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Documents

Managing government wealth in an aging society

The implications of fiscal rules

Abstract

This paper uses a large-scale overlapping generations model to assess the impact of fiscal rules in Norway. I apply up-to-date mortality and fertility rates, realistic projections for petroleum revenues and age profiles for government consumption. The model is used to study the economy's transition paths, starting in 2007, implied by the current and alternative fiscal rules. The current 4-percent rule is considered as the benchmark and I experiment with four alternatives: a growth-adjusted rule, a spending rule, constant tax rates and a wealth targeting rule. Due to large petroleum resources, alternative fiscal rules give rise to large differences in the timing and level of taxes and the welfare of different generations.

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1 Introduction

1.1 Background and motivation

A set of fiscal rules are, in general, economic constraints imposed on the fiscal authority in order to prevent excessive government deficit and debt accumulation. These fiscal measures are not economic bads, we worry about them because of their potential impact on other economic variables. Consequently, fiscal rules are usually motivated by some underlying objectives. For instance, Auerbach (2008a) notes the following: “There are at least three important long-term objectives that appear to be associated with concern about debt and deficit: intergenerational equity, economic performance, and fiscal sustainability”. Large deficits and persistent debt accumulation seems to be inconsistent with all those goals. To finance current spending by debt accumulation instead of taxes involve passing a financial burden onto future generations. The consequence could be an increase in future tax burden (or decline in spending), making current fiscal policy unsustainable. The implied fluctuations in tax rates and spending may in turn affect the performance of the economy. A well-known example of a fiscal policy constraining debt and deficit is the Stability and Growth pact of the EU. According to this rule, deficit and debt should not exceed 3 and 60 percent of GDP, respectively. If these criteria are not met, the state may be subject to sanctions.¹In the recent decades the US has experienced a number of rules, for instance deficit targets in the Gramm-Rudman-Hollings bill which was replaced by spending targets in the Budget Enforcement Act (Auerbach, 2008b).

In addition to long-term objectives, policymakers might also pay attention to practical and short-term issues. For instance, the rule should be transparent and easy to implement, time consistent, allow some deviations for the purpose of short-run stabilization, and prevent public failures e.g. deficit hikes from governments trying to be reelected. Moreover, governments acting in a discretionary manner may suffer from the dynamic inconsistency problem of optimal policy. Since the contribution of Kydland and Prescott (1977) it has been known that a policy which ties policymakers actions to a rule, rather than allowing full discretionary power, may lead to an improved outcome. Fiscal rules are often designed this way; a set of long term objectives is supposed to be reached by adhering to rules which are simple, time consistent, disciplining and flexible. The Norwegian rule provides us with a relevant illustration.

The current regime for economic policy in Norway dates back to 2001.² Norway’s large petroleum resource and cash flows generated from depletion was seen as both an opportunity and challenge. At least two main concerns dominated the discussion leading up to the creation of the fiscal rule. First, the expected combination

¹See the European Community Treaty (1997), article 99,104 and protocol 20

²The guidelines and the intentions behind them are found in Ministry of Finance (2001). Other useful references are Olsen and Skjæveland (2002) and Skancke (2003).

of gradually declining cash flows and sharp increase in age dependent expenditures would put pressure on public finances in the long run. Second, spending the entire cash flow each year would cause large fluctuations in overall public spending and presumably contribute to short run destabilization. It was believed that the large spending in turn would create large sectoral adjustment costs and a crowding out of the traded goods sector (these problems are familiar to all countries discovering large natural resources, and the problem is often referred to as Dutch disease). This would not be compatible with the declared fundamental objectives for fiscal policy: fiscal sustainability, fair distribution within and among generations, economic efficiency and short run stabilization. Moreover, the massive uncertainty attached to future reserves and prices called for cautious spending.

It was therefore decided to adopt a long-term strategy for petroleum wealth management, to serve both as a disciplining device and as guidelines for good economic policy. The key point was to separate generated cash flows from the spending over the budget. This would facilitate both a smooth phasing-in of wealth, thus avoiding short run fluctuations and Dutch disease, and wealth accumulation to meet future liabilities. According to the adopted fiscal rule the government's annual net cash flow from oil and gas activity is therefore invested abroad (The Government Pension Fund of Norway), while annual spending is limited to four percent of the fund, which coincides with an estimate of the expected real rate of return. Moreover, the fiscal budget must be balanced, allowing the government to run a non-petroleum budget deficit equal to the petroleum contribution. To incorporate flexibility, the rule permits some deviation over the business cycle. Even though the exact choice of four percent was a "pure coincidence, and not a general result.", as Martin Skancke (2003, p. 326) puts it, it is closely linked to the concept of expected return. However, it turned out that four percent also balanced the considerations of a stable, sustainable, fair and efficient development.

What then, should we expect from these fiscal rules? At the most basic level we would expect them to translate into actions and not simply remain a statement of the government's intentions. Furthermore, we expect it to perform reasonably well with respect to the long term objectives. Backward-looking measures such as debt and deficit do not contain all relevant information with regard to the fiscal position, and target trajectories for these measures are only means to an end. What we really care about is the policy's ultimate impact on the underlying objectives. Hence, any proposed target, such as the Norwegian fiscal rule, should be made subject to evaluation within the framework of a dynamic general equilibrium model. This allows us to be forward looking and explicitly and rigorously measure the outcomes for intergenerational distribution, economic efficiency and fiscal sustainability, taking behavioral responses and dynamics into account. Even though such models require a lot of simplifying and sometimes unrealistic assumption, and abstracts from several issues that were important concerns when designing the policy, it does not mean that we should not use these models to evaluate the performance of actual policies. These analyses are valuable inputs in any

discussion concerning the performance of current policy and how it should be redesigned, if at all.

In this paper I therefore develop and calibrate a large-scale overlapping generations model for Norway, and use it to investigate the long-term macroeconomic and welfare implications of simple fiscal rules. In order to preserve tractability, the model leaves out issues such as short-term stabilization, unemployment, uncertainty, Dutch disease and public failures. This study differs from previous work on Norwegian fiscal policy in terms of an up-to-date calibration, the problems I address, and the formal treatment of households with finite lives (see section 1.2). In particular, I apply mortality and fertility rates projections by Statistics Norway to create a realistic demographic transition. I also use realistic projections for petroleum revenues and age profiles for government consumption. The calibration makes sure that the current Norwegian fiscal rule was satisfied in 2006. The model is used to study the economy's transition paths, starting in 2007, implied by the current and alternative fiscal rules. The current 4-percent rule is considered as the benchmark and I experiment with four alternatives: a growth-adjusted rule, a spending rule, constant tax rates and a wealth targeting rule. In all scenarios tax rates adjust to satisfy the fiscal constraint. My main findings are:

- A continuation of the current fiscal rule gives rise to a short-term tax reduction and a long-term tax hike. Between 2006 and 2028 labor income taxes are reduced from 39 to 26 percent. From 2028 the tax rate is increased continually. The long run tax rate is 60 percent. These results are consistent with previous findings, e.g. Holmøy and Stensnes (2008).
- Alternative rules create very different tax rate dynamics. In the growth-adjusted scenario, which represents a more conservative regime, labor income tax rates are at most 7 percentage points larger in the short term, and 25 percentage points smaller in the long-term. The lowest sustainable constant tax rate is 36 percent. In the spending scenario, tax rate is negative the first two years, and grows rapidly until it reaches 55 percent at the end of this century.
- Since alternative fiscal rules create different tax paths, they necessarily redistribute welfare among generations compared to the benchmark scenario. In the growth-adjusted scenario, generations currently alive and those born in early transition years lose. The loss is the equivalent of 2.5 percent of life-time resources for some cohorts. In terms of consumption and leisure, this represents a 2.5 percent decrease in all remaining years alive. Future generations gain considerably. Cohorts born in 2050 gain 2.5 percent while the long-run gain is 17 percent. In the spending scenario, current generations gain up to 6.4 percent, while future generations lose up to 7 percent.
- In terms of macroeconomic performance different tax paths gives rise to different aggregate outcomes. Relative to the benchmark scenario, consumption and labor supply is increased (reduced) with up to

10 (5) percent in some scenarios. Savings is also very responsive. In particular, the spending scenario induces an 85 percent increase in aggregate wealth the first decade.

- Only the constant tax rate scenario generates efficiency gains relative to the benchmark. However, for baseline preference parameters, the gain is less than 0.1 of GDP in 2007. A combination of the growth-adjustment and spending scenarios gives a considerably larger gain, nearly 8 percent of 2007 GDP. Increasing the intertemporal elasticity of substitution from its baseline value, raises the efficiency gain of constant tax rates to 3.6 percent.

1.2 Related work

The pioneering work by Auerbach and Kotlikoff (1987) has been succeeded by a vast number of papers on macroeconomic and welfare implications of fiscal policy in large-scale overlapping generations models. Kotlikoff (1998) provides a fine summary of the model and its use. Among the studies that closely resembles mine, De Nardi et.al. (1999) is perhaps the most noteworthy example. They calibrate a large-scale overlapping generations model for the U.S, incorporating a realistic population projection. Their objective is to experiment with alternative fiscal responses to the coming aging of the population. They fix a long-run debt to GDP target and specify alternative financing schemes along the transition path to a new stationary equilibrium. The main finding is that, given the current U.S. social security system, a large future increase in either labor or consumption tax is necessary. Moreover, different strategies have vastly different implications for intergenerational welfare distribution. Nishiyama (2004) conducts a similar analysis for the U.S without fixing a debt to GDP ratio. His main result is that the welfare gains to future generations from an immediate increase in the payroll tax are much smaller than the welfare loss of current generations. Kotlikoff et.al. (2001) considers strategies for reforming the current U.S. social security system, under a constant debt per capita regime. In their baseline scenario, which assumes a continuation of current fiscal policy, the payroll tax increases with 77 percent over the first three decades. One particular social security reform seems to have a favorable appearance, namely switching to so-called advance funding. The real wage increases substantially relative to the baseline scenario, and it distributes the fiscal burden of population aging more evenly across generations. In a more relevant case for Norway, Jensen et.al. (2002) calibrate a Danish overlapping generations model. They consider two fiscal strategies for coping with population aging, namely tax smoothing and debt smoothing. By evaluating the outcome using a social welfare function, they find that tax smoothing is (marginally) preferred.

These contributions indicate that population aging renders current fiscal policies unsustainable, and the way we choose to cope with this problem has potentially major implications for intergenerational distribution

of welfare. But what about the case of Norway? There are not many attempts to use overlapping generations models for fiscal policy analysis in Norway. One contribution is Steigum and Thøgersen (1995). They investigate the welfare implications of consuming the entire petroleum wealth within 40 years. While being beneficial for the current generations, the consumption scenario has major negative welfare impact on future generations. This is illustrated by the path of wage tax. Compared to the benchmark scenario, in which wealth-GDP ratio is kept constant, the consumption strategy allows for short run tax reduction but necessitates a future tax hike. Their approach is relatively similar to mine. My contribution will be to investigate specific fiscal rules, including the current one. Moreover, I develop a more detailed model representing the current state of the Norwegian economy, applying the most up-to-date population and petroleum revenue projections. I also have a richer set of tax rates, all of which are distortionary (The wage tax in their model is non-distortionary, due to exogenous labor supply). This enables me to compare the efficiency effects of adopting different fiscal rules.

In an IMF working paper Jafarov and Leigh (2007) analyze the macroeconomic consequence of switching from the current fiscal rule to a growth-adjusted rule. This rule is more conservative than the current one, because it subtracts output growth rate from the 4 percent. The main result is that the growth adjusted rule stimulates long-run production, labor supply and private consumption compared with the 4 percent rule, because long-run tax rates are lower. Their paper addresses the same issue as mine, namely alternative fiscal rules. However, there are some differences. First, they do not carry out a welfare analysis. Second, they assume that all agents are credit constrained. Besides, it is not clear how they calibrate their model (They use a model called GIMF, developed at the IMF) to match the Norwegian economy, it is not even clear if they take population aging into account.

Projections of Norway's fiscal future have also been conducted in models which does not have the explicit OLG structure, for instance, in Holmøy and Stensnes (2008). In their simulation a combination of a large representative agent CGE model for Norway (MSG6) and a household microsimulation model (MOSART) is used to project the path of public finances under various fiscal policies. Their benchmark simulation indicates that a continuation of the current fiscal rule will lead to a short-term fiscal expansion (which in their model is used to reduce the payroll-tax rate) until about 2020, followed by successive contractions (increase in the payroll tax rate). The paper provides a good indication of Norway's fiscal position. The main difference between their approach and mine is the formal and unifying treatment of overlapping generations. MOSART has a rich household structure where agents live finite lives. In order to achieve consistency between MOSART and MSG6 the authors use an iterative procedure. Full consistency is however not achieved. Moreover, even though MOSART simulate the life cycle outcomes of different household, it does not contain any behavioral responses, and is not suitable for evaluating efficiency and intergenerational welfare effects of different fiscal

policies. The main feature of my model is the unified and formal treatment of household behavior and macroeconomic outcome, which provides an explicit and consistent analysis of macro variables, efficiency and welfare.

Finally, employing a standard generational accounts (GA) framework, several white papers from the Norwegian Ministry of Finance (e.g. Ministry of Finance, 2009) concludes that the primary budget deficit should be reduced by 4.75 – 7.25 percent of mainland GDP on an annual basis in order to make the current policy sustainable. The GA uses detailed projections of government income and expenditure components under the current fiscal regime, and employs net present value analysis to see if the policy satisfies the government's intertemporal budget constraint. On the other hand, it lacks economic behavior and general equilibrium mechanisms. Hence, it is a useful accounting tool, but the results should be supplemented with studies using formal economic models, such as the present one

The paper proceeds as follows. The next section summarizes the Norwegian fiscal outlook. In section 3, I develop a large-scale overlapping generations model calibrated to the Norwegian economy. In section 4, the model is used to simulate economic consequences of different policy scenarios. The final section concludes.

2 Fiscal outlook

2.1 Demographics and age-dependent expenditures

According to the most recent population projection published by Statistics Norway (see Brunborg et.al., 2008) Norway is expected to experience a substantial population growth. In their baseline scenario the population increases from 4.7 million in 2008 to 6.9 million in 2060. Moreover, the age structure is also changing, and the fraction of people older than 67 years is projected to grow from 0.614 million in 2008 to 1.5 million in 2060. The dependency ratio, the ratio of young (age 0-19) and old (age 67 and older) to the labor force (age 20-66), is projected to grow from 0.64 in 2008 to 0.80 in 2060. There are several factors contributing to this development. First, the fertility rates boomed in the 1950s and 60s with rates between 2.5 and 3. In the 1970s the rates dropped sharply to about 1.7 before it stabilized at around 1.8 in the 1990s. The projection is based on a constant fertility rate of 1.85 starting in 2012. Second, life expectancy at birth has grown considerably over the last century. Currently it is 78.24 for a newborn male and 82.66 for a female, which is 27 years higher than at the start on the 20th century. The population forecast assumes a continuation of this trend, with an annual increase of 0.14 and 0.15 years for men and women respectively. Consequently, life expectancy at birth is 86.3 and 90.2 in 2060. Finally we have net immigration. This component is the most important factor explaining population growth. In the baseline scenario it is assumed that annual net

immigration drops somewhat from the current level of about 40000 to 26000 in 2020 before it stabilizes at 20000 in 2040. To get an idea of the importance of this component, the scenario in which no net immigration occurs produces a population of only 5.07 million. In addition, the dependency ratio increases to 0.85.

