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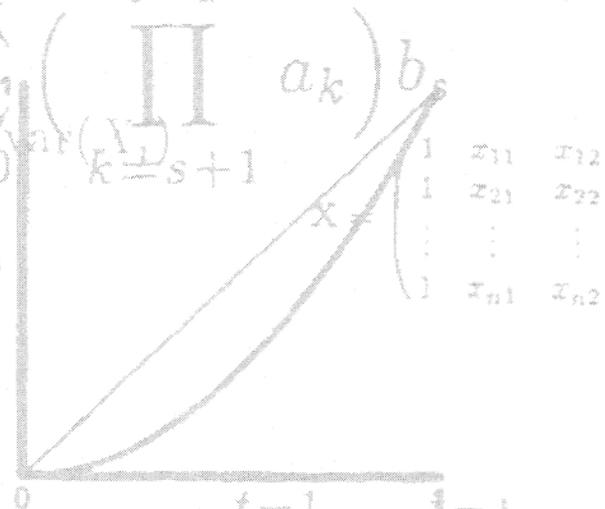
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and Jens Aune

Soil Wealth in Tanzania

# Discussion Papers

$$+ 2 \sum_{i>j} \sum_{j=1}^{n-1} \text{cov}_u(X_i, X_j)$$

$$\text{var}\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i^2 \text{var}(X_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n a_i a_j \text{cov}(X_i, X_j)$$



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## **Soil Wealth in Tanzania**

**Abstract:**

Many African countries are richly endowed with land, but the productive potential of the land base has been underutilised in farming systems with low intensity of external inputs and high intensity of labour. At the same time, mining and erosion of soils have been common features of rural Africa in the 1990s. National income, possibly of considerable size, is foregone in countries with pervasive poverty. This paper studies the income and wealth from the agricultural sector in Tanzania. The gains from a policy redesign are examined by formulating an intertemporal optimization problem where land degradation processes such as soil mining and erosion are taken explicitly into account. We show that land degradation processes, if dealt with in the optimal way, would deviate from the patterns that are currently observed.

Two versions of the model are presented. One considering only the nutrient stocks as determinant of land productivity. The other version also includes the effective rooting depth as determinant of land productivity. Using these models, we compute the soil wealth under the assumption that the opportunity cost of labour is equal to current wages, and under the assumption that opportunity cost of labour is zero. In both cases our estimates suggest that the potential gains from a change in agricultural management are considerable.

**JEL classification:** Soil mining, soil erosion, intertemporal optimization.

**Keywords:** Q10, Q24.

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# 1. Introduction

By the late 1980s, soil erosion and soil-mining (extraction of nutrients in excess of replacement rates), came to be regarded as severe threats to the long run potential of agriculture in sub-Saharan African countries (Stoorvogel and Smaling, 1990). Erosion and soil-mining problems are common on land cultivated by smallholders, and on communal land used for grazing of livestock. Analysis of national income from agricultural production should account for this depreciation of natural capital, and this study quantifies the on-site impacts of land degradation on national income in Tanzania<sup>1</sup>.

The impact of land degradation on macroeconomic indicators has been illustrated in standard computational general equilibrium models linked up with soil-modules. Franco et al. (1993) find that land degradation reduces annual economic growth in Nicaragua by 1.3 percentage points. A more recent study for Ghana with an improved description of the linkages between land degradation and productivity decline, indicates that annual GDP is reduced by 0.3 percentage points because of deterioration in soil quality (Alfsen et al., 1995).

Bojø (1994) has surveyed studies presenting estimates of national costs of land degradation in sub-Saharan Africa. Sutcliffe's (1993) work for the National Conservation Secretariat in Ethiopia relates productivity declines to erosion estimates based on the Universal Soil Loss Equation (USLE), use a soil life model, and a Water Requirements Satisfaction Index. He also calculates impacts on productivity of breaches in the nutrient cycle to assess the costs of nutrient extraction. This branch of the literature lacks a theory of optimal policy, making it difficult to present meaningful estimates of the costs of land degradation, or the costs imposed on the national economy when the optimal pattern of land degradation deviates from the current.

Agricultural policies in African countries have been biased against rural households (Bates, 1981, Dasgupta, 1993). Assessing the gains from agricultural policy redesign becomes important when poverty is widespread, agricultural productivity low and land quality declining. Policy recommendations should preferably internalise microeconomic insights about how institutions, financial incentives and dysfunctional markets influence resource allocation decisions made by farming households. There is already a rich microeconomic literature covering these issues in detail<sup>2</sup>. Among the contributions is Burt (1981), who determines the optimal area percentage of wheat at the farm level using depth of top-soil and percentage of organic matter in soils as state variables.

In this paper we study land management and agricultural policies with a similar methodology, but from a national rather than a farm-level perspective. The control variable in our model is input use, crop choice is exogenous and soil depth and nitrogen content in soils are state variables. This approach allows for comparison of national income losses caused by land degradation and national income losses caused by agricultural policy flaws, and for comparing achievements.

In this discussion, the modelling of the linkage between economic decisions and the status of the land base is a critical issue, requiring firm soil-scientific justification. This paper emphasises one important aspect of this linkage, frequently overlooked and often misrepresented in the economic literature on land degradation, the linkage between crop output per hectare and soil erosion. To highlight the importance

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<sup>1</sup> We purposely overlook off-site effects such as siltation of reservoirs.

<sup>2</sup> For an overview of this literature, see Grepperud (1995).

of this linkage, the cultivated area is exogenous, and constant throughout the simulations.

Land provides economic security for a majority of the people in many developing countries when opportunities for, or political will to redistribute income may be limited. The transition from the existing to a more efficient policy regime may thus be feasible only at high short-run social costs. A model claiming relevance for policies in developing countries should integrate these distributional concerns. We therefore consider an alternative model formulation where the objective is to maximize profit plus labour income. The existing policy regime moves closer to the optimal with this extended and more realistic policy objective, but there is still scope for considerable gains and more rational utilisation of resources.

## **2. Land degradation as a resource management problem**

A stock of renewable resources will recover unless threshold values are violated, while reduction in a stock of exhaustible resources over a reasonable time horizon is irreversible. Barber (1986) suggests that soil quality decline, belong somewhere between these two categories, and classifies soils or land as slowly renewable resources. This, however, is only part of the story.

