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ESTIMATING PRODUCTION FUNCTIONS

AND

TECHNICAL CHANGE FROM MICRO DATA.

An Exploratory Study of Individual Time Series of Norwegian Mining and Manufacturing Establishments.

By

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x I am indebted to Professors Zvi Griliches and Leif Johansen, Mr. Arne Amundsen and Mr. Steinar Strøm for comments and proposals for improvements. The work has been carried out in the Central Bureau of Statistics of Norway. Responsibility for contents and formulations rests with me.

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CHAPTER I

INTRODUCTION

Most econometric studies of production- and related or deducted functions are based on aggregate data of some kind or another. Very few are based on data for micro-units for which it is more relevant to speak of a production function. And those of the later kind are, most of them covering a very narrow range of activities. In fact, a quite recent study seems to be the first one that is both based on micro-data and covering most activities of a main production sector of a country, namely Manufacturing of Norway.[1)]

The purpose of that study was twofold. First to gain insight and experience in the use of large bodies of micro-data from sources like censuses of establishments in econometric studies, and second, as the title of the study suggests to analyse various properties of production functions in Norwegian manufacturing, particularly the scale-properties of that kind of relations.

As very little was known about the qualities and peculiarities of the data used in that study it became rather strongly coloured by experimenting. These experiments resulted finally in, among other things, the exclusion of very small units, units with characteristics missing or inconsistencies of the characteristics reported.

But even the characteristics of the units of the "samples" thus selected were subject of serious errors. This fact and the fact that data for one year only was available laid strong limitations on the scope of the analysis.

In light of these circumstances there are arguments of looking for another type of micro-data. Such data is available by the sample of establishments of large Norwegian firms of the years 1959-1967.[2)] And this is the empirical base of the present study.

This body of data does, in opposition to that of the Census-study[3)] contain a time-dimension and it consists of larger units on the average. Both of these properties are advantageous, the latter because some of the experiments of the Census-study suggested that the quality of the large units

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1) Z. Griliches and V. Ringstad: "Economies of Scale and the Form of the Production Function: An Econometric Study of Norwegian Establishment Data", <u>forthcoming</u>. This study is frequently referred to in the following as the Census-study.

2) For the sources of this body of data cf. Chapter II.

3) Cf. footnote 1 above.

was better than for smaller ones.[4)]

As the present study is an extension or a supplement to the Census study three of our aims are:

1) To get further insight and experience in the use of large bodies of micro-data in econometric studies of production functions.

2) To explore some of the central issues of the Census study and find out whether or not our data is a better empirical base for their investigation.

3) To compare, whenever possible, the results obtained about the properties of the production functions in Norwegian manufacturing with those of the Census study.

The first aim implies that we must carry out a rather detailed analysis of the data, particularly to figure out the importance and nature of the errors obviously present in them. The second implies that we to some extent at least should try the same models as analysed in the Census study while the third lays some constraints on the definition of variables and of industries of the present study. We do not, however, manage to operate with the same industry definitions as in the Census study, and thus some of the results of the two studies are hardly comparable.

To some extent the problems of each body of data are unique. Thus even if the experience of the Census study is a very valuable information some extent of experimenting is unavoidable in the present study. And therefore also this study is exploratory. In such studies there is a need for systematizing the experimenting and therefore we have a foruth aim for this study:

4) To try methods of "fishing" in data in a more systematic way than usually done in applied econometric studies.

This is done by applying multiple test procedures, and even if the scope of the analysis is rather narrow it throws some light on the problems present when trying to systematize "fishing" in data.

As we have a time dimension in our data, in opposition to the Census study, there are a few additional issues that can be explored. Thus we have also the following aims for this study:

5) To explore certain problems of measurement and methodology present in analyses of technical change and importance of various sources of growth by data like our cross section of time series.

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4) This is one of the reasons why the small units of the Census data were excluded from the analysis. Cf. above.

6) To investigate what the present data can tell us about the importance and nature of technical change in Norwegian mining and manufacturing industries.

As the present study is so closely related to the Census-study no extensive theoretical discussion of production functions etc. is presented here. Instead the theoretical parts of the Census-study will be referred to when necessary. Additional theoretical and statistical tools will be developed in due course, when they are needed for the exploration of particular issues.

We would like, however, to present here a model that will be used in most parts of the study and often referred to as the main model of the study:

$$(1.a) \qquad \ln V = a_o + \alpha \ln L + \beta \ln K + u$$

$$(1.b) \qquad \ln \frac{V}{L} = b_o + b \ln W + v$$

where V, L, K and W are value added, labour input, capital input and the real wage rate respectively. u and v are random errors.[5)]

The first one of these relations is the well known Cobb Douglas production function, while the second is based on the first order condition of profit maximum with respect to labour having a linear homogeneous CES relation and assuming perfect competition in the output and labour markets.[6)]

Thus 1.a and 1.b are based on slightly different sets of assumptions about the form of the production function. The former presumes a constant degree of returns to scale of any positive value but an elasticity of substitution constrained to one. The later presumes constant returns to scale while the elasticity of substitution may take any positive value.[7)]

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5) For a related model cf. G.S. Maddala and J.B. Kadane: "Some Notes on the Estimation of the Constant Elasticity of Substitution Production Function". The Review of Economics and Statistics, Feb. 1966.

6) This relation will frequently be referred to as the ACMS relation due to: K. Arrow, M.B. Chenery, B. Minhas and R. Solow: "Capital-labour substitution and economic efficiency". Review of Economics and Statistics, Aug. 1961.

7) The assumption of constant returns to scale for 1.b does not seem to have serious effects on the estimation of b even in cases when the scale-parameter is significantly different from one according to the results of the Cobb Douglas relation. This is for instance suggested by the results of the Census-study referred to in footnote 1) above. Some results of that study suggest also that the level of the factor-elasticities is fairly well determined by 1.a even if the results of the behaviour relation indicate that the elasticity of substitution is different from one.

Due to our assumptions the wage rate is an exogenous variable. And we will also argue that capital input is exogenous in our model. This assumption may be rather dubious, but as we shall show we will have other troubles with this variable that are more serious than those due to possible endogeneity.

There are three arguments for using the main model in our analysis in spite of the differencies in assumptions of the two relations of it. First, it will be a frame for the analysis. By constraining parameters in various ways the model will become "more consistent" and even simpler than the version of it presented above. Second,we may by means of 1.b investigate the assumption made about the substitution possibilities between labour and capital implied by 1.a. Third, we will in a particular context analyse the properties of the OLS estimators of the factor elasticities of the Cobb Douglas relation when the "true" production function is of CES-type and profit is maximized with respect to labour.[8)]

The plan of this study is as follows. In the next chapter we review the characteristics reported for the units of the study and other information available, the definitions of variables to be applied and the classification of industries. In a number of appendixes to that chapter particular issues concerning data are considered. Most of them deal with errors in data and how we have tried to solve various measurement problems present. Appendix II.7 may be of some methodological interest also as it deals with the problem of incomplete sets of data and particularly how to estimate the observations missing. Appendic II.8 is also somewhat more than just a description of data and data-problems as it aims to review the behaviour of our main variables in the samples of the various industries. Thus it is supposed to be a usefull supplement to the results and the discussion of them presented in later chapters.

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8) This implies that we use the Cobb Douglas production function as an approximation to the CES relation while we use the "true" behaviour relation. But we are not interested in the approximation errors as some experiments of the Census-study suggest that these are small. (Cf. footnote 7 above). Instead we would like to catch the effects due to simultaneous equations, of an elasticity of substitution different from one.

Chapter III deals with the problem of estimating the parameters of the main model. We show that due to particular errors in the data, methods that are usually applied on simultaneous equations models of that type do not work in our case. Instead we apply the OLS method on the production function and try to evaluate the seriousness of the biases present. And as they seem to be rather serious we try a method of estimation related to one applied in the Census-study. And this method seem to yield the more reasonable results also for our data. The analysis of this chapter is also in other respects very much related to a central part of the Census study. Thus this chapter does particularly refer to aim 2, but also to aim 3, as well as the general aim of this study; no. 1.

In Chapter IV the application of multiple test methods in econometrics is considered. It covers aim 4 of this study as we are particularly interested in the use of such methods when "fishing" in data for significant parameters. By means of covariance models we try to determine the nature of any variation of certain parameters of our main model across establishments and over time. The advantages of multiple test procedures, at least of those applied are that all alternative types of variation are specified a priori and we have a well defined strategy of choosing between them.

Chapter V covers aims 5 and 6 as it deals with the measurement of technical change and an exploration of its nature. But as we show there are errors in the data in addition to those making it difficult to obtain proper estimates on the parameters of the main model, that are particularly serious when trying to evaluate the importance of technical change. Some conclusions both about the importance and the nature of technical change are obtained, however. And the errors that may affect our findings more seriously are explicitly pointed out.

In a concluding chapter we present a summary of the study and try to figure out what we have learnt from it. In the main chapters we do not present any detailed discussion of the results for individual industries. Thus we add to the concluding chapter an appendix with a summary by industry of what we consider to be the more interesting findings.

Chapter II

THE EMPIRICAL BASE OF THE STUDY

1. Introduction.

In this chapter we present the empirical frame within which we have to work in the present study. The contents of the main part of it represent the bare minimum needed to understand what is done in the following chapters. In the next section, the data-sources, the sample selected and the industries to be analysed are presented. In Section 3 the information available is presented together with the measures applied for the main variables, and in Section 4 an evaluation of the data is attempted.

It should be strongly underlined, however, that a complete evaluation of the results is possible only if the main contents of the various appendixes of the present chapter are also known. Most of them concern empirical problems of the particular sources of data used. In these appendixes we try to figure out the nature of the problems and report on how we have tried to solve them.

In Appendix 1 we consider various causes of movements in the population of establishments available. Appendix 2 presents the construction of industries by four-digit industry-groups, and in Appendix 3 we report on some corrections of data carried out. In Appendix 4 we explain how missing values of subsidies and duties are calculated, and in Appendix 5 we have some remarks on the price-data applied for gross production and materials.

Appendix 6 and Appendix 7 concern the capital data and the capital input measure applied. In the first one we explain why it is not worth while to adopt a more refined capital input measure than the one actually applied. The second, a rather lengthy one, is a case study of the calculation of missing observations of a variable entering an econometric model, with capital as the one with observations missing. Finally in Appendix 8 some tables are presented, with a few comments on the "behaviour" of the main variables that enter the models analysed in the following. In this context Analysis of Variance is also applied, and the relevant statistics are deducted in Appendix 9.

2. The Units to be Studied.

a. The Data-Sources.

The units of this study are the establishments of "large" Norwegian firms in Mining and Manufacturing for the nine years 1959 through 1967. A large firm is in this context defined as one having at least 100 employees on the average in 1963 according to the Census of that year.[1] About 600 firms with about 1300 establishments in Mining and Manufacturing industries satisfy this criterion. The information for these establishments were for 1963 also obtained from the Census. For the other years one has used the information from the Annual Industrial Production Statistics. In addition price-data for gross production and input of materials and semiproducts are obtained from the national account system.[2] And to deflate the capital-stock data a price index based on current information on prices of new investment goods is applied.[3]

b. The Sample Selected.

In this study we will concentrate our efforts on complete time-series.[4] Thus, those establishments that according to their identification number did not exist in one or more of the years 1959-1967 were excluded.[5] And as we would like to have production establishments only, auxiliary units and so called investment establishments are excluded also.[6]

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1) Number of employees is defined as wage-earners (production workers) + salaried workers (non-production workers) + owners and unpaid family-members daily working in the establishment. Cf. Section 3.a.

2) Cf. Appendix II. 5.

3) Cf. Appendix II. 6.

4) In some contexts incomplete time-series are equally interesting as complete ones. Cf. F. Wedervang: *"Development of a Population of Industrial Firms"*, Universitetsforlaget, Bergen 1965. But as the high number of incomplete time-series in the present contexts seems to be a result of artificial births and deaths of establishments no attempt is made to analyse the structure of these units. Cf. Appendis II.1.

5) Cf. Appendix II. 1.

6) Investment establishments are such that are not "fully established" in the sense that they have not yet started production in the year for which the information is reported. Most of such units are, however, excluded as incomplete time-series. If one would like to analyse questions concerning "natural" births of establishments by means of this body of data, a look at these investment establishments seems to be the best point of departure.

Excluding incomplete time series, auxiliary units and investment establishments we have 913 complete time series for production units left. For different reasons six of these were also excluded.[7] The remaining 907 establishments are therefore the units selected for further analysis in this study.

By excluding the numerous incomplete time series for production units we have lost a substantial amount of information about the industries concerned.[8] But what is somewhat more worrying is the unknown number of "mongrel" units due to identification numbers refering to different physical units in different years.[9]

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7) Two of these were excluded because they obviously were investment establishments during 1959, even if they were reported to be ordinary production units for the whole period. Three establishments were excluded because they reported to have no employees for one or more years. The remaining one was excluded because of a complete break in production during one year.

8) The main reason why they are excluded, in spite of this fact, is the particular way the capital data are constructed. Namely by using the capital data available for 1959 and 1963 and the investment data of the period between to estimate a depreciation rate, and then interpolating and extrapolating to obtain capital data for the remaining years. (Cf. Appendix II. 7). For time-series that start running after 1963, no capital data are available or can be easily obtained. Those that start running after 1959 or stop before 1963 cannot be used in our estimation of the depreciation rate. This type together with the remaining type of incomplete time series; those that start running in 1959 but disappear between 1964 and 1967 could be included among the units subject for further analysis. But it was finally decided to exclude them as the fraction of missing capital values for 1959 and 1963 for these was higher than the one for complete time series. As such missing capital values cause a particular problem in our capital value computation we would probably have lost more than we gained by including the units concerned.

9) Cf. Appendix II. 1.

At least for some of the units concerned characteristics like output, inputs of labour and materials etc. will show a jump at one point of time, suggesting that "something has happened". We have not dared, however, to exclude units with such "jumps" as criterions for exclusion, as an inspection of the data suggested that we then quite clearly would have excluded a number of non-"mongrel" time series also.

In data like the present one with large errors of measurement such jumps due to mongrel time-series may not make much difference, provided that the main characteristics "jump together". But in our case we do not necessarily get jumps in the capital-values corresponding to those of the other characteristics, due to the particular way the capital data are constructed.[10)]

It isn't easy to figure out the importance of the errors introduced by these "mongrel" time-series. But thinking in terms of production functions the main effects of these are probably much the same as those of more or less random errors of measurement in capital input. Such errors are subject of further discussion in Appendix II. 7 and Chapter III.

c. *The Industries.*

The 907 units selected are divided into 15 "industries" for which results are reported separately during most parts of this study.

In Appendix II. 2 a table on the composition of these 15 industries is presented.[11)]

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10) Cf. Section II. 3. e. and Appendix II.7.

11) A few establishments were classified in different industry-groups in different years. To avoid ambiguity in the industry-group classification these units were classified in the industry-group to which they belonged in 1963. This is clearly a rather arbitrary procedure, but it concerns a very low number of units only.

Even if the presentation is by four-digit industry- groups, the base unit of the industry-construction is the two-digit group.[12] The divisions between the industries may in some cases look somewhat arbitrary. But if we are not going to rearrange two-digit industry-groups by our industry construction it looks for instance more convenient to "merge" industry groups 21 and 22 together with the main one; 20, than to merge them with the following industry group 23. But clearly then the notation Food Products is approximate only. The same is true for Basic Chemicals with two units of the 29 industry group, Leather products and 6 from the 30 industry group; Rubber products.[13]

12) In other contexts this detailed presentation of the composition of the industries is more important.
Cf. Appendix II.5, and Chapter III.

13) We cannot construct very homogeneous industries, provided that we shall cover all industry groups. We could for instance, have group 2311, Spinning and weaving of wool, and group 2710 Manufacture of Mechanical pulp as two of our industries, but what then with such groups as 2313, Spinning and weaving of hamp, jute and linen, and 2722, Manufacture of sulphate pulp? We could exclude them or merge them with the remaining groups of their respective two-digit industries. The first approach leads to a substantial reduction of units, and the second does not solve our problem of heterogenous industries. So we choose to go ahead with the industry construction presented in Appendix II.2.

3. *The Choice of Operational Definitions for the Main Variables.*

a. *The Characteristics Reported.*

In addition to general characteristics such as industry group, location, type of ownership, we get with the exceptions pointed out for each establishment for each of the nine years the following information that in one way or another will be applied in the study, mostly when constructing the variables on which the main part of the analysis is based:

X_1 Production on own account
X_2 Repairs
X_3 Contract work

M_1 Raw materials
M_2 Packing
M_3 Fuel
M_4 Auxiliary materials [14)]
M_5 Contract work [15)]

n_1 Number of wage earners (production workers)
n_2 Number of salaried employees (non-production workers)
n_3 Number of owners and family members

14) For the years 1959 and 1960 M_3+M_4 is reported instead of each component separately, and for the years 1965-67 M_4 is included in M_1. Thus only for the years 1961-64 do we get separate information on each of the components $M_1 - M_5$.

15) Except for 1959 and 1960 we have also information on traded goods bought and sold. There is an arguement for including these variables in the list of inputs and outputs. But as we do not have information for these characteristics for all years they are not used in the variable constructions, and therefore they are not included in the list either.

h Hours worked (in 1000) by wage earners

W_1 Wages, wage earners
W_2 Wages, salaried employees
W_3 Wages, home workers [16)]

U_1 Duties
U_2 Subsidies [17)]

I_1 Investments, purchased capital goods
I_2 Investments, repairs and maintenance

H_1 Inventories, raw materials
H_2 Inventories, goods in process
H_3 Inventories, finished goods

In addition to this information we have for the years 1959 and 1963 also information about:

K_1 Full fire insurance value of buildings
K_2 Full fire insurance value of machinery [18)]

By means of the characteristics above we will try to construct the variables needed for the present analysis. [19)]

16) Home workers are such who do not work on the premises of the establishment.

17) Information about duties and subsidies are not reported for 1959 and 1960. About the "estimation" of this information, see Appendix II.4.

18) For 1959, but not for 1963, we have information also about "other property".

19) Except for h, n_1, n_2 and n_3 all numbers are in 1000 (current) Norwegian kroner. About price-data, see below.

b. Gross Production, Materials and Value Added

As pointed out we would like to make the results of this study comparable to those of the Census-study and therefore we try to let the definitions of the main variables conform as closely as possible to those of the later study.

First we define gross production in current "factor prices" as:

(1) $Y' = X_1 + X_2 + X_3 + U_2 - U_1$

The input of materials is defined as all inputs "from outside" in buyers prices.

(2) $M' = M_1 + M_2 + M_3 + M_4 + M_5 + W_3$

As both Y' and M' are in current prices they are deflated with the price-indices discussed in Appendix II.5, and we get gross production and materials in constant (1961) prices as:[20]

(3) $Y = \frac{Y'}{P_Y}$

(4) $M = \frac{M'}{P_M}$

where P_Y and P_M are the two price-indices for gross production and materials respectively.

We have thus value added in current prices as:

(5) $V' = Y' - M'$

and in constant (1961) prices as:

(6) $V = Y - M$

Thus we have implicitly a price index with base in 1961 for value added as:

(7) $P_V = \frac{V'}{V} = \frac{P_Y Y - P_M M}{Y - M}$

20) Note that W_3, wages to home workers, are also included in our material input measure. Thus, W_3 will also be deflated with the price index for materials. This is strictly speaking not consistent as the numbers from the national accounts system used for the construction of price indexes to deflate current values of materials as defined in (II.3) do not contain this category W_3. But the error introduced by this is probably quite unimportant as the order of magnitude of W_3 itself is quite small.

c. *The Labour Input Measure*

The labour input measure to be applied is the following [21]:

$$(8) \quad L = \frac{h(W_1+W_2)}{W_1} + 2\, n_3$$

This measure implies that we calculate the number of hours worked by salaried employees in production workers equivalents. We also assume that owners and unpaid family members work 2 000 hours a year. This is approximately the average for production workers in 1963.

Some results of the Census study suggested that this measure on the average overstates the productive performance of owners and unpaid family-members. That is, we should rather have applied a coefficient of n_3 in (8) below 2. An alternative to this is to introduce an additional variable, for instance n_3/L to investigate if, or to what extent we in fact overstate the work done by this kind of labour. [22] Now, the interpretation of the results for such a variable is not quite straightforward as it may pick up effects other than wrong weighting of n_3 also. It is, for instance, mainly small establishments that have $n_3>0$. Thus n_3/L may work more or less like a dummy variable reflecting size-effects. But this suggests also that the problem of correct weighting of n_3 is less important in this study than in the Census study as the present one contains mostly large establishments, at least according to Norwegian standards. Therefore, nothing is done to eliminate this possible source of error. [23]

In the Census study both total number of employees N, and the two variables h and n_2+n_3 together were tried as labour input measures. Some

21) This measure is the same as the one applied in the Census study.

22) This is the way the validity of the weight given to n_3 in our labour input measure is investigated in the Census study, except that n_3/N is used instead.

23) In the Census study we also found that $n_3>0$ mostly for single unit firms and for a particular type of ownership, namely personal companies. Thus the results of n_3/L may contain both a size of firm effect and type of ownerships effect.

experiments showed that L as defined in (8) was generally superior to both of these alternatives. In light of these results we choose to go ahead with our input measure without further investigation of the validity of the aggregation of the components of this measure.

d. The "Real" Wage Rate

The price of labour input is measured as average wages per hour for production workers. That is;

$$(9) \quad W' = \frac{W_1}{h}$$

But as we in the present study rather are interested in the price of labour - price of output ratio we apply the following "real" wage rate:

$$(10) \quad W = \frac{W'}{P_V}$$

where P_V is defined in (7) above.

By means of the information available we could have constructed other wage-rate measures, but neither these nor the one to be used are quite good as measures of the price of labour as a factor of production.

The main drawback of our measure, as well as of the labour input measure, is that they both refers to the quantity component of labour. This property of the wage rate and labour input variables is subject of a rather lenghty discussion in Chapter III.

e. The Capital Input Measure

The information available for capital is as pointed out above full fire insurance values for two categories, namely buildings and machinery, but for the years 1959 and 1963 only. On the other hand, we have information about gross investment of two kinds for all years; for purchased capital goods and for repairs and maintenance. Thus, in principle it is possible to get sort of a capital measure for the remaining years too. But for various reasons such a capital measure construction is not so straightforward as it may look.

There are in the present context two types of problems that one has to consider when trying to construct a measure for the performance of capital as a factor of production. The first is the well-known question of concept, that is what is the correct measure of the productive performance of capital. And the second is the "behaviour" of the measure actually applied as compared to the presumed correct one.

In the present case we have, as pointed out full fire insurance values which are kind of market values af the capital stock. [24)] Quite clearly, whatever one believes is the correct measure of capital input, market values of the stock of capital could be blamed for numerous and serious weaknesses. But as we are in a take it or leave it - position it does not seem to pay to repeat a discussion that is well covered in the litterature. [25)] And as there are no real alternatives to this concept we will take it - or at least not leave it before we have seriously considered the possibilities for its application and investigated its performance. [26)]

24) The Census study contains a fairly detailed discussion the "contents" of "full fire insurance values" of capital.

25) See for instance: Z.Griliches: Capital Measures in Investment Functions, in Christ (ed.) Measurement in Economics, Stanford University Press, 1963.

26) In the Census study another measure was applied, without much success, however, namely the horse-power of the installed equipment as a measure of the capital's production capacity and the energy consumption (mainly electricity) per horse-power installation as a measure of the utilisation of this capacity.

Our main problem in the present context is, however, to get as reliable information as possible on the concept applied. Therefore we go ahead with the total full fire insurance values of buildings and machinery in constant (1961) prices as the capital input measure: Namely:

$$(11)\quad K = \frac{K_1' + K_2'}{P_K}$$

where P_K is a "price" index for total capital (buildings and machinery) based on price-data for new capital goods for Total Mining and Manufacturing.[27)]

4. *A Summary Evaluation of the Data*

The main sources of data applied in this study; the Census of 1963 and the Annual Production Statistics are meant to cover other needs than the one of empirical bases of econometric studies. And they may be good enough for the computation of sums and means of various central economic magnitudes such as production, materials, number of employees etc. But one runs into serious difficulties if trying to use these data in estimation of production and behaviour relations, which is the main purpose of the present study. This should be evident particularly from the contents of the various appendixes of this chapter. It was clearly demonstrated in the Census study also.

In the latter study the efforts were concentrated on analysing the effects of two types of errors that one presumed were the more important. Namely random errors of measurement in the capital input measure and the lack of a quality component of labour input.

27) A discussion of these price-data is presented in Appendix II.6. In Appendix II.7. it is shown how the missing capital values are computed.

Evidently these two types of errors are among the main ones in the present study also. Therefore we must pay proper attention to them. And in fact a whole chapter, the next one is devoted to the analysis of these errors with our "main model" as the frame.

But in addition we have a couple of other errors that at least may have serious impact on the results concerning "technical change". These are the deflators used for output and capital input, and also that we have no measure of the capacity utilization of the capital stock. And when discussing the importance and nature of technicl change we must also try to figure out to what extent and in what way these errors have affected our findings.

We will not argue that these errors are the only ones present or even that these are the only ones with any significant impact on the results. But they are clearly among the more serious. And they will be discussed explicitly in the following as these are the errors we may manage to say something more about than just that they are present in our data.

APPENDIX. 1.

ON NATURAL AND ARTIFICIAL BIRTHS AND DEATHS OF ESTABLISHMENTS.

There are significant movements in the reported number of establishments during the period covered by this study judged by the identification numbers of the establishments. These movements can clearly be divided into "natural" births and deaths of establishments and obviously "artificial" ones. "Natural births and deaths" are such as establishing of a completely new production unit and closing of a production unit previously in operation. More doubtful cases are movements into and out of the sample of establishments due to buying and selling of production units. Obviously artificial births and deaths are such as due to varying definitions over time of an establishment. As far as I know there has been no basic change in the definition of an establishment during the period considered, but rather some varying practicing of the definition (or definitions).

Generally this does not, however, seem to imply serious difficulties. A more disturbing cause of artificial movements in the number of establishments is the following: Due to a widening of the range of goods produced the CBS has considered it to be convenient to divide the activities of some establishments and classify them in different industry-groups. The opposite does also seem to have taken place to some extent; that two (or more) establishments of a firm are merged into one.

Judged by the identification number we may have in a "unmerging" case one complete time-series as usually the main branch of the production unit subject to unmerging gets its identification number. And thus we also get some " new " establishments the year the unmerging takes place. In case of a merging of two or more establishments of a firm, the merged unit usually gets the identification number of the more important of those establishments subject to merging. In this case one or more establishments "die" as their identification number disappears.

Merging and unmerging of firms have a related effect on movements in our population of establishments.

A change in location (municipality) may also lead to a change in identification, and thus lead to a break in time series. But this is a "less artificial" cause of movements compared with those mentioned previously, as it in this case is more reasonable to speak of a new production unit as a change in location probably also implies a basic change in "economic environments". Why move otherwise?

Thus, judged by the establishments' identification numbers there are substantial movements in the population of establishments during the period 1959-1967. But due to the causes pointed out above, much of these movements, probably most of them, are artificial. Or put in another way: The identification number is rather unreliable when tracing physical production units back into the past trying to construct time series for them. On the other hand, nothing better is available.

APPENDIX. 2.

COMPOSITION OF THE INDUSTRIES, BY FOUR-DIGIT INDUSTRY-GROUPS

Industry group	Name of industry-group	Number of units
1100	Coal mining	1
1210	Iron ore mining	4
1220	Pyrite and copper ore mining	5
1290	Metal mining not elsewhere classified	3
1410x	Stone quarrying	3
1510x	Limestone quarrying	5
1520x	Quartz and felspar quarrying	3
1590x	Mineral quarrying not elsewhere classified	2
	Total for Mining and Quarrying	26
2010	Slaughtering and preparation of meat	38
2021	Dairies	22
2022	Manufacture of condensed and dried milk	2
2023	Manufacture of ice-cream	2
2029	Milk collecting stations	9
2031	Canning of fruits and vegetables	1
2039	Other preserving of fruits and vegetables	3
2040	Canning of fish and meat	17
2051	Frozen fish	14
2052	Manufacture of prepared fish dishes etc.	3
2061	Local grain mills	1
2062	Commercial grain mills	8
2069	Other grain processing	1
2071	Manufacture of perishable bakery products	5
2072	Manufacture of bisquits etc.	2
2080	Man. of cocoa, chocolate and sugar confectionary	6
2091	Manufacture of margarine	5
2093	Manufacture of livestock feeds	3
2099	Manufacture of other food preparations	6
2110	Distilling, rectifying and blending of spirits	1
2130	Breweries and manufacturing of malt	9
2140	Soft drinks and carbonated water industries	2
2200	Tobacco manufactures	4
	Total for Food Products	164

1) For those industry-groups marked with a star (x) output is deflated by means of a price-index for inputs (labour included). Cf. Appendix II.5

Industry group	Name of industry-group	Number of units
2311	Spinning and weaving of wool	17
2312	Spinning and weaving of cotton and rayon	11
2313	Spinning and weaving of hemp, jute and linen ...	2
2314	Manufacture of narrow fabrics	3
2321	Manufacture of hosiery	2
2329	Other knitting mills	14
2330	Cordage, rope and twine industries	7
2392	Manufacture of impregnated textiles etc.	2
	Total for Textiles	58
2410	Manufacture of footwear	13
2431	Man. of garments of waterproof material	5
2432	Man. of work clothing	1
2433	Man. of men's and boys' garments	26
2434	Man. of women's, girls' and infants' garments ..	13
2443	Man. of hats and caps	3
2491	Man. of furnishing etc.	5
2499	Man. of other made-up textile goods	1
	Total for Clothing	67
2510	Saw mills and planing mills	25
2521	Wood preserving industries	4
2523	Prefabrication of wooden houses and structure ..	1
2525	Man. of wood-wool cement products	4
2529	Man. of other build. material of wood etc.	2
2532	Man. of casks	1
2599	Man. of wooden articles not elsewhere classified	1
2611	Man. of wooden furniture	5
2512	Man. of metal furniture	2
	Total for Wood Products	45
2710	Man. of mechanical pulp	22
2721	Man. of sulphite pulp	13
2722	Man. of sulphate pulp	5
2730	Man. of paper, paperboard and cardboard	40
2740	Man. of wallboards etc.	5
2751	Man. of paper and paperboard container	13
2759	Man. of other paper and paperboard prod.	5
	Total for Pulp and Paper	103

Industry group	Name of industry-group	Number of units
2821	x Printing of newspapers	23
2822	x Printing of books	5
2823	x Printing of commercial matter	6
2829	x Other printing activity	14
2830	x Bookbinding	9
2891	x Electrotyping and stereotyping	5
2899	x Other services incidental to printing	1
	Total for Printing	63
2910	Tanneries and leather finishing plants	1
2930	Man. of leather products, except footwear etc. .	1
3010	Manufacture of rubber products	6
3111	Man. of calcium carbide and cyanamide	5
3112	Man. of other fertilizers	4
3113	Man. of explosives	3
3114	Man. of synthetic fibres, resins etc.	3
3119	Man. of other basic ind. chemicals	19
3122	Herring oil and fish-meal factories	9
3123	Vegetable oil mills	1
3129	Other oil refineries etc.	3
3130	Man. of paints, varnishes and lacquers	5
3191	Man. of pharmaceutical preparations	3
3192	Man. of soap	4
3193	Man. of cosmetics etc.	1
3194	Man. of candles	1
3199	Man. of other chemical products	3
	Total for Basic Chemicals	72
3210	Man. of asphaltic felt	1
3290	Other coal and mineral oil processing	1
3310	Man. of structural clay products	4
3321	Man. of glass and glass prod. from raw. mat. ...	3
3329	Man. of glass products from purchased glass	1
3331	Man. of china and fine earthenware	5
3339	Man. of pottery and other earthenware	1
3340	Man. of cement (hydraulic)	3
3350	Man. of cement products	2
3391	Man. of abrasives	1
3393	Grinding of other non-metallic minerals	5
3394	Man. of cut-stone and stone products	3
3399	Man. of other non-metallic mineral products	6
	Total for Mineral Products	36

Industry group	Name of industry group	Number of units
3411	Man. of ferro-alloys	9
3412	Iron and steel works and rolling mills	6
3413	Iron and steel foundries	12
3420	Refinig of aluminium	6
3430	Man. of crude metals not elsewhere classified .	4
3491	Non-ferrous metal rolling mills	3
3492	Smelting anf refining of metals	2
	Total for Basic Steel	42
3511	Man. of wire and wire products	8
3512x	Man. of other metal building articles	5
3513x	Man. of steel structural parts	13
3520	Man. of metal shipping containers etc.	8
3530	Man. of metal household articles	5
3591x	Man. of metal equipment for offices and shops .	4
3592x	Man. of lighting fixtures	3
3593x	Man. of hand tools and implements	2
3594	Man. of metal fittings	3
3595x	Man. of arms and ammunition	4
3599x	Man. of other metal prod. not elsewhere classified	5
	Total for Metal Products	60
3610x	Man. of mining and industrial machinery	11
3620x	Man. of agricultural and forstry machinery	3
3680x	Machinery repair shops	4
3691x	Man. of household, office and shop machinery ..	5
3699x	Man. of other machinery	14
	Total for Non-El. Machinery	37
3711x	Man. of accumulators and batteries	2
3712	Man. of wires and cables	3
3713x	Man. of transformers, generators and electric motors	3
3719x	Man. of other distribution equipment	7
3720	Man. of signalling, radio and other telecom.equip.	11
3780x	Electro-technical repair shops	2
3791x	Man. of electric lamps	2
3799	Man. of other electrical products	4
	Total for El. Machinery	34

Industry group	Name of industry group	Number of units
3811*	Building and repairing of steel ships	33
3813*	Building and repairing of wooden ships	7
3814*	Man. of other marine machinery	2
3819*	Other services for ships	1
3821*	Man. of railroad cars and locomotives	3
3822*	Repairing of railroad cars and locomotives	13
3821*	Man. of bodies for motor vehicles	2
3839*	Man. of motor vehicles and parts not elsewhere classified	2
3840*	Repair of motor vehicles	17
3851*	Man. of motor-cycles and bicycles	1
3860*	Man. of aircraft	3
3890*	Man. of transport equipment not elsewhere classified	3
	Total for Transport equipment	87
3940	Man. of jewellery and related products	3
3991	Man. of brooms and brushes	1
3994	Man. of plastic products not elsewhere classified	8
3999	Man. of other products not elsewhere classified	1
	Total for Misc. Products	13
	Total for Mining and Manufacturing	[illegible]

APPENDIX 8.

SOME DATA-CORRECTIONS.

By inspecting the raw data for our 907 complete time series we found a few strange characteristics such as for instance an establishment reporting to have no wage earners (n_1) in 1964 while it reported to have paid more than 1.8 mill. N.kr. to this kind of employees. This is clearly inconsistent and as it reported 140 wage earners in 1963 and 146 in 1965 it is $n_1=0$ for 1964 that must be wrong.[1] In this case n_1 for 1964 was "interpolated" by means of n_1 for 1963 and 1965 and wages to wage earners (W_1) in 1963, 1964 and 1965. In related cases of obviusly erroneous characteristics related interpolations were carried out.

But only central characteristics such as wages to wage-earners, number of wage-earners, gross output and material input are subject to such corrections. The alternative would have been to exclude these units, but the "correction approach" was chosen from the point of view that it is better to correct an obvously incorrect information of a time series, than to exclude the whole time series and thus loose the information present by the other eight years. Evidently the exclusion approach could have been substantially less serious provided we accepted incomplete time series. But that is not done due to reasons pointed out elsewhere. Cf. footnote 8 of Chapter II and Appendix II.7.

1) There are several possible causes of such errors in data like bad reporting, mistakes in the revision and control of the forms and accidents in the transferring of data, particularly when cards are involved. It isn't at all easy to locate the stage at which each error enters the data, and thus no attempts are made to trace the sources of them.

APPENDIX 4.

CALCULATION OF MISSING VALUES FOR SUBSIDIES AND DUTIES.

As pointed out we don't have information about subsidies and duties for 1959 and 1960. Thus to get a measure of output that is comparable over years we've either to compute it in market prices, or in one way or another to "estimate" subsidies and duties for 1959 and 60 to obtain a measure in "factor-prices". For most industries it does not matter much whether we use market or factor-prices. But for a few, particularly those using inputs from Agriculture and Fishing there is a substantial difference between these two measures. Therefore the calculation approach is chosen.

We adopt an ad hoc procedure that is based on the assumption that there is a fixed ratio between subsidies and gross production and duties and gross production. And we calculate the missing values of subsidies and duties as:

$$U_{i,t} = \frac{Y'_t}{2}\left(\frac{U_{i,61}}{Y'_{61}} + \frac{U_{i,62}}{Y'_{62}}\right) \qquad \left\{\begin{array}{l} i=1,2 \\ t=59,60 \end{array}\right.$$

There are three matters that would make the "estimates" thus obtained basicly invalid.

Firstly, for some types of activities duties or subsidies are rather determined by input of materials than by gross production. This is presumably not very serious as in the short run we would expect a fairly stable proportionality between materials and gross production for each establishment. So it should not matter much which one we use in the formulas above. And as we would like to use only one of these variates, gross production was chosen as U_2 and U_1 for most industries is determined by this variable.

Secondly, a change in the product mix (or materials input mix) may have taken place during the period considered. This is clearly of importance only if duties or subsidies depend on certain kinds of outputs or inputs. But again, the period under consideration is rather short so that serious errors due to this arguement are unlikely.

Thirdly, there may have been changes in the policy of the Government as concerns duties and subsidies for Mining and Manufacturing industries. Now there are always some minor changes and adjustments in this policy. But for the period under consideration there are no changes that can make the missing observation "estimation" basicly invalid. Thus all in all this method of obtaining subsidies and duties for 1959 and 1960 should not be too bad.

And by an inspection of the "estimates" obtained the method seems to work quite well. The quality of the reporting on subsidies and duties looks surprisingly good, and the effects of what may be present of errors of observation we have tried to reduce by averaging information for 1961 and 1962.

APPENDIX 5.

THE "PRICE" DATA FOR GROSS PRODUCTION AND MATERIALS.

a. The "Price"-Data Used.

One important type of information is not provided by the Census of 1963 or the Annual Production Statistics, namely information on prices. Except, perhaps for labour input for which it is possible to get sort of a price variable by means of the information available. For output, materials and capital input we have, however, none. The lack of individual price-data for these central variables is certainly not just an empirical problem. It is equally much, or perhaps more a conceptual problem as it is not at all that clear how the price and quantity components of, for instance, gross output or value added shall be separated, provided also that these price and quantity components must be comparable across establishments.[1)]

Nothing can be done by these problems in the present context. On the other hand there are some possibilities to obtain price data to deflate the

1) This problem is basicly due to the approach adopted in this and related studies. Namely to squeese a multioutput multiinput problem into a one output "few" input frame. And also partly due to the fact that production units with widely different output and input mix are merged into one "industry". These, basicly index and aggregation problems, will not be discussed here as the data available do not make a suitable basis for an empirically interesting discussion.

variables of different years so that they refer to a common price base.

The best source available for such price-data seems to be the more disaggregated national account system. [2] By using the data for gross production and materials in current and constant f.o.b. and c.i.f. prices respectively, we have an implicit price-index to deflate the corresponding variables of the individual production units of our study, and thus we also get sort of a price-index for value added. [3] [4]

2) For Mining and Manufacturing there are about 85 sectors in this accounting system.

3) The base year of the national account system is now 1961, while it was previously 1955. By simple chaining we get indices with base in 1961 also for 1959 and 1960.

4) There is a difference between the gross production definition in the national account system and the one used by us, as the later is in "factor-prices". Data for a corresponding definition is not available in the national account system. But this discrepancy should not matter much.

b. An Evaluation of the "Price-Data"

This procedure to bring the variables on a common price base is clearly rather rough. On the other hand the numbers of the national accounts sectors under consideration are obtained from the same source of data as the empirical base for the present study, namely the Annual Production Statistics (and the Census in 1963). The main difference is that while our sample consists of establishments of so-called large firms only, the national accounts numbers are based on information for all units covered by the statistical sources under consideration. Apart from the pure aggregation problem which is present for all such index-numbers, the quality of our deflators depends critically on if the output-mix and materials input-mix of the units of our study are much different from the "average" mix of the production units of the national account sectors to which they belong. By selecting large firms we automatically also select the large establishments. The more important units not covered by our study are those of medium size single unit firms, that according to Norwegian standards are those with 50-100 employees. While the 907 units selected cover only about 5% of the number of establishments on the average for 1959 through 1967, these units have about 50% of total employment, gross output and materials input of the industries they cover. So the question of differences in output mix and materials input mix between units of this study and those making the base of the corresponding national account sectors can roughly be identified with such differences between small and large establishments. This suggests also that it probably is of highly varying importance for different sectors. It is presumably of less importance for sectors of the 20-industry group (Food Products) than for some of the sectors of the 27-industry group (Pulp and Paper Products) [5)] Thus for some industries at

5) The results for the 27-industry from the Census-study suggested that there are substantial differences between small and large establishments of this industry, both as concerns technology and kinds of activity.

least the gains of deflating the variables may be quite low.

The price-index of output is for some industries, however, quite misleading in another way, and this comes out to have serious effects on some of the results of the industries concerned. For some national accounts sectors the price-data are very spotty or generally of poor quality. For these sectors price-indices for output are computed by means of price-data for the inputs; deliveries from other sectors and labour. [6] As increased wages due to improved productivity of labour are not eliminated from the input-price data, the price-increase for the industries concerned is overstated and thus the growth in output "in constant prices" is understated. A further discussion of the particular problems this price-index computation causes for the interpretation of the results is presented in Chapter V. Cf. also Appendix II.8.

6) In Appendix II.2 industry groups for which the output price indices are computed in this way are marked with a star.

APPENDIX 6.

SOME REJECTED REFINEMENTS OF THE CAPITAL INPUT MEASURE.

a. The Role of Inventions.

It is not uncommon to include inventories in the capital input measure with an appropriate weight.[1] This component of capital input is presumed to reflect the costs of "optimal" stock of materials, semi-products and finished goods. The main problem with inventories in this context is that they very probably contain significant transitory components, reflecting transitory variations in demand for finished goods or supply of materials.[2] And as one should be very careful not to introduce more transitory elements into the capital input measure than is strictly speaking necessary, I decided to drop inventories as a component of this variable.

b. Weighting of Components.

In principle a slightly more "refined" capital measure could also be obtained by weighting the two components of K in (II.11) giving the larger weight to K_2, value of machinery, and thus taking care of a presumably higher depreciation rate for this kind of capital compared to buildings. But in one way or another we have to use the information of investments to obtain capital data for other years than 1959 and 1963. And as we do not have separate information about investments for the two kinds of capital, we must necessarily have adopted rather unreasonable assumptions to obtain a "weighted" capital measure for all years: For instance, that the composition of capital for those years we have investments only, is more or less the same as for those years we have information about capital. Thus the price of this refinement seems to be too high.

1) This is done in the Census study, for instance.

2) For instance Herring Oil and Meal Factories are subject to substantial variations of both types.

c. Separate Deflating of Components.

The weighting issue is related to another one that deserves some comments, namely the one of price-movements of capital over time. As for gross production and input of materials we would like to have eliminated the price-movements over time of the capital measure. Thus we have applied a common aggregate price-index for capital as shown in (11).

As there are price indices for different categories of capital available, we could have deflated buildings and machinery separately provided that we could have identified these components of the capital stock for other years than 1959 and 1963. If so one could have taken care of differences in the overall price-level of the capital stock of different establishments due to differences in the composition of the stock. This is clearly not so important if the price movements of the two kinds of capital had been roughly the same. In the present case this does not seem to be the case, however, as buildings seem to have become relatively more expensive compared to machinery during the period considered. The price-indices we use are based on information about prices of gross investments, for different kinds of capital. For buildings in Mining and Manufacturing the price index is about 133 in 1967, and the index for machinery is about 116, while the index we get for total capital by weighting these two indices, using capital stocks of the two kinds of capital as weights, is about 121, all indices with base in 1961.[3] Thus, even if this index may work fairly well on the average, it is very little establishment-specific. Therefore, quite likely there is a significant "transitory" component in the capital measure due to incomplete, or inaccurate deflating. But it looks unlikely that a significantly better approach could be adopted in this context.

3) As for gross production and raw materials the indices available for gross investments for 1959 and 1960 have 1955 as base. And correspondingly in this case, by simple chaining of corresponding indices with base in 1955 and 1961 we get price indices for gross investments with base in 1961 for 1959 and 1960 also.

APPENDIX 7.

THE TREATMENT OF MISSING OBSERVATIONS FOR CAPITAL

a. Introduction.

A very common empirical problem of econometric research is that of incomplete sets of data, or missing observations. This is particularly true for studies based on micro-economic data, as the missing values usually dissappears in aggregates. On the other hand such aggregates constructed of incomplete data at the micro-level may be subject to serious errors of measurement. This problem is well-known among the main suppliers of micro-economic data, the various national bureaus of statistics. And they have control and revision procedures on the current statistics by means of which obviously inaccurate data, among them also missing observations are corrected. This is, I believe, mostly done by getting correct information from the economic units concerned, at least for the more important caracteristics. But there seems also to be some amount of "guessing" with a fair room for personal judgement what are "reasonable" values. And after all "guessing" is generally better than to do nothing at all. Quite probably, the aggregates that usually are the output of such statistics become more reliable by such corrections.

But nevertheless, for some reason or another, after the correction and control has taken place, there are quite often a number of missing observations on important variables left. And if an econometrician is interested in analysing these data at the micro-level, he has to do something with them. Usually this problem is solved by excluding the units concerned, or not so often, he continues the work of the supplier of the data and "guestimates" values of the missing observations on ad hoc basis.

But evidently one should not be too satisfied with such ad hoc solutions. It is,however, probably impossible to obtain more satisfactory methods that are generally applicable on the whole range of missing observation problems in micro-economic data. On the other hand there exist methods that can solve partial missing observation problems.

One important property of such methods must be that it makes the "guestimation" look more like true estimation. That economic theory and statistical methods are applied to make the estimation of missing observations more systematic. By putting such computations into a econometric frame it may also become easier to evaluate what really happens to the data, and eventually to the results of analyses carried out on data with missing observations estimated by means of the observations reported.

In this Appendix we report on some attempts made to "estimate" missing observations for capital. Even if we are not very successful in these attempts, they seem to be interesting enough to deserve a fairly detailed presentation and discussion.

b. The Capital Data Missing

By inspecting the capital numbers reported for 1959 and 1963 a significant fraction of the establishments was found to report no buildings or no machinery for one or both of the years.

This suggests that the capital data are rather shaky. But they may look somewhat poorer than they really are. Firstly, by a closer examination of the numbers, it comes out that most of those establishments which reported one of the components of capital zero for one of the years, have lumped together both categories of capital and reported it as either buildings or machinery. This conclusion is based on the capital reported for the other year when the categories were reported separately, the investments in the period between the two years under consideration and price movements of that period. Secondly, some of those establishments which reported only buildings or only machinery for both years seemed to have lumped together the two categories for capital for both years and have reported it either as buildings or as machinery. This conclusion is also based on investments and price movements of capital, but in addition on the level of employment and value added of the units under consideration. For these units we accept the values of capital reported as representing total capital stock according to the definition in (11) above.

There are then 60 units left with missing or obviously incomplete information on capital for 1959 and there are 37 in 1963. The net number of units with incomplete information on capital is somewhat lower than the sum of these numbers, about 85, as there are 12 units with missing or incomplete information for both years.

As pointed out in section II.2 our 907 units are divided into 15 "industries".[1] In table A.II.1. the number of units of each industry is presented together with the number of missing observations for capital of each of the years 1959 and 1963.

Table A.II.1

The Number of Establishments and the Number of Missing Capital Values for 1959 and 1963 by Industry

Industry groups	Industry	Number of est.ments	Number of missing capital value 1959	1963
11 - 19	Mining and Quarrying	26	0	1
20 - 22	Food Products	164	9	10
23	Textiles	58	0	2
24	Clothing	67	6	6
25,26	Wood Products	45	1	1
27	Pulp and Paper	103	8	1
28	Printing	63	10	1
29 - 31	Basic Chemicals	72	4	3
32,33	Mineral Products	36	1	0
34	Basic Steel	42	1	3
35	Metal Products	60	3	3
36	Non-El. Machinery	37	4	1
37	El. Machinery	34	4	2
38	Transport Equipment	87	6	2
39	Misc. Products	13	3	1
	Total Mining and Manufacturing	907	60	37

1) In Appendix II.2 the composition of the industries is presented.

We note that the relative number of missing capital values is quite different for different industries. None are missing for Mining and Quarrying and Textiles in 1959 and Mineral Products in 1963, while almost 25% are missing for Misc. Products and about 16% for Printing in 1959. [2]

As we accept complete time-series only, we see from Table A.II.1 that it would, at least for some industries imply a substantial loss in number of degrees of freedom, to exclude the units with missing observations for capital. On the average the loss is almost 10% of the total number of degrees of freedom. If we on the other hand, in one way or another managed to estimate the missing observations we "loose" less than 2% of the total numbers of degree of freedom. Thus there is a strong argument for adopting the estimation approach in this case.

c. *The Model and the Basic Properties of the Method Applied.*

The point of departure in our attempts to "estimate" missing capital observations is the "main model" of this study. [3]

(1) $y = \alpha_0 + \alpha x + \beta z + u$

$y - x = b_0 + b\,w + v$

Where $y = \ln V$, $x = \ln L$, $z = \ln K$, and $w = \ln W$, and u and v are error of relation terms assumed to have zero means and constant variances. They are distributed independently and show no serial correlation.

In the litterature of how to treat missing observations in statistical research a number of methods are proposed. [4] The number of potential methods is,

2) The difference in the total number of missing capital observations for the two years suggests that there either has been an improvement in the reporting and/or the control of the data, or that the quality of Census data is better than that of the Annual Industrial Production Statistics.

