-work payment with an hours threshold: Labour supply and social welfare. *The Economic Record*, *81*(255), 367–377.

Creedy, J., & Duncan, A. (2002). Behavioural microsimulation with labour supply responses. *Journal of Economic Surveys*, *16*(1), 1–39.

Creedy, J., & Duncan, A. (2005). Aggregating labour supply and feedback effects in microsimulation. *Australian Journal of Labour Economics*, *8*(3), 277–290.

Creedy, J., & Hérault, N. (2012). Welfare-improving income tax reforms: A microsimulation analysis. *Oxford Economic Papers*, *64*(1), 128–150.

Creedy, J., & Kalb, G. (2005a). Behavioural microsimulation modelling for tax policy analysis in Australia: Experience and prospects. *Australian Journal of Labour Economics (AJLE)*, *8*(1), 73–110.

Creedy, J., & Kalb, G. (2005b). Discrete hours labour supply modelling: Specification, estimation and simulation. *Journal of Economic Surveys*, *19*(5), 697–734.

Creedy, J., Kalb, G., & Kew, H. (2003). Flattening the effective marginal tax rate structure in Australia: Policy simulations using the Melbourne institute tax and transfer simulator. *Australian Economic Review*, *36*(2), 156–172.

Creedy, J., Kalb, G., & Scutella, R. (2006). Income distribution in discrete hours behavioural microsimulation models: An illustration. *Journal of Economic Inequality*, *4*(1), 57–76.

Dagsvik, J. K. (1994). Discrete and continuous choice, max-stable processes, and independence from irrelevant attributes. *Econometrica*, *62*(5), 1179–1205.

Dagsvik, J. K. (2000). Aggregation in matching markets. *International Economic Review*, *41*(1), 27–57.

Dagsvik, J. K., Jia, Z., Kornstad, T., & Thoresen, T. O. (2014). Theoretical and practical arguments for modeling labor supply as a choice among latent jobs. *Journal of Economic Surveys*, *28*(1), 134–151.

Dagsvik, J. K., Locatelli, M., & Strøm, S. (2009). Tax reform, sector-specific labor supply and welfare effects. *Scandinavian Journal of Economics*, *111*(2), 299–321.

Dagsvik, J. K., & Strøm, S. (2006). Sectoral labour supply, choice restrictions and functional form. *Journal of Applied Econometrics*, *21*(6), 803–826.

De Luca, G., Rossetti, C., & Vuri, D. (2012). *In-work benefits for married couples: An ex-ante evaluation of EITC and WTC policies in Italy*. IZA Discussion Paper N. 6739.

Deaton, A., & Muellbauer, J. (1980). *Economics and consumer behavior*. Cambridge: Cambridge University Press.

Decoster, A., & Haan, P. (2014). Empirical welfare analysis with preference heterogeneity. *International Tax and Public Finance*. doi:10.1007/s10797-014-9304-5%I

Decoster, A., & Vanleenhove, P. (2012). In-work tax credits in Belgium: An analysis of the jobkorting using a discrete labour supply model. *Brussels Economic Review*, *55*(2), 121–150.

Del Boca, D. (2002). The effect of child care and part time opportunities on participation and fertility decisions in Italy. *Journal of Population Economics*, *15*(3), 549–573.

Del Boca, D., & Flinn, C. (2012). Endogenous household interaction. *Journal of Econometrics*, *166*(1), 49–65.

Del Boca, D., & Vuri, D. (2007). The mismatch between employment and child care in Italy: The impact of rationing. *Journal of Population Economics*, *20*(4), 805–832.

Diamond, P. (1998). Optimal income taxation: An example with a u-shaped pattern of optimal marginal tax rates. *American Economic Review*, *88*, 83–95.

Dickens, W. T., & Lundberg, S. J. (1993). Hours restrictions and labor supply. *International Economic Review*, *34*(1), 169–192.

Domencich, T. A., & McFadden, D. (1975). *Urban travel demand: A behavioral analysis*. Amsterdam: North-Holland.

Donaldson, D., & Weymark, J. A. (1980). A single-parameter generalization of the Gini indices of inequality. *Journal of Economic Theory*, *22*(1), 67–86.

