Arbeidsnotater

TATISTISK SENTRALBYRÅ

ronningensgt. 16, Oslo-Dep., Oslo 1. Tlf. 41 38 20, 41 36 60

WORKING PAPERS FROM THE CENTRAL BUREAU OF STATISTICS OF NORWAY

IO 72/6 23 March 1972

STUDIES IN THE STABILITY OF INPUT-OUTPUT RELATIONSHIPS EFFECTS OF AGGREGATION AND CHANGES IN COEFFICIENTS ON THE RESULTS OF INPUT-OUTPUT ANALYSES

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CONTENTS

I. INTRODUCTION

In the Central Bureau of Statistics of Norway a program of research into the stability of input-output coefficients has been in progress for several years. In an earlier paper[1] changes in input-output coefficients were studied on the basis of Norwegian data for the period 1949-60. Also the effects of aggregation on the stability of input-output coefficients were studied. However, it is not the stability or instability of the direct input-output coefficients themselves which is important, but the effects of possible instability on the precision in estimates based on input-output analyses.[2]

The present study is concerned with the effects of coefficient changes on the results of input-output computations. By using data for the same period, for which we have already studied the variations in the coefficients, we obtain some insight in the relationships between coefficient variation and precision in results. In particular, we study the effects on the precision in estimates of increases in the distance in time from the base year.

The data and methods used in this study are described in chapter II, and measures of the precision in estimates are developed in chapter III and more formally described in the appendix to that chapter.

The effects on precision in estimates of increases in the distance in time from the base year are of particular importance. In chapter IV it is shown that this effect is considerable, so that there is an increase in the errors of around 30 per cent of the first year error per year of distance between the base period and the estimation period. Instead of a coefficient matrix based on the observations for one single year, matrices giving average coefficients for a period of for example 3 years may be used. The result of such a procedure is a reduction of the random disturbances in the coefficients, but at the same time, the distance between (the center of) the basis period and the estimation period is increased. It is shown in chapter V that in our data the latter effect appears to dominate in the 4-8 years nearest to the basis period.

The effects of aggregation have been much discussed in literature on input-output analysis, but mainly with reference to a correct basis matrix. In chapter VI it is shown that when coefficients are used for a period outside the basis period, the effects of aggregation will depend on the changes that have occurred in the coefficients since the basis period[3] as well as on changes in the distribution of final deliveries between aggregate sectors and between detailed sectors within each aggregate sector. The argument, which is illustrated by the results of our computations, is set out formally in the appendix to chapter VI.

The sources of aggregation errors are further analysed in chapter VII, where it is shown that the relatively small magnitudes of aggregation errors in our data are due to the stability of final delivery proportions. Chapter VIII contains a number of special analyses of the errors in our test results. It is divided into 5 sections on A Sector by sector comparisons of errors, B size distributions of sector errors, C time patterns of errors, D identification and decomposition of large individual errors and E differences in error patterns for different time periods.

II. DATA AND METHOD

The data utilized for this study consisted of input-output accounts in constant, 1955-price-values, for 92 Norwegian sectors of production, 66 "import-sectors" and 9 "transfer" accounts for the period 1949-1960. Imports were grouped into sectors according to production sector of origin and the transfer accounts were used for certain summary adjustments.

Imports were treated as "structural", i.e. accounted for as separate deliveries from "foreign sectors" to the sectors of production where they were actually used as inputs and to the sectors of final use where they were actually consumed.

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1) Per Sevaldson: "The Stability of Input-Output Coefficients" in "Applications of Input-Output Analysis" Eds. A.P. Carter and A.Brody. Amsterdam, London 1969. Also as "Artikler" No. 32 from the Central Bureau of Statistics of Norway.

2) Per Sevaldson: "Changes in Input-Output Coefficients" in Ed. Tibor Barna: "Structural Interdependence and Economic Development", London 1963.

3) Compare also the reference in footnote 1) above.

These accounts were only available in purchasers' price values. In addition, input-output accounts in a specification of 133 Norwegian sectors of production, 118 import-sectors and 9 transfer sectors for the period 1959-1961 were also available in constant 1955-price-values. These accounts were available both in purchasers' and in producers' price values, but only the figures in purchasers' price values were used.

The 133 sector accounts for 1960 were aggregated successively to 92 sectors of production, 66 sectors of import and 9 transfer sectors, to 36 sectors of production, 32 sectors of import and 1 transfer sector, to 15 sectors of production, 12 sectors of import and 1 transfer sector and to 7 sectors of production, 6 sectors of import and 1 transfer sector. The aggregations were made on a gross basis, so that intrasector deliveries were not netted out in aggregation. The sector-specifications and their mutual relationships are given in appendix table I. When sectors are grouped, transfer sectors are treated as belonging to the group of import sectors.

From a practical point of view it would have been preferable that the aggregations had been step-wise, so that the 92 sector-specification could be obtained by consolidating the 133 sector-specification, the 36 sector-specification by consolidating the 92 sector-specification etc. up to the 7 sector-specification. However, the Central Bureau of Statistics has for various reasons earlier used particular 36, 15 and 7 sector-specifications, and it was thought advantageous to try out these particular specifications.

For each level of aggregation of the 1960-matrix we computed the matrix of direct input-output coefficients, i.e. direct requirements of intermediate deliveries from each Norwegian production sector per unit (krone) of product value in each sector of production (the A-matrix) and the corresponding matrix of direct requirements from each import and transfer sector per unit (krone) of product value in each sector of production (the B-matrix).

For each level of aggregation then the "inverse"-matrices could be computed ($(I-A)^{-1}$ and $B(I-A)^{-1}$), giving the direct plus indirect requirements for intermediate products, imports and transfers, all specified by sector of origin, per unit (krone) value of final delivery from each Norwegian sector of production in 1960.

In addition a set of "augmented accounts" for the period 1959-1961 was constructed by adding corresponding entries for the three years in the 133 sector accounts. On this basis average direct and direct plus indirect requirements matrices were computed in the 92-sector and in the 36-sector specifications.

We had thus the following sets of "inverse" or direct plus indirect requirements matrices:

Set no.	Number of production sectors[1)]	Number of import and transfer sectors	Base period
1	133	127	1960
2	92	75	1960
3	92	75	1959-1961
4	36	33	1960
5	36	33	1959-1961
6	15	13	1960
7	7	7	1960

1) In the sequel we will refer to specification levels either by the number of domestic production sectors or by both the number of domestic and the number of import and transfer sectors, with a / between these two figures.

By using these inverses on the accounts figures for final deliveries from each Norwegian sector of production in the appropriate sector specification in the period 1949-1960 we could obtain hypothetical estimates of intermediate deliveries from each production, import and transfer sector. These estimates could be compared with accounts figures, and measures of fit could be computed.

We also computed the total sum of all intermediate deliveries from domestic sectors and from import and transfer sectors as percentages of total final demand and of total gross national product in 1960. These percentages were then applied to total final demand ("Final demand blow-ups"), and total gross national product, ("GNP-blow-ups") in each of the years 1949-1960, and the estimates could be compared with the input-output estimates.

The accounts gave figures for intermediate and final deliveries in the period 1949-1960 in the 92-sector specification. However, we needed figures in the 133 sector specification, both if we were to use the 133-sector inverse, and because the aggregation to 36 sectors was based on the 133-sector-specification.

A breakdown of final deliveries into 133 sectors for each of the years 1949-1959, was constructed by assuming final deliveries from each of the 92 sectors each year to be composed in the same proportion of deliveries from sectors in the 133 specification as in 1960. Intermediate deliveries were broken down in the same way. This procedure, which was rather arbitrary, affects the results for our analysis in two ways: It introduces errors in the final demand estimates to which the sets of inverses are applied, and must thus be expected to lead to bigger errors in the hypothetical estimates of intermediate deliveries than what we would have obtained on the basis of correct figures for final deliveries. This type of errors may be important for our estimates at the 133 sector level, and to a limited extent at the 36 sector level, namely to the extent that these specifications cannot be obtained by aggregation from the 92 sector-specification. The other type of error is caused by the fact that we compare our hypothetical estimates of intermediate deliveries with erroneous "accounts" figures. This type of error will also affect our results at the 133 sector level and at the 36 sector level to the same extent as the first type[1].

III. MEASURES OF PRECISION

We are in this study <u>not</u> concerned with estimates of requirements for inputs in <u>individual</u> sectors of production. Estimates of such requirements are in the input-output model related to sector outputs, and should be studied on the basis of the direct requirement (input-output) coefficients.

In the present study we want to analyse the model's efficiency in predicting the total direct and indirect requirements for intermediate goods in the entire economy. We may study these estimates in total for each year, or the annual figures broken down by domestic and foreign origin and each of these again broken down by sector of origin in alternative sector-specifications. The limit for the sector breakdown is for each set of estimates determined by the sector-specification in the matrix used to compute the estimates. Thus estimates in the 133 sector-specification can be aggregated to each of the other specification levels up to totals for all sectors, and at each level we may compare estimates with accounting figures or with other estimates made at or aggregated to the same level of specification. Estimates in the 92 sector-specification cannot be aggregated quite so easily to the other specification levels, and estimates at the 36 sector level can be aggregated to the 7 sector level and to totals, but not to the 15 sector level. In some comparisons we use a 33 sector-specification, which is obtainable by aggregation both from the 92 sector and from the 36 sector-specifications.

The following table indicates for each set of estimates at which aggregation levels it can be compared to other estimates and to accounts.

Set of estimates			Base period	Specification levels for comparisons Production sectors/Import plus transfer sectors					
Set no.	Production sectors	Import, transfer sectors		92/75	36/33	33[1]/	15/13	7/7	1/1
1	133	127	1960	x	x	(x)	x	x	x
2	92	75	1960	x		x		x	x
3	92	75	1959-1961	x		(x)		(x)	x
4	36	33	1960		x	x		x	x
5	36	33	1959-1961		x	(x)		(x)	x
6	15	13	1960				x		x
7	7	7	1960					x	x
8[2]	1	1	1960						x
9[3]	1	1	1960						x

1) Whereas the 36 sector-specification level cannot be achieved by aggregation from the 92 sector level, a 33 sector-specification may be achieved by aggregation both from the 92 and the 36 sector-specification levels. 2) "Final demand blow-ups". 3) "GNP-blow-ups".

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[1] We return to this point in chapter VI appendix.

Comparisons can be made between estimates marked by an x or (x) in the same column, at the specification level indicated at the top of the column. The (x) indicates estimates which were not actually used in comparisons at the indicated specification level in the present study.

A direct measure of precision is the difference between the estimate and the corresponding "corret" accounting figure. These differences may be measured in absolute value, or as percentages of the corret values. These measures for the total of domestic intermediate deliveries and for the total of imported intermediate deliveries are given in tables 1 and 2 a and b.

Table 1.a. Errors in estimates of total domestic deliveries to intermediate uses per year. Million kroner at constant (1955-) prices

Year	Accounts figure		Error (= estimate minus accounts) for estimates based on								
	Total	Change from previous year	133 sector matrix 1960	92 sector matrix 1959-61	92 sector matrix 1960	36 sector matrix 1959-61	36 sector matrix 1960	15 sector matrix 1960	7 sector matrix 1960	Total final demand 1960	GNP 1960
1949	14 569	.	367	353	406	416	506	667	337	-1 018	-455
1950	15 246	677	625	587	645	706	797	791	557	-1 019	-434
1951	15 548	302	866	804	859	924	1 014	898	797	-582	63
1952	16 183	635	646	584	656	662	763	861	570	-921	-16
1953	16 812	629	572	511	591	582	689	748	416	-835	-6
1954	17 752	940	706	646	724	695	797	834	599	-759	-104
1955	18 613	861	169	82	160	120	229	193	98	-998	-564
1956	19 231	618	296	234	324	238	359	360	175	-595	-236
1957	19 409	178	358	250	348	257	380	353	285	-290	174
1958	19 274	-135	161	60	162	89	219	211	199	25	296
1959	20 122	848	355	248	353	271	405	394	400	7	2
1960	21 605	1 483	-	-96	-	-132	-	-	-	-	-
Averages[1]											
1949-1960 ..	17 864	664[2]	427	371	436	424	513	526	369	587	196
1949-1958 ..	17 264	553[2]	477	411	488	469	575	592	403	704	235
1949-1954 ..	16 018	637[2]	630	581	647	664	761	800	546	856	180
1955-1958 ..	19 132	448	246	157	249	176	297	279	189	477	318

1) Averages of numerical values. 2) From 1950.

Table 1.b. Errors in estimates of total domestic deliveries to intermediate uses. Per cent of accounts figures

Year	Error (= estimates minus accounts) for estimates based on								
	133 sector matrix 1960	92 sector matrix 1959-61	92 sector matrix 1960	36 sector matrix 1959-61	36 sector matrix 1960	15 sector matrix 1960	7 sector matrix 1960	Total final demand 1960	GNP 1960
1949	2.5	2.4	2.8	2.9	3.5	4.6	2.3	-7.0	-3.1
1950	4.1	3.9	4.2	4.6	5.2	5.2	3.7	-6.7	-2.8
1951	5.6	5.2	5.5	5.9	6.5	5.8	5.1	-3.7	.4
1952	4.0	3.6	4.1	4.1	4.7	5.3	3.5	-5.7	-.1
1953	3.4	3.0	3.5	3.5	4.1	4.4	2.5	-5.0	-
1954	4.0	3.6	4.1	3.9	4.5	4.7	3.4	-4.3	-.6
1955	.9	.4	.9	.6	1.2	1.0	.5	-5.4	-3.0
1956	1.5	1.2	1.7	1.2	1.9	1.9	.9	-3.1	-1.2
1957	1.8	1.3	1.8	1.3	2.0	1.8	1.5	-1.5	.9
1958	.8	.3	.8	.5	1.1	1.1	1.0	.1	1.5
1959	1.8	1.2	1.8	1.4	2.0	2.0	2.0	-	-
1960	-	-.4	-	-.6	-	-	-	-	-
Averages[1]									
1949-1960	2.5	2.2	2.6	2.5	3.1	3.1	2.2	3.5	1.1
1949-1958	2.9	2.5	2.9	2.9	3.5	3.6	2.4	4.2	1.4
1949-1954	3.9	3.6	4.0	4.2	4.8	5.0	3.4	5.4	1.2
1955-1958	1.3	.8	1.3	.9	1.6	1.5	1.0	2.5	1.7

1) Averages of numerical values.

Table 2.a. Errors in estimates of total import 1) deliveries to intermediate uses. Million kroner at constant (1955-) prices.

Year	Accounts figure		Error (= estimate minus accounts) for estimates based on								
	Total	Change from previous year	133 sector matrix 1960	92 sector matrix 1959-61	92 sector matrix 1960	36 sector matrix 1959-61	36 sector matrix 1960	15 sector matrix 1960	7 sector matrix 1960	Total final demand 1960	GNP 1960
1949	4 973	.	459	422	465	418	459	535	620	676	909
1950	5 395	422	490	445	492	441	484	566	559	535	778
1951	5 533	138	638	590	642	647	694	674	716	705	973
1952	5 645	112	599	556	603	614	659	709	756	716	1 093
1953	5 900	255	680	642	684	644	686	723	783	760	1 105
1954	6 613	713	379	335	385	379	428	432	541	470	742
1955	6 980	367	276	231	278	279	327	346	411	363	542
1956	7 174	194	447	410	453	479	527	520	552	594	743
1957	7 356	182	527	488	529	544	591	610	612	613	806
1958	7 435	79	455	419	457	394	441	495	519	609	721
1959	8 214	779	183	138	179	123	170	196	182	176	294
1960	9 005	791	-	-39	-	-51	-	-	-	-	-
Averages 2)											
1949-1960 ...	6 685	366 3)	428	393	431	418	456	484	521	518	726
1949-1958 ...	6 300	274 3)	495	454	499	484	530	561	607	604	841
1949-1954 ...	5 677	328 3)	541	498	545	524	568	607	663	644	933
1955-1958 ...	7 236	206 3)	426	387	429	424	472	493	524	545	703

1) Import and transfers. 2) Averages of numerical values. 3) From 1950.

Table 2.b. Errors in estimates of import 1) deliveries to intermediate uses. Per cent of accounts figures

Year	Error(=estimate minus accounts)for estimates based on								
	133 sector matrix 1960	92 sector matrix 1959-61	92 sector matrix 1960	36 sector matrix 1959-61	36 sector matrix 1960	15 sector matrix 1960	7 sector matrix 1960	Total final demand 1960	GNP 1960
1949	9.2	8.5	9.3	8.4	9.2	10.8	12.5	13.6	18.3
1950	9.1	8.3	9.1	8.2	9.0	10.5	10.4	9,9	14,4
1951	11.5	10.7	11.6	11.7	12.5	12.2	12.9	12.8	17.6
1952	10.6	9.9	10.7	10.9	11.7	12.6	13.4	12.7	19.4
1953	11.5	10.9	11.6	10.9	11.6	12.3	13.3	12.9	18.8
1954	5.7	5.1	5.8	5.7	6.5	6.5	8.2	7.1	11.2
1955	4.0	3.3	4.0	4.0	4.7	5.0	5.9	5.2	7.8
1956	6.2	5.7	6.3	6.7	7.3	7.2	7.7	8.3	10.4
1957	7.2	6.6	7.2	7.4	8.0	8.3	8.3	8.3	11.0
1958	6.1	5.6	6.2	5.3	5.9	6.7	7.0	8.2	9.7
1959	2.2	1.7	2.2	1.5	2.1	2.4	2.2	2.1	3.6
1960	-	-.4	-	-.6	-	-	-	-	-
Averages 2)									
1949-1960	6.9	6.4	7.0	6.7	7.4	7.9	8.5	8.4	11.9
1949-1958	8.1	7.5	8.2	7.9	8.6	9.2	10.0	9.9	13.9
1949-1954	9.6	8.9	9.7	9.3	10.1	10.8	11.8	11.5	16.6
1955-1958	5.9	5.3	5.9	5.9	6.5	6.8	7.2	7.5	9.7

1) Import plus transfers. 2) Averages of numerical values.

However, when we want to study comparative precision in sector detail, we also need more summary measures. As a convenient measure we have taken the square roots of means of squared errors. These "root-mean-square" or standard errors may be computed over the years for each sector in a given sector-specification, or they may be computed over the sectors in a given sector-specification for each year.

When this measure is taken over the years for individual sectors, we have in this study included all the 12 years. This implies that for the estimates based on 1960-matrices we include the base year, where the error is identically zero. For an assessment of the actual sizes of the errors, it might have been better to exclude the base year. This would imply multiplying all figures by 1.044, or an increase by about 4½ per cent in all standard errors.

For estimates based on the average 1959-61 matrices, including all twelve years implies inclusion of two of the base years, but in these years the errors do not vanish, even if they are generally small. In this case it is not a simple matter to estimate the effects on the computed standard errors of excluding the base years. But also in this case the standard errors would increase.

Standard errors might have been computed both on estimates in absolute (i.e. in 1955-kroner) values and on errors in per cent of the corresponding correct values. Our computations did only give the standard errors on the basis of errors in absolute values. However, when we compare standard errors taken over the 12-year period for different sectors, their magnitude will be influenced by the differences in value of the intermediate deliveries from different sectors. In order to be able to compare the errors for different sectors, we computed the standard errors as percentages of the averages over the 12 year period of the corresponding correct values. This gave us a sort of coefficients of variation. Correspondingly, for comparing standard errors taken over the sectors for individual years, we also computed these as percentages of the corresponding averages over the sectors of the correct values, thus correcting for the year to year real growth in the economy. To some extent (Ch. VI, Appendix) we have also used averages (over 11 years) of numerical errors (i.e. disregarding directions of the errors).

APPENDIX TO CHAPTER III.

Formulae for the measures of fit.

Assume that we have a full set of accounts:

x_{ij}^t = Value of deliveries from sector i to sector j in year t.

y_i^t = Value of deliveries to final uses from sector i in year t.

x_{kj}^{bt} = Value of import- or transfer-sector k-products delivered to sector j in year t.

y_k^{bt} = Value of import- or transfer-sector k-products delivered to final use in year t.

x_i^t = Value of total production in sector i in year t.

x_k^{bt} = Value of total of import- or transfer-sector k-products in year t.

$i,j = 1,2,\dots,n$
$k = 1,2,\dots,m$
$t = 1,2,\dots,T$

$$x_i^t = \Sigma_j x_{ij}^t + y_i^t \qquad (i = 1,2,\dots,n) \tag{1}$$

$$x_k^{bt} = \Sigma_j x_{kj}^{bt} + y_k^{bt} \qquad (k = 1,2,\dots,m) \tag{2}$$

We define:

$$v_i^t = \sum_{j=1}^{n} x_{ij}^t = x_i^t - y_i^t \qquad (i = 1,2,\dots,n) \tag{3}$$

$$v_k^{bt} = \sum_{j=1}^{n} x_{kj}^{bt} = x_k^{bt} - y_k^{bt} \qquad (k = 1,2,\dots,m) \tag{4}$$

$$a_{ij} = \frac{x_{ij}^{T'}}{x_j^{T'}} \qquad (i,j = 1,2,\dots,n) \tag{5}$$

$$b_{kj} = \frac{x_{kj}^{bT'}}{x_j^{T'}} \qquad (j = 1,2,\dots,n) \tag{6}$$

where T' is a base period of 1 or 3 years.

Marking estimated values with a "hat" ($\wedge$), we get:

$$\hat{x}_i^t = \hat{v}_i^t + y_i^t = \Sigma_j a_{ij} \hat{x}_j^t + y_i^t \qquad (i = 1,2,\dots,n) \tag{7}$$

$$\hat{x}_k^{bt} = \hat{v}_k^{bt} + y_k^{bt} = \Sigma_j b_{kj} \hat{x}_j^t + y_k^{bt} \qquad (k = 1,2,\dots,m) \tag{8}$$

With a_{ij} and b_{kj} given by (5) and (6), the system (7), (8) may be solved for $\hat{x}_i^t$, $\hat{x}_k^{bt}$, $\hat{v}_i^t$ and $\hat{v}_k^{bt}$, and we may find:

$$d_i^t = \hat{x}_i^t - x_i^t = \hat{v}_i^t - v_i^t \qquad \begin{matrix}(i = 1,2,\dots,n)\\(t = 1,2,\dots,T)\end{matrix}$$

$$d_k^{bt} = \hat{x}_k^{bt} - x_k^{bt} = \hat{v}_k^{bt} - v_k^{bt} \qquad \begin{matrix}(k = 1,2,\dots,m)\\(t = 1,2,\dots,T)\end{matrix}$$

$$d^t = \sum_{i=1}^{n} d_i^t \qquad (t = 1,2,\dots,T)$$

$$d^{bt} = \sum_{k=1}^{m} d_k^{bt} \qquad (t = 1,2,\dots,T)$$

We may also compute

$$s_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(d_i^t)^2} \qquad (i = 1,2,\ldots,n)$$

$$s_t = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d_i^t)^2} \qquad (t = 1,2,\ldots,T)$$

$$s_{bk} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(d_k^{bt})^2} \qquad (k = 1,2,\ldots,m)$$

$$s_{bt} = \sqrt{\frac{1}{m}\sum_{k=1}^{m}(d_k^{bt})^2} \qquad (t = 1,2,\ldots,T)$$

$$s_b = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(d^{bt})^2}$$

These "standard errors" per sector and per year may be taken as measures in million kroner of the average errors in estimates.

We may also relate these measures to the average size of the variables which are estimated:

$$p_i = 100\,\frac{s_i}{\frac{1}{T}\sum_t v_i^t} \qquad (i = 1,2,\ldots,n)$$

$$p_t = 100\,\frac{s_t}{\frac{1}{n}\sum_i v_i^t} \qquad (t = 1,2,\ldots,T)$$

$$p_{bk} = 100\,\frac{s_{bk}}{\frac{1}{T}\sum_t v_k^{bt}} \qquad (k = 1,2,\ldots,m)$$

$$p_{bt} = 100\,\frac{s_{bt}}{\frac{1}{m}\sum_k v_k^{bt}} \qquad (t = 1,2,\ldots,T)$$

$$p_b = 100\,\frac{s_b}{\frac{1}{T}\sum_t\sum_k v_k^{bt}}$$

IV. EFFECTS OF THE TIME FACTOR

A glance at the very aggregated tables 1.b. and 2.b. indicates that there are two important aspects of the time factor:

One is the number of years between the base year(s) and the year for which the estimates are made, and the other is that there are important differences in forecast precision between individual years. Both tables indicate a shift in the level of the percentage error around the middle of the period (a lower level of errors from 1955 and after in table 1 and from 1954 and after in table 2). This shift may be a merely statistical fenomenon, since accounts for years previous to 1955 originally were computed in fixed 1938-prices and subsequently converted to 1955-prices, whereas fixed price figures for 1955 and later years were computed directly by use of 1955-price data, and this may conceivably have resulted in more accurate accounts for the years after 1955.

Within each of the sub-periods 1949-1954 and 1955-1958 for domestic deliveries, and within 1949-1953 and 1954-1958 for imported deliveries, there do not seem to be systematic changes in the levels of errors, according to these two tables.

There remains then 1959, which corresponds to a one-year lag for the 1960-matrices, but which is within the base period for the average matrices. This year has a smaller error than the preceding years for imports, but not for domestic deliveries.

Within each sub-period there are considerable differences between the precision of estimates for individual years, and these differences appear to be of about the same direction and magnitude for a given year irrespective of the sector specification and base period of the matrix used in their computation.

The aggregate errors are the net results of diverging errors in the estimates for individual sectors. In tables 3 and 4.a. and b. we have reproduced the errors of estimates in the 7 sector-specification for estimates computed from three different coefficient matrices.

Table 3.a. Errors (estimate minus accounts) in estimates of domestic deliveries to intermediate uses from 7 sectors of production. Million kroner at constant (1955-) prices

Sector (and total delivery) Basis matrix (and error)	Year											
	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1. Agriculture etc.1), total.	3415	3483	3592	3706	3728	3871	3843	4240	4190	3972	4089	4278
133 sectors 1960, error ..	-324	-129	-126	-163	-181	-163	-40	-162	-216	-65	3	-
36 sectors 1960, error ..	-108	66	56	-34	-62	-46	47	-79	-156	-15	43	-
7 sectors 1960, error ..	-149	-24	-36	-86	-135	-51	33	-167	-168	-39	48	-
2. Mineral-, metal products,2) total	1501	1649	1723	1902	2109	2213	2334	2366	2478	2494	2729	2944
133 sectors 1960, error ..	434	365	322	278	262	318	210	163	195	176	48	-
36 sectors 1960, error ..	395	335	299	259	234	288	185	155	175	172	45	-
7 sectors 1960, error ..	390	338	305	262	229	279	182	152	168	161	43	-
3. Food, chemicals etc.3), total	1266	1361	1541	1611	1632	1796	2014	2072	2044	2063	2139	2453
133 sectors 1960, error ..	570	591	455	414	429	342	169	230	247	229	243	-
36 sectors 1960, error ..	591	625	500	456	451	363	187	273	259	222	246	-
7 sectors 1960, error ..	570	578	459	443	415	378	194	243	257	205	235	-
4. Wood and fiber products4), total	2413	2506	2396	2454	2649	2775	2911	2870	2941	2932	3014	3291
133 sectors 1960, error ..	-157	-101	76	41	-7	62	-57	28	48	-18	95	-
36 sectors 1960, error ..	-186	-119	68	47	9	89	-32	28	51	-33	83	-
7 sectors 1960, error ..	-138	-69	99	66	14	67	-35	48	46	-16	92	-
5. Construction, total	3	3	4	4	9	6	5	7	8	8	7	8
133 sectors 1960, error ..	2	2	1	1	-3	-	2	-	-	-	1	-
36 sectors 1960, error ..	2	2	1	1	-3	-	2	-	-	-	1	-
7 sectors 1960, error ..	1	2	1	1	-4	-	1	-	-1	-1	-	-
6. Trade and transportation, total	4154	4323	4385	4521	4691	4891	5145	5201	5200	5049	5352	5623
133 sectors 1960, error ..	-244	-206	-100	-149	-156	-107	-261	-97	-57	-57	-66	-
36 sectors 1960, error ..	-281	-213	-130	-163	-146	-127	-279	-129	-66	-23	-36	-
7 sectors 1960, error ..	-428	-357	-242	-308	-304	-282	-400	-217	-139	-35	-56	-
7. Services5), total	1817	1922	1907	1986	2067	2201	2362	2475	2548	2756	2791	3009
133 sectors 1960, error ..	85	104	238	224	229	254	145	132	141	-105	31	-
36 sectors 1960, error ..	93	102	219	196	217	230	120	111	117	-104	24	-
7 sectors 1960, error ..	90	90	211	192	201	209	123	116	121	-77	39	-

1) Agriculture, forestry, hunting and fishing. 2) Extraction and production of mineral and metal goods. 3) Production of food and tobacco, beverages, oils, fats and chemicals. 4) Products of wood, pulp and paper, printing, textiles, clothing, leather and rubber products. 5) All other activities.

Table 3.b. Errors (estimates minus accounts) in estimates of domestic deliveries to intermediate uses from 7 sectors of production. Errors in per cent of actual delivery.

