

# Discussion Paper

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## TAXING OR SUBSIDISING AN EXPORTING

### INDUSTRY ?

BY

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#### Abstract

This paper analyses whether a welfare maximizing government should tax or subsidize the home firms in an industry characterized by oligopolistic competition and differentiated products. The home firms are assumed to be pure exporters. It is shown that a symmetric, perfect Nash-equilibrium in the quantity setting game will involve an export subsidy if the industry is fairly concentrated, if the relative number of home firms is not too large and if the products are fairly homogenous. The paper presents a reduced form expression which makes this proposition precise. In the symmetric price setting game there is an unambiguous case for an export tax.

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## 1 Introduction

This paper presents a partial equilibrium analysis of whether a welfare maximizing government should tax or subsidize exporting firms in an imperfectly competitive industry. Dixit (1984) found that in the case of homogenous products and quantity competition, there is a case for an export subsidy if the number of home firms is not too large. Eaton and Grossman (1986) extended this analysis to the case of differentiated products. Assuming the firms have consistent conjectures, they found that there in general will be a case for an export tax. In this paper we also investigate the case of differentiated products, incorporating the case of identical products as a special case. In contrast to Eaton and Grossman, I employ the non-cooperative Nash equilibrium concept, investigating both the case where the firms regard quantities and prices as their strategy variables. I find that when the firms regard quantities as their strategy variable, it is ambiguous whether an optimal policy will involve a tax or a subsidy. The analysis shows that the answer to the principal question addressed in this paper will depend on three parameters in our model; *the elasticity of substitution between the goods in this industry, the total number of firms and the relative number of home firms*. In the case of competition in prices no ambiguity exists, and there is an unambiguous case for an export tax.

In the next section I formalize the government's policy problem. Section 3 outlines the model for the quantity setting game. In the next section, section 4, I present the price-setting game. In the last section I discuss some shortcomings of the present analysis.

## 2 The government's policy problem

In this section, I will formulate the government's policy choice in a full information two stage game framework. The industry I will investigate is characterized by monopolistic competition in differentiated products. The policy problem is viewed from the point of view of the home-government, which can levy a tax or a subsidy on the domestic producers. The domestic producers sell only on the world market, i.e. I limit the discussion to purely exporting industries. Furthermore, I will only consider cases where the government cannot influence the number of firms in the industry, i.e. there is a fixed number of foreign and domestic firms. I assume that the government is able to commit

itself to a policy action at the first stage of the game. In the second stage of the game, the firms choose simultaneously their actions subject to the policy determined at the first stage. I assume that all agents have full information. Abstracting from distortions in the rest of the economy, and assuming lump sum taxes are available, the policy problem is to maximize net revenue; tax income plus profit for the domestic producers. Total revenue ( $R$ ) can be expressed

$$R = \sum_{i \in N_A} \{(P_i - C_i)X_i - CF_i\} \quad (1)$$

where  $N_A$  denotes the set of domestic producers,  $P_i$  is the price facing the consumer for good  $i$ ,  $C_i$ ,  $X_i$  and  $CF_i$  are respectively the variable costs, total production and the fixed costs for firm  $i$ . I assume that there is only one firm producing each product. Let us investigate the impact on total revenue of a change in the export tax ( $\tau$ )

$$\frac{dR}{d\tau} = \sum_{i \in N_A} \left( \frac{dP_i}{d\tau} X_i + (P_i - C_i) \frac{dX_i}{d\tau} \right) \quad (2)$$

This simple equation incorporates several effects which will determine the sign on  $dR/d\tau$ . Basically there are two opposing forces when the government introduces a tax; the terms of trade will improve, whereas there is a loss of marketshares. If we look at these effects in more detail, we will see that they incorporate several issues. Firstly, we have to consider the tax incidence effect, i.e. to what extent will the tax be passed on to the foreign consumers and to what extent will the tax reduce the profit-margin of domestic producers. Secondly, when calculating the effect on the sales of the taxed producers, we have to consider to what extent the tax will influence the price charged by the producers which are not taxed. Since a tax change influencing some producers also will affect the market conditions for the other producers, we will in general expect the price charged by the non-taxed producers to change. Hence, the total effect on the sales of the taxed producers will be determined by a quite complicated set of effects.