Donaldson, D., & Weymark, J. A. (1983). Ethically flexible Gini indices for income distributions in the continuum. *Journal of Economic Theory*, *29*(2), 353–358.

Donni, O. (2003). Collective household labor supply: Nonparticipation and income taxation. *Journal of Public Economics*, *87*(5–6), 1179–1198.

Donni, O. (2007). Collective female labour supply: Theory and application. *Economic Journal*, *117*(516), 94–119.

Duncan, A., & Giles, C. (1996). Labour supply incentives and recent family credit reforms. *Economic Journal*, *106*(434), 142–155.

Duncan, A., & Harris, M. N. (2002). Simulating the behavioural effects of welfare reforms among sole parents in Australia. *The Economic Record*, *78*(242), 264–276.

Duncan, A., & Stark, G. (2000). *A recursive algorithm to generate piecewise linear budget constraints*. London, UK: Institute for Fiscal Studies. Retrieved from http://ideas.repec.org/p/ifs/ifsewp/00-11.html

Ebert, U. (1987). Size and distribution of incomes as determinants of social welfare. *Journal of Economic Theory*, 41(1), 23–33.

Eissa, N., & Hoynes, H. (2011). Redistribution and tax expenditures: The earned income tax credit. *National Tax Journal*, *64*(2), 689–729.

Eissa, N., & Hoynes, H. W. (2004). Taxes and the labor market participation of married couples: The earned income tax credit. *Journal of Public Economics*, *88*(9–10), 1931–1958.

Eissa, N., & Hoynes, H. W. (2006). Behavioral responses to taxes: Lessons from the EITC and labor supply. *Tax Policy and the Economy*, *20*, 73–110.

Eissa, N., Kleven, H. J., & Kreiner, C. T. (2008). Evaluation of four tax reforms in the United States: Labor supply and welfare effects for single mothers. *Journal of Public Economics*, *92*(3–4), 795–816.

Ericson, P., & Flood, L. (2012). A microsimulation approach to an optimal Swedish income tax. *International Journal of Microsimulation*, *2*(5), 2–21.

Euwals, R., & van Soest, A. (1999). Desired and actual labour supply of unmarried men and women in the Netherlands. *Labour Economics*, *6*(1), 95–118.

Fang, H., & Keane, M. P. (2004). Assessing the impact of welfare reform on single mothers. *Brookings Papers on Economic Activity*, *2004*(1), 1–95.

Figari, F. (2011). *From housewives to independent earners: Can the tax system help Italian women to work?* ISER Working Paper Series 2011–15. Institute for Social and Economic Research.

Fleurbaey, M. (2003). Social welfare, priority to the worst-off and the dimensions of individual well-being. IDEP Working Papers 0312. Institut d'economie publique (IDEP), Marseille, France.

Fleurbaey, M. (2008). *Fairness, responsibility and welfare*. Oxford: Oxford University Press.

