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ENGEL FUNCTIONS, PANEL DATA AND LATENT VARIABLES[1]

BY

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ABSTRACT

A system of consumer expenditure functions is estimated from Norwegian household budget data. Specific features of our approach are: (i) Panel data on individual households are used, which offer far richer opportunities for identification, estimation and testing than usual cross section data. (ii) Measurement errors are carefully modelled. Total consumption expenditure is modelled as a latent variable, purchase expenditures on different goods and two income measures are used as indicators of this basic latent variable. (iii) The distribution of latent total expenditure across households, and its evolution over time, is estimated and important properties tested. (iv) Individual differences in preferences, represented by individual, time invariant latent variables, are modelled, identified, estimated, and tested. (v) We test the hypothesis that preferences are uncorrelated with total consumption expenditure, which is basic to all cross section estimation of consumer demand functions. (vi) The model can be formalized as a special case of the LISREL model, and the maximum likelihood algorithm of the computer program LISREL VI is applied.

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CONTENTS

1. INTRODUCTION

Systems of consumer expenditure functions, or more specifically Engel functions, estimated from individual data has been in the focus of a substantial number of theoretical and empirical papers over the years. [See Deaton (1986) and Blundell (1988) for recent surveys.] Such expenditure systems are interesting for several purposes, including macro econometric model building and analysis of distributional policies. Of particular interest in this research are the Engel elasticities and the effect on consumption of socioeconomic characteristics. The vast majority of the empirical analyses of Engel functions have been based on cross section data with an income variable considered as observed without error, and often no distinction is made between income and total consumption expenditure.

In the present paper, the estimation of systems of consumer expenditure functions is reconsidered in a wider perspective. First, panel data from household budget surveys, with two observations from each respondent are used. It is well known that panel data in general offer a far richer opportunity for analyzing individual effects and, in particular, for controlling for individual 'nuisance' variables than conventional data types [cf. Mundlak (1978), Hausman and Taylor (1981), and Griliches and Hausman (1986)]. This is also the case for the data set used in the present study, even if the number of replications is the smallest possible. Second, in order to allow for imperfect measurement of income and consumption, they are considered as latent variables. Third, the distribution of latent total consumption expenditure across households, and its evolution over time, is identified and estimated simultaneously with the demand system. Fourth, individual differences in preferences, represented by individual time invariant latent variables, are allowed for. An important purpose of the investigation is to quantify these differences. Fifth, within this framework, an attempt is made to go one step further by investigating the possible correlation between income and preferences. The availability of data with more than one replication makes it possible to test for such correlation. As remarked by Griliches and Hausman (1986, p. 94), "in the panel data context, a variety of errors-in-variable models may be identifiable and estimable without the use of external instruments". See also Aigner et al. (1984, section 3.10).

The paper represents an extension of previous research by Biørn and Jansen (1982) and Aasness (1983). In the former, individual differences in consumption are analyzed by means of a complete demand system (including prices) with an error components specification of the disturbance vector, although with errors of measurement in income and consumption disregarded. The latter uses cross section data, thus neglecting the panel aspect, but focuses on the errors of measurement and identifies and estimates a distribution of latent total consumption expenditure across households simultaneously with a system of Engel functions. The present work integrates the two approaches, and extends them by, inter alia, incorporating information on observed incomes from tax records.

2. MODEL AND BASIC NOTATION

2.1 The Engel functions and the measurement equations

Let the consumption be divided into I commodity groups and assume that data from H households observed over T=2 years are available. The expenditure on the i'th group is assumed to be a linear function of the total expenditure which is considered as latent and M=2 observed demographic variables,

$$(1) \qquad \eta_{ith} = a_{it} + b_i\xi_{th} + c_{i1}z_{1h} + c_{i2}z_{2h} + \mu_{ih} , \qquad i = 1,\dots,I, \; t = 1, 2, \; h = 1,\dots,H,$$

where a_{it}, b_i, and c_{im} are coefficients and

η_{ith} = true expenditure on commodity i of household h in year t (latent),

ξ_{th} = true total expenditure of household h in year t (latent),

z_{1h}, z_{2h} = demographic variables for household h (observed),

μ_{ih} = a variable representing tastes, habits etc. attached to commodity i, household h (latent).

This function is denoted as the Engel function of commodity i. The time subscript on the constant term indicates that vertical shifts in the expenditure functions between the two years are allowed for. Since, by definition,

$$(2) \qquad \sum_{i=1}^{I} \eta_{ith} = \xi_{th} , \qquad t = 1,2, \; h = 1,\dots,H,$$

we know that the coefficients are subject to the adding-up restrictions

$$(3)\qquad \sum_{i=1}^{I} a_{it} = \sum_{i=1}^{I} c_{im} = 0\ , \quad \sum_{i=1}^{I} b_i = 1\ , \qquad t = 1,2,\ m = 1,2,$$

and that the taste variables add to zero,

$$(4)\qquad \sum_{i=1}^{I} \mu_{ih} = 0\ , \qquad h = 1,\ldots,H.$$

The latter restriction reflects the fact that the μ's represent 'relative preferences' with respect to the different commodities. In the aggregate, these effects cancel against each other.

The observed expenditure on commodity i of household h in period t is

$$(5)\qquad y_{ith} = \eta_{ith} + \nu_{ith}\ , \qquad i = 1,\ldots,I,\ t = 1,2,\ h = 1,\ldots,H,$$

where ν_{ith} is the measurement error. (The latter may also include a disturbance term in the expenditure function (1), which cannot be empirically distinguished from the measurement error.) In surveys of household expenditures, the observed expenditure (y_{ith}) will typically be purchase expenditures during a relatively short period. The true expenditures (η_{ith}) could be defined precisely within a specific consumer theory, and for a durable good it could for instance be the user cost of its service flow. The measurement error (ν_{ith}) will then contain the difference between the purchase expenditure during a short period and the true consumption expenditure.

Assume that K (observed) indicators of the latent total expenditure (e.g. taxable income) exist, and let w_{kth} be the value of the k'th of these indicators for household h in period t. Rather than considering the w's as 'extraneous' instruments, as is a common practice when using errors-in-variables models for cross sectional data, we formalize - in the spirit of the LISREL model - the correlation in the form of K linear relationships, denoted as the <u>measurement equations</u>,

$$(6)\qquad w_{kth} = d_{kt} + e_k\xi_{th} + f_{k1}z_{1h} + f_{k2}z_{2h} + \lambda_{kh} + \varepsilon_{kth}\ , \qquad k = 1,\ldots,K,\ t = 1,2,\ h = 1,\ldots,H,$$

where d_{kt}, e_k, and f_{km} are (unrestricted) coefficients and ε_{kth} is an error term. Since the relationships between these indicators and the total expenditure may also depend on socioeconomic variables and other individual characteristics, we include the z's as well as a latent variable λ_{kh} in the equations. Again, the time subscript on the constant term indicates that the equations are allowed to shift between the two years.

Of course, the interpretation of the d's, the e's, and the f's and of the λ's depend on the specific definition of the w's. If a w is an income variable recorded for tax purposes (there may be several, as is the case in for instance Norway), eq. (6) may represent, on the one hand, the savings behaviour of the household, on the other hand the definition of the taxable income in the tax code and (possibly also) the 'tax payment behaviour' of the household. The λ's may for instance represent both the thriftiness of the household and its attitude to (legal and illegal) tax evasion. Like the μ's, the λ's will be denoted as 'preference variables' in the following. It is then difficult to give (6) the status as structural relationships with a similar degree of autonomy as the expenditure functions (1). Rather they represent the reduced or semi-reduced form of a (possibly complex) model of the income distribution, the statutory tax system, and the spending, saving, and tax paying activity of the individual household.

From (1) and (5) it follows that the Engel functions expressed in terms of observed expenditures read

$$(7) \qquad y_{ith} = a_{it} + b_i\xi_{th} + c_{i1}z_{1h} + c_{i2}z_{2h} + \mu_{ih} + \nu_{ith}, \qquad i = 1,\dots,I,\ t = 1,2,\ h = 1,\dots,H.$$

These equations are similar to (6), so that we can formally consider w_{kth} and y_{ith} as K+I indicators of the latent total expenditure ξ_{th}, with individual specific observed and latent background variables. Note that the observed total expenditure,

$$(8) \qquad x_{th} = \sum_{i=1}^{I} y_{ith}, \qquad t = 1,2,\ h = 1,\dots,H,$$

will not be an independent indicator of the total expenditure in this model, as it is in the Friedman (1957) model of the Permanent Income Hypothesis, since (1)-(5) imply

$$(9) \qquad x_{th} = \xi_{th} + \nu_{th}, \qquad t = 1,2,\ h = 1,\dots,H$$

where

$$(10) \qquad \nu_{th} = \sum_{i=1}^{I} \nu_{ith}, \qquad t = 1,2,\ h = 1,\dots,H,$$

represents the error of measurement in the total expenditure. Eq. (9) is of the form (6), with $d_{kt}=0$, $e_k=1$, $f_{km}=0$ ($t=1,2$, $m=1,2$).

2.2. The model in matrix notation. Stochastic specification

We assume that all observations relating to different households are uncorrelated. Suppress then from now on the household subscript and let

$$y_t = (y_{1t} \dots y_{It})' , \qquad w_t = (w_{1t} \dots w_{Kt})' ,$$

$$\mu = (\mu_1 \dots \mu_I)' , \qquad \lambda = (\lambda_1 \dots \lambda_K)' , \qquad z = (z_1 z_2)' ,$$

$$\nu_t = (\nu_{1t} \dots \nu_{It})' , \qquad \varepsilon_t = (\varepsilon_{1t} \dots \varepsilon_{Kt})' ,$$

$$a_t = (a_{1t} \dots a_{It})' , \qquad d_t = (d_{1t} \dots d_{Kt})' ,$$

$$b = (b_1 \dots b_I)' , \qquad e = (e_1 \dots e_K)' ,$$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ c_{I1} & c_{I2} \end{bmatrix} , \qquad F = \begin{bmatrix} f_{11} & f_{12} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ f_{K1} & f_{K2} \end{bmatrix} .$$

The system of Engel functions (7) and of measurement equations (6) can then be written as

$$(11) \qquad y_t = a_t + b\xi_t + Cz + \mu + \nu_t ,$$

$$(12) \qquad w_t = d_t + e\xi_t + Fz + \lambda + \varepsilon_t ,$$

where $\iota' a_t=0$, $\iota' b=1$, $\iota' C=0_{12}$, ι being the $I\times 1$ vector of ones and 0_{12} being the 1×2 vector of zeros.