In general, one can formulate the pricing rule of the producers by a mark-up formula

$$P_i = M_i C_i \quad \forall i \in N \quad (3)$$

where  $N$  denotes the set of all producers, and  $M_i$  is the mark-up factor for firm  $i$ . It is well known that this mark-up factor can be related to the "perceived demand elasticity" facing firm  $i$  ( $\epsilon_i$ ) in the following way

$$M_i = \frac{1}{1 - 1/\epsilon_i} \quad \forall i \in N \quad (4)$$

Combining equations (2), (3) and (4) we obtain the expression

$$\frac{dR}{d\tau} = \sum_{i \in N_A} X_i \frac{dP_i}{d\tau} \left( 1 - \frac{\epsilon_{ir}}{\epsilon_i} \right) \quad (5)$$

where I have introduced a variable  $\epsilon_{i,r}$ ; which can be thought of as an elasticity of demand facing firm  $i$  under the given tax-reform:

$$\epsilon_{i,r} \equiv \frac{P_i dX_i/dr}{X_i dP_i/dr} \quad (6)$$

For the sake of argument, let us assume that all domestic producers are identical (i.e. they have the same cost-structure and face the same demand-conditions), except that they produce different products. In this case it is straight forward to state the condition which determines the direction of a welfare improving tax policy: From equation (5) we can see that it will be welfare improving to *tax* an exporting industry if

$$\epsilon_r < \epsilon \quad (7)$$

where I have dropped the subscripts referring to individual firms. In the same way one can conclude that it will be welfare improving to *subsidize* the domestic firms if the inequality sign is reversed in equation (7).

### 3 The quantity setting game

In this section I will analyse a special model in order to see how the ambiguity of whether to tax or subsidize can be referred back to the size of a few interesting parameters. The model I will use is the model of imperfect competition developed by Spence (1976) and Dixit-Stiglitz (1977). In this section I will look at the quantity-setting game, whereas the price-setting game is investigated in the next section <sup>1</sup>.

#### 3.1 The market equilibrium

Since this model is well known to practitioners in the field of international trade and industrial organization, I will only give a brief description here.

##### The demand side

Let us assume that the representative consumer in the rest of the world has an exogenous income ( $I$ ), and a utility-function of the log-linear/CES-type

$$U = (1 - \omega) \log Y + \omega \log \Gamma \quad (8)$$

where  $Y$  is an index representing the aggregate of all other goods, and  $\Gamma$  is a CES-index for the aggregate of the differentiated products produced in the industry in question <sup>2</sup>

$$\Gamma = \left[ \sum_{i \in N} X_i^\rho \right]^{1/\rho} \quad (9)$$

<sup>1</sup>In a recent paper Anderson, de Palma and Thisse (1987) show that there is an (or infact, infinitely many) address model (location model) and a (infinitely many) discrete choice model which will give the same market demand, pricing rule etc. as the Spence-Dixit-Stiglitz model presented above.

<sup>2</sup>The elasticity of substitution ( $\sigma$ ) between two goods in this aggregate is related to  $\rho$  by the expression  $\sigma = 1/(1 - \rho)$ .

The consumers budget constraint can be expressed as follows

$$I = P_Y Y + \sum_{i \in N} P_i X_i \quad (10)$$

where  $P_Y$  is a price index for all other goods. Maximizing (8) subject to (9) and (10), one obtains the implicit demand function for each differentiated product

$$X_i = \left[ \frac{\omega I}{P_i \Gamma^\rho} \right]^{\frac{1}{1-\rho}} \quad 0 < \rho \leq 1 \quad \forall i \in N \quad (11)$$

Equation (9) defines the demand for good  $i$  in an implicit way, since  $X_i$  appears both on the left and the right hand side of equation (11). However, I am considering the quantity-setting game in this section, so I want to keep quantities rather than prices of the other goods in the demand function.

