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The inequality effects of a dual income tax system

Abstract:
The overall inequality effects of a dual income tax (DIT) system, combining progressive taxation of labor income with proportional taxation of income from capital, are investigated. Simple examples show that correlations between distributions of wage and capital income, the degree of tax rate differentiation in the DIT, and reranking of tax-payers can be expected to complicate the analysis. We trace out what can be said definitively, obtaining sufficient conditions for unambiguous inequality reduction and identifying the nature of the implicit redistribution between labor and capital income which is involved, with the help of Norwegian income tax data.

Keywords: Personal income tax; dual income tax; redistributive effect

JEL classification: D31, D63, H31

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Sammendrag

Effekter på inntektsulikhet av av skattesystemer som kombinerer progressiv beskatning av lønn og flat beskatning av kapital (såkalte duale inntektsskattesystemer) diskuteres. Enkle eksempler viser at korrelasjoner mellom inntektsfordelinger for kapital og lønn, graden av differensiering i skattesatsene og omrangering av skattebetalere forventes å komplisere analysen. Vi redegjør for hva som kan sies med sikkerhet, presenterer tilstrekkelige betingelser for definitive ulikhetsreduksjoner og viser til den implisitte omfordelingen mellom arbeidsinntekt og kapitalinntekt som følger av et slikt system. De teoretiske analysene er supplert med empiriske illustrasjoner basert på norske inntektsdata.

1. Introduction

The “pure form” dual income tax (henceforth DIT) system combines progressive taxation of labor and transfer incomes with proportional taxation of income from capital at a level equal to the corporate income tax rate (Sørensen, 1994). The lower tax rate on capital income relative to that on labor income tends to undermine vertical equity, since income from capital tends to be concentrated in the upper income brackets. In addition, the DIT typically will cause reranking: wealthy recipients of labor incomes only can thus be worse off than modestly well-to-do recipients of capital income only.

Even though elements of the DIT have been introduced in several European countries (Genser and Reutter, 2007; Griffith, Hines and Sørensen, 2010), few have discussed how such tax systems influence overall income inequality. In this paper, we examine the vertical characteristics of a pure form DIT schedule. This paper was inspired by Kristjánsson’s (2010) study of the redistributive effect and progressivity of a DIT system in terms of Gini and concentration indices. Another analytical paper on DIT system redistributive effects has also recently appeared, see Calonge and Tejada (2011). Although Calonge and Tejada use redistribution indices to compare different tax cut policies, they refrain from considering inequality effects per se: “We do not investigate what conditions on the tax schedules are needed to reduce income inequality” (page 2). We return to these articles later in the paper.

The purpose of our paper is to trace out what can be said definitively about the inequality effects of a DIT system. There is not a great deal that can be said. In section 2 of the paper, we provide simple examples to show some of the complications involved, such as correlations between initial distributions of wage and capital income, the degree of tax rate differentiation in the two-rate schedule, and reranking of tax-payers in the transformation from gross to net income. In section 3 we obtain sufficient conditions for a DIT system to be unambiguously inequality reducing, extending a result of Calonge and Tejada (2011) in the process. Calonge and Tejada discuss redistribution when labor and capital incomes are perfectly aligned, i.e. there is a clear positive relationship; here results are also discussed for perfect inverse alignment. In Section 4 we describe the nature of the implicit redistribution between labor income and capital income which takes place in an inequality-reducing DIT system. Section 5 concludes.

2. Inequality effects of a dual income tax: illustrative examples

As the simple 3-person example below demonstrates, inequality reduction cannot be expected of a DIT system in general. We show 5 specimen configurations. The 3 persons have varying amounts of wage income in the 5 configurations, but unvarying capital income. The (fixed) dual tax system comprises a progressive labor income tax, at marginal rate 50% above a threshold wage $w = 4$, and a proportional tax at rate 25% on capital income in all cases.