Sector Basis matrix	Year											
	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1. Agriculture, etc.												
133 sectors 1960	-9.5	-3.7	-3.5	-4.4	-4.9	-4.2	-1.0	-3.8	-5.2	-1.6	0.1	-
36 sectors 1960	-3.2	1.9	1.6	-0.9	-1.7	-1.2	1.2	-1.9	-3.7	-0.4	1.1	-
7 sectors 1960	-4.4	-0.7	-1.0	-2.3	-3.6	-1.3	0.9	-3.9	-4.0	-1.0	1.2	-
2. Mineral-, metal products												
133 sectors 1960	28.9	22.1	18.7	14.6	12.4	14.4	9.0	6.9	7.9	7.1	1.8	-
36 sectors 1960	26.3	20.3	17.4	13.6	11.1	13.0	7.9	6.6	7.1	6.9	1.6	-
7 sectors 1960	26.0	20.5	17.7	13.8	10.9	12.6	7.8	6.4	6.8	6.5	1.6	-
3. Food, chemicals, etc.												
133 sectors 1960	45.0	43.4	29.5	25.7	26.3	19.0	8.4	11.1	12.1	11.1	11.4	-
36 sectors 1960	46.7	45.9	32.4	28.3	27.6	20.2	9.3	13.2	12.7	10.8	11.5	-
7 sectors 1960	45.0	42.5	29.8	27.5	25.4	21.0	9.6	11.7	12.6	9.9	11.0	-
4. Wood and fiber products												
133 sectors 1960	-6.5	-4.0	3.2	1.7	-0.3	2.2	-2.0	1.0	1.6	-0.6	3.2	-
36 sectors 1960	-7.7	-4.7	2.8	1.9	0.3	3.2	-1.1	1.0	1.7	-1.1	2.8	-
7 sectors 1960	-5.7	-2.8	4.1	2.7	0.5	2.4	-1.2	1.7	1.6	-0.5	3.1	-
5. Construction												
133 sectors 1960	66.7	66.7	25.0	25.0	-33.3	-	40.0	-	-	-	14.3	-
36 sectors 1960	66.7	66.7	25.0	25.0	-33.3	-	40.0	-	-	-	14.3	-
7 sectors 1960	33.3	66.7	25.0	25.0	-44.4	-	20.0	-	-12.5	-12.5	-	-
6. Trade and transportation												
133 sectors 1960	-5.9	-4.8	-2.3	-3.3	-3.4	-2.2	-5.1	-1.9	-1.1	-1.1	-1.2	-
36 sectors 1960	-6.8	-4.9	-3.0	-3.6	-3.2	-2.6	-5.4	-2.5	-1.3	-0.5	-0.7	-
7 sectors 1960	-10.3	-8.3	-5.5	-6.8	-6.6	-5.8	-7.8	-4.2	-2.7	-0.7	-1.0	-
7. Services												
133 sectors 1960	4.7	5.4	12.5	11.3	11.1	11.5	6.1	5.3	5.5	-3.8	1.1	-
36 sectors 1960	5.1	5.3	11.5	9.9	10.5	10.4	5.1	4.5	4.6	-3.8	0.9	-
7 sectors 1960	5.0	4.7	11.1	9.7	9.7	9.5	5.2	4.7	4.7	-2.8	1.4	-

Table 4.a. Errors (estimates minus accounts) in estimates of imports to intermediate use from 7 import sectors. Million kroner at constant (1955-) prices

Sector (and total delivery) Basis matrix (and error)	Year											
	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1. Agriculture, etc., total	682	770	730	712	701	749	857	799	655	707	760	863
133 sectors 1960, error	-20	-39	-13	-9	36	3	-96	-4	131	76	59	-
36 sectors 1960, error	-28	-41	38	25	32	44	-57	46	164	67	67	-
7 sectors 1960, error	-47	-84	-18	-8	6	10	-80	11	150	74	67	-
2. Mineral-,metal products,total	1496	1546	1581	1709	1824	2086	2112	2102	2258	2278	2386	2742
133 sectors 1960, error	38	82	106	48	84	-2	53	126	75	113	139	-
36 sectors 1960, error	18	60	101	47	62	-35	37	129	90	101	114	-
7 sectors 1960, error	98	145	188	134	149	49	107	184	137	134	135	-
3. Food, chemicals etc., total..	821	886	924	934	969	1155	1257	1339	1392	1447	1709	1789
133 sectors 1960, error	425	445	475	466	456	382	294	291	241	157	-11	-
36 sectors 1960, error	408	442	474	471	465	388	302	304	259	160	-17	-
7 sectors 1960, error	381	389	405	428	421	324	261	256	225	159	-16	-
4. Wood and fiber products,total	405	501	585	530	589	650	639	658	662	609	726	757
133 sectors 1960, error	139	96	27	62	36	10	34	38	36	51	-9	-
36 sectors 1960, error	117	77	2	46	21	-1	18	22	25	2	-7	-
7 sectors 1960, error	117	67	-2	39	9	-7	18	16	20	46	-20	-
6. Trade and transportation, total	48	60	61	62	59	75	72	80	78	77	90	88
133 sectors 1960, error	6	-1	3	3	9	1	5	2	4	1	-8	-
36 sectors 1960, error	9	4	5	7	7	6	5	5	2	4	2	-
7 sectors 1960, error	6	-2	-	-1	5	-6	-1	-4	-	-	-8	-
7. Services, total	1405	1512	1527	1567	1623	1760	1902	2046	2159	2155	2372	2573
133 sectors 1960, error	-141	-109	16	5	34	-47	-43	-30	9	47	-1	-
36 sectors 1960, error	-78	-69	56	49	78	7	-	7	26	62	8	-
7 sectors 1960, error	57	36	131	150	180	153	88	74	59	93	12	-
8. Transfers, total	116	121	125	130	135	138	142	151	151	161	170	194
133 sectors 1960, error	13	16	24	24	26	32	30	25	31	11	14	-
36 sectors 1960, error	13	15	21	21	22	28	27	23	28	13	14	-
7 sectors 1960, error	8	8	12	12	13	18	19	15	21	13	14	-

Table 4.b. Errors (estimates minus accounts) in estimates of imports to intermediate use from 7 import sectors. Errors in per cent of actual delivery

Sector Basis matrix and error	Year											
	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1. Agriculture etc.												
133 sectors 1960	-2.9	-5.1	-1.8	-1.3	5.1	0.4	-11.2	-0.5	20.0	10.7	7.8	-
36 sectors 1960	-4.1	-5.3	5.2	3.5	4.6	5.9	-6.7	5.8	25.0	9.5	8.8	-
7 sectors 1960	-6.9	-10.9	-2.5	-1.1	0.9	1.3	-9.3	1.4	22.9	10.5	8.8	-
2. Mineral-, metal products												
133 sectors 1960	2.5	5.3	6.7	2.8	4.6	-0.1	2.5	6.0	3.3	5.0	5.8	-
36 sectors 1960	1.2	3.9	6.4	2.8	3.4	-1.7	1.8	6.1	4.0	4.4	4.8	-
7 sectors 1960	6.6	9.4	11.9	7.8	8.2	2.3	5.1	8.8	6.1	5.9	5.7	-
3. Food, chemicals etc.												
133 sectors 1960	51.8	50.2	51.4	49.9	47.1	33.1	23.4	21.7	17.3	10.9	-0.6	-
36 sectors 1960	49.7	49.9	51.3	50.4	48.0	33.6	24.0	22.7	18.6	11.1	-1.0	-
7 sectors 1960	46.4	43.9	43.8	45.8	43.4	28.1	20.8	19.1	16.2	11.0	-0.9	-
4. Wood and fiber products												
133 sectors 1960	34.3	19.2	4.6	11.7	6.1	1.5	5.3	5.8	5.4	8.4	-1.2	-
36 sectors 1960	28.9	15.4	0.3	8.7	3.6	-0.2	2.8	3.3	3.8	0.3	-1.0	-
7 sectors 1960	28.9	13.4	-0.3	7.4	1.5	-1.1	2.8	2.4	3.0	7.6	-2.8	-
6. Trade and transportation												
133 sectors 1960	12.5	-1.7	4.9	4.8	15.3	1.3	6.9	2.5	5.1	1.3	-8.9	-
36 sectors 1960	18.8	6.7	8.2	11.3	11.9	8.0	6.9	6.3	2.6	5.2	2.2	-
7 sectors 1960	12.5	-3.3	-	-1.6	8.5	-8.0	-1.4	-5.0	-	-	-8.9	-
7. Services												
133 sectors 1960	-10.0	-7.2	1.0	0.3	2.1	-2.7	-2.3	-1.5	0.4	2.2	-	-
36 sectors 1960	-5.6	-4.6	3.7	3.1	4.8	0.4	-	0.3	1.2	2.9	0.3	-
7 sectors 1960	4.1	2.4	8.6	9.6	11.1	8.7	4.6	3.6	2.7	4.3	0.5	-
8. Transfers												
133 sectors 1960	11.2	13.2	19.2	18.5	19.3	23.2	21.1	16.6	20.5	6.8	8.2	-
36 sectors 1960	11.2	12.4	16.8	16.2	16.3	20.3	19.0	15.2	18.5	8.1	8.2	-
7 sectors 1960	6.9	6.6	9.6	9.2	9.6	13.0	13.4	9.9	13.9	8.1	8.2	-

These tables demonstrate that there are great variations in precision between different sectors, when the errors are measured in absolute values, as well as when they are measured as percentages of the correct (accounts) values.

Furthermore, the differences between the sectors in the level of errors seem to be stable throughout the period, and the direction of the error (over- or underestimate) tend to be the same for nearly all years for any given sector; except for sectors with small errors.

For these 7 sectors the relative level of the errors appear to be fairly constant for the years after 1955, whereas it generally increases as we go backwards from 1955 towards 1949.

For the more detailed sector specifications it is too cumbersome to study the errors for individual sectors for each year. Instead, we may look at the standard or "root-mean-square" deviations mentioned earlier. These measures are reproduced in the tables 5.a.-d. and summarized in table 6.

Table 5.a. Annual standard("root-mean-square")errors for estimates of deliveries to intermediate uses, computed on the basis of estimates and accounts aggregated to the 92 sector-specification

Year	Average delivery per sector Mill.kr.	Errors in estimates based on					
		133 sectors 1960		92 sectors 1959-61		92 sectors 1960	
		Mill.kr.	Per cent of average	Mill.kr.	Per cent of average	Mill.kr.	Per cent of average
From Norwegian sectors							
1949	158	55	35	57	36	57	36
1950	166	50	30	52	31	51	31
1951	169	45	27	45	27	46	27
1952	176	44	25	44	25	45	26
1953	183	47	26	46	25	48	26
1954	193	42	22	40	21	42	22
1955	202	36	18	31	15	36	18
1956	209	31	15	30	14	32	15
1957	211	35	17	33	16	35	17
1958	210	34	16	37	18	35	17
1959	219	23	11	25	11	23	11
1960	235	-	-	16	7	-	-
Average 1949-1958	187	42	23	42	23	43	23
Imports							
1949	66	47	71	45	68	47	71
1950	72	47	65	45	63	47	65
1951	74	39	53	36	49	39	53
1952	75	39	52	36	48	38	51
1953	79	43	54	39	49	42	53
1954	88	39	44	37	42	39	44
1955	93	33	36	31	33	33	36
1956	96	30	31	27	28	30	31
1957	98	28	29	27	28	28	29
1958	99	23	23	23	23	23	23
1959	110	17	15	20	18	18	16
1960	120	-	-	13	11	-	-
Average 1949-1958	84	37	46	35	43	37	46

Table 5.b. Annual standard("root-mean-square")errors for estimates of deliveries to intermediate uses, computed on the basis of estimates and accounts aggregated to the 36 and 33 sector-specifications

Year	36 sector-specification							33 sector-specification				
	Average delivery per sector Mill.kr.	Errors in estimates based on						Average delivery per sector Mill.kr.	Errors in estimates based on			
		133 sectors 1960		36 sectors 1959-61		36 sectors 1960			92 sectors 1960		36 sectors 1960	
		Mill. kr.	Per cent of average	Mill. kr.	Per cent of average	Mill. kr.	Per cent of average		Mill. kr.	Per cent of average	Mill. kr.	Per cent of average
From Norwegian sectors												
1949	405	96	24	103	25	109	27	441	105	24	116	26
1950	424	92	22	103	24	110	26	462	99	21	116	25
1951	432	86	20	101	23	111	25	471	92	20	116	25
1952	450	82	18	95	21	103	23	490	88	18	108	22
1953	467	85	18	87	19	95	20	509	91	18	101	20
1954	493	75	15	78	16	87	18	538	79	15	92	17
1955	517	61	12	68	13	77	15	564	64	11	80	14
1956	534	59	11	72	14	80	15	583	63	11	84	14
1957	539	62	12	59	11	68	13	588	65	11	72	12
1958	535	59	11	68	13	68	13	584	63	11	71	12
1959	559	43	8	47	8	51	9	610	46	8	53	9
1960	600	-	-	30	5	-	-	655	-	-	-	-
Average 1949-1958	480	76	16	83	18	91	20	523	81	16	96	19
Imports												
1949	151	74	49	63	42	68	45					
1950	162	73	45	64	40	70	43					
1951	168	64	38	54	32	62	37					
1952	171	63	37	52	30	60	35					
1953	179	62	35	52	29	59	33					
1954	200	54	27	46	23	47	23					
1955	212	47	22	39	18	41	19					
1956	217	45	21	36	17	42	19					
1957	223	41	18	39	18	41	18					
1958	225	32	14	35	16	33	15					
1959	249	21	8	31	12	21	8					
1960	273	-	-	25	9	-	-					
Average 1949-1958	191	56	31	48	27	52	29					

Table 5.c. Annual standard ("root-mean-square") errors for estimates of deliveries to intermediate uses, computed on the basis of estimates and accounts aggregated to the 15 sector-specification

Year	Average delivery per sector Mill.kr.	Errors in estimates based on 133 sectors 1960		15 sectors 1960	
		Mill.kr.	Per cent of average	Mill.kr.	Per cent of average
From Norwegian sectors					
1949	971	165	17	173	18
1950	1 016	153	15	162	16
1951	1 037	132	13	141	14
1952	1 079	120	11	133	12
1953	1 121	129	12	133	12
1954	1 183	110	9	121	10
1955	1 241	102	8	111	9
1956	1 282	78	6	95	7
1957	1 294	95	7	91	7
1958	1 285	99	8	94	7
1959	1 341	65	5	69	5
1960	1 440	-	-	-	-
Average 1949-1958	1 151	118	11	125	11
Imports					
1949	383	103	27	102	27
1950	412	104	25	107	26
1951	426	95	22	101	24
1952	434	90	21	105	24
1953	454	104	23	118	26
1954	509	84	17	97	19
1955	537	56	10	70	13
1956	552	65	12	82	15
1957	566	59	10	70	12
1958	572	55	10	62	11
1959	632	37	6	38	6
1960	693	-	-	-	-
Average 1949-1958	485	82	18	91	20

Table 5.d. Annual standard ("root-mean-square") errors for estimates of deliveries to intermediate uses, computed on the basis of estimates and accounts aggregated to the 7 sector-specification

Year	Average delivery per sector Mill.kr.	Errors in estimates based on 133 sectors 1960		92 sectors 1960		36 sectors 1960		7 sectors 1960	
		Mill.kr.	Per cent of average	Mill.kr.	Per cent of average	Mill.kr.	Per cent of average	Mill.kr.	Per cent of average
From Norwegian sectors									
1949	2 081	318	15	312	15	302	15	319	15
1950	2 178	284	13	281	13	287	13	290	13
1951	2 221	239	11	235	11	243	11	244	11
1952	2 312	224	10	219	9	222	10	242	10
1953	2 402	227	9	223	9	217	9	232	10
1954	2 536	215	8	211	8	205	8	224	9
1955	2 659	154	6	151	6	153	6	188	7
1956	2 747	138	5	135	5	139	5	157	6
1957	2 773	156	6	153	6	143	5	151	5
1958	2 753	108	4	119	4	114	4	97	4
1959	2 875	104	4	104	4	102	4	102	4
1960	3 086	-	-	-	-	-	-	-	-
Average 1949-1958.	2 466	206	9	204	9	203	9	214	9
Imports									
1949	710	178	25	177	25	164	23	158	22
1950	776	180	23	179	23	174	22	162	21
1951	790	185	23	184	23	185	23	176	22
1952	806	179	22	177	22	181	22	180	22
1953	843	177	21	176	21	180	21	182	22
1954	945	146	15	145	15	149	16	137	14
1955	997	121	12	120	12	118	12	116	12
1956	1 025	122	12	121	12	127	12	123	12
1957	1 051	109	10	109	10	122	12	117	11
1958	1 062	83	8	83	8	80	8	92	9
1959	1 173	58	5	56	5	51	4	58	5
1960	1 286	-	-	-	-	-	-	-	-
Average 1949-1958.	900	148	17	147	17	148	17	144	17

Table 6. Annual standard errors for estimates of deliveries to intermediate use. Per cent of average intermediate delivery per sector in corresponding sector specification. (Figures from tab.5a-d)

Year	Specification in computation of standard errors								
	92/75 sectors			36/33 sectors			7/7 sectors		
	Specification and year of basis matrix								
	133/127 sectors 1960	92/75 sectors 1959-61	92/75 sectors 1960	133/127 sectors 1960	36/33 sectors 1959-61	36/33 sectors 1960	133/127 sectors 1960	36/33 sectors 1960	7/7 sectors 1960
From Norwegian sectors									
1949	35	36	36	24	25	27	15	15	15
1950	30	31	31	22	24	26	13	13	13
1951	27	27	27	20	23	25	11	11	11
1952	25	25	26	18	21	23	10	10	10
1953	26	25	26	18	19	20	9	9	10
1954	22	21	22	15	16	18	8	8	9
1955	18	15	18	12	13	15	6	6	7
1956	15	14	15	11	14	15	5	5	6
1957	17	16	17	12	11	13	6	5	5
1958	16	18	17	11	13	13	4	4	4
1959	11	11	11	8	8	9	4	4	4
1960	-	7	-	-	5	-	-	-	-
Average 1949-58	23	23	23	16	18	20	9	9	9
Imports									
1949	71	68	71	49	42	45	25	23	22
1950	65	63	65	45	40	43	23	22	21
1951	53	49	53	38	32	37	23	23	22
1952	52	48	51	37	30	35	22	22	22
1953	54	49	53	35	29	33	21	21	22
1954	44	42	44	27	23	23	15	16	14
1955	36	33	36	22	18	19	12	12	12
1956	31	28	31	21	17	19	12	12	12
1957	29	28	29	18	18	18	10	12	11
1958	23	23	23	14	16	15	8	8	9
1959	15	18	16	8	12	8	5	4	5
1960	-	11	-	-	9	-	-	-	-
Average 1949-58	46	43	46	31	27	29	17	17	17

The figures are given in absolute values, i.e. million kroner at 1955-prices and also as percentages of the average delivery per sector to intermediate use in each year[1]. Not too much importance should be placed on the levels of these errors, neither the absolute nor the per cent figures, because there are very wide dispersions between individual sectors both in the levels of errors and in deliveries to intermediate use. However, the measures are convenient for comparisons between different years and between different sets of estimates for the same year.

All these series give an impression of increasing errors, as we move away from the base period for the coefficient matrix. In general, the impression of a somewhat slower increase in the years nearest to the base period cannot be said to be confirmed, even though some of the series (see for instance Norwegian deliveries in the 92 sector-specification) give a definite impression of a niveau in the relative levels of the errors between 1955 and 1958.

In order to eliminate as far as possible the influence of the differences that are specific of individual years, we have computed regression lines through the series of standard errors in per cent, outside the period of the basis matrix, i.e. for the years 1949-59 for the estimates based on coefficient matrices for 1960, and for the years 1949-58 for the estimates based on average coefficient matrices for 1959-61. The results are reproduced in tables 7.a. and b.

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1) It should be emphasized that these percentages are not the "root-mean-squares" of errors in per cent, but the "root-mean-square" errors as percentages of average deliveries. Taking percentages thus only serves to eliminate the effects of variations in average intermediate deliveries between the years.

Table 7.a. Regression lines of percentage standard errors. Production sectors

Sector specification in estimates	Errors in estimates based on								
	133 sectors 1960	92 sectors 1959-61	92 sectors 1960	36 sectors 1959-61	36 sectors 1960	15 sectors 1960	7 sectors 1960	Total 1960	GNP % 1960
92 sectors									
Error first year1)	11.37	12.82	11.41	.	.	.	.	.	.
Increase per year	2.13	2.22	2.19	.	.	.	.	.	.
36 sectors									
Error first year1)	7.91	.	.	10.51	9.50	.	.	.	.
Increase per year	1.53	.	.	1.64	1.81	.	.	.	.
33 sectors									
Error first year1)	.	.	7.64	.	8.82	.	.	.	.
Increase per year	.	.	1.53	.	1.80	.	.	.	.
15 sectors									
Error first year1)	4.64	..	..	.	.	4.50	.	.	.
Increase per year	1.09	..	..	.	.	1.23	.	.	.
7 sectors									
Error first year1)	2.86	..	2.85	..	2.64	..	3.09	.	.
Increase per year	1.08	..	1.06	..	1.11	..	1.09	.	.
Total									
Error first year1)	1.14	0.69	1.17	0.73	1.26	1.11	1.03	0.54	0.62
Increase per year	0.32	0.40	0.34	0.47	0.42	0.47	0.28	0.66	0.13

1) First year outside the base period.

Table 7.b. Regression lines of percentage standard error. Import sectors

Sector specification in estimates	Errors in estimates based on								
	133 sectors 1960	92 sectors 1959-61	92 sectors 1960	36 sectors 1959-61	36 sectors 1960	15 sectors 1960	7 sectors 1960	Total 1960	GNP % 1960
92 sectors									
Error first year1)	16.64	21.09	16.91	.	.	.	.	.	.
Increase per year	5.27	4.89	5.20	.	.	.	.	.	.
36 sectors									
Error first year1)	8.82	.	.	12.73	8.64	.	.	.	.
Increase per year	3.95	.	.	3.06	3.64	.	.	.	.
15 sectors									
Error first year1)	6.09	..	..	..	..	7.91	.	.	.
Increase per year	2.11	..	..	..	..	2.11	.	.	.
7 sectors									
Error first year1)	5.64	..	5.64	..	6.14	..	6.73	.	.
Increase per year	2.07	..	2.07	..	1.97	..	1.78	.	.
Total									
Error first year1)	4.10	5.04	4.17	5.62	4.56	4.64	4.80	4.90	6.52
Increase per year	0.69	0.54	0.70	0.51	0.70	0.79	0.89	0.86	1.29

1) First year outside the base period.

According to these computations the standard error would start at a level of about 11½ per cent of the average delivery to intermediate use for the year next to the base year in the 92 sector specification for domestic deliveries and around 16½ per cent for imports in the corresponding 75 sector specification of imports. The increase per year in error as we move away from the base year would be a little over 2 per cent of the average delivery for domestic deliveries and between 5 and 6 per cent for imports, that is roughly a deterioration per year amounting to about 20 and 30 per cent of the first year error respectively.

When we look at the results in more aggregate sector specifications, the average delivery per sector increases, and the level of the standard error as a percentage of average delivery is reduced. This applies both to the first year error and to the increase per year.

Our general conclusion about the importance of the time lag in input-output estimates must be that there is a rather strong deterioration in precision as the distance from the base period increases, so that the standard error 3-7 years from the base period is double that in the year closest to the base period.

The effects of differences in basis matrices will be discussed in the following chapters.

V. THE EFFECTS ON THE TIME FACTOR OF USING AN AVERAGE BASE YEAR MATRIX

In general the errors in estimates based on average coefficient matrices are smaller than the errors in estimates based on coefficient matrices for 1960 with the corresponding sector detail. (Tables 1, 2, 5 and 6). However, the differences are not great, and we have used as basis for the single year matrix the year in the middle of the period over which the averages are taken, whereas, if we were to use data for only one of these three years, it would make most sense to take the one closest to the years for which estimates were to be calculated. In our case, where we are "forecasting backwards", this would lead to the use of 1959 rather than 1960 as basis for the single year coefficient matrices.

In choosing 1960 as basis for the single year coefficient matrices, which for other reasons was convenient, we have thus made the evaluation of the merits of using the average coefficient matrices more complicated. As a consequence we will have to base our conclusions on the regression studies:

For the estimates based on average coefficient matrices the errors in the first year outside the base period according to the fitted regressions are in general higher than the first year errors in the estimates based on single year matrices, but not always. The reason for the higher values could be that the center of the base period is one year further away from the first year of estimate when we use a three year average base matrix, than when we use a one year basis. However, the way our computations were made, we cannot rule out the possibility that at least part of the difference may be due to peculiarities of the individual years or of the accounts for individual years, since the first year of estimate is 1959 when we use a one-year basis and 1958 when we use an average basis.

The increase in standard error per year is generally somewhat smaller when we use an average basis than when we use a one year basis in the same sector detail. Still the difference is in the "wrong" direction for Norwegian intermediate deliveries in the 92 sector-specification and not until we reach the 6th year away from (the nearest year of) the base period in the 36 sector-specification will we obtain better estimates of intermediate domestic deliveries according to our figures. For intermediate import deliveries the corresponding time distances are 13 years for the 75 sector-specification and 7 years for the 36 sector-specification.

For an evaluation of the effects of using an average base year matrix, it would, as already mentioned, have been preferable if our single year base matrix had been 1959, because then estimates for the same calendar year could have been compared. As it is, with 1960 as the single year base, the "first" estimates outside the base period is for 1959 with the single year base and for 1958 with the average base. We are consequently unable to assess to what extent differences particular to individual calendar years influence our results. Even when we base our conclusions on fitted trend lines, the differences in period may still have some influence. However, if we omit the year 1959 in both regression computations, then the only difference is that the period now starts two years away from the single year base period and one year away from the average base period. This change did, however, not materially change our conclusions. The results are given in table 8.

Table 8. Regression lines of percentage standard errors estimated for identical periods (1949-1958)

	Specification and basis matrix			
	92 sectors		36 sectors	
	1959-61	1960	1959-61	1960
Domestic deliveries				
Error first year	12.82	11.60	10.51	9.74
Increase per year	2.22	2.16	1.64	1.78
Number of years before average basis is best	..		6	
Imports				
Errors first year	21.09	17.33	12.73	8.93
Increase per year	4.89	5.14	3.06	3.59
Number of years before average basis is best	15		8	

We are brought to the conclusion that the reduction in random disturbances of the coefficients, which may be attained by using a three-year average base matrix appears to be outweighted by the necessary increase in the time lag between the year of the estimates and the center of the base period, at least over a period of 6 years starting from the first year outside the base period.

We may then ask if this result is due to the occurrence of a few, easily recognizable structural changes, which might perhaps have been identified from a study of the data, or on the basis of independent information, or if it is due to a large number of smaller structural changes, which cannot on inspection be distinguished from random disturbances.

We will look further into this when we analyse the results for individual sectors and groups of sectors. At present it seems justified to conclude that there does not appear to be any automatic gain in precision from a mechanical application of an average matrix, if this implies an increased distance between year of estimation and the centre of the base period.

VI. THE EFFECTS OF AGGREGATION AND COEFFICIENT CHANGE

It stands to reason, that as we go from a more to a less aggregated coefficient matrix in input-output analysis, there is a considerable probability that the precision in the more detailed estimates based on the detailed coefficient matrix will be relatively poorer than the precision in the aggregate estimates based on the aggregated coefficient matrix. But when the detailed estimates are aggregated, so that precision can be compared at the same level of detail, it seemes reasonable to expect that the estimates based on the most detailed coefficient matrix would prevail.

As is evident from our time series of standard errors (tables 1-7), analysed in the preceding section, this is also generally the case for our data, but the differences are in general only marginal, and there are some notable exceptions. We also notice, that if we are only interested in total intermediate deliveries from domestic sectors and import and transfer sectors respectively, we do not seem to lose much in precision by estimating these magnitudes as fixed percentages of total final demand or of total gross national product. For imports, judging from table 7.b., one might nearly as well conclude that there are no noticeable advantages from disaggregation, as far as precision in estimates is regarded.

We should, however, keep in mind that the "standard error per year" is a strange animal and withhold our conclusions until we have looked somewhat more closely into the matter.

Assume that we are trying to estimate intermediate deliveries in a given period from accounts figures for final demand in the period and two alternative input-output coefficient matrices, of which one is derived from accounts figures for some period outside the actual period of estimation and one is derived from the other by "gross" aggregation. The errors in estimates based on the detailed matrix will be the result of changes in coefficients from the base period. The errors in estimates based on the aggregated matrix will be a composite of these errors and what is usually referred to as "aggregation errors".

If the changes in detailed coefficients are not insignificant compared to the aggregation errors, there is no reason to believe that these two types of errors should be cumulative. They may as well be compensating, and in that case, estimates based on the aggregated matrix will be more precise than those based on the detailed matrix.

Normally, one might perhaps expect a mixture of cumulative and compensating effects, so that the errors for some sectors would be smaller and for others greater in estimates based on the aggregated matrix as compared to estimates based on the detailed matrix. This would tend to increase our measures of standard error for the estimates based on the aggregated matrix.

Obviously, if we from the errors of estimates based on some aggregated coefficient matrix deduct the corresponding errors of estimates based on a more detailed coefficient matrix, the difference will be that part of the error which is caused by aggregation from the detailed matrix. In this way we may decompose the errors in the estimates based on the 7 sector matrix into a) the error due to coefficient change in the 133 sector matrix and b) the error due to aggregation from the 133 sector-specification to the 7 sector-specification in the basis matrix. This latter aggregation error may again be decomposed into i) the error caused by aggregation from the 133 sector-specification to the 92 sector-specification ii) the error caused by aggregation from the 92 sector-specification to the 36 sector-specification and iii) the error caused by aggregation from the 36 sector-specification to the 7 sector-specification.

Such a decomposition of the errors in estimates based on the 7 sector matrix have been made in tables 9.a. and b.

Table 9.a. Decomposition of errors in estimates. Errors (estimates minus accounts) in estimates of domestic deliveries to intermediate uses from 7 sectors of production. Million kroner at constant (1955-)prices

Sector Type of error	Year										
	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
1. Agriculture etc.1)											
Error from coefficient change	-324	-129	-126	-163	-181	-163	-40	-162	-216	-65	3
Error from aggregation											
133 to 92 sectors	50	24	14	27	32	30	6	29	5	7	2
" " " 92 to 36 "	166	171	167	103	86	87	81	54	55	42	38
" " " 36 to 7 "	-42	-90	-92	-52	-72	-6	-14	-87	-12	-24	5
" " " 133 to 7 "	174	105	89	78	46	111	73	-4	48	25	45
2. Mineral-, metal products											
Error from coefficient change	434	365	322	278	262	318	210	163	195	176	48
Error from aggregation											
133 to 92 sectors	-26	-22	-19	-26	-29	-24	-17	-11	-16	-11	-9
" " " 92 to 36 "	-13	-8	-4	7	1	-7	-9	3	-4	6	6
" " " 36 to 7 "	-4	3	6	3	-5	-8	-2	-3	-7	-10	-3
" " " 133 to 7 "	-43	-27	-17	-16	-33	-39	-28	-11	-27	-15	-6
3. Food, chemicals etc.											
Error from coefficient change	570	591	455	414	429	342	169	230	247	229	243
Error from aggregation											
133 to 92 sectors	16	10	2	9	12	11	3	11	1	3	2
" " " 92 to 36 "	5	24	43	33	10	10	15	32	10	-10	-
" " " 36 to 7 "	-22	-47	-41	-13	-36	16	7	-30	-2	-17	-10
" " " 133 to 7 "	-1	-13	4	29	-14	37	25	13	9	-24	-8
4. Wood and fibre products											
Error from coefficient change	-157	-101	76	41	-7	62	-57	28	48	-18	95
Error from aggregation											
133 to 92 sectors	-	-	-1	-1	-3	-2	-2	-	-1	-2	-2
" " " 92 to 36 "	-29	-18	-7	7	19	29	26	-1	4	-13	-11
" " " 36 to 7 "	48	50	31	19	5	-23	-2	20	-5	17	10
" " " 133 to 7 "	19	32	23	25	21	4	22	19	-2	2	-3
5. Construction											
Error from coefficient change	2	2	1	1	-3	-	2	-	-	-	1
Error from aggregation											
133 to 92 sectors	-	-	-	-	-	-	-	-	-	-	-
" " " 92 to 36 "	-	-	-	-	-	-	-	-	-	-	-
" " " 36 to 7 "	-1	-	-	-	-1	-	-	-1	-1	-1	-
" " " 133 to 7 "	-1	-	-	-	-1	-	-	-1	-1	-1	-
6. Trade and transportation											
Error from coefficient change	-244	-206	-100	-149	-156	-107	-261	-97	-57	-57	-66
Error from aggregation											
133 to 92 sectors	-3	-1	-3	1	5	-18	2	-2	-	3	5
" " " 92 to 36 "	-35	-6	-27	-14	5	-2	-20	-30	-9	30	25
" " " 36 to 7 "	-147	-144	-112	-145	-158	-155	-121	-88	-73	-12	-20
" " " 133 to 7 "	-185	-151	-142	-158	-148	-175	-139	-120	-82	21	10
7. Sources											
Error from coefficient change	85	104	238	224	229	254	145	132	141	-105	31
Error from aggregation											
133 to 92 sectors	20	1	-1	1	1	1	-1	1	-1	-	-
" " " 92 to 36 "	-12	-3	-18	-28	-13	-25	-24	-23	-24	1	-7
" " " 36 to 7 "	-3	-12	-8	-4	-16	-20	3	6	4	27	15
" " " 133 to 7 "	5	-14	-27	-31	-28	-44	-22	-16	-21	28	8
Total											
Error from coefficient change	366	626	866	646	573	706	168	294	358	160	355
Error from aggregation											
133 to 92 sectors	57	12	-8	11	18	-2	-9	28	-12	-	-2
" " " 92 to 36 "	82	160	154	108	108	92	69	35	32	56	51
" " " 36 to 7 "	-171	-240	-216	-192	-283	-196	-129	-183	-96	-20	-3
" " " 133 to 7 "	-32	-68	-70	-73	-157	-106	-69	-120	-76	36	46

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1) See table 3.a, notes, for more comprehensive sector designations.

