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Dennis Fredriksen

Projections of Population, Education, Labour Supply and Public Pension Benefits

Analyses with the Dynamic Microsimulation Model MOSART

Statistisk sentralbyrå • Statistics Norway
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Preface

This study is based on research performed at Statistics Norway over the last decade. The MOSART project group has over the years included Leif Andreassen, Truls Andreassen, Helge Brunborg, Dennis Fredriksen, Andre Hval Hansen, Tone Heimdal, Arve Hetland, Nico Keilman, Gina Spurkland, Inger Texmon, Yngve Vogt and Jannecke Østervold. The contribution from each is only partly acknowledged through the references throughout the study. Especially Yngve Vogt and Gina Spurkland ought to be acknowledged for their programming of respectively the first and second version of the simulation model. The author would like to thank Nils Martin Stølen for commenting the study and Marit Berger Gundersen and Tone Veiby for editing.

The inclusion of public pension benefits in the model was financed by the Norwegian Research Council. Several Norwegian ministries have supported the project. Especially the Ministry of Finance has provided both financial support and been a valuable critical user of the results. The National Insurance Administration has provided important microdata on pension entitlements.

Most of the preparations for this study ended in june 1996, and more recent contributions to microsimulation or the MOSART model are not included, for example the extension of the MOSART model with household relations and more income variables.

Abstract

Dennis Fredriksen

Projections of Population, Education, Labour Supply and Public Pension Benefits

Analyses with the Dynamic Microsimulation Model MOSART

Social and Economic Studies 101 • Statistics Norway 1998

Norway, like most developed countries, is facing an ageing population from the beginning of the 21st century, and this may have large impacts on public pension expenditures. These relations are analysed with a dynamic cross-sectional microsimulation model called MOSART. The model simulates the life course of a representative sample of the Norwegian population with respect to demographic events, education, labour supply and public pension benefits. Changes in these subjects since 1960 are also reported, and the MOSART model is tested by its ability to reproduce the actual development in this period.

The base line alternative of the analyses is a situation where “everything continues as in 1993”. Consequences for the tax level are analysed by calculating a contribution rate given by dividing public pension expenditures by the sum of wages and half the public pension expenditures (pensioners pay less taxes than wage earners). This contribution rate was 15.6 per cent in 1993, and increases to 25 per cent by year 2040 with the base line alternative. The size of the population has stabilized by this time, and the projected contribution rate is the result of structural aspects of the individual life courses. These aspects include the average number of years each respectively participates in the labour force or is a pensioner, and the ratio between average pension benefits and wages. Improved benefits and longer life expectancy explain most of the growth in the contribution rate.

Systematic, but still moderate changes in the underlying assumptions on life expectancy, disability pension and labour force participation rates may change the conclusion of a growing contribution rate. Political decisions which may reduce and finally eliminate public supplementary pension schemes can also change the conclusion.

If the underlying assumptions turn out to be correct, simulation of historical data shows that the MOSART model is able to predict the actual development from 1960 and onwards reasonably well. A projection where all underlying assumptions are assigned the level in 1967 gives a surprisingly good prediction of the contribution rate in 1993. However, large changes in several components working in opposite direction are hidden behind this picture. The projections with the perspectives from respectively 1967 and 1993 are very different by the middle of the 21st century. Important changes in the underlying assumptions from 1967 to 1993 are lower fertility, larger propensities to enter disability pension, lower retirement age and a higher expected increase in life expectancy.

Keywords: Education, Labour supply, Microsimulation, Population projections, Public pension benefits.

Sammendrag

Dennis Fredriksen

Framskrivinger av befolkning, utdanning, arbeidsstyrke og folketrygdens pensjonsutgifter

Analyser utført med mikrosimuleringsmodellen MOSART

Sosiale og økonomiske studier 101 • Statistisk sentralbyrå 1988

Norge står overfor en situasjon med en aldrende befolkning i begynnelsen av det 21. århundre, og dette kan få store konsekvenser for offenlige utgifter til blant annet alders- og uførepensjon. Disse sammenhengene blir analysert med en dynamisk mikrosimuleringsmodell kalt MOSART. Modellen simulerer livsløpet for et representativt utvalg av befolkningen med hensyn til demografiske kjennetegn, utdanning, yrkesdeltaking og pensjonering. Utviklingen framover er analysert i lys av hva som har skjedd innenfor dette området siden 1960, og tester viser at MOSART i rimelig grad klarer å reprodusere historien dersom de underliggende forutsetningene holder.

Analysene har som utgangspunkt en referansebane hvor "alt fortsetter som i 1993". Betydningen for skattenivået analyseres ved å se på en beregnet pensjonsavgift gitt ved samlede pensjonsutgifter delt på summen av samlede lønnsinntekter og halvparten av samlede pensjonsutgifter (pensjonister skattlegges lempligere). Denne beregnede pensjonsavgiften var på 15,6 prosent i 1993, og vil med de forutsetninger som er gjort omkring befolkningsutvikling, yrkesdeltaking og regulering av grunnbeløpet stige til 25 prosent i år 2040. Dette høye nivået skyldes ikke negativ befolkningsvekst, men er et resultat av det antall år hver av oss vil være henholdsvis yrkesaktiv og pensjonist, samt det relative forholdet mellom pensjoner og lønninger.

Bildet av en sterk økning i den beregnede pensjonsavgiften vil med få unntak være upåvirket av endringer innenfor det som har vært observert de siste ti-årene for hver enkelt faktor. Imidlertid kan sammenfallende endringer påvirke bildet, og med yrkesdeltaking som i 1987, fortsatt økning i kvinners yrkesdeltaking, fortsatt lav tilgang av nye uførepensjonister og en mer moderat økning i levealder kan den beregnede pensjonsavgiften stabilisere seg under 18 prosent. En systematisk svakere vekst i grunnbeløpet enn i lønnsnivået kan også gi dette resultatet. Imidlertid vil økonomisk vekst i liten grad påvirke den beregnede pensjonsavgiften sålenge pensjonsytelsene indekseres i forhold til lønnsnivået.

Studien presenterer også framskrivinger med de perspektiver som man med lignende metoder ville hatt i 1967, det året folketrygden ble opprettet. Fram til idag gir dette en overraskende god prognose på veksten i den beregnede pensjonsavgiften, men bak dette bildet ligger store endringer i de underliggende forutsetningene som har påvirket avgiften i motsatt retning. Framskrivingene basert på perspektivene i henholdsvis 1967 og 1993 gir store forskjeller i utviklingen framover. Viktige endringer siden 1967 er en sterk nedgang i fruktbarheten, redusert effektiv pensjonsalder og at vi nå forventer en sterkere økning i levealderen i de neste ti-årene.

Emneord: Arbeidstilbud, befolkningsframskrivinger, mikrosimulering, prognoser, trygd, utdanning

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1. Introduction

Norway, like most developed countries, is facing an ageing population from the beginning of the 21st century. Furthermore, a system for general public pension benefits named the National Insurance Scheme was established in 1967. The average pension benefit from this scheme will continue to grow towards year 2035 as new pensioners have earned entitlements for a longer period than current pensioners. These two conditions, an increasing percentage of elders and improved pension benefits, *may* represent a challenge to the financing of public expenditures in the future. An adequate analysis of these phenomena requires an approach based on micro data (especially the supplementary pensions are calculated from the labour markets earnings in *each* year throughout the working career). This study presents a dynamic cross-sectional microsimulation model, called MOSART, that may be used to analyse the development of population size and composition, and the consequences for the educational level, labour supply and public pension benefits. One main topic is a discussion of microsimulation and the MOSART model as methods of analysis. However, the study also presents projections and analyses of the subjects presented in the title. An overview of the results from these projections can be achieved by reading the brief model description on page 13 and the following table 1.1, besides the parts of section 1.2 (summary of results) and chapters 4 to 8 of interest.

1.1. Summary of methods

Microsimulation

Orcutt (1957) was the first who suggested microsimulation modelling as a method in social planning, and the idea is to represent a socio-economic system by a sample of decision units, for example persons or firms. Behaviour et cetera is modelled at the micro level and then aggregated up to total quantities for the economic system, rather than modelling the relations between aggregated quantities directly. In some cases it is easier to model behaviour at the micro level, and it may often be inappropriate to model relations between the aggregated quantities due to heterogeneity between micro units, non-linearities and complexity. One example may be a non-proportional tax system, as most tax systems probably are. In this case total taxes will depend in a more or less complex manner on both income level and income distribution. An adequate analysis of the effects from changes in tax rules on tax revenue, income distribution and labour supply thus may require an approach at the micro level.

A main distinction between microsimulation models is whether they are *static* or *dynamic*. Static microsimulation models are usually based on a sample of persons with detailed information from for example income declaration forms. The sample is aged by reweighing and by indexing all income amounts, in such a way that the sample may be perceived as representative for the population in for example the forthcoming budget year. Two usual advantages of static microsimulation are moderate development costs combined with a very detailed representation of the tax and transfer system. However, static microsimulation is of little help in long term projections and analysis of life course aspects such as pension schemes.

In dynamic microsimulation the population is aged by drawing events dependent on each persons characteristics, and by implementing the effect of these events on the relevant characteristics of the person. It is thus possible and relevant to age a person for the full life span, and so make long term projections and follow each person over the life course. Dynamic microsimulation covers a large variety of life cycle models, and in this context the MOSART model is based on discrete time and a real sample with a cross-section of the Norwegian population. Furthermore, all aspects of the population in one year is simulated before the model starts simulating the next year, so-called *cross-sectional* simulation.
A disadvantage of (dynamic) microsimulation modelling is the development costs, and no major applications exist including a cross-section of both persons, firms and other institutions from a real sosio-economic system. Microsimulation models also require a large sample of decision units to achieve sufficiently small sample errors, and the subsequent computer runtimes may be unacceptable for several purposes. This may explain why it took more than twenty years from Orcutt (1957) until the first appearance of a dynamic microsimulation model which was practical to use. With less expensive hardware and more user-friendly software a number of dynamic microsimulation have appeared, and a model comparable to the MOSART model in complexity and persistence is the NEDYMAS model (see Nelissen 1995). Compared with this and some other dynamic cross-sectional models internationally the MOSART model is based on microdata of higher quality. The MOSART model includes fewer covariates for some of the transition probabilities, and the theoretical foundation may also be weaker. The model includes fewer characteristics because we have been reluctant to include characteristics where genuine microdata is unavailable. The number of subjects in the MOSART model are at average, but some comparable dynamic microsimulation models also include taxes, consumption and savings, and also a geographical dimension.

A discussion of methodological issues is found in section 2.1 and 2.2, including a comparison of microsimulation with other methods of analysing similar subjects (macroeconomic modelling and transition matrix modelling), and the MOSART model is compared with some other models in section 2.5 and 2.6.

History of the MOSART model

Statistics Norway has since the seventies projected population, education, labour force and public pension benefits for public planning. These subjects were analysed in different transition matrix models where the population is represented by a table with the number of persons for each combination of characteristics. This population table is aged from one period to the next by multiplying with the transition matrix; the percentages of each combination of characteristics that each year make a transition from this combination of characteristics to another. The transition matrix method imposes strict limitations on the number of characteristics within *one* model, because even a moderate number of multi-state characteristics can comprise a very large number of combinations of characteristics.

The computer used to project education and labour supply was closed down towards the end of the eighties, and rather than reprogramming the existing transition matrix models to a new computer, it was decided to integrate these models in one microsimulation model called MOSART. The MOSART model continues the traditional use of transition matrix models at Statistics Norway, and the transitional probabilities in the MOSART model are partly based on parameters from the preceding transition matrix models.

The first version of the MOSART model comprised demographic events, education and labour force, and was documented in Andreassen et al. (1993). The second version extended the MOSART model with public pension benefits and labour market earnings, and was documented in Fredriksen and Spurkland (1993). For the present analysis a revised version of Fredriksen and Spurkland (1993) is used, where the important changes go to drawing methods, computer programming and simulation of historical data. Section 2.3 and 2.4 provide a more detailed background and history of the MOSART model.

The MOSART model

MOSART is a dynamic microsimulation model with a cross-section of the Norwegian population and a comprehensive set of characteristics. The model starts with a representative sample of the population in a base year and simulates the further life course for each person in this *initial population*. The simulation is carried out by drawing if each person each year makes certain transitions from one state to another with *transition probabilities* depending on each person's characteristics. Each of these transition probabilities will normally constitute an event, and is estimated from observed transitions in a recent period. Events included in the simulation are migration, deaths, births, marriages, divorces, educational activities, retirement and labour force participation. Public pension benefits are calculated from labour market earnings and other characteristics included in the simulation. New-borns and new immigrants are added to the initial population each year. The result of the simulation is a *model population* with the life course of each individual.

The MOSART model projects the Norwegian population and its characteristics for the coming decades. Before these projections can be made, the model user must make some assumptions on how the behaviour et cetera underlying each event will develop in the future. The base line alternative in this study is called the *perspectives from 1993*, and its *underlying*

Table 1.1. **Main underlying assumptions with the perspectives from 1993**

Net immigration of 8 000 persons per year
Life expectancy at birth increases 4 to 5 years towards year 2050
Total fertility rate of 1.89
Propensity to study as in 1993
Entry into disability pension as average of 1989 to 1993
Retirement age remains at 67 years
Labour force participation rates as in 1993
Average labour market earnings as in 1993
Basic Pension Unit and Special Supplement as in 1993
All nominal amounts are measured in Norwegian kroner (NOK), deflated with the consumer price index with 1993 = 1.00[1]

[1] This can for most purposes be reinterpreted as if all nominal values is deflated by the wage level in 1993, and that the Basic Pension Unit and labour market earnings are growing at the same rate after 1993.

assumptions are summarized in table 1.1. The projections can be used in several ways, and one interpretation is that the base line alternative shows the development ahead if "everything continues as today". This interpretation presumes that the projections are a prediction of the development ahead given the underlying assumptions. Policies or other circumstances leading to changes in the underlying assumptions can be analysed by comparing two or more projections. The MOSART model may also analyse income distribution in a life course context.

The initial population, the accounting of persons and the transition probabilities carry the potential of the MOSART model as a relevant tool for social planning. Several personal characteristics are either stable or easy to predict, for example gender, age, educational attainment for adults and already earned pension entitlements. The initial population will therefore contain much information about the Norwegian population for several decades ahead. The modelling of behaviour at the micro level constitutes an accounting of persons; for example, the number of cohabiting men will automatically equal the number of cohabiting women. A microsimulation model is also a tool of presenting and synthesizing the knowledge that a set of transition probabilities contains about a socio-economic system.

Chapter 3 describes the MOSART model in more detail, while section 2.2 discusses some technical aspects of microsimulaion in general. An overview of technical aspects of the MOSART model is found in table 2.1, while table 3.1 gives a brief but compact presentation of the transition probabilities. Each of the subjects in the title of this study is handled throughout chapters 4 to 7, and this includes demography, education, labour supply and public pension benefits. The projections are summarized in chapter 8 by looking at a contribution rate given by the ratio between public pension expenditure and labour market earnings. These five chapters include roughly the same elements, and the structure is presented in the following: Each chapter starts out with a description of its part of the MOSART model and the development in the transition probabilities over the last decades. Furthermore, the simulation of historical data is compared with the actual level, before the projections are presented.

Testing of models and simulation of historical data

A main topic of this study is testing of models, see also section 2.7, and the hypothesis that is tested is:

(1.1) If the underlying assumptions in for example table 1.1 are correct, then the MOSART model describes the (future) development of the Norwegian population.

The hypothesis is tested by a *simulation of historical data*, where all transition probabilities are adjusted in such a way that aggregated measures for each transition probability match the actual levels for all subjects in table 1.1. The MOSART model is often used with a time horizon of 30 to 40 years, and this requires a test period of comparable length. We have for this reason used 1960 as the start year in the simulation of historical data, but only generated a synthetic initial population for this year (see appendix A). Furthermore, the simulation of historical data is based on the same transition probabilities used in the current version of the MOSART model. These transition probabilities are estimated over the period 1985 to 1993 and adjusted by simple methods, for example by proportional adjustment of all probabilities behind one event/transition.

The simulation of historical data may also be interpreted as a (weak) test of the MOSART model's quality in comparative statics, by the fact that alternative projections are generated by adjusting the transition probabilities with same techniques used in the simulation of historical data. However, the simulation of historical data does not test the relations between the transition probabilities and the explanatory variables, for example with respect to stability in covariates.

Discrepancies between the simulation of historical data and the actual level are referred to as *model errors*. This type of errors are most often caused by the model being to simple or changes in the effects of explanatory variables (covariates). The uncertainty generated by stochastic drawings is in this version of the MOSART model quite moderate due to the drawing method, see appendix B. We consider the uncertainty generated by stochastic drawing to be a small problem if computer runtimes up to one hour is acceptable for long term projections.

A projection where all transition probabilities are held at the level in 1967 is also presented, and discrepancies with the actual level are here referred to as *projection errors*. The contribution to the projection errors from changes in each underlying assumption is identified, and this emphasises the important changes since 1967. The projection errors combined with the sensitivity analyses may illuminate the strength of the presented conclusions with respect to changes in the underlying assumptions.

The simulation of historical data leads to a model population that may be used as a *simulated initial population*, and projections based on the simulated and the real initial population are compared.