The first and second order moments of the right hand side variables in (11) and (12) are assumed to satisfy

$$(13) \quad \left[\begin{array}{lll} E(\nu_t) = E(\mu) = 0_I , & E(\varepsilon_t) = E(\lambda) = 0_K , & \\ E(\xi_t) = \Phi_\xi^t , & E(z) = \Phi_z = (\Phi_z^1 \; \Phi_z^2)' , & t = 1,2, \end{array} \right.$$

$$(14) \quad \left[\begin{array}{lll} E(\nu_t \nu_s') = \delta_{ts}\Sigma_{\nu\nu} , & \Sigma_{\nu\nu} = (\sigma_{\nu\nu}^{ij}) , & \\ E(\varepsilon_t \varepsilon_s') = \delta_{ts}\Sigma_{\varepsilon\varepsilon} , & \Sigma_{\varepsilon\varepsilon} = (\sigma_{\varepsilon\varepsilon}^{kr}) , & \\ E(\nu_t \varepsilon_s') = 0_{IK} , & & t,s = 1,2, \end{array} \right.$$

$$(15)\quad \begin{cases} E(\mu\mu') = \Sigma_{\mu\mu} = (\sigma_{\mu\mu}^{ij}) , \\ E(\lambda\lambda') = \Sigma_{\lambda\lambda} = (\sigma_{\lambda\lambda}^{kr}) , \\ E(\mu\lambda') = \Sigma_{\mu\lambda} = (\sigma_{\mu\lambda}^{ik}) , \end{cases}$$

$$(16)\quad \begin{cases} E[(\xi_t-\Phi_\xi^t)(\xi_s-\Phi_\xi^s)] = \sigma_{\xi\xi}^{ts} , & t,s = 1,2, \\ E[(z-\Phi_z)(z-\Phi_z)'] = \Sigma_{zz} = (\sigma_{zz}^{mn}) , & \\ E[(\xi_t-\Phi_\xi^t)(z-\Phi_z)'] = \Sigma_{\xi z}^{(t)} = (\sigma_{\xi z}^{t1}\ \sigma_{\xi z}^{t2}) , & t = 1,2, \end{cases}$$

$$(17)\quad \begin{cases} E(\xi_t\mu') = \Sigma_{\xi\mu}^{(t)} = (\sigma_{\xi\mu}^{t1} \cdots \sigma_{\xi\mu}^{tI}) , & t = 1,2, \\ E(\xi_t\lambda') = \Sigma_{\xi\lambda}^{(t)} = (\sigma_{\xi\lambda}^{t1} \cdots \sigma_{\xi\lambda}^{tK}) , & t = 1,2, \\ E(z\mu') = \Sigma_{z\mu} = (\sigma_{z\mu}^{mi}) , & \\ E(z\lambda') = \Sigma_{z\lambda} = (\sigma_{z\lambda}^{mk}) . & \end{cases}$$

where δ_{ts} are Kronecker deltas. Furthermore, (ν_t,ε_t) are assumed to be uncorrelated with (ξ_t,z,μ,λ), t=1,2. This specification implies (i) zero expectations of errors and disturbances, (ii) constant variances of measurement errors and disturbances, (iii) measurement errors in expenditures uncorrelated with disturbances in measurement equations, and (iv) measurement errors and disturbances uncorrelated with the true total expenditure, with the socioeconomic variables, and with the preference variables. Allowance is, however, made for (contemporaneous) cross-correlation between the errors in the expenditures and between the disturbances in the measurement equations. Moreover, the specification allows for (v) correlation between the true total expenditure in the two years, (vi) correlation between the individual specific 'preference variables', i.e. the μ's and the λ's, and (vii) correlation between these preference variables on the one hand and the expenditure variables and socioeconomic variables on the other. This is a flexible set of assumptions which can be restricted in various ways and which open for testing of several interesting hypotheses. To ensure identification, some further restrictions on the second order moments may be needed (cf. section 2.5).

2.3. Restrictions on the second order moments

Two kinds of restrictions on the second order moments to obtain a more easily interpretable and a more parsimonious parametrization of the covariance structure (15)-(17) are presented below.

A. Restrictions on the distribution of latent total expenditure

We assume that the latent total expenditure is generated by

$$(18)\qquad \xi_1 = \xi + u_1 \,, \qquad \xi_2 = q_0 + q(\xi + u_2) \,,$$

where (ξ, u_1, u_2) are mutually uncorrelated, with expectations $(\Phi_\xi, 0, 0)$ and variances $(\sigma_{\xi\xi}, \sigma_{uu}, \sigma_{uu})$, (u_1, u_2) are uncorrelated with $(\xi, \mu, \lambda, z, v_t, \varepsilon_t)$ and q is a positive constant. Here ξ may be interpreted as a permanent component of consumption, i.e. common to both years, q_0 and q as its growth coefficients, and u_1 and u_2 as 'volatile' components, representing individual mobility in the distribution of consumption. From (13), (16)-(18) it follows that

$$(19)\qquad \Phi_\xi^1 = \Phi_\xi \,, \qquad \Phi_\xi^2 = q_0 + q\Phi_\xi \,,$$

$$(20)\qquad \sigma_{\xi\xi}^{11} = \sigma_{\xi\xi} + \sigma_{uu} \,, \qquad \sigma_{\xi\xi}^{12} = q\sigma_{\xi\xi} \,, \qquad \sigma_{\xi\xi}^{22} = q^2(\sigma_{\xi\xi} + \sigma_{uu}) \,,$$

$$(21)\qquad \Sigma_{\xi z}^{(2)} = q\Sigma_{\xi z}^{(1)} \,, \qquad \Sigma_{\xi\mu}^{(2)} = q\Sigma_{\xi\mu}^{(1)} \,, \qquad \Sigma_{\xi\lambda}^{(2)} = q\Sigma_{\xi\lambda}^{(1)} \,.$$

Regardless of the values of q_0 and q, the process determining latent total expenditure thus has a coefficient of (auto)correlation, $\sigma_{\xi\xi}/(\sigma_{\xi\xi}+\sigma_{uu})$, which is larger the smaller is the variance of its 'volatile' component in relation to the variance of its 'stable' component. If $q_0=0$, its coefficient of variation is constant. If $q_0=\sigma_{uu}=0$, the latent total expenditure increases with the same factor q for all households and if, in addition, $q=1$, then the latent total expenditure is constant for each household. In any case, (18) opens up for a more parsimonious parametrization of the covariances between the true total expenditure and the other variables, since (21) implies that the covariances, like the standard deviation of ξ_t increase by a factor q from year 1 to year 2.

B. Restrictions on the preference variables. A LES interpretation

So far, no restrictions, apart from the adding-up restriction (4) and the zero expectation assumption (13), have been imposed on the vector of preference variables. The former implies that $\Sigma_{\mu\mu}$ is singular, with rank at most I-1. Assume now that the system of Engel functions (1) is derived from an underlying Linear Expenditure System (LES)

$$\eta_t = \gamma_t + \beta(\xi_t - \iota'\gamma_t) \,, \qquad t = 1,2,$$

where γ_t is the I×1 vector containing the 'necessity consumption' in year t in value terms, β is the I×1 vector of marginal propensities to consume and ι is the I×1 vector of ones. Let γ_t be parametrized as

$$(22) \qquad \gamma_t = a_t^* + C^*z + \alpha \,, \qquad t = 1,2,$$

where C^* is a I×2 matrix representing the effect of the (observed) demographic variables on necessity consumption, α is a stochastic I×1 vector with zero mean, representing (unobserved) individual components in necessity consumption, and a_t^* is a (year specific) I×1 vector of constants (representing, inter alia, the effect of the price terms in the LES model). This implies

$$\eta_t = a_t + b\xi_t + Cz + \mu \,, \qquad t = 1,2,$$

where

$$(23) \qquad a_t = (I-\beta\iota')a_t^* \,, \quad b = \beta \,, \quad C = (I-\beta\iota')C^* \,, \quad \mu = (I-\beta\iota')\alpha \,, \quad t=1,2 \,.$$

This is the LES parametrization of the Engel functions (1), and since $\iota'\beta=1$, (2)-(4) will be satisfied automatically, regardless of the values of a_t^*, C^*, and α. The parameters a_t^* and C^* are not identifiable, since a_t and and C are invariant to replacing a_t^*, C^* by $a_t^*+k\beta, C^*+k\beta$ where k is an arbitrary scalar. The covariance matrix $\Sigma_{\mu\mu}$ is singular regardless of the distribution of α. From (23) we obtain

$$(24) \qquad \Sigma_{\mu\mu} = (I-b\iota')\,\Sigma_{\alpha\alpha}\,(I-\iota b') \,.$$

In the following, we assume that $\Sigma_{\alpha\alpha}$ is diagonal, which changes the number of covariances in the preference structure from I(I-1)/2 to I. This is an effective reduction as long as I>3. Then, we have

$$\sigma_{\mu\mu}^{ij} = \delta_{ij}\sigma_{\alpha\alpha}^{ii} - b_i\sigma_{\alpha\alpha}^{jj} - b_j\sigma_{\alpha\alpha}^{ii} + b_ib_j \sum_{j=1}^{I} \sigma_{\alpha\alpha}^{jj} \,,$$

where $\sigma_{\alpha\alpha}^{ii}$ is the i'th (diagonal) element of $\Sigma_{\alpha\alpha}$, and δ_{ij} is the Kronecker delta.

2.4. The model in incremental form

When panel data are available, one can control for individual specific effects - observable as well as unobservable - by measuring all variables from their individual means. In our context, with two replications only, this is equivalent to measuring all variables in terms of changes from the first to the second report. Or stated in the terminology the analysis of variance, all attention is given to the within household variation while neglecting the between household variation. Measuring the variables in this way, we do not need to be concerned with the possible bias in the estimated coefficients in the rest of the model caused by erroneous specification of its individual effects, in the present model represented by the observed z's and the unobserved μ's and λ's. From the point of view of robustness of the results, this must be considered an advantage, to the extent that the individual effects are not of interest per se. We do not, for instance, have to be bothered by the harmful effects on the estimation of the marginal budget shares caused by failing to properly account for correlation between the latent total expenditure and the latent preference variables. This data transformation, however, has its price. First, as already remarked, no parameter characterizing individual effects can be identified from within household variation. Second, the number of effective observations is reduced to one half, which, of course, entails a substantial loss of degrees of freedom. Third, the relative importance of the errors of measurement is considerably magnified when data are transformed from levels to first differences. Hence, no a priori verdict can be given about the relative merits of the two models.

Some attention will be given to the version of the model expressed in first difference form, denoted as its incremental version. Letting Δ denote the increase in any variable from the first to the second report, (11)-(17) imply the following incremental version of the Engel functions and the measurement equations

(25) $\Delta y = \Delta a + b\Delta\xi + \Delta\nu$,

(26) $\Delta w = \Delta d + e\Delta\xi + \Delta\varepsilon$,

where

$$(27)\quad\begin{aligned}&(a)\quad E(\Delta\nu) = 0_I\,, \qquad E(\Delta\varepsilon) = 0_K\,,\\ &(b)\quad E[(\Delta\nu)(\Delta\nu)'] = 2\Sigma_{\nu\nu}\,, \qquad E[(\Delta\varepsilon)(\Delta\varepsilon)'] = 2\Sigma_{\varepsilon\varepsilon}\,,\\ &(c)\quad E[(\Delta\nu)(\Delta\varepsilon)'] = 0_{IK}\,,\\ &(d)\quad E(\Delta\xi) = \Phi_\xi^2 - \Phi_\xi^1\,, \quad \mathrm{var}(\Delta\xi) = \sigma_{\Delta\xi\Delta\xi} = \sigma_{\xi\xi}^{11} - 2\sigma_{\xi\xi}^{12} + \sigma_{\xi\xi}^{22}\,.\end{aligned}$$

When the process generating latent total expenditure is parametrized as in (18)-(20), we get in particular

$$E(\Delta\xi) = q_0 + (q-1)\Phi_\xi\,,$$

$$\sigma_{\Delta\xi\Delta\xi} = (q-1)\sigma_{\xi\xi}^2 + (q^2+1)\sigma_{uu}\,.$$

2.5. Remarks on identification

The conditions for identification of linear interdependent models with latent variables are in general complicated. A necessary and sufficient condition for local identification of the parameters of such a model under the the assumption of normality are given in Aigner et al. (1984, p. 1366). See also Geraci (1976). Briefly stated, the condition is that a certain Jacobian matrix of the parameters in the model is of full rank. As remarked in the introduction, the conditions under which a latent variable model is identifiable are weaker in a panel data context than if only one observation from each individual is available, but they depend on the precise form of the model.

We confine ourselves to three brief remarks, summarizing our conclusions. (For details, see Aasness and Biørn (1988).)

(i) Without further restrictions on the distribution of (ξ,μ,λ,z), the complete model is not identifiable.

(ii) If the distribution of total expenditure is not restricted to be the same in the two years (i.e. if $q_1\neq 1$, $\sigma_{uu}\neq 0$), then full identification may be ensured if μ and λ are uncorrelated with z, i.e. if $\Sigma_{z\mu}=0_{2I}$, $\Sigma_{z\lambda}=0_{2K}$. This assumption will be made as a maintained hypothesis in the rest of the paper. If the distribution of total expenditure is the same in the two years (i.e. if q=1, $\sigma_{uu}=0$), then μ and λ must be uncorrelated not only with z but also with ξ, i.e. $\Sigma_{\xi\mu}^{(t)}=0_{1T}$, $\Sigma_{\xi\lambda}^{(t)}=0_{1K}$, for identification to be possible.

