

*Kyrre Stensnes*

**Equity versus Efficiency in Public  
Pension Schemes**  
Microsimulating the Trade-off

**Abstract:**

Any contribution to a pay-as-you-go pension system may be considered mandatory savings to the extent that it gives a claim to a future benefit. Contributors to the economic literature have argued that an increase in this savings component will lower implicit marginal tax rates, thereby reducing distortions in the labour market. However, the efficiency gain created by increasing the actuarial component of pensions may come at the cost of increased inequality in pension benefits. The trade-off between efficiency and equity is not easy to quantify in actual public pension schemes whose benefit functions intrinsically exhibit non-linear characteristics. This paper develops a framework to quantify this trade-off in a fully specified pension system using dynamic micro-simulation modelling. The methodology is then applied to five different pension schemes actually proposed for Norway. The results demonstrate the relevance of this study: The improvement of the equity-efficiency trade-off either does not materialise, as in one case, or is arguably driven by a different factor than advocated by policymakers.

**Keywords:** Pension reform, social security, equity, labour supply, and efficiency.

**JEL classification:** H53, H55, D31, J22

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# 1. Introduction

Are labour taxes levied to finance pay-as-you-go pension benefits different from other taxes? Specifically, are contributions to the actuarial dimension of the pension system only mandatory savings that do not distort labour supply? Disney (2004) argues that contributions to public pension programmes may differ from other taxes levied on households because these contributions also involve a claim to future benefits. A part of the contribution should thus be perceived by the individual and regarded by the analyst as savings. To the extent that a policy change can increase the savings part of pension contributions, distortions in the labour market can be reduced – a view outlined in the economic literature by among others Lindbeck and Persson (2003), Disney (2004) and Lindbeck (2006).

Recent pension reforms in Norway, Sweden, Finland, Italy, Latvia, and Poland, aiming to lower implicit tax rates on labour income, have tightened the link between earnings and pension entitlements. As long as the average contribution rate to the pension system remains unchanged, this offers a partial positive substitution effect on labour supply without any counteracting income effect. However, as is well known from the economic literature, efficiency improvements may come at the cost of increased inequality.

While the notions of efficiency and equity are simple in theory, they are not easy to quantify in actual public pension schemes. Most pension systems involve complex rules and non-linear features in the accumulation of entitlements. For instance, many countries calculate benefits on the basis of the  $n$  last years or  $m$  years with highest labour income. While such a rule is clearly non-actuarial, its redistributive effect is far from obvious. If such a rule were to be replaced by a more linear accumulation of entitlements, to what extent does it (1) imply a less egalitarian distribution, and (2) make the equity-efficiency trade-off more favourable, thereby enabling the realisation of Pareto improvement as suggested by the literature? Answers to such questions obviously depend on the specific features of the pension scheme in place.

This paper develops a framework to quantify the efficiency-equity (EE) trade-off in a fully specified pension system using dynamic micro-simulation modelling. This methodology allows us to combine necessary population heterogeneity with an exact replication of the detailed pension system rules. I apply the framework to answer the two questions posed above for five different pension schemes applied or being seriously discussed in a Norwegian context. Norway is a particularly interesting case since it recently approved a pension reform that aims to improve efficiency and maintain an egalitarian distribution of pension benefits while ensuring fiscal sustainability in the face of an ageing population.

The analysis thereby allows us to shed some empirical light on the questions raised by the theoretical literature.

The paper is organised as follows. Section 2 discusses analytically the main determinants of the EE trade-off within a simple model of overlapping generations. Section 3 briefly presents the microsimulation model employed, while section 4 presents the pension schemes under scrutiny. Section 5 presents simulation results and the estimated EE trade-off. Section 6 concludes.

## 2. Theoretical background

Lindbeck and Persson (2003) classify pension systems along two separate dimensions: the degree of funding, and the degree of actuarial fairness. In this article, I discuss the implications for efficiency and equity of changes in the actuarial properties of the pension system. Increased “efficiency” will in our context be equivalent to stronger labour supply incentives through lower effective marginal tax rates levied on labour income. By increased “equity” I mean a more compressed distribution of public old-age benefits. While the starting point of our discussion is a pay-as-you go pension system, the analysis and results are equally relevant to any system with funded elements. Changes in the actuarial properties will have implications for both the efficiency and equity aspects of the pension system.

### *Efficiency*

Using the notation from Lindbeck and Persson (2003), the efficiency dimension of a pay-as-you-go pension system can be analysed by a simple two-period overlapping generations model. The first period can be thought of as working age and the second period retirement, with periods being of equal length. The representative individual of generation  $t$  supplies labour  $l_t$  at wage rate  $w_t$  to obtain income  $y_t$  in period 1. A contribution rate  $\tau_t$  levied on labour income is paid to the government, and the individual receives a benefit  $b_t$  in period 2. For simplicity I assume that labour is not supplied in period 2 and that the representative individual holds no initial wealth. The return on pension contributions paid to government is then:

$$(1) \quad 1 + \text{return} = \frac{b_t}{\tau_t w_t l_t}$$

A continuously balanced budget would require  $n_t b_t = n_{t+1} w_{t+1} l_{t+1} \tau_{t+1}$  where  $n_t$  is the number of individuals in generation  $t$ . Inserting the balance requirement into (1) gives us:

$$(2) \quad 1 + \text{return} = \frac{\tau_{t+1} n_{t+1} w_{t+1} l_{t+1}}{\tau_t n_t w_t l_t} \equiv \frac{\tau_{t+1}}{\tau_t} (1 + G_{t+1})$$

where  $G_{t+1}$  represents the growth rate of the aggregate wage sum. If  $\tau_t = \tau_{t+1}$  it also denotes the return on the pay-as-you go assets. In this model environment,  $\tau$  is set exogenously and kept constant across generations. Assuming that they are constant over time, I will therefore drop subscripts on  $\tau$  and  $G$ . The consumption possibilities of the representative individual are given by the intertemporal budget constraint:

$$(3) \quad c_t^2 = [y_t(1-\tau) - c_t^1](1+R) + b_t$$

where  $R$  is the real interest rate. Subscripts denote generation whereas superscripts denote period. The pension benefit  $b_t$  paid to each individual will depend on the specific characteristics of the pension system in place. Most systems will in practice contain both a redistributive first-tier benefit and an earnings-based insurance benefit. In the general case, the benefit can be described as a function ( $\psi$ ) of earnings ( $y_t$ ), the contribution rate ( $\tau$ ), return ( $G$ ) and the actuarial characteristics of the system ( $\alpha$ ):

$$(4) \quad b_t = \psi(y_t, G, \alpha, \tau)$$

$\psi$  can for now be given the very simple functional form:

$$(5) \quad \psi(y_t, G, \alpha) = (1-\alpha)\bar{b} + \alpha\tau(1+G)y_t$$

$\alpha \in [0,1]$  where  $\alpha = 0$  denotes a non-actuarial system and  $\alpha = 1$  a fully actuarial pay-as-you-go system.  $\bar{b}$  represents a flat benefit set endogenously to keep the total benefit level at a constant fraction of the total economy. An exogenous increase in  $\alpha$  will give a more actuarial system and a corresponding endogenous decrease in  $\bar{b}$ . Substituting (4) and (5) into (3) yields

$$(6) \quad c_t^2 = \left[ y_t \left( 1 - \tau + \alpha\tau \frac{1+G}{1+R} \right) - c_t^1 \right] (1+R) + (1-\alpha)\bar{b}$$