The simulation of historical data points out weak sides of the MOSART model where improvements are wanted. Included in these improvements are a better simulation of historical data, for example by using more sophisticated methods of generating a synthetic initial population. The simulation of historical data may also start either earlier or later than 1960, the first increases the testing period, the latter makes the first year of the National Insurance Scheme more correct. Furthermore, the transition probabilities should be adjusted towards measures for probabilities rather than against the number of events each year (for example total fertility rate rather than the number of births). The simulation of historical data indicates that an analysis may have interest even when the initial population is generated partly or totally without genuine microdata.

The discrepancies between the simulation of historical and the actual development, the model errors, are reasonable small. However, the model errors point out some areas that need improvement, and some of the conclusions from the analyses should be drawn with caution. The transition probabilities are only weakly adjusted when simulating their own estimation period, indicating that the estimated relations are reasonable. Furthermore, the discrepancies between a projection with constant transition probabilities and the actual level, the projection errors, are moderate for the period 1967 to 1993. Two projections with base years in respectively 1967 and 1993 differ substantially in the long run.

1.2. Summary of results

Demographic events and population size

The simulation of demographic events comprises migration to and from Norway, deaths, births, marriages and divorces. The most striking change since 1967 is the dramatic reduction in fertility, with a drop in the total fertility rate[1] from 2.8 in 1967 to the current level of 1.89. Net immigration has increased in the same period and life expectancy is expected to increase more than earlier. With a total fertility rate of 1.89, net immigration of 8 000 persons per year and 4 to 5 years longer life expectancy than today, the Norwegian population will stabilize at 5.2 million persons towards the middle of the 21st century. The growth in the number of persons will mainly occur in age groups older than 55 years. The percentage of old persons in the population increases mainly due to lower mortality in the future than in the past.

The simulation of historical data shows, as expected, that aggregated measures for mortality, migration and fertility provide a good prediction on the number of persons by gender and age. However, the simulation of historical data reveals weaknesses in the modelling of demographic events with respect to the effects on some other subjects. For example, the absence of gross immigration and emigrations gives some problems with the number of Norwegian pensioners living abroad and pension entitlements earned by persons living abroad. Furthermore, mortality by sosio-economic status affects the ratio between pension expenditure and labour market earnings (the contribution rate).

Education

Educational activities and degrees are simulated depending on gender, age, educational level and last year's educational activities. The average period as pupil or student after the compulsory primary school increased from 2.2 years in 1960 to 6.9 years in 1993. With the

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[1] The average number of children a woman will give birth to if she lives until an age of 45 years.

current transition probabilities from 1993 the period as pupil or student will continue to increase and stabilize at a level of 8.7 years. The educational level has grown substantially over the last decades, and this growth will continue with any educational transition probabilities from the last decade. With the estimated propensity to study in 1993, more than 40 per cent of the labour force will have higher education in year 2030, compared with the current level of 20 per cent.

The simulation of historical data shows that a good prediction on the number of pupils and students leads to a good prediction of the educational level, even when the other aspects of the educational system are held constant. These aspects comprise the propensity to accomplish an educational activity, choice of subjects and the age distribution of pupils and students. However, some discrepancies point out areas that need improvement. A large percentage of young immigrants accomplish primary school after they enter Norway without being registered as pupils, and this should be included in the model. Furthermore, the projections should be presented with emphasis on more than the total number of pupils *and* students. This may include the percentage entering grammar school (by subject) after primary school, the extent of apprentices, the propensity to accomplish an ongoing educational activity, the number of pupils in secondary school and the number of students in higher education.

Public pension benefits

This version of the MOSART model comprises public pension schemes related to old age, disability and surviving spouses and the semi-public arrangement of early retirement[2]. The entry of new disability pensioners is emphasised since this transition is less determined by other events, such as age limits or widowhood. The risk of entering disability pension is represented by a logit function estimated on microdata, including covariates for gender, age, marital status, education and labour force participation. For example, the disability risk among male graduates is approximately one-tenth of the risk among males with just primary school.

Retirement age was reduced from 70 years to 67 years in 1973, and the extent of disability pension has increased substantially especially during the eighties. The recent development shows a declining entry of new disability pensioners, but an increase in the extent of rehabilitation schemes indicates that the development in disability pension may to some degree be a temporary decrease. The base line projection is therefore based on the average entry of new disability pensioners in the period 1989 to 1993.

The number of pensioners changes little towards year 2010 because new pensioners in the forthcoming decades will be recruited from the small birth cohorts from the thirties. After year 2010 the large birth cohorts from the post-war period approach retirement age, and life expectancy is expected to increase. The number of pensioners will therefore increase dramatically, and by year 2040 stabilize at a level 75 per cent higher than the current level.

The simulation of historical data shows that a good prediction on the number of new disability pensioners and general life expectancy leads to a good prediction on

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[2] In Norwegian: Avtalefestet pensjon.

the number of pensioners. Changes in mortality among disability pensioners may weaken this conclusion.

The National Insurance Scheme was established in 1967, and only labour market earnings after this year give entitlements for a Supplementary Pension from this scheme. Persons born in the period 1900 to 1936 are only partly compensated for the missing opportunities of earning full Supplementary Pensions, and this implies that the pension scheme will be fully matured first by year 2035 according to our projections. At this time the average old age pension benefit will be worth 98 000 NOK deflated with the wage level in 1993, compared with the value of 70 500 NOK in 1993.

All entitlements and benefits from the National Insurance Scheme are measured in Basic Pension Units, and changes in the value of this unit will index existing entitlements and decide the maximum Supplementary Pension. The base line projection assumes that the Basic Pension Unit grows at the same rate as wages, corresponding to the official policy. However, the Basic Pension Unit has historically grown less than wages, and if continued this will affect the National Insurance Scheme heavily. If the Minimum Pension Benefit grows at the same rate as wages, the maximum Supplementary Pension will end up being incorporated in the Minimum Pension Benefit. For a single pensioner the Minimum Pension Benefit was worth 60 000 NOK in 1993.

The simulation of historical data shows that a good prediction on the value of the Basic Pension Unit and average labour markets earnings (by gender) leads to a reasonable good prediction on the average pension benefits from the National Insurance Scheme. The MOSART model will probably underpredict the average pension benefit by 2 to 3 per cent with the underlying assumptions in the base line alternative.

Labour force

Labour supply is simulated in two steps in this version of the MOSART model. First, the number of persons in the labour force is projected by multiplying the number of persons with labour force participation rates (LFPR) depending on gender, age, children, education and pension status. The same procedure is repeated for the number of man-years versus average working hours. Second, labour market earnings are simulated for each person such that the number of income recipients is consistent with the projected labour force above. The individual labour market earnings depend on the same characteristics as the LFPRs above, but also include earlier years' labour market earnings.

The number of persons in the labour force has increased substantially after 1967 due to increased LFPRs among women. Male LFPRs have fallen in the same period, but not more than the increased percentage of students and disability pensioners. The base line projection assumes that LFPRs and working-hours remain constant at the level in 1993. Based on these assumptions the labour force continues to grow towards year 2020 mainly due to more persons in the age group 25 to 66 years. An increasing educational level also contributes to a larger labour force and smaller differences between male and female LFPRs.

The distribution of labour market earnings from the simulation of historical data is compared with the actual income distribution in the period 1967 to 1993. “Life course” proprieties are emphasized, and the MOSART model gives a reasonable realistic description of the income distribution. Variations in labour market earnings across the life course are understated for men, while the variations across men are overstated. However, the substantial problem in the simulation of labour supply is a weak theoretical foundation for the spesification and estimation of the model. Furthermore, our specification of unobserved heterogeneity makes labour market earnings unavailable as an explanatory variable for other transitions.

Contribution rates

Changes in the tax burden related to the ageing of the population are analysed in the MOSART model by calculation of a so-called *contribution rate*, here defined as:

$$(1.2)\ \text{Contribution rate} = \Sigma_i \text{Pension benefits} / (\Sigma_i \text{Labour market earnings} + 0.5 \times \Sigma_i \text{Pension benefits})$$

The calculation is based on the assumption that pensioners pay roughly half the tax rates of persons participating in the labour force, and that the matter of interest is the average tax rate. Other public expenditures related to old persons, for example health care, are disregarded. Other parts of the tax base, for example financial incomes, are also disregarded. However, the analyses have the advantage that a rich set of sensitivity tests related to any of the characteristics included in the MOSART model may be presented at a low cost.

The contribution rate increased from 7.3 per cent in 1967 to 15.6 per cent in 1993. If the entry of new disability pensioners remains at the current low level, the contribution rate will change little until year 2010. The contribution rate then grows rapidly towards year 2040 in the base line alternative, and stabilizes at a level just below 25 per cent. By this time the population size and the age distribution have also stabilized, and the high contribution rate cannot be explained by a declining population size. The level of the contribution rate in this situation is the result of structural aspects of the life courses, given by the average number of man-years across the life course, the average number of years as pensioner and the ratio between the average pension benefit and wage per man-year. The increasing contribution rate from 1967 until year 2040 is mainly caused by increasing pension benefits relative to wages and lower mortality in the future than in the past, each explaining roughly half the growth.

The result that the contribution rate will grow holds for changes in each underlying assumptions within the range observed over the last (turbulent) decades. Systematic changes in the underlying assumptions can however change the conclusions, even if each change is moderate. A more moderate increase in life expectancy, an entry of new disability pensioners at a low level as in 1993, male labour force participation rates as in 1987 and smaller gender differences in labour force participation may lead to a contribution rate stabilizing below 18 per cent. A systematic lower growth in the Basic Pension Unit may also lead to the same result. Economic growth, however, will only affect the contribution rate if economic growth also influences labour force participation rates, retirement age or the ratio between wages and average pension benefits.

The projection with the perspectives from 1967 gives a surprisingly good prediction of the contribution rate in 1993, but behind this result are large changes in several underlying assumptions counteracting each other. In the projection with the perspectives from 1967 the contribution rate will stabilize at the current level, and this indicates that the National Insurance Scheme was well funded in 1967. Important changes in the underlying assumptions from 1967 to 1993 are the higher expected increase in life expectancy, lower retirement age, larger propensities to enter disability pension and lower fertility. These changes have been counteracted by less generous rules for calculating Supplementary Pensions, increased female labour force participation and higher net immigration.

2. Some methodological aspects

We discuss in chapter 2 why we use microsimulation in our analyses and compare the MOSART model with other microsimulation models and other types of analyses. Section 2.1 and 2.2 discuss general aspects of microsimulation, while we in section 2.5 compare the MOSART model with other microsimulation models internationally and in section 2.6 with other planning models in Norway. The background and history of the MOSART model is presented in respectively section 2.3 and 2.4. Testing of models is examined in section 2.7. A discussion of methods related to the MOSART model is also found in Andreassen (1993).

2.1. Microsimulation versus other simulation techniques

Microsimulation modelling was first suggested by Orcutt (1957) as a tool for social planning, and a more recent discussion is found in Orcutt (1986). A simulation technique is used to solve a model or theory for the behaviour of an economy when an analytical solution is impossible to reach. The basic idea in *microsimulation modelling* is to represent a sosio-economic system by a sample of decision units, for example persons, and then model the behaviour of these primary units. Aggregated numbers can be reached by multiplying each unit with its sample weight and add across the sample. A second approach is *macroeconomic modelling* which is based on relations between aggregated numbers for different sectors of the economy. A third approach is *transition matrix modelling* where the population is represented by the number of persons for each combination of characteristics.

Orcutt (1957, 1986) mention the possibilities of testing and avoiding harmful aggregation as the main advantages of microsimulation modelling. General presentations of microsimulation models as a method are often vague on the disadvantages of microsimulation versus macroeconomic models and transition matrix models. Development costs and computer runtimes (or Monte Carlo Variability) can however be mentioned as two main disadvantages of microsimulation modelling. Lack of genuine micro data may also be a problem.

Aggregation

The problem of aggregation can be represented by a simple tax model with no interdependencies between households:

(2.1) $Y = \Sigma_{i \in I}(X_i)$

Where:
- Y is total taxes
- X_i is a vector of characteristics and tax parameters for household 'i'
- I is the population of households
- f(.) is the relation between the characteristics and the tax for a household

Microsimulation can solve this model depending on access to microdata with the characteristics included in X_i and a proper description of the functional forms f(.). All information in the initial situation are then utilized, for example the distribution of income across households. The specification of the functional forms f(.) may often be given a more intuitive interpretation at the micro level, for example by implementing the actual tax rules for households, rather than estimating the taxes as a function of average income et cetera. If the model includes behaviour, for example for the labour supply, these relations can be based directly on more basic microeconomic theory. An approach based on aggregated numbers (for Y and X) in equation 2.1 becomes problematic when the functional forms f(.) are non-linear and the characteristics are distributed such that:

(2.2) $Y = \Sigma_{i \in I} f(X_i) \neq N \times f(X)$

Where:
- N is the number of households
- X is a vector with the average value for each characteristic

An approximation between Y and X may be especially troublesome when f(.) is non-monotonic in one or several of the characteristics, as the relation between labour force participation and age. The problem of non-linearity can be solved with a transition matrix model where the population is represented by a vector where each element comprises the number of units with a given combination of characteristics. The charachteristics of the population are changed (aged) from one period to the next by multiplying the population vector with a transition matrix with the transition rates between each combination of characteristics:

(2.3)

$$P_{T+1} = P_T T$$

$$P_T = [p_{1T}, p_{2T}, \ldots. p_{MT}],$$

$$T = \begin{bmatrix} T_{11} \ldots\ldots\ldots t_{1N} \\ .. \\ T_{M1} \ldots\ldots\ldots t_{MM} \end{bmatrix}$$

Where:
- P is the population table
- p_{iT} is the number of units with the combination 'i' of characteristics in period 'T'
- M is the number of combinations of characteristics
- T is the transition matrix
- t_{ij} is the percentage of units with the combination 'i' of characteristics that will have the combination 'j' of characteristics in the next period

A transition matrix model can solve equation 2.1 by calculating the tax for each combination of characteristics, and then multiply with the number p_{iT} of persons (Andreassen 1993 refer to this as headship rate models).

A dynamic microsimulation model represents the population in equation 2.3 by a sample. The sample is aged by drawing if each unit experiences transitions for any characteristic with transition probabilities depending on characteristics of the unit simulated. If these transition probabilities are identical with the transition matrix T, the expectation values of the microsimulation model and the transition matrix model are identical. The choice between microsimulation and the transition matrix method is thus a question of which method that is most efficient at solving the model, where computer resources play a key role. If the numbers p_{iT} of units for each combination of characteristics are large, transition matrix models are usually far more efficient than microsimulation models. The number M of possible combinations of characteristics in the population vector P_T is given by the multiplied sum[3]:

(2.4) $M = \Pi_{j \in J} s_j$

Where:

- M is the number of possible combinations of characteristics
- s_j is the number of states for characteristic 'j'
- J is the set of characteristics

Even simple models can have a larger number of possible combinations of characteristics than the number of units in the population, and this usually disqualifies transition matrix models as an interesting approach. The number of possible combinations of characteristics will be very large with complex primary units and continuous state variables. An example of a complex primary unit is the cross-distribution of characteristics across all persons in a household. Another example is the life course of a person, and this unit is interesting, for example, when pension benefits depend on the full working career.

Truly continuous variables in the sense that each unit can have its own unique value of a characteristic cannot be represented in a numerical transition matrix model. Analyses of income distribution are often based on a set of truly continuous variables, income amounts, and will for this reason require a microsimulation approach. A characteristic that contains the number of years since an event occurred is almost continuous and can have a strong effect on the number of possible combinations of characteristics. An important example is age of youngest child and its effect on fertility and labour force participation. Modelling based on continuous time represents similar problems for a numeric transition matrix model, see also section 2.2.

Testing of models

Orcutt (1957, 1986) stresses testing of models as the main advantage of microsimulation. Arguments given are the limited number of observations in time series analyses that most macroeconomic models are based on, and the lacking possibilities of performing experiments at an aggregate level. Several other contributions on microsimulation also stresses testing, see for example Orcutt et al. (1986), Caldwell (1996) and Antcliff (1993). A suggested method of

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[3] The transition matrix T can be reduced by excluding transition rates that are zero (or approximately zero), in the same way as the transitional probabilities in a dynamic microsimulation model is simplified by excluding less important transitions or covariates. The transition matrix can therefore be less than the square of the number of combination of characteristics, or even the number of combination of characteristics if age is represented by one-year intervals. The transition matrix can also be reduced by representing it with a functional form rather than a table.

testing is to compare projections with the later actual development, and we interpret this type of tests with respect to the MOSART model in section 2.7.
Mot (1992, page 41) and Merz (1991) have found few examples of performed and published tests of microsimulation models. However, some tests of microsimulation models have been published since then.

Disadvantages: Development costs and computer runtimes

Dynamic microsimulation models are considered expensive to develop and require expensive microdata. To our knowledge there are no major applications that integrate a real sample of persons and firms in one microsimulation model. Present dynamic microsimulation models are therefore partial in the sense that these models neglect important parts of the socio-economic system and the interdependency between sectors of the economy. An example is possible dependencies between labour supply and labour demand. Technological progress in computers and soft-ware is however reducing the cost of developing and using microsimulation models.

Furthermore, stochastic drawing of events requires that the simulation comprises a large number of units or replications before the expectation value is found with sufficient precision. This can lead to computer runtimes that are unacceptable for certain purposes. Projections until year 2060 with one per cent of the Norwegian population require 45 minutes in this version of the MOSART model, and this leads to a relative standard deviation on most aggregate figures less than 0.1 per cent. See also appendix B for a discussion of how these problems are handled in the MOSART model.