(iii) If the income process is different in the two years, we can, by using data in incremental form, almost always identify b, e, $\sigma_{\Delta\xi\Delta\xi}$, $\Sigma_{\nu\nu}$, and $\Sigma_{\varepsilon\varepsilon}$ regardless of the assumptions made about the distribution of preferences.

In general, these conditions for identification are not restrictive, and are satisfied in all the model variants in the empirical part of the paper.

3. DATA AND ESTIMATION

Data are taken from the Norwegian Surveys of Consumer Expenditures in the period 1975-77. Detailed information is given in Appendix B. Our sample consists of 408 households with two replicated observations on expenditures and incomes. We have chosen a five commodity group classification (I=5), comprising the whole budget. The only "measurement variables" (w's) in this paper are two income variables (K=2), which we will comment upon in some detail.

The income variables are taken from the "tax file" which contains information from the individual tax returns for all the personal tax payers in Norway. Income is aggregated across all the individual tax payers in the household to get household income. The two income measures used are:

- Income measure 1 (w_1): Taxable income for the central government tax assessment minus taxes paid (i.e. net income).
- Income measure 2 (w_2): Income base for imposing social security premiums and pension rights in the public social security system. This includes wages and net entrepreneurial income, but excludes capital income (positive and negative, e.g. interests received and paid) and pensions.

From a formal point of view these two income measures can be looked upon as two different linear combinations of all the different items on the individual tax returns. Errors of measurement and individual effects connected to one item will in general be reflected in both of these two linear aggregates. Thus we will expect the errors of measurement and individual effects to be positively correlated for these two income variables, and we will take account of this in the specification of $\Sigma_{\varepsilon\varepsilon}$ and $\Sigma_{\lambda\lambda}$.

The covariance matrix of all the observed variables is given in table B1 in Appendix B. This is all the data input we use for our econometric analysis.

Let S symbolize the sample covariance matrix of our observed variables, with realized values given in table B1. Let $\Sigma(\theta)$ symbolize the theoretical covariance matrix of the observed variables (Σ) as a function of the unknown parameters (θ) in our model, as described in Appendix A and further specified in section 4. The estimates of θ are the values that minimize the function

$$(28) \qquad F = \ln|\Sigma(\theta)| + tr(S\Sigma(\theta)^{-1}) - \ln|S| - (2(I+K)+M) .$$

Minimization of F is equivalent to maximization of the likelihood function when assuming that all the observed variables (i.e. the y's, the w's and the z's) are multinormally distributed. This, however, is subject to the qualification that there is no information in the first order moments that could be used to estimate parameters θ appearing in the second order moments. This is true in our case since our 2(I+K)+M first order moments have to be used to estimate the 2(I+K)+M independent parameters ($a_1, a_2, d_1, d_2, \Phi_z, \Phi_\xi, q_0$) of which neither are contained in the covariance matrix $\Sigma(\theta)$. The estimation of these "first order parameters" can be done "recursively", i.e. as a second step after the estimation of θ. (If, however, we introduce restrictions on these "first order parameters", e.g. $q_0=0$, then the principle of maximum likelihood and the normality assumption strictly demands simultaneous estimation of all parameters from the first and second order sample moments).

If one is not willing to assume normality, which in the present context is a rather restrictive assumption, then the estimators derived from minimizing F above can be labeled quasi maximum likelihood estimators. Many of the properties of these estimators are robust w.r.t. important departures from normality, see e.g. Anderson, Fang and Hsu (1986), Shapiro and Brown (1987) and especially Anderson and Amemiya (1988).

Our model can be formalized as a special case of the LISREL model (cf. e.g. Jöreskog (1977)), and the computer program LISREL VI (cf. Jöreskog and Sörbom (1984)) is used to solve the numerical calculations. The only restriction we could not handle directly by applying LISREL VI, is the restriction that the b's which occur additively in (11) also enter multiplicatively in (24). This could, however, be handled by applying LISREL VI in a stepwise manner, where in each step we treat the b's in (11) as free parameters and the b's in (24) as fixed parameters with values taken from the estimates in the preceeding step, and continue until the b's in (11) and (24) converge to the same values. In this paper, we have not followed up this idea. However, for each model version using restriction (24) there exists, in our hierarchy of models (cf. section 4.1), a more general model without restriction (24) and equal in all other respects. The ML-estimator of b in this more general model will also be a consistent estimator of b in the more restictive model. We have used this estimate of b as a given parameter in (24) when estimating the other parameters of the restrictive model, including the estimation of b in (11). It turns out that these two estimates of b are quite close to each other, cf. table 8, and a more elaborate estimation procedure, imposing convergence, would probably not give substantially different results.

The LISREL program uses an algorithm based on the Davidon-Fletcher-Powell method. We got exactly the same estimates using different starting values and different LISREL formulations of the same econometric model. Furthermore, we have estimated 13 different models, cf. section 4, whose results are reasonable not only for each model separately but also when compared with the results of the other models. This has made us quite sure that we have reached a global minimum for each model.

LISREL VI minimizes the function F without imposing any constraints on the admissible values of the parameter vector θ. Thus the LISREL estimate of a parameter which we interpret as a variance may well turn out to be negative. This may be regarded as an important drawback of this computer program. However, if our model and its interpretation is correct the LISREL estimates should turn out to have the expected sign, apart from the sampling errors. Thus, if for a given model all the estimated variances are positive, and all the estimated covariance matrices are positive semidefinite, we will take this as a confirmation that the model has passed an important test. If we obtain negative estimates of variances, or negative definite "covariance matrices", this may be interpreted either that the model is misspecified or that the sampling errors in its estimates are substantial.

At the minimum of F, the information matrix is computed and used to estimate asymptotic standard errors and t-values.

We test a specific model 0 (the null hypothesis) against a more general model 1 (the maintained hypothesis) by a likelihood ratio test. Let

F_0 and F_1 be the minimum of F under model 0 and model 1, respectively, and let s be the difference in the number of parameters. It can be shown that minus twice the logarithm of the likelihood ratio is equal to $H(F_0-F_1)$, where H is the number of households. This statistic is thus, according to standard theory, approximately χ^2 distributed with s degrees of freedom. The χ^2 value given for each model given in table 2, is defined as HF_0, which can be interpreted as the test statistic above when the alternative hypothesis is an exactly identified model, giving a perfect fit to the sample covariance matrix and accordingly $F_1=0$. The test statistic $H(F_0-F_1)$ for an arbitrary pair of models can thus be computed by simply subtracting the corresponding pair of χ^2 values (given in table 2). The significance probability corresponding to the value of a test statistic, i.e. the probability of getting a χ^2 value larger than the value actually obtained given that the null hypothesis is true, is reported in table 3.

A standard measure of the goodness of fit of the entire model in LISREL is

(29) $$GFI = 1 - tr(\Sigma^{-1}S - I)^2/tr(\Sigma^{-1}S)^2 .$$

As a concept of goodness of fit of each equation (variable), we use, as exemplified by the variable y_i,

(30a) $$\varrho^i = \sigma^{ii}_{\eta\eta}/\sigma^{ii}_{yy} ,$$

i.e. the ratio between the variance of the latent 'structural' component of the equation and the variance of its observed left hand variable. This is analogous to the squared coefficient of multiple correlation in classical linear regression analysis. We consider this as a population parameter. It gives a measure of the signal/noise ratio for our observed variables, which all can be considered as indicators of latent total expenditure. In our model we have,

$$\sigma^{ii}_{yy} = \sigma^{ii}_{\eta\eta} + \sigma^{ii}_{\nu\nu} ,$$

since, by assumption, the ν's are uncorrelated with all the elements of the η's. Thus ϱ^i can also be written as,

(30b) $$\varrho^i = 1 - \sigma^{ii}_{\nu\nu}/\sigma^{ii}_{yy} ,$$

i.e. one minus the ratio of the variance of the measurement error and the variance of the observed variable.

We assume that the variance of the measurement error, σ_{vv}^{ii}, is constant over time (cf. (14). If the variance of expenditure, σ_{yy}^{ii}, increases (decreases) over time, it follows from (30b) that the squared coefficient of multiple correlation decreases (increases) over time.

The parameters ϱ^i, like all other population parameters, are estimated by the maximum likelihood method. (This contrast with the LISREL program which automatically produces a "squared multiple correlation", which can be looked upon as a "bastard" estimator of our parameter ϱ^i, using formula (30b) with the <u>ML-estimator</u> of σ_{vv}^{ii} and <u>the sample variance</u> of σ_{yy}^{ii}.)

4. EMPIRICAL RESULTS

4.1. Hierarchy of models

A lot of variants of the general model in section 2 can be estimated and tested with our data. We concentrate on some issues which we find particularly important. The classifications of the models is shown in table 1. Three "dimensions" of assumptions are selected, and for each dimension we have picked out 2 or 3 alternative assumptions of particular interest. The E dimension relates to the development of the distribution of latent total <u>expenditure</u> over time, the P dimension concerns the covariance structure of the <u>preference</u> variables (μ,λ), and the C dimension concerns the <u>covariation</u> between preference variables and latent total expenditure. Combining our assumptions in all possible ways, we obtain $3\times3\times2=18$ models, of which 2 are unidentified and 3 are equivalent to other models, leaving us with 13 specific models, as shown in table 2. Likelihood ratio test statistics of some important hypotheses are given in table 3, based on all possible pairwise combinations of the models involving these hypotheses. A complete record of the estimated parameters in all the models is given in tables 4 through 11. This makes it possible to examine systematically the robustness of the test and estimation results.

In all other respects, our 13 models have equal assumptions, and these are made as simple as possible, i.e. zero correlation if not otherwise explicitly stated. Note that we have assumed that the measurement errors of the different commodity groups are uncorrelated (i.e. Σ_{vv} is diagonal), since we have no specific a priori reason to believe they are correlated and we are not focusing on this aspect in this paper, while we allow the measurement errors of the income variables to be correlated (i.e. $\Sigma_{\varepsilon\varepsilon}$ is non-diagonal), since we have strong a priori reasons to believe that they are positively correlated, cf. section 3. As already remarked, the preference variables μ and λ are assumed to be uncorrelated with the demographic variables.

4.2. Likelihood ratio tests

Table 2 gives the likelihood ratio chi square test statistic for each model against a model with no restrictions on the covariance matrix. From these statistics we can easily perform the likelihood ratio chi square test for each model against an arbitrary more general model, cf. section 3. In this way, we can test a lot of different combinations of assumptions in different settings.

Table 3 gives the significance probabilities when testing each of the simple hypotheses in table 1 against the hypothesis one level higher in the hierarchy (e.g. E1 against E2), in all possible settings, i.e. for each combination of assumptions in the other dimensions. Choosing a significant level of 0.01, we can conclude that hypothesis E1 is rejected against hypothesis E2, E2 is rejected against E3, P1 is rejected against P2, P2 is not rejected against P3, and C1 is rejected against C2. These test results are the same in all settings, the only exception is that E2 is not rejected against E3 when assuming P1. However, P1 itself is clearly rejected in all settings. Thus we can safely conclude that E2 is rejected against E3 given our data and approach. Further implications of these test results will be discussed in the following subsections.

4.3. Distribution of latent total expenditure

The parameter estimates of the distribution of the latent total expenditure are given in table 4. All estimates but one have the expected positive sign, and they have a reasonable magnitude. The exception is the negative estimate of σ_{uu} in model E3P1C1, indicating a misspecified model, which is consistent with the fact that this model was strongly rejected by the likelihood ratio tests. The estimates of the covariances between latent total expenditure and the demographic variables are almost the same in all 13 models. The estimates of the other parameters vary somewhat, but are very robust within subclasses of the models.

