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A pooled GMM panel estimator

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Abstract:
Simultaneity represents a fundamental problem when estimating the elasticity of substitution between capital and labour. To overcome this problem, a wide variety of external instruments has been applied in the literature. However, the use of instruments may lead to wrong inference if they are either weak, or endogenous to the system being estimated. In this paper, we extend the widely used Feenstra (1994) estimator, which does not depend on external instruments, to make it applicable to the problem of estimating the elasticity of substitution between capital and labour. We propose a pooled GMM (P-GMM) estimator, examine its properties in a Monte Carlo study and apply it to a Norwegian sample of manufacturing firms. We identify the conditions under which P-GMM yields unbiased estimates and compare it to a fixed effects estimator which is unbiased when factor prices are exogenous – a typical assumption in the literature. We find that the fixed effects estimator is heavily downward biased in the presence of simultaneity. In contrast, the P-GMM estimator is nearly unbiased provided the number of time periods (T) is not too small (say, more than 10). In our application, with an unbalanced sample and T = 12, we estimate the elasticity of substitution to be 1.8 using P-GMM and 1.0 using a fixed effects estimator. Hence, neglecting simultaneity may lead to the conclusion that capital and labour are complements when, in fact, they are substitutes.

Keywords: Elasticity of Substitution, Simultaneity, Factor Demand, Non-Linear GMM, Pooled Estimator

JEL classification: C13, C15, C33, C51

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Sammendrag

De fleste virksomheter bruker både kapital og arbeidskraft for å produsere varer og tjenester. Endringer i prisen på kapital og/eller arbeidskraft vil kunne påvirke bruken av disse innsatsfaktorene. Til hvilken grad virksomheter kan substituere bruken av arbeidskraft med kapital måles ved substitusjonselastisiteten. Størrelsen på substitusjonselastisiteten har stor betydning for økonomiens virkemåte, herunder effektiviteten av selskapsbeskatning, økonomisk vekst, den funksjonelle inntektsfordelingen og næringssammensetningen. En god forståelse av økonomiens virkemåte fordrer derfor at man har et godt mål på substitusjonselastisiteten mellom arbeidskraft og kapital.

Simultanitet representerer et grunnleggende problem ved estimering av substitusjonselastisiteten mellom arbeidskraft og kapital. For å håndtere dette problemet har man i litteraturen brukt eksogene instrumentvariabler. Imidlertid kan bruken av svake instrumenter eller instrumenter som viser seg å være endogene i systemet medføre inferensfeil. I denne artikkelen videreutvikler vi estimatoren fra Feenstra (1994), som ikke avhenger av eksogene instrumenter, og anvender den nye estimatoren til estimering av substitusjonselastisiteten mellom arbeidskraft og kapital. Vi utleder en «pooled» GMM estimator (P-GMM), hvor vi først presenterer egenskapene til estimatoren ved hjelp av en Monte Carlo-analyse, og deretter bruker den for å estimere substitusjonselastisiteten for et utvalg av norske industriforetak. Videre sammenligner vi estimatoren med en fast effekt-estimator som er forventningsrett når faktorprisene er eksogene – en vanlig antakelse i litteraturen. Vi finner at fast effekt-estimatoren har systematisk forventningsskjevhet mot null ved simultanitet. P-GMM-estimatoren er derimot tilnærmet forventningsrett om antall tidsperioder (T) ikke er for liten (om lag 10 perioder). Ved estimering av substitusjonselastisiteten for utvalget av norske industriforetak, hvor $T = 12$, finner vi en elastisitet lik 1,8 ved bruk av P-GMM-estimatoren, mens fast effekt-estimatoren gir en elastisitet lik 1,0.

1 Introduction

An essential concept in economics is the elasticity of substitution between capital and labour, henceforth denoted σ. It represents how easily firms can substitute capital for labour and expresses the percentage change in the capital-labour ratio due to a one percent change in the price ratio of these inputs. The magnitude of σ is important for a broad range of issues, inter alia the effectiveness of corporate taxation (Chirinko, 2002), economic growth (de la Grandville and Solow, 2010), the direction of technical change (Acemoglu, 2002), the degree of sectoral transformation (Alvarez-Cuadrado et al., 2017) and the distribution of income between capital and labour (Karabarbounis and Neiman, 2014; Piketty and Zucman, 2014).

In a meta-analysis of 77 articles published between 1961 and 2017, Knoblach et al. (2020) found that the majority of empirical evidence suggests that σ is below the Cobb-Douglas value of unity. However, a key problem when estimating the elasticity of substitution is the potential endogeneity of regressors. The first-order conditions of profit maximization or cost minimization can be interpreted as describing firms' aggregate demand for capital and labour for given factor prices. If factor prices are endogenous, estimating σ based on first-order conditions may lead to simultaneous equation bias. To overcome simultaneity, it is common to apply an instrumental variable (IV) estimator, see e.g. Stock (2001). Knoblach et al. (2020) found that 15 of 77 studies applied IV techniques, while the majority applies OLS or fixed effects estimators. The list of external instruments used includes variables such as the U.S. population, wages in the government sector and the real capital stock owned by the government. However, the accuracy of the final estimates depends on how valid these external instruments are. It is well known that the use of external instruments may lead to wrong inference if the instruments are either weak, or not exogenous to the system being estimated.

In this paper, we provide an estimator of σ that relies on (internal) structural assumptions rather than (external) instruments for identification. We build on the approach developed by Feenstra (1994), which is becoming increasingly popular within the field of international economics, see e.g. Imbs and Mejean (2015), Broda et al. (2017), Feenstra et al. (2018) and Arkolakis et al. (2018). It has also been used to study price indices, see Broda and Weinstein (2010), Blonigen and Soderbery (2010) and Feenstra and Romalis (2014). The method overcomes simultaneity by utilizing the panel structure of the data set in combination with orthogonality restrictions on the error terms. In contrast to finding an external variable serving as an instrument, the system is rewritten in a form where variety indicators or dummy variables for each cross-section can be used as instruments. To incorporate parameter restrictions, Broda and Weinstein (2006) extended the framework in Feenstra (1994) using a grid search of admissible values if the first estimator yields inadmissible estimates, e.g. elasticities of the wrong sign. Adding to this literature, Soderbery (2015) created a hybrid estimator combining limited information maximum likelihood (LIML) with a restricted non-linear LIML routine. Brasch and Raknerud (2021a) created a pooled estimator which was shown in a Monte Carlo study to reduce the root-mean-square deviation compared to the Soderbery (2015) estimator by between 60 and 90 percent.

There are two distinct differences between the Feenstra (1994) framework, or some of the above-mentioned versions of it, and the current paper. First, in Feenstra (1994) it was assumed that the elasticity of substitution between varieties must exceed one. In contrast, the elasticity of substitution between capital and labour must be positive, i.e. $\sigma > 0$. Second, the structural econometric framework of demand and supply in Feenstra (1994) consists of prices and quantities of one variable only. In contrast, the structural econometric framework when estimating σ relies on prices and quantities of two variables, capital and labour.

We modify the Feenstra (1994) framework and create a pooled non-linear GMM (P-GMM) panel estimator for a system of equations with multiple quantity and price variables that handles the restriction $\sigma > 0$. While the Feenstra (1994) estimator involved complex hyperbolic restrictions on parameters, P-GMM handles parameter restrictions by simple re-parametrizations. To illustrate the properties of P-GMM, we carry out a Monte Carlo study and compare it with the fixed effects estimator often applied in the literature. The synthetic data are generated based on constant elasticity of substitution (CES) production technology where we allow for upward sloping supply schedules for capital and labour. We show both numerically and analytically that the fixed effects estimator has a negative bias in the presence of simultaneity. In our benchmark calibration where $\sigma = 1.5$, the fixed effects estimator yields estimates of around 0.6 and P-GMM around 1.5 in moderate or large samples. Thus P-GMM is unbiased also when there is simultaneity. This finding is relatively robust towards the number of cross-sections and time periods (T) in the simulations, provided T is not too small. We also demonstrate that the bias of the fixed effects estimator disappears when the supply elasticities go towards infinity, making both the wage rate and the rental price of capital exogenous. The Monte Carlo analysis thus shows the importance of using an estimator that allows for the possibility that there is simultaneity in the determination of factor inputs and factor prices.

To further illustrate the properties of the P-GMM estimator, we apply it to the case of manufacturing production in Norway. We merge micro data from the income statement for all limited liability companies with the registry of all employed workers and data from the national accounts. In total, our sample covers data for capital, labour, wages and rental prices of capital for nearly 8,000 firms spanning the years 2007 to 2018. For these firms, P-GMM yields the point estimate 1.8, which is significantly higher than one, and also much higher than what has been found in most of the literature, including studies on Norwegian macrodata, see e.g. Hungnes (2011) and Mallick (2012). In contrast, the fixed effects estimator yields a point estimate of 1.0, which is in line with many of the studies in the literature that have applied similar estimators. Our findings illustrate that it is important to take into account simultaneity when estimating σ, and that not doing so may lead to the conclusion that capital and labour are complements or can be described by Cobb-Douglas technology, when they in fact are substitutes.

The rest of the paper proceeds as follows. In Section 2, we provide the structural economic framework. In Section 3, we outline the P-GMM estimator and illustrate its properties by means of Monte Carlo analysis. The P-GMM estimator is applied to the case of Norwegian manufacturing production in Section 4. Section 5 provides a conclusion.

2 Structural Economic Framework

In the following we outline the demand and supply for labour and capital. Since the purpose of our study is to estimate σ, we will focus on the relative factor demand and the relative factor supply of these inputs. On the demand side, our point of departure is a firm (i) producing output (Y) at time t using capital (K) and labour (L), measured by hours worked, with constant elasticity of substitution (CES) technology:

$$Y_{it} = A_{it}\left(\alpha_{Ki}^{\frac{1}{\sigma}}(A_{Kit}K_{it})^{\frac{\sigma-1}{\sigma}} + \alpha_{Li}^{\frac{1}{\sigma}}(A_{Lit}L_{it})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma\kappa}{\sigma-1}} \tag{1}$$

where A_{it} represents factor-neutral technical change, α_{Ki} and α_{Li} are firm-specific share parameters, κ is the elasticity of scale, A_{Kit} and A_{Lit} represent factor-biased technical change of capital and labour, respectively, and $\sigma \in (0,\infty)$ is the elasticity of substitution between capital and labour. When $\sigma \to \infty$, the two factors are perfect substitutes and the production function is linear. The production function is Leontief in the limit when $\sigma \to 0$ (no substitution between capital and labour) and Cobb-Douglas when $\sigma = 1$.

