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Abstract:
This article introduces the concept of *co-non-linearity*. Co-non-linearity is an example of a *common feature* in time series (Engle and Koziciki, 1993, J. Bus. Econ. Statist.) and an extension of the concept of *common nonlinear components* (Anderson and Vahid, 1998, J. Econometrics). If some time series follow a non-linear process but there exists a linear relationship between the levels of these series that removes the non-linearity, then this relationship is said to be a co-non-linear relationship. In this article I show how to determine the number of such co-non-linear relationships. Furthermore, I show how to formulate hypothesis tests on the co-non-linear relationships in a full maximum likelihood framework.

Keywords: Common features, non-linearity, reduced rank regression

JEL classification: C32, E43

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Sammendrag

Artikkelen introduserer begrepet *co-non-linerity* (”ko-ikke-linearitet”). Dette er en egenskap som kan klassifiseres som en *common feature* (”felles egenskap”), se Engle and Koziciki (1993, J. Bus. Econ. Statist.). Samtidig er det en videreføring av *common nonlinear components* (”felles ikke-lineære komponenter”), se Anderson and Vahid (1998, J. Econometrics).

En *common feature* innebærer at flere tidsserier har en felles egenskap. Den felles egenskapen er slik at det er mulig å ta en (eller flere) lineære sammenhenger av tidsseriene slik at den felles sammenhengen forsvinner. *Cointegration* (”kointegrasjon”) er et eksempel. Her følger hver tidsserie en stokastisk trend, men det kan likevel finnes minst en lineær sammenheng mellom tidsseriene som ikke følger en stokastisk trend. Denne sammenhengen sies å utgjøre en kointegrerende sammenheng.

Tidsserier kan også følge ikke-lineære prosesser. Samtidig kan det finnes lineære sammenhenger mellom tidsseriene slik at ikke-lineariteten forsvinner. Ledighetsraten og rentesatser kan være eksempler på ikke-lineære tidsserier. Samtidig kan økonomisk teori tilsi at det er en lineær sammenhenger mellom rentesatser. Terminstrukturmodellen i Cox (1985, Econometrica) innebærer en lineær sammenheng mellom rentepapirer med forskjellig løpetid. Tilsvarende innebærer CAPM-modellen en lineær sammenheng mellom avkastningen på et verdipapir, avkastningen en portefølje av verdipapirer og renten.

I denne artikkelen sjekker vi om det er *co-non-linearity* mellom innskuddsrenten, utlånsrenten og pengemarkedsrenten i Norge. Vi finner indikasjon på at det er to slike sammenhenger. Den ene innebærer et en-til-en forhold mellom pengemarkedsrenten og utlånsrenten på lang sikt. Det innebærer at hvis pengemarkedsrenten øker med ett prosentpoeng vil vi tro at utlånsrenten vil øke med ett prosentpoeng også. Den andre sammenhengen er mellom pengemarkedsrenten og innskuddsrenten. Denne sammenhengen innebærer at innskuddsrenten øker med 0,8 prosentpoeng når pengemarkedsrenten øker med ett prosentpoeng. Grunnen til at innskuddsrenten endres mindre enn pengemarkedsrenten kan kanskje forklares med at innskuddsrenten ikke kan være negativ, og at innskuddsrenten dermed nødvendigvis må falle mindre enn pengemarkedsrenten ved rentenedgang ved tilstrekkelig lavt rentenivå. Samlet sett innebærer de to sammenhengene vi finner at rentemarginen – definert som forskjellen mellom utlåns- og innskuddsrente – er lavere når rentenivået er lavt enn når rentenivået er høyt.

1 Introduction

Economic time series may share similar types of properties. Non-stationary variables may be cointegrated and stationary variables (or non-stationary variables differentiated to be stationary) may share common cycles. Furthermore, variables may follow processes with many breaks but there might still exist co-breaking relationships among these variables. Here we will consider variables that follow non-linear processes, but where the number of independent non-linear processes is less than the number of variables. Hence, there exist linear relationships among these variables that are not explained by these non-linear processes. Following the usual terminology in the common feature literature, I will denote such relationships between the levels of these variables as co-non-linear relationships.

Many variables seem to follow non-linear processes. For example, interest rates and unemployment rates do not seem to be mean reverting; but at the same time they are bounded so they can not follow deterministic or stochastic trends.[1] Hence, such processes can be considered as non-linear processes. However, two interest rates might follow the same non-linear process such that there exist a linear relationship between the two interest rates that is not described by a non-linear process. Economic theory might imply such linear relationships. For example, the term structure model by Cox et al. (1985) implies a linear relationship between interest rates of different maturities. Similarly, the capital asset pricing model implies a linear relationship between excess return on one asset and the excess return of a market portfolio of assets.

Co-non-linearity is an example of *common features*, a concept introduced by Engle and Kozicki (1993). A feature is defined as common if the feature is present in a group of series, and there exists a nonzero linear combination of the series that does not have the feature. *Cointegration* (see Engle and Granger, 1987) is an example of a common feature, as the individual time series follow a stochastic trend but there exists at least one linear combination of the time series that does not follow a stochastic trend. *Com-*

[1]However, treating them as following stochastic trends may be a good approximation in empirical work.

mon cycles (see Vahid and Engle, 1993), *common seasonality* (see Engle and Hylleberg, 1996) and *co-breaking* (see Hendry and Massmann, 2007) are other examples of common features.[2]

Anderson and Vahid (1998) suggest a test for *common nonlinear components* in multiple time series. They provide a generalized methods of moments test in terms of canonical correlation (i.e., reduced rank) between multiple time series. Similarly, Bierens (2000) considers nonlinear cotrending among series that follow nonlinear trends. He suggests a nonparametric test for nonlinear cotrending.