Table 9.b. Decomposition of errors in estimates. Errors (estimates minus accounts) in estimates of imports to intermediate uses from 7 import sectors. Million kroner at constant (1955-) prices

Sector Type of error	Year										
	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
1. Agriculture etc.											
Error from coefficient change	-20	-39	-13	-9	36	3	-96	-4	131	76	59
Error from aggregation											
127 to 75 sectors	8	6	3	5	6	6	2	5	1	2	-
" " " 75 to 33 "	-16	-8	48	29	-10	35	37	44	32	-11	8
" " " 33 to 7 "	-19	-43	-56	-32	-26	-34	-23	-34	-14	7	-
" " " 127 to 7 "	-27	-45	-5	2	-30	7	16	15	19	-2	8
2. Mineral-, metal products											
Error from coefficient change	38	82	106	48	84	-2	53	126	75	113	139
Error from aggregation											
127 to 75 sectors	-	-	4	-	-2	-	-	-	1	-1	-4
" " " 75 to 33 "	-20	-22	-9	-1	-20	-33	-16	2	14	-11	-20
" " " 33 to 7 "	80	85	88	87	87	84	70	56	48	32	20
" " " 127 to 7 "	60	63	83	86	65	51	54	58	63	20	-4
3. Food, chemicals etc.											
Error from coefficient change	425	445	475	466	456	382	294	291	241	157	-11
Error from aggregation											
127 to 75 sectors	-4	-4	-4	-3	-3	-2	-2	-1	-1	-1	-1
" " " 75 to 33 "	-13	1	3	8	12	8	10	14	19	5	-5
" " " 33 to 7 "	-27	-53	-69	-42	-44	-64	-42	-47	-34	-2	1
" " " 127 to 7 "	-44	-56	-70	-37	-35	-58	-34	-34	-16	2	-5
4. Wood and fibre products											
Error from coefficient change	139	96	27	62	36	10	34	38	36	51	-9
Error from aggregation											
127 to 75 sectors	-	-	-	1	2	1	1	-	1	1	1
" " " 75 to 33 "	-22	-19	-25	-16	-17	-12	-17	-16	-13	-51	1
" " " 33 to 7 "	-	-10	-4	-8	-12	-6	-	-6	-4	44	-13
" " " 127 to 7 "	-22	-29	-29	-23	-27	-17	-16	-22	-16	-6	-11
6. Trade and transportation											
Error from coefficient change	6	-1	3	3	9	1	5	2	4	1	-8
Error from aggregation											
127 to 75 sectors	1	-	-	-	1	-	-	-	-	-	-
" " " 75 to 33 "	3	5	2	4	-3	5	-	3	-2	3	10
" " " 33 to 7 "	-4	-6	-5	-7	-1	-12	-6	-9	-2	-4	-10
" " " 127 to 7 "	-	-1	-3	-3	-3	-7	-6	-6	-4	-1	-
7. Services											
Error from coefficient change	-141	-109	16	5	34	-47	-43	-30	9	47	-1
Error from aggregation											
127 to 75 sectors	-	-	-	-	-	-	-	1	-	-	-
" " " 75 to 33 "	63	40	40	43	44	54	43	36	17	15	8
" " " 33 to 7 "	134	105	75	101	102	146	88	67	33	31	4
" " " 127 to 7 "	197	145	115	144	146	200	131	104	50	46	12
8. Transfers											
Error from coefficient change	13	16	24	24	26	32	30	25	31	11	14
Error from aggregation											
127 to 75 sectors	-	-	-	-	-	-	-	-	-	-	-
" " " 75 to 33 "	-	-1	-3	-3	-4	-4	-3	-2	-3	2	-
" " " 33 to 7 "	-5	-7	-9	-9	-9	-10	-8	-8	-7	-	-
" " " 127 to 7 "	-5	-8	-12	-12	-13	-14	-11	-10	-10	2	-

Corresponding decompositions could be made of estimates based on less aggregated matrices. Looking at the decomposition in the 7 sector-specification, we notice a difference in order of magnitude between the errors due to coefficient change and the errors due to aggregation. When we consider the entire jump from the 133/127 sector-specification to the 7/7 sector-specification, the numerical value of the aggregation error is on the average only 1/3 of the numerical value of the error due to coefficient change. This is perhaps most easily seen when we compute averages over the estimation period of the numerical values (disregarding signs) of the errors. Table 10.

Table 10. Decomposition of errors in estimates. Average numerical values 1949-1959 of errors (estimates minus accounts) in estimates of intermediate inputs from 7 sectors of production and 7 import sectors

	Average numerical value of error	Of this due to:					
			Aggregation				
		Change in coefficients	133/127 sectors to 92/75 sectors	92/75 sectors to 36/33 sectors	36/33 sectors to 7/7 sectors	133/127 sectors to 7/7 sectors	
	Million kroner at constant (1955-) prices						Pct.of change error
PRODUCTION SECTORS							
1. Agriculture etc.	85	143	21	95	45	73	51
2. Mineral-,metal products ...	228	252	19	6	5	24	10
3. Food, chemicals etc.	362	356	7	17	20	16	5
4. Wood and fibre products ...	63	63	1	15	21	16	25
5. Construction	1	1.1	-	-	-	0.5	45
6. Trade and transportation .	252	136	4	18	107	121	89
7. Services	134	153	3	16	12	23	15
All production sectors1) ..	161	158	8	24	30	39	25
IMPORT SECTORS							
1. Agriculture etc.	50	44	4	25	26	16	36
2. Mineral-,metal products ...	133	79	1	15	67	55	70
3. Foods, chemicals etc.	297	331	2	10	39	36	11
4. Wood and fibre products ...	33	49	1	19	10	20	41
6. Trade and transportation ..	3	4	-	4	6	3	75
7. Services	94	44	-	37	81	117	266
8. Transfers	14	22	-	2	6	9	41
All import sectors1)	89	82	1	16	34	37	45
All sectors2)	125	120	5	20	32	38	32

1) Averages of numerical errors for all 7 sectors.
2) Averages of numerical errors for all 14 sectors.

If we look at numerical averages of errors there are great variations from sector to sector among the 14 production and import sectors, both in the level of the error due to coefficient change and in the relations between this error and the various aggregation errors.

Only for one of the import sectors, Services, is the total aggregation error (127 to 7 sectors) greater than the error due to coefficient change. In two more import sectors and two production sectors, were the total aggregation errors more than half the error due to coefficient change. By and large the aggregation error appears to be relatively more important for import sectors (on the average 45 per cent of the error due to change) than for production sectors (aggregation error is here 25 per cent of the error due to change, on the average).

The two types of error are to a considerable extent compensating. For five domestic and four import sectors the average value of the total numerical error is not greater than that part of this error which is caused by coefficient change alone, and only for one sector of each type do the numerical components appear to be directly additive.

Many details concerning sectoral differences are lost at the 7 sector-specification level. It is therefore of interest also to study the decomposition of errors at the 36 sector-specification level. The numerical averages of components of the estimation errors for the 36 production sectors are given in table 11.

The impression of the dominance of the errors due to coefficient change at the detailed specification level over errors due to aggregations from this level is confirmed for all but a few sectors. For 26 sectors the aggregation error from 133 to 36 sectors was less than 5 million kroner, and only for 3 was it above 25 million kroner. For two of these latter sectors and for one more the aggregation error was bigger than the error due to coefficient change. Agriculture stands out with an exceptionally high aggregation error.

Table 11. Decomposition of errors in estimates. Average numerical values 1949-1959 of errors (estimates minus accounts) in estimates of intermediate inputs from 36 production sectors

Sector	Average numerical value of error	Of this due to			
		Change in coefficients	Aggregation		
			133 to 36 sectors	133 to 92 sectors	92 to 36 sectors
11. Agriculture	226	60	188	22	165
12. Forestry	154	140	14	3	13
13. Fishing, whaling	98	49	56	1	58
21. Mining	33	34	1	1	2
22. Non-metallic mineral products	98	95	2	1	3
23. Basic metal industries	27	44	21	17	4
24. Metal products	31	33	4	..	..
25. Machinery	5	5	-	..	..
26. Transport equipment	3	2	1	..	..
24, 25, 26. Iron and metal products	38	39	4	2	4
27. Ship-building industries	30	31	1	1	1
28. Electrical machinery etc.	51	51	-	-	-
29. Other manufacturing	34	34	-	-	-
31. Food industries	305	262	43	10	33
32. Tobacco and beverages	1	2	3	-	3
33. Products of oils and fats	66	50	25	2	26
34. Petroleum products	14	12	2	..	..
39, 49. Chemical products	125	124	2	..	..
34, 39, 49. Chemicals	139	136	3	7	3
41. Textiles	26	26	1	1	1
42. Clothing	9	9	-	-	-
43. Footwear, leather, fur	23	22	1	-	1
44. Wood and cork etc.	57	53	4	2	3
45. Pulp, paper and paper products	49	39	15	1	16
46. Printing and publishing	37	40	5	-	5
50. Construction	1	1	-	-	-
61. Wholesale and retail trade	109	113	15	2	13
62. Water transport	40	33	10	-	10
63. Land and air transport	19	19	2	-	1
64. Communications	16	22	6	-	6
71. Electricity, gas and water	56	53	4	-	4
72. Banking and insurance	27	25	2	-	2
75. Educational, health services	1	1	1	-	1
76. Personal services	2	3	1	-	1
77. Other services	12	15	4	-	4
78. Unspecified	112	125	15	-	15
Average[1]	53	46	13	2	12

[1] Sum for all sectors divided by 36.

If we want to compare the various components of the aggregation errors, we may do this too on the basis of the averages of numerical values. (Table 10.) As could be expected (given the way our figures were derived), the errors of aggregation from the 133 sector to the 92 sector-specification level, appear to be relatively insignificant. Errors from aggregating the 92 sector level to a 36 sector level are on the average two thirds of the error caused by aggregation from the 36 to the 7 sector level. However, there are considerable compensating effects between the various components of the aggregation error, for the aggregation error from the 36 to the 7 sector level is as much as 84 per cent of the total aggregation error from the 133 to the 7 sector level on the average, and greater than this error for 6 individual production and import sectors (when the errors are measured by their numerical averages).

We are thus left with the conclusion that errors due to coefficient change in our data a outweigh and partly are compensated by aggregation errors.

It is then pertinent to determine whether this result is due to some special characteristics of our data, or if they may be accepted as conclusions with general validity.

The size of the aggregation error will in general depend on the differences between sectors which are grouped together in the aggregation process both in regard to direct and indirect input-output coefficients and in regard to relative changes from the base period in final deliveries. If input-output coefficients for all the detailed sectors in an aggregate sector are identical, <u>or</u> if the changes in final deliveries from the base year, for which the coefficients were estimated, are such that relative changes in production levels would be the same in all detailed sectors belonging to the same aggregate sector provided there were no coefficient changes, then there will be no aggregation errors. This would i.a. occur if all final deliveries changed in the same proportion. The aggregation error can thus be seen as the combined effect of 1) dispersion in coefficients of detailed sectors within the aggregate sectors and 2) relative changes from the base period in final deliveries influencing the deliveries from detailed sectors within the same aggregate sectors.

In order to bring out more clearly the influences which are decisive for the elements in the vector of aggregation errors, we may try to break it down into separate components (see the appendix to this chapter). We must then focus on how the spread in input-output coefficients for different detailed sectors within the same aggregate sector combine with changes in proportions in final deliveries to determine the vector of aggregation errors. We have already mentioned that the aggregation errors depend on changes from the base year proportions in final deliveries. We will distinguish between changes from base year proportions in total final deliveries specified by aggregate sector of origin, and changes in the composition in terms of deliveries from detailed sectors, of final deliveries from each aggregate sector. It turns out then (see the appendix) that the vector of aggregation errors may be subdivided into two additive components, one associated with changes in aggregate final delivery-proportions, and one with changes in within aggregate sectors final delivery-proportions.

When we use an aggregated coefficient matrix, we are in principle interested in the effects of changes in aggregate final delivery proportions, whereas we are not particularly interested in the final delivery proportions of detailed sectors within each aggregate sector. For this reason the two additive components of the aggregation error vector are of different significance for the analysis, and deserve to be considered separately.

As we have indicated, the spread in input-output coefficients among sectors within the same aggregate sector are also decisive for the vector of aggregation errors, and this also applies to each of its two additive components. But the ways in which this influence works, are not quite the same: The component associated with changes in aggregate final delivery proportions is depending on a square matrix consisting of differences between on the one hand the inverse of the detailed coefficient matrix, aggregated with base year final delivery weights, and on the other hand the inverse of the aggregated coefficient matrix. This difference matrix is multiplied with the vector of a deviations from base year proportions of aggregate final deliveries to give the component of the error vector, associated with these deviations, D_{AA}.

The component associated with within aggregate sector changes in final delivery proportions is depending on a rectangular matrix, obtained as the differences of the elements in an inverse of the detailed matrix in which the lines, but not the columns have been aggregated, from their averages taken over columns belonging to the same aggregate sector and weighted by base year production weights. This difference matrix is multiplied with the vector of deviations from within - aggregate sector base year proportions of final deliveries, to give the error component associated with these deviations, D_{AD}.

Thus, whereas the first matrix was a difference between an aggregated inverse and the inverse of an aggregated matrix, the latter one is a matrix of the dispersion of columnwise (sub)aggregates of coefficients of the detailed inverse about their averages, when these averages are taken linewise over each aggregate sector.

In the next chapter we shall look somewhat further into the magnitudes of these difference matrices and vectors. Here we shall only consider the two additive components which are the results of the multiplication of each difference matrix with its corresponding difference vector.

Decomposition of the aggregation errors from the 36 sector-specification to the 7 sector-specification is given in table 12 and numerical averages over the test period are given in table 13.

Table 12. Decomposition of aggregation error 36-7 sectors of production

Sector Type of error		Year										
		1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
1. Agriculture etc.												
Total	$\bar{D}_A$	-42	-90	-92	-52	-72	-6	-14	-87	-12	-24	5
Aggregate proportions	D_{AA}	-48	-37	-34	-55	-30	-24	-20	-34	-18	-26	-17
of this: weighting[1]	D_{AW}	-6	-2	-2	-10	-5	-8	-3	-5	-3	-9	-5
Detailed proportions	D_{AD}	6	-53	-58	3	-42	18	6	-53	6	2	22
2. Mineral-, metal products												
Total	$\bar{D}_A$	-4	3	6	3	-5	-8	-2	-3	-7	-10	-3
Aggregate proportions	D_{AA}	-10	-8	-5	-10	-12	-10	-6	-3	-4	-4	-3
of this: weighting	D_{AW}	-7	-6	-4	-7	-7	-5	-3	-2	-3	-3	-2
Detailed proportions	D_{AD}	6	11	11	13	7	2	4	-	-3	-6	-
3. Food, chemicals etc.												
Total	$\bar{D}_A$	-22	-47	-41	-13	-36	16	7	-30	-2	-17	-10
Aggregate proportions	D_{AA}	-13	-8	-8	-15	-3	-	-1	-11	-4	-7	-4
of this: weighting	D_{AW}	-2	-2	-2	-2	-	-	-1	-2	-1	-1	-
Detailed proportions	D_{AD}	-9	-39	-33	2	-33	16	8	-19	2	-10	-6
4. Wood and fibre products												
Total	$\bar{D}_A$	48	50	31	19	5	-23	-2	20	-5	17	10
Aggregate proportions	D_{AA}	-2	3	3	-10	-4	1	0	-7	-4	-11	-4
of this: weighting	D_{AW}	8	16	18	-3	-2	5	5	1	-1	-12	-3
Detailed proportions	D_{AD}	50	47	28	29	9	-24	-2	27	-1	28	14
5. Construction												
Total	$\bar{D}_A$	-1	-	-	-	-1	-	-	-1	-1	-1	-
6. Trade and transportation												
Total	$\bar{D}_A$	-147	-144	-112	-145	-158	-155	-121	-88	-73	-12	-20
Aggregate proportions	D_{AA}	-98	-104	-88	-88	-74	-89	-70	-63	-38	-17	-12
of this: weighting	D_{AW}	-90	-80	-67	-63	-52	-65	-51	-48	-28	-8	-6
Detailed proportions	D_{AD}	-49	-40	-24	-57	-84	-66	-51	-25	-35	5	-8
7. Services												
Total	$\bar{D}_A$	-3	-12	-8	-4	-26	-20	3	6	4	27	15
Aggregate proportions	D_{AA}	-19	-23	-16	-17	-11	-18	-14	-16	-9	-1	2
of this: weighting	D_{AW}	-13	-17	-9	-11	-6	-12	-9	-12	-7	1	4
Detailed proportions	D_{AD}	16	11	8	13	-15	-2	17	22	13	28	13
All sectors, aggregate												
Total	$\bar{D}_A$	-171	-240	-216	-192	-293	-196	-129	-183	-96	-20	-3
Aggregate proportions	D_{AA}	-190	-177	-148	-195	-134	-140	-111	-134	-77	-66	-38
of this: weighting	D_{AW}	-110	-91	-66	-96	-72	-85	-62	-68	-43	-32	-12
Detailed proportions	D_{AD}	20	-63	-68	3	-158	-56	-18	-48	-18	47	35

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1) See p. 25.

Table 13. Decomposition of aggregation error 36-7 sectors of production. Average numerical values 1949-1959

	Average numerical value of error	Of this due to:				
		Changes in coefficient, 36 sector-specification	Aggregation 36 to 7 sectors			
			Total aggregation error	Changes in aggregate proportions	Weighting differences	Changes in detailed proportions
SECTOR	$\bar{D}_T$	D_C	$\bar{D}_A$	D_{AA}	D_{AW}	D_{AD}
1. Agriculture etc.	85	65	45	31	5	24
2. Mineral-, metal products .	228	231	5	7	4	6
3. Food, chemicals etc.	362	379	20	7	1	16
4. Wood, fibre products	63	68	21	4	7	24
5. Construction	1	1	-	-	-	-
6. Trade and transportation .	252	145	107	67	51	40
7. Services	134	139	12	13	9	14
Average for all production sectors in all years	161	147	30	22	11	17
All production sectors, aggregate	403	560	158	128	67	49

Apparently the two components of the aggregation error vector are of about the same magnitude in our data, when we aggregate from the 36 sector to the 7 sector–specification level, and they are partly compensating, since the sums of their numerical values normally exceed the total aggregation error. Still, the results would on the average be considerably improved if the error due to changes in aggregate final delivery proportions (D_{AA}) could be eliminated. This would be the case if we performed the aggregation not on the original direct input-output matrix but on the inverse of the detailed matrix (using base year final delivery proportions as aggregations weights. See the appendix).

We also note that the errors due to changes in aggregate proportions appear to have more of a cumulative bias than those due to changes in detailed proportions, as appears from the aggregate figures for all the sectors.

In the appendix we also show that the component of the vector of aggregation errors which is associated with changes in aggregate final delivery proportions may be further subdivided into one component vector, D_{AW} , depending on the differences between production proportions and final delivery proportions of detailed sectors within each aggregate sector in the base year, and an additive rest vector, D_{AR}. The first of these two components is also specified in tables 12 and 13, and for a majority of the sectors it turns out to be responsible for a considerable proportion of that part of the aggregation error which is associated with changes in aggregate final delivery proportions.

This error component (D_{AW}) is of particular interest. Even if we do aggregate in the basic matrix of direct coefficients, this error may be minimized by avoiding aggregation of sectors with large divergencies in the proportions between total production and final delivery.

APPENDIX TO CHAPTER VI

We write an input-output matrix in n sectors for a given year as

(1) $A = A^0 + \Delta$

with the elements

$a_{ij} = a_{ij}^0 + \delta_{ij}$ $\quad (i,j = 1,2,\dots,n)$

where $A^0 = (a_{ij}^0)$ is the corresponding matrix for a base year

and $\Delta = A - A^0$

We assume that n-element column vectors of production (X and X^0) and final deliveries (F and F^0) are given such that

(2) $(I_n - A)X = F$

(2^0) $(I_n - A^0)X^0 = F^0$

I_n is an n by n unit matrix.

Aggregation to m sectors ($m < n$) is now performed by help of two simple matrices:

a) An m by n "summation matrix":

(3) $\Pi = (\pi_{Ij})$ $\quad (I = 1,2,\dots,m\ ;\ j = 1,2,\dots,n)$

$$\pi_{Ij} = \begin{cases} 1 & \text{for } j \in I \\ 0 & \text{otherwise} \end{cases}$$

$j \in I$ means that sector j in the n-sector-specification belongs to sector I in the m-specification.

b) An n by m "averaging matrix":

(4) $\bar{\Gamma} = (\bar{\gamma}_{iJ})$ $\quad (i = 1,2,\dots,n\ ;\ J = 1,2,\dots,m)$

$0 < \bar{\gamma}_{iJ} \leqslant 1$ for $i \in J$

$\bar{\gamma}_{iJ} = 0$ otherwise

$i_n' \bar{\Gamma} = i_m'$,

where i_r is a column vector of r 1's, and i_r' its transpose.

It follows that

(5) $\Pi \bar{\Gamma} = I_m$

$\bar{\Gamma}$ is defined such that

(6) $X^0 = \bar{\Gamma}\, \Pi\, X^0$

(i.e.

$\bar{\Gamma} = \hat{X}^0\, \Pi' (\Pi \hat{X}^0)^{-1}$ $\quad$ i.e. simply $\bar{\gamma}_{iJ} = \begin{cases} \dfrac{x_i^0}{X_J^0} & \text{for } i \in J, \\ 0 & \text{for } i \text{ not } \in J \end{cases}$

when a hat (^) indicates a vector written in the form of a diagonal matrix with zero off-diagonal elements.)

We also define

$\bar{\bar{\Gamma}}$

in exactly the same way as $\bar{\Gamma}$, only that now

(7) $F^0 = \bar{\bar{\Gamma}}\, \Pi\, F^0$

and finally we define a column vector

(8) $K = (\kappa_I)$ $\quad (I = 1,2,\dots,m)$

such that

(9) $i_m' K = 1$

(10) $\Pi F^0 = K\, i_n' F^0$

i.e. K_I is the proportion that final deliveries from aggregate sector I represents of total final deliveries in the base year.

We can now aggregate the base year matrix, to get:

$\Pi\ A^0\ \bar{\Gamma}$, the aggregate direct matrix, production weights,

$\Pi\ A^0\ \bar{\bar{\Gamma}}$, the aggregate direct matrix, final delivery weights,

$(I - \Pi\ A^0\ \bar{\Gamma})^{-1}$, the inverse of the aggregate, production weights,

$(I - \Pi\ A^0\ \bar{\bar{\Gamma}})^{-1}$, the inverse of the aggregate, final delivery weights,

$\Pi\ (I - A^0)^{-1}\ \bar{\bar{\Gamma}}$, the aggregate inverse, final delivery weights.

And we may compute the following estimates of X and Π X:

$$X = (I_n - A)^{-1}F = (I_n - A^0 - \Delta)^{-1}\ F = (I_n - A^0)^{-1}\ F + (I_n - A^0)^{-1}\ \Delta X \tag{11}$$
$$= (I_n - A^0)^{-1}\ F + \left[(I_n - A^0 - \Delta)^{-1} - (I_n - A^0)^{-1}\right]\ F$$

$$\Pi X = \Pi(I_n - A)^{-1}\ F = \Pi(I_n - A^0 - \Delta)^{-1}\ F = \Pi(I_n - A^0)^{-1}\ F + \Pi(I_n - A^0)^{-1}\ \Delta X \tag{12}$$
$$= \Pi(I_n - A^0)^{-1}\ F + \Pi\left[(I_n - A^0 - \Delta)^{-1} - (I_n - A^0)^{-1}\right]\ F$$

$$Y = (I_n - A^0)^{-1}\ F \quad (= \text{estimate of } X) \tag{13}$$

$$\Pi Y = \Pi(I_n - A^0)^{-1}\ F \quad (= \text{estimate of } \Pi X) \tag{14}$$

$$\bar{Z} = (I_m - \Pi A^0\ \bar{\Gamma})^{-1}\ \Pi F \quad (= \text{estimate of } \Pi X) \tag{15}$$

$$\bar{\bar{Z}} = \Pi(I_n - A^0)^{-1}\ \bar{\bar{\Gamma}}\Pi F \quad (= \text{estimate of } \Pi X) \tag{16}$$

Here $\bar{Z}$ is the estimate, which is usually associated with the use of an aggregated matrix. In many cases A^0 cannot be estimated, but only the aggregate $(\Pi A^0 \bar{\Gamma})$, and in this case there is no alternative to using $\bar{Z}$ as an estimate of ΠX. However, if an estimate of A^0 is available, and the aggregation is to be performed for the sake of convenience, then the estimate $\bar{\bar{Z}}$ may have considerable advantages.

It follows from (2^0) that

$$Y^0 = (I - A^0)^{-1}\ F^0 = X^0 \tag{17}$$

and from (2^0) and (6) follow

$$\Pi(X^0 - A^0X^0) = \Pi F^0 = \Pi X^0 - \Pi A^0\ \bar{\Gamma}\Pi X^0 = (I_m - \Pi A^0\ \bar{\Gamma})\ \Pi X^0 \tag{18}$$

i.e. by (15)

$$\Pi X^0 = (I_m - \Pi A^0\ \bar{\Gamma})^{-1}\ \Pi F^0 = \bar{Z}^0 \tag{19}$$

From (7) and (16) we have

$$\bar{\bar{Z}}^0 = \Pi(I_n - A^0)^{-1}\ \bar{\bar{\Gamma}}\,\Pi\,F^0 = \Pi(I_n - A^0)^{-1}\ F^0 = \Pi X^0 \tag{20}$$

Now we define the error vectors:

$$\bar{D}_T = \Pi X - \bar{Z} = \Pi(I_n - A^0 - \Delta)^{-1}\ F - (I_m - \Pi A^0\ \bar{\Gamma})^{-1}\ \Pi F \tag{21}$$

and

$$\bar{\bar{D}}_T = \Pi X - \bar{\bar{Z}} = \Pi(I_n - A^0 - \Delta)^{-1}\ F - \Pi(I_n - A^0)^{-1}\ \bar{\bar{\Gamma}}\,\Pi\,F \tag{22}$$

We could term these vectors "the total error vector, direct aggregation" and"the total error vector, inverse aggregation" respectively. (21) and (22) may be subdivided to give:

$$\bar{D}_T = \left[\Pi(I_n - A^0 - \Delta)^{-1}\ F - \Pi(I_n - A^0)^{-1}\ F\right] + \left[\Pi(I_n - A^0)^{-1}\ F - (I_m - \Pi A^0\ \bar{\Gamma})^{-1}\ \Pi F\right] = D_C + \bar{D}_A \tag{23}$$

and

$$\bar{\bar{D}}_T = \left[\Pi(I_n - A^0 - \Delta)^{-1}\ F - \Pi(I_n - A^0)^{-1}\ F\right] + \left[\Pi(I_n - A^0)^{-1}\ F - \Pi(I_n - A^0)^{-1}\ \bar{\bar{\Gamma}}\,\Pi\,F\right] = D_C + \bar{\bar{D}}_A \tag{24}$$

$$D_C = \Pi\left[(I_n - A^0 - \Delta)^{-1} - (I_n - A^0)^{-1}\right]\ F = \Pi\,X - \Pi\,Y \tag{25}$$
= the vector of errors due to coefficient change

$$\bar{D}_A = \Pi(I_n - A^0)^{-1}\ F - (I_m - \Pi A^0\ \bar{\Gamma})^{-1}\ \Pi F = \Pi Y - \bar{Z} \tag{26}$$
= the vector of aggregation errors (direct aggregation)

$$\bar{\bar{D}}_A = \Pi(I_n - A^0)^{-1}\ F - \Pi(I_n - A^0)^{-1}\ \bar{\bar{\Gamma}}\,\Pi\,F = \Pi Y - \bar{\bar{Z}} = \Pi(I_n - A^0)^{-1}\ (F - \bar{\bar{\Gamma}}\,\Pi\,F) \tag{27}$$
= the vector of aggregation errors, inverse aggregation.

Let us first take a look at the coefficient change error, D_C (25). Expanding the inverse of the coefficient matrices in D_C by power series, we get

(28) $$D_C = \Pi\left[\Delta + A^0\Delta + \Delta A^0 + \Delta^2 + (A^0 + \Delta)^3 (I_n - A^0 - \Delta)^{-1} - (A^0)^3 (I_n - A^0)^{-1}\right] F$$

Assuming that the second and higher order terms within brackets in (28) are small, we see that D_C is dominated by

(29) $$\tilde{D}_C = \Pi\,\Delta\, F = (\Pi\, A - \Pi\, A^0)\, F$$

Terming the rectangular (m X n) matrices $\Pi\, A$ and $\Pi\, A^0$ "line aggregated" or "condensed" matrices, because they are obtained by adding together lines in the original matrices, we see that the D_C-errors, the errors due to coefficient change, are dominated by the linewise averages of the coefficient changes in the line aggregated matrices, weighted by the absolute levels of final deliveries (in the estimation year). (We would probably obtain a better approximation using

(28') $$\begin{aligned} D_C &= \Pi\left[(I_n - A^0 - \Delta)^{-1} - (I_n - A^0)^{-1}\right] F \\ &= \Pi(I_n - A^0 - \Delta)^{-1}\left[I_n - (I_n - A^0)(I_n - A^0)^{-1} + \Delta(I_n - A^0)^{-1}\right] F \\ &= \Pi(I_n - A^0 - \Delta)^{-1}\, \Delta(I_n - A^0)^{-1}\, F = \Pi(I_n - A^0 - \Delta)^{-1}\, \Delta\, Y \\ &= \Pi(I_n + A^0 + \Delta + (A^0 + \Delta)^2(I_n - A^0 - \Delta)^{-1})\, \Delta\, Y \end{aligned}$$

(29') $$\tilde{\tilde{D}}_S = \Pi\,\Delta\, Y$$

But since Y is in some sense a vector of hypothetical magnitudes, (29) may be preferable).

Turning now to $\bar{D}_A$ (26) we write it as

(30) $$\bar{D}_A = \left[\Pi\, (I_n - A^0)^{-1} - (I_m - \Pi\, A^0\, \bar{\Gamma})^{-1}\, \Pi\right]F$$

(Utilizing: $I_m\Pi = \Pi\, I_m = \Pi$ and $(I_m - \Pi\, A^0\, \bar{\Gamma})^{-1}(I_m - \Pi\, A^0\, \bar{\Gamma}) = I_m$)

We get:

(31) $$\begin{aligned} \bar{D}_A &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \left[(I_m - \Pi A^0\bar{\Gamma})\, \Pi(I_n - A^0)^{-1} - \Pi\right]F \\ &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \Pi\left[(I_n - A^0\bar{\Gamma}\Pi)(I_n - A^0)^{-1} - I_n\right]F \\ &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \Pi\left[(I_n - A^0)(I_n - A^0)^{-1} + (A^0 - A^0\bar{\Gamma}\Pi)(I_n - A^0)^{-1} - I_n\right]F \\ &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \Pi A^0(I_n - \bar{\Gamma}\Pi)(I_n - A_0)^{-1}\, F \\ &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \Pi A^0(I_n - \bar{\Gamma}\Pi)\, Y \end{aligned}$$

From (31) it is easily seen that if the classical condition for "horizontal" aggregation is fullfilled, i.e. detailed sectors, which are brought together in the same aggregate sector, have identical columns of input-coefficients, then $\bar{D}_A$ will vanish. Algebraically this condition is given by

(32) $$A^0 = A^0\bar{\Gamma}\Pi$$

where each column of A^0 on the right hand side has been replaced by an average of the columns in A^0 belonging to the same aggregate sector. From (31) we see that we may relax this requirement to demanding only that

(33) $$\Pi A^0 = \Pi A^0\bar{\Gamma}\Pi$$

since we have by insertion in (32).

(34) $$\begin{aligned} \bar{D}_A &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \left[(\Pi - \Pi A^0\bar{\Gamma}\Pi)(I - A^0)^{-1} - \Pi\right]F \\ &= (I_m - \Pi A^0\bar{\Gamma})^{-1}\, \left[(\Pi - \Pi A^0)(I - A^0)^{-1} - \Pi\right]F = 0 \end{aligned}$$

Thus: For the aggregation error to vanish, it is sufficient that the columns of the line aggregated matrix of direct input-output coefficients are identical for detailed sectors belonging to the same aggregate sector.

