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Odd Godal and Bjart Holtsmark

International emissions trading with endogenous taxes

Abstract:
Motivated by the climate problem, this paper examines some effects of international cap & trade when national quotas result from strategic choice. In contrast to the fairly optimistic tone of closely related literature, the tenor of our results is pessimistic. We find that though an international permit market may flourish, it will mainly redistribute income. As far as emissions reductions are concerned, the classical, rather inefficient, noncooperative outcome will prevail, regardless of the presence of cap & trade.

Keywords: International emissions trading, global externality, endogenous endowments, emissions taxes.

JEL classification: C72, D62, Q54

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Address: Bjart Holtsmark, Statistics Norway, Research Department. E-mail: bjj@ssb.no

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1 Introduction

Fundamental theorems on welfare economics provide good reasons for making rights to release greenhouse gases transferable. Quite simply, voluntary exchange can harm no trading party. However, because the initial allocation of pollution rights isn't determined by nature or any benevolent planner, the arguments are a bit more delicate. In particular, it complicates matters that the amount and distribution of permits must be approved by independent governments. Moreover, those bodies' demands could depend on whether trade is permitted. Presuming trade, this paper identifies some incentives and explores their implications.

Before embarking on analyses, we must take a stand regarding several issues that affect emission negotiations. Specifically, we must model governments' behaviors, constraints, decisions, and mode of interaction. We opt to view these items as elements of a noncooperative setting. The reasons are twofold. First, more than 15 years of intense negotiations have not limited emissions to levels that reflect true cooperation. Accordingly, we find it inappropriate or premature—and somewhat naive—to preclude strategic, noncooperative behavior. Second, concerning theory and its possible impacts, we note that several authors, in particular Carbone et al. [6], have offered rather optimistic conclusions regarding the benefits of cap & trade in a noncooperative setting. See also Copeland, Taylor [10] and Helm [18]. Therefore, following their lead, we retain the name and nature of the game. That is, governments first decide the national quota. Second, permits are transferred to domestic firms and traded internationally. A government may—or may not—foresee the effects of its choice. In any case, it enforces the rule that emissions be backed by permits.[1]

Our set-up generalizes, and goes somewhat beyond, that of Helm [18] by adding taxation of emissions generating activities as a governmental instrument. This expansion of the strategic arsenal fits with what governments can and actually do. Also, careful examination of equilibria in the Helm model [18] points to good reasons for making taxation part of the game.[2]

[1]Because of the specific nature of our results, the latter assumption doesn't really bite.

[2]Here we briefly explain why. Section 3 illustrates with an example. In a game *without* taxes, as in Helm, there will always be at least one country in equilibrium, notably a permit importer, which has a cost of reducing one more unit of emissions that is *less* than its private gain from the better climate. Clearly, such a country would benefit from lowering emissions further than the level that results at this equilibrium point. The reason why such reductions do not take place is because the only way it can be accomplished in that model is by demanding a smaller initial quota in the agreement. However, this will increase the permit price, and as the country is a permit buyer, its associated import bill will go up.

The paper complements Copeland, Taylor [10] and Carbone et al. [6], who, like Helm [18], limit the analysis to endogenous endowments followed by trade. However, the two former studies include general equilibrium responses. Like Helm [18], we retain a partial set-up and concentrate on the strategic effects. Our analysis also adds to Santore et al. [24] and Bréchet and Peralta [5], who focus on taxes, taking endowments as given.

While Helm [18] reaches ambiguous conclusions as to the effects of international cap & trade, Carbone et al. [6] offer more precise insights. Applying Helm's game in a computable general equilibrium environment, they find that trading is 'crucial'. In fact, with trade, an equilibrium agreement can double the emission reductions compared with the equilibrium without any trade, and it can achieve more than half of the Pareto-optimal abatements.[3]

Our results suggest little reason for optimism. Emissions trading may flourish but mainly yields income redistribution. Even with all countries on board, overall pollution abatement could prove negligible. In short, the only major effects of permit exchange are that taxes on domestic emissions are reduced and some income is redistributed.

Of course, it is not surprising that noncooperative behavior yields inefficient outcomes. But our results, spelled out below, challenge established views. Section 3 illustrates various instances with an example. It also offers some intuitions. Section 4 leaves taxation aside and discusses some features in closely connected literature. Section 5 comments on related studies, and Section 6 concludes.

2 The model and the results

There is a fixed and finite set I of jurisdictions, seen as countries. Each country $i \in I$ has a benefit $\pi_i(e_i)$ of releasing e_i units of emissions and is adversely affected by climate change $v_i(e)$, $e := \sum_{i \in I} e_i$. There are two decision makers in every country: a government and a representative agent, named a firm.

We begin with a situation where there *is* a global permit market in which trade occurs at unit price p. The case without trading then becomes special. Until further notice, decisions are made in accordance with the following timetable.

Thus, such a country has an incentive to implement a positive tax on domestic emissions.