Table 1. Example: wage income is taxed progressively, with a 50% marginal rate on income in excess of 4 units, and capital income is taxed at a proportional rate of 25%, for various joint distributions of pre-tax wages and capital income

A	Gross wage	4	8	12	Gross capital income	12	8	4	Gross income	16	16	16
	Net wage	4	6	8	Net capital income	9	6	3	Net income	13	12	11
B	Gross wage	4	12	20	Gross capital income	12	8	4	Gross income	16	20	24
	Net wage	4	6	8	Net capital income	9	6	3	Net income	13	14	15
C	Gross wage	4	14	18	Gross capital income	12	8	4	Gross income	16	20	22
	Net wage	4	9	11	Net capital income	9	6	3	Net income	13	15	14
D	Gross wage	14	19	22.5	Gross capital income	12	8	4	Gross income	26	27	26.5
	Net wage	9	11.5	13.25	Net capital income	9	6	3	Net income	18	17.5	16.25
E	Gross wage	4	9	14	Gross capital income	12	8	4	Gross income	16	17	18
	Net wage	4	6.5	9	Net capital income	9	6	3	Net income	13	12.5	12

Gross incomes are equal across persons in case A but net incomes are not, hence the dual tax system causes an inequality increase (from zero inequality) and introduces classical horizontal inequity (the unequal tax treatment of equals). In none of the other cases are there ties between gross or net income values, and hence there is no classical HI in any of the other cases. In case B, overall inequality is reduced by the DIT and there is no reranking. Overall inequality is exacerbated in case D (as well as in case A) and is reduced in cases C and E. There is reranking in all three cases, C, D and E.[1]

[1] The degrees of reranking HI can be compared in cases B – E using the "copula dominance test" introduced by Dardanoni and Lambert (2001). The copula is obtained from the familiar transition matrix, representing the impact of the tax system on people's ranks, by cumulation (from the top left corner outwards), but is not well-defined if there are income ties (gross and/or net). Copula *X* dominates copula *Y* if, entry by entry, each element of *X* is at least as big as the corresponding element of *Y*, and if *X* and *Y* are not the same. Dardanoni and Lambert (2001) show that *X* dominates *Y* if and only if the scenario from which *X* is derived has unambiguously less HI than does the scenario from which *Y* is derived. In the Appendix we show all possible transition matrices and copulas for a 3-person tax system with no income ties. The copulas for cases B, C, D and E of Table 1 can be found there, they are respectively C1, C2, C5 and C6, which are unambiguously ranked with C1 maximal. Hence reranking unambiguously increases from B (where there is none) to C, from C to D and from D to E. We do not consider horizontal inequity issues further in this paper; for much more, see Bø, Lambert and Thoresen (2011).

Other 3-person cases can easily be given which generate Lorenz curve crossings after tax, making for an ambiguous inequality effect of the dual tax system. As this very simple example shows, the scope for definitive analytical results on the inequality effects of DIT systems is quite limited.

3. Inequality reduction and the dual income tax: some analytics

Let pre-tax income be $x = w + d$ where w is wage income and d is capital income, and suppose that the wage income tax is incentive-preserving and progressive, and the capital income tax is proportionate. Let the post-tax values corresponding to w and d be v and k respectively, and let post-tax income be $y = v + k$. Given that the labor income tax is progressive and the dividend income tax is proportional, we have, from the Fellman (1976) and Jakobson (1976) theorem,

(1) $$L_V(p) \geq L_W(p) \quad \forall p \in (0,1) \quad \& \quad L_K \equiv L_D,$$

where p measures a person's rank in the gross (and net) wage distributions, and an obvious notation is adopted for pre- and post-tax Lorenz curves.

In Calonge and Tejada (2011) a "perfect alignment property" is assumed, according to which the pre-tax values w and d increase together from person to person. This is not the case in any of the specimen scenarios in our Table 1. We begin here with an "inverse perfect alignment" property, according to which gross wage income is increasing and gross capital income is falling from person to person, and overall gross and net incomes move together, i.e. there is no reranking. Scenario B of Table 1 follows this pattern. The inverse perfect alignment property may be representative of situations in which persons who are poor in labor income are rich in capital income and vice-versa. This question is (of course) ultimately an empirical one, but there are arguments, and also empirical evidence, suggesting that tax-payers trade-off wage income for less taxed capital income under a DIT, see Thoresen and Alstadsæter (2010). Calonge and Tejada (2011) report that for a large sample of taxpayers drawn from the Spanish Income Tax File Return of 2004, there is a "positive but not outstanding relationship between the ranks of labor and capital income" (with Spearman's rho and Kendall's tau-b rank correlation coefficients of 0.200 and 0.136 respectively). Estimates for Norway are lower;[2] for

[2] The measurement of such relationships depends on methodological choices, such as choice of measure of correlation, the treatment of members of the same household, sample selections, etc. Important in the Norwegian context is the treatment of capital income, or more specifically the taxation of dividends. As dividends are taxed at the firm level (as tax on profit) both before and after the tax reform of 2006, one may argue that this tax rather should be allocated to the individuals instead. It turns out that the empirical results do not depend on whether capital income (before and after tax) is included in the income concept or not.

instance we find values of 0.09 and 0.12 for 2000 (for Spearman and Kendall measures, respectively) and 0.07 and 0.10 for 2006.[3] Under the perfect alignment and perfect inverse alignment properties respectively, these correlations would be +1 and -1.

