1. What is seasonal adjustment?

Economic time series are often affected by events which recur each year at roughly the same time. These time series are said to be influenced by “seasonal effects”. For example, major household purchases undertaken prior to the Christmas holiday result in a seasonal effect whereby retail sales increase considerably from October to November and also from November to December. Similarly, the existence of widespread holidays in July contributes to a drop in production from June to July. The magnitude of seasonal fluctuations often complicates the interpretation and analysis of many statistics.

Time series are subjected to a process of seasonal adjustment in order to remove the effects of these seasonal fluctuations. Furthermore, any effects due to the number of trading/working days in a period (month or quarter) from one year to another are eliminated. Norway’s independence day (the 17th of May) is one such example: the amount of production in May can in certain industries vary depending on whether this holiday falls on a Sunday or a Monday. Similarly, Easter takes place in March and/or April and can thus affect both monthly and quarterly time series. Once data have been adjusted for seasonal effects, a clearer picture of the time series emerges. Statistics Norway employs mainly X-12-ARIMA when adjusting for seasonal effects.
Figure 1 above presents the retail sales volume index in the period from January 1979 to June 2008. The graph shows clear seasonal fluctuations that recur from year to year, as sales peak in December and reach their lowest point in February. The pattern illustrates that this time series is strongly influenced by seasonal effects and should therefore be subjected to seasonal adjustment.
2. Pre-treatment

2.1 Raw data
A time series is a series of observations or estimations (with the same statistical properties) carried out in a regular sequence, for instance monthly or quarterly, over a certain period of time. These time series are also referred to as raw data, non-adjusted series or original series. Figure 1 above shows a time series containing raw data from the retail sales volume index. This index is affected by strong fluctuations, and is a good example of a time series that is difficult to interpret in the absence of seasonal adjustment.

2.2 Objectives of pre-treatment
Raw data should, prior to the actual seasonal adjustment procedure, be subjected to pre-treatment, that is, an adjustment for variations caused by calendar effects and outliers. For time series that are influenced by these effects, the quality of the subsequent seasonal adjustment may deteriorate if pre-treatment is not performed.

2.3 Calendar adjustment
Calendar adjustment involves adjusting for the effects of working days/trading days and for moving holidays.

2.3.1 Working/trading day adjustment
Both the number of working days/trading days and their composition can vary from one month to another, and can have a big impact on monthly figures in many economic series. To illustrate, if two subsequent months consist of 20 and 21 working days respectively, and we fail to take into account the extra day, we might draw the unwarranted conclusion that productivity increased in the second month, when in fact productivity may not have changed or may even have decreased.

The composition of working days can also affect the data in ways that blur the actual tendency. Shopping intensity, for instance, is not distributed equally over the course of the week: we shop for clothes more on Thursdays and Saturdays. Turnover will therefore be higher for a clothing store in a month containing five Saturdays than in a month of only four Saturdays, even though the intensity of shopping may not actually have increased. The correction for the number and composition of working days is called working/trading day adjustment.

2.3.2 Correction for moving holidays
As is the case for working days, moving holidays like Easter can also affect economic activity, often both before and after the actual holiday itself. Easter can for instance fall in March one year but in April the following year, or partly in both. Since stores close and people travel during Easter, shopping will tend to decrease. These factors may consequently result in an increase in shopping in weeks prior to or following Easter, which furthermore can give rise to monthly variations that may not reflect a true growth or fall in economic activity, but instead indicate a shift.
2.3.3 National calendar
The use of national calendars is recommended. In other words, the calendar chosen for calendar adjustment should reflect the working days, public holidays and so forth specific to Norway.

2.4 Outlier treatment
The presence of extreme values, or outliers, should also be detected and corrected for during pre-treatment. These outliers are abnormal values of the series, and include additive outliers, level shifts and transitory changes.

An additive outlier is an extreme value that occurs in one period (i.e., a month or quarter) and then disappears in the following period. The effect of a strike, for instance, can result in this type of outlier.