There is considerable uncertainty related to such projections, and the above numbers reflect the so called middle alternative. It is possible that some or all components develop differently. The projection therefore includes scenarios with different assumption regarding the specific factors. One is already mentioned, the zero net immigration scenario. Others consist of various combinations of higher/lower growth in life expectancy, fertility rates and immigration. The projection concludes that population in 2060 will most likely be somewhere between 5.3 and 8.5 million in 2060, while the dependency ratio ends up between 0.75 and 0.85. Despite the uncertainty, it is widely acknowledged that the eventual aging of the population will have implications for public finances. The reason is that a large part of government spending is age related, such as old age pension benefits and health care. They are so-called entitlement spending, meaning that public spending is triggered whenever an agent reaches a particular state (age, health status, employment status, etc.). In some of these cases the age is the state. In others, the link is indirect. This is the case for e.g. health care. As the agent ages, he is more likely to reach a state which triggers health care spending. The main insight is that old agents triggers more spending than young, and hence an aging society would put pressure on public finances in the long run. Statistics Norway has illustrated the age and sex profile of public spending in Norway. They define a part of total government use of resources in 2004 as age and sex related, and then distributes this across agents according to age and sex. For instance, government age-related consumption was 62 percent of total government consumption in 2004. The corresponding government consumption profile for females (per individual) in 2004 is depict in figure 1³.

Now, using age-spending relationship and the population projections it is possible to get an idea of the size of government spending in 2050, given that public service standards are unchanged. The numbers are based on the benchmark scenario (see e.g. Holmøy and Nielsen, 2008) in an integrated simulation using the CGE model MSG-6 and the microsimulation model MOSART⁴, both developed at Statistics Norway. As a percentage of mainland GDP, public spending is expected to grow from 49 percent in 2008, to 53 percent in 2050. This is not overwhelming, but the assumption of unchanged standards is highly unlikely. They would presumably have to increase in the future, given that an increasingly wealthier population demands increasingly higher quality of public services.

³The data is provided by Pål Knudsen and Viebeke Oestreich Nielsen, section for public finances, Statistics Norway. The data comes from the Norwegian Generational Accounts, and is used in e.g. Norwegian Ministry of Finance (2009).

⁴The data is provided by Erling Kravik, section for public finances, Statistics Norway. It is the outcome of the baseline simulation in the most up to date calibration of MSG. For a description of the MSG model see, Heide et.al. (2004) For description of the integrated use of MSG and MOSART, and the main assumption in the baseline scenario see e.g. Holmøy and Stensnes (2008) and Heide et.al (2006).

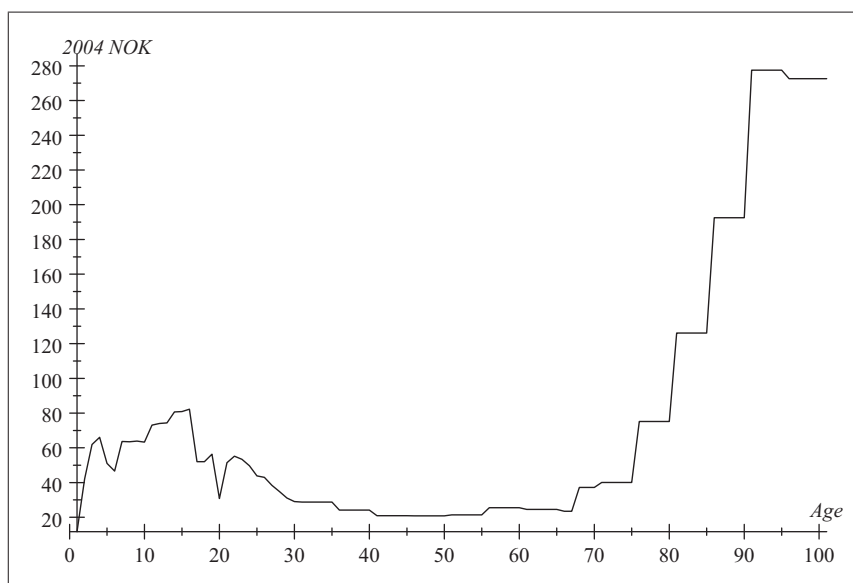


Figure 1: 2004 profile for government age-related consumption (per individual, females). In 1000 NOK (2004)

2.2 Petroleum cash flow and the fiscal rule

Cash flows from the petroleum sector represent a large part of government revenues. In the 2007 national budget (table 1), petroleum revenues amounted to about 30 percent of total revenues, while net cash flow was larger than the budget surplus. In accordance with the fiscal rule, this cash flow was invested in the Pension Fund, and the budget deficit was financed with transfers from the fund. Consequently, the growth in the fund consisted of petroleum cash flow plus interest and dividends from the fund minus the amount necessary to balance the budget⁵. At first glance it does not seem that the 4-percent rule was respected. However, as mentioned previously, the rule allows deviation over the business cycle. In fact, the 2007 structural petroleum-adjusted budget deficit, which removes revenue and expenditure variations attributed to short run fluctuations in economic activity, was NOK 58.9 B. The market value of the fund at the end of 2007 was NOK 1782 B. Although a little less than the permitted, 58.9 B is still fairly in line with the rule. Since the net cash flow is saved and spending limited to 4 percent, the fund is projected to grow over time. This implies that the 4-percent contribution also increases. However, as the petroleum resources are depleted, net cash flows decline. The factors determining the future path of revenue from the petroleum sector are essentially the planned depletion of oil and gas and the future prices of those commodities. Using numbers from the baseline projection mentioned above, based on constant 2007 oil and gas prices of NOK 418 per barrel of oil and NOK 1.97 per sm³ gas, the value of oil and gas production as a share of mainland GDP is projected to

⁵Let PF_t , CF_t , D_t and r_t denote period t Pension Fund, petroleum net cash flow, non-petroleum budget deficit and real rate of return, respectively. The fiscal rule requires $0.04PF_t = D_t$, and thus $PF_{t+1} = PF_t + CF_t + (r_t - 0.04)PF_t$

National Budget and Pension Fund	2007
Total revenues	1030.1
1. Revenues from petroleum activity	337.4
2. Other revenues	692.7
Total expenditures	715.1
1. Expenditures from petroleum activity	21.2
2. Other expenditures	694.0
Surplus before transfer to Pension Fund	315.0
– Net cash flow from petroleum activity	316.4
= Petroleum-adjusted surplus	–1.3
+ Transfer from Pension Fund	2.8
= National budget surplus	1.5
+ Net transfer to Pension Fund	313.6
+ Return on Pension Fund	78.4
= Total Surplus	393.5

Table 1: 2007 central government budget. In NOK billion. Source: table 3.2 Ministry of Finance (2008)

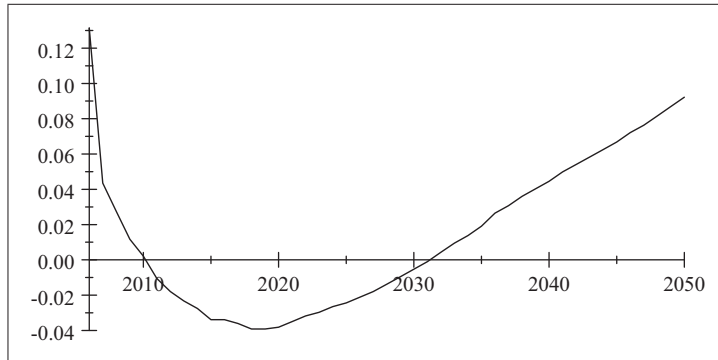


Figure 2: Social security payroll tax rate (the employer's contribution)

decline from 0.3 in 2008 to 0.02 in 2050⁶. The cash flow from petroleum activity is not identical to the value of petroleum production (the link is through tax on profit, indirect taxes and dividends from government owned petroleum firms). Nevertheless, declining petroleum activity inevitably leads to lower net cash flows.

2.3 Public finances

To get an idea of how public finances is affected by declining petroleum revenues and an aging population, I have to take into account the current fiscal rule. I can look at the baseline scenario in e.g. Holmøy and Nielsen (2008). In this simulation the government's annual budget constraint implied by the current fiscal rule is satisfied by adjusting the social security payroll tax paid by the employer. The tax path is displayed in figure 2 (a similar tax path is found in Holmøy and Stensnes, 2008). There are two notable features, the U shape, and 2050 tax rate. The path shows that it is possible to reduce tax rates in the short run, but

⁶Total petroleum production also include production of NGL and condensate. Value of total petroleum production is 0.34 of GDP in 2008 and 0.02 in 2050.

starting from 2020 rates must increase. In 2050 it is, nonetheless, still below the 2006 level. The dynamics is caused by the interplay between petroleum cash flow, demographics and the fiscal rule. Large cash flows in the first decades imply that the Pension Fund grows rapidly. Due to the fiscal rule, government revenues (including 4 percent of the Pension Fund) grows faster than government expenditures. This makes tax cuts possible. However, as the cash flow shrinks and the aging process kicks in, expenditures eventually grow faster than revenues. To satisfy the fiscal rule, tax rates are increased continuously from 2020 until 2050 when it still is about 4 percentage points below its 2006 level. This is consistent with the modest increase in government spending mentioned previously. So, at first glance the overall fiscal picture seems to be quite good, indicating that a future increase in tax burden is unnecessary. Still, Holmøy and Stensnes emphasize that we should not be too optimistic. As already mentioned, the standard in public welfare schemes are held constant. Assuming growth in these standards would shift the tax path upwards. Moreover, there are no reason to believe that the growth in tax rates would not continue beyond 2050. Hence, tax burden must eventually increase relative to the current level.

The summary of Norway’s fiscal outlook illustrates the need for my kind of analysis. With explicit modelling of overlapping generations, my setup enables me to focus on intergenerational welfare implications of the current, and alternative, fiscal rules.

3 Overlapping generations model for the small open economy

The setup follows closely the Auerbach-Kotlikoff OLG model (AK-model). A detailed description of the original model is found in Auerbach and Kotlikoff (1987). My model economy includes consumption and leisure choice, endogenous retirement, lifespan uncertainty and secular growth. It also incorporates a direct link between population composition and government expenditures. A complete description of exogenous and endogenous variables, how to solve the model and how it is calibrated is found in the appendix.

3.1 The model

The household sector consists of a number of co-existing cohorts which maximize utility by choosing leisure and consumption over the life cycle subject to a suitable budget constraint. The production sector consists of a profit maximizing representative firm producing an aggregate macro good using labor and capital as input. Finally, the government levies taxes and issues debt in order to finance its expenditures.

The household sector

At each period⁷ t a cohort of size L_t is born. Members of any cohort has a maximum age of 90, but

⁷One period equals one year. Single subscript refers to period, while double subscript refers to period and age. E.g. $c_{t,j}$ is

faces a probability of premature death at the end of each period. In the first 19 periods, the agent is a child, and for simplicity, entirely supported by the government. When reaching age 19 the agent maximizes utility over the life cycle by choosing consumption, c , and leisure, l , in each period, constrained by its lifetime budget. Due to uninsurable mortality risk, agents die with positive wealth, and thus leave unintended bequest. The representative agent of a cohort reaching adulthood in year t faces the problem of maximizing a time separable nested CES function given perfect foresight regarding all relevant economic variables. The problem is to maximize

$$\frac{1}{1 - \frac{1}{\gamma}} \sum_{j=t}^{t+71} \beta^{j-t} P_{j,j-t+19}^{19} u(c_{j,j-t+19}, l_{j,j-t+10})^{(1-\frac{1}{\gamma})}, \quad (1)$$

subject to a life time budget constraint defined by

$$\begin{aligned} s_{t,j} &= r^* (1 - \tau_t^s) (a_{t,j} + inh_{t,j}) + (1 - \tau_t^l) w_t e_{t,j} (E_t - l_{t,j}) - (1 + \tau_t^c) c_{t,j} + b_{t,j} \\ a_{t+1,j+1} &= a_{t,j} + inh_{t,j} + s_{t,j} \\ a_{t,19} &= 0, \quad a_{t+72,91} = \tilde{a}_{t+72} \\ l_{t,j} &\leq E_t. \end{aligned} \quad (2)$$

γ is the intertemporal elasticity of substitution, β is the discount factor derived from the pure rate of time preferences, $\theta = \frac{1}{\beta} - 1$, while $P_{t,j}^{19}$ is the agent's probability of living to reach age j in period t , conditional on reaching adulthood. The agent's savings in period t is $s_{t,j}$ and $a_{t+1,j+1}$ is the beginning of period $t + 1$ wealth. At the beginning of each period he also receives inheritance $inh_{t,j}$ which is invested in the capital market. Borrowing and lending takes place on a perfect international capital market with exogenous interest rate r^* . $\tau_t^s, \tau_t^l, \tau_t^c$ are proportional tax rates on capital income, labor income and consumption respectively. The wage rate per efficiency unit is w_t , while the agent's human capital profile (his efficiency, earnings ability) is given by $e_{t,j}$. The total available time to allocate between labor and leisure in period t is E_t . $b_{t,j}$ is the old-age pension benefit. The instantaneous utility takes the CES form

$$u(c_{t,j}, l_{t,j}) = \left(c_{t,j}^{(1-\frac{1}{\rho})} + \alpha l_{t,j}^{(1-\frac{1}{\rho})} \right)^{\frac{1}{(1-\frac{1}{\gamma})}}, \quad (3)$$

α is the relative preference weight on leisure and ρ is the intratemporal elasticity of substitution between consumption and leisure.

consumption by cohort of age j in period t , while C_t is aggregate consumption in period t .

To incorporate secular growth, and at the same time allow the economy to reach a balanced growth path, I cannot use the standard approach of labor augmenting technological change. This is because of the preference specification. I therefore assume time augmenting technical change. This imply that each generation's time endowment grows at the rate g

$$E_{t+1} = (1 + g)E_t . \quad (4)$$

As a consequence, the wage rate w_t will be constant over time (which can be seen from equations 14 and 15). The agent's age-earnings profile is given by

$$e_{t+1,j+1} = (1 + g)h_{t,j}e_{t,j} . \quad (5)$$

$h_{t,j}$ increases with age. This is reasonable due to the fact that wages tend to grow over time as the agent acquire human capital. However, I include an old age productivity drop, implying that $h_{t,j}$ drops when the agent reaches age 65. The presence of g is to ensure that the wage profile includes secular growth, which comes on top of growth in human capital⁸.

There are no borrowing constraints. To avoid that agents die with negative wealth, I would preferably include a bequest motive. However, this is not trivial in a model with risk of premature death. Therefore, I simply assume that agents dying after reaching age 90 must leave positive wealth, $a_{t,91} = \tilde{a}_t > 0$ and $\tilde{a}_{t+1} = (1 + g)\tilde{a}_t$. This could be interpreted as follows: The agent receives infinite dissutility of leaving bequest at age 90 less than \tilde{a}_t , and zero marginal utility of any additional bequest over and above. Hence, \tilde{a}_t serves as an imperfect proxy for a bequest motive. A sufficiently big value will prevent negative asset positions.

The public old-age pension systemt is a defined benefit, pay-as-you-go system, in which pension benefits depend on work history but is independent of retirement choice. All agents qualify for pension transfer when they reach the exogenous eligibility age 65. They receive a transfer $b_{t,j}$ in each period from then on. The transfer is related to average earnings for the first 46 years of adulthood.

$$b_{t,j} = \frac{f}{46} \sum_{i=19}^{64} w_{t-j+i} e_{t-j+i,i} (E_{t-j+i} - l_{t-j+i,i}) , \quad (6)$$

where $0 \leq f \leq 1$.