[3] They also provide, in a supplement, some results with current but *exogenous* taxes. They report that their main results are insensitive to this extension. We will not compare our results with those of Copeland and Taylor [10] until Section 4. Their more recent paper [12], is less relevant to our study, as permits in that study are given exogenously.

Date 1: Each and every government demands ω_i units of permits that it finds acceptable—in fact optimal for itself. These permits are then transferred to the domestic firm—precisely how is left unspecified. At the same time, the government levies a tax t_i on domestic emissions.
Date 2: The firm chooses its emissions level e_i, pays amount $t_i e_i$ to the government, and because compliance is assumed enforced, its choice of e_i defines its action in the global permit market.[4]

We put no restrictions on the sign of any variable of choice: negative emissions mean that more carbon is captured from the atmosphere than is released. Plainly, this is an unlikely equilibrium outcome with current technologies. Negative allocation of permits entails committing to being more than a 'carbon neutral' society. That is, more permits must be bought than the emissions that can occur. This may well happen for reasonable parameters in some games. A negative tax is simply a subsidy and will not be precluded in our analysis.

It is commonplace and convenient to assume that $\pi_i(e_i)$ and $-v_i(e_i)$ are strictly monotone, strictly concave, smooth, etc. Such assumptions are not sufficient to guarantee equilibrium existence in several of the games to be discussed, including our own. The upshot is that they aren't really necessary either. In any case, equilibrium existence will not be addressed here.[5]

Standing assumption in this section (*On equilibrium existence*) In every game considered, a Nash equilibrium exists.

We next define equilibria with and without trade.

Definition 1
• (*Tax-and-trade equilibrium*) A collection $(\omega_i, t_i, e_i)_{i \in I}$ and a permit price p

[4] One could of course consider other timetables, in particular a three-stage game where taxes are chosen second. While such a scenario seems perfectly reasonable, the analysis becomes so complicated that we have nothing to offer. In particular, great difficulty would result if we insisted on strategic behavior, subgame perfection and not sacrificing the generality of π_i and v_i.

[5] The main reason why equilibrium existence becomes a delicate issue is because we know little more about the endogenous price curve p other than that it slopes downwards. Thus, the same issues that arise in a standard Cournot game also arise here. In addition, to complicate matters further we have players on both sides of the market. Nevertheless, in a quadratic–linear environment as in Section 3, equilibrium existence appears to be granted.

is declared a *tax-and-trade* equilibrium iff (ω_i, t_i)

$$\text{maximizes}\left\{\pi_i(e_i) - v_i(e) + p \cdot (\omega_i - e_i)\right\} \tag{1}$$

for each government $i \in I$. Moreover, e_i

$$\text{maximizes}\{\pi_i(e_i) - t_i e_i + p \cdot (\omega_i - e_i)\} \tag{2}$$

for each firm $i \in I$, and p clears the emissions market:

$$\sum_{i \in I} e_i = \sum_{i \in I} \omega_i. \tag{3}$$

When aiming for (1), the government takes the choices of other governments as given. How they behave is detailed below. Each and every firm treats ω_i, t_i and p parametrically.

• (*No-trade equilibrium*) A profile $(t_i, e_i)_{i \in I}$ is a *no-trade* equilibrium iff it is a *tax-and-trade* equilibrium with decision variable ω_i removed from (1) together with market revenues in (1) and (2), as well as dispensing with the clearing condition (3).

Note that by (1) a government has no particular interest in tax revenues or expenses—it merely cares for the domestic product.

The *no-trade* equilibrium yields the classical $\pi_i'(e_i) = v_i'(e)$ for all $i \in I$, which will be referred to as the **no-trade condition**. In our partial framework, such an equilibrium may come about as a result of policies other than taxes, for instance via a family of closed domestic permit markets. We next relate our game to some established ones in the literature.

Definition 2

• (*No-tax equilibrium*) A collection $(\omega_i, e_i)_{i \in I}$ and a permit price p is a *no-tax* equilibrium iff it is a *tax-and-trade* equilibrium for *fixed* $t_i = 0$ for all $i \in I$.

• (*Exogenous-endowment equilibrium*) A profile $(t_i, e_i)_{i \in I}$ and a permit price p is an *exogenous-endowment* equilibrium iff it is a *tax-and-trade* equilibrium for *exogenous* $(\omega_i)_{i \in I}$.

• (*Pareto optimum*) Emissions $(e_i)_{i \in I}$ are *Pareto optimal* iff $\pi_i'(e_i) = \sum_{j \in I} v_j'(e)$ for all $i \in I$.

The *no-tax* equilibrium is none other than Helm's [18] and—in terms of governments' strategy sets—also that of Carbone et al. [6] and Copeland and Taylor [10]. Helm [18] also has a no-trade equilibrium that is essentially the same as ours. The *exogenous-endowment* equilibrium is fairly close to that of Santore et al. [24, Section 3.2] and Bréchet and Peralta [5]. The *Pareto optimality* condition is because of Samuelson [23] and is included for the record.

