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*Bjart J. Holtsmark and Dag Einar Sommervoll*

## **International emissions trading in a non-cooperative equilibrium**

**Abstract:**

Linkage of different countries' domestic permit markets for pollution rights into a single international market alters governments' incentives, and may trigger adjustments of the number of allocated permits. First, this work finds that in a non-cooperative equilibrium, international emissions trading is likely to increase the total emissions. Second, although trading will give a more efficient cross-country allocation of emissions, efficiency may nevertheless fall, because an already inefficiently low abatement level is likely to be further reduced. Third, we find that large countries are likely to experience losses from linking their permit markets to the permit markets of smaller countries.

**Keywords:** Emissions trading, efficiency, non-cooperative games.

**JEL classification:** C72, F53, Q54.

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**Address:** Bjart J. Holtsmark, Statistics Norway, Research Department.  
E-mail: [bjart.holtsmark@ssb.no](mailto:bjart.holtsmark@ssb.no)

Dag Einar Sommervoll Statistics Norway, Research Department.  
E-mail: [dag.einar.sommervoll@ssb.no](mailto:dag.einar.sommervoll@ssb.no)

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## 1 Introduction

The literature on international environmental agreements gives reasons to be pessimistic about the prospects for an effective international climate agreement with broad participation and deep emission reductions. Incentives for free-riding are significant, while stable coalitions tend to be small. For example, see Barrett (1994), Carraro and Siniscalco (1993), and Hoel (1992). Moreover, after more than 15 years of negotiations, an effective international climate agreement is far from reality. After the withdrawal of the USA, the Kyoto Protocol will not provide significant emissions reductions, cf. Böhringer (2002), and others. At the same time, the negotiations on a follow-up agreement to the Kyoto Protocol have showed slow progress during recent years.

With poor prospects for comprehensive international cooperation for climate control, the world may converge towards a non-cooperative equilibrium. That would imply inefficiently small abatement efforts that are inefficiently allocated across countries.<sup>1</sup>

Therefore, an important question is whether the efficiency of the non-cooperative solution can be improved by integration (linking) of national permit markets into a single international market. Such an integration would imply that abatement efforts are efficiently allocated across countries. Stern (2006), and a number of other studies, for example, see Anger (2006), Blyth and Bosi (2004), Ellis and Tirpak (2006), and IETA (2006), recommend this type of international cooperation. However, surprisingly enough, these studies ignore how such linking of national permit markets would influence national governments' incentives.

In contrast to these studies, Helm (2003) took seriously the idea that international trading will alter national governments' incentives. Helm's starting point is that there is no "*central authority with the power to determine the initial allocations of tradable allowances. Hence, the allocation is chosen by interdependent, yet sovereign, states, and the possibility of trading will affect their allowance choices*" (Helm, 2003, pp. 2737–2738).

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<sup>1</sup>Hoel (2005) points to a third source of inefficiency of uncoordinated policies. If governments apply emission taxes in a non-cooperative game, then they have incentives to differentiate these taxes across sectors to influence emissions in other countries through the effect on international trade. Hence, Hoel (2005) concludes that a non-cooperative equilibrium, where all countries use tradable permits as their domestic environmental policy instead of taxes, may be a better solution, as this solution would avoid this third source of inefficiency. Hoel's point is that if all other countries apply quotas, then their emissions are fixed, and consequently, the argument for differentiated taxes is removed.

Therefore, the question is whether international emissions trading will increase total allocation, and consequently, reduce the total abatement. Helm (2003) concludes that the overall effect on emissions is ambiguous and states that "*emissions in a regime with allowance trading may exceed those in a regime without trading*" (Helm, 2003, p. 2742).

This paper further investigates the non-cooperative equilibrium with linked permit markets. Our conclusions differ from Helm's conclusions in a number of points. Most important, we find that the normal case will be that international trading will lead to higher emissions. Although there may be exceptions to this, we basically conclude that international emissions trading gives incentive structures that, in total, lead in the direction of increased emissions.

The reason why we draw a stronger conclusion than Helm at this point is related to the observation that the size of the countries involved is likely to vary considerably, and that the benefits from abatement and some characteristics of the abatement cost functions will vary in a systematic way between large and small countries. We find that trading will give small countries incentives for more generous allocations, while large countries will have a tendency to reduce their number of allocated permits. However, the smaller countries' incentives towards more generous allocations more than outweigh the larger countries' incentives for tightening their allocations.

It follows that large countries are likely to lose if they link their domestic permit markets to the permit markets of smaller countries. Therefore, large countries may, after all, be less willing to be involved in such linking processes.

Helm concludes that even in cases where "*emissions are higher with trading, the welfare of each individual country may improve due to efficiency gains on the permit market*". We find that this is not possible, and that not all countries will be better off in cases with increased total emissions due to trading. For example, in the two country case, increased emissions necessarily implies that one of these countries would be worse off with trading.

The other main question is, how international emissions trading affects efficiency. Because abatement in the unlinked case is inefficiently low, Helm (2003) finds that efficiency *may* be reduced when permit markets are linked. This corresponds with our findings. We applied two models in this work. In the most general model, we found that the effect on efficiency was ambiguous. However, within the more restrictive theoretical model, we found that efficiency was *always* reduced through linking of national permit markets.

Due to both the result that large countries are likely to lose from link-

ing and that overall efficiency may be reduced, we conclude that linking of national permit markets may turn out to be less important than was concluded in studies such as Anger (2006), Blyth and Bosi (2004), Ellis and Tirpak (2006), and IETA (2006). These studies ignored how linking of permit markets would influence the governments' incentives to revise their respective allocations. Hence, their conclusions that there are always efficiency gains from linking of national permit markets relies on a challengeable assumption on governments' behavior.

An analysis of how incentives are affected by integration of national permit markets is relevant to the current political situation. In recent years, a number of national governments, and some state governments in the US, are designing and/or implementing national abatement policies on a unilateral basis. Recently, it has been considered to be a promising opportunity to link national emissions trading schemes. For example, the EU ETS and the emerging ETS in California and other US states. Moreover, a number of north-eastern states in the US are planning to link their planned state-based ETS from 2009 under the Regional Greenhouse Gas Initiative (RGGI). Stern (2006) recommends this type of linking.

The next section provides an introduction to this work's main results using graphs to illustrate the two country case. The subsequent section takes an analytical approach, and deals with the  $n$ -country case and presents the main results. The final section provides a conclusion.

## 2 An introduction to the model and the main results

Consider two countries. Both cause emissions of a transboundary pollutant. Abatement is considered to be a pure collective good. Marginal abatement costs are linearly increasing. The lines  $Be_1^o$  and  $Be_2^o$  shown in Figure 1 denote the marginal abatement costs of country 1 and 2, respectively. Country 2 has a steeper marginal abatement cost curve than Country 1 does.