This paper builds on Anderson and Vahid (1998) and use reduced rank regression in order to determine the number of non-linear processes. However, instead of using generalized methods of moments, I use a maximum likelihood approach, as described in Johansen (1996). Anderson and Vahid (1998) show that the test statistic based on generalized methods of moments and maximum likelihood has the same asymptotic distribution.

This paper extends Anderson and Vahid (1998) by deriving the co-non-linear relationships between the levels of the time series. Furthermore, I show how to formulate hypothesis tests on the co-non-linear relationships. The test is formulated in terms of a fully specified co-non-linear space, but can be generalized to less restrictive assumptions.

The paper is organized as follows: The econometric method is described in Section 2. An empirical example — using Norwegian interest rates — illustrates the method in Section 3. Section 4 concludes and points out directions for future work.

2 Econometric method

This Section formulate the system of equations and defines the co-non-linear vectors. I show how to test for restrictions on these co-non-linear vectors and suggest how to approximate the non-linear part of the system. I also show how the approach can

[2]See also Anderson et al. (2006) and Urga (2007) for more overview, or special issues of *Journal of Econometrics 132(1)* in 2006 and *Journal of Business & Economic Statistics 25(1)* in 2007.

be modified if the time series are non-stationary but cointegrated. The estimation involves reduced rank regression, see Anderson (1951) and Johansen (1996).

2.1 Identifying the co-non-linear space

Consider the non-linear system

$$Y_t = \Pi X_t + \sum_{i=1}^{\ell} \Lambda_i Y_{t-i} + \Phi D_t + \varepsilon_t, \tag{1}$$

where Y_t is a vector of n variables; X_t is a vector of m variables that contain the non-linear components; and D_t is a vector of n_D deterministic variables. The non-linear components in X_t can be non-linear functions of the variables in Y. Furthermore, Π, Λ_i $(i = 1, \ldots, \ell)$ and Φ are coefficient matrixes (of dimension $n \times m$, $n \times n$ and $n \times n_D$, respectively). The errors are assumed to be Gaussian white noise ($\varepsilon_t \sim N.i.i.d.\,(0, \Omega)$).

In order to investigate reduced rank for the non-linear part, let $\Pi = \xi\eta'$ where ξ is of dimension $n \times s$ and η is of dimension $m \times s$ with $0 \leq s \leq min\{n, m\}$.[3] The term $\eta' X_t$ expresses the *common nonlinear components*, see Anderson and Vahid (1998).[4]

We may re-write (1) in difference form with one level-lag (also known as 'error-correction form' in the cointegration literature) to make it easier to identify and restrict the co-non-linear space.

$$\Delta Y_t = \xi\eta' X_t + \Psi Y_{t-1} + \sum_{i=1}^{\ell-1} \Gamma_i \Delta Y_{t-i} + \Phi D_t + \varepsilon_t, \tag{2}$$

where $\Psi = \sum_{i=1}^{\ell} \Lambda_i - I_n$ and $\Gamma_i = \sum_{j=i+1}^{\ell} \Lambda_j$. Imposing the reduced rank on (2) implies imposing $(n - s)\,(m - s)$ independent coefficient restrictions. The likelihood ratio statistic for the reduced rank is χ^2-distributed with $(n - s)\,(m - s)$ degrees of freedom when the variables are stationary, see Anderson (1951). Also when Y_t and X_t

[3]Note that if $m < n$ then Π will always have reduced rank. This will imply — as shown below — that there are co-non-linear relationships between the series.

[4]The coefficient matrices ξ and η are not unique. If η is post-multiplied with an $s \times s$ matrix of full rank and ξ is post-multiplied with the inverse of the same matrix, the product of these two new matrices will yield the same Π as with the original ξ and η.

are $I(1)$ variables — i.e., stationary after differencing them once — this statistic is χ^2-distributed when no restrictions are imposed on Ψ, see Section 2.4.

The non-linear term in (2) can be removed by pre-multiplying with $\xi'_\perp$, where $\xi_\perp$ is the orthogonal complement to ξ and $\xi'_\perp$ is the transposed of the orthogonal complement. This matrix has dimension $(n-s) \times n$ and has the property that $(\xi, \xi_\perp)$ has full rank and $\xi'_\perp \xi = 0$. (For mathematical convenience, I define $\xi_\perp = 0$ if ξ has full rank n and that $\xi_\perp$ is any full rank $n \times n$ matrix if ξ has rank 0.) Pre-multiplying with $\xi'_\perp$ yields

$$\xi'_\perp \Delta Y_t = \Psi^* Y_{t-1} + \sum_{i=1}^{\ell-1} \Gamma_i^* \Delta Y_{t-i} + \Phi^* D_t + \varepsilon_t^*, \tag{3}$$

where $\Psi^* = \xi'_\perp \Psi$, $\Gamma_i^* = \xi'_\perp \Gamma_i$, $\Phi^* = \xi'_\perp \Phi$ and $\varepsilon_t^* = \xi'_\perp \varepsilon_t$.

The coefficient matrix $\xi_\perp$ is not uniquely identified as the matrix resulting from pre-multiplying $(\xi_\perp)'$ with any $(n-s) \times (n-s)$ matrix of full rank will also be the orthogonal complement to ξ. Only the space spanned by this coefficient matrix is uniquely identified.[5] Hence, it may be convenient to normalize $\xi_\perp$ by letting

$$c'_\xi \xi^o_\perp = I_{n-s}, \tag{4}$$

where I use the top-script 'o' to indicate that the coefficient matrix is normalized; and c_ξ is an $n \times (n-s)$ matrix of identifying restrictions. One natural choice of c_ξ could be $c_\xi = \left(I_{n-s}, 0_{(n-s)\times s}\right)'$ as this will normalize $\xi_\perp$ to $\xi_\perp = (I_{n-s}, \varphi)'$: then (3) can be written as $\Delta Y_{1,t} = -\varphi \Delta Y_{2,t} + \Psi^* Y_{t-1} + \sum_{i=1}^{\ell-1} \Gamma_i^* \Delta Y_{t-i} + \Phi^* D_t + \varepsilon_t$, where $Y_{1,t}$ are the first $n-s$ variables in Y_t and $Y_{2,t}$ are the remaining s variables in Y_t.