When it comes to the behavior of governments, we examine two scenarios:

Definition 3 (*On behavioral mode*) A government is said to be **strategic** if it fully accounts for the effects on p of its own ω_i and t_i, i.e., the terms-of-trade effects. It is said to be **price taking** if it consistently treats p as an exogenous parameter.

Our main result follows.

Theorem (On emissions) *Suppose all governments are strategic and that* $(e_i)_{i\in I}$ *is an emissions profile in a tax-and-trade equilibrium. This profile thus satisfies the no-trade condition:* $\pi_i'(e_i) = v_i'(e)$ *for all* $i \in I$.

The proof of this result, together with those for most of the others, is in the appendix. Before comparing this result to those in the literature, an implication merits mention.

Corollary (On welfare) *Suppose all governments are strategic and that an equilibrium emissions profile is unique, both with trade and without. Then the benefits and costs of emissions in every country are unaffected by the presence of trade.*

It's most natural to compare our main result with those in Helm [18]. He finds that the effects of cap & trade on aggregate emissions are data dependent. Moreover, the welfare effects can go either way. Our result only depends on the existence of equilibrium. Furthermore, because of the nature of our result, the global welfare implications of having an international permit market are unambiguous. There are none.

One may perhaps wonder whether there would be any trade in our *tax-and-trade* equilibrium. Indeed, when countries differ in π_i and v_i, the outcome typically involves trade (see also Section 3). Such trade will of course promote efficiency for the given endowments, but not compared with the policies that would arise without trade.

We next inquire how robust our main result is against some governments being price takers, believing that $\frac{\partial p}{\partial \omega_i} = \frac{\partial p}{\partial t_i} = 0$; see Definition 3.

Proposition 1 (On the effects of market power) *Suppose at least one gov-*

ernment acts as price taker and that $(e_i)_{i\in I}$ *is an emissions profile in an equilibrium with tax and trade. Such a profile satisfies the no-trade condition:* $\pi_i'(e_i) = v_i'(e)$ *for all* $i \in I$.

This result shows that the first assumption in our theorem may be relaxed, and it implies that there are no real effects of market power other than a possible redistribution of income. However, the tax *levels* may be affected by behavioral mode, as the permit price, in the presence of price takers, becomes nil (see the proof). Nevertheless, the price for emissions faced by firms remains unaffected. At first sight, this result appears identical to Copeland and Taylor's [10, Proposition 8] although they are in a general equilibrium environment without taxes. As it turns out, this isn't quite so; see Section 4.

Now, following Carbone et al. [6] and Helm [18] we ask: Which countries, if any, would find it interesting to participate in a trading regime? To address that question, we add a stage to the game at **Date 0**. Here, each government makes a noncooperative regime choice $r_i \in \{g, l\}$ of qualitative nature. If the government chooses $r_i = g$ for 'global', then it decides to participate in an international trading regime and belongs to set G. On the other hand, if it chooses $r_i = l$ for 'local', it stands alone and belongs to set L. Thus, I is the disjoint union of G and L. Temporarily replace I with G in the market clearing condition (3), and write $\Pi_i(r_i, r_{-i})$ for the value function associated with a government objective when $r_i \in \{g, l\}$. Here, and elsewhere, $-i := I \diagdown \{i\}$.

Definition 4 (*An equilibrium agreement*) A collection $(r_i^*)_{i\in I}$ is declared a Nash *equilibrium agreement* iff

$$\Pi_i\left(r_i^*, r_{-i}^*\right) \geq \Pi_i\left(r_i, r_{-i}^*\right) \tag{4}$$

for all $r_i \in \{g, l\}$ and all $i \in I$.

Proposition 2 (On an equilibrium agreement) *For every given* $(r_i)_{i\in I}$ *suppose there exists a unique equilibrium in the game occurring at Date 1. Then, if some* $(r_i)_{i\in I}$ *is an equilibrium agreement, then the associated emissions profile* $(e_i)_{i\in I}$ *satisfies the no-trade condition:* $\pi_i'(e_i) = v_i'(e)$ *for all* $i \in I$.

Whether a government gains or loses by belonging to *any* list G depends merely on whether it ends up as a permit seller or buyer. If indeed there *is* trade among those listed in G, at least one agent must be a buyer, having $p \cdot (\omega_i - e_i) < 0$. If this country was listed in L, it would get exactly the same emissions benefits and climate impacts, but market expenses would disappear. Hence, an *equilibrium* list G cannot contain any permit buyers and is therefore not compatible with an active market. Finally, opening up for multiple regional permit markets clutters notation considerably, with no gain in insight of which we are aware.

The rest of the paper returns to the timetable starting at **Date 1,** and assumes that all governments take part in international permit exchange. Two minor results remain in this section. With one exception, notably (9), economic intuition and established literature are merely confirmed, mostly without mention. The results follow from the proof of our theorem and may be skipped or postponed.