To make the model tractable, we make the simplifying and commonly applied assumption that these countries have constant marginal benefits from abatement equal to  $b_i$ , see Figure 1. Moreover, we assume that Country 1 has higher marginal benefits from abatement than Country 2 does.

The assumptions made so far may be interpreted that Country 1 is larger than Country 2. To see that larger countries, as a general rule, have higher benefits from abatement than smaller countries do, consider two equally-sized countries that are merged into a single country. The benefits of abatement of the new, larger country will be the sum of the benefits to the two former, smaller countries.

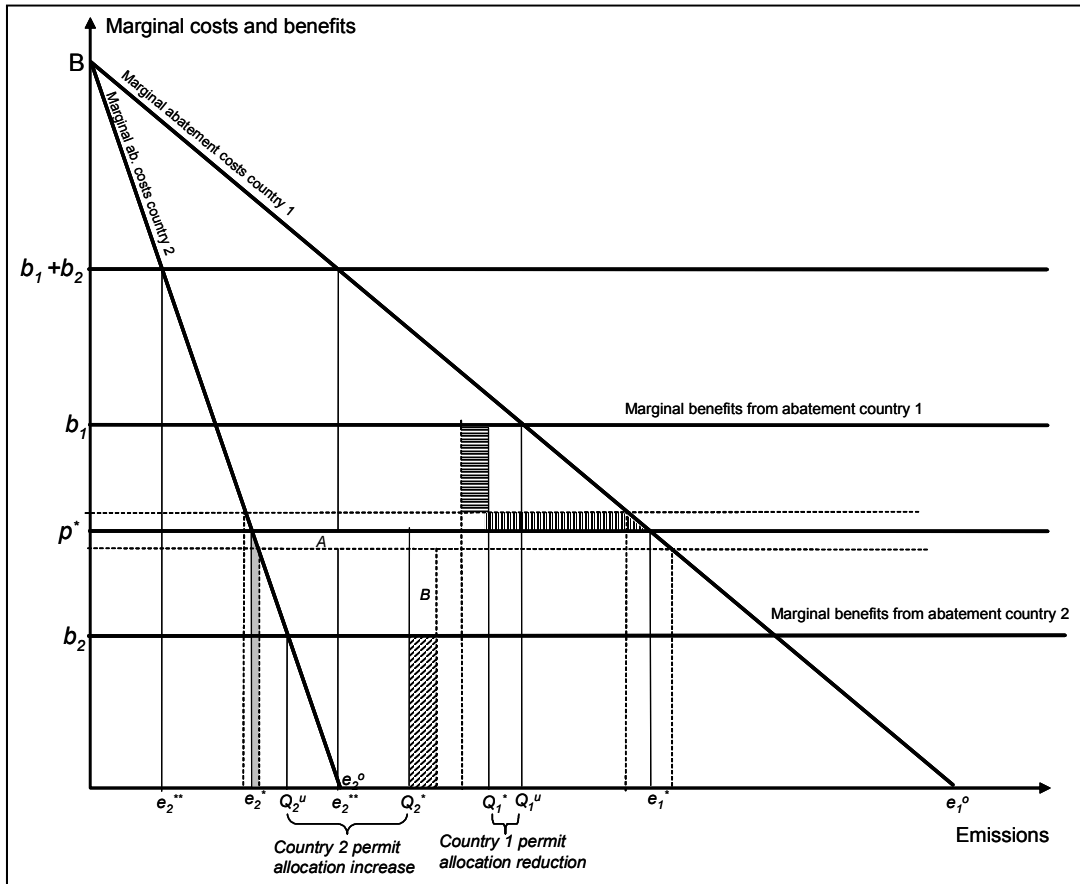


Figure 1: A two-country case with increased emissions from linking.  $e_1^{**}$  and  $e_2^{**}$  are Pareto-efficient emission levels.  $Q_1^u$  and  $Q_2^u$  are allocations in the non-linked equilibrium, while  $Q_1^*$  and  $Q_2^*$  are allocations in the linked equilibrium

In the case described in Figure 1 the large country has a flatter marginal abatement cost curve compared to the smaller country. As aggregation of marginal cost curves always gives less steep curves (due to horizontal summation), it is reasonable that the smaller Country 2 has a steeper marginal abatement cost curve compared to the larger Country 1.

The two countries could establish a Pareto-efficient agreement. In that case, abatement in both countries is set at the levels where the marginal abatement costs are equal to the sum of the two countries' marginal benefits of abatement. That would give the emission levels  $e_1^{**}$  and  $e_2^{**}$ , as indicated in Figure 1. However, an ambitious international climate agreement may be difficult to establish due to the well-known

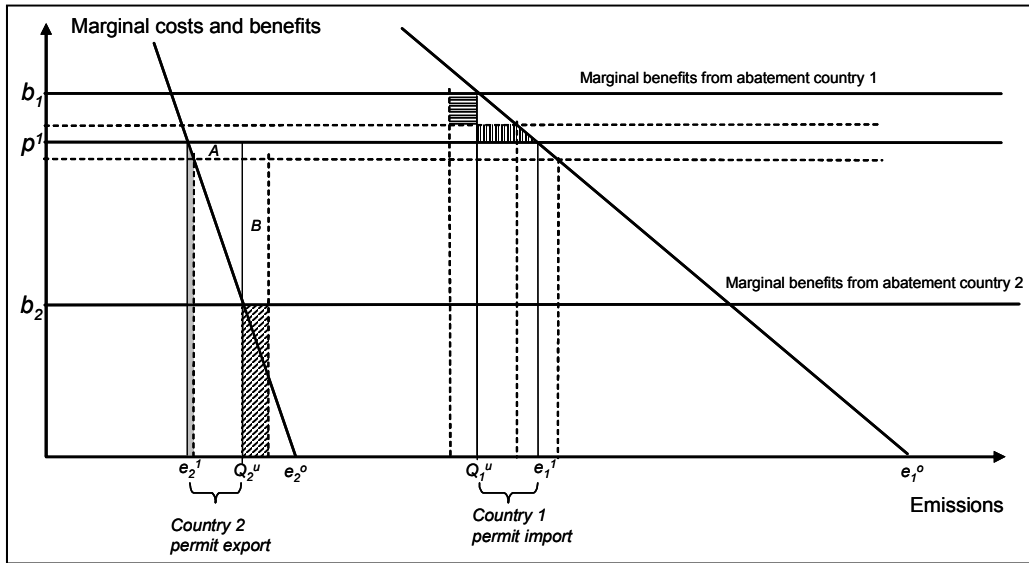


Figure 2: Linking of two countries' permit markets with permit allocations as in the unlinked case. Because the the area of rectangle B is bigger than the area of paralellogram A, country 2 would be better off increasing its allocation.

result that stable coalitions are small and the strong incentives for free-riding. Therefore, we studied non-cooperative equilibria only.