2.2. Different types of micro simulation

A classification of microsimulation models is found in Mot (1992) including a table with a list of standard proprieties that easily shows the content of different microsimulation models. Mot (1992) is based on visits to institutions that develop microsimulation models, because published material was insufficient, and this indicates the problems of a standard presentation of this type of models. Table 2.1 presents the MOSART model in a similar way as the tables in Mot (1992), and compare the model with alternative approaches. Some topics are however omitted and discussed later, this includes development costs in section 2.5 and administrative settings and computer resources in section 3.5. The transition probabilities of the MOSART model are later presented in table 3.1.

Dynamic and static modelling

The main distinction between microsimulation models is between *static* and *dynamic* modelling. Static microsimulation models describe the population for a single year or date, and the original sample is aged to a future year or date by reweighing the sample and by indexing nominal amounts. The advantage of static microsimulation models is moderate development costs (if genuine microdata is available), and they will often comprise a rich set of characteristics based on a sample survey including interviews. The disadvantage is the absence of life course aspects, and the ageing may be weak where cohort differences are more important than life course proprieties (for example where the current 63 years olds are more informative about next year’s 64 years olds than the current 64 years olds).

Table 2.1. **Some methodological aspects of microsimulation models**

Status of the MOSART model	Alternatives
Dynamic microsimulation, population is aged by drawing of events	Static microsimulation, population is aged by reweighing and indexing of nominal values et cetera
Full cross-section of the population	A birth cohort or other sub-populations
Subjects: Migration, mortality, fertility, nuptiality, education, labour supply and income, public pension benefits	Other subjects in person models: Geographic mobility, household formation, social insurance in general, taxation, consumption/savings/wealth
Simulation unit: Persons	Other simulation units: Households, firms
Micro-macro link: None	Inclusion of both persons, firms and other institutions in one microsimulation model, *or* Iterative simulation of a microsimulation model and a macroeconomic model, *or* Constraining the projections against labour demand et cetera from a macroeconomic model
Discrete time with the calendar year as time unit	Continuous time, *or* Discrete time with other time units
Simulates one event at a time in a fixed order with conditional transitional probabilities, often called *recursive simulation*	Simulate all events for a person (in one period) simultaneously, *or* Simulate one event at a time in a random order
Simulates the whole population year-by-year, often called *cross-sectional simulation*	Simulate the full life course of one person at a time, often called *longitudinal simulation*
Real initial population based on administrative data	Synthetic initial population, *or* Real initial population based on interview data
Transition probabilities estimated with observed rates	Transition probabilities estimated based on explicit theory, often mentioned as *behavioural responses*
Mean-constrained drawing method that handles heterogeneous binomial probabilities in an efficient way	Random number method, *or* other methods of constraining the number of events to the expectation value of the model

Static microsimulation models are typically used for analyses of tax systems with focus on the forthcoming budget year, while dynamic microsimulation models typically are used for analyses of pension schemes. Merz (1991) gives an overview of static microsimulation models in Europe, while the static microsimulation model LOTTE for the Norwegian tax system is briefly presented later in this chapter.

An important distinction between dynamic microsimulation models is the *sample* underlying the model, either a sample with a full cross-section of the population or a sample from a sub-population such as a single birth cohort. Dynamic *cross-sectional* models are based both on a full cross-section of the population and an approach where the entire population at one point in time is simulated before entering the next point in time (for example calendar year). *Cohort models* start out with a number of blank newborns, and save the cost of constructing an initial population with a complex distribution of characteristics. Cohort models can be used for analyses of income distribution across the life course and between persons, and for these purposes cohort models are more cost efficient than dynamic microsimulation models based on a cross-section of the population. Wolfson (1988) presents a cohort model for Canada. Harding and Falkingham (1996) perform a comparative analysis of taxes and transfers in Australia and England based on two cohort models for these two countries. Conclusions drawn are that the British system transfers more across the life course (social insurance), while the Australian transfers more between persons (social assistance).

Some research institutions that develop microsimulation models have both a static, a cohort and a dynamic cross-sectional microsimulation model based on the same microdata, econometric analyses and computer soft-ware. Mot (1992) mentions the Sfb3 model in Germany as an example, and to some less degree DYNASIM II in USA. NATSEM in Australia is also an example. Statistics Norway also covers these three types of microsimulation with the static microsimulation model LOTTE and the dynamic microsimulation model MOSART, where the latter model also can be used for cohort analyses. However, the LOTTE model and the MOSART model arise from two different traditions, the LOTTE model from tax analyses, and the MOSART model from population projections based on transition matrix models. At present the LOTTE model and the MOSART model are thus not much integrated.

Other aspects of dynamic microsimulation models

Some *subjects* or events/characteristics are found in most dynamic microsimulation models where persons are the *simulation unit*. A minimum set of demographic events are required, and this comprises births, deaths and migration to and from the country. Furthermore, natural extensions are household formation, educational activities, labour supply and pension schemes. The more advanced models for persons also include taxation, consumption, wealth and migration within the country.

An important technical aspect is the *measurement of time*, either as continuous or discrete with for example the calendar year as time unit. While discrete time models draw if an event occurs during the time unit, the continuous time model draw the length of time until an event occurs. Most dynamic microsimulation model is based on discrete time since this is presumed to simplify the programming. Discrete time combined with cross-sectional simulation makes it possible to use exogenous constraints on the number of events within each time unit. Discrete time is however an aggregation, and can cause problems if the time unit is too large relative to the processes modelled. Sick leave and unemployment are examples where the time spells are too short to be captured properly with any time unit longer than a week. Discrete time models will also have problems with events that can happen during the same unit of time. Most discrete time models are based on *recursive simulation* where one event is simulated at a time, see section 3.4 for a discussion. Continuous time models handle this problem of competing risk far more elegant because each event can be handled independent of the other events, see Schweder (1988). No major dynamic microsimulation models with continuous time is accomplished in the sense that projection results have been published, but two projects can be mentioned. DYNAMOD is presented in Antcliff (1993) and is now operational. The plans for a dynamic microsimulation model for the Norwegian social security system, TRYGD, are presented in Østervold and Bragstad (1995).

Dynamic microsimulation models usually draw if certain events occurs for each person in the sample with probabilities depending on each person's characteristics. An alternative approach is so-called *statistical matching* of panel data with observations for at least two consecutive years, and the method is here described for the two-year case. In statistical matching the model starts with a random person at the lowest age level in the sample and matches this initial person with a random person with the same age in the first year as the initial person will have in the second year. Besides age, the matching can also include other characteristics present in the first and last years. The matching is repeated successively with a person who is one year older each time until the process reaches the maximum age or a person who died in the second year. The advantage of statistical matching is the possibility of generating life course data with a rich set of characteristics with less costs than the ordinary method of drawing events. The disadvantage is that all transitions are limited to the development between the first and second year in the original panel data, and the very limited possibilities of changing the assumptions underlying the simulation. An example of a cohort model based on statistical matching is found in Swedish Ministry of Finance (1994), and a cross-sectional dynamic microsimulation model based on statistical matching is found in Haga (1992).

Some of the microsimulation models are referred to as general in Mot (1992), but this is misguiding as pointed out by Mot (1992) herself, since none of the models comprise both demand and supply in any markets, for example the labour market. This aspect is referred to as a *micro-macro link*, and can be solved by including both firms, institutions and persons in the same microsimulation model. Solutions considered less expensive are a simulation with some exogenous constraints or a recursive simulation of a micro- and macro-simulation model. The latter is included in the plans for some of the microsimulation models presented in section 2.5 and may already have been implemented. We have no plans for a micro-macro link in the MOSART model. A further discussion of micro-macro links can be found in Caldwell (1986).

The empirical foundation of a microsimulation model is given by its *initial population* and *transition probabilities*. Several characteristics at micro level are either given or very stable, for example gender, age, educational attainment of adults and already earned pension entitlements. The initial population is therefore important for the quality of the projections. The same is obvious for the transition probabilities. Dynamic microsimulation based on discrete time can use *drawing methods* that constrain the simulated number of events to the expectation value of the model, in most cases given by the transition probabilities. See appendix B for a discussion of drawing methods.

2.3. Models preceding the MOSART model at Statistics Norway

Statistics Norway has since the sixties and seventies published long term projections of population, labour force, education and public pension benefits. These projections have been used in analyses by Statistics Norway and for various purposes in public planning, including white papers from the Norwegian government. The MOSART model continues this tradition, and is to some degree based on input from the earlier models. An overview of these models are given in table 2.2, while the content of the models is described in the rest of this section. Andreassen et al. (1993) give a survey of white papers using long term projections on labour supply and education from Statistics Norway, and also compare these projections with the later development and the first projections with the MOSART model.

A transition matrix model for population projections was developed at Statistics Norway towards the end of the sixties under the name of BEFREG. This model comprises mortality, fertility and external and internal migration, and the population is projected by gender, age and municipality. The BEFREG model is still used for making the official population projections from Statistics Norway, and the last version/projection is documented in Statistics Norway (1994). A test of the BEFREG model by comparing earlier projections with the later actual development is found in Texmon (1992). The demographic transition probabilities in the MOSART model are to a large degree based on parameters from the BEFREG model.

A transition matrix model for projections of educational characteristics was developed at Statistics Norway towards

Table 2.2. **Projection models preceding the MOSART model**

Model	Method	State variables	Documentation
BEFREG	Transitional matrix	Gender, age and municipality	Statistics Norway (1994)
MONS	Transitional matrix	Gender, age and educational activities and attainment	Hernæs (1986)
MAKE	Transitional matrix	Gender, age and marital status	Kravdal (1986)
MATAUK	Headship rate	Population, labour force and man-years by gender, age marital status and educational, activities and attainment	Sørlie (1985)
MAFO	Transitional matrix	Old age pensioners and benefits by age and gender	Koren (1979)
Household projections	Transitional matrix	Gender, age and household position	Keilman and Brunborg (1995)

the end of the seventies under the name of MONS. The last version of the MONS model was used as input to the labour force projections, and projected the population by gender, age and educational activities and attainment. The MONS model comprised educational transitions and the population size was based on projections with the BEFREG model. The MOSART model simulates educational characteristics similar to the MONS model, except that education is more aggregated in the MOSART model.

Projections of the population by marital status was performed with different models in the period 1972 to 1984, and the last version was a transition matrix model called MAKE. The transitions comprised mortality, external migration, fertility and nuptiality, and it was possible to make the projections consistent with the BEFREG model. The simulation of marital status in this version of the MOSART model is based on the same transition rates used in the last version of the MAKE model (Kravdal, 1986), but will be replaced in 1996 with a simulation of household formation.

Projections of labour force was formalized towards the end of the seventies in a model called MATAUK. The first part of the MATAUK model generates a projection of the population by gender, age, marital status and education based on projections with the models BEFREG, MONS and MAKE. The compound population projection is then multiplied with labour force participation rates and average working hours depending on the same characteristics. This second part of the MATAUK model is often referred to as headship rate models, see Andreassen (1993) for a discussion. This version of the MOSART model projects total labour supply by adding up each person's labour force participation rate and expected working hours. Since these parameters are independent of earlier labour force participation, the MOSART model has the same proprieties as the MATAUK model regarding this aspect. However, the MOSART model also includes characteristics such as children and disability pension. Furthermore, labour market earnings are simulated for each person dependent on earlier labour market earnings in such a way that total labour market earnings is consistent with projected total labour supply.

A model for old age pension benefits and expenditures was developed at Statistics Norway towards the end of the seventies under the name MAFO. The projection of the number of old age pensioners by gender and age is based on a transition matrix model, while pension benefits are projected for new cohorts of old age pensioners by a simple accomplishment of existing pension entitlements. Koren (1979) suggests that persons with high labour income continue at their current income level until retirement, while those with low income gradually leave the labour force before retirement. This method performed well for short term projections, but is clearly insufficient for projections with a time horizon longer than 20 years.

Statistics Norway has recently developed a transition matrix model that projects the population by gender, age and household position. Parameters estimated for this model will also be used in the MOSART model to simulate household formation, see Keilman and Brunborg (1995) for details.

2.4. History of the MOSART model

The computer used for the models MONS and MATAUK was closed down towards the end of the eighties, and this occurred together with a turnover in the staff. At this background the model system of MONS and MATAUK was reconsidered. An approach where demography, marital status, education and labour supply were integrated in *one* model seemed beneficial, and this was not possible with the transition matrix method. Rather than reprogramming the existing transition matrix models, a dynamic cross-sectional microsimulation model called MOSART was developed. The MOSART model was later extended with public pension benefits and labour market earnings, and both versions are described later in this section.

The development of the first version of the MOSART model started in 1988 and was accomplished in 1991 with Andreassen and Fredriksen (1991). This version comprised demographic events, marriage, education and labour supply, and was later fully documented in Andreassen et al. (1993). This study also includes some tests of the MOSART model. Projections with the MOSART model are compared with the projections with the earlier mentioned models MATAUK/MONS regarding the actual development in labour force and education. The study also calculates empirical standard deviation on replicated simulations and presents sensitivity analyses with respect to changes in labour force participation rates, migration and educational activity.

The development of the second version of the MOSART model started in 1991 and was accomplished with Fredriksen and Spurkland (1993), later documented in English in Andreassen et al. (1996). The second version extended the MOSART model with labour market earnings and public pension benefits. The latter depends on previous labour market earnings. Fredriksen and Spurkland (1993) tested the model in two ways. The projected distribution of labour market earnings was compared with the actual income distribution in the period 1967 to 1989, as a test of the ability to predict Supplementary Pensions. The study also included projections that showed the sensitivity of the projection results with respect to a large variety of changes in the underlying assumptions.

A comparison with the preceding transition matrix models

Andreassen et al. (1988) analyse redistribution of income between age groups in the population and the consequences of an ageing population. Besides a theoretical discussion of pay-as-you-go systems, this study includes projections with all of the transition matrix models BEFREG, MAKE, MONS, MATAUK and MAFO. Especially a pension fee until year 2020 is estimated as the ratio between current public old age pension expenditure and current total labour market earnings. The pension fee from Andreassen et al (1988) is compared with similar estimates from this study in table 2.3, and both approaches report the same expected development ahead. This is reasonable

Table 2.3. **Pension fee, 1990-2020**[1]

Year	Andreassen et al. (1988)	This study
1990	11.0	10.5
1995	11.6	11.6
2000	11.7	11.8
2007	11.7	12.1
2020	17.0	16.8

[1] Calculated as the ratio between public old age pension expenditure and total labour market eranings.

considering that the underlying assumptions are roughly the same in 1993 as they were in 1988.

This version of the MOSART model still represents an improvement compared with Andreassen et al. (1988) for several reasons. The model system is extended with several important characteristics, for example children and disability pensions, and the computer system itself in a microsimulation model imposes few restrictions on the number of characteristics. The long term projections of public pension benefits are clearly improved, and extend the projection horizon beyond year 2020, where a large increase in the pension fee is expected. The main improvements are however lower maintenance costs and the possibilities for performing comparative statics. While Andreassen et al. (1988) only present sensitivity analyses for wage growth and indexing of the Basic Pension Unit, sensitivity analyses are one of the main purposes of the MOSART model.

The current version of the MOSART model

This study documents a revised version of the MOSART model presented in Fredriksen and Spurkland (1993), where most of the changes goes to the presentation of the model and some important technical progresses. The presentation of the MOSART model is in this study extended with simulations of historical data, and this may be useful when evaluating the relevance of the MOSART model in forecasting and analyses of the Norwegian sosio-economic system.

The technical progresses relates to computer programmes and the method of drawing events. The computer programmes have been simplified to make it easier to use and change the MOSART model. Combined with improved hardware this have reduced computer runtimes to one-tenth, see Fredriksen (1995A) for documentation of the technical aspects. A drawing method that reduces the standard deviation induced by stochastic drawing with 70 to 90 per cent is also implemented in the model, see appendix B for details.

The MOSART model now includes early retirement[4] and also persons below 16 years. It is now easy to implement time variations in several underlying assumptions of the projections, while only constant transition probabilities were available earlier. The remaining changes in the content of the MOSART model are mainly minor improvements of transition probabilities.

The underlying assumptions of the projections are changed since Fredriksen and Spurkland (1993), and some of the results in this study are earlier published in Fredriksen (1995B). Net immigration is increased from 5 000 persons per year to 8 000, and life expectancy at birth increases with 4 to 5 years instead of 1 to 2 years. Entry of new disability pensioners is reduced from the high level in 1989 to the average of the period 1989 to 1993, which is approximately 25 per cent lower. The propensity to study is increased, while other underlying assumptions are almost unchanged. The main effect of the changed assumptions is that the Norwegian population now will stabilize at 5.2 million persons after year 2050, while earlier projections predicted a declining population from year 2030 and onwards. The effect on the contribution rate, the ratio between pension expenditure and total labour market earnings, is however

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[4] In Norwegian: Avtalefestet pensjon.

small since life expectancy increases 3 years more than earlier.

By 1996 we plan to extend the MOSART model with household formation and rehabilitation schemes. The initial population and the underlying assumptions will also be updated for use in the coming four-year plan from the Norwegian government. Over the two next years we plan to include taxation, consumption, savings and wealth in the MOSART model.