Level shifts refer to events which result in permanent effects on the level of the series. A considerable increase in production or production capacity following a technological innovation, for example, can generate a level shift.

Transitory changes are events which cause a considerable change in one period, with gradually decreasing effects.

2.5 Model selection
The process of pre-treatment and seasonal adjustment requires a selection of a regARIMA model, that is, a regression model where noise or the residual is modelled by an ARIMA model. This type of model can be expressed mathematically as:

\[ O_t = \sum \beta_i x_{it} + \epsilon_t, \]

where

- \( O_t \) is the original data in period \( t \)
- \( \beta_i \) is the effect of working days, moving holidays or outliers etc., 'i' is the index for the parameters to be modelled, and \( x \) is a dummy variable which indicates whether the effect is present or not in period \( t \).
- \( \epsilon_t \) is the residual modelled by an ARIMA model. For a description of ARIMA models, see Dinh Quang Pham: Innføring i tidsserier - sesongjustering og X-12-ARIMA, Notater 2001/2, Statistisk sentralbyrå

Seasonal adjustment requires a projection of the time series, in order to achieve a moving average. Models can be selected automatically by X-12-ARIMA, manually, or based on a set of pre-defined models. Another decision that needs to be made is whether data should be log-transformed or not.

When adjusting for seasonal effects it is possible to select the period in which the estimation of the model and of correction factors is to occur. Correction factors refer to factors used to perform pre-treatment and seasonal adjustment.
3. Seasonal adjustment

Once pre-treatment has been carried out, the process of seasonal adjustment can begin. Statistics Norway uses the seasonal adjustment software X-12-ARIMA for this purpose. The time series is split into three components: seasonal, irregular (random variation) and trend.

3.1 Seasonal component, trend component and irregular component:

Seasonal component

The seasonal component refers to the variation in the time series that occurs within one year. These movements are more or less consistent from year to year in terms of placement in time, direction and magnitude. Seasonal fluctuations can occur for many reasons, including natural conditions like the weather, administrative factors like the beginning and end of school holidays as well as various social/cultural/religious traditions (for example Christmas holidays). The term seasonal effect is often used to describe the effects of these factors. Some time series are strongly influenced by these effects, while other series are not affected at all.

The X-12-ARIMA program estimates a seasonal factor (that is, an estimate of the strength of the seasonal effect) for each month or period. The time series of these seasonal factors make up the seasonal component (see figure 2 below).

Figure 2. Retail sales volume index. Seasonal component.

Seasonally adjusted figures are a series of data where seasonal and calendar effects have been removed. Outliers are ordinarily included in these figures.
Figure 3. Retail sales volume index. Seasonally adjusted figures

**Trend (trend component):**
The trend represents the long-term tendency in the data. The trend is a better source of information than the seasonally adjusted series in cases where the raw data are characterised by a large degree of random variation.

Figure 4. Retail sales volume index. Trend

**Irregular component (random variation):**
The irregular component is the part of the observed values not included in the trend or seasonal components (or dealt with during pre-treatment). The irregular component is non-systematic and cannot be predicted. It is often referred to as unexplained variation.
3.2 Decomposition scheme

It is often assumed that a time series can be written either as the sum or product of three components, in one of the following ways:

\[ O_t = S_t + T_t + I_t \] (additive scheme)
\[ O_t = S_t \times T_t \times I_t \] (multiplicative scheme)
\[ \log(O_t) = \log(S_t) + \log(T_t) + \log(I_t) \] (log-additive scheme)
\[ O_t = T_t(S_t + I_t) \] (pseudo-additive scheme)

Where
- \( O_t \) – raw data or pre-adjusted data
- \( S_t \) – seasonal component
- \( T_t \) – trend component
- \( I_t \) – irregular component

A seasonally adjusted time series is \( A_t = T_t + I_t \)

The selection of additive, multiplicative, log-additive or pseudo-additive decomposition schemes is determined first, followed by an estimation of the three components through an iterative process.