This expression shows how the marginal tax on labour income depends on the actuarial properties of the pension system. By a tax, measured either on average or on the margin, I mean transfers to government net of pension benefits received. In the case where  $\alpha = 0$ , the tax wedge in the labour market is simply  $\tau$ . When  $\alpha = 1$  the marginal tax rate is  $\tau(R-G)/(1+R)$ . In a fully actuarial pay-as-you-go system, a tax is imposed only by forcing the individual to save at a lower rate of return than the market interest rate as we assume that the economy is dynamically efficient. A more actuarial system clearly reduces the tax wedge and distortions in the labour market. Although the marginal tax rate depends on the actuarial properties of the system, it is worth noting that the average tax rate net of

pension benefits is  $\tau(R - G)/(1 + R)$  for all  $\alpha$ . A more actuarial system will therefore improve work incentives through the substitution effect without any counteracting income effect, unambiguously increasing labour supply. Finally, we note that the general case where (4) does not have a specified functional form, the marginal tax rate is expressed by:

$$(7) \quad \tau - \frac{\psi'(y_t)}{1 + R}$$

### *Equity*

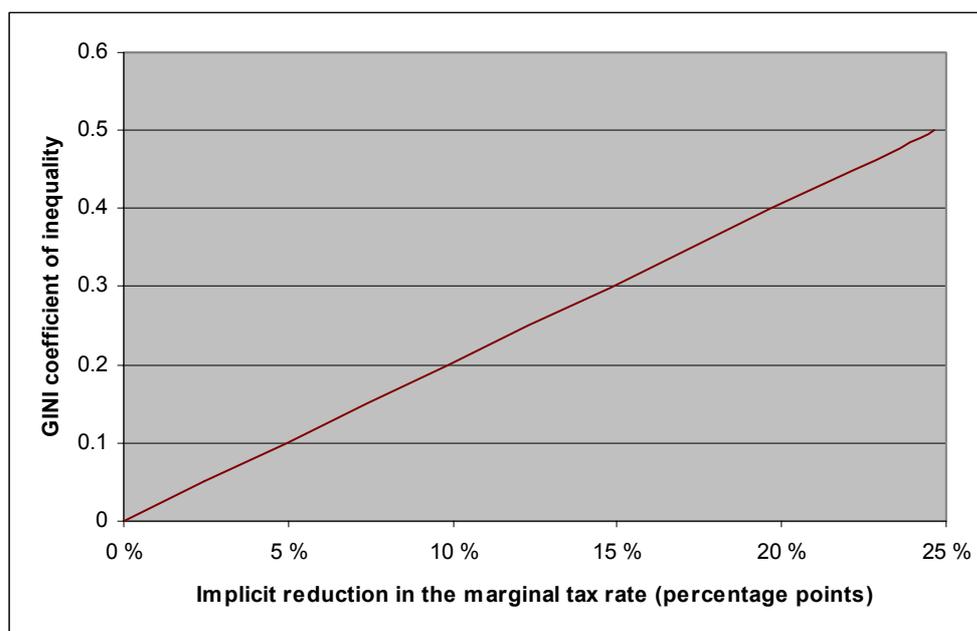
Reaping efficiency gains by making the system more actuarial may come at an equity cost since tightening the link between earnings and benefits limits the scope for intra-generational redistribution. This EE trade-off can relatively simply be illustrated by introducing some simplistic heterogeneity in a population faced with a pension system as specified by equation (5). We assume that the population is divided in two equally large groups. Only the group/type one individuals have labour income in period 1. The consumption of the group/type two individuals only comes through the flat pension benefit  $\bar{b}$ , financed by the contribution of the type one individuals in generation  $t + 1$ . The credit market is perfect, and future benefits may serve as collateral for lending. Increasing  $\alpha$  in from 0 to 1, combined with plausible parameter values<sup>1</sup>, gives an equity-efficiency trade-off as illustrated by figure 1. The horizontal axis contains the implicit reduction in the marginal contribution rate represented by the expression  $\alpha\tau(1 + G)/(1 + R)$ . This is equivalent to the second term of equation (7). Note that the marginal tax rate reduction does not completely outweigh the contribution rate even when  $\alpha = 1$  since  $R > G$ . The vertical axis illustrates the GINI coefficient for pension benefits.<sup>2</sup> For a population with only two types of individuals, its maximum value is 0.5.

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<sup>1</sup> We assume  $\tau = 0.25$  and a net interest rate of  $(1 + R)/(1 + G) - 1 = 1.5\%$ .

<sup>2</sup> In the second period, type 2 individuals receive  $\bar{b} = 0.5\tau(1 - \alpha)y_t(1 + G)$  and type 1 individuals receive  $\bar{b} + \tau\alpha y_t(1 + G)$ . Insertion into the GINI-coefficient formula gives  $\frac{F(1 + R)}{2\tau(1 + G)}$  where  $F$  is the implicit reduction in the marginal tax rate indicated by the horizontal axis of figure 1.

**Figure 1: The equity-efficiency trade-off in a stylised model**



It is worth noting that a reduction in  $\alpha$  does not automatically imply more equity in more complex pension systems. Many OECD systems contain non-actuarial elements that turn out to be regressive. For instance, the present Norwegian system calculates benefits on the basis of the 20 best income years of each individual. An argument for upholding this rule has been to increase benefits for women with low labour supply in periods with childcare responsibilities. In practice, however, the rule also works to the advantage of career-oriented men with steep age-earning profiles (Koren, 2003). Accordingly, pension reform is not limited to movements along the EE-curve. The curve itself may shift, depending on the specific benefit function in place (i.e. the functional form of equation (4)).

Unrealistically, in this framework labour supply is not affected in the trade-off illustrated by Figure 1. If labour supply were made endogenous, the curve is not likely to maintain its linear shape. However, the purpose of this paper is confined to describe the labour supply incentives created by the pension system. An evaluation of responses should be undertaken as a study in optimal taxation, since it also would be necessary to consider the level of other taxes in the economy.

### **3. Methodology for estimating the EE trade-off**

Despite the straightforward underlying theory, actually estimating the EE trade-off in a real-world pension scheme is an intimidating task. A fully specified pension system typically involves complicated benefit rules. The main challenge therefore lies in specifying the pension function  $\psi$  in all relevant details. The following three examples illustrate characteristic non-linear elements:

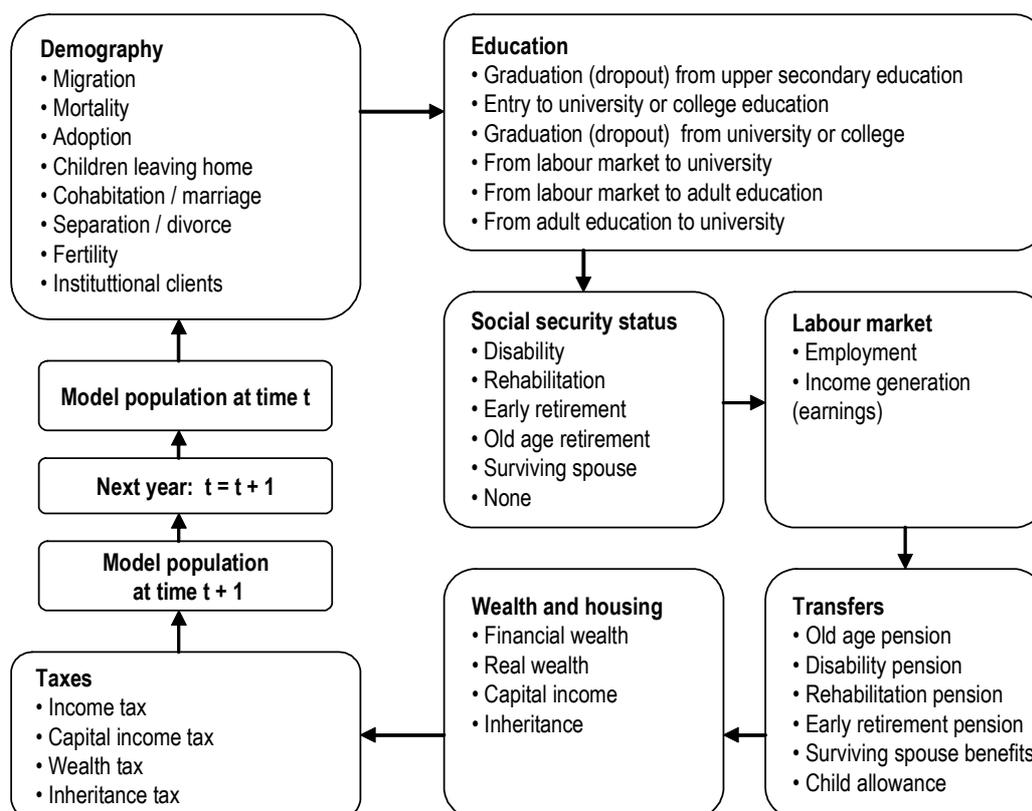
1. Public pension schemes that combine redistributive and insurance benefits, usually apply means-testing to target benefits. Even a simple means-test would make the specification in (7) discontinuous. In fully specified systems, the interplay between earnings-dependent benefits and guaranteed benefits is quite complex.
2. The simple overlapping generations model above treats all period 1 labour income equally. In practice, the timing of income during the life cycle is important because benefits are typically calculated on the basis of annual income. Rules that place a cap on accumulation years, calculations based on best or most recent income years, and annual ceilings for pension-eligible income introduce non-linearities in the function.
3. Public pension schemes may also impute pension entitlements for military service, childcare responsibility, disability periods, unemployment benefits etc. Such non-income based entitlements further complicate the benefit structure.