When the main reason for land degradation is nutrient loss, soil quality can be enhanced from the use of fertilisers or manure, or by investing in soil improving measures that secure a continuous flow of nutrients to the relevant piece of land. Agroforestry systems typically have this property. Similarly, the most common response to soil fertility declines in African farming systems is to allow nutrient stocks to rebuild by leaving land fallow for a suitable period of time. Over a short time horizon, this property of regeneration refers to the nutrient stock dimension of soil quality.

Topsoil and soil physical structures, on the other hand, can be described as slowly renewable resources. Over a reasonable time horizon erosion-induced losses of topsoil and soil physical structures will be irreversible. Again, precautionary measures to reduce or arrest erosion processes, such as terracing, are available to farmers.

Extraction of nutrients, denoted as soil mining, can thus occur and significantly affect land productivity without posing an irreversible long run threat because remedies are available not only to arrest but also to compensate for the nutrient loss ex post. As noted above, erosion or processes that remove top-soil and damage soil physical structures can be arrested through precautionary measures, but beyond a point nature and human beings have limited capacity to correct the damage. This property gives the problem a different flavour. Pure declines in nutrient stocks, and losses of topsoil and physical structures, are therefore to some extent parallels to the distinction between renewable and exhaustible resources.

In the first of two model formulations, land productivity (soil quality) is regarded as a function of nutrient stocks. This stock will change if fertilisers are added, when nutrients are extracted through the crop, or when nutrients are lost (or rather transported and deposited) because of soil erosion. A simplified description of the nitrogen cycle in soils is used to illustrate the dynamics of soil fertility. This formulation has its strength in cases where soil erosion represents less of a threat to soil physical properties, and shortage of and declines in nutrient stocks is the main constraining factor on land productivity. This version is denoted the soil mining model and is presented in section 3. We use it to devise empirical wealth estimates in section 4.

To enrich our discussion and capture other essential elements of land degradation problems we reformulate the model to a two-dimensional description of soil-quality in section 5. Apart from accounting for nutrient extraction, this model also captures the negative impact of erosion on rooting depth, i.e. the depth of soil that crop roots are able to utilise for extraction of nutrients and water. Unlike nutrient extraction, root depth reductions are not reversible.

### 3. The soil mining model

To model soil-mining, we assume that production on a hectare (ha) of arable land can be described as

$$(1) \quad Q_t = f(K_t, L_t, N_t),$$

where  $K_t$  is capital input,  $L_t$  is labour input and  $N_t$  denotes nutritional content of the soil. The access to land is assumed to be fixed, and hence we assume decreasing returns to scale in the production function. The dynamics of real capital is

$$(2) \quad \dot{K}_t = I_t - \delta K_t,$$

where  $I_t$  is investment and  $\delta K_t$  denotes depreciation of the capital stock in year  $t$ . Soil quality and land productivity is a function of the nutrient stock. In a simplified form the dynamics of this nutrient stock can be described as

$$(3) \quad \dot{N}_t = F_t - Q_t n - \beta E_t,$$

where  $F_t$  is input of nutrients from a nutrient source that is interpreted as fertilisers. In a more generalised framework the  $F_t$  term has other possible interpretations including the flow of nutrients from agroforestry systems or fallow.  $n$  is the unit content of nutrients in the cultivated crop, and  $\beta E_t$  is the loss of nutrients caused by soil erosion.

The magnitude of soil erosion on cultivated land depends on several factors including rainfall, cropping pattern, crop yield, land slope, soil type and farmers use of conservation measures. In flat terrain, other than under arid conditions, soil erosion will have limited influence on nutrient losses, and the soil-mining model has its strength when the linkage between land productivity and nutrient losses are not complicated by negative impacts of soil erosion on soil physical structures. Equation (3) captures the most important elements in the nitrogen cycle, but gives a simplified description of nitrogen flows. More realism could be gained at the cost of a more complicated model, but the loss from keeping the structure of the analysis simple is limited.

According to the Soil Loss Estimation Model for Southern Africa (SLEMSA), developed by Elwell and Stocking (1982), crop canopy has significant influence on the magnitude of erosion. An increase in crop canopy reduces the kinetic energy with which raindrops hit the ground and thus the damage of rainfall. While a hike in crop yield also increases the binding capacity of the root system, making the soil less susceptible to erosion. Root structures and protective plant cover vary between crops, so does the linkage between crop yields and crop canopy.

To add some empirical substance to the importance of crop canopy in explaining erosion rates and land

degradation, consider the case of maize, the most important and most area-consuming crop in Tanzania's agricultural sector. With existing yields, the annual loss of topsoil in an average maize farm in Tanzania is 1,5 mm. An increase in yields from the extremely low current to a feasible level of 1600-2500 kgs, could reduce annual erosion rates by 12-25 %. This suggests prospects for considerable environment gains from more intensive agricultural production in Tanzania.

The level of erosion following SLEMSA can now be expressed as a function

$$(4) \quad E_t = \phi \cdot \exp(-bQ_t),$$

The parameters  $\phi$  and  $b$  in (4) depend on slope, rainfall intensity and other factors unique for each crop, and in the following, we assume that these are constant and cropspecific. According to (4) smallholders can manipulate erosion rates by altering cropping patterns, or by influencing yields via the control variable in the optimisation problem, nutrient input through fertilisers.

This model indicates that soil erosion is decreased with increased production. This is related to the way production is increased in this model. Expansion of cultivated area normally implies increased erosion because forest and grasslands gives a better protection of the soil than annual crops (Roose 1977). On the other hand, increasing yield of annual crops reduces soil erosion because high yield implies a more developed canopy. It has been observed that soil erosion can be more than twice as high under low yields compared to under high yields (Roose 1977, Young 1990). It therefore makes sense to increase soil yield in order to reduce soil erosion.

Notice that our formulation will remove two options from farmer decision-making; the choice of which crops to grow and the option to expand or reduce land area. We focus on crop yield-erosion linkage by keeping these variables exogenous<sup>3</sup>. Investments in soil conservation measures such as terracing are not considered in this analysis. But the analysis presented is a prerequisite for economic analysis of different types of conservation investments.