To find the size of any generation I need to adjust for deceased members. I specify lifespan uncertainty in terms of mortality probabilities $mor_{t,j}$, i.e. as the probability of not surviving to period t and reach age

⁸The need for time augmenting growth and steepening of wage profile is described in detail in Auerbach et.al (1989)

j , given that you survived to period $t - 1$. The relationship between P and mor is

$$P_{t,j}^q = \prod_{i=q+1}^j (1 - mor_{t-j+i,i}) . \quad (7)$$

Let $\Gamma_{t,j}$ denote the size of age j cohort in period t . Assuming all deaths occur at the end of the period, cohort size is given by

$$\Gamma_{t,j} = L_{t-j+1} \prod_{i=1}^j (1 - mor_{t-j+i,i}) . \quad (8)$$

To compute the size of a newborn generation I calculate the number of children born by household in fertile age groups. I let agents in age group 15 to 49 give birth to a fraction of a child. The fraction is determined by period and age specific fertility rates $fer_{t,j}$. L_t is thus given by⁹

$$L_t = \sum_{i=15}^{49} fer_{t,i} \Gamma_{t,i} . \quad (9)$$

Bequest is shared equally among households between age 24 and 58. Let BEQ_t denote total bequest from period $t - 1$ deceased agents, then the inheritance is given by

$$\begin{aligned} BEQ_t &= \sum_{i=19}^{90} a_{t,i} \Gamma_{t-1,i-1} mor_{t,i} + a_{t,T+1} \Gamma_{t,T} \\ inh_{t,j} &= \frac{1}{\sum_{i=24}^{58} \Gamma_{t,i}} BEQ_t . \end{aligned} \quad (10)$$

To find aggregate demand and supply in any period t I sum over all living households, weighted by cohort size. Total labor supply in efficiency units and consumption is

$$N_t = \sum_{i=19}^{90} (E_t - l_{t,i}) e_{t,i} \Gamma_{t,i} . \quad (11)$$

Aggregate household consumption, saving, wealth, inheritance and social security transfer is

$$C_t = \sum_{i=19}^{90} c_{t,i} \Gamma_{t,i}, \quad S_t^p = \sum_{i=19}^{90} s_{t,i} \Gamma_{t,i}, \quad A_t^p = \sum_{i=19}^{90} (a_{t,i} + inh_{t,i}) \Gamma_{t,i}, \quad B_t = \sum_{i=19}^{90} b_{t,i} \Gamma_{t,i}, \quad INH_t = \sum_{i=19}^{90} inh_{t,i} \Gamma_{t,i} . \quad (12)$$

The firm

There is only one good in this economy. Consequently, a country exports and imports the same good. All gains from trade comes from the ability of the country as a whole to smooth consumption and leisure intertemporally. The good is produced by a representative firm employing labor and capital. The production

⁹Note that $\Gamma_{t,j}$ consists of children for $j < 19$

function is constant returns to scale Cobb-Douglas

$$Y_t = K_t^\phi N_t^{1-\phi} . \quad (13)$$

Since capital is internationally mobile, it will locate where the rate of return is highest. Hence, equilibrium requires equalization of domestic and world rate of return¹⁰.

$$r^* = \phi \left(\frac{K_t}{N_t} \right)^{\phi-1} - \delta . \quad (14)$$

From competitive labor markets I get

$$w_t(1 + \tau_t^p) = (1 - \phi) \left(\frac{K_t}{N_t} \right)^\phi . \quad (15)$$

Here, τ^p is a wage tax levied on the firm (corresponds to the employers' contribution to the social security payroll tax). To find the desired capital stock, and the corresponding investment demand, I calculate aggregate labor supply at the going wage w_t . Given capital stock I deduce the corresponding investment demand

$$I_t = K_{t+1} - (1 - \delta)K_t . \quad (16)$$

The government

The government issues bonds at a rate equal to the world interest rate and collects taxes on capital income, labor income and consumption. Although a petroleum sector is not modeled explicitly, the government receives a net cash flow each period CF_t , interpreted as a windfall of income from abroad. This is used to finance exogenous age-dependent consumption G_t , regular net expenditures X_t , old-age pension benefits B_t , and disability benefits Q_t ¹¹

$$G_t = \sum_{i=0}^{90} d_{t,i} \Gamma_{t,i}, \quad Q_t = \sum_{i=0}^{90} q_{t,i} \Gamma_{t,i}, \quad X_t = x_t \sum_{i=0}^{90} \Gamma_{t,i}, \quad (17)$$

where $d_{t,i}$ and $q_{t,i}$ is consumption and disability benefits per individual of age i in period t . Both d and q increases with a factor of $(1+g)$ over time. I specify regular net expenditures in per capita terms, x_t , and assume that it grows with rate g over time, implying that X_t increases with a factor of $(1+g)$ times

¹⁰This means that all capital income taxation is resident based.

¹¹Both X_t and Q_t consists, in part, of transfers between the private and the public sector. However, non of these transfers show up in the household's budget constraint, and so they have no impact on the allocation of consumption and leisure. They thus represents unproductive, non-utility generating consumption

population growth. Government savings and wealth accumulation is then

$$\begin{aligned} S_t^G &= r^* A_t^G + \tau_t^s A_t^p + (\tau_t^l + \tau_t^p) w_t N_t + \tau_t^c C_t - G_t - Q_t - B_t - X_t + C F_t \\ A_{t+1}^G &= A_t^G + S_t^G . \end{aligned} \quad (18)$$

Starting in period t , the paths of tax rates and expenditures are constrained by the non-ponzi game condition

$$\lim_{T \rightarrow \infty} A_{T+t}^G (1 + r^*)^{-T} \geq 0 . \quad (19)$$

This prevents the government from continually issuing new debt in order to meet interest payments on existing debt.

Connecting all three sectors gives the economy's net foreign asset position. Any excess (shortage) of public and private wealth over national capital stock, corresponds to the nation's net claims (liabilities) on foreigners. The current account is defined as the change in this position between two periods

$$C A_t = S_t^G + S_t^p - (I_t - \delta K_t) = A_{t+1}^G - A_t^G + A_{t+1}^p - A_t^p - (K_{t+1} - K_t) . \quad (20)$$

3.2 Calibration

Table 2 summarizes the main numerical values. This section contains a brief comment on the calibration strategy, while a more detailed discussion of the parameterization is found in the appendix. I start the simulations outside steady state in 2007. To do so, an initial distribution of asset is needed. I let the model create this distribution by starting the simulation from a balanced growth path in 1979. Along this path the demographics are stable with constant 1979 levels on mortality and fertility rates. The tax rates and pension system is also constant. From 1980 I use actual mortality and fertility rates, and run the model to 2006 keeping tax rates constant. This gives me an initial distribution of assets in 2007. I calibrate the fiscal structure to match certain key facts from the national accounts for 2006. The demographic variables (fertility and mortality rates) are collected from Statistics Norway's population forecast (Brunborg et.al. 2008). Finally, regarding the preference parameters I fix a value for γ while the rest are calibrated to match some key features of household behavior. These features are consumption and leisure growth over the life cycle and time allocated to labor income generating activities. As can be seen from table 2, a large negative pure rate of time preference is needed. Why is this necessary? The answer lies in the combined choice of γ , r^* and the targets for consumption growth and labor supply. To see this, we must first look at the household's

Calibration	Value	
Preferences		
γ	0.35	Intertemporal substitution elasticity
θ	-0.05	Pure rate of time preference
ρ	0.5	Intratemporal substitution elasticity
α	2.2	Relative preference for leisure
e		Human capital profile, Statistics Norway
Demographics		
mor		mortality rates, Statistics Norway
fer		fertility rates, Statistics Norway
Technology		
ϕ	0.3	Capital share of GDP
g	0.017	Productivity growth rate
δ	0.05	Capital depreciation rate
r^*	0.04	Exogenous world interest rate
Fiscal structure		
τ^l	0.39	Labor income tax rate
τ^p	0.13	Payroll tax rate, levied on the employer
τ^s	0.28	Capital income tax rate
τ^c	0.19	Consumption tax rate
f	0.56	Pension benefit (share of average labor income)
d, q		Age-profiles, government spending, Statistics Norway
G_{2006}/Y_{2006}	0.162	Age related spending (2006 share of GDP)
Q_{2006}/Y_{2006}	0.032	Disability benefits (2006 share of GDP)
X_{2006}/Y_{2006}	0.176	Regular net spending (2006 share of GDP)
A_{2006}^g/Y_{2006}	0.88	Pension fund (2006 share of GDP)
CF_{2006}/Y_{2006}	0.223	Petroleum net cash flow (2006 share of GDP)

Table 2: Summary of calibration

γ	0.25	0.35	0.6	0.9	2	3.7
θ	-0.075	-0.051	-0.029	-0.0197	-0.004	0.001

Table 3: Intertemporal elasticity of substitution and pure rate of time preference

Euler equation (see appendix equation 26, irrelevant variables omitted)

$$c_{j+1} = c_j \left[\frac{1 + (w_j e_j)^{(1-\rho)}}{1 + (w_{j+1} e_{j+1})^{(1-\rho)}} \right]^{\frac{(\rho-\gamma)}{(\rho-1)}} \left[\frac{1 + r^*}{1 + \theta} \right]^\gamma. \quad (21)$$

First, increasing the value of γ will unambiguously tilt the consumption path upwards, due to the assumption of growing human capital. This will, other things being equal, increase the required θ . Table 3 reports different values for γ and the corresponding θ .¹² From this it is obvious that I need an equally unrealistic γ to create a positive time preference. Estimates for γ range between 0.2 and 0.75 (see e.g. Fehr, 1999). In addition, this leads to retirement also when young and a consumption hike at old age retirement. I choose my initial values because they create more realistic labor supply and consumption profiles, and is more in line

¹²Changing γ , leaves ρ essentially unaffected

with the literature. For instance, a negative time preference of 5.5 and 10.5 percent is used in Auerbach et.al. (1989). Second, I could simply alter the targets. The most natural candidate is the consumption growth target, due to the fact that reliable empirical evidence on household age-consumption profiles is scarce. However, this does not seem to help that much. If I reduce the target to 1.02 it will only increase time preference to -0.041 given baseline value of γ . And moreover, it requires that relative preference for leisure takes the astounding value of 9. Disregarding the target entirely, and simply set a positive time preference would lead to the equally unrealistic outcome of households holding negative wealth over the entire life cycle.

The reason I have to choose among extremes is the exogenous interest rate. For chosen parameter values the interest rate would adjust upwards in a closed economy setting. This reflects the fact that for given preferences and tax structure, households save too little in order to finance the required national capital stock. From the Euler equation it is evident that an increase in interest rate leaves room for a larger time preferences.

3.3 Some comparative dynamics

In this section I will look more closely at individual and aggregate responses to changes in three key variables, namely wage rate, consumption tax rate and demographics. In order to interpret the simulation results it is essential to understand these effects. The analysis will be conducted in partial equilibrium, i.e. I do not consider the feedback effects from changes in labor supply and consumption on tax rates (factor prices are constant, due to the small open economy assumption). I will consider both the short run (immediate) and the long-run effects.

The micro effect of aging can be found by looking at equation 26. The effective discount factor between two succeeding periods becomes $(1 - mor)/(1 - \theta)$. Reduced mortality tilts the consumption (and leisure) path upwards, and thus boost saving. Replacing 1979 mortality rates with 2080 mortality, increases the average life cycle asset of a representative household by a factor of 1.78. Moreover, the household now spends on average 44 percent of available time on labor activity, a 4 percentage points increase. Since retirement age is unchanged this in turn must boost household life cycle consumption. The macro effect takes into account the implied change in population structure. To single out the combined effect of changes in behavior and population structure induced by the models demographic transition, I compare two partial equilibrium transition paths. The first simulates a transition starting with the 1979 demographic shock, keeping tax rates constant. The second replicates the population growth implied by the demographic transition, but keeps mortality and fertility rates constant (e.g. a population growth of 1 percent is created by increasing the size of all cohorts by 1 percent). Thus, the latter path ignores the aging effect on household behavior

and population structure, implying that all aggregate variables grow at the trend rate of the economy, i.e. population growth times technology growth. From 2007 to 2100 the average trend growth is 1.61 percent. The average labor supply growth is 1.49 percent. Even though individual labor supply increases, growth in aggregate labor supply is on average smaller than trend growth. The reason for this is the same as the reason behind increased dependency ratio, i.e. an older population. Throughout the transition aggregate household wealth and consumption is higher in the first than in the second simulation. The average growth between 2007 and 2100 is 2.06 and 1.71 percent for wealth and consumption respectively.

The response to wage changes is analyzed by computing the wage elasticity of aggregate labor supply implied by the initial balanced growth path. When doing this I keep tax rates and demographics constant. The immediate elasticity is -0.12 percent, and decreases continually until it stabilizes at -0.29 after 30 years. This can be explained by income and substitution effects. A higher wage increase the relative price of leisure, inducing households to consume less leisure and more of other goods. This is the substitution effect. It has a positive impact on labor supply. On the other hand, a higher wage increases the value of time endowment. Since net supply of time never turns negative (leisure cannot exceed time endowment), the household becomes wealthier. This is the income effect, and it makes the household want to consume more of all goods, including leisure. Hence, it has a negative impact on labor supply. Starting from the calibrated balanced growth path, the income effect is larger than the substitution effect. To understand the short run effect note that the relative importance of income effects is smaller the closer the household is to retirement age. Since the immediate effect consists of the combined response of newborns and generation alive at the time of wage increase, aggregate income effects are smaller in the short run than in the long run. When it comes to consumption and wealth the impact is always positive, and one percent wage increase raises long-run aggregate wealth and consumption by 0.77 and 0.71 percent. Finally, a one percent reduction in the consumption tax rate induce a short-run (long-run) aggregate consumption increase of 0.13 (0.11) percent. Aggregate labor supply and wealth decreases with 0.05 and 0.03 percent respectively, both in the short and long run. Hence, the income effect on leisure dominates the substitution effect slightly.

When discussing behavioral responses in the policy experiments, there are essentially three things to keep in mind. First, the individual effect of gradually declining (or increasing) labor income tax rates. From the perspective of an individual, declining tax path stimulate current leisure since households shift labor supply to future periods when after-tax wages are high. The effect on consumption growth is positive since $\rho > \gamma$, which can be confirmed by the Euler equation A5 in appendix A. When we observe a declining tax path we would thus expect faster growth in household labor supply and consumption. On the other hand, when tax rates fall, future generations will have smaller lifetime tax burden than current generations. In turn, this leads to lower future aggregate labor supply and higher future aggregate saving and consumption, according

to the findings in this section. The final thing to keep in mind is the aggregate effect of the demographic transition. This boosts growth in household wealth and consumption and reduce labor supply growth.