The hypotheses E1 and E2 can be tested by Wald tests based on the standard deviations of q and σ_{uu}, respectively in table 4. The results are the same as for the likelihood ratio tests in table 3. We conclude that both E1 and E2 are too simple, there has both been a general growth in latent total expenditure (q>1) and a change in the ranking of the households by total expenditure ($\sigma_{uu}>0$) in Norway in 1975-77. This fact enables us to identify and estimate the covariance between preferences and total expenditure from our data, cf. section 2.5. Note, however, that σ_{uu} is small compared to $\sigma_{\xi\xi}$. Thus the coefficient of autocorrelation in the process determining latent total expenditure is high (the estimate is larger than 0.9 in all models), cf. section 2.3.A.

Although it seems very safe to conclude that E1 and E2 are definitely wrong in Norway in 1975-77, it is interesting to note that many of the results from models based on these assumptions are quite robust, cf. tables 2 through 12.

The sample variance of observed total expenditure (x) is 507.29 i period 1 and 665.59 in period 2, which is considerably higher than the estimated variances of latent total expenditure in the corresponding periods (which can be calculated from table 4, using (20)). Thus the parameters of the expenditure functions can be heavily biased if observed total expenditure is used as the explanatory variable (cf. Aasness (1983) for an explicit analysis of these biases in a similar but simpler model). The coefficient of variation of latent total expenditure, which can be calculated from table 4 and the first order moments given in Appendix B, varied around 0.5. The hypothesis that the coefficient of variation is constant over time (i.e. $q_0=0$, cf. section 2.3.A) could not be rejected.

4.4. Distribution of preferences

The distribution of preferences across the population of consumers is, in one interpretation of our model, equivalent to the distribution of the individual effects in the Engel functions (μ) and in the income measurement equations (λ). The variances and covariances of these preference variables are given in table 5 for the P3 models, and in table 6 for the P2 models. (Table 6 reports the variances of the independent α variables from which the whole covariance matrix of the dependent μ variables can be computed, cf. (24)).

Note that:

i) All the estimated variances of the preference variables are positive, and the estimated covariance matrices of any combinations of preference variables are positive semidefinite, which was not imposed as constraints on the estimation procedure.

ii) We cannot find any parameter estimate of unreasonable size.

iii) The hypothesis of no variation in preferences (P1) is strongly rejected, cf. tables 2 and 3.

iv) The results are clearly robust with respect to model specification.

Thus our econometric interpretation of the statistical model makes sense, and confirms that this model represents a fruitful approach to consumer econometrics.

Since the μ's add to zero they will tend to be negatively correlated. Indeed, there are no significantly positively correlated pairs of μ's in table 5. There are three pairs of goods for which the μ's are significantly negatively correlated: Food, beverages and tobacco vs. Travel and recreation, Clothing and footwear vs. Housing, fuel and furniture, and Housing, fuel and furniture vs. Travel and recreation. However, the estimated covariance matrices of μ based on the estimated variances of α in table 6, using (24), show a similar pattern (not tabulated). This, and the result that the P2 models are not rejected against the P3 models, cf. section 4.2, make us conclude that the P2 hypothesis gives a reasonably good approximation to the covariance structure of the preference variables in our model with five broad commodity groups. This result may prove very useful if the model is extended to a more detailed commodity grouping where an unrestricted $\Sigma_{\mu\mu}$ will be very demanding in terms of degrees of freedom, while a block diagonal $\Sigma_{\alpha\alpha}$ could be appropriate.

Observe that the individual effects of the two income measures are significantly positively correlated, and the covariance is of a large magnitude, in accordance with our a priori considerations in section 3.

4.5. Covariation of preferences and latent total expenditure

A maintained hypothesis in most empirical work of consumer behavior is independence of preferences and income (or total expenditure). Our panel data with two replications makes it possible to subject this to formal tests. The likelihood ratio tests in table 2 reject the hypothesis (C1 against C2). From table 7 we see that preference variable for Food, beverages and tobacco is significantly positively correlated with latent total expenditure, while the opposite is true for Travel and recreation. Furthermore, the λ-variables are significantly positively correlated with latent total expenditure, for both income measures. One possible interpretation of this result is that the preferences for savings are positively correlated with total expenditure and income. However, this correlation could also be explained by a progressive tax system, or by a negative correlation between the degree of tax evasion activities and income, cf. section 2.1.

4.6. Engel elasticities

The marginal budget shares (b) are presented in table 8, and the corresponding Engel elasticities are given in table 9. The estimates are rather robust with respect to model specification, although they vary somewhat between the three main groups of models and within the most "sophisticated" group, but are very robust within the two simplest groups of models.

The estimated Engel elasticity for Food, beverages and tobacco is larger than zero and significantly less than one for all models, confirming once again Engel's law, now in a framework with errors in variables and with preferences allowed to be correlated with total expenditure. The Engel elasticity for Clothing and footwear, and for Housing, fuel and furniture, are rather close to one in all models. While Travel and recreation, and Other goods and services, are classified as luxuries by all the models. These results are in broad agreement with much of the earlier empirical results on Engel elasticities.

Note that the estimates of the Engel elasticity for Food, beverages and tobacco, are lower in the C2 models than in the C1 models. This is in agreement with the result that the preference variable for this commodity group is positively correlated with total expenditure in C2 models, which accounts for some of the covariance between the consumption for Food, beverages and tobacco and latent total expenditure. Correspondingly, the preference variable for Travel and recreation is estimated to be negatively correlated with total expenditure, and thus the estimated Engel elasticity should be expected to be larger in the C2 models than in the C1 models, which is in fact the case.

Estimates of e_1 and e_2, which are reduced form parameters reflecting saving behavior and other effects, cf. section 2.1, are also presented in table 8. All the estimates of e_1 and e_2 are significantly positive, confirming our hypothesis that both income measures are good indicators of latent total expenditure. Viewed as measures of "true income", both these income measures contain not only random, but also systematic measurement errors, making it difficult to give clear interpretations about saving behavior from the sizes of the parameters. The estimates of e_k vary considerably between the three main groups of models, the largest estimates are obtained in the P1 models (with no preference variables) and the smallest estimates are obtained in the C2 models (with preference variables that can be correlated with latent total expenditure).

4.7. Demographic effects

The estimated effects on consumption of household size and composition are given in table 10. Again, the results are quite robust, although the estimates vary somewhat between the main groups of the models.

The effect on food consumption of an additional child or of an additional adult, given the level of latent total expenditure, is significantly positive in all models, in agreement with Engel's law. The estimated effects on Clothing and footwear are also positive in all models, both for children and adults, but these effects are small in magnitude and not significantly different from zero. The effect of the demographic variables on the expenditure on Housing, fuel and furniture is negative in all models, and significantly so for adults. An additional child significantly decreases the consumption of Travel and recreation, given total expenditure, according to all models. The estimated effects on the consumption of Travel and recreation of an additional adult, vary between the models, but are not significantly different from zero according to C2 models. The number of adults and children affect expenditure on Other goods and services negatively in all models, but not significantly, and the magnitudes are small.

4.8. Measurement errors

Table 11 gives the estimates of the covariance matrices of measurement errors, i.e. Σ_{vv} which is assumed to be diagonal and $\Sigma_{\varepsilon\varepsilon}$ which is a full 2×2 matrix. The estimates of the variances and the covariance are much larger in the models which do not permit individual differences in preferences (P1) than in the models that allow for this (P2 and P3). A plausible explanation is that with no allowance for differences in preferences, the estimated variances (and covariance) of measurement errors also pick up variation (and covariation) which is due to preference differences. Within these two classes of models the estimates are very robust with respect to the model specification.

For all our models we obtain significantly positive estimates of the variances of the measurement errors, which again confirms our econometric interpretation of the statistical models (cf. section 3). The covariance between the measurement errors attached to the two different income measures ($\sigma_{\varepsilon\varepsilon}^{12}$) is also significantly positive in all our models, which is congruent with our a priori considerations (cf. section 3).

Table 12 presents the estimates of the squared coefficient of multiple correlation, as defined in (30a). This is one type of measure of the relative variance of the measurement error, cf. (30b). The squared coefficient of multiple correlation are much lower in P1 models than in the P2 and P3 models, but within these two classes of models the estimates of these population parameters are very robust to the model specification.

A ranking of the observed consumption and income variables according to the squared coefficient of multiple correlation is informative since this can be considered as a measure of the quality of these variables as indicators of latent total expenditure. Income measure 2 (income base for imposing social security premiums) is number 1, Income measure 1 (net income after tax) is number 2, and expenditure on Food, beverages and tobacco is number 3, according to this ranking criterion irrespective of model specification. Within the class of the P2 and P3 models, Housing, fuel and furniture is number 4, Other goods and services is number 5, while Clothing and footwear and Travel and recreation are number 6 or 7.

The squared coefficient of multiple correlation always increases from the first to the second report, since our estimates of the variances of the observed variables increases, cf. (30b).

4.9. The incremental form of the model

The estimation results for the incremental form are reported in Tables 13, 14 and 15. For the purpose of comparison, corresponding estimates from some of the level models are also given. The parameters of the matrices $E[(\Delta v)(\Delta v)']$, $E[(\Delta\varepsilon)(\Delta\varepsilon)']$ and the scalar parameter $var(\Delta\xi)$ (cf. equation (27)) can be estimated from the level versions. The estimates are not directly obtainable from the LISREL output, but can easily be deduced from the estimates reported above for the level specifications. It is of great interest to compare the estimates from the level specifications with those from the incremental version. Large discrepancies in parameter estimates between these two estimation procedures may indicate misspecification and may indicate in what part of the model the misspecification occurs.

From Table 13 it appears that there are important differences in estimated marginal budget shares. The marginal budget share for Housing, fuel and furniture is very low in the incremental version compared with the estimates from the level formulations, while the opposite is the case for Travel and recreation. This is somewhat disturbing and indicates a need for further research. Below we attempt an explanation which can give guidelines for such studies.

One possible explanation of the conflicting evidence may be that the IM model, depending solely on the time series information in the data, give a negatively biased estimator for the marginal budget share for Housing, fuel and furniture and a positively biased estimator for the marginal budget share for Transport and recreation, due to the design of the survey and the method of registration. The survey does not pick up households who change dwelling from the first to the second period, thus underestimating the response from total expenditure on this part of expenditure on Housing, fuel and furniture. The survey represents consumption expenditure on cars by the purchase expenditures, thus the measurement errors of expenditure on cars reflects the difference between the purchase (stock) and consumption (service flow) during the year of observation. This may create no bias in the cross section part since we model the errors in variables, but in the time series part we may expect a positive correlation between the change in the latent total expenditure and the measurement errors for cars, since those households which have experienced a high increase in latent total expenditure also will have a tendency to invest in a new car and thus get a high positive measurement error in expenditure on cars.

We may get some information on the sizes of these possible biases by reestimating our models with a more detailed commodity grouping. More satisfactory measurement methods and a more explicit modeling of consumer durables seems to be an important area for future research with these types of models.

Table 14 reveals a close correspondence between the incremental form and the level estimates as far as the variances and covariance of the measurement errors is concerned. The only important difference occurs for Travel and recreation where the estimated variance is somewhat lower in the incremental form. This may be due to the no autocorrelation assumption of the measurement errors and the assumption that the variance of the measurement error in the level form is equal for both periods, which may be inappropriate for this commodity group.

The estimated variance of the increase in total expenditure from the incremental form and from the level formulations are reported in Table 15. The former is higher than the latter. The difference is most pronounced when the incremental specification is compared to those level versions in which correlation between preference variables and total expenditure is disregarded.

5. CONCLUSIONS

In this paper, a new econometric model with a system of consumer expenditures for each household in the population, has been presented and applied successfully on Norwegian panel data from 1975-77. Thirteen systematically selected specifications of the model have been estimated and tested, and the results are compared across models. Many of the substantial results are robust with respect to model specification.

The estimated variances of all the latent variables (total expenditure, preference variables, measurement errors) are positive in all our models (with one unimportant exeption), which was not used as constraints in the estimation procedure. Thus our econometric interpretation of the statistical models of the observed consumption and income variables has passed an important test. The estimated parameters are of reasonable magnitude, the results make sense, and confirm our view that this is a fruitful approach to consumer econometrics.