Proposition 2.1 *Under the assumption that $c'_\xi \xi_\perp$ has full rank, the normalization in* (4) *implies*

$$\xi^o_\perp = \xi_\perp \left(c'_\xi \xi_\perp\right)^{-1} \tag{5}$$

Proof. Pre-multiplying (5) with c'_ξ yields (4) if $c'_\xi \xi_\perp$ has full rank. ■

[5] Two matrices, say A and B, are said to span the same space if all vectors in A can be constructed as linear combinations of the vectors in B and all vectors in B can be constructed as linear combinations of the vectors in A.

The $(n-s)\times n$ coefficient matrix Ψ^* in (3) describes the co-non-linear space. It may be convenient to normalize the co-non-linear space. In order to do so we can decompose Ψ^* into two matrices by the equality $\Psi^* = \zeta v'$, where ζ is $(n-s)\times(n-s)$ and v is $n \times (n-s)$. Similarly, we use Ψ^{*o} to indicate that this coefficient matrix is normalized when $\xi_\perp$ is normalized (i.e., $\Psi^{*o} = (\xi_\perp^o)'\Psi$). Now v can be normalized by

$$c_v' v^o = I_{n-s}, \tag{6}$$

where c_v is an $n\times(n-s)$ matrix of known coefficients. (Also here a natural choice could be $c_v = \left(I_{n-s}, 0_{(n-s)\times s}\right)$.) The vectors in v^o express the *co-non-linear* relationships among the variables in Y.

Below (Corollary 2.1) I will show that v is independent of the normalization of $\xi_\perp$ in (4). This is an important results, as if v had not been independent of (4) it would be difficult to give it a meaningful interpretation. In contrast, the results in Corollary 2.2 show that ζ depends on the normalization in both (4) and (6) and — hence — it is more difficult to give this coefficient matrix a meaningful interpretation.

Proposition 2.2 *Under the assumption that $c_v'v$ has full rank, the normalization in* (6) *implies*

$$v^o = \Psi^{*o\prime}\left(c_v'\Psi^{*o\prime}\right)^{-1} \tag{7}$$

Proof. Pre-multiplying (7) with c_v' yields (6) if $c_v'v$ has full rank. ■

Corollary 2.1 *v^o is independent of the normalization in* (4).

Proof. Inserting for $\Psi^{*o\prime}$ in (7) yields

$$\begin{aligned} v^o &= \Psi'\xi_\perp^o\left(c_v'\Psi^{*\prime}\xi_\perp^o\right)^{-1} \\ &= \Psi'\xi_\perp\left(c_\xi'\xi_\perp\right)^{-1}\left(c_v'\Psi^{*\prime}\xi_\perp\left(c_\xi'\xi_\perp\right)^{-1}\right)^{-1} \\ &= \Psi'\xi_\perp\left(c_\xi'\xi_\perp\right)^{-1}\left(c_\xi'\xi_\perp\right)\left(c_v'\Psi^{*\prime}\xi_\perp\right)^{-1} \\ &= \Psi'\xi_\perp\left(c_v'\Psi^{*\prime}\xi_\perp\right)^{-1}. \end{aligned}$$

The equality $\Psi'\xi^o_\perp \left(c'_v\Psi^{*\prime}\xi^o_\perp\right)^{-1} = \Psi'\xi_\perp \left(c'_v\Psi^{*\prime}\xi_\perp\right)^{-1}$ shows that the expression in (4) is independent of the chosen normalization on $\xi_\perp$ as long as $c'_\xi\xi_\perp$ has full rank. ■

Proposition 2.3 *Under the assumption that $c'_\xi\xi_\perp$ and $c'_v v$ have full rank, the normalization in* (4) *and* (6) *implies*

$$\zeta^o = \Psi^{*o}c_v \tag{8}$$

Proof. Postmultiplying (8) with v^o yields $\zeta^o v^{o\prime} = \Psi^{*o}$. ■

Corollary 2.2 *ζ^o depends on the normalization in both* (4) *and* (6).

Proof. From the equality $\Psi^{*o} = \zeta^o v^{o\prime} = \zeta^o \left(\Psi^{*o}c_v\right)^{-1}\Psi^{*o}$ it follows that $\zeta^o = \Psi^{*o}c_v$. It implies from this expression that ζ^o depends on the normalization of v^o (through c_v) and the normalization of $\xi^o_\perp$ (through Ψ^{*o}). ■

Imposing the decomposition of Ψ^* in (3) yields

$$\left(\xi_\perp\right)'\Delta Y_t = \zeta v' Y_{t-1} + \sum_{i=1}^{\ell-1}\Gamma^*_i\Delta Y_{t-i} + \Phi^* D_t + \varepsilon_t, \tag{9}$$

where v is uniquely defined given (6) and ζ is uniquely defined given both (4) and (6).