If there are two unlinked, competitive domestic permit markets within each of the two countries, then the two governments would maximize national welfares by issuing  $Q_1^u$  and  $Q_2^u$  emissions permits, respectively, see Figure 1. The permit prices would settle at the levels of the two countries' marginal benefits from the abatement, i.e.,  $p_i = b_i$ ,  $i = 1, 2$ .

However, the two countries could integrate the two domestic permit markets into a single market to collect gains from efficient cross-border abatement allocations. Assume that such integration takes place.

Consider the hypothetical situation where, after integration, the two governments of the two permit markets still issue  $Q_1^u$  and  $Q_2^u$  emissions permits, respectively. Then, the permit price would settle somewhere between  $b_1$  and  $b_2$ , but closer to  $b_1$  than to  $b_2$ . This situation is described in Figure 2.

Country 1 experiences a permit price drop compared to the unlinked case while, Country 2 experiences a permit price increase. This implies increased abatement in Country 2 and a reduced abatement in Country 1 compared to the unlinked case. Country 1 becomes a permit importer, while Country 2 becomes a permit exporter.

Unadjusted permit allocations, as shown in Figure 2, do not define a Nash equilibrium. If Country 2 increases its number of permits, as indicated in Figure 2, the income from permit exports is increased equal to the sum of the areas of the hatched rectangle and Rectangle B, minus the areas of Parallelogram A and the grey parallelogram. Parallelogram A represents the price drop effect on income from permit sales, while the grey parallelogram reflects that a permit price drop gives lower domestic abatement, and consequently, a reduced number of permits is available for sale. In addition, the grey parallelogram represents reduced abatement costs. Hence, the grey area can be ignored.

The reduced benefits from abatement have to be subtracted (hatched rectangle). Consequently, Country 2's change in payoff from an allocation extension is equal to the difference in areas of A and B. At the point where A and B occupy equally-sized areas, then a small country cannot gain from changing the number of permits.

Correspondingly, Country 1 cannot gain from changing its allocation when the rectangle with horizontal lines has the same area as the parallelogram with vertical lines.

In Figure 2, the area of B is significantly larger than the area of A, and consequently, Country 2 will gain from inflating its permit allocation. Although it is not equally evident from Figure 2 that Country 1 would be better off through the allocation of a smaller number of permits, it is clear that the situation with unchanged permit allocations is not a Nash equilibrium.

The steepness of the abatement cost curves plays a crucial role in the adjustment process towards a Nash equilibrium. As Country 1 increases the number of permits, then the permit price drops. This is followed by a reduced abatement in both countries, but the abatement reduction is largest in Country 1. The abatement reduction moderates the price drop, and consequently, Country 2 has to carry out a relatively large allocation increase before the new equilibrium is reached.

Country 1 (the larger country) will adjust the number of allocated permits downwards. Increased permit price and increased abatement in both countries follows.

The new equilibrium gives the permit price,  $p^*$ , see Figure 1. As is shown in the next section, the equilibrium price,  $p^*$ , is equal to the average of  $b_1$  and  $b_2$ . The larger country becomes the permit importer, while the smaller country exports permits. The total amount of emissions is increased.

This result rests on the assumption that the country with lowest benefits from abatement has the steepest marginal abatement cost curve. It is simple to show that instead, if the country with the highest benefits



had the steeper marginal cost curve, then trading would give lower total emissions. However, as argued above, the normal case will be that if  $b_1 > b_2$ , then  $c_1 < c_2$ , i.e., where the smaller country has the steeper marginal abatement cost function and lower marginal benefits from abatement.

Here, it is important to take into account that the size of countries varies significantly. Hence, if, for example, a two-digit number of countries of different sizes is involved, then it is very unlikely that we will see a pattern where the countries with high marginal benefits from abatement also have the steeper marginal abatement cost functions.

The insight from this is that international emissions trading will tend to give incentives towards more generous allocations of permits, and consequently, a reduced total abatement. As total abatement already at the outset was inefficiently low, increased emissions represents an efficiency loss that counteracts the efficiency gains from efficient cross-border abatement allocation.

This section provides an informal introduction to this work's main result. The next section analyzes this within an 'n-country' setting, and also explores under what conditions linking of emissions trading schemes will either improve or reverse efficiency and how gains and losses are distributed between the participating countries.

### 3 A model of transboundary pollution

Consider a world containing  $n$  countries, with  $N = \{1, \dots, n\}$  denoting the set of all countries. Emission abatement is thought of as a pure public good that benefits each and every country. Each country  $i$ 's periodic payoff is given by

$$\pi_i = b_i \sum_{j=1}^n a_j - \frac{c_i}{2} a_i^2, \quad i = 1, \dots, n, \quad (1)$$

where  $a_i$  is the abatement of greenhouse gas emissions in country  $i$ , the positive parameter  $b_i$  represents country  $i$ 's marginal benefit from abatement, while  $(c_i/2)a_i^2$  represents the total abatement costs of country  $i$ , where  $c_i$  is a positive parameter.

This model of national payoffs is frequently applied in the literature on international environmental agreements. For example, in Barrett (1994).

#### 3.1 The non-cooperative solution with domestic permit trading only

Assume that each country has a competitive emissions trading system, and consequently, that governments do not control abatement directly,

only the number of allocated permits. The  $n$  governments interact in a one-period simultaneous game by choosing the number of allocated permits.

This paper considers two cases. In the first case, the governments introduce independent (unlinked) national emissions trading markets. In the second case, the governments link the emissions trading market into a single global market. In this last case, permits issued by one government are approved in all other countries. In both the linked and the unlinked cases permit markets are competitive. This subsection considers the case with  $n$  unlinked permit markets, while the next subsection considers the linked case.

The governments maximize the national payoffs taking into account that their behavior alters the permit price. The national payoff is maximized when marginal abatement costs equal marginal benefits, i.e. when  $b_i = c_i a_i$ , cf. (1). The abatement levels follow:

$$a_i^u = \frac{b_i}{c_i}. \quad (2)$$

Hence, irrespective of the other countries' behavior it is a dominant strategy to issue

$$Q_i^u = e_i^o - \frac{b_i}{c_i} \quad (3)$$

permits, where  $e_i^o$  is the "business as usual" emissions. There is a unique non-cooperative Nash equilibrium where each government issues permits  $Q_i^u$ . The permit price  $p_i^u$  equals the marginal abatement costs  $c_i a_i$ . Hence, from (2) it follows that  $p_i^u = b_i$ . In the unlinked case, the global permit supply,  $Q$ , is:

$$Q^u = \sum_{i=1}^n e_i^o - \sum_{i=1}^n \frac{b_i}{c_i}. \quad (4)$$

### 3.2 The non-cooperative solution with international permit trading

Consider then the case with international emissions trading meaning that all governments approve all other countries' emission permits in their domestic permit markets. As in Hoel (2005) and Helm (2003), we take into account that there is no global authority that determines national allocation of permits. This decision is left to the governments that act in a simultaneous game, taking into account that their permit allocations influence the global permit price. Governments enforce the permit system domestically meaning that domestic emissions after abatement equals the number of permits held nationally. It follows that the global