Fleurbaey, M., & Maniquet, F. (2006). Fair income tax. *Review of Economic Studies*, *73*(1), 55–84.
Flood, L., Hansen, J., & Wahlberg, R. (2004). Household labor supply and welfare participation in Sweden. *Journal of Human Resources*, *39*(4), 1008–1032.
Flood, L., & MaCurdy, T. (1992). Work disincentive effects of taxes: An empirical analysis of Swedish men. *Carnegie-Rochester Conference Series on Public Policy*, *37*(1), 239–277.
Flood, L., Wahlberg, R., & Pylkkänen, E. (2007). From welfare to work: Evaluating a tax and benefit reform targeted at single mothers in Sweden. *Labour*, *21*(3), 443–471.
Fortin, B., Truchon, M., & Beausejour, L. (1993). On reforming the welfare system: Workfare meets the negative income tax. *Journal of Public Economics*, *51*(2), 119–151.
Fraker, T., & Moffitt, R. (1988). The effect of food stamps on labor supply: A bivariate selection model. *Journal of Public Economics*, *35*(1), 25–56.
Fuest, C., Peichl, A., & Schaefer, T. (2008). Is a at tax reform feasible in a grown-up democracy of Western Europe? A simulation study for Germany. *International Tax and Public Finance*, *15*(5), 620–636.
Gong, X., & Van Soest, A. (2002). Family structure and female labor supply in Mexico city. *Journal of Human Resources*, *37*(1), 163–191.
Grogger, J. (2003). The effects of time limits, the EITC, and other policy changes on welfare use, work, and income among female-headed families. *Review of Economics and Statistics*, *85*(2), 394–408.
Grogger, J., & Karoly, L. A. (2005). *Welfare reform: Effects of a decade of change*. Cambridge, MA: Harvard University Press.
Gurgand, M., & Margolis, D. (2008). Does work pay in France? Monetary incentives, hours constraints, and the guaranteed minimum income. *Journal of Public Economics*, *92*(7), 1669–1697.
Gustafsson, S., & Stafford, F. (1992). Child care subsidies and labor supply in Sweden. *Journal of Human Resources*, University of Wisconsin Press, 27(1), 204–230.
Haan, P., & Myck, M. (2007). Apply with caution: Introducing UK-style in-work support in Germany. *Fiscal Studies*, *28*(1), 43–72.
Haan, P., & Steiner, V. (2005). Distributional effects of the German tax reform 2000 – A behavioral microsimulation analysis. *Zeitschrift für Wirtschafts- und Sozialwissenschaften*. [*Schmollers Jahrbuch: Journal of Applied Social Science Studies*.], *125*(1), 39–49.
Haan, P., & Steiner, V. (2008). Making work pay for the elderly unemployed – Evaluating alternative policy reforms for Germany. *FinanzArchiv/Public Finance Analysis*, *64*(3), 380–402.
Haan, P., & Wrohlich, K. (2010). Optimal taxation: The design of child-related cash and in-kind benefits. *German Economic Review*, *11*, 278–301.

Haan, P., & Wrohlich, K. (2011). Can child care policy encourage employment and fertility?: Evidence from a structural model. *Labour Economics*, *18*(4), 498−512.

Hall, R. (1973). Wages, income, and hours of work in the U.S. labor force. In G. Cain & H. Watts (Eds.), *Income maintenance and labor supply* (pp. 102−162). Chicago, IL: Markham.

Ham, J. C. (1982). Estimation of a labour supply model with censoring due to unemployment and underemployment. *Review of Economic Studies*, *49*(3), 335−354. doi:10.2307/2297360

Hammond, P. J. (1991). Interpersonal comparisons of utility: Why and how they are and should be made. In J. Elster & J. E. Roemer (Eds.), *Interpersonal comparisons of well-being* (pp. 200−254). Studies in Rationality and Social Change. Cambridge, USA: Cambridge University Press.

Harris, M. N., & Duncan, A. (2002). *Intransigencies in the labour supply choice*. Melbourne Institute of Applied Economic and Social Research, University of Melbourne.

Hausman, J. A. (1979). The econometrics of labor supply on convex budget sets. *Economics Letters*, *3*(2), 171−174.

Hausman, J. A. (1980). The effect of wages, taxes, and fixed costs on women's labor force participation. *Journal of Public Economics*, *14*(2), 161−194.

Hausman, J. A. (1985a). Taxes and labor supply. In A. J. Auerbach & M. Feldstein (Eds.), *Handbook of public economics* (1st ed., Vol. 1, pp. 213−263). Amsterdam: Elsevier.

Hausman, J. A. (1985b). The econometrics of nonlinear budget sets. *Econometrica*, *53*(6), 1255−1282. doi:10.2307/1913207

Hausman, J. A., & Ruud, P. (1984). Family labor supply with taxes. *American Economic Review*, *74*(2), 242−248.

Hausman, J. A., & Wise, D. A. (1980). Discontinuous budget constraints and estimation: The demand for housing. *Review of Economic Studies*, *47*(1), 75−96. doi:10.2307/2297104

Heckman, J. J. (1974a). Shadow prices, market wages, and labor supply. *Econometrica*, *42*(4), 679−694. doi:10.2307/1913937

Heckman, J. J. (1974b). Effects of child-care programs on women's work effort. *Journal of Political Economy*, *82*(2), S136−S163.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica*, *47*(1), 153−161. doi:10.2307/1912352

Heckman, J. J., & MaCurdy, T. E. (1986). Labor econometrics. In Z. Griliches (Ed.), *Handbook of econometrics* (pp. 1917−1977). Amsterdam: North-Holland.