Let the wage rate and the rental price of capital be denoted by W and R, respectively. The factor demand for capital and labour may be written as:

$$\begin{aligned} l_{it} &= -\sigma w_{it} + \ln(\alpha_{Li}) + (\sigma-1)\ln(A_{Lit}) + \frac{1}{\kappa}(\ln Y_{it} - \ln A_{it}) + \sigma \ln(c_{it}) \\ k_{it} &= -\sigma r_{it} + \ln(\alpha_{Ki}) + (\sigma-1)\ln(A_{Kit}) + \frac{1}{\kappa}(\ln Y_{it} - \ln A_{it}) + \sigma \ln(c_{it}) \end{aligned} \tag{2}$$

where $l_{it} = \ln(L_{it})$, $k_{it} = \ln(K_{it})$, $w_{it} = \ln(W_{it})$, $r_{it} = \ln(R_{it})$ and

$$c_{it} = \left[\alpha_{Ki}\left(\frac{R_{it}}{A_{Kit}}\right)^{1-\sigma} + \alpha_{Li}\left(\frac{W_{it}}{A_{Lit}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

It follows that the *relative factor demand* equation for capital and labour in the case of cost minimisation can be written compactly as:

$$q_{it} = \sigma(r_{it} - w_{it}) + \varepsilon_{Lit} - \varepsilon_{Kit} \tag{3}$$

where $q_{it} = l_{it} - k_{it}$ and

$$\varepsilon_{Nit} = \ln(\alpha_{Ni}) + (\sigma-1)\ln(A_{Nit}) \text{ for } N = L, K \tag{4}$$

Note that the additive variable representing factor-neutral technical change, $\ln(A_{it})$, does not enter the relative factor demand for labour and capital since its coefficient is the same in both equations $(1/\kappa)$. The same holds for the log-level of output, $\ln(Y_{it})$, and the log-price index, $\ln(c_{it})$. It follows from Equation (3) that the percentage change in the capital-labour ratio due to a one percent change in the price ratio is given by the elasticity of substitution, σ. A key identifying assumption in relation to our proposed estimator below is that σ is assumed constant across firms.

A challenge when estimating σ is that the log-price difference, $w_{it} - r_{it}$, is not likely to be exogenous. The first reason is that a change in wage rates may influence labour supply at the firm level. As noted by

Manning (2003), finite labour supply elasticities at the firm level may be present because of frictions in the labour market. These frictions give rise to a monopsonistic labour market where firms possess wage setting power. If a firm chooses to lower wage rates, all workers will not instantaneously find another job and the labour supply elasticity is thus not infinite. A large and increasing literature has analysed the labour supply elasticity at the firm level. In a meta-study of 52 empirical studies, Sokolova and Sorensen (2021) found strong evidence for monopsonistic competition and finite labour supply elasticities. The variation across studies, comprising 1,320 estimates, is large with a mean of 7.1 and a median of 1.7. In line with this literature, we allow for an upward sloping labour supply function:

$$l_{it} = \gamma_L w_{it} + \phi_{Lt} + u_{Li} + \gamma_L \xi_{Lit}, \tag{5}$$

where $\gamma_L \in (0, \infty)$ is the labour supply elasticity, assumed equal across firms, and ϕ_{Lt} and u_{Li} represent fixed time- and firm-effects.[1] Moreover, ξ_{Lit} is a structural labour supply shock, for example due to a change in the workers' level of wealth, preferences, income, or other reasons that are independent of the demand shocks to the firm. The scaled error term, $\gamma_L \xi_{Lit}$, incorporates the limiting case where $\gamma_L \to \infty$, i.e. when w_{it} is exogenous with $w_{it} = -\xi_{Lit}$ (of course, ϕ_{Lt} and u_{Li} could be scaled similarly to retain the time- and firm effect in the limit).

The second reason why the log-price difference, $w_{it} - r_{it}$, may not necessarily be exogenous in Equation (3) is an upward sloping supply curve for capital. We consider an economy with at competitive rental market for capital where the supply of capital is given by:

$$k_{it} = \gamma_K r_{it} + \phi_{Kt} + u_{Ki} + \gamma_K \xi_{Kit}, \tag{6}$$

where $\gamma_K \in (0, \infty)$ is the capital supply elasticity, ϕ_{Kt} and u_{Ki} represent, respectively, firm- and time-effects and ξ_{Kit} is a structural capital supply shock, i.e. independent of the demand shock for firms. The scaled error term, $\gamma_K \xi_{Lit}$, incorporates the case where $\gamma_K \to \infty$, i.e. r_{it} is exogenous. Some studies point to an upward sloping supply of capital. Goolsbee (1998) found a short-run supply elasticity of around one based on U.S. data. However, as pointed out by Hassett and Hubbard (2002), while it is implausible that the supply function for most individual capital goods manufacturers is perfectly elastic, the effective supply might be highly elastic in the long run if the world market for capital goods is open (p. 1329). In a closed economy, however, where investments equal savings, the flip-side of the capital elasticity is the saving elasticity with respect to the interest rate. Several studies have estimated a saving elasticity ranging from values close to 0 up to 0.4, see Bernheim (2002).[2]

[1] Note that the labour supply function we employ also encompass a wide variety of labour supply models analysing the aggregate extensive margin, see e.g. Blundell and MaCurdy (1999). The estimated aggregate labour supply elasticities varies considerably in the literature, but is typically positive and less than one in most micro based studies, see e.g. Jäntti et al. (2015) and the meta-study by Evers et al. (2008). In macro-based studies, labour supply elasticities are generally found to be higher than one, see e.g. Chetty et al. (2012).

[2] Adding to this literature, Kasamatsu and Ogawa (2020) find in an infinitely repeated game of tax competition with endogenous capital supply that the capital supply elasticity is lower when countries are less integrated into the international capital market, and vice versa.

To identify σ it is important to allow for the possibility that the supply elasticity of capital and/or labour is positive. Seemingly, since both the labour-capital ratio, the price of labour and the price of capital enter Equation (3) and Equations (5)-(6), σ is not identified without further restrictions. However, by utilizing the panel structure of the data, we can rewrite this system in a manner where σ can be identified.

3 Estimation and Identification

In the presence of an upward sloping supply curve for labour and/or capital, both the OLS and fixed effects estimator of σ based on Equation (3) – which are the estimators of σ most often applied in the literature – will be biased. To handle the simultaneity problem in a demand and supply system, Leamer (1981) considered time series estimation in absence of instruments. Leamer (1981) showed that when the errors in the demand and supply equations are uncorrelated, the true demand elasticity (assumed > 1) and supply elasticities (> 0) lie on a hyperbola defined by the theoretical second moments of the log-price- and log-quantity variables. Feenstra (1994) utilized this insight to estimate the demand elasticity. The approach we outline in this paper modifies the case considered in Feenstra (1994) to the case of estimating σ and yields a non-linear pooled GMM (P-GMM) estimator that does not involve parameter restrictions in the form of hyperbolas. Before we outline P-GMM, we first examine the bias of the OLS estimator naively applied to Equation (3).

3.1 Bias of the OLS Estimator

The basis for OLS estimation is the following statistical specification of Equation (3):

$$q_{it} = -\sigma\left(w_{it} - r_{it}\right) + \alpha_i + \gamma_t + \varepsilon_{it} \tag{7}$$

where we have assumed that $\varepsilon_{Lit} - \varepsilon_{Kit}$ (see Equation (4)) has a two-way error structure:

$$\varepsilon_{Lit} - \varepsilon_{Kit} = \alpha_i + \gamma_t + \varepsilon_{it} \tag{8}$$

When deriving a formula for the bias of the OLS estimator, $\widehat{\sigma}_{OLS}$, we will assume that $w_{it}, r_{it}, \varepsilon_{Lit}$ and ε_{Kit} are weakly stationary variables, i.e., with constant (finite) mean and variance across firms and over time. In that case, dropping subscripts i and t for simplicity of notation, the asymptotic limit is:

$$\text{plim } \widehat{\sigma}_{OLS} = \sigma + \frac{\text{cov}(r-w, \varepsilon_L - \varepsilon_K)}{\text{var}(r-w)} \tag{9}$$

From Equation (2):

$$l = -\sigma w + \mu + \varepsilon_L$$
$$k = -\sigma r + \mu + \varepsilon_K$$

where

$$\mu_{it} = \frac{1}{\kappa}(\ln Y_{it} - \ln A_{it}) + \sigma \ln(c_{it})$$

In order to determine μ_{it}, additional assumptions are needed as Y_{it} is an endogenous variable. Appendix A gives a closed form expression for μ_{it} in the case where $\kappa = 1$ and Y_{it} is determined by profit maximization with monopolistic competition.

In our simulations, we will assume that all random terms are independent with $\text{var}(\varepsilon_N) = \lambda_N^2$ and $\text{var}(\xi_N) = \nu_N^2$ for $N = L, K$ and $\gamma_L = \gamma_K = \gamma$. Then we show in Appendix B that the asymptotic bias of $\widehat{\sigma}_{OLS}$ is:

$$\text{bias}(\widehat{\sigma}_{OLS}) = -\frac{(\sigma+\gamma)(\lambda_K^2+\lambda_L^2)}{\lambda_K^2+\lambda_L^2+\gamma^2(\nu_K^2+\nu_L^2)} < 0$$

That is, the bias is always negative but tends to zero as $\gamma \to \infty$, i.e. when both factor prices are exogenous. If α_i and γ_t in Equation (7) are *fixed effects* – instead of stationary random variables – this could bring in additional sources of bias related to either a finite number of time periods (T) or a finite number of firms (I) as TI tends to infinity. Although such fixed effects can be handled by applying a fixed effects estimator, this estimator will not remove the part of the bias that is due to the genuine random terms. We will address these issues below by means of simulation studies.

3.2 The Pooled GMM Panel Estimator (P-GMM)

As just discussed, the factor supply in Equations (5)-(6) and relative factor demand in Equation (7) may include fixed time and firm-effects. Similar to Feenstra (1994), we eliminate these effects by means of double differencing, i.e. differencing first with respect to time and then with respect to a fixed reference firm, m. Formally, the double difference operator is defined as:

$$\Delta^{(m)} z_{it} = \Delta z_{it} - \Delta z_{mt}$$

for any variable z_{it} and reference firm m. Applying this operator to Equation (3) and Equations (5)-(6), assuming that demand equals supply, we can reformulate the system of equations compactly as:

$$\begin{aligned} \text{Demand} \quad &: \quad \Delta^{(m)} q = -\sigma \Delta^{(m)}(w-r) + \Delta^{(m)} \varepsilon \\ \text{Supply} \quad &: \quad \Delta^{(m)} l = \gamma_L \Delta^{(m)} w + \gamma_L \Delta^{(m)} \xi_L \\ &\quad\;\; \Delta^{(m)} k = \gamma_K \Delta^{(m)} r + \gamma_K \Delta^{(m)} \xi_K \end{aligned} \tag{10}$$

where, again, we have dropped the subscripts i and t for simplicity of notation. Thus, for example, $\Delta^{(m)} q = \Delta^{(m)} q_{it} = \Delta^{(m)}(l_{it} - k_{it})$.