Lemma 1 (Characterization of buyers and sellers) *Suppose all governments are strategic. Then, in an equilibrium with trade, the following are equivalent: country i*
- *is a permit exporter,* $e_i < \omega_i$,
- *has a lower marginal abatement cost than permit price,* $\pi_i'(e_i) < p$,
- *and subsidizes domestic emissions,* $t_i < 0$.

The same results apply to a permit buyer if the inequalities are reversed and the wording changed accordingly. Applying our main result, the second bullet point states that a permit exporter has marginal climate damage that is below the permit price, similar to Helm [18]. The equivalency between the first and last bullet points is similar to that in Santore et al. [24].

In preparation for what comes next, write

$$s_i := \frac{1}{\pi_i''(e_i)} \text{ and } S := \sum_{i \in I} s_i \tag{5}$$

evaluated at equilibrium e_i.

Lemma 2 (Characterization of behavioral mode) *For given* $(\omega_i, t_i)_{i \in I}$, *suppose there is a unique* $(e_i)_{i \in I}$ *and clearing price* p, *and that* $\pi_i(e_i)$ *is strictly concave and twice continuously differentiable in the neighborhood of optimal* e_i. *Then, and with apologies for abusing notations, a strategic government* i *behaves consistently with setting*

$$\frac{\partial p}{\partial \omega_i} = \frac{1}{S} < 0 \text{ and } \frac{\partial p}{\partial t_i} = -\frac{s_i}{S} \in (-1, 0); \tag{6}$$

$$\frac{\partial e_i}{\partial \omega_i} = s_i \frac{\partial p}{\partial \omega_i} \in (0,1) \;\; and \;\; \frac{\partial e_i}{\partial t_i} = s_i \left(\frac{\partial p}{\partial t_i} + 1 \right) < 0; \tag{7}$$

$$\sum_{j \in I} \frac{\partial e_j}{\partial \omega_i} = S \frac{\partial p}{\partial \omega_i} = 1 \;\; and \;\; \sum_{j \in I} \frac{\partial e_j}{\partial t_i} = s_i + S \frac{\partial p}{\partial t_i} = 0. \tag{8}$$

Moreover, a price-taking government i behaves in accordance with the knowledge that

$$\frac{\partial e_i}{\partial \omega_i} = 0, \; \frac{\partial e_i}{\partial t_i} = s_i, \; \sum_{j \in I} \frac{\partial e_j}{\partial \omega_i} = 0 \;\; and \;\; \sum_{j \in I} \frac{\partial e_j}{\partial t_i} = s_i. \tag{9}$$

(6)–(8) and the first two equalities in (9) are standard and in line with economic intuition. What appears to contrast with the findings of Copeland and Taylor [10] is the right hand side of the third equality in (9). That is, a price-taking government does not understand that demanding another permit from a treaty will have an effect on global emissions. To see why this is so, it is most expedient to look at the origin of the first result in (8).

A firm $j \in I$ chooses emissions $e_j = (\pi'_j)^{-1} (p + t_j)$. Differentiating aggregate emissions with respect to ω_i, one gets $\sum_{j \in I} \frac{\partial e_j}{\partial \omega_i} = \sum_{j \in I} \frac{1}{\pi''_j (e_i)} \frac{\partial p}{\partial \omega_i}$. Anyone consistently seeing p as a parameter must believe that the last sum vanishes. In economic terms, emissions adapt in fact because firms around the globe will face lower prices and therefore adjust. But when the price effect is not understood by the government, the right hand side of the third equality in (9) is the result. Finally, the last part of (9) suggests that a price-taking government behaves consistently by believing that global emissions fall if its domestic tax is raised. Those readers interested in some generalizations may consult [13].

Remark (*On second stage uniqueness*) When it comes to the key hypothesis on uniqueness in Lemma 2, one may ask: How strong is it? Recall that we are in a quasilinear and externality-free environment at the second stage of the game. Therefore, the fixed point problem of finding a profile $(e_i)_{i \in I}$ and clearing price p is reduced to the *single* optimization problem

$$\max_{(e_i)_{i \in I}} \left\{ \sum_{i \in I} \{ \pi_i (e_i) - t_i e_i \} : \sum_{i \in I} e_i = \sum_{i \in I} \omega_i \right\}$$

with p being the familiar Lagrange multiplier; see, for example, Shapley and Shubik [25, Footnote 3] or Qin et al. [27] for more recent material. Hence, a sufficient but not necessary condition for uniqueness is that all $\pi_i(e_i)$ are strictly concave and at least one of them is continuously differentiable. Clearly, if we do not have uniqueness at the second stage, then our first stage game isn't even well defined. Nevertheless, these issues will not be addressed further here (see also Section 5).

3 An example

This section offers an example for the sole purpose of illustration. There are two countries with strategic governments. Emissions benefits $\pi_i(e_i)$ are quadratic, climate costs $v_i(e)$ are linear, and the parameters have been selected to facilitate exposition.

Table 1. The example.