4 Policy analysis

The above model is used to study the economy's transition path to a long-run stationary equilibrium under five different fiscal policy scenarios. (1) the current 4-percent rule, (2) a growth-adjusted rule, (3) a no-rule scenario, (4) constant tax rate scenario and (5) a combination of (1) and (2). In each scenario I consider *a*) endogenous labor income tax and *b*) endogenous consumption tax. The benchmark scenario is the 4-percent rule. Formally, under the various rules the endogenous (labor income) tax rate and government wealth satisfy the following equations for all t :

1a

$$\begin{aligned}\tau_t^l &= \frac{G_t + Q_t + B_t + X_t - r^* A_t^G - \tau_t^s A_t^p}{w_t N_t} - \tau_t^p \\ A_{t+1}^G &= A_t^G + CF_t\end{aligned}$$

2a

$$\begin{aligned}\tau_t^l &= \frac{G_t + Q_t + B_t + X_t - (r^* - z_{t+1})A_t^G - \tau_t^s A_t^p}{w_t N_t} - \tau_t^p \\ A_{t+1}^G &= (1 + z_{t+1})A_t^G + CF_t \\ z_{t+1} &= g + \lambda_{t+1} + g\lambda_{t+1} \\ g &\equiv \text{productivity growth rate} \\ \lambda_{t+1} &\equiv \text{population growth rate between period } t \text{ and } t + 1\end{aligned}$$

3a

$$\begin{aligned}\tau_t^l &= \frac{G_t + Q_t + B_t + X_t - r^* A_t^G - CF_t - \tau_t^s A_t^p}{w_t N_t} - \tau_t^p \\ A_{t+1}^G &= A_t^G\end{aligned}$$

4a

$$\tau_t^l = \hat{\tau}^l$$

Year	Tax rate τ^l	Current account ^a CA/Y	Pension Fund ^a A^g/Y	Consumption ^b (C)	Labor supply ^b (N)	Private wealth ^b (A^p)	Pension Fund ^b (A^g)	Old-age pension ^b (B)	Gov. cons. (G)	Gov. exp. (X)
2007	0.41	0.13	1.18							
2017	0.30	0.15	2.57	0.26	0.27	0.33	1.75	0.33	0.23	0.20
2027	0.27	0.14	3.16	0.24	0.18	0.57	0.45	0.31	0.21	0.19
2037	0.29	0.12	3.43	0.21	0.13	0.44	0.22	0.29	0.22	0.18
2047	0.31	0.03	3.48	0.19	0.14	0.25	0.15	0.23	0.22	0.17
2057	0.32	0.05	3.20	0.18	0.20	0.24	0.10	0.15	0.19	0.16
2067	0.34	0.02	2.84	0.16	0.16	0.22	0.03	0.20	0.16	0.16
2077	0.41	-0.02	2.51	0.17	0.13	0.16	0.00	0.23	0.21	0.17
2087	0.43	-0.03	2.10	0.14	0.19	0.06	0.00	0.16	0.18	0.16
2097	0.45	-0.03	1.80	0.12	0.17	0.07	0.00	0.18	0.15	0.15
<i>longrun</i>	0.60	-0.03	0.00	0.15	0.15	0.15	0.00	0.15	0.15	0.15

Table 4: Key macroeconomic variables. Benchmark scenario - endogenous labor income tax. a: Share of GDP. b: Ten-year growth rates

where the constant tax rate $\hat{\tau}^l$ satisfies the government's intertemporal budget constraint

$$A_{2007}^G(1+r^*) = - \sum_{t=2007}^{\infty} \frac{1}{(1+r^*)^{t-2007}} \left[\tau_t^s A_t^p + (\hat{\tau}^l + \tau_t^p) w_t N_t + \tau_t^c C_t - G_t - Q_t - B_t - X_t + CF_t \right]$$

5a Follow 1a until petroleum resources are depleted ($CF_t = 0$), then switch to 2a

The government can alternatively satisfy its intertemporal budget by adjusting spending. I do not consider this in my simulations. Instead, I focus on alternative strategies for financing a given government spending arrangement (welfare state design). The policy is introduced at the end of 2006, as an unanticipated event.¹³ Regarding the current rule this might seem surprising, given that it was introduced in 2001. However, since I start my simulation in 1979 with constant tax rates, I implicitly assume a pre-2007 policy with endogenous spending, and that the households did not anticipate the 2007 switch to endogenous tax rates.

4.1 Scenario (1a) - the current fiscal rule

Table 4 reports the macroeconomic implications of the current rule. The second column shows the tax rate dynamics (also to be found in figure 3, graph 1a), the third and fourth the current account and Pension Fund dynamics, while the rest present 10-year growth rates in all government budget components (except for disability pension). The Norwegian 4-percent fiscal rule leaves large room for fiscal expansion until about 2025; labor income tax rates can be reduced by 32 percent from current levels. From then on, tax rates must be raised continually. The long run tax rate implied by the fiscal rule is 60 percent. A huge current account surplus is eventually replaced by a deficit, while a large short-run wealth accumulation is replaced by a constant Pension Fund when petroleum cash flows disappears in 2061. Due to secular growth in output, Pension Fund as a share of GDP converges to zero in the long run.

¹³Regarding the capital stock it is worth noting that I assume that capital stock adjusts to equalize rate of returns also in 2007. If this was not the case (i.e. 2007 capital stock was determined prior to policy announcement) then domestic factor prices would be endogenous in 2007.

To understand the dynamics, I have decomposed government budget growth into the growth of its various components. The last column in table 4 represents what I, in the previous section, referred to as trend growth. This term deserves a thorough explanation. Imagine that we are on a balanced growth path, with population growing at gross rate z and technology growing at gross rate d . Aggregate variables grow at rate zd . Now, suppose population growth suddenly becomes \hat{z} . As long as this growth comes about by increasing the size of all cohorts by the the same factor \hat{z} (e.g. population growth through immigration, in which all immigrants are identical to the locals in terms of assets, human capital, preferences etc.), neither household behavior nor required tax rate change. Hence, all per capita variables remain unaffected. In my simulation both behavior and tax rates do change. First of all, the demographic transition boosts growth in government age dependent expenditures G and B . This tend to increase the required tax rate. On the other hand, the rapid growth in the Pension Fund brings about a large 4-percent contribution. Over the course of the coming two decades this overshadows growth in expenditures, setting the stage for lower tax rates. In addition, both consumption and wealth grows substantially during the first decades, contributing to further tax cuts. Labor supply grows rapidly the first decade, but is on average smaller than trend growth. Eventually, as the petroleum cash flows shrink, the Pension Fund stabilizes. Because the Fund by now is quite large, its relative size in the government budget is also large. Thus, any slowdown in growth will have large impact on required tax rates. In addition, the relative size of age dependent expenditure components increase and consumption and wealth growth declines. In total this renders the small tax burden unsustainable and the fiscal rule calls for continually increasing tax rates. However, around 2045 taxes decline for about 5 years. This is caused by the post-1979 fertility drop. Large cohorts from the initial balanced growth path dies, while relatively small cohorts enter retirement age. Hence, the economy experience a decade with declining old-age dependency ratio.

The dynamics in aggregate consumption, wealth and labor supply is partly explained by the demographic transition and partly by tax rate dynamics itself. As explained in section 3.3, the aging process tend to make consumption and wealth grow faster than trend. As mortality rates eventually stabilize, the growth will fall back towards trend. This partly explains the high but steadily declining growth. For labor supply the opposite is true. It also displays quite volatile growth. The initial high growth rate of 27 percent per decade can be explained by the fact that labor supply drops in 2007 (explaining the immediate tax increase). This is attributed to intertemporal substitution. The households anticipate a large tax reduction in the near term, and so they choose to consume more leisure and goods immediately, which reduce savings. This explains the relatively slow growth in household wealth the first decade. At the end of the century growth in consumption and wealth (labor supply) is below (above) trend growth. This is not due to demographics, rather it results from the continually increase in taxes. Consequently, on an aggregate level, increasing tax

rates reduce (increase) consumption and wealth (labor supply) growth due to steadily growing lifetime tax burden. One could wonder why then, the initial decline in tax rates does not reduce labor supply growth. The reason is presumably that the initial decline is only temporary, and so the substitution effect is much stronger than the effect of a small reduction in lifetime tax burden.

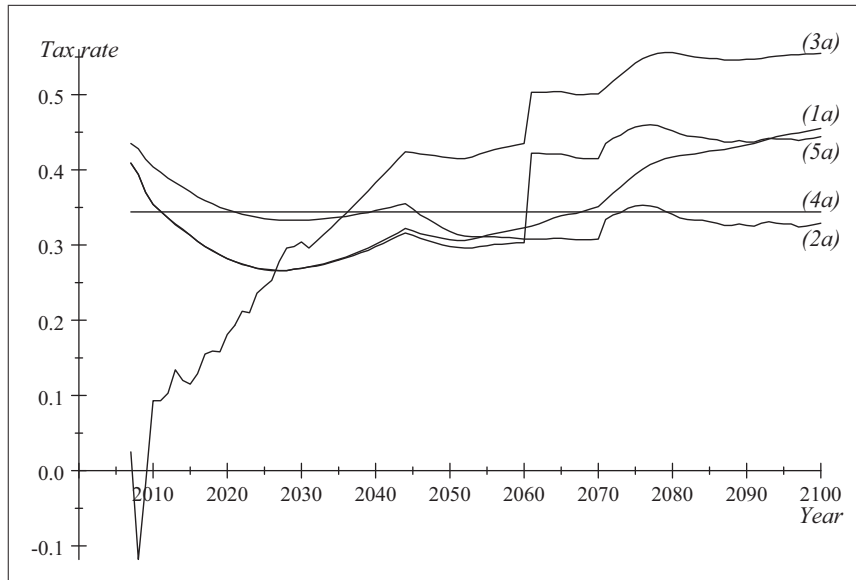


Figure 3: Labor income tax rates. Scenario (1a) - (5a).

Compared to Norway's fiscal outlook, and in particular the tax rate path in figure 2, my benchmark scenario yields essentially the same dynamic picture. The difference is that the persistence of tax cuts is larger. Tax rates do not reach pre-reform levels until 2075. The explanation is related to the relative importance of petroleum revenues. My model abstracts from immigration, which, in population projections is expected to account for the main share of future population growth. Insofar as immigrants only affect population growth, and leave the relative size of different cohorts unaffected, they only have an impact on trend growth. I could then replicate immigration simply by scaling up the size of the economy. The only thing not scaled up would be the Pension Fund. Consequently, its relative size would decrease, and its ability to create tax cuts diminish. Therefore, the assumption of no immigration creates a bias towards a favorable fiscal position. Regarding the fiscal policy comparisons I conduct, these demographic imprecisions will be present in all scenarios, and therefore not affect the qualitative conclusions.

4.2 Scenario (2a) - growth-adjusted rule

The growth-adjusted rule involve a constant long-run government wealth to GDP ratio. In particular, it subtracts trend growth from the four percent real return. As a result, the Pension Fund will have a long-

Year	Tax Rate τ^l	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.43	0.17	1.15	0.97	1.03	1.00	1.00	1.00
2017	0.36	0.18	2.88	0.97	1.01	0.94	1.13	1.00
2027	0.33	0.18	3.98	0.96	1.01	0.87	1.26	1.00
2037	0.34	0.17	4.85	0.96	1.01	0.83	1.42	1.01
2047	0.34	0.10	5.49	0.97	1.01	0.81	1.59	1.02
2057	0.31	0.14	5.67	0.98	1.01	0.83	1.78	1.02
2077	0.35	0.08	6.13	1.02	0.98	0.89	2.40	1.02
2097	0.32	0.10	5.96	1.07	0.97	1.04	3.22	1.00
2147	0.34	0.08	6.00	1.20	0.93	1.53	6.62	0.95
<i>longrun</i>	0.34	0.09	6.00	1.35	0.89	2.82	∞	0.89

Table 5: Macroeconomic effect of growth-adjusted rule - endogenous labor income tax. a: Value relative to benchmark scenario 1a

term impact on the economy. This can be seen from table 5. The fund grows rapidly the first five decades, before it stabilizes at 6 times GDP, causing a permanent reduction in tax rates relative to the benchmark scenario. In fact, a 44 percent long-term reduction is made possible. The downside comes in the near term. Growth-adjusting the fiscal rule leads to higher tax rates the first five decades, it is the inevitable consequence of a more conservative fiscal rule. In terms of macroeconomic performances, short-term substitution effects are still important. However, the incentive to both work and save less for the future is not as strong as in 1a. In addition tax burden is heavier. This triggers larger short- and medium-term labor supply relative to benchmark, while the opposite holds for aggregate consumption and household wealth. As tax burden eventually becomes smaller, consumption and savings are stimulated, but labor supply dampened. This is in line with the behavioral response to wage changes discussed in section 3.3. The welfare effect is summarized in figure 4

Following e.g. Fehr and Habermann (2008) and Auerbach Kotlikoff (1987), I measure the welfare effect as the proportional change in consumption and leisure that would make the individual as well off in the benchmark scenario as in the reform scenario. The household is better off under the alternative regime if the change is positive. In figure 4,¹⁴ a welfare index value of 1 represents unchanged utility, while e.g. 1.01 imply that the household is better off in the reform scenario than in the benchmark scenario. It would take a one percent increase in consumption and leisure in the benchmark scenario, in order to be indifferent. In terms of wealth, this represents a one percent increase in full lifetime resources.¹⁵ The graph denoted (2a) displays the welfare effect of the growth-adjusted rule. Cohorts who are retired at the time of reform, are not affected. The initial young generations lose up to 2.5 percent of full lifetime resource. Early transition

¹⁴An individual's age at the time of reform is 19 minus the number on the horizontal axis

¹⁵Full lifetime resources consists of the households current asset position plus the present value of time endowment. In the appendix I show that an increase in full lifetime resources lead to proportional increase in optimal consumption and leisure.

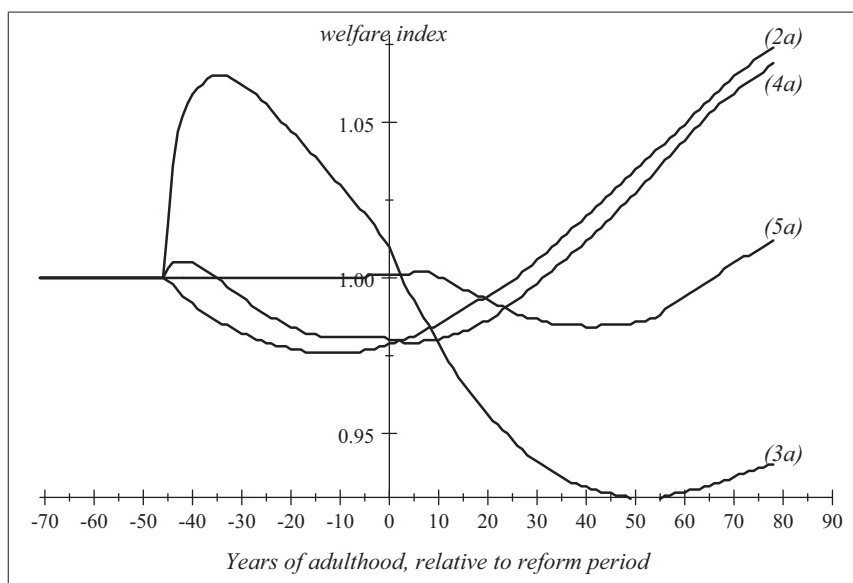


Figure 4: Welfare effects of alternative fiscal rules. Endogenous labor income tax. Scenario (2a)-(5a)

generations also lose, while future generations gain. A cohort born in 2050 gains 2.5 percent of full lifetime resource. The long run gain is much larger, in fact 17 percent. However, the long run literally means long run (at least 300 years), so it should not be at the center of our attention. In any case, the reform unambiguously redistributes welfare from current towards future generations.

4.3 Scenario (3a) - no rule

By no rule I mean that instead of setting the petroleum cash flow aside, the government spends the entire amount each period, in addition to earned interest on current assets. By doing this the Pension Fund remains constant at its 2006 level. This scenario is interesting, not because of its practical relevance, but because it gives us an idea of how a frictionless economy is affected by huge short-term expansionary policy. As mentioned earlier when discussing the introduction of the current rule, policymakers emphasized the stabilization aspect of fiscal rules, implicitly referring to a Keynesian-type economy and issues such as unemployment and sectoral adjustment costs. In my simulation there are no such issues. Instead I focus on fluctuations in macroeconomic variables and welfare brought about by a short-term expansionary policy. The results are reported in table 6 .