We show in Appendix C that from the expressions for $\gamma_L \Delta^{(m)} \varepsilon \Delta^{(m)} \xi_L$ and $\gamma_K \Delta^{(m)} \varepsilon \Delta^{(m)} \xi_K$, we obtain the following two-equation system:

$$Y_N = \theta_{N1} X_{N1} + \theta_{N2} X_{N2} + \theta_{N3} X_{N3} + U_N \text{ for } N = K, L \tag{11}$$

where

$$\begin{aligned}
Y_L &= \Delta^{(m)} w \Delta^{(m)} (w-r), Y_K = \Delta^{(m)} r \Delta^{(m)} (w-r), \\
X_{L1} &= -\Delta^{(m)} q \Delta^{(m)} w,\ X_{K1} = -\Delta^{(m)} q \Delta^{(m)} r, \\
X_{L2} &= \Delta^{(m)} l \Delta^{(m)} (w-r),\ X_{K2} = \Delta^{(m)} k \Delta^{(m)} (w-r), \\
X_{L3} &= \Delta^{(m)} q \Delta^{(m)} l,\ X_{K3} = \Delta^{(m)} q \Delta^{(m)} k
\end{aligned}$$

and

$$U_N = -\Delta^{(m)} \varepsilon \Delta^{(m)} \xi_N \text{ for } N = L, K$$

Moreover,

$$\theta_{N1} = \frac{1}{\sigma}, \theta_{N2} = \frac{1}{\gamma_N}, \theta_{N3} = \frac{1}{\sigma \gamma_N}$$

from which it follows that $\theta_{N3} = \theta_{N1}\theta_{N2}$ and $\theta_{L1} = \theta_{K1} = 1/\sigma$. In the special case where r is exogenous $\theta_{K2} = \theta_{K3} = 0$. In the case where *both* prices are exogenous, Equation (11) reduces to the first order conditions for the ordinary least squares estimation of $\theta_{L1} = 1/\sigma$ under the restrictions $\theta_{L1} = \theta_{K1}$ and $\theta_{N2} = \theta_{N3} = 0$ and no intercepts. The need for additional moment restrictions arises only when (at least one of) the factor prices are endogenous.

All parameter restrictions can be satisfied by the following re-parametrisation: $\theta_{N1} = \exp(\tau)$, $\theta_{N2} = \exp(\tau_N)$ and $\theta_{N3} = \exp(\tau + \tau_N)$ – with no constraints on (τ, τ_L, τ_K). That is, Equation (11) can be re-formulated as:

$$Y_N = \exp(\tau) X_{N1} + \exp(\tau_N) X_{N2} + \exp(\tau + \tau_N) X_{N3} + U_N \text{ for } N = K, L \tag{12}$$

with

$$\sigma = \exp(-\tau) \text{ and } \gamma_N = \exp(-\tau_N) \tag{13}$$

To estimate these three parameters we use observed data sequences $y_N = \{Y_{Nit}\}_{it}$ and $x_{Nk} = \{X_{Nk}\}_{it}$ for $i = 1, \ldots, I$ and $t = 1, \ldots, T_i$ (allowing firm-specific number of time periods, T_i). The sequence x_{Nk} is constructed from volume- and price indices for $(L_{it}, K_{it}, R_{it}, W_{it})$, as described in Section 4. Under the identifying assumptions of Feenstra (1994), the idiosyncratic error terms ε_{Nit} and ξ_{Nit} are assumed to be independent for any i, t and $N = K, L$ implying that $E(U_{Nit}) = 0$.

Equation (12) is *not* a valid (non-linear) regression equation for estimating τ and τ_N because the regressors X_{Nk} are correlated with U_N. Instead, we propose to estimate parameters using the generalized method of moments, building on Feenstra's 2SLS estimator, the Feenstra/Soderbery estimator and its more efficient refinement proposed by Brasch and Raknerud (2021a), involving a pooling of GMM estimates across several reference units. Technically, the GMM estimator can be seen as a non-linear IV estimator with I firm indicators as instruments, see Feenstra (1994, p. 164).

We use the $2I$ orthogonality conditions:

$$E(\sum_{t=1}^{T_i} U_{Nit}) = 0 \text{ for } N = K, L \text{ and } i = 1, ..., I \tag{14}$$

as the basis of our non-linear GMM estimation algorithm, equipped, for computational accuracy and efficiency, with the following analytical derivatives:

$$\frac{\partial U_{Nit}}{\partial \tau} = -\exp(\tau) X_{N1it} - \exp(\tau + \tau_N) X_{N3it}$$
$$\frac{\partial U_{Nit}}{\partial \tau_N} = -\exp(\tau_N) X_{N2it} - \exp(\tau + \tau_N) X_{N3it}$$

for $N = K, L$. In the estimation, we first randomly split the sample into 10 equally sized subsamples by firm identifiers. Then, for each subsample, k, we randomly draw M reference firms and estimate the parameters (τ, τ_L and τ_K) from the $2I$ orthogonality conditions in Equation (14) for each reference firm, say m. We allow for 50 iterations using a one-step GMM estimator with the weight matrix being the identity matrix, i.e. all the $2I$ orthogonality conditions in Equation (14) have equal weight in the estimation. For given l and m, the estimated structural parameters are denoted $\widehat{\sigma}_{lm}$ and $\widehat{\gamma}_{Nlm}$ (for $N = K, L$) (see Equation (13)). For given subsample l, we then calculate the mean estimated elasticity of substitution across the M reference firms as: $\bar{\sigma}_l = (1/M)\sum_{m=1}^{M} \widehat{\sigma}_{lm}$.

To arrive at an overall pooled estimator, $\widehat{\sigma}$, we calculate a weighted average across the 10 subsamples:

$$\widehat{\sigma} = \sum_{l=1}^{10} w_l \bar{\sigma}_l.$$

where

$$w_l = \text{var}(\widehat{\sigma}) \frac{1}{\text{var}(\bar{\sigma}_l)}$$

are optimal weights in the sense that:

$$\text{var}(\widehat{\sigma}) = \left(\sum_{l=1}^{10} \frac{1}{\text{var}(\bar{\sigma}_l)} \right)^{-1}$$

is the minimum attainable variance.

For given subsample, l, we expect the different estimates, $\widehat{\sigma}_{lm}$, to be highly correlated as the same firm-years are used many times with different reference firms, m. An upper bound expression is: $\text{var}(\overline{\sigma}_l) \leq \frac{1}{M}\sum_{m=1}^{M} \text{var}(\widehat{\sigma}_{lm}) = \overline{\text{var}}(\overline{\sigma}_l)$, where $\text{var}(\widehat{\sigma}_{lm})$ can be estimated using the usual delta method: $\widehat{\text{var}}(\widehat{\sigma}_{lm}) = \exp(-2\hat{\tau}_{lm})\widehat{\text{var}}(\widehat{\tau}_{lm})$ with $\widehat{\text{var}}(\widehat{\tau}_{lm})$ being a robust variance estimator (clustered by firm). The associated upper bound of the standard error of $\widehat{\sigma}$ is:

$$\text{SE}(\widehat{\sigma}) \leq \left(\sum_{l=1}^{10} \frac{1}{\overline{\text{var}}(\overline{\sigma}_l)} \right)^{-1/2}$$

3.3 Monte Carlo Analysis

To shed light on the properties of the P-GMM estimator, we conduct a Monte Carlo analysis and compare it with the fixed effects estimator of Equation (7). We start by outlining the simulation algorithm before we present our benchmark results. The benchmark results are based on a particular set of parameter values. In the final part of this section, we illustrate how the benchmark result changes when changing the set of parameter values. The details of the Monte Carlo simulation algorithm are found in Appendix D.

Benchmark Results

The benchmark CES model is simulated based on the calibrated values $\sigma = \gamma_K = \gamma_L = 1.5$, $I = 100$ (cross-sections), $T = 20$ (time periods) and $M = 10$ (reference firms). Table 1 shows both the estimated value of σ when using a fixed effects estimator and when using P-GMM as outlined above. The fixed effects estimator yields an estimated value of 0.59, well below the true value of 1.5 underlying the simulations. In contrast, P-GMM yields an estimate of 1.5. P-GMM also provides estimates for the capital- and labour supply elasticities that are close to the true values in the simulations. Note that σ is more precisely estimated than both γ_K and γ_L. This is because σ enters both the auxiliary equations in Equation (12) and that the cross-equation restriction allows for more precise estimates as more variation in the data is utilised to identify this parameter.

Robustness

Figure 1 shows how the estimates of σ change for both the fixed effects estimator and P-GMM when applying different values of σ in the simulations, ranging from 0 to 5. All of the other parameters are at their benchmark values. There is a clear negative bias for the fixed effects estimator, as shown analytically in Section 3.1. In contrast, the P-GMM estimator is close to the true value, at least for σ ranging from 0.5 to 3. For $\sigma < 0.5$ there seems to be a small upward bias in the estimator and for $\sigma > 3$ there is a small downward bias, but these biases are negligible compared with the bias from the fixed effects estimator. In the robustness analysis below we illustrate what the source of this bias is.

Figure 2 shows the robustness of P-GMM when changing some key benchmark values. The upper left panel shows how the estimated value of σ changes when changing the common values of γ_K and γ_L. P-GMM yields estimates close to the true value of σ for most values of γ_K and γ_L, but there seems to be larger variation in the estimates when $\gamma_K = \gamma_L$ is close to zero. This panel also shows the bias of the fixed effects estimator, which is large in absolute value for values of $\gamma_K = \gamma_L < 8$. When γ_K and γ_L are close to zero, the fixed effects estimator yields an estimated value of σ close to zero although the true value is $\sigma = 1.5$. The bias of the fixed effects estimator goes towards zero for large values of the capital supply and labour supply elasticities since wage rates and the rental price of capital in Equation (3) become near exogenous when γ_K and γ_L are large.

Table 1: Benchmark CES model

	True value	Estimate
Fixed effects estimator		
Elasticity of substitution (σ)	1.5	0.59
		(0.08)
P-GMM estimator		
Elasticity of substitution (σ)	1.5	1.50
		(0.05)
Capital supply elasticity (γ_K)	1.5	1.36
		(0.13)
Labour supply elasticity (γ_L)	1.5	1.85
		(0.23)

Source: Authors' own calculations.
Note: Standard errors clustered on firm level in parenthesis. The benchmark CES model used to simulate the data is based on the following parameters: $\sigma = 1.5$, $\gamma_L = \gamma_K = 1.5$, $T = 20$ (time periods), $M = 10$ (reference firms) and $I = 100$ (cross-sections). Convergence is defined as (i) identification of the auxiliary parameters (τ and τ_{m2} for $m = K, L$) by means of convergence, with (ii) each implied fundamental parameter estimate ($\hat{\sigma}$, $\hat{\gamma}_L$ and $\hat{\gamma}_K$) having a p-value less than 0.90.

The three box-plots in the panel shows the distribution for the estimated value of σ for different values of reference firms, cross-sections and time periods, respectively. Each box plot is based on 500 repeated simulations and estimations. The rectangles represent the estimates that lie within the interquartile range (IQR), i.e. the 50 percent estimates closest to the median in rank. The median is depicted using a line through the center of the box while the mean is drawn using the circle symbol. The whiskers represent the estimates that are within $\pm 1.5 \times IQR$.