3) Cf. Chapter I and Chapter III.

4) For a survey of the litterature and discussion of the different methods cf. Elashoff, R.M. and Afifi, A.A.: "Missing Observations in Multivariate Statistics," Journal of the American Statistical Association. Part I; "Review of the Litterature", 1966, Part II; "Point Estimation in Simple Linear Regression", 1967, Part III; "Large Sample Analysis of Simple Linear Regression", and Part IV; "A Note on Simple Linear Regression", 1969.

however, substantially lower in the present case, as we consider a particular situation; when observations for only one variable are missing. And among these methods only one will be considered, namely the one that presumably is the more appealing intuitively. The contents and implications of it are also easy to understand. But to be sure a fairly detailed deduction of it is presented below, also as this clearly shows under what conditions a couple of ad hoc methods do not work, under what conditions they may work, and also under what conditions one of them may be better than the more "refined" one.

To illustrate the basic properties of the method we are going to use, let us for a moment assume that labour input is not subject to profit maximization, that is; the behaviour relation in (1) is invalid and also that the variables are correctly measured so that x and z are two true exogenous variables in the production relation. Then we know that the ordinary least square method on this relation gives best linear unbiased estimators for α and β.

Now we have n sets of observations of which $n_1 \leq n$ are complete. Thus there are $n - n_1$ "unknown" values of z and we are going to estimate these values along the same lines as α and β. We can write the sum of squares function to be minimized as:

$$
\begin{aligned}
(2) \quad U^2 = & \sum_{i=1}^{n_1} (y_{1i}-\bar{y}-\alpha(x_{1i}-\bar{x}) -\beta(z_{1i}-\bar{z}))^2 + \\
& + \sum_{i=n_1+1}^{n} (y_{2i}-\bar{y}-\alpha(x_{2i}-\bar{x}) - \beta(z_{2i}-\bar{z}))^2
\end{aligned}
$$

where the subscripts 1 and 2 of the variables refer to complete and incomplete sets of observations respectively and the averages $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$ refer to all sets of observations, which implies that for $\bar{z}$ also the $n - n_1$ unknown values of z are included.

Minimizing (2) with respect to the unknown z - values gives the $n - n_1$ 1. order conditions for minimum as :

$$
\begin{aligned}
(3) \quad \frac{\partial U^2}{\partial Z_j} = & \frac{2\beta}{n} \sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta(z_{1i} - \bar{z})) + \\
& + \frac{2\beta}{n} \sum_{i=n_1+1}^{n} (y_{2i} - \bar{y} - \alpha(x_{2i} - \bar{x}) - \beta(z_{2i} - \bar{z})) - \\
& - \frac{2\beta}{n} (y_{2j} - \bar{y} - \alpha(x_{2j} - \bar{x}) - \beta(z_{2j} - \bar{z})) = 0 \quad (j = n_1+1, \ldots\ldots, n)
\end{aligned}
$$

As the sum of the two first terms of (3) is zero due to the properties of the least square method we get:

(4) $$(y_{2j} - \bar{y}) - \alpha(x_{2j} - \bar{x}) - \beta(z_{2j} - \bar{z}) = 0 \quad (j = n_1+1,\ldots..n)$$

That is, each unit with a missing observation gets a value of z which gives an error of relation of zero for the unit concerned; in other words the error is "absorbed" in the estimate of z. This is a property of the method subject to further comments below.

Now, the formula in (4) cannot be used directly to estimate the missing z-values as it includes $\bar{z}$. But $\bar{z}$ is found in the following way:

From (4) we have that:

(5) $$\sum_{j=n_1+1}^{n} (y_{2j} - \bar{y} - \alpha(x_{2j} - \bar{x}) - \beta(z_{2j} - \bar{z}) = 0$$

Due to the properties of the least square method this implies that

(6) $$\sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta(z_{1i} - \bar{z})) = 0$$

And so we get:

(7) $$\bar{z} = -\frac{1}{\beta n_1} \sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta z_{1i})$$

or

(8) $$\bar{z} = -\frac{1}{\beta} (\bar{y}_1 - \bar{y} - \alpha(\bar{x}_1 - \bar{x}) - \beta\bar{z}_1)$$

where $\bar{k}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} k_{1i} \quad (k_1 = y_1, x_1, z_1)$

Inserting (8) into (4) yields;

(9) $$(y_{2j} - \bar{y}_1) - \alpha(x_{2j} - \bar{x}_1) - \beta(z_{2j} - \bar{z}_1) = 0 \quad (j = n_1+1,\ldots\ldots.n)$$

But estimating the $n - n_1$ missing capital values by means of (9) does also imply that the second term of (2) disappears. [5)]

5) There are "no degrees of freedom left" for this part of the sum of squares function.

Thus to estimate α and β we are left to minimize the sum of squares of the complete sets of data. In this expression $\bar{z}$ enters also. But inserting (8) into (2) yields:

$$(10) \quad U^2 = \sum_{i=1}^{n_1} (y_{1i} - \bar{y}_1 - \alpha(x_{1i} - \bar{x}_1) - \beta(z_{1i} - \bar{z}_1))^2$$

Thus this least square method of estimating missing observations is, not surprisingly, separable in the sense that we first can estimate the parameters of the relation concerned by means of the complete sets of data and then use a relation like (9) to estimate the missing variate-values.

d. Modifications of the Method I

The procedure of estimating missing values for capital deducted in the previous section is based on assumptions that imply consistent estimates from the ordinary least square method on the production relation. Now the main model tells us that profit is maximized with respect to labour. And we know that the observed capital data are of rather poor quality, containing a substantial, but presumably random error-component. As is shown in Chapter III this implies inconsistent estimates on the factor-elasticities when applying the ordinary least square method. From (9) we see that this does also imply inconsistent estimates on the missing capital values.[6)]

Therefore we need a method taking care of both the simultaneity of y and x and the errors of measurement of z. Such a method is discussed in Chapter III. It implies that the elasticity of labour is estimated by a particular factor share method, assuming the elasticity of substitution equal to unity, and that the elasticity of capital is estimated by a size- dummies instrumental variable method. [7)]

The adoption of another estimation method than simple least squares to estimate α and β has no consequences for the "algebra" of estimating the missing capital values as deducted above. To estimate α we now can apply all sets of

6) Given $\bar{y}_1$, $\bar{x}_1$, z_1, y_{2j} and x_{2j} we get the probability limit of $\hat{z}_{2j}$ as: $\text{plim}\ z_{2j} = \bar{z}_1 + \frac{1}{\beta + \text{bias}\ \hat{\beta}} (y_{2j} - \bar{y}_1) - \frac{\alpha + \text{bias}\ \hat{\alpha}}{\beta + \text{bias}\ \hat{\beta}} (x_{2j} - \bar{x}_1)$

when having estimated α and β by ordinary least squares. And under reasonable assumptions we have bias $\alpha > 0$ and bias $\beta < 0$. This implies that we overstate the deviations of y_{2j} from the mean of this variable for the complete observations and thus also the transitory components in output. We also overstate the importance of the deviation of labour from its mean of the complete observations.

7) Cf. Section III.4.

data, while the complete sets only enters when β is estimated. Relation (9) is still valid, as the only way of estimating the intereept of the production relation is by means of the complete sets of data.

e. *Two Ad Hoc Methods to Compute Missing Capital Values.*

Having estimated α and β we get the capital estimates from (9) as:

$$(11) \quad \hat{z}_{2j} = \bar{z}_1 + \frac{1}{\hat{\beta}}(y_{2j} - \bar{y}_1) - \frac{\hat{\alpha}}{\hat{\beta}}(x_{2j} - \bar{x}_1)$$

When working with incomplete data one may be tempted to "estimate" the missing values of a variable by means of the average of this variable for the complete sets of data. That would in our case be equivalent to ignoring the two last terms of the right side of (11). But even if there may be substantial transitory variation in y_{2j} this method is not recommendable in the present case as it ignores completely differences in size of the units.

We can, however, write (11) as:

$$(12) \quad z_{2j} - x_{2j} = \bar{z}_1 - \bar{x}_1 + \frac{1}{\hat{\beta}}(y_{2j} - x_{2j} - \bar{y}_1 - \bar{x}_1) + \frac{(1-\hat{\alpha}-\hat{\beta})}{\hat{\beta}}(x_{2j} - \bar{x}_1)$$

And we see from (12) that another ad hoc method may work fairly well provided we have approximately constant returns to scale. Namely by using the geometric mean of the capital labour ratio for the complete sets of observation to compute the capital-labour ratio for the incomplete sets of data. In that case the last term of (12) is ignorable and the difference between this ad-hoc method and the least square method is that the latter takes care of the difference between the average productivity of labour of each of the units with incomplete data and the average for the complete sets of data. Thus in case of large transitory variation in output between units, or in other words a large standard deviation of the residual, the ad hoc procedure may give more reasonable results than the least square method.

f. Modifications of the Method, II.

As we for our kind of data may expect rather poor fit, we should adopt a mixed method of estimation: We estimate the missing capital values by means of the "consistent" method described in section d above. If these estimates are within certain limits they are accepted. If not a modified version of the average capital/labour ratio for complete sets of data-method is applied.

The limits of "the region of acceptance" are determined by the average capital/labour ratio for the industry concerned so that the lower limit is one third of the average and the upper is three times the average.[8] But for units with missing capital values for only one year an estimate for this missing value outside the region is accepted provided that the observed capital/labour ratio for the other year is also outside the corresponding region for that year, and outside on the same side as the estimate. This implies that we consider each of the two years separately in the first stage of the estimation procedure.

With the exception mentioned estimates below the lower limit or above the upper limit is set equal to the corresponding limits. This seems to be better than to estimate them by means of the average capital/labour ratio as extreme estimates may be "true". In a sense this last step in our procedure corresponds to the method of "Wisorizing" samples in errors of variables situations.[9]

Thus the main part of this method of estimating missing observations is theoretically fairly well founded, but "the empirical reality" forces us to adopt ad hoc coloured modifications. And the results of these experiments show that this is necessary.

g. The Results

In Table A.II.2. the estimates on α and β are presented, and also estimates on their standard deviations according to formulas presented in Section III.4.

8) This region may look too wide. But I think the limits are reasonable as the "probability" of rejecting an estimate that is "correct" should be low. As we then also obviously accept a number of "wrong" estimates conforms quite well with the quality of the observed data that evidently contains substantial errors of measurement.

9) Cf. Tukey, J.W.; "The Future of Data Analysis" in The Annals of Mathematical Statistics vol 33, March 1962, pp. 17-19.

The mean square error of the residual obtained from ordinary least squares on the Cobb Douglas relation applied on the complete sets of data is also presented to give an idea of the fit, or rather the unexplained variation in output. 10)

As we see from this table, the fit is poor and we have not obtained a "sharp" determination of the parameter-values. Thus, as expected, we get a number of "wild shots" when "estimating" missing capital values by mears of our method. A total number of 21 of 93 "estimates" are outside "the regions of acceptance" discussed above. This is no unreasonable number judged by the contents of Table A.II.2. And four of these "estimates" are not necessarily so "wild" as they may look as the capital/labour ratio for the other year for which capital is reported is also outside the region of acceptance. Thus these estimates are accepted also. For the 17 remaining "wild shots" we present in Table A.II.3. their distribution on industry and year and if they are "too low" or "too high".

10) No results are presented for Misc. Products as the method could not be applied on this industry due to degrees of freedom problems.

TABLE A.II.2.

Output Elasticities of Labour and Capital Estimated by the Complete Sets of Data in 1959 and 1963 [x]

Industry	1959 $\hat{\alpha}$	1959 $\hat{\beta}$	1959 $\hat{\alpha}+\hat{\beta}$	1959 $\hat{\sigma}_u^2$	1963 $\hat{\alpha}$	1963 $\hat{\beta}$	1963 $\hat{\alpha}+\hat{\beta}$	1963 $\hat{\sigma}_u^2$
Mining and Quarrying	-	-	-	0,172	0,605 (0,047)	0,367 (0,035)	0,972	0,163
Food Products	0,641 (0,034)	0,324 (0,037)	0,965	0,367	0,649 (0,036)	0,243 (0,037)	0,892	0,373
Textiles	-	-	-	0,161	0,587 (0,025)	0,278 (0,043)	0,865	0,114
Clothing	0,625 (0,029)	0,322 (0,060)	0,947	0,112	0,615 (0,022)	0,346 (0,067)	0,961	0,118
Wood Products	0,749 (0,045)	0,296 (0,052)	1,045	0,173	0,753 (0,051)	0,364 (0,055)	1,116	0,223
Pulp and Paper	0,526 (0,020)	0,373 (0,030)	0,899	0,120	0,635 (0,025)	0,233 (0,032)	0,868	0,144
Printing	0,721 (0,034)	0,367 (0,060)	1,088	0,147	0,770 (0,029)	0,232 (0,036)	1,002	0,133
Basic Chemicals	0,568 (0,045)	0,431 (0,053)	0,999	0,390	0,657 (0,061)	0,327 (0,054)	0,984	0,510
Mineral Products	0,643 (0,050)	0,410 (0,058)	1,053	0,241	-	-	-	0,164
Basic Steel	0,547 (0,034)	0,604 (0,043)	1,151	0,146	0,621 (0,041)	0,420 (0,048)	1,041	0,171
Metal Products	0,680 (0,035)	0,286 (0,045)	0,966	0,144	0,608 (0,028)	0,382 (0,041)	0,990	0,107
Non-El. Machinery	0,686 (0,037)	0,371 (0,039)	1,057	0,089	0,726 (0,053)	0,319 (0,059)	1,045	0,166
Electr. Machinery	0,687 (0,047)	0,267 (0,058)	0,954	0,148	0,655 (0,049)	0,576 (0,072)	1,231	0,197
Transport Equipm.	0,858 (0,037)	0,227 (0,022)	1,085	0,124	0,778 (0,028)	0,280 (0,019)	1,058	0,089

[x] Cf. Section III.4.d about the method of estimation applied.

TABLE A.II.3.

Estimates on Missing Capital Values Outside the "Region of Acceptance"

Industry	*Number of "wild shots"*				
	1959		*1963*		
	Too low	*Too high*	*Too low*	*Too high*	*Total*
Mining and Quarrying	-	-	0	0	0
Food Products	0	2	0	0	2
Textiles	-	-	0	0	0
Clothing	0	1	0	0	1
Wood Products	0	0	0	0	0
Pulp and Paper	0	1	1	0	2
Printing	2	0	0	0	2
Basic Chemicals	0	2	2	0	4
Mineral Products	0	0	-	-	0
Basic Steel	0	0	0	0	0
Metal Products	0	1	1	0	2 1)
Non-El. Machinery	0	0	1	0	1
Electr. Machinery	0	1	0	0	1
Transport Equipm.	1	1	0	0	2
Total	3	9	5	0	17

1) Refers to the same unit

We note that Basic Chemicals has 4 wild shots. And this is a rather poor result as this industry has 7 missing capital values only. But as it also has the highest mean square error among our industries, this result is not too surprising even if more than 50% wild shots is somewhat more than one would expect. On the other hand, Food Products which also has a high mean square error behaves fairly well as only 2 of 19 estimates are wild. We also note that for Pulp and Paper and Non-El. Machinery we have only one estimate in 1963 and both are wild, even if the mean square errors of these industries are relatively low. Finally we see from these computations that the 59 data are of poorer quality than those of 1963. 1959 has more than 60% of the missing observations, and it has an even higher fraction of the wild shots, about 70%. This higher fraction of wild shots than of missing observations may be explained, at least partly by the fact that we have a lower number of degrees of freedom in the second stage of our estimation procedure for 1959 compared with 1963 just because the former year has the majority of missing values.

As pointed out the missing capital values for those units for which we get "wild shots" are estimated by setting them equal to the upper or lower limit of the "region of acceptance" depending on whether the wild shot is above or below this region. This implies among other things that the capital values thus estimated are in some cases quite "inconsistent" with information of the other year for which capital is reported, taking investments, price movements and depreciation into consideration. This may clearly also be the case for estimates within the region of acceptance. But such obvious inconsistences can also be observed quite frequently among those units with complete sets of data.

h. Calculation of Capital Values for Other Years than 1959 and 1963.

h.i The Information Needed

We have, in principle, information on all characteristics, but one of those necessary for the computation of capital values of other years than 1959 and 1963. We have capital values for 1959 and 1963 and we have gross investments for all years. And we have also a price-index that makes it possible to eliminate even if in a rather approximate way the price movements over time in these two variables.[11] What we need in addition is information on depreciation. Adopting the simplifying, and also rather dubious assumption that the capital stock as measured by us is reduced by a constant fraction during one year due to depreciation,[12] we could either apply the "official" depreciation ratios, for instance those applied by the CBS, or we could try to estimate them. The simplest would clearly be to accept the former, but as they look unreasonably low, about 5%-6% on the average, the latter approach is preferable as it also may serve

11) Cf. Appendix II.6.

12) Evidently a constant depreciation ratio in the sense that the initial value of the capital was reduced by a constant fraction each year is preferable in the present context. But as we then would need to know the age-distribution of the capital stock to compute the depreciation each year this concept is not operational in the present case.

as a check on the validity of the former. [13)]

h.ii. On the Consistency of the Capital and Investment Information.

Before we try to estimate the depreciation rate there is a particular issue that deserves a few comments, namely the degree of consistency of the capital and investment data applied.

We know that the capital measure of 1959 and 1963 refers to full fire insurance values at the end of these years, while the investment measure is the accumulated flows of repairs and maintenance and of purchased capital goods during the year. The question is now if, or to what extent the investment during one year is reported as part of the capital stock at the end of the year. The results of a recent study suggest that there in fact, and not unexpectedly is a lag or kind of sluggishness between reported investments and reported capital. [14)] The lag may occur because current outlays on investments are reported even if some investment projects are not completed before the next year (or even later, still) while the reported capital stock is adjusted for completed investment projects only. In addition there may be a sluggishness in adjusting the full fire insurance values even for completed investment projects.

If the lag-hypothesis is valid, this is in fact an improvement of our capital measure as incompleted investment-projects usually do not add to the production capacity of capital. But on the other hand, when computing capital values by means of current investments we clearly should know how much the outlays on incompleted investment projects make of the total outlays in investments. And such information is not available. Any sluggishness in adjusting the fire insurance values for new capital goods makes matters difficult in another way as this implies that the reported capital values of 1959 and 1963 are

13) For such "official" depreciation ratios see for instance: Johansen, L: "A Multisectoral Study of Economic Growth". N. Holland Publ. Co. 1961. And also Heli, H and Johansen A.D.: "Investeringer i norsk industri." Memorandum from the Institute of Economics, University of Oslo, 1961.

14) Ringstad V, and Griliches Z., "A Method of Analyzing the Consistency of Capital and Investments." The Review of Income and Wealth, No. 4. 1968.

generally too low.

Clearly there may also be substantial individual variations as concerns "lags and sluggishness". but we can at most take care of the average of these effects. We could try to take care of them by adding lagged instead of current investments to depreciated capital of the previous year to get the capital-value of a year.[15] This is, however, rather arbitrary even if it seems to have some support in the study refered to above.[16] But what one considers less arbitrary in this context is a question of taste. One could even compromize by weighting lagged and current investment in the computations of capital and perhaps also try to find out something about the average lag. But such an approach does not look particularly promising in the present context.

As a choice has to be made I am inclined to ignore the problem of possible "lag and sluggishness". And in the computations of capital values this is done. But in a particular context below we also refer a few results obtained using lagged instead of current investment.[17]

h. iii. Estimation of the Depreciation Ratio

By means of the information available about capital and investments we get the capital values of the years 1960 through 1963 as:

$$\begin{aligned} K_{60} &= (1-\Delta)\,K_{59} + I_{60} \\ K_{61} &= (1-\Delta)^2 K_{59} + (1-\Delta)\,I_{60} + I_{61} \\ K_{62} &= (1-\Delta)^3 K_{59} + (1-\Delta)^2 I_{60} + (1-\Delta) I_{61} + I_{62} \\ K_{63} &= (1-\Delta)^4 K_{59} + (1-\Delta)^3 I_{60} + (1-\Delta)^2 I_{61} + (1-\Delta) I_{62} + I_{63} \end{aligned} \tag{13}$$

15) Both in this context and later we speak of capital and investments data which have a common price base, namely 1961.

16) Ringstad V., and Griliches Z., op. cit.

17) As concerns capital we would also like to have an average for the year instead of the stock at the end of the year. In this context this problem is of minor importance. And it is ignored also as we would need information about capital at the end of 1958 to obtain an average for 1959, otherwise we would have to exclude 1959 from the analysis.

Where Δ is the depreciation ratio and K and I are capital and investment respectively in constant 1961-prices.

The last relation will be used to estimate the depreciation ratio by fitting it to the data involved. This may look like a rather complex optimization problem as there are non-linear constraints on the parameters. But as the relation considered has no intercept, the following relation must hold approximately for an optimal value of Δ.

$$(14) \qquad \bar{K}_{63} - \bar{I}_{63} = (1-\Delta)^4\bar{K}_{59} + (1-\Delta)^3\bar{I}_{60} + (1-\Delta)^2\bar{I}_{61} + (1-\Delta)\,\bar{I}_{62}$$

where the barred variables are averages across establishments. And as it for the present data does not make much sense to apply an expensive optimization method to get an estimate on Δ with many decimal places, we use instead a "scanning" procedure to get a much cheaper, but also somewhat rougher estimate.

For different values of Δ the difference between the left and the right side of (14) was computed and the value of Δ that gave the lowest absolute value of this difference was chosen as the optimum value. For a relation like (14) there is clearly no problem of local optimums, as the difference between the left and the right side increases monothonicly from negative to positive values with an increasing Δ.[18)]

The search was made for values of Δ between - 10% and + 20% with step 0,1%. For Total Mining and Manufacturing we got an optimum value of Δ of 7,7%.[19)] [20)] This estimate looks quite reasonable and it also suggests that the CBS depreciation ratios are somewhat too low.

18) Thus the meansquare error has the absolute and only minimum of zero when this difference is zero.

19) Clearly by "scanning" the region 7,6 - 7,8% in one or more stages using smaller steps we could get as many decimals in our estimate as we liked. But as pointed out above this does not seem to be worth while.

20) An attempt to apply this method on the individual industries did not work quite well as we got unreasonably large variations in the optimum value of Δ across industries with about 18% for Mining and Quarrying and 2% for Basic Steel Products as extremes. While the former is not completely unreasonable it is difficult to believe in the latter. However, these results give additional evidence of the poor quality of the data involved and that we in fact need averages for a fairly large number of units to obtain reasonable answers on the kind of questions we ask in this context.

A related scanning procedure for the same values of Δ as above on the last relation of (13) when all constraints except that the intercept is zero were taken into account gave an estimate on Δ of 5%.[21] This implies that the intercept estimate is negative. And it may lend some support to the "lags and sluggishness" hypothesis. Using lagged instead of current investments in the sense pointed out above on a relation related to (14) we got by our scanning procedure an estimate on Δ of 6,8%.

h. iv. Calculation of Capital Values by Means of Investments and the Estimated Depreciation Ratio

Even if there is some evidence of a lag between the reported investments and capital data, we choose to compute the missing capital data using the depreciation ratio estimated by means of (14) with current and not lagged investments. And as the results for the individual industries as concerns Δ look rather unreliable it seems to be better to use the result obtained for "Total Mining and Manufacturing" for all industries.

Thus for $\Delta = 0{,}077$ we compute capital data for 1960, 1961 and 1962 by means of the three first relations of (13) and correspondingly we get the estimates on capital for the years after 1963 as:

$$
\begin{aligned}
K_{64} &= 0{,}923K_{63} + I_{64} \\
K_{65} &= (0{,}923)^2K_{63} + 0{,}923I_{64} + I_{65} \\
K_{66} &= (0{,}923)^3K_{63} + (0{,}923)^2I_{64} + 0{,}923I_{65} + I_{66} \\
K_{67} &= (0{,}923)^4K_{63} + (0{,}923)^3I_{64} + (0{,}923)^2I_{65} + 0{,}923\, I_{66} + I_{67}
\end{aligned}
\tag{15}
$$

Thus in this way we get capital data of all of the 907 establishments for the 9 years 1959 - 1963. But whatever standard is used for the judgement of the quality of these data, the conclusion must be that they are extremely shaky. The consequences of this fact when they are applied in econometric analysis is, however, subject for investigation in another context.[22]

21) In this case we use explicity the meansquare error as the criterion of fit.

22) Cf. Chapter III.

i. Some Concluding Remarks

Even if we in these attempts to estimate missing capital values have tried to apply systematic analysis, they are strongly coloured by ad hocery, based on personal judgement, taste and intuition. In econometric research one can probably never expect to become completely independent of ad hoc solutions of empirical problems, but one should, however, try to use more satisfactory solutions whenever possible. That is solutions based on firmly founded econometric methods. The field under discussion in this Appendix has so far been highly dominated by such ad hoc solutions, but we have not been very successful in our attempts to systematize the estimation of missing observations. There is one obvious reason for this, namely the quality of the data. For capital the missing values is quite clearly one among several indications of that the information for this variable is generally poor. To some extent this may be true also for the other variables entering the production function. And as the quality of the reported investment data is generally considered to be of a even poorer quality than the capital data the second stage of our estimation becomes difficult also. Thus the main conclusion of this analysis is quite obvious: Shaky reported observations imply shaky estimates on missing observations whatever method is applied. On the other hand I believe that even if this is true "estimation" is better than exclusion of the units concerned for reasons pointed out previously.

APPENDIX 8

Some Basic Characteristics of the Main Variables.

a. Introduction

In this Appendix we present in a series of tables some characteristics of the main variables of this study. The variables are transformed in a way that should make the contents of these tables more easily comparable to the results obtained in the following chapters.

The variables for which such statistics are presented are labour input, the average "value added" - productivity for labour, the capital-labour ratio, the materials-labour ratio, the "real" wage rate i.e. the current wage rate divided by the "price"-index for value added, the share of labour in value added and finally materials' share in gross production. All, except the two latter variables are transformed to logs.

In addition to the mean and standard-deviation we are basicly interested in the variation of these variables along the "main dimensions" of our data; across establishments, with time and with "size".

To figure out the significance of the systematic variation of the variables along the two former dimensions we use the analysis of variance approach. [1)] We also run regressions with time as the independent variable to have an idea of the average growth rates of these variables. We should note that the standard-deviation presented concerns variation of growth rates both across establishments and over years. [2)]

1) Thus we consider the variables concerned to be random and normally distributed. The statistics applied are deducted in Appendix II.9.

2) Later the variation over years of average (per establishment) growth rates is considered. (Cf. Table V.2)
The OLS method on $X_{it} = a + b_x t + u_{it}$ must necessarily yield the same estimate on the growth rate for X, b_x as the OLS-method on $\bar{X}_t = a + b_x t + \bar{u}_t$ where $\bar{X}_t = \frac{1}{I} \sum_{i=1}^{I} X_{it}$ and $\bar{u}_t = \frac{1}{I} \sum_{i=1}^{I} u_{it}$. For the first relation we have :

$$\hat{b}_x = \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} (X_{it} - \bar{X})(t - \bar{t})}{\sum_{i=1}^{I}\sum_{t=1}^{T} (t - \bar{t})^2} = \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} (X_{it} - \bar{X}_t)(t - \bar{t}) + I\sum_{t=1}^{T}(\bar{X}_t - \bar{X})(t - \bar{t})}{I\sum_{t=1}^{T}(t - \bar{t})^2}$$

But the first term of the numerator must be zero as $\sum_{i=1}^{I} (X_{it} - \bar{X}_t) = 0$ for each t, and therefore

$$\hat{b}_x = \frac{\sum^{T} (\bar{X}_t - \bar{X})(t - \bar{t})}{\sum^{T} (t - \bar{t})^2}$$

which is also the OLS-estimate on b_x from the second relation above.

Concerning size we try to figure out the importance of this dimension by running regressions on ln N, where N= total number of employees, is the criterion of "size". This is done, both when imposing a common intercept for all units for all years, and when allowing different intercepts for different units and years.

Even if the contents of the tables speak for themselves it may be worth while to summarize what seems to be the more interesting findings.

b. Labour Input and Total Number of Employees.

Table A.II.4. tells us that even if the units selected belong to large firms, i.e. those with 100 employees or more in 1963, there are in our sample quite a few small establishments. The median value of N for all units is, we note, only slightly above 100, and one third of the units have 67 employees or less. As could be expected such industries as Food Products, Wood Products and Printing have mostly small establishments. At the other end of the scale we have the more heavy industries like Pulp and Paper and Basic Steel. More surprisingly the samples for both Textiles and Clothing contain mostly large units. We also note that industries like Mining and Quarrying, Basic Chemicals, Mineral Products and Transport Equipment cover a rather wide range of size.

Judged by the Analysis of Variance statistics there is for labour input even a more marked difference between the significance of the variation across establishments and over time than one perhaps would expect. And the results of the regression of lnL on t tells us also that labour input is on the average fairly stable over time. But the large standard errors of the estimates on the growth rate suggest that there are probably large individual variations concerning the growth of labour.

c. Average "Value Added" Productivity of Labour

The Analysis of Variance statistics of Table A.II.5 tells us that there are significant variations in the average productivity for all industries both across establishments and over time. The growth rates must be fairly uniform across establishments as the standard-deviation of the estimated growth rate is fairly low for most industries. And judged by the ordinary t-test at 5% level the growth rate is significant for all industries. Now, for some

TABLE A.II.4.

Basic Characteristics for lnL and N [x]

Industry	Mean and st. dev. for lnL	Growth rate for lnL	Analysis of Variance for lnL F_C	F_T	Median Value for N	1/3-Fractiles for N Lower	Upper
Total Mining and Manufacturing	5,309 (1.218)	,0058 (,0052)	228,45	11,97	113	67	160
Mining and Quar.	5,144 (1,383)	-,0161 (,0351)	470,15	1,82	105	33	183
Food Products	4,729 (1,245)	,0089 (.0126)	214,36	4,73	60	32	102
Textiles	5,763 (,795)	-,0001 (,0135)	126,33	3,56	159	119	218
Clothing	5,446 (,713)	-,0073 (,0113)	65,55	2,30	121	102	147
Wood Products	4,615 (1,102)	,0126 (,0212)	160,82	2,01	50	34	79
Pulp and Paper	5,657 (,943)	-,0160 (,0120)	308,72	10,42	144	106	191
Printing	4,607 (,866)	,0021 (,0141)	170,35	1,29	43	30	70
Basic Chemicals	5,213 (1,457)	-,0018 (,0222)	230,81	1,48	105	45	168
Mineral Products	5,430 (1,168)	-,0033 (,0252)	236,36	0,82	135	78	214
Basic Steel	6,475 (,858)	,0257 (,0171)	151,66	5,25	294	195	418
Metal Products	5,577 (,977)	,0221 (,0163)	172,15	4,92	125	98	155
Non-El. Machinery	5,563 (,984)	,0038 (,0209)	139,03	2,42	130	91	182
El. Machinery	5,884 (,986)	,0296 (,0218)	191,84	6,71	128	105	183
Transport Equipm.	5,531 (1,443)	,0178 (,0200)	260,77	3,51	143	96	247
Misc. Products	4,768 (1,290)	,0623 (,0460)	120,20	4,24	75	35	138

[x] See Notes on p.61.

TABLE A.II.5

Basic Characteristics for $\ln \frac{V}{L}$*

Industry	Mean and st. dev.	Growth-rate	Slope-co. from reg. on lnN: No dummies	E dummies	E and T dummies	Analysis of Variance: F_C	F_T
Total Mining and Manufacturing	2,612 (,576)	,0421 (,0024)	-,0054 (,0052)	-,0655 (,0172)	-,1288 (,0165)	16,13	104,16
Mining and Quar.	2,821 (,477)	,0613 (,0114)	,0406 (,0226)	-,4859 (,0968)	-,3331 (,0856)	20,20	11,64
Food Products	2,726 (,740)	,0625 (,0073)	-,1477 (,0149)	-,0951 (,0494)	-,2070 (,0466)	16,65	27,69
Textiles	2,414 (,422)	,0285 (,0070)	-,0253 (,0232)	,1620 (,0672)	,1461 (,0674)	9,13	4,19
Clothing	2,311 (,372)	,0215 (,0058)	,0012 (,0223)	-,3182 (,0452)	-,3062 (,0447)	9,25	4,92
Wood Products	2,388 (,558)	,0274 (,0107)	,0959 (,0246)	,1927 (,0708)	,1326 (,0697)	14,12	5,10
Pulp and Paper	2,751 (,451)	,0733 (,0052)	-,0478 (,0157)	-,5362 (,0712)	-,3850 (,0601)	10,94	54,41
Printing	2,407 (,362)	-,0173 (,0059)	,0604 (,0173)	-,2898 (,0551)	-,2562 (,0551)	10,51	3,98
Basic Chemicals	2,940 (,725)	,0671 (,0107)	-,0333 (,0196)	,1026 (,0619)	,0809 (,0570)	19,74	15,50
Mineral Products	2,741 (,534)	,0245 (,0114)	,1625 (,0243)	-,1938 (,0647)	-,1885 (,0633)	27,01	2,95
Basic Steel	2,979 (,476)	,0607 (,0090)	,1266 (,0280)	,0323 (,0789)	-,2500 (,0716)	14,35	15,11
Metal Products	2,587 (,424)	,0314 (,0070)	-,0633 (,0185)	-,0435 (,0661)	-,1784 (,0673)	6,44	4,52
Non-El. Mach.	2,565 (,379)	,0302 (,0079)	,0862 (,0207)	,0911 (,0619)	,0773 (,0610)	10,18	5,02
El. Machinery	2,585 (,497)	,0415 (,0108)	,0523 (,0288)	,0824 (,0826)	-,1525 (,0864)	14,54	5,50
Transport Equipm.	2,337 (,400)	,0194 (,0055)	,0662 (,0097)	-,0482 (,0439)	-,0958 (,0447)	6,00	2,84
Misc. Products	2,637 (,737)	,0883 (,0252)	-,0751 (,0516)	,1379 (,1345)	-,1851 (,1440)	8,63	3,38

* See Notes on p. 61.

TABLE A.II.6.

Basic Characteristics for $\ln \frac{K}{L}$ [x]

Industry	Mean and st. dev.	Growth rate	Slope-coef. from regr. on lnN: No dummies	E-dummies	E and T dummies	Analysis of Variance: F_C	F_T
Total Mining and Manufacturing	3,288 (,823)	,0266 (,0035)	-,0098 (,0075)	-,5291 (,0145)	-,5777 (,0145)	47,12	49,34
Mining and Quar.	3,366 (,751)	,0566 (,0187)	,1845 (,0336)	-,9020 (,1142)	-,8112 (,1091)	31,85	5,77
Food Products	3,424 (,705)	,0225 (,0071)	-,1233 (,0143)	-,5859 (,0326)	-,6346 (,0321)	31,35	6,90
Textiles	3,198 (,506)	,0285 (,0085)	,0624 (,0277)	-,4103 (,0517)	-,4328 (,0493)	32,61	8,85
Clothing	2,148 (,621)	,0301 (,0097)	-,0281 (,0371)	-,7885 (,0477)	-,7906 (,0466)	24,53	4,94
Wood Products	3,061 (,703)	,0280 (,0135)	,0039 (,0316)	-,4305 (,0687)	-,4828 (,0676)	26,17	3,25
Pulp and Paper	4,030 (,594)	,0491 (,0074)	,1555 (,0201)	-,6309 (,0697)	-,5493 (,0660)	22,69	21,46
Printing	3,342 (,641)	,0065 (,0104)	,1198 (,0305)	-,5395 (,0719)	-,5666 (,0734)	23,53	0,35
Basic Chemicals	3,923 (,762)	,0160 (,0116)	-,0914 (,0203)	-,4409 (,0469)	-,4369 (,0470)	34,15	2,17
Mineral Products	3,399 (,679)	,0341 (,0145)	,1798 (,0315)	-,3690 (,0635)	-,3617 (,0602)	46,63	5,04
Basic Steel	3,765 (,646)	,0354 (,0128)	,2632 (,0366)	-,4280 (,0786)	-,6553 (,0770)	26,09	3,82
Metal Products	3,113 (,532)	,0326 (,0088)	-,0417 (,0233)	-,5451 (,0612)	-,7195 (,0574)	16,36	8,21
Non-El. Mach.	3,018 (,554)	,0438 (,0115)	,0747 (,0308)	-,6055 (,0651)	-,6678 (,0580)	22,63	8,67
El. Machinery	2,823 (,668)	,0221 (,0148)	,0895 (,0386)	-,4909 (,0689)	-,7191 (,0693)	40,89	2,83
Transport Equipm.	2,795 (,661)	,0051 (,0092)	-,0782 (,0162)	-,4080 (,0473)	-,4407 (,0487)	21,72	0,31
Misc. Products	3,072 (1,019)	-,0083 (,0366)	-,0496 (,0717)	-,5133 (,0935)	-,5930 (,1098)	40,96	0,75

[x] See Notes on p. 61.

TABLE A.II.7

Basic Characteristics for $\ln \frac{M}{L}$ ¤

Industry	Mean and st. dev.	Growth rate	Slope-coeff. from regr. on lnN: No dummies	E-dummies	E and T dummies	Analysis of Variance: F_C	F_T
Total Mining and Manufacturing	2,720 (1,233)	,0403 (,0053)	-,0564 (,0111)	-,0692 (,0187)	-,1328 (,0182)	85,92	77,14
Mining and Quar.	1,024 (,654)	,0213 (,0165)	,0451 (,0310)	-,6409 (,1406)	-,6316 (,1440)	12,40	1,39
Food Products	3,639 (1,216)	,0293 (,0122)	-,4786 (,0219)	-,3243 (,0277)	-,3879 (,0265)	182,51	15,39
Textiles	2,417 (1,007)	,0265 (,0171)	,2447 (,0544)	,1072 (,0765)	,1122 (,0774)	68,56	2,98
Clothing	2,181 (1,460)	,0124 (,0230)	,5262 (,0845)	-,2768 (,1039)	-,2888 (,1060)	43,89	0,37
Wood Products	3,046 (,600)	,0169 (,0115)	-,1056 (,0265)	,0381 (,0422)	,0046 (,0422)	64,85	3,18
Pulp and Paper	3,424 (,621)	,0456 (,0078)	,0676 (,0216)	-,4325 (,0439)	-,3568 (,0373)	96,65	50,42
Printing	1,889 (,860)	,0408 (,0139)	,2833 (,0398)	-,1914 (,0832)	-,2852 (,0807)	42,11	6,73
Basic Chemicals	2,940 (,921)	,0808 (,0137)	,0826 (,0248)	-,0529 (,0607)	-,0880 (,0529)	44,52	25,44
Mineral Products	1,980 (1,231)	,0414 (,0264)	,1681 (,0592)	-,3618 (,0933)	-,3638 (,0909)	81,49	3,22
Basic Steel	3,153 (1,001)	,0624 (,0197)	,2359 (,0593)	,3069 (,0727)	,0356 (,0666)	121,79	18,39
Metal Products	2,486 (,993)	,0267 (,0165)	-,0211 (,0437)	,2034 (,0850)	,0958 (,0885)	39,81	2,96
Non-El. Mach.	2,324 (,866)	,0520 (,0182)	,3539 (,0445)	,1491 (,0945)	,0956 (,0928)	34,53	5,36
El. Machinery	2,592 (,770)	,0628 (,0167)	,0921 (,0446)	,3811 (,0848)	,0666 (,0859)	47,80	11,03
Transport Equipm.	1,983 (1,202)	,0507 (,0166)	,1873 (,0292)	,2944 (,0784)	,1832 (,0793)	32,25	5,37
Misc, Products	2,864 (,687)	,0646 (,0240)	-,3030 (,0394)	-,1322 (,0751)	-,4630 (,0610)	48,30	5,95

¤ See Notes on p. 61.

TABLE A.II.8

Basic Characteristics lnW *

Industry	Mean and st.dev.	Growth rate	Slope-coeff. from regr. on lnN: No dummies	E-dummies	E and T dummies	Analysis of Variance: F_G	F_T
Total Mining and Manufact.	1,984 (,300)	,0458 (,0012)	,0308 (,0026)	,0130 (,0111)	-,0544 (,0095)	9,33	362,89
Mining and Quar.	2,162 (,191)	,0454 (,0038)	,0225 (,0090)	-,0961 (,0527)	,0295 (,0338)	13,37	42,80
Food Products	1,888 (,363)	,0645 (,0033)	-,0031 (,0075)	-,0077 (,0336)	-,1134 (,0281)	5,34	78,72
Textiles	1,787 (,191)	,0344 (,0029)	,0062 (,0105)	-,0923 (,0330)	-,0958 (,0258)	10,43	42,06
Clothing	1,748 (,220)	,0308 (,0032)	,0065 (,0131)	-,1017 (,0314)	-,0774 (,0278)	6,46	22,72
Wood Products	1,949 (,188)	,0360 (,0032)	,0341 (,0083)	,1129 (,0289)	,0383 (,0223)	10,87	37,87
Pulp and Paper	2,098 (,337)	,0960 (,0029)	,0400 (,0117)	-,2751 (,0681)	-,0734 (,0410)	4,89	205,28
Printing	2,010 (,218)	-,0133 (,0035)	-,0062 (,0105)	-,0330 (,0300)	-,0118 (,0289)	17,02	8,67
Basic Chemicals	2,041 (,345)	,0705 (,0045)	,0357 (,0093)	,0328 (,0419)	,0070 (,0319)	7,18	56,00
Mineral Products	2,067 (,207)	,0292 (,0042)	,0431 (,0098)	,1111 (,0363)	,1276 (,0306)	10,97	14,97
Basic Steel	2,214 (,214)	,0593 (,0030)	,0521 (,0127)	,1874 (,0460)	-,0679 (,0265)	9,30	106,16
Metal Products	2,027 (,191)	,0321 (,0029)	,0243 (,0083)	,0541 (,0292)	-,0747 (,0246)	10,12	33,54
Non-El. Machinery	2,058 (,163)	,0312 (,0030)	,0081 (,0091)	-,0080 (,0297)	-,0196 (,0215)	12,05	39,45
El. Machinery	2,062 (,295)	,0475 (,0059)	,0632 (,0168)	,2796 (,0543)	,0720 (,0510)	11,58	18,95
Transport Equipm.	2,008 (,224)	,0083 (,0031)	,0094 (,0056)	-,0224 (,0202)	-,0441 (,0205)	12,85	3,64
Misc. Products	1,995 (,375)	,0767 (,0114)	,0606 (,0259)	,2806 (,0688)	,0378 (,0608)	10,45	13,89

* See Notes on p. 61.

TABLE A.II.9

Basic Characteristics for $\frac{WL}{V}^{x}$

Industry	Mean and st. dev.	Growth coeff.	Slope-coeff. from regr. on lnN: No dummies	E-dummies	E and T dummies	Analysis of Variance: F_C	F_T
Total Mining and Manufact.	,6016 (,3867)	,0017 (,0018)	-,0066 (,0035)	-,0230 (,0152)	-,0270 (,0154)	5,09	4,58
Mining and Quar.	,5728 (,2980)	-,0101 (,0075)	-,0064 (,0142)	,3091 (,0650)	,2936 (,0670)	11,73	1,28
Food Products	,5236 (,4466)	-,0009 (,0045)	,0336 (,0092)	-,0995 (,0392)	-,1037 (,0401)	4,86	0,38
Textiles	,5719 (,2425)	,0041 (,0041)	-,0061 (,0134)	-,3252 (,0418)	-,3148 (,0430)	4,92	1,50
Clothing	,5997 (,2017)	,0047 (,0032)	-,0204 (,0120)	,1013 (,0251)	,1097 (,0254)	8,75	1,15
Wood Products	,7532 (,8751)	,0281 (,0168)	-,1033 (,0390)	-,3750 (,1586)	-,4400 (,1610)	2,27	1,52
Pulp and Paper	,5577 (,2287)	,0117 (,0029)	,0415 (,0079)	,1130 (,0346)	,1387 (,0341)	10,23	10,04
Printing	,7041 (,2203)	,0001 (,0036)	-,0435 (,0105)	,1217 (,0352)	,1187 (,0358)	8,86	1,50
Basic Chemicals	,5011 (,4044)	,0017 (,0061)	,0200 (,0109)	-,0324 (,0429)	-,0307 (,0433)	7,86	0,85
Mineral Products	,5539 (,2247)	,0033 (,0048)	-,0737 (,0101)	,1347 (,0290)	,1417 (,0291)	20,45	0,99
Basic Steel	,5082 (,2206)	-,0005 (,0044)	-,0419 (,0132)	,0889 (,0423)	,1020 (,0467)	5,88	1,65
Metal Products	,6283 (,4293)	-,0028 (,0072)	,0371 (,0188)	,0504 (,0760)	,0572 (,0802)	2,88	0,78
Non-El. Machinery	,6385 (,2109)	,0011 (,0045)	-,0486 (,0115)	-,0499 (,0335)	-,0548 (,0347)	10,24	0,83
El. Machinery	,6468 (,3396)	,0006 (,0075)	-,0140 (,0198)	,1034 (,0654)	,1164 (,0730)	7,39	1,05
Transport Equipm.	,7728 (,3518)	-,0111 (,0049)	,0518 (,0086	,0147 (,0419)	,0378 (,0429)	3,79	2,21
Misc. Products	,6127 (,3816)	-,0138 (,0137)	,0354 (,0267)	-,0461 (,0817)	-,0070 (,0979)	2,87	0,38

x See Notes on p. 61.

TABLE A.II.10

Basic Characteristics for $\frac{M'}{Y'}$ [x]

Industry	Mean and st. dev.	Growth-coeff.	Slope-coef. from regr. on lnN: No dummies	E-dummies	E and T dummies	Analysis of Variance: F_C	F_T
Total Mining and Manufact.	,5197 (,2120)	-,0039 (,0009)	-,0111 (,0019)	-,0085 (,0037)	-,0031 (,0037)	59,08	17,99
Mining and Quar.	,1611 (,0995)	-,0064 (,0025)	,0025 (,0047)	,0042 (,0225)	-,0171 (,0223)	12,84	3,23
Food Products	,6780 (,2141)	-,0057 (,0022)	-,0682 (,0041)	-,0357 (,0074)	-,0265 (,0074)	78,71	9,32
Textiles	,4998 (,1456)	-,0104 (,0024)	,0286 (,0079)	-,0372 (,0187)	-,0248 (,0183)	19,76	7,60
Clothing	,4962 (,1753)	-,0090 (,0027)	,0603 (,0101)	-,0121 (,0144)	-,0169 (,0140)	34,63	7,02
Wood Products	,6293 (,1362)	-,0051 (,0026)	-,0341 (,0059)	-,0327 (,0148)	-,0254 (,0149)	21,82	2,92
Pulp and Paper	,6585 (,1338)	,0003 (,0017)	,0317 (,0046)	,0046 (,0108)	-,0022 (,0110)	55,30	3,74
Printing	,3586 (,1545)	-,0028 (,0025)	,0392 (,0073)	,0020 (,0149)	,0079 (,0150)	40,30	2,64
Basic Chemicals	,4977 (,1920)	-,0009 (,0029)	,0257 (,0051)	-,0284 (,0104)	-,0310 (,0105)	52,34	0,84
Mineral Products	,3402 (,1725	-,0038 (,0037)	-,0049 (,0084)	-,0363 (,0177)	-,0342 (,0177)	40,68	1,22
Basic Steel	,5448 (,1984)	,0004 (,0040)	,0254 (,0119)	,0371 (,0178)	,0383 (,0191)	56,11	1,35
Metal Products	,4686 (,1555)	-,0083 (,0026)	-,0112 (,0068)	,0141 (,0175)	,0473 (,0177)	20,44	5,62
Non-El. Machinery	,4395 (,1568)	-,0019 (,0033)	,0485 (,0084)	-,0037 (,0215)	-,0023 (,0223)	16,63	0,73
El. Machinery	,4997 (,1423)	,0015 (,0032)	,0067 (,0083)	,0649 (,0183)	,0684 (,0205)	25,47	0,70
Transport Equipm.	,4138 (,1740)	-,0003 (,0024)	,0161 (,0043)	,0482 (,0141)	,0516 (,0146)	17,04	0,27
Misc. Products	,5172 (,1214)	-,0071 (,0043)	-,0452 (,0075)	-,0653 (,0189)	-,0655 (,0225)	11,13	0,83

[x] See Notes on the next page

NOTES TO TABLES A.II.4-10.

a) The growth rates are determined as the OLS- estimate on b from the relation:

$$X_{it} = a + bt + u_{it} \quad (X = \ln L,\ \ln \frac{V}{L},\ \ln \frac{K}{L},\ \ln \frac{M}{L},\ \ln W,\ \frac{WL}{V},\ \frac{M'}{Y'})$$

b) The slope-coefficients from regressions on lnN are determined as the estimates on c_1, c_2 and c_3 from the relations:

$$X_{it} = a + c_1 \ln N_{it}$$

$$X_{it} = a_i + c_2 \ln N_{it} \qquad (X = \ln L,\ \ln \frac{V}{L},\ \ln \frac{K}{L},\ \ln \frac{M}{L},\ \ln W,\ \frac{WL}{V},\ \frac{M'}{Y'})$$

$$X_{it} = a_i + b_t + c_3 \ln N_{it}$$

where a_i are establishment-specific coefficients and b_t year-specific coefficients taken care of by socalled E-dummies and T-dummies respectively.

c) F_C and F_T are defined in Appendix II.9

industries the growth rate of average productivity of labour is quite probably underrated. This is at least evident for Printing where the growth rate is significantly negative. [3)]

There is a substantial difference between industries concerning the variation of the average productivity of labour with "size". Imposing the same intercept for all units for all years in the regression on ln N, we get a significantly positive slope-coefficient for six industries and a significantly negative one for three. And allowing the intercept to vary across units, or both across units and over time we get widely different results. Generally the estimate on the slope-coefficient becomes lower. In the latter case there are now one significantly positive slope-coefficient only (for Textiles) and eight significantly negative ones. This finding tastes strongly of errors in variables. But it is probably not due to "errors of reporting" rather it is an effect of transitory variation in labour input. N instead of L was used as the "size" variable just to avoid distorted slope-coefficients due to errors of measurement in labour input. It comes out, however, that these two variables do not yield very different results when used as measures of size. Having eliminated the systematic variation of both average productivity of labour and number of employees both across establishments and over time, the slope-coefficient is dominated by the negative correlation between the non-systematic components of - lnL and lnN.