Tax rates drop significantly the first year. Labor income tax decrease from 39 to 2 percent and is further reduced to -12 percent the following year. Due to steady decline in petroleum revenue, the initial low levels are far from being sustainable. Taxes instantly start to grow, and reach pre-reform levels in 2041. Relative to benchmark, consumption and labor supply increase by 10 and 11 percent respectively the first

Year	Tax Rate τ^l	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.02	-0.01	1.07	1.10	1.11	1.00	1.00	1.00
2017	0.16	0.12	0.95	1.08	0.99	1.85	0.36	1.00
2027	0.28	0.08	0.84	1.06	0.95	1.72	0.25	0.99
2037	0.35	0.02	0.73	1.03	0.96	1.44	0.20	0.97
2047	0.42	-0.07	0.63	1.00	0.98	1.23	0.18	0.96
2057	0.43	-0.02	0.50	0.96	1.02	1.07	0.16	0.96
2077	0.55	-0.08	0.38	0.91	1.03	0.79	0.16	1.00
2097	0.55	-0.05	0.27	0.88	1.05	0.65	0.16	1.04
2147	0.59	-0.04	0.13	0.91	1.03	0.64	0.16	1.05
<i>longrun</i>	0.60	-0.03	0.00	1.00	1.00	1.00	0.16	1.00

Table 6: Macroeconomic effect of no rule - endogenous labor income tax.a: Value relative to benchmark scenario (1a)

year. Consumption remains relatively high for the next three decades while labor supply drops below baseline path. After 50 years consumption becomes relatively low, while labor supply is high. The dynamics can be explained by looking at tax rates. When rates are lower (higher) than in benchmark scenario, consumption is higher (lower). The opposite is true for labor supply. The immediate increase, and the subsequent drop, in labor supply is caused by a strong positive substitution effect (since rates are expected to grow substantially from its initial low level). In scenario (1a) the substitution effect was negative. Small initial tax burden boosts saving and consumption. Because it is an immediate and very temporary tax cut, there is a particularly strong incentive to save. By 2017 household wealth is 85 percent larger. However, as tax rates continually increase over time, lifetime tax burden increase and eventually become larger than in the benchmark scenario. Consequently, consumption and wealth (labor supply) decline (increase) and eventually become smaller (larger). In the long run, the no-rule scenario is identical to the current fiscal rule, reflecting the fact that neither strategies preserve the Pension Fund as a constant share of GDP (i.e. petroleum wealth eventually becomes irrelevant). Welfare effects are large. The "no-rule" rule correspond to a large distribution of welfare from future generations towards the current young and adult cohorts. If you are 35 years old at the time of the reform your welfare gain is the equivalent of 6.5 percent increase in consumption and leisure in all remaining years. However, if you are born 50 years after the reform the welfare loss is the equivalent of a 7 percent reduction. It is interesting to note that the initial reduction of government wealth is paralleled by an increase in household wealth. In benchmark scenario the ratio of government to household wealth is 1.34 in 2017. Hence, in absolute terms the reduction in government wealth is almost entirely matched by increased household wealth. However, due to overlapping generations, the increase is only temporary while the decrease in government wealth is permanent.

Year	Tax Rate τ^l	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.34	0.20	1.11	0.99	1.07	1.00	1.00	1.00
2017	0.34	0.18	2.64	0.98	1.00	1.11	1.03	1.00
2027	0.34	0.16	3.71	0.97	0.99	0.98	1.16	1.00
2037	0.34	0.16	4.51	0.97	1.00	0.88	1.32	1.00
2047	0.34	0.11	5.06	0.97	1.01	0.87	1.47	1.01
2057	0.34	0.13	5.37	0.98	1.00	0.85	1.68	1.01
2077	0.34	0.08	5.82	1.01	1.00	0.85	2.33	1.02
2097	0.34	0.09	5.80	1.06	0.97	1.00	3.13	1.01
2147	0.34	0.08	5.82	1.19	0.93	1.51	6.43	0.95
<i>longrun</i>	0.34	0.08	5.81	1.34	0.90	2.77	∞	0.89

Table 7: Macroeconomic effect of constant tax rates - endogenous labor income tax. a: Value relative to benchmark scenario (1a)

4.4 Scenario (4a) - constant tax rates

Suppose for a moment that the Norwegian fiscal rule does not exist. Is the current level of tax rates and welfare state design unsustainable, in particular, will a continuation of today's tax rates violate of the government's intertemporal budget constraint? Within my stylized economy the answer is no. Not only is a continuation of 2006 tax rate level feasible, we can in fact reduce labor income tax rates with 13 percent. The lowest feasible level of tax rates, such that the government's budget constraint is satisfied with equality, is 34 percent. In the short run tax burden is thus smaller than in benchmark, in the medium run its larger, while in the long run its again smaller. The short-term expansion of government wealth is somewhat smaller than in the growth-adjusted scenario, causing a slightly larger tax burden after two decades. Relative to the 4-percent rule, a constant tax reform induce a redistribution of welfare from current young to current old working households and future generations. In the long run, households are better off due to permanently lower tax burden, the welfare gain is the equivalent of a 16 percent increase in lifetime consumption and leisure. On the macro side, due to constant tax rates, there is no initial incentive to substitute labor supply intertemporally. Both labor supply and savings is therefore stimulated in the short run, relative to benchmark. Since a fairly long period of high tax rates follows, consumption and wealth drop below benchmark paths. In the medium and long run the development is approximately equal to scenario (2a). This should not come as a surprise. In figure 3, disregarding the temporary drop in graph (2a) in the 50s and 60s, the growth adjusted tax path fairly soon (in 2020) stabilizes around the constant tax rate.

My result which shows that maintaining the 2006 tax rates is a sustainable fiscal policy, is rather different from the conclusions drawn in several white papers from the Norwegian Ministry of Finance (e.g. Ministry of Finance, 2009). Employing a standard generational accounts (GA) framework the Revised National Budget 2009 concludes that the primary budget deficit should be reduced by 4.75 – 7.25 percent of mainland GDP

Year	Tax Rate τ^l	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.41	0.13	1.18	1.00	1.00	1.00	1.00	1.00
2017	0.30	0.15	2.58	1.00	1.00	1.00	1.00	1.00
2027	0.27	0.13	3.16	1.00	1.00	1.00	1.00	1.00
2037	0.29	0.12	3.41	1.00	1.00	1.00	1.00	1.00
2047	0.31	0.04	3.43	1.00	1.01	1.03	1.00	1.00
2057	0.30	0.08	3.11	0.99	1.03	1.08	1.00	1.00
2077	0.46	-0.02	3.26	0.98	0.99	0.92	1.28	1.00
2097	0.44	0.02	3.07	0.99	1.01	0.87	1.72	1.01
2147	0.46	0.02	3.06	1.06	0.98	1.09	3.54	0.99
<i>longrun</i>	0.47	0.03	3.05	1.18	0.94	1.95	∞	0.94

Table 8: Macroeconomic effect of wealth targeting - endogenous labor income tax. a: Value relative to benchmark scenario (1a)

on an annual basis in order to make the fiscal policy sustainable. There are several reasons why my result differs from the result obtained by the Ministry of Finance. Probably, the most important one is that the GA analysis includes what is presently regarded as the most plausible projection for net immigration, whereas my study has neglected immigration. This difference makes the population in my projections significantly smaller compared to the population forecast used in the GA. In 2060 the difference is about 1.9 millions, i.e. 38 percent. In both my model and in the GA a proportional increase in the size of all cohorts contributes to reduce the primary budget surplus simply because the petroleum wealth per capita decreases. It is important to stress that incorporating a more realistic demographic projection, especially of immigration, may show that the sustainable constant tax rate would be larger than 0.39.

4.5 Scenario (5a) - wealth target

The rule is a combination of (1a) and (2a), in particular it involves following the current fiscal rule until petroleum cash flow disappears in 2060, and then switch to rule (2a). In effect, this rule maintains a constant government wealth to GDP ratio, starting in 2060. The results are reported in table 8.

Due to short- and medium-term equivalence to the benchmark scenario, nothing happens during the first three decades. In 2060, when switching from rule (1a) to (2a), tax rates instantly jump (see graph (5a) in figure 3). A period of relatively high tax rates follows. This enables the government to permanently lower the long-run tax burden. There are two notable aspects. First, the tax hike induces households to substitute labor supply intertemporally, thus the increase in labor supply and wealth prior to 2060. For the next three decades tax rates are higher than in benchmark scenario, leading to lower consumption and wealth and a small increase in labor supply. In the long run the effects are qualitatively the same as in scenario (2a), albeit of a smaller magnitude. Moreover, as long as we switch to a growth adjusted rule sometime in the future,

we can avoid passing a large fiscal burden onto future generations. In my simulation long run tax rate is 22 percent lower than in benchmark. Compared to the other reforms, this one therefore entail less redistribution of welfare.

4.6 Endogenous consumption tax

Table 9-13 reports the macroeconomic effect of scenario (1b) to (5b) with endogenous consumption tax. Figure 5 and 6 display tax dynamics and welfare effects. Regarding tax rates, the results are qualitatively the same as under endogenous labor income tax. The long-run tax rate implied by current fiscal rule is 55 percent, but rates do not reach pre-reform levels until 2070. The lowest sustainable constant tax rate is 16 percent. Under the growth adjusted rule, tax rates stabilize around the constant tax rate in 2020 and beyond , except for the temporary drop in the 50s and 60s. The no-rule scenario is still the most extreme rule, with tax rates dropping to -39 percent two years after the reform, followed by rapid and persistent growth.

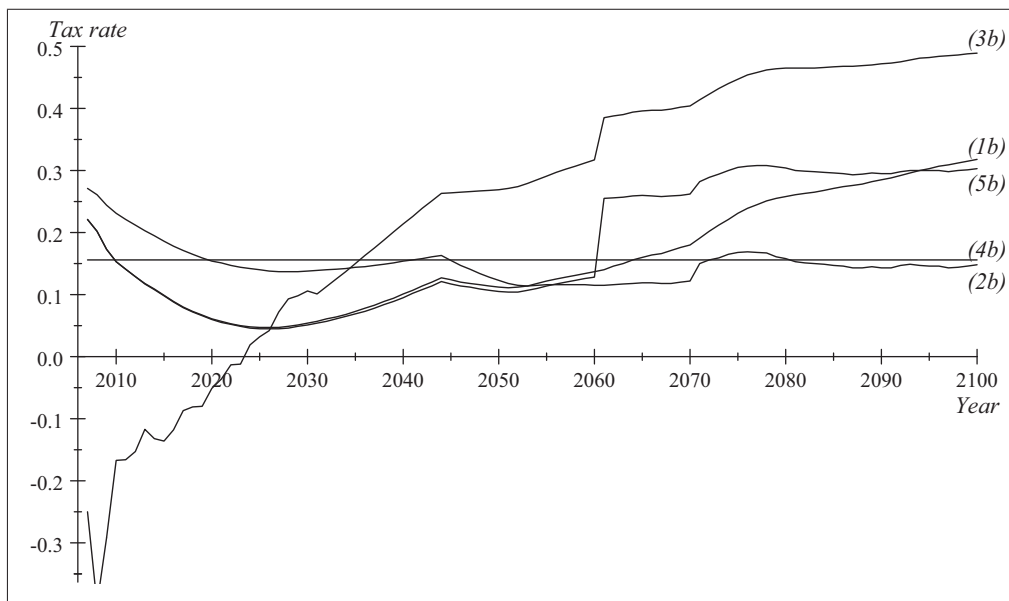


Figure 5: Consumption tax rates. Scenario (1b) - (5b).

Concerning welfare, one major difference between consumption and labor income tax adjustments is that the former also affects households who are retired at the time of reform. Initial old and young cohorts benefit a lot in the no-rule scenario. In fact, the gains range from 20 to 30 percent of remaining life time wealth for age group 85 to 90 in 2007. These gains come partly at the expense of future transition generations who experience a welfare loss of up to 5 percent. Scenario (4b), with its constant tax rates, also involve large welfare gains for initial old cohorts. In this case, future transition generations also gain. The losers are

the initial working cohorts. Fiscal rule (2b) involve a large welfare improvement for households born during transition and in the new stationary equilibrium. The long-term utility gain is 12 percent. Both current young and old cohorts lose. Finally, wealth targeting still induce relatively little welfare redistribution.

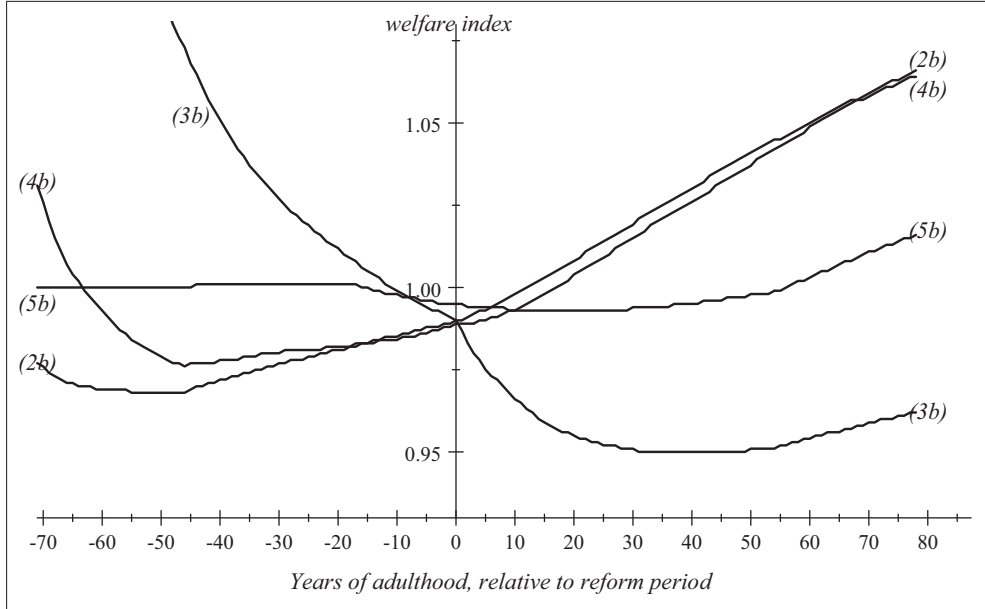


Figure 6: Welfare effects of alternative fiscal rules. Endogenous consumption tax. Scenario (2b)-(5b)

On the macroeconomic side, the Pension Fund continue to be the main driving force behind tax cuts and hikes. Again, there is some feedback from behavioral responses. In contrast to scenario (1a), the immediate action in the benchmark scenario (see table 9) is greatest in consumption. It grows with 30 percent over the first decade, 10 percent more than trend growth. Growth remains higher than trend the next two decades. The main reasons, in addition to an increase in life expectancy, is the downward trend in tax rates. It has two effects, both of which contribute to relatively high growth rates. First, falling consumption tax rates induce households to postpone consumption. Second, from the perspective of a newborn, lifetime tax burden declines over the course of the first two or three decades. Thus, early transition cohorts have lower lifetime tax burden than cohorts alive at the time of reform. This tends to make aggregate consumption grow, since consumption increases in response to a permanently lower tax burden. When tax rate starts to increase, the revers holds, partly explaining the slowdown in consumption growth. The other reason is again that the aging process also slows down.

Regarding the reforms, whenever tax rates are high (low), consumption tend to be low (high), relative to benchmark scenario. The revers holds for labor supply and aggregate wealth. This is in line with the behavioral response to permanent tax cuts found section 3.3. The exception is the no-rule reform. Even

though tax burden is relatively small initially, it is expected to grow substantially. In this case, the incentive to save for the future in order to smooth consumption and leisure is so strong that wealth in 2017 in fact is 35 percent higher than in benchmark scenario.