permit supply equals the global emissions:

$$\sum_{i \in N} Q_i = \sum_{i \in N} e_i, \quad (5)$$

where  $e_i$  is the emissions from country  $i$ . By definition  $e_i = e_i^o - a_i$ . Because the permit markets are competitive the permit price  $p$  equals the marginal abatement costs in all countries:

$$p = c_i a_i. \quad (6)$$

With linked permit markets, the countries collect a (positive or negative) permit income  $p \cdot (Q_i + a_i - e_i^o)$ . Hence, the national payoffs are:

$$\pi_i = b_i \sum_{j=1}^n a_j - \frac{c_i}{2} a_i^2 + p \cdot (Q_i + a_i - e_i^o). \quad (7)$$

where  $a := \sum_{j=1}^n a_j$ . The governments maximize their national payoffs  $\pi_i$  with respect to their permit allocations,  $Q_i$ , subject to the  $n$  restrictions in (6) and the equilibrium condition of the permit market (5). It is assumed that all governments take other countries' permit allocations as given.

Solving the  $n$  countries' maximization problems simultaneously yields the unique non-cooperative Nash equilibrium (see appendix):

$$Q_i^* = Q_i^u - \left( \sum_{j \neq i} \frac{1}{c_j} \right) (b_i - \bar{b}), \quad (8)$$

where  $\bar{b} = \frac{1}{n} \sum_{j \in N} b_j$ .

In the previous section we argued that there will be a general pattern where small countries have a small  $b_i$  and large countries have a large  $b_i$ . It follows from (8) that small countries will tend to increase the allocated number of permits, while large countries will tend to tighten their allocations when the permit markets become linked, cf. the discussion in the previous section. Moreover, from (8) we see that the less steep other countries' marginal abatement cost curves are, i.e. the smaller other countries'  $c_j$  are, then the more inflated a small country's permit allocation is, when the permit markets become linked.

In the linked equilibrium, the permit price equals the average of the marginal benefits of abatement (see the appendix):

$$p^* = \bar{b}, \quad (9)$$

and the global permit supply becomes:

$$Q^* = Q^u + \sum_{i \in N} \left[ \frac{1}{c_i} (b_i - \bar{b}) \right]. \quad (10)$$

Define the vectors  $\mathbf{b} := (b_1, \dots, b_n)$  and  $\mathbf{c}^{-1} := (\frac{1}{c_1}, \dots, \frac{1}{c_n})$ . Then (10) can be reformulated to:

$$Q^* = Q^u + n \text{cov}(\mathbf{c}^{-1}, \mathbf{b}), \quad (11)$$

which constitutes the basis for Proposition 1.

### 3.3 Effects on emissions

**Proposition 1** *Assume that the  $n$  countries link their domestic permit markets. If  $b_i = b_j$ , for all  $i, j \in N$ , then the behavior of all countries is unaltered by linking. Hence, both global abatement and individual countries' abatement are unaltered and there are no efficiency gains from linking. If  $c_i = c_j$  for all  $i, j \in N$ , then global abatement is unaltered from the unlinked case.*

**Proof.** It follows from (8) that if  $b_i = b_j$  for all  $i$  and  $j$ , then  $Q_i^* = Q_i^u$  for all  $i \in N$ . With unaltered behavior by all Countries, there cannot be any efficiency gains, and it follows that  $Q^* = Q^u$ . If  $c_i = c_j$  for all  $i$  and  $j$  then  $\text{cov}(\mathbf{c}^{-1}, \mathbf{b}) = 0$ . From (11) it follows that  $Q^* = Q^u$ . ■

Proposition 1 underlines that in a non-cooperative solution, efficiency gains from trading arise when countries value abatement differently, and not from different abatement cost functions. If countries value abatement equally, i.e. if  $b_i = b_j$  for all  $i$  and  $j$ , there are no efficiency gains from trading even when abatement cost functions are different.

**Proposition 2** *Assume that the  $n$  countries link their domestic permit markets. If there is a positive (negative) covariance between  $\mathbf{b}$  and  $\mathbf{c}^{-1}$ , then global abatement is lower (higher) in the case with linked permit markets compared to the unlinked case.*

**Proof.** Proposition 2 follows directly from (11). ■

Proposition 2 has important implications if it can be argued that  $\text{cov}(\mathbf{c}^{-1}, \mathbf{b})$  is positive. We have already given some arguments for this, but we return to the question of whether this is the case below.

To provide intuition to Proposition 2, the two country case is considered analytically in the following discussion. This supplements the intuition given in the Section 2.

Recall that with unlinked permit markets the national governments issue  $Q_i = e_i^o - b_i/c_i$  permits and the permit prices equal the marginal benefits of abatement and marginal abatement costs, i.e.  $p_i^u = b_i = c_i a_i$ .

Assume that the two permit markets are linked, and assume temporarily that the number of allocated permits in both countries is fixed at the levels of the unlinked case. Private sector behavior will lead to abatement levels,  $a_i^1$ , that give marginal costs equal to the new, global permit price that we label  $p^1$ . Hence, we have that  $a_i^1 = p^1/c_i$ . Using (3) and (5) gives:

$$p^1 = \frac{c_2}{c_1 + c_2} b_1 + \frac{c_1}{c_1 + c_2} b_2. \quad (12)$$

Hence, with unchanged allocations the new permit price will settle at a level between  $b_1$  and  $b_2$ . If  $c_2 > c_1$ , then  $p^1$  is closer to  $b_1$  than to  $b_2$ . The country with the steepest abatement cost curve will experience the largest price change with unchanged permit allocations.

However, the number of allocated permits fixed at the levels of the unlinked case is not a Nash equilibrium. Using that  $da/dQ_i = -1$  and that  $p - c_i a_i = 0$ , the derivation of (7) gives that:

$$\frac{d\pi_i}{dQ_i} = p - b_i - (Q_i - e_i) \frac{dp}{dQ_i}. \quad (13)$$

The first term on the right hand side ( $p$ ) represents the income gain from an additional permit. In the case of Country 2, this effect is represented by the sum of the areas of B and the hatched rectangle in Figure 2. The second term,  $-b_i$ , represents the reduction of country  $i$ 's benefits following an increase in  $Q_i$ , cf. the area of the hatched rectangle in Figure 2. (Recall that the benefits of abatement are equal to  $b_i (a_1 + a_2)$ , and that  $da_1 + da_2 = dQ_i$ .) The last term on the right hand side of (13) represents the income loss from permit sales following the permit price drop. This last effect is illustrated by the area of A in Figure 2.