A related arguement seems to be valid for the other ratio variables also, where lnL enters.

d. *The Capital Labour Ratio*

There are significant differences in the capital-labour ratio across establishments for all industries, judged by the F_C-statistics. And the F_T-statistics tell us that except from four industries, Printing, Basic Chemicals, Transport Equipment and Misc. Products there are also significant differences in this variable over time. These four do also rank lowest with respect to growth rate over time. The latter industry is the only one with a negative growth rate. [4)] Among the remaining eleven industries all except one have a

3)Cf. Appendix II.2 and Appendix II.5.

4)But this industry has a substantial growth in both factors. From Table II.4 we know that the growth in labour input is 6,2%. And thus the growth rate of capital input is 5,4%. Both growth rates, particularly the one for labour is substantially above the average for Total Mining and Manufacturing.

significantly positive growth-rate. The more heavy industries like Mining and Quarrying, Pulp and Paper and Non-El. Machinery are those with the fastest growth in the capital - labour ratio.

There are eight industries with a significantly positive slope-coefficient in the regression of $\ln \frac{K}{L}$ on the size-variable lnN, when imposing the same intercept for all units for all years, while there are three industries with a significantly negative one. But when allowing the intercept to vary between units, or both between units and over years, the slope-coefficient shows an even sharper drop than for $\ln \frac{V}{L}$. Both when E-dummies and when E and T-dummies are introduced, the slope-coefficient is significantly negative for all industries. [5)]

e. The Materials - Labour Ratio

The systematic variation of the materials-labour ratio is somewhat "more significant" than for the capital-labour ratio both across establishments and over time. All F_C- statistics are above the corresponding upper 5% fractile and the same is true for all but one for the F_T - statistics. The exception is Mining and Quarrying, which is also the one among our industries with the lower value of the F_C - statistics. [6)]

The trend of the materials-labour ratio is positive for all industries, and it is significantly positive for nine. Concerning the variation with size there are substantial differences between industries. When imposing the same intercept for all units and all years ("no dummies" included) there are ten industries with a significantly positive slope-coefficient in the regression on lnN, while there are three industries with a significantly negative one. Thus there are only two industries with a slope coefficient not significantly different from zero, namely Mining and Quarrying and Metal Products. The extremes are Food Products and Clothing with coefficients of approximately minus and plus. 5 respectively.

5) Cf. the later part of Section c of this Appendix for a probable explanation of these results.

6) Strictly speaking a comparison of F-statistics over industries is not directly possible due to different degrees of freedom. But these [illegible] differences do not matter very much as concerns the fractiles of the corresponding F-distributions. Cf. Appendix II.9

The drop in the slope-coefficient when E, or E and T dummies are included is substantially less pronounced and uniform for the materials-labour ratio than for the capital-labour ratio. In fact some of the industries have a higher slope coefficient when these dummies are introduced. But when both E and T dummies are present there are only two industries with a significantly positive slope-coefficient while there are now seven industries with a significantly negative one. Thus evidently the correlation between the "transitory" components of lnL and lnN seems to play an important role for these results too. [7)]

f. The "Real" Wage Rate

Not unexpectedly the main dimension of the variation of the real wage rate is over years. The F_T - values are quite high for most industries. But evidently, as the wage rate as defined by us is deflated with the price-index for value-added, the growth of this "real" wage rate is under-rated for some industries in the same way as the growth in value added (or average value added productivity of labour) is under-rated. [8)] This seems to be more serious for Printing which has a significantly negative growth rate, and Transport Equipment which has a positive, but not significant growth rate. For the other industries the growth rate is significantly positive.

According to the results of Table A.II.8 large production units seem in general to pay a higher real wage rate than smaller ones. The coefficient of lnN when a common intercept is imposed for all units for all years is significantly positive for nine industries. It is negative, but not significant for two, namely Food Products and Printing. It is somewhat more difficult to explain for this variable than for the previous ones the general drop in the slope-coefficient when E and T dummies are introduced. The explanation may

7) Cf. the later part of Section c of this Appendix.

8) Cf. Appendix II.2 and Appendix II.5

be the way the wage-rate is defined, namely as wages paid to production workers divided by total number of hours worked by this type of employees. The denominator is clearly positively correlated with the systematic parts of N, and presumably also with the more "transitory" part of this variable. This seems to be true for some of the industries at least, as there are six industries with a significantly negative slope-coefficient of lnN when E and T - dummies are included, and only one industry with a significantly positive one, namely Mineral Products.

g. Labour's Share in Value Added

According to the Analysis of Variance results there are for all industries significant differences across establishments in labour's share in value added. But there are two industries only with significant differences over years, namely Pulp and Paper and Transport Equipments. And these two industries are also the only ones with significant growth coefficients,[9] a positive one for Pulp and Paper and a negative one for Transport Equipments.

Not surprisingly there are also some differences across industries concerning the level of labour's share. For Basic Chemicals and Basic Steel it is about .5 while for Wood Products and Transport Equipments it is about .75 and .77 respectively.

Labour's share does also show a significant variation with size for some industries. For three it has a significantly negative slope-coefficient and for five a significantly positive slope-coefficient in the regression on lnN with a common intercept for all units for all years. When intercepts are allowed to differ the results are rather puzzling with change of sign for a number of industries etc.

Generally the results suggest that our samples for the different industries are rather heterogenous; that labour's role in production may be widely different even for units belonging to the same two-digit industry-group.[10]

9) In opposition to the previous variables absolute and not relative changes are studied for the two share-variables. Therefore the term "growth rate" is avoided.

10) Looking at the composition of our industries we see that this is not very surprising. Cf. Appendix II.2.

h. Materials' Share in Gross Production

The heterogeneity of the samples is still more apparent in the results of materials' share in gross production presented in Table II.10. The F_c statistics are quite high for most industries and the across differences are significant for all industries, while there are significant differences over years for eight industries. There is a quite uniform downward trend in materials' share over time. [11)] For the five industries, Mining and Quarrying through Wood Products and for Metal Products the trend-coefficient is significantly negative. There are also substantial differences across industries in the level of materials' share. For Mining and Quarrying it is as low as .16, while at the other extreme it is about .68 for Food Products and about .66 for Pulp and Paper.

The heterogeneity of the samples is underlined by the results of the regressions on lnN. For eight industries the coefficient of lnN is significantly positive while it is significantly negative for three when a common intercept is imposed for all units for all years. But as for labour's share in value added the results turn out to be rather different when allowing the intercept to vary between units, or between units and over years.

11) As for labours share in value added we consider absolute changes in materials' share. Thus we avoid the term "growth rate".

APPENDIX 9

Analysis of Variance of Variables in Cross-Section Time-Series Data [1)]

Having a random variable y_{it}, where the subscripts represent establishment and time respectively, we may have the hypothesis that it shows systematic variation along the two dimensions in the following way:

$$(1) \quad y_{it} = \alpha+\beta_i +\gamma_t+ \varepsilon_{it} \qquad \begin{matrix}(i = 1, \ldots\ldots I)\\(t = 1, \ldots\ldots T)\end{matrix}$$

where α, β_i and γ_t are non-random magnitudes while ε_{it} is a random variable presumed to be normally distributed with zero mean and constant standard deviation σ. I is the number of establishments and T is the number of years. This model corresponds to an Analysis of Variance model with a two way classification without any interaction effect and with one unit per cell.

The total sum of squares of deviation from the mean can be decomposed in the following way:

$$(2) \quad S^2 = \sum_{i=1}^{I} \sum_{t=1}^{T} (y_{it} - \bar{\bar{y}})^2 =$$

$$\sum_{i=1}^{I} \sum_{t=1}^{T} (y_{it} - \bar{y}_{i.} - \bar{y}_{.t} + \bar{\bar{y}})^2 + T \sum_{i=1}^{I} (\bar{y}_{i.} - \bar{\bar{y}})^2 + I \sum_{t=1}^{T} (\bar{y}_{.t} - \bar{\bar{y}})^2$$

or

$$(3) \quad S^2 = S_o^2 + S_c^2 + S_T^2$$

Provided that $\sum_{i=1}^{I} \beta_i = \sum_{t=1}^{T} \gamma_t = 0$ which implies no loss of generality we have that

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1) For a detailed discussion Analysis of Variance models see: Scheffé, H.: "The Analysis of Variance", J. Wiley & Sons, Inc., New York 1959.

$$\bar{y}_{i.} = \frac{1}{T}\sum_{t=1}^{T} y_{it} = \alpha + \beta_i + \bar{\varepsilon}_{i.}$$

(4) $$\bar{y}_{.t} = \frac{1}{I}\sum_{i=1}^{I} y_{it} = \alpha + \gamma_t + \bar{\varepsilon}_{.t}$$

$$\bar{y} = \frac{1}{I \cdot T}\sum_{i=1}^{I}\sum_{t=1}^{T} y_{it} = \alpha + \bar{\varepsilon}$$

where

$\bar{\varepsilon}_{i.}$ is normally distributed $(0, \frac{\sigma}{\sqrt{T}})$

(5) $\bar{\varepsilon}_{.t}$ " " " $(0, \frac{\sigma}{\sqrt{I}})$

$\bar{\varepsilon}$ " " " $(0, \frac{\sigma}{\sqrt{IT}})$

It can now be shown that under our assumption we have:

$$E(S_o^2) = (I - 1)\ (T - 1)\sigma^2$$

(6) $$E(S_c^2) = T \sum_{i=1}^{I} \beta_i^2 + (I - 1)\sigma^2$$

$$E(S_T^2) = I \sum_{t=1}^{T} \gamma_t^2 + (T - 1)\sigma^2$$

And due to our assumptions $\frac{S_o^2}{\sigma^2}$ is χ^2-distributed with (I - 1) (T - 1) degrees of freedom, and S_o, S_c and S_T are distributed independently. And therefore, provided that $\sum_{i=1}^{I} \beta_i^2 = 0$ which implies that all across effects are zero we have that

(7) $$F_c = \frac{S_c^2}{S_o^2}(T - 1)$$

is F-distributed with (I-1) and (T-1) (I-1) degrees of freedom.

And provided that $\sum_{t=1}^{T} \gamma_t^2 = 0$, which implies that all year-effects are zero we have that:

(8) $$F_T = \frac{S_T^2}{S_o^2}(I-1)$$

is F-distributed with (T-1) and (I-1) (T-1) degrees of freedom.

Therefore, by means of (6) we can test the hypothesis:

$$H_{Co}: \sum_{i=1}^{I} \beta_i^2 = 0$$

against

$$H_{C1}: \sum_{i=1}^{I} \beta_i^2 > 0$$

And by means of (7) we correspondingly can test the hypothesis:

$$H_{To}: \sum_{t=1}^{T} \gamma_t^2 = 0$$

against

$$H_{T1}: \sum_{t=1}^{T} \gamma_t^2 > 0$$

We get tests with level ε if we reject H_{Co} when we observe

$F_C > F_{1-\varepsilon,(I-1,\ (I-1)\ (T-1))}$ and reject H_{To} when we observe

$F_T > F_{1-\varepsilon,(T-1,\ (I-1)\ (T-1))}$. For our data with T = 9 and I = 13 at least and I = 164 at most (except for Tot. Min. and Man. where I = 907) we have $F_{0.95,\ (I-1,\ 8(I-1))}$ approximately between 1.90 and 1.25 and

$F_{0.95,\ (8,8(I-1))}$ approximately between 2.05 and 1.95

CHAPTER III

ESTIMATION OF PRODUCTION FUNCTION PARAMETERS IN CASE OF SIMULTANEOUS EQUATIONS AND ERRORS OF MEASUREMENT.

1. *Introduction*

In applied econometric analyses there are generally two main problems. First, what is the proper specification of the model, particularly which variables are endogenous and which ones are exogenous. Second, what is the importance and nature of any differences between the variable-measures and their theoretical counterparts, or the problem of measurement errors in wide sense. It is fair to say that the first one has caught much more attention than the second, which is also usually assumed away when the former is discussed. On the other hand any simultaneity problems are usually ignored when errors of measurement problems are handled.

In this chapter we will try to treat these problems in a more simultaneous way. The theoretical frame is the following model. [1)]

$$(1) \qquad \begin{aligned} y &= \alpha x + \beta z + u \\ y - x &= bw + v \end{aligned}$$

where $y = \ln V$, $x = \ln L$, $Z = \ln K$, $w = \ln W$. u and v are error terms, with zero means and constant variances, and they show no serial correlation.[2)] In this model y and x are endogenous variables, while z and w are assumed to be exogenous. It is not easy to evaluate the validity of the latter assumption. Clearly, both z and w may be considered as endogenous by the establishments, subjects of their economic behaviour. We will argue, however, that other problems than possible endogeneity are the main ones concerning these variables, namely errors of measurement. Capital input contains a large, but presumably random error component, while w and also x are more systematicly wrong as they both refer to the quantity component of labour input ignoring the quality component.

1) Cf. Chapter I

2) All variables are computed from their means.

If the variables of (1) were correctly measured both relations are exactly identified and thus a number of text-book methods yield consistent estimates on the parameters. In the next section we show, however, that the presence of the two kinds of measurement errors mentioned makes such methods generally worthless. Evidently they are very little roboust towards such errors. On the other hand it is shown that ordinary least squares on the production function, if yielding estimates on the factorelasticities that are subject to both simultaneous equations bias and errors of measurement biases is generally preferable, just because of its roboustness in the present context. Thus a main conclusion of the next section is that in case of errors of measurement we may pay a quite unreasonable price for the elimination of biases due to simultaneity.

Having two or more cross-sections for the same units it has been argued that it is possible to reduce or eliminate the effects of simultaneity by means of covariance analysis. The argument runs as follows; In cross section data the error term of a production function like (1) has to catch differences in management and "environments" between units. And as more well-managed units with favourable environments tend to use more of the inputs than poorly managed ones with less favourable environments there is a positive correlation between the error term and the inputs and thus the OLS estimates are subject to sort of simultaneous equations bias. But in case we have more than one observation per unit we can eliminate the across-specific component of the error term by means of covariance analysis. If the time series of cross-sections is not too long the differences in management and environments across units are presumably fairly stable, and having eliminated them from the error term we have also presumably eliminated the main source of simultaneity bias of the OLS-estimates.

But we show in the third section of this chapter that this method of eliminating simultaneous equations bias is neither very roboust towards measurement errors. Thus also in this context we may pay an unreasonable price for obtaining estimates "free" of simultaneous equations bias.

We have therefore to look for other methods of estimation. If we accept the assumptions on which (1) is based and thus constrain the elasticity of substitution to unity, we may estimate the elasticity of labour as that factor's share in output. This is probably the best estimate on that parameter obtainable in the present context as none of the two kinds of errors affect it.

This is subject of discussion in Section 4 of this chapter. And we also discuss the various possibilities to estimate the elasticity of capital. By constraining the elasticity of scale to unity one has an estimate with the same properties as the one on the elasticity of labour. But as we would like the scale-elasticity to be a free parameter a few other methods are tried. And we are finally converging towards a method of estimation that seems to be the best one given the kind of data we are working on.

In this section we also report on additional attempts made to estimate the elasticity of substitution. And in a concluding section we present a short summary of the findings of the present chapter.

2. *THE PROPERTIES OF THE ILS AND OLS METHODS OF ESTIMATION IN THE PRESENT CONTEXT.*

a. The ILS-method.

In this section we will try to figure out if, or to what extent the two main errors under consideration, making some simplifying assumptions about their nature, affect the results of two well known methods of estimation, namely indirect least squares (ILS) and ordinary least squares (OLS). First we consider the ILS-method.

The reduced form of (1) consists of the second relation of that model together with: [3)]

(2) $$x = \pi_1 w + \pi_2 z + r$$

where $\pi_1 = -\dfrac{b}{1-\alpha}$, $\pi_2 = \dfrac{\beta}{1-\alpha}$ and $r = \dfrac{u-v}{1-\alpha}$

Provided that there are no errors of measurement we can that obtain consistent estimates on b from the behaviour relation in (1) as well as for the parameters of (2), and thus we get consistent estimates of the factor elasticities as : [4)]

3) We could solve the system with respect to y instead of x, but this does not make any difference.

4) Other methods like the two-stage least square methods and the instrumental variable method do not yield exactly the same estimators for the factor-elasticities as the ILS-method. But asympthoticly they yield the same results both when the error-terms are assumed to be uncorrelated with the exogenous variables and in the errors in variables-cases discussed later.

$$(3) \quad \hat{\alpha} = \frac{\hat{\pi}_1 + \hat{b}}{\hat{\pi}_1} \qquad \hat{\beta} = -\frac{\hat{b}\hat{\pi}_2}{\hat{\pi}_1}$$

And we get the corresponding estimate on the scale-elasticity as:

$$(4) \quad \hat{\varepsilon} = \frac{\hat{\pi}_1 + (1 - \hat{\pi}_2)\hat{b}}{\hat{\pi}_1}$$

We know, however, that there are particular types of errors of measurement present and we would like to know how they may affect the estimates deducted above.

Concerning labour input our measure refers to the quantity of that factor. Now, there are obviously some variations in the quality of labour both across establishments and over time. As the relevant measure of the productive performance of labour is " quantity times quality", variations in the quality-component in our sample makes a potential cause of inconsistent estimates when labour input is measured by the quantity component only

Quite probably the quality component of "total" labour input does not show a quite random variation, as the observed wage-rate,that also refers to the quantity-component of labour, may be positively correlated with it. To say something more about what can happen to our ILS-estimates when there are such variations in labour-quality we adopt the rather extreme assumption that all observed differences in the wage-rate are due to these. [5)]

5) This assumption conforms to the one made about quality-differences between production and non-production workers when constructing the labour-input measure applied. Cf. Section II.3.c. But the rather approximate nature of the assumption of perfect correlation between the wage-rate and quality of labour should be evident. For instance as we apply a "real" wage rate, i.e. the ratio between the current wage-rate and the "price"-index of value-added, our wage-rate is clearly affected by the prices obtained on output (and also the prices of materials).

As is shown in Section a of Appendix III.1 we get under these assumptions that: [6)]

$$\text{plim } \hat{b} = 1$$

$$\text{(5)} \quad \text{plim } \hat{\alpha} = 0$$

$$\text{plim } \hat{\beta} = \frac{\beta}{1-\alpha}$$

This is hardly no surprising result as there is by assumption no "real" variation in one of our identifying variables, namely w. Thus neither of the relations of our model is identifyable. What we manage to estimate is a ratio that should be approximately equal to one provided that there is constant returns to scale.

Now, as pointed out the assumptions underlying the deductions in (5) are rather extreme. But, matters need not be that bad to make indirect least squares worthless: There are quality differences in labour input across establishments in our sample and they are to some extent at least, correlated with the wage rate. Therefore indirect least squares must give very poor results. Another reason why the ILS method does not work is the substantial errors of measurement in the capital data. [7)] Assuming that these errors are completely random we show in Section b of Appendix III.1 that the

6) "plim" denotes as usual the probability limit, i.e. the limit-value of the estimates when number of observations approaches infinity. Basicly it should not matter in our case whether this concerns number of units (I) or number of years (T). But in some cases there may be arguments for the following kind of probability limit $I \to \infty$ and $T \to \infty$ but $\frac{I}{T}$ is constant equal to the value of this ratio in the sample.

7) Cf. Appendix II.6 and Appendix II.7.

asympthotic biases of the ILS-estimates on the factor-elasticities are;

$$(6)\quad \begin{aligned} \text{bias } \hat{\alpha} &= \frac{(1-\alpha)\, b_{zw} B}{b_{zw} B - b} \\ \text{bias } \hat{\beta} &= \frac{(b - \beta\, b_{zw}) B}{b_{zw} B - b} \end{aligned}$$

and the bias in the implied estimate on the elasticity of scale is:

$$(7)\quad \text{bias } \hat{\varepsilon} = \frac{((1-\varepsilon) b_{zw} + b) B}{b_{zw} B - b}$$

or in case of constant returns to scale:

$$(8)\quad \text{bias } \hat{\varepsilon} = \frac{bB}{b_{zw} B - b}$$

where $B = \beta k^2/(1 - r^2_{zw})$, $k^2 = (\frac{\sigma_e}{\sigma_z})^2$ is the error to total variance ratio of capital input, b_{zw} is the regression coefficient of w from the auxiliary regression of z on w and r_{zw} is the simple correlation coefficient between z and w.

Provided the elasticity of substitution is not too low both biases in (6) are presumably negative as it is reasonable to assume that $b_{zw}>0$. But we see that the denominator may be positive implying a positive bias for $\hat{\alpha}$, but also in this case it is reasonable to believe that the bias for $\hat{\beta}$ is negative.

Anyhow the nature of the denominator is such that even for moderate error-variance ratios the biases may be quite serious. And in our case we must expect the present method of estimation to give generally poor results, also because of errors of measurement in capital.

As the two kinds of errors of measurement discussed are largely independent we could easily have analysed them simultaneously. But this does not add anything new to the findings so far. We have above enough evidence to conclude that the present method of taking the simultaneity of the model into account is presumably quite worthless due to errors of measurement.

b. The OLS Method.

Using the OLS method instead of the ILS method on the production function there is an additional source of bias, namely simultaneous equations, or in our case, the endogeneity of labour input.

In the Census study the biases due to simultaneous equations, errors of measurement in labour input and errors of measurement in capital when using the OLS method are deducted and analysed.

Under fairly general assumptions we can show that the biases due to simultaneous equations are:

$$\text{bias } \hat{\alpha} = \frac{(1-\alpha)\,\sigma_u^2}{D_1}$$

(9)

$$\text{bias } \hat{\beta} = \frac{-(\beta - b\, b_{wz})\,\sigma_u^2}{D_1}$$

And therefore the bias in the estimate on the elasticity of scale is:

$$\text{(10) bias } \hat{\varepsilon} = \frac{((1-\varepsilon) + b\, b_{wz})\,\sigma_u^2}{D_1}$$

where b_{wz} is the coefficient of z in the auxiliary regression of w on z and:

$$\text{(11)} \quad D_1 = b^2 \sigma_w^2 (1-r_{zw}^2) + \sigma_u^2 + \sigma_v^2$$

We note that the denominator will always be positive, and provided that $\alpha < 1$ the bias in the estimate on α due to simultaneous equations will always be positive too. b_{wz} is presumably also positive and therefore we cannot determine the sign of the bias of the estimate on β. Having constant or decreasing returns to scale, the estimate on the scale-elasticity will be biased upwards. This is also true if we have slightly increasing returns to scale. We note that provided the "identifying" variables w,z and v have large variances compared to the variance of the error term, u of the production function,the simultaneous equations biases need not be too bad. We also note that a large elasticity of substitution helps us to identify the parameters, and that this is particularly the case for α.

Concerning errors of measurement in labour input it could be shown that under the same assumptions about the behaviour of these errors as we have adopted in the ILS case, we get the following biases when applying the OLS-method:

$$\text{bias } \hat{\alpha} = \frac{\alpha(b_{wx} - b_{wz}\, b_{zx})}{D_2}$$

$$\text{(12)} \quad \text{bias } \hat{\beta} = \frac{\alpha(b_{wz} - b_{wx}\, b_{xz})}{D_2}$$

$$\text{bias } \hat{\varepsilon} = \frac{\alpha((1-b_{xz})b_{wx} + (1-b_{zx})b_{wz})}{D_2}$$

where b_{wx}, b_{wz}, b_{xz} and b_{zx} are the slope-coefficients of the simple "auxiliary" regressions of w on x, w on z, x on z and z on x respectively. And

$$\text{(13)} \quad D_2 = 1-r^2_{xz}$$

Correspondingly we get for the case when having errors of measurement in capital only:

$$\text{bias } \hat{\alpha} = \frac{\beta\, b_{zx}}{D_3} k_1^{\;2}$$

$$\text{(14)} \quad \text{bias } \hat{\beta} = \frac{-\beta}{D_3} k_1^{\;2}$$

$$\text{bias } \hat{\varepsilon} = \frac{\beta(b_{zx}-1)}{D_3} k_1^{\;2}$$

where:

$$\text{(15)} \quad D_3 = 1 - r^2_{x,z-x}$$

with $r_{x,z-x}$ as the correlation coefficient between the logs of labour and the capital labour ratio and $k_1^{\;2} = (\frac{\sigma_e}{\sigma_{z-x}})^2$ is the ratio between variance of the error of the capital measure and the variance of the observed capital labour ratio.

We note from (12) that we cannot determine the sign of the biases due to the kind of errors of measurement in labour input we are considering. This is somewhat easier for the biases due to errors of measurement in capital. We note that the bias of $\hat{\beta}$ is always negative and except of quite peculiar situations the bias of $\hat{\alpha}$ is always positive. If the coefficient b_{zx} is near one, we also note that the bias of $\hat{\varepsilon}$ is ignorable.

These bias-computations are, however, partial and this is not quite satisfactory as what we would like to know is the simultaneous effect of them. But such a simultaneous analysis is very complicated in the present context and so partial analyses are carried out as they give a suggestion at least, of the importance of these errors. Later we try to "estimate" these biases to get an idea of their magnitudes.

But even if OLS in opposition to ILS do not seem to give completely "wild" estimates, the biases of the former are presumably of a magnitude that makes it rather poor in any case and we should look for something better. There is, however, one possibility to "save" this method that should be investigated. That is to combine it with analysis of covariance.

3. ANALYSIS OF COVARIANCE.

a. Causes of Differences in Results Obtained from Different Kinds of Data.

In applied econometrics one has two main data-dimensions; cross section and time series. The first refers to different units of one kind or another at one point of time or for one period, while the later refers to one unit at different points of time or for different periods. If one has at least two cross-sections of the same units (and then necessarily at different points of time or for different periods) or time-series of at least two units one has so called combined cross-section of time-series data. [8)]

Even if the later kind of data has been applied to some extent an overwhelming fraction of econometric research is based on either pure cross-section or pure time-series data. A problem in this context that hardly has caught proper attention is the often rather puzzling differences

8) Clearly, by aggregating such data we can get one cross-section or one time-series.

in the results when applying the same model on the two kinds of data. [9]

The reasons for this are numerous, but presumably the following are the more important. First, one can say a priori that one kind of data is better suited for the investigation of a particular problem than the other. [10] Second, the two kinds of data usually refer to essentially different kinds of unit: The cross-section data are often based on more disaggregated units than time-series data. Thus differences in the results may simply be the result of aggregation, and apparently different results may suggest that there are aggregation errors in the time-series results. Third, results obtained by means of the two kinds of data may be quite different, but in spite of this both tell the "truth". In this category comes the argument that, at least for some kinds of relations, such as production functions, results from cross-section data tell us about the long run effect of changes in the explanatory variables while results from time-series tell us about the short run effect

9) Some aspects of these problems are discussed in for instance:
Kuh. E. and Meyer, J.R.: "How Extraneous are Extraneous Estimates?", The Review of Economics and Statistics Nov. 1957.
Kuh, E.: "Capital Stock Growth: A Micro-Econometric Approach." North Holland Publ. Co., Amsterdam 1963.
Nerlove, M.: "Recent Empirical Studies of the CES and Related Production Functions," in Brown, M. (ed.) The Theory and Empirical Analysis of Production, Studies in Income and Wealth vol. 31, NBER, New York 1967.

10) A well-known subject of econometric analyses could serve as an example in this context, namely the one of estimating damand functions for consumption goods: It is difficult to get reliable estimates on price-elasticities by means of cross-section data even if there is some variation in prices across units. Instead time-series data are used for this purpose. On the other hand, it is asserted that it is easier to get reliable estimates on the income-elasticities from cross-section data than from time-series data. This has clearly to do with the "behaviour" of the variables that are necessary for the identification of particular properties of the relation(s) under consideration.

from such changes.[11)]

Finally there is, in this context an argument of underlining quite strongly differences in errors of specification and measurement for the two kinds of data. We may, for instance manage to deflate the output and wage-rate variables applied in a model like (1) using pure time-series data, while it is virtually impossible in cross-section data, at least of the present kind. Even if the market price is approximately the same for all establishments the net-price that is relevant for the behaviour of these units may show at least some variation. And if this price-variation is correlated with the observed wage-rate, we may get seriously biased estimates when using cross section data. While using properly deflated time-series data may give more consistent estimates.
For time-series data we may also succeed in getting an index measuring the quality of labour input and thus be able to take care of this component of the production performance of labour input. This is much more difficult when having a cross-section of micro-units. And as is shown in the previous section ignoring the quality component of labour input may have serious effects on our estimates.[12)]

And generally the effects of non-deflated output and wage-rate variables or quality-variations in labour input may be quite different for the two kinds of data if prices or quality shows a different degree of variation across units than over time, or if they otherwise behave different in the two kinds of data.

11) This again has basicly to do with the "behaviour" of variables, as this argument refers to variables that have a wide variation in cross-section data but much smaller variation in time-series data. Having for instance establishments of widely different sizes, we can use the results obtained by means of these data to "predict" what will happen to the endogenous variable-values of a small establishment that got time enough to expand into an upper size-class. A related argument may be valid for a sample of households with widely different incomes. For time-series data the differences of the explanatory variables are rather small and we can observe what happens to the endogeneous variables from one period to the next due to changes in the exogeneous variables; how they are adjusted in the short run.

12) It could be shown that under certain assumptions the biases of the OLS-estimates on the production function parameters due to non-deflated output and wage-rate variables are the same as those due to quality-variations in labour input.

We should also mention in this context variations in the capacity utilization of the capital stock which is presumably more important in time-series than in cross-sections data. In a sense we may, however, have a related variation along the across dimension due to transitory variation in demand that is more or less establishment-specific. [13)]

Finally, errors due to variations in management and "environments" should be pointed out. As this characteristic presumably is fairly stable over time even over a nine-year period, it operates mainly across establishments in our sample. The argument why such variations may lead to poor results is as pointed out previously that units with good management and favourable "environments" tend to use more of the inputs than those with poor management and unfavourable "environments". In other words, a relevant variable is left out of the production relation and thus enters the error term of that relation. And as it is correlated with the inputs it makes the OLS estimates of the production function parameters inconsistent. But clearly this is an aspect of the simultaneous equations problem and provided that problem is properly solved the problem of variations in management and environments is too.

b. *Analysis of Covariance of the Relations of the Main Model.*

Some econometricians working on combined cross-section time-series data for production units have tried to reduce or eliminate the effects of variations in management and environments by means of covariance analysis. [14)]

13) We may also have transitory variation in supply of materials implying variations in the capacity utilization along the across dimension. For instance this seems to be a serious problem in the Census-study for the identification of the production function parameters for the industry: Fish and Herring Oil and Meal Factories. As we use value added as the output measure transitory variations in the supply of materials and in the demand for the final products have generally the same effect on the results.

14) See for instance Mundlack, J. 1961: "Empirical Production Function Free of Management Bias". Journal of Farm Economics, February 1961. Reprinted in Zellner, A. (ed.): Economic Statistics and Econometrics. Litte, Brown and Co. Hoch, I.1955: "Estimation of Production Function Parameters and Testing for Efficiency". Econometrica 1955 and Hoch, I 1957 "Estimation of Agricultural Resource Productivities Combining Time Series and Cross Section Data." Unpubl. dissertation, Chicago March 1957.

They have obtained some puzzling results, particularly that the estimate on the elasticity of scale is substantially lower when having "eliminated the management bias". But this can hardly be the only reason; there are generally other errors in operation than just variation in management. To be more specific about this let us consider what happens when applying covariance analysis on our time-series cross-section data. [15)]

We assume that the error term of the production function can be partitioned in the following way:[16)]

16) $U_{it} = a_i + b_t + u_{it}$ $\qquad i = 1 \ldots\ldots I$, $t = 1 \ldots\ldots T$

where a_i and b_t are assumed to be non-random across and time "effects" respectively.[17)] And u_{it} is a random term which with no loss of generality can be assumed to have zero mean so that $EU_{it} = a_i + b_t$. In addition we assume that the variance-covariance matrix of U and thus also of u is equal to $\sigma^2 I$ where I is a (I xT) x (IxT) identity matrix and σ^2 is the common variance of the individual residuals.

When ignoring one or both of the effects, what we consider to be the variance covariance matrix is a "mongrel" matrix consisting of moments of a_i or b_t (or both) in some of the off-diagonal elements and a mix of moments of these "effects"and the variance of the residual in the diagonal elements.[18)] And if, in addition the across and/or time effects are correlated with the inputs we clearly may commit serious errors by ignoring them.

15) On the application of covariance analysis on such kind of data see: Mundlack, Y. 1963: "Estimation of Production and Behaviour Functions from a Combination of Cross-Section and Time-Series Data" in Christ, C.F. (ed.) Measurement in Economics, Stanford University Press 1963.

16) Cf. Scheffé, H. "The Analysis of Variance", John Wiley & Sons, Inc. N.Y. 1959.

17) In the next chapter we carry out some experiments by means of multiple tests to find out something about the importance and nature of such effects. This is done for the behaviour relation of (1) also.

18) Cf. Balestra, P. and Nerlove. M:"Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas". Econometrica No. 3, 1966.

a_i and b_t can also be considered as "shifts" in the production function across establishments and over time. And if we manage to estimate them we have also eliminated from the "gross" residual U_{it} its systematic variation along the two dimensions, so that the presumed wellbehaved "net" residual u_{it} is left only.

And this is just what we can do by means of covariance analysis. But as will be evident later the costs of this procedure may be too high as we then in fact must use all the systematic variation along both dimensions of the variables entering the production function. Thus we will stay more flexible and consider the outcome of four situations.

A) When no effects are eliminated.

B) When time-effects are eliminated.

C) When across-effects are eliminated.

D) When both effects are eliminated.

Case A) implies that we use the gross variation of the variables to estimate the slope-coefficients of the production function, while B) implies that the systematic variation over time of the variables is eliminated before the slope-coefficients are estimated. Correspondingly case C) implies that the systematic variation across establishments is eliminated and as pointed out D) implies that the systematic variation along both dimensions is eliminated. [20] The results for the capital-and scale-elasticities for the four cases are presented in Table III.1. [21]

20) Case B) is the one which in principle yield estimates on the slope-coefficients of a more long-run nature, but as the across-dimension is the dominating one in our samples this is also to some extent true for case A). As the systematic across-variation is eliminated in cases C) and D) the estimates obtained by means of these are of a more short run nature. But as we shall show the differences between the results obtained for the various "cases" are mainly due to other properties than long run/short run-ness of the estimates.

21) Multiple correlation coefficients are not presented as those computed are not comparable across "cases". Neither are intercepts presented as they have little interest in this context.

We carry out the same procedure for the behaviour relation of (1) and the results of the estimates on the elasticity of substitution are presented in Table III.2. [22)]

The results obtained for Total Mining and Manufacturing when no effects are eliminated suggest that on the average for the industries concerned there are constant returns to scale as well as an elasticity of substitution of unity. The results are basicly the same when eliminating the time effects. But when eliminating the across effects the results suggest that there are decreasing returns to scale as well as an elasticity of substitution below unity. These are also the main findings for most of the individual industries, even if there are some striking differences in the results of some of them.

We could argue that this general pattern of the results is due to the "fact" that short run elasticities are smaller than long run ones. [23)] We could also argue along the same lines as I. Hoch and Y. Mundlack [24)] that when having eliminated the across-effects we have also eliminated the main cause of simultaneous equations bias. This is not unreasonable for the scale-elasticity as this kind of bias is presumably positive in our case. [25)] It is more difficult to accept the general drop in the estimate of the capital-elasticity when eliminating across and time-effects. It is hard to believe that the short run elasticity of capital is close to zero for most of the industries. The <u>differences</u> between the industries concerning the

22) About arguements for using covariance analysis on a behaviour relation like the present one, cf. Mundlack, Y. (1963) op. cit.

23) Cf. footnote 20) above.

24) Cf. Hoch. I. (1955) <u>op. cit.</u> and Mundlack, Y.(1961) <u>op. cit.</u>

25) Cf. (10) above.

Table III.1.

Analysis of Covariance of the Cobb-Douglas Production Function.

Industry	No effects eliminated			Time-effects eliminated			Across-effects eliminated			Both effects eliminated		
	El. of scale	El. of capital	MSE	El. of scale	El of capital	MSE	El. of scale	El. of capital	MSE	El. of scale	El. of capital	MSE
Tot. Min and Man.	0.994	0.272	0.281	0.993	0.263	0.273	0.895	0.178	0.113	0.799	0.076	0.102
	(0.005)	(0.007)		(0.005)	(0.007)		(0.018)	(0.013)		(0.018)	(0.013)	
Mining and Quar.	0.988	0.281	0.187	0.997	0.247	0.176	0.739	0.211	0.072	0.773	0.079	0.056
	(0.022)	(0.040)		(0.040)	(0.040)		(0.110)	(0.056)		(0.099)	(0.054)	
Food Products	0.888	0.372	0.445	0.883	0.353	0.424	0.758	0.155	0.189	0.557	-0.020	0.159
	(0.014)	(0.025)		(0.014)	(0.025)		(0.055)	(0.041)		(0.053)	(0.039)	
Textiles	0.946	0.293	0.155	0.946	0.282	0.154	1.052	0.295	0.082	1.000	0.211	0.080
	(0.022)	(0.034)		(0.022)	(0.035)		(0.071)	(0.060)		(0.074)	(0.065)	
Clothing	0.993	0.080	0.137	0.995	0.070	0.134	0.663	0.060	0.058	0.648	0.017	0.056
	(0.021)	(0.024)		(0.021)	(0.024)		(0.051)	(0.039)		(0.051)	(0.039	
Wood Products	1.092	0.188	0.285	1.089	0.179	0.281	1.194	0.192	0.117	1.102	0.123	0.109
	(0.024)	(0.038)		(0.024)	(0.038)		(0.076)	(0.053)		(0.076)	(0.054)	
Pulp and Paper	0.896	0.300	0.170	0.923	0.237	0.147	0.315	0.254	0.082	0.415	0.079	0.061
	(0.015)	(0.023)		(0.014)	(0.023)		(0.066)	(0.032)		(0.058)	(0.029)	
Printing	1.041	0.146	0.121	1.04	0.147	0.119	0.706	0.103	0.053	0.718	0.113	0.051
	(0.017)	(0.023)		(0.017)	(0.023)		(0.057)	(0.032)		(0.056)	(0.032	
Basic Chemicals	0.893	0.195	0.503	0.982	0.182	0.481	1.020	0.077	0.174	0.999	0.001	0.145
	(0.019)	(0.037)		(0.019)	(0.037)		(0.066)	(0.053)		(0.061)	(0.049)	
Mineral Products	1.110	0.322	0.205	1.112	0.313	0.208	0.938	0.394	0.057	0.922	0.349	0.057
	(0.023)	(0.039)		(0.023)	(0.040)		(0.068)	(0.054)		(0.068)	(0.058)	
Basic Steel	1.074	0.200	0.201	1.069	0.173	0.185	1.041	0.109	0.097	0.609	-0.142	0.069
	(0.029)	(0.038)		(0.028)	(0.037)		(0.083)	(0.052)		(0.078)	(0.048)	
Metal Products	0.944	0.129	0.171	0.923	0.104	0.167	0.914	0.147	0.100	0.690	-0.009	0.094
	(0.018)	(0.034)		(0.018)	(0.034)		(0.071)	(0.048)		(0.077)	(0.053)	
Non-El. Machinery	1.078	0.028	0.138	1.081	-0.007	0.133	1.128	0.234	0.064	1.046	0.112	0.061
	(0.021)	(0.037)		(0.021)	(0.037)		(0.070)	(0.054)		(0.074)	(0.062	
El. Machinery	1.023	0.111	0.242	1.017	0.098	0.237	1.060	0.238	0.094	0.674	-0.013	0.082
	(0.029)	(0.043)		(0.029)	(0.042)		(0.093)	(0.072)		(0.105)	(0.077)	
Transp. Equipm.	1.070	0.091	0.149	1.069	0.089	0.148	0.944	0.107	0.091	0.899	0.089	0.089
	(0.010)	(0.021)		(0.010)	(0.021)		(0.045)	(0.033)		(0.045)	(0.033)	
Misc. Products	0.938	0.355	0.410	0.911	0.367	0.354	1.243	0.319	0.285	0.877	0.238	0.236
	(0.046)	(0.058)		(0.043)	(0.055)		(1.153)	(0.134)		(0.163)	(0.127)	

Table III.2.

Analysis of Covariance of the ACMS-Relation.

Industry	No effects eliminated		Time-effects eliminated		Across-effects eliminated		Both effects eliminated	
	El. of subst.	MSE	El. of subst.	MSE	El. of subst.	MSE	El. of subst.	MSE
Tot. Min. and Man.	0.992 (0.018)	0.243	1.006 (0.020)	0.243	0.894 (0.014)	0.079	0.885 (0.017	0.078
Mining and Quarr.	0.989 (0.150)	0.193	0.811 (0.193)	0.194	1.008 (0.107)	0.061	0.563 (0.066)	0.057
Food Products	1.114 (0.045)	0.385	1.153 (0.051)	0.386	0.858 (0.031)	0.131	0.800 (0.038)	0.130
Textiles	1.082 (0.084)	0.135	1.185 (0.097)	0.135	0.950 (0.079)	0.067	1.088 (0.104)	0.067
Clothing	0.870 (0.059)	0.102	0.895 (0.065)	0.103	0.898 (0.048)	0.042	0.953 (0.056)	0.042
Wood Products	1.433 (0.129)	0.239	1.641 (0.149)	0.234	0.974 (0.111)	0.101	1.069 (0.148)	0.098
Pulp and Paper	0.798 (0.035)	0.131	0.863 (0.053)	0.129	0.788 (0.024)	0.049	0.874 (0.040)	0.047
Printing	0.885 (0.056)	0.094	0.876 (0.060)	0.094	0.916 (0.069)	0.045	0.891 (0.073)	0.044
Basic Chemicals	1.047 (0.072)	0.396	1.098 (0.086)	0.399	0.892 (0.046)	0.111	0.868 (0.062)	0.111
Mineral Products	1.790 (0.104)	0.148	1.962 (0.111)	0.144	1.022 (0.081)	0.047	1.102 (0.097)	0.047
Basic Steel	0.906 (0.104)	0.189	0.840 (0.155)	0.190	0.910 (0.073)	0.069	0.779 (0.136)	0.067
Metal Products	0.714 (0.091)	0.162	0.658 (0.102)	0.163	1.091 (0.085)	0.079	1.178 (0.107)	0.079
Non-El. Machinery	0.853 (0.119)	0.124	0.807 (0.143)	0.126	1.119 (0.097)	0.048	1.296 (0.140)	0.048
El. Machinery	0.991 (0.078)	0.163	1.022 (0.088)	0.165	0.950 (0.064)	0.056	1.003 (0.080)	0.056
Transp. Equipm.	0.709 (0.059)	0.135	0.695 (0.059)	0.135	1.123 (0.066)	0.068	1.105 (0.068)	0.068
Misc. Products	1.334 (0.135)	0.296	1.426 (0.170	0.313	1.228 (0.127)	0.164	1.318 (0.192)	0.173

impact on the results of eliminating the various effects are also difficult to explain by short run/long run nature of elasticities estimated or by differences between industries of the importance of management and environments as a "left out variable".

Therefore we will seek other explanations of these findings, namely by the errors present in our data.

c. Bias-Computations for the Covariance Analysis-Estimates.

The analysis of covariance of the previous subsection implies that the OLS method is used for the estimation of the various elasticities. [26]
Thus the estimates obtained are subject to the three kinds of biases discussed in section 2 of this chapter. In this subsection we will try to investigate to what extent these biases may explain the differences in the results obtained from the four "cases". Thus we have to quantify the biases. We do this by using sample statistics of the various components entering these biases. This is clearly quite rough as the biases deducted are asympthotic. But such computations may yield some suggestion of the importance of the various biases. [27]

In the computation of the simultaneous equations biases we use for each of the four cases the corresponding estimates on the mean square errors from the production and behaviour relation of (1) for σ_u^2 and σ_v^2 respectively. [28]

Concerning the production function parameters entering the bias-formulas, we have not yet managed to identify them. For the factor elasticities we use, however, a particular kind of factor share estimates to be considered later, as one of the conclusions of this chapter is that those are at least "less inconsistent" than the estimates on these parameters considered so far. [29]

26) We need not necessarily use the OLS method for the estimation of the slope-coefficients in a covariance analysis of the kind considered in this chapter. But in the present context it does not seem to be worth while to try alternatives to the OLS method.

27) Cf. the Census-Study.

28) We know that due to simultaneous equations the OLS-estimate on the MSE of the production function is biased downwards. On the other hand the MSE has a positive bias due to errors of measurement in capital, and the later bias is presumably more important than the former. Thus the way we estimate σ_u^2 probably overstates the residual error of the production relation, and therefore we also probably overstate the simultaneous equations biases.

29) This method presumes constant returns to scale. From (10) we see that this simplifies the bias of the scale-elasticity due to simultaneous equations. About this method of estimation, see the next section.

Concerning the elasticity of substitution we use the OLS-estimates presented in Table III.2. For the computation of the biases due to the errors of measurement in capital we need one additional information, namely the ratio between the variance of the error component of the capital input measure and the observed capital labour ratio. [30] Such an information is hardly obtainable, however, and therefore we assume that $k_1 = 0{,}5$ or $k_1^2 = 0{,}25$ which seems to be reasonable on the average for the four "cases". [31]

The results of the bias-computations are presented in Table III.3. [32]

Considering the simultaneous equations bias first the computations of case A) suggest that there is a positive bias in the OLS-estimate on the elasticity of scale if the assumption of constant returns to scale is true. But for most industries it seems to be quite unimportant even if our computations understate this bias if we really have decreasing returns to scale. Only for Mineral Products and Misc. Products this bias is of some magnitude, but we should note that as the former according to our OLS estimates seems to have increasing returns to scale, our computations may overstate this bias. [33]

30) We consider this ratio rather than the ratio between the error-variance and the capital measure-variance to avoid inconsistencies. Due to our assumptions we must have:

$$k_1^2 \quad \frac{\sigma_e^2}{\sigma_{z-x}^2} = \frac{\sigma_e^2}{\sigma_{z^*-x}^2 + \sigma_e^2} \leq 1 \qquad \text{We have } k_1^2 = 1$$

only in the case when there is no variation in the "true" capital labour ratio.

31) It is probably too high when no effects are eliminated and almost certainly too low when both time and across effects are eliminated. We shall have some further comments on this later.

32) The estimates of the slope-coefficients from the auxiliary-regressions are presented in Appendix III.2.

33) Cf. formula (10) above.

When the time-effects are eliminated the simultaneous equations biases are about the same as for the previous case, while when the across effects or both time and across effects are eliminated we get somewhat different results. Particularly for the former kind of data the bias in the scale-elasticity seems to be more serious. We should, however, be aware that the bias-computations for the two later kinds of data may be quite misleading. There is clearly some truth in the argument that taking out across effects the main source of simultaneity is eliminated. [34] That is in the present context; the systematic across effects of the residual is transferred to the behaviour relation, and back into the production relation via x, while it is less likely that the systematic effect along the time-dimension and particularly the random component of the error term are so transferred. Therefore, quite probably, the simultaneous equations biases computed for case C and D overstate the "true" biases. In any case the simultaneous equations bias of the scale-elasticity can not be seriously negative for the two later cases, and therefore this kind of bias does not explain the sharp drop in the estimate on this elasticity for most industries when the across effects are eliminated.

Concerning the second kind of bias, the one due to errors of measurement in labour input it is very much related to the simultaneous equations bias due to the way we compute it, assuming a perfect correlation between the observed wage-rate and the quality variations in the labour power. But we note that this kind of bias behaves differently. Except for Electrical Machinery in case A and B and four industries (but not Electrical Machinery;) in case D this kind of error seems to lead to a positive bias in the estimate on the capital elasticity. When eliminating the across effects and both time and across-effects the bias in the estimate on the elasticity of scale is for most

34) Cf. Y. Mundlack (1961) op. cit. and sub-section 3.b. above.

Table III.3.

Bias Computations of Covariance Analysis Estimates on the Capital- and Scale-Elasticities.