Hernæs, E., Jia, Z., & Strøm, S., (2001). *Retirement in non-cooperative and cooperative families*. CESifo Working Paper Series 476.

Hey, J. D., & Lambert, P. J. (1980). Relative deprivation and the Gini coefficient: Comment. *Quarterly Journal of Economics*, *95*(3), 567−573.

Horstschräer, J., Clauss, M., & Schnabel, R. (2010). *An unconditional basic income in the family context: Labor supply and distributional effects*. ZEW Discussion Papers 10-091. ZEW – Zentrum für Europäische Wirtschaftsforschung [Center for European Economic Research].

Hotz, V. J., & Miller, R. A. (1988). An empirical analysis of life cycle fertility and female labor supply. *Econometrica*, *56*(1), 91–118.

Hotz, V. J., & Scholz, J. K. (2003). The earned income tax credit. In R. Mofitt (Ed.), *Means-tested transfer programs in the United States*. Chicago, IL: University Chicago Press.

Hoynes, H. (1996). Welfare transfers in two-parent families: Labor supply and welfare participation under AFDC-UP. *Econometrica*, *64*(2), 295–332.

Hurwicz, L. (1962). On the structural form of interdependent systems. In E. Nagel, P. Suppes, & A. Tarski (Eds.), *Logic, methodology and philosophy of science* (pp. 232–239). Stanford, CA: Stanford University Press.

Ilmakunnas, S., & Pudney, S. (1990). A model of female labour supply in the presence of hours restrictions. *Journal of Public Economics*, *41*(2), 183–210.

Imbens, G. W. (2010). Better late than nothing: Some comments on Deaton (2009) and Heckman and Urzua (2009). *Journal of Economic Literature*, *48*(2), 399–423.

Immervoll, H., Kleven, H. J., Kreiner, C. T., & Saez, E. (2007). Welfare reform in European countries: A microsimulation analysis. *Economic Journal*, *117*(516), 1–44.

Kalb, G. R. (2000). *Labour supply and welfare participation in Australian two-adult households: Accounting for involuntary unemployment and the 'cost' of part-time work*. Victoria University, Centre of Policy Studies/IMPACT Centre. Retrieved from http://ideas.repec.org/p/cop/wpaper/bp-35.html

Kalb, G. R. (2009). Children, labour supply and child care: Challenges for empirical analysis. *Australian Economic Review*, *42*(3), 276–299.

Kapteyn, A., Kooreman, P., & van Soest, A. (1990). Quantity rationing and concavity in a flexible household labor supply model. *Review of Economics and Statistics*, *72*(1), 55–62.

Keane, M., & Moffitt, R. (1998). A structural model of multiple welfare program participation and labor supply. *International Economic Review*, *39*(3), 553–589.

Keane, M. P. (1995). A new idea for welfare reform. *Quarterly Review,* Federal Reserve Bank of Minneapolis, (Spring), 2–28.

Keane, M. P. (2010). A structural perspective on the experimentalist school. *Journal of Economic Perspectives*, *24*(2), 47–58.

Keane, M. P. (2011). Labor supply and taxes: A survey. *Journal of Economic Literature*, *49*(4), 961–1075.

Keane, M. P., Todd, P. E., & Wolpin, K. I. (2011). The structural estimation of behavioral models: Discrete choice dynamic programming methods and applications. In O. Ashenfelter & D. Card (Eds.), *Handbook of labor economics* (1st ed., Vol. 4, No. 4, pp. 331–461). Amsterdam: Elsevier.

Keane, M. P., & Wasi, N. (2013). *The structure of consumer taste heterogeneity in revealed vs. stated preference data*. Economics Papers 2013-W10, Economics Group, Nuffield College, University of Oxford.

Keane, M. P., & Wolpin, K. I. (2002a). Estimating welfare effects consistent with forward-looking behavior. Part I: Lessons from a simulation exercise. *Journal of Human Resources*, *37*(3), 570–599.