#### The firms' decision problem

The firms maximize profit

$$\Pi_i = (P_i - C_i) X_i - C F_i \quad \forall i \in N \quad (12)$$

I am looking for a Nash-Cournot equilibrium, and I assume that the firms have constant marginal costs. This equilibrium is characterized by the set of first order conditions

$$\frac{d\Pi_i}{dX_i} = \rho P_i (1 - \beta_i) - C_i = 0 \quad \forall i \in N \quad (13)$$

where I have introduced the auxillary variable

$$\beta_i \equiv \frac{X_i d\Gamma}{\Gamma dX_i} \quad (14)$$

Employing the Nash-Cournot equilibrium concept, it is straight forward to show that <sup>3</sup>

$$\beta_i = \frac{X_i^\rho}{\sum_{j \in N} X_j^\rho} \quad (15)$$

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<sup>3</sup>Performing this exercise, it is quite widely claimed that this involves some sort of miscalculation on behalf of the firms, since each firm seems to neglect the impact changes in its strategy choice has on the strategy choices of all other firms. This is the startingpoint of the "conjectural variation" approach. However, according to Johansen (1982) the non-cooperative Nash-equilibrium concept involves no miscalculation on behalf of the firms. Johansen explains how the set of first order conditions arises because the firms are highly rational and have full information. The firms use the first order conditions in order to determine the optimal strategy choices for all firms, i.e. the set of first order conditions identifies the point in the (joint) strategy space, where no firm has any incentive to *unilaterally* deviate. This interpretation of the Cournot equilibrium concept was also identified by Schumpeter (1954) who wrote: "It is not true that (Cournot's) duopolist are supposed to act on an assumption ... that each (firm) takes the quantity offered by the other as constant .... No such assumption is implied. All that is required is that each chooses this particular method in order to find out *how the other will react* ..." (p.980).

which can be interpreted as a marketshare-indicator. From equations (4) and (13) it is straight forward to show that the mark-up factor can be expressed

$$M_i = \frac{1}{\rho(1 - \beta_i)} \quad (16)$$

From this expression I can see that there are two sources to a positive profit margin. Firstly, since the firms products are inhomogenous, the firms can exploit their monopolistic situation. Notice that as we approach the case of identical products ( $\rho$  approaches unity), this source or pure profit will vanish. Secondly, since the firms compete in quantities there will be a positive margin since each firm can influence their terms of trade on their marginal, but finite, market-share.

The market equilibrium is completely characterized by the equations (3), (11), (15) and (16).

### 3.2 The impact of tax-changes on market equilibrium

In this section I will assume that all firms have the same marginal costs. The only differences between the firms is whether they are located in the home-country or in the rest of the world, and that they produce different products. I will in this section investigate the impact of the introduction of a tax (or subsidy) on the domestic firms, starting from *laissez faire* (no taxes).

In the Appendix A I have shown how to derive reduced form expressions for the changes of the market equilibrium when an export-tax is introduced. We need the following expressions

$$\frac{1}{P_A} \frac{dP_A}{dr} = \frac{(n-1)(1-\rho) + \rho\nu}{n-1-\rho(n-2)} \quad (17)$$

where I have introduced the subscript  $A$  in order to refer to changes in the variables for firms located in the home country. I have also introduced the auxillary variables  $\nu$ , where  $\nu \equiv n_A/n$ ;  $n_A$  is the number of domestic firms, and  $n$  is the total number of firms in the industry.

From equation (17) we can analyse the tax-incidence question: The tax is to a larger extent passed over to the consumers the less homogenous the products are. Furthermore, we find that the tax is passed over to the consumer to a larger extent the larger is the relative number of domestic producers.

In Appendix A, I have also identified the impact of taxation on the sales by domestic firms:

$$\frac{1}{X_A} \frac{dX_A}{dr} = -\frac{n-1-\rho\nu(n-2)}{n-1-\rho(n-2)} \quad (18)$$

From this equation I can see that the the loss of sales for the domestic producers will be larger, the higher is the degree of homogeneity between the products in the industry. The larger is the relative number of domestic firms in the industry, the less will the taxed firms lose their sales.