Case	A: No Effects Eliminated						B: Time Effects Eliminated					
Bias Due To	Simult. Equat.		Errors in Labour		Errors in Capital		Simult. Equat.		Errors in Labour		Errors in Capital	
Industry	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$
Tot. Min. and Man.	-0.160	0.024	0.066	0.016	-0.099	-0.001	-0.162	0.022	0.059	0.015	-0.099	-0.001
Mining and Quar.	-0.185	0.010	0.026	0.007	-0.123	0.024	-0.191	0.007	0.004	0.013	-0.125	0.024
Food Products	-0.197	0.016	0.075	0.004	-0.125	-0.016	-0.201	0.013	0.065	0.001	-0.125	-0.016
Textiles	-0.182	0.019	0.083	-0.005	-0.108	0.005	-0.184	0.018	0.069	-0.003	-0.108	0.005
Clothing	-0.190	0.009	0.031	-0.002	-0.100	-0.002	-0.192	0.007	0.022	0.000	-0.100	-0.001
Wood Products	-0.098	0.022	0.035	0.020	-0.063	-0.001	-0.100	0.021	0.026	0.018	-0.063	-0.001
Pulp and Paper	-0.185	0.018	0.081	0.003	-0.117	0.017	-0.194	0.015	0.027	0.017	-0.118	0.018
Printing	-0.136	0.006	0.055	-0.016	-0.075	0.008	-0.135	0.006	0.056	-0.017	-0.075	0.008
Basic Chemicals	-0.227	0.021	0.029	0.018	-0.131	-0.013	-0.228	0.020	0.022	0.018	-0.131	-0.013
Mineral Products	-0.152	0.043	0.079	0.009	-0.122	0.021	-0.151	0.045	0.071	0.011	-0.123	0.022
Basic Steel	-0.211	0.020	0.034	0.017	-0.139	0.037	-0.220	0.012	0.017	0.014	-0.139	0.036
Metal Products	-0.170	0.015	0.075	0.015	-0.096	-0.005	-0.175	0.011	0.062	0.011	-0.096	-0.006
Non.-El. Mach.	-0.164	0.012	0.078	-0.003	-0.091	0.006	-0.167	0.009	0.059	-0.001	-0.091	0.006
El. Mach.	-0.172	0.004	-0.057	0.037	-0.090	0.007	-0.178	-0.001	-0.067	0.032	-0.090	0.007
Transp. Equipm	-0.105	0.005	0.039	0.007	-0.059	-0.005	-0.105	0.005	0.034	0.006	-0.059	-0.005
Misc. Products	-0.100	0.067	0.119	0.043	-0.097	-0.004	-0.102	0.068	0.126	0.030	-0.097	-0.004

Table III.3 (cont.).

Case	C: Across Effects Eliminated						D: Both Time and Across Effects Eliminated					
Bias Due To	Simult. Equat.		Errors in Labour		Errors in Capital		Simult. Equat.		Errors in Labour		Errors in Capital	
Industry	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$	Bias $\hat{\beta}$	Bias $\hat{\varepsilon}$
Tot. Min. and Man.	-0.149	0.046	0.084	-0.016	-0.123	-0.076	-0.197	0.002	0.017	-0.078	-0.127	-0.082
Mining and Quar.	-0.131	0.079	0.085	0.002	-0.144	-0.143	-0.203	0.005	0.012	0.009	-0.139	-0.122
Food Products	-0.207	0.025	0.057	-0.065	-0.164	-0.116	-0.277	-0.047	-0.037	-0.168	-0.170	-0.126
Textiles	-0.166	0.044	0.107	-0.050	-0.139	-0.080	-0.225	-0.013	0.027	-0.083	-0.145	-0.084
Clothing	-0.181	0.006	0.030	-0.089	-0.154	-0.120	-0.211	-0.022	-0.012	-0.099	-0.154	-0.118
Wood Products	-0.072	0.052	0.087	0.060	-0.075	-0.043	-0.114	0.011	0.025	-0.021	-0.077	-0.047
Pulp and Paper	-0.099	0.097	0.171	-0.235	-0.134	-0.115	-0.189	0.016	0.036	-0.135	-0.127	-0.090
Printing	-0.142	-0.002	0.008	-0.081	-0.085	-0.055	-0.140	0.000	0.013	-0.082	-0.085	-0.056
Basic Chemicals	-0.240	0.012	0.032	-0.024	-0.150	-0.074	-0.267	-0.015	-0.008	-0.037	-0.148	-0.072
Mineral Products	-0.105	0.100	0.133	0.085	-0.131	-0.065	-0.136	0.073	0.081	0.074	-0.132	-0.062
Basic Steel	-0.153	0.095	0.099	0.128	-0.133	-0.062	-0.250	-0.013	-0.011	-0.056	-0.149	-0.102
Metal Products	-0.111	0.078	0.100	0.039	-0.117	-0.076	-0.150	0.008	0.024	-0.078	-0.136	-0.109
Non-El. Mach.	-0.076	0.103	0.142	0.041	-0.126	-0.087	-0.137	0.043	0.062	-0.007	-0.141	-0.102
El. Mach.	-0.054	0.125	0.161	0.190	-0.116	-0.073	-0.157	0.018	0.029	-0.010	-0.142	-0.118
Transp. Equipm.	-0.116	-0.003	0.013	-0.045	-0.063	-0.027	-0.121	-0.009	0.006	-0.062	-0.064	-0.029
Misc. Products	-0.051	0.142	0.102	0.208	-0.126	-0.069	-0.125	0.064	0.069	0.040	-0.128	-0.081

industries negative and for some it seems to be quite serious. And we should note that according to our computations those industries which have the sharpest drop in the estimate on this elasticity when the effects concerned are eliminated, do also have the more serious bias in this estimate due to variations in the quality of the labour input. [35)]

Finally we consider the computed biases due to random errors of measurement in capital input. For the capital elasticity it is negative for all industries for all four "cases", while for the scale-elasticity it is negative for all industries for cases C and D. We note that also for this kind of error the computed biases of the scale-elasticity in C and D cases are more serious for those industries with the sharpest drop in the estimate on this parameter when across effects or both across and time effects are eliminated. [36)] It should be underlined, however, that these biases are computed for the same value of $k_1 = \frac{\sigma_e}{\sigma_{z-x}} = .5$ for all cases. This ratio may be too high when no effects are eliminated, but it is certainly too low when the across effects or both the time and across effects are eliminated. There is not much variation in the "true" capital-labour ratio left when the two main components of this variable are eliminated. Therefore the biases due to errors of measurement in case A and possibly also B are probably too high, while they are presumably too low for cases C and D.

Even if these computations are of a very tentative nature I think they provide sufficient evidence to conclude that to use covariance analysis to eliminate the management bias (or simultaneous equations bias) in the OLS-estimates of the production function may be subject to very high costs in terms of seriously increased biases due to errors of measurement in the factors of production. [37)] In the present case they certainly are too high.

35) Cf. the results for Food Products and Pulp and Paper for instance.

36) Cf. for instance the results for Mining and Quarrying and Food Products.

37) We will not argue that this is necessarily true for data of the present kind. For instance it does not seem to be generally true for the data applied by: Krishna, K.L.: Production Relations in Manufacturing Plants: An Exploratory Study, unpublished. Ph.D. Dissertation, Chicago, May 1967. This study is based on establishment data for three U.S. manufacturing industries; Blast Furnaces and Steel Mills, Steel Foundries and Hydraulic Cement for the period 1954-1963. For the two former industries the estimates on the scale-elasticity are not much changed when eliminating across and time effects, while the estimate on this parameter for the later one shows a drop of about .09. For all industries, however, the estimates on the factor-elasticities are somewhat twisted when across and time effects are eliminated. Generally, the labour elasticity becomes larger while the capital elasticity is reduced.

4. *A SEARCH FOR ESTIMATION METHODS THAT ARE ROBOUST TOWARDS ERRORS OF MEASUREMENT IN CASE OF SIMULTANEOUS EQUATIONS*

a. *A Factor Share Estimate on the Labour Elasticity.*

So far we have not had much success in our attempts to obtain consistent estimates on the production function parameters. In this section some other methods of estimation are considered.

In the bias-computations of the previous section particular estimates on the factor-elasticities were applied, namely factor-shares.[38)] For the elasticity of labour this estimate is computed as:

$$(17)\qquad \tilde{\alpha} = S_L = \exp\left(\bar{s} + \frac{n-1}{2n}\sigma_s^2\right)$$

where $s = \ln \frac{WL}{V}$ and n the number of observations.[39)] S_L is an unbiased estimate on the average factor-share assuming the individual factor-shares log-normally distributed. It is also an unbiased estimate on α provided free competition and that the units maximize profits on the arithmetic rather geometric average and provided also that the elasticity of substitution is one. An approximate standard-deviation of $\tilde{\alpha}$ is:[40)]

$$(18)\qquad \sigma_{\tilde{\alpha}} = \alpha\,\sigma_s\sqrt{\frac{2+\sigma_s^2}{2n}}$$

We note that $\tilde{\alpha}$ is not subject to any of the biases discussed in the previous section. We have taken the simultaneity problem into consideration, and errors of measurement in capital does clearly not matter for this estimate. Neither does the particular kind of error in labour input matter.[41)] Therefore, even if the assumptions on which this factor share estimate is based are not completely realistic, particularly the one of perfect competition, this estimate seems to be more reliable than those discussed previously.

38) Cf. the Census-study.

39) Thus $\bar{s}$ and σ_s^2 are the mean and variance of s.

40) See Aitchison, J. and Brown, J.A.C.: "The Lognormal Distribution". Cambridge University Press, 1963 p. 46

41) We have $\ln \frac{W^* L^*}{V}$ where $L^* = LQ$ and $W^* = \frac{W}{Q}$ where Q is the quality-index of labour input. $(q = \ln Q)$.
Therefore $s = \ln \frac{W^* L^*}{V} = \ln \frac{WL}{V}$. Correspondingly will incomplete deflating of the output and wage-rate variables have no effect on s, $s = \ln \frac{W^* L}{V^*} = \ln \frac{WL}{V}$ as $W^* = \frac{W}{P}$ and $V^* = \frac{V}{P}$.

b. Estimates on the Capital and Scale-Elasticities Free of Simultaneous Equations Bias.

Concerning the elasticity of capital there is now one obvious way of estimating it. Namely by assuming constant returns to scale and thus obtaining:

(19) $\tilde{\beta} = 1-\tilde{\alpha}$

The results so far suggest that this assumption about an elasticity of scale of one is not too bad for most industries. The effects of the different kinds of errors present are, however, rather unpredictable, and we should therefore not rely too heavily on these results.

If we are not willing to accept the assumption of constant returns to scale, at least without further investigations, we may estimate β from the relation

(20) $v - S_L x = a + \beta z + u'$

by means of ordinary least squares. The estimate on the capital and scale elasticities thus obtained are not subject to simultaneous equations bias, provided that the assumptions on which the estimation of α as S_L are true. But these estimates are, however, subject to errors of measurement biases of both kinds discussed above.

c. An Evaluation of the Effects of Simultaneous Equations on the OLS-Estimates

An investigation of the results of (20) and a comparison of these with the OLS-estimates obtained previously should make a good base for an evaluation of the importance of the simultaneous equations bias of our previous results. This is done for the capital elasticity as it is almost similiar for the scale-elasticity.

In Table III.4 the results of (20) are presented, with various treatment of the systematic across and time components (or effects) of the residual.

First of all we note that when no effects, or if only time-effects are eliminated the estimate on β is substantially higher as compared to the corresponding results of Table III.1. But when across-effects are eliminated, either alone or together with the time-effects the estimate on β drops sharply, and on the average for all industries they are not far from the corresponding estimates obtained when the simultaneity bias is ignored.

This suggests that the errors of measurement biases are more serious in this case when analysis of covariance is applied than previously. An attempt to verify this assertion is made by computing these biases in a corresponding way as previously.

It is easily shown that the bias of the estimate on β from (20) due to random errors of measurement in capital is:[42)]

$$\text{(21)} \qquad \text{bias } \hat{\beta} = -\beta \frac{\sigma_e^2}{\sigma_z^2}$$

And we can show that the bias due to errors of measurement in labour input of the kind previously discussed is:

$$\text{(22)} \qquad \text{Bias } \hat{\beta} = S_L b_{wz}$$

where b_{wz} is the "auxiliary" regression coefficient of w on z[43)]. Assuming as previously that the "true" β is equal to the capital's share in value added, S_K and that $\sigma_e^2 = 0.25\ \sigma_{z-x}^2$ we get the "estimates" of the biases from (21) and (22) as presented in Table III.5.

We note from this table that when no effects or time-effects only are eliminated, the two kinds of biases, under the assumptions made, tend to balance each other for most industries, in opposition to the bias-computations presented in Table III.3 where the errors of measurement in capital-bias seems to be more important than the one due to errors of measurement in labour.

Thus the differences between the estimates on β in the case when simultaneous equations errors are present and when they are not, look generally too small. For Total Mining and Manufacturing, when no effects are eliminated the difference is -.082 while the corresponding simultaneous equations bias computed is -0.160 or almost the double.

There seems to be one reasonable explanation on this, namely that the biases due to simultaneous equations are highly overstated by our computations in Table III.3, even in the case when no effects are eliminated. And this is evidently due to the fact that the computed mean square error applied in the bias-calculations contains components of measurement-errors of the factors and that these

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42) As z according to our assumptions is a random variable this is an asympthotic bias.

43) Under reasonable assumptions it can be shown that these two biases are additive: That is the joint effect of the two kinds of errors of measurement is, bias $\hat{\beta} = -\beta \frac{\sigma_e^2}{\sigma_z^2} + S_L b_{wz}$

The biases are computed separately, however, to make an evaluation of the importance of each kind of error possible.

Table III.4.

OLS-Estimates on β from the Relation $v - S_L x = \beta z + u'$.

Industry	No effects eliminated		Time effects eliminated		Across effects eliminated		Both effects eliminated	
	$\hat{\beta}$	MSE	$\hat{\beta}$	MSE	$\hat{\beta}$	MSE	$\hat{\beta}$	MSE
Tot. Min. and Man.	0.354 (0.004)	0.288	0.351 (0.004)	0.281	0.202 (0.012)	0.114	0.101 (0.012)	0.103
Mining and Quar.	0.379 (0.016)	0.192	0.376 (0.016)	0.185	0.210 (0.056)	0.072	0.085 (0.054)	0.056
Food Products	0.365 (0.013)	0.445	0.358 (0.013)	0.424	0.171 (0.040)	0.189	-0.010 (0.038)	0.160
Textiles	0.351 (0.018)	0.156	0.348 (0.018)	0.155	0.357 (0.056)	0.083	0.291 (0.060)	0.081
Clothing	0.258 (0.017)	0.158	0.257 (0.017)	0.158	0.060 (0.038)	0.058	0.023 (0.038)	0.056
Wood Products	0.313 (0.021)	0.295	0.309 (0.021)	0.292	0.249 (0.051)	0.120	0.174 (0.052)	0.112
Pulp and Paper	0.327 (0.011)	0.170	0.319 (0.010)	0.150	0.234 (0.033)	0.088	0.060 (0.029)	0.062
Printing	0.265 (0.013)	0.129	0.266 (0.013)	0.127	0.090 (0.032)	0.054	0.100 (0.031)	0.051
Basic Chemicals	0.439 (0.019)	0.547	0.437 (0.019)	0.529	0.221 (0.050)	0.186	0.166 (0.047)	0.160
Mineral Products	0.483 (0.017)	0.218	0.483 (0.017)	0.222	0.390 (0.049)	0.057	0.355 (0.052)	0.056
Basic Steel	0.418 (0.020)	0.223	0.406 (0.019)	0.210	0.195 (0.050)	0.104	-0.109 (0.048)	0.071
Metal Products	0.276 (0.017)	0.179	0.269 (0.017)	0.177	0.174 (0.046)	0.101	0.002 (0.052)	0.094
Non-El. Machinery	0.328 (0.019)	0.172	0.325 (0.019)	0.173	0.295 (0.053)	0.067	0.194 (0.061)	0.065
El. Machinery	0.291 (0.023)	0.261	0.283 (0.023)	0.257	0.285 (0.068)	0.095	-0.007 (0.076)	0.081
Transp. Equipment	0.273 (0.010)	0.166	0.272 (0.010)	0.166	0.126 (0.030)	0.091	0.100 (0.030)	0.091
Misc. Products	0.336 (0.037)	0.407	0.324 (0.035)	0.354	0.435 (0.124)	0.294	0.245 (0.120)	0.234

Table III.5.

Biases of the Capital Elasticity Estimates Due to Errors of Measurement in Labour and Capital.

	No effects eliminated		Time-effects eliminated		Across-effects eliminated		Both effects eliminated	
	Bias $\hat{\beta}$ due to		Bias $\hat{\beta}$ due to		Bias $\hat{\beta}$ due to		Bias $\hat{\beta}$ due to	
Industry	Err. of labour	Err. of capital	Err. of labour	Err. of capital	Err. of labour	Err. of capital	Err. of labour	Err. of capital
Tot. Min. and Man.	0.031	-0.032	0.029	-0.031	0.063	-0.113	-0.003	-0.117
Min. and Quar.	0.013	-0.019	0.010	-0.018	0.085	-0.144	0.011	-0.138
Food Products	0.017	-0.036	0.013	-0.035	0.032	-0.154	-0.063	-0.161
Textiles	0.022	-0.029	0.018	-0.028	0.054	-0.119	-0.013	-0.122
Clothing	0.012	-0.044	0.010	-0.043	0.008	-0.147	-0.029	-0.146
Wood Products	0.024	-0.018	0.020	-0.018	0.081	-0.067	0.015	-0.071
Pulp and Paper	0.028	-0.026	0.020	-0.025	0.154	-0.133	0.022	-0.124
Printing	0.010	-0.023	0.010	-0.023	-0.003	-0.082	0.001	-0.082
Basic Chemicals	0.020	-0.032	0.018	-0.032	0.014	-0.125	-0.017	-0.123
Mineral Products	0.030	-0.022	0.029	-0.022	0.117	-0.110	0.078	-0.107
Basic Steel	0.023	-0.033	0.015	-0.033	0.105	-0.119	-0.018	-0.142
Metal Products	0.027	-0.024	0.021	-0.023	0.089	-0.110	0.011	-0.132
Non-El. Machinery	0.019	-0.020	0.014	-0.019	0.118	-0.116	0.042	-0.130
El. Machinery	0.006	-0.025	-0.001	-0.025	0.169	-0.105	0.023	-0.138
Transp. Equipm.	0.011	-0.011	0.010	-0.011	-0.004	-0.053	-0.012	-0.054
Misc. Prod.	0.071	-0.039	0.066	-0.039	0.142	-0.104	0.061	-0.116

components are only partly balanced by the negative bias in this estimate on the mean square error due to simultaneous equations.

We also note that in this case the bias in $\hat{\beta}$ due to errors of measurement in capital is less important when no effects or time-effects only are eliminated, compared to the previous case when the simultaneous equations errors were ignored, but that in case the across-effects are eliminated, alone or together with the time effects this kind of bias seems to be of the same order of magnitude in the present and the previous case[44]. To a less extent this seems also to be true for the bias due to errors of measurement in labour.

As pointed out the two kinds of error-biases computed tend to balance eachother in the present case, when no effects or time effects only are eliminated. This is, however, not generally true when the across effects are eliminated also. Thus as we now observe a sharper drop in the estimates on β when covariance analysis is applied, as compared to the case where the simultaneity errors were ignored, we have two explanations for this: When no effects are eliminated the generally higher level of the estimates on β is a joint effect of the elimination of the simultaneous equations bias and reduced net-effect of the errors of measurement biases. But when applying covariance analysis (both across- and time-effects eliminated) the net effect of the two errors is about the same as when the simultaneity errors are ignored.

d. Estimates on the Capital and Scale-Elasticities Free of Both Simultaneous Equations Bias and Errors of Measurement in Capital-Bias.

Even if the net effect of the two types of error of measurement seems to be quite unimportant in the case when no effects are eliminated we should not feel too satisfied with the results obtained. The bias calculations are very tentative and thus we really do not know what in fact <u>is</u> the net effect of the errors. Thus in this section we will go a step further and try to reduce or eliminate one of the errors of measurement biases, namely the one due to errors in our capital measure.

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44) As pointed out in the previous section we probably overstate the error of measurement in capital by assuming $\sigma_e^2 = 0.25\,\sigma_{z-x}^2$, in the case no effects are eliminated, but that we certainly underestimate this error in the case across- and time-effects are eliminated.

Considering relation (20) we have, when ignoring errors of measurement in labour a classical error of measurement problem as concerns z. From the litterature a number of methods are available to "solve" this problem. We will consider only one class of methods, namely instrumental variables[45]. In the present case the number of potential instruments that could serve our purpose is, however, very low. We can in fact think of only one ordinary variable, namely the wage-rate. But this one has clearly to be rejected as it is subject to a particular kind of measurement error itself. Therefore we turn to another method known as grouping of data[46], that in a special case is equal to a particular application of dummy-variables as an instrument for a right-side variable subject to error.

As capital input clearly is correlated with size, we rank the establishments per year according to total employment and define $r_1 = 1$ for the lower third of the units and $r_1 = 0$ otherwise, and $r_2 = 1$ for the upper third and $r_2 = 0$ otherwise[47]. Using $r_2 - r_1$, as an instrument for z in (20) we get the corresponding estimate on β as:[48]

$$\tilde{\beta} = \frac{\bar{v}_2 - \bar{v}_1 - s_L(\bar{x}_2 - \bar{x}_1)}{\bar{z}_2 - \bar{z}_1} \qquad (23)$$

where the bars indicate means and the subscripts indicate size-groups.

45) Cf. for instance Sargan, J.D. "The Estimation of Economic Relationships using Instrumental Variables". *Econometrica*, July 1958.

46) Cf. Wald, A. "Fitting of straight lines if both variables are subject to error", *Annals of Mathematical Statistics*, vol. 11 1940, and A. Madansky; "The fitting of straight lines when both variables are subject to error", *Journal of the American Statistical Association*, 1959. The properties of this method in various applications are considered in Gabrielsen, A. "Grupperingsmetoden", *Memorandum from the Institute of Economics*, University of Oslo, 15 April 1969.

47) The findings of some studies indicate that under rather wide conditions the efficiency of the estimates obtained by the method of grouping is best when about one third of the units of each extreme of the observations are included in the way done by us in the present context. Cf. Bartlett, M.S. "Fitting a straight line when both variables are subject to error" *Biometrics*, 1949, Gibson, W.M. and Jowett, G.H. ""Three-group" regression analysis, Part I; Simple regression analysis", *Applied Statistics*, 1957, Nair, K.R. and Shrivastava, M.P. "On a simple method of curve fitting", *Sankhya*, 1942 and Theil, H. and Yzeren, J. "On the efficiency of Wald's method of fitting straight lines", Revue de l'Institut International de Statistique, 1956.

48) A related method of estimation is applied in the Census-study.

And we get a standard error of this estimate as:[49)]

$$(24) \qquad \sigma_\beta = \frac{\sigma_u}{\sqrt{\frac{n}{6}\,(\bar{z}_2 - z_1)^2}}$$

where σ_u is the standard deviation of the error term and n is the number of observations of the sample.

Having estimated the elasticity of labour by means of the factor-share method described in subsection a above we have now, by means of (23) also an estimate on the elasticity of scale free of both simultaneous equations bias and errors of measurement in capital-bias.[49b)]

e) A Comparison of the Various Estimates Obtained on the Capital and Scale-Elasticities.

The results for the method deducted in the previous sub-section are presented in Table III.6 together with the results for the capital elasticity and the scale elasticity obtained by means of the methods previously discussed[50)]. The first set of estimates presented in this table are subject to all three kinds of biases under discussion. The second set of estimates (containing the estimates of β only) are free of all three kinds of biases, but may be subject to other errors due to the assumption made about constant returns to scale. The third set is subject to biases due to errors of measurement, while the final one is subject to bias due to errors of measurement in labour input only. In light of the bias computations above the differences between the different sets of estimates are as expected.

On the average for our industries the factor-share estimates on the capital elasticity is about 50% higher than the pure OLS-ones. But for a number of industries, such as Clothing, Metal Products, Non-El. Machinery, El. Machinery and Transport Equipment the difference between the two capital-elasticity estimates

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49) Cf. Goldberger, A.S.; *"Econometric Theory"*, Wiley Publications in Statistics, New York 1964, section 6.4.

50) Concerning the instrumental variable method the estimate on the standard error of the estimate on the capital-elasticity is approximate as we use the mean square error obtained by means of the OLS-method as an estimate on the variance of the error term (σ_u^2). But an inspection of this approximation for Total Mining and Manufacturing suggested that it does not understate unduely the "true" estimate of this variance - (implied by the estimates obtained on the factor elasticities, S_L and β).

49b) This method of estimating the factor elasticities will be referred to as the Klein Wald method in the following (Klein for the factor-share method and Wald for the grouping method).

Table III.6.

Estimates on the Capital-Elasticity and the Elasticity of Scale from the Cobb-Douglas Relation.

Industry	OLS		Factor share for α Factor share for β		Factor share for α OLS for β		Klein Wald method	
	$\hat{\beta}$	$\hat{\varepsilon}$	$\hat{\beta}$	ε 1)	$\hat{\beta}$	$\hat{\varepsilon}$	$\tilde{\beta}$	ε
Tot. Min. and Man.	0.272 (0.007)	0.994 (0.005)	0.397 (0.003		0.354 (0.004)	0.957	0.433 (0.006)	1.036
Mining and Quar.	0.281 (0.040)	0.988 (0.022)	0.431 (0.017)		0.379 (0.016)	0.924	0.389 (0.018)	0.958
Food Products	0.372 (0.025)	0.888 (0.014)	0.475 (0.009)		0.365 (0.013)	0.890	0.420 (0.018)	0.945
Textiles	0.293 (0.034)	0.946 (0.022)	0.429 (0.010)		0.351 (0.018)	0.922	0.380 (0.023)	0.951
Clothing	0.080 (0.024)	0.933 (0.021)	0.400 (0.008)		0.258 (0.017)	0.858	0.453 (0.030)	1.053
Wood Products	0.188 (0.038)	1.092 (0.024)	0.251 (0.019)		0.289 (0.021)	1.038	0.393 (0.027)	1.142
Pulp and Paper	0.300 (0.023)	0.896 (0.015)	0.443 (0.007)		0.327 (0.011)	0.884	0.367 (0.016)	0.924
Printing	0.146 (0.023)	1.041 (0.071)	0.295 (0.009)		0.265 (0.013)	0.970	0.336 (0.017)	1.041
Basic Chemicals	0.195 (0.037)	0.893 (0.019)	0.504 (0.013)		0.439 (0m019)	0.935	0.527 (0.024)	1.023
Mineral Products	0.322 (0.023)	1.110 (0.023)	0.444 (0.013)		0.483 (0.017)	1.039	0.520 (0.021)	1.076
Basic Steel	0.200 (0.029)	1.074 (0.029)	0.488 (0.012)		0.418 (0.020)	0.930	0.543 (0.024)	1.055
Metal Products	0.129 (0.034)	0.944 (0.018)	0.380 (0.011)		0.276 (0.017)	0.896	0.335 (0.023)	0.955
Non-El. Machinery	0.028 (0.037)	1.078 (0.021)	0.359 (0.003)		0.328 (0.019)	0.969	0.393 (0.023)	1.034
El. Machinery	0.111 (0.043)	1.023 (0.029)	0.357 (0.015)		0.290 (0.023)	0.933	0.464 (0.033)	1.107
Transp. Equipment	0.091 (0.021)	1.070 (0.010)	0.228 (0.011)		0.273 (0.010)	1.045	0.304 (0.011)	1.076
Misc. Products	0.355 (0.046)	0.938 (0.046)	0.386 (0.034)		0.336 (0.037)	0.950	0.309 (0.059)	0.923

1) ε = 1 per assumption.

is several hundred precents.

As expected the factor-share /OLS method yields generally lower estimates on the capital elasticity than the pure factor-share method. There are three industries for which the opposite is true , namely Wood Products, Mineral Products and Transport Equipment. Thus these do also have an estimate on the elasticity of scale above one. But when eliminating the error of measurement in capital - bias there are nine industries with an estimate on the scale-elasticity above one. Seven of these do also have an OLS estimate on the scale-elasticity above one. In the former case there are thus six industries with an estimate on the capital elasticity below the factor-share estimate on that parameter.

f. An Attempt to Eliminate the Effects of Quality Variations in Labour Input.

All types of estimates on the capital and scale-elasticities, except those obtained by the pure factor-share method, are subject to one or more of the three biases under discussion in the present chapter. The last set of estimates presented in Table III.6 is, however, subject to biases due to quality variations in the labour input measure only.

We have previously argued that the assumption made about the behaviour of this error is rather extreme, namely that the quality component of labour input is perfectly correlated with the observed wage-rate. The results of the ACMS-relation suggest that this cannot be true for all industries. But having adopted this assumption in the bias-computations we may take the full consequenses of it and measure labour input correspondingly, as it implies that the proper labour input measure is WL and not L. By doing this we should in the case factor share for α and OLS for β is applied get the same estimate as by subtracting the computed bias in $\hat{\beta}$ due to this kind of error(cf. Table III.5 when no effects are eliminated) from the corresponding estimate on β when L is applied as input measure[51].

In Table III.7 the results of our methods of estimation when WL is applied as the labour input measure are presented. The first set of estimates is free of errors of measurement in labour-bias only, in the second set the simultaneous

51) When the "pure" OLS-method is applied, we can "predict" that the difference between the two estimates on β, when L and WL is applied as measures of labour input is larger than the computed bias as this bias is computed by assuming $\alpha = S_L$ which is smaller than the OLS estimate on this parameter.

Table III.7.

A Comparison of Different Estimators of the Capital Elasticity and the Elasticity of Scale from the Relation $v = \alpha(w+x) + \beta z + u''$.

Industry	OLS		Factor share for α OLS for β		Klein Wald Method	
	β	ε	β	ε	$\hat{\beta}$	$\hat{\varepsilon}$
Tot. Min. and Man.	0.178 (0.007)	0.977 (0.004)	0.323 (0.004)	0.926	0.410 (0.004)	1.013
Mining and Quar.	0.241 (0.038)	0.981 (0.020)	0.342 (0.015)	0.911	0.382 (0.017)	0.951
Food Products	0.199 (0.023)	0.899 (0.012)	0.348 (0.012)	0.873	0.422 (0.016)	0.947
Textiles	0.159 (0.034)	0.961 (0.019)	0.329 (0.017)	0.900	0.373 (0.021)	0.944
Clothing	0.042 (0.021)	0.987 (0.018)	0.246 (0.016)	0.846	0.436 (0.025)	1.036
Wood Products	0.120 (0.035)	1.076 (0.035)	0.289 (0.019)	1.018	0.394 (0.025)	1.123
Pulp and Paper	0.188 (0.018)	0.897 (0.011)	0.299 (0.009)	0.856	0.388 (0.013)	0.895
Printing	0.087 (0.020)	1.053 (0.014)	0.255 (0.012)	0.960	0.346 (0.014)	1.052
Basic Chemicals	0.108 (0.032)	0.959 (0.017)	0.420 (0.018)	0.916	0.505 (0.021)	1.001
Mineral Products	0.123 (0.036)	1.117 (0.018)	0.453 (0.016)	1.011	0.502 (0.017)	1.058
Basic Steel	0.161 (0.036)	1.034 (0.024)	0.395 (0.019)	0.907	0.524 (0.023)	1.036
Metal Products	0.043 (0.034)	0.909 (0.017)	0.250 (0.017)	0.870	0.319 (0.021)	0.939
Non-El. Machinery	-0.097 (0.037)	1.076 (0.019)	0.310 (0.019)	0.951	0.391 (0.021)	1.032
El. Machinery	0.144 (0.029)	0.996 (0.021)	0.286 (0.019)	0.929	0.398 (0.025)	1.041
Transp. Equipment	0.076 (0.020)	1.057 (0.009)	0.262 (0.009)	1.034	0.297 (0.011)	1.069
Misc. Products	0.152 (0.056)	0.917 (0.035)	0.264 (0.031)	0.878	0.287 (0.050)	0.901

equations bias is eliminated also, and the final set of estimates should be "free" of all three kinds of bias[52]. But due to the extreme assumption made about the error in the labour input measure we will rather argue that the last set of estimates presented in Table III.7 represents kind of lower limits of the unbiased estimates of the capital and scale elasticities that we could have obtained by the Klein Wald -method if the labour input was correctly measured. And in the same way the corresponding set of estimates of Table III.6 represents the same kind of upper limit of these estimates.[53]

g) Errors of Measurement and the Estimation of the Elasticity of Substitution.

The estimates obtained for the elasticity of substitution by means of the behaviour relation of our model are seriously distorted due to quality variations in labour input.[54] An alternative to this method worth considering is the socalled Kmenta-approximation which is a Taylor expansion of the CES relation around the value of the elasticity of substitution of one which corresponds to the Cobb-Douglas case.[55] Excluding terms of third and higher orders we get this approximation as:

$$(25) \qquad v = \gamma_1 x + \gamma_2 z + \gamma_3 (z-x)^2.$$

The factor-elasticities implied by this relation depend on the capital-labour ratio. But we get directly the values of these elasticities as α and β for the

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52) The standard error of $\hat{\beta}$ is computed by means of formula (20) using the estimated standard deviation of the error term obtained from the OLS regression when WL is applied as the labour input measure. Thus this standard error of β is approximate in the same way as the standard error of this estimate when L is applied as input measure.

53) We should note, however, that for two industries this interpretation of the two sets of estimates does not hold, namely for Food Products and Printing as when using WL as the labour input measure yields somewhat higher estimates than when using L.

54) It is shown in Appendix III.1 that under our assumptions this estimate is biased towards one.

55) See: Kmenta, J.: "On the Estimation of the CES Production Function" *International Economic Review VIII (2)* 1967, 180-189.

mean of the log capital labour ratio by rewriting the relation as:

$$(26) \qquad v = \alpha x + \beta z + \gamma(z-x-(\bar{z}-\bar{x}))^2$$

And it can be shown that the elasticity of substitution for the mean of the log capital labour ratio is [56]:

$$(27) \qquad b = \frac{1}{1 - \frac{2\gamma(\alpha+\beta)}{\alpha\beta}}$$

But evidently as there are serious problems present when trying to obtain reliable estimates on the factor elasticities from the Cobb-Douglas relation it must be even more difficult to obtain reliable estimates on the elasticity of substitution from (26) as this method implies both a squared variable in the regression equation and an indirect estimation by (27) of the parameter of interest.

We must therefore consider the effects of the three kinds of errors previously discussed on the estimation of the elasticity of substitution by means of (26) and (27). The simultaneous equations biases will primarely have the effect that the product of $\hat{\alpha}$ and $\hat{\beta}$ in formula (27) is biased downwards. As shown previously the estimate on the scale elasticity is fairly roboust towards errors due to simultaneous equations and is about one. But while the product of the two factor share estimates for Total Mining and Manufacturing for instance is $\sim .24$ the product of the OLS-estimates is $\sim .14$. Thus, even if the effect on the estimate on γ of simultaneous equations is rather unpredictable, this kind of error probably bias the estimate on the elasticity of substitution away from one. For the same reason this does also seem to be the main effect of errors of measurement of labour input. Concerning errors of measurement in capital input it has been shown that the OLS estimates both on β and particularly on γ are seriously biased downwards.[57] Generally $\hat{\alpha}$ is biased upwards which implies that this kind of error has two opposite effects on the estimate on the elasticity of substitution. It is biased towards one because $\hat{\gamma}$ is biased towards zero, while it is biased

56) The Kmenta approximation does not have a constant elasticity of substitution, but it is convenient to compute the value of this parameter for the sample-mean of the variable on which it depends, namely $z-x = \ln \frac{K}{L}$. But as it depends on the capital/labour ratio only, the Kmenta-approximation is a homothetic production function.

57) Cf. Griliches, Z. and Ringstad, V.: "Error-in-the-Variables Bias in Non-Linear Contexts." *forthcoming in Econometrica.*

away from one because $\hat{\alpha}\hat{\beta}$ is biased towards zero. But if we adopt the assumptions of the study referred to in footnote 57) it can be shown that the net effect is a bias of b towards one. We have for large samples $\hat{\beta} \sim \beta(1-\lambda)$ and as constant returns to scale is assumed, $\hat{\alpha} \sim \alpha + \beta\lambda$, and $\hat{\gamma} \sim \gamma(1-\lambda)^2$, where $\lambda = \frac{\sigma_e^2}{\sigma_{z-x}^2}$, the ratio of the error variance to the variance of the measured log capital/labour ratio. Therefore

$$\text{(28)} \qquad \hat{b} \sim \frac{1}{1 - \frac{2\gamma(1-\lambda)}{\beta(\alpha+\beta\lambda)}} \quad ..$$

which clearly implies that $\hat{b}$ is biased towards one.

We try to investigate the importance of the different kinds of biases in a related way as for the factor elasticities. First, we estimate b by means of the OLS-method on (26). This is done both when the elasticity of scale is unconstrained and when it is constrained to one. And it is done both when L and WL is applied as the labour input measure. Second we estimate γ when the means of the factor elasticities are constrained to be equal to these factors shares in value added. Thus we have:

$$\text{(29)} \qquad v - S_L x - S_K z = \gamma(z-x-(\bar{z}-\bar{x}))^2$$

and

$$\text{(30)} \qquad v - S_L(w+x) - S_K z = \gamma(z-(w+x) - (\bar{z}-(\bar{w}+\bar{x}))^2$$

Third the size-dummies-instrumental variable method is applied on (29) by ranking the units according to the size of the right side variable. This is done with L as the labour input only.

In Table III.8 the results of these computations are presented for Total Mining and Manufacturing. We note first that all results from the Kmenta-relation implies that the elasticity of substitution is above one. This does not correspond quite well to the results of the ACMS relation which suggests that the elasticity of substitution is below one.[58] This divergence leads us to try the

58) Neither do these results correspond quite well to the results obtained for these two relations for Total Man. in the Census-study. Even if these two relations showed highly different results for most of the individual industries the results for Total Man. were approximately the same. $\hat{b}$= .871 from the Kmenta relation and $\hat{b}$= .950 from the ACMS relation.

Table III.8.

*Estimates on the Elasticity of Substitution from the Kmenta Relation and The ACMS Relation for Total Mining and Manufacturing.**

	Method of Estimation	$\hat{\gamma}$	$\hat{b}$
Kmenta-relation	Unconstrained OLS (I)	0.054 (0.006)	2.144
	Constrained OLS (I)	0.054 (0.006)	2.140
	Unconstrained OLS (II)	0.026 (0.005)	1.573
	Constrained OLS (II)	0.029 (0.005)	1.641
	Factor share/OLS (I)	0.067 (0.006)	2.255
	Factor share/OLS (II)	0.045 (0.005)	1.593
	Factor share/S.d.i.v. (I)	0.070 (0.011)	2.337
ACMS Relation	OLS	-	0.992 (0.016)
	S.d.i.v.	-	0.980 (0.016)

* I refers to L as labour input measure while II refers to WL as labour input measure. S.d.i.v. refers to size-dummies instrumental variable method (or Wald's method of grouping).

size-dummies-instrumental-variable method also for the ACMS relation by ranking the units by the size of W. After all W is computed as a ratio between two characteristics that both may be subject to errors. But according to the results of this estimation this does not seem to matter. In fact the estimate on b shows a drop compared to the one obtained by means of the OLS-method.

The constraining of the elasticity of scale to one does not matter for the results when L is used as the labour input measure, and it does not matter much in the case WL is applied either. We note that the effect of the elimination of the simultaneous equations bias on b depends on if we have eliminated the errors of measurement in labour bias or not. If not $\hat{\gamma}$ is slightly bigger and if so it is slightly smaller compared to the constrained OLS results. The elimination of the errors of measurement bias leads as expected to a somewhat higher value of b. But rather surprisingly, neither the simultaneous equations bias nor the error of measurement in capital-bias seems to be very important for the estimation of b from the Kmenta-approximation.

The errors of measurement in labour bias seem to be more important but again we should remember that we presumably overstate this kind of bias by our computations.

5. *Summary.*

The findings of this chapter provide sufficient evidence to conclude that in econometric studies like the present one, one should be very careful when interpreting the results without a thorough investigation of the "behaviour" of the variables involved.

To be more specific the following conclusions seem to be apparent.

1. In general one should never ignore possible errors in the measures of the right side-variables of a relation when trying to deal with the problem of simultaneous equations.
2. In particular indirect least squares does definitely not work in our case due to errors of measurement.
3. Neither does analysis of covariance work as this method also is very little roboust towards errors of measurement.
4. Using the factor share method to estimate the elasticity of labour and combining this with the ordinary least square method to estimate the elasticity of capital does not only eliminate the simultaueous equations bias, it does also seem to reduce the importance of the errors of measurement biases.
5. Estimating the elasticity of labour by means of the factor share method and the elasticity of capital by means of the size-dummies-instrumental variables method seems to yield the more reliable estimates for the present kind of data.
6. The results obtained by means of this method suggest that on the average there are slightly increasing returns to scale in Norwegian Mining and Manufacturing. That we in this case generally get a somewhat lower estimate on the scale-elasticity than in the Census-study can probably be explained by the fact that on the average the units of the present study are larger than those of the Census-study and as the results of the later study suggest that there in Norwegian Manufacturing is a decrease of the scale-elasticity with scale.[59)]
7. The results concerning the elasticity of substitution are rather inconclusive.

59) In the Census-study a slightly different size-group method was applied in the estimation of the capital elasticity, with a higher number of units in the lower group than in the upper one. When using L as labour input measure the estimate on the scale-elasticity for Total Manufacturing was, $\tilde{\varepsilon} = 1.050$ while using WL yielded $\tilde{\varepsilon} = 1.025$.

Appendix III.1.
Deduction of Biases of the ILS-Estimates on the Production Function Parameters in case of Errors of Measurement.

a. The Effects of Quality-Variations in Labour Input.

The model is;

$$(1)\qquad \begin{aligned} y &= \alpha x + \beta z + u \\ y-x &= bw + v \end{aligned}$$

where $y = \ln V$, $x = \ln L$, $z = \ln K$ and $w = \ln W$[1].
The "correct" model is, however:

$$(2)\qquad \begin{aligned} y &= \alpha x^{*} + \beta z + u' \\ y-x^{*} &= bw^{*} + v' \end{aligned}$$

where $x^{*} = x + q$ and $w^{*} = w - q$ where $q = \ln Q$ and Q is the quality-level of labour input. And we get

$$(3)\qquad \begin{aligned} u &= u' + \alpha q \\ v &= v' + (1-b)q \end{aligned}$$

We assume that indirect least squares when applied on (2) yields consistent estimates on the parameters, assuming y and x to be endogeneous, and z and w exogeneous.

Provided now that the wage rate is perfectly correlated with the quality-index we get the following results when indirect least squares is applied on (1):

$$(4)\qquad \hat{b} = \frac{\Sigma(y-x)w}{\Sigma w^{2}} = b + \frac{\Sigma(v'+(1-b)q)w}{\Sigma w^{2}}$$

---

1) The variables are computed as deviations from their means.

Due to our assumptions:

(5) $$\text{plim } \hat{b} = 1$$

From the second reduced form equation:

(6) $$x = -\frac{b}{1-\alpha} w + \frac{\beta}{1-\alpha} z + \frac{u-v}{1-\alpha}$$

or

(7) $$x = \pi_1 w + \pi_2 z + r$$

we get

$$\hat{\pi}_1 = \frac{\Sigma xw \Sigma z^2 - \Sigma xz \Sigma wz}{D_1} = \pi_1 + \frac{\Sigma rw\ z^2 - \Sigma rz \Sigma wz}{D_1}$$

(8) $$\hat{\pi}_2 = \frac{\Sigma xz \Sigma w^2 - \Sigma xw \Sigma wz}{D_1} = \pi_2 + \frac{\Sigma rz \Sigma w^2 - \Sigma rw \Sigma wz}{D_1}$$

Where:

(9) $$D_1 = \Sigma w^2 \Sigma z^2 - (\Sigma wz)^2$$

As

(10) $$r = \frac{u' - v' - (1-\alpha-b)q}{1-\alpha}$$

we get

(11) $$\text{plim } \hat{\pi}_1 = -1$$
$$\text{plim } \hat{\pi}_2 = \pi_2 - \frac{\beta}{1-\alpha}$$

Thus:

(12) $$\text{plim } \hat{\alpha} = 0$$
$$\text{plim } \hat{\beta} = \frac{\beta}{1-\alpha}$$

b. The Effects of Errors of Measurement in Capital.

We apply the model in (1) while the "correct" model now is:

$$y = \alpha x + \beta z^* + u^*$$

(13)

$$y - x = bw + v$$

where the "true measure" of capital z^* is equal to our measure minus an error term e with constant variance and which is distributed randomly:

That is:

(14) $$z^* = z - e$$

Indirect least squares applied on (13) is assumed to give consistent estimates.

From (1) and (13) we get that

(15) $$u = u^* - \beta e$$

We note that the estimation of b is not affected of the error e, and clearly we have in this case;

(16) $$Eb = b$$

As we now have:

(17) $$r = \frac{u^* - v - \beta e}{1 - \alpha}$$

we get, by means of ordinary least squares on (7), using formulas (8) and (9) and (17):

$$\hat{\pi}_1 = \pi_1 + \frac{\frac{1}{1-\alpha}(\Sigma(u^*-v-\beta e)w\Sigma z^2 - \Sigma(u^*-v-\beta e)z\Sigma zw}{D_1}$$

(18)

$$\hat{\pi}_2 = \pi_2 + \frac{\frac{1}{1-\alpha}(\Sigma(u^*-v-\beta e)z\Sigma w^2 - \Sigma(u^*-v-\beta e)w\Sigma zw}{D_1}$$

And we therefore get that:

$$\text{plim}\ \hat{\pi}_1 = \pi_1 + \frac{\beta}{1-\alpha}\ \frac{b_{zw}}{1-r_{zw}^2}\, k^2 \tag{19}$$

$$\text{plim}\ \hat{\pi}_2 = \pi_2 - \frac{\beta}{1-\alpha}\ \frac{1}{1-r_{zw}^2}\, k^2$$

where b_{zw}, $\frac{\sigma_{zw}}{\sigma_w^2}$ and r_{zw} is the correlation coefficient between z and w and $k^2 = (\frac{\sigma_e}{\sigma_z})^2$ is the error to total variance-ratio of the capital input measure.

Thus we have that:

$$\text{plim}\ (\hat{\alpha}-\alpha) = \frac{(1-\alpha)b_{zw}B}{b_{zw}B-b} \tag{20}$$

$$\text{plim}\ (\hat{\beta}-\beta) = \frac{(b-\beta b_{zw})B}{b_{zw}B-b}$$

where

$$B = \beta k^2/(1-r_{zw}^2) \tag{21}$$

Appendix III.2.

Table A.III.1

Coefficients from Auxiliary Regressions Applied in the Bias-Computations of Section III.3. *

A: No Effects Eliminated.

Industry	b_{xz}	b_{wx}	b_{wz}	b_{zx}	b_{zw}	$b_{x,z-x}$
Tot. Min. and Man.	0.6893	0.0252	0.0521	0.9902	1.2365	-0.0214
Mining and Quar.	0.7102	0.0213	0.0221	1.1915	1.9373	0.6496
Food Products	0.8169	-0.0102	0.0324	0.8753	0.4081	-0.3885
Textiles	0.6990	-0.0014	0.0381	1.0462	0.9889	0.1142
Clothing	0.5702	-0.0047	0.0204	0.9819	0.3676	-0.0239
Wood Products	0.7137	0.0265	0.0326	0.9915	1.5542	-0.0208
Pulp and Paper	0.6792	0.0261	0.0501	1.1439	0.6617	0.3632
Printing	0.6290	-0.0152	0.0143	1.1015	0.3938	0.1853
Basic Chemicals	0.8372	0.0307	0.0400	0.9021	0.7666	-0.3580
Mineral Products	0.6962	0.0413	0.0546	1.1756	2.9454	0.5203
Basic Steel	0.6029	0.0499	0.0456	1.2668	1.5340	0.4701
Metal Products	0.7971	0.0169	0.0433	0.9434	1.3439	-0.1908
Non-El. Machinery	0.7344	0.0042	0.0292	1.0691	1.5491	0.2175
El. Machinery	0.6678	0.0504	0.0089	1.0784	0.1616	0.1711
Transp. Equipm.	0.8760	0.0050	0.0142	0.9202	0.6179	-0.3798
Misc. Products	0.6229	0.0606	0.1164	0.9552	0.2119	-0.0719

B: Time Effects Eliminated.

Industry	b_{xz}	b_{wz}	b_{wz}	b_{zx}	b_{zw}	$b_{x,z-x}$
Tot. Min. and Man	0.6911	0.0241	0.0477	0.9897	1.3359	-0.0228
Mining and Quar.	0.7147	0.0239	0.0181	1.1949	2.5898	0.6909
Food Products	0.8190	-0.0131	0.0242	0.8749	0.3919	-0.3928
Textiles	0.7035	0.0008	0.0322	1.0480	1.0955	0.1219
Clothing	0.5734	0.0000	0.0159	0.9863	0.3393	-0.0183
Wood Products	0.7170	0.0236	0.0268	0.9908	1.7292	-0.0229
Pulp and Paper	0.6858	0.0379	0.0361	1.1502	1.0651	0.3988
Printing	0.6291	-0.0153	0.0147	1.1020	0.9330	0.1862
Basic Chemicals	0.8378	0.0312	0.0370	0.9029	1.0028	-0.3572
Mineral Products	0.6985	0.0423	0.0521	1.1769	3.3085	0.5341
Basic Steel	0.6060	0.0364	0.0300	1.2603	2.1778	0.4656
Metal Products	0.8075	0.0119	0.0338	0.9390	1.2857	-0.2143
Non-El. Machinery	0.7430	0.0050	0.0226	1.0702	1.6947	0.2340
El. Machinery	0.6694	0.0420	-0.0011	1.0760	-0.0242	0.1668
Transp. Equipm.	0.8461	0.0045	0.0136	0.9200	0.6011	-0.3809
Misc. Products	0.6191	0.0406	0.1080	0.9608	2.9144	-0.0621

* $x = \ln L$, $z = \ln K$, $w = \ln W$

C: Across Effects Eliminated.