For each $p \in [0,1]$, let *x(p)* be the pre-tax income of a person at rank *p*, and let *w(p)* and *d(p)* be the pre-tax wage and capital income of that person, where, under our assumptions, *w(p)* increases with *p* and *d(p)* decreases with *p*. Thus

(2) $$L_X(p) = \frac{\int_0^p x(q)dq}{\bar{x}} = a\frac{\int_0^p w(q)dq}{\bar{w}} + (1-a)\frac{\int_0^p d(q)dq}{\bar{d}} = aL_W(p) + (1-a)\left[1 - L_D(1-p)\right]$$

where $a = \frac{\bar{w}}{\bar{x}}$ (because the lowest 100p% of *d*-values are the values *d(q)* for *q* > 1 - *p*). In the absence of reranking, we similarly have

(3) $$L_Y(p) = bL_V(p) + (1-b)\left[1 - L_K(1-p)\right].$$

where $b = \frac{\bar{v}}{\bar{y}}$. Now subtract (2) from (3), using (1):

(4) $$L_Y(p) - L_X(p) = \left[bL_V(p) - aL_W(p)\right] + (a-b)\left[1 - L_D(1-p)\right]$$

Proposition 1

Under the perfect inverse alignment property, and in the absence of reranking, the following holds:

(a) if $a = b$, the DIT is inequality-reducing regardless of Lorenz configuration;

(b) if $L_V(p) \geq 1 - L_D(1-p) \geq L_W(p) \quad \forall p \in [0,1]$, the DIT is inequality-reducing regardless of the relativity between *a* and *b*;

[3] For 2004 we observe somewhat higher degrees of correlation, but data of 2004 is influenced by capital income timing effects, as this year is the last before introducing a revision of the DIT of Norway: see Sørensen (2005) for further details about the Norwegian tax reform of 2006 (phased-in in 2005). This revised version of a dual income tax system may be characterized as a “semi-dual” tax system, as there is taxation of dividends above a normal rate of return at the individual level, implying that the marginal tax rates on dividends and wage income are approximately equal.

(c) if $a < b$ and $L_V(p) + L_D(1-p) \geq 1 \quad \forall p \in [0,1]$, the DIT is inequality-reducing; and

(d) if $a > b$ and $L_W(p) + L_D(1-p) \leq 1 \quad \forall p \in [0,1]$, the DIT is inequality-reducing.

Proof

Immediate from the following consequences of (4):

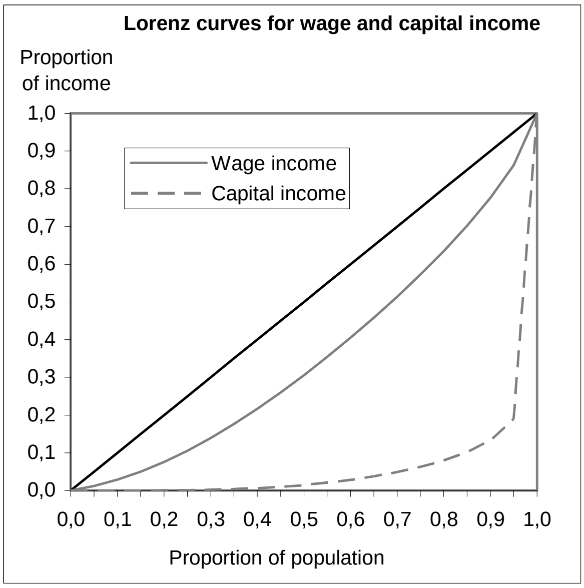
$$b > a \Rightarrow L_Y(p) - L_X(p) \geq (b-a)\left[L_V(p) + L_D(1-p) - 1\right]$$
$$b = a \Rightarrow L_Y(p) - L_X(p) = b\left[L_V(p) - L_W(p)\right] > 0 \quad \forall p \in (0,1)$$
$$b < a \Rightarrow L_Y(p) - L_X(p) \geq (a-b)\left[1 - L_D(1-p) - L_W(p)\right]$$