The upper right panel illustrates that the variance of the estimates from P-GMM is markedly reduced when the number of reference firms increases from 1 to 10. The marginal gain of increasing the number of reference firms is however decreasing and there is not much gain from increasing the number of reference firms beyond 10. This finding is consistent with the conclusion in Brasch and Raknerud (2021a) for the extended Feenstra (1994) estimator. Although there is an efficiency gain in increasing the number of reference firms from 1 to 10, there is still a small upward bias in the estimates. The lower left panel illustrates the source of this bias. It shows both how the variance and the mean of the estimates change when changing the number of time periods (T). When the number of time periods is small, i.e. 5 or 10, there is a small upward bias and the variance of the estimates is large. Both the variance and the bias are decreasing in the num-

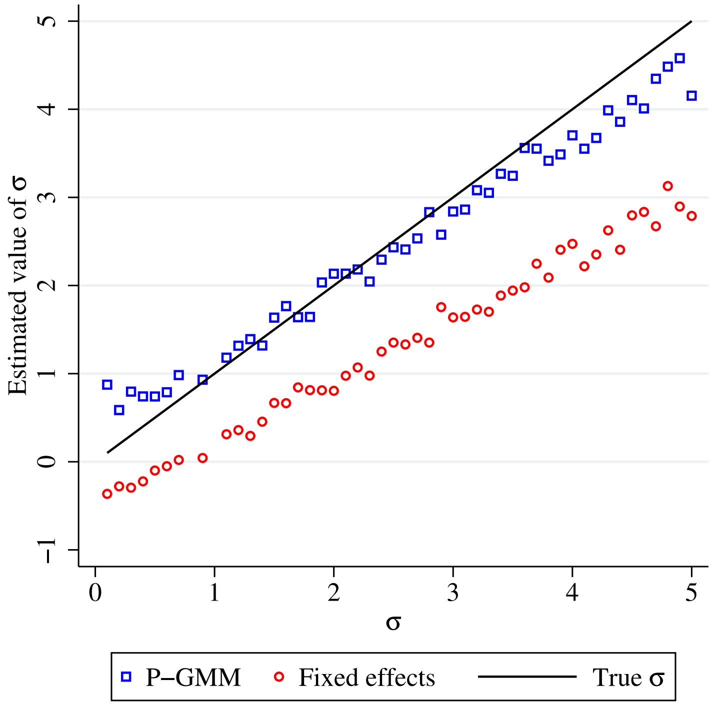

Figure 1: Estimated values of σ. The P-GMM and fixed effects estimator.

Source: Authors' own calculations.

Note: The simulated data are based on the benchmark parameter values: $\gamma_K = \gamma_L = 1.5$, $I = 100$, $T = 20$ and $M = 10$.

ber of time periods, and the bias is almost nonexistent for $T = 50$. The lower right panel shows how the variance and the mean changes when changing the number of cross-sections (I). Since these variations are made around $T = 20$, the small upward bias in the estimates remains when increasing the number of cross-sections. However, there is a marked lowering of the variance when going from $I = 20$ to $I = 100$, but no significant further reduction in the variance when doubling I from 100 to 200.

Three main conclusions can be drawn from this Monte Carlo study. First, the fixed effects estimator is heavily downward biased when simultaneity is present, even for relatively large supply elasticities of capital and labour. This shows the importance of using an estimator that allows for one or both of the factor prices being endogenous. Second, P-GMM identifies σ accurately for most parameter variations, but when T is small, there is an upward bias in the estimator. Third, there is a large efficiency gain from using more than one reference firm, but there is not much efficiency gain from using more than 10 reference firms. In the following, we will utilise these insights when applying P-GMM to the case of manufacturing firms in Norway.

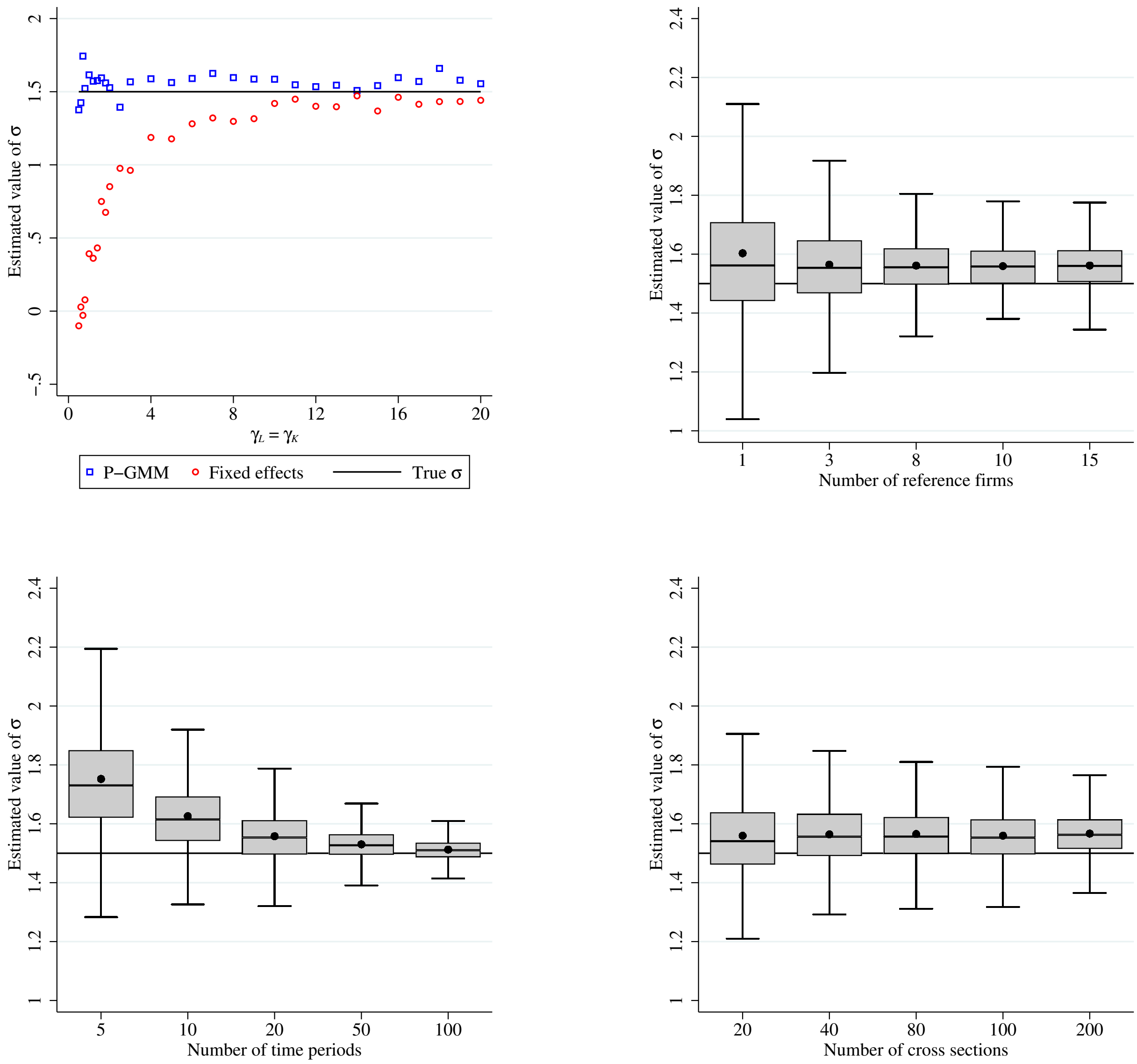

Figure 2: Robustness of P-GMM

Source: Authors' own calculations.

Note: Parameter variations are made with other parameters fixed at their benchmark values: $\sigma = \gamma_K = \gamma_L = 1.5$, cross-sections: $I = 100$, time periods: $T = 20$ and reference firms: $M = 10$. Each boxplot is based on 500 simulations and estimations.

4 Empirical Analysis

4.1 Data Sources

Our main source of data is the income statement form submitted by all limited liability companies and enterprises who keep accounts according to the IFRS (International Financial Reporting Standard). The form (*Næringsoppgave 2*) contains information about the firms' balance sheets, and most importantly for our purpose: the firms' balance of fixed assets. This allows us to measure the value of capital ($R \times K$) for each firm. We use data for the whole population of firms within the manufacturing sector reporting the income statement, spanning the years 2007 to 2018, and restrict attention to incumbent firms that were 3 years or older in 2007. We also use the registry of all employed workers (*Arbeidstakerregisteret/a-meldingen*) sampled in November of each year to derive the number of contracted working hours in each firm (L), as well as the hourly wage rate (W) derived from the firm' total wage bill ($W \times L$) and number of working hours.[3] Also these data span the years 2007 to 2018. In order to measure the rental price of capital (R), we use the estimated prices from National Accounts, which is common to all firms but specific for each capital type. The price is of the Jorgensen type, depending on the tax-adjusted interest rate, investment price and depreciation rate of capital (see Sandmo 1974 and Brasch et al. 2021 for details about formulas). In other words, we attribute a rental-equivalent capital price for each fixed asset of the firm.

4.2 Descriptive Statistics: Firms

The construction of the sample of firm-year observations we use is described in Table 2 showing the number of firm-year observations and unique firms for each of the sample selection steps. Our final sample consists of 62,170 firm-year observations across 7,977 firms within the manufacturing sector.

Table 2: Number of firms and firm-years after each sample selection step

	Firm-years	Firms
The income statement data set (full population)	2,733,934	426,340
Subset present in register of employed workers	1,501,769	204,753
Subset with valid industry code	1,497,569	204,517
Subset with at least one employee	1,133,901	201,589
Subset with positive capital stock	910,636	163,337
Subset operating within manufacturing	82,531	12,810
Subset of incumbent firms (at least 3 years old in 2007)	62,170	7,977

Source: Authors' own calculations using data from Statistics Norway.
Note: The register of employed workers is restricted to workers (i) aged 18-70 years, (ii) not registered as (full-time) unemployed, (iii) registered with positive values for both earnings and working hours. Furthermore, employment relationships where the calculated hourly wage rate is below NOK 100 or above NOK 50,000 (CPI-adjusted to 2018-NOK) are dropped.

Most of the manufacturing firms in our sample operate within fabrication of metal products (14 percent) and manufacturing of food products (14 percent), and the mean age of firms is 16.5 years. Most of the

[3] Prior to 2015, the registry was only sampled in November each year, while in 2015 and onwards the sampling occurs each calendar month. We only sample November in 2015 and onwards to increase comparability across all years in our sample.

manufacturing firms are fairly large in terms of employment, with the mean number of employees (measured as head count) equal to 33, with a standard deviation of 130.

4.3 Descriptive Statistics for Labour and Capital

Since neither labour nor capital are homogeneous inputs in the production process, we partition these into six and five categories, respectively. Labour is partitioned into male and female and three levels of education: compulsory schooling, secondary school or university education.[4] Capital is partitioned into 5 categories: real estate, vehicles, machinery, R&D, and boats and planes. Each category (j) has its own price, either based on the firm i's hourly wage rate for labour type j in year t (W_{ijt}) or the price the firm faces in the rental market of capital type j in year t (R_{jt}).[5]

Figure 3 shows the distribution of contracted working hours across the six categories. The largest category is men with secondary education (36 percent), followed by men with compulsory education (28 percent) and men with a university degree (16 percent). Only 20 percent of contracted working hours refer to women. The average hourly wage rate for the various worker types is weakly increasing over time, with stable differences between categories, as shown in Figure 4. The hourly wage rate has the expected gradient in terms of gender and educational level, with university-educated men on average earning about 30 percent more than men with secondary education *and* women with university degree, and 60 percent more than men with compulsory education.[6] In terms of the value of the capital stock of firms, the largest share consists of machinery (61 percent) and real estate (32 percent), see Figure 5 for the distribution of capital stock value across all firm-year observations. More detailed descriptive statistics on the sample of manufacturing firms are deferred to Appendix E.