Industry	b_{xz}	b_{wx}	b_{wz}	b_{zx}	b_{zw}	$b_{x,z-x}$
Tot. Min. and Man.	0.2124	-0.1133	0.1040	0.3772	0.1915	-0.3081
Mining and Quar.	0.0029	-0.1452	0.1493	0.0084	0.6551	-0.2554
Food Products	0.2075	-0.1998	0.0602	0.2919	0.0534	-0.3884
Textiles	0.3364	-0.1944	0.0946	0.4283	0.2304	-0.4036
Clothing	0.1827	-0.1871	0.0134	0.2175	0.0300	-0.4458
Wood Products	0.1827	0.0142	0.1075	0.4293	0.5961	-0.2811
Pulp and Paper	0.0404	-0.6840	0.2773	0.1435	0.2583	-0.2006
Printing	0.1279	-0.1232	-0.0045	0.3526	-0.0255	-0.2121
Basic Chemicals	0.3224	-0.0803	0.0274	0.5039	0.0392	-0.3190
Mineral Products	0.3200	0.0339	0.2113	0.5061	0.7003	-0.3147
Basic Steel	0.2055	0.1591	0.2053	0.5362	0.6378	-0.1828
Metal Products	0.1831	-0.0417	0.1437	0.3510	0.6256	-0.2930
Non-El. Mach.	0.2390	-0.0883	0.1839	0.3087	0.8589	-0.4129
El. Mach.	0.2621	0.1393	0.2622	0.3744	0.3849	-0.3724
Transp. Equipm	0.2884	-0.0661	-0.0051	0.5672	-0.0330	-0.2362
Misc. Products	0.3764	0.2475	0.2310	0.4516	0.4348	-0.4230

D: Both Time and Across Effects Eliminated

Industry	b_{xz}	b_{wx}	b_{wz}	b_{zx}	b_{zw}	$b_{x,z-x}$
Tot. Min. and Man.	0.2141	-0.1481	-0.0051	0.3528	-0.0120	-0.3333
Mining and Quar.	0.0476	-0.0029	0.0202	0.1255	0.2033	-0.2584
Food Products	0.1981	-0.2674	-0.1204	0.2624	-0.1450	-0.4098
Textiles	0.3676	-0.1729	-0.0233	0.4204	-0.0834	-0.4449
Clothing	0.2009	-0.1503	-0.0487	0.2318	-0.1373	-0.4546
Wood Products	0.2216	-0.0438	0.0202	0.3891	0.1851	-0.3089
Pulp and Paper	0.0837	-0.2890	0.0389	0.2903	0.0967	-0.1826
Printing	0.1244	-0.1275	0.0011	0.3476	0.0074	-0.2105
Basic Chemicals	0.3283	-0.0662	-0.0344	0.5169	-0.0866	-0.3136
Mineral Products	0.3618	0.0644	0.1406	0.5288	0.6007	-0.3356
Basic Steel	0.1391	-0.0950	-0.0343	0.3119	-0.2912	-0.2628
Metal Products	0.1339	-0.1575	0.0172	0.2025	0.0856	-0.3785
Non-El. Mach.	0.2791	-0.0808	0.0661	0.2778	0.4673	-0.5016
El. Mach.	0.1415	-0.0522	0.0362	0.1700	0.0584	-0.4458
Transp. Equipm.	0.2737	-0.0844	-0.0160	0.5445	-0.1042	-0.2397
Misc. Products	0.2641	-0.0068	0.0997	0.3642	0.3441	-0.3852

CHAPTER IV.

ON TESTING OF MULTIPLE HYPOTHESES.

1. *Introduction*

In econometric studies testing of hypotheses is a valuable statistical tool to investigate the importance of various "causes", the validity of models specified etc. But quite often a fairly high number of tests are carried out, where frequently later tests are directly or indirectly based on the outcome of former ones. Particularly this is a common feature of exploratory studies based on data about which little is known a priori.

The tests thus carried out are usually partial, that is each test-situation is treated separately. And having carried out such a process of data-snooping one may in fact wonder what the conclusions finally obtained are worth. Generally the statistics of the final test(s) may be quite misleading. Thus, clearly one should rather try to consider the multitude of tests as a whole. And by deciding a priori what to do under different outcomes of the individual tests the prospects are better of a proper evaluation of the conclusions obtained.

There are, however, two basic problems when trying to apply such an approach. First, the issues subject to investigation may be of widely different natures. Thus one may be interested in the testing of a varity of hypotheses that are not all related in a way that makes an overall multiple test procedure applicable. But even in such cases something could be done if one managed to divide the hypotheses into groups so that multiple test procedures could be applied on each group separately.

Multiple testing is a fairly new branch of theoretical statistics and thus the second basic difficulty one runs into when trying to apply multiple test methods on particular problems in econometrics is just to find an appropriate method for which the properties are known.[1] This is also a main

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1) About the litterature cf. Scheffé, H: *op.cit.*, Miller jr. R.G. *"Simultaenous Statistical Inference"* Mc. Graw Hill Series in Probability and Statistics, 1966, Gabriel, K.R.: "Simultaneous test procedures - some theory of multiple comparisons". *The Annals of Mathematical Statistics*, February 1969. See also Malinvaud, E.: *"Statistical Methods of Econometrics"*, North Holland Publ. Co., Amsterdam 1966, ch. 7 §3.

problem in the present context. What we manage to do is to determine an upper limit of the level of the overall tests. As will be shown this will be done in two ways, by slightly different methods of testing.

2. The Testing Scheme.

The basis of our illustration is the two relations of our main model, namely:

$$\text{(1a)} \quad \ln V_{it} = \alpha \ln L_{it} + \beta \ln K_{it} + u_{it} \qquad \left| \begin{array}{l}(i=1\ldots I)\\(t=1\ldots T)\end{array}\right.$$

$$\text{(1b)} \quad \ln (\tfrac{V}{L})_{it} = b \ln W_{it} + v_{it}$$

where, for a moment we assume the error terms to be distributed independently, with no serial correlation, zero means and constant variances.

There are numerous possible errors of specification in this model. To mention a few of the presumably more important ones:[2] L and K may enter in a much more complicated way, that is, the production function is of a more complex nature; perhaps we should have used gross production as the output measure instead of value added and with materials as an "independent" factor of production alongside labour and capital; and perhaps also the specification of the error-terms is wrong. Such questions and related ones could be investigated, but not easily at the same time. In this context we will consider only one, namely the specification of the error terms. This is clearly a "partial" analysis as the other doubts we may have about the validity of 1a and 1b is not subject for discussion or investigation More precisely, we are going to study the assumption made above about the error means; that they are zero for all units of observation.

In the covariance-analysis of the previous chapter we asserted that the error means might vary both across establishments and over time. This presumption about the behaviour of the error means could clearly serve as one-hypotheses when testing the validity of the ones of zero means.

Thus we could have the following test-situation for the production relation[3][4]

2) We ignore the deliberate inconsistencies between 1a) and 1b) pointed out in Chapter I.

3) The test-situation for the behaviour relation is clearly the same.

4) The "contents" of the null-hypotheses is that the means are constant. If these constants are zero or not is trivial provided that we are not particularly interested in the identification of the intercept.

$$\text{(2a)} \qquad H_o : Eu_{it} = 0 \qquad \Big| \begin{array}{l} i=1\ldots I \\ t=1\ldots T \end{array}$$

$$\text{(2b)} \qquad H_1 : Eu_{it} = a_i + b_t$$

And assuming the error-term is normally distributed one can apply an ordinary F-statistics to test the null-hypothesis.[5)]

In case H_o is rejected, however, we do not know if it is due to the across or the time component of the mean, or both. In case we would like to know that, we should rather carry out two tests, with the null-hypotheses:

$$\text{(3a)} \qquad H'_o : Eu_{it} = a_i$$

$$\text{(3b)} \qquad H''_o : Eu_{it} = b_t \qquad \Big| \begin{array}{l} i=1\ldots I \\ t=1\ldots T \end{array}$$

with (2b) as the common one-hypothesis.

But in the present case we are interested in an even further investigation of the nature of any variation of the error mean. In our two-way classification there is one observation per cell only. And the one-hypothesis above implies that each cell may have its "own" error mean. But if one or both of the null-hypotheses above are rejected it may quite well be due to a more "constrained" variation of the error mean. It could be true that for each of our industries the error mean varies *between* sub-industries only, while it is constant *within* sub-industries. And it could also be true that the variation of the error mean over time is equal to a trend.

This is the frame within which we will work in the search for the "nature" of any variation of the error mean. In Table IV.1 we present the various potential types of error mean variation implied by this frame, with an explanation of their contents and a notation to be used for them in the following.

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5) In this case the F-statistics would be:

$$F_{obs} = \frac{\overset{TI}{\Sigma\Sigma}\hat{u}_{it} - \overset{TI}{\Sigma\Sigma}\hat{u}'^{2}_{it}}{\overset{TI}{\Sigma\Sigma}\hat{u}'^{2}_{it}} \cdot \frac{(I-1)(T-1)-2}{I+T-1}$$

where $\hat{u}_{it}$ is the estimated residual for the production function for "cell" (i,t) under the null hypothesis and $\hat{u}'_{it}$ the estimated residual under the one hypothesis ($u'_{it} = u_{it} - a_i - b_t$).

Table IV.1

*List of Types of Error Mean Variation.**

No.	Type of Variation	Explanation	Notation
1	$Eu_{it} = a_i + b_t$	Error-mean may vary between establishments and show an unconstrained variation over years	E and T
2	$Eu_{it} = c_j + b_t$ $(c_1=a_1=a_2=\ldots=a_{n_1},\ldots$ $\ldots c_J=a_{n_{J-1}+1}=\ldots=a_{n_J})$	Error-mean may vary between sub-industries but is constant within sub-industries, and it may show an unconstrained variation over time	J and T
3	$Eu_{it} = a_i + bt$	Error-mean may vary between establishments, and any variation over time is constrained to a trend.	E and t
4	$Eu_{it} = b_t$	No variation of the error mean across establishments while it may show an unconstrained variation over time.	T
5	$Eu_{it}=c_j+bt$ $(c_1=a_1=a_2=\ldots=a_{n_1},\ldots$ $\ldots,c_J=a_{n_{J-1}+1}=\ldots=a_{n_J})$	Error-mean may vary between sub-industries but is constant within sub-industries, and any variation over time is constrained to a trend.	J and t
6	$Eu_{it} = a_i$	Error-mean may vary between establishments but it shows no variation over time.	E
7	$Eu_{it} = bt$	No variation of the error-mean across establishments and any variation over time is constrained to a trend.	t
8	$Eu_{it} = c_j$ $(c_1=a_1=a_2=\ldots=a_{n_1}\ldots$ $\ldots c_J=a_{n_{J-1}+1}=\ldots=a_{n_J})$	Error-mean may vary between sub-industries that is constant within sub-industries, and it shows no variation over time	J
9	$Eu_{it} = 0$	No variation of the error mean either across establishments or over time.	0

* $i=1\ldots I$, I is the number of establishments.
$j=1\ldots J$, J is the number of sub-industries (cf. Appendix II.2).
$t=1\ldots T$, T is the number of years.

These "types of variation" presented in Table IV.1 make an hierarchy with the more general type (1) on top and the less general one, (9) - no variation - at bottom. It is not unique, however. In Fig. IV.1 we see that there are five "levels" in this hierarchy with (1) in the first, (2) and (3) in the second, (4), (5) and (6) in the third, (7) and (8) in the fourth and (9) in the fifth.

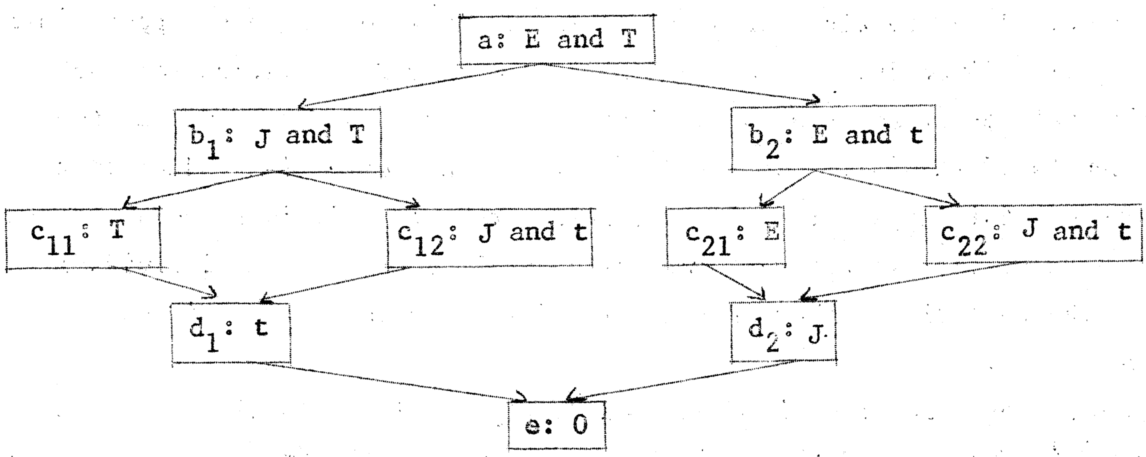

Fig. IV.1

The Hierarchy of Types of Variation of Error Means.

Fig. IV.1 will serve as a scheme of testing of the nature of the error mean's variation. And the strategy of the testing is the following: In the first round the two alternatives at the b-level (b_1 and b_2) serve as null-hypotheses and they are each tested with "a" as the one-hypothesis. If both are rejected there is evidence of establishment-specific and year-specific differences in the error mean. If either b_1 or b_2 or both cannot be rejected the testing is continued, with the c-level types of variation as null-hypotheses. If, say b_1 is rejected but not b_2, no further testing is carried out along the left main-branch of the hierarchy in Fig. IV.1. In that case we test c_{21} and c_{22} against b_2.

If both are rejected b_2 is the "optimal" type of error-mean variation. If, say c_{22} is not rejected while c_{21} is, we test d_2 against c_{22}. If d_2 is rejected c_{22} is the "optimal" type of error-mean variation. If not a final test is carried out, e against d_2. And either d_2 or e is the "optimal" type, depending on the rejection or non-rejection of e by this test.

But clearly if b_1 really is rejected when tested against a it is not likely that we will come further than to the c-level along the right main branch of the hierarchy. If we do, we run into problems of interpretation, as b_1 versus a and d_2 versus c_{21} concern the same parameters. There are also other possibilities of inconsistencies, as we may get more than one optimal type of error-mean variation. But such problems we do not run into in this context.[6)]

The individual tests are carried out by means of F-statistics. As pointed out one do not manage to determine an exact level of the overall test.[7)] Instead we apply two methods of testing for which it is possible to determine upper limits of the level. The first one implies the use of ordinary F-statistics for each of the individual tests, and the upper limit of the level is determined as the sum of the levels of the individual tests.[8)] If the number of individual tests is high, however, this upper limit is of little interest as it is presumably far off the "true" level.[9)] In this case we use a level of .5% for the individual tests and as the potential number of tests is 12 we have an upper limit of the overall level of 6%.

The second test procedure is developed by Spjötvoll and is also based on F-statistics.[10)] For each of the individual tests we use the "modified" F-statistics[11)]

$$F'_{obs} = \frac{Q_i^2 - Q_j^2}{Q_k^2} \; \frac{(I-1)(T-1)-a}{I+T-1}$$

where Q_i^2 and Q_j^2 are the sums of squares of the residual under the null-hypothesis and one-hypothesis respectively and Q_k^2 is the sum of squares of the residual for the type of error mean variation of the more general kind, i.e. the one at the top

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6) They are, however, apparent in the next chapter when trying to apply a related scheme of testing on the nature of technical change. Cf. Section V.3. Cf. also Section IV.4.

7) The overall level of the multiple test is the probability of accepting a particular one-alternative when any one of the other alternatives specified in the testing scheme is right.

8) Cf. Malinvaud. E; *op.cit.*

9) Cf. Section 4 of this chapter.

10) Spjøtvoll, E: "Multiple Comparison of Regression Functions". Unpublished paper, 1969.

11) a is the number of slope-coefficients, and thus a=2 for the production relation and a=1 for the behaviour relation.

of the hierarchy. Both Q_k^2 and the degrees of freedom, which are those of the ordinary F-statistics of the tests of 2a) versus 2b), are common for all individual tests.

Now, if we carry out the same testprocedure as for the first method of testing but reject each null-hypothesis for which $F'_{obs} \geq F_{(1-\varepsilon),(I+T-1),((I-1)(T-1)-a)}$ we have an upper limit of the overall test of ε. We choose $\varepsilon = 5\%$ to have an upper limit of the overall level of this test roughly comparable to the one discussed first.

3. *The Results.*

The test-procedures sketched above are applied on 1a) and 1b) separately, and thus we ignore the simultaneous equations problem of the estimation of the parameters of 1a). The outcome of the tests is presented by industry in Table IV.2. And to give an idea of the magnitudes of the F-values computed we present for for Food Products in Tables IV.3 and IV.4 the values of the ordinary F-values (F_{obs}) as compared to the corresponding upper .5% fractiles ($F_{0.995}$) as well as Spjøtvolls F-statistics. (F'_{obs}.) In the later case the fractile used for testing is the same (5%) for all individual tests. The industry Food Products is selected as this is the one with the highest number of establishments as well as the highest number of sub-industries.

From Table IV.2 we note first that the two methods of testing yield somewhat different results. Generally the Spjøtvoll method is rougher towards the time-components of the error mean, as compared to the ordinary F-statistics method. This is not so surprising as the former method does not take care of the highly varying number of parameters in the individual tests. Thus it does not seem to suit quite well to the test-situation considered in this section.

According to the results obtained by ordinary F-statistics we have for the Cobb-Douglas relation that the optimal type of error mean variation is the same for all except one of the fifteen industries, namely individual variation across establishments and a trend variation over time. For Mineral Products the optimal type is individual across-variations.

For the ACMS-relation the results are also the same for all but two industries, namely individual variation across establishments with no variation over time. Rather surprisingly the optimal type of error mean variation of the ACMS-relation for Pulp and Paper is the more general one, while the optimal type of error mean

Table IV.2

The Results of Multiple Tests of the Error Means of the Cobb-Douglas and ACMS Relations

Industry	Optimal Type of Error Mean Variation				No. of establ.	No. of sub-ind.*
	Cobb-Douglas		ACMS			
	Ord. F-statistics	Spjøtv. F-statistics	Ord. F-statistics	Spjøtv. F.statistics		
Mining and Quar.	E and t	E and t	E and t	E	26	7
Food Prod.	E and t	E and t	E	E	164	22
Textiles	E and t	E	E	E	58	7
Clothing	E and t	E	E	E	67	7
Wood Products	E and t	E	E	E	45	8
Pulp and Paper	E and t	E and t	E and t	E	103	6
Printing	E and t	E	E	E	63	6
Basic Chemicals	E and t	E and t	E	E	72	16
Mineral Prod.	E	E	E	E	36	12
Basic Steel	E and t	E and t	E	E	42	6
Metal Prod.	E and t	E	E	E	60	10
Non-El. Mach.	E and t	E	E	E	37	4
El. Mach.		E	E		34	7
Transp. Equip.	E and t	E	E	E	87	11
Misc. Prod.	E and t	E	E	E	13	3

* The sub-industries are two, three and four digit industry groups according to the Norwegian version of the ISIC-code. Cf. Appendix II.2.

Table IV.3

Testing Scheme for the Error Mean of the Cobb-Douglas Relation for Food Products

a: E and T effects

- a → b₁: F_{obs} 7.01; $F_{0.995} = 1.40$; $F'_{obs} = 5.70$
- a → b₂: F_{obs} 2.71; $F_{0.995} = 2.85$; $F'_{obs} = 0.11$

b$_1$: J and T effects

- b$_1$ → c$_{11}$: $F_{obs} = 34.05$; $F_{0.995} = 2.00$; $F'_{obs} = 6.89$
- b$_1$ → c$_{12}$: $F_{obs} = 1.86$; $F_{0.995} = 2.85$; $F'_{obs} = 0.12$

b$_2$: E and t effects

- b$_2$ → c$_{21}$: $F_{obs} = 221.15$; $F_{0.995} = 8.00$; $F'_{obs} = 1.30$
- b$_2$ → c$_{22}$: $F_{obs} = 9.10$; $F_{0.995} = 1.40$; $F'_{obs} = 5.69$

c$_{11}$: T effects

- c$_{11}$ → d$_1$: $F_{obs} = 1.21$; $F_{0.995} = 2.85$; $F'_{obs} = 0.12$

c$_{12}$: J and t effects

- c$_{12}$ → d$_1$: $F_{obs} = 33.89$; $F_{0.995} = 2.00$; $F'_{obs} = 6.88$

c$_{21}$: E effects

- c$_{21}$ → d$_2$: $F_{obs} = 8.38$; $F_{0.995} = 1.40$; $F'_{obs} = 5.56$

c$_{22}$: J and t effects

- c$_{22}$ → d$_2$: $F_{obs} = 127.22$; $F_{0.995} = 8.00$; $F'_{obs} = 1.17$

d$_1$: t effects

- d$_1$ → e: $F_{obs} = 71.21$; $F_{0.995} = 8.00$; $F'_{obs} = 0.98$

d$_2$: J effects

- d$_2$ → e: $F_{obs} = 30.30$; $F_{0.995} = 3.00$; $F'_{obs} = 6.69$

e: No effects

$F'_{0.950} \approx 1.20$

Table IV.4

Testing Scheme for the Error Mean of the ACMS Relation for Food Products

a: E and T effects

- F_{obs} 7.50; $F_{0.995}$ = 1.40; F'_{obs} = 6.30 → **b_1: J and T effects**
  - F_{obs} 40.69; $F_{0.995}$ = 2.00; F'_{obs} = 8.60 → **c_{11}: T effects**
    - F_{obs} 0.21; $F_{0.995}$ = 2.85; F'_{obs} = 0.02 → **d_1: t effects**
  - F_{obs} 0.44; $F_{0.995}$ = 2.85; F'_{obs} = 0.03 → **c_{12}: J and t effects**
    - F_{obs} 40.77; $F_{0.995}$ = 2.00; F'_{obs} = 8.59 → **d_1: t effects**
- F_{obs} 0.74; $F_{0.995}$ = 2.85; F'_{obs} = 0.03 → **b_2: E and t effects**
  - F_{obs} 5.06; $F_{0.995}$ = 8.00; F'_{obs} = 0.03 → **c_{21}: E effects**
    - F_{obs} 9.41; $F_{0.995}$ = 1.40; F'_{obs} = 6.29 → **d_2: J effects**
  - F_{obs} 9.41; $F_{0.995}$ = 1.40; F'_{obs} = 6.30 → **c_{22}: J and t effects**
    - F_{obs} 2.21; $F_{0.995}$ = 8.00; F'_{obs} = 0.02 → **d_2: J effects**

d_1: t effects
- F_{obs} 2.79; $F_{0.995}$ = 8.00; F'_{obs} = 0.04 → **e: No effects**

d_2: J effects
- F_{obs} 36.83; $F_{0.995}$ = 2.00; F'_{obs} = 8.61 → **e: No effects**

e: No effects

$F'_{0.950} \approx 1.20$

variation for Mining and Quarrying is individual variations across establishments and a trend over time.

Thus when applying the ordinary F-statistics method the results are ~~fairly~~ uniform for each relation across industries. The difference between the relations is easily explained by the fact that if the elasticity of substitution is close to one, which is just the result obtained by the behaviour relation for most industries, the neutral shifts over time in the intercept of this relation should not be significant.[12] On the other hand if there are technical changes of some importance over time, not accounted for by the input measures we should have significant shifts over time in the production function.[13] Using the Spjøtvoll method these shifts are significant only for the more heavy industries Mining and Quarrying, Pulp and Paper, Basic Chemicals and Basic Steel together with Food Products, while this method yields significant shifts over time in the behaviour relation for no industry. Shifts in the two relations over time are subject of further discussion in the next chapter.

4. *Results of a More Complex Test.*

In the multiple test schemes studied above even the more general alternative (E and T) is quite restrictive, as it presumes that the slope coefficients of the relations concerned are constant both across establishments and over time. We will in this section consider a more complex multiple test situation where differences both in error-means and slope-coefficients are involved.

The analysis is carried out for Total Mining and Manufacturing only. For a given year we assume that all parameters (error-means included) are constant within each industry, while they may be different for different industries.[14] Over time the parameters are allowed to have trends, common for all units.

Thus in the less restrictive case we have the two relations as:[15]

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12) Cf. Section V.3.

13) Cf. Section V.2.

14) The across variation is slightly more constrained than this as the coefficients are assumed to be the same for El. Machinery and Non-Electr. Machinery, and for Transport Equipment and Misc. Products. This is done due to certain capacity problems of the program applied in the computations.

15) Note that we here redefine the error terms so that we get differences in the intercept instead of differences in error-means. The contents of the relations are clearly not changed by this reformulation.

(4a) $\ln V_{it} = a_j + bt + (\alpha_j + \gamma t)\ln L_{it} + (\beta_j + \mu t)\ln K_{it} + \varepsilon_{it}$

(4b) $(\ln \frac{V}{L})_{it} = c_j + dt + (\theta_j + \eta t)\ln W_{it} + v_{it}$

$i = 1 \ldots 907$
$j = 1 \ldots 13$
$t = 1 \ldots 9$

where ε_{it} and v_{it} are the "pure" error terms assumed to have zero means, constant variances and no serial correlation.

For the production function (4a) is now on the top of the hierarchy of alternatives, while at the bottom we have the same as the one in the case discussed in the previous section. In this case the number of alternatives is, however, much higher. As there in (4a) are six "effects",[16] the first round of testing implies partial tests of six null-hypotheses against the common one-hypothesis in (4a).

These six alternatives are in turn one-hypotheses for 15 null-hypotheses. In the third round these 15 are one-hypotheses for 20 null-hypotheses which in turn are one-hypotheses for 15 null-hypotheses which in turn are one-hypotheses for 6 null-hypotheses which are one-hypotheses with a common null-hypothesis, namely when all parameters are constant over the sample. For the ACMS-relation we have a related scheme.

It is easily shown that the number of alternatives in a testing scheme like the present ones is:

(5) $$\sum_{i=0}^{m} \binom{m}{i}$$

where m is the number of "effects" subject to testing, in this case six for the Cobb-Douglas relation and four for the ACMS relation. Thus we get 64 alternatives for the former and 16 for the later relation. The number of tests is, however, substantially higher, as it is given by:

16) The across and a time "effect" for each of the intercept, the labour elasticity and the capital elasticity.

Table IV. 5

*Results for the Cobb-Douglas relation for the "optimal combinations" of across and time variation of the coefficients**

Industry	Inter-cept	ln L	ln K	t ln K	El.s of capital and scale 1963: El. of capital	El. of scale	Inter-cept	t	ln L	ln K	El. of scale
Mining and Quarrying	-0.044	0.743	-0.012		0.253	0.996	-0.041		0.745	0.251	0.996
	(0.189)	(0.060)	(0.049		(0.046)	(0.025)	(0.189)		(0.060)	(0.046)	(0.025)
Food Products (Base)	2.036	0.525	0.095		0.360	0.885	-0.245		0.526	0.360	0.886
		(0.019)	(0.025)		(0.019)	(0.011)			(0.019)	(0.019)	(0.011)
Textiles	-0.175	0.683	+0.0 1)		0.265	0.948	-0.174		0.681	0.266	0.947
	(0.222)	(0.053)	(0.046)		(0.043)	(0.027)	(0.222)		(0.053)	(0.043)	(0.027)
Clothing	0.159	0.929	-0.190		0.066	0.995	0.166		0.934	0.061	0.995
	(0.194)	(0.043)	(0.036)		(0.033)	(0.028)	(0.194)		(0.043)	(0.033)	(0.028)
Wood Products	-0.591	0.915	-0.092		0.174	1.089	-0.595		0.915	0.175	1.091
	(0.177)	(0.041)	(0.039)		(0.035)	(0.023)	(0.177)		(0.041)	(0.035)	(0.022)
Pulp and Paper	0.201	0.649	-0.007	0.0042	0.258	0.907	0.186	0.0362	0.643	0.263	0.906
	(0.162)	(0.037)	(0.033)		(0.028)	(0.018)	(0.162)		(0.037)	(0.028)	(0.018)
Printing	-0.285	0.898	-0.124	(0.0003)	(0.141	1.039	-0.290	(0.0021)	0.898	0.142	0.940
	(0.173)	(0.043)	(0.036)		(0.033)	(0.024)	(0.173)		(0.043)	(0.033)	(0.024)
Basic Chemicals	0.249	0.793	-0.076		0.190	0.983	0.254		0.794	0.188	0.982
	(0.163)	(0.027)	(0.030)		(0.026)	(0.014)	(0.164)		(0.027)	(0.026)	(0.014)
Mineral Products	-0.925	0.814	0.034		0.299	1.113	-0.940		0.812	0.302	1.114
	(0.190)	(0.055)	(0.046)		(0.043)	(0.025)	(0.190)		(0.055)	(0.043)	(0.025)
Basic Steel	-0.179	0.894	-0.089		0.176	1.070	-0.197		0.889	0.181	1.070
	(0.231)	(0.061)	(0.045)		(0.042)	(0.032)	(0.231)		(0.061)	(0.042)	(0.032)
Metal Products	0.644	0.826	-0.164		0.101	0.927	0.648		0.827	0.100	0.927
	(0.206)	(0.044)	(0.044)		(0.040)	(0.022)	(0.206)		(0.044)	(0.040)	(0.022)
Non.El. Mach. }	0.095	0.999	-0.214		0.052	1.051	0.098		0.998	0.052	1.050
El. Machinery }	(0.168)	(0.039)	(0.036)		(0.032)	(0.020)	(0.168)		(0.039)	(0.032)	(0.020)
Transp. Equipm. }	-0.396	0.874	-0.093		0.172	1.046	-0.396		0.875	0.171	1.046
Misc. Products }	(0.135)	(0.024)	(0.028)		(0.023)	(0.012)	(0.135)		(0.024)	(0.023)	(0.012)
	R = 0.930	MSE = 0.2447						R = 0.929	MSE = 0.2450		

* Note that in the computations t = 59,60...67.

1) Less than $5 \cdot 10^{-6}$.

$$(6) \qquad \sum_{i=0}^{m-1} \binom{m}{i} (m-i)$$

which gives 192 tests for the Cobb-Douglas relation and 32 for the ACMS relation. Thus even with a level of 0.001 for the individual tests the upper limit of the level of the overall test when using the ordinary F-statistics method becomes quite high, at least for the former relation.

Table IV.6

Results for the ACMS relation for the "optimal combination" of variation of the coefficients.

Industry	Intercept	lnW
Mining and Quarrying	0.058 (0.348)	0.989 (0.157)
Food Products (Base)	0.624	1.114 (0.033)
Textiles	-0.143 (0.200)	1.082 (0.105)
Clothing	0.166 (0.163)	0.870 (0.085)
Wood Products	-1.029 (0.246	1.433 (0.122)
Pulp and Paper	0.454 (0.115)	0.798 (0.045)
Printing	0.003 (0.190)	0.885 (0.089)
Basic Chemicals	0.179 (0.126)	1.047 (0.052)
Mineral Products	-1.583 (0.265)	1.790 (0.124)
Basic Steel Prod.	0.350 (0.254)	0.906 (0.110)
Metal Products	0.515 (0.220)	0.714 (0.104)
Non. El. Machinery El. Machinery	-0.020 (0.173	0.957 (0.077)
Transp. Equipm. Misc. Products	-0.024 (0.140)	0.885 (0.062)

R = 0.603, MSE = 0.2122

For the Spjøtvoll method we can, however, as previously determine the level independent of the number of tests, and also in this case we use 5%.

Carrying out the multiple test by the two methods we run into a "dual optimum" problem for the Cobb-Douglas relation; the same one for both methods. Both optimums imply industry-specific intercepts and factor-elasticities, but while one implies a trend in the capital elasticity the other implies a trend in the intercept. Our testing procedures do not allow us to choose between these optimums. But we note from Table IV.5 that the former optimum yield slightly better fit. Thus if anything that one is more "optimal" than the later one.[17] We also note from Table IV.5 that the two optimums yield approximately the same results concerning the factor- and scale-elasticities.[18] Thus also in this respect the two optimums are almost perfect substitutes.

For the ACMS relation we get, however, a unique optimum. And also in this case the result is the same for the two methods. It implies that there are differences across industries both as concerns the intercept and the elasticity of substitution, while no trends are present (cf. Table IV.6). Thus this finding supports the results of the previous section suggesting that the error mean generally varies along the across-dimension only. In addition this optimum suggest that there is no trend in the elasticity of substitution over time, at least not when imposing the same trend coefficient for all industries as done here.

5. *Concluding Remarks.*

The intension with this chapter on the application of multiple tests is to show how this statistical tool can be used to analyse the nature of possible differences in parameters of structural relations along certain dimensions of a sample. As an illustration simple production and behaviour relations are used with combined cross section time-series data as the empirical base.

17) Cf. Section V.3.

18) To make the results of the two alternatives comparable we have, in the case of a trend in the capital-elasticity to compute the estimates for the average of t, $\bar{t}$ which is 63 in the computations.

But neither of the methods applied are quite satisfactory. The one based on ordinary F-statistics yields an upper limit of the overall test that is completely uninteresting in the case the number of alternatives is high. This is confirmed by formulas (5) and (6). By the other one we are in cases with a high number of alternatives at least, able to determine a less conservative level of the test. But on the other hand it has a basic weakness as it does not suite quite well in situations where the number of parameters under test is much different in the different parts of the testing scheme.

But even if the methods applied are not quite satisfactory we have dared to present some illustrations of the application of multiple tests in econometrics. And in the next chapter we have some further examples of applications, related to the ones presented in the present chapter.

Chapter V.

ON THE ESTIMATION OF TECHNICAL CHANGE; SOME PROBLEMS OF METHOD AND MEASUREMENT.

1. *Introduction*

As should be fairly evident from the discussion in the previous chapters the present empirical base is not particularly well suited for a discussion of the importance and nature of technical change and related topics. In addition to the general weaknesses such as more or less random errors of measurement and heterogenous production units etc., there are four of particular relevance in this context. 1. The time-dimension is "too short" to make an investigation of technical change by means of pure time series data possible. 2. The price data applied have some apparent weaknesses that may affect quite strongly the conclusions obtained, particularly of the degree of technical progress. 3. We have applied a common, and constant depreciation ratio for capital for all industries, and thus do not allow for differences due to the capital mix or the "recentness" of the capital stock. 4. We have no measure of the degree of utilization of the capital stock.

But provided that we don't forget these weaknesses, I think we can throw at least some light on certain aspects of technical change in Norwegian Mining and Manufacturing by means of the present empirical base. In addition we'll also try to figure out how and to what extent the data problems mentioned may have affected the results obtained.

The basic relation of all studies of technical change is, explicitly or implicitly the production function, and technical change is usually defined in the following way. Having the production function:[1)]

$$(1) \quad Y_t = f_t(X_{1t}, \ldots, X_{nt})$$

where Y is output and $X_1 \ldots X_n$ are inputs and the index t denotes period

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1) A good review of the theory of technical change and its application is: M. Brown: "On the Theory and Measurement of Technological Change". Cambridge University Press 1966. Another famous and very stimulating study is W.E.G. Salter: "Productivity and Technical Change", Cambridge University Press 1966 (Second ed). A survey (that regrettably contains a number of confusing errors) of econometric studies is: L.B. Lave: Technological Change: Its Conception and Measurement, Prentice Hall, N.J. 1966. A recent econometric study that considers a number of different specifications of the nature of technical change is: M.J. Beckmann and R. Sato: "Aggregate Production Functions and Types of Technical Change: A Statistical Analysis". The American Economic Review, March 1969. Cf. also "The Residual Factor and Economic Growth", OECD-Report, Paris 1964.

of time, technical change is identified as shifts in the function "f_t" over time in opposition to movements along the production function due to changes in the factors of production. Thus the nature of technical progress can be identified with the way in which "f_t" shifts.[2)] At this stage three main problems have to be dealt with. First, what is the proper specification of "f", second how should the output and the inputs be measured and third given a certain functional form of the production relation, what is the proper way of estimating the parameters.

We have in the previous chapters applied the Cobb Douglas relation and partly also the CES relation. This will be done in this context also. But this choice precludes a number of possible types of technical change. On the other hand, the prospects of analysing more complex types of technical change by means of the present empirical base are equally poor as for the analysis of more complex types of production functions. Instead we use various "types" of Cobb Douglas and CES-relations to investigate the nature of technical change.

The question of proper estimation of the parameters of a Cobb Douglas relation is dealt with in Chapter III and in the next section we try to figure out the effects of inproper parameter estimation on the estimate of the shift in the production function as well as on the estimates of the contributions to growth from labour and capital.

In these calculations we use the measures of output, labour and capital as defined in Chapter II.

There is a particular problem connected to the use of combined cross-section time series data for the study of technical change, namely concerning aggregation over the across-dimension. Using individual establishment-data in log-linear relations implies another type of aggregation than when using pure time-series data available for the different industries in that kind of relations. In the second part of the next section we consider how the results obtained using the two methods of aggregation conform.

2) Cf. M.J. Beckmann and R. Sato: op.cit.

In the third section of this chapter various approaches are explored to determine the nature of technical change. We are particularly interested in the neutrality/non-neutrality aspect. First we try a CES relation with factor-augmenting technical change for this purpose. Second we apply a Cobb Douglas production function with a generalized trend allowing non-neutral shifts, and third we explore the embodyment hypothesis by means of a rather rough ad hoc method. In this section we have also some computations where materials enter more explicitly into the production function and we present finally some computations in an attempt to "explain" the differences in degree of technical change between establishments. In an appendix to this chapter we present the results of some tentative calculations carried out in an investigation of two issues; namely the relevance of transitory variations in demand and costs of change.

2. *Separating Movements along, from Shifts in - the Production Function*

a. *The Effects of Biased Estimation of Factor Elasticities.*

Accepting the conventional value added Cobb Douglas relation;

$$(2) \qquad V_t = \Gamma_t L_t^{\alpha} K_t^{\beta}$$

we have that the relative growth in output is determined as;

$$(3) \qquad \frac{\dot{V}_t}{V_t} = \frac{\dot{\Gamma}_t}{\Gamma_t} + \alpha \frac{\dot{L}_t}{L_t} + \beta \frac{\dot{K}_t}{K_t}$$

where the dots as usual denote partial derivatives of the variables with respect to time. This relation is more conveniently written as

$$(4) \qquad v_t = \gamma_t + \alpha l_t + \beta k_t$$

where αl_t and βk_t are the contributions to growth in output from labour and capital respectively. Together they account for the movements along the production function while γ_t - the residual - represents the shift in the production function.

When trying to calculate these three components of growth and thus also to separate the shifts in from movements along the production function we evidently face two basic problems: What is the proper method of estimation of the parameters of the production function (in this case the factor elasticities) and what is the proper way of measuring output and the factors of production.[3]

To analyse the effects of biased estimation of parameters and growth rates of factor inputs we write the contributions to growth of labour and capital in the following way:[4]

$$5\text{ a)}\quad \widehat{\alpha l_t} = \alpha^* l_t^* = \alpha l_t + \alpha(l_t^* - l_t) + (\alpha^* - \alpha)l_t + (\alpha^* - \alpha)(l_t^* - l_t)$$

$$5\text{ b)}\quad \widehat{\beta k_t} = \beta^* k_t^* = \beta k_t + \beta(k_t^* - k_t) + (\beta^* - \beta)k_t + (\beta^* - \beta)(k_t^* - k_t)$$

Consequently we get the estimated contribution to growth from "other factors", or the shift in the production function as

$$6\text{ a)}\quad \hat{\gamma}_t = \gamma_t - \left[\alpha(l_t^* - l_t) + (\alpha^* - \alpha)l_t + (\alpha^* - \alpha)(l_t^* - l_t) + \beta^*(k_t^* - k_t) + (\beta^* - \beta)k_t^* + (\beta^* - \beta)(k_t^* - k_t)\right] + (v_t^* - v_t)$$

or

$$6\text{ b}\quad \hat{\gamma}_t = \gamma_t - (\alpha^* l_t^* - \alpha l_t) - (\beta^* k_t^* - \beta k_t) + (v_t^* - v_t)$$

On the right side of the second equation of 5 a) the first term is the "true" contribution to growth of labour input, the second term is the bias of the estimate due to biased growth rate estimation of labour input, the third is the bias due to biased estimation of the labour elasticity and the fourth is the "cross"-effect of biased growth rate- and labour elasticity estimation, a term that is zero if any one of the former two biases is zero. The interpretation of 5 b) for capital is

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3) The later one is not generally independent of the first as what is "proper estimation depends on (among other things) what are the measures applied. (Cf. Chapter III). Thus the measures may have two effects on the estimated contributions to growth; one direct via v_t, l_t and k_t and one indirect via the impact on the estimates of the factor elasticities.

4) For a related discussion of this topic see Zvi Griliches, "Production Functions in Manufacturing. Some Preliminary Results". In M. Brown (ed.) "The Theory and Empirical Analysis of Production". NBER, Studies in Income and Wealth No. 31, N.Y. 1967.

similar. (6) tells us that the net effect of these biases has to appear in the estimate on the shift in the production function.[5),6)]

In Chapter III we discussed the problem of consistent estimation of factor-elasticities. It was shown that the ordinary least square method yields highly inconsistent estimates on the factor elasticities, while a mixed method with factor-share estimation of the labour elasticity and a certain instrumental variable method for the estimation of the capital-elasticity yields "more consistent" estimates. We'll now consider the results as concerns estimated contributions to growth implied by these two methods of estimation.

The calculations are based on data for the period 1959-1967 as a whole so that all growth-rates (and thus also the various "contributions") are averages per year. By applying the OLS-method on the Cobb-Douglas relation with a "trend" we get the average percentage shift per year in the production function directly. For the Klein/Wald method the factor-elasticities and the average shift cannot easily be estimated simultaneously. Instead we accept the estimates of the factor elasticities as previously obtained and estimate the trend from the estimated residual error-values of the production function.[7)8)]

When applying the OLS-method we get a residual trend, or an average annual shift in the production function of about 3.5% for Total Mining and Manufacturing and this estimated shift is highly significant judged by a conventional t-test at 5% level. As the average annual growth in value added for the period under consideration is about 4.8% we must conclude that according to the OLS-results, shifts in the production function account for

5) (6) does also show the effect on the estimated shift of biased growth rate estimation of output.

6) Cf. D. Jorgenson and Z. Griliches: "The Explanation of Productivity Change". The Review of Economic Studies 1967.

7) That is, we estimate γ by means of the OLS-method on $\ln V - \tilde{\alpha} \ln L - \tilde{\beta} \ln K = a + \gamma t + u$ where $\tilde{\alpha}$ and $\tilde{\beta}$ are the Klein/Wald estimates on the labour and capital elasticities respectively.

8) This non-symmetric estimation of the trend of the two methods may have some impact on the outcome of the comparison of the two methods' results concerning estimated contributions to growth. Comparing the results of Table V.1 for the factor-elasticities when the OLS method is applied with the corresponding results of Table III.1 we find that including a trend the estimate on the capital elasticity becomes generally somewhat lower. But it seems to be quite unimportant for other than a few industries such as Mining and Quarrying, Pulp and Paper and Non-El. Machinery. But for these industries at least the estimated "biases" presented later in Table V.2 are presumably too large.

more than 70% of total growth in output.[9)]

For 13 of the individual industries the residual trend is positive and significant, while it is positive but not significant in one case, for Mineral Products. And it is significantly negative for one, for Printing.[10)11)] Apart from the two later industries the residual trend varies from about 9.7% for Misc. Products to about 1.8% for Transport Equipment. It is also quite high for the three more heavy industries Pulp and Paper, Basic Chemicals and Basic Steel. It is also rather high for Mining and Quarrying and Food Products.

From Table V.2 we learn that there are six industries with a decrease in labour input over the period considered. For three of these the residual trend is greater than the growth rate of output, namely for Mining and Quarrying, Clothing and Pulp and Paper.[12)] For Basic Chemicals which is also among those with a drop in labour input over time almost all of the growth in output is accounted for by the residual trend. In fact, only for one industry movements along the production function can explain more than half of the growth in output, namely Transport Equipment for which shifts

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9) The growth rates are calculated as b_x from the relation $x_{it} = a + b_x t + u_{it}$ $(x = v, l, k)$. But as shown in footnote 2 of Appendix II.8 we get the same growth rates whether we use the relation above or $\bar{x}_t = a + b_x t + \bar{u}_t$ where $\bar{x}_t = \frac{1}{I} \sum_{i=1}^{I} x_{it}$ and $\bar{u}_t = \frac{1}{I} \sum_{i=1}^{I} u_{it}$.

10) Note that the results are based on individual establishments data, and not on means per year (cf. footnote 9) above). Therefore the estimated standard deviation of $\hat{\gamma}$ contains also variation of the growth-rates between establishment, while when using geometric means it expresses the variation of the growth-rate over time only. And as $\hat{\gamma}$ is the same whether individual data or means are applied the t values and therefore probably also the degree of significance will be substantially higher when the later kind of data are applied. To be sure, it should be added that these two t values are not "contradictory", they simply concern different kinds of hypotheses. The first concerns a common shift for all establishments for all years while the latter concerns a common average per establishment-shift for all years.

11) This rather puzzling result for Printing is presumably caused by an overestimation of the output price growth over time by the official price-index for this industry. Cf. Appendix II.5.

12) For Printing $\hat{\gamma}/v$ is also above one. But as both v and $\hat{\gamma}$ are negative this implies that the total effect on the growth in output of labour and capital is positive, as is also seen from columns 4 and 5 of Table V.2.

TABLE V.1

Results for the Relation $\ln V = a + \alpha \ln L + \beta \ln K + \gamma t$ with the OLS - and the Klein/Wald/OLS - Methods of Estimation

Industry	OLS $\hat{\alpha}$	OLS $\hat{\beta}$	OLS $\hat{\gamma}$	OLS MSE	Klein/Wald/OLS[x)] $\tilde{\alpha}$	Klein/Wald/OLS[x)] $\tilde{\beta}$	Klein/Wald/OLS[x)] $\tilde{\gamma}$	Klein/Wald/OLS[x)] MSE
	0.730	0.263	0.03511		0.603	0.433	0.03036	
Tot.Min. and Man.	(0.008)	(0.007)	(0.00225)	0.272	(0.003)	(0.004)	(0.00233)	0.294
	0.756	0.242	0.04762		0.569	0.389	0.03864	
Mining and Quar.	(0.051)	(0.040)	(0.01081)	0.173	(0.017)	(0.013)	(0.01081)	0.182
	0.531	0.353	0.05564		0.525	0.420	0.05360	
Food Products	(0.026)	(0.025)	(0.00660)	0.425	(0.009)	(0.013)	(0.00661)	0.430
	0.669	0.278	0.02060		0.571	0.380	0.01770	
Textiles	(0.042)	(0.034)	(0.00670)	0.153	(0.010)	(0.016)	(0.00667)	0.155
	0.924	0.070	0.01935		0.600	0.453	0.00826	
Clothing	(0.032)	(0.024)	(0.00583)	0.134	(0.008)	(0.021)	(0.00690)	0.191
	0.910	0.180	0.02122		0.729	0.418	0.01385	
Wood Products	(0.044)	(0.038)	(0.01029)	0.283	(0.019)	(0.019)	(0.01076)	0.312
	0.674	0.238	0.06018		0.557	0.367	0.05403	
Pulp and Paper	(0.029)	(0.022)	(0.00502)	0.147	(0.007)	(0.011)	(0.00498)	0.153
	0.893	0.147	-0.01833		0.705	0.336	-0.01955	
Printing	(0.030)	(0.023)	(0.00560)	0.119	(0.009)	(0.012)	(0.00593)	0.133
	0.799	0.183	0.06417		0.496	0.527	0.05875	
Basic Chemicals	(0.038)	(0.036)	(0.01052)	0.477	(0.013)	(0.017)	0.01120)	0.541
	0.797	0.314	0.01416		0.556	0.520	0.00702	
Mineral Products	(0.051)	(0.039)	(0.00982)	0.204	(0.013)	(0.015)	(0.01011)	0.221
	0.896	0.173	0.05283		0.512	0.543	0.04009	
Basic Steel	(0.053)	(0.037)	(0.00861)	0.183	(0.012)	(0.017)	(0.00609)	0.237
	0.823	0.105	0.02960		0.620	0.335	0.02151	
Metal Products	(0.036)	(0.034)	(0.00689)	0.166	(0.011)	(0.016)	0.00706)	0.180
	1.080	-0.001	0.02991		0.641	0.393	0.01282	
Non.El.Mach.	(0.044)	(0.020)	(0.00789)	0.132	(0.003)	(0.016)	(0.00893)	0.177
	0.916	0.100	0.03885		0.643	0.464	0.02810	
El.Mach	(0.053)	(0.042)	(0.01075)	0.233	(0.015)	(0.023)	(0.01219)	0.303
	0.980	0.089	0.01767		0.772	0.304	0.01645	
Transp.Equipm.	(0.022)	(0.021)	(0.00531)	0.147	(0.011)	(0.008)	(0.00565)	0.166
	0.555	0.358	0.09661		0.614	0.309	0.09561	
Misc.prod.	(0.067)	(0.054)	(0.02136)	0.350	(0.034)	(0.042)	(0.02109)	0.357

x) α is estimated by the factor-share method, β by the size-dummies instrumental variable method, and γ is estimated by applying the OLS method on the residual:
$\ln V - \tilde{\alpha} \ln L - \tilde{\beta} \ln K = a + \gamma t + u'$

TABLE V.2. Growth Rates for Value Added, Labour and Capital. Estimated Contribution to Growth from Labour, Capital and "Shifts".[x)]

Industry	v	l	k	$\hat{\alpha}l$	$\hat{\beta}k$	$\hat{\gamma}$	$\frac{\hat{\gamma}}{v}100$	$\tilde{\alpha}l$	$\tilde{\beta}k$	$\tilde{\gamma}$	$\frac{\tilde{\gamma}}{v}100$	$(\hat{\alpha}-\tilde{\alpha})l$	$(\hat{\beta}-\tilde{\beta})k$	$\hat{\gamma}-\tilde{\gamma}$ [1)]
Tot. Min. and Man.	4.79 (0.38)	0.58 (0.30)	3.23 (0.37)	0.42	0.85	3.51	73	0.35	1.40	3.04	63	0.07	-0.55	0.47
Mining and Quar.	4.52 (0.65)	-1.61 -(0.35)	4.05 (0.95)	-1.22	0.98	4.76	105	-0.92	1.58	3.86	85	-0.30	-0.60	0.90
Food Products	7.14 (0.89)	0.89 (0.49)	3.13 (0.26)	0.47	1.10	5.56	78	0.47	1.31	5.36	75	0.01	-0.21	0.20
Textiles	2.85 (0.77)	-0.01 (0.71)	2.84 (0.73)	-0.01	0.79	2.06	72	-0.01	1.08	1.77	62	-0.00	-0.29	0.29
Clothing	1.42 (0.61)	-0.73 (0.58)	2.28 (0.43)	-0.67	0.16	1.94	137	-0.44	1.03	0.83	58	-0.23	-0.87	1.11
Wood Products	4.00 (1.57)	1.26 (0.58)	4.06 (0.59)	1.15	0.73	2.12	53	0.92	1.70	1.39	35	0.23	-0.97	0.73
Pulp and Paper	5.73 (0.65)	-1.60 (0.35)	3.31 (0.78)	-1.08	0.79	6.02	105	-0.89	1.21	5.40	94	-.019	-0.93	0.62
Printing	-1.52 (0.63)	0.21 (0.38)	0.86 (0.22)	0.19	0.13	-1.83	120	0.15	0.29	-1.96	129	0.04	-0.16	0.13
Basic Chemicals	6.53 (0.61)	-0.18 (0.56)	1.42 (0.22)	-0.14	0.26	6.42	98	-0.09	0.75	5.88	90	-0.05	-0.49	0.54
Mineral Products	2.12 (0.60)	-0.33 (0.46)	3.08 (0.38)	-0.26	0.97	1.42	67	-0.18	1.60	0.70	33	-0.08	-0.63	0.72
Basic Steel	8.64 (0.64)	2.57 (0.25)	6.11 (0.41)	2.30	1.06	5.28	61	1.32	3.32	4.01	46	0.99	-2.26	1.27
Metal Products	5.35 (0.40)	2.21 (0.23)	5.47 (0.91)	1.82	0.57	2.96	55	1.37	1.83	2.15	40	0.45	-1.26	0.81
Non-El. Mach.	3.40 (0.91)	0.38 (0.86)	4.76 (1.03)	0.41	-0.00	2.99	88	0.24	1.87	1.28	38	0.17	-1.87	1.71
El.Mach	7.11 (0.92)	2.96 (0.66)	5.17 (0.67)	2.71	0.52	3.89	55	1.90	2.40	2.81	40	0.81	-1.88	1.08
Transp.Equipm.	3.71 (0.35)	1.78 (0.30)	2.29 (0.12)	1.74	0.20	1.77	48	1.37	0.70	1.65	44	0.37	-0.49	0.12
Misc. Products	15.06 (2.20)	6.23 (1.44)	5.40 (1.05)	3.46	1.93	9.66	64	3.83	1.67	9.56	63	-0.37	0.26	0.10

x) All numbers are percentages. About growth rates cf. footnote 12.