Country	Benefits, $\pi_i(e_i)$	∂Benefits, $\pi_i'(e_i)$	Damages, $v_i(e)$
1	$4e_1 - \frac{1}{2}(e_1)^2$	$4 - 1e_1$	$1(e_1 + e_2)$
2	$8e_2 - \frac{3}{2}(e_2)^2$	$8 - 3e_2$	$3(e_1 + e_2)$

For interpretation, country 1 may be seen as a high-emissions country, having four units of business-as-usual emissions, not greatly affected by climate change and with a marginal cost of reducing emissions that grows fairly slowly. Conversely, country 2 has lower business-as-usual emissions but is more severely affected by climate change and has a marginal cost of reducing emissions that grows more rapidly.

Pareto optimum Here, each government sets a tax that reflects the social cost, yielding $\pi_i'(e_i) = v_1'(e) + v_2'(e)$ for both $i = 1, 2$. The results are as follows.

Table 2. Pareto optimum.

Country	t_i	e_i	$\pi_i'(e_i)$	$\pi_i(e_i)$	$-v_i(e)$	Total payoff
1	4	0	4	0	$-1\frac{1}{3}$	$-1\frac{1}{3}$
2	4	$1\frac{1}{3}$	4	8	-4	4
Total		$1\frac{1}{3}$		8	$-5\frac{1}{3}$	$2\frac{2}{3}$

As is well known, this situation isn't robust against individual deviations. Therefore, we continue with the classical noncooperative outcome.

No-trade equilibrium In our version of the game, each government sets a tax that maximizes domestic welfare implying $\pi_i'(e_i) = t_i = v_i'(e)$ for each $i = 1, 2$. Alternatively, it caps national emissions via a domestic permit market.

Table 3. No-trade equilibrium.

Country	t_i	e_i	$\pi_i'(e_i)$	$\pi_i(e_i)$	$-v_i(e)$	Total payoff
1	1	3	1	$7\frac{1}{2}$	$-4\frac{2}{3}$	$2\frac{5}{6}$
2	3	$1\frac{2}{3}$	3	$9\frac{1}{6}$	-14	$-4\frac{5}{6}$
Total		$4\frac{2}{3}$		$16\frac{2}{3}$	$-18\frac{2}{3}$	-2

Alternatively, with separate domestic permit markets instead of taxes, one gets $\omega_1 = 3$ and $\omega_2 = 1\frac{2}{3}$. As usual, it follows that the aggregate emissions are inefficiently allocated, because they produce benefits at different margins. This motivates bringing in a permit market as in Helm [18].

No-tax equilibrium Here taxes are exogenously fixed to zero and each government decides noncooperatively on an initial quota ω_i, understanding that there will be subsequent exchange. Together with market clearing, the first-order optimality conditions are given by

$$\pi_i'(e_i) + \frac{\partial p}{\partial \omega_i}(\omega_i - e_i) - v_i'(e) = 0 \text{ and } \pi_i'(e_i) = p, \tag{10}$$

with $\frac{\partial p}{\partial \omega_i} = -\frac{3}{4}$ for our specific parameters. The outcome follows next.

Table 4. No-tax equilibrium, Helm [18]. The permit price equals 2.

Country	ω_i	e_i	$\pi_i'(e_i)$	$\pi_i(e_i)$	$p(\omega_i - e_i)$	$-v_i(e)$	Total payoff
1	$3\frac{1}{3}$	2	2	6	$2\frac{2}{3}$	-4	$4\frac{2}{3}$
2	$\frac{2}{3}$	2	2	10	$-2\frac{2}{3}$	-12	$-4\frac{2}{3}$
Total	4	4		16	0	-16	0

Compared with the *no-trade* case, the low-damage country 1 increases its endowment; country 2 reduces it. Emissions are allocated efficiently. Moreover, that total falls and climate damage is less, leaving total welfare change positive. But, as pointed out by Helm [18], these results are parameter sensitive, and for others, total emissions as well as welfare may show the opposite tendency. Moreover, in accordance with Helm [18, Proposition 1] the country less affected by climate change becomes the permit exporter.

Note that in the *no-tax* equilibrium, the permit importing country 2 has marginal damage that is greater than its own cost of reducing a ton of emissions. From (10), we see that this is a general feature. Hence, ceteris paribus, country 2 would benefit from lowering its emissions further. However, in the absence of an emissions tax, the only way this can be accomplished is by cutting its quota. That raises the import bill. This is why it doesn't pay off at an equilibrium point.

Tax-and-trade equilibrium Finally, each government decides on both a quota and a tax. Then firms exchange permits in a common marketplace and pay a tax on their own emissions to their respective governments.

Table 5. Tax-and-trade equilibrium. The permit price equals $2\frac{1}{2}$.