Keane, M. P., & Wolpin, K. I. (2002b). Estimating welfare effects consistent with forward-looking behavior. Part II: Empirical results. *Journal of Human Resources*, *37*(3), 600–622.

Keane, M. P., & Wolpin, K. I. (2007). Exploring the usefulness of a nonrandom holdout sample for model validation: Welfare effects on female behavior. *International Economic Review*, *48*(4), 1351–1378.

King, M. A. (1983). Welfare analysis of tax reforms using household data. *Journal of Public Economics*, *21*(2), 183–214.

Klevmarken, N. A. (1997). *Modelling behavioural response in EUROMOD*. Faculty of Economics, University of Cambridge. Retrieved from http://ideas.repec.org/p/cam/camdae/9720

Kornstad, T., & Thoresen, T. O. (2006). Effects of family policy reforms in Norway: Results from a joint labour supply and childcare choice microsimulation analysis. *Fiscal Studies*, *27*(3), 339–371.

Kornstad, T., & Thoresen, T. O. (2007). A discrete choice model for labor supply and childcare. *Journal of Population Economics*, *20*(4), 781–803.

Kosters, M. H. (1966). *Income and substitution effects in a family labor supply model*. P-3339. The Rand Corporation.

Kosters, M. H. (1969). Effects of an income tax on labor supply. In A. C. Harberger & M. C. Bailey (Eds.), *The taxation of income from capital* (pp. 301–332). Washington, DC: Studies of Government Finance, Brookings Institution.

Labeaga, J., Oliver, X., & Spadaro, A. (2008). Discrete choice models of labour supply, behavioural microsimulation and the Spanish tax reforms. *Journal of Economic Inequality*, *6*(3), 247–273.

Li, J., & O'Donoghue, C. (2013). A survey of dynamic microsimulation models: Uses, model structure and methodology. *International Journal of Microsimulation*, *6*(2), 3–55.

Löffler, M., Peichl, A., & Siegloch, S. (2013). *Validating structural labor supply models*. Annual Conference 2013 (Duesseldorf): Competition Policy and Regulation in a Global Economic Order 79819, VereinfürSocialpolitik [German Economic Association].

Lokshin, M. (2004). Household childcare choices and women's work behavior in Russia. *Journal of Human Resources*, *39*(4), 1094–1115.

Lucas, R. Jr. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, *1*(1), 19–46. Elsevier.

MaCurdy, T., Green, D., & Paarsch, H. (1993). Assessing empirical approaches for analyzing taxes and labor supply. *The Journal of Human Resources*, *25*(3), 415–490.

Manski, C. (2012). *Identification of income-leisure preferences and evaluation of income tax policy*. Centre for Microdata Methods and Practice, Institute for Fiscal Studies. Retrieved from http://ideas.repec.org/p/ifs/cemmap/07-12.html

Marschak, J. (1953). Economic measurements for policy and prediction. In W. Hood & T. Koopmans (Eds.), *Studies in econometric method* (pp. 1–26). New York, NY: Wiley.

Matzkin, R. L. (2013). Nonparametric identification in structural economic models. *Annual Review of Economics*, *5*(1), 457–486.

McElroy, M. B., & Horney, M. J. (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International Economic Review*, *22*(2), 333–349.

McFadden, D. (1974). The measurement of urban travel demand. *Journal of Public Economics*, *3*(4), 303–328.

McFadden, D. (1978). Modeling the choice of residential location. In A. Karlqvist, L. Lundqvist, F. Snickars, & J. Weibull (Eds.), *Spatial interaction theory and planning models* (pp. 75–96). Cambridge, MA: Harvard University Press.

McFadden, D. (1984). Econometric analysis of qualitative response models. In Z. Griliches & M. D. Intriligator (Eds.), *Handbook of econometrics* (1st ed., Vol. 2, pp. 1395–1457). Amsterdam: Elsevier.

McFadden, D., & Train, K. (2000). Mixed MNL models for discrete response. *Journal of Applied Econometrics*, *15*(5), 447–470.

Meghir, C., & Phillips, D. (2008). *Labour supply and taxes*. IZA Discussion Paper No. 3405.

Meyer, B. D., & Holtz-Eakin, D. (2002). *Making work pay*. New York, USA: Russell Sage Foundation.