Sometimes it may be informative to include some or all of the deterministic variables in the co-non-linear relationships (as also will be shown in Section 3). Let $\Phi^* D_t = \Phi^*_0 D_{0,t} + \Phi^*_1 D_{1,t}$ where $D_{0,t}$ are the deterministic variables included in the co-non-linear relationship. Equation (9) can then be reformulated as

$$\left(\xi_\perp\right)'\Delta Y_t = \zeta v^{e\prime} Y^e_{t-1} + \sum_{i=1}^{\ell-1}\Gamma^*_i\Delta Y_{t-i} + \Phi^*_1 D_{1,t} + \varepsilon_t, \tag{10}$$

where $Y^e_{t-1} = \left(Y'_{t-1}, D'_{0,t}\right)'$ (with the top-script 'e' used for extended). For example, if a constant is the only deterministic variable included in the system, this constant can be included in the co-non-linear relationships if the level of these relationships are interesting to identify. If both a constant and a trend are included in the system, the trend can be included in the co-non-linear relationships so that the drift of these relationships is identified. (The constant, however, will in this case not have a direct interpretation, as it will be a function of both the level of the co-non-linear relationships and the drift in the variables.) Note that moving some of the deterministic variables into the co-non-linear relationships does not involve any restriction on the system since $\zeta v^{e\prime}$ has full rank.

2.2 Hypothesis restrictions on the co-non-linear space

For given estimates of $\xi_\perp$ restrictions on the co-non-linear space v can be tested by the formulation in (9) or (10) with techniques developed for testing the cointegrating space in the cointegration literature, see e.g., Johansen (1996, Chap. 7) and Boswijk and Doornik (2004). Below I will consider how to perform tests on the co-non-linear space in a full information maximum likelihood setting (i.e., without conditioning on any estimated coefficient matrix).

2.2.1 Testing for a fully specified co-non-linear space

Here we only consider restrictions of the form where v is fully specified, i.e. $v = H$, where H is an $n \times (n-s)$ matrix of full rank where all elements are known. By using the property $I_n = H\left(H'H\right)^{-1}H' + H_\perp\left(H'_\perp H_\perp\right)^{-1}H'_\perp$ equation (2) can be written as

$$\Delta Y_t = \xi\eta' X_t + \Psi_H\left[H'Y_{t-1}\right] + \Psi_{H_\perp}\left[H'_\perp Y_{t-1}\right] + \sum_{i=1}^{\ell-1}\Gamma_i \Delta Y_{t-i} + \Phi D_t + \varepsilon, \qquad (11)$$

where $\Psi_H = \Psi H\left(H'H\right)^{-1}$ and $\Psi_{H_\perp} = \Psi H_\perp\left(H'_\perp H_\perp\right)^{-1}$. If the hypothesis is correct then ξ and $\Psi_{H_\perp}$ span the same space and the term $\Psi_{H_\perp} H'_\perp Y_{t-1}$ can be included in the reduced rank part together with X_t:

$$\Delta Y_t = \xi\left(\eta' X_t + \kappa\left[H'_\perp Y_{t-1}\right]\right) + \Psi^*_H\left[H'Y_{t-1}\right] + \sum_{i=1}^{\ell-1}\Gamma_i \Delta Y_{t-i} + \Phi D_t + \varepsilon, \qquad (12)$$

where κ is an $s \times s$ matrix of coefficients. Pre-multiplying (12) with $(\xi_\perp)'$ yields (9) with $(\xi_\perp)'\Psi^*_H = \zeta$ and $H = v$. Hence, the system in (12) is equal to the system in (2) with $v = H$ imposed. The restrictions on the co-non-linear relationship can be tested with a likelihood ratio test statistic which under the null hypothesis is χ^2-distributed with $(n-s)s$ degrees of freedom (when X_t is known).

2.2.2 Testing for a partly restricted co-non-linear space

The approach for testing restriction on v above can be extended to situations where some of the elements in H are unknown. The restriction that we consider is $v = H(\phi)$ where ϕ is a k-dimensional vector of the unknown elements in H. We assume that $H(\phi)$ is formulated such that the vector ϕ is identifiable.[6]

The hypothesis can be tested with a likelihood ratio test where the system under the restriction is estimated with (11) with ϕ that is maximizing the likelihood of this system. This implies that the system in (11) must be estimated with some maximization technique. Under the null hypothesis the likelihood ratio test statistic is χ^2-distributed with $(n-s)s-k$ degrees of freedom.

2.3 Approximating non-linear relationships

The non-linearity can be estimated directly, or an approximation can be applied. Here I choose the latter. One advantage for choosing an approximation is that the approximation might work well for different non-linearities, and I do not need to identify the type of the non-linearity. Another advantage is that estimation becomes easier: for many types of non-linearities the estimation will involve some kind of optimization techniques which can be rather complicated in a large system of equations.

The approximation I use is based on the theory on smooth transition regression (STR) models, see e.g., Teräsvirta et al. (2010). However, I will also mention some alternatives.

2.3.1 X as a non-linear function of the variables in Y

Here I will consider a generalization of a STR model to a vector of variables. The specification of the non-linearity is closely related to logistic vector STR model suggested

[6] Identifiability implies that the order and rank condition must be satisfied for $H(\phi)$. If not, the degrees of freedom in the χ^2-distribution must be adjusted.

by Rothman et al. (2001) and Anderson and Vahid (1998). An important difference, however, is that Rothman et al. (2001) also consider non-linearity in the short run dynamics, whereas I consider if there are common shifts in the mean of the variables and the adjustment speed towards those levels.[7]

Consider the following expression of the s common non-linear components;

$$\eta' X_t = G(\gamma, c; z_t)\left[\Theta_0 + \Theta_1 Y_{t-1}\right], \tag{13}$$

where Θ us an $s \times n$ coefficient matrix, and

$$G(\gamma, c; z_t) = \text{diag}\left\{g_1(\gamma_1, c_1; z_{1,t}), \ldots, g_s(\gamma_s, c_s; z_{s,t})\right\},$$

and where $g_i(\gamma_i, c_i; z_{i,t})$ is a continuous bounded function between 0 and 1 with $z_{i,t}$ as the transition variable. If Ψ has full rank — i.e., the variables are stationary — a non-zero value of (any of the s element in) Θ_0 will imply level-shift of one or more of the variables in the system, whereas a non-zero value of Θ_1 implies a shift in the speed of adjustment towards those mean levels.