### 3.1. Linking nutrient stocks and productivity declines

Equation (3) describes the net loss of nutrients, and through (1) this loss is translated into productivity declines. Let  $w_K$ ,  $w_L$  and  $w_F$  denote the factor prices of investment goods, labour and fertilisers in the wealth maximization problem

$$(5) \quad \max \int_0^{\infty} (P_t Q_t - w_F F_t - w_L L_t - w_K I_t) e^{-rt} dt,$$

The Hamiltonian of this system is

$$(6) \quad H = P_t Q_t - w_F F_t - w_K I_t - \mu_t (I_t - \delta K_t) + v_t (F_t - n Q_t - \beta E_t),$$

and to simplify these expressions, we introduce the net price on the product. Let

$$\gamma = -\frac{\beta \partial E(Q)}{\partial Q} = \beta b \phi \exp(-bQ),$$

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<sup>3</sup> For a discussion of the importance of crop-choice and cropping patterns for erosion rates, see Aune and Lal (1996).

be the marginal effect on erosion from a marginal increase in crop yields. Since the crop sells at market prices  $P$ , and will remove  $n - \gamma$  units of nutrients from the soil, at a shadow value  $v$ , the net price will be  $p = P - v(n - \gamma)$ . The first order conditions for optimal input of labour investments and fertilisers are

$$(7) \quad \begin{aligned} f_L P &= w_L \\ \mu_t &= w_K \\ v_t &= w_F \end{aligned}$$

and the dynamics of the adjoint variables

$$(8) \quad \begin{aligned} \dot{\mu}_t &= (r + \delta)\mu_t - f_K P \\ \dot{v}_t &= r v_t - f_N P \end{aligned}$$

Combining the first order condition for investment with the differential equation for  $\mu$ , we note that

$$(9) \quad p f_K = w_K \left( r + \delta - \frac{\dot{w}_K}{w_K} \right)$$

which claims that investments should be chosen so that the marginal productivity of capital equals the user cost of capital. Similarly for investment in nutrient stock

$$(10) \quad p f_N = w_F \left( r - \frac{\dot{w}_F}{w_F} \right),$$

and the marginal productivity of soil quality in optimum should equal the “user cost of soil capital”. Combining the first order conditions (7), (9) and (10) with the production function, we have four equations to determine  $Q_t$ ,  $L_t$ ,  $N_t$  and  $K_t$ . Since these first order conditions uniquely determine the state variables, the optimal policy is immediately to adjust to the optimal level of soil quality and capital input, and decide the corresponding input of labour.

In most cases in Tanzania, there are good reasons for believing that soil quality will be below the optimal level, and external inputs (here fertilisers), can be used to adjust the level of nutrient stocks from the current to the optimal level. As the model stands, this adjustment can occur instantaneously. This implication will later be modified because nutrient stocks can be built up only gradually. Now, let  $\pi_t$  denote the net profit at the optimum

$$(11) \quad \pi_t = P_t Q_t^* - w_K I_t^* - w_L L_t^* - w_F F_t^*,$$

where  $F_t^* = \dot{N}_t^* + Q_t^* n + bE(Q_t^*)$ . Profit can be decomposed into a normal return to capital  $(r + \delta)K_t$ , and the scarcity rent on land  $\pi_t - (r + \delta)K_t$ . The total soil wealth is defined as the net present value of the scarcity rent, minus the initial investments in soil quality,  $w_F(N_0^* - N_0)$ . The initial adjustments reflect jumps in the state variables, and have to be accounted for separately. After a simplification, we can rewrite the soil wealth as

$$(12) \quad W = \int_0^{\infty} \pi_t e^{-rt} dt - w_K K_0^* - w_F (N_0^* - N_0).$$

If all prices are constant, the optimal production and factor use will be constant, too. In this case, profit is constant and equals

$$(13) \quad \pi_t = \pi = PQ^* - w_F(Q^*n + \beta E(Q^*)) - w_L L^* - w_K \delta K^*,$$

where  $(Qn + \beta E(Q))$  is the amount of fertiliser or external nutrients supply required to keep  $N$  constant. The expression for soil wealth now simplifies to

$$(14) \quad W = \frac{\pi}{r} + w_F (N_0 - N_0^*) - w_K K^*,$$

where the first term is the present value of future rent. The second term is the value of excess soil quality - most likely negative - while the last term is the initial investment including a capitalisation of future returns to capital.

To derive a closed form solution and calculate the soil wealth estimate, we use a Cobb-Douglas production function  $Q = AK^{\alpha_1} L^{\alpha_2} N^{\alpha_3}$  where the associated parameters  $\alpha_i$  for  $i = 1, \dots, 3$ , are the output elasticities for capital, labour and nutrients, respectively. Since we assume decreasing returns to scale,  $\sum \alpha_i < 1$ . More details about the specification are presented in appendix 1.

## 4. Empirical estimation of soil wealth

### 4.1. Model calibration

The agricultural policies followed by the Tanzanian government complicates the standard procedure for calibration of the parameters in a Cobb-Douglas production function, which is to use cost-shares to calibrate the elasticities  $\alpha_i$ , and base year quantities to calibrate the constant  $A$ . Following the standard procedure would produce biased estimates because the input use has been restricted and thus deviate from the optimum<sup>4</sup>. To avoid this problem, the output elasticity for nutrient can be calibrated from soil scientific analyses. The yield function can be reasonably approximated by an exponential function expressing how different variables affect future yields. The function has been calibrated from data from field experiments (see Aune and Lal (1996)) and is given by

$$(15) \quad Q_t = Q_0 [\exp((F - nQ - E)tk) + X(1 - \exp((F - nQ - E)tk))],$$

where  $Q(t)$  is yield in year  $t$  and  $F$  is the quantity of nitrogen added through organic and inorganic fertilisers.  $nQ$  is nitrogen extraction through the harvested crop, while  $E$  represents half the total loss of nitrogen because of soil erosion.  $k$  is a calibrating parameter and  $X$  accounts for differences between species in their susceptibility to soil erosion, soil buffer capacity and water regime. If  $F - nQ - E$  is constant over time, soil quality is given as  $N_t = N_0 + (F - nQ - E)t$ . Under this assumption (15) can be rewritten as a

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<sup>4</sup> One possible exception to this pattern, is the Southern Highlands, an area that has benefited from selective government policies and consumed between 60 and 70 per cent of the total fertilizer consumption in the country.

function of  $N_t$ , and estimates from the calculator is used to assess reasonable intervals for the size of the output elasticity of nutrient contents. Output elasticities for labour and capital are guesstimates.