2.5. Some dynamic microsimulation models internationally

A presentation of microsimulation models in the Netherlands, Germany and North America is found in Mot (1992). Another general presentation of microsimulation models is found in Merz (1991), however with emphasis on static microsimulation models. Orcutt et al. (1986) includes several contributions with documentation of and analyses with microsimulation models. Harding (1996) comprises papers from a conference on microsimulation and public policy held by IARIW in 1993. Dynamic microsimulation models with a cross-section of a population and a comprehensive set of characteristics still in use are presented below.

The DYNASIM model

One of the first major applications of dynamic microsimulation models was the DYNASIM model developed by Orcutt and others between 1969 and 1976 at the Urban Institute, but lack of computer resources made the DYNASIM model impractical to use (Mot 1992). For this reason a second version, the DYNASIM2 model, was developed and a documentation is found in Wertheimer II et al. (1986). The demand for computer resources is reduced in the DYNASIM2 model by dividing the simulation into two models, one dynamic cross-sectional model and one longitudinal model. The cross-sectional model is run first, and comprises (demographic events), family formation, geographic mobility, education, disability pensions, labour force participation, labour market earnings, taxes and transfers. The longitudinal model is based on simulated life courses from the cross-sectional model, and adds employer, retirement and pension benefits. Dividing the model into two parts prevents interdependencies between the events/characteristics. Fredriksen and Spurkland (1993) used a similar approach for the MOSART model by calculating pension benefits after the simulation was accomplished, and all information for each life course was accessible from one sequential file. Both Mot (1992) and Merz (1991) refer to tests of DYNASIM, and mention that a comparison of projection results with panel data revealed weaknesses in the simulation of labour supply. Several North American microsimulation models are descending from the DYNASIM model, for example the CORSIM model presented in Caldwell (1996).

The Sfb3 model

The first major application of dynamic microsimulation in Europe is the Sfb3 model in Germany, and a documentation is found in Galler and Wagner (1986). The Sfb3 model comprises three versions, one dynamic cross-sectional, one longitudinal and one static model, all three build from the same microdata, econometric analyses and computer programmes. The dynamic cross-sectional model is recursive and comprises demographic events, household formation, education, labour supply, incomes, taxes, transfers, consumption, savings and wealth. Merz (1991) refers to a test of the Sfb3 model where microdata from 1969 is aged by dynamic microsimulation until 1978, and then compared with aggregate statistics. The discrepancies were surprisingly small, but the underlying assumptions of the projection are not mentioned in Merz (1991). According to Mot (1992) the Sfb3 model is now maintained only on ad hoc basis.

The NEDYMAS model

A more recent application similar to the MOSART model is the NEDYMAS model in the Netherlands, last documented and applied in Nelissen (1995). The NEDYMAS model comprises demographic events, household formation, labour supply, taxes and transfers. The initial population of the NEDYMAS model is simulated due too lack of proper microdata, with techniques similar to the simulation of historical data in this study. The NEDYMAS model is tested by comparing the simulated initial population with aggregate statistics, and some documentation of this is found in Nelissen (1991) and Nelissen (1993). The number of characteristics is very large in the NEDYMAS model, and this makes an initial population based on genuine microdata prohibitively expensive.

The DYNAMOD model

A project differing from other dynamic microsimulation models is the DYNAMOD model in Australia, see Antcliff (1993). The reason is that this model is based on continuous time. The model population is still handled simultaneously similar to the year-by-year simulation in table 2.1. Antcliff (1993) stresses that testing the model is an important part of the project, and mentions test methods such as comparison of the projection results with other (types of) models, simulation of historical data and calculation of empirical standard deviation from replicated simulations.

Comparison with the MOSART model

Compared with the applications presented in this section the MOSART model is based on better and more genuine microdata for the initial population and probably also for the estimated transition probabilities. The theoretical foundation for the transition probabilities in the MOSART model is weaker, and includes fewer interdependencies between the events/characteristics (for example from labour supply to demographic events such as fertility). The specification of some characteristics is more crude in the MOSART model, especially transfers. Among the models mentioned above and in Mot (1992) and Merz (1991), MOSART and DYNAMOD are the only microsimulation models programmed in an object oriented language. Development and use of the MOSART model is also characterised by a close co-operation with the Norwegian ministry of Finance.

Mot (1992) reports the development costs of the documented microsimulation models. The comparison is problematic because development very often includes important elements of use and maintenance of a model. The older applications also required resources for programming that currently is included in standard software or made abundant by better hardware. Comparable projects are the MOSART and NEDYMAS models, where development costs of the latter was 4.5 man-years. The first version of the MOSART model required 6 man-years (Andreassen et al. 1993) while the second version required 3 man-years (Fredriksen and Spurkland 1993). We have probably spend more resources on preparing microdata, to a larger extent estimated our own transition probabilities and included documentation that may be perceived as use of the model.

2.6. Other related models in Norway

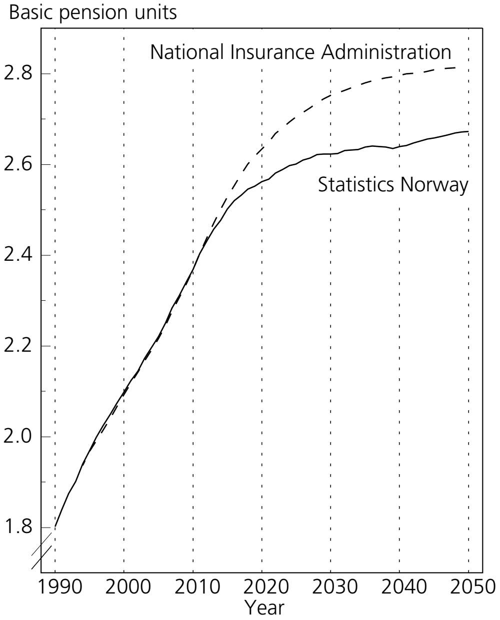
Some Norwegian models that may be used for analysing the same subjects as the MOSART model are presented in this section.

Planning models at the National Insurance Administration

The National Insurance Administration develops and uses models for budgeting and analyses of the National Insurance Scheme (NIS). See chapter 6 for a general description of NIS. The Supplementary Pensions from NIS are projected with a microsimulation model called TRAP, see Haga (1992). Projected Supplementary Pensions from the TRAP model are used as input to transition matrix models covering disability pensions and old age pensions. In comparison, the MOSART model includes other relevant characteristics in a more integrated, flexible and explicit manner. Examples are demographic events, education and labour supply.

Supplementary Pensions from NIS are calculated from annual labour market earnings from the age of 17 years until 69 years. The National Insurance Administration has a register by person for labour market earnings for each year since 1967, the year NIS was established. The TRAP model completes the histories of labour market earnings for a sample of the population by so-called statistical matching. Each simulated person is matched with a random person who six years earlier had the same gender, age, pension status and deflated labour market earnings as the simulated person has at present. The following five year history of labour market earnings from the chosen random person is added to the simulated person. The matching is repeated with the new characteristics at the end of these added five years, which implies that the simulated person is five years older and has a new labour market earning. The matching is repeated until each person reaches an age of 69 years or becomes a pensioner. Supplementary Pensions are then calculated from the simulated histories of labour market earnings. The TRAP model is tested by comparison of the male distribution of labour market earnings in the projections with actual panel data from the period from 1967 to 1989.

Figure 2.1. **Average old age pension benefits**



Average old age pension benefits projected by the two models TRAP and MOSART are compared in figure 2.1. The differences until year 2010 are less than 0.5 per cent, indicating that both models have captured in the same way important elements of already earned pension entitlements. From year 2010 an increasing discrepancy occurs, and after year 2030 the average old age pension benefit is 5 to 6 per cent higher in the TRAP model than in the MOSART model. Half of the discrepancy can be explained by different underlying assumptions. Labour market earnings per man-year is presumed constant in the MOSART model, and since gender differences decrease, the average labour market earnings drops by 3 per cent for men. The average labour market earnings for men are constant in the TRAP model, and increase for women. The other half of the discrepancy may be explained by too small variations in labour market earnings across the life course in the MOSART model, see sections 6.3 and 7.3.

Entry of new disability pensioners are in the models at the National Insurance Administration simulated depending on gender and age, while the MOSART model also includes covariates such as education. The educational level increases in the projections, and this explains why projections with the MOSART model result in a lower number of disability pensioners than the projections with the National Insurance Administration.

The Norwegian Ministry of Health and Social Affairs, the National Insurance Administration and Statistics Norway have co-operated on the creation of a micro-data file called KIRUT[5]. This file comprises 10 per cent of the population in Norway in the age group 16 to 66 years. Relevant covariates and participation in any social insurance scheme are registered on a monthly basis from 1989 and onwards. See National Insurance Administration (1992) for details. The National Insurance Administration plans to use KIRUT in a dynamic microsimulation model based on continuous time called TRYGD, see Østervold and Bragstad (1995).

Microeconometric research at Statistics Norway

Statistics Norway has a comprehensive microeconometric research related to income distribution analyses, taxation, consumption, labour supply and behaviour of firms. Some of these results are implemented in microsimulation models, however, none of these are dynamic models with a representative sample of the population. In this sense these models cannot substitute the MOSART model, but they may deliver parameters and/or knowledge of interest. For example, the inclusion of taxes, consumption and savings in the MOSART model will be based on one of these microeconometric projects.

The tax model LOTTE

A major static microsimulation model in Norway is the tax model LOTTE, see Arneberg et al. (1995) for documentation. At the core of this model is a detailed description of the Norwegian tax system for persons and a sample of tax returns supplemented with information from other administrative registers and from interviews. The sample is projected to an analysis year, often the forthcoming budget year, by reweighing the sample and indexing of all nominal amounts. Pension entitlements are projected based

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[5] KIRUT is an acronym for “Klientstrømmer Inn i, Ut av og Rundt i Trygdesystemet”, or freely translated, “Gross flows of clients Into, Out of and Within the Social insurance system”.

on projected cohort information from the MOSART model. The LOTTE model is used in budget work by both the Ministry of Finance and the opposition in the Parliament. The LOTTE model is tested by comparing the projected data with aggregate statistics. Around the LOTTE model are several other models used for analyses of consumption, public pension benefits and income distribution. An overview of and analysis of the tax system for pensioners are found in Arneberg and Gravningsmyhr (1994). An analysis of taxation of pensioners when the National Insurance Scheme is fully developed is found in Gravningsmyhr (1995). The sample in the LOTTE is here projected to year 2030 by reweighing and indexing based on a projection of population and pension benefits from the MOSART model.

The two models LOTTE and MOSART are generally used for different purposes like most static and dynamic microsimulation models. The co-operation on methods, data and programming is however also small, contrary to some other research institutions that develop both a static and a dynamic microsimulation model.

Macroeconometric models at Statistic Norway

Statistics Norway has developed two macroeconometric models, one based on quarterly data called KVARTS, and one based on yearly data called MODAG. Documentation and testing of these two models can be found in respectively Bowitz and Eika (1989) and Cappelen (1992). Both models give a general description of the Norwegian economy, and capture interdependencies between the different sectors of the economy and the economic behaviour of (representative) decision units. The main data source for the models are the National accounting system for Norway with quarterly time series data and a detailed representation of commodities and industries. Population figures are represented by an exogenous population projection multiplied with behavioural relations. For example, the labour force is projected by multiplying the number of persons by gender and coarse age groups with labour force participation rates depending on gender, age, wages, unemployment et cetera.

The two models are tested by their ability to reproduce historical time series data, especially for the years after the period the model parameters are estimated, so-called post-sample simulations. The ability to describe the functioning of the economy is tested with post-sample simulation where all exogenous variables[6] are given their correct historical values. The discrepancies between the post-sample simulations and the historical data are not aggregated into *one* formal measure for fitness. The simulation of historical data in this study may to some degree be compared with these type of post-sample simulations with macro-econometric models. One exception is that we simulate the years before the period where the parameters are estimated, because the time series with historical data after the oldest year with microdata are too short to constitute an adequate test. Furthermore, the exogenous variables in a macroeconometric model will usually have less influence on the simulation results than the underlying assumptions in the MOSART model.

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[6] Variables that are decided outside the model, contrary to the variables that follow from the model.

The MODAG model is used for analyses of medium term aspects of the economy, and one important subject is transfers and social security. The entry of new disability pensioners is modelled by time series econometrics, and the risk of disability depends on the wage-benefit ratio, unemployment and female labour force participation. Compared with the MOSART model, the MODAG model may analyse interdependencies between labour supply and labour demand. One propriety is that increased unemployment leads to increased entry into disability pension, and these persons are later lost for the labour market. At the other hand the representation of the population is coarse in the MODAG model, and the use of time series econometrics is problematic versus the entry of new disability pensioners. Especially is the aggregated time series used for time series econometrics strongly affected by administrative changes in the National Insurance Administration throughout the eighties (see also simulation of historical data in section 6.1). The average pension benefits are exogenously given in the MODAG model, and the MOSART model projects these benefits.

Cappelen and Stølen (1994) match the labour supply by educational attainment from the MOSART model with labour demand from the MODAG model. This is used to analyse possible future mismatches in the Norwegian labour market.

General equilibrium models at Statistics Norway

Statistics Norway has developed a general equilibrium model called MSG based on the same data as the two models MODAG and KVARTS above. The population is also represented by the similar techniques as in the two models MODAG and KVARTS. An application of the MSG model towards public pension expenditure is found in Holtsmark (1993), including a projection of pension benefits based on income profiles for coarse income groups. The growth in pension benefits is calculated to be far smaller than the results from the models MOSART and TRAP, probably because its simple approach is unable to capture the effects from income variations across the life course on Supplementary Pensions. Holtsmark (1993) shows that a quite moderate growth in productivity may prevent consumption from decreasing even if tax rates increase due to growing expenditure on public pension benefits and health care for old persons. He also shows that a price indexing of the Basic Pension Unit combined with economic growth may prevent tax rates from increasing, as this study also shows.

Generational accounting

Current public pension benefits financed through current taxes combined with decreasing fertility may lead to substantial redistribution of income between generations, and this subject is analysed by so-called *generational accounting* for Norway in Auerbach et al. (1993). This approach assumes that all individual public services, transfers and taxes by gender and age remain at the same level as in a chosen base year, with some exceptions. One exception is that public pension benefits increase, and the MOSART model is used to project these benefits. Furthermore, all future birth cohorts is presumed to be taxed such that the present value of all future public incomes equals the expenditures. The differences in taxes between the current, especially the youngest, and the future generations are interpreted as the redistributional effect of public services and transfers between generations. However, the approach is static and excludes past services, transfers and taxes, and disregards that the transition to a balanced budget may come gradually. The premise that constant public services, transfers and taxes is “constant policy” may neither be the case.

Another approach is so-called *overlapping generations models*, and one example for Norway is found in Steigum (1993). This analysis is based on a general equilibrium model with a more aggregated set of commodities and industries than the above mentioned MSG model, but includes intertemporal behaviour for the decision units. The population is represented by a transition matrix model with five year age groups, and the average public pension benefit are presumed to constitute 70 per cent of the average wage per man-year. Steigum (1993) analyses redistribution of income between generations related to an ageing population and the consequences of a full funding of the National Insurance Scheme.

The MOSART model may analyse redistribution of income in a life course context, and compared with Auerbach et al. (1993) and Steigum (1993), the MOSART model may also analyse redistribution of income between persons. However, the MOSART model lacks important parts of the public expenditures and incomes, for example health care. Furthermore, substantial parts of the tax base are placed outside the household sector, for example by the presence of the oil wealth in the Nordic Sea. An example of the MOSART model used to analyse redistribution of income is found in Andreassen et al. (1996).

2.7. Testing of models

Several contributions on microsimulation focus on simulation of historical data to test the relevance of a model and replicated simulation to test the uncertainty generated by stochastic drawing, see for example Orcutt et al. (1986), Caldwell (1996) and Antcliff (1993). The simulation of historical data is interpreted below, while drawing methods and uncertainty generated by stochastic drawing are discussed in appendix B.

Hypothesis

Before any tests can be performed, it is necessary to clarify the hypothesis to be tested. The MOSART model projects the population, and the results are used as conditional predictions and to analyse the consequences of changes in the underlying assumptions. Three hypotheses can be expressed:

(2.5) If the underlying assumptions in for example table 1.1 or 3.2 are correct, then the MOSART model describes how the Norwegian population will develop.

(2.6) If the underlying assumptions change, measured by aggregate quantities as in table 1.1 or 3.2, then the MOSART model describes the effect on how the Norwegian population will develop.

(2.7) The approach of constant transition probabilities describes reasonable well how the Norwegian population will develop.

Hypothesis 2.5 is important because this is how we present the projections with the MOSART model. However, it is impossible to create a situation where all (or any of) the transition probabilities are constant. A test can still be executed by what we refer to as a *simulation of historical data*, where all transition probabilities are adjusted

Table 2.4. **Simulation of historical data and adjusted transition probabilities**[1]

Event	Adjustment targets
Migration	Net immigration in number of persons
Mortality	Periodic life expectancy at birth by gender
Fertility	Number of births
Educational activities	Number of pupils and students, by gender
Disability pension	Number of new disability pensioners, by gender
Old age pension	Retirement age
Pension benefits	Basic Pension Unit, Special Supplements and other rules for calculating pension entitlements and benefits
Labour force participation	Number of persons in the labour force by gender, total labour market earnings by gender

[1] All transition probabilities for each event are adjusted proportionally et cetera each year to make the simulated number of events consistent with the actual level.

such that they correspond to the one or a few aggregate measures for each event. Neither this is sufficient, because the starting year of the projections are currently 1989, while the projections are often used with a time horizon of 40 to 60 years. A comparison with seven years of actual data until 1996 is in this context inadequate as a test. The simulation of historical data therefore starts in 1960, giving a time span of 33 years to test the projections. However, it is beyond the scope of this study to create a real initial population in 1960 or re-estimate all the transition probabilities with microdata from 1960. Instead a synthetic initial population is generated by using aggregate statistics from 1960, see appendix A. Furthermore, the available transition probabilities in the MOSART model are adjusted by simple means, for example proportionally, such that the transitions are consistent with an aggregate measure for each event each year. See table 2.4 for an overview and chapters 4 to 7 for details on methods of adjustment. Several of the transition probabilities are adjusted towards the number of events rather than an aggregate measure for the transitions, for example the number of births rather than total fertility rate. This has probably little effect on the results due to the close relation between the number of events and the aggregated measure for the behaviour when the number of persons are more or less given. Discrepancies between the simulation of historical data and the actual level is discussed later in this section.