These non-linearities inhibit a specification of  $\psi$  as a continuous and differentiable function. For a fully specified pension system, my analysis of the equity-efficiency trade-off is therefore limited to numerical point estimates. Specifically, to approximate the implicit reduction in the tax rate given by the second term of equation (7), I investigate the impact on  $b_t$  of a marginal increase in  $y_t$ . Such estimates require a detailed modelling of the pension system rules. In addition, the estimation framework must capture population heterogeneity as different parts of the population may face different rules, and there may be substantial problems of aggregation in calculating the total effect on government budgets of changes in tax or pension systems. To overcome these problems microsimulation models, as advocated among others by Orcutt *et al.* (1986), have become increasingly used in the last few decades for analysing the effects of different social and financial policies. The basic idea in microsimulation modelling is to represent a socio-economic system by a sample of decision units (e.g. persons), and then model the behaviour of these primary units. Contrary to what is possible in models with few representative agents, the detailed and complicated tax and benefit rules may be exactly reproduced.

I analyse the present Norwegian public pension, as well as four alternatives that were discussed in the preparation of the pension reform approved by the parliament in 2007. To this end I employ the dynamic microsimulation model MOSART, which is especially designed to analyse what Gruber and Wise (2004) refer to as mechanical effects on individual pension entitlements, benefits and government pension expenditures of changes in the pension system. By mechanical effects I mean effects ignoring behavioural responses and general equilibrium effects. The model simulates the life courses of a representative cross-section of the Norwegian population, emphasizing what is relevant

for individuals' accumulation of public pension entitlements. It captures the following events: migration, deaths, births, marriages, divorces, educational activities, retirement and labour force participation. Transitions between states over the life course depend on individual characteristics, and the transition probabilities have been estimated from observations in a recent period.<sup>3</sup> The model includes an accurate description of the pension rules and captures all relevant details of the population dynamics, as well as the heterogeneity of individual age-earnings profiles and individual public pension entitlements. For detailed model documentation, see Fredriksen (1998).

**Figure 2: Structure of the dynamic microsimulation model MOSART**



<sup>3</sup> The analyses in this paper are based on a representative sample from 1993 that is mainly calibrated to the situation in 2001. The demographic assumptions are based on Statistics Norway's demographic projections from December 2005. A total fertility rate of 1.8 and a net immigration of 16,000 persons each year imply a gradually diminishing population growth measured by the size of each birth cohort. By 2050 the subsequent generation will still be approximately 7 per cent larger. The aggregate population may however increase more as a result of a further increase in life expectancy at birth of around 6-7 years in the same period, and then a further increase towards 2100. Assumptions about participation in the labour force and working hours are based on 2001-observations, while the necessary information about distribution of incomes between individuals over the life cycle is based on observations from a longer period. Our exogenous assumptions are fully in line with those described in detail by Stensnes, Stølen, and Texmon (2007).

## 4. The pension schemes to be evaluated

I estimate the EE trade-off in five different pension schemes applied in the Norwegian context. In line with Scandinavian welfare state traditions, Norway has a pay-as-you-go public pension system with relatively generous replacement rates. The after-tax replacement ratio is about 65 per cent for a person with with 40 years of labour market earnings and a steady, normal income level. However, ageing combined with the increase in average benefits that come with a maturing pension scheme, will put severe pressure on public financed in the decades to come. In April 2007, the Norwegian Parliament therefore approved a pension reform that aims to reduce the growth in government pension expenditures, improve efficiency by stimulating labour supply, while maintaining an egalitarian distribution of pension benefits. This makes Norway a case of particular interest. If the reform succeeds with these three intentions, it would seem that policy makers have actually been able to accomplish a Pareto improvement by shifting the EE-curve of figure 1 outwards. In addition to evaluating the present and reformed pension system, I analyse three rejected proposals that were heavily debated in the pension reform process. The proposals differ in their emphasis on the redistributive versus insurance-based pension components, universal as opposed to means-tested benefits, and the specific accrual scheme for entitlements in place.<sup>4</sup> In other words: The schemes are intended to have different EE trade-offs. My ambition is to examine to what extent such trade-off differences prevail when one accounts for all relevant available information.

### *The present scheme*

The present pension system combines a first-tier redistributive benefit with a second-tier earnings-based benefit. The former contains both a universal benefit and a means-tested guarantee that work together to provide income security in old age, and the minimum benefit is presently NOK 120 000 annually<sup>5</sup>. The latter earnings-based benefit is intended to provide income replacement and based on the entitlements each person earns through his or her own working career. In addition, imputed entitlements are awarded on the basis of unemployment, disability, and caring responsibilities until the youngest child reaches school age. All benefits, entitlements, and entitlement accumulation brackets are indexed to average wage growth.

The official retirement age is 67 years, but a majority of workers leave the labour force earlier by means of a disability pension, or through an early retirement scheme that encompasses 60 per cent of the workforce. A number of non-linear elements in the present scheme weaken the link between

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<sup>4</sup> The proposals are also different with respect to retirement incentives. The present system provides heavy subsidies to early retirement. In contrast, the reform proposals all aimed to align the social and private costs of retirement through an actuarial mechanism that keeps the present value of expected pension benefits independent of retirement age. I leave these feature aside in this paper since I only aim to analyse the implicit tax rates on the intensive margin.

<sup>5</sup> At present, 1 EUR is approximately 8 NOK.

earnings and entitlement accumulation. In particular, the maximum possible number of accrual years is limited to 40, and the second-tier benefit is calculated using the average of the 20 years with highest labour market earnings. Details on the rules for accumulating entitlements and calculating pension are given in an appendix to this paper.

### *Principles for pension reform*

The pension reform approved by the Norwegian Parliament is based on the following main principles:

- The minimum pension benefit is kept at the present level, but entirely transformed into a means-tested guarantee.
- To limit expenditures if life expectancy increases, an actuarial mechanism is introduced to reduce annual benefits as the expected length of retirement spells increases. It is possible for each individual to counteract the lower benefits by postponing retirement.
- The statutory retirement age of 67 is replaced by a flexible retirement scheme starting at age 62. Annual benefits will actuarially reflect retirement age.
- Entitlements are indexed by wage growth, as in the present system. However, benefits in payment are to be indexed by the average of wage and price growth. A special indexation rule linked to life expectancy implies that the income guarantee over time will lose value relative to the earnings-based benefit.
- There is to be a tighter link between earnings and pension benefits to improve work incentives. Lifelong accumulation of entitlements replaces the present cap on accrual years and the principle of calculating benefits on the basis on the best 20 years.