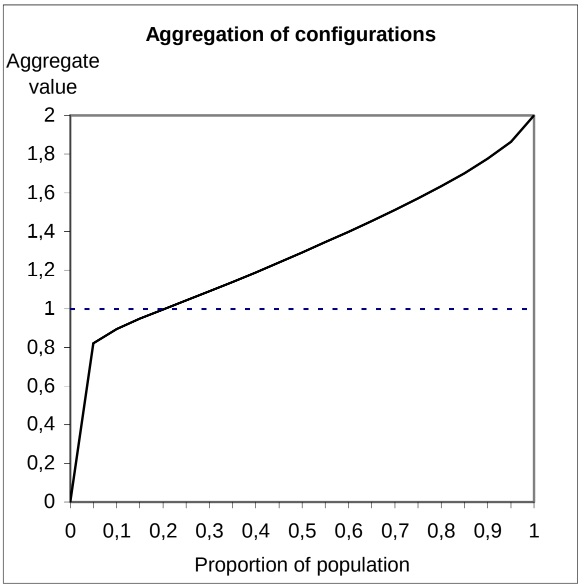
QED.

The inequality reduction we observed in case B in Table 1 is explained by part (d) of this proposition, since $0.6 = a > b = 0.57$ and $L_W(p) + L_D(1-p) = 11/18$ for $p = 1/3, 2/3$. If the Lorenz configurations are such that Proposition 1 cannot be used to establish inequality reduction, analysis of (4) may still be helpful. For example, for Norway before the tax reform of 2006, $a > b$, which means that that part (d) of Proposition 1 is relevant. However, as $L_W(p) + L_D(1-p) > 1$ for large parts of the Lorenz configurations, the second part of the condition is not met. Figure 1 shows the Lorenz curves L_W and L_D for Norway for 2000 (left panel), $L_W(p) + L_D(1-p)$ is also plotted (right panel). The tax reform of 2006 means that dividends (the most important part of capital income) not only are now taxed at the firm level, but also a tax is levied on individual dividend incomes above a rate-of-return allowance, that is, on profits above a risk-free rate of return. Our data show that this "semi-dual" tax schedule (see footnote 4), as signified by data from 2006, is rather close to what is expected for a comprehensive schedule, i.e. $a = b$. Then according to Proposition 1, the schedule is inequality reducing regardless of the patterns of the Lorenz curves.[4]

[4] The Norwegian DIT system redistributes income both before and after the tax reform of 2006; see Lambert and Thoresen (2009) for evidence on the period leading up to the reform in 2006, and see Lambert, Nesbakken and Thoresen (2010) for Norwegian redistribution in an international context. Even though summary measures of redistribution are used in these studies, one will find that the Lorenz curve of the post-tax income distribution is everywhere above the curve for pre-tax income.

Figure 1. Lorenz-curves and aggregation of configurations





We turn now to the case of perfect alignment, as assumed by Calonge and Tejada (2011). In this case, $w(p)$ and $d(p)$ are both increasing with p and therefore there can be no reranking. In this case,

(5) $$L_X(p)=\frac{\int_0^p x(q)dq}{\overline{x}}=a\frac{\int_0^p w(q)dq}{\overline{w}}+(1-a)\frac{\int_0^p d(q)dq}{\overline{d}}=aL_W(p)+(1-a)L_D(p),$$

And

(6) $$L_Y(p)=bL_V(p)+(1-b)L_K(p)$$

Now subtract (5) from (6), again using (1):

(7) $$L_Y(p)-L_X(p)=\left[bL_V(p)-aL_W(p)\right]+(a-b)L_D(p)$$

Proposition 2

Under the perfect alignment property,

(a) if $a=b$, the DIT is inequality-reducing regardless of Lorenz configuration;

(b) if $L_V \geq L_D \geq L_W$ the DIT is inequality-reducing regardless of the relativity between a and b;

(c) if $a<b$ and $L_V \geq L_D$, the DIT is inequality-reducing; and

(d) if $a > b$ and $L_D \geq L_W$ the DIT is inequality-reducing.

Proof

Immediate from the following consequences of (7):

$$b > a \Rightarrow L_Y(p) - L_X(p) \geq (b-a)\left[L_V(p) - L_D(p)\right]$$
$$b = a \Rightarrow L_Y(p) - L_X(p) = b\left[L_V(p) - L_W(p)\right] > 0 \quad \forall p \in (0,1)$$
$$b < a \Rightarrow L_Y(p) - L_X(p) \geq (a-b)\left[L_D(p) - L_W(p)\right]$$

QED.