However, there are also other possibilities for the aggregation error to vanish:

If

(35) $$Y = \bar{\Gamma}\,\Pi Y, \quad \bar{D}_A = 0$$

If the Y-vector for some reason is proportional to the X^0-vector:

(36) $$Y = \rho\, X^0$$

where ρ is a scalar, we have by (6)

(37) $$Y = \rho\, X^0 = \rho\, \bar{\Gamma}\,\Pi X^0 = \bar{\Gamma}\,\Pi Y$$

so that in this case the aggregation error, direct aggregation is zero.

Now we also have, by (7) that if the F-vector for some reason is proportional to the F^0-vector:

(38) $$F = \rho\, F^0$$

then

(39) $$Y = (I_n - A^0)^{-1}\, F = (I_n - A^0)^{-1}\, \rho F^0 = \rho Y^0 = \rho \bar{\Gamma}\,\Pi Y^0 = \bar{\Gamma}\,\Pi Y$$

Thus, proportionality in the Y-vector may be caused by proportionality in the F-vector. (35) will also be satisified if the classical condition for "vertical aggregation" is fullfilled. In this case a group of "supporting" sectors deliver all their products to other sectors in the group or to one and the same sector outside the group.[1] As long as coefficients remain constant, then, the production levels in the supporting industries will remain in the same proportions to each other and to the production level in the sector which receives products from the group and (35) will be satisfied.

If there are r supporting sectors ($r < n$) with production levels given by the column vector X_R of dimension ($r \times 1$) and the matrix of intragroup input-output coefficients A_{RR} (a submatrix of A^0) of dimension ($r \times r$) and if the sector outside the group drawing on its production is sector k with production level X_k and column vector of input coefficients for deliveries from the supporting group a_{Rk}, then we have

$$(I - A_{RR})\, X_R - a_{Rk}\, X_k = 0 \qquad \text{i.e.}$$

(40) $$X_R = (I - A_{RR})^{-1}\, a_{Rk}\, X_k = (I - A_{RR})^{-1}\, a_{Rk}\, X_k^0 \cdot \frac{X_k}{X_k^0} = \frac{X_k}{X_k^0}\, X_R^0$$

i.e. output of all the detailed sectors which we want to aggregate are changed in the same proportion, which is the condition for (35) to be satisfied.

For $\bar{\bar{D}}_A$ we get (Cfr. (27)):

(41) $$\bar{\bar{D}}_A = \Pi(I_n - A_0)^{-1}\, (F - \bar{\bar{\Gamma}}\,\Pi F)$$

and if

(42) $$F = \bar{\bar{\Gamma}}\,\Pi F, \quad \text{then } \bar{\bar{D}}_A = 0.$$

Again if

(43) $$F = \rho\, F^0$$

then by (7)

(44) $$F = \rho\, F^0 = \rho\, \bar{\bar{\Gamma}}\,\Pi F^0 = \bar{\bar{\Gamma}}\,\Pi F$$

So that if $F = \rho F^0$, then both $\bar{D}_A = 0$ and $\bar{\bar{D}}_A = 0$

Conceivably relative production levels may be determined by relative capacities and final deliveries may be adjusted, e.g. through changes in net exports (exports minus competitive imports) from each sector. It will be seen then from (35) (assuming no coefficient changes) that it is sufficient that production levels change in the same proportion within each aggregate sector, for the aggregation error, direct aggregation, to be zero (because then the base year proportions $\bar{\Gamma}$ between detailed production levels within each aggregate sector will remain unchanged, and (35) will hold). However, this presupposes that all final deliveries change in the same proportion from the base year, or that there are compensating changes, which may affect a large number of items. The more usual assumption, at least in connection with input-output-analysis, is that relative production levels are determined by final delivery proportions.

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1) This may be another production sector or a "sector of final uses", as long as the proportions between inputs from the given group of sectors remains constant.

Then it is seen from (42) that it is sufficient for the aggregation error, inverse aggregation, to vanish that final delivery proportions for each detailed sector within each aggregate sector remains constant.

We shall now give an alternative break-down of $\bar{D}_A$, noting first that we may write:

$$F = \bar{\bar{\Gamma}} K i'_n F + \bar{\bar{\Gamma}} (\Pi F - K i'_n F) + (F - \bar{\bar{\Gamma}} \Pi F) \tag{45}$$

Remembering that $\Pi \bar{\bar{\Gamma}} = I_m$ (by (5)), we have now

$$\begin{aligned} \bar{D}_A &= \Pi(I_n - A^0)^{-1} F - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi F \\ &= \Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} K i'_n F - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi \bar{\bar{\Gamma}} K i'_n F \\ &+ \Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} (\Pi F - K i'_n F) - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi \bar{\bar{\Gamma}} (F - K i'_n F) \\ &+ \Pi(I_n - A^0)^{-1} (F - \bar{\bar{\Gamma}} \Pi F) - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi(F - \bar{\bar{\Gamma}} \Pi F) \\ &= \Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} K i'_n F - (I_m - \Pi A^0 \bar{\Gamma})^{-1} K i'_n F \\ &+ \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \right] (\Pi F - K i'_n F) \\ &+ \left[\Pi(I_n - A^0)^{-1} - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi \right] (F - \bar{\bar{\Gamma}} \Pi F) \end{aligned} \tag{46}$$

The first term in the last expression may be written

$$\begin{aligned} \Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} K i'_n F &= \Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} K i'_n F^0 \cdot \frac{i'_n F}{i'_n F^0} \\ = \Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} \Pi F^0 \cdot \frac{i'_n F}{i'_n F^0} &= \Pi(I_n - A^0)^{-1} F^0 \frac{i'_n F}{i'_n F^0} = \Pi X^0 \cdot \frac{i'_n F}{i'_n F^0} \end{aligned} \tag{47}$$

by (10), (7) and (20).

The second term gives in the same way by (10), (7) and (19)

$$\begin{aligned} -(I_m - \Pi A^0 \bar{\Gamma})^{-1} K i'_n F &= -(I_m - \Pi A^0 \bar{\Gamma})^{-1} K i'_n F^0 \cdot \frac{i'_n F}{i'_n F^0} \\ = -(I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi F^0 \frac{i'_n F}{i'_n F^0} &= - \Pi X^0 \frac{i'_n F}{i'_n F^0} \end{aligned} \tag{48}$$

so that these two terms sum to zero. We then have:

$$\begin{aligned} \bar{D}_A &= \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \right](\Pi F - K i'_n F) \\ &+ \left[\Pi(I_n - A^0)^{-1} - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \Pi\right](F - \bar{\bar{\Gamma}} \Pi F) \end{aligned} \tag{49}$$

Here, again, the last term may be simplified

$$\begin{aligned} &\left[\Pi(I_n - A^0)^{-1} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\Pi\right] (F - \bar{\bar{\Gamma}} \Pi F) \\ &= \left[\Pi(I_n - A^0)^{-1} - \Pi(I_n - A^0)^{-1} \bar{\Gamma}\Pi + \Pi(I_n - A^0)^{-1} \bar{\Gamma} \Pi - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\Pi\right] \cdot (F - \bar{\bar{\Gamma}} \Pi F) \\ &= \left[\Pi(I_n - A^0)^{-1} - \Pi(I_n - A^0)^{-1} \bar{\Gamma} \Pi\right](F - \bar{\bar{\Gamma}} \Pi F) \\ &+ \left[\Pi(I_n - A^0)^{-1} \bar{\Gamma} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right] \Pi (F - \bar{\bar{\Gamma}} \Pi F) \end{aligned} \tag{50}$$

But here the last addend vanishes, since

$$\Pi F - \Pi \bar{\bar{\Gamma}} \Pi F = \Pi F - \Pi I_m F = 0 \quad \text{(by (5) etc.)}$$

so that we have

$$\begin{aligned} \bar{D}_A &= \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1} \right](\Pi F - K i'_n F) \\ &+ \left[\Pi(I_n - A^0)^{-1} - \Pi(I_n - A^0)^{-1} \bar{\Gamma} \Pi\right](F - \bar{\bar{\Gamma}} \Pi F) \end{aligned} \tag{51}$$

(51) gives a decomposition of the vector of aggregation errors, direct aggregation, into two additive components, both of which are generated by the multiplication of a "difference matrix" with a "difference vector".

The first component is associated with changes from the base year in aggregate final delivery proportions, and has as its "difference vector" the deviations from proportionality with base year values of aggregated final deliveries. The corresponding difference matrix is square and is the difference between the "final delivery aggregated" inverse of the base year detailed coefficient matrix and the inverse of the aggregated, direct aggregation, base year matrix. It could be seen as a vector of several weighted averages of the differences from base year proportions in aggregate final delivery composition, the weights being given by the lines of the difference matrix. Alternatively, it might be seen as a vector of weighted averages of the lines of the difference matrix, the weights being the same for all the lines, and made up of the elements of the difference vector. (In no case will all the weights normally be non-negative.)

The second component is associated with changes from the base year in within aggregate sector final delivery proportions. It has as its difference vector the deviations from proportionality with base year values within each aggregate sector of final deliveries from detailed sectors. The corresponding difference matrix is rectangular and is constituted by the differences of the columns for the detailed sectors in the line-aggregated inverse of the base year detailed coefficient matrix from the columns for the corresponding aggregate sectors in the final delivery aggregated inverse. This component could be seen as a vector of several weighted averages of the differences from within aggregate sectors base year proportions of final deliveries, or as a vector of weighted averages of the lines of the difference matrix.

Noting that (5)

(52) $$\Pi F - \Pi \bar{\bar{\Gamma}} \Pi F = 0$$

and using (41) we may develop (51) further to get:

(53) $$\begin{aligned} \bar{D}_A &= \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K i_n' F) + \Pi(I_n - A^0)^{-1}(F - \bar{\bar{\Gamma}} \Pi F) \\ &= \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K i_n' F) + \bar{\bar{D}}_A \\ &= D_{AA} + D_{AD} = D_{AA} + \bar{\bar{D}}_A \end{aligned}$$

where

(54) D_{AA} = vector of errors due to changes in aggregate final delivery proportions

and

(55) $D_{AD} = \bar{\bar{D}}_A$ = vector of errors due to changes in within aggregate sectors final delivery proportions

= vector of aggregation errors, inverse aggregation (27).

It is thus established that the vector of aggregation errors, direct aggregation is equal to the vector of aggregation errors, inverse aggregation plus a component, D_{AA}, the vector of errors due to changes in aggregate final delivery proportions. Since we can make no statements about the signs of the elements of these vectors, we cannot flatly assume that the sum will generally be greater than each of the two addends, but at least we have established a close connection.

Examining D_{AA} further, using the expansion of inverses by power series, we have:

(56) $$\begin{aligned} D_{AA} &= \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K i_n' F) \\ &= \left[\Pi(I_n - A^0)^{-1}(\bar{\bar{\Gamma}} - \bar{\Gamma}) + \Pi(I_n - A^0)^{-1} \bar{\Gamma} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K i_n' F) \\ &= \left[\Pi(I_n - A^0)^{-1}(\bar{\bar{\Gamma}} - \bar{\Gamma}) + I_m + \Pi A^0 \bar{\Gamma} + \Pi(A^0)^2(I_n - A^0)^{-1} \bar{\Gamma} \right. \\ &\quad \left. - I_m - \Pi A^0 \bar{\Gamma} - (\Pi A^0 \bar{\Gamma})^2(I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K i_n' F) \\ &= \left[\Pi(I_n - A^0)^{-1}(\bar{\bar{\Gamma}} - \bar{\Gamma}) + \Pi(A^0)^2(I_n - A^0)^{-1} \bar{\Gamma} - (\Pi A^0 \bar{\Gamma})^2(I_m - \Pi A^0 \bar{\Gamma})^{-1}\right] \cdot (\Pi F - K i_n' F) \end{aligned}$$

It appears from (56) that the dominating term in the weighting matrix will depend on the difference matrix ($\bar{\bar{\Gamma}} - \bar{\Gamma}$). We may conclude from this, that if we aggregate in the usual (direct) way, we should avoid lumping together sectors in such a way that this difference matrix becomes big, i.e. if $\frac{F_i^0}{X_i^0} - \frac{F_j^0}{X_j^0}$ is large, sectors i and j should not be combined in aggregation (except in cases where the conditions for vertical integration are close to being met).

Writing now

$$(57) \qquad D_{AW} = \Pi(I_n - A^0)^{-1} (\bar{\bar{\Gamma}} - \bar{\Gamma})(\Pi F - K i_n' F),$$

error due to difference between production weights and final delivery weights

$$(58) \qquad D_{AR} = \left[\Pi(I_n - A^0)^{-1} \bar{\Gamma} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K i_n' F),$$

rest of D_{AA}

$$(59) \qquad D_{AA} = D_{AW} + D_{AR},$$

error due to change in aggregate final delivery proportions we have:

$$(60) \qquad \bar{D}_T = D_C + \bar{D}_A = D_C + \bar{\bar{D}}_A + D_{AA} = D_C + D_{AD} + D_{AA} = D_C + D_{AD} + D_{AW} + D_{AR} = \bar{\bar{D}}_T + D_{AA}$$

We have thus succeeded in breaking down the total estimation error at any particular level of sector specification into one part, which is caused by coefficient changes in a more detailed sector specification, and one which is caused by the aggregation from the detailed specification. Obviously, if the aggregation is done in stages, so that we for instance go from the 92 sector level to the 36 sector level and then to the 7 sector level, we would obtain the corresponding breakdowns of the aggregation error, so that we may decompose the total estimation error at the 7 sector level into one part which is due to coefficient changes at the 92 sector level, one part which is caused by aggregation from the 92 sector level to the 36 sector level, and, finally, one part which is caused by aggregation from the 36 sector level to the 7 sector level. The aggregation error may again be decomposed into one part (D_{AA}) which is caused by deviations from base year aggregate final delivery proportions and one part ($\bar{\bar{D}}_A$) which is caused by deviations from the base year in final delivery proportions within aggregated sectors. From the first of these two parts we may again separate that component which is due to base year differences between production proportions and final delivery proportions within aggregate sectors. Our analytic expressions also reveal which proportions in the basic data are decisive for the size of the various components of the estimation error vectors. Thus it is possible to study both the error components themselves, and the corresponding variations in the basic proportions.

Before we go on to discuss empirical findings, we may give a more precise interpretation of our computations at the 133 sector specification level: Using the same notation as above, and referring specifically to equation (13). We see that in computing the vector Y in 133 sectors we used not F, but $\bar{\bar{\Gamma}}$ (ΠF), when Π is the summation and $\bar{\bar{\Gamma}}$ the final delivery averaging matrix from the 133 to the 92 sector level, so that we have:

$$(59) \qquad D = X - (I_n - A^0)^{-1} \bar{\bar{\Gamma}} \Pi F = X - (I_n - A^0)^{-1} F$$

$$+ (I_n - A_0)^{-1} (F - \bar{\bar{\Gamma}} \Pi F) = D_C + (I_n - A_0)^{-1} (F - \bar{\bar{\Gamma}} \Pi F)$$

where D is the vector of observed errors in the 133 sector estimates. This means that we cannot obtain a correct breakdown of the total error at the 92 sector specification level into the component due to coefficient change at the 133 sector level and the component due to aggregation from this level.

With this reservation, we can use our data in the following way:

1) We can obtain estimates of the errors due to aggregation from more detailed coefficient matrices, simply by subtracting the aggregated errors of the more detailed estimates from the total errors of the aggregate estimates. Figures giving the decomposition of errors in the seven sector specification into errors due to change and to aggregation from the 133 specification level to the 92, 36 and 7 sector levels for each year of the test period are given in tables 9 a and b

and the numerical averages over the years are given in table 10. Figures giving the numerical averages over the years for the decomposition of errors in the 36 sector specification into errors due to change and to aggregation from the 133 sector specification level to the 92 and 36 sector levels are given in table 11.

2) We can break down a vector of aggregation errors into its three components in the following way:

a) Compute $D_{AA} = \left[\Pi(I_n - A^0)^{-1} \bar{\bar{\Gamma}} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K\, i_n' F)$

b) Find $D_{AD} = \bar{D}_A - D_{AA}$

c) Compute $D_{AR} = \left[\Pi(I_n - A_0)^{-1} \bar{\Gamma} - (I_m - \Pi A^0 \bar{\Gamma})^{-1}\right](\Pi F - K\, i_n' F)$

d) Find $D_{AW} = D_{AA} - D_{AR}$

Decomposition of the aggregation errors from the 36 sector specification to the 7 sector specification are given in table 12 and numerical averages over the test period are given in table 13.

3) We can study the variability in our basic data of the components which are decisive for each component of the estimation errors:

a) Variability of input-output coefficients over time are decisive for the errors due to coefficient change. These variations have to some extent been studied elsewhere[1].

b) The part of the aggregation errors, which we have associated with changes in aggregate final delivery proportions (53) is determined by these proportions and by the difference between the aggregated base year inverse and the inverse of the aggregated base year matrix. These sources of variation are analysed in the text of chapter VII.

c) The sizes of the weighting errors of aggregation (51) are determined by the differences between production weights and final delivery weights (base year proportions) and by changes in aggregate final delivery proportions. These sources of variation are studied in the text of chapter VII.

d) The part of the aggregation error, which we have associated with changes in detailed final delivery proportions (55) is determined by these proportions and by the differences between the columns of the line aggregated inverse of the base year matrix and the corresponding columns of the inverse of the aggregated base year coefficient matrix (49) or by the differences between the columns of the line aggregated inverse and the corresponding columns of the aggregated inverse (51). The two expressions are equivalent in their effects. Also these sources of variation are analysed in the text of chapter VII.

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[1] Per Sevaldson, op.cit. 1969.

VII. SOURCES OF AGGREGATION ERRORS

We have shown (Appendix to chapter VI) that the vector of aggregation errors may be subdivided into two additive main components: The first component is associated with changes from the base year in aggregate final delivery proportions, i.e. the proportion of total final deliveries coming from each of the aggregate sectors. It is obtained by applying a matrix of "weights"[1) to the vector of differences between actual final deliveries in the aggregate sector specification and the deliveries as they would have been if the same total sum of deliveries had been divided between aggregate sectors in the same proportions as in the base year. The matrix of weights[1) is the difference between the aggregated inverse of the detailed matrix (using final delivery aggregation weights) and the inverse of the aggregated direct coefficient matrix.

The second component of the vector of aggregation errors is associated with changes from the base year in final delivery proportions within each aggregate sector. It is obtained by applying a matrix of weights[1) to the vector of differences between actual final deliveries in the detailed sector specification and the corresponding deliveries as they would have been if actual final deliveries from each aggregate sector had been distributed on detailed sectors in the same proportions as in the base year. The matrix of weights for this component is the matrix of differences between the items of the "line aggregated" inverse of the detailed matrix and the corresponding items of the aggregated inverse or the inverse of the aggregated matrix. (Either of the two latter matrices may be used. Corresponding means here that all the columns of the detailed sectors in the first matrix are compared with that column in the second matrix, which represents the aggregate sector to which they belong.)

From the first component vector of aggregation errors we may also distinguish a "sub-component" which is associated with the differences between production proportions and final delivery proportions within the aggregate sectors in the base year. This sub-component is derived from the same difference vector as the total component, but the matrix of weights is now the inverse of the detailed matrix, aggregated by using the base year difference between final delivery proportions and production proportions as aggregation weights.

Thus each component and sub-component of the aggregation error is the product of a matrix of differences with a vector of differences. Each product is of course dependent on both its factors.

In chapter VI we studied the observations on the additive components which could be derived from our data. We will now look into the evidence on variations in the difference matrices and the difference vectors which in combination are the origins of the error components. In our time-series data the difference matrices will be constant over time and at each point in time only one line of the difference matrix will be associated with each element in the error vector for that point of time. On the other hand, the difference vectors will vary over time, but at a given point of time the entire difference vector influences all the elements in the error vector.

The variations over time and over the sectors of the elements in the error vector depend on the sizes of the elements in the difference matrices and the difference vectors as well as on their interactions: When the components of the aggregation errors in our data are relatively small, the reason may be:

a) that the elements of the difference matrices are small. (Only sectors with similar coefficients are grouped together in aggregation.)

b) that the elements of the difference vectors are small. (Final delivery proportions are stable.)

c) that elements of both difference matrices and difference vectors are moderately small

d) that elements of neither difference matrices nor difference vectors are particularly small, but that their interactions are such that the resulting error components are small.

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1) The weights are not non-negative and they do not sum to unity.

As long as we have no a priori standard for what is "small", "moderately small" and "not small", this is not very helpful.

We may of course study the data themselves or try to develop more or less comprehensive measures. This will enable us to say that with these characteristics of the data, the results for the error vectors were those which we have already discussed. On this basis it may be found possible to draw some conclusions.

It is, however, also possible to make some relative measurements and tests against simple hypotheses.

Let us start by examining the most detailed difference matrix, which gives the differences of the columns of the line aggregated inverse from the corresponding columns in the inverse of the aggregated matrix. In table 14 we have juxtaposed the elements of the line aggregated inverse of

Table 14. Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 36 sector and 7 sector matrices for 1960. Per cent of changes in final demand.

	1. Agriculture, forestry, hunting and fishing				
	7 sector specification	36 sector specification			
	1	11	12	13	
	Agriculture etc.	Agriculture	Forestry	Fishing Whaling	Average
Final deliveries 1949-1960. Bill. 1960-kroner	2 168	1 466	172	530	
Effects of changes in final deliveries on production in					
1. Agriculture etc.	129.7	148.8	104.6	102.1	133.6
2. Mineral and metal goods	0.9	1.4	0.1	0.5	1.1
3. Food etc. chemicals	14.9	24.7	0.8	1.7	17.1
4. Wood, pulp, textile etc.	1.2	2.4	1.0	2.2	2.2
6. Trade, transport	16.7	18.2	12.0	22.5	18.8
7. Services	4.4	5.7	3.3	5.9	5.6
Total	167.8	201.2	122.0	134.8	178.4

	2. Extraction and production of mineral and metal goods										
	7 sector specification	36 sector specification									
	2	21	22	23	24	25	26	27	28	29	
	Mineral and metal goods	Mining	Mineral products	Basic metal	Metal products	Machinery	Transport equipments	Ship-building	Electrical machinery	Other manufacture	Average
Final deliveries 1949-1960. Bill. 1960-kroner	4 803	212	148	1 203	462	527	552	924	479	296	
Effects of changes in final deliveries on production in											
1. Agriculture etc.	1.2	0.2	1.1	0.2	0.6	0.5	0.3	0.8	0.9	1.7	0.6
2. Mineral and metal goods	116.8	102.8	118.7	115.9	121.0	119.3	108.4	128.4	114.8	104.3	117.0
3. Food etc., chemicals	2.4	1.9	1.0	2.7	2.1	2.2	2.2	2.6	1.6	7.8	2.6
4. Wood, pulp, textile etc.	2.9	1.2	6.1	1.4	4.0	3.2	2.5	4.6	5.7	5.2	3.4
5. Construction	0.1	-	-	-	-	-	-	0.8	-	-	0.2
6. Trade, transport	13.5	13.4	26.3	6.4	19.1	16.5	8.7	10.7	18.7	34.5	13.7
7. Services	6.1	5.7	8.2	9.1	7.7	7.2	5.9	5.2	6.8	5.7	7.0
Total	143.0	125.2	161.4	135.7	154.5	148.9	128.0	153.1	148.5	159.2	144.5

Table 14 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 36 sector and 7 sector matrices for 1960. Per cent of changes in final demand

	3. Production of food and beverages, oils, fats and chemicals						
	7 sector specification	36 sector specification					
	3 Food, etc., chemicals	31 Food industries	32 Tobacco, beverages	33 Oils and fats	34 Petroleum products	39/49 Chemical products	Average
Final deliveries 1949-1960. Bill. 1960-kroner	6 788	4 072	1 259	464	37	956	
Effects of changes in final deliveries on production in							
1. Agriculture etc.	38.9	66.4	1.9	40.7	1.1	2.3	43.3
2. Mineral and metal goods	3.4	3.0	1.5	2.8	4.4	6.2	3.2
3. Food etc., chemicals ..	123.4	134.7	104.2	116.1	106.5	113.1	124.6
4. Wood, pulp,textile etc.	4.6	5.6	3.6	3.9	6.6	10.0	5.7
6. Trade, transport	26.4	29.5	43.5	20.3	26.2	18.8	30.0
7. Services	6.6	7.8	6.1	7.6	5.2	11.0	7.9
Total	203.3	247.0	160.8	191.4	150.0	161.4	214.7

	4. Products of wood, pulp and paper, printing, textiles, clothing, leather and rubber products							
	7 sector specification	36 sector specification						
	4 Wood, pulp, textile etc.	41 Textiles	42 Clothing	43 Footwear, leather, fur	44 Wood and cork	45 Pulp, paper and products	46 Printing, publishing	Average
Final deliveries 1949-1960. Bill. 1960-kroner	5 009	726	1 076	1 001	542	1 283	381	
Effects of changes in final deliveries on production in								
1. Agriculture etc.	16.6	5.9	1.5	3.7	22.4	23.4	4.1	10.6
2. Mineral and metal goods	2.2	1.0	1.2	2.5	4.2	2.2	1.2	2.0
3. Food etc., chemicals ..	3.7	5.0	1.5	3.4	1.8	1.9	2.0	2.6
4. Wood, pulp,textile etc.	128.3	116.7	126.6	118.2	120.1	136.0	157.4	127.5
6. Trade, transport	20.3	23.9	35.8	29.5	21.7	11.2	19.2	23.7
7. Services	7.1	6.7	5.3	10.7	6.6	7.2	19.2	8.3
Total	178.2	159.2	171.9	168.0	176.8	181.9	203.1	174.7

	5. Construction	
	7 sector specification	36 sector specification
	5 Construction	50 Construction
Final deliveries 1949-1960. Bill. 1960-kroner	5 176	5 176
Effects of changes in final deliveries on production in		
1. Agriculture etc.	7.3	6.4
2. Minerals and metal goods	32.0	32.6
3. Food etc., chemicals ...	6.1	5.1
4. Wood, pulp, textile etc.	19.3	19.0
5. Construction	100.0	100.0
6. Trade, transport	10.0	11.5
7. Services	5.5	6.4
Total	180.2	181.0

Table 14 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 36 sector and 7 sector matrices for 1960. Per cent of changes in final demand

	6. Trade and transportation					
	7 sector specification	36 sector specification				
	6 Trade, transport	61 Trade	62 Water transport	63 Land, air transport	64 Communications	Average
Final deliveries 1949-1960. Bill. 1960-kroner	6 811	257	5 593	822	139	
Effects of changes in final deliveries on production in						
1. Agriculture	0.5	0.7	0.2	0.5	0.2	0.3
2. Mineral and metal goods	0.4	0.6	0.3	0.5	0.2	0.3
3. Food etc., chemicals	0.5	0.4	0.8	0.5	0.5	0.7
4. Wood, pulp, textile etc. ...	1.8	4.4	0.8	1.8	3.0	1.1
6. Trade, transport	109.3	116.8	103.8	102.2	109.3	104.2
7. Services	6.0	8.4	3.7	10.7	12.8	4.9
Total	118.5	131.3	109.6	116.2	126.0	111.5

	7. All other activities (Services)									
	7 sector specification	36 sector specification								
	7 Services	71 El., gas, water	72 Bank, insurance	73 Buildings	74 Government, defence	75 Education, health	76 Personal services	77 Other services	78 Unspecified	Average
Final deliveries 1949-1960. Bill. 1960-kroner	6 252	342	478	1 193	983	1 200	715	1 017	324	
Effects of changes in final deliveries on production in										
1. Agriculture etc. ..	1.2	0.1	0.3	-	-	0.2	0.2	0.1	2.4	0.2
2. Mineral and metal goods	0.5	0.2	0.3	-	-	0.3	0.2	0.2	2.1	0.3
3. Food etc.,chemicals	1.0	0.1	0.5	0.1	0.3	1.7	1.5	0.2	3.6	0.8
4. Wood, pulp, textile etc.	5.8	1.1	4.3	0.4	-	2.1	1.8	1.9	32.6	3.1
6. Trade, transport ..	7.2	1.3	6.9	0.6	-	2.2	2.8	2.8	31.9	3.6
7. Services	114.3	130.6	119.4	103.2	100.0	109.3	114.0	110.4	136.6	110.7
Total	130.0	133.4	131.7	104.3	100.3	115.8	120.5	115.6	209.2	118.7

the 36 sector inverse, their linewise averages1) and the elements of the 7 sector inverse (1960), and in table 15 we have reproduced the 80 odd coefficients of the 36 sector inverse which were as large as 0.05 or more (1.05 for diagonal items) and the corresponding items from the line aggregated 92 sector inverse (1960) as well as the linewise averages of these items1). The latter table only allows comparisons for 80 out of the total 1 296 items in a 36 sector inverse, but these items will dominate the effects. For the remaining elements, both the effects and the differences must be small.

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1) 1949-1960 final delivery weights have been used in averaging.