The price drop following an increase in  $Q_2$  implies a lower abatement in both countries due to the price drop. Country 2's reduced abatement costs are illustrated by the grey area in Figure 2. However, the abatement cost reduction is equal to a corresponding income reduction from permit for sales due to reduced abatement. Recall here that  $p = c_i a_i$ . Hence, at the margin, these two cost and income effects cancel each other out. Consequently, in the case of Country 2, an increased permit allocation is profitable if the area of B is larger than A. Correspondingly, Country 1 would gain from a reduced permit allocation if the area with the horizontal lines is larger than area with the vertical lines. (Recall that here, Country 1 is a permit importer.)

Figures 1 and 2 represent a case where  $c_1 < c_2$  and  $b_1 > b_2$ , i.e.

$cov(\mathbf{c}^{-1}, \mathbf{b}) > 0$ . According to Proposition 2 this implies that integration of the two permit markets would give increased emissions. This is in accordance with the findings in Section 2. Why is this so?

Figure 2 can provide some intuition here. It shows that, at the outset, when the number of permits is as in the unlinked case, the area of B is significantly larger than the area of A. Recall that the height of B represents the difference between the permit price and Country 2's marginal benefit from abatement. Hence, the area of B represents the additional net benefit to Country 2 of additional permits on the market. To obtain the net effect on Country 2's payoff we have to subtract the area A. The length of A represents Country 2's permit export, while the height of A represents the price drop following an increase in  $Q_2$ . It follows that the area of A represents the income loss due to the price drop that follows from the increased number of permits on the market. Because the area of B is larger than the area of A, Country 2 would be better off by increasing its number of permits, cf. the discussion in Section 2.

As the number of permits is increased the permit price falls, while the rectangle B moves to the right and becomes lower. Hence, the height of B is reduced while A becomes longer. Country 2 will increase its number of permits until the area of B equals the area of A.

The argument is turned around with respect to Country 1. The area with horizontal lines represents the difference between Country 1's marginal benefit of abatement and the permit price. The area with vertical lines represents the import bill increase following the reduced allocation of permits. Hence, Country 1 would gain by reducing its number of permits if the area with horizontal lines is larger than the area with vertical lines. As Country 1's number of permits is reduced the permit price increases and the height of the area with horizontal lines decreases while the length of the area with vertical lines increases.

Proposition 2 has important implications if  $cov(\mathbf{c}^{-1}, \mathbf{b})$  is positive. This is the case if the countries with small marginal benefits from abatement have steep marginal abatement cost functions, and vice versa. Although there will be important exemptions, it is obvious that there will be a general pattern where large countries have large marginal benefits from abatement. For example, consider the case that the world consists of only a single country. This single country's marginal benefit of abatement would reflect the global collective benefit of abatement. Hence, its marginal benefit from abatement would be large. On the other hand, if the world consisted of a large number of small countries, each country would have relatively small marginal benefits from abatement, reflecting that a small country will have to endure only a small fraction of total

damage from climate change.

Therefore the question is whether there is a general pattern where small countries have steep marginal abatement cost functions. In the previous section we argued that this will be the normal case because aggregation of marginal cost curves implies horizontal summation. Moreover, recall that quadratic abatement cost functions can be considered as approximations to the abatement costs that increase in steps, where each step in the cost functions represents certain types of projects with defined cost levels. In a small country, there are likely to be fewer projects within each step compared to a large country.

We do not claim that it will always be the case that the smallest of two countries has the steeper marginal abatement cost curve and the lower marginal benefit from abatement. Especially, when the countries are of the same magnitude, there will be many exceptions to this stylized rule. However, here, it is important to take into account that the world contains countries and states of a wide range of sizes. For example, the seven US states included in the RGGI<sup>2</sup>, vary from Vermont with a population of 623,000 and annual CO<sub>2</sub>-emissions of approximately 23 million tonnes, to New York with a population of 19.3 millions and annual CO<sub>2</sub>-emissions of 170 million tonnes. Hence, for example, it appears likely that Vermont has a considerably steeper marginal abatement cost curve compared to New York. With respect to the marginal benefits from abatement, an average Vermont citizen should have very high benefits from abatement compared to an average New York citizen if Vermont's marginal benefits from abatement are not to be smaller than the benefits to New York. Regarding the original Kyoto Protocol it was signed by countries with a population of more than 300 million (USA) and those with less than 0.35 million (Iceland).

Of course, there may be important exemptions to a rule saying that  $cov(\mathbf{c}^{-1}, \mathbf{b})$  is positive. Nevertheless, we find it likely that  $cov(\mathbf{c}^{-1}, \mathbf{b})$  will be positive in most cases. Hence, it is more likely that integration of permit markets will give increased emissions than the opposite case.

### 3.4 Effects on gains and losses

In this subsection we study how gains and losses from linking of permit markets are distributed between participating countries.

Equations (1) and (2) give that

$$\pi_i^u = b_i \sum_{j \in N} \frac{b_j}{c_j} - \frac{1}{2} \frac{b_i^2}{c_i}. \quad (14)$$

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<sup>2</sup>Connecticut, Delaware, Maine, New Hampshire, New Jersey, New York, and Vermont.

Equations (6) and (9) shows that  $a_i^* = \bar{b}c_i$ . Upon substitution of (3) and (8) into (7) gives:

$$\pi_i^* = \bar{b}^2 \left( \sum_{j \in N} \frac{1}{c_j} - \frac{1}{2} \frac{1}{c_i} \right). \quad (15)$$

This gives:

$$\pi_i^* - \pi_i^u = \left( \bar{b} \sum_{j \in N} \frac{\bar{b}}{c_j} - b_i \sum_{j \in N} \frac{b_j}{c_j} \right) + \frac{1}{2} \left( \frac{b_i^2}{c_i} - \frac{\bar{b}^2}{c_i} \right) \quad (16)$$

Consider the case where  $b_i > \bar{b}$  and  $cov(\mathbf{c}^{-1}, \mathbf{b}) \geq 0$ . Then the expression in the first bracket on the right hand side of (16) is negative. However, the second bracket is positive. Hence, in general, we cannot state whether country  $i$  will lose or win from linking although we assume that  $b_i > \bar{b}$  and  $cov(\mathbf{c}^{-1}, \mathbf{b}) > 0$ .

To simplify the discussion, consider the two country case. Then, we have:

$$\pi_1^* - \pi_1^u = \left( \frac{b_1 - b_2}{8c_2c_1} \right) (b_1(2c_1 - 3c_2) - b_2(2c_1 + c_2)) \quad (17)$$

$$\pi_2^* - \pi_2^u = \left( \frac{b_1 - b_2}{8c_1c_2} \right) (b_1(c_1 + 2c_2) + b_2(3c_1 - 2c_2)) \quad (18)$$

Then we could state the following result:

**Proposition 3** *Assume that  $n = 2$ , and that  $b_1 > b_2$ . Then, Country 2 will always gain from linking. Country 1 will gain from linking, if and only if:*

$$c_1 > \left( 1 + \frac{b_1 + 3b_2}{2(b_1 - b_2)} \right) c_2.$$

**Proof.** The Proposition follows directly from (17) and (18). ■

From Proposition 3 it follows that when  $b_1 > b_2$  in the two country case, Country 1 will lose from linking when  $c_1 < c_2$ , and even in some cases where  $c_1 > c_2$ . This implies that in all cases where  $cov(\mathbf{c}^{-1}, \mathbf{b})$  is positive, i.e. in all cases where emissions are increased by linking, and even in some cases where  $cov(\mathbf{c}^{-1}, \mathbf{b})$  is negative, the country with the largest benefit from abatement will lose from linking.