1) We should have $(\hat{\alpha}-\tilde{\alpha}) + (\hat{\beta}-\tilde{\beta}) = -(\hat{\gamma}-\tilde{\gamma})$, but as the right and left side are computed independently there are small differences due to rounding errors.

in the production function account for 48% "only".[13]

These are the main findings concerning shifts in and movements along the production function when the OLS-method is applied. Turning now to the Klein/Wald/OLS method of estimation we know that the results must be somewhat different. We showed in Chapter III that compared to the OLS-method the Klein/Wald/OLS method yielded smaller estimates on the labour elasticity and bigger estimates on the capital elasticity.[14] This must necessarily lead to a generally lower shift as for all industries except one capital has grown faster than labour (which as pointed out has shown a drop for six industries).

We note, however, from the results of the Klein/Wald/OLS method presented in Table V.1 that for most industries the reduction of the estimated shift is not very impressive. For Total Mining and Manufacturing the trend is 3% per year as compared to 3.5% according to the OLS method. Still, the shift in the production function accounts for almost two thirds, or about 63% of the growth in output. But for a few individual industries there are some quite notable differences compared to the results yielded by the OLS method.[15] For Non-Electrical Machinery the contribution to growth from "shifts" drops from 88% to 38%. For Clothing the corresponding percentages are 137 and 58. The drop is also substantial for Mineral Products. On the other hand, the differences are rather unimportant for Food Products, Textiles, Basic Chemicals, Transport Equipment and Misc. Products. But even if the drop in the estimated shift is quite low or moderate for most industries there are now, at least, seven industries for which movements along the production function account for more than half of the growth in output.

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13) But this "low" contribution of technical change is, however, quite likely a result of that the output price growth is overstated, and consequently that the growth in output is understated. Cf. Appendix II.2 and Appendix II.5

14) There is also another difference as the mixed method yields a higher estimate on the scale-elasticity. But we have not separated the difference due to different levels of the scale elasticity from the difference due to a pure twist in the factor-elasticity estimates. As we are primarily interested in the total contribution to growth in output from each of labour and capital, we do not compute their contribution provided constant returns to scale with the effect of degree of returns to scale as separate component.

15) According to the conventional t-test at 5% level we have now that the shift is significantly positive for 10 industries, and positive but not significant for 4. These later are, in addition to Mineral Products; Clothing, Wood Products and Non-El. Machinery. The shift is still negative for Printing.

If we believe in the later set of estimates as a consistent one we get the OLS biases in the estimated contributions to growth of labour and capital as $(\hat{\alpha} - \tilde{\alpha})l$ and $(\hat{\beta} - \tilde{\beta})k$. They are presented in columns 12 and 13 of Table V.2. As the growth in labour input has been quite low, the bias in the estimated contribution to growth due to inconsistent estimation of the labour elasticity is also fairly low for most industries. It is more important for the two industries Basic Steel and Electrical Machinery which rank third and second respectively as concerns growth in labour input.[16)]

The bias due to inconsistent estimation of the capital elasticity is generally much more important. This is particularly the case for the industry groups 34-37, or the industries Basic Steel, Metal Products, El.- and Non-El. Machinery.[17)] Thus, evidently, consistent estimation of the factor elasticities is of decisive importance for correct evaluation of the contributions to growth of labour and capital. It is in our case somewhat less important for the problem of separation of shifts in- from movements along the production function.

b. A Problem of Aggregation.

The way the calculations of the previous section are carried out implies a particular type of aggregates, namely geometric means. Thus the implied aggregate production function is:

$$(7) \qquad \overline{\ln V_t} = a + \alpha \overline{\ln L_t} + \beta \overline{\ln K_t} + \gamma t$$

$$\text{where} \quad \overline{\ln X_t} = \frac{1}{I} \sum_{i=1}^{I} \ln X_{it} \qquad (X = V, L, K)$$

And therefore the aggregate growth rates applied by us are also the growth rates of the aggregates in (7). Thus these growth rates are unweighted means of the individual growth rates, as the growth rate of X_t, x_t is equal to $\frac{1}{I} \sum_{i=1}^{I} x_{it}$.

16) The industry that rank highest, Misc. Products, has a negative bias due to the fact that the "consistent" method of estimation leads to a bigger estimate on the labour elasticity than the OLS-method.

17) But at least for Non-El. Machinery this bias may be "overestimated" due to the non-symmetrical estimation of the residual trend. Cf. footnote 8) above.

In this section we will investigate what happens if we instead use the same method of aggregation as used to obtain the numbers published for output, labour and capital etc. by industry in for instance the Annual Industrial Production Statistics. These aggregates are arithmetic sums, and differentiating these with respect to time yields:

$$(8) \qquad x'_t = \frac{\sum_{i=1}^{I} X_{it}\, x_{it}}{\sum_{i=1}^{I} X_{it}} \qquad\qquad (X = V, L, K)$$

Thus the growth rates of these arithmetic sums are weighted averages of the individual growth rates. If there is a positive correlation between X_{it} and its growth rate x_{it} our unweighted growth-rate x_t understates the total growth of X of a sector judged by the weighted index x'_t. And clearly the opposite is true if X_{it} and x_{it} are negatively correlated.[18)]

This method of aggregation does also have some effects on the price-index for output. Measuring aggregate real output as $V_t = \sum_{i=1}^{I} V_{it}$ and having correspondingly output in current prices as $V^*_t = \sum_{i=1}^{I} V^*_{it}$ we have an aggregate price index that, to be consistent must be equal to $P_{Vt} = \frac{V^*_t}{V_t}$, which corresponds to using a Paasche price index formula.[19)]

The separation of the price and quantity components when using weighted indices is the same for gross production and materials as for value added. To figure out the price movements of gross production and materials for the different sectors in Mining and Manufacturing the weighted price-indices and their trends are presented in Table V.3.[20)] And in Table V.4 the weighted index of value added in constant prices and the corresponding

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18) In Appendix V.2 correlation coefficients between $\overline{\ln V_i}$, $\overline{\ln L_i}$, $\overline{\ln K_i}$ and their trends are presented.

19) This is evident also because the computation of V_t corresponds to using the Laspeyre quantity index.

20) The price indices of gross production and materials for Mining and Manufacturing are 114 and 110 respectively in 1967 according to the National Accounts aggregates. Thus they are somewhat higher than those computed by us for Total Mining and Manufacturing.

TABLE V.3

Price Indices for Gross Production and Materials (Base 1961)

Industry	Gross Production									Materials									Trends in % p.a.	
	59	60	61	62	63	64	65	66	67	59	60	61	62	63	64	65	66	67	Gr.Pr.	Mat.s.
Tot. Min. and Man.	98	98	100	101	100	104	107	109	110	100	100	100	100	100	103	106	107	107	1.54 (0.16)	1.05 (0.20)
Mining and Quarr.	101	103	100	101	100	106	112	115	110	105	111	100	100	102	103	104	105	108	1.60 (0.43)	0.09 (0.46)
Food Products	102	100	100	105	102	109	110	113	116	103	100	100	102	103	111	112	114	116	1.84 (0.29)	2.00 (0.34)
Textiles	95	95	100	102	104	107	108	108	108	99	101	100	100	101	103	100	98	96	1.82 (0.23)	-0.29 (0.25)
Clothing	95	95	100	103	105	107	111	113	113	100	97	100	100	101	103	105	105	104	2.42 (0.18)	0.87 (0.18)
Wood Products	93	94	100	102	100	106	112	114	115	90	94	100	101	101	104	112	112	110	2.80 (0.25)	2.66 (0.31)
Pulp and Paper	99	100	100	99	98	100	103	102	101	100	99	100	99	100	101	107	106	105	0.36 (0.15)	1.01 (0.25)
Printing	92	93	100	105	113	119	127	136	145	97	98	100	103	105	106	110	112	115	5.92 (0.22)	2.15 (0.08)
Basic Chemicals	101	101	100	98	99	103	105	106	99	105	103	100	95	98	104	108	110	104	0.29 (0.36)	0.63 (0.59)
Mineral Prod.	99	95	100	101	101	101	101	102	104	109	108	100	100	101	101	98	96	100	0.80 (0.21)	-1.20 (0.35)
Basic Steel	96	99	100	98	91	98	102	103	104	98	99	100	99	97	101	102	104	105	0.81 (0.45)	0.82 (0.21)
Metal Prod.	95	96	100	102	102	106	110	111	108	97	100	100	98	96	99	102	103	103	1.97 (0.21)	0.60 (0.27)
Non-El. Mach.	93	92	100	105	105	107	110	114	116	97	97	100	101	99	102	104	105	106	2.91 (0.27)	1.13 (0.14)
El. Mach.	101	97	100	102	107	108	112	118	116	101	106	100	100	101	104	109	116	113	2.37 (0.31)	1.65 (0.44)
Transp. Equipm.	97	97	100	105	109	113	120	122	125	95	96	100	102	100	103	107	108	109	3.58 (0.19)	1.75 (0.17)
Misc. Prod.	117	112	100	104	105	105	106	107	109	113	109	100	98	94	98	99	100	98	-0.50 (0.61)	-1.37 (0.58)

TABLE V.4

Volume and Price Indices and Trends for Value Added. (Base 1961).

Industry	Volume indices of $V_t = \sum^{I} V_{it}$									Price indices computed as $P^*_v = V^*_t/V_t$									Trends in % p.a.[1]	
	59	60	61	62	63	64	65	66	67	59	60	61	62	63	64	65	66	67	P_v	P^*_v
Tot. Min. and Man.	90	97	100	103	112	120	126	131	138	96	97	100	103	101	105	109	112	114	2.97 (0.22)	2.24 (0.17)
Mining and Quarr.	96	100	100	113	121	126	132	136	140	100	101	100	101	100	107	114	117	110	2.87 (0.22)	1.89 (0.46)
Food Products	74	80	100	96	105	104	116	127	124	101	101	100	111	101	105	106	110	117	1.31 (0.65)	1.54 (0.49)
Textiles	95	107	100	108	107	113	114	112	114	91	88	100	104	107	112	117	120	122	4.54 (0.37)	4.17 (0.37)
Clothing	98	105	100	107	108	114	108	115	123	90	93	100	105	109	113	119	123	123	4.54 (0.37)	4.25 (0.27)
Wood Products	74	100	100	101	122	131	145	142	138	101	93	100	105	100	108	112	117	124	3.68 (0.53)	2.89 (0.53)
Pulp and Paper	100	104	100	98	106	125	134	132	135	98	103	100	97	95	97	93	92	93	-2.21 (0.43)	-1.11 (0.27)
Printing	98	107	100	107	98	109	104	96	96	89	90	100	107	121	130	142	161	179	8.88 (0.44)	9.07 (0.41)
Basic Chemicals	95	96	100	101	119	132	139	146	161	97	98	100	102	100	100	101	101	93	0.72 (0.60)	-0.12 (0.38)
Mineral Prod.	91	105	100	107	112	132	138	138	158	91	87	100	102	101	102	106	111	111	4.06 (0.32)	2.73 (0.45)
Basic Steel	93	106	100	104	120	129	143	145	152	91	100	100	96	83	92	101	101	102	0.81 (0.60)	0.80 (0.92)
Metal Prod.	99	92	100	111	131	120	119	140	148	93	92	100	106	108	115	119	120	125	4.05 (0.25)	4.01 (0.25)
Non El.Mach.	85	99	100	110	109	119	124	117	124	90	87	100	109	110	112	115	125	128	4.76 (0.50)	4.67 (0.48)
El. Mach.	75	94	100	101	111	121	120	121	130	102	90	100	103	113	111	116	120	120	3.03 (0.49)	3.19 (0.62)
Transp. Equipm.	90	89	100	100	99	105	106	116	132	99	98	100	109	117	124	136	141	147	6.78 (0.32)	5.73 (0.42)
Misc. Products	62	72	100	105	105	122	158	155	163	121	115	100	110	117	113	114	115	118	-0.43 (1.16)	0.28 (0.26)

1) P_v is the unweighted price-index, and P^*_v is the weighted one.

TABLE V.5

Trends of Aggregates for Value Added Labour and Capital and Contributions to Growth from Labour, Capital and "Shifts".

Industry	Trends in % p.a.			Contributions to Growth*			The % of growth due to shift
	ΣV^{I}_{it}	ΣL^{I}_{it}	ΣK^{I}_{it}	Labour	Capital	"Shifts"	
Tot. Min. and Man.	5.33 (0.21)	0.70 (0.15)	3.50 (0.41)	0.42	1.52	3.39	63.6
Min. and Quarr.	5.11 (0.38)	-1.50 (0.38)	2.78 (0.59)	-0.85	1.08	4.88	95.5
Food Products	6.39 (0.78)	0.93 (0.47)	3.77 (0.04)	0.49	1.58	4.32	67.6
Textiles	1.88 (0.46)	-0.50 (0.55)	1.98 (0.71)	-0.29	0.75	1.42	75.5
Clothing	2.31 (0.41)	-0.63 (0.32)	0.93 (0.33)	-0.38	0.42	2.27	98.3
Wood Products	7.57 (1.19)	1.69 (0.14)	6.68 (0.81)	1.23	2.79	3.55	46.9
Pulp and Paper	4.54 (0.84)	-1.60 (0.39)	3.42 (0.84)	-0.89	1.26	4.17	91.8
Printing	-0.53 (0.64)	0.77 (0.46)	3.31 (0.68)	0.54	1.11	-2.18	-
Basic Chemicals	7.16 (0.57)	0.53 (0.23)	-1.10 (0.28)	0.26	-0.58	7.48	104.5
Mineral Prod.	6.46 (0.61)	0.37 (0.29)	6.39 (0.51)	0.21	3.32	2.93	45.3
Basic Steel	6.36 (0.63)	1.73 (0.16)	6.29 (0.90)	0.89	3.42	2.05	31.6
Metal Prod.	5.50 (0.87)	2.62 (0.26)	5.51 (0.84)	1.62	1.85	2.03	36.9
NonEl.Mach.	4.19 (0.67)	2.93 (1.14)	3.51 (0.80)	1.88	1.38	0.93	22.2
El. Mach.	5.82 (0.85)	0.28 (0.79)	4.57 (0.27)	0.18	2.12	3.52	60.5
Transp. Equipm.	4.15 (0.61)	2.18 (0.28)	2.81 (0.26)	1.68	0.85	1.62	39.0
Misc. Products	12.04 (1.24)	6.07 (1.22)	6.14 (0.97)	3.73	1.90	6.41	53.2

* Computed by means of the Klein/Wald estimates on the factor-elasticities of the Cobb-Douglas production function presented in Table III.6.

price index are presented, together with the trends of the unweighted and weighted price indices.[21)]

For Total Mining and Manufacturing the trend of the weighted price-index is somewhat lower than that of the unweighted one implying that on the average smaller units have a somewhat more rapid price growth than larger ones.[22)] There are, however, substantial differences between industries in this respect, but generally the difference between the two price-trends goes in the same direction as for the total.[23)]

In Table V.5 the weighted growth rates of value added in constant prices, labour and capital are presented. By comparing them to the unweighted growth-rates presented in Table V.2 we find for Total Mining and Manufacturing that the individual growth rates for all three variables must be positively correlated with their weights, or in other words the level of the corresponding variables.[24)] But as the differences of the weighted and unweighted growth rates go in the same direction for both output and the inputs it has little impact on the relative position of the computed contributions to growth from the three sources, labour, capital and "shift". The later source accounts for 63.6% of the growth in output, while using unweighted growth rates this percentage is 63.5% or approximately the same.[25)] The corresponding percentages are for labour 7.9 and 7.3 and thus for capital 28.5 and 29.2.

Even if it does not matter much what kind of aggregates we use for the total, it really makes a substantial difference for some of the individual industries. The more notable differences we have for: Clothing due to a substantially lower weighted than unweighted capital growth; Basic Chemicals which have negative growth in labour input and a positive growth in capital input when using unweighted growth rates, while the

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21) According to the National Accounts data the volume and price indices for value added of Mining and Manufacturing are 139 and 120 in 1967. The volume index is very close to the one computed for value added by us, while the price-index is somewhat higher.

22) Cf. (8) above.

23) There are also substantial differences between industries as concerns the level of the price-trend whether based on a weighted or an unweighted price-index. Some of these differences are, however, presumably a result of the way the price-indices for value added of some of the national account sectors are computed. Cf. Appendix II.5.

24) Cf. also Appendix V.2.

25) As pointed out in Table V.10 the computations are carried out by means of the Klein/Wald estimates on the factor elasticities.

opposite is the case when using weighted growth rates; for Non-El. Machinery with a substantially higher weighted than unweighted growth rate for labour; and for El.Machinery for which the opposite is true.

The main conclusion of this section is therefore that when calculating the contributions to growth from labour, capital and "shifts" by our data for an industry we should use the Klein/Wald estimates on the factor elasticities and the weighted growth rates of output and the inputs. This seems to be the best we can manage to do. But as pointed out even the calculated "contributions" thus obtained are for some industries rather misleading due to problems with the separation of the price and the quantity components of output in current prices.

3. *On the Nature of Technical Change.*

a) *Introduction*

Even if the direct results of our regressions based on combined cross-section time-series data may be misleading concerning the importance of technical change when identified as shifts in the production function, that kind of results may be useful when trying to analyse the nature of technical change. And in this second main section of this chapter we will among other things try to analyse the nature of the technical change in Norwegian Mining and Manufacturing by some further regression results.

In this analysis we will consentrate our efforts on the issue whether technical change is neutral or non-neutral. Adopting the Hicksian definition, we must have that the marginal rate of technical substitution

$$(9)\quad \frac{\dfrac{\partial V_t}{\partial L_t}}{\dfrac{\partial V_t}{\partial K_t}} = \frac{m_L}{m_K}$$

is constant over time in case of neutrality. That is:

$$(10)\quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = 0$$

where the dot indicates derivative with respect to time.

It is easily shown that this is the case for a Cobb Douglas relation with a "traditional" residual trend: Technical change is neutral or purely product augmenting. If (10) is negative, technical change is non-neutral

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26) Cf. Appendix II.5.

and of the labour-saving type, as the marginal productivity of capital has increased as compared to that of labour. And if (10) is positive we have correspondingly non-neutral and capital-saving technical change.

We try two different approaches to analyse this issue. First we apply a CES-function, without much success, however. Second we apply a generalized Cobb Douglas relation with trends both in the intercept and in the factor-elasticities. In this context we also try multiple test procedures to determine an "optimal" combination of trends in the parameters of the Cobb Douglas relation.

We also try to investigate the relevance of the embodyment hypothesis by an ad-hoc method of testing. And we have a separate sub-section of the role of materials concerning technical change. Finally in the present section we try to explain why the degree of technical change varies between establishments.

b) *Technical Change and the CES Relation.*

Assuming that both labour and capital consist of a quality and a quantity component and that the later ones are properly measured by L and K respectively and denoting the quality components as Q_L and Q_K we have the CES relation. 27)

$$(11) \qquad V_t = \left[(Q_{Lt}L_t)^{-\rho} + (Q_{Kt}K_t)^{-\rho} \right]^{-\frac{1}{\rho}}$$

Assuming that the quality components grow exponentially over time, we have:

$$(12) \qquad V_t = \left[(Q_{LO}e^{q_L t} L_t)^{-\rho} + (Q_{KO}e^{q_K t} K_t)^{-\rho} \right]^{-\frac{1}{\rho}}$$

And assuming now in addition that profit is maximized with respect to both factors we get 28),

$$(13) \qquad \frac{K_t}{L_t} = \frac{Q_{LO}}{Q_{KO}} \left(\frac{S_{Lt}}{S_{Kt}}\right)^{\frac{\rho}{1-\rho}} e^{(q_L - q_K)t}$$

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27) Cf. P.A. David and Th. van de Klundert; "Biased Efficiency Growth and Capital-Labour Substitution in the U.S. 1899-1960". American Economic Review 1965.

28) Having constant returns to scale this assumption does clearly not hold if there are perfect competition in all markets. There are, however, various ways of "saving" this assumption, for instance that the elasticity of scale in fact is below one and therefore (11) is an approximation to the true production function.

or in logs,

$$(14) \qquad \ln \frac{K_t}{L_t} = \ln \frac{Q_{LO}}{Q_{KO}} + \frac{\sigma}{1-\sigma} \ln \left(\frac{S_L}{S_K}\right) + (q_L - q_K)t$$

where S_L and S_K are the shares of labour and capital respectively.[29]

We have now that the relative change in the marginal rate of technical substitution is determined as:

$$(15) \qquad \left(\frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K}\right) = \frac{\sigma-1}{\sigma} (q_L - q_K)$$

This implies that if the rate of growth in labour quality is higher than that of capital, technical change is of the labour saving type provided the elasticity of substitution is below one. Technical change is also labour saving if the growth rate of the quality of capital is above that of labour and the elasticity of substitution is above one. Thus technical change is capital saving if the rate of growth in labour quality is higher than that of capital and the elasticity of substitution is above one or if the growth rate of the quality of capital is above that of labour and the elasticity of substitution is below one.

We will try to determine the sign and size of (15) by estimating the parameters of relation (14). The basic assumptions for obtaining unbiased estimates on the parameters of this relation by means of the OLS-method are not fullfilled, however. By the assumption made $\ln \frac{S_L}{S_K}$ is no exogeneous variable as S_L is equal to $\frac{WL}{V}$ and thus $S_K = 1 - \frac{WL}{V}$ where both V and L are endogenous. In addition the estimate of $\frac{\sigma}{1-\sigma}$ may be distorted by spurious correlation due to errors of measurement in labour quantity input L.[30]

There are various ways to reduce the effects of these errors, and to investigate the performance of (14) and try to figure out the importance of the errors involved, the OLS-method was applied on the following kinds of data:

a) Pooled cross-section time-series
b) First differences
c) Pure time series

In addition the size-dummies instrumental variable method was applied on (14) without a trend, for the pooled cross-section time-series data.[31]

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29) We should note that this relation breaks down if the production function is of Cobb-Douglas type, that is $\sigma = 1$.

30) These errors of measurement will tend to bias the estimate on $\frac{\sigma}{1-\sigma}$ downwards, but the magnitude of the bias is not easily figured out due to the rather complex way L enters $\ln S_L/S_K$.

31) In this case as well as in the case when first differences were applied the effect of the t-variable has to be computed from the residuals.

And for all of these four cases both L and N, the number of employees, were applied as the input-measure.

The results of these experiments can be summarized in the following way. All types of data gave generally negative point-estimates on $\frac{\sigma}{1-\sigma}$, but not less than minus 1. And this implies that the point-estimate on σ is negative. Clearly a negative σ does not make much sense, and as expected the pooled cross-section time-series data gave the poorest results. First differences behaved much better, particularly when N was applied as labour-input measure. Thus it "helps" both to eliminate the cross-sectional level of the variables and to introduce a labour input variable that is measured independently from labour's share. We didn't gain anything by using the size-dummy method on the pooled data, either when L on N was used, as compared to the use of first differences. Finally, pure time-series gave very shaky results, and even for that method the point estimate on σ is negative for Total Mining and Manufacturing, when using L as labour input measure.

However, for those kinds of data where the errors of different kinds are less important (first differences and pure time series with N as labour input measure) $\frac{\sigma}{1-\sigma}$ isn't significantly different from zero at 5% level for most industries.[32] This may allow us to conclude that the short run elasticity of substitution is in fact very low. This is supported by the results of the ACMS-relation implied by (12).

$$\ln \frac{V}{L} = a + \sigma \ln W + (1-\sigma)q_L t \tag{16}$$

that for pure time series data for Total Mining and Manufacturing yields estimates on σ of .075, and on $(1-\sigma)q_L$ of .0387. But none of the parameters are significantly positive at 5% level.

The results for the trend of (14) are very little affected of what kind of data is applied. Its coefficient is, with a few exceptions significantly positive. For the pure time series data when L is applied as the labour input measure we get for Total Mining and Manufacturing $q_L - q_K = .0278$. This result together with the result of the trend of (16) implies that $q_L = 4.17\%$ and $q_K = 1.39\%$. The total growth of labour and capital input is according to these results $\ell + q_L = 0.58\% + 4.17\% = 4.75\%$

32) For pure time series it was significantly different from zero for none. But clearly the estimate concerned is very little efficient due to the low number of degrees of freedom.

and $k + q_K = 3.23\% + 1.39\% = 4.62\%$ respectively, for Total Mining and Manufacturing. And accepting that the elasticities of labour and capital are about .6 and .44 respectively we get their average contribution to growth as 2.85% and 2.03% while the growth in value added is 4.79%, which implies that the residual trend is -0.09%.

As we generally have $q_L - q_K$ significantly positive we have evidence to conclude that the technical change is of labour-augmenting type. The results of (14) and (16) do also suggest that the short run elasticity of substitution is quite low. These results are, however, very shaky and they are of little or no help for us when trying to figure out more exactly what is the probable level of the short run elasticity of substitution. But I think they provide sufficient evidence to conclude that this parameter is below one. This implies that the labour-augmenting technical change also is of labour saving type. Thus, at least tentatively, the findings of this section lènd support to the findings of the nature of technical change of the following section.

c) The Results of a Generalized Cobb Douglas Relation.

The calculations in section 2 of this chapter are based on the assumption of constant factor-elasticities. If we adopt the following generalized Cobb-Douglas relation;

$$(17) \qquad V_t = \Gamma_o L_t^{(\alpha + \gamma_1 t)} K_t^{(\beta + \gamma_2 t)} e^{\gamma_o t}$$

it is possible to study more complex types of shifts in the production function. This will be done by estimating the parameters by the OLS method. This is easily done as the relation is linear in the parameters when transformed to logaritms:

$$(18) \quad \ln V_t = \ln \Gamma_o + \alpha \ln L_t + \beta \ln K_t + \gamma_1 t \ln L_t + \gamma_2 t \ln K_t + \gamma_o t$$

The marginal rate of technical substitution is now:

$$(19) \qquad \frac{m_L}{m_K} = \frac{\alpha + \gamma_1 t}{\beta + \gamma_2 t}$$

And the relative change over time in this rate is

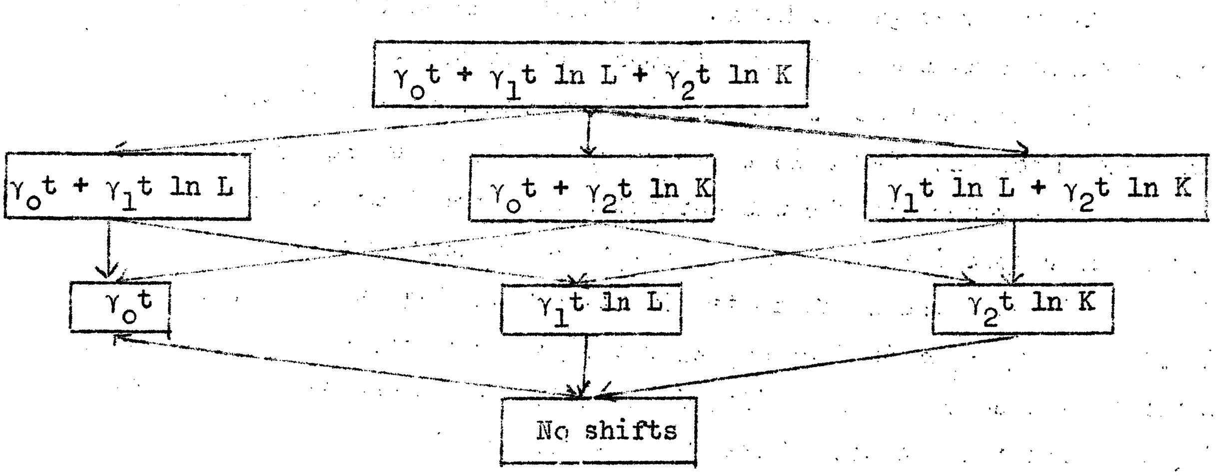
$$(20) \qquad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = \frac{\gamma_1 \beta - \gamma_2 \alpha}{(\alpha + \gamma_1 t)(\beta + \gamma_2 t)}$$

Thus, provided that the elasticities of labour and capital are always positive, for all t, or in our case that $\alpha + 59\gamma_1$, $\alpha + 67\gamma_1$, $\beta + 59\gamma_2$ and $\beta + 67\gamma_2$ are all positive, the non-neutrality of technical change is determined by the sign of $\gamma_1\beta - \gamma_2\alpha$. If it is positive technical change is capital saving and if it is negative technical change is labour saving. Neutrality is always secured if $\gamma_1 = \gamma_2 = 0$, or the ordinary Cobb Douglas relation with (or without) a "traditional" residual trend.[33] Given the assumption above about positive factor elasticities of labour and capital we have always a capital saving bias if $\gamma_1 > 0$ and $\gamma_2 = 0$ or $\gamma_1 = 0$ and $\gamma_2 < 0$ and labour saving bias if $\gamma_1 = 0$ and $\gamma_2 > 0$ or $\gamma_1 < 0$ and $\gamma_2 = 0$.

To analyse the nature of technical change by means of (18) isn't straightforward, however, as the estimation of the parameters of that relation almost certainly will lead to point-estimates that imply either labour saving or capital saving technical change even in cases when the hypothesis of purely neutral technical change cannot be rejected at any reasonable level of significance. Therefore, another approach is adopted, but the results of that one is compared to the results obtained by means of the estimates of (18) with no constraints on the parameters. This approach is to apply a multiple test-procedure related to the one applied in chapter IV. The point of departure is the testing scheme presented in fig. V.1.

Fig. V.1

A Scheme for Multiple Tests of Types of Shifts in the Production Function



33) There are clearly an infinite number of other parameter values that yield neutral technical change. But provided that we shall have positive factor elasticities they must satisfy the following condition $a(\alpha + \gamma_1 t) = \beta + \gamma_2 t > 0$ where a is any positive number.

In the same way as previously we carry out the tests downwards - from the more to the less general type of shift. In this case, when the number of parameters under test is the same in all stages, namely one, the ordinary F-statistics method and the Spjotvoll F-statistics method yield the same results, given roughly comparable levels of the two tests. For the first method we choose .5% level of the individual tests and thus the corresponding upper limit of the overall test is 12%. We then get the same results as for the second method with level 10%.[34)]

This is no roboust procedure of analysing the nature of technical change, however, as we for only 6 of the individual industries and for Total Mining and Manufacturing get a unique optimal type of shift. The results about the optimums found are presented in Table V.6 and they tell us that for most industries different types of shifts implying different conclusions about the nature of technical change are equivalent or almost equivalent as concerns fit to the data. But accepting the one that yields the lowest mean square error in case of more than one optimum we have in Table V.7 summarized the findings about the nature of technical change obtained from the multiple test-procedure. In this table the corresponding findings when using the unconstrained generalized Cobb Douglas relation are also presented.

Even if the uniqueness of the results is not too apparent they suggest quite strongly that the shift in the production function is substantially more complex than assumed for the previous computations when analysing the residual factor. Only for a few industries neutrality has some support from the present computations.

On the average for Mining and Manufacturing the results suggest that technical change is of the labour saving type.

But as the results are obtained by means of the OLS-method there are some important biases in these results. We know from Chapter III

that generally the OLS estimates of α and β are biased upwards and downwards respectively. Thus $\gamma_1\beta - \gamma_2\alpha$ is presumably biased downwards. That is, in case the shift is truely labour saving it is overstated. Also if it is neutral, or in fact capital saving we may estimate it to be labour saving. The denominator of (10) is also biased downwards as the OLS-method implies that the product of the factor-elasticities, and in our case also their sum are biased downwards. Apparently this tend to make the biases still worse, except in the case when the estimated shift in the production function is capital saving.

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34) Cf. Chapter IV.

TABLE V.6

Results of Multiple Tests of Types of Shifts in the Production Function over Time[1)]

Industry	"Optimal" Types of Shifts in the Prod. Functions		
Tot. Min. and Man.	-.0108 t ln L + .0108 t ln K (.0032) (.0020)		
Mining and Quarr.	.0476 t (.0108)	.0091 t ln L* (.0020)	.0055 t ln K (.0012)
Food Products	.0556 t (.0066)		
Textiles	.0206 t* (.0067)	.0034 t ln L (.0012)	.0021 t ln K (.0008)
Clothing	.0703 t ln L (.0243)	.0039 t ln K* (.0011)	
Wood Products	-.1117 t + .0290 t ln L* (.0426) (.0090)	-.1586 t + .0235 t ln K (.0614) (.0079)	
Pulp and Paper	.1599 t - .0103 t ln K (.0394) (.0040)		
Printing	.0365 t ln L - .0236 t ln K (.0115) (.0067)		
Basic Chemicals	.0071 t ln K (.0011)		
Mineral Prod.	None		
Basic Steel	-.0707 t ln L + .0497 t ln K (.0201) (.0127)		
Metal Prod.	.0296 t (.0069)	.0054 t ln L* (.0012)	.0035 t ln K (.0008)
Non El.Mach.	.0299 t (.0079)	.0051 t ln L (.0014)	.0035 t ln K* (.0009)
El. Mach.	.0389 t (.0108)	.0068 t ln L* (.0018)	
Transp. Equipm.	.0177 t (.0053)	.0032 t ln L* (.0009)	.0021 t ln K (.0006)
Misc. Products	.0966 t* (.0214)	.0114 t ln K (.0027)	

1) When more than one type of shift is reported, the one that has the lowest MSE value is marked with a star; *

TABLE V.7

The Nature of the Shift in the Production Function Analysed by the Generalized Cobb Douglas Production Function and the "Optimal" Type of Shift

Industry	Nature of Shift	
	Generalized Prod. Funct.	"Optimal" Type of Shift
Total Mining and Manufacturing	Labour Saving	Labour Saving
Mining and Quar.	Capital "	Capital " *
Food Products	Labour " 1)	Neutral
Textiles	Capital "	Neutral*
Clothing	Capital "	Labour Saving*
Wood Products	Labour "	Capital " *
Pulp and Paper	Capital "	Capital "
Printing	Capital "	Capital "
Basic Chemicals	Labour "	Labour "
Mineral Prod.	Labour "	None
Basic Steel	Labour "	Labour Saving
Metal Prod.	Labour "	Capital " *
Non-El. Mach.	Labour "	Labour " *
El. Mach.	Capital "	Capital " *
Transp. Equipment	Capital "	Capital " *
Misc. Prod.	Labour "	Neutral*

* No unique optimum. Cf. Table V.6.

1) $\gamma_1\beta - \gamma_2\alpha$ is very low, -.00073, while for Total Mining and Manufacturing we get -.01310.

Now, clearly if the assumptions of functional form and of profit maximization with respect to labour are true, factor saving technical change should also show up in the factor-share of labour. But investigating this question we find that neither for Total Mining and Manufacturing nor for any of the individual industries there is a trend over time of the average share of labour in value added that is significant at 1% level. At 5% level there is one, namely Pulp and Paper with a significantly positive trend. Therefore, if carrying out a multiple test based on ordinary F-statistics with 1% level of the individual tests when the trend in the labour-elasticity is estimated by the trend in labour's share, we can get an optimum shift that implies capital saving technical change only if the trend in the capital elasticity is significantly negative.[35)]

Carrying out the analysis on the relation

$$(21) \qquad \ln V_t - S_{Lt} \ln L_t = a + (\beta + \gamma_2 t) \ln K_t + \gamma_0 t$$

we have that the test procedure when applied on Total Mining and Manufacturing still gives an optimum that implies labour saving technical change, even if a neutral trend is also an optimum, which, however, yields poorer fit. The same outcome do we get by applying the results of (21) as it is. The magnitude of (20) implied by the estimates obtained for the general and optimal shifts are, however, surprisingly close, -.63% and -.64% respectively.[36)] [37)]

As for Total Mining and Manufacturing we get for all of the individual industries except two a dual optimum type of trend, either t or t ln K.[38)] Using the MSE-value to choose between them we get neutral

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35) This is so as all individual tests about the trend of the labour elastisity in the multiple test scheme is the same, as this trend is estimated separately from the other trends.

36) The percentages presented are computed for the year 1963. As in this case both γ_1 and γ_2 are positive the estimated degree of labour saving decreases with time. But this decrease is quite ignorable, however. For instance, the magnitude of (20) implied by the general shift varies from -.61% in 1959 to -.65% in 1967.

37) When the OLS-method is applied, the magnitude of (20) is -6.3% when it is computed for the general shift and -5.6% for the optimal one. This shows that the estimation-bias when applying the OLS-method may be quite substantial, even if this is hardly the only cause of the differences between these percentages and those obtained when using the factor-share for labour input.

38) For Printing the optimal shift comes out to be capital saving and for Mineral products the optimum is as previously "no shift". Cf. Tables V.4 and V.5.

technical change for six and labour saving technical change for seven. All in all the results of these experiments are very mixed. Our difficulties are primarily caused by the fact that as concerns the fit to data the three components of the general trend, t, t ln L and t ln K are almost perfect substitutes. But we can draw two tentative conclusions of the results above. First the shift in the production function seems to be of a much more complex type than the one implied by the residual trend. Second there is some evidence for that the shift is generally labour saving, or that the marginal productivity of capital grows faster than that of labour.

It does not seem possible to come much further than this by the present approach. We will, however, return to it in a particular context, namely when analysing materials position in a process of technical change. But first we will discuss the results of a different approach in the analysis of the nature of technical change-issue.

d) A Tentative Test of the Embodyment Hypothesis.

To some extent the analysis of the nature of technical change has to do with the quality-components of the inputs, and their behaviour over time. This is also true for the so-called embodyment-hypothesis advanced by R. Solow.[39] The basic idea of this hypothesis is that capital of recent vintages is more productive than capital of older ones, due to technical progress "embodied" in new capital goods.

With the empirical base available in this study there is one possible way to analyse the validity of the embodyment hypothesis, namely by investigating the performance of variables expressing the "recentness" of capital. And we do this by introducing into the Cobb Douglas production function the following one:[40]

$$E_t = \frac{(1-\Delta)^3 I_{t-3} + (1-\Delta)^2 I_{t-2} + (1-\Delta) I_{t-1}}{K_t} \tag{22}$$

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39) Cf. R. Solow: Investment and Technical Progress in K.J. Arrow, S. Karlin and P. Sappors (editors) Mathematical Methods in Social Sciences , Stanford 1960. Cf. also M. Brown (1966) op.cit. pp. 77-81.

40) For studies where the embodyment hypothesis is analysed in a similar way see: E. Berglas: "Investment and Technological Change." In The Journal of Political Economy 1965, and K.L. Krishna op.cit. See also, Z. Griliches: "Production Functions in Manufacturing: Some Preliminary Results", in M. Brown (ed.) op.cit.

where the numerator expresses what is assumed by us to be left in year t of the latest three vintages.[41)]

Now, if the embodyment hypothesis is true, that capital goods of recent vintages are more productive than those of older vintages it should show up in the results as a significantly positive coefficient of E. There are, however, a number of reasons why this must be a rather weak test. To point out two of the more important ones: First we have assumed a declining balance depreciation formula to be valid in the computations of the physical detoriation of the capital goods. If, say, the productive performance of capital goods less than four years old is unchanged the undepreciated values of I_{t-i} $i=1,2,3$ should enter the capital measure. And if we in that case include E in the production function with K as the capital input we may get a significantly positive coefficient of E even if the embodyment hypothesis is invalid.

Second, we should in our case take into consideration the poor quality of the investment data. As pointed out in Chapter II there is clearly a substantial amount of bad reporting in our data, both assuming capital and investments. Even if an establishment need not buy investment goods each year, it is quite hard to believe that the high fraction of zeros reported for this information is real. If this in fact is a result of bad reporting then the results of E may become highly distorted. And to guard against such effects a dummy-variable defined in the following way

$$(22.b) \qquad F_E = \begin{array}{ll} 1 & \text{when } E = 0 \\ 0 & \text{"} \quad E > 0 \end{array}$$

is introduced into the production function together with E.

The coefficient of F_E will presumably also catch any differences in the level of productivity between units with bad and not so bad reporting of the investment information. As there are reasons to believe that poorly managed units also tend to have a poor quality in the reporting of their activities, we will expect the coefficient of F_E to be negative. But as we by F_E also catch stagnant units its coefficient may become negative also for that reason. But a negative coefficient of F_E may also lend support to the hypothesis under test as it implies just that - as well as a positive coefficient of E.

41) As embodied technical change is initiated through purchased investment goods, only this cathegory of investments is included in E. We have, however, not included current investments in E. This is done of two reasons. First that incompleted investment projects may be reported while these do not add to the production performance of the capital of that period.(Cf. Section h.ii of Appendix II.7). Second, current investments may reflect "costs of change", and thus have a negative impact on output. (Cf. Appendix V.1).

All in all the interpretation of the results of both E and F_E is rather difficult. But on the other hand, they both have to do with misspecifications and mismeasurement and as such, an analysis of their performance and effects is interesting. There are three aspects of these variables we would like to investigate. First if their effect on output is the one expected and if their coefficients are significant; that is, the coefficients of E and F_E significantly positive and negative respectively. Second if, in particular E, leads to a reduction of the residual trend, or the disembodied technical change. And third if their presence in the production function leads to substantially different estimates on the factor-elasticities. About the first there is not much more to say a priori. About the second it is reasonable to believe that as there is presumably little variation of E along the time-dimension it cannot catch much of the effect of t; in our sample the coefficient of E must be determined mainly by the across-dimension. About the third one can at least "predict" that if the coefficient of E has the expected sign the estimate on the capital-elasticity will become lower, as there is then a positive effect of parts of the capital stock in addition to the "main" capital input variable.

Turning now to the empirical findings we should note that due to the way our "recentness" variable is constructed we "loose" one third of the degrees of freedoms available. Thus to make a complete analysis of the effects of E and F_E we re-run the Cobb-Douglas relation with purely disembodied technical change for the truncated sample. In addition the results of two other regressions are presented, when the "recentness" variables only are included together with the ordinary factors of production, and when they are included together with the residual trend. The results of these three regressions are presented in table V.8.[42)]

42) Another regression was also run, namely $\ln V - S_L \ln L - \tilde{\beta} \ln K = a + \gamma t + \mu_1 E + \mu_2 F_E$, where S_L is the share of labour and $\tilde{\beta}$ is the size-dummies instrumental variable estimate on the capital elasticity, both obtained from the complete sample. (Cf. Ch. III). This relation provides a test of the performance of the trend and the recentness-variables when imposing presumably more consistent estimates on the factor-elasticities than those implied by the OLS-method. But this approach does not take care of the sample-truncation, neither of the possible effects of the technical change variables on the estimates of the factor elasticities. But in spite of this the relation above did not yield results for E and F_E basicly different from those obtained by means of the OLS method. Concerning the effects on t of consistent estimation of the factor elasticities, cf. section 2.b above.

TABLE V.8. Results of the Cobb Douglas Relations with Embodied and Disembodied Technical Change.*

Industry	ln L	ln $\frac{K}{L}$	t	MSE	ln L	ln $\frac{K}{L}$	E	F_E	MSE	ln L	ln $\frac{K}{L}$	t	E	F_E	MSE	Number of F_E=1 in %
Tot. Min. and Man.	-.007	.280	.0382	.282	-.021	.285	.213	-.105	.283	-.021	.280	.0379	.216	-.100	.279	12.2
	(.006)	(.009)	(.0042)		(.006)	(.009)	(.038)	(.025)		(.006)	(.009)	(.0042)	(.038)	(.024)		
Min. and Quarr.	.022	.211	.0613	.182	.026	.245	.091	.114	.193	.028	.223	.0608	.109	.091	.184	12.8
	(.026)	(.050)	(.0202)		(.028)	(.053)	(.184)	(.128)		(.028)	(.052)	(.0203)	(.179)	(.125)		
Food Products	-.137	.354	.0579	.448	-.160	.335	.046	-.231	.452	-.162	.319	.0603	.034	-.250	.442	14.3
	(.018)	(.031)	(.0125)		(.019)	(.033)	(.093)	(.069)		(.018)	(.032)	(.0125)	(.092)	(.068)		
Textiles	-.065	.253	.0158	.162	-.111	.253	.921	-.119	.149	-.111	.244	.0159	.914	-.125	.149	11.2
	(.028)	(.043)	(.0128)		(.031)	(.043)	(.192)	(.085)		(.031)	(.044)	(.0123)	(.192)	(.085)		
Clothing	.027	.056	.0263	.147	-.027	.050	.100	-.213	.144	-.023	.043	.0252	.117	-.204	.143	13.4
	(.027)	(.033)	(.0113)		(.030)	(.033)	(.126)	(.065)		(.027)	(.033)	(.0111)	(.125)	(.064)		
Wood Products	.134	.196	.0154	.308	.109	.191	.663	-.003	.294	.109	.188	.0137	.661	-.003	.294	23.0
	(.031)	(.045)	(.0199)		(.033)	(.044)	(.185)	(.092)		(.033)	(.045)	(.0194)	(.185)	(.092)		
Pulp and Paper	-.105	.178	.0701	.108	-.129	.221	.060	-.067	.121	-.111	.191	.0697	.070	-.019	.108	12.9
	(.014)	(.023)	(.0078)		(.017)	(.025)	(.077)	(.048)		(.016)	(.024)	(.0079)	(.072)	(.045)		
Printing	.050	.098	-.0117	.109	.050	.095	-.116	-.014	.109	.050	.096	-.0128	-.120	-.013	.108	9.8
	(.020)	(.027)	(.0100)		(.020)	(.027)	(.063)	(.060)		(.020)	(.027)	(.0099)	(.063)	(.060)		
Basic Chemical	-.016	.235	.0625	.449	-.052	.225	.130	-.397	.440	-.051	.227	.0543	.128	-.369	.432	16.4
	(.023)	(.044)	(.0189)		(.024)	(.044)	(.238)	(.098)		(.024)	(.043)	(.0186)	(.236)	(.097)		
Mineral Prod.	.145	.318	.0188	.169	.152	.314	.710	.238	.182	.153	.308	.0153	.704	.232	.183	4.2
	(.027)	(.046)	(.0178)		(.027)	(.044)	(.168)	(.161)		(.027)	(.044)	(.0172)	(.169)	(.161)		
Basic Steel	.004	.234	.0653	.181	-.008	.264	.278	-.149	.189	-.009	.252	.0654	.283	-.026	.177	0.8
	(.035)	(.045)	(.0158)		(.037)	(.046)	(.106)	(.311)		(.035)	(.045)	(.0156)	(.102)	(.303)		
Metal Prod.	-.053	.123	.0244	.149	-.032	.176	.567	.241	.143	-.034	.169	.0233	.560	.240	.142	10.3
	(.021)	(.041)	(.0120)		(.021)	(.041)	(.145)	(.072)		(.021)	(.041)	(.0117)	(.145)	(.072)		
Non El.Mach.	.078	.016	.0388	.151	.045	.073	.468	.146	.150	.052	.053	.0370	.476	-.115	.147	5.9
	(.027)	(.049)	(.0155)		(.030)	(.051)	(.198)	(.126)		(.030)	(.051)	(.0154)	(.196)	(.125)		
El.Mach.	.041	.052	.0438	.271	.060	.037	-.294	-.004	.276	.061	.027	.0448	-.309	.015	.271	2.0
	(.039)	(.056)	(.0214)		(.043)	(.059)	(.240)	(.272)		(.043)	(.059)	(.0214)	(.238)	(.270)		
Transp. Equipm.	.077	.079	.0263	.152	.083	.102	.359	.123	.152	.083	.099	.0266	.361	.124	.150	17.1
	(.012)	(.027)	(.0100)		(.014)	(.028)	(.122)	(.056)		(.014)	(.028)	(.0099)	(.122)	(.056)		
Misc. Prod.	-.160	.387	.793	.297	-.158	.358	1.295	.234	.296	-.153	.369	.0791	1.141	.359	.282	9.0
	(.048)	(.068)	(.0362)		(.049)	(.076)	(.533)	(.242)		(.048)	(.074)	(.0370)	(.525)	(.243)		

* Dependent variable is ln $\frac{V}{L}$; Method of estimation: OLS.