Table 15. Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	11. Agriculture				
	36 sector specification	92 sector specification			
	11 Agriculture	111 Agriculture	112 Agricultural capital	113 Hunting	Average
Final deliveries 1949-1960. Bill. 1960-kroner	1 466	1 325	77	64	
Effects of changes in final deliveries on production in					
11. Agriculture	145.7	149.3	100.0	100.1	145.6
31. Food industries	17.2	18.2	-	-	16.8
39. Chemical products	5.3	5.8	-	0.3	5.4
61. Trade	15.1	10.8	-	38.0	10.8

	12. Forestry			
	36 sector specification	92 sector specification		
	12 Forestry	121 Forestry	122 Standing forests	Average
Final deliveries 1949-1960. Bill. 1960-kroner	172	141	31	
Effects of changes in final deliveries on production in				
61. Trade	9.9	10.3	-	8.4

	13. Fishing, whaling			
	36 sector specification	92 sector specification		
	13 Fishing, whaling	131 Fishing etc.	132 Whaling	Average
Final deliveries 1949-1960. Bill. 1960-kroner	530	314	216	
Effects of changes in final deliveries on production in				
61. Trade	17.9	22.6	0.9	13.8

	21. Mining				
	36 sector specification	92 sector specification			
	21 Mining	211 Coal mining	212 Metal mining	213 Quarrying and mining n.e.c.	Average
Final deliveries 1949-1960. Bill. 1960-kroner	212	18	160	34	
Effects of changes in final deliveries on production in					
61. Trade	11.2	23.1	3.8	22.1	8.4

	22. Non-metallic mineral products	
	36 sector specification	92 sector specification
	22 Non-metallic mineral	220 Non-metallic mineral
Final deliveries 1949-1960. Bill. 1960-kroner	148	148
Effects of changes in final deliveries on production in		
21. Mining	7.1	7.2
22. Non-metallic mineral	110.2	110.2
61. Trade	22.0	22.9

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[1] 105 or more for diagonal coefficients.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	23. Basic metal industries							
	36 sector specification	92 sector specification						
	23 Basic metal	231 Ferro-alloys	232 Iron, steel works	233 Iron, steel foundries	234 Refining of aluminium	235 Other non-ferrous metals	236 Non-ferrous metal foundries	Average
Final deliveries 1949-1960. Bill. 1960-kroner	1 203	294	96	40	248	520	5	
Effects of changes in final deliveries on production in:								
23. Basic metal industries	109.1	103.8	110.8	106.8	100.2	115.1	135.6	108.7
61. Trade	5.2	2.9	6.2	11.1	1.5	5.6	13.9	4.4
71. Electricity, gas,water	6.9	12.7	6.3	2.5	12.5	3.0	2.8	7.6

	24. Metal products and 25. Machinery and 26. Transport equipment				
	36 sector specification				92 sector specification
	24 Metal products	25 Machinery	26 Transport equipment	Average	24/25/26 Iron and metal products
Final deliveries 1949-1960. Bill. 1960-kroner	462	527	552		1 541
Effects of changes in final deliveries on production in:					
23. Basic metal industries	13.2	7.6	1.9	7.1	8.7
24. Metal products	106.4	3.1	2.0		
25. Machinery	-	105.6	0.2	105.8	106.9
26. Transport equipment	-	0.1	103.2		
61. Trade	15.6	13.4	6.8	11.5	12.8

	27. Ship-building industries	
	36 sector specification	92 sector specification
	27 Ship-building	270 Ship-building
Final deliveries 1949-1960. Bill. 1960-kroner	924	924
Effects of changes in final deliveries on production in:		
23. Basic metal industries	8.4	8.5
27. Ship-building	114.2	114.3
61. Trade	8.6	8.8

	28. Electrical machinery etc.	
	36 sector specification	92 sector specification
	28 Electrical machinery	280 Electrical machinery
Final deliveries 1949-1960. Bill. 1960-kroner	479	479
Effects of changes in final deliveries on production in:		
23. Basic metal industries	5.4	5.5
28. Electrical machinery	105.6	105.6
61. Trade	15.3	15.4

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1) See note 1 page 38.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	29. Other manufacturing	
	36 sector specification	92 sector specification
	29 Other manufacturing	290 Other manufacturing
Final deliveries 1949-1960. Bill. 1960-kroner	296	296
Effects of changes in final deliveries on production in:		
39/49. Chemical products	7.1	7.4
61. Trade	29.3	29.7

	31. Food industries										
	36 sector specification	92 sector specification									
	31 Food industries	311 Slaughtering	312 Dairy	313 Margarine	314 Canning	315 Fish processing	316 Grain mills	317 Bakery	318 Chocolate	319 Other food	Average
Final deliveries 1949-1960.Bill.1960-kroner	4 072	819	838	256	301	677	132	478	379	192	
Effects of changes in final deliveries on production in:											
11. Agriculture	55.0	84.5	135.4	1.7	8.6	1.3	55.0	17.9	2.9	30.5	50.1
13. Fishing, whaling.	10.6	1.0	1.7	14.2	21.3	51.4	3.4	1.4	0.1	1.2	11.8
31. Food industries .	126.4	135.6	147.2	102.4	113.7	112.8	124.5	136.5	103.5	109.3	123.6
33. Products of oils and fats	5.7	1.6	2.6	61.0	3.2	0.1	11.1	4.9	0.3	1.0	6.0
61. Trade	24.6	25.5	29.9	26.6	28.5	30.4	-6.5	14.3	28.5	35.0	25.4

	32. Tobacco and beverages				
	36 sector specification	92 sector specification			
	32 Tobacco and beverages	321 Distilling etc. of spirits	322 Breweries, soft drinks	323 Tobacco	Average
Final deliveries 1949-1960. Bill. 1960-kroner	1 259	432	320	507	
Effects of changes in final deliveries on production in:					
61. Trade	36.9	75.0	16.5	22.1	38.8

	33. Products of oils and fats				
	36 sector specification	92 sector specification			
	33 Products of oils and fats	331 Herring oil, fish meal	332 Vegetable oil mills	333 Other oil refineries	Average
Final deliveries 1949-1960. Bill. 1960-kroner	464	203	7	254	
Effects of changes in final deliveries on production in:					
13. Fishing, whaling	38.8	59.1	0.4	38.6	47.0
33. Products of oils and fats	111.6	100.7	100.9	126.3	114.7
61. Trade	16.3	24.8	9.8	10.0	16.5

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[1] See note 1 page 38.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	34. Petroleum products	
	36 sector specification	92 sector specification
	34 Petrol products	348/398 Chemicals and products[2]
Final deliveries 1949-1960. Bill. 1960-kroner	37	(37)
Effects of changes in final deliveries on production in:		
34. Petroleum products	105.2	108.5
45. Pulp, paper, products	5.6	6.8
61. Trade	22.2	21.2

	39/49. Chemical products				
	36 sector specification	92 sector specification			
	39/49 Chemical products	391 Fertilizers etc.	348/398 Chemicals and products[2]	491 Rubber products	Average
Final deliveries 1949-1960. Bill. 1960-kroner	956	164	448(-37)	381	
Effects of changes in final deliveries on production in:					
39/49. Chemical products	111.2	114.5	110.4	104.7	108.8
45. Pulp, paper, products	7.9	13.9	6.8	2.3	6.2
61. Trade	15.3	4.5	21.2	21.0	18.3
71. Electricity, gas, water	5.1	11.9	1.8	1.0	3.2

	41. Textiles				
	36 sector specification	92 sector specification			
	41 Textiles	411 Spinning, weaving	412 Knitting mills	413 Cordage, rope,twine	Average
Final deliveries 1949-1960. Bill. 1960-kroner	726	317	312	97	
Effects of changes in final deliveries on production in:					
11. Agriculture	5.4	7.1	4.2	0.6	5.0
41. Textiles	113.3	110.1	119.0	114.4	114.5
61. Trade	19.8	14.3	30.2	22.0	22.2

	42. Clothing	
	36 sector specification	92 sector specification
	42 Clothing	420 Clothing
Final deliveries 1949-1960. Bill. 1960-kroner	1 076	1 076
Effects of changes in final deliveries on production in:		
41. Textiles	22.2	21.6
61. Trade	30.4	29.4

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1) See note 1 page 38.

2) The sector Petroleum products is not specified in the 92 sector specification.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more1) in the 36-sector inverse. Per cent of changes in final deliveries

	43. Footwear, leather, fur			
	36 sector specification	92 sector specification		
	43 Footwear, leather, fur	431 Footwear, repair, fur	432 Leather and products	Average
Final deliveries 1949-1960. Bill. 1960-kroner	1 001	408	593	
Effects of changes in final deliveries on production in:				
43. Footwear, leather, fur	111.5	113.3	104.0	107.8
61. Trade	24.0	24.5	22.4	23.3
78. Unspecified	7.4	8.5	3.3	5.4

	44. Wood and cork etc.			
	36 sector specification	92 sector specification		
	44 Wood and cork	441 Sawmills planing	442 Other wood products	Average
Final deliveries 1949-1960. Bill. 1960-kroner	542	67	475	
Effects of changes in final deliveries on production in:				
12. Forestry	21.3	50.3	7.9	13.2
44. Wood and cork etc.	115.8	113.3	116.9	116.5
61. Trade	18.0	14.6	19.6	19.0
78. Unspecified	4.0	5.1	3.7	3.9

	45. Pulp, paper and paper products					
	36 sector specification	92 sector specification				
	45 Pulp, paper and products	451 Wood pulp	452 Paper, paperboard	453 Wallboards etc.	454 Paper products	Average
Final deliveries 1949-1960.Bill.1960-kroner	1 283	611	593	26	53	
Effects of changes in final deliveries on production in:						
12. Forestry	22.3	38.0	18.7	13.3	7.0	27.3
45. Pulp, paper and products	133.6	100.7	150.9	104.3	155.9	126.2
61. Trade	9.1	5.7	8.4	11.9	16.4	7.5

	46. Printing and publishing			
	36 sector specification	92 sector specification		
	46 Printing, publishing	461 Publishing etc.	462 Printing, bookbinding	Average
Final deliveries 1949-1960. Bill. 1960-kroner	381	329	52	
Effects of changes in final deliveries on production in:				
45. Pulp, paper, products	22.8	18.7	42.2	21.9
46. Printing, publishing	134.0	145.6	105.1	140.1
61. Trade	14.7	17.8	6.9	16.3
77. Other services	7.2	9.9	1.4	8.7
78. Unspecified	9.1	8.8	10.1	9.0

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1) See note 1 page 38.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	50. Construction	
	36 sector specification	92 sector specification
	50 Construction	500 Construction
Final deliveries 1949-1960. Bill. 1960-kroner	5 176	5 176
Effects of changes in final deliveries on production in:		
22. Non-metal mineral products	9.7	9.7
23. Basic metal industries	5.1	4.9
24. Metal products	7.7	9.8
25. Machinery	2.1	
26. Transport equipment	0.1	
28. Electrical machinery	5.5	5.5
44. Wood and cork etc.	14.8	14.8
61. Trade	7.8	7.7

	61. Trade	
	36 sector specification	92 sector specification
	61 Trade	610 Trade
Final deliveries 1949-1960. Bill. 1960-kroner	257	257
Effects of changes in final deliveries on production in:		
63. Land and air transport	10.9	11.3

	63. Land and air transport						
	36 sector specification	92 sector specification					
	63 Land and air transport	631 Railway transport	632 Tram-ways	633 Land transport n.e.c.	634 Air transport	635 Services to transport	Average
Final deliveries 1949-1960. Bill. 1960-kroner	822	199	68	364	188	3	
Effects of changes in final deliveries on production in:							
78. Unspecified	6.0	7.5	7.7	1.4	0.8	44.0	3.4

	71. Electricity, gas and water				
	36 sector specification	92 sector specification			
	71 Electricity, gas, water	711 Electricity	712 Gas	713 Water	Average
Final deliveries 1949-1960. Bill. 1960-kroner	342	297	12	33	
Effects of changes in final deliveries on production in:					
71. Electricity, gas and water	126.0	126.4	101.1	100.0	123.0

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[1] See note 1 page 38.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	72. Banking, insurance							
	36 sector specification	92 sector specification						
	72 Banking, insurance	721 Bank of Norway	722 State banks etc.	723 Other banks	724 Life insurance	725 Non-life insurance	726 Social insurance	Average
Final deliveries 1949-1960. Bill. 1960-kroner	478	16	15	292	67	54	34	
Effects of changes in final deliveries on production in:								
78. Unspecified	14.2	24.8	20.1	14.5	27.9	11.6	-	15.5

	75. Educational, health services			
	36 sector specification	92 sector specification		
	75 Educational, health	751 Educational services	752 Medical, veterinary	Average
Final deliveries 1949-1960. Bill. 1960-kroner	1 200	538	662	
Effects of changes in final deliveries on production in:				
78. Unspecified	6.3	0.2	11.8	6.6

	76. Personal services				
	36 sector specification	92 sector specification			
	76 Personal services	761 Domestic services	762 Hotels, restaurants	763 Laundry and other	Average
Final deliveries 1949-1960. Bill. 1960-kroner	715	273	286	156	
Effects of changes in final deliveries on production in:					
73. Business buildings	6.4	-	10.4	4.9	5.2
78. Unspecified	5.6	-	7.4	8.7	4.9

	77. Other services							
	36 sector specification	92 sector specification						
	77 Other services	771 Central gvmt.	772 Local gvmt.	773 Religious, welfare	774 Non-business organisations	775 Legal etc. services	776 Recreation	Average
Final deliveries 1949-1960. Bill. 1960-kroner	1 017	183	295	123	130	63	223	
Effects of changes in final deliveries on production in:								
78. Unspecified	6.4	-	-	3.0	14.1	5.2	18.9	6.6

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1) See note 1 page 38.

Table 15 (cont.). Comparisons of input-output "inverse coefficients" (direct plus indirect effects per unit change in final demand) in 92-sector and 36-sector matrices for 1960. Coefficients of 5 per cent or more[1] in the 36-sector inverse. Per cent of changes in final deliveries

	78. Unspecified					
	36-sector specification	92-sector specification				
	78 Unspecified	781 Unspecified office supplies	782 Unspecified energy	783 Unspecified services	784 Unspecified transport	Average
Final deliveries 1949-1960. Bill. 1960-kroner	324	109	-	215	-	
Effects of changes in final deliveries on production in:						
45. Pulp, paper, prod. ..	8.0	28.1	0.1	0.1	0.1	9.8
46. Printing, publishing.	23.2	65.7	0.1	1.2	0.9	22.9
63. Land, air transport .	8.4	1.5	0.3	6.4	75.9	4.8
64. Communications	16.7	1.1	0.2	28.9	0.5	19.6
71. Electricity, gas, water	5.4	10.1	32.9	0.4	0.9	3.7
72. Banking, insurance ..	6.3	0.5	0.1	10.7	0.9	7.3
77. Other services	12.0	3.3	0.1	18.9	0.4	13.7
78. Unspecified	108.8	106.0	101.0	111.8	103.8	109.9

[1] See note 1 page 38.

By inspection of tables 14 and 15 it is easily seen that there are in general big differences between the columns of the line aggregated, detailed inverses on the one hand and both the aggregated inverses (the averages)and the inverses of the aggregated matrices on the other.

Using tables 14 or 15, we could easily compose schedules of changes in final deliveries, which would lead to large errors, if they were applied to the aggregated matrix. A 10 million kroner increase in final deliveries of Agricultural capital (sector 112) would according to the 92-sector inverse (Table 15) have no indirect effects. But if we use the 36 sector inverse, such a change cannot be distinguished from changes in final deliveries from the 92 specification sectors Agriculture (111) and Hunting (113), and the effects in the 36 sector specification would be calculated to be an extra 4.6 million kroner induced production increase in Agriculture (in addition to the original 10), 1.7 million kroner in Food industries, 0.5 million kroner in Chemicals and 1.5 million in Trade. The effects in the seven sector specification if we compare the 36 and 7 sector inverses (Table 13) will be 4.9 and 3 million kroner respectively extra in Agriculture etc., 2.5 and 1.5 million kroner in Food etc. and chemicals and so on. up to a total of 10.1 and 6.8 million kroner respectively in indirect effects. (We have here compared with inverses of aggregate matrices, but the results would not differ materially if we compared with aggregates of inverses.)

Does this mean that our aggregations are entirely random from the point of view of "horizontal aggregation"? In other words, are the coefficients in the line aggregated inverse, of sectors which are combined in the same aggregate sector no more similar than the coefficients of random groups of sectors? We may assess this problem on the basis of a comparison of the dispersions of coefficients on a line in the line aggregated inverse, belonging to the same aggregate sector with the total dispersion of all the coefficients on the same line.

A measure of the total spread in coefficients on a line in a line aggregated inverse is their variance about an unweighted average of all the coefficients on this line. If we were to estimate the effects of a one unit change in final deliveries from a sector choosen at random, and not identified to us, we would have to use this average coefficient, and the expected variances of our estimate would be the variance of the coefficients about this average.

The total variance may be subdivided into one part which measures the individual coefficients' (average squared) deviations from averages taken within each aggregate sector and one part, which is an average of the squared deviations of these average coefficients for the aggregate sectors from the average coefficient for the entire line, when these latter deviations are weighted by the number of detailed sectors in each aggregate sector. The first of these two component variances is also the variance we should expect on our estimates of the effects of a one unit change in final deliveries from a sector chosen at random, when the identity of the aggregate sector to which the chosen sector belongs can be identified, so that the effects may be estimated by the average coefficient for that aggregate sector.

This component of the total variance, the variance of individual, detailed sectors about the averages for the aggregate sectors, is in a way a measure of the information in the detailed inverse matrix which is ignored by aggregation, whereas the remainder is a measure of the information which is preserved in the aggregation procedure.

Table 16. Variances (mean square deviations) of coefficients of the 7 by 36 sector line aggregated inverse for 1960, subdivided into variances of individual coefficients about aggregate sector averages and (weighted1)) variance of aggregate sector averages about total line averages

Sector	Variance about line average	Of this:		Average coefficient	Mean square percentage aggregation error 1949-1959
		Variance about aggregate sector averages	Variance of aggregate sector averages about line average 1)		
1. Agriculture etc.	.0223	.0151	.0072	.069	.0023
2. Mineral-, metal products	.0071	.0016	.0055	.056	.0001
3. Food, chemicals etc.	.0050	.0028	.0022	.044	.0029
4. Wood and fibre products	.0149	.0059	.0090	.088	.0011
5. Construction	-	-	-	-	-
6. Trade and transportation	.0124	.0066	.0058	.161	.0066
7. Services	.0050	.0038	.0012	.093	.0004
Average (excluding 5)	.0111	.0060	.0052	.085	.0022

1) Weighted by numbers of detailed sectors in each aggregate sector.

In table 16 these variances have been computed for the 7 x 36 sector line aggregated inverse in relation to the 7 sector aggregation and in table 17 the corresponding variances have been

Table 17. Variances (mean square deviations) of coefficients of 12 lines of the 36 by 92 sector line aggregated inverse for 1960, subdivided into variances of individual sectors about aggregate sector averages and (weighted1)) variances of aggregate sector averages about total line averages

Sector	Variance about line average	Of this:		Average coefficient	Mean square percentage aggregation error 1949-1959
		Variance about aggregate sector averages	Variance of aggregate sector averages about line average 1)		
11. Agriculture	.03305	.02049	.01256	.0473	.00564
12. Forestry	.00477	.00160	.00317	.0191	.00038
22. Non-metallic mineral products ...	.00022	.00002	.00020	.0040	.00006
23. Basic metal industries	.00240	.00121	.00119	.0122	.00010
27. Ship-building industries	.00023	.00001	.00022	.0025	.00005
31. Food industries	.00621	.00249	.00372	.0241	.00198
32. Tobacco and beverages	.00010	.00006	.00004	.0004	.00446
45. Pulp, paper and paper products ..	.00866	.00391	.00475	.0385	.00047
46. Printing and publishing	.00672	.00445	.00227	.0238	.00025
61. Wholesale and retail trade	.01625	.00580	.01045	.1058	.00002
64. Communications	.00105	.00078	.00027	.0121	.00070
78. Unspecified	.00428	.00269	.00159	.0544	.00027
Average of 12 sectors	.00700	.00363	.00337	.0287	.00120

1) See note 1 table 16.

computed for 12 out of the 36 lines of the 36 x 92 sector line aggregated inverse relating to the 36 sector aggregation. We have also computed these variances for the first of the seven lines Agriculture etc. of the 7 x 92 sector line aggregated inverse. Here we got the following figures: Total variance is .0449 of this .0307 is the variance of individual coefficients about the aggregate sector averages, the part which is ignored by aggregation and .0142 is the weighted variance of aggregate sector averages about the total line average, i.e. the part of total variance which is not ignored through aggregation from 92 to 7 sectors.

(Computations for the remaining 20 lines of the 36 by 92 sector line-aggregated inverse and the remaining 5 non-zero lines of the 7 by 92 sector line-aggregated inverse have been omitted because of the computational burden involved. For the same reason the effects on import and transfer sectors have been omitted.)

Looking at our figures, we find that the variance of individual coefficients about aggregate sector averages, the information ignored in aggregation dominates for some sectors and the residual, the information not ignored dominates for others both in the 7 x 36 sector table and in the 36 x 92 sector table, but by and large the two components appear to be of roughly the same order of magnitude, with a slight dominance for the information ignored by aggregation in both tables. Taking simple averages of the figures for the sectors in each of the tables (ignoring sector 5, Construction in table 5) we get .0060 for the ignored variance and .0052 for the not ignored variance in the 7 x 36 sector aggregation and .00363 and .00337 respectively in the 36 x 92 sector aggregation. For sector 1, Agriculture etc. the figures are somewhat atypical with .0151 ignored and .0072 not ignored, i.e. 2 to 1 in the 7 x 36 sector aggregation. This corresponds to .0307 ignored and .0142 not ignored, or again roughly 2 to 1 in the figures for the 7 x 92 sector aggregation quoted above. (Again the .0307 ignored in aggregation from 92 to 7 sectors can be subdivided into .021 ignored in 92 to 36 sector aggregation and .010 ignored in 36 to 7 sector aggregation. These figures do not correspond to the figures for 36 to 7 sector aggregation in table 16, since that table starts from the inverse of the 36 sector aggregate matrix, whereas the present computations are based on successive averages of the inverse of the 92 sector matrix.)

We conclude from this analysis, that in the 92 to 36 sector aggregation as well as in the 36 to 7 sector aggregation something of the order of half the variation in coefficients on a given line in the inverse is ignored, and if it were not for the structural stability in the final demand dispersion on detailed sectors within each aggregate sector, we should expect something like a doubling of the mean square errors as compared to the foregoing level for each of our aggregations 92 to 36 and 36 to 7 sectors.

We thus come to the conclusion that our aggregation procedures suppress a considerable amount of information on inter-sector coefficient differences. If this was not made less important through regularities in final delivery proportions, we should have found greater aggregation errors. Indeed, if final delivery proportions varied quite irregularly, we should expect to find the biggest errors in the estimates of effects on sectors for which the dispersion of coefficients within aggregate sectors were greatest.

In tables 16 and 17 we have given in the last columns of each table, the means of the squared aggregation errors in per cent of the correct intermediate delivery figures for each sector in the observation period. Any tendency to covariation with the size of the variance about aggregate sector averages is at least extremely weak.

We will now investigate the second difference matrix, the differences of the elements of the inverse of the aggregated matrix from the elements of the aggregated inverse of the detailed matrix, when final delivery weights are used in the latter aggregation. These figures are given in table 18 for the 36 - 7 sector aggregation.

Table 18. Matrix of differences 7 sector inverse minus 7 sector aggregation of 36 sector inverse, aggregated by final delivery weights 1960

Difference in effects on deliveries from	Differences of effects of changes in final deliveries from: I Agriculture etc.	II Mineral-, metal products	III Food, chemicals etc.	IV Wood and fibre products	V Construction	VI Trade and transportation	VII Services
1. Agriculture etc.	-.039	.006	-.044	.060	.009	.002	.010
2. Mineral-, metal products	-.002	-.002	.002	.002	-.006	.001	.002
3. Food, chemicals etc. .	-.022	-.002	-.012	.011	.010	-.002	.002
4. Wood and fibre products	-.010	-.005	-.011	.008	.003	.007	.027
5. Construction	-	-.001	-	-	-	-	-
6. Trade and transportation	-.021	-.002	-.036	-.034	-.015	.051	.036
7. Services	-.012	-.009	-.013	-.012	-.009	.011	.036

Again we need a standard by which to evaluate the difference. Here we choose to compare the mean square difference between corresponding items for each line in the two matrices with the variance of the same items about the line average in the aggregated inverse. The results of these computations for the 7 lines of the 36 to 7 sector aggregation are given in table 19 and the results for 12 of the 36 lines of the 92 to 36 sector aggregation are given in table 20. It appears from these

Table 19. Linewise mean square difference between 7-sector inverse and 7-sector aggregation of 36 sector inverse, final delivery weights (1960) and linewise variance and average of 7 sector aggregation of 36 sector inverse

Sector	Mean square difference	Variance 7 sector aggregation of 36 sector inverse	Average coefficient	Mean square difference in per cent of variance %
1. Agriculture etc.	.00104	.02661	.136	3.9
2. Mineral-, metal products	.00008	.01297	.081	.6
3. Food, chemicals etc.	.00012	.00763	.076	1.6
4. Wood and fibre products	.00015	.00889	.089	1.7
5. Construction	-	-	.000	-
6. Trade and transportation	.00100	.00814	.151	12.3
7. Services	.00029	.00026	.073	111.5

Table 20. Linewise mean square difference between 36-sector inverse and 36-sector aggregation of 92 sector inverse, final delivery weights (averages 1949-1960), and linewise variance and average of 36-sector aggregation of 92 sector inverse, 12 out of 36 sectors

Sector	Mean square difference	Variance 36 sector aggregation of 92 sector inverse	Average coefficient	Mean square difference in per cent of variance
11. Agriculture	1.075	128.32	3.65	.8
12. Forestry	2.832	26.03	1.79	10.9
22. Non-metallic mineral products	.002	5.49	.77	.04
23. Basic metal industries	.089	6.96	1.23	1.3
27. Ship-building industries	.006	5.99	.48	.1
31. Food industries	.259	23.65	1.47	1.1
32. Tobacco and beverages	.002	.40	.13	.5
45. Pulp, paper and paper products	1.860	31.85	3.40	5.8
46. Printing and publishing	1.221	58.00	2.78	2.1
61. Wholesale and reatil trade	2.179	111.04	11.16	2.0
64. Communications	.340	11.36	1.34	3.0
78. Unspecified	.513	8.86	4.55	5.8

tables, that the items in these difference matrices are very small; in general the linewise mean square differences are less than a tenth of the variances on the corresponding lines in the aggregated inverse. Also compared to the coefficient differences ignored within the aggregate sectors, and measured by the variances about the aggregate sector averages in the line aggregated inverses (Tables 16 and 17), the present differences are close to negligible.

Our next subject of investigation must then be the dispersion in final deliveries. We consider first the stability of distribution of final deliveries from detailed sectors within each aggregate sector.

As a measure of the variatiability of the proportions within each aggregate sector we have taken the standard deviation (or root mean square deviation) of the distribution percentages from their respective means in the observation period,

(i.e. $\sigma = \left[\frac{1}{12k} \sum_{i=1}^{k} \sum_{t=1}^{12} (p_{it} - \frac{1}{12} \sum_{t=1}^{12} p_{it})^2 \right]^{\frac{1}{2}}$

where σ is the measure of variability for a given aggregate sector, comprising k detailed sectors, p_{it} is the percentage of final deliveries from the aggregate sector in year t originating from detailed sector i and the average over the observation period of

$p_{it} : \bar{p}_i = \frac{1}{12} \sum_{t=1}^{12} p_{it}$)

Since these standard deviations are in terms of percentages of annual total final deliveries from each aggregate sector, and since the levels of these deliveries vary between the aggregate sectors, these measures are not readily comparable. In order to facilitate comparisons we have "normalized" these percentage standard deviations" by applying them to the corresponding averages of final deliveries from the aggregate sectors in the observation period in terms of values. This is the standard deviations in kroner, which we would get if the total delivery from each aggregate sector in each year had been equal to the average over the observation period, but distributed over detailed sectors in the proportions actually observed for each year. The figures for the 36 to 7 sector aggregation are given in table 21 and for the 92 to 36 sector aggregation in table 22. The tables also give number of detailed sectors and average final delivery over the observation period for each aggregate sector. Finally, the tables give a measure, which we have termed "constructed coefficient of variation". This is the average standard deviation of percentages divided by 50 for aggregate sectors with two detailed sectors, by 33.3 for aggregate sectors with three detailed sectors, by 25 for aggregate sectors with four detailed sectors a.s.o. 50, 33, 3, 25 etc. are of course the average percentage for detailed sectors in the respective aggregate sectors. The standard deviations both in per cent and in kroner are remarkably small. For comparison it may be mentioned that if all percentage distributions had the same probability, the percentage standard deviation would be around 29 per cent when there are two detailed sectors,

Table 21. Dispersion in final delivery proportions within aggregate sectors 36 - 7 sector aggregation (1949-1960)

Sector	Number of detailed(36-classification)sectors within aggregate sectors	Average standard deviation of distribution percentages	Average final delivery 1949-1960 in million 1955-kroner	Standard deviation "normalized" to million 1955-kroner Col(2) x Col(3)	Constructed coefficient of variation1)
1. Agriculture etc.	3	2.55	1806.5	46.1	.0765
2. Mineral-, metal products ...	9	1.24	4002.6	49.6	.1116
3. Food, chemicals etc.	5	1.21	5520.9	66.8	.0605
4. Wood and fibre products	6	1.52	3884.4	59.0	.0912
5. Construction	1	..	4313.5	..	..
6. Trade and transportation ...	4	.76	5674.6	43.1	.0304
7. Services	8	.85	5210.2	44.3	.0680
Total	36	1.32	4344.7	57.4	.

1) See the text.

27 per cent when there are three and 23 per cent when there are 4 detailed sectors. With all but one percentages less than 2 for the 7 aggregate sectors and all but one less than 9 for the 36 aggregate sectors our data are at least far from this "structureless" situation.

Table 22. Dispersion in final delivery proportions within aggregate sectors 92 - 36 sector aggregations (1949-1960)

Sector	Number of detailed(92-classification)sectors within aggregate sectors	Average standard deviation of distribution percentages	Average final delivery 1949-1960 in million 1955-kroner	Standard deviation "normalized" to million 1955-kroner Col(2) x Col(3)	Constructed coefficient of variation1)
11. Agriculture	3	.57	1221.6	7.0	.0171
12. Forestry	2	20.65	143.1	29.6	.4130
13. Fishing, whaling	2	8.46	441.7	37.4	.1692
21. Mining	3	4.63	176.5	8.2	.1389
22. Non-metallic mineral products.	1	..	123.3	..	..
23. Basic metal industries	6	3.29	1002.5	33.0	.1974
24. Metal products	1	..	385.2	..	..
25. Machinery			439.3	..	..
26. Transport equipment			459.9	..	..
27. Ship-building industries	1	..	770.1	..	..
28. Electrical machinery etc.	1	..	403.3	..	..
29. Other manufacturing	1	..	246.6	..	..
31. Food industries	9	1.00	3394.0	33.9	.0900
32. Tobacco and beverages	3	2.32	1049.5	24.3	.0696
33. Products of oils and fats	3	8.81	386.3	34.0	.2643
34/39/49. Chemicals	3	3.50	828.1	29.0	.1050
41. Textiles	3	3.39	605.3	20.6	.1017
42. Clothing	1	..	896.3	..	..
43. Footwear, leather, fur	2	2.40	406.9	9.8	.0480
44. Wood and cork etc.	2	5.71	452.3	25.8	.1142
45. Pulp, paper and paper products	4	1.92	1064.4	20.4	.0768
46. Printing and publishing	2	2.11	317.3	6.7	.0422
50. Construction	1	..	4313.5	..	..
61. Wholesale and retail trade ...	1	..	213.8	..	..
62. Water transport	3	.39	4661.2	18.2	.0107
63. Land and air transport	5	3.84	684.0	26.3	.1920
64. Communications	1	..	115.6	..	..
71. Electricity, gas and water ...	3	2.48	284.6	7.1	.0744
72. Banking and insurance	6	.38	398.4	1.5	.0228
73. Business buildings, dwellings.	2	.20	994.2	2.0	.0040
74. Government, defence	2	6.43	819.4	52.6	.1286
75. Educational, health services .	2	2.40	1000.1	24.0	.0480
76. Personal services	3	4.74	595.2	28.2	.1422
77. Other services	6	.77	848.7	6.5	.0462
78. Unspecified	4	3.17	270.6	8.6	.1268
Total	92	4.49	868.9	39.0	.

1) See the text.

We must now finally consider the distribution of total final deliveries on aggregate sectors. Again we compute standard deviations of percentages, but now the percentages are final deliveries from each aggregate sector in per cent of total final deliveries from all sectors, and the normalisation to kroner is effected by multiplying the standard deviations in per cent by the average of total final deliveries (from Norwegian sectors) in the observation period.

Figures are given for the 7-sectors in table 23 and for 36 sectors in table 24.

Table 23. Dispersion in final delivery proportions for aggregate sectors. 7 sectors 1949-1960

Sector	Average percentage of total final deliveries	Standard deviation of percentages 1949-1960	Standard deviation "normalized" to million 1955-kroner	Coefficient of variation	Average final delivery 1949-1960 for each sector in million 1955-kroner
1. Agriculture etc.	6.0	.07	21.3	.012	1806.5
2. Mineral-, metal products	13.0	1.34	407.5	.103	4002.6
3. Food, chemicals etc.	18.3	1.02	310.2	.056	5520.9
4. Wood and fiber products	12.9	.73	222.0	.057	3884.4
5. Construction	14.2	.92	279.8	.065	4313.5
6. Trade and transportation	18.5	1.41	428.8	.076	5674.6
7. Services	17.1	.66	200.7	.039	5210.2
Total	100.0	1.08	328.4	.076[1)]	4344.7

1) 1.08 ÷ 14.29.