Here, our result differs from the a result in Helm's Proposition 4. Helm claims that "*even if overall emissions are higher with trading, all countries may consent to it because their welfare without trading would be lower*" (Helm, 2003, p. 2744). Although Helm refers to the  $n$ -country case, the somewhat unclear proof is based on the two country case.



### 3.5 Effects on efficiency

The non-cooperative Nash equilibrium with unlinked permit markets is not Pareto efficient because the abatement levels are inefficiently low, and abatement allocations are not cost-effectively distributed across countries. With fixed allocations, linking would obviously imply an increased efficiency. However, it is an open question whether total efficiency is improved if an inefficiently low abatement level is reduced further.

Define:

$$\begin{aligned}\pi^u &:= \sum_{i \in N} \pi_i^u, \\ \pi^* &:= \sum_{i \in N} \pi_i^*.\end{aligned}$$

Using (14) gives:

$$\pi^u = \sum_{i=1}^n b_i \left( \sum_{j=1}^n \frac{b_j}{c_j} \right) - \frac{1}{2} \sum_{i=1}^n \frac{(b_i)^2}{c_i}. \quad (19)$$

Using (15) gives:

$$\pi^* = \left( n - \frac{1}{2} \right) \bar{b}^2 \left( \sum_{j=1}^n \frac{1}{c_j} \right). \quad (20)$$

It follows that

$$\pi^* - \pi^u = \frac{1}{2} \sum_{i=1}^n \frac{b_i^2}{c_i} - \sum_{i=1}^n b_i \left( \sum_{j=1}^n \frac{b_j}{c_j} \right) + \left( n - \frac{1}{2} \right) \bar{b}^2 \left( \sum_{j=1}^n \frac{1}{c_j} \right) \quad (21)$$

The right hand side of (21) could be either positive or negative. Hence, with no restrictions on the relationship between  $b_i$  and  $c_i$  there can be either an efficiency gain or loss from linking. However, we can state the following result:

**Proposition 4** *Assume that all countries have equal marginal benefits from abatement, i.e. that  $b_i = b_j, \forall i, j \in N$ . Then, the overall efficiency is unchanged by linking.*

**Proof.** Insert  $b_i = b_j, \forall i, j \in N$  into (21). Then it follows that  $\pi^u = \pi^*$ . ■

Proposition 4 follows from (8), which implies that if  $b_i = b_j, \forall i, j \in N$ , then allocations in all countries are unaltered from the unlinked case.

Moreover, because  $p_i^u = b_i$ , and  $p^* = \bar{b}$ , it follows that there will be no permit trade if  $b_i = b_j, \forall i, j \in N$ .

Consider again the two country case. From (21) we have the effect on efficiency:

$$\pi^* - \pi^u = \frac{1}{8c_1c_2} (b_1 - b_2) (3b_1c_1 - b_1c_2 - 3b_2c_2 + b_2c_1) \quad (22)$$

Now we are ready to state the following result:

**Proposition 5** *Consider the two country case. Assume that  $b_1 > b_2$ . Then, efficiency will be improved by linking if, and only if:*

$$c_1 > \left(1 - \frac{2(b_1 - b_2)}{3b_1 + b_2}\right) c_2.$$

**Proof.** Proposition 5 follows directly from (22). ■

### 3.6 Summing up results on emissions, gains and losses and efficiency

In the  $n$ -country case, it is difficult to provide explicit results regarding which countries will lose, and which countries will gain from linking and how efficiency will be affected. However, the definite results in the two country case can serve as an indicator on how linking of permit markets may affect both emissions, incentives, gains, losses, and efficiency.

Figure 5 provides an overview of our findings regarding the two country case when  $b_1 > b_2$ . If  $c_1 < c_2$ , then  $cov(\mathbf{c}^{-1}, \mathbf{b}) > 0$ , and it follows from (11) and Proposition 2 that linking gives increased emissions. Moreover, if

$$c_1 < \left(1 - \frac{2(b_1 - b_2)}{3b_1 + b_2}\right) c_2, \quad (23)$$

then there will be an efficiency loss.

In the interval

$$\left(1 - \frac{2(b_1 - b_2)}{3b_1 + b_2}\right) c_2 < c_1 < \left(1 + \frac{b_1 + 3b_2}{2(b_1 - b_2)}\right) c_2, \quad (24)$$

Country 1 will lose while Country 2 will profit from linking. Only if

$$c_1 > \left(1 + \frac{b_1 + 3b_2}{2(b_1 - b_2)}\right) c_2, \quad (25)$$

will both countries profit from linking.

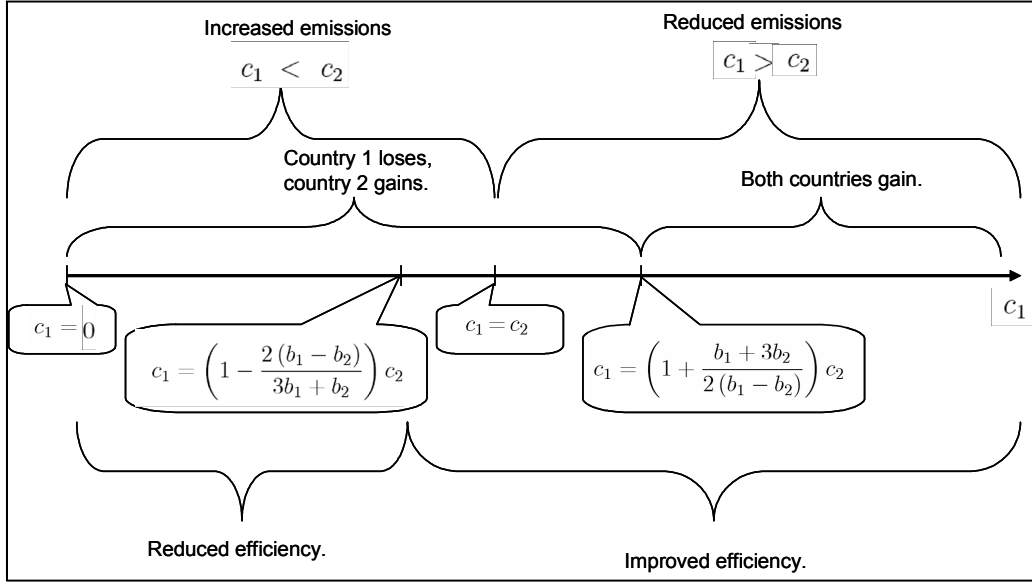


Figure 3: Illustrates the two country case when  $b_1 > b_2$  and considers how the relationship between the parameters influences gains, losses, emissions, and the overall efficiency.