4.7 Efficiency vs. redistribution

Since tax rates distort behavior, any redistribution of welfare can partly be ascribed to efficiency gains and losses. In order to single out these effects, and thus identify whether a reform represents a potential Pareto improvement, I introduce a Lump Sum Redistribution Authority (LSRA) (see Auerbach and Kotlikoff, 1987, Nishiyama and Smetters, 2005, and Fehr and Habermann, 2008). The task of this authority is to collect and distribute lump sum taxes and transfers such that all households enjoy the same utility level in the reform scenario as in the benchmark scenario. In the first transition period, all cohorts currently alive pay a lump sum tax (transfer, if negative) such that their utility is exactly equal to benchmark utility. Likewise, in each subsequent period, all households pay a lump sum tax upon reaching adulthood. Consequently, all cohorts who are worse off under the alternative fiscal regime is compensated with a lump sum transfer, while all those who are better off must pay a lump sum tax. In this new LSRA scenario, no redistribution of welfare occurs. If the net present value of required taxes over transfers is positive, the reform is Pareto efficient after lump sum redistribution. All households enjoy unchanged utility, and yet there is a surplus of resources. The LSRA is free to distribute this surplus to any particular agent, making at least someone strictly better off. A negative surplus implies that the LSRA must levy lump sum taxes on some agents, making them strictly worse off. In that case, the reform reduces efficiency.

Regarding scenario (2a) – (5a), the only fiscal rule which increase efficiency is the constant tax rule. However, the efficiency gain is only minor, and the LSRA surplus amounts to less than 0.1percent of 2007 GDP. The growth-adjusted rule involves an efficiency loss of 7 percent of GDP, while the no-rule scenario generates a 1.3 percent loss. In addition to the above policy scenarios I experiment with a combination of (2a) and (3a). In this scenario the government spends the growth-adjusted return on the Fund, in addition to 40 percent of the annual cash flow each year. This rule leaves room for reduction in tax burden both in the short and long term relative to the benchmark scenario. In the medium term tax burden is higher. The efficiency gain is 8 percent of 2007 GDP, indicating that an efficient time pattern for tax rates at least has the following properties: Lower long-term tax burden and an immediate, and temporary, tax reduction relative to the benchmark scenario.

These results are presumably conditional on the preference parameters chosen. For instance, I have a fairly low intertemporal substitution elasticity, which might explain why constant tax rates do not give rise

to a larger efficiency gain. Indeed, redoing all simulations with $\gamma = 0.6$ and $\theta = -0.075$ (see table 3) increases the efficiency gain induced by constant labor income tax rates to 3.6 percent of 2007 GDP. This is not surprising. The more sensitive behavior is to time-varying prices, the greater are the distortions arising from fluctuating tax rates.

5 Concluding remarks

In this paper I have developed a large-scale overlapping generations (OLG) model for Norway, and used it to illustrate the implications of different fiscal rule specifications. Due to large petroleum resources, alternative fiscal rules give rise to large differences in the timing and level of taxes and the welfare of different generations. Since I endogenize tax rates rather than government expenditures, the simulation results indicate how public finances evolve over time given a continuation of today's welfare state design. The current Norwegian rule is partly motivated by long-term concerns such as fiscal sustainability and, ultimately, intergenerational welfare distribution. My results show in what way a fiscal reform would affect these objectives. The issue of what rule to prefer would in turn depend on the particular weight policymakers put on the welfare of different generations, as well as other concerns not addressed in my model (such as ensuring that the policy is transparent, operational, time consistent, provides short-term stabilization, avoids large sectoral adjustment costs and Dutch disease and other issues that were important concerns when designing the current rule, see e.g. Ministry of Finance, 2001)

The current fiscal rule gives rise to a short- to medium-term tax reduction, followed by successive increments in the tax rates until a new balanced growth path is reached. More precisely, under the current fiscal regime tax rates can be kept below today's level until 2075, implying that the fiscal position is fairly sound. However it does not represent what I define to be a sustainable fiscal policy, since long-run tax burden must increase relative to today's level. In other words, the rule does not generate tax smoothing. Rather, tax rates are highly time varying. Regarding the former issue, both the growth- adjusted rule and the constant-tax rule indicate that a fiscal policy reform which provides more prefunding through stronger accumulation government wealth in the short run, will prevent a large future tax increase. The basic reason why the current rule implies non-sustainability is that it allows for intermediate tax cuts. My results indicate that today's tax burden is more than sustainable, meaning that it exceeds the simulated permanent tax rate in scenario 4a.

However, these results are conditioned on at least two assumptions, namely that of no immigration and the somewhat arbitrary assumptions determining the convergence to a long run balanced growth path. The comparisons between my simulated tax paths (and today's tax burden) must therefore be interpreted with

caution. For instance, since I leave out immigration I neglect the fact that more than 85 percent of the projected 2.2 million population increase from 2007 to 2060 is explained by net immigration (see Brunborg et.al 2008). Since a large population, *cet. par.*, reduces the petroleum wealth per capita, my simplifying assumption of no immigration overstates the positive impact petroleum wealth has on public finances, and it might also overstate the quantitative differences in tax paths of the alternative fiscal rules considered in this paper. It is conceptually straightforward to include immigration in the model, but due to time limitation it has been left out of the study. It would thus be a natural topic for future research. Moreover, both the constant tax rate (in policy 4) and long run tax rates (in policy 1, 2, 3, 5) are dependent on the balanced growth assumption. In particular, I ensure long run balanced growth by stopping the aging process in year 2080. A sensitivity analysis with respect to this stopping period is beyond the scope of this paper, but the assumption (and zero immigration) presumably creates a bias towards a favorable fiscal position. Nevertheless, the main objective of this paper has been to develop a large-scale OLG model for Norway and use it for a comparative study of alternative fiscal rules. And since the bias is present in all policy experiments, it does not necessarily alter the qualitative properties of my simulation results

Regarding tax smoothing, the growth adjusted rule provides less time varying tax rate, while constant tax rate provides perfect tax smoothing. However, my results show a surprisingly small (even negligible) gain from constant tax rates, even though the scope for tax smoothing is substantial. This may be surprising in light of the emphasis given to this source of inefficiency in the literature following Barro (1979). To investigate the causes behind this result is an interesting topic for future research.

Even though the model contains many simplifying and unrealistic assumptions and abstracts from considerable uncertainty, my stylized perfect foresight OLG model does provide results that deserve to be interpreted a first approximation. Petroleum revenues, demographics and fiscal rules, will be effects of first order, no matter how I choose to model the economy. And even though some of my assumptions create a bias towards a favorable fiscal position, the model still provides insight into how alternative fiscal rules affect the economy.

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A Appendix

The appendix contains a detailed description of the exogenous and endogenous variables, equilibrium definition, and how to solve and calibrate the model.

Summary of exogenous and endogenous variables

This section summarizes the exogenous specification of the model, i.e. the variables treated as input to the model, and endogenously determined variables (time and cohort subscripts are dropped).

- The demographic structure: mor, fer
- Preference parameters: $\rho, \gamma, \theta, \alpha$
- Household age-earnings profile: e, h
- Time endowment: E
- Interest rate: r^*
- Secular growth rate: g
- Production function parameter: ϕ
- Capital depreciation: δ
- Government age-consumption profile: d
- Disability pension age profile: q
- Government net expenditures per capita: x
- Petroleum cash flow: CF
- Earnings dependent pension benefit parameter: f
- Bequest requirement at age 91: \tilde{a}
- Initial distribution of household and government wealth

In addition I need to do some normalizations.

- Normalize initial population size
- Normalize initial time endowment
- Normalize age-earnings at age 19

The remaining input needed before the model can be solved, is how the governments intertemporal budget constraint is met. In the benchmark setup the government specifies a fiscal rule which ensures that the government wealth develops according to

$$A_{t+1}^G = A_t^G + CF_t$$

implying an annual budget constraint given by

$$r^* A_t^G + \tau_t^s A_t^p + (\tau_t^l + \tau_t^p) w_t N_t + \tau_t^c C_t - G_t - Q_t - B_t - X_t = 0$$

In the benchmark scenario this constraint is met by adjusting τ_t^l , adding the following variables to the list of exogenous specifications

- Capital income tax rate, consumption tax rate and employer's payroll tax rate: τ^s, τ^c, τ^p

The variables endogenously determined by the model, is then

- Individual and aggregate labor supply, consumption and private wealth: $(E - l), c, a, N, C, A^p$
- Individual and aggregate pension benefit: b, B
- Aggregate bequest and individual and aggregate inheritance: BEQ, inh, INH
- Age-dependent consumption: G
- Regular net expenditures: X
- Disability pension: Q
- Labor income tax rate and government wealth: τ^l, A^G
- wage rate and capital stock: w, K
- Current account: CA

Some of these variables are easily determined.

- Given r^* and τ^p , w is determined from equation 15
- Given q, d, x and demographic structure, G, X and Q are determined from equation 17
- Given aggregate labor supply, aggregate capital stock is derived from profit maximizing capital-labor ratio determined from equation 14

- Given aggregate capital stock and private and government wealth, CA is determined from equation 20
- All other aggregate variables (N, C, A^p, B, INH) are calculated from individual variables, using equations 11 and 12
- Given petroleum cash flow and an initial value for government wealth, A^G is determined

The remaining variables are individual allocations of consumption, leisure, wealth, inheritance, aggregate bequest (c, l, a, inh, BEQ) and labor income tax rate (τ^l)

Competitive equilibrium

Definition 1 Let $\widehat{c}_t = (\widehat{c}_{t,19}, \dots, \widehat{c}_{t,T})$ and correspondingly for $\widehat{l}_t, \widehat{a}_t, \widehat{b}_t, \widehat{inh}_t, \dots$. An equilibrium in this economy is a sequence of (i) individual allocations $\left\{ \widehat{c}_t, \widehat{l}_t, \widehat{a}_t, \widehat{b}_t, \widehat{inh}_t \right\}_{t=1}^{\infty}$, (ii) aggregate labor supply, labor demand, capital stock, bequest, inheritance and pension benefits $\left\{ \widehat{N}_t, N_t^*, \widehat{K}_t, \widehat{BEQ}_t, \widehat{INH}_t, \widehat{B}_t \right\}_{t=1}^{\infty}$, (iii) prices $\left\{ \widehat{w}_t, \tau_t^* \right\}_{t=1}^{\infty}$, (iv) tax rates, expenditures and government wealth $\left\{ \widehat{\tau}_t^c, \widehat{\tau}_t^l, \widehat{\tau}_t^s, \widehat{\tau}_t^p, \widehat{G}_t, \widehat{Q}_t, \widehat{X}_t, \widehat{A}_t^g \right\}_{t=1}^{\infty}$, and (v) initial distribution of individual assets \widehat{a}_0 and government wealth A_0^g , such that

- 1) $\left\{ \widehat{c}_t, \widehat{l}_t, \widehat{b}_t, \widehat{a}_t \right\}_{t=1}^{\infty}$ solve the household's problem given prices, tax rates and inheritance
- 2) $\left\{ N_t^*, \widehat{K}_t \right\}_{t=1}^{\infty}$ solve the firm's problem given prices and tax rates
- 3) Labor market clears

$$\widehat{N}_t = N_t^*$$

- 4) Aggregate inheritance equals aggregate bequest

$$\widehat{INH}_t = \widehat{BEQ}_t$$

- 5) The government satisfies its intertemporal budget constraint.

Since neither individual agents nor the government violates their constraints, I do not have to check that the country as a whole is not running a ponzi scheme. And since savings does not have to equal investment I do not have a capital market clearing condition.

Balanced growth path

When the economy is on a balanced growth path a variable is either constant or grows at a constant rate. Specifically, all tax rates, factor prices, mortality and fertility rates are constant. Facing the same prices and tax rates, the only difference between two succeeding generations is that the younger agents have $(1 + g)$ times more full lifetime resources¹⁶. This, together with the fact that CES utility function is homothetic, causes consumption, labor supply, wealth (and bequest) for a cohort of age j to grow with rate g over time,

¹⁶Full lifetime resources is the present value of time endowments, inheritance and bequest requirement at age 91

e.g. $c_{t+1,j} = (1+g)c_{t,j}$. Moreover, since fertility and mortality rates are constant, the population dynamics are stable, implying that each cohort, and thus the entire population, grows at a constant rate λ over time. Consequently, aggregate variables grow at rate $z = g + \lambda + g\lambda$, e.g. $C_{t+1} = (1+z)C_t$. Balanced growth will be feasible so long as the growth of exogenous government expenditures does not exceed $(1+z)$. An expenditure growth rate exceeding z , together with constant tax rates, will lead to a violation of the government's budget constraint. Furthermore, if expenditure growth is smaller than $(1+g)(1+\lambda)$ the ratio of these expenditures to GDP will converge to zero. The assumptions implicit in my model, ensure that government expenditure components grow at rate z along a balanced growth path. The assumption implies that X_t and Q_t grow at rate z , while $d_{t,i}$ and $q_{t,i}$ must grow at rate g . The cash flow CF_t is temporary, and is thus irrelevant for the existence of a balanced growth path. I confirm the balanced growth property below

Solving the balanced growth path

I use a Gauss-Seidel algorithm to solve the model numerically.¹⁷ Variants of this has been described in e.g. Auerbach and Kotlikoff (1987) and Altig et.al. (2001). The idea is basically to make a guess on some of the endogenous variables, then calculate the remaining endogenous variables, and finally check for consistency. A very simple example provides the basic intuition. Assume we have a system of 2 equations in 2 unknowns (x, y) .

$$\begin{aligned} a_1x + b_1y &= c_1, \quad (i) \\ a_2x + b_2y &= c_2, \quad (ii) \end{aligned}$$

rewrite to get

$$\begin{aligned} y &= \frac{c_1}{b_1} - \frac{a_1}{b_1}x, \quad (i)' \\ x &= \frac{c_2}{a_2} - \frac{b_2}{a_2}y, \quad (ii)' \end{aligned}$$

The following algorithm will find the solution. Make an initial guess in $x = x^g$ and calculate $y = y^g$ using $(i)'$. Use $(ii)'$ to calculate the $x = x^i$ value implied by y^g . If $x^g = x^i$ the solution is found. If $x^i < (>)x^g$ decrease (increase) the guessed value, e.g. $\hat{x}^g = \lambda x^g + (1-\lambda)x^i$, $\lambda \in (0,1)$, and repeat with $x^g = \hat{x}^g$ as the updated guess on x . Continue until the guessed x^g is sufficiently close to x^i .

The system of equations determining the endogenous variables In the OLG model is a bit more complex than a simple 2-by-2 linear system. The algorithm is, however, exactly the same. To solve for the balanced growth path I first make a guess on labor income tax rate and aggregate bequest. Since all exogenous variables

¹⁷All simulations were carried out in Matlab. I checked the uniqueness of the solutions by altering initial conditions.

are given numerical values, all information needed to solve the household problem is known. The only thing I do not need to specify exogenously is the entire distribution of household wealth across cohorts. All I need is wealth upon reaching adulthood, which is assumed to be zero. The reason for this is that even though there are multiple cohort coexisting along the balanced growth path, I only need to solve the household problem for one cohort (the age 19 cohort). Optimal allocations for other cohorts (including their wealth) are found by using the balanced growth property described above. Below I will give a detailed description of how to solve for the household problem. Now, assume that c^* is the optimal planned consumption for age 19 cohort one period ahead. To find optimal consumption for e.g. age 20 cohort, I use the property $c_{t+1,20} = c^* = (1+g)c_{t,20}$, and likewise for leisure and wealth. Let $\hat{c}_t = (c_{t,19}, c_{t,20}, \dots, c_{t,90})$ be the distribution of consumption across cohorts along the balanced growth path. It is clear that $\hat{c}_{t+1} = (1+g)\hat{c}_t$. To calculate aggregate variables I use the stable population composition implied by the constant fertility and mortality rates (with total population size normalized). I can then calculate the implied aggregate bequest and labor income tax rate. If initial guess and implied values are sufficiently close, I have found the solution, otherwise, I update my guesses.