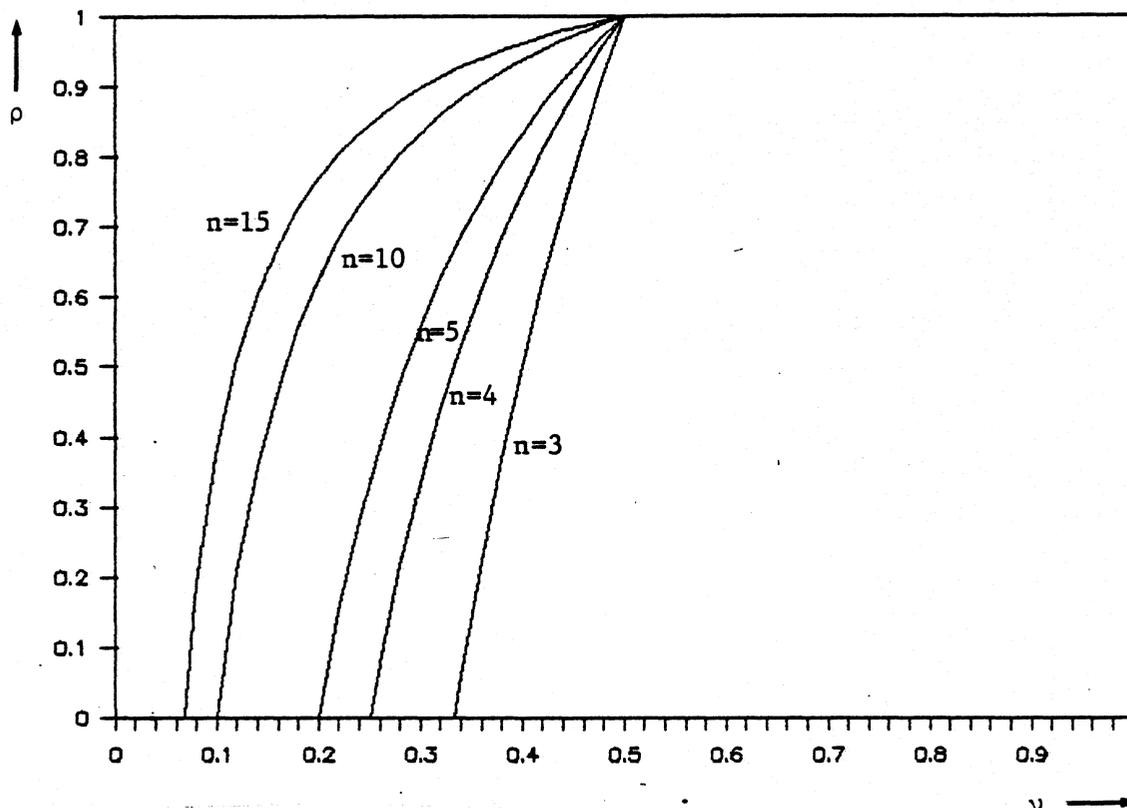


Figure 1: The  $T(\nu, \rho; n)=0$  lines for different values of  $n$

### 3.3 Taxing or subsidizing?

We have now the results we need in order to address the primary question posed in the title of this paper: Should a welfare-maximizing government tax or subsidize an exporting industry? By combining equations (4), (16), (17) and (18), one can show that this depends on whether

$$T(\nu, \rho; n) = (\nu - 1/n)(n - 1) - \rho\nu(n - 3 + 2/n) \quad (19)$$

is positive or negative (remember that  $0 < \nu, \rho \leq 1, n \geq 3$ ). This function has been plotted in figure 1. From figure 1 we can draw several rules of thumb characterizing the answer to the principal problem we set out to answer. Notice that there is a case for an export subsidy north-west of the " $T = 0$ "-line in the figure, and a case for an export tax in the south-eastern part of the figure.

**Proposition 1** *The higher the relative number of domestic firms in the industry, the more is an optimal policy pulled in the direction of an export tax.*

**Remark 1** The explanation for this result follows from equations (17) and (18). There are two effects to take into consideration: Firstly, a tax has a negative effect on sales for domestic firms. This effect will be smaller the larger is the number of taxed firms, i.e. the number of home firms. Secondly, the tax will to a larger extent be passed over to the consumers, i.e. improve the terms of trade, the larger is the number of home firms.

**Remark 2** This result is in accordance with Dixit (1984), who showed that in the case of homogenous products, the case for an export subsidy depends on the number of home firms.

**Proposition 2** *In industries where there is a high elasticity of substitution (relatively homogenous products), there is a case for an export subsidy and vice versa.*

**Remark 3** The driving force behind this proposition is the profit-shifting motive: An export subsidy raises domestic revenue by transferring industry profit to the domestic firms. In the case of duopoly this argument has been analysed by Brander-Spencer (1985) and Eaton-Grossman (1986).