Table 24. Dispersion in final delivery proportions for aggregate sectors. 36 sectors 1949-1960

Sector	Average percentage of total final deliveries	Standard deviation of percentages 1949-1960	Standard deviation "normalized" to million 1955-kroner	Coefficient of variation	Average final delivery 1949-1960 for each sector in million 1955-kroner
11. Agriculture	4.07	.42	127.7	.103	1221.6
12. Forestry	.50	.25	76.0	.500	143.1
13. Fishing, whaling	1.47	.16	48.7	.108	441.7
21. Mining	.58	.07	21.3	.121	176.5
22. Non-metallic mineral products	.41	.05	15.2	.122	123.3
23. Basic metal industries	3.21	.71	215.9	.221	1002.5
24. Metal products	1.24	.11	33.5	.089	385.2
25. Machinery	1.42	.12	36.5	.084	439.3
26. Transport equipment	1.50	.14	42.6	.093	459.9
27. Ship-building industries	2.52	.19	57.8	.075	770.1
28. Electrical machinery etc.	1.30	.16	48.7	.123	403.3
29. Other manufacturing	.81	.03	9.1	.036	246.6
31. Food industries	11.27	.71	215.9	.063	3394.0
32. Tobacco and beverages	3.48	.28	85.2	.081	1049.5
33. Products of oils and fats	1.31	.34	103.4	.260	386.3
34. Petroleum products	.10	-	-	-	828.1
39/49. Chemicals	2.60	.16	48.7	.062	
41. Textiles	2.02	.29	88.2	.143	605.3
42. Clothing	2.94	.17	51.7	.058	896.3
43. Footwear, leather, fur	1.38	.26	79.1	.189	406.9
44. Wood and cork etc.	1.49	.13	39.5	.087	452.3
45. Pulp, paper and paper products	3.51	.19	57.8	.054	1064.4
46. Printing and publishing	1.03	.07	21.3	.068	317.3
50. Construction	14.26	.89	270.6	.063	4313.5
61. Wholesale and retail trade	.72	.08	24.3	.111	213.8
62. Water transport	15.15	1.38	419.7	.091	4661.2
63. Land and air transport	2.26	.07	21.3	.031	684.0
64. Communications	.40	-	-	-	115.6
71. Electricity, gas and water	.92	.12	36.5	.131	284.6
72. Banking and insurance	1.32	.07	21.3	.053	398.4
73. Business, buildings, dwellings	3.29	.08	24.3	.024	994.2
74. Government, defence	2.69	.18	54.7	.067	819.4
75. Educational, health services	3.29	.04	12.2	.012	1000.1
76. Personal services	1.97	.26	79.1	.131	595.2
77. Other services	2.77	.08	24.3	.029	848.7
78. Unspecified	.89	.17	51.7	.191	270.6
Total	100.09	.36	109.8	.130	868.9

If we try to compare the stability of these "intersector", percentages from tables 23 and 24 with the corresponding "intrasector" percentages from tables 21 and 22, there is no marked difference. If we compare the coefficients of variation with the "constructed coefficients of variation" the levels appear to be about the same, apart from the fact that there is of course no variation in the intrasector composition of deliveries from those "aggregate" sectors in the 36 sector specification which consist of only one detailed sector each. However, when we consider the standard deviations normalized to kroner, there are marked differences in levels. These differences are of course due to the fact that intrasector standard deviations are normalized by multiplying by the average final deliveries from the respective aggregate sectors, whereas intersector standard deviations are normalized by multiplication by the average grand total of all final deliveries. Since we compute our aggregation errors in (1955-)kroner values, it is the normalized standard deviations which are indicative of the effects of final delivery variations.

We are thus in a position to conclude, that we have found a very marked stability in both intersector and intra aggregate sector proportions of final deliveries in our data for a 12 year period. There is no evidence of less variation in proportions within aggregate sectors than between aggregate sectors, indicating that we have not grouped together in our aggregates groups of sectors which have more parallell movements in their final deliveries than average. We must, however, qualify this by remembering that in the 92-36-sector aggregation we have been able to retain the detailed specification for a considerable number of sectors through the aggregation of others. For these sectors the final delivery proportions are all constant (= 100) and they cannot contribute to the aggregation error. When we consider the variability in terms of kroner values, we find the variability of intersector shares to be of a much higher order than the variability of intrasector shares. In the previous chapter we found the aggregation error associated with changes in intersector final delivery proportions to be of about the same order of magnitude as the aggregation error due to changes in intra aggregate sector final delivery proportions, and both error components being small compared to the errors caused by coefficient changes. We may now conclude that the small sizes of both types of errors seem to be explained by the relative stability of both intersector and intrasector final delivery proportions. But whereas the deviations from proportionality measured in (constant) kroner value are relatively large for intersector shares, the matrix of coefficient differences - which together with these differences determine the component of the aggregation error associated with variation in intersector final delivery proportions - is composed of only small items. The situation is the opposite, when we consider the component of the aggregation error associated with variation in intrasector final delivery proportions: Here the deviations from proportionality in final deliveries are relatively small measured in (1955-)kroner values, but the matrix of coefficient differences, (measuring the differences between line-aggregated inverse coefficients and weighted means of such coefficients) are considerable. The net effects are, as we have already mentioned, to give small errors of about the same order of magnitude for both components.

We shall finally take a look at the difference between base year proportions in final deliveries as compared to toal production proportions within aggregate sectors. Figures which indicate the magnitudes of these differences in our aggregate groupings are given in tables 25 for the proportions in the 36 sector specification within the 7 most aggregated sectors and in table 26 for the proportions (in the 92 sector specification) within the 36 sectors.

Table 25. Differences between total production distributions and final delivery distributions on detailed sectors within aggregate sectors 1960. Numerical differences of percentages 7 aggregate sectors

Sector	Number of detailed sectors	Numerical differences	
		Biggest	Smallest
		Pct.	Pct.
1. Agriculture etc.	3	9.5	2.4
2. Mineral-, metal products	9	5.7	.9
3. Food, chemicals etc.	5	4.6	1.2
4. Wood and fibre products	6	9.7	.9
5. Construction	1	-	-
6. Trade and transportation	4	38.8	.4
7. Services	8	15.7	.3

Table 26. Differences between total production distributions and final delivery distributions on detailed sectors within aggregate sectors 1960. Numerical differences of percentages. 36 aggregate sectors

	Number of detailed sectors	Numerical differences	
		Biggest	Smallest
		Pct. points	Pct. points
11. Agriculture	3	5.1	2.5
12. Forestry	2	39.0	39.0
13. Fishing, whaling	2	11.1	11.1
21. Mining	3	15.0	-
22. Non-metallic mineral products	1	-	-
23. Basic metal industries	6	7.6	.4
24. Metal products 25. Machinery 26. Transport equipment	1	-	-
27. Ship-building industries	1	-	-
28. Electrical machinery etc.	1	-	-
29. Other manufacturing	1	-	-
31. Food industries	9	10.2	.5
32. Tobacco and beverages	3	2.2	.6
33. Products of oils and fats	3	14.0	-
34/39/49. Chemicals	3	8.4	4.1
41. Textiles	3	21.4	5.0
42. Clothing	1	-	-
43. Footwear, leather, fur	2	9.5	9.5
44. Wood and cork etc.	2	25.0	25.0
45. Pulp, paper and paper products	4	9.5	.8
46. Printing and publishing	2	29.6	29.6
50. Construction	1	-	-
61. Wholesale and retail trade	1	-	-
62. Water transport	3	7.3	3.2
63. Land and air transport	5	14.5	2.7
64. Communications	1	-	-
71. Electricity, gas and water	3	4.9	.3
72. Banking and insurance	6	20.5	.8
73. Business buildings, dwellings	2	17.1	17.1
74. Government, defence	2	-	-
75. Educational, health services	2	.3	.3
76. Personal services	3	2.9	.4
77. Other services	6	15.7	1.0
78. Unspecified	4	8.7	4.9

We found in chapter VI (tables 12 and 13) that the error due to differences between total production proportions and final delivery proportions constituted a considerable part of the aggregation error component associated with changes in aggregate proportions, at least for the aggregation from 36 to 7 sectors. The figures in tables 25 and 26 show that there are considerable differences both for the distributions within the 7 sectors and, perhaps even more, within the 36 sectors. A regrouping of the sectors in the various aggregations should thus offer a promising prospect of improving the estimates.

CHAPTER VIII. ESTIMATION ERRORS AND COEFFICIENT CHANGE

The analysis in chapter VI clearly showed that the errors due to coefficient change at the detailed levels of sector specification dominate our results. The improvements in results following from successive disaggregations from a specification level of only 7 sectors of production to 36 sectors and further to 92 and 133 sectors are relatively moderate, compared to the magnitudes of errors which remain. We are led to ask what is the ultimate cause or causes of these remaining errors? Is there a chance that they would be more drastically reduced if we could push the disaggregation even further than to the 133 production sector level, and in that case, to what level would we have to go? The ultimate answer can only, if at all, be found by a thorough study of the causes of changes in direct input-output coefficients.

However, we may arrive at some conclusions by studying our test results for the period 1949-1960 in some more detail. The rest of this chapter is divided into 5 sections (A-E), where we draw to light various results of such further studies: Section A is concerned with a comparison,sector by sector,of results based on alternative coefficient matrices and it is shown that the magnitudes of the estimation errors vary considerably from sector to sector for estimates based on the same coefficient matrix, but variations for the same sector are usually quite small when alternative coefficient matrices are used. The subject of section B is the size distribution of individual sector errors, which is found to be very peaked, and characterized by large errors for a small number of sectors, and small errors for the majority of sectors.

Section C takes up the behavious of estimation errors over time, and a classification of the sectors according to the time patterns of estimation errors is effected.

Section D gives a further study of some of the largest errors and connect them with changes in important direct coefficients.

Section E is devoted to a study of the estimation errors for individual sectors in alternative sector specifications for periods of varying distance from the base year.

Section A. Comparisons of estimation errors by sectors

Average intermediate deliveries from each sector in the period 1949-1960 and standard errors (over the 12 year period) in the 92-, 36- and 7-sector specifications are given in Appendix table I. The standard errors are given for estimates based on six alternative coefficient matrices:

the 133-sector matrix for 1960
the 92-sector average matrix for 1959-1961
the 92-sector matrix for 1960
the 36-sector average matrix for 1959-1961
the 36-sector matrix for 1960 and
the 7-sector matrix for 1960

If we compare the standard errors for estimates based on the 133-sector matrix and the 92-sector matrix for 1960, they turn out to be very nearly the same in nearly all sectors. Of 79 Norwegian production sectors only 7 had a standard error for estimates based on the 92-sector matrix exceeding the standard error for estimates based on the 133 sector matrix by as much as 0.5 million kroner and 7 had differences of corresponding magnitudes in the opposite direction. For 65 of the sectors the difference was less than 0.5 million kroner. The corresponding figures for 60 import and transfer sectors are 4 with larger standard errors for estimates based on the 92-sector matrix and 8 with smaller standard errors. For 48 import and transfer sectors the difference was less than 0.5 million kroner. In the evaluation of these results it should be remembered that accounting figures for final demand in the 133-sector specification were not available, and estimates were produced by a rather crude process of splitting up the 92-sector accounts figures (Chapter II).

When we compare the estimates based on the single year 92-sector matrix for 1960 with those based on the 92-sector average matrix for 1959-1961, we obtain by the same standard as above 36 sectors with greater standard errors for the single year based estimates, 21 with smaller and 23 with differences of less than ½ million kroner in the standard error for the total of 79 Norwegian sectors with non-zero intermediate deliveries. Correspondingly, there are 26 sectors with greater standard errors, 17 with smaller and 17 with less than ½ million kroner in difference for the 60 import and transfer sectors. (The reservations about direct comparisons of the standard errors, as we have computed them, have been mentioned earlier and should be kept in mind.)

The tendency noticed earlier to improved estimates when the average basis matrix is used is thus by far not uniform for all sectors but is due to a slight dominance of the sectors for which there is improvement over those for which there is deterioration. Thus there are 15 Norwegian sectors and 12 import sectors for which there is as much as a 20 per cent decrease in the standard error against 9 Norwegian and 12 import sectors for which there is as much as a 20 per cent increase in the standard error when we go from the single year base estimates to the average year base estimates.

In the 36 sector specification we may study the effects of aggregating the base year matrix from the 133 and 92 sector levels to the 36 sector level. For the sake of convenience the standard errors in the 36 sector specification have been reproduced in table 27. We will restrict our analysis to a comparison of the estimates based on the 92 sector and the 36 sector matrices for 1960. Again it is the similarities in the standard errors for estimates based on different basis matrices which is most striking. Among the 31 Norwegian and 28 import sectors for which comparisons can be made there are 8 Norwegian and 10 import sectors for which the difference was less than ½ million kroner. 13 Norwegian and 10 import sectors for which the 92 sector base gave the best estimates and 10 Norwegian and 8 import sectors for which the 36 sector base gave the best estimates.

The sectors where the 36 sector base gave particularly noticeable improvements in the estimates were for the Norwegian sectors: No. 17 Basic metal industries, No. 31 Communications, No. 35 Other services and No. 36 Unspecified. The first of these delivers its products to the Iron and metal products industries, which are specified into 3 sectors in the 36 sector matrix but is only one in the 92 sector matrix. However, there is even greater improvement from the 133 sector base to the 36 sector base for Basic metal industries. The other three sectors deliver products that one can easily imagine do not vary in strict proportion with production in the receiving sectors.

Among the import sectors the 36 sector base gave the most noticeable reductions in the standard error for No. 1 Agriculture, No. 13 Products of oils and fats and No. 37 Transport expenditures abroad.

At the 7-sector specification level (table 28) we may compare estimates based on base year matrices both with 133, 92, 36 and 7 sectors all for 1960. Again, the most striking feature is the similarities in the levels of magnitude of the standard deviations, and the apparent randomness in the determination of which basic matrix gives the smallest, the second smallest etc. standard error.

Table 27 a. Standard errors in per cent for individual domestic sectors in the 36/33 sector specification for estimates based on different coefficient matrices

Sectors	Average intermediate delivery 1949-1960	Basis matrix			
		133 sectors 1960	92 sectors 1960	36 sectors 1959-61	36 sectors 1960
	Million 1955-kroner	Pct.	Pct.	Pct.	Pct.
Norwegian production sectors					
11 Agriculture	2 401	2.7	3.3	7.9	9.4
12 Forestry	806	17.4	17.6	19.1	18.9
13 Fishing, whaling	661	9.9	9.7	15.5	16.0
21 Mining	198	19.2	19.0	17.9	19.2
22 Non-metallic mineral products	403	25.1	25.0	25.2	25.6
23 Basic metal industries	519	9.1	6.3	7.4	5.6
24 Metal products	524	7.6	..	7.9	6.8
25 Machinery	121	4.7	..	4.2	4.7
26 Transport equipment	26	9.2	..	14.3	10.4
24/ 25/ 26 Iron and metal products	671	7.0	6.8	..	6.5
27 Shipbuilding industries	115	26.3	25.9	16.8	26.0
28 Electrical machinery etc.	236	22.2	22.3	20.9	22.1
29 Other manufacturing	61	59.3	59.0	63.7	59.0
31 Food industries	885	29.0	30.2	31.2	33.8
32 Tobacco and beverages	49	5.1	5.3	8.4	3.5
33 Products of oils and fats	347	15.5	15.4	15.5	20.9
34 Petroleum products	87	15.4	..	104.4	16.8
39/ 49 Chemical products	506	25.2	..	21.2	25.4
34/ 39/ 49 Chemicals	592	..	23.0	..	24.1
41 Textiles	336	8.5	8.4	8.3	8.4
42 Clothing	36	28.0	27.8	39.8	27.8
43 Footwear, leather, fur	74	36.7	36.7	25.7	37.8
44 Wood and cork etc.	902	6.9	7.1	10.1	7.3
45 Pulp, paper and paper products	972	2.0	5.0	7.0	6.4
46 Printing and publishing	404	10.3	10.3	7.5	9.7
50 Construction	6	21.7	21.7	21.7	21.7
61 Wholesale and retail trade	3 362	3.6	3.6	2.6	3.6
62 Water transport	462	7.8	7.8	12.9	9.6
63 Land and air transport	774	2.7	2.7	3.5	2.6
64 Communications	273	8.7	8.7	7.0	6.5
71 Electricity, gas and water	501	11.2	11.2	10.6	11.9
72 Banking and insurance	217	11.7	11.6	13.4	12.3
75 Educational, health services	8	17.5	16.3	11.3	12.5
76 Personal services	60	4.5	4.5	3.5	3.7
77 Other services	207	8.0	8.0	6.1	6.2
78 Unspecified	1 107	12.0	12.1	6.8	10.7

Table 27 b. Standard errors in per cent for individual import sectors in the 36/33 sector specification for estimates based on different coefficient matrices

Sectors	Average intermediate delivery 1949-1960	Basis matrix 133 sectors 1960	92 sectors 1960	36 sectors 1959-61	36 sectors 1960
	Million 1955-kroner	Pct.	Pct.	Pct.	Pct.
Import sectors					
11 Agriculture	634	10.6	10.0	8.2	8.6
12 Forestry	105	67.4	67.2	60.7	65.8
13 Fishing, whaling	9	77.8	77.8	66.7	80.0
21 Mining	307	32.9	34.2	27.0	37.6
22 Non-metallic mineral products	104	14.4	14.4	15.1	14.5
23 Basic metal industries	1 008	7.4	6.5	6.5	7.6
24 Metal products	129	21.1	..	27.2	20.7
25 Machinery	100	17.2	..	16.4	17.2
26 Transport equipment	124	17.4	..	17.7	17.2
24/25/26 Iron and metal products	352	16.3	16.9	..	16.4
27 Shipbuilding industries	69	20.7	19.7	25.5	19.7
28 Electrical machinery etc.	125	35.4	38.3	38.8	38.0
29 Other manufacturing	45	71.8	70.5	114.4	72.3
31 Food industries	77	35.7	36.2	35.3	40.5
32 Tobacco and beverages	16	25.0	25.0	13.1	17.5
33 Products of oils and fats	84	143.8	145.6	107.8	125.0
34 Petroleum products	730	16.4	..	11.8	15.9
39/49 Chemical products	326	31.0	..	39.0	31.8
34/39/49 Chemicals	1 057	..	20.4	..	20.7
41 Textiles	396	5.8	5.7	7.5	6.0
42 Clothing	7	77.1	77.1	92.9	80.0
43 Footwear, leather, fur	45	28.9	28.9	28.9	30.2
44 Wood and cork etc.	79	16.1	16.2	24.7	16.5
45 Pulp, paper and paper products	63	30.0	30.0	33.5	32.3
46 Printing and publishing	3	358.6	358.6	282.7	358.6
61 Wholesale and retail trade	57	12.5	12.5	9.7	9.1
62 Water transport	3	100.0	100.0	84.4	110.0
63 Land and air transport	2	61.9	61.9	57.1	61.9
64 Communications	9	28.1	28.1	24.7	28.1
71 Electricity, gas and water	93	14.9	15.0	14.4	16.4
72 Banking and insurance	31	5.5	5.5	6.8	8.4
77 Other services	127	54.2	54.2	43.9	50.2
79 Transport expenditures abroad	1 632	7.1	7.1	5.0	4.7
Transfer accounts	145	15.5	15.5	..	14.0

Table 28. Standard errors for individual sectors in the 7 sector specification for estimates based on different coefficient matrices

	Average intermediate delivery 1949-1960	Basis matrix			
		133 sectors 1960	92 sectors 1960	36 sectors 1960	7 sectors 1960
	Million 1955-kroner	Pct.	Pct.	Pct.	Pct.
Norwegian production sectors					
1. Agriculture, forestry, fishing, etc.	3 867	4.1	3.5	1.8	2.5
2. Minerals. metals and products	2 203	11.8	11.0	10.8	10.7
3. Production of food, drink, chemicals	1 833	20.0	20.4	20.8	20.2
4. Products of wood, fibres etc.	2 763	2.6	2.6	2.9	2.5
5. Construction	6	21.7	21.7	21.7	23.4
6. Trade and transport	4 872	3.0	3.0	3.3	5.5
7. Other activities (services)	2 320	7.0	7.0	6.3	6.0
Import sectors					
1. Agriculture, forestry, fishing, etc.	749	7.6	7.6	8.5	8.5
2. Minerals, metals and products	2 011	4.2	4.2	3.8	6.5
3. Production of food, drink, chemicals	1 218	28.3	28.1	28.2	25.3
4. Products of wood, fibres etc.	609	9.6	9.7	7.5	7.2
6. Trade and transport	71	6.5	6.6	7.3	5.6
7. Other activities (services)	1 883	3.1	3.1	2.5	5.4
8. Transfer accounts	145	15.5	15.5	14.0	9.7

Section B. The size distributions of estimation errors

The standard errors vary between the sectors. In Appendix table I we have also computed the standard errors for estimates based on the coefficient matrices for 1960 as percentages of the average intermediate deliveries from each sector. Even these percentages vary widely between sectors. To some extent they appear to depend on the absolute size of the average intermediate delivery from the sector. (Table 29). But there are great dispersions also in the standard errors for sectors with the same average size of intermediate deliveries.

It is of some interest to examine whether the precision differs between different types of sectors. For this purpose we have excluded the 4 unspecified sectors, the sector Agricultural capital formulation and Electricity from the 92-sector specification and divided the rest into a group of commodity producing and a group of service producing sectors.

Table 29. Standard error in per cent of average intermediate delivery by size of delivery for estimates in the 92 sector specification, based on the 92 sector coefficient matrix for 1960

Size of average intermediate delivery Million kroner	Domestic production sectors			Import sectors		
	Number of sectors	Average standard error		Number of sectors	Average standard error	
		Mill. kroner	Per cent		Mill. kroner	Per cent
.1 - .9	2	0.7	161.1	3 2) (2)	2.7 2) (1.1)	362.5 2) (122.3)
1.0 - 4.9	5	1.9	104.5	9	4.1	176.3
5.0 - 24	13	2.6	24.6	17	13.8	88.4
25 - 49	9 1) (8)	16.3 (10.9)	49.8 1) (31.7)	6	17.8	39.9
50 - 149	19	24.3	27.5	14	30.7	31.2
150 - 1 020	29	45.8	12.9	6	73.2	12.9
2 000 -	2	109.5	3.9	1	116.2	7.1
Total	79 1) (78)	27.9 (27.5)	31.9 1) (29.8)	56 2) (55)	24.5 (24.9)	88.2 (74.4)

1) One sector had a particularly high standard error, namely 194.5 per cent. Figures in parantheses are exclusive of this sector. 2) One sector had a particularly high standard error, 842.9 per cent. Figures in parantheses are exclusive of this sector.

The commodity producing sectors include all from no. 111 Agriculture up to and including no. 491 Rubber products, with the exclusion of no. 461 Publishing etc. and no. 462 Printing, bookbinding etc. The two latter together with the rest of the sectors are then grouped as service producing. The results for the two groups are given in table 30. There does not appear to be significant differences in precision.

The errors in estimates should be considered in relation to the variance in direct input-output coefficients. In the study of coefficient stability1) we found group averages of standard deviations about average coefficients over the period 1949 to 1960 of the order of 1 to 3 per cent points, depending on the size and type of coefficients. The dispersions of standard deviations for individual coefficients within each size group were rather peaked, with the majority close to the average, but with a small percentage of very large standard deviations.

Table 30. Standard error in per cent of average intermediate delivery by size of delivery for estimates in the 92 sector specification, based on the 92 sector coefficient matrix for 1960. Commodity producing and service producing domestic sectors

Size of average intermediate delivery Million kroner	Commodity producing sectors			Service producing sectors		
	Number of sectors	Average standard error		Number of sectors	Average standard error	
		Mill. kroner	Per cent		Mill. kroner	Per cent
.1 - .9	2	.7	161.1	-	-	-
1.0 - 4.9	5	1.9	104.5	-	-	-
5.0 - 24	6	3.1	25.0	7	2.1	24.4
25 - 49	6 (5)1)	23.2 (11.2)	59.5 (32.8)	3	10.3	29.9
50 - 149	13	25.5	27.4	3	21.0	25.7
150 - 1 000	17	57.3	15.6	9	19.0	8.1
2 000 -	1	98.0	4.2	1	121.0	3.6
Total	50	31.0	39.6	23	17.4	15.2

1) One sector had a particularly high standard error, namely 194,5 per cent. Figures in parantheses are exclusive of this sector.

Let us now look at the dispersion in errors of estimates obtained for this period by the use of the matrix of coefficients for one of its years (1960). We take as the subject of our study the estimates in the 36-sector specification obtained by using the 36-sector coefficient matrix for 1960 and the estimates in the 92-sector specification obtained by using the 92-sector coefficient matrix for 1960. We omit the 7-sector estimates, since they are so few in number and the 133-sector estimates, since they are not quite "straight forward" in the way they were derived (See chapter II). For each of our two sets of estimates we take the standard error for each individual sector as a percentage of the average intermediate delivery from that sector in the observation period. For the 36-sector estimates we get 34 such percentages (2 sectors had zero average intermediate deliveries) and for the 92-sector estimates we get 79 percentages (13 sectors had zero average intermediate deliveries). Correspondingly we get respectively 32 and 56 intermediate import estimates. Characteristics of the distributions of these percentages are given in table 31 a and b, and figures in constant (1955)-kroner are given in table 32 a and b. As we have already noticed, the standard errors as percentages of average intermediate deliveries are quite considerable, 15 per cent on the average for the 36-sector estimates and 32 per cent for the 92-sector estimates. However, 59 per cent of the sectors in the 36-sector specification and 72 per cent in the 92-sector specification have percentage standard errors below the average and only 12 and 9 per cent respectively have standard errors above the average plus the standard deviation (against 16 per cent for the normal distribution). Then again there are some extremely large standar errors (percentagewise): 3 per cent and 4 per cent respectively are above the average plus three times the standard deviation (against .2 per cent for the normal distribution). The median error is only 10½ per cent for the 36 sector estimates and 19 per cent for the 92 sector estimates.

The figures in parantheses in tables 29 and 31 a, illustrate the reduction in dispersion when the most extreme standard errors are omitted.

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1) Sevaldson: Op.cit. 1969.

Table 31 a. Characteristics of the distributions of standard errors of estimates for individual production sectors, taken as percentages of average intermediate deliveries 1949-1960 for each sector. Estimates based on 36-sector and 92-sector coefficient matrices for 1960

	36-sector estimates[1]	92-sector estimates[2]	Normal distribution
Average of standard error in per cent of average intermediate delivery	15.4 (14.1)	31.9 (22.9)	.
Standard deviation about average percentage	11.8 (9.2)	39.9 (18.8)	.
Per cent of units below average minus standard deviation	11.8 (15.2)	- (4.1)	15.85
Per cent of units between average and average minus standard deviation	47.0 (45.4)	72.2 (60.8)	34.15
Per cent of units between average and average plus standard deviation	29.4 (21.2)	19.0 (21.6)	34.15
Per cent of units between average plus one and two times standard deviation	8.9 (12.1)	2.5 (8.1)	13.55
Per cent of units between average plus two and three times standard deviation	- (6.1)	2.5 (4.1)	2.15
Per cent of units above average plus three times standard deviation	2.9 (-)	3.8 (1.3)	.15

1) Figures in parenthesis for the 36-sector estimates have been computed after ommission of the greatest percentage standard error (59.0 per cent). 2) Figures in parenthesis for the 92-sector estimates have been computed after ommission of the 5 greatest percentage standard errors (200.0, 194.5, 183.3, 122.2 and 120.0).

Table 31 b. Characteristics of the distributions of standard errors of estimates for individual import sectors, taken as percentages of average intermediate deliveries 1949-1960 for each sector. Estimates based on 36-sector and 92-sector matrices for 1960

	36-sector estimates[1]	92-sector estimates[2]
Average of standard error in per cent of average intermediate delivery	45.5 (35.4)	88.1 (58.9)
Standard deviation about average percentage	63.8 (30.6)	130.5 (51.7)
Per cent of units below average minus standard deviation	- (3.2)	- (5.8)
Per cent of units between average and average minus standard deviation	71.9 (61.3)	73.2 (55.7)
Per cent of units between average and average plus standard deviation	18.7 (19.4)	19.6 (21.2)
Per cent of units between average plus one and two times standard deviation	6.3 (9.7)	1.8 (7.7)
Per cent of units between average plus two and three times standard deviation	- (6.4)	3.6 (9.6)
Per cent of units above average plus three times standard deviation	3.1 (-)	1.8 (-)

1) Figures in parenthesis for the 36-sector estimates have been computed after ommission of the greatest percentage standard error (385.6 per cent). 2) Figures in parenthesis for the 92-sector estimates have been computed after ommission of the 4 greatest percentage standard errors (842.9, 379.2, 358.6 and 294.0).

Table 32 a. Characteristics of the distributions of standard errors of estimates for individual production sectors, in constant price (1955-)kroner for the period 1949-1960. Estimates based on 36-sector and 92-sector coefficient matrices for 1960

	36-sector estimates 1)	92-sector estimates 2)	Normal distribution
Average standard error in (1955-)kroner	57.3 (50.2)	27.8 (21.2)	.
Standard deviation about average standard error	65.0 (50.8	33.5 (22.3)	.
Per cent of units below average minus standard deviation	- (-)	- (-)	15.85
Per cent of units between average and average minus standard deviation	65.7 (64.8)	65.8 (63.5)	34.15
Per cent of units between average and average plus standard deviation	22.8 (14.7)	24.1 (23.0)	34.15
Per cent of units between average plus one and two times standard deviation	5.7 (14.7)	1.3 (9.5)	13.55
Per cent of units between average plus two and three times standard deviation	2.9 (2.9)	6.3 (-)	2.15
Per cent of units above average plus three times standard deviation	2.9 (2.9)	2.5 (4.0)	.15

1) Figures in parenthesis for the 36-sector estimates have been computed after ommission of the greatest standard error (299.0 kroner). 2) Figures in parenthesis for the 92-sector estimates have been computed after ommission of the 5 greatest standard errors (145.9, 142.2, 120.8, 110.6 and 106.1 kroner).

Table 32 b. Characteristics of the distributions of standard errors of estimates for individual import sectors in constant price (1955-)kroner for the period 1949-1960. Estimates based on 36-sector and 92-sector coefficient matrices for 1960

	36-sector estimates 1)	92-sector estimates 2)
Average standard error in (1955-)kroner	35.4 (32.8)	24.4 (18.1)
Standard deviation about average standard error	35.5 (30.6)	31.4 (19.6)
Per cent of units below average minus standard deviation	- (-)	- (-)
Per cent of units between average and average minus standard deviation	68.7 (70.9)	71.4 (69.2)
Per cent of units between average and average plus standard deviation	12.5 (9.7)	10.7 (15.4)
Per cent of units between average plus one and two times standard deviation	12.5 (9.7)	14.3 (3.9)
Per cent of units between average plus two and three times standard deviation	6.3 (9.7)	1.8 (11.5)
Per cent of units above average plus three times standard deviation	- (-)	1.8 (-)

1) Figures in parenthesis for the 36-sector estimates have been computed after ommission of the greatest standard error (116.0 kroner). 2) Figures in parenthesis for the 92-sector estimates have been computed after ommission of the 4 greatest standard errors (163.7, 116.2, 80.3 and 70.6 kroner).

Similar characteristics as those found for estimation errors for domestic intermediate deliveris are found for the distributions of estimation errors for intermediate imports in table 31 b, only the average percentages are much larger. The medians are 24.4 for the 36-sector estimates and 41.3 for the 92-sector estimates.

The distributions of errors in absolute constant kroner values in tables 32 a and b show much of the same characteristics as the distributions of percentages, but difference in the level of magnitude between domestic and imports are now in the opposite direction from what they are when the standard errors are taken in per cent of average deliveries.

In million kroner the median standard deviation is 35.6 for domestic and 11.3 for imported intermediate deliveries in the 36-sector specification and 16.6 and 11.0 respectively in the 92-sector specification.

In table 33 we have described the distributions of errors in terms of constant price values for the standard errors as well as the absolute values of errors for three individual years (1949, 1955 and 1959) and the maximum numerical errors for each sector, all for the 92-sector estimates based on the 92-sector coefficient matrix for 1960. According to this table 71 out of the 79 sectors or 9 out of 10 have standard errors below 60 million kroner. (In comparison 46 of the 79 sectors had average intermediate deliveries above 60 million kroner). Half of the sectors had standard errors of less than 18 million kroner. Figures for the absolute numerical values of the errors in the three years 1949, 1955 and 1959 and for the maximum error over the whole period for each sector, do not add very much to the picture. The tendency for errors to increase as we move away from the base year comes out in these figures also.

Deplorable as they are, large (standard) errors measured in kroner value, are less disturbing when they are small in relation to the figures to be estimated, whereas we will not be much concerned with errors in the estimates of small figures, even if they are large percentagewise. There is consequently a need to see the errors and the figures to be estimated in connection in some more detail than what is revealed by the standard errors in per cent of average deliveries.