Here I will assume that the transition variables are linear functions of the variables in Y_{t-1}. The logistic STR model of first order (LSTR1) this transition function is given by

$$g_i(\gamma_i, c_i; z_{i,t}) = \frac{1}{1 + e^{-\gamma_i (z_{i,t} - c_i)}}, \; \gamma_i > 0.$$

The test of s common linear components is $H_0 : (\gamma_{s+1}, \ldots, \gamma_n) = 0$ where n common non-linear components is the alternative hypothesis (i.e., Π has full rank). This involves $n - s$ parameter restrictions. It is shown for single equation models that this test does not have a standard asymptotic χ^2-distribution under the null hypothesis, see e.g. Davies (1977, 1987, 2002). Hence, Saikkonen and Luukkonen (1988) suggest to circumvent this problem by testing for linearity by using a Taylor-approximation. In the system I consider, the first order Taylor approximation for the auxiliary regression

[7] Rothman et al. (2001) also allows for mean-level shifts and shifts in the adjustment speed, but does not consider that such shifts can be common for the differen variables or cointegrating vectors.

gives the following formulation of the non-linear part

$$\eta' X_t = \Theta^* \text{diag}\{z_{1,t}, \ldots, z_{s,t}\} Y_{t-1} + l(Y_{t-1}), \tag{14}$$

where $l(Y_{t-1})$ is an s dimensional vector where each element is a linear function of the variables in Y_{t-1} (plus a term picking up the approximation error). To apply this approximation for testing the number of common non-linear components we replace $\eta' X_t$ in (2) with $\Theta^* \text{diag}\{z_{1,t}, \ldots, z_{s,t}\} Y_{t-1}$ (as the linear part given by $l(Y_{t-1})$ only affects the estimates of Ψ and the intercept in ΦD_t). This is the same as including all cross products between the variables in Y_{t-1} (including their square) in X_t.

The formulation in (14) assumes that we know which variables are the transitory variables. An alternative is that the transitory variables are unknown linear functions of the variables in Y_{t-1} — i.e., $z_{i,t} = a_i' Y_{t-1}$ — where the parameters in the vector a_i are unknown.[8] For the auxiliary regression we have

$$\eta' X_t = \sum_{i=1}^{s} \sum_{j=i}^{s} \Theta_{ij}^{**} y_{i,t-1} y_{j,t-1} + l(Y_{t-1}), \tag{15}$$

where Θ_{ij}^{*} are coefficient vectors (of dimension s).

However, there is one problem by using the first order approximations in (14) or (15) to test for non-linearity. As shown for the single equation equivalents, see Luukkonen et al. (1988), the test has no power in the case where a shift in the intercept is the only non-linear element (i.e., where $\Theta_0 \neq 0$ and $\Theta_1 = 0$). An alternative is to use the third order Taylor approximation of (13):

$$\begin{aligned} \eta' X_t \quad = \quad & \sum_{i=1}^{s} \sum_{j=i}^{s} \pi_{1ij} y_{i,t-1} y_{j,t-1} + \sum_{i=1}^{s} \sum_{j=1}^{s} \pi_{2ij} y_{i,t-1} y_{j,t-1}^2 \\ & + \sum_{i=1}^{s} \sum_{j=i}^{s} \pi_{2ij} y_{i,t-1} y_{j,t-1}^3 + l(Y_{t-1}), \end{aligned}$$

[8]Normally a is assumed to be an indicator function, i.e., that all elements are zero except one that is unity. For the test of linearity this restriction is not necessary. Here we only assume that not all elements are zero (because if they are zero (13) is linear). However, if the non-linearity shall be estimated, such restriction will be helpful as it reduces the number of non-linear parameters. In the present paper I do not estimate the actual non-linear part of the model.

where π_{kij} are vectors with s elements. This approximation involves that $2p^2s$ extra parameter must be estimated. However, as Luukkonen et al. (1988) show in the single equation version, of these extra parameters only π_{2ii} ($i = 1, \ldots, p$) are functions of the level parameters in Θ_0. Hence, if (15) is extended with the terms involving π_{2ii} ($i = 1, \ldots, p$), this new auxiliary model has explainable power for all types of parameter shifts in (13). They label this the 'economy version' since it does not involve all the terms of the third order Taylor approximation. In our system the corresponding approximation yields

$$\eta' X_t = \sum_{i=1}^{s} \sum_{j=i}^{s} \pi_{1ij} y_{i,t-1} y_{j,t-1} + \sum_{i=1}^{s} \pi_{2ii} y_{j,t-1}^3 + l(Y_{t-1}). \tag{16}$$

Based on (16) we include all the cross products between the variables (including the square) plus the variables in the third power in X. This can be expressed as $X_t = \left((\text{vech}\,(Y'_{t-1} \otimes Y_{t-1}))', (Y_{t-1} \odot Y_{t-1} \odot Y_{t-1})' \right)'$, where $\otimes$ is the *Kronecker product* operator, $\odot$ is the *Hadamard product* operator and vech is the operator for vectorizing the lower diagonal of a matrix.[9] For example, if there are two variables in Y, we have $X_t = \left(y_{1,t-1}^2,\ y_{1,t-1} y_{2,t-1}, y_{2,t-1}^2, y_{1,t-1}^3, y_{2,t-1}^3 \right)'$. With three variables in Y, we have $X_t = \left(y_{1,t-1}^2, y_{1,t-1} y_{2,t-1}, y_{1,t-1} y_{3,t-1}, y_{2,t-1}^2, y_{2,t-1} y_{3,t-1}, y_{3,t-1}^2, y_{1,t-1}^3, y_{2,t-1}^3, y_{3,t-1}^3 \right)'$. In general, X_t will contain $m = n(n+1)/2 + n$ elements.