Proposition 2 in Calonge and Tejada (2011) establishes part (a) of this Proposition. If there are Lorenz intersections in the pair(s) $\{L_V, L_D\}$ and/or $\{L_D, L_W\}$, Proposition 2 does not apply, but analysis of (7) may nevertheless be helpful. Even though the perfect alignment property is somewhat closer to the Norwegian case, as signified by the correlation estimates reported above, Proposition 2 does not provide much guidance. According to the dual income tax schedule of 2000,[5] where $a > b$, we see that $L_D < L_W$, and not the opposite which is the condition for part (d) of Proposition 2 to be met. The general case is not amenable to such analysis, due to the complications arising when different rankings are involved for different income concepts.[6]

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[5] We suggest using data as far back as in 2000 because more recent data sets reflect less clear-cut dual income tax systems (from 2006 and onward) or are strongly affected by timing effects due to the (announced) change in dividend taxation in 2006 (years prior to 2004). It should be also noted that there was a (non-announced) temporary tax on dividends in 2001.

[6] Rietveld (1990) has shown that in the general case, overall inequality is no more than in the most unequal component. For our model this means that $L_X(p) \geq aL_W(p) + (1-a)L_D(p) \geq \min\{L_W(p), L_D(p)\}$ and that $L_Y(p) \geq bL_W(p) + (1-b)L_D(p) \geq \min\{L_W(p), L_D(p)\} \ \forall p \in [0,1]$, but this does not place any obvious restrictions on how $L_X(p)$ and $L_Y(p)$ are configured. See on, however.

4. Implicit redistribution between labor income and capital income

If an income tax system is inequality-reducing then, relative to an equal yield flat tax, the rich pay more and the poor pay less, that is, there is implicit redistribution from rich to poor; similarly, in a family income tax, there may be implicit redistribution from the single to the married, from the married childless to those with children, etc. (see Lambert, 2001, p. 39 and chapter 10, on all of this). Here we examine the implicit redistribution which takes place between labor income and capital income sources in the case of an inequality-reducing DIT system.

Let the total tax ratios (fractions of all relevant income taken in tax) be g, g_W and g_D so that $\bar{y} = (1-g)\bar{x}$, $\bar{v} = (1-g_W)\bar{w}$ and $\bar{k} = (1-g_D)\bar{d}$. When Rietveld's result for net incomes specified in footnote 7 is multiplied through by $\bar{y}$, it comes down to $GL_Y(p) \geq (1-g_W)GL_W(p) + (1-g_D)GL_D(p)\ \forall p \in [0,1]$, where, in general, if μ is the mean of a distribution with Lorenz curve $L(p)$, $GL(p) = \mu L(p)$ is the generalized Lorenz curve, and it measures cumulated income per capita up to percentile *p*. A sufficient condition for $L_Y(p) \geq L_X(p)\ \forall p$, equivalently for $GL_Y(p) \geq (1-g)GL_X(p)\ \forall p$, is thus that

(8) $$(1-g_W)GL_W(p) + (1-g_D)GL_D(p) \geq (1-g)GL_X(p)\ \forall p$$

Consider now a hypothetical 3-step process, in which *(a)* all capital incomes and all wages are taxed at the common rate *g*, *(b)* specific flat rates g_W and g_D are introduced, and *(c)* progression is introduced into the tax on wages. Let $\Omega(p)$ be the set of persons who are the $100p\%$ poorest in wages, and let $\Delta(p)$ be the set of persons who are the $100p\%$ poorest in capital incomes. We cannot say anything *a priori* about $\Omega(p) \cap \Delta(p)$ or $\Omega(p) \cup \Delta(p)$ or about the relative source income magnitudes. The sufficient condition (8) for inequality reduction asks that after step (*b)*, the wages of $\Omega(p)$ plus capital incomes of $\Delta(p)$ exceed the total income after step *(a)* of the $100p\%$ overall poorest (for every $p \in (0,1)$). If (8) holds, then so does

(9) $$(1-g_W)GL_W(p) + (1-g_D)GL_D(p) \geq (1-g)GL_W(p) + (1-g)GL_D(p)\,,$$

which is in general a weaker condition than (8). If $g_W > g > g_D$, this says $(g-g_D)GL_D(p) \geq (g_W - g)GL_W(p)$ so that the move from step *(a)* to step *(b)* would cause wage losses for $\Omega(p)$ which are exceeded by the capital income gains for $\Delta(p)$. In the move to step *(c),* wage losses in $\Omega(p)$ would be mitigated and capital income gains for $\Delta(p)$ would be enhanced. If $g_W < g < g_D$ then $(g-g_W)GL_W(p) \geq (g_D - g)GL_D(p)$ with a similar interpretation.