The new system thus means that annual labour incomes below an annual income ceiling are accumulated as fictitious capital and converted into an annuity at retirement. The actuarial mechanism will, for a given level of entitlements, keep the present value of total pension benefits independent of retirement age.<sup>6</sup> This is one implementation of what Lindbeck (2006) identifies as an “automatic rule mimicking the functioning of actuarially fair private income insurance systems”. The divisor is further outlined and discussed in Stensnes and Stølen (2007) and in the detailed pension rules appendix. The new system is calibrated so that individuals from 1943-cohort, retiring at age 67 at the introduction of the reform in 2010, will receive the same pension benefit as in the existing system. However, life expectancy adjustment and lower indexation in payment imply that retirees over time receive lower annual benefits than in the present system, unless retirement is postponed.

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<sup>6</sup> However, special rules imply deviations from an exact actuarial adjustment. For instance, the annual benefits and pension premium are independent of gender and other observable characteristics correlated with life expectancy.

### *Alternative accrual schemes for pension reform*

All the alternative proposals seek to preserve both the social security and income replacement dimensions of the present system, building on the modifying principles described by the bullet points above. The accrual schemes all include a contribution-based pension component whereby annual labour income below a threshold is accumulated as fictitious capital, wage indexed and converted into an annuity at retirement. Common for the schemes is also a minimum pension continued at the level defined by present system. However, the different alternatives are based on a different mix of universal benefits, means testing and the income pension that follows from accumulated entitlements. Table 1 summarises the model proposals, and further details are given in the appendix.

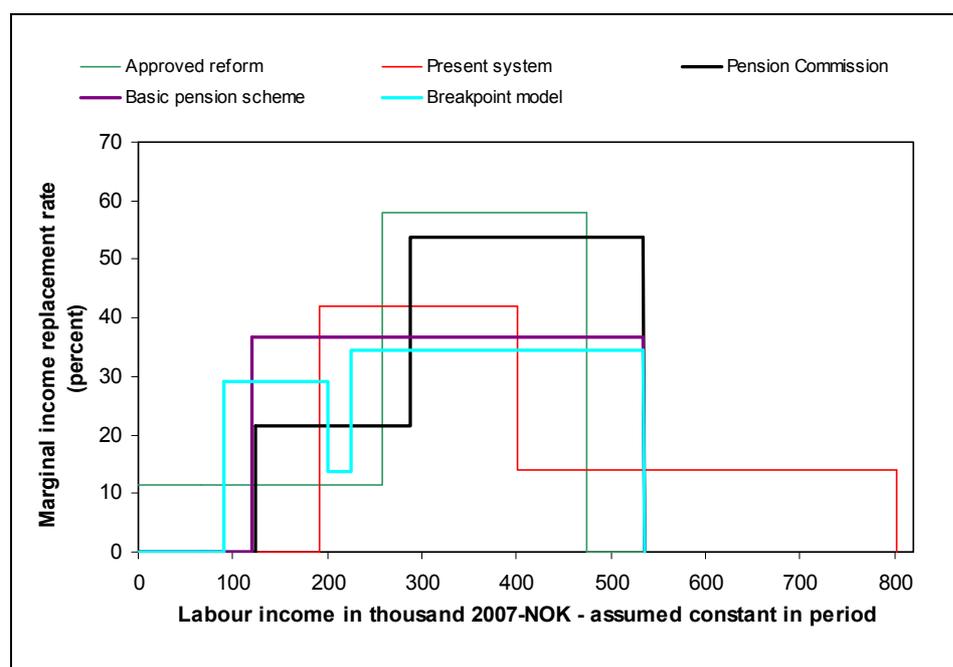
The **Pension Commission proposal** suggests that the minimum pension should be given as a targeted guarantee, and reduced against the income-based pension. For pension benefits above 66 thousand NOK, the Commission recommended that the reduction rate be softened to 60 percent to ensure that also individuals with low income receive pensions in excess of the guaranteed level. The **approved reform** adapts the principles of a targeted guarantee and a uniform accrual rate from the Pension Commission proposal, but narrows the benefit range in both ends. At the lower end of the scale, tapering of the guaranteed benefit starts already at the first unit of income-earned pensions. Only individuals with no accumulated entitlements will therefore become minimum pensioners. At the other end of the scale, the annual income ceiling is lowered at the expense of high-income earners. In part, this finances an increase in the accrual coefficient. Compared to the Pension Commission and Government proposals, the **basic pension scheme** introduces more equity by way of a universal minimum pension benefit. There is no means-testing and accumulated income-based pension benefits will always supplement the minimum pension. The **breakpoint model** is another approach to a more egalitarian distribution of pension incomes. A breakpoint is introduced in the accrual coefficient for income pensions, so that the accumulation of entitlements is more weighted towards the lower end of the income scale. The targeted, guaranteed minimum pension with soft means-testing is adopted from the Pension Commission.

Figure 2 gives an illustration of the incentives faced at different income intervals for the very stylised individual with a constant (wage-indexed) income for 43 years. A unit increase in annual income in the entire accumulation period, will for different income intervals give an annual increase in pension as indicated by the vertical axis. For instance, a marginal income increase each of the 43 years will, starting at NOK 300 000 under the Approved Reform, give an increase of the annual pension benefit with 0.58 per income unit. Please note that the models may compare differently with a non-constant income and fewer accrual years.

**Table 1. A summary of the alternative pension schemes**

	<i>Minimum benefit (NOK 120 000)</i>		<i>Earnings-based benefit Numbers in per cent and 1000 NOK</i>				
	Universal or means-tested?	Tapering rate (average)	Lifelong earnings	Accrual coefficient R = regressive	Annual income interval covered	Actuarial retirement mechanism	Indexation in payment
Present system	Mix	100 %	No	R: 1.05 / 0.35 %	8-101	No	Wage
Approved reform	MT	80 %	Yes	1.35 %	0-60	Yes	Price/Wage
Pension commission	MT	82 %	Yes	1.25 %	0-67	Yes	Price/Wage
Basic pension scheme	Uni	n/a	Yes	0.85 %	15-67	Yes	Price/Wage
Breakpoint model	MT	82 %	Yes	R: 1.70 / 0.80 %	0-67	Yes	Price/Wage

**Figure 2: Labour incentives at different income intervals with 43 accumulation years**



## 5. Efficiency-Equity trade-offs

### 5.1. The effective tax rate element

I employ microsimulation to estimate accurately the contribution to the effective marginal tax rate levied on labour income from the pension system. Specifically, I expose a cross-section of the model population to a small and transitory income shock in one specific period (when  $t=j$ ).<sup>7</sup> Once implemented, I run the model to let the individual life courses unfold keeping all individual characteristics constant *as if* the income shock had not taken place. This provides a *ceteris paribus* evaluation of the income shock's effect on future benefits. Each year of retirement, for every individual in the model population, I calculate public pension benefits  $b_t$  and  $b_t'$  in parallel. These respectively represent the retirement benefit in *absence* and *presence* of the transitory income shock that the individual was exposed to during working age. The difference between  $b_t$  and  $b_t'$  is measured in percentage points, and the marginal tax rate given by equation (7) is accordingly approximated by:

$$(8) \quad \tau - \frac{1}{N} \sum_{i=1}^N \sum_{t=j+1}^{\infty} \left( \frac{1+W}{1+R} \right)^t (b_t' - b_t).$$

$N$  is the number of individuals in the model population who participate in the labour force at time  $j$ .<sup>8</sup>  $W$  here indicates the growth rate of wages. As above, the discount factor  $(1+R)/(1+W) - 1$  is set to 1.5 per cent. Choosing a higher discount factor would partially increase the marginal tax rate. Few defined benefit systems operate with mechanisms that automatically guarantee the contribution rate to be completely fixed across time and invariant to changes in benefit and entitlement schedules. To ensure the comparability of the five pension schemes under scrutiny, aggregate pension expenditures have been aligned the five schemes using the present system as a numeraire. Details of this alignment methodology are given in appendix B. The calculated implicit reduction in the marginal tax rates are given by table 2.