2.3.2 Alternative approximations

Above I have considered an approximation of the non-linearity based on the smooth transition regression literature. Standard switching models — see e.g., Tong (1990) — are special cases of as special cases of the smooth regression for large γ_i's.

However, other non-linearities and approximations might be chosen. Instead of using one of the included variables as a transition variable — as above — time could be used as a transition variable. Markov-switching regression models — see e.g., Lind-

[9] In our case the *Kronecker product* $Y'_{t-1} \otimes Y_{t-1}$ gives an $n \times n$ matrix where the i'th column is given by $y_{i,t-1} Y_{t-1}$; and the *Hadamard product* $Y_{t-1} \odot Y_{t-1} \odot Y_{t-1}$ gives an vector of n elements where the i'th element is given by $y_{i,t-1}^3$. Since $Y'_{t-1} \otimes Y_{t-1}$ is symmetric we use the vech operator to vectorize it and thereby ignoring the upper diagonal elements.

gren (1978) — are special versions of standard switching models where the switching variables are given by unobservable discrete stochastic variables. These models are not special cases of the smooth transition regressions since the latter assumes the transition variable to be observable. Hence, applying a Markov-switching type of non-linearity would be an alternative to the one chosen here.

Furthermore, the non-linearity could be based on neural network modeling, see e.g. Gonzalez and Teräsvirta (2008) for a process with shifting mean. Anderson and Vahid (1998) also considers neural network model when testing for common non-linear components.

Non-linearity in the error process — such as ARCH (autoregressive conditional heteroskedasticity), see Engle (1982), and extensions such as GARCH (generalized ARCH), see Bollerslev (1986) and Taylor (1986) — does not fit into the framework considered in this paper. Nor does state space models — see e.g., Durbin and Koopman (2001) — where the parameters are time-varying.

2.4 Cointegration and co-non-linearity

Above it is assumed that the variables are — except for the non-linear part — stationary. However, if the variables are non-stationary $I(1)$ variables — i.e., stationary after differencing them once — we can follow the approach with minor modifications. Here I will consider two alternatives. One where the cointegrating rank is established first and another where the common nonlinear components and the co-non-linear relationships are identified before the cointegrating rank test is conducted.

2.4.1 Testing the cointegrating rank first

Here I consider testing reduced rank of Ψ in (2) when no rank-restriction is imposed on Π. The problem with this approach is how the non-linear term, X, affects the critical values. If the non-linearity is approximated as suggested in Section 2.3 where the variables are raised to the third power and some of these variables are following a drift, then this has the same effect as to including cubic trend in the vector autoregressive model. Critical values for reduced rank in such systems are not simulated.

The inclusion of variables in the third power in X is not done to allow the system to have a cubic trend (and — since this trend is not restricted to lie in the cointegrating space — the Granger represetation theorem for such a system would show that the variables in it would follow a quartic trend — i.e., a trend in the forth power — if the rank is not full); they are included to approximate non-linearities within the sample we are considering. However, the interpretation of this non-linearity is important for determining the critical values for the cointegrating rank test. This means that it is necessary to restrict the functional form of the non-linearities first so that appropriate critical values can be simulated.

2.4.2 Testing the cointegrating after the non-linearities are removed

An alternative to imposing additional restriction in the non-linear processes is to test the cointegrating rank after the non-linearities are removed. This implies testing the cointegrating rank of Ψ^* in (3) where the estimates of $\zeta_\perp$ (through the estimates of ζ) are used to reduce the system from a dimension of n down to a dimension of $n - s$. Since the system dimension is reduced, we can not necessary identify all the cointegrating vectors in (2), but only those that remain in the common nonlinear components of the system. This implies that we are not identifying the cointegrating vector that involves non-linear terms.

The system in (3) is a conditional (or partial) system where the processes of the $n - s$ variables in $(\zeta_\perp)' Y_t$ are conditioned on the s variables in $\zeta' Y_t$. The theory for the asymptotic distribution is discussed in Harbo et al. (1998) and approximated in Doornik (1998). Tables for critical values are reported in Doornik (2003).

Without imposing restrictions on the deterministic variables the asymptotic distribution of the rank test will involve nuisance parameters. The decomposition of the deterministic variables described in Section 2.1 must therefore be used.

The asymptotically distribution of the test statistic for determining s — i.e., the number of non-linear components — is χ^2-distributed also for $I(1)$ variables.[10] This test is conducted before testing for cointegration. Hence, the number of stochastic trends in the system is equal to n — i.e., the number of variables in Y_t — both under the null hypothesis (of reduced number of non-linear components) and the alternative hypothesis (of equal many non-linear components as n). Restriction that involve reduction in the number of non-linear components corresponds to restrictions on the cointegraed space for a given cointegration rank in standard cointegrated vector autoregressive models. These tests on the cointegrated space are known to be asymptotically χ^2-distributed.[11]

3 Co-non-linearity among Norwegian interest rates

We consider the three variables: $rDep$, the average deposit rate for household; $rLend$, the average lending rate for households; and r, the three months money market interest rate (given by NIBOR): $Y_t = (rDep_t, rLend_t, r_t)'$. The data series are Norwegian quarterly data, see Figure 1.

To approximate the non-linearity we use the 'economy version', see (16), i.e.,

$$\hat{X}_t = \left(rDep^2, (rDep * rLend), (rDep * r), rLend^2, (rLend * r), r^2, rDep^3, rLend^3, r^3\right)'_{t-1}$$

The estimation period is 1990q1-2009q4, i.e., 80 observations. Due to the use of two lags, I use data from 1989q3.