Meyer, B. D., & Rosenbaum, D. T. (2001). Welfare, the earned income tax credit, and the labor supply of single mothers. *The Quarterly Journal of Economics*, *116*(3), 1063–1114.

Miller, R., & Sanders, S. (1997). Human capital development and welfare participation. *Carnegie-Rochester Conference Series on Public Policy*, *46*, 1–44.

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies*, *38*(114), 175–208.

Moffitt, R. (1983). An economic model of welfare stigma. *American Economic Review*, *73*(5), 1023–1035.

Moffitt, R. (1986). The econometrics of piecewise-linear budget constraints: A survey and exposition of the maximum likelihood method. *Journal of Business & Economic Statistics*, *4*(3), 317–328.

Moffitt, R. (2006). Welfare work requirements with paternalistic government preferences. *Economic Journal*, *116*(515), F441–F458.

O'Donoghue, C. (2001). Dynamic microsimulation: A methodological survey. *Brazilian Electronic Journal of Economics*, *4*(2). Retrieved from http://econpapers.repec.org/article/bejissued/v_3a4_3ay_3a2001_3ai_3a2_3acathal.htm

Orcutt, G., Greenberger, M., Korbel, J., & Rivlin, A. (1961). *Microanalysis of socioeconomic systems: A simulation study*. New York, NY: Harper & Row.

Orcutt, G. H. (1957). A new type of socio-economic system. *Review of Economics and Statistics*, 39(2), 116–123.

Pacifico, D. (2013). On the role of unobserved preference heterogeneity in discrete choice models of labour supply. *Empirical Economics*, *45*(2), 929–963.

Peichl, A., & Siegloch, S. (2012). Accounting for labor demand effects in structural labor supply models. *Labour Economics*, *19*(1), 129–138.

Revesz, J. T. (1989). The optimal taxation of labour income. *Public Finance*, *44*(3), 453–475.

Ribar, D. C. (1995). A structural model of child care and the labor supply of married women. *Journal of Labor Economics*, *13*(3), 558–597.

Roemer, J. E. (1998). *Equality of opportunity*. Cambridge, MA: Harvard University Press.

Rosen, H. S. (1976). Taxes in a Labor supply model with joint wage-hours determination. *Econometrica*, *44*(3), 485–507.

Rosen, S. (1974). Effects of child-care programs on women's work effort: Comment. *Journal of Political Economy*, *82*(2), 164–169.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies*, *68*(1), 205–229.

Saez, E. (2002). Optimal income transfer programs: Intensive versus extensive labor supply responses. *Quarterly Journal of Economics*, *117*(3), 1039–1073.

Saez, E., Slemrod, J., & Giertz, S. H. (2012). The elasticity of taxable income with respect to marginal tax rates: A critical review. *Journal of Economic Literature*, *50*(1), 3–50.

Scutella, R. (2004). *Moves to a basic income-flat tax system in Australia: Implications for the distribution of income and supply of labour*. Melbourne Institute Working Paper Series wp2004n05. Melbourne Institute of Applied Economic and Social Research, The University of Melbourne.

Sen, A. (1974). Informational bases of alternative welfare approaches: Aggregation and income distribution. *Journal of Public Economics*, *3*(4), 387–403.

Stern, N. (1986). On the specification of labour supply functions. In R. Blundell & I. Walker (Eds.), *Unemployment, search and labour supply* (pp. 143–189). Cambridge: Cambridge University Press. ISBN: 0521320275.

Swann, C. A. (2005). Welfare reform when recipients are forward-looking. *Journal of Human Resources*, *40*(1), 31–56.

Tobin, J. (1958). Estimation of relationships for limited dependent variables. *Econometrica*, *26*(1), 24–36. doi:10.2307/1907382

Todd, P. E., & Wolpin, K. I. (2006). Assessing the impact of a school subsidy program in Mexico: Using a social experiment to validate a dynamic behavioral model of child schooling and fertility. *American Economic Review*, *96*(5), 1384–1417.

Todd, P. E., & Wolpin, K. I. (2008). Ex ante evaluation of social programs. *Annales d'Économie et de Statistique*, (91–92), 263–291.