3.3 Seasonal adjustment of aggregated series

There are two choices when seasonally adjusting a time series consisting of several aggregates. A direct approach is employed if the time series for the total and the time series for the corresponding sub-aggregates are separately seasonally adjusted.

An indirect approach is used for the total if the time series for the corresponding sub-aggregates are first directly seasonally adjusted and then the total is made by aggregation.

3.4 Time horizon

When performing seasonal adjustment it is possible to select the period in which the estimation of the correction factors is to be used. Ordinarily, the entire time series is used, but the period can in some cases be shorter. Seasonal adjustment should only be considered if the monthly time series consists of at least 7-8 years.
3.5 Revisions

Both seasonally adjusted figures and the trend can often undergo revisions in later publications. The addition of a new observation in a raw series can affect previously adjusted figures. In some cases, this type of revision results in a change in the rate of growth for the previous month and possibly in the overall tendency of the time series. The way revisions are handled varies from statistic to statistic.

4. Quality of seasonally adjusted data

Seasonal adjustment is a complicated data procedure and requires accurate monitoring before the results can be accepted and disseminated. In order to ensure that seasonally adjusted data are of good quality they should be validated by a wide range of quality measures. For instance, the absence of seasonal and/or calendar effects in the irregular component should be assessed. Another important property in the use and interpretation of seasonally adjusted figures is the stability of seasonal factors. X-12-ARIMA produces results from many different tests and can provide insights when attempting to evaluate quality. Individually and collectively, these tests can describe the qualitative properties of a series.

This type of evaluation can also be guided by graphical depictions of the properties of a series, for instance in the form of spectrum analysis. These graphical depictions can contribute to an extended understanding of the properties of the series. They are primarily of value to people who perform seasonal adjustment as part of their work and are hence not published.

4.1 Quality measures

This paragraph presents a list of important measures which collectively describe the quality of seasonally adjusted series (see below). These quality measures can be published along with seasonally adjusted figures. A description of the interpretation, estimation and boundary values for each measure follows the list.

- Period of quality estimations (period)
- Multiplicative or additive decomposition (Method)
- Choice of ARIMA model (standard or alternative model)
- The relative contribution of the irregular component to the variance of the stationary portion of the series (M2)
- The amount of stable seasonality present relative to the amount of moving seasonality (M7)
- The size of seasonal component fluctuations in recent years (M10)
- The size of linear movement in the seasonal component in recent years (M11)
- A collective measure of quality in X-12-ARIMA (Q)
- Sliding span tests (S(%) and MM(%) )
- Average absolute revisions in seasonally adjusted series (ASA)
- Average absolute revisions in monthly (or quarterly) changes in seasonally adjusted series (ACH)
- Stability in trend and seasonally adjusted series (STAR)
- Analysis of variance (Anova)
- Trading day effects (TD)
- Easter – moving holidays/holidays in March and April (Easter effects)

Period
The quality measures are often updated yearly. The measures may therefore refer to a different period than the published data.

**Method**
Method specifies whether adjustment is direct or indirect. When estimating the seasonal component of directly adjusted time series, there are four different decomposition schemes: Multiplicative (MULT), Additive (ADD), Log-additive (LADD) and Pseudo-additive (PADD) decomposition.

**Choice of ARIMA model**
The model is used to forecast the series can either be selected automatically by X-12-ARIMA or manually. X-12-ARIMA tests the degree to which different models are suitable for a specific series. If all models are rejected, then a model is selected manually (that is, a default solution).

**4 important quality measures from X-12-ARIMA and the collective measure Q:**
The four quality measures selected from the 11 M-measures are M2, M7, M10 and M11, and are produced automatically by X-12-ARIMA. Q is a weighted average of all 11 M-measures.

**The relative contribution of the irregular component to the variance of the stationary portion of the series (M2)**
M2 measures whether the amount of random variation in the data is small enough for an estimation of trend and seasonal components. The values can vary from 0.0 to 3.0, and should ideally be close to zero.