Solving the transition path

In order to solve for a transition path following a shock in period t (e.g. in mortality and fertility rates) the same algorithm is used. The only difference is that I now have to *i*) guess on the entire path of tax rates and bequest from the new to the old balanced growth path, and *ii*) solve the household problem for all cohorts alive at the time of the shock and born along the transition path. When guessing on the entire path of tax rates and bequest I assume that after, say 300 periods, the economy is sufficiently close to the new balanced growth path so that forcing the economy to be exactly on that path after 300 periods gives rise only to a small approximation error. Let $\hat{\tau}^l = (\hat{\tau}_{t+1}^l, \dots, \hat{\tau}_{t+371}^l)$ be the path of tax rate guesses. If I know the tax rate value on the new balanced growth path I simply set $\left\{ \hat{\tau}_{t+i}^l \right\}_{i=300}^{371}$ equal to this value. If I don't know the value on the new balanced growth path (due to path dependency, i.e. the new balanced growth path depends on the transition path), I make a guess on the sequence $\left\{ \hat{\tau}_{t+i}^l \right\}_{i=1}^{300}$ and set $\hat{\tau}_{t+i}^l = \hat{\tau}_{t+300}^l$ for $i = 301, \dots, 371$. For the generations born after the shock, the optimization problem is exactly the same as in the balanced growth case (except for the fact that tax rates and bequest (inheritance) is time-varying). Generations alive at the shock, re-optimize. The problem is exactly the same, except for the fact that they have a shorter time horizon and a non-zero initial wealth (consisting of financial wealth and old-age pension wealth)

Solving the household problem and confirming balanced growth path

The problem for a cohort reaching adulthood is (dropping time subscripts on individual variables, i.e. period equals age, and age 1 is first year of adulthood)

$$\max_{\{c_t, l_t\}_{t=1}^{72}} \frac{1}{1 - \frac{1}{\gamma}} \sum_{t=1}^{72} \beta^{t-1} P_t u(c_t, l_t)^{(1 - \frac{1}{\gamma})} \quad s.t. \quad (22)$$

$$\sum_{t=1}^{72} \left[\prod_{i=1}^t (1 + r^*(1 - \tau_i^s))^{-1} \right] \left[(1 - \tau_t^l) w_t e_t (E_t - l_t) + (1 + r^*(1 - \tau_t^s)) i n h_t - (1 + \tau_t^c) c_t + b_t \right] = \prod_{i=1}^{72} (1 + r^*(1 - \tau_i^s))^{-1} \tilde{a} \quad (23)$$

$$\begin{aligned} a_1 &= 0, a_{73} = \tilde{a} \\ c_t &> 0 \\ 0 &< l_t \leq E_t \\ \mu_t &\geq 0, l_t < E_t \Rightarrow \mu_t = 0 \\ t &\in [1, 72] \end{aligned}$$

rewriting the budget constraint

$$\sum_{t=1}^{72} \left[\prod_{i=1}^t (1 + r^*(1 - \tau_i^s))^{-1} \right] [\psi_t (E_t - l_t) (1 + r^*(1 - \tau_t^s)) i n h_t - (1 + \tau_t^c) c_t - 1_t \tilde{a}] = 0$$

where

$$\psi_t = \left[1 - \tau_t^l + \frac{f}{46} \sum_{j=47}^{72} \prod_{i=t+1}^j (1 + r^*(1 - \tau_i^s))^{-1} \right] w_t e_t, \quad \text{for } 1 \leq t < PA$$

$$\psi_t = (1 - \tau_t^l) w_t e_t, \quad \text{for } PA \leq t \leq T$$

$$b_{t,j} = \frac{f}{46} \sum_{i=1}^{46} w_{t-j+i} e_{t-j+i, i} (E_{t-j+i} - l_{t-j+i, i})$$

$$1_t = 0 \text{ if } t < 72, \quad 1_T = 1$$

The utility function satisfies the Inada-conditions, so we can ignore the non-negativity constraints on

leisure and consumption. Let η be the Lagrangian multiplier with respect to the budget constraint. The first order conditions with respect to period j consumption and leisure becomes

$$\begin{aligned} c_j & : \beta^{j-1} P_j \left(c_j^{(1-\frac{1}{\rho})} + \alpha l_j^{(1-\frac{1}{\rho})} \right)^{\frac{(1-\frac{1}{\rho})}{(1-\frac{1}{\rho})}-1} c_j^{\frac{-1}{\rho}} \frac{1}{(1+\tau_j^c)} \prod_{i=1}^j (1+r^*(1-\tau_i^s)) = \eta \\ l_j & : \beta^{j-1} P_j \left(c_j^{(1-\frac{1}{\rho})} + \alpha l_j^{(1-\frac{1}{\rho})} \right)^{\frac{(1-\frac{1}{\rho})}{(1-\frac{1}{\rho})}-1} \frac{\alpha l_j^{\frac{-1}{\rho}}}{\psi_j + \mu_j} \prod_{i=1}^j (1+r^*(1-\tau_i^s)) = \eta \end{aligned} \quad (24)$$

Hence, the intratemporal optimality condition is

$$l_j = \left(\frac{(1+\tau_j^c)\alpha}{\psi_j + \mu_j} \right)^\rho c_j \quad (25)$$

From this we can identify ρ as the percentage change in the ratio l_j to c_j in response to a one percent increase in the wage rate. Eliminate leisure in consumption FOC

$$c_j \left[1 + \alpha^\rho \left(\frac{(1+\tau_j^c)}{\psi_j + \mu_j} \right)^{(\rho-1)} \right]^{\frac{(\rho-\gamma)}{(\rho-1)}} \left[\frac{\beta^{j-1} P_j}{(1+\tau_j^c)} \right]^{-\gamma} \prod_{i=1}^j (1+r^*(1-\tau_i^s))^{-\gamma} = \eta^{-\gamma}$$

Forwarding one period we get

$$c_{j+1} \left[1 + \alpha^\rho \left(\frac{(1+\tau_{j+1}^c)}{\psi_{j+1} + \mu_{j+1}} \right)^{(\rho-1)} \right]^{\frac{(\rho-\gamma)}{(\rho-1)}} \left[\frac{\beta^j P_{j+1}}{(1+\tau_{j+1}^c)} \right]^{-\gamma} \prod_{i=1}^{j+1} (1+r^*(1-\tau_i^s))^{-\gamma} = \eta^{-\gamma}$$

equalize with period j first order condition to get the intertemporal optimality condition

$$c_{j+1} = c_j \left[\frac{1 + \alpha^\rho \left(\frac{(1+\tau_j^c)}{\psi_j + \mu_j} \right)^{(\rho-1)}}{1 + \alpha^\rho \left(\frac{(1+\tau_{j+1}^c)}{\psi_{j+1} + \mu_{j+1}} \right)^{(\rho-1)}} \right]^{\frac{(\rho-\gamma)}{(\rho-1)}} \left[\beta \frac{(1 - mor_{j+1})(1+\tau_j^c)}{(1+\tau_{j+1}^c)} (1+r^*(1-\tau_{j+1}^s)) \right]^\gamma \quad (26)$$

From this we identify γ as the percentage change in the ratio c_{j+1} to c_j in response to a percentage increase in the after-tax gross return $(1+r^*(1-\tau_{j+1}^s))$. In optimum, if time constraint is binding, i.e. agent retires, the shadow wage must satisfy

$$\mu_j = (1+\tau_j^c)\alpha \left(\frac{c_j}{E_j} \right)^{\frac{1}{\rho}} - \psi_j \quad (27)$$

To confirm the balanced growth properties, note that from the intertemporal and intratemporal optimality conditions we can solve for period 2.. T consumption and period 1.. T leisure as linear functions of period 1 consumption. These functions has variables which are stationary along the balanced growth path as

parameters. Let $c_j = c_1 D_j$ and $l_j = c_1 H_j$, $j = 1..T$ denote these functions, with $D_1 = 1$. Due to the linearity, we can solve for period 1 consumption using the lifetime budget constraint.

$$c_1 = \frac{\sum_{t=1}^{72} \prod_{i=1}^t (1 + r^*(1 - \tau_{j+1}^s))^{-1} [\psi_t E_t + (1 + r^*(1 - \tau_t^s)) inh_t - 1_t \tilde{a}]}{\sum_{t=1}^{72} \prod_{i=1}^t (1 + r^*(1 - \tau_{j+1}^s))^{-1} [\psi_t H_t + (1 + \tau_t^e) D_t]} \quad (28)$$

The numerator is the agent's full lifetime resource. Note that this is the closed form solution for consumption under the maintained assumption of no retirement. When some time constraints bind, the problem is complicated by the presence of shadow wages in the denominator, and we would have to solve for c_1 numerically. Let c_1^* , denote the unique optimal solution. Suppose life time resources is increased by a factor $(1 + g)$, and that this comes about by increasing time endowments (E_t), inheritance (inh_t) and bequest requirement (\tilde{a}) with the same factor. My claim is that this lead to proportional increase in consumption and leisure at all ages of $(1 + g)$. Clearly, if neither $\{H_i\}_{i=1}^T$ nor $\{D_j\}_{j=1}^T$ change, c_1 and $\{c_j, l_i\}_{j=2, i=1}^{T, T}$ will increase by the factor $(1 + g)$. The only way H and D can change is if shadow wages change. But from 27, we see that shadow wages are unaffected since the ratio c_j/E_j is unchanged. Hence, a proportional increase in c^* and l^* of $(1 + g)$ satisfies 28, and is therefore the optimal solution.

Now, by assumption E_t, \tilde{a}_t grows with rate g over time . Let's assume that the same is true for inh_t . (reintroducing time subscripts on individual variables, $inh_{t,j} = inh_{t-1,j}(1 + g)$) The only difference between generations born in period t and $t + 1$ is that the latter has $(1 + g)$ more full lifetime resources than the former. Then due to the linearity between c_1 and (c_j, l_j, a_j) , aggregate wealth, consumption, labor supply grow with rate $z = g + \lambda + g\lambda$. Moreover, this also imply that aggregate bequest grows with rate z . Hence, inh_t will in fact grow with rate g . Consequently, the balanced growth path is an equilibrium.

I will now explain how I solve for the household problem. First I make a guess on the sequence of shadow wages (e.g. the zero vector), Then I calculate optimal consumption in the first year of adulthood from equation 28, and use H and D to find the optimal sequence of consumption and leisure. For the years in which leisure exceeds time endowment I calculate a new shadow wage using equation 27. With these new shadow wages I find a new optimal sequence of consumption. If the new and old sequences of consumption (or alterantively, new and old sequence of shadow wages) are sufficiently close, I have found the solution. If not, I update my shadow wage guess and repeat.

Calibration (Parameterization)

In this section I illustrate an approach of calibrating the model. First, I briefly explain the benchmark simulation for which this model is used. The Idea is to simulate the economy's transition path, starting in

2007, to a new balanced growth path implied by the above fiscal rule, demographic transition and temporary petroleum cash flow. An important choice I have to make is whether I start outside balanced growth or not. One approach to assume that the economy is on a balanced growth path in, say 2007, when an external shock hits the economy. Alternatively, I can assume that the economy is outside the balanced growth path. This is an important distinction, because in the latter case I need an initial distribution of financial and pension wealth for the cohorts alive in 2007. In the following parameterization I take the latter approach. I let the model create the distribution by starting the simulation from a balanced growth path in 1979. Along this path the demographics are stable with constant 1979 levels on mortality and fertility rates. The tax rates and pension system is also constant. On the initial balanced growth path, the government uses expenditures (except for old-age pensions) to satisfy the annual budget constraint implied by the fiscal rule, assuming government wealth is zero¹⁸. From 1980 I use actual mortality and fertility rates, and run the model to 2006, keeping tax rates constant. This gives me an initial distribution of assets in 2007. I then simulate the transition to a new balanced growth path using endogenous labor tax rates. Hence, the only external shock hitting the economy in 2007 is a switch from endogenous expenditures to endogenous tax rates¹⁹. As an alternative approach, I could have estimated (or calibrated) the 2007 age-profile for household assets. One problem with this approach is that it is hard to tell whether the subsequent dynamics is due to the external shocks or to the fact that the 2007 asset distribution is not consistent with household optimization. There is one notable advantage however, namely that this approach allows for an exact reproduction of the 2007 population composition. When I start with a 1979 balanced growth I have to use the implied 2007 population composition to be consistent with the pre-2007 simulation, and this composition is not necessarily equal to the actual population composition. If I use the actual 2007 population composition I implicitly impose another shock to the model, namely a 2007 shock in cohort sizes. In the following parameterization I use the population structure implied by the 1979 demographic shock.

Demographics

Mortality and fertility rates from 1979 to 2007 are estimated by Statistic Norway. From 2008 to 2080 the numbers are based on the middle alternative in the population forecast, also from Statistic Norway²⁰. From 2080 the rates are held constant in order to reach a new balanced growth path. I set the mortality rates for age 91 equal to 1, thus capping maximum age. I also adjust rates for the fact that my model has a representative agent in each cohort, while the rates are sex specific. I simply sum the mortality rates for

¹⁸This is consistent as long as X turns out to be non-negative. The interpretation is that the surplus created by $S = \tau^s A^p + (\tau^l + \tau^p) wN + \tau^c C - B$ is entirely spend on government consumption. $S = Q + G + X > 0$. However, if pension benefits exceed tax income, $S < 0$, the deficit must be financed through a lump sum taxation.

¹⁹The demographic and petroleum shock occurred in 1979, i.e. starting in 2007 the post-2007 demographics and petroleum cash flow dynamics is fully anticipated in this setup.

²⁰The numbers are provided by Inger Texmon, section for demographics at Statistics Norway. Brunborg et.al (2008) employ the numbers to make population projections

women and men and divide by two, and divide the fertility rates by two. This is a crude approach, but still sufficient for my purpose. In the stable 1979 population age 19 cohort is normalized to 1. The population can be summarized by e.g. its old-age dependency ratio, meaning the number of individuals older than 64 divided by the number of individuals in age group 20 to 64. Since population aging is an important feature of my model it is essential that my calibration at least gets the order of magnitude of this ratio correct. However, the problem with starting from a stable 1979 population is that this misses the baby boom in the 50s and 60s economy. The implied dependency ratio is 0.4 in 2007 and 0.5 in 2060. In terms of growth, this is too small compared to the projections in Brunborg et.al (2008). In their forecast the dependency ratio is predicted to increase from about 0.22 in 2007 to 0.45 in 2060, i.e. a 100 percent increase. To improve the accuracy of the model's population structure I therefore raise the 1979 fertility rates with a factor of 1.3. The dependency ratio then becomes 0.30 in 2007 and 0.48 in 2060. It reaches 0.55 in 2080 and remains fairly stable from then on. When doing this I actually implicitly assume a baby boom that was not really a boom, that is, the relatively high fertility rates prior to my 1979 demographic shock had existed all along. Moreover, I abstract from immigration.

Tax rates

There are four tax rates: a capital income tax τ^s , two labor income tax rates τ^l, τ^p , and a consumption tax τ^c . I set capital income tax rate equal to 28 percent which is the current flat capital income tax rate in Norway.²¹ The employer's social security payroll tax rate is 13 percent.²² For τ^l and τ^c I compute an average tax rate based on the 2006 National Account.²³ Total household labor taxes divided by total household labor income yields $\tau^l = 0.39$, and total household indirect taxes divided by total household consumption yields $\tau^c = 0.19$.