The variance of the latent total expenditure is substantially lower than the variance of its observed counterpart. An implication of this is that the parameters of expenditure functions can be seriously biased if observed total expenditure is used as the explanatory variable. There has been a general growth in latent total expenditure in Norway 1975-77. The coefficient of autocorrelation of this variable is high, but there has been a significant change in the ranking of the households by latent total expenditure. We found no significant change in the inequality of latent total expenditure measured by its coefficient of variation, in the period under investigation.

There is a substantial variation in preferences across households, conditional on latent total expenditure and the demographic variables. Our model can be interpreted as relating to a population of households, each having a Stone-Geary utility function. An assumption of independently distributed "necessity quantities" places strong restrictions on the covariance structure of the "preference variables" in our econometric model, but these restrictions are not rejected. This independence assumption thus seems to be a good approximation in our setting with five broad commodity aggregates.

A fundamental assumption in all cross-section estimation of demand functions is that preferences and total expenditure are uncorrelated across households. This hypothesis is tested and clearly rejected. The preference variable for Food, beverages and tobacco is positively correlated with latent total expenditure, implying a positive bias in the estimator of the Engel elastisicity when assuming zero correlation. The preference variable for Travel and recreation is negatively correlated with total expenditure, implying a negative bias in the estimator of the Engel elasticity when assuming zero correlation. These results should not be taken too far, but they indicate a fruitful area for future research.

Table 1

Classification of models

A spesific model is labeled EiPjCk, which means that the model is based on assumption Ei w.r.t. the distribution of latent total expenditure (ξ), assumption Pj w.r.t. the distribution of preference variables (μ,λ) and assumption Ck w.r.t. the covariation between latent total expenditure and preference variables.

Assumptions w.r.t. the distribution of latent total expenditure (ξ)

Label	Parameter restrictions $\sigma_{\xi\xi}$	q	σ_{uu}	Interpretation
E3	free	free	free	No restrictions
E2	free	free	0	Equal growth rate (q) for all consumers
E1	free	1	0	Constant latent total expenditure over time for each consumer

Assumptions w.r.t. the distribution of preference variables (μ,λ)

Label	Parameter restrictions $\Sigma_{\mu\mu}$	$\Sigma_{\lambda\lambda}$	Interpretation
P3	free[1]	free	No restrictions on covariances between preference variables (μ)[1]
P2	$\Sigma_{\alpha\alpha}$ = free[2]	free	LES interpretation with independently distributed necessity quantities
P1	0	0	No individual differences in preferences

[1] Except for the restrictions from the adding-up condition

[2] $\Sigma_{\mu\mu} = (I - bl')\,\Sigma_{\alpha\alpha}\,(I - lb')$, $\Sigma_{\alpha\alpha}$ = diagonal

Assumptions w.r.t. covariation between latent total expenditure and preference variables

Label	Parameter restrictions $\Sigma_{\xi\mu}$	$\Sigma_{\xi\lambda}$	Interpretation
C2	free	free	Preference variables (μ,λ) are correlated with latent total expenditure (ξ)
C1	0	0	Preference variables (μ,λ) are uncorrelated with latent total expenditure (ξ)

Table 2

Overview of fitted models[a)b)]

Covariation between preferences and total expenditures	Total expenditure distribution	Preference distribution P3	P2	P1
C2	E3	E3P3C2 χ^2=147.99 df=83 GFI=0.957	E3P2C2 χ^2=159.19 df=88 GFI=0.954	E3P1C2 c)
	E2	E2P3C2 χ^2=167.71 df=84 GFI=0.951	E2P2C2 χ^2=179.09 df=89 GFI=0.947	E2P1C2 c)
C1	E3	E3P3C1 χ^2=175.17 df=89 GFI=0.950	E3P2C1 χ^2=183.52 df=94 GFI=0.947	E3P1C1 χ^2=1005.84 df=102 GFI=0.740
	E2	E2P3C1 χ^2=190.53 df=90 GFI=0.945	E2P2C1 χ^2=199.19 df=95 GFI=0.943	E2P1C1 χ^2=1007.83 df=103 GFI=0.737
	E1	E1P3C1 χ^2=207.14 df=91 GFI=0.939	E1P2C1 χ^2=215.69 df=96 GFI=0.936	E1P1C1 χ^2=1021.52 df=104 GFI=0.734

a) The models are generated from all possible combinations of assumptions in dimension E, P and C, see table 1 for definitions. However, models E1PjC2, j=2,3, are not identifiable and are thus not fitted

b) Chi square statistics (χ^2), degrees of freedom (df) and Goodness of fit index (GFI) in comparison to a model with no restrictions on the co-variance matrix, of section 3

c) Model EiP1C2 is equivalent to model EiP1C1 for i=1,2,3

Table 3

Significance probabilities in likelihood ratio tests[a]

I. Tests of E-hypotheses

Maintained assumptions	Null and alternative hypotheses	
	E1 against E2	E2 against E3
P3, C2	b)	0.000009
P2, C2	b)	0.000008
P3, C1	0.000046	0.000089
P2, C1	0.000049	0.000075
P1, C1	0.000216	0.158341

II. Tests of P-hypotheses

Maintained assumptions	Null and alternative hypotheses	
	P1 against P2	P2 against P3
E3, C2	0.000000	0.047556
E2, C2	0.000000	0.044335
E3, C1	0.000000	0.137972
E2, C1	0.000000	0.123419
E1, C1	0.000000	0.128416

III. Tests of C-hypotheses

Maintained assumptions	Null and alternative hypotheses
	C1 against C2
E3, P3	0.000134
E2, P3	0.000859
E3, P2	0.000454
E2, P2	0.002658

a) See table 1 and 2 for detailed definitions of hypotheses and models

b) Since models E1P2C2 and E1P3C2 are not identified the test can not be performed

Table 4

Distribution of latent total expenditure[a]

	MODEL												
	Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
Parameter	E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
$\sigma_{\xi\xi}$	341.96	346.76	359.41	358.84	380.02	380.26	381.12	381.45	420.96	421.12	266.98	262.14	275.98
	(34.19)	(34.42)	(34.19)	(33.97)	(33.68)	(33.70)	(33.16)	(33.17)	(34.93)	(34.92)	(25.05)	(24.70)	(25.72)
q	1.168	1.160	1.152	1.159	1.104	1.104	1.106	1.106	1[b]	1[b]	1.052	1.051	1[b]
	(0.048)	(0.048)	(0.044)	(0.044)	(0.030)	(0.030)	(0.027)	(0.027)			(0.014)	(0.014)	
σ_{uu}	35.20	34.81	0[b]	0[b]	15.15	15.25	0[b]	0[b]	0[b]	0[b]	-3.30	0[b]	0[b]
	(9.63)	(9.61)			(4.60)	(4.61)					(2.01)		
$\sigma^1_{\xi z}$	8.52	8.55	8.64	8.59	8.80	8.80	8.82	8.82	9.19	9.19	9.07	9.06	9.19
	(1.36)	(1.37)	(1.37)	(1.37)	(1.40)	(1.40)	(1.40)	(1.40)	(1.46)	(1.46)	(1.27)	(1.27)	(1.30)
$\sigma^2_{\xi z}$	9.70	9.74	9.77	9.74	10.00	10.01	9.99	9.99	10.52	10.52	10.23	10.23	10.52
	(1.07)	(1.07)	(1.07)	(1.07)	(1.09)	(1.09)	(1.09)	(1.09)	(1.13)	(1.13)	(1.00)	(1.00)	(1.02)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

b) A priori restrictions

Table 5

Variances and covariances of preference variables in P3 models[a)]

Commodity/ Income measure	Parameter	MODEL: Preference variables correlated with total expenditure		Preference variables uncorrelated with total expenditure		
		E3P3C2	E2P3C2	E3P3C1	E2P3C1	E1P3C1
Food, beverages and tobacco	$\sigma_{\mu\mu}^{11}$	7.90 (1.41)	9.57 (3.42)	6.23 (0.84)	6.00 (0.84)	5.96 (0.84)
Clothing and footwear	$\sigma_{\mu\mu}^{22}$	3.20 (0.76)	3.36 (1.14)	3.01 (0.73)	2.82 (0.74)	2.77 (0.74)
Housing, fuel and furniture	$\sigma_{\mu\mu}^{33}$	8.20 (1.52)	7.00 (1.47)	7.74 (1.46)	7.27 (1.46)	7.19 (1.46)
Travel and recreation	$\sigma_{\mu\mu}^{44}$	15.75 (4.26)	15.82 (8.13)	10.32 (2.57)	9.24 (2.60)	8.98 (2.62)
Other goods and services	$\sigma_{\mu\mu}^{55}$	1.43 (0.34)	1.25 (0.37)	1.32 (0.33)	1.19 (0.33)	1.20 (0.33)
Food etc. vs. Clothing etc.	$\sigma_{\mu\mu}^{12}$	0.06 (0.73)	1.15 (1.54)	-0.20 (0.51)	-0.18 (0.51)	-0.17 (0.51)
Food etc. vs. Housing etc.	$\sigma_{\mu\mu}^{13}$	-0.12 (1.07)	-1.10 (1.50)	-0.80 (0.76)	-0.83 (0.76)	-0.88 (0.76)
Food etc. vs. Travel etc.	$\sigma_{\mu\mu}^{14}$	-7.85 (2.08)	-9.71 (4.95)	-4.94 (1.11)	-4.70 (1.12)	-4.61 (1.12)
Food etc. vs. Others	$\sigma_{\mu\mu}^{15}$	0.01 (0.50)	0.09 (0.79)	-0.29 (0.35)	-0.29 (0.35)	-0.30 (0.35)
Clothing etc. vs. Housing etc.	$\sigma_{\mu\mu}^{23}$	-2.11 (0.72)	-2.32 (0.87)	-2.19 (0.69)	-2.19 (0.69)	-2.18 (0.69)
Clothing etc. vs. Travel etc.	$\sigma_{\mu\mu}^{24}$	-1.33 (1.31)	-2.46 (2.50)	-0.76 (0.93)	-0.60 (0.93)	-0.58 (0.94)
Clothing etc. vs. Others	$\sigma_{\mu\mu}^{25}$	0.18 (0.33)	0.27 (0.43)	0.14 (0.31)	0.15 (0.31)	0.16 (0.31)
Housing etc. vs. Travel etc.	$\sigma_{\mu\mu}^{34}$	-5.46 (2.05)	-2.81 (2.38)	-4.10 (1.52)	-3.57 (1.53)	-3.43 (1.54)
Housing etc. vs. Others	$\sigma_{\mu\mu}^{35}$	-0.51 (0.49)	-0.77 (0.48)	-0.65 (0.46)	-0.68 (0.46)	-0.70 (0.46)
Travel etc. vs. Others	$\sigma_{\mu\mu}^{45}$	-1.11 (0.90)	-0.84 (1.26)	-0.52 (0.63)	-0.37 (0.64)	-0.36 (0.64)
Income measure 1	$\sigma_{\lambda\lambda}^{11}$	202.38 (19.88)	196.49 (22.52)	192.57 (16.68)	190.29 (16.78)	189.54 (16.77)
Income measure 2	$\sigma_{\lambda\lambda}^{22}$	799.31 (76.39)	749.35 (91.96)	721.53 (58.80)	708.11 (59.19)	701.91 (59.18)
Income measure 1 vs. Income measure	$\sigma_{\lambda\lambda}^{12}$	304.33 (35.60)	287.19 (42.37)	276.53 (27.72)	271.08 (27.91)	268.92 (27.90)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

Table 6

Variances and covariances of preference variables in P2 models[a)]