4.4 Aggregation of Labour and Capital

Each of the categories of labour and capital are aggregated to a measure of labour (L) and capital (K) at the firm-year level, using a Feenstra volume index (for $N = L, K$):

$$\Delta \ln (N_{it}) = \sum_{j \in C} \bar{S}^N_{ijt} \Delta \ln (N_{ijt}) + \left(\frac{\varphi^N_i}{1 - \varphi^N_i} \right) \ln \left(1 - s^N_{Et}\right) - \left(\frac{\varphi^N_i}{1 - \varphi^N_i} \right) \ln \left(1 - s^N_{Xt}\right) \tag{15}$$

[4] Individuals with missing education level are assigned as having compulsory schooling.

[5] Note that the rental prices of capital are not firm-specific, hence no subscript i.

[6] The raw wage differential between men and women, on average over the sample period, is 14.5 percent. The magnitude corresponds to the gender wage gap for the Norwegian manufacturing sector reported in Bøler et al. (2018).

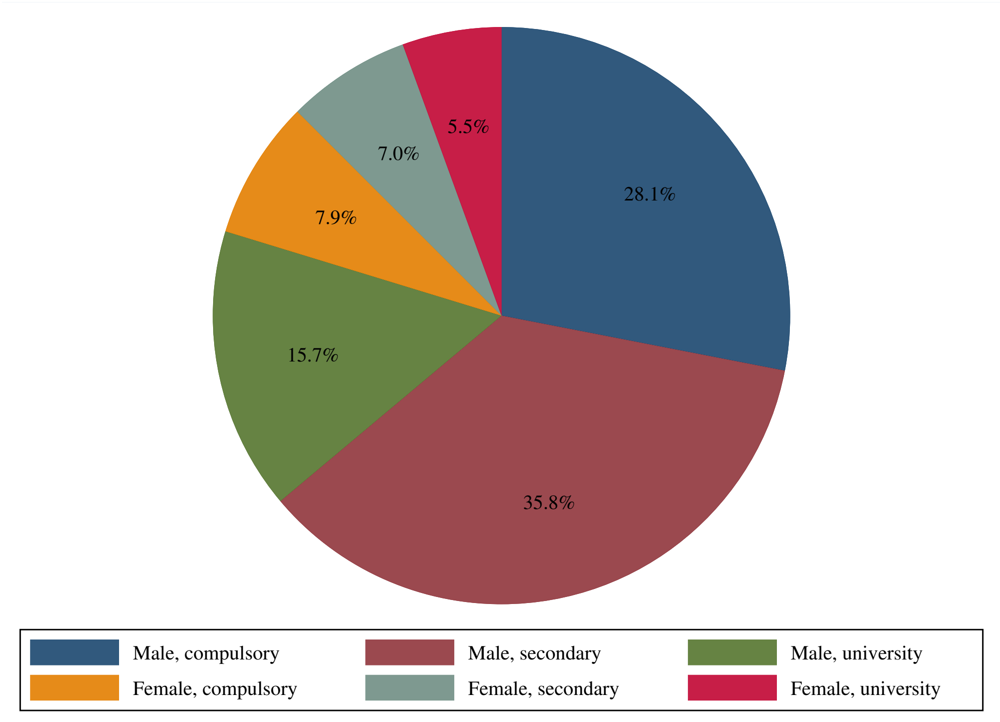

Figure 3: Distribution of contracted working hours, by worker type, manufacturing firms

Source: Authors' own calculations using data from Statistics Norway.

where φ_i^N is the elasticity of substitution between the categories, and the value shares ($\bar{S}_{ijt}^N$, s_{Et}^N and s_{Xt}^N) are defined as follows in terms of the value of the input factor (V_{ijt}^N):

$$\bar{S}_{ijt}^N = \frac{\frac{S_{ijt}^N - S_{ij,t-1}^N}{\ln\left(S_{ijt}^N\right) - \ln\left(S_{ij,t-1}^N\right)}}{\sum_{b \in C} \frac{S_{ibt}^N - S_{ib,t-1}^N}{\ln\left(S_{ibt}^N\right) - \ln\left(S_{ib,t-1}^N\right)}}$$

$$S_{ijt}^N = \frac{V_{ijt}^N}{\sum_{b \in C} V_{ibt}^N}$$

$$s_{Et}^N = \frac{\sum_{b \in E} V_{ibt}^N}{\sum_{b \in C} V_{ibt}^N + \sum_{b \in E} V_{ibt}^N}$$

$$s_{Xt}^N = \frac{\sum_{b \in X} V_{ib,t-1}^N}{\sum_{b \in C} V_{ib,t-1}^N + \sum_{b \in X} V_{ib,t-1}^N}$$

Here C denotes the set of continuing types of the input factor (present for firm i in both year t and $t-1$), E denotes the set of entering types of the input factor (present for firm i in year t, but not year $t-1$), and X denotes the set of exiting types of the input factor (present for firm i in year $t-1$, but not year t). See Feenstra (1994) for more details on how these indices are constructed and Brasch and Raknerud (2021b)

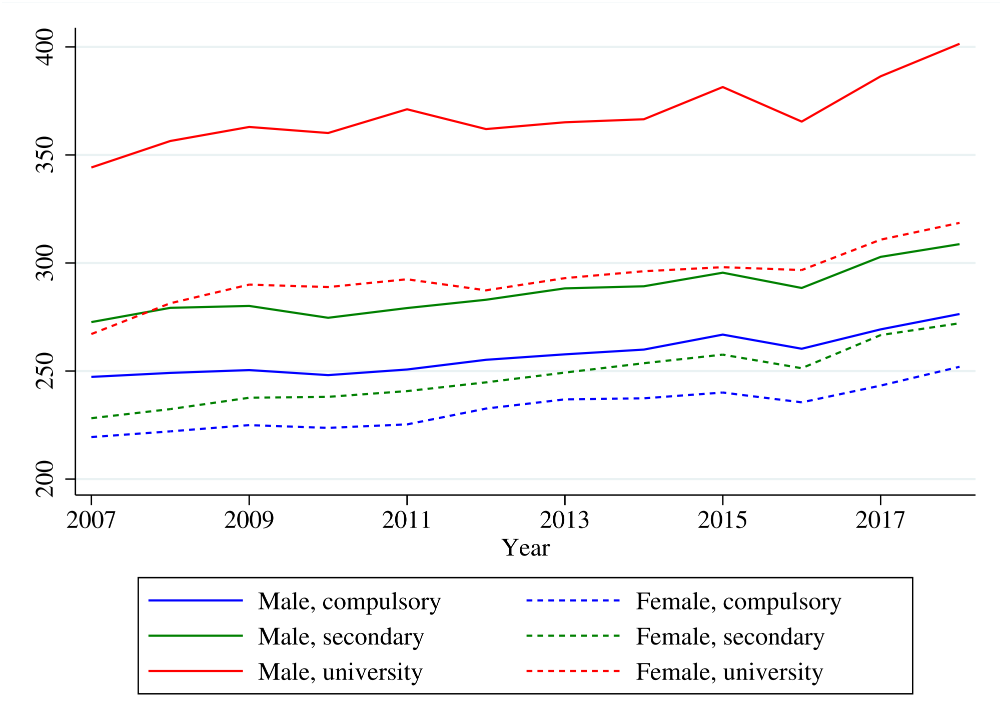

Figure 4: Average hourly wage rate, by worker type and year, manufacturing firms
Source: Authors' own calculations using data from Statistics Norway.
Note: The vertical axis shows the calculated mean hourly wage rate, measured in 2018-NOK (deflated by the CPI).

for an empirical application.

Following the lines of Brasch et al. (2018), we assume that each category of capital input are perfectly substitutable for each other at the firm level, and equivalently for the labour input categories, i.e. $\varphi_i^N \to \infty$ and Equation (15) simplifies to:

$$\Delta \ln(N_{it}) = \sum_{j \in C} \bar{S}_{ijt}^N \Delta \ln(N_{ijt}) - \ln(1 - s_{Xt}) + \ln(1 - s_{Et})$$

In contrast to the traditional approach of summing up hours worked as an index for labour services, which implicitly assumes that workers are both perfect substitutes and of equal quality, the above index is more general since it only assumes that workers are perfect substitutes, with wage rate differences reflecting quality differences. We define the corresponding price indices R_{jt} and W_{ijt} implicitly from the product rule which states that the ratio of value aggregates for two time periods equals the product of a volume index and a price index, see Fisher (1911, p. 418) and Frisch (1930, p. 399).

4.5 Estimation Results

Table 3 shows the estimation results for both the P-GMM and fixed effects estimator. The first pair of columns shows the estimated elasticity of substitution from P-GMM and the associated standard error. The second pair of columns show the fixed effects estimates of Equation (7). The latter indicates an elastic-

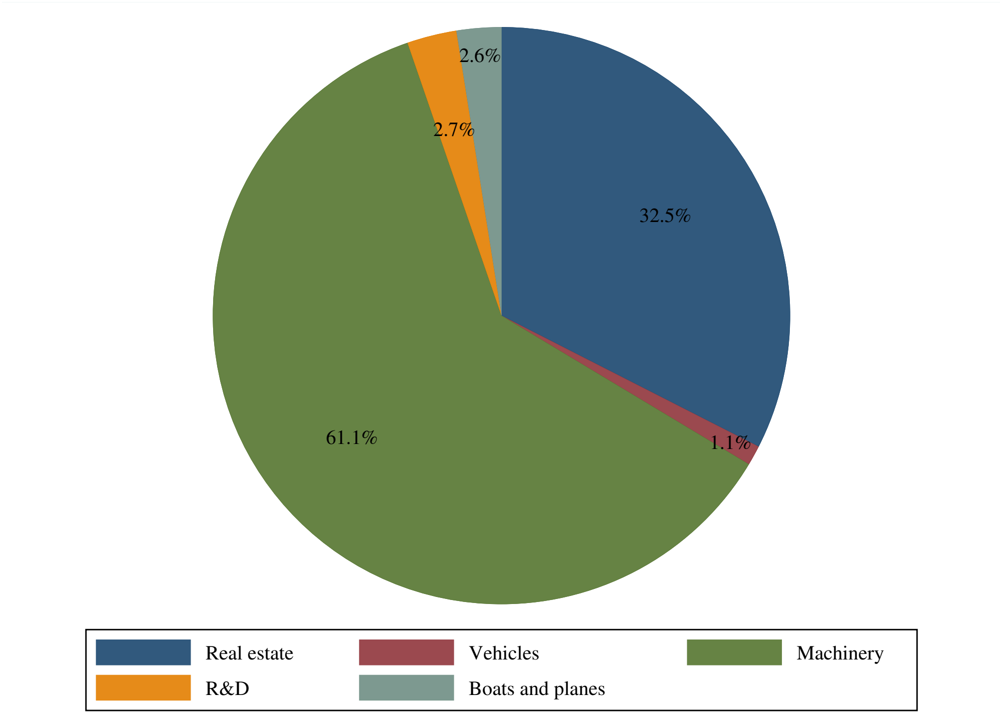

Figure 5: Value of capital stock, by capital type, manufacturing firms
Source: Authors' own calculations using data from Statistics Norway.

ity of substitution between capital and labour of 1.04, which is consistent with Cobb-Douglas technology. In contrast, P-GMM yields an estimate of 1.84, indicating a substantially higher elasticity of substitution than the fixed effects estimate. In contrast to the estimate of the elasticity of substitution, the estimates of the capital- and labour-supply elasticities are quite uncertain. However, the estimate for the capital supply elasticity of 103.43 is as expected, probably reflecting that Norwegian firms have access to the world market for capital goods where prices are exogenously given. This result must also be seen in conjunction with the fact that our data on capital prices are not firm-specific, which may explain the large standard error of this coefficient. In contrast, our data hold firm-specific wage rates, and the results show a finite labour supply elasticity of 6.97. This is on par with the mean estimate of labour supply elasticities found in the meta study by Sokolova and Sorensen (2021).