**The amount of stable seasonality present relative to the amount of moving seasonality (M7)**
The formula for M7 is as follows:

\[
M7 = \frac{1}{2} \left( \frac{7}{F_S} + \frac{3F_M}{F_S} \right)
\]

F_S refers to the relative contribution of the stable part of the season, while F_M refers to the contribution of the moving part of the season.

The M7 value is a common quantity in the evaluation of the setup and routines in use. The value of M7 for series with a stable seasonal pattern is usually well below 1 – and the closer this value is to 0, the more stable the seasonal pattern becomes. The formula for estimating this quantity is rather complicated and difficult to interpret intuitively. The M7 test is often more robust than the total quality measure (Q, see below) as well as the other M tests produced by X-12-ARIMA.

**The size of seasonal component fluctuations in recent years (M10)**
M10 measures the amount of fluctuation in the seasonal component in recent years. Fluctuation is too large, and seasonal adjustment no longer stable, if M10 >1.

**The size of linear movement in the seasonal component in recent years (M11)**
M11 measures the degree of linear movement in the seasonal component in recent years. Fluctuation is not random if M11 >1.

**Q value (collective measure of quality in X-12-ARIMA – Q)**
The Q value is a weighted average of the eleven M tests in X-12-ARIMA. The weights reflect the importance assigned to the various tests by the developers of X-12-ARIMA.
The closer $Q$ is to 0, the higher the quality of seasonal adjustment becomes. The M-measures should be reassessed when $Q$ is greater than 1, in order to determine whether the variation in the irregular or the seasonal component is too large.

**Sliding spans (percentage observations/growth rates revised beyond a given cut-off)**

This quality measure compares the results of seasonally adjusted data estimated for overlapping segments of a series. 3-4 overlapping segments are common. The test results show the percentage of observations undergoing revisions beyond a given cut-off value (usually 3 percent). Test results are estimated both for $S(\%)$ – percentage of months (quarters) where seasonal adjustment is defined as unstable, and for $MM(\%)$ – percentage of months where the growth rate is unstable. A rule of thumb suggests that the result for $S(\%)$ should be lower than 15 per cent and for $MM(\%)$ below 40 per cent. The closer these test results are to zero, the more stable the seasonally adjusted data become. For more information, see chapter 4.2.10 in Dinh Quang Pham: Innføring i tidsserier - sesongjustering og X-12-ARIMA, Notater 2001/2, Statistisk sentralbyrå.

**Average absolute revisions in seasonally adjusted series – ASA**

ASA measures average revisions of the seasonally adjusted series based on empirical simulations. The measure gives a number in per cent and disregards the sign of revisions.

$$ASA = \frac{1}{N} \sum_{t=1}^{N} R_t$$

$$R_t = \frac{A_{gt} - A_{gt}}{A_{gt}}$$

For a given series $y_t$ where $t=1, \ldots, T$, $A_{gt}$ is defined as a seasonally adjusted value of $y_t$ estimated from the series $y_1, y_2, \ldots, y_n$, where $t \leq n \leq T$. The seasonally adjusted value for observation $t$ is $A_{gt}$, and the final seasonally adjusted value for the same observation is $A_{gT}$. ASA is estimated only for those series whose component is multiplicative.

ASA is roughly equal to STAR/2 when time series are of adequate length. ASA can also be used to estimate a “rough” confidence interval for seasonally adjusted value.

**Average absolute revisions in monthly (quarterly) changes in seasonally adjusted series – ACH**

ACH gives the average revisions in monthly (quarterly) growth rates for seasonally adjusted data based on empirical simulations. The ACH value is expressed as a percentage and disregards the sign of revisions.

$$ACH = \frac{1}{N} \sum_{t=1}^{N} R_t$$

$$R_t = \frac{c_{gt} - c_{gt}}{c_{gt}}$$

For a given series, $y_t$ where $t=1, \ldots, T$, $C_{gt}$ is defined as the growth rate of seasonally adjusted data for $y_t$ estimated from the series $y_1, y_2, \ldots, y_n$, where $t \leq n \leq T$. The growth rate of seasonally adjusted data for observation $t$ is $C_{gt}$, and the final growth rate of seasonally adjusted data for the same observation is $C_{gT}$. ACH is estimated only for series whose component is multiplicative.