The projections with other underlying assumptions are normally made by adjusting the transition probabilities with the same simple methods used for the simulation of historical data. If the simulation of historical data is unable to reproduce the actual development, this gives reason to reject hypothesis 2.6 that the MOSART model describes the effect of changes in the underlying assumptions. This is however a weak test since it is easy to imagine situations where a correlation at micro level may not hold when policies are changed. An example can be that the risk of entering disability pension decreases with increasing educational level, also controlled for age, gender and income. If more resources are put into the educational sector, the MOSART model predicts fewer disability pensioners. This is plausible if skills changes the motivation to work or the content of present jobs, for example by replacing manual work with machines. This is however implausible if education is only a sorting procedure of persons to the jobs with least demands for strong health. Testing if the covariates of the transition probabilities are autonomous to a change in policy is beyond the scope of this study. The covariates in the MOSART model can still be defended by the fact that they capture important distributional aspects, for example that groups with a high educational level have both lower mortality, higher labour market earnings and lower risk of entering disability pension. Furthermore, the interpretation of constant transition probabilities may include the effect of any covariates, for example that constant risk of entering disability pension is conditional on the educational level.

Hypothesis 2.7 that projections must have some ability to predict the actual development is generally unpopular among researchers. However, a policy advice based on changes in the underlying assumptions has no value if we are absolutely unable to predict the condition of the economy at the time the policy has an effect. The MOSART model's ability to predict the actual development is illuminated by comparing a projection with constant transition probabilities as in 1967 with the later actual development. Furthermore, this study includes a large variety of sensitivity analyses, and compared with the actual changes in the transition probabilities over the last decades, this may also illuminate the strength of the presented conclusions.

Discrepancies between simulation of historical data and actual data

An ordinary projection with the MOSART model may deviate from the actual development[7] for several reasons, and table 2.5 is an attempt of making an exhaustive list of reasons. Starting at the bottom of the list, the computer programmes in the MOSART model comprise roughly 10 000 lines. It is quite sure that this system contains errors in the sense that the computer programme some places does other things than intended. By thorough comparison with earlier projections and search for inconsistencies in output and simulated life courses we believe that programming errors seldom influence the conclusions.

Dynamic microsimulation models are usually based on stochastic drawings and this generates an extra uncertainty in the projections. This type of error is however easy to control by performing replicated projections where the only difference is the sequence of random numbers. The MOSART model uses a mean-constrained drawing method that reduces these problems substantially, and we consider this as a small problem. See appendix B for a presentation of the drawing method and table B.1 for an estimate on the uncertainty generated by stochastic drawings.

Transition probabilities are estimated from a limited number of observations, and the applied estimates will generally not represent the precise "true" values due to ordinary sample errors. In the MOSART model the transition probabilities are mainly estimated from administrative registers comprising at least 10 per cent of the Norwegian population. our intuition is therefore that estimation weaknesses due

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[7] We interpret the "actual development" as the what official statistics reports for each subject, except in cases where we know of "errors" in these statistics.

Table 2.5. **Sources for discrepancies between a projection and the actual level**

Specification of the model
Changes in underlying assumptions
Changes in covariates
Errors in the initial population
Errors in the estimation of parameters
Stochastic drawing
Programming errors

to sample errors are moderate in the MOSART model, but this is not tested.

The initial population is based on a sample and the quality of the underlying administrative registers may differ. The sample errors are here easy to control because different samples are available, and table B1 in appendix B reports the sample uncertainty on the labour force projections. This type of errors are large in this version of the model, but the next version of the initial population will reduce these problems through better stratification of the sample. Furthermore, projections with different underlying assumptions may use the same initial population, and this reduces the problems of sample errors. This may be tested by performing the same sensitivity analyses for different samples. The quality of the administrative registers are tested by comparing the initial population with aggregate statistics in the base year, and this occurs to be a small problem. The simulation of historical data is based on a synthetic initial population, and this may be a substantial source to discrepancies with the actual level and thereby reduce the value of the testing.

The usual assumption of constant transition probabilities is here divided between what we refer to as underlying assumptions and covariates (see table 1.1 or 3.2 for an example of what we include in underlying assumptions in this version). The distinction between underlying assumptions and covariates is made because we usually present the projections with emphasis on a few aggregate measures for each event, for example total fertility rate for the number of births. This is especially the case for projections with other underlying assumptions. Errors in the specification of the model includes for example the simplifications of the dependencies between events.

Discrepancies between the simulation of historical data and the actual level are referred to as *model errors*, and this may include any sources of errors in table 2.5 except changes in the underlying assumptions. In some cases the underlying assumptions will be very comprehensive with respect to the projection results, for example the relation between vital statistics and the number of persons in the population, and this should increase the demand for precision.

Discrepancies between an ordinary projection with constant transition probabilities and the actual level are referred to as *projection errors*, and this includes the mentioned model errors and changes in the underlying assumptions. our intuition before this study was that changes in the underlying assumptions were the far dominant part of the projection errors. The simulation of historical data show that model errors constitute as much as one-third of the projection errors over the last 25 years related to the contribution rate, see later table 8.3.

We have not included any formal measures on the discrepancies between the simulation of historical data and the actual level, but only reported the level of discrepancies for the different subjects of the projections. The reason for doing this is that a compact measure is difficult to construct for a composite model such as MOSART, and that such measures only have interest when choosing between different models or versions of a model. The simulation of historical data in this study also suffers from the weakness that the best model when simulating the sixties or seventies may not be best at simulating the forthcoming decades.

3. The MOSART model

The MOSART model starts with a representative sample of the Norwegian population in a base year and simulates the further life course of each person in this sample. The model simulates migration, deaths, births, marriage, education, retirement and labour force participation. Public pension benefits are calculated from labour market earnings and other characteristics included in the simulation. Table 3.1 and 3.2 give a brief description of each event, while chapters 4 to 7 give a more detailed discussion of each subject. The result of the simulation is a model population with the life course of each person.

3.1. Initial population

This version of the MOSART model starts with a 1 per cent random sample of the population in Norway in 1989. This initial population comprises 40 000 persons with actual information on marriage, birth histories, educational level and activities, pension status and pension entitlements in the National Insurance Scheme. The information is gathered from registers run by the Directorate of Taxes, the National Insurance Administration and Statistics Norway. The sample is stratified by gender and age, and the sample includes the spouse of all married persons. Ten disjunct 1 per cent samples are available as initial populations for the MOSART model.

The initial population will be improved and updated in 1996, see Fredriksen (1996A). The new data comprises 12 per cent of the population in 1993, and the stratification is improved. New characteristics are included such as rehabilitation schemes, special pension entitlements for civil servants, wealth and household status. The latter includes to some degree relations to spouses, parents and children.

The *real* initial population in the MOSART model contains much information about the future because many personal characteristics are given or easy to predict. Examples are gender, age, educational level for adults and already earned pension entitlements. The simulation of historical data is based on a *synthetic* initial population corresponding to 1 per cent of the population in Norway in 1960, see appendix A for details. The model population in 1993 from the simulation of historical data is referred to

Table 3.1. **Transition probabilities in the MOSART model**

Event	Estimation methods and periods	Covariates
Migration	Observed 1989-rates	Sex and age
Mortality	Observed 1993-rates	Sex, age, marital status, educational attainment and disability pension
Fertility	Observed 1989-rates	Mother's age, number of children and age of youngest child
Nuptiality	Observed 1984-rates	Woman's age, children and marital status
Educational activities	Observed 1987-rates	Sex, age and educational activities and attainment
Entry into disability pension	Logit function, 1986-1989	Sex, age, marital status, educational attainment, pension status and labour force participation
Other transitions in pension status	Observed rates, 1986-1989	Sex, age, pension status, educational attainment, labour force participation, widow(er)hood
Labour force participation	Logit function etc, 1985-1988, 1991	Sex, age, children, marital status, educational activities and attainment, pension status and previous year's labour force participation

as a *simulated* initial population when used for projections beyond 1993.

3.2. Transition probabilities

A dynamic microsimulation model with discrete time is based on the assumption that each person in each period has certain (transition) probabilities of experiencing transitions from one state to another. Each of these transition probabilities will normally constitute an event, and depends on each person's characteristics. The model is then simulated with stochastic drawing, often called the Monte Carlo technique. Usually this implies that the computer generates a so-called random number with uniform distribution (0,1), and if this random number is smaller than the transition probability, the transition occurs. The drawing method in the MOSART model is similar, except that the uncertainty generated by stochastic drawing is lower. See appendix B for a discussion of drawing methods.

If the simulation comprises a sufficient number of persons or replications the model will describe the population or groups of the population, confer the central limit theorem. The transition probabilities in the MOSART model are mainly estimated using event history analysis on actual events in a recent period, varying from 1984 until 1993. Important transition probabilities in the MOSART model are summarized in table 3.1, including covariates and estimation periods and methods. Andreassen et al. (1993) and Fredriksen and Spurkland (1993) give a more detailed description of how the different transition probabilities are estimated.

Table 3.2. **Underlying assumptions with the perspectives from 1967 and 1993**[1]

	Perspectives from 1967	Perspectives from 1993
Net immigration per year	0	8 000 persons
Life expectancy at birth		
- Men	71 years	74 years increasing to 79 years
- Women	77 years	80 years increasing to 85 years
Total fertility rate	2.80	1.89
Average number of years as pupil or student after primary school		
- Men	3.7 years	8.1 years
- Women	2.4 years	8.6 years
Average number of years as disability pensioner[2]		
- Men	2.1 years	3.9 years
- Women	3.0 years	4.1 years
Retirement age	70 years	67 years
Average number of years in the labour force		
- Men	49.0 years	41.7 years
- Women	25.1 years	38.5 years
Average labour market earnings (in 1993)		
- Men	115 000 NOK	161 000 NOK
- Women	30 000 NOK	91 000 NOK
Basic Pension Unit (see section 6.3)	32 000 NOK	37 000 NOK

[1] All assumptions correspond to the actual level in respectively 1967 and 1993, except that entry of new disability pensioners is set to the average of the period 1989 to 1993 in the perspectives from 1993. Norwegian kroner (NOK) are in general deflated with Statistics Norway's consumer price index to the level in 1993. One US Dollar was worth 6.30 NOK at 26th of February 1996.

[2] Retirement age was reduced from 70 years to 67 years in 1973, and this reform reduces the number of years as disability pensioner by roughly 2 years in 1993.

3.3. Underlying assumptions

The model user must make some *underlying assumptions* on how the transition probabilities will develop in the future, before any projections can be made. The projections may then be interpreted as a prediction of the expected development given the underlying assumption. A base line alternative will typically let all probabilities remain at the same level as in the latest year with available data. This may also be interpreted as the expected population changes if "everything continue as now". The base line alternative is often used as a reference track when making alternative projections with other underlying assumptions. The base line alternative in this study is called the *perspectives from 1993*, and is summarized in table 3.2. A similar description of the perspectives from 1967 is also given in this table.

The number of events in the simulation of historical data equals the actual numbers each year for each subject in table 3.2, and this is accomplished by adjusting the transition probabilities from table 3.1. The simulated number of events with the drawing method in the MOSART model is the sum of the transition probabilities. The transition probabilities from table 3.1

are then adjusted by simple techniques, for example by changing proportionally all transition probabilities for an event. Adjustments of the transition probabilities in their own estimation year or period in the simulation of historical data are reported throughout chapters 4 to 7. The transition probabilities in projections with alternative underlying assumptions are usually generated by the same adjustment techniques.

3.4. Technical notes

The MOSART model simulates the whole population each year before entering into the next year. This year-by-year simulation makes it is easy to maintain relations between persons in the model population, for example between spouses.

The MOSART model is based on, as mentioned, discrete time with the calendar year as time unit. We have found this more practical in relation to our needs, especially when most of the events seldom happen more than once a year. Concerning event history analysis, the econometric part, the choice between discrete or continuous time is a matter of convenience if the time unit is sufficiently small, see Allison (1984).

However, some of the different events can happen in the same year, and this gives a large number of combinations of events. If the events in table 3.1 are dichotomous, the number of possible combinations of events will be 128. Even this is in a practical sense impossible to handle simultaneously with discrete time. A multinomial logit function with 128 states will lead to unacceptable computer runtimes, if at all possible. Simultaneous probabilities represented by one large cross-table will comprise at least several million probabilities with the complexity of the MOSART model.

Our approach is to simulate one event at the time in a fixed order, often called *recursive simulation*. This gives in the example above 8 binomial logit functions, which is easy to handle. The 8 transitions can also be represented by 8 tables where only the interesting covariates are represented. Simulating one event at a time combined with the year-by-year simulation also makes it more easy to adjust the projections against any exogenous constraint on the number of events. If the probabilities are properly conditioned on the results of events drawn previously for the same year, as the intention is, recursive simulation will as a principle not affect the result using the identities:

$$(3.1) \quad P(A,B|X) \equiv P(A|X) \cdot P(B|A,X) \equiv P(B|X) \cdot P(A|B,X)$$

Where:
- A and B describes changes in two arbitrary characteristics during the calendar year
- X is a vector of personal characteristics at the beginning of the calendar year

Limitations on resources, available micro-data and simplifications of how probabilities are represented mathematically may prevent us from estimating the proper conditional probabilities that equation 3.1 requires. The choice between discrete/continuous time and simultaneous/recursive simulation is therefore more important than equation 3.1 suggests. When we exclude some covariates or use simple mathematical functions, these are however intended simplifications. Moreover, the problems of estimating the proper parameters will also

be present in models with discrete time and simultaneous probabilities and in models with continuous time.
The order of the events is the same as in table 3.1, and the sequence reflects several considerations. For practical reasons, demographic events are simulated first to define each year's population before the other events are simulated. Furthermore, we have considered that provisionally some of the transition probabilities are estimated unconditionally. And last, the order reflects our beliefs on the causality between the events, and this eases the interpretation of the conditional probabilities. For example, we find it more natural to say that a person went out of the labour force because he became a disability pensioner, than the opposite.

3.5. Computer resources and administrative settings

This version of the MOSART model is run on a Sun Sparc work station 10 model 61, with UNIX as operating system. The simulation model is written in the object-oriented language Simula, see Kirkerud (1989) for a presentation of Simula. A simulation with 1 per cent of the population, 40 000 persons, requires 22 Mb RAM. A projection until year 2060 with only standard tables will usually have a computer runtime of 45 minutes. Special tables require that the model population is generated as an ASCII-file and handled in a suitable statistics programme after the simulation. This will increase the mentioned computer runtime and require a hard-disk up to 1 Gb. Fredriksen (1995A) gives a technical documentation of the model, except for some improvements on the production of the model population.

The MOSART model is at present only operated at Statistics Norway, but this may change as the computer programmes are cured for its teething problems and computer hard-ware becomes less expensive. We aim at making the model user-friendly for a *trained* researcher or user, either the purpose is development, maintenance or analyses (developing and maintaining an interface that is user-friendly for the beginner is usually expensive, and will often be nothing more than an obstacle to the trained user).

The results from the MOSART model is mainly used in analyses by Statistics Norway and by the Norwegian government in white papers et cetera. Development and maintenance of the model is partly financed through general funding of Statistics Norway, partly through special projects financed by the Norwegian Research Council or the Norwegian government.

4. Demographic events and population size

The modelling of demographic events and the underlying demographic assumptions of the MOSART model are to a large degree based on the official population projections from Statistics Norway. See Statistics Norway (1994) and Texmon (1992) for details. Each demographic event is documented in sections 4.1 through 4.4, while a summary is given in section 4.5 by reporting changes in the size and age composition of the population. Some future effects of variations in migration, mortality and fertility seen through the last decades are also reported in chapter 8.

4.1. Migration

Migration is included in the MOSART model at present only to make the development in population size more accurate. Two major simplifications are made. First, the MOSART model includes only net immigration rather than gross immigration and emigration. This means that an exogenous number of "net immigrants" by gender and age are added to the model population each year. Second, net immigrants are mainly assigned characteristics as the average Norwegian population. The only exception is educational attainment that is set to unknown for new immigrants as in the educational registers. This also reflects the actual distribution of educational attainment among "net immigrants" in 1993. Unknown educational attainment implies higher risk of disability and lower labour force participation rates than the average population, see appendix C for details. In age groups with net emigration a given number of random persons are removed corresponding to the exogenous level of net immigration.

The development in net immigration over the last 45 years is reported in figure 4.1. An important component behind the tendency towards larger positive net immigration is immigrants from third world countries in the seventies, and later refugees. The large changes in the years 1960 and 1970 can be a result of poor quality in the statistics. The abrupt changes in net immigration since 1985 are mainly caused by large and opposite changes in unemployment in Norway and Sweden. Changes in the flow of refugees and the policy towards these persons have also contributed to variations in net immigration in recent years.