**Proposition 3** *The more firms in the industry, the more is an optimal policy pulled in the direction of an export tax.*

**Remark 4** The intuition behind this result goes back to the simple fact that the more firms in the industry the less profit (cfr. eq. (16)). Hence, the more firms in the industry the less important is the profit-shifting motive referred to above.

## 4 The case of price-setting firms

### 4.1 Nash-Bertrand market equilibrium

In this case I want to have the competitors' prices in the demand function facing each firm, rather than quantities (cfr. eq. (11)). It is well known that in this case the demand function can be expressed

$$X_i = \frac{\omega IP_i^{-\sigma}}{\sum_{j \in N} P_j^{1-\sigma}} \quad (20)$$

where  $\sigma = \frac{1}{1-\rho}$  (, and  $\sigma > 1$ ).

To identify the Nash-Bertrand equilibrium in this case we have to find the set of prices which solves the set of first order conditions

$$\frac{d\Pi_i}{dP_i} = X_i + (P_i - C_i) \frac{dX_i}{dP_i} = 0 \quad \forall i \in N \quad (21)$$

Following the same procedure as in section 3 it is straight forward to show that

$$M_i = \frac{\sigma - (\sigma - 1)\kappa_i}{(\sigma - 1)(1 - \kappa_i)} \quad (22)$$

where I have introduced

$$\kappa_i \equiv \frac{P_i^{1-\sigma}}{\sum_{j \in N} P_j^{1-\sigma}} \quad (23)$$

### 4.2 A case for taxing!

In Appendix B I have shown how to derive the demand elasticity  $\varepsilon_{i,r}$  (defined by equation (6)) in the case where all firms have the same marginal costs. I have furthermore shown that in this case  $\varepsilon_r < \varepsilon$ , hence I have established the following proposition:

**Proposition 4** *In the case where firms compete in prices there is an unambiguous case for an export tax.*

## 5 Shortcomings of this analysis

In this paper I have given some rules of thumb for whether a welfare maximizing government will tax or subsidize an exporting industry. In the case of quantity competition, the answer depends on the degree of homogeneity of the products in the industry, the relative number of home firms and the absolute number of firms in the industry. In the case of price competition, there is an unambiguous case for taxing.

However, there are several shortcomings of the present analysis. Firstly, one would like to investigate the case of endogenous number of firms. However, an analytical treatment of this case usually requires a large number of firms, so equilibrium profit can be set approximately equal to zero. This eliminates the profit-shifting motive for subsidizing, and the problem becomes trivial. To treat the whole problem, one probably has to use numerical analysis, along the lines of e.g. Venables and Smith (1987).

Secondly, this analysis has not incorporated the possibility of home consumption. This is considered outside the scope of the present paper (see Eaton and Grossman (1986) for a discussion of the significance of home consumption).

Thirdly, one might argue that the partial equilibrium approach is a weak spot. In order to elucidate this point, let us first stick to the case where there is no other distortions in the economy and lump-sum taxes are available. In this case the limitations of the present analysis is not important, since I have only addressed the qualitative question of whether to tax or subsidize. The new effects from a general equilibrium analysis should be fairly easy to predict in this case. The main effect would be that factor prices would be influenced by tax-changes, i.e. that one cannot assume that unit costs will remain constant as I did above. However, this effect will only influence the magnitude of the optimal tax or subsidy, and not the direction of the tax-reform. The limitations of the partial equilibrium approach becomes more important when one takes into consideration the distortions in the rest of the economy. Take the case considered by Dixit and Grossman (1984), where there are more imperfectly competitive industries in the economy. Their point is that a subsidy to one industry will draw resources out of the other, which, however might also be inclined to receive a subsidy according to the partial equilibrium analysis above. In this case one has to consider the quantitative differences between the sectors in question.<sup>4</sup> Another pervasive kind of distortion in actual economies is taxes on factors of production. A satisfactory analysis of the problem would involve shadow-prices of the factors of production faced by the industry. This is probably analytically intractable in most cases and indicate the need for simulation approaches even to obtain only qualitative results.