For each farming system in Tanzania data on nutrient extraction are derived from Stoorvogel and Smaling (1990), and modified by the crop grown using the factor for soil cover in the Universal Soil Loss Equation (USLE). 50 % of the total nitrogen loss is relevant because nitrogen pools in soils have different turnover times. Productivity declines for maize is assumed to be 50 % lower on a soil with high buffer capacity, because agroforestry experiments with maize indicate that yield declines are less pronounced on volcanic soils, typically found in the Kilimanjaro area (Kamasho, 1995).

Perennial crops are less susceptible to soil erosion because of their deeper rooting system, enabling trees to extract water and nutrients from a larger soil volume than annual crops. Legumes can be ranked somewhere between annual and perennial crops due to a combination of nitrogen fixing capability and shorter growing cycle. With regard to buffer capacity, the scientific basis for assigning a value to tree crops compared to cereals is weak. Young (1990) indicates that annual crops are twice as susceptible to soil erosion as trees. We assume a similar magnitude.

Calculation of the soil wealth requires calibration of the initial level of soil quality. Estimates of the net extraction of nutrients from soil in different farming systems in Tanzania are available. Similarly the annual (relative) loss of productive capacity on a hectare of land, denoted  $\varepsilon$  are estimated from (15).

Formally,  $\varepsilon = \alpha_3 \rho$ , where  $\rho = \frac{\dot{N}}{N}$ . Thus

$$(16) \quad N_t = \frac{\alpha_3}{\varepsilon} \dot{N}_t = \frac{\alpha_3}{\varepsilon} (F_t - Q_t n - \beta E_t),$$

and equation (16) is used to calibrate  $N_0$ . The soil wealth calculations are based on three alternative values on output elasticities. The user cost of capital and material input is aggregated to one input with cost share  $\alpha_1$ . The elasticities for the three input categories in three alternative scenarios are given in Table 1.

**Table 1.** Output elasticities for three scenario-alternatives

	$K$	$L$	$N$
	$\alpha_1$	$\alpha_2$	$\alpha_3$
Alt. 1	0.15	0.15	0.15
Alt. 2	0.20	0.20	0.20
Alt. 3	0.15	0.12	0.25

## 4.2. The agricultural sector

The model with the assumptions of exogenous crop choice and cultivated area, allows us to study land areas of a specific quality where one type of crop is cultivated. The wealth for each crop type and area is computed separately, and the national soil wealth is the aggregated wealth for all hectares of cultivated land. The soil wealth will take on different values contingent on the crops grown, and the optimal combination of crops is the combination that maximizes wealth. The choice of which crop to grow is, however, exogenous to the model, and the wealth calculations are based on the assumption that the same crops are cultivated on the same land areas. The above assumptions make the estimated gains from

redesigned agricultural policies too low because important dimensions of flexibility in farmers decision making are removed.

Soil properties, use and allocation of labour, fertiliser and material inputs show systematic and unsystematic variation across the countryside in rural Tanzania. The major food and export crops in the major farming systems have been included in the soil wealth calculations. Total output and estimates of area under each crop for the agricultural season 1990/91 are used as inputs to the model and presented in Table 2.

### 4.3. Soil wealth estimates

Technologies, use of inputs, yields and impacts on land degradation differ between crops, and average figures for use of capital and labour and production per hectare are based on figures from official Tanzanian sources<sup>5</sup>. In table 3 inputs other than fertilisers and labour have been classified as capital. This means that agrochemicals, typically an annual input, is integrated in the capital concept. Table 3 depicts data on input use and production figures together with the net extraction of nutrients calculated from "representative" levels of use of fertilisers, crops composition, yields and soil erosion. The last column indicates the expected annual productivity declines derived from equation (15).

Using the different output elasticities in the three scenarios from table 1, we have derived the optimal quantities of production for the major food and export crops in Tanzania. The results of the calculations are reported in Table 4, where  $Q^*(i)$  for  $i=1,..,3$  is the optimal quantity in each scenario. The results indicate the order of magnitude of the deviations between current and optimal production, and the gains from a policy redesign. A more detailed elaboration is required before specific policy recommendations can be made.

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<sup>5</sup> Marketing Development Bureau in the Ministry of Agriculture in Dar es Salaam publishes separate cropwise reports for coffee, cotton and tobacco annually and summary reports including the major food crops. The figures have been derived mainly from these reports. Supplementary sources are the monthly bulletins from the Crop Monitoring and Early Warning Unit of the Ministry of Agriculture.

**Table 2. Key figures in agricultural production in Tanzania 1990/91**

	Yield, tonnes	Area, ha	US \$ pr tonne
Maize	2.111.000	1.848.300	120
<b>Farming system</b>			
Southern Highlands	944.553	551.947	
w/beans	252.794	208.075	500
Other	913.471	1.088.277	
Cassava	1.777.000	632.000	180
Sorghum	591.000	475.900	106
Beans	311.000	560.000	300
Paddy	370.000	310.400	200
Coffee	58.000	242.060	
<b>Farming system</b>			
Arabica w/bananas	26.146	87.600	1.280
Arabica	15.589	64.300	
Robusta	13.624	90.000	730
Tobacco	16.447	31480	
<b>Farming system</b>			
Flue cured (Large scale)	1.126	1.251	2.340
Flue cured (Small scale)	9.592	16.488	
Fire cured	5.728	13.736	1.819
Cotton	261.900	424.860	500
<b>Farming system</b>			
Oxen	52.357	145.000	
Typical hand	9.321	29.000	
Improved hand	30.241	57.900	
Improved hand plus hired labour	169.979	193.000	
Tea	16.000	12.500	1300