Figure 4.1. **Net immigration**

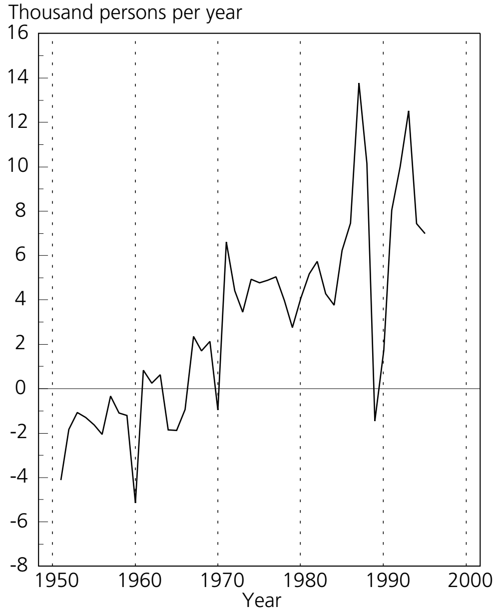
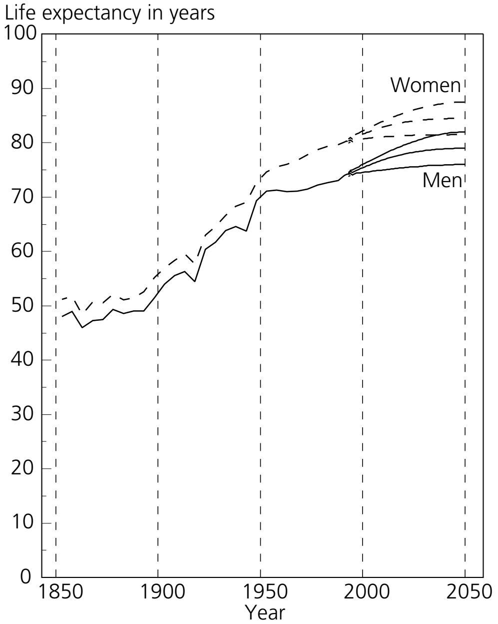


Figure 4.2. **Life expectancy at birth**



The projection with the perspectives from 1993 is based on a net immigration of 8 000 persons per year, roughly the average of the period 1987 to 1993. The projection with the perspectives from 1967 has no migration, similar to the population projections from the same period. Net immigration is set to its actual level each year in the simulation of historical data, while distribution across gender and age is held at the level in 1989. Any negative number of net immigrants in years with net emigration are transferred to the nearest year with positive net immigration. This version of the MOSART model is unable to handle net emigration.

4.2. Mortality

Mortality depends on gender and age, and the base line mortality rates from 1989 are adjusted proportionally corresponding with life expectancy at birth each year[8]. Furthermore, mortality is higher in the simulation for single, disabled and those with a low educational level. The covariates are roughly estimated from various sources of mortality statistics, and included because these differences are of significance for the pension expenditures (see later table 8.2). Simultaneously the base line mortality is adjusted to ensure that these covariates do not influence the average mortality by gender and age. If a married person dies, the surviving spouse is made a widow or widower.

The development in life expectancy at birth throughout the last 150 years and in the projections is reported in figure 4.2. Brunborg (1992) gives a discussion on

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[8] Periodic life expectancy at birth is how old a person is expected to become if this person experiences that period’s mortality rates throughout his or her lifetime.

Table 4.1. **Changes in life expectancy at birth by gender and period, decomposed by lower mortality in different age groups**[1]

	Men			Women		
Contribution by age group	1893-1953	1953-1993	Remaining potential[2]	1893-1953	1953-1993	Remaining potential[2]
All	22.0	3.1	..	22.1	5.6	..
Infant mortality	3.9	1.4	0.4	3.5	1.2	0.4
1 to 19 years	6.1	0.7	0.4	6.6	0.4	0.3
20 to 59 years	6.4	1.0	3.3	6.2	1.0	2.0
60 years and over	1.1	-0.1	..	1.4	2.6	..
Cross-effects	4.6	0.1	0.1	4.4	0.3	0.0

[1] The effect on life expectancy at birth from lower mortality within each age group is calculated as if all other mortality rates remained constant. The cross-effects occur because lower mortality at any age is more important the larger the probability of surviving until this age and the longer the remaining life expectancy are.
[2] The "remaining potential" is calculated with no mortality below an age of 60 years.

mortality in Norway throughout the last 200 years, while Mamelund and Brunborg (1996) present statistics on cohort and period mortality for the period 1846 until 1994. From 1890 until 1950 life expectancy increased with 22 years, or with an annual growth of 0.4 years. Life expectancy has increased less since 1950, especially for men in the first decades after the Second World War.

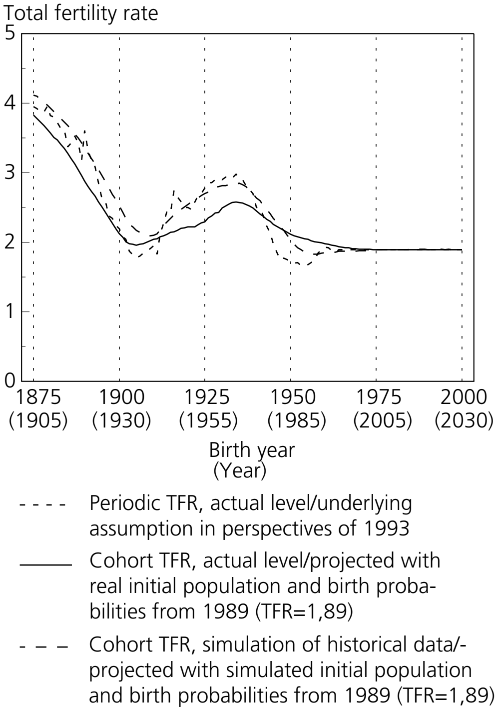
The increase in life expectancy from lower mortality in different age groups is decomposed in table 4.1. This increase has until now mainly been a result of lower mortality below an age of 60 years, with roughly one-quarter from lower infant mortality. Further increase in life expectancy depends on lower mortality among persons older than 50 to 60 years, while mortality in this age group has increased slightly for men over the last 40 years. The proportional reduction in mortality for all ages in the perspectives from 1993 implies an increase in life expectancy at age 67 with 3 to 4 years towards year 2050, from a level of 15 years today.

Life expectancy has increased with 4 years throughout the last 25 years, while it will increase with 3 years in the first 25 years in the projection with the perspectives from 1993. Towards year 2050 life expectancy will increase with 4 to 5 years. The alternatives on mortality later used in chapter 8 are also shown in figure 4.2; an increase in life expectancy of respectively 1-2 years and 7-8 years. The projection with the perspectives from 1967 has no increase in life expectancy, similar to the population projections from this period. This may seem reasonable considering that male life expectancy remained at the same level from the beginning of the fifties until the end of the sixties. Furthermore, female life expectancy increased little compared with the first half of this century. In the simulation of historical data the base line mortality from 1993 is adjusted proportionally each year to make mortality consistent with each year's life expectancy at birth.

4.3. Fertility

Births are simulated for women with fertility rates from 1989 depending on age, number of children and age of the youngest child. Each time a birth occurs, a child is added to the model population. In the simulation of historical data fertility rates from 1989 are adjusted proportionally each year to let the simulated number of births be equal with the actual level each year. This adjustment is roughly the same as adjusting the fertility rates against periodic total fertility rate (TFR)[9]. Fertility rates were adjusted by less than 1 per cent in 1989 in the simulation of historical data, indicating that the estimated fertility rates give a proper description of its own estimation year 1989. The fertility rates from 1989 are also used to simulate the number of children for the initial population in 1960. The fertility rates are also here adjusted proportionally to be consistent with periodic TFR each year before 1960.

Figure 4.3. **Total fertility rates**



The development in TFRs over the last 100 years and in the projections is reported in figure 4.3, see Kravdal (1994) and Brunborg and Mamelund (1994) for more details. The actual cohort TFRs are estimated from real data for female birth cohorts born before 1940. "Actual" cohort TFRs for later birth cohorts are estimated from actual data extrapolated with fertility rates from 1989. The large decrease in fertility from the end of the nineteenth century is the so-called demographic transition most developed countries have experienced with first lower mortality and then lower fertility. The increased fertility from 1935 until 1946 can probably be explained by births postponed due to the economic depression between the two World Wars.
The cohort TFR for females born in 1935 is 2.6, while projections based on fertility rates from 1989 will give a TFR of 1.89. The reduction is mainly caused by fewer women giving birth to three children or more. Among women born in 1935 are 9 per cent childless, while first child fertility rates in 1989 lead to 16 per cent childless women.

The actual cohort TFRs are poorly predicted by the simulation of historical data. One explanation may be that decreasing TFRs are caused by lower fertility among women with two children or more, while fertility rates here are adjusted independent of the number of children. This is specially problematic since most women stop at two children

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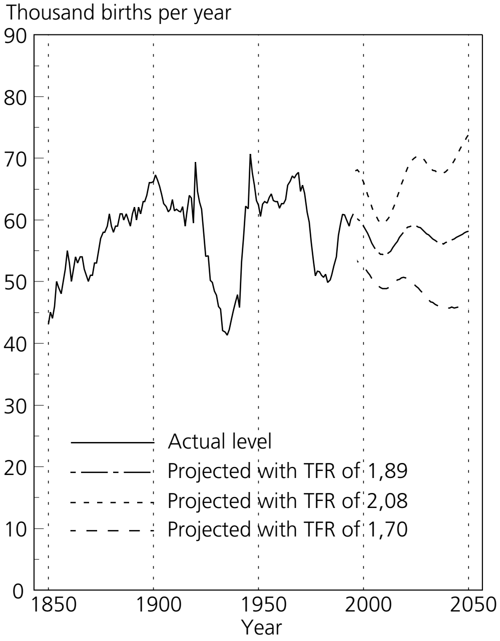
[9] Total fertility rate, abbreviated TFR, is the number of children a woman is expected to give birth to if she lives until an age of 45 years. Disregarding differences in mortality by number of children, cohort TFRs can be estimated as the average number of children in each female birth cohort at an age of 45 years. Periodic TFRs are estimated as the sum of each period's age-specific fertility rates. Fertility is said to reproduce the population when each new-born girl on average will give birth to one girl. With current mortality and proportion of new-borns that are girls, reproduction requires a TFR of 2,08.

Figure 4.4. **Births**



with the current fertility rates. Another explanation may be that periodic variations in fertility have a different impact on contemporary birth cohorts of fertile women, while we adjust all fertility rates in one year with the same factor independent of age.

The large variations in the annual number of births as reported in figure 4.4 are an important key to understand the dynamics of the projections with the MOSART model. One aspect of the figure is the low number of births between the two World Wars, probably due to the mentioned economic depression. This will give a low or none growth in the number of old persons towards year 2010, contrary to many other developed countries.

The projection with the perspectives from 1993 is based on a TFR of 1.89, which is near today's periodic TFR. The projection with the perspectives from 1967 is based on a TFR of 2.8 corresponding to the level in periodic TFR at that time. A TFR of 2.8 is way above reproduction level, and will with the actual age distribution of fertility lead to a population growth of 1.1 per cent per year. The alternatives on fertility later used in chapter 8 are also shown in figure 4.4, TFRs of respectively reproduction level and the lowest periodic TFR observed in Norway (we have here for convenience only adjusted first child fertility rates proportionally, while fertility rates for second child or more are kept unchanged).

4.4. Nuptiality

Marital status and choice of spouse are simulated with focus on women. Marriages are simulated for not married women depending on her age and marital status (unmarried, widow, divorced). If a marriage occurs, the age of her coming husband is drawn depending on her own age, and a not married man with this age is chosen at random. Divorce is simulated for married women, and if a divorce occurs also her husband is divorced.

Marriages and divorces are simulated with transition probabilities from Kravdal (1986) for the year 1984 with no adjustments. Nuptiality has changed since 1984, but the intention is to use marriage as a proxy for the total number of cohabiting couples, and this may have changed less. The simulation of nuptiality will be replaced in 1996 by a simulation of household status and relations, including marriages and divorces. Focus will be on two-sex couples and their children. Household formation will be implemented in the MOSART model based on a project described in Keilman and Brunborg (1995).

4.5. Population size

Simulation of historical data: Population size

The simulation of historical data is made consistent with aggregate vital statistics, comprising fertility, net immigration and male and female mortality. We would then expect only small discrepancies between the simulated population and the actual level even with a projection period of 33 years. Potential sources of discrepancies are changes in the distribution of events by gender and age and other included covariates. The simulated and the actual population by age in 1993 are reported in figure 4.5. The aggregated number of persons is overpredicted by approximately 0.6 per cent. In some age groups the number of persons deviates by 2 to 3 per cent. The reason for the latter is probably changes in the age distribution of net immigrants, with older immigrants in the beginning of the simulation period than the age distribution from 1989 used here. The number of men and women is equally well (or bad) predicted.

A simulation with constant transition probabilities from 1967 is also included in figure 4.5, see table 3.2 for an overview of the underlying assumptions. The number of persons older than 25 years is well predicted, while the number of young persons is poorly predicted. The number of persons 67 years and older is underpredicted by 14 per cent due to an increase in life expectancy in the same period. The number of persons aged 25 to 66 years, the potential labour force, is underpredicted by 3 to 4 per cent due to the exclusion of migration. The number of persons 16 years and younger is overpredicted with more than 50 per cent, showing the dramatic reduction in fertility in the period 1967 to 1977.

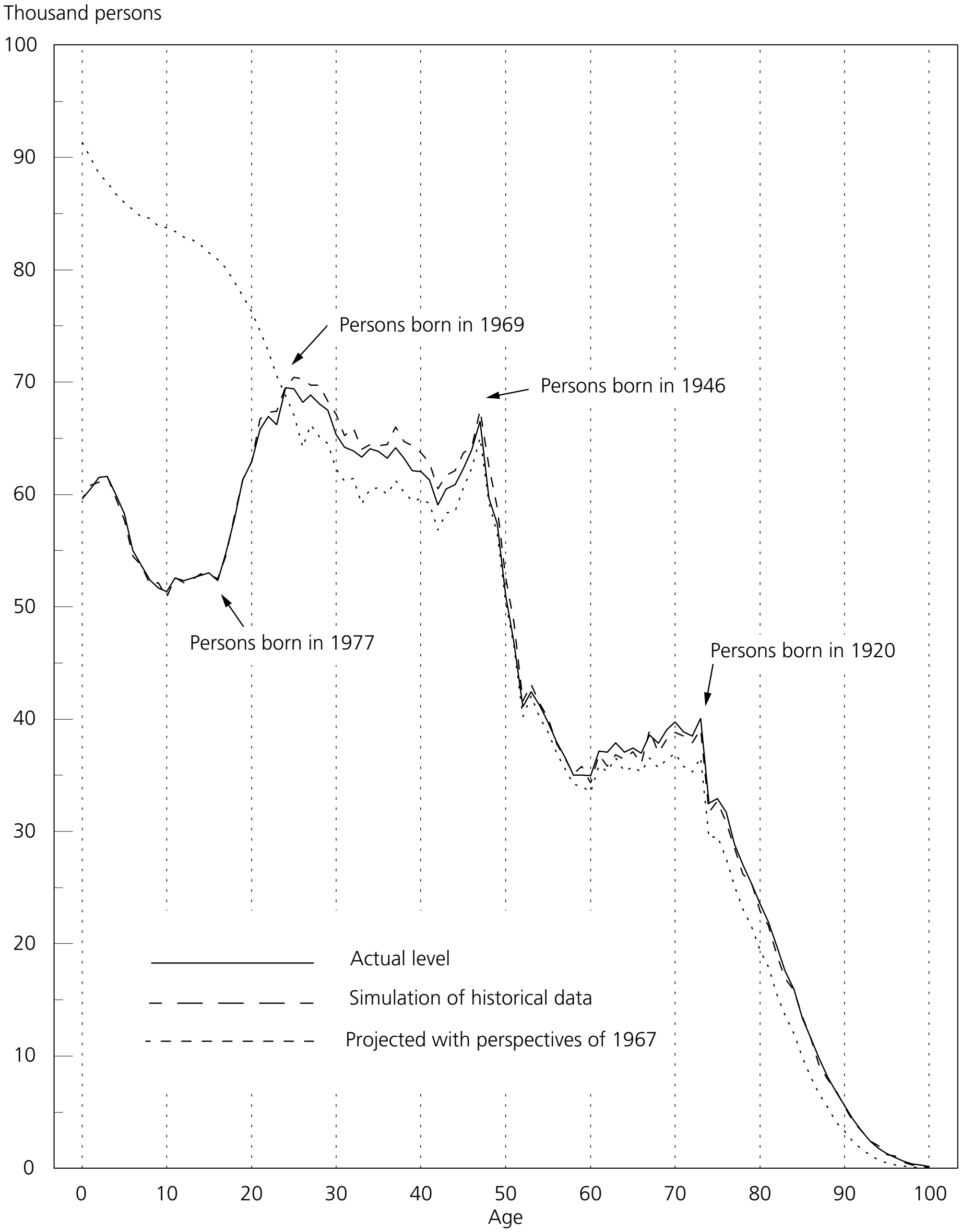
Projections: Population size

The combination of a constant fertility below reproduction level, a constant net immigration in number of persons and no further increase in life expectancy will lead to a stable population in the long run, both in number of persons and age composition. This will happen after a transitional period with either sustained population growth or decrease that may last for several hundred years if fertility is close to reproduction level. The population size will be proportional with the number of net immigrants, and the age composition of the population will be decided by the total fertility rate and to some degree by the age distribution of net immigrants.