Lastly; I will stick to a widespread tradition in this field; it is a quite common practice to withdraw from the conclusions carefully obtained in the very last moment. Economists seem to be strongly attracted to the non-intervention position in this field, despite the numerous results obtained indicating that it might be welfare improving for a country to intervene in imperfectly competitive trading industries. The justification

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<sup>4</sup>Notice that the government might be in the fortunate position that it would be welfare improving to tax some industries and subsidize others. In such a situation a tax-reform in one industry could also have indirect beneficial effects in other industries.

is that intervention by one government will be at the expense of the producers or/and consumers in other countries. The total outcome of unilateral welfare optimization by each individual country would most likely be inferior to the cooperative welfare optimum.<sup>5</sup> Hence one could argue that the primary achievement of the kind of analysis outlined above is to show the need for international agreements in order to offset the incentives I have identified to introduce unilateral trade restrictions.

## References

- [1] Anderson, S.P., A. de Palma and J.-F. Thisse (1987): "Demand for differentiated products, discrete choice models, and the address approach". *CORE Discussion Paper, Universite Catholique de Louvain, Louvain-la-Neuve, Belgium*.
- [2] Brander, J.A. and B.J. Spencer (1985): "Export subsidies and International Market Share Rivalry". *Journal of International Economics*
- [3] Dixit, A.K. (1984): "Internatinal trade policy for oligopolistic industries". *Economic Journal Supplement*
- [4] Dixit, A.K. and G.M. Grossman (1984) "Targeted export promotion with several oligopolistic industries" *Woodrow Wilson School Discussion Paper No. 71*
- [5] Dixit, A.K. and J. Stiglitz (1977): "Monopolistic competition and optimum product diversity". *American Economic Review*
- [6] Eaton, J. and G.M. Grossman (1986): "Optimal Trade Policy under Oligopoly". *Quartly Journal of Economics*
- [7] Johansen, L.(1982): "On the status of the non-cooperative Nash equilibrium concept". *Scandinavian Journal of Economics*
- [8] Schumpeter, J.A. (1954): *History of Economic Analysis*. Allen & Unwin (Publ.) Ltd.
- [9] Selten, R. (1975): "Re-examination of the perfectness concept for equilibrium points in extensive games". *International Journal of Game Theory*
- [10] Venables, A.J. and A. Smith (1987): "Trade and industrial policy under imperfect competition". *Economic Policy*
- [11] Spence, A.M. (1976): "Product selection, fixed costs and monopolistic competition". *Review of Economic Studies*

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<sup>5</sup>Notice that this proposition is not without ambiguity: Intervention by subsidising an exporting industry will reduce profit in the rest of the world, but it will benefit the consumers in the rest of the world. Since a subsidy might bring consumerprices closer to production costs in imperfectly competitive industries, overall welfare might be improved after intervention.

## Appendix A: Tax-changes in the quantity setting game

In this appendix I will derive the impact on equilibrium in the quantity-setting game, of the introduction of an export tax on the home producers. I assume in this case that the home government chooses its tax policy in the first stage of the game, and that the firms in this industry simultaneously chooses their level of production in the second stage of this game. I am looking for the direction of the government tax-policy, i.e. whether it should tax or subsidize the home-firms, in a perfect Nash equilibrium (cfr. Selten (1975)) in this two stage game.

Let us start by modifying the price setting rule to incorporate an *ad valorem* tax at rate  $\tau$ :

$$P_i = \frac{C_i(1 + \tau)}{\rho(1 - \beta_i)} \quad (24)$$

In order to obtain the final results, I will logarithmically differentiate the set of market equilibrium conditions, consisting of equations (11) and (24):

$$dx_i(1 - \rho) + dp_i + \rho d\tau = 0 \quad (25)$$

$$dp_i = \frac{d\tau}{1 + \tau} + \frac{d\beta_i}{1 - \beta_i} \quad (26)$$

where I have introduced the notational convention that lower case letters corresponds to the logarithm of the corresponding upper case variable. I will investigate the case of deviations from the no tax situation, assuming furthermore that all firms have the same marginal costs of production. In this case the only interesting distinction between the firms will be whether they belong to the set of home firms (denoted by  $N_A$ ), or foreign firms (the set denoted by  $N_B$ ). I will also use the notation that the subscript  $A$  refers to home firms, whereas the subscript  $B$  refers to foreign firms. From equation (26) one then obtains