The estimated elasticity of substitution from the P-GMM procedure for the 10 subsamples, each estimated separately for 10 randomly drawn reference firms, are shown in the upper panel of Figure 6. The estimates for σ varies between 1.23 and 3.99. The lower panel shows the average estimated elasticity of substitution for each of the 10 subsamples.

Table 3: Estimation results, manufacturing firms

	P-GMM		Fixed effects	
	Estimate	Standard error	Estimate	Standard error
Elasticity of substitution (σ)	1.84	0.08	1.04	0.04
Capital supply elasticity (γ_K)	103.43	30.00	–	–
Labour supply elasticity (γ_L)	6.97	3.10	–	–
Number of observations	53,207		53,207	
Number of firms	7,261		7,261	

Source: Authors' own calculations.

Note: For the fixed effects specification, the analytical standard errors are clustered on the firm level. 10 reference firms have been used in P-GMM, where we require the reference firm to be present in all years. Convergence is defined as (i) identification of the auxiliary parameters (τ and τ_{m2} for $m = K, L$) by means of convergence, with (ii) each implied fundamental parameter estimate ($\hat{\sigma}$, $\hat{\gamma}_L$ and $\hat{\gamma}_K$) having a p-value less than 0.90.

5 Conclusion

In this paper, we have developed and implemented a new pooled GMM (P-GMM) panel estimator for the elasticity of substitution (σ) between capital and labour. The P-GMM estimator is a refinement of the Feenstra (1994) estimator, which handles simultaneity without the use of external instruments. In contrast to the Feenstra estimator, which was based on $\sigma > 1$ and consists of prices and quantities of one variable only, our refined estimator allows for $\sigma > 0$ and consists of prices and quantities of multiple variables. Moreover, P-GMM is much simpler to apply than the Feenstra estimator as it does not involve hyperbolic parameter restrictions, with the complicating possibility of boundary estimates. To illustrate the properties of P-GMM, we conducted a Monte Carlo analysis and compared P-GMM with the fixed effects estimator often applied in the literature, and which assumes that wage rates and the rental price of capital are determined exogenously. We showed that P-GMM clearly outperforms the fixed effects estimator, which is heavily downward biased. In our benchmark simulation, where $\sigma = 1.5$, the estimate obtained from P-GMM was 1.5, while the fixed effects estimate was 0.6. Moreover, we applied both these estimators to the case of manufacturing firms in Norway. P-GMM gave an estimate of 1.84. In contrast, the fixed effects estimate was 1.04, which indicates Cobb-Douglas technology. Our study thus shows the importance of using an estimator that allows for simultaneity: not considering simultaneity may lead to the conclusion that capital and labour are complements or described by Cobb-Douglas technology, when they in fact are substitutes.

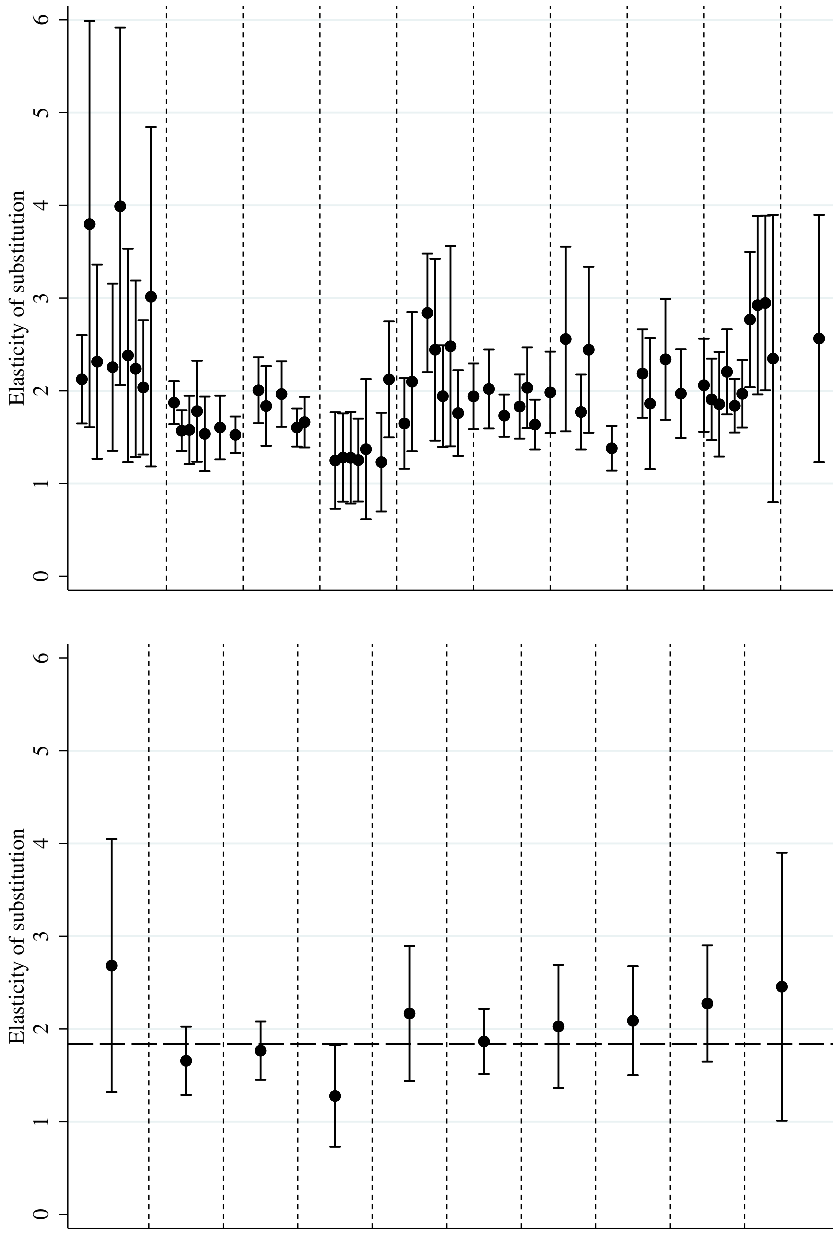

Figure 6: P-GMM estimation results for the elasticity of substitution

Source: Authors' own calculations using data from Statistics Norway.

Note: Upper panel shows estimated σ across all reference firms and subsamples. Lower panel shows the average estimated σ for each subsample. The vertical dashed lines in both panels indicate the 10 separate subsamples. Capped lines show 95 percent confidence intervals. The estimated elasticity of substitution of 1.84 is derived as the weighted sum of the estimates in the lower panel, with weights defined in Section 3.2.

References

ACEMOGLU, D. (2002): "Directed technical change," *The Review of Economic Studies*, 69, 781–809.

ALVAREZ-CUADRADO, F., N. VAN LONG, AND M. POSCHKE (2017): "Capital-labor substitution, structural change, and growth," *Theoretical Economics*, 12, 1229–1266.

ARKOLAKIS, C., A. COSTINOT, D. DONALDSON, AND A. RODRÍGUEZ-CLARE (2018): "The elusive pro-competitive effects of trade," *The Review of Economic Studies*, 86, 46–80.

BERNHEIM, B. D. (2002): *Chapter 18 Taxation and saving*, vol. 3, Elsevier Masson SAS.

BLONIGEN, B. A. AND A. SODERBERY (2010): "Measuring the benefits of foreign product variety with an accurate variety set," *Journal of International Economics*, 82, 168–180.

BLUNDELL, R. AND T. MACURDY (1999): *Chapter 27 Labor supply: A review of alternative approaches*, vol. 3 PART, Elsevier Masson SAS.

BØLER, E. A., B. JAVORCIK, AND K. H. ULLTVEIT-MOE (2018): "Working Across Time Zones: Exporters and the Gender Wage Gap," *Journal of International Economics*, 122–133.

BRASCH, T. V., Å. CAPPELEN, H. HUNGNES, AND T. SKJERPEN (2021): "Modeling RD spillovers to productivity: The effects of tax credits," *Economic Modelling*, 101.

BRASCH, T. V., D.-C. IANCU, AND A. RAKNERUD (2018): "Productivity growth, firm turnover and new varieties," Statistics Norway: Discussion Papers (872).

BRASCH, T. V. AND A. RAKNERUD (2021a): "A two-stage pooled panel data estimator of demand elasticities," Statistics Norway: Discussion Papers (951).

——— (2021b): "The impact of new varieties on aggregate productivity growth," *The Scandinavian Journal of Economics*.

BRODA, C., J. GREENFIELD, AND D. E. WEINSTEIN (2017): "From groundnuts to globalization: A structural estimate of trade and growth," *Research in Economics*, 71, 759–783.

BRODA, C. AND D. E. WEINSTEIN (2006): "Globalization and the gains from variety," *Quarterly Journal of Economics*, 121, 541–585.

——— (2010): "Product creation and destruction: evidence and price implications," *American Economic Review*, 100, 691–723.

CHETTY, R., A. GUREN, D. MANOLI, AND A. WEBER (2012): *Does indivisible labor explain the difference between micro and macro elasticities? A meta-analysis of extensive margin elasticities*, vol. 27.

CHIRINKO, R. S. (2002): "Corporate taxation, capital formation, and the substitution elasticity between labor and capital," *National Tax Journal*, 55, 339–355.

DE LA GRANDVILLE, O. AND R. M. SOLOW (2010): "Capital-labour substitution and economic growth," in *Economic Growth - A Unified Approach*, Cambridge: Cambridge University Press, chap. 5, 114–155.

EVERS, M., R. MOOIJ, AND D. VUUREN (2008): "The wage elasticity of labour supply: A synthesis of empirical estimates," *Economist*, 156, 25–43.

FEENSTRA, R. (1994): "New product varieties and the measurement of international prices," *American Economic Review*, 84, 157–177.

FEENSTRA, R., P. LUCK, M. OBSTFELD, AND K. N. RUSS (2018): "In search of the Armington elasticity," *The Review of Economics and Statistics*, 100, 135–150.

FEENSTRA, R. C. AND J. ROMALIS (2014): "International prices and endogenous quality," *The Quarterly Journal of Economics*, 129, 477–527.

FISHER, I. (1911): *The Purchasing Power of Money*, London: Macmillan.

FRISCH, R. (1930): "Necessary and sufficient conditions regarding the form of an index number which shall meet certain of Fisher's tests," *Journal of the American Statistical Association*, 25, 397–406.

GOOLSBEE, A. (1998): "Investment tax incentives, prices, and the supply of capital goods," *Quarterly Journal of Economics*, 113, 120–148.