Country	ω_i	t_i	e_i	$\pi_i'(e_i)$	$\pi_i(e_i)$	$p(\omega_i - e_i)$	$-v_i(e)$	Total payoff
1	$3\frac{1}{2}$	$-1\frac{1}{2}$	3	1	$7\frac{1}{2}$	$1\frac{1}{4}$	$-4\frac{2}{3}$	$4\frac{1}{12}$
2	$1\frac{1}{6}$	$\frac{1}{2}$	$1\frac{2}{3}$	3	$9\frac{1}{6}$	$-1\frac{1}{4}$	-14	$-6\frac{1}{12}$
Total	$4\frac{2}{3}$		$4\frac{2}{3}$		$16\frac{2}{3}$	0	$-18\frac{2}{3}$	-2

As in Helm [18], the low-damage country 1 will export permits. Also, its initial quota is adjusted upward compared with the *no-trade equilibrium.* Finally, emissions there become subsidized. The converse is true for country 2. Most importantly is that the variables of prime economic interests—emissions, benefits and costs—remain unchanged compared with the *no trade* case (Table 3).

If at least one of the governments were modeled as a price taker when choosing t_i and ω_i, then the outcome would also return to the *no-trade* case given in Table 3 with $\omega_1 + \omega_2 = 4\frac{2}{3}$ and illustrating Proposition 1. Lastly, returning to strategic governments, and illuminating Proposition 2, according to Table 5 the permit importing country 2 is worse off compared with the *no-trade* equilibrium. Hence, if we include a decision at **Date 0** dealing with participation in international trade, then country 2 would chose not to participate—a decision that once again brings us back to the *no-trade* equilibrium.

4 Some remarks on the related literature

This section returns to the case without taxes. Some properties emerge that we find a bit discomforting in the most closely related literature, notably Helm [18], Carbone et al. [6] and Copeland and Taylor [10]. The problems addressed arise if not all governments are strategists.

4.1 Partial equilibrium

Helm [18] presumes that all governments are strategic. In practical terms, this means that all countries involved—and irrespective of their size and other characteristics—fully account for the effects on the international permit price by choice of quota. We next inquire how price-taking behavior affects equilibrium.

Proposition 3 (On price-taking behavior without taxes) *Consider a no-tax game as given in the first item of Definition 2 and taken from Helm [18]. Suppose, as in Helm's study, that $\pi_i'(e_i) > 0$ for all $i \in I$, and that at least one government is a price taker. Then, no equilibrium exists.*

The proof of this result fits best here. Together with market clearing, the first-order *necessary* optimality conditions in a no-tax equilibrium at the first and second stages, respectively, read

$$\pi_i'(e_i)\frac{\partial e_i}{\partial \omega_i} + \frac{\partial p}{\partial \omega_i}(\omega_i - e_i) + p\left(1 - \frac{\partial e_i}{\partial \omega_i}\right) - v_i'(e)\sum_{j\in I}\frac{\partial e_j}{\partial \omega_i} = 0 \tag{11}$$

and

$$\pi_i'(e_i) = p. \tag{12}$$

Because the first and third equality in (9) are not affected by the presence of taxes, they can be directly plugged into (11) to get $p = 0$. By assumption, $\pi_i'(e_i) > 0$, which contradicts (12). $\square$

It may well be that equilibrium existence can be reestablished if the assumption $\pi_i'(e_i) > 0$ is replaced with the weaker $\pi_i'(e_i) \geq 0$. Indeed, such a modification is quite reasonable as a permit is a right to pollute, not an obligation. Having more than enough will add nothing to profits. But if e_i is so large that $\pi_i'(e_i) = 0$, then $\pi_i''(e_i)$ naturally vanishes as well. If strategic agents are also present, the key assumption in Lemma 2 may no longer hold, so that the inverse function theorem does not immediately apply. Where this will lead, we do not know.

The above result has made use of $\sum_{j\in I} \frac{\partial e_j}{\partial \omega_i} = 0$ for a price taker; refer to (9). If one instead follows Copeland and Taylor [10] in setting $\sum_{j\in I} \frac{\partial e_j}{\partial \omega_i} = 1$ for a price taker, then we get from (11) that $v_i'(e) = p$, which by (12) also must $= \pi_i'(e_i)$ for every nonstrategist. Now suppose that v_i is affine but not the same for all governments that ignore their market power. Then, few reasonable results seem to appear.

4.2 General equilibrium

In contrast to Helm [18], Carbone et al. [6] do *not* assume that all governments are strategic. Their general equilibrium computations exclude all but five strategic players from the negotiations and subsequent trade. They argue that many of the countries in this excluded 'rest of the world' are unlikely to pursue strategic climate change policies. However, as we are in a pure public good situation, we may become worried about getting rid of most participants from the outset, as well as aggregating many countries into regions before playing the game.[6] Nevertheless, if we insist that governments of 'small' countries are nonstrategic, then it could be posited that they take prices as given. If this is done in Carbone et al. [6, p. 272, system (11)] we get the same qualitative result as for Helm [18] discussed above.