The rank reduction tests in Table 1 indicate a rank equal to 1 or 2. (The reduction

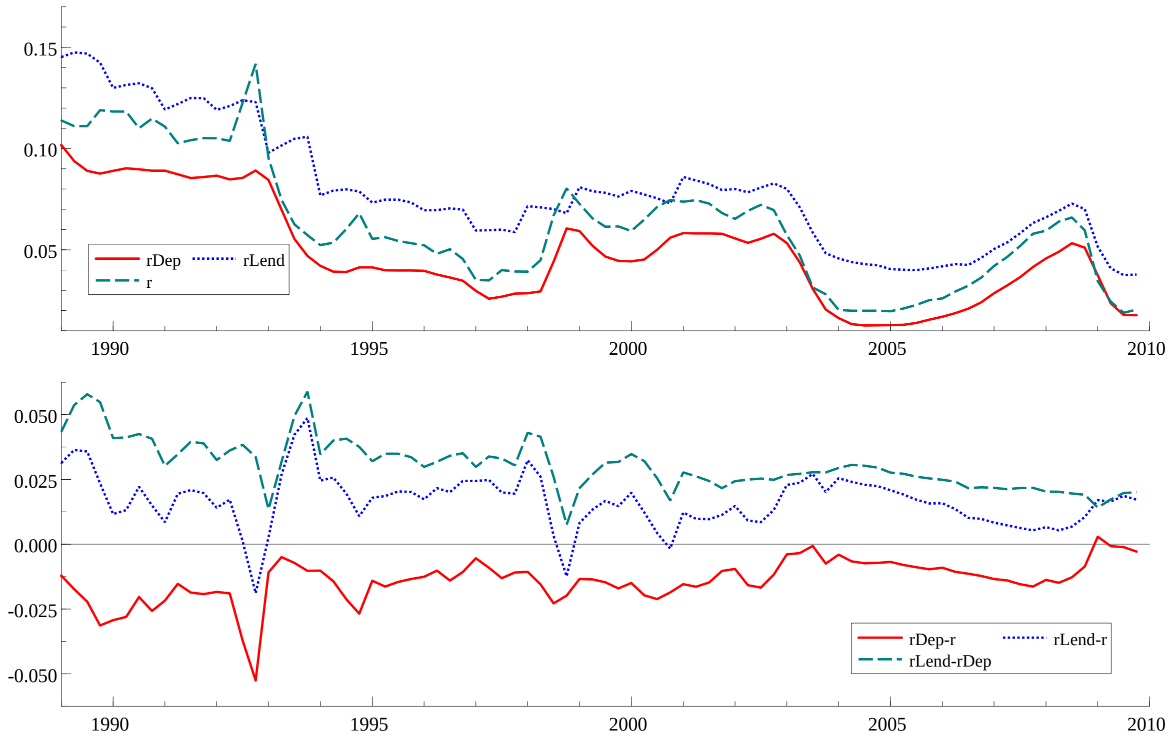
[10]For many types of non-linear processes (such as the LSTR1 in (13)) the X_t will have the same integration-order as Y_t, even though the approximations of that non-linear process might not have the same integration-order as Y_t.

[11]To see that restrictions on Π can be formulated as restrictions in the cointegrated space in a conditional cointegraded vector autoregressive model, equation (2) can be reformulated as

$$\Delta Y_t = \Psi\left(Y_{t-1} + \xi^* \eta' X_t\right) + \sum_{i=1}^{\ell-1} \Gamma_i \Delta Y_{t-i} + \Phi D_t + \varepsilon_t, \tag{17}$$

where $\xi^* = \Psi^{-1}\xi$. Restrictions on the rank of $\Pi = \xi\eta'$ can be formulated as restrictions on the rank of $\Psi^{-1}\Pi = \xi^*\eta'$. These are restrictions inside the cointegrating space given by $Y_{t-1} + \xi^*\eta' X_t$, and such restrictions are known to have a standard distribution.

Figure 1: Norwegian interest rates



rDep: average nominal interest rate on deposits in banks, *rLend*: average nominal interest rate on lending from banks to households, *r*: 3 months money market interest rate.

Table 1: Test of rank of Π (the number of independent non-linear processes)

rank	$-(T/2)\|\Omega\|$	LR (vs full rank)	LR (vs r+1)
3	1366.1		
2	1363.5	5.192 (7) [0.637]	5.192 (7) [0.637]
1	1354.1	23.989 (16) [0.090]	18.797 (9) [0.027]*
0	1332.3	67.577 (27) [0.000]**	43.588 (11) [0.000]**

Degrees of freedom in parentheses and p-values in square brackets. One asterisk denote significance at a 5 per cent level, two asterisks denote significance at a 10 per cent level.

from rank 3 to 2 is not rejected, the reduction to 1 is a borderline, and the reduction to a rank equal to zero — i.e., no non-linearity — is clearly rejected). I continue with a rank of 1.

In the case of a rank equal to 1 there is one common non-linear component

$$\hat{\zeta}' = \begin{pmatrix} 1 & 2.1208 & 2.5860 \end{pmatrix}$$

Table 2: Hypotheses tests

$H' =$	$-(T/2)\|\Omega\|$	LR (d.f.) [p-val]	$\hat{\phi}$
	1354.1		
$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$	1346.1	15.8911 (2) [0.0004]**	N.A.
$\begin{pmatrix} 1 & 0 & \phi \\ 0 & 1 & -1 \end{pmatrix}$	1353.7	0.7165 (1) [0.3973]	-0.79902
$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & \phi \end{pmatrix}$	1346.2	15.7387 (1) [0.0001]**	-0.93066
$\begin{pmatrix} 1 & 0 & \phi \\ 0 & 1 & \phi \end{pmatrix}$	1351.1	5.9813 (1) [0.0145]*	-0.82102

Notes: See Table 1.