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<sup>7</sup> For convenience, we choose to give the marginal income shock by setting  $j=2050$ . Choosing a date so far into the future ensures that our results are not affected by the pension reform transition rules. Simulations are then carried out to 2150. The income shock is set as large as NOK 10 000 to avoid errors in rounding numbers.

<sup>8</sup> Labour force participation is negatively defined as being above 20 years and not being a recipient of any benefit from the National Insurance Scheme. Such benefits are provided for disability, rehabilitation, work-qualifying training and old age. The model is run on a representative sample of 1% of the Norwegian population with  $N$  approximately equal to 40 000.

**Table 2: Implicit reduction in marginal tax rates. Percentage points**

Approved reform	15.7 %
Present system	10.1 %
Basic pension scheme	8.1 %
Breakpoint model	13.8 %
Pension Commission proposal	16.2 %

We observe that the approved reform and the Pension Commission proposal give the largest reduction in the marginal tax rate. The Breakpoint model and the present system proposal follow suit, whereas the Basic pension model provide the least reductions. The accrual coefficient intervals and magnitudes illustrated by figure 2 are important to understanding outcomes. The Pension Commission proposal and approved reform are the most efficient because they have the highest accrual coefficient in the income range of most full-time workers, keeping in mind that Norway has a compressed wage structure relative to many other countries. The Pension Commission scores slightly than the approved reform because it has a higher income ceiling for entitlement accumulation. The breakpoint model has a lower accrual coefficient in the most densely populated income interval, and accordingly scores lower. On the other hand, efficiency is improved for those who are faced with a very high coefficient in the lower income bracket. This latter element helps insure that the model scores better than the basic pension scheme. In addition, in the basic pension scheme, the large income deduction in the entitlement accumulations combined with a universal pension benefit represents an inefficient way of providing the minimum income guarantee. The poor score of the present system is in part explained by a low accrual coefficient in the income range of most full-time workers. Furthermore, the non-linearities of the present system contribute towards inefficiency.

## 5.2. Effects on equity

My evaluation of the consequences for the intra-generational distribution of old-age benefits is based on three important qualifications. First, I consider only pension benefits, neglecting taxes and pension premiums. With Norwegian old age pensions being fully funded by general tax receipts, it is not possible to identify specific contributions to the system. Adjustments of tax and contribution rates are nevertheless likely to have much smaller distributional effects compared to the changes in benefits. Second, I analyse only the direct effects of pension reform for a given labour supply, both at the intensive and extensive margins. Neglecting indirect income effects caused by behavioural changes can be justified when focusing on changes in utility level. According to the envelope theorem, the utility of marginal income increases from increased labour supply is neutralized by the utility loss of reduced leisure, as long as the individual is free to choose hours worked. Third, I report the

distributional effects measured as annual benefits in a cross-section of pensioners without accounting for the actuarial adjustment through the flexible pension scheme. This scheme effectively replaces a fixed retirement age with an individual retirement choice above 62 years, and the retirement decision will influence both retirement age and length. If I included the actuarial implications of this choice, annual benefits would be a poor approximation for pension wealth because they would indicate both (relevant) changes in pension wealth and (for my purpose irrelevant) changes in retirement spells.

**Figure 3: Distributional indicators**

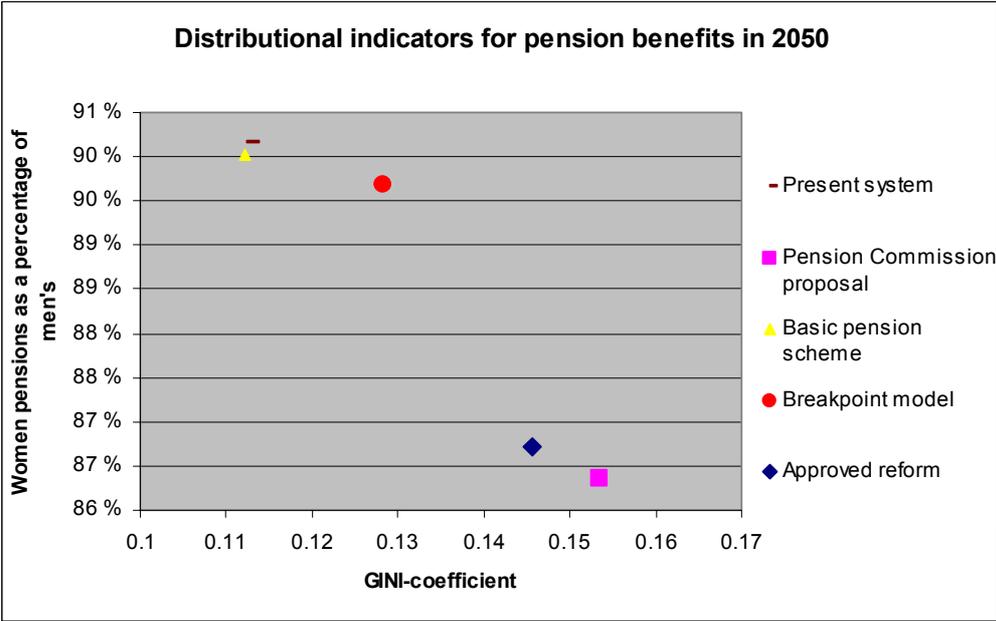
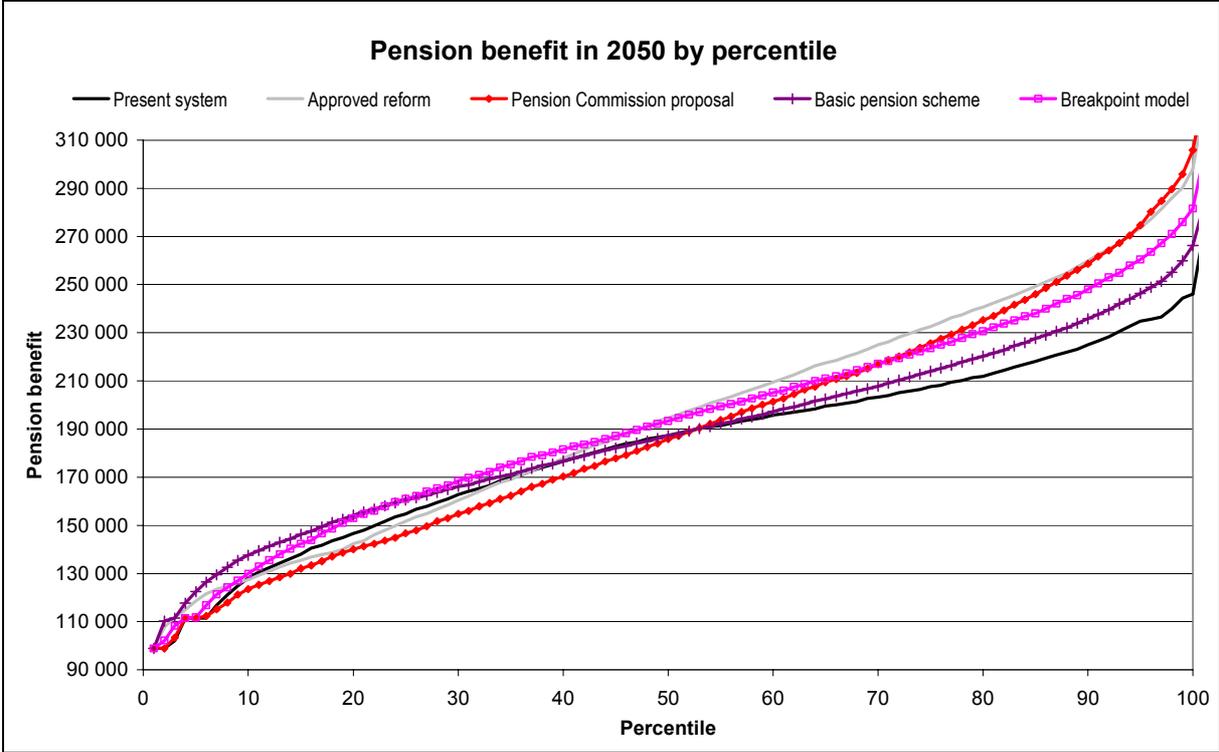


Figure 3 gives a first impression of how different accrual schemes distribute income between individuals at system maturity in 2050. The horizontal axis measures the GINI-coefficient of inequality and the vertical axis shows women’s pensions as a share of men’s, on average. Along the two dimensions, the upper left quadrant would therefore indicate more equality and lower right more inequality. The public old age pension scheme is redistributive, having a much lower GINI-coefficient than the cross section of Norwegian labour incomes, due to components such as a minimum benefit and the annual income ceiling on accumulating pension entitlements.