Although Helm [18] and Carbone et al. [6] do not consider price-taking governments, Copeland and Taylor [10] do. They report in Section IV C, page 727, that with such governments

"... allowing trade in pollution permits has no effect on production, incomes,

[6] Their five strategic players are: USA, Japan, China, western Europe and the former Soviet Union. Only the first three are sovereign states.

pollution, or welfare. [...] Despite the fact that pollution permits are potential revenue-generators for governments, opening up international trade in pollution permits does not create an incentive to increase their supply beyond the levels of the pure goods-trading equilibrium. This is because pollution is a pure public bad."

The formal analysis on this issue appears in their Section VII, where governments understand the terms-of-trade effects. The first-order condition for a government taking care of its market power when choosing its permit endowment is given by differentiating the indirect utility function in their display (25) to get

$$\frac{1}{L_i + \rho E_i}\left(\rho + E_i \frac{\partial \rho}{\partial E_i}\right) - \bar{\theta}\frac{1}{\rho}\frac{\partial \rho}{\partial E_i} - \beta \left(E^w\right)^{\gamma-1} \sum_{j \in I} \frac{\partial E_j}{\partial E_i} = 0. \tag{13}$$

What matters here, in terms of notation, is that ρ is the permit price, E_i the chosen permit allocation by government i, E^w the global emissions, and $\beta \left(E^w\right)^{\gamma-1}$ the marginal impact of climate change in the country under consideration, β and γ being parameters, the same for all. If replacing $\sum_{j \in I} \frac{\partial E_j}{\partial E_i}$ in (13) with 1, just as in (8), we then indeed obtain their stated condition (on page 731) that

$$\rho - \beta \left(E^w\right)^{\gamma-1} I_i - \frac{\theta_i - \bar{\theta}}{E^w} I_i = 0, \tag{14}$$

where the last term captures the terms-of-trade effects. As they point out, that effect vanishes if governments are price takers. It also vanishes when aggregating (14) across all countries when governments are strategic, and the conclusion is reached on page 732 that "the level of world pollution is unaffected by the recognition of terms of trade effects!". If we now adopt the logical consequence of being a price taker in not understanding that emissions by all firms adapt to lower prices, see Lemma 2 (with comments), then it seems that this will correspond to setting both $\sum_{j \in I} \frac{\partial E_j}{\partial E_i}$ and $\frac{\partial \rho}{\partial E_i} = 0$ throughout (13). If we do this, the abovementioned condition reduces to $\rho = 0$, just as in Helm [18] and Carbone et al. [6]. Conversely, if price-taking governments treat $\sum_{j \in I} \frac{\partial E_j}{\partial E_i} = 1$, then $\rho = \beta \left(E^w\right)^{\gamma-1} I_i$ is obtained for each and every price-taking government i. This can only be possible if all countries have the same income $I_i = L_i + \rho E_i$, which appears to be the case in Copeland and Taylor's two-type country world.

To sum up, we are not sure which message we should take home from Copeland and Taylor's [10] analysis, regardless of the definition of price-taking behavior.

5 On other related literature

The strand of literature this paper belongs to—Helm [18], Carbone et al. [6] and Copeland and Taylor [10]—has an international permit market at center stage and brushes away many of the issues that are at the core of other material on international environmental agreements, whether in some form of coalition or partition. See, for example, Barrett [3], Chander and Tulkins [8], Finus [14] and Hoel [19] for early material; see Asheim et al. [1], Chander [7] and Germain et al. [17] for more recent.

In terms of the tax instrument, our analysis is most closely related to Santore et al. [24] and Bréchet and Peralta [5], which both work in a partial equilibrium framework. Some papers study both taxes and quotas, although separately, such as Ishikawa and Kiyono [20], but they have no environmental externality. In terms of transboundary pollution and taxes within a general equilibrium environment; see Copeland and Taylor [9], [11].

In the broader perspective, but without environmental externalities, our paper draws on two strands of literature. One—well known—deals with tariffs and international trade. The other, also well-established but seemingly lesser known, concerns pure exchange economies manipulated via endowments. Of particular relevance to our paper are Aumann and Peleg [2], Gabszewicz [15] and Postlewaite [21]. In that family of studies, however, everyone has an endowment given by nature. But when it can be manipulated on the market, the analytical apparatus shares many traits with ours.

The reader may recall that we have not dealt with the fundamental issue of equilibrium existence. This is a delicate matter, although quasilinear utilities, separable climate impacts and other convenient assumptions we have made are likely to help. When it comes to the—presumably easier—case of pure, externality-free, exchange economies manipulated via endowments, there are few results available. One is Safra [22], but he considers large economies. Bonnisseau and Florig [4] is another, but they limit attention to linear exchange economies. Peck et al. [26] is a third, but they consider Shapley and Shubik's strategic market game [25], which is very differently formulated from ours. Existence of equilibrium in tariff games is addressed in Wong [28]. It appears though that if one is willing to broaden the notion of equilibrium beyond standard Nash, see for example [4] and references therein, then equilibrium existence may become easier to guarantee.