### 3.7 Efficiency in a model with identical firms

To investigate further the relationship between linking of permit markets and efficiency, this section introduces a more restrictive model. As in Golombek and Hoel (2008), we assume that each country has a varying number of identical firms  $m_i$ . A firm  $j$  in country  $i$  has abatement costs:

$$\frac{\gamma}{2} (a_{ji})^2, \quad (26)$$

where  $\gamma$  is a positive parameter and  $a_{ji}$  is the abatement carried out by the firm. If the abatement is carried out efficiently, country  $i$  now has abatement costs:

$$\min_{a_{ij}} \left[ \sum_{j=1}^{m_i} \frac{\gamma}{2} (a_{ij})^2 \right] = \frac{\gamma}{2m_i} a_i^2. \quad (27)$$

Moreover, we assume that the benefits that countries gain from the abatement are proportional to the size of their economy reflected by their respective numbers of firms. Then, country  $i$  has the following benefit from abatement:

$$\beta m_i a, \quad (28)$$

where  $\beta$  is a positive parameter.

It follows that:

$$b_i = m_i \beta, \quad (29a)$$

$$c_i = \gamma / m_i. \quad (29b)$$

From (7), (6) and (5) we have:

$$\pi^u = \frac{\beta^2}{\gamma} \left( \sum_{i=1}^n m_i \left( \sum_{j=1}^n m_j^2 \right) - \frac{1}{2} \sum_{i=1}^n m_i^3 \right), \quad (30)$$

$$\pi^* = \left( n - \frac{1}{2} \right) \frac{\beta^2}{\gamma n^2} \left( \sum_{i=1}^n m_i \right)^3. \quad (31)$$

Define:

$$M_1 := \sum_{i=1}^n m_i, \quad (32)$$

$$M_2 := \sum_{i=1}^n m_i^2, \quad (33)$$

$$M_3 := \sum_{i=1}^n m_i^3. \quad (34)$$

It follows that:

$$\pi^* - \pi^u = \frac{\beta^2}{2\gamma} \left( \frac{2n-1}{n^2} M_1^3 - 2M_1 M_2 + M_3 \right) \quad (35)$$

Now we are ready to state the following result:

**Proposition 6** *Assume that each country has  $m_i$  identical firms with abatement costs  $(\gamma/2)(a_{ji})^2$ , and assume that the countries' marginal benefits of abatement are proportional to the numbers of firms in the different countries, such that  $b_i = m_i \beta$ . Assume that at least two of the  $n$  countries are different, i.e., that  $m_i \neq m_j$ ,  $i, j \in N$ . If the  $n$  countries link their originally unlinked domestic permit markets, then the efficiency is reduced.*

**Proof.** Proposition 3 claims that if there exists an  $i$  and  $j$ , such that  $m_i \neq m_j$ ,  $i, j \in N$  then  $\pi^u > \pi^*$ . The proof is carried out by induction. First, consider the two country case. It follows from (35) that in the two country case we have:

$$\pi^* - \pi^u = -\frac{1}{4} (m_1 + m_2) (m_2 - m_1)^2 \quad (36)$$

It follows that in the  $n = 2$  case  $\pi^* < \pi^u$  if  $m_1 \neq m_2$ ,  $m_1 > 0$ ,  $m_2 > 0$ . Hence, linkage of Country 1 and Country 2 gives a smaller total payoff  $\pi_{1,2}^*$  compared to the unlinked payoff  $\pi_{1,2}^u$  if  $m_1 \neq m_2$ .

The  $n$  country case follows directly by an iterative use of the  $n = 2$  case. To see this, let  $m = (m_1, \dots, m_n)$  and consider first linkage of country 1 and 2. From the  $n = 2$  case above we know that

$$\pi_{1,\dots,n}^u = \pi_{1,2}^u + \pi_{3,\dots,n}^u \geq \pi_{1,2;3,\dots,n}^* := \pi_{1,2}^* + \pi_{3,\dots,n}^u \quad (37)$$

This inequality is strict if  $m_1 \neq m_2$ .

Define  $\pi_{\bar{1},\bar{2}}^u$  as the payoff of two equally sized countries with  $\bar{m}_1 = \bar{m}_2 = 1/2(m_1 + m_2)$ . It follows directly from (36) that  $\pi_{1,2;3,\dots,n}^* = \pi_{\bar{1},\bar{2}}^* + \pi_{3,\dots,n}^u$ . In short, we have proved the following:

$$\pi_{1,\dots,n}^u = \pi_{1,2}^u + \pi_{3,\dots,n}^u \geq \pi_{1,2}^* + \pi_{3,\dots,n}^u = \pi_{\bar{1},\bar{2}}^u + \pi_{3,\dots,n}^u = \pi_{\bar{1},\bar{2},3,\dots,n}^u \quad (38)$$

That is  $\pi_{1,\dots,n}^u$ , which corresponds to the payoff of  $m = (m_1, \dots, m_n)$ , is at least the payoff of  $\acute{m} = (\bar{m}_1, \bar{m}_2, m_3, \dots, m_n)$ . Now the result follows from iteration of this step: Apply the same reasoning to any two countries in  $\acute{m} = (\bar{m}_1, \bar{m}_2, m_3, \dots, m_n)$  (say  $\bar{m}_2$  and  $m_3$ ), call this  $\pi_{\bar{1},\bar{2},\bar{3},4,\dots,n}^u$ , and get a chain of inequalities:

$$\pi_{1,\dots,n}^u \geq \pi_{\bar{1},\bar{2},3,\dots,n}^u \geq \pi_{\bar{1},\bar{2},\bar{3},4,\dots,n}^u \geq \pi_{\bar{1},\bar{2},\bar{3},\bar{4},5,\dots,n}^u \geq \dots \quad (39)$$

This chain converges to  $\pi_{\acute{1},\dots,\acute{n}}^u$  where  $\acute{m} = (\acute{m}_1, \dots, \acute{m}_n)$  where  $\acute{m}_i = 1/n \sum m_i$ . In other words,

$$\pi_{1,\dots,n}^u \geq \pi_{\acute{1},\dots,\acute{n}}^u = \pi_{\acute{1},\dots,\acute{n}}^* \quad (40)$$

where the final equality stems from that in the case of  $\acute{m}_i = 1/n \sum m_i$  is  $\pi_{\acute{1},\dots,\acute{n}}^u = \pi_{\acute{1},\dots,\acute{n}}^*$  which is trivial to check. In fact, since  $m_i \neq m_j$  we know that (at least one) of the inequalities in the chain is strict, and we know that the inequality:

$$\pi_{1,\dots,n}^u > \pi_{\acute{1},\dots,\acute{n}}^u = \pi_{\acute{1},\dots,\acute{n}}^* \quad (41)$$

is strict. ■

Some intuition to Proposition 6 can be reached by recalling that it follows from (35) that  $\pi^u = \pi^*$  if  $m_i = m_j$  for all  $i$  and  $j$  in  $N$ . Hence, the efficiency in the linked case is the same as in the the unlinked case when the countries are of equal size. At the same time, (31) shows that efficiency in the linked case is independent of the relative size of the countries. However, in the unlinked case, the relative size of the countries matters.