By comparing the results of the first regression of table V.8 with the first one in table V.1 we get an impression of the effects of the sample-truncation, as the first one is based on data for the years 1962-67 while the later is based on data for the whole period 1959-1967. The main difference between the two sets of results is that the trend seems to be of greater importance for the truncated sample, suggesting that the trend is not constant but increasing over time. This effect is more notable for Mining and Quarrying, Pulp and Paper, Basic Steel, Non.El.Machinery and Transport Equipment, or generally rather heavy industries. The level of the capital elasticity is somewhat reduced for Total Mining and Manufacturing, but this is "compensated" by an increase in the elasticity of labour. But for the individual industries there is no uniform tendency of a reduced capital elasticity due to the increased trend. It is true that the estimate on the capital elasticity is lower for industries like Mining and Quarrying and Pulp and Paper. But on the other hand, it is higher for Basic Steel and Non El. Machinery and only slightly lower for Transport Equipment.

The results of the second regression tell us that at least for Total Mining and Manufacturing the coefficients of E and F_E both have the expected signs and they are both significant at conventional levels. Thus the embodyment hypothesis seems to get some support by these results. But for none of the individual industries the findings are equally uniform. For eight of the fifteen industries the coefficient of E is significantly positive. For only three the coefficient of F_E is significantly negative, and none of these are among those with a significantly positive coefficient of E. [43)] On the other hand, we get for two industries, Metal Products and Transport Equipment the rather peculiar result that <u>both</u> coefficients are significantly positive.

The third regression of Table V.8 tells us that our variables expressing embodied and disembodied technical change are largely independent. Compared to the results of the first regression we see that the residual trend is approximately of the same magnitude. And compared to the results of the second regression we can conclude that the estimates of the"recentness" variables are also virually unaffected by introducing a trend.

43) In the last column of Table V.8 the percentage of observations with $F_E = 1$ (or E = 0) are presented. We note that this percentage varies widely between industries, also suggesting that the quality of the reporting is substantially different. We should note also that as E covers a period of three years the percentages of zeros reported on purchased investment goods are much higher than those presented in Table V.8.

This confirms our a priori "predictions" of the results. Our "recentness" variables are as pointed out mainly determined of the across dimension and therefore they work more or less like dummy-variables for establishments. This is probably also the main reason why the labour elasticity seems to be more affected by these variables than the capital elasticity. The former is almost solely determined by the across dimension while for the later the time-dimension is of somewhat larger importance.[44)] But all in all our "recentness" variables do not have any serious impact on the estimates of the factor-elasticities. Thus having ignored them in the previous analysis of the levels of these parameters does not make this analysis basicly invalid.[45)]

The main conclusion of this section is therefore that the embodyment-hypothesis seems to have some support in our data. But the introduction of variables taking care of that quality-component of capital has little impact on the main production function parameters. Neither do they affect the residual trend significantly.

e) Technical Change and the Role of Materials.

Basicly there are three factors of production (or rather three groups of factors) in operation when manufacturing a final product, namely labour, capital <u>and</u> materials. The treatment of these is, however, generally rather asymmetrical, as the later one is usually subtracted from output to obtain a net output measure, value added.[46)]

So far that approach is adopted also in this study. But in this section we will analyse if a more symmetrical treatment of the three factors of production leads to different conclusions concerning the importance and nature of technical change.

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44) Cf. Tables II.1 and II.3.

45) In a similar way as for the embodyment hypothesis attempts were made to investigate two other hypotheses, namely "costs of change" and "transitoty variation in demand." The results were rather inconclusive as concerns the importance and validity of these hypotheses. On the other hand, we found, as for the embodyment hypothesis that the results of the "main factors" were virtually unchanged. A summary of these computations are presented in Appendix V.1.

46) Cf. the Census study, ch. V.

As we have mainly operated with a Cobb Douglas relation for labour and capital an obvious way of treating all factors symmetrically is to adopt a three factor Cobb Douglas relation with gross production as the output measure. And assuming neutral, or purely disembodied technical change, we have

$$(23) \qquad \ln Y = A\, L^{\alpha'} K^{\beta'} M^{\mu}\, e^{\gamma' t}$$

or

$$(24) \qquad \ln Y = \ln A + \alpha' \ln L + \beta' \ln K + \mu \ln M + \gamma' t$$

or

$$(25) \qquad \ln \frac{Y}{L} = \ln A + (\alpha' + \beta' + \mu - 1) \ln L + \beta' \ln \frac{K}{L} + \mu \ln \frac{M}{L} + \gamma' t$$

Estimating the parameters of this relation for Total Mining and Manufacturing we have:

$$(26) \quad \ln \frac{Y}{L} = .906 - \underset{(.003)}{.055} \ln L + \underset{(.005)}{.132} \ln \frac{K}{L} + \underset{(.003)}{.491} \ln \frac{M}{L} + \underset{(.00141)}{.01764}\, t \qquad \begin{matrix} R = 0.896 \\ MSE = 0.107 \end{matrix}$$

Both the labour and the capital elasticities as well as the residual trend are much lower for this relation than those obtained for the value added Cobb Douglas relation.[47) 48)] But as we now have a different output-measure these are not comparable.

A kind of comparability can be obtained, however, by writing (23) in a slightly different manner. We may write it as:

$$(27) \qquad Y = (A\, L^{\alpha^*} K^{\beta^*} e^{\gamma^* t})^{1-\mu}\, M^{\mu}$$

where

$$(28) \qquad \begin{aligned} \alpha^* &= \alpha'/(1-\mu) \\ \beta^* &= \beta'/(1-\mu) \\ \gamma^* &= \gamma'/(1-\mu) \end{aligned}$$

Taking logs, subtracting $\mu \ln M$ and dividing through by $1-\mu$ yields the relation

$$(29) \quad (\ln Y - \mu \ln M)/(1-\mu) = \ln A + \alpha^* \ln L + \beta^* \ln K + \gamma^* t$$

where the left side is a "geometric value added".[49)] (29) can also be written in a way that corresponds to (25).

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47) The estimate on α' implied by the estimates of (26) is .322.

48) Cf. Table V.1.

49) Cf. E. Domar: "On the Measurement of Technological Change", The Economic Journal, Dec. 1961.

$$(30)\quad (\ln \frac{Y}{L} - \mu \ln \frac{M}{L})/(1-\mu) = \ln A + (\alpha^{*} + \beta^{*} - 1) \ln L + \beta^{*} \ln \frac{K}{L} + \gamma^{*}t$$

Now, there are two ways of estimating α^{*}, β^{*} and γ^{*}. Either by using the estimates obtained by (25) together with (28), or by using (29) or (30), having estimated μ independently.

The first method yields $\alpha^{*} = .634$, $\beta^{*} = .259$ and $\gamma^{*} = .03466$. And by estimating μ as the arithmetic average of materials share in gross production, $S_M = .520$ we get by applying (30),

$$(31)\quad (\ln \frac{Y}{L} - S_M \ln \frac{M}{L})/(1-S_M) = \underset{}{1.914} - \underset{(.011)}{.111} \ln L + \underset{(.009)}{.243} \ln \frac{K}{L} + \underset{(.00295)}{.03509}\, t$$

$$R = 0.360$$
$$MSE = 0.470$$

Thus concerning the magnitude of technical change measured by the residual trend it is approximately the same whether ordinary value added or the geometric value added measure is applied. But as concerns the factor-elasticities as well as the scale-elasticity there is a striking difference between the results obtained by means of (30) and the ordinary value added relation. The estimate on α^{*} implied by (31) is .646 while $\hat{\beta}^{*}$ is as we note .243. The corresponding estimates from the ordinary value added relation are .730 and .263. Thus, particularly the labour elasticity is substantially lower when using the geometric value added measure. And this implies also as we see from (31) that we have significantly decreasing returns to scale.[50)] [51)]

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50) This finding is quite different from the one obtained in the Census study where almost the same estimate on the scale-elasticity was obtained for Total Manufacturing when using the geometric value added measure as when using the ordinary value added measure.

51) Constraining the labour elasticity to its share in (ordinary) value added, and using the size-dummies-instrumental variable method to estimate β^{*} leaves the estimate on the scale-elasticity virtually unchanged. We obtained $\hat{\beta}^{*} = .304$ and as $S_L = .603$ we have the estimate on the scale-elasticity as .907 as compared to .889 obtained by OLS on (30). Even when using the geometric value added measure it may be convenient, as done here, to use the share of labour in ordinary value added as an estimate on α^{*}. Alternatively we could have used $\tilde{\alpha}^{*} = \frac{S_{LY}}{1-S_{MY}}$ where S_{LY} and S_{MY} are the shares of labour and materials respectively in gross production. There is, however, a close relationship between this estimate and the former; $\hat{\alpha}^{*}_{WL} = S_L$. S_{LY}, S_{MY} and S_L are based on the variables $\frac{WL}{Y}$, $\frac{M}{Y}$ and $\frac{WL}{V}$ and we have that $\frac{WL}{Y}/1-\frac{M}{Y} = WL/Y-M = WL/V$. But clearly $\tilde{\alpha}^{*}$ and $\hat{\alpha}^{*}$ will generally be different.

TABLE V.9

The Share of Materials in Gross Production and its Trend over Time

Industry	$\overline{S_{Mt}}$	OLS on S_{Mt} = a + bt		
		$\hat{b}$	R	MSE*
Tot. Min. and Man.	.520 (.011)	-.00387 (.00039)	.967	.001
Mining and Quar.	.160 (.022)	-.00603 (.00202)	.748	.024
Food Products	.678 (.016)	-.00572 (.00071)	.950	.003
Textiles	.496 (.033)	-.00983 (.00255)	.824	.039
Clothing	.496 (.026)	-.00893 (.00111)	.950	.007
Wood Products	.629 (.019)	-.00513 (.00179)	.734	.019
Pulp and Paper	.658 (.010)	.00023 (.00134)	.066	.011
Printing	.359 (.013)	-.00285 (.00152)	.579	.014
Basic Chemicals	.498 (.008)	-.00095 (.00108)	.314	.007
Mineral Products	.340 (.014)	-.00385 (.00124)	.760	.009
Basic Steel	.545 (.009)	.00037 (.00117)	.118	.008
Metal Products	.469 (.027)	-.00832 (.00189)	.857	.021
Non El. Mach.	.439 (.014)	-.00192 (.00173)	.387	.018
El. Mach.	.500 (.011)	.00148 (.00138)	.377	.011
Transp. Equipm.	.414 (.005)	-.00023 (.00075)	.117	.003
Misc. Prod.	.517 (.021)	-.00717 (.00116)	.920	.008

* Multiply these entries by 10^{-2}

The virtually unchanged trend estimate together with reduced factor-elasticity-estimates implies that when using the geometric value added measure, shifts in the production function account for a higher fraction of growth in output than when using the ordinary value added measure. The growth rate of the geometric value added measure is 4.67% as opposed to 4.79% for ordinary value added. In the present case movements along the production function account for 1.15% or 24.7% of the growth in output while it was 1.27% or 26.5% of the growth in case ordinary value added was applied.[52)]

Therefore, it is misleading to conclude from the results of (26) that we manage to explain more of the growth in net output by means of movements along the production function by treating materials as a factor of production in the same way as the two other factors. It is true, of course, that the shifts in the production function are less important both absolutely and relatively for the gross production function with all three factors of production, than for the ordinary value added relation. But the point is that the importance of the shift of the value added relation implied by our gross production function is equally large or larger than for the ordinary value added relation.

This conclusion is obtained, however, by assuming that the share of raw materials in gross production is constant over time. The computations presented in Table II.5 suggest that there is a drop in this share. In Table V.9 we present the average share of raw materials in gross production and its trend over time. For eight of the industries there is a significant drop in this share, as well as for Total Mining and Manufacturing. Considering it as an estimate on the elasticity of materials this finding implies that, at least for the eight industries pointed out, there is a kind of non-neutral technical change, provided of course that the production function is correctly specified. To this topic we will return later.

What we would like to know first is if this movement of the elasticity of raw materials alters the conclusions obtained by means of (30) about the role of shifts in, as opposed to movements along the production function, compared to the corresponding results obtained for ordinary value added.[53)]

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52) The use of a presumably more consistent method of estimation does not alter this picture. Accepting the factor share instrumental variable-estimates referred to in footnote 51) above we have that 28.5% of the growth in output can be explained by movements along the production function as opposed to 36.5% when ordinary value added is applied.

53) The computations are carried out as previously for Total Mining and Manufacturing only.

Having a drop in S_M over time implies that the average growth in the geometric value added measure must be lower too. In the case the materials share is constant the growth rate is determined simply as $(y-S_M m)/(1-S_M)$ where y and m are the growth rates of gross production and materials respectively. Having a certain (absolute) trend of S_M, $\dot{S}_M$ we get the growth rate of geometric value added as:

$$(32) \qquad (y-S_M m)/(1-S_M) + (\dot{S}_M/(1-S_M)^2) \ln \frac{Y}{M}$$

As for Total Mining and Manufacturing $S_M = .520$, $\dot{S}_M = -.00387$ and $\overline{\ln \frac{Y}{M}} = 0.7756$ we have that the mean of the later term of (32) is equal to -1.30%. And as the first term was found to be equal to 4.67% we have a growth rate of geometric value added with a variable share of raw materials of about 3.37%. This drop in the growth rate of output does presumably have the more significant impact on the residual trend as the factor-elasticities are mainly determined by the across-dimension. This is confirmed by running (30) with μ estimated separately for each year as the share of materials in gross production of that year. Then we get:

$$(33) \quad (\ln \frac{Y}{L} - S_{Mt} \ln \frac{M}{L})/(1-S_{Mt}) = 2.735 - \underset{(.006)}{.111} \ln L + \underset{(.009)}{.244} \ln \frac{K}{L} + \underset{(0.00294)}{0.02200}\, t$$

$$R = 0.347$$
$$MSE = 0.468$$

The estimates on the elasticities of labour and capital are even more unaffected by introducing a variable elasticity of materials than expected. The whole effect of the new geometric value added measure as compared to the previous one is absorbed by the trend. Thus the relative importance of the trend is also reduced. In opposition to the previous distribution of 24.7% and 75.3% of the growth in output due to movements along - and shifts in the production function respectively we have now 34.5% and 65.5%.

But assuming a variable elasticity of materials implies that technical change may be non-neutral. And the contents of Table V.9 suggests quite strongly that this is generally true. And for the eight industries with a significant trend of materials share in gross production the technical change seems to be of the value added using or materials saving type. The issues we would like to explore next are thus; if this finding is supported by the direct regression results on a "generalized" version of the three-factor Cobb-Douglas relation and if the conclusion about the nature of technical change obtained previously using the ordinary value added relation is supported by the results of the three-factor Cobb Douglas relation.

The "generalized" production function is in this case:

$$\ln Y = \ln A + \alpha \ln L + \beta \ln K + \mu \ln M + \gamma_0 t + \gamma_1 t \ln L + \gamma_2 t \ln K + \gamma_3 t \ln M \tag{34}$$

we have now two "independent" marginal rates of substitution.[54)]

$$\begin{aligned} \frac{m_L}{m_K} &= \frac{\alpha + \gamma_1 t}{\beta + \gamma_2 t} \\ \frac{m_K}{m_M} &= \frac{\beta + \gamma_2 t}{\mu + \gamma_3 t} \end{aligned} \tag{35}$$

And their relative change over time is given by:

$$\begin{aligned} \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} &= \frac{\gamma_1 \beta - \gamma_2 \alpha}{(\alpha + \gamma_1 t)(\beta + \gamma_2 t)} \\ \frac{\dot{m}_K}{m_K} - \frac{\dot{m}_M}{m_M} &= \frac{\gamma_2 \mu - \gamma_3 \beta}{(\beta + \gamma_2 t)(\mu + \gamma_3 t)} \end{aligned} \tag{36}$$

Estimating the parameters of (34) by means of ordinary least squares for Total Mining and Manufacturing we have that[55)]

$$\begin{aligned} \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} &= -.02472 \\ \frac{\dot{m}_K}{m_K} - \frac{\dot{m}_M}{m_M} &= .03235 \end{aligned} \tag{37}$$

which implies that

$$\frac{\dot{m}_L}{m_L} - \frac{\dot{m}_M}{m_M} = .00763 \tag{38}$$

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54) The third one, for labour and raw materials can easily be computed from those two presented.

55) γ_3 is significantly negative at 1% level while γ_0, γ_1 and γ_2 are positive but not significant at that level.

Applying a similar multiple test procedure in this case as for the value added Cobb Douglas relation yields an "optimal" shift of

$$\underset{(.00109)}{.00475}\, t \ln K - \underset{(.00116)}{.00295}\, t \ln M$$ [56)]

And the relative growth in the marginal rates of substitution is

$$\frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -\frac{\dot{m}_K}{m_K} = -.00601$$

(39)

$$\frac{\dot{m}_K}{m_K} - \frac{\dot{m}_M}{m_M} = .04208$$

And thus:

(40) $$\frac{\dot{m}_L}{m_L} - \frac{\dot{m}_M}{m_M} = -\frac{\dot{m}_M}{m_M} = .03607$$

Thus, the previous finding of value added using or materials saving technical change is supported by these direct production function regression results. And they do also suggest that technical change is more labour saving than capital saving, thus supporting the findings of section V.2.c.

These findings are also supported by the results obtained by means of (34) when constraining the elasticity of materials to its share in gross production. Assuming it to be constant over time we get by means of unconstrained estimation of the trend-parameters γ_0, γ_1 and γ_2 that

(41) $$\frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -.00239$$

And allowing materials share to vary over time we have that unconstrained estimation of γ_0, γ_1 and γ_2 yields

(42) $$\frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -.00421$$

All in all there are sufficient evidence to conclude that treating all three factors symmetrically does not alter the main conclusion obtained previously about the magnitude and nature of technical change at the value added level.

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56) Cf. Fig. V.1. The optimum obtained is, however, not "unique".
Cf. Table V.4. The number of individual tests are 20 and thus we get an upper limit of the level of the overall test when using the ordinary F-statistics method of 20% as we choose a level of the individual tests of 1%. The Spjøtvoll F-statistics method yields the same result as the other one both when choosing a level of 10% and of 25%. (A tabulation of the upper 20% fractiles of the F-distributions was not available.)

f) On the Differences in Degree of Technical Change between Establishments.

In this section we'll try to attack the problem of determining the nature of technical change by a somewhat different approach. We know from our previous results that technical change measured by the residual shift shows a substantial variation between units. This may partly be due to the fact that we have imposed factor elasticities common to all units of an industry. Thus variation in the residual may reflect also variation in the factor-elasticities But due to too short time-series we do not manage to estimate the parameters of the production function separately for each establishment with any reasonable degree of accuracy.[57] Therefore we accept the estimates on the individual shifts γ_i obtained by assuming common factor-elasticities for all units.

What we would like to know is if the variation of γ_i can be "explained" by the variation of other characteristics of the units. This excursion will be divided into two parts. First we analyse if γ_i shows any correlation with the estimated level of efficiency. Thinking in terms of an analysis of covariance model we have the Cobb Douglas production function:

$$(43) \qquad \ln V_{it} = \alpha \ln L_{it} + \beta \ln K_{it} + u_{it}$$

where the residual u_{it} is decomposed into three parts;

$$(44) \qquad u_{it} = \mu_i + \gamma_i t + \varepsilon_{it}$$

---

57) Attempts were made in this direction for Mining and Quarrying by an investigation of the performance of a method proposed by P. Balestra and M. Nerlove: "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas". Econometrica, vol. 34, No 3. July 1966. Cf. p. 607. This method implies among other things that α and β are estimated separately for each establishment and then estimating the "industry" elasticities as $\hat{\alpha} = \frac{1}{I}\sum_{i=1}^{I} \hat{\alpha}_i$ and $\hat{\beta} = \frac{1}{I}\sum_{i=1}^{I} \hat{\beta}_i$.

But as this method yielded insensible results even for the "averages" $\hat{\alpha}$ and $\hat{\beta}$, no further attempts were made along these lines.

where ε_{it} is a random error with zero mean and constant variance.[58)]
Without loss of generality we can assume that:[59)]

$$(45)\qquad \frac{1}{I}\sum_{i=1}^{I}\mu_i = \frac{1}{T}\sum_{t=1}^{T}\gamma_i t = 0$$

Evidently μ_i is the establishment specific level of efficiency and we get an estimate on it as:

$$(46)\qquad \hat{\mu}_i = \overline{\ln V_i} - \tilde{\alpha}\,\overline{\ln L_i} - \tilde{\beta}\,\overline{\ln K_i}$$

where the estimates on the factor-elasticities are obtained by the Klein Wald method and the bars indicate averages over years.

We are interested in the estimate on a_1 in the relation

$$(47)\qquad \hat{\gamma}_i = a_o + a_1\hat{\mu}_i$$

A priori we would expect a_1 to be positive as it is not unreasonable to believe that units with high efficiency and good management also tend to have a faster technological progress.

But even if our estimates on the factor elasticities were unbiased there are reasons to believe that they may be somewhat misleading as "weights" of the factor inputs in the expression of $\hat{\mu}_i$ in (46) when using $\hat{\mu}_i$ to "explain" $\hat{\gamma}_i$. For Total Mining and Manufacturing we have found some evidence for labour saving technical change. And as the levels of output and the inputs are positively correlated with their growth rates,[60)] we will expect that when running the regression;

$$(48)\qquad \hat{\gamma}_i = b_o + b_1\,\overline{\ln V_i} + b_2\,\overline{\ln L_i} + b_3\,\overline{\ln K_i}$$

we get $\hat{b}_2 < \tilde{\alpha}\,\hat{a}_1$ and $\hat{b}_3 > \tilde{\beta}\,\hat{a}_1$, (if $\hat{a}_1 > 0$).

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58) This is a slightly more complex covariance analysis model than the one discussed in Ch. III, as we now have sort of an interaction term $\gamma_i t$. But in this case we have no "pure" time-term.

59) We get $\frac{1}{T}\sum^{T}\gamma_i t = 0$ simply by re-scaling t so that t=0 for the year 1963.

60) Cf. Table A.V.3.

The second part of this excursion is an investigation of the variation of γ_i with the level and trends of factor shares, to obtain further tests on the nature of technical change. The two shares that will be considered are as previously the share of materials in gross production and the share of labour in value added.

Concerning the growth rate of the share of materials it isn't very interesting in this context as it must almost necessarily have a negative correlation with $\hat{\gamma}_i$. But we include it together with the level of the share of materials to catch the simultaneous effect of these two variables. This is analysed by applying the OLS-method on

$$(49) \quad \hat{\gamma}_i = a_o + a_1 S_{Mi} + a_2 \dot{S}_{Mi}$$

The results of the impact on $\hat{\gamma}_i$ of the share of labour and its growth rate provide a direct test on the nature of technical change. If it is true that technical change is labour saving this should show up in the results of:

$$(50) \quad \hat{\gamma}_i = b_o + b_1 S_{Li} + b_2 \dot{S}_{Li}$$

as significantly negative coefficients of both right-side variables: Those units with a high and increasing share of capital will presumably have the fastest technical progress.

The results of (47) does not lend conclusive support to the assumption about a positive impact on the rate of technical progress of the level of efficiency. For Total Mining and Manufacturing we have $a_1 = \underset{(.0064)}{.0124}$ and thus a_1 isn't significantly positive at 5% level. For two industries the presumed relationship between μ_i and γ_i seems to be fullfilled, namely for Textiles and El.Machinery: For both industries a_1 is significantly positive. On the other hand we have a_1 significantly negative for one industry, namely Pulp and Paper.

By means of OLS on (48) and thus letting the regression itself determine the weights of the three components of μ_i in a relationship with γ_i we get for Total Mining and Manufacturing:

$$(50) \quad \gamma_i = -.0396 + \underset{(.0065)}{.0206}\,\overline{\ln V_i} - \underset{(.0063)}{.0339}\,\overline{\ln L_i} + \underset{(.0040)}{.0101}\,\overline{\ln K_i}$$

And we get $-.0339 = \hat{b}_2 < \hat{a}_1\hat{\alpha} = -.0075$ and $.0101 = \hat{b}_3 > \hat{a}_1\hat{\beta} = -.0054$ as expected. Thus on the average for Mining and Manufacturing we have evidence from these results also that technical change is labour saving.

TABLE V.10

Differences in Technical Change Between Units Due to Differences in Level and Trend of Factor Shares

Regression	$\hat{\gamma}_i = a_o + a_1 S_{Mi} + a_2 \dot{S}_{Mi}$					$\hat{\gamma}_i = b_o + b_1 S_{Li} + b_2 \dot{S}_{Li}$					$\hat{\gamma}_i$
Industry	a_1	a_2	a_o	R	MSE	b_1	b_2	b_o	R	MSE	
Tot. Min. and Man.	0.0739	-1.7275	-0.0147	0.388	0.0057	-0.0455	-0.3643	0.0700	0.595	0.0043	0.0304
	(0.0127)	(0.1477)				(0.0057)	(0.0164)				(0.0818)
Min. and Quarr.	0.1601	-2.6637	-0.0042	0.698	0.0026	0.0210	-0.7400)	0.0322	0.615	0.0031	0.0386
	(0.1305)	(0.5824)				((0.0427)	(0.1984)				(0.0677)
Food Prod.	0.1118	-2.8187	-0.0383	0.453	0.0075	-0.0309	-0.6524	0.0805	0.747	0.0042	0.0536
	(0.0337)	(0.4689)				(0.0140)	(0.0464)				(0.0964)
Textiles	-0.0386	-1.6741	0.0195	0.448	0.0035	0.0584	-0.8742	0.0135	0.820	0.0014	0.0177
	(0.0647)	(0.4512)				(0.0332)	(0.0916)				(0.0646)
Clothing	-0.0899	-1.6902	0.0372	0.446	0.0040	0.0781	-0.8789	-0.0086	0.787	0.0019	0.0083
	(0.0496)	(0.4836)				(0.0344)	(0.1034)				(0.0697)
Wood Products	-0.1047	-3.1072	0.0640	0.710	0.0026	0.0458	-0.2835	-0.0065	0.557	0.0037	0.0139
	(0.0671)	(0.5133)				(0.0319)	(0.0789)				(0.0713)
Pulp and Paper	0.2118	-3.3522	-0.0845	0.585	0.0029	-0.0321	-0.5583	0.0660	0.731	0.0020	0.0540
	(0.0461)	(0.4840)				(0.0286)	(0.0727)				(0.0655)
Printing	-0.1670	-1.2845	0.0367	0.467	0.0031	0.1432	-1.0373	-0.0613	0.782	0.0015	-0.0196
	(0.0501)	(0.4802)				(0.0270)	(0.1067)				(0.0619)
Basic Chemicals	0.1644	-1.3285	-0.0242	0.309	0.0109	-0.0720	-0.3142	0.0937	0.550	0.0084	0.0588
	(0.0705)	(0.7277)				(0.0312)	(0.0785)				(0.1081)
Mineral Prod.	0.0114	-1.0054	-0.0007	0.293	0.0049	0.0833	-1.3722	0.0003	0.804	0.0019	0.0070
	(0.0778)	(0.5872)				(0.0390)	(0.1874)				(0.0710)
Basic Steel	0.0961	-2.0343	-0.0115	0.445	0.0062	0.0716	-2.1244	0.0116	0.793	0.0029	0.0401
	(0.0660)	(0.7932)				(0.0533)	(0.2628)				(0.0856)
Metal Prod.	0.0356	-1.9352	-0.0113	0.494	0.0037	-0.0150	-0.5604	0.0477	0.693	0.0026	0.0215
	(0.0608)	(0.4570)				(0.0272)	(0.0819)				(0.0691)
Non.El.Mach.	0.0217	-0.3840	0.0026	0.149	0.0031	0.0353	-1.2839	0.0327	0.800	0.0011	0.0128
	(0.0729)	(0.4407)				(0.0351)	(0.1703)				(0.0543)
El.Mach.	-0.0726	-0.7338	0.0655	0.185	0.0048	0.0218	-0.4100	0.0281	0.607	0.0031	0.0281
	(0.0976)	(0.8569)				(0.0405)	(0.1293)				(0.0683)
Transp.Equipm.	0.0097	-1.6468	0.0120	0.615	0.0022	0.0526	-0.7451	0.0070	0.796	0.0013	0.0165
	(0.0355)	(0.2317)				(0.0156)	(0.0618)				(0.0589)
Misc. Prod.	0.0079	-5.7874	0.0500	0.907	0.0036	-0.1518	-0.8442	0.1856	0.817	0.0067	0.0956
	(0.1882)	(0.8759)				(0.0715)	(0.1942)				(0.1300)

But like the results for a_1 the results of b_1, b_2 and b_3 for the individual industries are rather poor and will not be subject of further discussion.[61)]

The results of (48) and (49) are presented in Table V.10 together with the mean and standard-deviation of $\hat{\gamma}_i$.

As expected the coefficient of S_{Mi} is negative for all industries, and significant for all but four, Basic Chemicals, Mineral Products, Non-El. Machinery and El. Machinery. More surprisingly the coefficient of S_{Mi} tends to be positive, and it is significantly positive for Total Mining and Manufacturing and three of the individual industries; Food Products, Pulp and Paper, and Basic Chemicals. It is significantly negative for one; Printing. A priori we would rather think that a_1 should be negative as the general tendency of reduced S_M could be considered as a move from a less to a more profitable position of the production units. And also that those with high S_M also had a lower rate of technical change than those with low S_{Mi} values.[62)]

The results of the second regression presented in Table V.10 show that for all industries those units with a decreasing share of labour in value added have a higher rate of technical change than those with a stable or increasing share; b_2 comes out to be significantly negative at 5% level for all industries. Again we've added some more evidence of labour saving technical change. We should note, however, that trend-like variations in the capacity utilization of capital may have affected these results. It has the same effect on b_2 as labour saving technical change. The results of the level of labour's share in value added are rather mixed. For Total Mining and Manufacturing its coefficient is significantly negative as expected a priori. On the other hand, we get the same outcome for only three of the individual industries: Food Products, Basic Chemicals and Misc. Products, while its coefficient is significantly positive for four industries: Clothing, Printing, Mineral Products and Transport Equipments.

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61) The variation of $\hat{\gamma}_i$ with size, measured as $\bar{N}_i = \sum_{t=1}^{T} N_{it}$ was also investigated. It is not unreasonable to believe that γ_i and N_i are positively correlated; that large units show a higher rate of technical change than small ones. For Total Mining and Manufacturing we get, however, a negative but not significant coefficient of $\bar{N}_i$ in a regression with $\hat{\gamma}_i$ as dependent variable. For the individual industries it is significantly positive for Wood Products and Mineral Products and significantly negative for Textiles and Basic Steel.

62) In Appendix V.2 simple correlation coefficients between the two factor shares and their growth rates are presented.

All in all the results of this section lead basicly to the same conclusions as the previous ones about the nature of technical change, namely that it is labour saving. But even if we now have obtained much the same conclusions about the nature of technical change from a number of various approaches it is evident that they are all vulnerable towards measurement errors. Those based on more or less purely direct production function regressions may be affected of the way capital stock is deflated, and those based on factor shares may be affected of trend like capacity variations of capital.

But accepting the data as they are we have obtained rather uniform conclusions. And after all we have added at least some evidence of that technical change is not neutral, but to be more conclusive about that issue we evidently need a better empirical base. It does not look possible to come much further with the present one.

APPENDIX V.1.
TENTATIVE TESTS OF TRANSITORY VARIATION IN DEMAND AND COSTS OF CHANGE.

a) Introduction

Basicly the two issues considered in this appendix have to do with proper specification of the model. We have made a number of simplifying assumptions when constructing the main model subject of analysis. This is primarily a result of "empirical necessity" as the possibilities are quite modest to investigate empirically the performance of more complex models.

If possible we would have analysed the importance of transitory variation in demand and costs of change by means of a model-specification taking these aspects explicitly into account. Instead we have to rely on an ad-hoc procedure of the same kind as the one applied in the analysis of the embodyment hypothesis in section V.3.d, namely by adding presumably relevant variables to the production relation and estimate the parameters of that relation by ordinary least squares.

And as for the embodyment - hypothesis two aspects of these variables are of particular interest. First if their coefficients have the expected signs and are significant and second if their presence in the production function alters the estimates on the main coefficients. In a sense the later aspect is the more important as it indicates the seriousness of the specification errors due to the presence of any transitory variations in demand or of costs of change.

b) On Transitory Variation in Demand

We may expect that in the short run the establishments have adjusted themselves to what they consider to be normal or "permanent" demand for goods.[1] The actual demand may, however, show short run variation that are not easily predictable. To some extent inventories can serve as a buffer towards such variations, but its absorbing capacity is generally limited. If a slack in the demand cannot be

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1) Some establishments may rather have adjusted themselves to a normal supply of materials. This is presumably true for units which get materials from primary production, or such industries as Slaughtering and Preparation of Meat, Dairies, Canning of Fruit and Vegetables, Canning of Fish and Meat, Fish and Herring Oil and Meal Factories etc.

absorbed by inventories it must necessarily result in a reduction of the capacity utilization.[2)]

There are, however, no information available to us about differences in the capacity utilization, either across establishments or over time. The question is, therefore, if any of those characteristics actually available are affected by transitory variation in demand so that any variations in the capacity utilization could be traced indirectly. Clearly, as pointed out, variations in the inventories of finished goods is one such variable. But that one is of minor interest in this context as it reflects that part of transitory variation in demand that does not imply variations in the capacity utilization. There is, however, another information; repairs and maintenance, that may tell us something about variations in demand that cannot be absorbed by inventories.

Some current repairs and maintenance have always to be carried out to "keep the wheels going". These will be assumed to be proportional to the capital stock.[3)] But for some of the repairs and maintenance there is a general flexibility as concerns when to carry them out. Particularly if they imply a break in production it is profitable to carry them out, if possible, in a recession so that current demand for a while can be dealt with by means of inventories. This is a fortiori true as establishments often prefer to let it be done by their own labour power that otherwise is engaged in pure production activities.

This leads us to try repairs and maintenance, or more precisely $T=I_R/K$ as a variable taking care of variations in the capacity utilization due to transitory variations in demand. Provided that the assumption about the role of this variable is true, and provided that it does not reflect other misspecifications we'll expect it to get a significantly negative

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2) Variations in the capacity utilization due to variation in the demand are usually considered to be a time-series phenomena. But no doubt, having production units with different locations, we may quite well have differences in the capacity utilization across units due to factors that affect the net price of their goods differently. And this is obviously true for some units which have adjusted themselves to a normal supply of materials such as Fish and Herring Oil and Meal Factories.

3) They do, however, quite probably depend on the age-distribution of the capital stock, But this effect cannot be taken properly care of by the present kind of data.

coefficient in case transitory variation in demand is of some magnitude. Even if the first presumption is true this is, however, a very weak test as the second quite probably is not. This is subject of some comments in section d) of this appendix where the results of our experiments of the repairs and maintenance variable are presented.

c) On Costs of Change.

If an establishment wants to hire more workers, or in particular to expand the capital stock (or both), resources like organization and administration etc. have to be allocated to this purpose, resources that otherwise could have been used for current production. This is roughly the basic idea of the theory of adjustment costs or costs of change; that there are specific costs of changing the scale of operation.[4) 5)]

The ad hoc procedure adopted to investigate the importance of this theory for our model specification we introduce into the production function ratio-variables expressing change in the scale of operation.[6)] The variables that we can think of in this context are $(I_p - \Delta SK)/K$ and $(N_t - N_{t-1})/N_{t-1}$, where K and N are capital stock and number of employees respectively, I_p is new investment goods and Δ is the estimated depreciation ratio.[7)] According to the presumed role of these variables, we should expect that, when including them in the production function their coefficients should get significantly negative coefficients provided that the costs of change hypothesis is valid. The results of these computations are presented in the next section.

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4) Cf. R. Lucas: "Adjustment costs and the Theory of Supply", The Journal of Political Economy, No. 4, 1967. See also M. Nerlove: "Estimation and Identification of Cobb Douglas Production Functions". North Holland Publ. Co. 1965, and C.D. Hodgins: "On Estimating the Economics of Large Scale Production, Some Tests on Data for the Canadian Manufacturing Sector. Ph.D. dissertation, Chicago 1968.

5) In addition to an assumption of costs of change a long run profit function is introduced. Thus it is assumed that resources are allocated to the purpose of changing the scale of operation to an extent that maximizes the long run (or multi-period) profit, if necessary on the expence of the short run (or one-period) profit.

6) Both $I_p - \Delta SK$ and $N_t - N_{t-1}$ were tried but didn't yield results superior to the ratio-variables.

7) Cf. Section h.iii) of Appendix II.7.

d) The Results

To reduce the distortion of the estimates of the parameters of the variables under consideration, due to poor quality of the reporting we introduce a couple of dummy-variables. When analysing the effect of $T = {}^{I_R}/K$ we also include:

$$F_T = \begin{cases} 1 \text{ when } I_R = 0 \\ 0 \text{ when } I_R > 0 \end{cases}$$

In Table A.V.1 where the results of these variables are presented, the percentage of $F_T = 1$ (or $I_R = 0$) is also reported. In the same way, when analysing the effects of $C = \frac{I_p - \Delta K}{K}$ we introduce the dummy-variable:[8)]

$$F_C = \begin{cases} 1 \text{ when } I_p = 0 \\ 0 \text{ when } I_p > 0 \end{cases}$$

The findings of these variables can be summarized in the following way: We get little support of the transitory variation in demand-hypothesis, and no support for the costs of change hypothesis. We get in fact that the coefficient of T is significantly positive for eight of the individual industries as well as for Total Mining and Manufacturing. It is negative and significant for one industry only, namely Electrical Machinery. On the other hand F_T is significantly positive for seven industries and for Total Mining and Manufacturing. The results of that variable thus yield a slight indication of that there are some variation in the capacity utilization due to transitory variation in demand that can be traced by means of repairs and maintenance.

The results of T are rather puzzling as we in addition to a negative effect due to transitory variation in demand also would expect a negative effect as units with predominantly old capital have more costs of repairs

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8) There is an argument of defining this dummy-variable in the following way,
$F_C = \begin{cases} 1 \text{ when } (I_p - \Delta K) \leq 0 \\ 0 \text{ when } (I_p - \Delta K) > 0 \end{cases}$ as then at least, partly it could have absorbed the effects of any asymmetry of the costs of change function. That is, a cost function that is positive when C is positive, but zero when C is zero or negative. But quite probably this does not matter much for the results.

TABLE A.V.1

Results for the Cobb Douglas Relation with Variables Presumed to Reflect Transitory Variation in Demand.*

Industry	ln L	$\ln \frac{K}{L}$	T	F_T	MSE	Number of F_T = 1,in %
	-0.009	0.286	1.722	0.077		
Tot. Min. and Man.	(0.005)	(0.007)	(0.146)	(0.016)	0.276	20.9
	-0.001	0.309	0.794	0.242		
Mining and Quar.	(0.022)	(0.041)	(0.417)	(0.086)	0.182	19.2
	-0.112	0.416	2.632	0.148		
Food Products	(0.015)	(0.027)	(0.450)	(0.047)	0.435	24.6
	-0.085	0.280	3.767	-0.062		
Textiles	(0.022)	(0.035)	(0.832)	(0.054)	0.146	19.3
	-0.033	0.097	1.770	-0.059		
Clothing	(0.022)	(0.024)	(0.430)	(0.041)	0.131	20.6
	0.121	0.223	1.047	0.268		
Wood Products	(0.025)	(0.040)	(0.784)	(0.073)	0.277	26.2
	-0.115	0.336	1.035	0.006		
Pulp and Paper	(0.015)	(0.025)	(0.219)	(0.044)	0.166	11.9
	0.029	0.153	1.985	-0.030		
Printing	(0.017)	(0.024)	(0.898)	(0.040)	0.119	25.0
	-0.027	0.237	4.339	0.105		
Basic Chemicals	(0.019)	(0.038)	(0.916)	(0.087)	0.487	14.7
	0.104	0.339	3.772	0.239		
Mineral Prod.	(0.022)	(0.037)	(0.650)	(0.077)	0.187	15.7
	0.052	0.314	4.950	0.434		
Basic Steel	(0.027)	(0.039)	(0.688)	(0.072)	0.174	13.2
	-0.047	0.134	0.167	0.139		
Metal Prod.	(0.019)	(0.034)	(0.629)	(0.051)	0.169	22.8
	0.080	0.037	0.134	0.056		
Non. El. Mach.	(0.021)	(0.041)	(0.836)	(0.061)	0.138	18.3
	0.062	0.037	-4.184	0.079		
El. Machinery	(0.028)	(0.044)	(0.839)	(0.080)	0.219	16.0
	0.084	0.089	0.065	0.082		
Transp. Equipm.	(0.012)	(0.021)	(0.408)	(0.040)	0.148	31.8
	-0.057	0.376	5.140	0.269		
Misc. products	(0.048)	(0.062)	(3.392)	(0.187)	0.408	34.2

* Method of estimation: Ordinary least squares.

TABLE A.V.2

Results for the Cobb Douglas Relation with Variables Presumed to Reflect Costs of Change*

Industry	ln L	ln $\frac{K}{L}$	C	F_C	MSE	no of $F_C = 1$ in %
	-0.014	0.272	0.175	-0.042		
Tot.Min.and Man.	(0.005)	(0.007)	(0.042)	(0.016)	0.280	22.1
	-0.006	0.280	0.097	0.039		
Mining and Quar.	(0.026)	(0.040)	(0.296)	(0.087)	0.188	22.7
	-0.112	0.375	0.105	0.006		
Food Products	(0.016)	(0.026)	(0.064)	(0.045)	0.445	26.6
	-0.081	0.281	0.465	-0.137		
Textiles	(0.024)	(0.034)	(0.241)	(0.055)	0.152	16.3
	-0.053	0.069	0.054	-0.161		
Clothing	(0.024)	(0.024)	(0.173)	(0.044)	0.134	20.4
	0.097	0.178	0.719	0.072		
Wood Products	(0.028)	(0.039)	(0.288)	(0.070)	0.282	38.8
	-0.130	0.291	-0.164	-0.131		
Pulp and Paper	(0.017)	(0.023)	(0.181)	(0.039)	0.168	23.4
	0.036	0.144	0.050	-0.025		
Printing	(0.018)	(0.023)	(0.151)	(0.039)	0.121	24.5
	-0.063	0.188	0.488	-0.272		
Basic Chemicals	(0.022)	(0.037)	(0.292)	(0.074)	0.489	27.6
	0.104	0.303	0.983	0.017		
Mineral Products	(0.027)	(0.039)	(0.297)	(0.090)	0.199	15.7
	0.105	0.204	0.338	0.452		
Basic Steel	(0.029)	(0.037)	(0.242)	(0.102)	0.192	5.8
	-0.044	0.137	0.396	0.141		
Metal Products	(0.020)	(0.034)	(0.192)	(0.053)	0.169	18.7
	0.062	0.044	0.503	-0.060		
Non.El.Mach.	(0.024)	(0.038)	(0.267)	(0.071)	0.136	13.8
	0.028	0.108	-0.214	0.001		
El.Mach.	(0.030)	(0.043)	(0.367)	(0.127)	0.244	5.6
	0.082	0.090	0.054	0.076		
Transp.Equipm.	(0.011)	(0.022)	(0.155)	(0.040)	0.149	25.0
	-0.023	0.394	0.054	0.283		
Misc.Products	(0.052)	(0.063)	(0-770)	(0.127)	0.407	23.9

* Method of Estimation: Ordinary least squares.

and maintenance than those with new capital; that is, kind of an adverse embodyment effect. Presumably the positive coefficients of T reflect a positive correlation between good management and good maintenance and that this effect overshadows completely any negative effects due to transitory variations in demand.

The results of C are basicly the same as those for T, but there are only three industries with a significantly positive coefficient of C. And there are four industries with a significantly negative coefficient of F_C. Therefore, except for Basic Steel and Mineral Products that have a significantly positive coefficient of F_C, there is no support at all of the costs of change hypothesis.[9] And even for the two industries mentioned the evidence of the validity of this hypothesis is rather weak.

The generally positive effect of C may be explained in two ways. First there may be an embodyment effect of current investments.[10]

And therefore the inclusion of current investments in our "recentness"variable of section V.d would probably distort the results. Second, as the effect of C is mainly determined by the across dimension of our data the results of that variable too may reflect variation in management across establishments; units with good management may have a higher long-run profit-maximising growth rate than those with poorer management.

Finally we note that by comparing the results of Tables A.V.1 and A.V.2 with the OLS-metod results of Table III.6 we can conclude that the variables introduced into the production function to analyse transitory variation in demand and costs of change have very little impact on the estimates of the factor-elasticities. The main effect of them seems to be that T and F_T twist the estimates slightly; for most industries the estimate on the capital elasticity is somewhat higher, but the estimate on the estimate on the elasticity of labour is correspondingly lower, leaving the elasticity of scale approximately unaffected. Therefore, whatever is the proper interpretation of the variables analysed they seem at least to have little or no importance for the results of our main model.

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9) The results of $\frac{N_t - N_{t-1}}{N_{t-1}}$ are not presented here as the performance of that variable is even poorer than that of C and F_C. And its results yield even less support to the hypothesis under consideration.

10) But this does not invalidate the argument put forward previously that some of the current investments reported do not add to the productive capacity of capital of the same year. Cf. Section h.ii of Appendix II.7.

APPENDIX V.2

TABLE A.V.3

Simple Correlation Coefficients Between the Level and Trend of Some Central Variables

Industry	Simple Correlation Coefficients Between				
	$\overline{\ln V_i}, v_i$	$\overline{\ln L_i}, l_i$	$\overline{\ln K_i}, k_i$	$S_{Mi}, \dot{S}_{Mi}$	$S_{Li}, \dot{S}_{Li}$
Tot.Min.and Man.	0.1029	0.0613	0.0787	0.0839	-0.2891
Mining and Quar.	0.2755	0.1986	-0.0057	0.0601	0.0677
Food Products	-0.0342	0.0348	0.1059	0.1703	0.0133
Textiles	-0.1454	-0.1741	-0.1340	-0.1369	0.5818
Clothing	0.2784	0.1040	-0.1745	0.0348	0.7259
Wood Products	0.4482	0.1374	0.2905	0.1475	0.8053
Pulp and Paper	-0.2207	0.0807	0.0986	0.3931	-0.7681
Printing	0.1033	0.1214	0.2273	-0.0909	0.5784
Basic Chemicals	0.0327	0.1254	-0.1127	0.2144	-0.9268
Mineral Prod.	0.5701	0.3684	0.4652	0.3159	0.5787
Basic Steel	-0.3355	-0.2018	0.0671	-0.1091	0.0603
Metal Prod.	0.0810	-0.0412	-0.0699	-0.0205	0.2590
Non.El.Mach.	-0.0329	-0.0460	-0.1804	0.2352	0.3648
El.Mach.	-0.0216	-0.3779	0.0538	-0.1708	0.7565
Transp.Equipm.	0.1593	0.1425	0.1392	0.1470	0.3214
Misc.Prod.	-0.1288	0.2043	-0.1767	-0.2270	-0.2463

Chapter VI.
SUMMARY AND CONCLUSIONS.

As has been pointed out in Chapter I three of the main aims of this study are; to gain experience and insight in the use of large bodies of micro data in econometric studies of production models, in addition to what is obtained by the Census study; to explore some of the central issues of that study by the empirical base available to us in the present one; and to compare the findings of the two studies to find out whether they conform or not. The outcome of the latter aim is left to an appendix of this chapter where the results are reviewed by industry.

Due to the first aim, which is also the general one a fairly detailed analysis of data is carried out in this study. And due to aims two and three we will here review the main empirical problems of this study and compare the data quality of this study with that of the Census study.

The great advantages of the present data as compared to the Census-study data are supposed to be that we have observations for more than one year for each production unit and that the present study covers larger units, the data for which are considered to be of generally better quality than those of smaller ones. But there are a variety of reasons why our data are not significantly better, nevertheless:

a) As pointed out in Appendix II.1 there are obviously a number of time series refering to different physical units at different times.

b) As compared to the Census study we use a much rougher classification of industries. Thus our samples are more heterogeneous.[1)]

c) We have made a few corrections of the characteristics reported, in cases when the units would have been excluded if having adopted the same approach as in the Census study.[2)]

d) Information about subsidies and duties are missing for 1959 and 1960. As we would like to have an output measure in factor-prices to let it conform as closely as possible to the output-measure of the Census study we have calculated the missing values of subsidies and duties.[3)]

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1) Cf. Appendix II.2 for the industry classification used, and cf. Section II.c about the reasons why this industry classification is adopted.
2) Cf. Appendix II.3.
3) Cf. Appendix II.4.

e) Time series data requires information that is not considered to be so demanding in pure cross-section data, namely about prices. The price data used are, however, rather poor of two reasons: First they are indices for production sectors covering a wide range of goods and not neccesarily the same as the mix of goods of the units of this study. Second for some sectors the price index of output is constructed by price data for inputs of materials and labour without correction of changes in the price of the latter input due to improvements in efficiency. This leads to an overvaluation of the growth in output price and using this index to deflate output in current prices implies that we undervalue the growth in "real" output.[4)]

f) The time-dimension does also cause troubles when trying to obtain a measure of capital input. First, as for output and inputs of materials we need price data to deflate the capital data so that they refer to a common price base. For this purpose a price index of new capital goods is applied. And this index tends therefore to overstate the true increase in prices of total capital stock as the price index is not corrected for quality improvements. Thus by using this index to deflate our capital data in current prices we tend to undervalue the growth of capital over time. Second, we need a depreciation rate to obtain capital data for all years.[5)] And we use a common depreciation rate for all units, and thus it is neither establishment - nor even industry - specific. Third we have little information of capacity utilization and its possible variation over time. Fourth we use generally a rougher capital input measure than the one of the Census study. We use capital stock (in "constant prices") while in the Census study a more establishment specific measure was applied, as machinery and buildings were given different weights. In addition inventories and computed value and operation costs of cars (with proper weights) were included in the capital measure of that study.[6)7)]

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4) Cf. Appendix II.2 and II.5.
5) Cf. Appendix II.7.
6) Cf. Appendix II.6.
7) It should be added here that the calculation missing capital values presented in Appendix II.7 does not in itself lead to poorer capital data. It is true that the "estimates" obtained are poor, but as is shown this is mainly due to poor quality of the data reported. Thus we probably neither gain nor loose anything as far as the quality of the capital data is concerned. But all in all there is a net gain by calculating missing capital values as we get more degrees of freedom.