Table 33. Distributions of errors of estimates for individual sectors. Based on 92 sector coefficient matrix 1960. Million kroner. Standard errors 1949-1960 and absolute errors 1949, 1955 and 1959

	Standard errors	Absolute errors			Maximum errors
		1949	1955	1959	
a. Production sectors					
Maximum error is	145.9	220.6	198.4	109.0	220.6
75 sectors or 95 per cent have errors below	106.2	161.8	75.9	49.4	184.0
71 " " 90 " " " " "	60.5	106.1	60.6	33.5	134.0
59 " " 75 " " " " "	31.7	46.3	27.2	16.8	55.1
39 " " 50 " " " " "	17.5	11.8	13.4	4.9	29.8
26 " " 33.3 " " " " "	8.3	4.4	6.1	2.0	14.0
Arithmetic mean	27.8	35.0	23.2	13.9	46.8
Median	17.4	14.0	13.9	4.8	30.5
b. Import sectors					
Maximum error is	163.7	231.8	123.0	85.5	246.8
53 sectors or 95 per cent have errors below	70.7	100.5	99.1	37.4	125.5
50 " " 90 " " " " "	66.1	73.0	66.9	24.9	119.5
42 " " 75 " " " " "	31.8	33.5	22.7	10.2	49.0
28 " " 50 " " " " "	10.9	9.5	10.0	4.2	18.5
19 " " 33.3 " " " " "	6.0	3.8	3.7	2.4	12.5
Arithmetic mean	24.4	28.7	25.3	10.6	40.9
Median	11.0	9.6	10.5	4.2	18.9

In table 34 we have grouped the sectors according to the size of the standard error in per cent of average intermediate delivery. For each size group of percentage standard errors we have given the number of sectors and the sum of average intermediate deliveries from each of them. These sums are then found as percentages of the sum total of average intermediate deliveries from all sectors (with non-zero average intermediate deliveries) and as averages per sector in each group.

The basis matrix is the 92 sector matrix for 1960 and the estimates are in the 92 sector specification. According to this table half the production sectors have a standard error of less than one fifth of the average intermediate delivery, but these sectors cover four fifths of total average intermediate deliveries. There is a clear negative correlation between the percentage standard error and the size of average intermediate deliveries. 58 per cent of average intermediate deliveries are estimated with standard errors of less than 10 per cent.

For import sectors, one third of the sectors have a standard error of less than one fifth of the average intermediate delivery and these sectors have four fifths of average intermediate import deliveries. The negative correlation between percentage standard error and average size of intermediate deliveries is clear, 32 per cent of intermediate deliveries are estimated with less than 10 per cent standard error.

Table 34. Percentage standard errors and average intermediate deliveries 92 sectors 1949-1960. Basic matrix 92 sectors 1960

Sectorwise standard error in per cent of average intermediate delivery	Number of sectors	Sum of average intermediate deliveries		Average of average intermediate deliveries per sector
		Million kroner	Per cent of total	
a. Production sectors				
3.6 - 4.9	7	7 185	40.1	1 026
5.0 - 9.9	13	3 161	17.7	243
10.0 - 14.9	10	2 198	12.3	220
15.0 - 19.9	10	1 844	10.3	184
20.0 - 24.9	9	1 430	8.0	159
25.0 - 29.9	7	799	4.5	114
30.0 - 39.9	7	377	2.1	54
40.0 - 49.9	4	709	4.0	177
50.0 - 59.9	3	99	.6	33
60.0 - 200.0	9	76	.4	8.5
Total	79	17 878	100.0	226
b. Import sectors				
5.5 - 9.9	4	2 138	32.0	535
10.0 - 14.9	5	1 377	20.6	275
15.0 - 19.9	5	1 867	28.0	373
20.0 - 24.9	4	217	3.5	54
25.0 - 29.9	3	77	1.0	26
30.0 - 39.9	7	407	6.1	67
40.0 - 59.9	4	162	2.4	41
60.0 - 79.9	8	331	5.0	41
80.0 - 99.9	2	2	-	1
100.0 - 149.9	5	42	.6	8
150.0 - 199.9	5	41	.6	8
294 - 380	3	25	.4	8
842.9	1	1	-	1
Total	56	6 687	100.0	119

Section C. The behaviour of estimation errors over time

Erroneous estimates of the intermediate deliveries from a sector may be due to one or both of the following causes:

1) erroneous forecasts of production levels in receiving sectors
2) use of erroneous input-output coefficients in the computations of deliveries to one or more of the receiving sectors.

The more substantial errors will in general be associated with the second cause, i.e. errors due to changes in "direct" coefficients. In our data for the period 1949-1960 we might expect the following types of coefficient behaviour:

a) Coefficients in the test period fluctuate about a base year level. If the demand for intermediate deliveries from a sector is dominated by coefficients of this type, we may expect deviations of varying signs, and with a small average.

b) Coefficients in the test period and through the base year follow a trend. If this type of coefficients dominate the demand, we will expect to observe deviations which are all of the same sign and which increase in magnitude as we move away from the base year.

c) Coefficients fluctuate about a stable level, but the base year coefficient happens to be atypical. In this case the deviations should tend to have the same sign and show no trend. Estimates based on an average of several years instead of one single base year should be expected to improve the precision materially.

d) There has been a change in the coefficient in the test period from one level to another, of which one applies in the base year. In this case, deviations should have the same sign from a certain number of years away from the base period and onwards, and the size of deviations in this sub-period should be approximately stable. Estimates based on average coefficients should not improve precision.

We must expect all these causes to be in operation simultaneously.

Going through the estimates in the 92-sector specification, based on the 92-sector coefficient-matrix for 1960, we found that of the 79 domestic production sectors with non-zero intermediate deliveries there were 31 for which the estimates deviated in the same direction in all the 11 test years 1949-1959, 14 for which the deviations were in the same direction in all but one year, and of these 9 had the deviation of opposite sign in one of the two years closest to the base period. Alltogether there were 45 sectors for which the estimates deviated in the same direction in all but one or both of the two years closest to the base period. Only 19 sectors, or roughly 1/4, did not have the same direction of deviations in all the 5 years farthest away from the base period.

The corresponding figures for the 56 import sectors with non-zero intermediate deliveries were 28 with the same direction of deviations in all test years, 7 with the same direction in all but one test year and for 3 of these the break in pattern occured in the year closest to the base year. There were 34 import sectors which had all deviations in the same direction in the 9 years farthest away from the base year, and 15 or less than 1/3 which did not have the deviations in the same direction in all the 5 years farthest away from the base year. (Table 35.)

These patterns are not materially changed when we consider deviations for estimates based on the average 92 sector coefficient matrix for the years 1959-1961. Thus, among the domestic production sectors 23 of the 31 sectors which have deviations in the same direction in all years for estimates based on the 1960 matrix, also have deviations in the same direction in all years for estimates based on the average matrix, and of the remaining 8, six had deviations in the same direction in at least all the 7 years farthest away from the base period also when estimates were based on the average matrix. Similarly, 23 of the 28 import sectors which have deviations in the same direction in all years for estimates based on the 1960 matrix also have deviations in the same direction for the entire period 1949-1959 when estimates are based on the average matrix, and the remaining 7 had deviations in the same direction in at least all the 7 years farthest away from the base period also when estimates were based on the average matrix. There is thus an indication that the errors in the estimates are dominated by permanent changes in the coefficients.

The errors in the estimates of intermediate deliveries from a sector may fluctuate from one year to the next, and it must be a matter of individual judgement to decide whether there is a gradual trend in the size of the errors through the test period, or if they fluctuate about one or two relatively stable levels, with more marked or "structural" changes only when there is a shift from one level to another. Bearing this in mind, I have gone through the listed deviations for each sector,

Table 35. The direction of errors in estimates based on the 92 sector coefficient matrix for 1960. Number of sectors in the 92 sector specification

Sectors with same direction of deviations	Period of years farthest away from the base year							Total number of sectors
	5 years (49-53)	6 years (49-54)	7 years (49-55)	8 years (49-56)	9 years (49-57)	10 years (49-58)	11 years (49-59)	
in all years of the period:								
Domestic sectors	60	55	52	51	45	37	31	79
Import sectors	41	39	38	37	34	31	28	56
in all or all but one year of the period:								
Domestic sectors	70	65	62	60	55	49	39	79
Import sectors	46	42	41	40	37	36	32	56

for estimates based on the 92 sector coefficient matrix for 1960 and tried to characterize the sectors according to the nature of the deviations into the following three classes:

a) Those with a trend in deviations
b) those with deviations fluctuating about one or two levels and
c) those where none of these simple characteristics applied.

Among the domestic production sectors it was in my judgement reasonable to speak of a trend only for 10 sectors. For 27 sectors there appeared to be one level different from zero, about which the deviations fluctuated all or part of the time. Before this level was reached, there might be a period of numerically increasing deviations, or a period of fluctuations about zero and then numerically increasing deviations up to the new level. For 28 sectors there appeared to me to be two levels different from zero and for 14 sectors the fluctuations appeared to have a more random character.

If we for each of these groups of sectors compute the average of the standard error taken as a percentage of average intermediate delivery in the period, we get the following result:

Sectors with	Number	Average standard error in per cent
trend in error	10	22.9
2 changes in level of errors	28	41.3
of these with average intermediate delivery greater than 1.5 million 1955-kroner	22	23.3
1 change in level of errors	27	29.9
random errors	14	8.6

The results of my classification are given in table 36. Other persons might arrive at different classifications, but the main impression of the groupings would probably be much the same: Very few of the sectors display a pattern of gradually increasing errors from year to year as the estimates are referred to years in increasing distance from the base year. The "normal" pattern is that there occurs one or two shifts in the levels about which the errors fluctuate. Some of these shifts may be said to occur from one year to the next, others may take a couple of years. In my classification I got the numbers of "shift years" given in table 37.

It appears from this that the level of errors as a function of the distance from the base period is dominated by a limited number of shifts in the level of errors for individual sectors, shifts which in the majority of cases take effect over periods of 1 to 2 years.

Table 36. Classification of sectors according to the nature of the fluctuations in errors in estimates based on the 1960-coefficient matrix. 92 sector specification

Character of errors	Domestic production sectors	Import sectors
Trend or fluctuations about more than two non-zero levels	10	6
Fluctuations about:		
zero and 2 non-zero levels	4	4
2 non-zero levels	24	14
zero and one non-zero level	12	6
one non-zero level	15	20
zero	9	4
Other	5	2
Total number of sectors with non-zero intermediate deliveries	79	56

Table 37. Sectors distributed according to the number of years of changes in the level of errors in estimates based on the 1960-coefficient matrix. 92 sector classification

Number of years with changes in error level[1]	Number of shifts in error levels			Sectors with trend	Total
	0	1	2		
Domestic production sectors					
0	9	-	-	-	9
1	-	17	-	-	17
2	-	4	16	-	20
3	-	4	8	2	14
4	-	2	2	1	5
5	-	-	1	1	2
6	-	-	1	4	5
7	-	-	-	2	2
No pattern	5	-	-	-	5
Total	14	27	28	10	79
Import sectors					
0	4	-	-	-	4
1	-	14	-	-	14
2	-	8	8	-	16
3	-	3	7	3	13
4	-	1	3	-	4
5	-	-	-	1	1
7	-	-	-	1	1
10	-	-	-	1	1
No pattern	2	-	-	-	2
Total	6	26	18	6	56

1) Some changes continue over more than one year.

It is of some interest to investigate to what extent the shifts in error levels are associated with particular years in our test period. We get the following picture:

Years	Sectors with changes in levels of errors		
	Domestic	Import	Total
60/59	47	36	83
59/58	15	26	42
58/57	15	11	26
57/56	13	7	20
56/55	9	8	17
55/54	17	11	28
54/53	13	7	20
53/52	10	1	11
52/51	7	9	16
51/50	11	3	14
50/49	12	2	14

There is a remarkable concentration of changes in the level of errors in the years closest to the base year. To a considerable extent this may be caused by a tendency to overlook minor changes from one level to another of the same sign, whereas changes from zero (in the base year) are more easily registered. There may also have been significant changes in the statistical recording process in the years 1959-1961, and finally there are indications of a more rapid rate of structural change in the Norwegian economy from the end of the 1950-ies onwards. We also note the relatively large number of changes between 1954 and 1955, remembering that 1955 is the base year for the fixed-price calculations, and that figures in fixed prices for earlier years had to be recalculated from previous estimates. Apart from this, the changes appear to be spread relatively evenly over the years 1949-1957.

The fact that errors seem to be characterized by shifts in error levels for individual sectors makes it probable that they are associated with major shifts in specific input-output coefficients, either there might be a shift in the input coefficient for a major use of a product, e.g. due to technical change or a change in product mix, or there may be simultaneous changes in the input coefficients for all sectors using a given product, e.g. due to substitution. This type of substitution we would mainly expect between imports and domestic products.

The conclusion of this should be that if we could keep track of major structural changes in dominating input coefficients and major changes in market proportions between the base period and the period for which we try to make predictions, we would be able to improve the precision in input-output computations materially, and this should be a more promising approach than the estimation of time trends in input-output ratios. It is also possible, that by constructing a coefficient matrix partly from average coefficients for a base period of 2-3 years, and partly from coefficients from the year closest to the period of estimation, we might be able to considerably improve the precision in input-output estimates.

Section D. Errors for individual sectors

In table 38 we have listed the sectors with abnormally high standard errors compared to the average for the size group to which they belong according to table 29. (Sectors with standard errors in per cent exceeding the average for the group with more than the standard deviation for the percentages in the group.)

For the 14 domestic and 11 import sectors we have listed 44 deliveries of intermediate inputs in excess of 10 per cent of the deliveries from any one of these sectors in 1960. There were altogether 26 different sectors which received these deliveries and of these, dummy sectors for unspecified, and the sectors Agriculture, Grain mills products and livestock feed, Other oil refineries and Construction stood as receivers of 24 deliveries. 1 of these last mentioned deliveries and 4 more were intra-sector deliveries. In all the sectors mentioned there are reasons to believe that input-output coefficients may be particularly unstable, due to changes in product mix and particular substitutability among inputs (Grain mills and Oil refineries). They are also in general, so large that small changes in coefficients will give large changes in inputs. There is also evidence that the registration of intra-sector deliveries is susceptible to changes which are not real but merely caused by changes in registration practice. Some of the delivering sectors are also rather special. Whaling and Coal mining have had strong declines in production, and their products have been substituted as inputs by other commodities. The reverse applies for Air transport.

Table 38. Sectors with relatively large standard errors of estimates in the 92 sector specification based on the 92 sector coefficient matrix for 1960

Sector		Average inter-mediate delivery Mill.kr.	Standard error		Main users		Per cent of deliveries 1960
			Mill. kr.	Per cent			
Domestic production sectors							
220	Non-metallic mineral products	408	101	25	500	Construction	74.1
					220	Own sector	11.4
3x8[1]	Chemicals	408	91	22	500	Construction	27.3
					3x8	Own sector	14.2
316	Grain mill products and livestock feed	364	146	40	111	Agriculture	66.6
					316	Own sector	18.0
280	Electrical machinery etc.	236	53	22	500	Construction	75.5
					280	Own sector	15.2
391	Fertilizers etc.	166	41	25	111	Agriculture	56.6
					391	Own sector	29.4
331	Herring oil and fish meal	142	61	43	333	Other oil refineries etc.	53.8
					316	Grain mills, etc.	28.7
132	Whaling	107	49	46	333	Other oil refineries etc.	91.3
461	Publishing etc.	96	43	45	781	Unspecified, office supplies	100.0
290	Other manufacturing	61	36	59	500	Construction	20.2
					2x9	Iron and metal products	11.8
634	Air transport	34	20	59	784	Unspecified transport	78.3
					621	Ocean water transport	11.8
211	Coal mining	32	24	75	782	Unspecified energy	18.3
315	Fish processing	31	60	194	315	Own sector	71.4
					131	Fishing etc.	11.0
721	Bank of Norway	5	3	64	783	Unspecified services	100.0
318	Cocoa, chocolate and sugar confectionary	1	2	183	317	Bakery products	46.1
					318	Own sector	28.2
Import sectors							
773	Unspecified services	118	66	56	783	Unspecified services	100.0
211	Coal mining	108	80	74	631	Railway transport	21.2
					712	Gas supply	17.7
					782	Unspecified energy	11.5
121	Forestry	105	71	67	451	Wood pulp	69.4
					491	Rubber products	12.9
332	Vegetable oil mills	49	37	75	111	Agriculture	57.4
					316	Grain mill products	19.1
290	Other manufacturing	45	32	70	452	Paper, paperboard	13.3
					500	Construction	11.6
391	Fertilizers etc.	21	37	175	111	Agriculture	62.1
					391	Fertilizers etc.	24.2
331	Herring oil and fish meal	20	59	294	333	Other oil refineries etc.	98.8
333	Other oil refineries etc.	15	29	192	333	Other oil refineries etc.	51.7
					500	Construction	41.5
462	Printing, book-binding etc.	3	10	359	781	Unsp. office supplies	77.3
					461	Publishing etc.	22.7
131	Fishing etc.	2	9	379	331	Herring oil etc.	74.1
					432	Leather and prod.	19.0
711	Electricity supply	1	6	842	711	Electricity supply	100.0

1) 348 and 398.

It is possible to push the analysis of the biggest errors one step further: By applying the 1960 direct input-output coefficients to the estimated production levels in each sector, we are able to specify the implicit estimates of individual deliveries between the sectors. These may be compared with accounts figures for the same deliveries, and thus used to derive errors for these items.
In table 39 such errors have been derived for the years 1949, 1955 and 1959 for deliveries from the sectors with the biggest errors in estimates of total intermediate deliveries. Only errors for items corresponding to input-output coefficients above 5 per cent in 1960 have been specified. 9 of the 10 sectors with the greatest standard errors in kroner have been covered. The sector Trade has been omitted, because it has a large number of deliveries with large coefficients. With some exceptions, a small number of big errors, corresponding to variations in the biggest input-output coefficients,

are responsible for the major part of the total error for a sector.

The errors for individual items may be looked upon as composed of two parts, namely the error caused by the error in estimated output level for the receiving sector, and the error that would be caused by using an erroneous input-output coefficient, even if the output estimate had been correct.

Table 39. Decomposition of errors in estimates of intermediate deliveries from production sectors. 92 sector specification estimates based on 92 sector coefficient matrix 1960

Delivering sector	Receiving sector	Coefficient 1960	1949		1955		1959	
			Intermediate delivery	Error	Intermediate delivery	Error	Intermediate delivery	Error
		Pct.			Million kroner			
316 Grain mills	111 Agriculture	9.1	201.0	91.0	313.0	-4.0	386.4	65.6
	316 Grain mills	14.9	3.0	80.2	1.0	86.4	90.1	5.1
	317 Bakery	20.1	95.0	-11.8	91.0	-7.8	76.7	.4
	Sum	..	299.0	159.4	405.0	74.6	453.2	71.1
	All other	..	6.0	2.5	7.0	1.2	5.2	3.1
	Total	..	305.0	161.9	412.0	75.8	458.4	74.2
121 Forestry	441 Sawmills	44.4	290.0	-109.0	303.0	-66.5	255.2	-32.2
	451 Woodpulp	37.8	353.0	-97.0	404.0	-31.0	464.5	-68.5
	453 Wallboards	11.6	13.0	-7.3	7.0	1.1	12.1	-2.3
	Sum	..	656.0	-213.3	714.0	-96.4	731.8	-103.0
	All other	..	92.1	-7.3	99.4	3.6	113.7	-6.0
	Total	..	748.1	-220.6	813.4	-92.8	845.5	-109.0
441 Sawmills	441 Sawmills	11.6	63.0	-15.6	66.0	-4.1	65.1	-6.7
	442 Wood ind.	9.7	130.0	-63.9	83.0	-1.5	85.0	.7
	500 Construction	5.6	271.0	-70.5	320.0	-66.0	304.9	-37.4
	Sum	..	464.0	-150.0	469.0	-71.6	455.0	-43.4
	All other	..	89.1	-39.5	91.5	-26.9	75.8	-3.7
	Total	..	553.1	-189.5	560.5	-98.5	530.8	-47.1
783 Unspecified	623 Service water transport	11.6	39.0	-17.1	42.0	-10.1	33.3	2.6
	635 Service transport..	32.0	27.0	-3.9	37.0	-4.3	41.4	-4.0
	721 Bank of Norway	10.1	1.0	.3	2.0	-.6	2.3	-.1
	722 State banks	16.9	1.0	.7	2.0	1.2	2.3	.3
	724 Life insurance	17.4	9.0	-1.1	8.0	1.3	11.8	-
	776 Recreation	11.5	21.0	-3.3	22.0	.4	27.0	-1.4
	Sum	..	98.0	-24.4	113.0	-12.1	118.1	-2.6
	All other	..	357.3	76.0	422.3	135.8	622.3	2.6
	Total	..	455.3	51.6	535.3	123.7	740.4	-
220 Mineral products	220 Mineral products ..	9.2	15.0	31.6	46.0	13.5	56.7	4.3
	500 Construction	8.6	206.0	101.0	342.0	47.5	410.4	.6
	Sum	..	221.0	132.6	388.0	61.0	467.1	4.9
	All other	..	38.0	17.4	60.0	7.2	69.8	6.3
	Total	..	259.0	150.0	448.0	68.2	536.9	11.2
111 Agriculture	111 Agriculture	28.8	890.3	34.7	959.2	17.8	1 082.4	31.6
	311 Slaugthering	45.2	332.0	-15.0	445.0	-21.0	453.5	-.5
	312 Dairy	69.5	597.0	-22.0	686.0	-28.0	781.7	18.3
	316 Grain mills	31.3	46.0	129.0	97.0	86.5	135.0	64.0
	319 Other food	18.7	20.0	16.9	29.0	8.0	37.6	-1.0
	Sum	..	1 885.3	143.6	2 216.2	63.3	2 490.2	112.4
	All other	..	117.0	-32.6	98.0	-4.3	105.3	-11.0
	Total	..	2 002.3	111.0	2 314.2	59.0	2 595.5	101.4

Table 39 (cont.). Decomposition of errors in estimates of intermediate deliveries from production sectors. 92 sector specification estimates based on 92 sector coefficient matrix 1960

Delivering sector	Receiving sector	Coefficient 1960	1949		1955		1959	
			Intermediate delivery	Error	Intermediate delivery	Error	Intermediate delivery	Error
		Pct.			Million kroner			
3x8 Chemicals	411 Spinning mills	4.7	5.0	21.3	25.0	1.2	23.0	5.2
	3x8 Chemicals	7.5	23.0	22.7	71.0	-4.1	76.0	-1.3
	234 Aluminium	5.1	-	5.8	6.0	5.1	6.0	17.3
	290 Other manufacturing	6.1	3.0	12.5	10.0	10.8	19.1	6.0
	500 Construction	3.3	58.0	59.8	124.0	25.8	158.4	-.6
	Sum	..	89.0	122.1	236.0	38.8	282.5	26.6
	All other	..	126.0	39.6	232.0	-7.0	242.3	14.6
	Total	..	215.0	161.7	468.0	31.8	524.8	41.2
331 Herring oil	333 Oil refinery	16.6	46.0	7.1	96.0	-35.6	67.6	-17.9
	All other	..	44.0	-8.6	62.0	-25.8	32.5	+5.0
	Total	..	90.0	-1.5	158.0	-61.4	100.1	-12.9
315 Fish processing	315 Fish processing	10.0	2.0	46.2	3.0	67.1	.7	69.0
	All other	..	21.0	-2.2	28.0	-4.4	22.5	-.2
	Total	..	23.0	44.0	31.0	62.7	23.2	68.8
711 Electricity	781 Unspecified	6.9	15.2	10.4	32.8	.3	40.6	-4.4
	782 Unspecified	21.7	13.0	-1.2	14.0	1.3	20.2	-3.0
	391 Fertilizer	8.1	12.0	20.3	33.0	8.4	36.9	14.6
	231 Ferro alloys	9.5	15.0	3.7	21.0	4.0	24.7	5.6
	234 Aluminium	9.8	10.0	1.2	15.0	6.2	33.0	11.7
	632 Tramway	7.4	3.0	1.9	4.0	.7	3.7	.2
	Sum	..	68.2	36.3	119.8	20.9	159.1	24.7
	All other	..	249.0	57.4	369.3	39.6	462.8	5.1
	Total	..	317.2	93.7	489.1	60.5	621.9	29.8

Such a decomposition has been effected in table 40 for some of the bigger individual errors from table 39.

It is perhaps not surprising that by far the largest parts of the errors are in general associated with the direct effects of changes in the input-output coefficients. The indirect effects of such changes are quite moderate, and relate to inputs in sectors which themselves have large errors in the output estimates. In table 40 we have decomposed 12 errors of individual estimates for each of the three years 1949, 1955 and 1959. There are 9 out of the 36 cases, where the error due to erroneous output estimates are larger than the error due to change in direct coefficient for the same item, out of these 9 items, 6 concern inputs to sectors which also have large standard errors of total intermediate output.

We are again brought to the conclusion that it is the big errors in the input-output coefficients relating to large inter- or intra-sector deliveries which are the main "trouble makers". We need not be equally concerned about the smaller items, individually or combined.

Table 40. Decomposition of large errors for individual deliveries 1949, 1955 and 1959, 92 sector estimates based on 92 sector matrix 1960. Million kroner

Delivering sector	Receiving sector	1949			1955			1959		
			Of this due to			Of this due to			Of this due to	
		Total error	Change in direct coefficient	Error in output level	Total error	Change in direct coefficient	Error in output level	Total error	Change in direct coefficient	Error in output level
111 Agriculture	316 Grain mills	129.0	78.2	50.8	86.5	68.8	17.7	64.0	41.5	22.5
121 Forestry	441 Sawmills	-109.0	-25.3	-83.7	-66.5	-23.0	-43.5	-32.2	-11.3	-20.9
220 Mineral products	500 Construction	101.0	102.0	-1.0	47.5	48.0	-.5	.6	1.6	-1.0
121 Forestry	451 Wood pulp	-97.0	-92.3	-4.7	-31.0	-34.0	3.0	-68.5	-74.5	6.0
316 Grain mills	316 Grain mills	80.2	56.1	24.1	86.4	75.1	11.3	5.1	-6.1	11.2
441 Sawmills	500 Construction	-70.5	-71.0	.5	-66.0	-66.0	-	-37.4	-36.9	-.5
315 Fish processing	315 Fish processing	46.2	41.8	4.4	67.1	60.8	6.3	69.0	62.1	6.9
3x8 Chemicals	500 Construction	59.8	60.0	-.2	25.8	25.5	.3	-.6	-.4	-.2
331 Herring oil	333 Oil refinery	7.1	.7	6.4	-35.6	-37.1	1.5	-17.9	-18.3	.4
111 Agriculture	111 Agriculture	34.7	2.7	32.0	17.8	1.8	16.0	31.6	1.6	30.0
220 Mineral products	220 Mineral products	31.6	17.9	13.7	13.5	7.2	6.3	4.3	3.4	.9
111 Agriculture	312 Dairy	-22.0	-53.0	31.0	-28.0	-50.5	22.5	18.3	6.3	12.0
Average of numerical values: for 12 items		65.7	50.1	21.0	47.6	41.5	10.7	29.1	22.0	9.4

Section E. Effects of the time element and the basis matrix specification on estimates for individual sectors

In order to investigate how the time element may effect the comparative precision in estimates for aggregated and disaggregated matrices we examined the results for the following periods at the 36 and 7 sector specification level: The four years 1949-1952, the four years 1953-1956 and the three years 1957-1959. For each sector in the 36 sector specification in each period the aggregate of the numerical values of the errors in each year of the period was found for estimates based on the 92-sector 1960 base matrix, on the 36 sector 1960 base matrix and the 36 sector 1959-61 base matrix. The results for the two latter sets of estimates were then taken as percentages of the former (tables 41 and 42).

Comparing the estimates based on the 92 and 36 sector matrices for 1960 (Table 41) we find that among the 32 estimates for Norwegian sectors the 36 sector basis matrix gave poorer estimates than the 92 sector matrix (percentages above 100) in all the three periods for 10 sectors better estimates in all periods for 6 sectors and (nearly) the same estimates for 5 sectors. We thus have a consistent ranking of the estimates in all the periods for 21 out of 32 sector estimates. For 6 sectors the 92 sector basis gave the best estimates in two consecutive periods and for 4 sectors they gave the poorest estimates in two consecutive periods whereas for one sector the 92 sector basis gave the best estimate in the middle period and the poorest in the first and the last.

Looking at each period in succession, we find the best estimates for the 92 sector based estimates in 16 sectors in 1949/52 in 17 sectors in 1953/56 and in 13 sectors in 1957/59. Correspondingly the figures for the numbers of sectors with best estimates based on the 36 sector matrix are 10, 10 and 13 sectors respectively and the sectors with indifferent estimates (99.1 - 100.9 %) are 6, 5 and 6.

Table 41. Ratios between cumulated numerical values of errors in estimates based on the 36-sector 1960-matrix and the 92-sector 1960-matrix for the sub-periods 1949-52, 1953-56 and 1957-59. 36-sector specification

		Errors in 36-sector estimates as percentages of errors in 92-sector estimates			
		1949 -52	1953 -56	1957 -59	Difference 1949 -52 - 1957 -59
Domestic sectors of production					
11	Agriculture	288.1	711.7	177.7	110.4
12	Forestry	104.3	116.6	101.6	2.7
13	Fishing, whaling	136.7	659.6	308.2	-171.5
21	Mining	101.6	101.5	72.6	29.0
22	Non-metallic mineral products	102.2	102.8	107.1	-4.9
23	Basic metal industries	93.2	84.0	101.5	-8.3
24/ 25/ 26	Iron and metal products (aggregate)	93.6	104.7	109.7	-16.1
27	Shipbuilding industries	100.4	100.4	100.2	.2
28	Electrical machinery etc.	99.5	99.1	99.4	.1
29	Other manufacturing	99.8	100.7	100.4	- .6
31	Food industries	112.4	113.3	109.4	3.0
32	Tobacco and beverages	27.4	106.8	157.9	-130.5
33	Products of oils and fats	102.1	145.5	147.6	-45.5
34/ 39/ 49	Chemicals (aggregate)	104.1	107.4	103.0	1.1
41	Textiles	99.1	91.8	99.8	- .7
42	Clothing	100.5	99.7	99.0	1.5
43	Footwear, leather, fur	102.6	104.4	104.1	-1.5
44	Wood and cork etc.	103.4	104.2	111.8	-8.4
45	Pulp, paper and paper products	112.4	155.2	98.4	14.0
46	Printing and publishing	90.9	97.0	85.8	5.1
50	Construction	100.0	100.0	100.0	-
61	Wholesale and retail trade	104.4	97.1	77.1	27.3
62	Water transport	121.5	150.6	70.6	50.9
63	Land and air transport	97.6	107.7	95.7	1.9
64	Communications	78.3	65.9	77.0	1.3
71	Electricity, gas and water	102.8	113.1	93.8	9.0
72	Banking and insurance	107.0	117.0	100.9	6.1
73	Business buildings, dwellings	71.6	92.2	91.3	-19.7
75	Educational, health services	74.6	75.0	105.2	-30.6
76	Personal services	127.1	33.9	78.6	48.5
77	Other services	75.9	74.0	80.8	-4.9
78	Unspecified	87.1	86.7	93.5	-6.4

Table 42. Ratios between cumulated numerical values of errors in estimates based on the 36-sector 1959-61 matrix and the 92-sector 1960-matrix for the sub-periods 1949-52, 1953-56 and 1957-59. 36-sector specification

		Errors in 36-sector (average matrix)-estimates as percentages of errors in 92-sector (1960-matrix)-estimates			
		1949 -52	1953 -56	1957 -59	Difference 1949 -52 - 1957 -59
Domestic sectors of production					
11	Agriculture	250.1	564.7	124.3	125.8
12	Forestry	105.7	118.5	103.3	2.4
13	Fishing, whaling	133.1	640.7	256.0	-122.9
21	Mining	95.9	87.9	92.1	3.8
22	Non-metallic mineral products	100.4	100.4	100.7	- .3
23	Basic metal industries	141.5	120.9	77.8	63.7
24/ 25/ 26	Iron and metal products (aggregate)	96.6	98.9	119.8	-23.2
27	Shipbuilding industries	73.6	45.3	42.6	31.0
28	Electrical machinery etc.	94.7	90.0	90.7	4.0
29	Other manufacturing	105.6	112.7	119.8	-14.2
31	Food industries	105.3	103.3	96.9	8.4
32	Tobacco and beverages	100.0	278.7	379.0	-279.0
33	Products of oils and fats	89.5	101.2	103.9	-14.4
34/ 39/ 49	Chemicals (aggregate)	128.3	153.4	176.3	-48.0
41	Textiles	127.1	64.8	70.4	56.7
42	Clothing	155.5	164.0	132.3	23.2
43	Footwear, leather, fur	97.6	92.2	97.5	.1
44	Wood and cork etc.	139.3	161.5	225.7	-86.4
45	Pulp, paper and paper products	126.8	177.4	118.6	8.2
46	Printing and publishing	73.0	57.5	48.0	25.0
50	Construction	79.3	90.6	166.7	-87.4
61	Wholesale and retail trade	70.0	63.3	106.1	-36.1
62	Water transport	155.6	238.1	69.4	86.2
63	Land and air transport	127.8	135.6	123.9	3.9
64	Communications	86.5	75.3	76.5	10.0
71	Electricity, gas and water	93.2	97.9	70.4	22.8
72	Banking and insurance	114.2	130.2	112.6	1.6
73	Business buildings, dwellings	62.7	78.3	74.6	-11.9
75	Educational, health services	62.7	64.3	110.5	-47.8
76	Personal services	87.0	35.4	78.6	8.4
77	Other services	77.4	72.1	74.1	3.3
78	Unspecified	35.0	32.0	107.3	-72.3

These results might indicate a sligt tendency for the advantages of the 92 sector matrix to increase as we move away from the base year. A closer study of the changes in the errors in the 36 sector base estimates as percentages of the errors in the 92-sector base estimates does not appear to confirm this. There were 15 sectors for which the error in the 36 sector base estimates were relatively larger in the period farthest away from the base year and 12 for which they were smaller. However, there were exactly 11 sectors in each of these groups, for which the difference was more than 2 per cent points.