**Table 3. Input structure, yields and productivity declines in farming systems in Tanzania in 1990/91. (per ha)**

	Capital stock (TZS)	Labour (Mandays)	Yield (kg)	Nitrogen extraction (kg)		Fertiliser (kg)	Annual prod. decline (%)
				Crop	Erosion		
<b>Maize</b>							
Southern Highlands	207800	136	2000	48.8	10.0	50 (SA)	1.3
w/beans	238440	132		24.4	7.5	25 (SA)	1.3
Other	162120	76		19.5	10.0	10 (U)	2.0
Cassava	15080	120	2800	30.8	5.0	0	0.6
Sorghum	20960	193	1250	35.0	10.0	0	3.0
Beans	138540	120	555	17.0	5.0	20 (SA)	3.0
Paddy	83300	217	1200	26.0	0.0	0	0.6
<b>Coffee</b>							
Arabica w/bananas	448000	184	297	11.7	2.5	20(NPK)	0.7
Arabica	892940	183	242	9.5	4.0	20 (NPK)	0.6
Robusta	277660	128	151	6.0	4.0	0	
<b>Tobacco</b>							
Flue cured LS	1184540	536	900	50.3	7.5	650(NPK)	0.2
Flue cured SS	1067280	533	581	32.5	7.5	650 (NPK)	0.2
Fire cured	717820	428	417	23.3	7.5	250 (SA)	
<b>Cotton</b>							
Oxen	289500	136	361	21.2	6.5	0	2.6
Typical hand	243500	132	321	18.7	6.5	0	
Improved hand	263300	138	522	19.6	6.5	30 (SA)	
TH + IH	350300	126	880	45.2	6.5	30 (SA)	1.1
Tea	115000	178	1200	42.0	4.0	30 (NPK)	0.9

As indicated in table 4, the deviations between current production and optimal levels are considerable for two crops; cassava and tea. Cassava is a root crop and the response to increased use of fertiliser is low;  $\alpha_3$  as it now stands is overestimated for cassava. The "true" optimal level is therefore lower than the table suggests. The input of capital in tea production in table 3 is low compared to other crops; tea is also one of the main consumers of fertilisers in Tanzania. We may have underestimated the costs of intermediate inputs for tea, and overestimated the optimal quantity. Also, the nutrient extraction figures for tobacco do not incorporate that production of one ton of tobacco requires the equivalent of two ha of miombo woodlands (Mascarenhas, 1991). The price of tobacco inputs do not reflect these social costs of tobacco production.

**Table 4. Optimal quantity for each farming system in three scenarios, tonnes**

	Current Q	Q*(1)	Q*(2)	Q*(3)
<b>Maize</b>				
Southern Highlands	944.543	433.860	364.197	508.434
w/Beans	252.974	133.319	120.359	159.435
Other	913.741	407.396	319.953	471.994
Cassava	1.777.000	3.600.258	9.994.121	5.924.769
Sorghum	591.000	300.461	263.248	376.953
Beans	311.000	169.068	156.365	199.048
Paddy	370.000	265.175	309.302	352.251
<b>Coffee</b>				
Arabica w/bananas	26.145	19.427	23.351	27.996
Arabica	15.589	7.442	6.187	9.605
Robusta	13.624	4.026	2.245	4.134
<b>Tobacco</b>				
Flue cured LS	1.126	1.834	4.241	4.057
Flue cured SS	9.592	11.663	21.135	23.214
Fire cured	5.278	5.201	7.390	8.979
<b>Cotton</b>				
Oxen	52.357	26.802	23.617	32.028
Typical hand	9.321	4.873	4.267	5.737
Improved hand	30.241	21.389	25.637	28.959
TH + IH	169.979	166.176	251.066	245.329
Tea	16.000	66.559	336.149	131.565

#### 4.4. Soil Wealth with current and optimal policy

A benchmark value for the soil wealth is calculated as the present value of future production assuming that current policies are maintained. We assume that use of labour, fertiliser, capital and material inputs is constant and do not allow for adjustments by the policy maker or by farmers at a future point of time, when soil quality has deteriorated.

Analyses of soil quality changes are quite complex because soil deterioration reduces crop yields and the extraction of nutrients, while erosion increases when crop canopy declines (Elwell and Stocking, 1976). Numerical simulation of the total effect is straightforward, but calculations are more transparent if we assume that the sum of the two effects makes relative land degradation constant, in other words that

$\rho = \frac{\dot{N}}{N}$ . In this case yields will decline at an annual rate  $\alpha_3\rho$ . With constant prices, the net present value of future production will then be

$$(17) \quad W_0 = \frac{1}{r + \alpha_3\rho} P_0 Q_0 - \frac{1}{r} (w_K I_0 + w_L L_0 + w_F F_0) - w_K K_0$$

To draw some inferences from equation (17), we assume that the real rate of interest is 5 %. According to table 3, the crop- and region-specific annual productivity declines caused by soil erosion and soil-mining in Tanzania are in the range 0.5 - 3.0 per cent. If we assume that the average annual decline (i.e.

$\alpha_3\rho$ ) is 1 per cent, the soil wealth becomes negative if production costs exceed 5/6 of the value of output. Using (17), existing practices and the current price structure, Tanzania's soil wealth should equal US \$ 4.2 billion. The return on this wealth is US \$ 0.2 billion.

The value of the agricultural product in the 1991 Tanzanian GDP was US \$ 1.5 billion. Besides the land rent, GDP includes the return to labour without subtracting for depreciation of capital and nutrients. The estimate therefore indicates that these components account for a large share of current GDP. Calculation of the soil wealth using the optimal levels for each crop (alternative three in table 3 above) and the corresponding optimal input use, increases total wealth to US \$ 14.7 billion. This is more than threefold the benchmark value, and indicates that there may be considerable economic gains from a more rational utilisation of Tanzania's land resources.

The focus on profit is reasonable for analysis of optimal management of exhaustible resources such as oil in industrial economies. Agricultural production and land resources have other important functions in developing countries. Provision of employment and income for poor people with few alternative options, and often with governments unwilling or unable to redistribute income, suggests that restructuring of production to reach the optimal path may involve substantial social costs of adjustment. These costs have not been included in the wealth estimate. Policy redesigns that fit the social context of developing countries require that due attention is paid to this problem. We will discuss how the analysis can be modified to account for this concern in 5.3.1.

The model presented so far has used nutrient stocks as the main determinant of land productivity. This soil mining model is applicable and relevant to land areas and regions where soil fertility declines represent the major constraint on productivity, given that erosion has negligible impacts on soil physical characteristics. Since the model recommends instant adjustment of nutrient stocks, and adjustment is feasible only by using excessive amounts of fertilisers over the first years, it is likely that our use of fertiliser prices underestimates the real investment costs required to increase the stock of nutrients from the current to the optimal level. Moreover the economic viability of alternatives to fertiliser application that ensure regular supply of nutrients needs to be examined separately and thoroughly.