Train, K. (2003). *Discrete choice methods with simulation*. Online economics textbooks, SUNY-Oswego, Department of Economics.

Train, K. E., McFadden, D. L., & Ben-Akiva, M. (1987). The demand for local telephone service: A fully discrete model of residential calling patterns and service choices. *The RAND Journal of Economics*, *18*(1), 109–123.

Triest, R. K. (1990). The effect of income taxation on labor supply in the United States. *Journal of Human Resources*, *25*(3), 491–516.

Tummers, M., & Woittiez, I. (1991). A simultaneous wage and labour supply model with hours restrictions. *Journal of Human Resources*, *26*, 393–423.

Tuomala, M. (1990). *Optimal income tax and redistribution*. Oxford University Press.

Tuomala, M. (2010). On optimal non-linear income taxation: Numerical results revisited. *International Tax and Public Finance*, *17*(3), 259–270.

Van Klaveren, C., & Ghysels, J. (2012). Collective labor supply and child care expenditures: Theory and application. *Journal of Labor Research*, *33*(2), 196–224.

Van Soest, A. (1995). Structural models of family labor supply: A discrete choice approach. *Journal of Human Resources*, *30*(1), 63–88.

Van Soest, A., Das, M., & Gong, X. (2002). A structural labour supply model with flexible preferences. *Journal of Econometrics*, *107*(1–2), 345–374.

Van Soest, A., Woittiez, I., & Kapteyn, A. (1990). Labor supply, income taxes, and hours restrictions in the Netherlands. *The Journal of Human Resources*, *25*(3), 517–558.

Varian, H. R. (2012). Revealed preferences and its applications. *Economic Journal*, *122*(560), 332–338.

Vermeulen, F. (2005). And the winner is … An empirical evaluation of unitary and collective labour supply models. *Empirical Economics*, *30*(3), 711–734.

Vermeulen, F., Bargain, O., Beblo, M., Beninger, D., Blundell, R., Carrasco, R., … Myck, M. (2006). Collective models of labor supply with nonconvex budget sets and nonparticipation: A calibration approach. *Review of Economics of the Household*, *4*(2), 113–127.

Wales, T. J., & Woodland, A. D. (1976). Estimation of household utility functions and labor supply response. *International Economic Review*, *17*(2), 397–410.

Wales, T. J., & Woodland, A. D. (1979). Labour supply and progressive taxes. *Review of Economic Studies*, *46*(1), 83–95.

Watanatada, T., & Ben-Akiva, M. (1979). Forecasting urban travel demand for quick policy analysis with disaggregate choice models: A Monte Carlo simulation approach. *Transportation Research*, *13A*, 241–248.

Weymark, J. A. (1981). Generalized Gini inequality indices. *Mathematical Social Sciences*, *1*(4), 409–430.

Wolpin, K. I. (1996). Public-policy uses of discrete-choice dynamic programming models. *American Economic Review*, *86*(2), 427–432.

Wolpin, K. I. (2007). Ex ante policy evaluation, structural estimation, and model selection. *American Economic Review*, *97*(2), 48–52.

Wrohlich, K. (2008). The excess demand for subsidized child care in Germany. *Applied Economics*, *40*(10), 1217–1228.

Yaari, M. E. (1987). The dual theory of choice under risk. *Econometrica*, *55*(1), 95–115.

Yaari, M. E. (1988). A controversial proposal concerning inequality measurement. *Journal of Economic Theory*, *44*(2), 381–397.

Zabalza, A. (1983). The CES utility function, non-linear budget constraints and labour supply: Results on female participation and hours. *Economic Journal*, *93*(37), 312–330.

Zabalza, A., Pissarides, C., & Barton, M. (1980). Social security and the choice between full-time work, part-time work and retirement. *Journal of Public Economics*, *14*(2), 245–276.

Statistics Norway

Postal address:
PO Box 8131 Dept
NO-0033 Oslo

Office address:
Akersveien 26, Oslo
Oterveien 23, Kongsvinger

E-mail: ssb@ssb.no
Internet: www.ssb.no
Telephone: + 47 62 88 50 00

ISSN: 1892-753X

Statistisk sentralbyrå
Statistics Norway

Design: Siri Boquist