These are sources of errors present in our data, but not present in the data of the Census study. Thus the advantages of the present data seem to be more than outweighted by the disadvantages due to more serious measurement errors.

In the Census study two types of errors were considered to be particularly serious, namely quality variations in labour input not accounted for by the measure applied and presumably random errors of measurement in capital. As the same labour input measure is applied by us the first error is also present in our data. But there are no particular reasons to believe that this error is more serious in this study than in the Census study. There are, however, reasons to believe that the errors in the capital measure are more serious in our data. We have the advantage of larger units with presumably better quality of reporting, also concerning the capital data. But on the other hand we have the errors pointed out in f) above that quite likely outweight that advantage.

The more or less random errors in output and wages do also seem to be more serious in our data, while the quality of the labour input measure may be better due to better reporting by the units of our study.

But all in all, as far as the estimation of production function parameters is concerned there is no basic difference between the data problems of the two studies. The main errors are in this study also quality variations in labour input and random errors in the capital measure. The random errors in output and wages are not so much more serious that this fact could be altered.[8)]

Chapter III of this study is devoted to the analysis of the effects of the two main errors with the main model; the Cobb-Douglas production function and the ACMS behaviour relation as the frame for the discussion.[9)] If the variables were measured without errors full information methods like indirect least squares would yield consistent estimates of the parameters of that model, namely the

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8) As a summary presentation of the behaviour of the main variables we have in Appendix II.8 computed some basic characteristics for them. These sample-statistics are also of some use as a supplement to the results in the following chapters, as they are also supposed to tell us quite a lot about the structure of the industries covered by our study. Thus we have in the appendix to this study, where the findings are surveyed by industry, also included some of the contents of Appendix II.8.

9) Cf. Chapter I.

factor-elasticities (and thus also the elasticity of scale) and the elasticity of substitution. Now it is shown in that chapter that the errors present lead to highly biased full information estimates. It is also shown that ordinary least squares on the production function generally will yield better estimates, even if these estimates are biased both due to the errors present and due to simultaneous equations.

An alternative to full information methods to take care of the simultaneity of the model is analysis of covariance of the production function. But it is demonstrated in Chapter III that this method is also extremely vulnerable towards measurement errors. This is shown in two ways. First by pointing out the unreasonable differences in the estimates obtained by using covariance analysis as compared to the OLS estimates. And second by tentative computations of the biases due to errors of measurement in labour and capital, showing that these biases are substantially more serious when covariance analysis is applied as compared to those present when OLS is applied.

The second half of Chapter III is devoted to various ways of eliminating the OLS-biases of the estimates on the factor-elasticities due to simultaneity and errors of measurement, and we end up with a slightly modified version of a method applied in the Census study. This method implies that the elasticity of labour is estimated by a particular factor share method and given this estimate, the elasticity of capital is estimated by using size-dummies as an instrumental variable for capital. By this method we are able to reduce or eliminate the effects of simultaneity and errors of measurement in capital, but not the effects of the errors in labour. We try to take care of these effects too by adopting a labour input measure that is consistent with the assumption made in the bias-computations of the OLS estimates about the behaviour of the errors in the labour measures. Namely that the quality component of labour (which in our case is equal to the error component) is perfectly correlated with the observed wage rate. But even if this assumption may be good enough for tentative bias calculations it is rather extreme to adopt when trying to obtain estimates "free" of biases due to errors in labour also. Therefore we are inclined to believe more in the factor share size-dummies results when not taking errors of labour into account. It should be noted, however, that for most of our industries it does not matter much what we do, as the effects of errors in labour are rather small for these industries, provided we have "eliminated" the effects of simultaneity and errors in capital.

Due to the particular type of errors of measurement in labour we have also serious problems in identifying the elasticity of substitution from the ACMS relation. It is easily shown that these errors tend to bias the estimate on that parameter towards our. Therefore we also try another method of estimating the elasticity of substitution, namely by the socalled Kmenta approximation which is a Taylor-expansion of the CES-relation around the value of one of the elasticity of substitution, corresponding to the Cobb-Douglas case. As the square of the capital-labour ratio enters this relation errors of measurement in capital are even more serious in this relation. By some experiments with this relation carried out for Total Mining and Manufacturing we try to figure out the importance of both simultaneity and errors of measurement in labour and capital. The results rather uniformly suggest that the elasticity of substitution is fairly high. On the other hand this does not correspond quite well either with the ACMS relation results or the results obtained for that relation as well as the Kmenta approximation in the Census study.

Apart from the results of the Kmenta relation for the elasticity of substitution the results of the two studies concerning the production function parameters conform fairly well. There are deviations for some industries, but these can be easily explained by differences in the empirical bases of the two studies.

Chapter IV covers aim 4; to explore the possibilities to use multiple testing procedures when "fishing" in data. As illustrations we apply such procedures to determine the nature of any variation of the error mean across establishments and over time within the frame of an analysis of covariance model for each of the two relations of our main model. The outcome of these tests strongly underline the heterogeneity of our samples previously pointed out. For all industries, and both for the production relation and the behaviour relation the error mean is establishment-specific; it does not vary between subindustries only. And we do also find that for most industries differences over time in the error mean of the production function can be represented by a trend. But except for a few industries there are no variations over time of the error mean of the behaviour relation.

Multiple test procedures are also tried in more complex situations where not only the error mean (or the intercept) but also the slope coefficients are allowed to differ. The analysis is carried out for Total Mining and Manufacturing and the findings suggest that there really are differences in the production function parameters across industries. There are also some suggestions of non-neutral technical change in these results, but this is an issure dealt with more in details in Chapter V.

Chapter V covers aims 5 and 6 as it deals with the problems present when trying to identify the importance and nature of technical change in Norwegian mining and manufacturing industries by our data. Particularly due to problems of deflating outputs and input of capital, variations in capacity utilization not taken into account and imperfect knowledge of depreciation our calculations must be rather tentative. But in this chapter we manage to demonstrate, I think, certain important aspects of the measurement of technical change, and we also demonstrate some methods to determine the nature of technical change that may be of some use in situations related to the present one.

First we show the importance of consistent estimation of factor elasticities in the computation of technical change. Here the advantages of a cross section of time series are apparent. Having aggregate data for the variables used i.e. pure time series data we would have had even more serious difficulties in getting reliable estimates on the parameters.[11] At least we could not have used the method that seems to be the better one given the variable measures available.[12]

Second we show that using the direct regression results of the production function with a residual trend when having a cross section of time series implies a not quite reasonable method of aggregation of the variables. That is, a growth rate of a certain variable of an establishment has the same weight in the total whether the establishment is small or large. A more conventional method of aggregation is to use arithmetic sums over establishments for the variable values. This method of aggegation implies that the aggregate growth rates are weighted sums of the individual growth rates with weights equal to the shares of the respective variables in the corresponding aggregate. For an industry as a whole the later method of aggregation is the more reasonable, and it is also the one pure time series data usually is based on.

It is demonstrated that at least for some industries it makes a substantial difference whether we use unweighted or weighted growth rates. And we conclude that the later ones together with the factor share size-dummies instrumental variable method of estimating the factor elasticities yield the more reliable results both concerning the contributions to growth from labour and capital and the importance of technical change.

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11) Some experiments carried out on pure time series (i.e. aggregates of our data) suggest that for that kind of data the pure factor share method is the only one that works. This method need not be too bad, but we loose the effects of any increasing or decreasing returns to scale that in fact seem to be **important** for a number of our industries.

12) Cf. Chapter III.

Third some further experiments are carried out with the CES relation trying to determine the nature of technical change. We are not very successful, however. And we try an alternative approach by carrying out some calculations on a generalized Cobb-Douglas relation, i.e. when both the intercept and the factor-elasticities are allowed to vary with time. In this context it is of particular interest to find out whether technical change is capital or labour saving rather than neutral. We also experiment with multiple test procedures related to those applied in Chapter IV. The outcome of the calculations show that for most industries technical change does not seem to be neutral. But for some of the industries the findings are rather ambiguous concerning the nature of non-neutrality.

Fourth we try to attack the issue of the nature of technical change by investigating the importance of "recentness" of the capital stock. This is a tentative test of the embodiment hypothesis as, if it is true recent vintages of the capital stock are more productive than older ones. The outcome of this part of the analysis is that for most industries there is some support at least, for this hypothesis.

Fifth it is shown that by analysing value added production functions only we may loose information about some of the technical change taking place in Norwegian mining and manufacturing-industries. By using a production function with gross production rather than value added as output measure we may analyse the role of materials and any changes of this factor's position over time.

Judged by materials' share in gross production there is a significant tendency of materials saving technical change for about half the industries as for these the share of materials shows a significantly negative trend. Using a generalized production function of a related type to the one used in the value added case this finding is confirmed for Total Mining and Manufacturing.

Sixth we try to explore the differences in degree of technical change between units by differences in level of efficiency and differences in the levels and trends of the factor shares. Even if we get significant coefficients in most cases, we do not really manage to explain much of the differences in degree of technical change between units. Evidently errors of measurement account for much of these differences.

Seventh and finally we have tried to trace the effects of transitory variation in demand and costs of change by some tentative calculations. We are not very successful, however, due to errors of measurement and certain problems of interpretation of the results.

All in all, if the success or failure of this study should be measured by number of unambiguous conclusions concerning the production structure, economic behaviour of production units and importance and nature of technical change in Norwegian mining and manufacturing industries we are at least closer to a failure than to a success. Nevertheless at the present state of applied econometric research it must be of interest and value to learn about the qualities and defects of new bodies of data. This is probably the only way open if we will seriously try to bridge the gap between economic theory and the realities around us.

We have demonstrated that even for very simple models we are in serious troubles when trying to identify the parameters. In the first part of this concluding chapter it is implicitly summarized the improvements of data that would be necessary, or in some cases at least desireable to manage to carry out a successful econometric analysis of the models presented.

We must admit, however, that our models may be undue simplifications of reality. Particularly there is a need of introducing dynamics, both in the assumptions of the production structure and in those concerning behaviour. But we know that such models are even more demanding concerning data. Thus at present and probably for a long time the rate of development of applied econometric analysis of production models is primarely determined by the rate of improvement of data.

Appendix VI.1.

SUMMARY OF THE MAIN FINDINGS BY INDUSTRY.

a. Introduction.

As we have carried out most parts of our explorations for fifteen individual industries there may be a need for a summary of the findings by industry. It is, however, rather difficult to present the results in tables by industry as the calculations carried out are of widely different natures. Thus, instead another method of presentation is tried.

In this appendix it is of main interest to throw some further light on the differences between the industries. Thus issues explored which lead to largely similar results for the various industries, such as the outcome of the multiple tests in chapter IV, are ignored in this context. And instead of reproducing the estimates or various numbers calculated we make a ranking of them, from 1 through 15. In case the estimates or numbers themselves are of particular interest they will be refered to in the text.

In five tables rankings of the results considered to be of most interest are presented. In this way we summarize in Table A.VI.1 the results of Appendix II.8, namely the mean values, growth rates and slope-coefficients from regressions on lnN of the seven main variables of this study. In Table A.VI.2 we have a corresponding ranking of the estimates on the capital and scale elasticities of the Cobb-Douglas relation obtained by the OLS method and the Klein Wald method of estimation. In the same way we have in Table A.VI.3 a ranking of the estimates on the elasticity of substitution obtained by covariance analysis of the ACMS relation. After all these are the only estimates obtained on that parameter that make sense as the results of the Kmenta-relation came out to be generally very poor. We cannot argue that one of the four sets of estimates reported is "better" than the others. But an evaluation of the four estimates together may allow us to conclude something for some industries about the probable level of the elasticity of substitution.

The two concluding tables refer to results obtained in chapter V. The first one, Table A.VI.4 presents a ranking of the unweighted and weighted growth rates of value added, labour and capital and in the final one, Table A.VI.5 we

Table A.VI.1.

Ranking of Some Main Characteristics.[1)]

Characteristic	Mean Value							Slope-coeff. from regr. on t[2)]				Slope-coeff. from regr. on lnN					
Industry:	$\ln L$	$\ln\frac{V}{L}$	$\ln\frac{K}{L}$	$\ln\frac{M}{L}$	$\ln W$	$\frac{WL}{V}$	$\frac{M'}{Y'}$	$\ln\frac{M}{L}$	$\ln W$	$\frac{WL}{V}$	$\frac{M'}{Y'}$	$\ln\frac{V}{L}$	$\ln\frac{K}{L}$	$\ln\frac{M}{L}$	$\ln W$	$\frac{WL}{V}$	$\frac{M'}{Y'}$
Mining and Quar.	10	3	6	15	2	9	15	13	7	13	11	8	2	11	9	8	10
Food Products	13	6	4	1	13	13	1	10	4	11	10	15	15	15	14*	5	15
Textiles	3	11	8	9	14	10	6	12	9	4	15	10*	8	4	13	7*	5
Clothing	8	15	15	11	15	8	9	15	12	3	14	9	10*	1	12	10	1
Wood Products	14	13	11	4	12	2	3	14	8	1	9	3	9	13	7	15	13
Pulp and Paper	4	4	1	2	3	11	2	7	1	2	3	12	4	10	5	2	4
Printing	15	12	7	14	9	3	13	9	15*	9	7	6	5	3	15	12	3
Basic Chemicals	11	2	2	5	7	15	8	1	3	6	5	11	14	9	6	6	6
Mineral Prod.	9	5	5	13	4	12	14	8	13	5	8	1	3	7	4	14	11*
Basic Steel	1	1	3	3	1	14	4	4	5	10*	2	2	1	5	3	11	7
Metal Prod.	5	8	9	8	8	6	10	11	10	12	13	13	11	12*	8	3	12
Non-El. Mach.	6	10	12	10	6	5	11	5	11	7	6	4	7	2	11	13	2
El. Mach.	2	9	14	7	5	4	7	3	6	8	1	7	6	8	1	9	9
Transp. Equipm.	7	14	13	12	10	1	12	6	14	14	4*	5	13	6	10	1	8
Misc. Prod.	12	7	10	6	11	7	5	2	2	15	12	14	12	14	2	4	14

1) Cf. Tables A.II.8.1-7.
2) Rankings of the growth rates of value added, labour and capital are presented in Table A.VI.4.
*) First negative number in the rank.

Table A.VI.2.
Ranking of OLS and Klein Wald Estimates of the Capital and Scale-Elasticities.[1)]

Industry	OLS est.		Klein Wald est.	
	Capital-el.	Scale-el.	Capital-el.	Scale-el.
Mining and Quar.	6	8*	9	10*
Food Products	1	15	6	13
Textiles	5	9	10	12
Clothing	14	12	5	6
Wood Products	9	2	7	1
Pulp and Paper	4	13	11	14
Printing	10	6	12	7
Basic Chemicals	8	14	2	9
Mineral Prod.	3	1	3	3
Basic Steel	7	4	1	5
Metal Prod.	11	10	13	11
Non-El. Mach.	15	3	8	8
El. Mach.	12	7	4	2
Transp. Equipm.	13	5	15	3
Misc. Products	2	11	14	15

1) Cf. Table III.6

* First estimate in the rank below one.

Table A.VI.3.

Ranking of the Covariance Analysis Estimates of the Elasticity of Substitution from the ACMS Relation.[1)]

Industry	No eff. elimin.	T-eff. elimin.	E-eff. elimin.	E and T eff. elimin.
Mining and Quar.	8	12	6	15
Food Products	4	5	14	13
Textiles	5	4	8	6
Clothing	11	8*	12	9*
Wood Products	2	2	7*	7
Pulp and Paper	13	10	15	11
Printing	10	9	10	10
Basic Chemicals	6	6	13	12
Mineral Prod.	1	1	5	5
Basic Steel	9	11	11	14
Metal Prod.	14	15	4	3
Non-El. Mach.	12	13	3	2
El. Mach.	7*	7	8	8
Transp. Equipm.	15	14	2	4
Misc. Products	3	3	1	1

1) Cf. Table III.2

* First estimate in the rank below one.

Table A.VI.4.
Ranking of Unweighted and Weighted Growth-Rates for Value Added, Labour and Capital.[1)]

	Unweighted growth rates			Weighted growth rates		
Industry	Value added	Labour input	Capital input	Value added	Labour input	Capital input
Mining and Quar.	8	15	7	9	14	12
Food Products	3	7	9	5	7	7
Textiles	12	10*	11	14	12*	13
Clothing	14	13	13	13	13	14
Wood Products	9	6	6	2	6	1
Pulp and Paper	6	14	8	10	15	9
Printing	15*	9	15	15*	8	10
Basic Chemicals	5	11	14	3	9	15*
Mineral Prod.	13	12	10	4	10	2
Basic Steel	2	3	1	6	5	3
Metal Prod.	7	4	2	8	3	5
Non-El. Mach.	11	8	5	11	2	8
El. Mach.	4	2	4	7	11	6
Transp. Equipm.	10	5	12	12	4	11
Misc. Prod.	1	1	3	1	1	4

1) The unweighted growth rate is computed as the OLS estimate on "a" from the regression $\ln X_{it} = a_o + at + u_{it}$ while the weighted growth rates are computed as the OLS estimate on b from the regression $\ln(\sum_i X_{it}) = b_o + bt + u_t$. Cf. Section V.2.

* First negative number in the rank.

Table A.VI.5

Ranking of Calculated Contributions to Growth from Labour, Capital and "Shifts".

	Unweighted growth-rates						Weighted growth-rates		
	OLS-method of estimation			Klein Wald-method of estimation			Klein Wald of estim.		
Industry	Labour	Capital	"Shifts"	Labour	Capital	"Shifts"	Labour	Capital	"Shifts"
Mining and Quar.	15	4	6	15	8	6	14	11	3
Food Products	7	2	4	7	9	4	8	7	4
Textiles	10*	6	11	10*	11	9	12*	13	13
Clothing	13	13	12	13	12	13	13	14	9
Wood Products	6	8	10	6	5	11	5	3	6
Pulp and Paper	14	6	3	14	10	3	15	9	5
Printing	9	14	15*	9	15	15*	7	10	15*
Basic Chemicals	11	11	2	11	13	2	9	15*	1
Mineral Prod.	12	5	14	12	7	14	10	2	8
Basic Steel	3	3	5	5	1	5	6	1	10
Metal Prod.	4	9	9	3	4	8	4	6	11
Non-El. Mach.	8	15*	8	8	3	12	2	8	14
El. Mach.	2	10	7	2	2	7	11	4	7
Transp. Equipm.	5	12	13	3	14	10	3	12	12
Misc. Prod.	1	1	1	1	6	1	1	5	2

1) Cf. Tables V.1-3.

* First negative number in the rank.

have a ranking of the calculated contributions to growth from labour capital and "shifts" according to the results obtained when using unweighted growth rates and the OLS and Klein Wald methods of estimation and when using weighted growth rates and the latter method of estimation.

In addition we will also refer to the findings concerning the nature of technical change.

To some extent our findings about the production function parameters will be compared with the corresponding results of the Census study, for industries covering approximately the same industry groups in the two studies. There is a number of reasons why even for these industries the results may be quite different. But anyway such a comparison may be of some interest.

b. Mining and Quarrying.

Mining and Quarrying is that of our fifteen industries which has the lowest mean value of the materials labour ratio and materials share in gross production. On the other hand it ranks third concerning average value added productivity of labour and second concerning wages. It has also the lowest (and negative) unweighted growth rate of labour input while only one has a lower weighted growth rate of that variable. The growth rate of capital input drops from rank 7 to rank 12 when turning from unweighted to weighted growth rates. The (unweighted) growth rates of the materials labour ratio, labour's share in value added and materials share in gross production are also quite low, with rank 13, 13 and 11 respectively. We also note that Mining and Quarrying ranks second concerning the growth of the capital labour ratio with size. Probably there is a basic difference between Mining on one side and Quarrying on the other not accounted for in our analysis.

The OLS estimate of the capital elasticity has a rank slightly below the mean while the Klein Wald estimate has a higher rank. The estimate on the scale-elasticity is slightly below one for both methods.

The covariance analysis estimates of the elasticity of substitution suggest that this parameter is below one. When eliminating year-effects the estimates are among the lower ones obtained for any one industry. In fact when both year and establishment effects are eliminated it ranks lowest. In that case the elasticity of substitution also is significantly less than one.

The calculated contributions to growth imply that labour has the lowest rank when using unweighted growth rates and the second lowest when using weighted growth rates. Capital's contribution has a fairly high rank when using the OLS method of estimation and unweighted growth rates. It is lower using the Klein Wald method of estimation for the same growth rates and it has a fairly low rank when using that method of estimation and weighted growth rates. In the latter case contributions from shifts rank third among our fifteen industries.

The significantly negative trend of materials' share in gross production suggest that there is a materials saving type of technical change in this industry. At the "value added level" the results about the nature of technical change does not tell us much. There is, however, a slight suggestion of capital saving technical change. The embodiment hypothesis has no support in our computations for this industry.

c. Food Products.

This industry is rather heterogeneous, covering widely different activities. In the Census-study seven of the twentyseven industries were from the 20- and 21-industry groups. The results for these industries were rather different, and as we also have a few units from group 22 in our Food Products industry we should expect rather poor fit of our relations. And this is proved by the computations carried out. Only one industry, Basic Chemicals has a higher mean square errors of the two main relations; the Cobb-Douglas production function and the ACMS behaviour relation.

Table A.VI.1 tells us that this industry consists of mostly small units, it pays low wages and has a low share of labour in value added. On the other hand it is the one having the highest average materials labour ratio and the highest share of materials in gross production.

There is nothing particularly note-worthy about the growth rates computed except perhaps that this industry,if paying low wages rank four concerning growth of wages over time, and also that it ranks third concerning unweighted growth rate of value added and fifth concerning the weighted growth rate of that variable.

But the variation of the main variables along the size-dimension is rather peculiar, except for labour's share in value added. Both average productivity of labour, the capital labour ratio the wage rate and materials share in gross production vary inversely with size. And as we see from Table A.VI.1 the slope coefficient for the wage rate rank 14 while the others rank 15. This does also suggest that our Food Products industry is quite inhomogeneous.

Rather surprisingly, this industry has the highest estimate of the capital elasticity when using the OLS method. The estimate is also high, but has a lower rank when using the Klein Wald method. The estimates on the scale elasticity suggest that decreasing returns to scale rules in this industry. The OLS-estimate ranks 15, and the elasticity of scale is significantly below one according to the results of that method. The Klein Wald estimate on that parameter does also have a low rank, namely 13.

There is a basic difference in level of the covariance analysis estimates on the elasticity of substitution when eliminating establishment effects and when not. In the first case the estimates have a very low rank and the elasticity of substitution is according to both sets of results significantly below one, while in the latter case the estimates have a fairly high rank and the elasticity of substitution is significantly above one.

As pointed out this industry is pretty heterogeneous and thus it is reasonable to believe that taking out establishemt effects yields better estimates. Thus, if anything our results suggest that the elasticity of substitution of this industry is below one.

Due to the high OLS-estimate on the capital elasticity the contribution to growth of capital using this method of estimation rank second. We also note from Table A.VI.5 that irrespective of method of estimation and type of growth rates contribution to growth form shifts has a high rank.

Also for this industry there are suggestions in our results of materials saving technical change, while there is no strong evidence against neutrality of the technical change at the value added level. But there is some support for the embodiment hypothesis in the results of Section V.3.c for this industry.

d. Textiles.

On the average Textiles has fairly large units as the mean value of labour input rank third among our fifteen industries. But the average productivity of labour is low and the mean value of the wage rate is in fact the second lowest. But in spite of the low wages in this industry the growth in the wage rate is moderate. On the other hand labour's share in value added shows a fairly strong positive trend.

Textiles is the industry with the sharpest decrease in materials' share in gross production over time. This is also reflected in the low rank of the growth rate of the materials labour ratio. These two variables do also show a rather strong positive covariation with size, while this is not true for the other main variables, except for the capital labour ratio which also shows a positive correlation with size.

Using the OLS method the estimate on the capital elasticity ranks fairly high while the estimate on the scale-elasticity has a rank below the average. According to these results the elasticity of scale is significantly below one, and the Klein Wald method yields almost the same point-estimate. Thus there is evidence of decreasing returns to scale in this industry. This does not correspond quite well with the results of the Census study which suggested increasing returns to scale for this industry.

Three of the four covariance analysis estimates on the elasticity of substitution are above one. But none of the results imply that the elasticity of substitution is significantly different from one at conventional levels of the tests. These results at least correspond quite well with those of the Census study.

The contents of Table A.VI.4 tell us that Textiles is a stagnant industry as the growth rates computed for this industry all have a low rank. So is true for the contributions to growth of Table A.VI.5 too.

As pointed out there is a sharp decrease over time in materials' share in gross production suggesting materials saving technical change also for this industry. At the value added level the findings about the nature of technical change are ambiguous. The results of the multiple test of Section V.3.b. indicate, if anything that technical change is neutral. The results of Section V.3.c. lend fairly strong support to the embodiment hypothesis.

e. Clothing.

Like Textiles, Clothing is also a low-wage industry. In fact it is the one having the lowest average wage rate according to our computations. It does also have the lowest average productivity of labour and the lowest average capital labour ratio. Like Textiles, Clothing has a sharp decline over time in materials' share in gross production and a low (the lowest) growth rate of the materials labour ratio.

There is no tendency of an equalization over time of the wages of this industry and the other industries as it ranks 12 concerning the growth of the wage rate. In spite of this Clothing ranks third concerning the growth of labour's share in value added.

This industry is somewhat peculiar in another way also as it ranks first concerning the slope-coefficient of the size-variable lnN both for the materials labour ratio and materials' share in gross production.

According to the results of the Cobb-Douglas relation when applying the OLS-method this industry has very low elasticities of capital and scale as the estimates on them rank second and fourth lowest respectively. But this is one of the industries for which the net effect of the OLS-biases discussed in Chapter III seems to be most important. When using the Klein Wald method instead of OLS the estimates on the elasticities under discussion are substantially increased. While the results of the OLS method imply that the elasticity of scale is significantly below one the results of the latter method suggest that it is above one. The latter results correspond better to those of the Census-study.

The four covariance analysis estimates of the elasticity of substitution are all below one. And when no effects or E effects are eliminated this elasticity is significantly below one. Thus these results lend relatively strong support to a conclusion that in Clothing the elasticity of substitution is fairly low. The Census study does, however, yield quite different results on this point.

Table A.VI.4 tells us that like Textiles, Clothing is a stagnant industry. There is no basic difference between unweighted and weighted growth rates, and Table A.VI.5 tells us that the calculated contributions to growth are low whatever method of estimation and type of growth rates are used.

The findings about the nature of technical change-issue suggest strongly that it is materials saving, while at the value added level they are largely ambiguous. But there is a slight support of the embodiment hypothesis in the results of Section V.3.c.

f. Wood Products.

This industry consists of mostly small units. It has also a low average productivity for labour, a low capital/labour ratio and pays fairly low wages on the average. But it ranks first concerning the growth over time in labour's share in value added. On the other hand the materials labour ratio shows a fairly stable pattern over time as the growth rate of this variable is the second lowest.

From the regressions of the main variables on the size-variable we note that both the materials labour ratio and materials' share in gross production rank third lowest, while labour's share in value added ranks lowest. The findings of the two share variables suggest that there are some heterogeneity along the size-dimension in this industry, both at the gross production level and at the value added level.

The estimates of the capital elasticities have medium ranks while the estimates on the scale elasticity rank very high. Using the OLS method we have that the scale elasticity is significantly above one. The corresponding estimate ranks second and using the Klein Wald method it ranks first. These findings correspond fairly well with those of the Census-study.

From the covariance-analysis results of the ACMS relation we note that there is a basic difference between the estimates on the elasticity of substitution when across effects are eliminated and when they are not. In the first case they are close to one and the elasticity of substitution is not significantly different from one. When across effects are not eliminated the estimates are much higher and the elasticity of substitution is significantly above one at 5% level. If anything, these results suggest therefore that the elasticity of substitution is fairly high. But on the other hand this does not correspond quite well with the results obtained for the corresponding industries of the Census-study.

The unweighted growth rates of value added and capital and labour input have medium ranks while the weighted growth rate of value added rank second and the corresponding growth rate of capital rank first. This leads to, as we note from Table A.VI.5, that the calculated contribution to growth from capital get a higher rank when using weighted growth rates, but due to the higher weighted growth rate of value added, the rank of shifts is higher when using that type of growth rates. The findings about the nature of technical change suggest that also in this industry it is materials saving. The results are inconclusive about the issue of capital or labour saving, or neutral technical change. Finally we note that there is evidence from our results of embodied technical change in Wood Products.

g. Pulp and Paper.

This is the first heavy industry of those so far considered. It ranks four concerning average labour input and also concerning average productivity of labour. It ranks first concerning the average capital labour ratio and third concerning average wages. It is fairly materials intensive as both materials labour ratio and materials' share in gross production rank second. And it is also fairly capital intensive as labour's share in value added has a rather low rank.

Wages have a high growth rate. In fact this industry has the highest growth rate of that variable. We should note, however, that this is partly due to decreasing prices of output, as our wage-rate variable is deflated with a price index of output price. Pulp and Paper is one of the few industries with a fairly stable share of materials in gross production over time. On the other hand the share of labour in value added has a relatively strong growth over time as its trend coefficient ranks second. Rather surprisingly Pulp and Paper ranks second also concerning the growth with size of labour's share in value added. This is peculiar as this industry has a high rank concerning the capital labour ratio's growth with size. And it is also somewhat peculiar that this industry rank as low as 12 concerning the growth of average productivity of labour with size and as high 4 concerning the growth of materials' share in gross production with size. All in all these findings taste of an heterogeneous sample. This was also the conclusion of some computations in the Census study, particularly that there are basic differences between small and large units of this industry. Perhaps we could have reduced heterogeneity by dividing this industry into two, namely the Pulp industry and the Paper industry But no calculations are tried in this direction.

The capital elasticity is fairly high according to the OLS method as the estimate on this parameter ranks fourth. The Klein Wald estimate is somewhat higher but has a substitutionally lower rank. The estimates in the scale-elasticity are low for both methods. The OLS estimate is the third lowest obtained for any of the fifteen industries and the Klein Wald estimate ranks as the second lowest. They are both below one and according to the results of the former method the elasticity of scale is significantly below one at any reasonable levels of the test. Thus there is evidence of decreasing returns to scale for this industry. And this finding is supported by the results of the Census study.

The estimates on the elasticity of substitution are also rather low. According to all four sets of results the elasticity of substitution is significantly below one at 5% level. On the other hand the results of the Census study suggested, if anything at all, that this parameter is above one for Pulp and Paper.

This industry has a relatively sharp decline in labour input over time. The unweighted growth rate for this variable ranks second lowest while the weighted one ranks lowest. The same rank has the corresponding computed contribution to growth from labour. And in both cases the contributions to growth from shifts rank high; as third when unweighted growth rates are used and as fifth when weighted ones are used. This difference is mainly due to a lower weighted than unweighted growth rate of value added.

In opposition to the previous industries there are no indications of materials saving technical change in this industry. On the other hand there are fairly strong indications of a capital saving type of technical change. And we note from Section V.3.c that there is no support for the embodiment hypothesis in this industry.

h. Printing.

As pointed out in the previous chapters we have a serious data-problem for this industry due to an over-rating of the price-change over time and a corresponding under-rating of the growth in output. This makes some of our results as good as worthless. For instance are the growth rates of output and wages negative and rank lowest among our industries. The same is true for the contributions to growth from "shifts". There are, however, a few other results that should not be seriously affected by the particular data problem for this industry.

Printing is the industry with the lowest average size of the units. Thus it does also have the lowest rank concerning labour input. It does also have a low materials labour ratio and a low share of materials in gross production. It is, however, rather labour-intensive as labour's share in value added is the third highest. The growth rates over time are fairly "normal" except those depending on the price-index of output.

From the results of the regressions on the size-variable, lnN we note that the larger units of this industry tend to be more materials intensive than smaller ones while they seem to be less labour intensive. The wage rate tends to be slightly lower for large units.

Printing seems to have a rather low capital elasticity. The two estimates on the scale-elasticity are exactly the same and slightly above one, but according to the OLS-results this parameter is not significantly above one at any reasonable levels of the test. These results conform fairly well with those of the Census-study.

The estimates on the elasticity of substitution are all below one, but only one set of results (when no effects are eliminated) implies an elasticity of substitution significantly below one. But in general the results suggest relatively strongly that this parameter is below one for Printing. And this conforms quite well with the results of the Census-study.

Printing has a very low growth in capital input judged by the unweighted growth rates. The weighted growth rate for this variable is somewhat higher and when using the Klein Wald method of estimation capital's contribution to growth in Printing rank as ten, while when using unweighted growth rates it ranks as the last one using the same method of estimation, and second last using the OLS method and unweighted growth rates. Both the growth rates of labour and this factor's contribution to growth are fairly "normal" as compared to the other industries.

There is no support of the embodiment hypothesis in the results for this industry. Neither are there indications of materials saving technical change. But there is some evidence of capital saving technical change in the Printing industry.

i. Basic Chemicals.

This industry has a lower average size of the units than one might expect, as one would usually consider it as a rather heavy industry. But average productivity and the capital/labour ratio are quite high, however, as they both rank second. Their high rank conforms quite well with the low (in fact the lowest) rank of labour's share in value added. Basic Chemicals has a relatively high growth rate of wages and it ranks first concerning the growth in the material's labour ratio.

Even if the mean square errors of the relations estimated are quite high there are no strong suggestions of heterogeneous sample along the size-dimension. The more surprising finding from the regressions of the main variables on the size-criterion lnN is the negative growth rate with size of the capital labour ratio.

The OLS-biases of the Cobb-Douglas production function seem to be quite serious for this industry: The OLS estimate on the capital elasticity ranks eight while the Klein Wald one ranks second. The OLS-estimate on the scale-elasticity ranks second last and this method of estimation yields an elasticity of scale significantly below one. The Klein Wald estimate has a much higher rank and it is slightly above one. Thus in this case the OLS results are strongly misleading, and we should not conclude that the elasticity of scale really is below one.

When not eliminating across effects the covariance analysis estimates on the elasticity of substitution is slightly above one

while the elasticity of substitution is significantly below one when the across effects are eliminated. Thus there is a very slight suggestion of an elasticity of substitution below one in these results.

While the growth rates of value added are fairly high for Basic Chemicals the growth rates of labour are rather low and those of the capital input are very low, in fact the weighted growth rate of that variable is negative. This implies low ranks of the calculated contributions to growth from the ordinary factors of production and a high rank of "shifts". This is confirmed by the contents of Table A.VI.5.

The findings of Section V.3.b suggest that technical change is labour saving, and those of Section V.3.c. yield a slight support of the embodiment hypothesis.

j. Mineral Products.

The only things worth nothing from the mean values computed are that this industry seems to be rather capital intensive with low ranks both of labour's share in value added and of materials' share in gross production and that it pays fairly high wages. On the other hand the growth rate of wages over time is rather low. From the results of the regressions of the main variables on lnN we note that large units are more capital intensive than smaller ones as the regression coefficient of the size-regression of labour's share in value added is significantly negative and ranks as second last. This conforms quite well with the high rank of the "size"-regression coefficient of the capital labour ratio and it may be the main explanation of the high "size"-regression coefficient of the average productivity of labour, which in fact ranks as one.

Mineral Products seems to have both a high elasticity of capital and a high elasticity of scale. The OLS estimate on the latter has rank one, and the elasticity of scale is significantly above one according to these results. The Klein Wald estimate on that parameter is also quite high. The estimate on the capital elasticity ranks third for both methods.

The elasticity of substitution does also seem to be quite high. When the across effects are not eliminated the estimates in fact rank first. While when across effects are eliminated the estimates are lower but they are still above one. According to the two former sets of results the elasticity of substitution is significantly above one.

The results both concerning the elasticity of scale and the elasticity of substitution conform fairly well with those of the Census-study.

There is a basic difference between the unweighted and weighted grwoth rates of value added and capital for Mineral Products. The unweighted ones are rather low while the weighted ones rank fourth and second respectively. This implies that the large establishments also have larger growth rates of these two variables than small ones. This may suggest that what we estimate as increasing returns to scale is a basic difference between small and large units concerning the <u>level</u> of the scale-elasticity due to a difference in the level of the capital elasticity; That large units also tend to have a large capital-elasticity and scale-elasticity. This is also suggested by the "size"-regressions. But this issue has not been subject of further investigation.

The calculated contribution to growth from capital when using weighted growth rates ranks second. But due to the high weighted growth rate of value added the "shifts" have also a much higher rank than when unweighted growth rates are used.

The results of Chapter V suggest that technical change is materials saving, and at the value added level that it is, if anything labour saving. We get some support of the embodiment hypothesis as the coefficient of the "main embodiment variable", E is significantly positive as it should in case the embodiment hypothesis is true. On the other hand the coefficient of the dummy-variable, F_E is also significantly positive while it should rather be negative to be "consistent" with the results obtained for E.

k. *Basic Steel.*

This is a typically heavy industry. And it ranks first among our fifteen industries both concerning average hours worked per establishment and the average productivity of labour. It does also rank first concerning the level of wages and it has a high rank of the capital labour ratio. In addition we note from the mean values computed that this industry is rather capital intensive and also fairly materials intensive.

There are no significant trends in the share-variables in the period overed by the data while there is a significantly positive trend with size of materials share in gross production and a significantly negative trend with size of labour's share in value added. Large units does also tend to pay higher wages and they have definitely higher capital labour ratios on the average and also higher average productivity of labour.

It makes a substantial difference which method of estimation is applied on the production function for this industry. Using the OLS method the estimate on the capital-elasticity ranks seven, while using the Klein Wald method it ranks first. The difference between the two estimates on the scale-elasticity is substantially less. It is above one for both methods and according to the OLS method the elasticity of scale is significantly above one at 5% level. Thus, even if the Klein Wald estimate is slightly lower there is evidence of increasing returns to scale in Basic Steel.

The covariance analysis estimates of the elasticity of substitution suggest that this parameter is fairly low, but due to large standard errors we cannot reject the hypothesis of an elasticity of substitution of one for this industry.

The unweighted growth rates of value added, labour and capital are quite high as they rank 2, 3 and 1 respectively. The weighted growth rates are also high but with a somewhat lower rank. Using the Klein Wald method of estimation the calculated contribution to growth from capital ranks as one both when unweighted and when weighted growth rates are used. The contribution from labour does also have a fairly high rank for Basic Steel, while when using unweighted growth rates and the Klein Wald method of estimation the shifts have a fairly low rank.

About the nature of technical change our results suggest that it is labour saving. And there is also some support for the embodiment hypothesis in the results for Basic Steel.

l. Metal Products.

Judged by the mean values computed Metal Products is a rather "normal" industry. The only things worth nothing are that the average size of the units is slightly above the average for our fifteen industries and that materials' share in value added is somewhat below. The trend over time of the labour variable is significantly negative while labour's share in value added seems to be rather stable over time. The results of the size-regressions suggest that there are only small differences between small and large establishments.

Both the capital elasticity and the scale elasticity seem to be rather low for this industry. The estimate of the latter is less than one both when using the OLS and the Klein Wald method. And according to the results of the former the elasticity of scale is significantly less than one. The Klein Wald estimate is only slightly higher, and thus there is some evidence of decreasing returns to scale in this industry. On the other hand the results of the Census study suggested that there are increasing returns to scale in the Metal Products industry.

Concerning the analysis of covariance estimates of the elasticity of substitution it makes a basic difference whether across effects are eliminated or not. In the first case the estimates are above one but the elasticity of substitution is not significantly above one according to these results. If across effects are not eliminated the estimates are below one and the elasticity of substitution is according to these results significantly less than one. The results of the Census study lend support to the later results. Thus, if anything the results indicate that the elasticity of substitution is rather low for Metal Products.

The growth rates for value added are of moderate magnitude while those of the inputs are fairly high. The calculated contribution to growth from labour has a fairly high rank whatever method of estimation and type of growth rates are applied: The "shifts" have a low rank in case we use the Klein Wald method of estimation and weighted growth rates. We should note, however, that for this industry we probably underrate the growth in output due to an overrating of the growth in prices. Thus we also underrate the contribution to growth from "shifts".

The rather sharp decrease of materials' share in gross production suggests that technical change is materials saving in Metal Products. The results of the value-added relations are, however, inconclusive about whether technical change is labour or capital saving. Concerning the embodiment hypothesis we get basicly the same results for this industry as for Mineral Products; The coefficient of the main "embodiment variable", E is significantly positive as it should if the embodiment hypothesis is valid. But the coefficient of the dummy-variable F_E is also significantly positive which should not be the case if the embodiment hypothesis is valid. But even if the results are "inconsistent" there is, all in all more evidence for than against the embodiment hypothesis.

m. *Non-Electrical Machinery.*

Judged by the mean values there are small differences between this industry and the previous one. The main differences are that the average productivity of labour and the capital labour ratio are somewhat lower for the present one. Concerning growth rates the differences are somewhat more apparent as this industry has a stronger positive trend in the materials labour ratio, it has a positive trend in labour's share in value added and it has a less pronounced negative trend in materials share in gross production. The size-regressions suggest that larger units are more materials intensive and capital intensive than smaller ones, that large units pay about the same wages as small ones and that average productivity of labour is increasing with size.

For this industry too it matters quite a lot which method of estimation is applied on the production function. According to the OLS-method the capital elasticity is not significantly different from zero at 5% level, and its estimate is the lowest obtained for any industry. The Klein Wald estimate on this parameter is much higher and does also have a much higher rank. According to the OLS method there are increasing returns to scale as the scale parameter is significantly above one. The Klein Wald estimate is somewhat lower, and does also have a lower rank, but it is still above one. Thus, if anything, the results suggest that there in fact are increasing returns to scale in this industry. This finding is supported by the results of the Census-study.

The covariance analysis results of the ACMS relation are very ambiguous concerning the level of the elasticity of substitution. When the across effects are not eliminated the estimates on that parameter are below one and when they are eliminated they are above one. But in no case do the results yield an elasticity of substitution significantly different from one. The results of the Census study conform closer to the former results.

There is a basic difference between the unweighted and weighted growth rate of labour input, the latter being much larger than the former. The former has rank 2 and so has the calculated contribution to growth when that kind of growth rates are used, together with the Klein Wald method of estimation. We should also note that the calculated contribution to growth from shifts has a low rank irrespective of type of growth rates when the Klein Wald method of estimation is used. This may, however, partly be a result of an upward bias in the price index computed and a corresponding downward bias in the growth rate of output.

About the nature of technical change there are indications of a labour saving type. We also note that the embodiment hypothesis seems to get some support from the calculations of Section V.3.c.

n. *Electrical Machinery.*

Electrical Machinery ranks second concerning the average labour input. This does not conform quite well with the median of number of employees of this industry according to which it ranks seven. There is a number of possible reasons for this discrepancy, and probably the more important is the wage differences between production and non production workers and the role they play in our computations of hours worked by non production workers.

We note that the share variables are fairly stable over time and they do not vary much with size either.

As for Non-Electrical Machinery it matters much whether we use the OLS or Klein Wald method of estimation. The estimates both on the capital and the scale elasticity are much larger and have much higher ranks when the later method is applied. In fact there is only one among our fifteen industries that has a higher Klein Wald estimate on the scale elasticity. Thus even if the OLS results do not yield an elasticity of scale significantly above one there seems to be enough evidence to conclude that there are increasing returns to scale in this industry. The results of the Census study seem to support this finding.

The covariance analysis estimates on the elasticity of substitution are about one. The Census study results lend some support to the conclusion of a rather low elasticity of substitution. But the standard error of the estimates are large and the elasticity of substitution is not significantly below one according to those results.

There are some differences between unweighted and weighted growth rates of output and labour input as the later ones are substantially lower, particularly for labour input. The weighted growth rate of capital is also somewhat lower than the unweighted one. This does also affect the rank of the calculated contribution to growth from labour as shown in Table VI.5. But we also note from this table that the rank of the contributions to growth from "shifts" is not affected, either of method of estimation or type of growth rates.

About the nature of technical change there are some indications of a capital saving type, while there are no indications of a materials saving type, unlike a number of other industries. There are no support of the embodiment hypothesis either in our computations.

o. Transport Equipment.

This industry has a low average productivity of labour, low capital labour - and materials labour ratios and a rather low wage rate. In addition we learn from the mean values computed that Transport Equipment is labour intensive as it has the highest share of labour in value added among our industries, while there are only three other industries with a lower share of materials in gross production.

This industry does also have a very low growth rate of wages, but we should note that this finding and partly also the mean values of the average productivity of labour and the wage rate are effected of the undervaluation of the growth in output, due to an overvaluation of the growth in prices. The share-variables are not affected of these errors in data, and we note that neither of them show any substantive trend-like variation over time.

But the results of the regression of labour's share of value added on our size-variable lnN unveil a rather surprising difference between large and small units as the first evidently tend to be more labour intensive than the later ones. In fact the slope coefficient of this regression for Transport Equipment ranks first. This greater labour intensivity of large units is also evident in the negative slope coefficient from the size-regression of the capital labour ratio.

According to both OLS and the Klein Wald methods of estimation the capital elasticity is rather low while the elasticity of scale is fairly high. The Klein Wald estimate of the capital elasticity for Transport Equipment is the lowest among the fifteen industries while there are only two industries with a higher Klein Wald estimate on the scale elasticity. And as the OLS results yield an elasticity of scale significantly above one there is evidence of increasing returns to scale for Transport Equipment. This finding is supported by the Census study results.

The covariance analysis results of the ACMS relation are rather ambiguous concerning the elasticity of substitution. When the across effects are not eliminated it is significantly below one while when across effects are eliminated we get estimates above one. But the standard deviations are rather large so that we do not get an elasticity of substution significantly above one. The results of the Census study yield strong support to the former results, and thus there are more evidence for than against an elasticity of substitution below one for Transport Equipment.

There are no basic differences between unweighted and weighted growth rates for this industry; those of value added and capital are rather low while those of labour are relatively higher. Labour does also get a fairly high rank concerning calculated contributions to growth, while capital's contribution has a low rank irrespective of method of estimation and type of growth rates. The low rank of "shifts", as well as the low rank of the growth rate of output may be due to the errors of data pointed out previously; that the price growth is overvalued implying a corresponding undervaluation of growth in output in constant prices.

If anything the results about the nature of technical change suggest that it is of capital saving type. There is also some evidence for the validity of the embodiment hypothesis. But the results are not unambiguous as the coefficient of the dummy-variable F_E is significantly positive, while it should be significantly negative to be consistent with the finding of a significantly positive coefficient of the "main" embodiment variable, E.

p. *Misc. Products.*

The size of the units of Misc. Products is rather low on the average. It is also by far the smallest industry measured by number of units.

It covers 13 units, 8 of which are engaged in various plastic products activities and they are responsible, to a large extent for some of the peculiar findings of this industry, particularly those concerning growth rates.

The mean values computed are fairly normal while some of the growth rates computed are widely different from those of the other industries of this study. Misc. Products ranks first concerning the growth rates of both value added and labour input, irrespective if we use weighted or unweighted growth rates. But in spite of this it ranks as high as second concerning the growth rate of materials labour ratio. It does also rank second concerning the growth rate of wages, but due to the high growth rate of output, the growth rate of labour's share in value added is negative and it has the lowest possible rank. And we also note that even if the value of materials must be growing quite fast, gross production is growing even faster leading to a fairly strong negative trend in materials' share in gross production.

The size-regression results tell us that both average productivity of labour, the capital labour ratio, the materials labour ratio and materials share in gross production is lower for large units than for smaller ones. On the other hand large units seem to pay higher wages and they do also seem to be more labour intensive than smaller units.

In opposition to all other industries the Klein Wald method yields a lower estimate on the capital elasticity than the OLS method. Thus, as we note there is an almost maximum possible difference in rank between these two estimates. The Klein Wald estimate on the scale elasticity is also somewhat lower than the OLS estimate. Both estimates are below one, but according to the later method the scale-elasticity is not significantly below one at conventional levels of the test.

The results from the covariance analysis of the ACMS relation suggest that the elasticity of substitution is above one. In case across effects are not eliminated the elasticity of substitution is significantly above one. In case the across effects are eliminated the estimates are still above one, but in spite of the fact that they rank first we cannot reject the hypothesis of an elasticity of substitution of one at conventional levels of the tests. But anyway the results rather uniformly suggest a relatively high elasticity of substitution.

Due to the high growth rates both of output and inputs the rank of the calculated contributions to growth rank very high. All rank first except capital when the Klein Wald method is applied and "shifts" in case that method together with weighted growth rates are applied.

Finally we note that for this industry we have evidence of materials saving technical change, while the results at the value added level are ambiguous. But there is some support for the embodiment hypothesis in our computations for Misc. Products.

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