Figure 4 illustrates the benefit profiles of the different accrual schemes more in detail. For the **lowest 25 percentiles**, pension incomes will be decided in a complex interplay of minimum benefits and means-testing, combined with the accrual coefficient that links lifetime earnings to the income pension benefit. The exact combination will influence the curves’ intersection with the vertical axis. The continuation of each benefit curve for the **middle percentiles** should be more transparent as the relative slope is proportional to the accrual coefficient. In the breakpoint model, the curve reaches an

inflection point just below the 40<sup>th</sup> percentile as higher incomes are exposed to a reduced accrual coefficient.

**Figure 4: Pension benefits in 2050 for different accrual schemes by pension benefit percentile. Real 2006-wages in Norwegian kroner serve as a numeraire for the wage-indexed benefits**



The annual income ceiling for pension entitlements will, in combination with the accrual coefficient, determine pension benefits for the **highest 25 percentiles**. The basic pension scheme and the breakpoint model have the smallest accrual coefficient, and accordingly the lowest pension benefits for this group. The Pension Commission proposal and the Approved reform are the most generous, with the former on top. Because the Government has a lower annual income ceiling for the accumulation of entitlements than the Pension Commission proposal, the curves cross at the 90<sup>th</sup> percentile. Because of its low accrual coefficient for high incomes, combined with a high deduction in the conversion of income to entitlements, the present system provides the lowest benefits for the uppermost deciles. For all schemes the curves are convex for the top deciles, because they exhibit an increasing gap between the average incomes.

### 5.3 The EE trade-off

Figure 5 summarises the results by combining the equity and efficiency dimensions. Whereas the stylised model allows for a two-dimensional representation of the EE trade-off, as shown in figure 1, the actual systems are far too complex to permit such a description. Instead, each pension scheme is represented by a single point estimate. In the actual pension schemes under scrutiny, the variations in actuarial properties are determined by the joint effect of 10 different parameter values and 2 equations (see appendix A for details). No single parameter can be varied to illustrate the EE curves of each system. Academic exercises often leave the impression that equity and efficiency are tangible notions simply represented by a two-dimensional trade-off. Any solution to the optimal choice problem is then a matter of specifying preferences over just these two variables quickly delegated to academics or politicians equipped with a set of preferences. The analysis in this paper demonstrates that such a simplification may leave a severely misleading impression of the complexities involved in even identifying the EE trade-off. The heterogeneity in the range of individual income histories, combined with the numerous parameters that determine the pension benefits, is a telling example of this complexity. The stylised curve in figure 1 is an effective way of communicating the main ideas, but it simply does not have an empirical counterpart. This reflects a more fundamental reality: The more heterogeneous are the reform effects on individual labour incentives and incomes, the harder it is design reforms that can shift the EE-curve downwards.

Since the assessments of efficiency and equity are limited to point estimates, it is not possible to precisely identify shifts in the curve and the corresponding Pareto improvements discussed in the literature. Nevertheless, the exercise justifies at least four valuable conclusions. First, the estimates verify the expected correlation between inequality and inefficiency: A move towards a more actuarial pension scheme reduces intra-generational redistribution. An interesting exception is the basic pension scheme, which could be replaced by the more efficient present system without any increase in the GINI-coefficient.

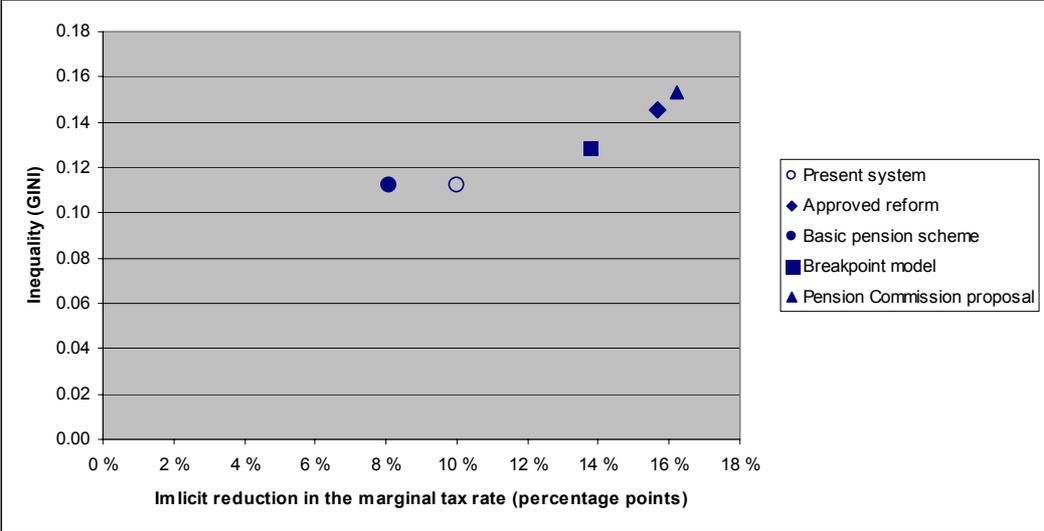
Second, there is evidence to suggest that the principal cause of inefficiency in the basic pension scheme is the strong universal benefit of NOK 120 thousand. The basic pension scheme expands the universal pension element found in the present system by removing means-testing entirely. Despite eliminating many of the non-linearities in the entitlement accrual function of the present system, in particular the cap on accumulation years and the principle of basing pension on the 20 best entitlement years, the basic pension scheme delivers lower efficiency than the present system with a universal benefit of NOK 67 thousand. Because a universal benefit gives such a strong negative impact on efficiency, it seems plausible that its replacement by a fully means-tested income guarantee in the three remaining schemes is the main reason why these schemes perform more efficiently.

Accordingly, removing the entitlement accumulation non-linearities by introducing the principle of

lifelong entitlement accumulation may have a smaller impact on our efficiency measure than emphasised by policymakers.<sup>9</sup>

Third, if the universal benefit is the main driver of inefficiency, Pareto improvements may be realised by delivering the income guarantee as a means-tested benefit and using the freed revenue to create incentives in the income intervals relevant to large groups of earners. This would be the case for the breakpoint model, the Pension Commission proposal, and the Approved reform. A natural extension of the view is the notion that the latter schemes lie closer to the EE frontier than the other the present system and the basic pension scheme. Fourth, these results show the importance of estimating the equity and efficiency dimensions of a fully specified pension system: Once heterogeneity and pension rules are fully accounted for, the predicted Pareto improvements of reform do not materialise in the case of the basic pension scheme. In the other schemes, gains seem to stem from a different source than advocated by policymakers.

**Figure 5: The EE trade-off in five pension schemes**



<sup>9</sup> The non-linearities may negatively impact on efficiency measured on the *extensive* margin if they distort retirement decisions.