A comparison of estimates based on the 92 sector matrix for 1960 and the 36 sector matrix 1959-61 gives very similar results. (Table 42.) The 36 sector average basis matrix gave poorer results than the 92 sector 1960 matrix in all the three periods for 11 sectors, better for 10 and the same for 1 sector. For 4 sectors the 92 sector basis gave the best estimates in two consecutive periods and for 6 the poorest in two consecutive periods.

The 92 sector based estimates were best in 14 sectors in 1949/52, in 15 sectors in 1953/56 and in 17 sectors in 1957/59. The 36 sector average based estimates are improved in relation to the 92 sector 1960-based estimates for 19 sectors and deteriorated for 11 if we move from the period closest to the base period)1957/59) to the one farthest away (1949/52).

Appendix table I. Sector specification. Average intermediate deliveries and standard errors of estimates. Individual sectors.

A. 133, 92 and 7 sectors

Code1)	Sector	Specification, in basis matrix, cols. 4-6 Norwegian sectors	Average intermediate deliveries 1949-60	Standard error			
				Basis matrix			
				133/127 sectors 1960	Average 1959-61	1960	
			Mill.kr.	Mill.kr.	Mill.kr.	Mill.kr.	Pct.
		1	2	3	4	5	6
	Domestic production sectors						
1112	Agriculture	92	2 345	80.9	87.3	97.5	4.2
112	Agriculture capital formation	92	52	20.5	20.8	20.5	39.5
113	Hunting etc.	92	4	2.4	2.2	2.4	66.7
11x	Agriculture, aggregate	92)	2 401	64.7	(..	78.8	3.3
11	Agriculture	36)			(188.0	225.4	9.4
121	Forestry	92	806	140.2	143.5	142.2	17.6
122	Standing forests	92	-	-	-	-	-
12	Forestry	36	806	140.2	154.1	152.0	18.9
131	Fishing etc.	92	553	30.1	19.6	29.3	5.3
132	Whaling	92	107	49.0	59.4	48.9	45.7
13x	Fishing, whaling, aggregate	92)	660	65.1	(..	64.0	9.7
13	Fishing, whaling	36)			(102.6	105.6	16.0
1xx	Agriculture, forestry, fishing, etc. aggregate	) 92)	3 867	158.5	((..	135.8	3.5
1x	Agriculture, forestry, fishing, etc. aggregate	) 36)			((..	71.2	1.8
1	Agriculture, forestry, fishing, etc.	7)			(..	97.1	2.5
211	Coal mining	92	32	24.1	23.9	24.0	75.0
212	Metal mining	92	78	15.6	14.2	15.1	19.4
213	Quarrying and mining n.e.c.	92	89	7.9	9.4	7.8	8.8
21x	Mining, aggregate	92)	198	38.1	(..	37.6	19.0
21	Mining	36)			(35.4	38.0	19.2
2207	Non-metallic mineral products	92)	403	101.1	(99.0	100.8	25.0
22	Non-metallic mineral products	36)			(101.3	103.3	25.6
231	Ferro alloys	92	22	4.7	4.8	4.8	21.8
232	Iron and steel works and rolling	92	141	32.8	31.9	29.3	20.8
233	Iron and steel foundries	92	90	13.5	18.5	13.9	15.4
234	Refining of aluminium	92	52	8.5	14.2	12.2	23.5
235	Other non-ferrous metals	92	203	11.8	11.9	10.9	5.4
236	Non-ferrous metal foundries	92	12	2.0	2.2	1.9	15.8
23x	Basic metal industries, aggregate	92)	519	47.1	(..	32.5	6.3
23	Basic metal industries	36)			(38.4	29.2	5.6
2x912	Iron and metal products	92	671	46.6	47.6	45.8	6.8
24 (96)	Metal products	36	524	39.7	41.4	35.6	6.8
25 (92)	Machinery	36	121	5.7	5.1	5.7	4.7
26 (94)	Transport equipment	36	26	2.4	3.7	2.7	10.4
24,25,26	Iron and metal products aggregate	36	671	46.6	..	43.5	6.5
2703	Shipbuilding industries	92)	115	30.3	(19.6	29.8	25.9
27	Shipbuilding industries	36)			(19.3	29.9	26.0
2806	Electrical machinery etc.	92)	236	52.4	(49.8	52.6	22.3
28	Electrical machinery etc.	36)			(49.3	52.3	22.2
2903	Other manufacturing	92)	61	36.1	(38.6	36.0	59.0
29	Other manufacturing	36)			(38.8	36.0	59.0
2xx	Minerals, metals and products, aggregate	92)	2 203	260.5	(..	241.6	11.0
2x	Minerals, metals and products, aggregate	36)			(..	238.3	10.8
2	Minerals, metals and products	7)			(..	236.5	10.7
311	Slaughtering and preparation of meat	92	181	28.5	29.5	28.7	15.8
312	Dairy products	92	260	28.9	31.6	30.1	11.6
313	Margarine	92	17	3.5	3.7	3.5	21.1
314	Canning of fish and meat	92	1	1.1	.9	1.1	122.2
315	Fish processing	92	31	59.6	41.3	59.7	194.5
316	Grain mill products and livestock feed ..	92	364	137.4	144.9	145.9	40.1
317	Bakery products	92	.1	.2	.5	.2	200.0
318	Cocoa, chocolate and sugar confectionary	92	1	2.2	1.6	2.2	183.3
319	Other food preparations	92	30	10.4	9.8	10.5	35.0
31x	Food industries, aggregate	92)	885	256.8	(..	267.0	30.2
31	Food industries	36)			(276.0	299.0	33.8
321	Distilling, rectifying and blending of spirits	92	40	3.3	2.6	3.5	8.8
322	Breweries and soft drink production	92	8	1.6	1.4	1.6	21.3
323	Tobacco	92	1	1.2	1.1	1.2	120.0
32x	Tobacco and beverages, aggregate	92)	49	2.5	(..	2.6	5.3
32	Tobacco and beverages	36)			(4.1	1.7	3.5
3312	Herring oil and fish meal	92	142	60.9	46.4	60.5	42.6

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1) The fourth and fifth digit in the code indicates the number of sub-sectors in the 133-sector classification.

Appendix table I (cont.). Sector specification. Average intermediate deliveries and standard errors of estimates. Individual sectors. A. 133, 92, 36 and 7 sectors

Code	Sector	Specification, in basis matrix, cols. 4-6 Norwegian sectors	Average intermediate deliveries 1949-60	Standard error			
				Basis matrix			
				133/127 sectors 1960	Average 1959-61	1960	
			Mill.kr.	Mill.kr.	Mill.kr.	Mill.kr.	Pct.
		1	2	3	4	5	6
332	Vegetable oil mills	92	102	10.7	7.7	10.3	10.1
333	Other oil refineries etc.	92	104	31.4	30.0	31.9	30.7
33x	Products of oils and fats, aggregate	92)	347	53.9	(..	53.3	15.4
33	Products of oils and fats	36)			(54.0	72.6	20.9
3912	Fertilizers etc.	92	151	43.3	28.7	40.9	27.1
348+3987	Chemicals and products of chemicals	92	401	92.9	153.2	90.8	22.6
491	Rubber products	92	40	6.5	5.3	6.9	17.3
348,39x+491	Chemicals, aggregate	92	592		..	136.1	23.0
39+49	Chemical products	36	506	127.3	107.3	128.2	25.4
34	Petroleum products	36	87	13.4	90.6	14.6	16.8
34+39+49	Chemicals, aggregate	36	592	..	..	142.4	24.1
3xx	Production of food, drink, chemicals, aggregate1)	92	1 833	366.0	..	373.6	20.4
3x+49	Production of food, drink, chemicals, aggregate	36	1 873	..	..	389.4	20.8
3	Production of food, drink, chemicals1)	7	1 833	..		369.8	20.2
4112	Spinning and weaving	92	323	27.1	25.5	27.0	8.4
412	Knitting mills	92	4	2.3	2.5	2.3	52.3
413	Cordage, rope and twine	92	9	3.2	2.5	3.3	37.5
41x	Textiles, aggregate	92)	336	28.5	(..	28.3	8.4
41	Textiles	36)			(27.9	28.1	8.4
420	Clothing	92)	36	10.1	(14.3	10.0	27.8
42	Clothing	36)			(14.3	10.0	27.8
431	Footwear and repair, fur goods etc.	92	1	1.3	1.4	1.3	100.0
432	Leather and leather products	92	73	27.7	26.6	27.7	37.9
43x	Footwear, leather, fur, aggregate	92)	74	27.2	(..	27.2	36.8
43	Footwear, leather, fur	36)			(26.4	27.9	37.8
441	Sawmills, planing mills. etc.	92	536	108.0	119.3	110.6	20.6
4423	Other wood and cork products	92	366	51.9	36.7	53.2	14.5
44x	Wood and cork etc., aggregate	92)	902	62.5	(..	63.7	7.1
44	Wood and cork etc.	36)			(90.6	65.5	7.3
4512	Wood pulp	92	388	16.9	15.6	16.6	4.3
452	Paper, paperboard and cardboard	92	288	15.2	20.5	15.2	5.3
453	Wallboards etc.	92	52	8.2	8.8	8.3	16.0
454	Paper and paperboard products	92	244	31.6	30.2	31.6	13.0
45x	Pulp, paper and paper products, aggregate	92)	972	19.5	(..	48.2	5.0
45	Pulp, paper and paper products	36)			(68.3	61.7	6.4
461	Publishing etc.	92	96	43.1	40.5	43.1	44.9
462	Printing, bookbinding, etc.	92	307	11.7	11.6	11.7	3.8
46x	Printing and publishing, aggregate	92)	404	41.6	(..	41.6	10.3
46	Printing and publishing	36)			(30.3	39.0	9.8
4xx	Products of wood, fibres etc., aggregate2)	92	2 763	71.7	..	71.4	2.6
4x	Products of wood fibres etc., aggregate	36	2 722	..	..	79.5	2.9
4	Products of wood, fibres etc.2)	7	2 763	..	..	68.9	2.5
500	Construction	92)	6	1.3	(1.3	1.3	21.7
50	Construction	36)			(1.3	1.3	21.7
5	Construction	7)			(	1.4	23.4
610	Trade	92)	3 362	121.2	(80.0	120.8	3.6
61	Wholesale and retail trade	36)			(88.4	120.2	3.6
621	Ocean water transport	92	32	8.6	10.3	8.6	26.9
622	Coastal water transport	92	231	17.4	19.1	17.4	7.5
623	Services related to water transport	92	199	23.2	27.8	23.2	11.7
62x	Water transport, aggregate	92)	462	36.2	(..	36.2	7.8
62	Water transport	36)			(59.7	44.3	9.6
631	Railway transport	92	274	29.0	37.4	29.0	10.6
632	Tramway and suburban railway transport	92	6	1.6	1.8	1.6	28.6
633	Land transport n.e.c.	92	365	5.6	13.3	17.5	4.8
634	Air transport	92	34	20.0	18.1	20.0	58.8
635	Services related to transport and storage	92	95	4.8	4.5	4.8	5.1
63x	Land and air transport, aggregate	92)	774	21.1	(..	21.1	2.7
63	Land and air transport	36)			(26.9	20.3	2.6
640	Communications	92)	273	23.7	(22.8	23.8	8.7
64	Communications	36)			(19.1	17.7	6.5
6xx	Trade and transport, aggregate	92)	4 872	146.6	(..	147.5	3.0
6x	Trade and transport, aggregate	36)			(..	159.7	3.3
6	Trade and transport	7)			(..	268.7	5.5

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1) Excluding 491 Rubber products.

2) Including 491 Rubber products.

Appendix table I (cont.). Sector specification. Average intermediate deliveries and standard errors of estimates. Individual sectors. A. 133, 92, 36 and 7 sectors

Code	Sector	Specification, in basis matrix, cols. 4-6 Norwegian sectors	Average intermediate deliveries 1949-60	Standard error			
				Basis matrix			
				133/127 sectors 1960	Average 1959-61	1960	
			Mill.kr.	Mill.kr.	Mill.kr.	Mill.kr.	Pct.
		1	2	3	4	5	6
711	Electricity supply	92	488	59.2	52.7	59.1	12.1
712	Gas supply	92	13	4.3	4.8	4.2	32.3
713	Water supply	92	-	-	-	-	-
71x	Electricity, gas and water, aggregate	92)	501	56.0	(..	55.9	11.2
71	Electricity, gas and water	36)			(53.0	59.5	11.9
721	Bank of Norway	92	5	3.4	3.5	3.4	64.2
722	State banks and loan associations	92	-	.2	.2	.2	..
723	Other banks etc.	92	56	15.2	13.5	15.2	27.1
724	Life insurance	92	-	-	-	-	-
725	Non-life insurance	92	156	8.2	13.8	8.2	5.3
726	Social insurance	92	-	-	-	-	-
72x	Banking and insurance, aggregate	92)	217	25.3	(..	25.2	11.6
72	Banking and insurance	36)			(29.1	26.6	12.3
731	Commercial buildings	92	221	20.9	18.1	20.9	9.5
732	Dwellings	92	-	-	-	-	-
73	Business buildings, dwellings	36	221	20.9	14.8	17.3	7.8
741	Government administration	92	-	-	-	-	-
742	Military defence services	92	-	-	-	-	-
74	Government,defence	36	-	-	-	-	-
751	Educational services	92	-	-	-	-	-
752	Medical and veterinary services	92	8	1.4	1.1	1.3	17.1
75	Educational, health services	36	8	1.4	.9	1.0	12.5
761	Domestic services	92	-	-	-	-	-
762	Hotel and restaurant services	92	42	1.7	1.7	1.7	4.0
763	Laundry, cleaning, other personal services	92	18	1.7	1.5	1.7	9.4
76x	Personal services, aggregate	92)	60	2.7	(..	2.7	4.5
76	Personal services	36)			(2.1	2.2	3.7
771	Central government consumption capital	92	-	-	-	-	-
772	Local government consumption capital	92	-	-	-	-	-
773	Religious and welfare activities	92	-	-	-	-	-
774	Non-business org. and institutions	92	19	3.5	2.7	3.5	18.4
775	Legal, techn. and business services	92	180	18.9	16.3	19.0	10.6
776	Recreation services	92	8	.9	1.0	.9	11.1
77x	Other services, aggregate	92)	207	16.5	(..	16.6	8.0
77	Other services	36)			(12.7	12.9	6.2
781	Unspecified, office supplies	92	376	18.5	36.8	18.5	4.9
782	Unspecified, energy supplies	92	77	10.3	10.3	10.3	13.4
783	Unspecified services	92	558	106.1	75.2	106.1	19.0
784	Unspecified transport	92	96	35.5	24.8	35.5	37.0
78x	Unspecified, aggregate	92)	1 107	133.0	(..	134.0	12.1
78	Unspecified	36)			(75.0	118.5	10.7
7xx	Other activities (services), aggregate	92)			(..	162.4	7.0
7x	Other activities (services), aggregate	36)	2 320	161.2	(..	146.3	6.3
7	Other activities (services)	7)			(	139.1	6.0
	Import sectors						
1114	Agriculture	92	622	71.0	60.7	66.7	10.7
112	Hunting etc.	92	13	5.0	4.9	5.0	38.5
11x	Agriculture, aggregate	92)	634	67.2	(..	63.0	9.9
11	Agriculture	36)			(52.0	54.2	8.5
1212	Forestry	92)	105	70.8	(66.0	70.6	67.2
12	Forestry	36)			(63.8	69.2	65.9
131	Fishing etc.	92	2	9.1	4.8	9.1	379.2
132	Whaling	92	7	7.7	6.4	7.7	113.2
13x	Fishing, whaling, aggregate	92)	9	7.0	(..	7.0	77.8
13	Fishing, whaling	36)			(6.0	7.2	80.0
1xx	Agriculture, forestry, fishing etc., aggregate	92)			(..	57.0	7.6
1x	Agriculture, forestry, fishing, etc., aggregate	) 36)	749	57.0	((..	63.5	8.5
1	Agriculture, forestry, fishing, etc.	7)			(..	63.7	8.5
211	Coal mining	92	108	80.4	80.4	80.3	74.4
2122	Metal mining	92	122	45.5	35.8	44.9	36.8
2132	Quarrying and mining n.e.c.	92	78	24.0	71.8	24.0	30.8
21x	Mining, aggregate	92)	307	101.2	(..	100.5	32.7
21	Mining	36)			(82.9	115.2	37.5
2207	Non-metallic mineral products	92)	104	15.0	(15.1	14.9	14.3
22	Non-metallic mineral products	36)			(15.6	15.1	14.5

Appendix table I (cont.). Sector specification. Average intermediate deliveries and standard errors of estimates. Individual sectors. A. 133, 92, 36 and 7 sectors

Code	Sector	Specification, in basis matrix, cols. 4-6 Norwegian sectors	Average intermediate deliveries 1949-60	Standard error			
				Basis matrix			
				133/127 sectors 1960	Average 1959-61	1960	
			Mill.kr.	Mill.kr.	Mill.kr.	Mill.kr.	Pct.
		1	2	3	4	5	6
231	Ferro alloys	92	1	1.3	2.0	1.3	92.9
232	Iron and steel works and rolling	92	501	53.3	56.9	57.9	11.6
233	Iron and steel foundries	92	14	16.9	17.9	16.9	120.7
234	Refining of aluminium	92	81	20.8	27.5	19.3	23.8
2352	Other non-ferrous metals	92	411	70.4	15.1	67.9	16.5
236	Non-ferrous metal foundries	92	-	-	-	-	-
23x	Basic metal industries, aggregate	92)	1 008	74.4	(..	65.3	6.5
23	Basic metal industries	36)			(65.1	76.1	7.5
2x916	Iron and metal products	92	352	57.2	15.5	59.3	16.8
24 (97)	Metal products	36	129	27.2	35.1	26.7	20.7
25 (92)	Machinery	36	100	17.2	16.4	17.2	17.2
26 (97)	Transport equipment	36	124	21.5	21.9	21.3	17.2
24,25,26	Iron and metal products, aggregate	36	352		..	57.6	16.4
2703	Shipbuilding industries	92)	69	14.3	(18.3	13.6	19.7
27	Shipbuilding industries	36)			(17.6	13.6	19.7
2807	Electrical machinery, etc.	92)	125	44.3	(15.5	47.9	38.3
28	Electrical machinery, etc.	36)			(48.5	47.5	38.0
2903	Other manufacturing	92)	45	32.3	(50.8	31.7	70.4
29	Other manufacturing	36)			(51.5	32.5	72.2
2xx	Minerals, metals and products, aggregate	92)	2 011	84.1	(..	83.9	8.4
2x	Minerals, metals and products, aggregate	36)			(..	76.6	7.6
2	Minerals, metals and products	7)			(..	131.8	13.1
311	Slaughtering and preparation of meat	92	14	10.6	8.0	10.6	75.7
312	Dairy products	92	-	.7	.7	.7	..
313	Margarine	92	-	.1	.1	.1	..
314	Canning of fish and meat	92	-	-	-	-	-
315	Fish processing	92	3	5.7	5.3	5.7	183.9
316	Grain mill products and livestock feed	92	14	7.9	8.4	7.9	56.4
317	Bakery products	92	-	-	-	-	-
318	Cocoa, chocolate and sugar confectionary	92	2	3.0	3.1	3.0	142.9
319	Other food preparations	92	44	17.1	18.7	17.3	39.3
31x	Food industries, aggregate	92)	77	27.5	(..	27.9	36.2
31	Food industries	36)			(27.2	31.2	40.5
321	Distilling, rectifying and blending of spirits	92	15	2.5	1.9	2.5	16.7
322	Breweries and soft drink production	92	1	.7	1.0	.7	77.8
323	Tobacco	92	-	1.3	.6	1.3	..
32x	Tobacco and beverages, aggregate	92)	16	4.0	(..	4.0	25.0
32	Tobacco and beverages	36)			(2.1	2.8	17.5
331	Herring oil and fish meal	92	20	58.7	50.7	58.8	294.0
332	Vegetable oil mills	92	49	34.0	35.0	36.6	74.7
333	Other oil refineries etc.	92	15	28.8	15.7	28.8	192.0
33x	Products of oils and fats, aggregate	92)	84	120.8	(..	122.2	145.5
33	Products of oils and fats	36)			(90.5	105.0	125.0
3912	Fertilizers etc.	92	21	37.3	30.4	36.7	174.8
348+3987	Chemicals and products of chemicals	92	1 020	167.3	143.7	163.7	16.0
491	Rubber products	92	16	18.5	16.0	18.9	118.1
348+39x+ 491	Chemicals, aggregate	92	1 057		..	216.0	20.4
39+49	Chemical products	36	326	100.9	127.3	103.8	31.8
34	Petroleum products	36	730	119.8	86.2	116.0	15.9
34+39+49	Chemicals, aggregate	36	1 057	..	..	218.7	20.7
3xx	Production of food, drink, chemicals, aggregate1)	92	1 218	344.9	..	342.6	28.1
3x	Production of food, drink, chemicals, aggregate	36	1 234	..	..	347.8	28.2
3	Production of food, drink, chemicals1)	7	1 218	..	..	307.7	25.3
4112	Spinning and weaving	92	394	23.2	26.9	22.9	5.8
412	Knitting mills	92	1	1.5	2.9	1.5	166.7
413	Cordage, rope and twine	92	1	1.1	1.2	1.1	84.6
41x	Textiles, aggregate	92)	396	22.8	(..	22.5	5.7
41	Textiles	36)			(29.6	23.8	6.0
420	Clothing	92)	7	5.4	(6.5	5.4	79.4
42	Clothing	36)			(6.5	5.6	80.0
431	Footwear, products and repair	92	-	-	-	-	-
4322	Leather and leather products	92)	45	13.0	(12.6	13.0	28.9
43	Footwear, leather, fur	36)			(13.0	13.6	30.2
441	Sawmills, planing mills. etc.	92	50	11.4	14.5	11.3	22.6

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1) Excluding 491 Rubber products.

Appendix table I (cont.). Sector specification. Average intermediate deliveries and standard errors of estimates. Individual sectors. A. 133, 92, 36 and 7 sectors

Code	Sector	Specification, in basis matrix, cols. 4-6 Norwegian sectors	Average intermediate deliveries 1949-60	Standard error			
				Basis matrix			
				133/127 sectors 1960	Average 1959-61	1960	
			Mill.kr.	Mill.kr.	Mill.kr.	Mill.kr.	Pct.
		1	2	3	4	5	6
4423	Other wood and cork products	92	29	5.6	7.5	5.9	20.3
44x	Wood and cork etc., aggregate	92)	79	12.7	(..	12.8	16.2
44	Wood and cork etc.	36)			(19.5	13.0	16.4
4512	Wood pulp	92	25	13.8	13.2	13.8	55.2
452	Paper, paperboard and cardboard	92	16	5.1	7.6	5.1	31.9
453	Wallboards etc.	92	-	.3	.3	.3	..
454	Paper and paperboard products	92	23	6.6	7.1	6.6	28.7
45x	Pulp, paper and paper products, aggregate	92)	63	18.9	(..	18.9	30.9
45	Pulp, paper and paper products	36)			(21.1	20.3	32.2
461	Publishing etc.	92	-	-	-	-	-
462	Printing, bookbinding etc.	92)	3	10.4	(8.4	10.4	358.6
46	Printing and publishing	36)			(8.2	10.4	358.6
4xx	Products of wood, fibres etc., aggregate1)	92	609	58.4	..	58.8	9.7
4x	Products of wood, fibres etc., aggregate	36	593	..	..	44.3	7.5
4	Products of wood, fibres etc.1)	7	609	..	..	44.1	7.2
610	Trade	92)	57	7.1	(7.3	7.1	12.5
61	Wholesale and retail trade	36)			(5.5	5.2	9.6
620	Services related to water transport	92)	3	3.2	(2.6	3.2	100.0
62	Water transport	36)			(2.7	3.3	110.0
630	Railway transport	92)	2	1.3	(1.3	1.3	61.9
63	Land and air transport	36)			(1.2	1.3	61.9
640	Communications	92)	9	2.5	(2.3	2.5	28.1
64	Communications	36)			(2.2	2.5	28.1
6xx	Trade and transportation, aggregate	92)	71	4.6	(..	4.7	6.6
6x	Trade and transportation, aggregate	36)			(..	5.2	7.3
6	Trade and transportation	7)			(..	4.0	5.6
711	Electricity supply	92	1	5.9	3.3	5.9	842.9
712	Gas supply	92	93	10.7	13.6	10.8	11.6
71x	Electricity, gas and water, aggregate	92)	93	13.8	(..	13.9	14.9
71	Electricity, gas and water	36)			(13.4	15.2	16.3
720	Non-life insurance	92)	31	1.7	(1.8	1.7	5.5
72	Banking and insurance	36)			(2.1	2.6	8.4
771	Central government import, military	92	-	-	-	-	-
772	Central government import, civilian	92	-	-	-	-	-
773	Unspecified services	92	118	65.9	54.4	66.0	55.9
774	Non-business organisations and institutions	92	-	-	-	-	-
775	Legal, technical and business services	92	-	-	-	-	-
776	Recreation services	92	9	3.0	3.0	3.0	35.3
77x	Other services, aggregate	92)	127	68.8	(..	68.8	54.2
77	Other services	36)			(55.7	63.7	50.2
780	Whaling, water and air transport expenditures abroad	92	-	-	-	-	-
78	Whaling, water and air transport expenditures abroad	36	-	-	-	-	-
790	Transport expenditures abroad	92)	1 632	116.2	(120.5	116.2	7.1
79	Transport expenditures abroad	36)			(81.5	76.8	4.7
7xx	Other activities (services), aggregate	92)	1 883	58.2	(..	58.8	3.1
7x	Other activities (services), aggregate	36)			(..	47.4	2.5
7	Other activities (services)	7)			(..	102.3	5.4
801	Sundry transfers	92	-	-	-	-	-
802	Transfer account, military government consumption	92	5	2.3	1.8	2.3	43.4
803	Transfer account, civil government consumption	92	57	12.0	9.1	12.0	21.1
804	Transfer account, local government consumption	92	81	6.8	6.0	6.8	8.4
805	Transfer account, capital to government consumption	92	1	2.0	1.8	2.0	181.8
806	Deliveries from capital formation to export	92	-	-	-	-	-
807	Deliveries from capital formation to production or government consumption	92	-	-	-	-	-
809	Installation and repair work	92	-	-	-	-	-
80x	Transfer accounts, aggregate	92)	145	22.5	(..	22.5	15.5
80	Transfer accounts, aggregate	36)			(..	20.3	14.0
8	Transfer accounts	7)			(..	14.0	9.7

1) Including 491 Rubber products.

Appendix table I. Sector specification. Average intermediate deliveries and standard errors of estimates.
B. 14 sectors

Code	Sector	Sectors in the 92-sector specification	Average intermediate deliveries 1949-60	Standard error		
				133 sector basis matrix 1960	14-sector basis matrix 1960	
			Mill.kr.	Mill.kr.	Mill.kr.	Pct.
	Domestic production sectors					
1	Agriculture and dairy products	111,112,312	2 657	86.4	210.0	7.9
2	Food industries excl.: dairy products, canning of fish and meat and fish processing	311,313,316,317,318,319,321, 322,323	642	175.0	182.5	28.4
3	Textiles, footwear, other wearing apparel and made-up textile goods	411,412,413,420,431	373	32.3	26.7	7.2
4	Other industries producing commodities for consumption ...	113,211,213,220,290,332,348, 398,432,461,462,491,712	1 621	214.9	202.0	12.5
5	Manufacturing of investment goods	232,233,236,2x9,270,280	1 265	166.9	155.9	12.3
6	Forestry, wood, pulp, paper and products	121,122,441,442,451,452,453,454	2 680	174.6	148.1	5.5
7	Fishing etc., whaling and processing of fish	131,132,314,315,331,333	938	43.2	78.3	8.3
8	Metal mining, metals, ferro alloys, fertilizers, carbide ..	212,231,234,235,391	506	28.6	20.0	4.0
9	Water transport	621.622	263	22.2	21.7	8.3
10	Construction	500	6	1.3	1.4	23.3
11	Electricity supply	711	488	59.2	63.4	13.0
12	Real estate,dwellings	731,732	221	20.9	20.4	9.2
13	Trade and transport excl. water transport	610,623,631,632,633,634,635,640	4 608	138.9	138.9	3.1
14	Service industries n.e.c.	713,721,722,723,724,725,726,741, 742,751,752,761,762,763,771,772, 773,774,775,776	492	25.8	26.9	5.5
15	Unspecified	781,782,783,784	1 107	132.9	121.6	11.0
	Import sectors					
1	Agriculture and dairy products	111,312	622	71.5	84.6	13.6
2	Food industries excl.: dairy products, canning of fish and meat and fish processing	311,313,316,317,318,319,321, 322,323	90	28.8	30.8	34.2
3	Textiles, footwear, other wearing apparel and made-up textile goods	411,412,413,420,431	403	24.6	33.5	8.3
4	Other industries producing commodities for consumption ...	112,213,220,290,332,398,432,461, 462,491,712	1 466	203,0	205,5	14,0
5	Manufacturing of investment goods	232,233,236,2x9,270,280	1 061	77.3	70.5	6.6
6	Forestry, wood, pulp, paper and products	121,441,442,451,452,453,454	248	93.1	100.2	40.5
7	Fishing etc. whaling and processing of fish	131,132,314,315,331,333	47	92.8	73.1	155.5
8	Metal mining, metals, ferro alloys, fertilizers, carbide ..	211,212,231,234,235,391	744	86.8	126.5	17.0
9	Electricity supply	711	1	5.9	5.9	590.0
10	Trade and transport excl. water transport	610,620,630,640	71	4.6	4.6	6.5
11	Service industries n.e.c.	720,774,775,776	40	2.0	2.7	6.8
12	Imports n.e.c.	771,772,773,780,790	1 750	63.3	44.6	2.6
13	Transfers	801,802,803,804,805,806,807, 808,809	145	22.5	20.3	14.1