$$dp_A = d\tau + \frac{d\beta_A}{1 - 1/n} \quad (27)$$

noticing that

$$\beta_i = 1/n \quad \forall i \in N \quad (28)$$

in the no tax, symmetric case.  $n$  is the number of firms in the industry. Furthermore

$$dp_B = \frac{d\beta_B}{1 - 1/n} \quad (29)$$

Using the definition of  $\beta_i$  (cfr. equation (15)) it is straight forward to show that

$$d\beta_A = \frac{\rho(1 - \nu)}{n} (dx_A - dx_B) \quad (30)$$

and

$$d\beta_B = \frac{\rho\nu}{n} (dx_B - dx_A). \quad (31)$$

where I have introduced

$$\nu \equiv \frac{n_A}{n} \quad (32)$$

From the definition of  $\Gamma$  (cfr. equation (9)) one obtains

$$d\gamma = \nu dx_A + (1 - \nu) dx_B \quad (33)$$

From the set of demand equations (cfr. eq. (11)), we obtain the two last equations:

$$dx_A(1 - \rho) + dp_A + \rho d\gamma = 0 \quad (34)$$

$$dx_B(1 - \rho) + dp_B + \rho d\gamma = 0 \quad (35)$$

The set of equations (27), (29), (30), (31), (33), (34) and (35) determines completely the set of unknowns  $dx_A, dx_B, dp_A, dp_B, d\beta_A, d\beta_B$  and  $d\gamma$ . After some manipulations one obtains the final results (when  $n \geq 3$ )

$$\frac{dx_A}{d\tau} = -\frac{n-1-\rho\nu(n-2)}{n-1-\rho(n-2)} \quad (36)$$

Moreover

$$\frac{dp_A}{d\tau} = \frac{(n-1)(1-\rho) + \rho\nu}{n-1-\rho(n-2)} \quad (37)$$

## Appendix B: Impact of tax changes in the price setting game

In this appendix, I will show that condition (7) is always fulfilled in the symmetric price setting game. By totally differentiating the (implicit) demand function defined by equation (20), it is straight forward to show that

$$\varepsilon_\tau = 1 + (\nu + \sigma - \nu\sigma) \left(1 - \frac{dp_B/d\tau}{dp_A/d\tau}\right) \quad (38)$$

From equations (4) and (22) one can see that

$$\varepsilon_i = \sigma(1 - \kappa_i) + \kappa_i \quad (39)$$

where

$$\kappa_i \equiv \frac{P_i^{1-\sigma}}{\sum_{j \in N} P_j^{1-\sigma}} \quad (40)$$

In the symmetric equilibrium  $\kappa_i = 1/n$ . Furthermore, by totally differentiating (40) one obtain

$$\frac{d\kappa_A}{d\tau} = 1/n(\sigma - 1)(1 - \nu) \left(\frac{dp_B}{d\tau} - \frac{dp_A}{d\tau}\right) \quad (41)$$

and

$$\frac{d\kappa_B}{d\tau} = 1/n(\sigma - 1)\nu \left(\frac{dp_A}{d\tau} - \frac{dp_B}{d\tau}\right) \quad (42)$$

From equation (22) it follows that

$$\frac{dm_i}{d\tau} = \frac{d\kappa_i}{(1 - 1/n)(\sigma(1 - 1/n) - 1/n)} \quad i \in A, B \quad (43)$$

Lastly, in the same way as (27) and (29)

$$\frac{dp_A}{dr} = 1 + \frac{dm_A}{dr} \quad (44)$$

and

$$\frac{dp_B}{dr} = \frac{dm_B}{dr} \quad (45)$$

Combining equations (41), (42), (43), (44) and (45), one can show that

$$\frac{dp_B/dr}{dp_A/dr} = \frac{\nu(\sigma - 1)}{(n - 1)\sigma - (\sigma - 1)(1 - \nu - 1/n)} \quad (46)$$

Inserting (38) and (39) into equation (7) and replacing  $\sigma$  by  $1/(1 - \rho)$  one obtain the following condition for a tax to be welfare improving

$$\frac{\rho(1 - 1/n)}{1 - \nu\rho} > 1 - \frac{\nu\rho}{n - 1 - \rho(1 - \nu - 1/n)} \quad (47)$$

which indeed is true for all permissible values for  $\rho$  and  $\nu$ .

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