The projection with the perspectives from 1993 has these proprieties, and implies a stabilization in population size and age composition already by the middle of the 21st century, where the Norwegian population will reach 5.2 million persons. The Norwegian population comprised 4.3 million persons in 1993, and the growth in population size towards year 2050 will mainly be in the numbers of persons 55 years and older, especially after year 2000. Both larger birth cohorts and lower mortality will contribute to the growth in the number of older persons after year 2010, see also figure 6.4 and 6.5.

With the perspectives from 1967 the Norwegian population would have grown with 1.1 per cent per year, and reached 10 million persons by year 2060 and its first billion before year 2500

Figure 4.5. **Population by age in 1993**



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5. Education

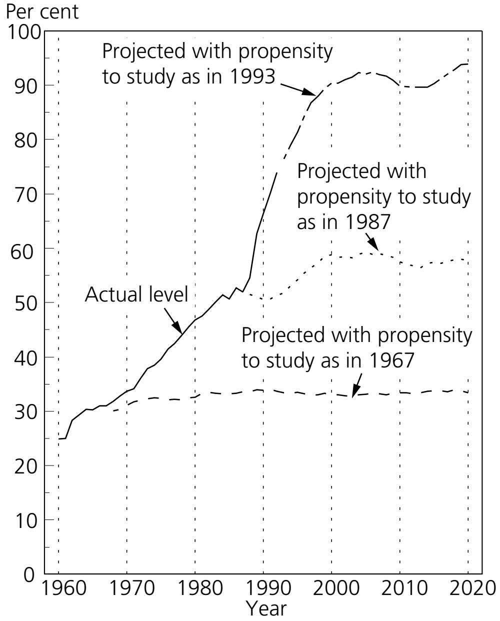
The MOSART model simulates educational activities and examinations for each person depending on gender, age and last year's educational activity and attainment. The current educational transition probabilities are estimated from all transitions in the Norwegian educational system between the school years 1986/87 and 1987/88. See Andreassen et al. (1993) for details, while appendix C gives a description of how education is classified in the MOSART model. The educational model reported here is planned replaced by educational transition probabilities also depending on unemployment, relative wages and capacity in the educational system. Cappelen and Stølen (1994) match the labour supply by educational attainment from the MOSART model with labour demand from the macroeconometric model MODAG. This is used to analyse possible future mismatches in the Norwegian labour market.

Education is an important covariate for other events in the MOSART model, such as mortality, disability and labour force participation. An aspect of education as a covariate is the fact that most adults keep the same educational attainment throughout his or her life course. A person with a low or high educational level will therefore have a systematic low or high labour force participation across his or her life course in the MOSART model.

5.1. Educational activities

The development in the number of pupils and students measured relatively to the number of persons aged 16 to 24 years old is reported in figure 5.1. Pupils in primary school are excluded from the figures, while students older than 24 years are included. The latter can cause the level to pass 100 per cent. Implicit figure 5.1 gives a rough estimate on the amount of *additional educational years*, the number of years a person chooses to spend in the educational system after the compulsory primary school. Measured on the cross-section of the population, the amount of additional educational years increased from 2.2 in 1960 to 4.7 in 1987. Through 1988 unemployment increased from 2 per cent to almost 5 per cent of the labour force, and this may explain the abrupt increase in the number of pupils and students since 1988. In 1993 the amount of additional educational years had reached a level of 6.9 measured on the cross-section of the population.

Figure 5.1. **Number of pupils and students as percentage of number of persons aged 16 to 24 years. Primary schools are excluded**



Simulation of historical data: The number of pupils and students

The probabilities of being a pupil or a student are adjusted to make the simulation of historical data consistent with the actual number of pupils and students each year[10]. A proportional adjustment won't do since many of these probabilities are close to 1, especially among persons who were pupils and students in the previous year. The base line probabilities p_i^0 of either continuing or becoming a pupil or student are instead adjusted by the following:

$$(5.1) \quad p_i^{adjusted} = p_i^0 \cdot (1+r)/(1+ p_i^0 \cdot r), \quad r \geq -1$$

The adjustment in equation 5.1 can be interpreted as changing the constant term of a logit function[11]. The adjustment factor 'r' is calculated by an iteration over a first order Taylor polynomial each year. This is based on that the probabilities $p_i^{adjusted}$ are a function of 'r' and that the number of events equals the sum of the transition probabilities with the drawing method in the MOSART model. For small probabilities the adjustment by equation 5.1 is quite similar to a proportional adjustment.

Choices between accomplishing/dropping out of an education are roughly held constant as in the school year 1986/87. The probability of accomplishing certain educations without formally being a pupil or student, mostly apprentices, are also held at the level in the school year 1986/88. Choices of educational subjects are held constant as the distribution from the school year 1987/88.

In the simulation of historical data the probabilities of being a pupil or student are adjusted with a factor 'r' of -13 per cent for men and -2 per cent for women in 1987, the estimation year for the educational transition probabilities. With the large increase in educational activities in the preceding decade we consider this a moderate adjustment. The adjustment factors 'r' by gender from the years 1967 and 1993 are used in the projections, and interpreted as the propensity to be a student or pupil in those years. The effect on the number of pupils and students is shown in figure 5.1.

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[10] A better target would be the number of persons that are a pupil or student, and this figure is roughly 6 000 lower due to some persons participating in more than one educational activity. Apprentices, roughly 20 000 persons, should also have been excluded from the number of pupils and students since they also are excluded from the current version of the educational transitions.

[11] $p^0 = \exp(X\beta)/(1+ \exp(X\beta))$,
$p^{adjusted} = \exp(X\beta+\delta)/(1+ \exp(X\beta+\delta))$,
$r = \exp(\delta)-1 > -1$.

The rough method of adjusting the propensity to study in equation 5.1 gives some errors in the distribution of pupils and students already in 1993. The projection with a real initial population predicts 217 000 pupils in secondary school in 1993, while the actual level was 252 000 thousand. The similar figures for higher education are a prediction of 211 000 students against an actual level of 177 000 students. Furthermore, the number of 17 to 21 years old pupils and students is underpredicted, while the number of students older than 25 years is overpredicted. All together this indicates that the projection with the perspectives from 1993 overpredicts the percentage that will accomplish a higher education compared with the actual educational transitions for 1993 (which we do not have yet). The number of pupils and students by educational subjects has also grown unequal, with less growth of pupils and students in crafts, technology and economics.

Projections: The number of pupils and students

The number of pupils and students continues to increase after 1993 even with a “constant” propensity to study as in 1993. One explanation may be that we have adjusted gross flows into and out of the educational system with a large net inflow in 1993. The number of pupils and student must then continue to increase before the gross outflow, as a percentage of the total stock, will match the constant and high gross inflow. Another reason may be that a strongly increasing educational level will lead to a higher propensity to study among persons older than 25 years.

The propensity to study in 1993 will give in the long run an amount of additional educational years of 8.7, and this figure by gender is reported in table 3.2. Since the efficiency in the educational system is low, this does not imply that the average Norwegian will reach graduate level. The inefficiency is to a large degree caused by pupils and students changing subjects, instead of pursuing the education they initially started. In secondary school this was (in 1987) to a large degree caused by excess supply of basic courses and insufficient supply of advanced courses. This may have changed due to recent reforms in the educational system.

Additional education measured by the level of the educational attainment with the highest level, will on average only reach 4.5 years with the propensity to study as in 1993. Of the total of 8.7 additional educational years 4.2 years are “wasted” related to the expected progression through the educational system. Using the educational transitions unadjusted, the level in 1987, additional educational years are 5.2, while additional education measured by educational attainment is 3.5 years.

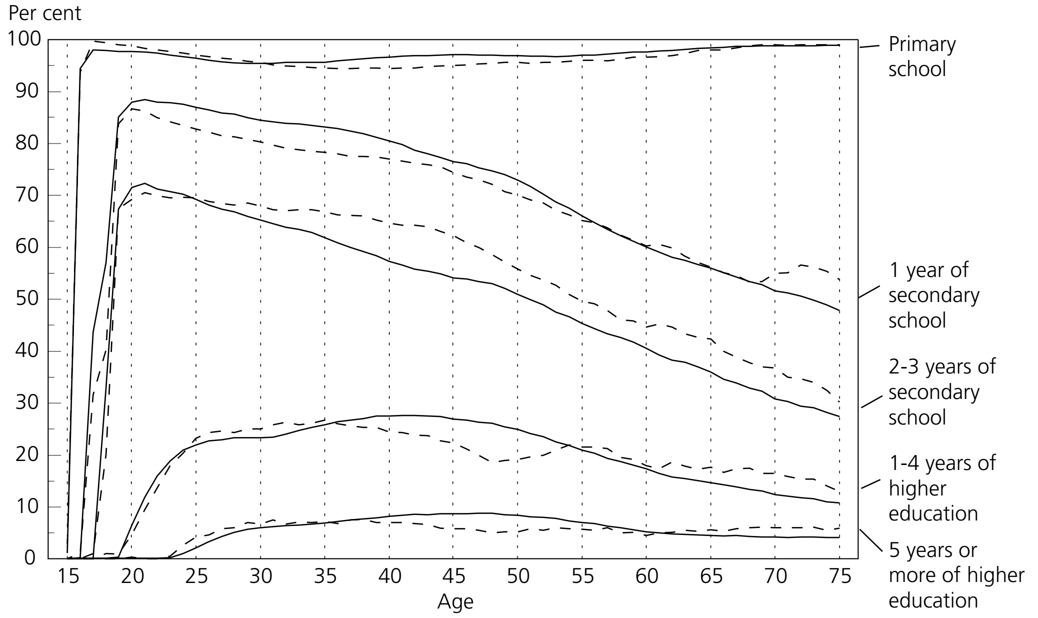
5.2. Educational attainment

How well (or bad) the simulation of historical data predicts the educational level, especially in 1993, is examined in this section. The effect on the percentage with higher education in the future with the propensity to study in 1967, 1987 and 1993 is also reported.

Simulation of historical data: Educational attainment in 1993

With figures 5.2 and 5.3 we are primarily interested in the percentage that at least have accomplished a given educational level. The presumption is that for example 4 years of higher education is “closer” to graduate level than primary school. In this

Figure 5.2. **Educational level, men 1993. Percentage with at least mentioned educational level**



——— Actual level

- - - - - Simulation of historical data

Graphs are smoothed by moving average

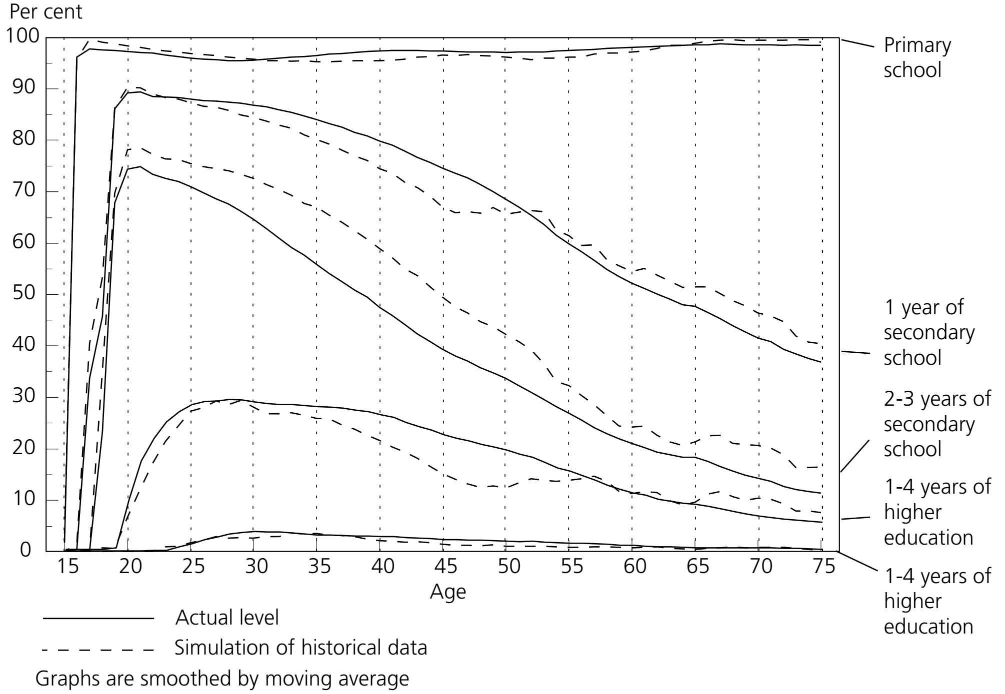
sense the discrepancies are given as the distance between corresponding lines from the simulation of historical data and the actual level in 1993. Anyhow, the vertical distance between a line and the line under from the same data reports the percentage with this specified educational level.

The shape of the lines in figure 5.2 and 5.3 is a combination of age effects and cohort effects. Young persons have not yet finished their educational activities, while older persons had lesser opportunities of achieving educational attainment. An adjustment of only the propensity to study seems to give a proper description of the educational level in 1993, even if the number of additional educational years tripled from 1960 to 1993. Persons who are most affected by the simulation of historical data are persons 16 years old in 1961, or 48 years in 1993. The educational attainment for persons older than 23 years in 1960, or 55 years in 1993, are mainly decided by the initial values in 1960, and not the simulation as such.

Unknown education

Persons with unknown educational attainment comprise youths who have not yet accomplished primary school and (new) immigrants. In the MOSART model all youths are assumed to have accomplished primary school before the age of 17, while 2 per cent do this later (mostly young immigrants). This explains why too few young persons have unknown educational attainment in simulation of historical data. All “net immigrants” are assigned unknown education as their educational attainment, and this is

Figure 5.3. **Educational level, women 1993. Percentage with at least mentioned educational level**



consistent with actual net migration by education in 1993. However, many young immigrants accomplish primary school after they have come to Norway, without being registered as a pupil or student. The MOSART model does not capture this transition, and this explains why too many persons older than 30 years have unknown education in the simulation of historical data.

Primary school

Too many persons in the age group 25 to 50 years end up with primary school or less as their educational attainment in the simulation of historical data. The reason for this is not too few entering secondary school after primary school in the simulation of historical data. One might expect that a “proportional” adjustment of all educational transitions would have a too large impact on the high probability of continuing to be a pupil after primary school (90 per cent in 1987). However, by equation 5.1 it is the small probability of not continuing (10 per cent) that is changed “proportionally”, and this gives in the simulation of historical data too many 16 years old pupils. This excludes the transition from primary school to secondary as an explanation of why too many have stopped at primary school in these birth cohorts. The remaining explanation is that implemented propensities to accomplish an educational activity underpredicts the actual probability of accomplishing an educational activity contrary to dropping out. This seems probable considering that the amount of additional educational years doubled from 1960 until the estimation year 1987. Furthermore, the simple adjustment of the propensity to study indirectly affects the propensity to accomplish an educatinoal activity, most often with a reduction at the lower levels.

Secondary school

The largest discrepancies between the simulation of historical data and the actual level in 1993 are related to the transition from one year of secondary school (basic courses) to three years of secondary school (skilled craft). The percentage with at least 2 to 3 years of secondary school is too high in the simulation of historical data. The reason for this is that many persons achieve their craftsmanship as apprentices, and these transitions are held constant at the level in 1987. Apprentices are also older than pupils in secondary school, and this is not taken into account when constructing the initial population in 1960 (see appendix A). This explains why there are too many persons with more than one year secondary school among persons older than 55 years in 1993.

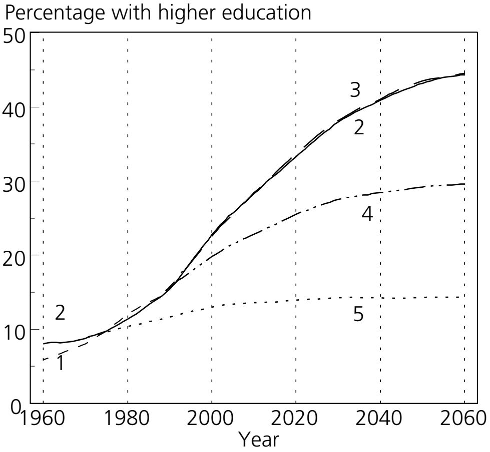
Higher education

A smaller percentage of new secondary school pupils selected grammar school in the school year 1987/88 than in the previous decades. This may explain why too few persons 45 to 50 years old in 1993 have achieved a higher education in the simulation of historical data.

Single educational subjects

Inspecting single educational subjects, the simulation of historical data makes some severe errors especially for women. This implies that only adjusting the propensity of being a student, while keeping the choice of subject constant, is inadequate for predicting the distribution across educational subjects. However, the projections may identify educational subjects where the production of candidates is too large or too small to persist.