## 5. Soil quality in the two-dimensional model

### 5.1. Model structure

The above model has a too simple structure to capture the dynamics of land degradation processes in areas where both soil mining and soil erosion are common. While erosion removes soil and hence nutrients, the perhaps most important element of erosion is the removal of top soil and destruction of soil physical structures. Unlike in the soil mining model, this process cannot be reversed by adding fertilisers or other nutrient supplying sources. Based on Aune and Lal (1995) we use a multiplicative soil quality index,  $S_t = g_1(N_t)g_2(D_t)$ , where  $N_t$  denotes the nutrient content and  $D_t$  denotes the root-depth. The soil quality is interpreted as a productivity index, and the production function is linear in  $S_t$ . The production function is now

$$(18) \quad Q_t = f(K_t, L_t, N_t, D_t)$$

The development in nutrient content is the same as described in (3), while development in root depth is:

$$(19) \quad \dot{D}_t = -E_t$$

Net-profit is unchanged, but the Hamiltonian of the system now becomes

$$(20) \quad H_t = \pi_t + \mu_t(I_t - \delta K_t) + \gamma_t(F_t - nQ_t - \beta E_t) - \lambda_t E$$

The first order conditions are the same, except that the net price of the crop equals  $P - nv_t + \gamma(v_t + \lambda_t) = p + \gamma\lambda$ . We repeat these for completeness;

$$(21) \quad \begin{aligned} f_L(p + \gamma\lambda) &= w_L \\ \mu_t &= w_K \\ v_t &= w_F \end{aligned}$$

The dynamics of the adjoint variables can be written as

$$(22) \quad \begin{aligned} \dot{\mu}_t &= r\mu_t - (f_K(p + \gamma\lambda) - \delta) \\ \dot{\gamma}_t &= r\gamma_t - f_N(p + \gamma\lambda) \\ \dot{\lambda}_t &= r\lambda_t - f_D(p + \gamma\lambda) \end{aligned}$$

Note that  $\mu$  and  $v$  are determined by the first order conditions, it remains to determine  $\lambda$  to identify the optimal policy. To proceed we make more specific assumptions about the technology, and again we use a Cobb-Douglas specification. The details are elaborated in appendix 2.

## 5.2. Empirical results

To study the effect of this two dimensional specification of soil quality, we have focused on maize production in the Southern Highlands in Tanzania. The rooting depth in this area is assumed to be about 30 cm for maize. When the root depth is below 8.5 cm, the soil becomes unproductive. The effective root depth is therefore the depth exceeding 8.5 cm. Currently, erosion removes about 1.5 mm annually, and the soil should remain productive for 140 years. This prediction is modified by the feedback effect, since lower root depth decreases soil productivity, which by reducing yields and crop canopy, boosts erosion. With the current input structure, the soil will remain productive for about 110 years.

Another important observation is that while the first 11 cm of root depth lost reduces productivity by 30 %, the next 10.5 cm will completely wipe out the productive potential of the soil. Since the timespan before the first 11 cm are lost is about 70 years, the important consequences of soil erosion will be visible only in the last part of the simulation period.

The theory of resource management prescribes that it may be optimal in this model, to completely exhaust a renewable resource if the rate of regeneration is lower than the interest rate. Erosion can be reduced only by expanding yields, and some level of erosion is unavoidable. To highlight these effects we use the slightly lower interest rate of 4 % in the simulations.

The benchmark value is the present value of future land rent under the current policy. This policy is defined by a constant input mix throughout the simulation period, with a proportional reduction in input quantities as root depth declines. The wealth per hectare in the benchmark scenario is US \$ 385. If the Government alternatively followed the optimal policy, the wealth would increase to US \$ 1940.

The optimal policy prescribes a sharp reduction in the use of labour input, from 136 to 26 mandays pr ha. The nitrogen content of the soil should increase from 1950 kg/ha to 2473 kg/ha. Capital input should increase from US \$ 24.5 per ha to US \$ 55.5 per ha. Yields are reduced from 2.17 tons/ha to 1.28 ton/ha. This reduction will increase erosion from 1.5 mm/year to 1.67 mm/year. Note that these results are highly dependent on the elasticities, and these are highly uncertain, especially the elasticities of labour and capital.

The optimal solution is thus to produce less of the staple crop in the Tanzanian diet; maize. The adjustment will make more labour available for other productive activities, at the given wage rate, but the recommended policy can be optimal only if the redundant labour force can be gainfully employed elsewhere. The required scale of adjustment suggests that this is not a very realistic description for a policy redesign apart from perhaps in the long run.

The shadow price of root depth is estimated to US \$ 64/cm X ha, and drops to US \$ 45/ha X cm in hundred years. A measure that would reduce annual erosion on one ha of land by 0.1 mm annually, is worth the present value of an income stream starting at US \$ 0.64 and dropping to US \$ 0.45 over hundred years, with present value approximately US \$ 15. The present value of a measure that completely arrests erosion on a ha of land in Southern Highlands is worth approximately US \$ 250.

The value of the annual loss of root depth is about US \$ 9.6/ha, and the total loss for the maize areas in Southern Highlands US \$ 5.3 mill. If the shadow price on lost root depth is compared for the return to wealth of US \$ 77.6/ha, the indicated loss proxies 12 %. The contribution to GDP includes the value of about 26 mandays/ha at cost of US \$ 31.7/ha. The total contribution to GDP is thus US \$ 109.3/ha. The losses caused by erosion correspond to about 9 % of the total contribution to GDP under the optimal policy.

### 5.3. Some extensions

#### 5.3.1. Exogenous labour supply

Wages are exogenous in the model specification above. This is reasonable if labour has alternative employment opportunities. With the elasticities used in the previous simulation, demand for labour is far below the current levels; and it is not likely that labour will find alternative employment at the going wage rate. An equally realistic assumption is that the labour force is given as  $L_t = \bar{L}$ . The production function now becomes

$$(23) \quad Q_t = (AL^{\alpha_2}) K_t^{\alpha_1} N_t^{\alpha_3} D_t^{\alpha_4}$$

The optimal production with this technology is denoted with a hat, and wages are now endogenous. By the equilibrium condition for labour,

$$w_L = \frac{\alpha_2 p Q}{\bar{L}}$$

and labour cost is  $\alpha_2 p Q$ . Moreover, the net profit is

$$(24) \quad \pi_t = \hat{\pi} = (P - \alpha_2) \hat{Q} - w_F (\hat{Q}n + E(\hat{Q})) - w_K \delta \hat{K},$$

and the corresponding wealth

$$(25) \quad W = \frac{\hat{\pi}}{r} - w_F(\hat{S} - S_0) - w_K \hat{K}$$

We now assume that the input of labour is exogenous. Endogenous and downward flexibility in agricultural wages when the incidence of poverty is high, is also a somewhat artificial assumption.