## 6. Conclusions

I have used a microsimulation framework to quantify the efficiency and equity dimensions of five fully specified pension schemes, applied in a Norwegian context. While all combine a redistributive first-tier benefit with and an earnings-based insurance benefit, the schemes place different emphasis on these components and differ with respect to the specific accrual scheme in place. Keeping the contribution rate constant, a move towards a quasi-actuarial pension system will permit a reduction in the effective marginal tax rate because contributions involve a claim to future benefits. Efficiency is evaluated by estimating the average size of this claim for a marginal increase in labour income, discounted to present value. Equity is represented by the GINI-coefficient calculated on the basis of pension benefits. My estimates show that increased efficiency in all schemes comes at the expense of increased inequality. The results indicate that the pension benefit return to a marginal increase in income to an important extent is driven by the way the minimum income guarantee is delivered. Even though the GINI-coefficient remains unchanged, the pension scheme with a fully universal benefit has lower efficiency than a scheme incorporating an element of means-testing. While the two systems in question also differ along other dimensions, these differences a priori work to increase the efficiency of the poorly performing scheme. Keeping with this line of argument, a shift towards means-testing at the expense of universal benefits should improve the equity-efficiency trade-off. In contrast to the notions advocated by policymakers, the principle of lifelong earnings implemented to reduce non-linearities in scheme for accumulating pension entitlements, seems to be less important. Without the detailed modelling of income heterogeneity and benefit rules made possible by our dynamic microsimulation approach, these insights would not have been materialised.

Even though I have identified the marginal tax rate reductions in different pension schemes, the consequences for labour market distortions remain contingent on a number of factors. First, labour supply will depend on the other taxes in the economy and need to be addressed in an optimal taxation framework. To some extent, this analysis needs to address relevant heterogeneity in marginal tax rates and labour supply elasticities. Second, actuarial pension contributions represent a claim on a future asset subject to individual survival and political risk. Uncertainty may therefore be an important element in future analyses. Third, one fundamental justification for public pensions in old age is to provide income support to individuals whose myopic behaviour would cause too little savings in absence of a pension system. Accordingly, it may seem inconsistent to base efficiency evaluations on the discount rates of rational agents. More work is needed before the theoretical labour supply effects can be quantified with confidence.

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## Pension scheme specifications

Both in the present and reformed system, the Norwegian public pension benefit for old-age pensioners will combine a first-tier redistributive element ( $\bar{b}$ ) with a second-tier earnings-based insurance element ( $\hat{b}$ ). Specified as an annual benefit ( $b_t$ ) in year  $t$  it is given by the equation:

$$(A1) \quad b_t = \mu_t (\bar{b} + \hat{b})$$

where  $\mu_t$  is a wage index parameter<sup>10</sup> relative to a base year wage  $W_0$ , and defined by:

$$(A2) \quad \mu_t = \frac{W_t}{W_0}$$

The redistributive benefit ( $\bar{b}$ ) can be given as a combination of a universal benefit ( $u$ ) and an income guarantee ( $g$ ) that is means-tested against the earnings-based benefit ( $\hat{b}$ ). The minimum pension awarded to a person without any accumulated entitlements is then given by the sum  $u + g$ .  $\bar{b}$  is calculated by:

$$(A3) \quad \bar{b} = \lambda_t u + \max \left[ 0, \lambda_t g - \rho \max(0, \hat{b} - d) - \min(\hat{b}, d) \right]$$

where  $\lambda_t$  is an indexation parameter,  $\rho$  represents the tapering rate of the means-tested guarantee, and  $d$  a deduction that is fully means tested (i.e. with a tapering rate equal to unity). The earnings-based benefit ( $\hat{b}$ ) is based on annual labour-market earnings ( $y_t$ ) in the accumulation period. Earnings are each year transformed to a year-specific entitlement ( $\hat{y}_t$ ) through the formula:

$$(A4) \quad \hat{y}_t = \gamma_1 \max \left[ 0, \min \left( \frac{y_t}{\mu_t}, c_1 \right) - f_1 \right] + \gamma_2 \max \left[ 0, \min \left( \frac{y_t}{\mu_t}, c_2 \right) - f_2 \right]$$

The terms on the RHS represent two entitlement brackets with accrual coefficients  $\gamma_1$  and  $\lambda_2$  respectively, with  $\gamma_1 > \gamma_2$ . The bracket intervals are defined by the floor ( $f$ ) and ceiling ( $c$ ) parameters. The bracket intervals and annual entitlement ( $\hat{y}_t$ ) are, by means of the wage indexation parameter  $\mu_t$ , defined in terms of wages in the base year. This is fully equivalent to the parameters  $c, f$  and  $\hat{y}_t$  being wage indexed.

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<sup>10</sup> Wage indexation is the political intention. Historically, however, indexation has been implemented through discretionary political decisions whose outcome has tended to favour raising the first-tier benefit (through increased  $g$ ) at the expense of full wage indexation.

In the present system, the benefit is based on the 20 best income years of each individual. 40 years of positive earnings are sufficient to achieve maximum accumulation. Let the positive annual entitlements of an individual  $(\hat{y}_t \forall t)$  be ordered from high to low in a vector. Let the average of the first 20 elements be given by  $\bar{V}$  and the number of elements by  $V$ . In the present system,  $\hat{b}$  is then defined by the expression:

$$(A5) \quad \hat{b} = \min(40, V) \bar{V} \quad (\text{present scheme})$$

In the reformed system, the earnings-based benefit is based on lifelong earnings and given by the formula:

$$(A6) \quad \hat{b} = \frac{\sum_{t=0}^{A-1} \hat{y}_t}{(1 + \pi)^{t-A} \partial_{K,A}} \quad (\text{reformed schemes})$$

where  $A$  represents retirement year and  $K$  the cohort to which an individual belongs. Even though entitlements are wage indexed through the parameter  $\mu_t$ , second-tier benefits in payment are indexed by less than wages. The element of price indexation is measured by a fixed deduction  $\pi$  measured in percentage points.  $\partial$  is a parameter relating to the adjustment of annual pension to retirement age and longevity. Pension reform will replace a statutory retirement age of 67 years with a flexible retirement system starting at age 62. Furthermore, annual benefits in the new system will be lowered (increased) if the expected duration of retirement is longer (shorter) than the 1943-cohort retiring at age 67 in 2010 when the reform is implemented. For a given entitlement level, the present value of expected pension benefits are then independent of retirement age and increases in longevity. Specifically, the parameter  $\partial$  is defined by:

$$(A7) \quad \partial_{K,A} = \frac{\Phi_{K,A}}{\Phi_{K=1943, A=67}}$$

where  $\Phi$  is an expression for the present expected value of a first-year pension benefit of unity for an individual of cohort  $K$  who retires at age  $A$ .  $\Phi$  is determined as follows:

$$(A8) \quad \Phi_{K,A} = \sum_{x=A}^{\infty} p_{K,x} \left( \frac{1+W-\pi}{1+R} \right)^{x-A}$$

$W$  is expected wage growth and  $R$  the expected interest rate.  $p$  represents the probability that a person of cohort  $K$  will survive until the end of the year  $x$ , conditional on the individual being alive at age 62. These probabilities are to be based on cohort averages, irrespective of sex, and are to be estimated and fixed when each cohort is 60 years old (well below the first possible retirement age). Finally, the first-

tier benefit ( $\bar{b}$ ) is wage indexed, but with an elements that corrects for changes in life expectancy in the reformed system. The gradual reduction of  $\bar{b}$  relative to  $\hat{b}$  is meant to provide an additional stimulus for postponing retirement. The indexation parameter is formally defined by:

$$(A9) \quad \lambda_t = \frac{\Phi_{K=t-67, A=67}}{\Phi_{K=1943, A=67}} \quad (\text{reformed schemes})$$

In the present system, there is no correction for life expectancy such that the indexation parameter is merely given by:

$$(A10) \quad \lambda_t = 1 \quad (\text{present scheme})$$

The pension schemes analysed in paper may then be defined by the parameter values given in table 1. In addition, all schemes provide imputed entitlements for unemployment, disability, and caring responsibilities until the youngest child reaches school age. Details of these arrangements may be found in Stensnes et al (2007).