Commodity/ income measure	Parameter	MODEL: Preference variables correlated with total expenditure E3P2C2	E2P2C2	Preference variables uncorrelated with total expenditure E3P2C1	E2P2C1	E1P2C1
Food, beverages and tobacco	$\sigma^{11}_{\alpha\alpha}$	7.86 (1.23)	7.91 (1.69)	7.19 (1.11)	6.93 (1.12)	6.90 (1.12)
Clothing and footwear	$\sigma^{22}_{\alpha\alpha}$	3.45 (0.91)	3.16 (0.87)	3.24 (0.92)	3.04 (0.92)	2.98 (0.93)
Housing, fuel and furniture	$\sigma^{33}_{\alpha\alpha}$	11.57 (2.56)	11.71 (2.95)	11.22 (2.55)	10.90 (2.58)	10.87 (2.59)
Travel and recreation	$\sigma^{44}_{\alpha\alpha}$	16.34 (9.91)	5.25 (9.79)	17.72 (5.87)	15.25 (5.72)	14.74 (5.73)
Other goods and services	$\sigma^{55}_{\alpha\alpha}$	1.28 (0.40)	1.13 (0.41)	1.12 (0.41)	1.04 (0.41)	1.04 (0.41)
Income measure 1	$\sigma^{11}_{\lambda\lambda}$	210.08 (21.92)	209.09 (25.45)	193.33 (16.71)	191.14 (16.82)	190.45 (16.81)
Income measure 2	$\sigma^{22}_{\lambda\lambda}$	851.86 (85.36)	842.05 (111.48)	725.02 (58.96)	712.14 (59.37)	706.29 (59.36)
Income measure 1 vs. income measure 2	$\sigma^{12}_{\lambda\lambda}$	324.22 (40.12)	321.22 (49.78)	278.16 (27.79)	272.94 (27.99)	270.93 (27.98)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

Table 7

Covariances between preference variables and latent total expenditure[a)]

Commodity/ income measure	Parameter	MODEL: E3P3C2	E3P2C2	E2P3C2	E2P2C2
Food, beverages and tobacco	$\sigma^{1}_{\xi\mu}$	20.78 (5.95)	15.83 (4.78)	28.73 (12.16)	19.64 (6.91)
Clothing and footwear	$\sigma^{2}_{\xi\mu}$	4.73 (4.88)	2.26 (3.83)	11.35 (7.51)	2.75 (3.65)
Housing, fuel and furniture	$\sigma^{3}_{\xi\mu}$	6.60 (7.00)	-1.08 (5.40)	-0.36 (9.20)	-6.96 (5.63)
Travel and recreation	$\sigma^{4}_{\xi\mu}$	-36.38 (10.20)	-19.10 (8.95)	-44.00 (19.07)	-16.48 (9.69)
Other goods and services	$\sigma^{5}_{\xi\mu}$	4.27 (3.31)	2.09 (2.44)	4.28 (4.41)	1.05 (2.42)
Income measure 1	$\sigma^{1}_{\xi\lambda}$	67.07 (19.84)	77.74 (22.59)	60.79 (25.86)	74.06 (27.60)
Income measure 2	$\sigma^{2}_{\xi\lambda}$	163.97 (36.89)	200.14 (39.43)	138.07 (54.08)	188.67 (54.72)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

Table 8

Marginal budget shares and savings parameters[a)]

Commodity/ income measure	Parameter	MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	b_1	0.082 (0.025)	0.108 (0.020)	0.047 (0.052)	0.091 (0.030)	0.162 (0.012)	0.165 (0.012)	0.164 (0.012)	0.168 (0.012)	0.166 (0.012)	0.169 (0.012)	0.124 (0.016)	0.120 (0.016)	0.121 (0.016)
Clothing and footwear	b_2	0.106 (0.021)	0.117 (0.016)	0.076 (0.033)	0.113 (0.016)	0.122 (0.011)	0.121 (0.011)	0.122 (0.011)	0.121 (0.011)	0.121 (0.011)	0.120 (0.011)	0.102 (0.015)	0.101 (0.015)	0.101 (0.015)
Housing, fuel and furniture	b_3	0.237 (0.031)	0.269 (0.024)	0.279 (0.040)	0.305 (0.026)	0.268 (0.016)	0.267 (0.016)	0.276 (0.016)	0.275 (0.016)	0.279 (0.016)	0.277 (0.016)	0.301 (0.022)	0.297 (0.022)	0.297 (0.022)
Travel and recreation	b_4	0.487 (0.044)	0.408 (0.031)	0.507 (0.082)	0.386 (0.032)	0.343 (0.021)	0.341 (0.021)	0.331 (0.022)	0.330 (0.022)	0.328 (0.022)	0.327 (0.022)	0.373 (0.027)	0.384 (0.027)	0.382 (0.027)
Other goods and services	b_5	0.088 (0.015)	0.098 (0.011)	0.091 (0.019)	0.105 (0.011)	0.105 (0.007)	0.106 (0.007)	0.107 (0.007)	0.106 (0.007)	0.106 (0.008)	0.107 (0.007)	0.100 (0.013)	0.098 (0.011)	0.099 (0.011)
Income measure 1	e_1	0.275 (0.080)	0.228 (0.086)	0.307 (0.118)	0.234 (0.120)	0.514 (0.053)	0.510 (0.053)	0.527 (0.055)	0.523 (0.055)	0.522 (0.055)	0.518 (0.055)	1.172 (0.077)	1.207 (0.079)	1.190 (0.079)
Income measure 2	e_2	0.524 (0.136)	0.393 (0.135)	0.660 (0.239)	0.419 (0.227)	1.110 (0.100)	1.103 (0.100)	1.163 (0.103)	1.156 (0.103)	1.188 (0.105)	1.180 (0.105)	3.142 (0.169)	3.232 (0.173)	3.213 (0.173)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

Table 9

Engel elasticities[a)]

		MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
Commodity	Parameter	E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages and tobacco	E_1	0.32	0.42	0.18	0.35	0.63	0.64	0.64	0.65	0.65	0.66	0.48	0.47	0.47
Clothing and footwear	E_2	0.99	1.09	0.71	1.06	1.14	1.13	1.14	1.13	1.13	1.12	0.95	0.94	0.94
Housing, fuel and furniture	E_3	0.95	1.08	1.12	1.23	1.08	1.07	1.11	1.11	1.12	1.11	1.21	1.20	1.20
Travel and recreation	E_4	1.56	1.31	1.63	1.24	1.10	1.09	1.06	1.06	1.05	1.05	1.20	1.23	1.22
Other goods and services	E_5	1.16	1.29	1.20	1.38	1.38	1.39	1.41	1.39	1.39	1.41	1.32	1.29	1.30

a) See table 1 and 2 for model descriptions

Table 10

Demographic effects[a)]

		MODEL												
		Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
Commodity/ income measure	Parameter	E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
	No. of children													
Food, beverages and tobacco	c_{11}	1.328	1.193	1.513	1.280	0.906	0.888	0.895	0.875	0.891	0.872	1.108	1.129	1.126
		(0.191)	(0.171)	(0.315)	(0.209)	(0.148)	(0.146)	(0.149)	(0.146)	(0.149)	(0.146)	(0.149)	(0.149)	(0.149)
Clothing and footwear	c_{21}	0.194	0.141	0.355	0.161	0.109	0.115	0.113	0.117	0.121	0.124	0.216	0.221	0.222
		(0.163)	(0.145)	(0.213)	(0.147)	(0.132)	(0.131)	(0.132)	(0.132)	(0.132)	(0.132)	(0.140)	(0.141)	(0.140)
Housing, fuel and furniture	c_{31}	-0.167	-0.336	-0.394	-0.527	-0.330	-0.327	-0.373	-0.368	-0.382	-0.376	-0.503	-0.482	-0.480
		(0.237)	(0.216)	(0.274)	(0.223)	(0.192)	(0.193)	(0.193)	(0.194)	(0.193)	(0.194)	(0.204)	(0.205)	(0.204)
Travel and recreation	c_{41}	-1.253	-0.841	-1.356	-0.721	-0.492	-0.487	-0.429	-0.423	-0.430	-0.425	-0.650	-0.709	-0.713
		(0.324)	(0.271)	(0.494)	(0.276)	(0.253)	(0.253)	(0.256)	(0.255)	(0.256)	(0.256)	(0.257)	(0.256)	(0.255)
Other goods and services	c_{51}	-0.102	-0.157	-0.118	-0.193	-0.193	-0.189	-0.206	-0.201	-0.200	-0.195	-0.171	-0.159	-0.155
		(0.111)	(0.099)	(0.130)	(0.100)	(0.090)	(0.090)	(0.090)	(0.091)	(0.090)	(0.090)	(0.118)	(0.101)	(0.101)
Income measure 1	f_{11}	-0.126	0.119	-0.302	0.087	-1.384	-1.367	-1.461	-1.442	-1.401	-1.381	-4.877	-5.052	-4.883
		(0.739)	(0.765)	(0.867)	(0.882)	(0.673)	(0.673)	(0.679)	(0.679)	(0.679)	(0.680)	(0.727)	(0.739)	(0.735)
Income measure 2	f_{21}	2.963	3.651	2.333	3.506	-0.122	-0.087	-0.416	-0.377	-0.471	-0.430	-10.885	-11.345	-11.022
		(1.366)	(1.390)	(1.704)	(1.690)	(1.270)	(1.271)	(1.283)	(1.285)	(1.286)	(1.288)	(1.583)	(1.615)	(1.611)
	No. of adults													
Food, beverages and tobacco	c_{12}	1.551	1.235	1.981	1.441	0.569	0.526	0.545	0.500	0.523	0.477	1.037	1.084	1.073
		(0.360)	(0.309)	(0.674)	(0.412)	(0.236)	(0.232)	(0.238)	(0.234)	(0.239)	(0.235)	(0.256)	(0.259)	(0.259)
Clothing and footwear	c_{22}	0.245	0.121	0.618	0.170	0.048	0.060	0.057	0.067	0.072	0.080	0.297	0.308	0.308
		(0.308)	(0.255)	(0.437)	(0.259)	(0.210)	(0.209)	(0.212)	(0.211)	(0.212)	(0.211)	(0.242)	(0.244)	(0.244)
Housing, fuel and furniture	c_{32}	-1.145	-1.540	-1.666	-1.978	-1.526	-1.518	-1.621	-1.609	-1.658	-1.643	-1.924	-1.876	-1.886
		(0.446)	(0.383)	(0.546)	(0.398)	(0.307)	(0.309)	(0.310)	(0.311)	(0.311)	(0.313)	(0.352)	(0.354)	(0.355)
Travel and recreation	c_{42}	-0.649	0.312	-0.899	0.580	1.122	1.135	1.258	1.271	1.301	1.314	0.747	0.613	0.636
		(0.620)	(0.483)	(1.057)	(0.495)	(0.409)	(0.408)	(0.414)	(0.414)	(0.417)	(0.417)	(0.443)	(0.442)	(0.444)
Other goods and services	c_{52}	-0.002	-0.128	-0.034	-0.213	-0.213	-0.203	-0.239	-0.229	-0.238	-0.228	-0.157	-0.129	-0.131
		(0.209)	(0.173)	(0.262)	(0.177)	(0.143)	(0.142)	(0.145)	(0.144)	(0.145)	(0.146)	(0.206)	(0.174)	(0.175)
Income measure 1	f_{12}	12.402	12.967	12.010	12.903	9.474	9.512	9.314	9.356	9.372	9.418	1.459	1.044	1.192
		(1.288)	(1.357)	(1.663)	(1.692)	(1.066)	(1.067)	(1.081)	(1.081)	(1.087)	(1.088)	(1.257)	(1.282)	(1.282)
Income measure 2	f_{22}	18.291	19.883	16.633	19.565	11.111	11.191	10.471	10.559	10.157	10.254	-13.630	-14.718	-14.627
		(2.315)	(2.335)	(3.323)	(3.227)	(2.011)	(2.012)	(2.042)	(2.044)	(2.060)	(2.062)	(2.737)	(2.797)	(2.806)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

Table 11

Variances and covariances of measurement errors[a]