The wealth for maize production in the Southern Highlands now increases to US \$ 3380 pr ha, compared to the estimate of US \$ 1940 when the labour force was endogenous. The main reason is that wages are forced down to US \$ 0.38 manday the first year, compared to the current level of US \$ 1.22 /manday. With reduced factor prices and constant output prices, the rent goes up. Moreover, the scale of production increases from the current level of 2.17tons/ha, to 2.62 tons/ha reducing erosion to 1.44 mm/year. The optimal content of nitrogen increases by about two tons/ha; the optimal level of nutrients in the soil is thus much higher than the current level. Compared to the current policy scenario the increase in wealth from US \$ 385 to US \$ 3380 is formidable.

When a farmer works his own land, profit and wages are not separable. But, a wage decline represents a severe adverse effect on vulnerable households in an agriculturally based economy where wage labour is widespread. Such a decline will therefore be socially unacceptable, and a narrow interpretation of the wealth concept can take us astray, and lack relevance for practical policy purposes.

There are various ways to overcome this barrier. One possibility is to include the value of human capital, defined as the present value of future income, in the wealth concept. In other words to maximize profit plus labour income. With the current policy, the estimate of this extended wealth becomes US \$ 3730/ha. If we maximize the alternative wealth concept with exogenous labour, wealth per hectare increases to US \$ 4325/ha. The change remains significant. The total surplus (rent plus wages) pr manday increases by about 16 %. The shadow price on soil depth is US \$ 175/cm X ha, while the value of annual loss of rooting depth is about US \$ 25/ha, compared to a return to the wealth of USD 173/ha. The value of lost root depth equals about 15 % of the return on wealth.

### 5.3.3. Endogenous food prices

Exogenous crop prices is a reasonable assumption for export commodities, but for crops like maize mainly sold in domestic markets, the level of production affect prices. For the single farmer the producer price remains exogenous, and the previous analysis will hold under the additional assumption that  $P_t = P(Q_t)$ .

The assumption that crop prices are endogenous is most important in the case of two dimensional soil quality. Since erosion of root depth is an irreversible process, output will eventually decline. As will be seen, the corresponding increase in crop prices may have considerable impacts on the shadow price of root depth and on the profitability of measures that arrest or reduce erosion. In a more realistic scenario, we have made a simulation based on the assumption that the elasticity of demand is -0.4, i.e.  $P_t = \text{konst } Q_t^{-0.4}$ . The equilibrium is now very close to the current level of prices and output, with output declining only slightly over time. Since prices are somewhat higher than in the reference scenario, the wealth and shadow prices are higher, too. The soil wealth estimate under this assumption is US \$ 2900/ha. While this is significantly higher than wealth with exogenous prices, the cost of living also increases in this

scenario. The maize price starts at US \$ 129/ton and rises to US \$ 200/ton, while the current price of maize is US \$ 120/ton.

Perhaps more important to observe is the high shadow price on root depth and therefore the high returns to investments in measures that reduce or arrest erosion. While the initial price increases by only 7.5 %, the shadow price on root depth more than doubles from US \$ 64/cm to US \$ 150/cm; with the value of the annual loss of root depth of US \$ 23/ha. Moreover, this loss increases over time since both the shadow value and erosion increases. After 100 years the annual loss is more than US \$ 60/ha.

#### 5.4. Hicksian income

Hicks' definition of income is the amount a nation can spend during a year and remain as well off at the end of the year as it was when the year started. With constant interest rate and prices, the Hicksian income is equivalent to the return earned on national wealth. Since land rent is decreasing over time in some of our simulations, and large initial investments are required in other simulations, the observed income can deviate significantly from income as interpreted in the Hicksian sense<sup>6</sup>.

When labour supply is exogenous, the permanent income of the soil wealth including future labour income, is about US \$ 216/ha. Due to heavy initial investment the first year, the cash-flow is -717 US \$/ha in addition to US \$ 51/ha in labour income. To spend permanent income, it is necessary to borrow US \$ 882/ha. The second year cash-flow including labour income is US \$ 275/ha, of which US \$ 239/ha remains after the payment of interest on the loan. An additional US \$ 22/ha should be saved to maintain income when productivity declines because of reduced root depth.

The figures are even more striking with the current policy. The permanent income (including labour income) is US \$ 145/ha. The first year cash-flow including labour cost is US \$ 180/ha. Thus, to be equally well off at the end of the period, US \$ 35/ha should be saved. Since the cash-flow in this case is close to the contribution to GDP<sup>7</sup>, the model prescribes a savings ratio of about 20 %. Saving can either be in domestic capital or in foreign bonds. Note that in most scenarios in this paper the capital stock in the agricultural sector is optimised. Thus a further investment in agricultural capital is not profitable, except under a sub-optimal policy.

#### 5.5. Optimal crop choice

The optimal production of each crop for each technology or region has been computed. For this computation, we assumed that the areas used for production of different crops are unchanged over the simulation period. This is unrealistic because farmers have options in deciding which crops to grow. A more resource-demanding approach is to optimize the share of output for each crop in the national agricultural portfolio. This extension would, however, be more information-intensive because land is not a homogeneous input to production. Different agroecological constraints to production would have to be accounted for, and areas used for production of cassava today are not necessarily suited for production of maize or coffee tomorrow. Using the model above, we could derive the wealth for each hectare of land,

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<sup>6</sup> In a scenario where an initial investment makes the cashflow  $\pi_t$  negative the first year, and positive the second year, we compute the first year current income as follows. Suppose that the second year cash-flow is sustainable, then the cash flow would be,  $\pi_1, \pi_2, \dots$ . If we spend  $x$  each year we would have to borrow  $x - \pi_1$  the first year, with an annual interest payment  $r(x - \pi_1)$ . This policy is sustainable if  $\pi_2 - r(x - \pi_1) = x$ , i.e. if  $x = (\pi_2 + r\pi_1)/(1 + r)$ .