**Table 1: Pension scheme parameters. Please note that the parameters  $u$  and  $g$  will be somewhat reduced for married and cohabiting couples. Values marked with \* are given in Norwegian 2007-NOK, rounded to the nearest thousand. 2007 also serves as the base year. At present, 1 EUR is approximately equal to 8 NOK**

	Present system	Approved reform	Pension Commission proposal	Basic pension scheme	Breakpoint model	
<b>First-tier</b>	$u^*$	67 000	0	0	120 000	0
	$g^*$	53 000	120 000	120 000	0	120 000
	$u+g^*$	120 000	120 000	120 000	120 000	120 000
	$\rho$	1	0.8	0.6	0	0.6
	$d^*$	0	0	67 000	0	67 000
	Indexation equation	A10	A9	A9	A9	A9
<b>Second-tier</b>	$\gamma_1$	0.0105	0.0135	0.0125	0.085	0.017
	$f_1^*$	67 000	0	0	120 000	0
	$c_1^*$	401 000	474 000	534 000	534 000	200 000
	$\gamma_2$	0.0035	0	0	0	0.008
	$f_2^*$	401 000	0	0	0	200 000
	$c_2^*$	802 000	0	0	0	534 000
	Benefit equation	A5	A6	A6	A6	A6
Common parameters	N/A	W=R=1.5%		$\pi=0.75$		

## Expenditure alignment when the contribution rate is endogenous

The model presented in section 2 of this paper assumes that contribution rate  $\tau$  is exogenous and the aggregate benefit level endogenously determined as a function of the contribution rate ( $\tau$ ) and wage sum growth ( $G$ ). This assumption does not hold in many real-world pension systems. Instead, benefits are determined exogenously whereas the efficient contribution rate, required to keep the budget in continual balance, is endogenous. As Disney (2004) points out, even official contribution rates are misleading if they do not cover the full costs of the pension scheme. Three main reasons will lead to contributions rates becoming endogenous:

1. Pensions are financed by general tax receipts with no specific contributions made to the system. This is the case in for instance Norway, Australia, and New Zealand.
2. Many countries subsidise the public pensions by budgetary transfers. In some cases this serves to finance redistributive first-tier benefits such as income guarantees or imputed pension entitlements for unemployment, military service, caring obligations etc.
3. The pension budget is not continually balanced either because of pre-funding or benefit levels that are unsustainable over time. In practice, it is rare to see pay-as-you-go pension systems operate with a continually balanced budget.

Any of these three factors imply that the effective contributions rate is endogenously determined according to the balanced budget requirement (B1). Since we abstract from labour supply responses, the only practical restriction on  $\tau$  is that its maximum value is unity.

$$(B1) \quad \tau_{t+1} = \frac{n_t b_t}{n_{t+1} w_{t+1} l_{t+1}} \leq 1$$

An endogenous  $\tau$  has two implications. First, it means that  $\tau$  is likely to vary over time and across pension systems. One source of such variation is demographic though changes in the support ratio ( $n_{t+1}/n_t$ ), ageing being a relevant example. Another source is shifts in the growth rate of average per capita labour income ( $w_{t+1} l_{t+1}/w_t l_t$ ). A further source is modifications to the benefit function itself. Unless such shocks are perfectly outweighed by continual adjustments the average benefit level ( $b_t$ ), the contribution rate cannot be held. However, a process of political fine-tuning seems highly unlikely.

The second implication of an endogenous  $\tau$  relates to the marginal tax rate of the pension system as measured by equation (7): Unless the contribution rate is exogenous, the implicit reduction in tax rates

cannot be compared over time and across systems. To see why, consider the case where the simple benefit function (5) is inserted into the expression for the marginal tax rate (7):

$$(B2) \quad \tau_t - \tau_{t+1} \alpha \frac{1+G}{1+R}$$

We observe that marginal tax rate on the labour of income of generation  $t$  is determined by the contribution rates for both generation  $t$  and  $t+1$ . The case where  $\tau_{t+1} > \tau_t$  illustrates the expansion of the public pension system financed by the working generation at time  $t+1$ , equivalent to the classical case where the first generation positively benefits from the introduction of a PAYGO system. The expansion will, however, increase the value of the second term even if the actuarial properties of the system remain unchanged ( $\alpha$  constant). Unless corrected for, this will seriously undermine the value of the calculations in the paper, since only the second term can be observed as long as actual contributions in period one are not explicitly defined (neither the tax rate  $\tau_t$  nor the tax base are defined in the Norwegian system). If we rely on the information embodied in the second term only and the contribution rate were not held constant, a system expansion will appear to lower the marginal tax rate. If we had full information to the complete expression, we would see that the marginal tax rate actually increases in the latter case.<sup>11</sup> In summary: Unless the contribution is kept constant across time and pension schemes, system generosity and the savings share cannot be disentangled over time or by only looking at the second term of equation (B2). The equation, and in particular its second term, would therefore be an incorrect measure of the marginal tax rate and therefore system efficiency: Any measure of the actuarial properties of a specific benefit function ( $\psi$ ) should be invariant to a scalar transformation of that function.

This result does not mean we must abandon the ambition to empirically compare the efficiency of different pension systems, at least as long as the contribution rate is not too different. The calculations in this paper employ a method whereby the expenditures of different systems are aligned to keep  $\tau$  constant. Specifically, if we want to compare pension system  $j$  to scheme  $i$  at time  $t$ , we define an alignment factor  $\delta$  as follows:

$$(B3) \quad \delta_{t+1,j} = \frac{\tau_{t+1,i}}{\tau_{t+1,j}}$$

This factor is then applied to all computed pension benefits for system  $j$  ( $b_{t,j}$ ) such that actual payments from the system are given by:

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<sup>11</sup> This holds for any dynamically efficient economy. The derivative of equation (B2) with respect to  $\tau$  can only be non-positive if  $\alpha(1+G)/(1+R) \leq 0$ .

$$(B4) \quad \tilde{b}_{t,j} = \delta_{t+1,i} b_{t,j}$$

Replacing (B4) and (B3) into (B1) for pension system  $j$  gives aligns the contribution rates to system  $j$  with that of system  $i$ :

$$(B5) \quad \tau_{t+1,i} = \partial_{t+1,j} \tau_{t+1,j} = \frac{n_{t,j} \tilde{b}_{t,j}}{n_{t+1,j} w_{t+1,j} l_{t+1,j}}$$

We also note that the approach allows for different support ratios and average per capita earnings across systems, which is convenient if such variations result directly from differences in pension system rules. Finally, we may now calculate an expression for the marginal tax rate that is constant across systems the two systems, where  $k = i, f$  :

$$(B6) \quad \tau_{t,k} = \frac{\partial_{t+1,k} \psi'_k(y_t)}{1+R}$$

It is important to make two clarifications with respect to this methodology. First, the method implicitly aligns pension benefits of system  $j$  to meet the budget requirements of system  $i$  through a uniform cut/increase in benefits. If this process were to take place, it is neither obvious nor likely that the constraint would be met by uniform cuts. This method is chosen, however, as not to change the balance between the savings component and redistributive part of the pension benefits. The GINI coefficient of inequality is also invariant to this linear transformation. Second, the methodology only corrects for differences in  $\tau$  across systems, not over time. It is therefore still possible to undertake a continual expansion of the pension system successively financed by coming generations. Increased generosity in the actuarial part of benefits at time  $t$  will reduce the marginal tax rate at time  $t-1$  even if pension benefit formula itself remains unchanged. However, the alignment methodology ensures that the same expansion over time takes place in all pension schemes under scrutiny, thereby permitting a comparison of their actuarial properties.