HASSETT, K. A. AND R. G. HUBBARD (2002): "Chapter 20 Tax policy and business investment," *Handbook of Public Economics*, 3, 1293–1343.

HUNGNES, H. (2011): "A demand system for input factors when there are technological changes in production," *Empirical Economics*, 581–600.

IMBS, J. AND I. MEJEAN (2015): "Elasticity optimism," *American Economic Journal: Macroeconomics*, 7, 43–83.

JÄNTTI, M., J. PIRTTILÄ, AND H. SELIN (2015): "Estimating labour supply elasticities based on cross-country micro data: A bridge between micro and macro estimates?" *Journal of Public Economics*, 127, 87–99.

KARABARBOUNIS, L. AND B. NEIMAN (2014): "The global decline of the labor share," *Quarterly Journal of Economics*, 129, 61–103.

KASAMATSU, S. AND H. OGAWA (2020): "International capital market and repeated tax competition," *Journal of Public Economic Theory*, 22, 751–768.

KNOBLACH, M., M. ROESSLER, AND P. ZWERSCHKE (2020): "The Elasticity of Substitution Between Capital and Labour in the US Economy: A Meta-Regression Analysis," *Oxford Bulletin of Economics and Statistics*, 82, 62–82.

LEAMER, E. (1981): "Is it a demand curve, or is it a supply curve? Partial identification through inequality constraints," *The Review of Economics and Statistics*, 63, 319–327.

MALLICK, D. (2012): "The role of the elasticity of substitution in economic growth: A cross-country investigation," *Labour Economics*, 19, 682–694.

MANNING, A. (2003): *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton SN: Princeton University Press.

PIKETTY, T. AND G. ZUCMAN (2014): "Capital is back: Wealth-income ratios in rich countries 1700-2010," *Quarterly Journal of Economics*, 129, 1255–1310.

SANDMO, A. (1974): "Investment Incentives and the Corporate Income Tax," *Journal of Political Economy*, 82, 287.

SODERBERY, A. (2015): "Estimating import supply and demand elasticities: Analysis and implications," *Journal of International Economics*, 96, 1–17.

SOKOLOVA, A. AND T. SORENSEN (2021): "Monopsony in Labor Markets: A Meta-Analysis," *ILR Review*, 74, 27–55.

STOCK, J. (2001): "Instrumental variables in statistics and econometrics," in *International Encyclopedia of the Social & Behavioral Sciences*, ed. by N. J. Smelser and P. B. Baltes, Elsevier, 7577–7582.

Appendix A: Reduced Form of the CES Model with Monopolistic Competition

From the first-order condition of cost minimization based on the factor demand equations in Equation (2) and the supply Equations (5)-(6), we can write the system of supply and demand compactly as:

$$\begin{aligned}
\text{demand:} \qquad & l = -\sigma w + \mu + \varepsilon_L \\
& k = -\sigma r + \mu + \varepsilon_K \\
\text{supply:} \qquad & l = \gamma_L w + \xi_L \\
& k = \gamma_K r + \xi_K
\end{aligned}$$

where μ is an appropriately defined "neutral" term to be defined below. For now we treat μ as exogenously given. Later we will specify μ in accordance with the CES model, where it is endogenous and determined simultaneously with (w,r) in a non-linear 2×1 system of equations.

On reduced form, with fixed μ, the above system can be written:

$$\begin{bmatrix} 1 & 0 & \sigma & 0 \\ 0 & 1 & 0 & \sigma \\ 1 & 0 & -\gamma_L & 0 \\ 0 & 1 & 0 & -\gamma_K \end{bmatrix} \begin{bmatrix} l \\ k \\ w \\ r \end{bmatrix} = \begin{bmatrix} \mu + \varepsilon_L \\ \mu + \varepsilon_K \\ \xi_L \\ \xi_K \end{bmatrix}$$

That is:

$$Ax = y$$

with solution

$$x = A^{-1} y$$

where

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} \gamma_L \gamma_K + \sigma \gamma_L & 0 & \sigma^2 + \sigma \gamma_K & 0 \\ 0 & \gamma_L \gamma_K + \sigma \gamma_K & 0 & \sigma^2 + \sigma \gamma_L \\ \sigma + \gamma_K & 0 & -\sigma - \gamma_K & 0 \\ 0 & \sigma + \gamma_L & 0 & -\sigma - \gamma_L \end{bmatrix}$$

with

$$|A| = \gamma_L \gamma_K + \sigma \gamma_L + \sigma \gamma_K + \sigma^2 = (\gamma_L + \sigma)(\gamma_K + \sigma)$$

Assuming $|A| \neq 0$, which is an identifying condition, it follows that, on reduced form:

$$\begin{bmatrix} l \\ k \\ w \\ r \end{bmatrix} = \frac{1}{|A|}\left\{ \begin{bmatrix} \gamma_L\gamma_K+\sigma\gamma_L \\ \gamma_L\gamma_K+\sigma\gamma_K \\ \sigma+\gamma_K \\ \sigma+\gamma_L \end{bmatrix}\mu + \begin{bmatrix} \gamma_L\gamma_K+\sigma\gamma_L \\ 0 \\ \sigma+\gamma_K \\ 0 \end{bmatrix}\varepsilon_L \right.$$
$$\left. + \begin{bmatrix} 0 \\ \gamma_L\gamma_K+\sigma\gamma_K \\ 0 \\ \sigma+\gamma_L \end{bmatrix}\varepsilon_K + \begin{bmatrix} \sigma^2+\sigma\gamma_K \\ 0 \\ -\sigma-\gamma_K \\ 0 \end{bmatrix}\xi_L + \begin{bmatrix} 0 \\ \sigma^2+\sigma\gamma_L \\ 0 \\ -\sigma-\gamma_L \end{bmatrix}\xi_K \right\}$$

Now let us examine ε_L, ε_K and μ. Assume monopolistic competition in the CES model with $\varepsilon = 1$ and demand function $Q_{it}^D = \Phi_{it}P_{it}^{-e}$ (with $e > 1$ and profit maximization) and define $A_{it}^* = \Phi_{it}^{1/(e-1)}A_{it}$. Then

$$\begin{aligned} \mu &= (e-1)\ln A_{it}^* + (\sigma-e)\ln c \\ &= a^* + (\sigma-e)\ln c \end{aligned}$$

where

$$a^* = (e-1)\ln A_{it}^*$$

Moreover,

$$\begin{aligned} \ln c &= \frac{1}{1-\sigma}\ln\left[\alpha(\frac{R}{A_{Kit}})^{1-\sigma} + (1-\alpha)(\frac{W}{A_{Lit}})^{1-\sigma}\right] \\ &= \frac{1}{1-\sigma}\ln\left[\exp(\varepsilon_K+(1-\sigma)r)+\exp(\varepsilon_L+(1-\sigma)w)\right] \end{aligned}$$

Thus (w,r) can be found by solving a non-linear 2×1 system of equations:

$$\begin{aligned} \begin{bmatrix} w \\ r \end{bmatrix} &= \frac{\sigma-e}{|A|(1-\sigma)}\begin{bmatrix} \sigma+\gamma_K \\ \sigma+\gamma_L \end{bmatrix}\ln\left[\exp(\varepsilon_K+(1-\sigma)r)+\exp(\varepsilon_L+(1-\sigma)w)\right] \\ &\quad + \frac{1}{|A|}\begin{bmatrix} (\sigma+\gamma_K)a^*+(\sigma+\gamma_K)\varepsilon_L-(\sigma+\gamma_K)\xi_L \\ (\sigma+\gamma_L)a^*+(\sigma+\gamma_L)\varepsilon_K-(\sigma+\gamma_L)\xi_K \end{bmatrix} \end{aligned} \tag{16}$$

with respect to (w,r). Given the solution, μ can be calculated from the definition:

$$\mu = a^* + \frac{\sigma-e}{1-\sigma}\ln\left[\exp(\varepsilon_K+(1-\sigma)r)+\exp(\varepsilon_L+(1-\sigma)w)\right] \tag{17}$$

and (l,k) from:

$$\begin{bmatrix} l \\ k \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} \gamma_L\gamma_K+\sigma\gamma_L \\ \gamma_L\gamma_K+\sigma\gamma_K \end{bmatrix}\mu + \frac{1}{|A|}\begin{bmatrix} (\gamma_L\gamma_K+\sigma\gamma_L)\varepsilon_L+(\sigma^2+\sigma\gamma_K)\xi_L \\ (\gamma_L\gamma_K+\sigma\gamma_K)\varepsilon_K+(\sigma^2+\sigma\gamma_L)\xi_K \end{bmatrix} \tag{18}$$

Appendix B: Bias of OLS Estimator

We have:

$$\text{bias}(\widehat{\sigma}) = \frac{\text{cov}(r-w, \varepsilon_L - \varepsilon_K)}{\text{var}(r-w)}$$

with

$$r - w = \frac{\varepsilon_K - \xi_K}{\sigma + \gamma_K} - \frac{\varepsilon_L - \xi_L}{\sigma + \gamma_L} + \frac{(\gamma_L - \gamma_K)\mu}{(\sigma + \gamma_L)(\sigma + \gamma_K)}$$

Hence

$$\text{cov}(r-w, \varepsilon_L - \varepsilon_K) = -\left\{ \frac{\tau_K^2}{(\sigma + \gamma_K)} + \frac{\tau_L^2}{(\sigma + \gamma_L)} \right\}$$

Then

$$\text{bias}(\widehat{\sigma}) = -\frac{\frac{\tau_K^2}{(\sigma+\gamma_K)} + \frac{\tau_L^2}{(\sigma+\gamma_L)}}{\frac{\tau_K^2+\gamma_K^2 v_K^2}{(\sigma+\gamma_K)^2} + \frac{\tau_L^2+\gamma_L^2 v_L^2}{(\sigma+\gamma_L)^2} + \frac{(\gamma_L-\gamma_K)^2\tau_\mu^2}{(\sigma+\gamma_L)^2(\sigma+\gamma_K)^2}}$$

Assume $\gamma_K \to \infty$. Then

$$\text{bias}(\widehat{\sigma}) \to -\frac{\tau_L^2(\sigma + \gamma_L)}{v_K^2(\sigma + \gamma_L)^2 + \tau_L^2 + \gamma_L^2 v_L^2 + \tau_\mu^2}$$

Appendix C: Derivation of the System of Equations in Equation (11)

By straightforward calculations, using Equation (10):

$$\begin{aligned}\gamma_L \Delta^{(m)}\varepsilon\,\Delta^{(m)}\xi_L &= \Delta^{(m)}q + \sigma(\Delta^{(m)}w - \Delta^{(m)}r)(\Delta^{(m)}l - \gamma_L\Delta^{(m)}w) \\ &= \Delta^{(m)}q\Delta^{(m)}l - \gamma_L\Delta^{(m)}q\Delta^{(m)}w + \sigma\Delta^{(m)}w - \Delta^{(m)}r\Delta^{(m)}l - \sigma\gamma_L\Delta^{(m)}w - \Delta^{(m)}r\Delta^{(m)}w\end{aligned}$$

which can be reformulated as:

$$\Delta^{(m)}w\,\Delta^{(m)}(w-r) = \frac{1}{\sigma} - \Delta^{(m)}q\Delta^{(m)}w + \frac{1}{\gamma_L}\Delta^{(m)}l\,\Delta^{(m)}(w-r) + \frac{1}{\sigma\gamma_L}\Delta^{(m)}q\Delta^{(m)}l - \Delta^{(m)}\varepsilon\,\Delta^{(m)}\xi_L$$

Similarly, we obtain:

$$\Delta^{(m)}r\Delta^{(m)}(w-r) \quad = \quad \frac{1}{\sigma} - \Delta^{(m)}q\Delta^{(m)}r + \frac{1}{\gamma_K}\Delta^{(m)}k\Delta^{(m)}(w-r) + \frac{1}{\sigma\gamma_K}\Delta^{(m)}q\Delta^{(m)}k - \Delta^{(m)}\varepsilon\,\Delta^{(m)}\xi_K$$

and Equation (11) follows.