ACH can also be used as a reference to estimate a “rough” confidence interval for growth rates of seasonally adjusted data.

**Analysis of variance (ANOVA)**

This test shows the portion of the changes in seasonally adjusted data that can be accounted for by the trend.
where $T_t =$ trend value in period $t$ and $A_t =$ seasonally adjusted value in period $t$

A test value between 0 and 1 is common for a series. The test value will ordinarily be considerably higher for monthly than for quarterly series. The value can be interpreted as a percentage. An ANOVA value close to 1 means that there are only minor differences between trend and the seasonally adjusted series, or, stated differently, that the seasonally adjusted series is stable in that it is hardly affected by the irregular component. Trend and the seasonally adjusted series are identical when ANOVA is equal to 1. An ANOVA value close to 0 indicates that the seasonally adjusted series in large part is characterised by the irregular component.

Note also that ANOVA is based on the relationship between two variances. Therefore, the ANOVA value in itself does not speak to the development of the two series involved. It can still be a good indicator if the purpose of an analysis is to inspect a group of series in a quality table. Another function of ANOVA is as a measure of expected revisions as new observations are added to a series.

This quality measure is only of interest if it is part of a larger table with multiple series, all of the same frequency (monthly, quarterly and so on).

**Stability in trend and seasonally adjusted series – STAR**

This test gives a measure of average absolute change in the irregular component, that is a quantity of how "large" the irregular component is in a series. STAR is only estimated for time series with a multiplicative decomposition scheme.

$$\text{STAR} = \frac{1}{N^2 - 1} \sum_{t=2}^{n} \left| I_t - I_{t-1} \right|$$

$I_t =$ data for the irregular component in period $t$, and $N =$ number of observations.

STAR can be a useful measure of the amount of revisions for the most recent seasonally adjusted figure when a new observation is added. According to Eurostat's manual, the amount of revision is ordinarily about half of the estimated STAR value, in other words STAR/2.

A STAR value close to 0 indicates that the irregular component is small (that is, less noise in the series). A rule of thumb is that STAR should be smaller than 2 for monthly series and smaller than 1 for quarterly series.

**Trading day effects (TD)**

This measure reveals whether the series has been corrected for trading day effects. Trading day effects are rarely significant for quarterly series. Note that this test takes into account the number of different weekdays in a month (quarter) but not other holidays. The TD routine is primarily carried out to facilitate the process of estimating and interpreting twelve-monthly growth rates.

**Easter – moving holidays/holidays in March and April**
This test aims to explain whether series in March/April are affected by Easter. The Easter holiday has a major impact on many statistics published by Statistics Norway. The process of identifying just how strong this impact is can furthermore be fairly complicated. Seasonally adjusted figures for March (first quarter) can undergo substantial revisions in series with significant Easter effects when data for April (second quarter) are released.

5. Publications and other links to seasonal adjustment:

Documents from Statistics Norway:
Dinh Quang Pham: Ny metode for påskekorrigerings for norske data, Notater 2007/43, Statistisk sentralbyrå.
Ole Klungsøyr: Sesongjustering av tidsserier. Spektralanalyse og filtrering, Notat 2001/54, Statistisk sentralbyrå
Dinh Quang Pham: Innføring i tidsserier - sesongjustering og X-12-ARIMA, Notater 2001/2, Statistisk sentralbyrå

International manuals and recommendations:
ESS Guidelines on seasonal adjustments

Links to websites with information about seasonal adjustment:
USA:
http://www.census.gov/srd/www/x12a/

Australia:
http://www.abs.gov.au/Websitedbs/d3310114.nsf/4a256353001af3ed4b2562bb00121564/70611eabf8a97acca256ce10018a0d3fOpenDocument

New Zealand:

"Newsletter" from Singapore:

OECD definitions:
www.oecd.org/dataoecd/25/56/33722473.ppt