Government expenditures, petroleum cash flow and wealth

Exogenous government expenditures G_t, Q_t , government wealth A_t^g and petroleum cash flow CF_t , are calibrated to match data in the 2006 National Accounts. Endogenous budget components (labor, capital and consumption tax revenue, and old age pension benefit) are determined by the 1979 simulation. The remaining component, X_t , is a residual calibrated to ensure that the fiscal rule is satisfied in 2006. Since I assume that exogenous expenditures per capita grow with the rate of time endowment, I simply use the population projections to find the corresponding 2007 values. Regarding the definition of public finances and the Norwegian fiscal rule there are a couple of points worth noting. First of all, the government can in

²¹see tabel 4.4, Ministry of Finance (2008)

²²The actual tax rate is differentiated according to geographical zones, with values ranging from 14.1 for the big cities and percent to 0 percent for some regions in the periphery, see Ministry of Finance (2008). The value 13.1 is in line with the MSG calibration mentioned in section 2

²³I use total household labor income, and labor income taxes. For consumption taxes i use total household consumption and total household indirect taxes. See Statistics Norway (2007)

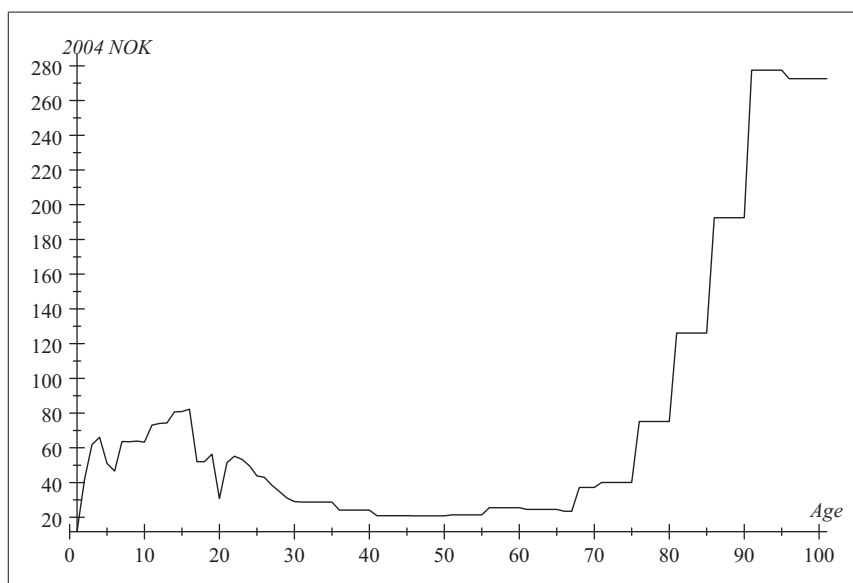


Figure 7: 2004 profile for government age-related consumption (per individual, females). In 1000 NOK (2004)

general be divided into central and local governments. However, the fiscal rule only applies for the central government. This means, that in principle, the local governments can run large deficits, leaving the entire government with a deficit larger than what the rule allows. In reality, the local government sector is prohibited from running large deficits. Actually, at the end of 2006, the entire local government debt was 3.8 percent of mainland GDP, while for the central government financial wealth it was 191 percent²⁴. Furthermore, the fiscal rule only applies to the part of government financial wealth which is invested in the Pension Fund. The focus is on the Pension Fund, petroleum cash flows and fiscal rules, so I simply abstract from other financial assets²⁵ and local governments.

According to the National accounts I find that government consumption was 26.2 percent of mainland GDP in 2006. Statistics Norway has illustrated the age and sex profile of public spending in Norway. They define a part of total government use of resources in 2004 as age and sex related, and then distributes this across agents according to age and sex. For instance, government age-related consumption was 62 percent of total government consumption in 2004. The corresponding government consumption profile for females (per individual) in 2004 is depict in figure 7²⁶. To find the 2006 age-related part, I use the ratio from 2004, i.e.

²⁴The relevant GDP measure for my purpose is mainland GDP. In 2006 it was 1580 B, see http://www.ssb.no/english/subjects/09/01/nr_en/tabc-09.html This measure ignores petroleum activity. For financial wealth see table 07018 (National account and external trade) in Statbank Norway (www.ssb.no)

²⁵The non-oil deficit in the national budget includes interest and dividends on non-petroleum financial wealth. Since I calibrate public finances to be consistent with the fiscal rule in 2006, those payments will be included in the residual X . It would be inconsistent to also include the assets in the definition of financial wealth A^g , since the return on those assets then would be counted twice. Hence, by convention, these financial assets are treated in the same manner as the oil resource, namely as a cash flow

²⁶The data is provided by Pål Knudsen and Viebeke Oestreich Nielsen, section for public finances, Statistics Norway.

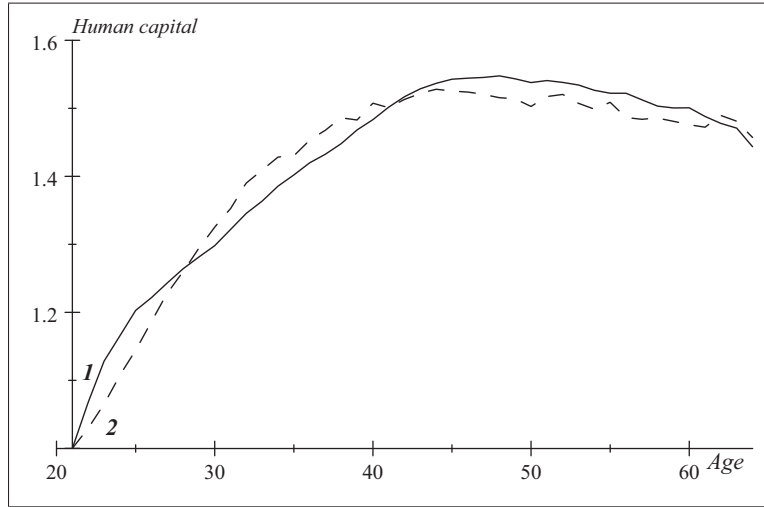


Figure 8: Human capital profiles, age 21 to 64. 1: Labor income. 2: Hourly wage. Age 21 normalized to 1

age-related consumption is 62 percent of total consumption. Total disability pensions (see Statistics Norway, 2009) was 3.2 percent of output. I distribute this across age according to the 2004 age-profiles. The Pension Fund at the start of 2006 was 88 percent of output²⁷. When it comes to the path of petroleum cash flow I employ the Ministry of Finance baseline projection in St.meld No. 9, 2009 (Ministry of Finance, 2009). It is based on a constant 2009 oil price of 400 NOK per barrel. The cash flow was 22.3 percent of GDP in 2006.

Human capital profile

The profile, h , is estimated using a 2005 Norwegian cross section data set. The estimated equation in vector form is

$$\log(y_i) = \text{constant} + \alpha \text{sex}_i + \beta \text{age}_i + \gamma \text{edu}_i \quad (29)$$

Labor income, y_i , in 2005 is regressed on a set of sex, age and education dummies. The observations are conditioned on the individual being employed full time in 2005 and in age group 21 to 64.²⁸ When calculating the age profile I extrapolate the growth rates linearly back to age 19. In addition, using hourly wage data I can calculate the growth rate in hourly wage for the same age group (males only).²⁹ The two age profiles are depicted in figure 8. The close match is reassuring. I continue with the labor income estimated profile, i.e. equation 29. Choosing the other lead only to small changes in life cycle profiles for consumption and leisure. To account for secular growth in wages over the life cycle, I multiply the estimated profile with the (constant) productivity parameter. Finally, I set the productivity growth rate equal to 0.02 when the individual reaches

²⁷Value of pension fund was 1390 B, see table 3.2 Ministry of Finance (2007)

²⁸The estimates are provided by Torbjørn Hægeland, and use Norwegian register data for all full-time, full-year employees in Norway in 2005. The earnings measure is annual labour earnings. See e.g. Hægeland and Kirkebøen (2007) for more details on the data.

²⁹2005 cross section for hourly wage and weekly working hours. Data set is provided by Manudeep Bhuller, Statistics Norway.

age 65. I thereby "force" agents to retire at age 65. If this was not done, my model would stimulate more old age labor supply than what is reasonable.³⁰ Moreover, I assume that the profile remains constant over time.

Preferences

The model has four preference parameters ($\gamma, \rho, \theta, \alpha$). There are basically two approaches for choosing values for these parameters. The first is to look at empirical estimates. The second is to choose values such that the model is consistent with some observed features of the economy. In the literature, starting with Auerbach and Kotlikoff (1987), a combination of the two approaches is often used. Altig et.al. (2001), Kotlikoff et.al. (2001) and Fehr et.al. (2005) set the inter- and intratemporal elasticities (γ and ρ) based on available empirical estimates, and then choose time and leisure preference parameters (θ and α) to create realistic lifecycle behavior and macroeconomic outcomes. Fehr (1999), uses ρ and α to create realistic outcomes, and sets γ and θ based on empirical estimates. I set the value for γ equal to 0.35, which is within the range of commonly used values in the AK-model literature (see Fehr and Habermann, 2008). The value for ρ is set equal to 0.5 to be consistent with the fact that labor supply is fairly unchanged during prime working age.³¹ The value for time preferences is -0.051 . It is chosen such that, on the initial balanced growth path, the ratio of per household consumption in age group 40 – 49 to household consumption in age group 30 – 39 is about 1.15³². Finally, the leisure parameter is set equal to 2.2, such that, on average, a household allocates about 40 percent of available time to labor income generating activity during the 46 working years.

other parameters

The fraction of labor income received as pension benefit is set equal to 0.56. The bequest requirement at age 91 is set equal to 0.1 for an individual reaching adulthood in 1979. This is a relatively small number, reflecting the fact that individuals' asset positions is positive throughout retirement period, hence a small bequest motive is needed in order to avoid negative wealth.³³ The exogenous interest rate and time-augmenting technology growth rate is 4 and 1.7 percent respectively. Annual capital depreciation rate is 5 percent, while capital's share of output is 1/3. The implied capital-output ratio is 3.3.

³⁰Both secular growth adjustment and old age productivity drop is included in e.g. Kotlikoff et.al. (2001).

³¹The abovementioned data set on 2005 weekly working hours, indicate that labor supply remains constant between age 38 and 48

³²Using age-consumption data from 1997 to 2007, I find a ratio of average household consumption in the two age groups equal to 1.1. The data is found in Statistic Norway's StatBank, subject 05, table 04970.

³³Households do in fact have a small negative asset position in their first years as adults. However, their mortality rates are essentially zero, so at the aggregate level they leave an insignificant negative bequest.

Year	Tax rate τ^c	Current account ^a CA/Y	Pension Fund ^a A^g/Y	Consumption ^b (C)	Labor supply ^b (N)	private wealth ^b (A^P)	Pension Fund ^b (A^g)	Old-age pension ^b (B)	Gov. cons. (G)	Gov. exp. (X)
2007	0.22	0.17	1.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2017	0.08	0.13	2.59	0.30	0.21	0.30	1.75	0.33	0.23	0.20
2027	0.05	0.10	3.19	0.23	0.18	0.40	0.45	0.30	0.21	0.19
2037	0.08	0.09	3.40	0.18	0.15	0.35	0.22	0.30	0.22	0.18
2047	0.12	0.03	3.40	0.17	0.15	0.25	0.15	0.24	0.22	0.17
2057	0.13	0.04	3.14	0.17	0.19	0.25	0.10	0.16	0.19	0.16
2067	0.17	0.02	2.80	0.15	0.16	0.24	0.03	0.21	0.16	0.16
2077	0.25	-0.01	2.42	0.15	0.15	0.20	0.00	0.24	0.21	0.17
2087	0.28	-0.01	2.05	0.15	0.18	0.14	0.00	0.16	0.18	0.16
2097	0.31	0.00	1.77	0.13	0.16	0.15	0.00	0.18	0.15	0.15
<i>longrun</i>	0.55	-0.01	0.00	0.15	0.15	0.15	0.00	0.15	0.15	0.15

Table 9: Key macroeconomic variables. Benchmark scenario - endogenous consumption tax. a: share of GDP. b: Ten year growth rates

Year	Tax Rate τ^c	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.27	0.20	1.11	0.96	1.02	1.00	1.00	1.00
2017	0.17	0.18	2.89	0.95	1.01	1.00	1.13	1.00
2027	0.14	0.15	4.00	0.95	1.01	0.96	1.26	1.01
2037	0.15	0.15	4.82	0.97	1.00	0.92	1.42	1.01
2047	0.14	0.11	5.43	0.99	1.00	0.89	1.59	1.01
2057	0.12	0.12	5.66	1.01	0.99	0.88	1.78	1.01
2077	0.17	0.08	5.96	1.04	0.98	0.87	2.40	1.00
2097	0.14	0.08	5.91	1.09	0.96	0.88	3.22	0.98
2147	0.15	0.08	5.93	1.16	0.94	0.91	6.62	0.95
<i>longrun</i>	0.15	0.08	5.92	1.24	0.92	0.94	∞	0.92

Table 10: Macroeconomic effect of growth-adjusted rule - endogenous consumption tax. a: Value relative to benchmark scenario (1b)

Year	Tax Rate τ^c	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	-0.25	0.00	1.14	1.32	1.00	1.00	1.00	1.00
2017	-0.09	0.04	0.94	1.11	1.00	1.34	0.36	0.99
2027	0.07	0.03	0.80	1.00	1.00	1.37	0.25	0.99
2037	0.18	0.03	0.69	0.97	1.01	1.29	0.20	0.99
2047	0.27	-0.02	0.59	0.94	1.02	1.23	0.18	1.00
2057	0.30	0.01	0.49	0.92	1.03	1.17	0.16	1.00
2077	0.46	-0.02	0.37	0.90	1.03	1.03	0.16	1.03
2097	0.49	-0.01	0.27	0.91	1.03	0.98	0.16	1.03
2147	0.53	-0.01	0.13	0.95	1.02	0.97	0.16	1.03
<i>longrun</i>	0.55	-0.01	0.00	1.00	1.00	1.00	0.16	1.00

Table 11: Macroeconomic effect of no rule - endogenous consumption tax. a: Value relative to benchmark scenario (1b)

Year	Tax Rate τ^c	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.16	0.19	1.11	1.01	1.03	1.00	1.00	1.00
2017	0.16	0.17	2.66	0.96	1.01	1.07	1.04	1.00
2027	0.16	0.15	3.74	0.94	1.01	1.01	1.18	1.01
2037	0.16	0.15	4.56	0.96	1.00	0.95	1.35	1.01
2047	0.16	0.10	5.12	0.98	1.00	0.93	1.50	1.01
2057	0.16	0.12	5.44	0.99	0.99	0.90	1.72	1.01
2077	0.16	0.08	5.89	1.04	0.98	0.86	2.38	1.00
2097	0.16	0.08	5.88	1.08	0.97	0.87	3.21	0.99
2147	0.16	0.08	5.89	1.16	0.94	0.91	6.58	0.95
<i>longrun</i>	0.16	0.08	5.90	1.24	0.92	0.95	∞	0.92

Table 12: Macroeconomic effect of constant tax rates - endogenous consumption tax. a: Value relative to benchmark scenario (1b)

Year	Tax Rate τ^c	Current account CA/Y	Pension Fund A^g/Y	Consumption ^a	Labor supply ^a	Private wealth ^a	Pension Fund ^a	Old-age Benefit ^a
2007	0.22	0.17	1.14	1.00	1.00	1.00	1.00	1.00
2017	0.08	0.13	2.58	1.00	1.00	1.00	1.00	1.00
2027	0.04	0.10	3.18	1.00	1.00	1.01	1.00	1.00
2037	0.08	0.09	3.38	1.00	1.00	1.02	1.00	1.00
2047	0.11	0.04	3.38	1.00	1.01	1.04	1.00	1.00
2057	0.12	0.05	3.13	1.00	1.01	1.06	1.00	1.00
2077	0.31	0.03	3.11	0.98	1.00	0.98	1.28	1.01
2097	0.30	0.04	3.06	1.00	1.00	0.93	1.72	1.00
2147	0.32	0.03	3.06	1.06	0.98	0.93	3.54	0.99
<i>longrun</i>	0.32	0.04	3.06	1.13	0.96	0.97	∞	0.96

Table 13: Macroeconomic effect of wealth targeting- endogenous consumption tax. a: Value relative to benchmark scenario (1b)