Appendix D: Simulation Algorithm

The estimators are invariant to the realized fixed time- and firm effects. Hence, when we simulate data we assume without loss of generality that all firm- and time-specific effects equal 0: $u_{Ni} = \gamma_{Nt} = \ln(\alpha_{Ni}) = 0$ (of course, we make no such assumptions when *estimating* the model on the simulated data).

Algorithm for Monte Carlo simulations:

For every $i = 1, ..., I$, $t = 1, \dots, T$ and $N = L, K$ (given θ):

1. Draw τ_{Ni}^2 from Gamma$(\nu, 1)$ and ν_{Ni}^2 from Gamma$(\nu, 1)$
2. Draw $\widetilde{\varepsilon}_{Nit}$ from $N(0, 1)$ and $\widetilde{\xi}_{Nit}$ from $N(0, 1)$
3. Set $\varepsilon_{Nit} = \sqrt{\kappa}\tau_{Ni}\widetilde{\varepsilon}_{Nit}$ and $\xi_{Nit} = \nu_{ni}\widetilde{\xi}_{Nit}$
4. Solve the non-linear system of equations in Equation (16) with respect to w_{it} and r_{it}
5. Calculate μ_{it} from Equation (17) and then (l_{it}, k_{it}) from Equation (18)

In all our reported simulation results, we use $\nu = 0.4$ and $\kappa = 1.4$, which are the averages of estimated values (over different goods) using the data described in Brasch et al. (2018).

Appendix E: Descriptive Statistics

See Figure 7 for the distribution of firms across the 11 largest subindustries within manufacturing.

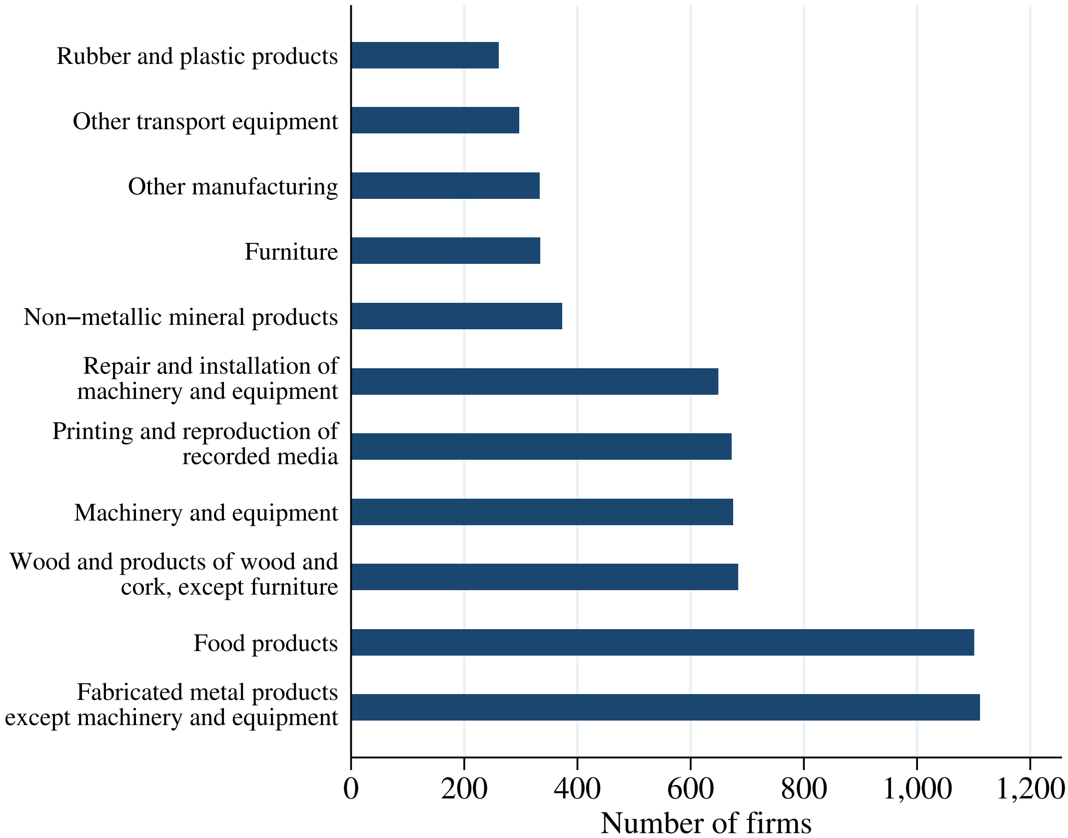

Figure 7: Distribution across subindustries, manufacturing firms

Source: Authors' own calculations using data from Statistics Norway.

Note: Industry is defined by the Standard Industrial Classification 2007 (SIC 2007).

The distribution of the age of the manufacturing firms in 2007 is shown in Figure 8, while Figure 9 shows the distribution of number of employees across all firm-year observations.

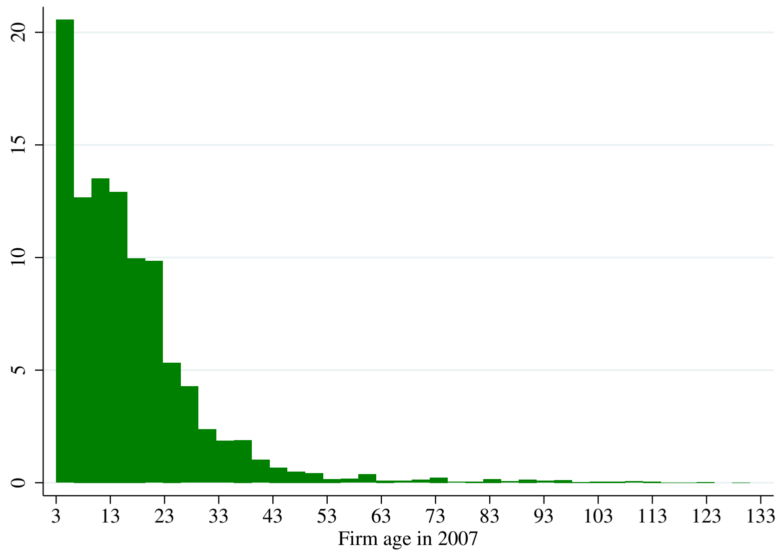

Figure 8: Age in 2007, manufacturing firms

Source: Authors' own calculations using data from Statistics Norway.

Note: Age of the firm is measured in 2007, and the sample is contingent on the firm being at least three years old in 2007.

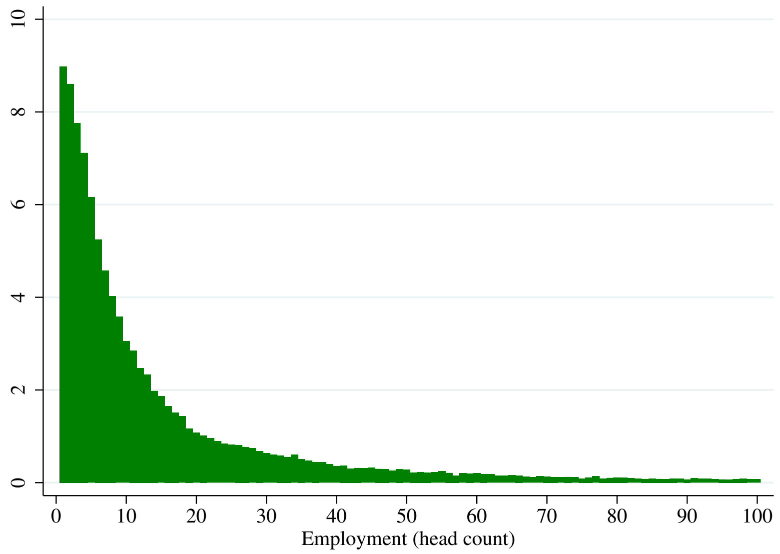

Figure 9: Number of employees, manufacturing firms

Source: Authors' own calculations using data from Statistics Norway.

Note: The vertical axis show the percent of total employment (head count) within the bin. The distribution is right-censored at 100 employees for the sake of exposition.

Firms mostly employ male workers, with an average of 45,864 annual contracted working hours for men and 11,751 annual contracted working hours for women, see Table 4 for the average and median number of annual contracted working hours by gender and educational level.

Table 4: Annual working hours, by worker type, manufacturing firms

	Working hours, average	Working hours, median
Male	45,864	11,696
Male, compulsory	16,193	4,710
Male, secondary	20,636	4,420
Male, university	9,036	312
Female	11,751	1,950
Female, compulsory	4,579	390
Female, secondary	4,030	0
Female, university	3,142	0
Total	57,615	14,469

Source: Authors' own calculations using data from Statistics Norway.
Note: Annual contracted working hours are measured at the firm level.

This distribution of the hourly wage rates for men and women is shown in Figure 10. The mean hourly wage rate for men is NOK 284, while the mean hourly wage rate for women is NOK 248.

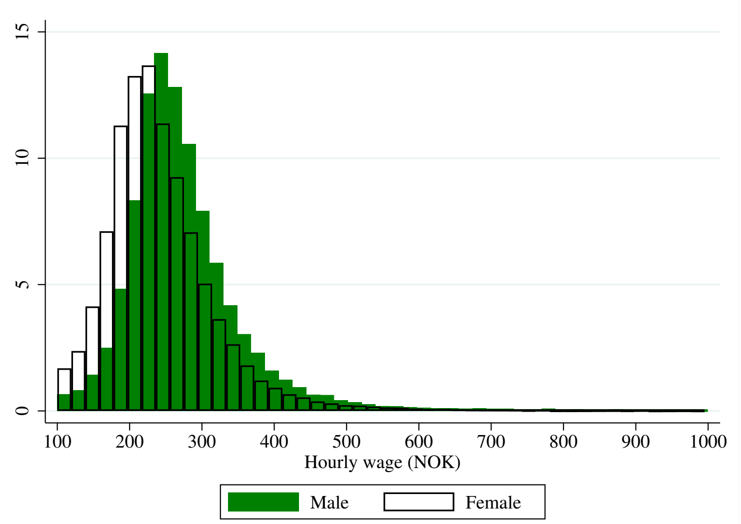

Figure 10: Hourly wage rate, by gender, manufacturing firms
Source: Authors' own calculations using data from Statistics Norway.
Note: The calculated mean hourly wage rate is measured in 2018-NOK (deflated by the CPI).