Commodity/ income measure	Parameter	MODEL: Preference variables correlated with total expenditure				Preference variables uncorrelated with total expenditure						No individual differences in preferences		
		E3P3C2	E3P2C2	E2P3C2	E2P2C2	E3P3C1	E3P2C1	E2P3C1	E2P2C1	E1P3C1	E1P2C1	E3P1C1	E2P1C1	E1P1C1
Food, beverages	σ^{11}_{w}	9.77	9.57	10.15	10.10	9.82	9.82	10.31	10.34	10.32	10.34	21.58	21.70	21.68
and tobacco		(0.71)	(0.71)	(0.71)	(0.71)	(0.72)	(0.72)	(0.72)	(0.72)	(0.72)	(0.72)	(1.07)	(1.08)	(1.08)
Clothing and	σ^{22}_{w}	12.81	12.73	13.49	13.48	13.15	13.15	13.57	13.57	13.70	13.70	19.01	19.05	19.05
footwear		(0.93)	(0.93)	(0.95)	(0.94)	(0.93)	(0.93)	(0.95)	(0.95)	(0.95)	(0.95)	(0.94)	(0.95)	(0.95)
Housing, fuel	σ^{33}_{w}	26.54	26.18	28.00	28.18	26.92	26.90	27.66	27.65	27.65	27.62	44.55	44.95	44.90
and furniture		(2.05)	(2.07)	(1.96)	(1.97)	(1.96)	(1.96)	(1.92)	(1.92)	(1.92)	(1.92)	(2.23)	(2.25)	(2.25)
Travel and	σ^{44}_{w}	87.51	90.24	98.59	97.97	89.02	88.63	93.32	92.88	94.29	93.88	114.29	113.75	113.86
recreation		(6.78)	(6.70)	(6.88)	(6.83)	(6.16)	(6.14)	(6.24)	(6.22)	(6.30)	(6.29)	(5.70)	(5.67)	(5.68)
Other goods	σ^{55}_{w}	5.19	5.11	5.48	5.46	5.32	5.31	5.47	5.47	5.51	5.51	8.48	8.55	8.55
and services		(0.39)	(0.39)	(0.38)	(0.38)	(0.39)	(0.38)	(0.38)	(0.38)	(0.39)	(0.39)	(0.42)	(0.43)	(0.43)
Income measure 1	$\sigma^{11}_{\varepsilon\varepsilon}$	58.83	60.04	62.05	62.25	57.44	57.47	61.85	61.86	63.27	63.27	181.03	175.45	178.58
		(4.49)	(4.47)	(4.35)	(4.37)	(4.44)	(4.44)	(4.34)	(4.34)	(4.44)	(4.44)	(9.77)	(9.31)	(9.47)
Income measure 2	$\sigma^{22}_{\varepsilon\varepsilon}$	101.09	106.15	111.84	112.48	92.82	92.96	111.71	111.74	114.64	114.64	167.81	128.98	135.55
		(8.72)	(8.33)	(7.85)	(7.89)	(9.13)	(9.10)	(7.85)	(7.86)	(8.04)	(8.04)	(22.79)	(8.90)	(9.33)
Income measure 1	$\sigma^{12}_{\varepsilon\varepsilon}$	57.14	59.68	62.96	63.41	53.57	53.64	62.70	62.71	64.92	64.92	114.61	99.89	105.32
vs. income measure 2		(5.54)	(5.42)	(5.18)	(5.21)	(5.59)	(5.58)	(5.17)	(5.17)	(5.31)	(5.31)	(11.41)	(8.27)	(8.57)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

Table 12

Squared coefficient of multiple correlation[a)]

Commodity/ income measure	MODEL 1																									
	Preference variables correlated with total expenditure								Preference variables uncorrelated with total expenditure												No individual differences in preferences					
	E3P3C2 Period		E3P2C2 Period		E3P3C2 Period		E2P2C2 Period		E3P3C1 Period		E3P2C1 Period		E2P3C1 Period		E2P2C1 Period		E1P3C1 Period		E1P2C1 Period		E3P1C1 Period		E2P1C1 Period		E1P1C1 Period	
	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
Food, beverages and tobacco	0.70	0.72	0.71	0.73	0.70	0.71	0.69	0.71	0.70	0.72	0.70	0.72	0.68	0.71	0.68	0.71	0.70	0.70	0.70	0.70	0.36	0.37	0.36	0.37	0.36	0.36
Clothing and footwear	0.42	0.47	0.43	0.47	0.40	0.43	0.39	0.44	0.41	0.45	0.41	0.45	0.40	0.43	0.40	0.43	0.41	0.41	0.41	0.41	0.17	0.18	0.17	0.18	0.18	0.18
Housing, fuel and furniture	0.51	0.57	0.52	0.58	0.49	0.55	0.48	0.55	0.51	0.56	0.52	0.56	0.50	0.55	0.51	0.55	0.53	0.53	0.53	0.53	0.22	0.25	0.22	0.24	0.23	0.23
Travel and recreation	0.39	0.48	0.38	0.45	0.32	0.40	0.33	0.40	0.41	0.45	0.41	0.45	0.38	0.42	0.39	0.42	0.40	0.40	0.40	0.40	0.25	0.27	0.26	0.28	0.27	0.27
Other goods and services	0.49	0.54	0.50	0.55	0.46	0.51	0.46	0.52	0.48	0.52	0.48	0.52	0.47	0.51	0.47	0.51	0.49	0.49	0.49	0.49	0.20	0.22	0.19	0.21	0.20	0.20
Income measure 1	0.89	0.89	0.89	0.89	0.88	0.89	0.88	0.89	0.89	0.89	0.89	0.89	0.88	0.89	0.88	0.89	0.88	0.88	0.88	0.88	0.65	0.67	0.66	0.68	0.66	0.66
Income measure 2	0.94	0.94	0.94	0.94	0.93	0.94	0.93	0.94	0.94	0.95	0.94	0.95	0.93	0.94	0.93	0.94	0.93	0.93	0.93	0.93	0.90	0.91	0.92	0.93	0.92	0.92

[a)] See table 1 and 2 for model descriptions. See section 3 for definition of the squared coefficient of multiple correlation

Table 13

Marginal budget shares and saving parameters[a)]

Commodity/ Income measure	Parameter	MODEL IM[b)]	E3P3C2	E3P2C2	E3P3C1	E3P2C1
Food, beverages and tobacco	b_1	0.101 (0.035)	0.082 (0.025)	0.108 (0.020)	0.162 (0.012)	0.165 (0.012)
Clothing and footwear	b_2	0.147 (0.043)	0.106 (0.021)	0.117 (0.016)	0.122 (0.011)	0.121 (0.011)
Housing, fuel and furniture	b_3	0.039 (0.052)	0.237 (0.031)	0.269 (0.024)	0.268 (0.016)	0.267 (0.016)
Travel and recreation	b_4	0.657 (0.077)	0.487 (0.044)	0.408 (0.031)	0.343 (0.021)	0.341 (0.021)
Other goods and services	b_5	0.056 (0.025)	0.088 (0.015)	0.098 (0.011)	0.105 (0.007)	0.106 (0.007)
Income measure 1	e_1	0.329 (0.112)	0.275 (0.080)	0.228 (0.086)	0.514 (0.053)	0.516 (0.053)
Income measure 2	e_2	0.510 (0.160)	0.524 (0.136)	0.393 (0.135)	1.110 (0.100)	1.103 (0.100)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

b) IM is the incremental model

Table 14

Variances and covariances of measurement errors in the incremental form[a)]

Commodity/ Income measure	Parameter	MODEL IM[b)]	E3P3C2	E3P2C2	E3P3C1	E3P2C1
Food, beverages and tobacco	$\sigma^{11}_{\Delta v \Delta v}$	19.09 (1.48)	19.54 (1.42)	19.14 (1.42)	19.64 (1.44)	19.64 (1.44)
Clothing and footwear	$\sigma^{22}_{\Delta v \Delta v}$	24.57 (2.05)	25.62 (1.86)	25.46 (1.86)	26.30 (1.86)	26.30 (1.86)
Housing, fuel and furniture	$\sigma^{33}_{\Delta v \Delta v}$	55.96 (3.94)	53.08 (4.10)	52.36 (4.14)	53.84 (3.92)	53.80 (3.92)
Travel and recreation	$\sigma^{44}_{\Delta v \Delta v}$	150.26 (21.46)	175.02 (13.56)	180.48 (13.40)	178.04 (12.32)	177.26 (12.28)
Other goods and services	$\sigma^{55}_{\Delta v \Delta v}$	10.64 (0.79)	10.38 (0.78)	10.22 (0.78)	10.64 (0.78)	10.62 (0.76)
Income measure 1	$\sigma^{11}_{\Delta \varepsilon \Delta \varepsilon}$	113.88 (9.85)	117.66 (8.98)	120.08 (8.94)	114.88 (8.88)	114.94 (8.88)
Income measure 2	$\sigma^{22}_{\Delta \varepsilon \Delta \varepsilon}$	198.86 (18.57)	202.18 (17.44)	212.30 (16.66)	185.64 (18.26)	185.92 (18.20)
Income measure 1 vs. income measure 2	$\sigma^{12}_{\Delta \varepsilon \Delta \varepsilon}$	110.22 (12.21)	114.28 (11.08)	119.36 (10.84)	107.14 (11.18)	107.28 (11.16)

a) See table 1 and 2 for model descriptions. Standard errors in parenthesis

b) IM is the incremental model

Table 15

Variance of total expenditure in the incremental form[a)]

Parameter	IM[b)]	E3P3C2	E3P2C2	E3P3C1	E2P3C1
$\sigma_{\Delta \xi \Delta \xi}$	116.93 (33.08)	92.87 (23.09)	90.52 (22.92)	37.72 (10.45)	37.95 (10.48)

a) See table 1 and 2 for model descriptions. Standard deviations in parenthesis

b) IM is the incremental model

APPENDIX A: RELATIONSHIPS BETWEEN FIRST AND SECOND ORDER MOMENTS OF OBSERVED AND LATENT VARIABLES

In this appendix, we give the explicit expressions for the first and second order moments of the observed variables in the level and incremental versions of the Engel function model.

a. Level version

From (11)-(13) it follows that the expected values of the observed vectors y_t and w_t are given by

$$\text{(A.1)}\qquad \begin{aligned} E(y_t) &= \Phi_y^t = a_t + b\Phi_\xi^t + C\Phi_z\,, \qquad && t = 1,2,\\ E(w_t) &= \Phi_w^t = d_t + e\Phi_\xi^t + F\Phi_z\,, \qquad && t = 1,2, \end{aligned}$$

Since $\iota' a_t=0$, $\iota' b=1$, $\iota' C=0_{12}$, we know that the expected total expenditure is [cf. (3), (8)-(10)]

$$\text{(A.2)}\qquad E(x_t) = E(\iota' y_t) = \Phi_\xi^t\,, \qquad t = 1,2.$$

From (11), (12), (14)-(17), and the assumption of zero correlation between (ν_t,ε_t) and (ξ_t,z,μ,λ) it follows that the second order moments of the observed variables (the subscripts and the superscripts indicating the variables and the periods, respectively) are given by

$$\begin{aligned}
\Sigma_{yy}^{ts} &= b\sigma_{\xi\xi}^{ts}b' + C\Sigma_{zz}C' + \Sigma_{\mu\mu} + b\Sigma_{\xi z}^{(t)}C' + C\Sigma_{\xi z}^{(s)\prime}b' + b\Sigma_{\xi\mu}^{(t)} + \Sigma_{\xi\mu}^{(s)\prime}b'\\
&\quad + C\Sigma_{z\mu} + \Sigma_{z\mu}'C' + \delta_{ts}\Sigma_{\nu\nu}\,,\\
\Sigma_{ww}^{ts} &= e\sigma_{\xi\xi}^{ts}e' + F\Sigma_{zz}F' + \Sigma_{\lambda\lambda} + e\Sigma_{\xi z}^{(t)}F' + F\Sigma_{\xi z}^{(s)\prime}e' + e\Sigma_{\xi\lambda}^{(t)} + \Sigma_{\xi\lambda}^{(s)\prime}e'\\
&\quad + F\Sigma_{z\lambda} + \Sigma_{z\lambda}'F' + \delta_{ts}\Sigma_{\varepsilon\varepsilon}\,,\\
\text{(A.3)}\qquad \Sigma_{yw}^{ts} &= b\sigma_{\xi\xi}^{ts}e' + C\Sigma_{zz}F' + \Sigma_{\mu\lambda} + b\Sigma_{\xi z}^{(t)}F' + C\Sigma_{\xi z}^{(s)\prime}e' + b\Sigma_{\xi\lambda}^{(t)} + \Sigma_{\xi\mu}^{(s)\prime}e'\\
&\quad + C\Sigma_{z\lambda} + \Sigma_{z\mu}'F'\,,\\
\Sigma_{yz}^{t} &= b\Sigma_{\xi z}^{(t)} + C\Sigma_{zz} + \Sigma_{z\mu}'\,,\\
\Sigma_{wz}^{t} &= e\Sigma_{\xi z}^{(t)} + F\Sigma_{zz} + \Sigma_{z\lambda}'\,, \qquad t,s = 1,2.
\end{aligned}$$