and, hence, two common co-linear components

$$(\hat{\zeta}_{\perp})' = \begin{pmatrix} 1 & 0 & -0.38670 \\ 0 & 1 & -0.82031 \end{pmatrix}.$$

The normalizing of the co-non-linear relationships can be written as

$$\begin{aligned} \hat{v}^{e\prime} Y_t &= \begin{pmatrix} 1 & 0 & -0.79829 & 0.00209 \\ 0 & 1 & -0.95585 & -0.01702 \end{pmatrix} \begin{pmatrix} rDep_t \\ rLend_t \\ r_t \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} rDep_t - 0.79829 r_t + 0.00209 \\ rLend_t - 0.95585 r_t - 0.01702 \end{pmatrix} \end{aligned} \quad (18)$$

Next we test some restrictions on the co-non-linear space, v, see Table 2. In the first line of Table 2 we test if the difference between any two of the interest rates expresses a co-non-linear relationship. The restriction matrix H is formulated as a test of $rDep - r$ and $rLend - r$ jointly being two independent co-non-linear relationships. If they are, it follows that also $rDep - rLend$ is a co-non-linear relationship (as it is only a linear combination of tho two former). This gives $-(T/2)|\Omega| = 1346.1$ and a LR test value of $2(1354.1 - 1346.1) = 15.8911$. The test is χ^2-distributed with 2 degrees of freedom, and the hypothesis is clearly rejected. (The corresponding p-value is 0.0004).

In the last three lines in Table 2 we test if one of the interest rate difference expresses a co-non-linear relationship. We see that the test of if $rLend_t - r_t$ is a co-non-linear relationship is not rejected (p-value of 0.40), whereas the tests of $rDep_t - r_t$ and $rDep_t - rLend_t$ are both rejected.[12]

The estimated co-non-linear space with intercept included for the hypothesis that was not rejected in Table 2 is given by

$$\hat{v}^{e\prime} Y_t = \begin{pmatrix} rDep_t - 0.79902 r_t + 0.00213 \\ rLend_t - r_t - 0.01440 \end{pmatrix} \tag{19}$$

The coefficient for the intercept in the second co-non-linear vector implies that over time the lending rate is 1.44 percentage points higher than the money market rate.

The coefficient of about 0.8 for the money market interest rate in the first co-non-linear vector implies that the deposit interest rate increases by 0.8 percentage points when the money market rate increases by one percentage point.[13] The lack of a one-to-one relationship between these two interest rates might be due to the fact that the deposit interest rate can not be negative. Hence, when the money market interest rate decreases, the deposit rate might not be able to fall equally much. Should the money market interest rate fall to zero, the intercept in the first co-non-linear relationship implies a deposit rate of 0.2 per cent — implying a negative deposit margin for banks. However, the intercept is probably not significantly different from zero.[14]

Under the (non-rejected) restriction that $rLend_t - r_t$ is a co-non-linear relationship,

[12]The latter restriction could also be written as $H' = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & \phi \end{pmatrix}$ or $H' = \begin{pmatrix} 1 & 0 & \phi \\ 1 & -1 & 0 \end{pmatrix}$, as all these matrices are spanning the same space.

[13]Raknerud et al. (2011) also find this relationship between the money market rate and the deposit rate in Norway. However, they can not reject the hypothesis of a one-to-one relationship between the lending rate and the deposit rate. One reason for this can be that they are using a shorter data set (2001q2-2010q3).

[14]The hypothesis that this coefficient for the intercept is zero can be tested with (12) where Y_{t-1} is replaced by Y^e_{t-1}; D_t is replaced by $D_{1,t}$ (which in this case does not involve any series); and $H = \begin{pmatrix} 1 & 0 & \phi_1 & 0 \\ 0 & 1 & -1 & \phi_2 \end{pmatrix}$ (or any other H-matrix spanning the same space).

Table 3: Test of rank of Ψ^* (the number of remaining cointegrating vectors)

rank	$-(T/2)\lvert\Omega\rvert$	LR (vs full rank)	95% quantlie
2	860.77		
1	857.32	6.90 [0.3328]	12.28
0	845.88	29.79 [0.0138]*	25.57

Notes: See Table 1. P-values are based on Doornik (1998).

the estimated common co-linear component is

$$(\hat{\zeta}_\perp)' = \begin{pmatrix} 1 & 0 & -0.38956 \\ 0 & 1 & -0.89459 \end{pmatrix}.$$

We can test for remaining cointegrated vectors under this co-non-linear restriction by pre-multiplying the system with this estimated co-linear component matrix. The cointegrating rank tests are reported in Table 3. The results show that we can not reject the hypothesis of a rank equal to 1. However, a rank equal to 0 is rejected (at a 5 per cent significance level).

4 Conclusions

This article introduces the concept of co-non-linearity. The naming of the property follows the convention in the common feature literature. For example, as cointegration (CI(1,1)) involves linear relationships of integrated series such that the integrated process vanishes, co-non-linear relationships involve linear relationships of non-linear processes such that the non-linearity disappears.

In the common feature literature (or co-feature literature, see Ericsson, 1993) each feature can normally be studied from two different angels. The focus can be on the features that are in common, such as common stochastic trends or common non-linearities. Alternatively, the focus can be on the relationships that remove the common feature (i.e., the focus is on a co-feature), such as cointegration or co-non-linearity.

Co-non-linearity can be seen both as an alternative and as a complement to cointegration. A non-linear process can be used as an alternative to an integrated process.

At the sam time, non-linear processes and integrated processes can be combined in the same system, as this article shows.

In the present paper I have not modeled the non-linearity directly. This could be a natural extension. Then it would be possible to distinguish between non-linearity in the short run dynamics and non-linearity in the relationships between the levels of the series. If a linear relationship between two or more variables only involves non-linearity in the short run dynamics, we could choose to also define such a relationship as a co-non-linear relationship.

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