When tested on Norwegian data for 2000, with $g_w = 0.28$, $g_D = 0.19$ and $g = 0.27$, we find that neither of the conditions (8) and (9) hold. Thus, according to an "implicit redistribution" assessment, the separate schedule has increased inequality. This is influenced by capital income being rather extremely concentrated among people at the high end of the income distribution in the Norwegian case (see Figure 1); one may find other patterns in other countries.

5. Index comparisons

When dominance conditions fail, one can look to index measures of the inequality impact of a DIT system, and seek to understand outcomes in terms of the inputs, or ingredients, which are the distributions of pre-tax labor and capital incomes and the DIT system parameters. A huge stride was recently made in this respect by Kristjánsson (2010), and is indeed the motivation for the present paper.

Kristjánsson was able to develop a Reynolds-Smolensky (1977) index of redistributive effect (reduction in the Gini coefficient from the pre-tax to the post-tax overall income distribution) for a DIT system such as ours,[7] by using Shorrocks's (1982, 1983) suggested "natural" Gini/concentration index decomposition by income source along with Pfähler's (1990) decomposition of redistributive effect into base and rate effects. As a result, he obtains a three-part decomposition of the Reynolds-Smolensky index, into components capturing: the direct redistributive effect of labor taxation on the labor income distribution and its contribution to overall inequality; a similar term for the capital income tax; and a term measuring the (indirect) effect of the difference in tax level between the different income components, which determines how the component sub-distributions "fit together" post-tax. Kristjánsson also shows how to accommodate effects of classical horizontal inequity and

[7] Kristjánsson based his model on the Icelandic dual income tax, which has a proportional capital income tax and a tax on labor income with a flat rate, made progressive by a lump-sum allowance (and negligible income-related deductions).

reranking into his analysis, and investigates some partial effects of changes in tax parameters and the composition of gross income on overall redistribution.

We see little likelihood of improving upon this achievement at present, but would note, as Kristjánsson himself does, that the computation of partial effects may not be straightforward in cases where changes in the joint income distribution and/or tax parameters cause the ordering of persons by their overall gross income to vary. This brings us back to the examples in our Table 1, in which the joint income distribution changes from row to row even though the tax parameters do not, and the overall inequality effects considerably. In this paper, we have determined what can be said about the impact of a DIT system in the most transparent cases only, those which involve inequality dominance and no reranking.

6. Conclusion

In this paper we have explored the overall inequality effects of a pure-form dual income tax system. We have shown by simple examples that correlations between distributions of wage and capital income, the degree of tax rate differentiation in the DIT, and reranking of tax-payers can all be expected to complicate a general analysis. With the help of Norwegian income tax data for 2000-2006, we have traced out what can be said definitively, obtaining sufficient conditions for unambiguous inequality reduction and identifying the nature of the implicit redistribution between labor and capital income which is involved.

According to an “implicit redistribution” assessment, the DIT system of Norway prior to 2006 has, we showed, increased inequality. This is due to the lower tax rate on capital income relative to that on labor income, which undermines vertical equity, since income from capital tends to be concentrated in the upper income brackets. One may find other patterns in other countries with DIT systems. Elements of the DIT have been introduced in several European countries, but how such tax systems influence overall income inequality has not been much discussed, save for the two studies based on redistribution indices which are cited here. Of course, redistribution indices rest on particularized value judgments. The question of designing a DIT with the strong property that overall net incomes Lorenz dominate overall gross incomes remains open for further study. The sufficient conditions charted out in this paper are a beginning.

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Appendix

With three persons, and no ties in either gross or net incomes, the possible transition matrices, $T = (t_{ij})$ where $t_{ij} = 1$ if the person whose gross income has rank *i* (with *i* = 1 being the poorest) has net income at rank *j*, and $t_{ij} = 0$ otherwise, are these six:

$$\text{T1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \text{T2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \ \text{T3} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \text{T4} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \ \text{T5} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ \text{T6} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and the corresponding cumulated transition matrices (from the top left corner outwards) are these:

$$\text{C1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ \text{C2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ \text{C3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ \text{C4} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ \text{C5} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ \text{C6} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

It can readily be checked that $C1 \prec \{C2 \text{ or } C3\} \prec \{C4 \text{ or } C5\} \prec C6$ (there are four strings here). The comparisons between $C2$ and $C3$, and betweeen $C4$ and $C5$, are ambiguous.