6 Concluding remarks

We hesitate in drawing policy implications from our exercise. What is notable is that our results—in contrast to those in Helm [18]—are sharp, and they're markedly different from the specific and rather optimistic ones in Carbone et al. [6]. In any case, it seems worth emphasizing that all the good properties of cap & trade programs when applied to environmental problems confined to a single jurisdiction do not immediately carry over to an international setting, in neither theory nor practice thus far.

Appendix

Proof of Lemma 2 The first-order necessary optimality conditions at the second stage of the game are

$$\pi_i'(e_i) = p + t_i \text{ for each firm } i \in I \text{ and} \sum_{i \in I} e_i = \sum_{i \in I} \omega_i. \tag{A.1}$$

Under the assumed conditions, there exists, locally, a differentiable function $f_i := (\pi_i')^{-1}$ for each $i \in I$ such that

$$e_i = f_i(p + t_i) \tag{A.2}$$

with $f_i' = \frac{1}{\pi_i''}$. From (A.2), it then follows that

$$\frac{\partial e_i}{\partial \omega_i} = \frac{1}{\pi_i''(e_i)} \frac{\partial p}{\partial \omega_i} \text{ and } \frac{\partial e_i}{\partial t_i} = \frac{1}{\pi_i''(e_i)} \left(\frac{\partial p}{\partial t_i} + 1 \right). \tag{A.3}$$

Market clearing requires

$$\sum_{j \in I} f_j(p + t_j) = \sum_{j \in I} \omega_j. \tag{A.4}$$

Differentiating the last equality throughout with respect to ω_i yields

$$\frac{\partial p}{\partial \omega_i} = \frac{1}{\sum_{j \in I} \frac{1}{\pi_j''(e_j)}} \text{ and } \frac{\partial p}{\partial t_i} = -\frac{\frac{1}{\pi_i''(e_i)}}{\sum_{j \in I} \frac{1}{\pi_j''(e_j)}}, \tag{A.5}$$

which takes care of (6). Apply (A.5) in (A.3) to get (7). Statement (8) follows by differentiating the left hand side of (A.4) with respect to ω_i and t_i, respectively. Finally, we obtain (9) by setting $\frac{\partial p}{\partial \omega_i}$ and $\frac{\partial p}{\partial t_i} = 0$ in (7) and (8), respectively. □

Proof of Lemma 1 By the necessary first-order optimality conditions at the first stage of the game, a profile $(\omega_i, t_i)_{i \in I}$ satisfies

$$\pi_i'(e_i) \frac{\partial e_i}{\partial \omega_i} + \frac{\partial p}{\partial \omega_i} (\omega_i - e_i) + p \left(1 - \frac{\partial e_i}{\partial \omega_i} \right) - v_i'(e) \sum_{j \in I} \frac{\partial e_j}{\partial \omega_i} = 0 \tag{A.6}$$

and

$$\pi_i'(e_i)\frac{\partial e_i}{\partial t_i} + \frac{\partial p}{\partial t_i}(\omega_i - e_i) + p\left(0 - \frac{\partial e_i}{\partial t_i}\right) - v_i'(e)\sum_{j \in I}\frac{\partial e_j}{\partial t_i} = 0 \tag{A.7}$$

for each government $i \in I$. Combining this with Lemma 2 and definition (5), it then follows that (A.6) and (A.7) reduce to

$$(\pi_i'(e_i) - p)\frac{s_i}{S} + p + \frac{1}{S}(\omega_i - e_i) - v_i'(e) = 0 \tag{A.8}$$

and

$$(\pi_i'(e_i) - p)\left(1 - \frac{s_i}{S}\right) - \frac{1}{S}(\omega_i - e_i) = 0, \tag{A.9}$$

respectively. As $S < 0$ and $\left(1 - \frac{s_i}{S}\right) \in (0, 1)$, it follows from (A.9) that a permit seller must have $\pi_i'(e_i) < p$, which takes care of the first equivalency. Combine that with $\pi_i'(e_i) - p = t_i$ from (A.1) to get the last. □

Proof of Theorem Add the left hand sides of (A.8) and (A.9). □

Proof of Proposition 1 The first-order necessary optimality conditions for strategic agents, if any, given in (A.6) and (A.7) are not affected by the presence of other agents acting as price takers. For anyone of the latter type, plug (9) into (A.6) and (A.7). The last two conditions then reduce to

$$p = 0 \text{ and } \pi_i'(e_i) - p - v_i'(e) = 0, \tag{A.10}$$

respectively, which completes the proof. □

Proof of Proposition 2 A country $i \in L$ gets a payoff $\Pi_i(l, \cdot)$ that satisfies $\pi_i'(e_i) = v_i'(e) = t_i$. Next, replace I with G in (3), (5), (8), (A.1), (A.4) and (A.5), but importantly not in the definition of $e := \sum_{i \in I} e_i$, appearing in (A.6) and (A.7). One then gets, by the same argument as in the proof of Theorem 1, that $\pi_i'(e_i) = v_i'(e) = p + t_i$, for all $i \in G$. □

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