The overall covariance matrix of the observed variables is the $(2I+2K+M)\times(2I+2K+M)$ matrix

$$(A.4)\qquad \Sigma = \begin{bmatrix} \Sigma^{11}_{yy} & \Sigma^{12}_{yy} & \Sigma^{11}_{wy} & \Sigma^{12}_{wy} & \Sigma^{1}_{zy} \\ \Sigma^{21}_{yy} & \Sigma^{22}_{yy} & \Sigma^{21}_{wy} & \Sigma^{22}_{wy} & \Sigma^{2}_{zy} \\ \Sigma^{11}_{yw} & \Sigma^{12}_{yw} & \Sigma^{11}_{ww} & \Sigma^{12}_{ww} & \Sigma^{1}_{zw} \\ \Sigma^{21}_{yw} & \Sigma^{22}_{yw} & \Sigma^{21}_{ww} & \Sigma^{22}_{ww} & \Sigma^{2}_{zw} \\ \Sigma^{1}_{yz} & \Sigma^{2}_{yz} & \Sigma^{1}_{wz} & \Sigma^{2}_{wz} & \Sigma_{zz} \end{bmatrix},$$

where $\Sigma^{st}_{wy}=\Sigma^{ts}_{yw}{}'$, $\Sigma^{t}_{zy}=\Sigma^{t}_{zy}{}'$, $\Sigma^{t}_{zw}=\Sigma^{t}_{wz}{}'$.

b. Incremental version

From (25)-(27) it follows that the changes in the observed endogenous vectors y and w from period 1 to period 2 have expectations given by

$$(A.5)\qquad \begin{aligned} E(\Delta y) &= \Delta a + b\Phi_{\Delta\xi}\,, \\ E(\Delta w) &= \Delta d + e\Phi_{\Delta\xi}\,, \end{aligned}$$

where $\Phi_{\Delta\xi}=\Phi^{2}_{\xi}-\Phi^{1}_{\xi}=E(\Delta\xi)$. Furthermore,

$$(A.6)\qquad E(\Delta x) = E(\iota'\Delta y) = \Phi_{\Delta\xi}\,.$$

The covariance matrix of the observed endogenous variables reads

$$(A.7)\qquad \Sigma_{\Delta} = \begin{bmatrix} \Sigma_{\Delta y\Delta y} & \Sigma_{\Delta w\Delta y} \\ \Sigma_{\Delta y\Delta w} & \Sigma_{\Delta w\Delta w} \end{bmatrix},$$

where

$$(A.8)\qquad \begin{aligned} &(a)\quad \Sigma_{\Delta y\Delta y} = b\sigma_{\Delta\xi\Delta\xi}b' + 2\Sigma_{\nu\nu}\,, \\ &(b)\quad \Sigma_{\Delta w\Delta w} = e\sigma_{\Delta\xi\Delta\xi}e' + 2\Sigma_{\varepsilon\varepsilon}\,, \\ &(c)\quad \Sigma_{\Delta y\Delta w} = b\sigma_{\Delta\xi\Delta\xi}e' = \Sigma'_{\Delta w\Delta y}\,, \end{aligned}$$

with $\sigma_{\Delta\xi\Delta\xi} = \sigma^{11}_{\xi\xi}-2\sigma^{12}_{\xi\xi}+\sigma^{22}_{\xi\xi} = var(\Delta\xi)$.

APPENDIX B: DATA

The Norwegian Surveys of Consumer Expenditures comprise the data base for this study. These sample surveys have been performed continuously since January 1, 1973 and give data in the incomplete cross-section/time series format. The sampling method is a three-stage stratified design, giving a selfweighted random sample of all the private households in Norway. The rate of nonresponse averages about 30%. It is lower for households asked to give their second report than for those asked the first time. We have taken data from the data base described in Biørn and Jansen (1980), and have used data on 408 households for which two reports exist in the years 1975-1977, one half observed in 1975 and 1976 and the other half in 1976 and 1977. This is the same data set as used in Biørn and Jansen (1982), with a few modifications as noted below, and with the addition of income data taken from tax files.

Purchase expenditures on consumption goods are recorded by a combination of of bookkeeping and interviews. Each household is asked to keep detailed accounts of its expenses over a period of two weeks. For commodities with a low purchase frequency, expenses during the last 12 months are registered in a concluding interview at the end of the accounting period. Housing expenses are measured by rent (including maintenance and repairs), whereas other durable goods are represented by the value of last year's total purchases.

The expenditure data are collected evenly throughout the year, 1/26 of the households participating in a particular year are observed beetween 1st and 14th of January, another 1/26 between 15th and 28th of January, and so on. We have deflated the expenditures by price indexes constructed from the basic data used in calculating the official Norwegian Consumer Price Index. Each series of monthly Laspeyres indexes is smoothed to a periodicity of 14 days to make the periodicity coincide with the length of the accounting period for consumption expenses, see Biørn and Jansen (1980) for details. Thus consumption are measured at constant prices, and effects of the gradual changes in relative prices and unobserved individual differences in prices (due e.g. to geographical price variations) are captured partly by the period specific constant term, partly by the individual latent variables and partly by the residuals. (An explicit modelling of relative price effects in our latent variable framework would be feasible, but is quite complicated and might be the topic for a separate research project.) The households who report twice do so with an interval of exactly one year. By constructing annual aggregates, we get two annual reports from H=408 households, which we formally treat as if it were a two period balanced panel, although the two time periods are not exactly the same for all households.

The income variables are defined and commented on in section 3. The demographic variables used to characterize the size and composition of the household are:

- the number of children (z_1), i.e. persons with age $\leq$ 15 years,
- the number of adults (z_2), i.e. persons with age $\geq$ 16 years.

We have used the observations on these variables from the first period and analyze the data as if they were the same in both periods, i.e. individual specific. (The effects of one year ageing of each household member will then be captured by the period specific constant term.) For ten households. the number of household members (z_1+z_2) changed by more than one beetween the two periods. These where deleted from our sample, since they could hardly be considered as the same household in both periods and might also dominate the results to an undesired degree when transforming the model to its incremental form. (This was not done by Biørn and Jansen (1982) which explains why they used 418 households versus our 408 households.)

The covariance matrix of all the observed variables are given in table B1. This is all the data input we use for our econometric analysis. We have used the same five commodity group classification, aggregated from a more detailed grouping with 28 groups, as described in detail in Biørn and Jansen (1980).

The covariance matrix did not look like the one in table B1 at the initial stages of our calculations. An inspection showed that the covariances, especially those for the incremental variables, were extremely sensitive to a few extreme observations. Some sort of robust procedure was needed, cf. Huber (1981) or Hampel et al. (1986). We found the idea of winsorizing the observations, i.e. to replace all observations of expenditure and incomes larger (lower) than an upper (lower) bound with this bound, particularly promising in our setting. We chose to winsorize moderately, by setting the maximum value of each variable equal to the 9th largest observation (i.e. an estimate of the 0.98 quantile) in the original sample. (It was not necessary to introduce a minimum bound on the variables since zero was such a lower limit in the original data.) This procedure was followed for all the basic expenditure and income variables in our data file, for each of the two periods, while all the derived variables were defined, as before, as functions of the basic variables. Thus our expenditure data was winsorized by applying the above rule routinely on each of the commodities in the most detailed grouping (28 groups) in each period. Then we aggregated over groups and calculated first differences etc. Our econometric analysis in this paper is based on the distribution of these modified observations.

The first order sample moments of our observed variables are, y_1: (10.5581,4.3886,9.8937,11.9299,2.9802), y_2: (10.5655,4.4365,10.5787, 13.8448,3.3036), w_1: (38.0961,55.1871), w_2: (41.9946,58.5761), z: (0.7672,2.2377).

Table B1
Covariance matrix of the observed variables[a)]

	Food, beverages and tobacco		Clothing and footwear		Housing, fuel and furniture		Travel and recreation		Other goods and services	
	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}	y_{41}	y_{42}	y_{51}	y_{52}
y_{11}	33.866399									
y_{12}	23.931881	34.277795								
y_{21}	9.771840	9.894488	22.692229							
y_{22}	10.362220	12.385392	9.556499	23.523477						
y_{31}	16.573568	17.280461	7.069731	11.628316	54.839144					
y_{32}	17.656239	19.497959	6.757210	13.738555	30.243543	61.781544				
y_{41}	21.432868	21.064339	14.633581	14.944365	24.033823	22.290424	131.351104			
y_{42}	26.029033	32.328722	15.823002	26.467252	34.579921	31.770744	55.026045	179.365061		
y_{51}	6.995257	6.929788	5.202998	5.008590	7.746768	8.136831	14.012352	14.429180	9.590906	
y_{52}	7.414759	9.006666	4.324339	6.445988	9.069422	10.505322	11.609091	15.243447	5.218842	11.862711
w_{11}	51.588551	48.147343	32.549316	31.866398	59.113226	60.929099	127.655839	116.312647	29.988223	28.188578
w_{12}	57.612568	58.135318	35.658711	40.761958	63.376137	64.759473	133.288826	149.462801	32.373895	30.914929
w_{21}	107.289966	97.009444	64.431758	67.096449	140.906382	133.627689	238.857808	234.022386	57.959994	50.511329
w_{22}	113.944568	108.692940	69.767003	78.773507	141.262465	136.863114	235.569617	278.668461	62.880572	55.233889
z_1	2.912353	3.029978	1.318979	1.289102	1.469587	2.181751	1.699158	3.172564	0.540539	0.760104
z_2	2.197420	2.294813	1.334488	1.339847	1.310812	1.752556	4.525261	4.460283	0.836724	0.995516

a) Measurement unit: 1000 Norwegian 1974 kroner

Table B1 (cont.)
Covariance matrix of the observed variables

	Income measure 1		Income measure 2		Number of children	Number of adults
	w_{11}	w_{12}	w_{21}	w_{22}	z_1	z_2
w_{11}	487.977561					
w_{12}	467.259940	573.080053				
w_{21}	766.034038	774.003609	1626.371358			
w_{22}	752.773693	890.587960	1624.143500	1851.203952		
z_1	2.281886	4.351582	9.495276	12.433148	1.579200	
z_2	12.660054	13.595467	20.156360	21.558213	0.078527	0.826605

Table B2
Covariance matrix of the observed variables for the incremental version[a)]

	Food, beverages and tobacco	Clothing and footwear	Housing, fuel and furniture	Travel and recreation	Other goods and services	Income measure 1	Income measure 2
	Δy_1	Δy_2	Δy_3	Δy_4	Δy_5	Δw_1	Δw_2
Δy_1	20.280431						
Δy_2	1.900523	27.102707					
Δy_3	1.134827	2.422760	56.133601				
Δy_4	6.668212	10.333467	-1.065778	200.664076			
Δy_5	1.657375	2.316057	1.045836	3.217528	11.015934		
Δw_1	5.963958	5.786165	-0.432537	27.517167	0.340679	126.537734	
Δw_2	5.028894	6.341813	2.879342	47.934266	-0.198018	129.844696	229.288310

a) Measurement unit: 1000 Norwegian 1974 kroner

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