## 4 Conclusions

Linking of emissions trading schemes can release efficiency gains. However, this paper has proved that such linking is likely to increase total emissions. The reason is that international emissions trading is likely to provide incentive structures where governments of small countries will have incentives towards more generous allocation of permits while governments of large countries will have incentives to tighten up their allocations. However, more generous allocations from the small countries are likely to outdo the allocation reductions from larger countries', and, consequently there will be a reduced total abatement.

A second finding is that large countries, are likely to lose when their permit markets are linked to the permit markets of smaller countries with lower benefits from abatement.

It remains uncertain how linking of emissions trading schemes will affect the overall efficiency. We conclude that efficiency may increase or decrease when permit markets are linked, as found in Helm (2003). Linking will give more efficient allocation of abatement efforts. On the other hand, an inefficiently low abatement level is likely to be even lower. This represents an efficiency loss. The net effect on efficiency is uncertain.

The question of efficiency was analysed using a more restrictive model. In the restrictive model, the emissions were caused by a set of identical firms. The number of firms varied among countries, and the countries' benefits from abatement are assumed to be proportional to their number of firms. Within this model international emissions trading reduces both global abatement and efficiency.

Our conclusion is that linking of national emission trading schemes will turn out to be less attractive in future in international climate policy than often assumed. Not least important is the result that large countries are likely to experience reduced payoffs, and therefore, will be less willing to enter into this type of cooperation.

## A Appendix

### A.1 Derivation of the non-cooperative Nash equilibrium

The  $n$  governments' maximization problems with linked permit markets are:

$$\max_{Q_i} \pi_i = b_i \left( \sum_{j \in N} a_j \right) - \frac{c_i}{2} a_i^2 + p \left( \sum_{j \in N} Q_j \right) \cdot (Q_i - e_i^o + a_i) \quad (42)$$

s.t.

$$c_j a_j = p \left( \sum_{j \in N} Q_j \right), \quad \forall j \in N, \quad (43)$$

$$(44)$$

$$\sum_{j \in N} Q_j = \sum_{j \in N} e_j^o - \sum_{j \in N} a_j. \quad (45)$$

Substitution from (6) the equilibrium condition becomes:

$$\sum_{j \in N} Q_j = \sum_{j \in N} e_j^o - p \left( \sum_{j \in N} Q_j \right) \cdot \sum_{j \in N} \frac{1}{c_j}. \quad (46)$$

Defining

$$\alpha = - \frac{1}{\sum_{j \in N} \frac{1}{c_j}}, \quad (47)$$

it follows from (6) and (5) that the equilibrium permit price is:

$$p \left( \sum_{j \in N} Q_j \right) = \alpha \left( \sum_{j \in N} Q_j - \sum_{j \in N} e_j^o \right). \quad (48)$$

Substitution from (6) into (7), the payoff become:

$$\pi_i = b_i p \sum_{j \in N} \frac{1}{c_j} - \frac{c_i}{2} \left( \frac{p}{c_i} \right)^2 + p \left( Q_i - e_i^o + \frac{p}{c_i} \right). \quad (49)$$

Upon substitution from (47) we have:

$$\pi_i = \left( Q_i - e_i^o - \frac{b_i}{\alpha} \right) p + \frac{1}{2c_i} p^2. \quad (50)$$

Hence, we have:

$$\frac{d\pi_i}{dQ_i} = p + \left( Q_i - e_i^o - \frac{b_i}{\alpha} \right) \frac{dp}{dQ_i} + \frac{p}{c_i} \frac{dp}{dQ_i}. \quad (51)$$

It follows from (48) that:

$$\frac{dp}{dQ_i} = \alpha. \quad (52)$$

Hence, we have:

$$\frac{d\pi_i}{dQ_i} = p + \left( Q_i - e_i^o - \frac{b_i}{\alpha} \right) \alpha + \frac{p}{c_i} \alpha. \quad (53)$$

Substitution from (48) and rewriting give that:

$$\frac{d\pi_i}{dQ_i} = \alpha Q_i - b_i - \alpha e_i^o + \left( \alpha + \frac{\alpha^2}{c_i} \right) \left( \sum_{j \in N} Q_j - \sum_{j \in N} e_j \right). \quad (54)$$

The first order conditions  $d\pi_i/dQ_i = 0$  yields the reaction curve of country  $i$ :

$$Q_i = \frac{1}{2\alpha + \frac{\alpha^2}{c_i}} \left( b_i + \alpha e_i^o - \left( \alpha + \frac{\alpha^2}{c_i} \right) \left( \sum_{j \neq i} Q_j - \sum_{j \in N} e_j^o \right) \right), \quad (55)$$

which could be rewritten to:

$$Q_i = e_i + \frac{c_1 b_1}{\alpha(\alpha + 2c_1)} - \frac{\alpha + c_1}{\alpha + 2c_1} \left( \sum_{j \neq 1} Q_j - e_i^o \right). \quad (56)$$

(56) represents the  $n$  reaction curves and constitutes  $n$  equations in the  $n$  unknown variables  $Q_j$ ,  $j = 1, \dots, n$ . Solving these  $n$  equations, using that  $Q_i^u = e_i^o - a_i^u$  and substituting from (47) gives the number of permits allocated by the government of country  $i$  in the non-cooperative Nash equilibrium in the non-cooperative case with linked permit markets:

$$Q_i^* = Q_i^u - \left( \sum_{j \neq i} \frac{1}{c_j} \right) \left( b_i - \frac{1}{n} \sum_j b_j \right). \quad (57)$$

It should here be added that derivation of (57) is not straightforward. We applied (56) to the three and four country cases and used the results to find the solution of the  $n$ -country case.

The aggregate allocation  $Q^*$  given in (10) follows by summing from (57).



## A.2 Derivation of the permit price

In the following it will be shown that  $p = \bar{b}$ . Insert (10) into (48):

$$p = \alpha \left( \sum_{i \in N} Q_i^u + \sum_{i \in N} \left[ \frac{1}{c_i} \left( b_i - \frac{1}{n} \sum_j b_j \right) \right] - \sum_{j \in N} e_j^o \right) \quad (58)$$

Using that  $Q_i^u = e_i^o - a_i^u$  and some rewriting gives that  $p = \bar{b}$ .

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