<sup>7</sup> Actually, the cash-flow is closer to NDP, since capital depreciation is subtracted. Capital depreciation is less than US \$ 5/ha.

and among all feasible crop categories select the wealth maximizing one. Adding the wealth with optimal crop choice for each hectare, will determine the total wealth.

## 5.6. Summary of the results

A summary with estimates of wealth and shadow prices are given in table 5, where  $W$  is the wealth in US \$/ha; the present value of future resource rent, and  $W+I$  is wealth included the present value of future labour income. Finally  $P$  is the shadow price of rooting depth in US \$ per cm X ha. We note that the difference between the current and optimal policy attains the largest value for pure wealth estimates. But, focus on wealth alone presupposes that the labour force has alternative uses and disregards the important role the agricultural sector plays for provision of employment and food security for a large share of the population in Tanzania. When the income to the labour force is included, the current policy is much closer to optimum both in absolute and relative terms. Notice, however, that this is valid for the Southern Highlands, an area that is atypical for the agricultural sector in Tanzania, because agricultural policies have been less distorting there than elsewhere. Note also that in the main model, the shadow price does not reflect the generation of income to the labour force. Assuming either that there is no alternative use of labour even in the long run, or that the prices on agricultural products rises as if the Tanzanian economy was closed, more than doubles the shadow price of rooting depth.

**Table 5.** Summary of main results

	W	W + I	P
Current	385	2900	
Main model	1940	3730	64
Exogenous labour	3380	4325	175
Endogenous price	2900		150

## 6. Conclusions

We have analysed the soil wealth and shadow prices of soil under optimal production in two versions of an intertemporal model. The relevance and applicability of the models under different agroecological conditions have been briefly discussed. The calculations demonstrate that the optimal content of nutrients in Tanzanian soils may be much higher than the current levels. This conclusion is very sensitive to the elasticities of nutrient content in the production function. The conclusion therefore requires further empirical investigation before being translated into detailed practical policy recommendations.

The total soil wealth with the current policy was estimated to US \$ 4.2 billion. In a model where the effect of soil mining, but not loss of rooting depth is integrated, the optimal wealth was calculated to be US \$ 14.7 billion. This indicates that there may be considerable gains to improved management of the soil wealth in Tanzania, and suggests that investments to enhance the productive potential of land indeed are good investments.

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## Appendix 1

To derive a closed form solution we use a Cobb-Douglas production function,

$$A.1.1 \quad Q_t = A_t K_t^{\alpha_1} L_t^{\alpha_2} N_t^{\alpha_3}$$

$A_t$  is a scaling parameter incorporating technological progress. Since the production is restricted to a limited area of land, we assume decreasing returns to scale, i.e.  $\alpha = \alpha_1 + \alpha_2 + \alpha_3 < 1$ . Assuming  $Q > 0$ , the optimality conditions (8) can be rewritten as

$$\alpha_1 p Q = \left( r - \frac{\dot{w}_K}{w_K} + \delta \right) w_K K$$

$$A.1.2 \quad \alpha_2 p Q = w_L L$$

$$\alpha_3 p Q = \left( r - \frac{\dot{w}_F}{w_F} \right) w_F N$$

To simplify, we assume that prices are constant, in other words that  $\dot{w}_K = \dot{w}_F = 0$ . Substituting the first order conditions into the production function, optimal production  $Q^*$  satisfies

$$A.1.3 \quad Q^* = \eta \cdot Q^{*\alpha}$$

$$\text{where } \eta = A_t \left[ \frac{p\alpha_1}{(r+\delta)w_K} \right]^{\alpha_1} \left[ \frac{p\alpha_2}{w_L} \right]^{\alpha_2} \left[ \frac{p\alpha_3}{rw_F} \right]^{\alpha_3}.$$

This equation has two solutions,  $Q = 0$  and the strictly positive solution, but A.1.2 only applies with  $Q > 0$  since  $Q = 0$  does not solve the first order conditions. Hence,

$$Q^* = \eta^{1/(1-\alpha)}.$$

Notice that  $\eta$  depends upon the net price  $p$ , which in turn depends upon  $\gamma$ , which again depends upon  $Q^*$ . Thus  $\eta$  is a function of  $Q^*$ , and hence  $Q^*$  appears on both sides of the equation.

## Appendix 2

To derive a closed form solution, in the two dimensional formulation of soil quality, we repeat the use of a Cobb-Douglas production function,

$$A.2.1 \quad Q_t = A_t K_t^{\alpha_1} L_t^{\alpha_2} N_t^{\alpha_3} D_t^{\alpha_4},$$

and again assume constant prices and decreasing returns to scale. Combining the first order conditions and the dynamic equations for  $\mu$  and  $\nu$ , the optimality conditions can be rewritten as

$$\alpha_1 (p + \gamma\lambda) Q = (r + \delta) w_K K$$

$$A.2.2 \quad \alpha_2 (p + \gamma\lambda) Q = w_L L$$

$$\alpha_3 (p + \gamma\lambda) Q = r w_F N$$

Note that this equation involves the shadow price of root depth  $\lambda$  since the net price on output depends

on the shadow value of soil erosion that is avoided when crop yields increase marginally. Once  $\lambda$  is known, the optimal policy can be computed. Let now  $\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$ , then the optimal solution is

$$A.2.3 \quad Q^* = \xi^{1/(1-\alpha)},$$

$$\text{where } \xi = A_t \left[ \frac{(p + \gamma\lambda)\alpha_1}{(r + \delta)w_K} \right]^{\alpha_1} \left[ \frac{(p + \gamma\lambda)\alpha_2}{w_L} \right]^{\alpha_2} \left[ \frac{(p + \gamma\lambda)\alpha_3}{rw_F} \right]^{\alpha_3} D_t^{\alpha_4}.$$

The optimal use of fertiliser and optimal investment are again not directly determined.. But since optimal capital and nutrient level are determined, the optimal policy follows from the dynamic equations for capital and nutrient content.

Once the initial level of  $\lambda$  is established, it is straightforward to compute the optimal policy and the path for  $\lambda$ . Hence, a procedure for wealth computation is to make a guesstimate for the value of  $\lambda$ , and simulate the optimal development given this value. If the value is too high, production is excessive while erosion is below the optimal level. The opposite conclusion holds for a too low value for  $\lambda$ . Root depth will be eroded away, and production drop to zero too rapidly. From the set of alternative paths, the path with the highest wealth estimate is selected.

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