Figure 5.4. **Population 16 years and older by educational level**



1. Estimated actual level
2. Simulation of historical data/projected with simulated initial population and propensity to study as in 1993
3. Projected with real initial population and propensity to study as in 1993
4. Projected with propensity to study as in 1987
5. Projected with propensity to study as in 1967

Simulation of historical data and projections: The percentage with higher education

The development in the extent of higher education in the population 16 years and older is reported in figure 5.4. The simulation of historical data is compared with an estimate[12] on the actual level until

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[12] We do not have access to comparable statistics in this period due to changes in definitions and contents of the educational surveys. In 1990 unknown education was replaced with the actual educational attainment for several persons in the educational registers, and we have removed this effect. Prior to 1986 we have interpolated the population censuses, and adjusted these to new definitions of education, extrapolating the discrepancies in the period 1986-1993.

1993. The initial population in 1960 is generated on the assumption that all persons accomplished their current educational attainment successively after primary school and with expected progression through the educational system. In reality some persons may have used longer time and accomplished their current educational attainment later than 1960. This explain why the educational level is too high in 1960. However, most of the persons wrongly assigned a higher education were inside the educational system and would have accomplished a higher education in the sixties. This explains why the simulation of historical data so soon catches up with the actual level. The percentage with higher education is well predicted in 1993.

The extent of higher education in the projections with a propensity to study in 1967, 1987 and 1993 is also reported in figure 5.4. The latter projection may overpredict the extent of higher education, since the number of students in 1993 are overpredicted and the number of pupils and students continues to grow. Still, the percentage with higher education varies in the long run between 14 per cent and 45 per cent, showing the dramatic growth in educational activities over the last 25 years. See also table 7.2 later in chapter 7 for a presentation of the labour force by educational attainment.

6. The National Insurance Scheme

The National insurance scheme (NIS) was established in 1967 and handles all general public pension benefits in Norway. See Norwegian Ministry of Health and Social Affairs (1995) for an overview. The MOSART model simulates entry into public pension schemes based on old age, disability, widow(er)hood and early retirement[13]. Disability pensioners are recruited from a wide age group, and the risk of disability has shown large variations during the last 15 years. The entry into disability is therefore an important transition in the MOSART model, and section 6.1 reports more on this event. Other transitions in pension status, including exits, are either more rare or related to other events such as age limits or widowhood. These transitions are more briefly presented in section 6.2.

The MOSART model calculates NIS pension benefits for pensioners in the model population based on the simulated labour force participation and other characteristics included in the simulation. Section 6.3 describes how pension entitlements are calculated, while section 6.4 reports how pension benefits have developed and may develop in the future.

Several important benefits are not included in this version of the MOSART model. Persons with severe health problems often participate in a rehabilitation programme for several years before entering into disability pension. We plan to include rehabilitation in the model to give a better description of the transition from labour force participation to disability pension. This version of the MOSART model include labour force participation, but not absence due to unemployment and sickness (at the personal level). Anyhow, unemployment and sickness spells are often too short to be handled properly with the time unit in the MOSART model, the calendar year. Many persons, typically white collar workers, supplement their NIS pension benefits with other pension schemes. Among these are the pension scheme that guarantees all civil servants a pension benefit included NIS of 66 per cent of the final salary. Pension schemes outside the NIS are not included in this version of the MOSART model.

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[13] In Norwegian: Avtalefestet pensjon.

6.1. Disability pension

Persons with a permanently reduced ability to work due to disease or accident can be granted a disability pension from the National Insurance Scheme (NIS). The ability to work must be reduced by at least 50 per cent. The benefit equals the old age pension benefit this person would have received if he or she had continued to work until retirement age. Persons who are not working at present, may be granted a disability pension on basis of house work or possible future jobs. Still, most disability pensioners were participating in the labour force when disability *occurred*. After one year of sick leave and often an attempt of rehabilitation, the disabled can apply the National Insurance Administration for disability pension. If the benefit is granted, projections with the MOSART model indicate that more than 95 per cent of the disabled remain in disability pension until retirement age (67 years) or they die.

The social insurance system in Norway has until recently lacked a general possibility of early retirement, and disability pension may have served as a flexible retirement age. The extent of different pension schemes in 1993 by gender and age for persons below the retirement age of 67 years is reported in figures 6.1 and 6.2. At the retirement age of 67 years 45 per cent of the population were disability pensioners. Survivors' pension comprised 7 per cent, implying that 52 per cent of the population received a NIS pension benefit before entering old age pension. Early retirement comprised however only 4 per cent of the population at an age of 66 years in 1993.

Figure 6.1. **Percentage of men who are pensioners in 1993**

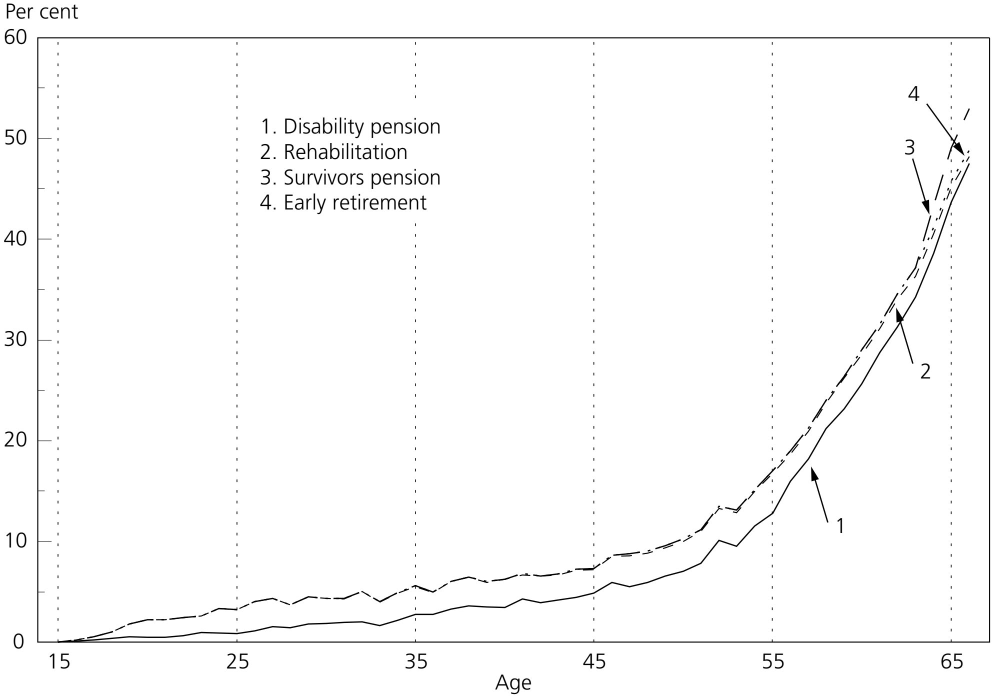
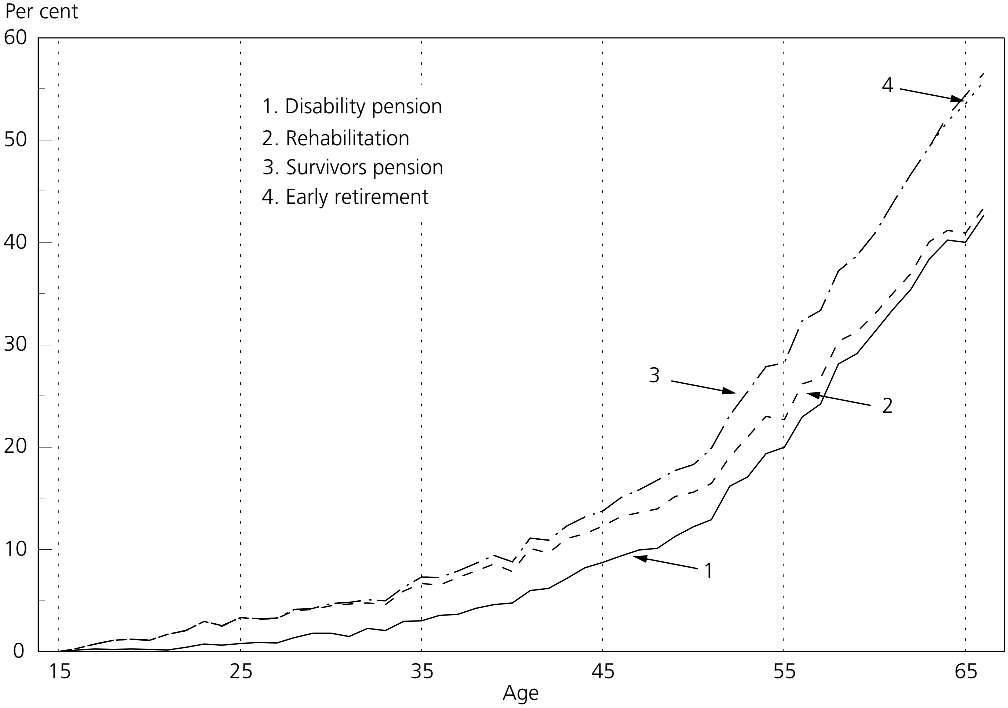


Figure 6.2. **Percentage of women who are pensioners in 1993**



See NOU (1990:17) for a broad discussion on disability pension in Norway.

Transition probabilities for disability pension

For all persons 16 to 66 years old who are not already disabled, the model draws if this person enters disability pension. For persons who are disability pensioners, the model draws the degree of disability and entries back to the labour force et cetera (rare). The risk of disability depends on gender, age, year, marital status, educational attainment and labour force participation. The covariates are estimated using maximum likelihood over a logit function separately for men and women, based on microdata from the period 1986 to 1989:

(6.1) $p_{it} = xp(X_{it}\beta+\delta_t)/(1+\exp(X_{it}\beta+\delta_t))$,

Where:

- p_{it} is the probability that person ‘i’ will enter disability pension in year ‘t
- X_{it} is a row-vector of characteristics for person ‘i’ in year ‘t’
- is column-vector of covariates for each characteristic
- δ_t is an adjustment factor/dummy variable for each year ‘t’

As figure 6.1 and 6.2 indicate, gender and age are important covariates. Educational level is also important, persons with only primary school have a risk of entering disability pension that is ten times higher than for those at graduate level. See Fredriksen and Spurkland (1993) for a more detailed presentation of equation 6.1.

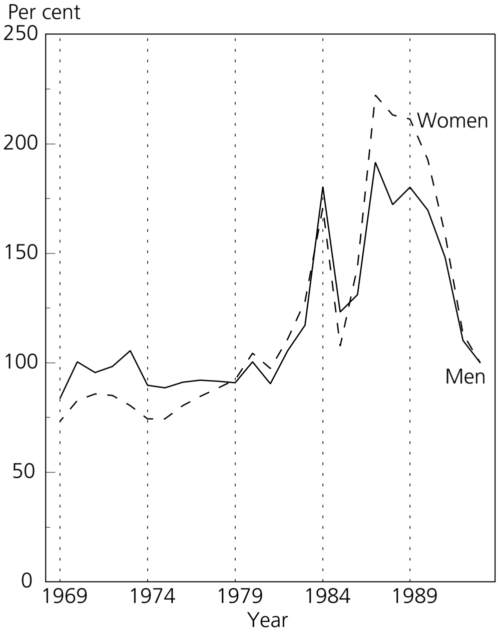
Simulation of historical data: Entry of new disability pensioners

The risk of entering disability pension is adjusted each year in the simulation of historical data such that the expected number of new disability pensioners is consistent with the actual number. This is achieved by adjusting the "constant" term δ_t in equation 6.1 each year separately for men and women, and the values for δ_t are here calibrated ahead of the simulation.

The annual adjustment factors for disability in the simulation of historical data is in figure 6.3 measured relative to the risk of disability in 1993. This is as an available and compact measure on how the risk of entering disability pension has developed. Note that a model without educational attainment would have reported a small decrease in the risk of entering disability pension from 1970 to 1980 and from 1980 *to* 1993. A large percentage of the fluctuations around 1985 can be explained by administrative causes, first a build-up of unhandled applications for disability pension, and then a rush of fast and generous handling of the accumulated applications.
Bowitz (1993) reports that increased female labour force participation, better benefits and increasing unemployment may explain the trend of increased entry of new disability pensioners in the eighties.

The reduction in the entry of new disability pensioners since 1989 occurred after unemployment increased dramatically and before pension benefits was reduced in 1992 along with more restrictive criterias for granting disability pension. Westin (1993) discusses this, and suggests that potential disability pensioners, doctors and the National Insurance Administration may have adapted their behaviour on basis of the political debate ahead of the reform in 1992. Other explanations are an increased use of rehabilitation schemes and a possible drainage of potential new disability pensioners during the period 1987 to 1989.

Figure 6.3. **Relative risk of disability. Estimated from the simulation of historical data. 1993=100**



The risk of entering disability pension in the projections with the perspectives from 1993 equals the average entry of new disability pensioners in the period 1989 to 1993. Two alternative projections presented in chapter 8 use the risk in respectively 1989 and 1993. These two years have a risk of entering disability pension that is respectively 30 to 35 per cent higher and lower than the average of the period 1989 to 1993, and this will also roughly be the effect on the total number of disability pensioners in the long run.

In the simulation of historical data, the risk of entering disability pension is adjusted within a margin of +/- 5 per cent of the estimated levels in the estimation period 1986 to 1989.

Simulation of historical data: The number of disability pensioners

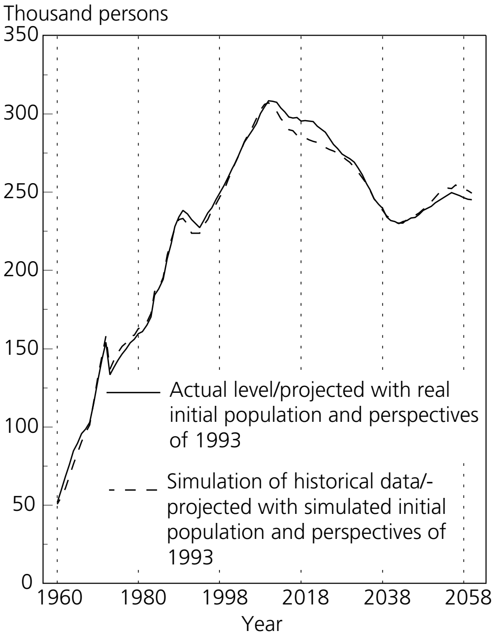
Discrepancies in the total number of disability pensioners between the simulation of historical data and the actual level must here be explained by differences in exit from disability. Exits comprise transfers to old age pension at age 67, deaths and entries back to the labour force et cetera (rare). The two main components influencing the exit rate are the age distribution of new disability pensioners and the degree of higher mortality among disability pensioners as a group. The total number of disability pensioners in the simulation of historical data is compared with the actual level in figure 6.4. The sudden drop in the number of disability pensioners in 1973 is caused by a reduction in the retirement age from 70 years to 67 years.

The number of disability pensioners is well predicted in the simulation of historical data, except for an under-prediction in the last years towards 1993. The discrepancy is caused by too many deaths among male disability pensioners in the simulation of historical data, and an obvious reason may be that the degree of higher mortality among male disability pensioners has fallen in recent years.

Projections: The number of disability pensioners

Two projections with respectively the real and a simulated initial population are also compared in figure 6.4. The assumed risk of disability is the average of the period 1989 to 1993. The number of disability pensioners continues to grow due to an increasing number of persons in the age group 55 to 66 years. When the first post-war birth cohort reach retirement age in year 2010, the number of both 55 to 66 years old and disability pensioners stops to grow. The increase in educational level explains the decrease in the projected number of disability pensioners in the decades after year 2010. This presumes that education has a real effect on a person's risk of disability, not being a mere selection process.

Figure 6.4. **Disability pensioners**



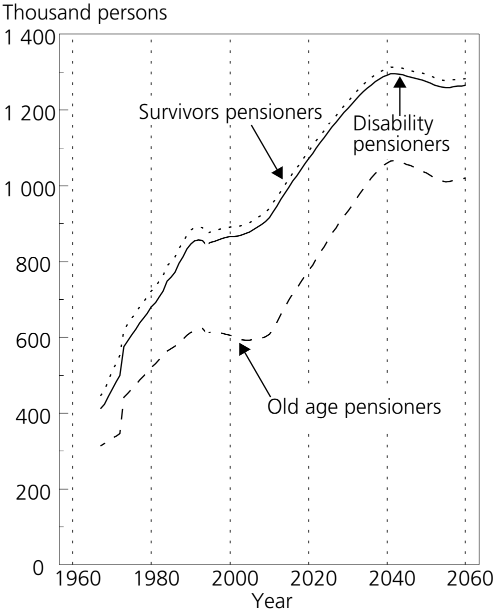
The number of disability pensioners is around year 2015 lower in the projections based on a simulated rather than the real initial population. The discrepancy may be explained by the fact that in birth cohorts born around 1950 too many have an educational level of more than 1 year of secondary school in the simulation of historical data, and thus too low risk of entering disability pension. When these birth cohorts reach the age groups where disability pension is important, around year 2015, the number of disability pensioners becomes too low in the projections with a simulated initial population.

6.2. Old age pension, survivors' pension and early retirement

The general retirement age is 67 years in Norway, and the pension benefit is tested against any labour market earnings the pensioner may have until he or she reaches an age of 70 years. After 70 years the benefit is granted unconditionally, and employees are often obliged to retire at this age, for example in public services. The retirement age was reduced from 70 years to the current level of 67 years in 1973. All persons that already receive a pension benefit or have no labour market earnings are assumed to enter old age pension when they are 67 years old. The MOSART model draws if old employees retire at age of 67 years, and two out of three do this in the simulation.

The development in the number of old age pensioners, both actual and in the projections with perspectives from 1993, is reported in figure 6.5. The number of old age pensioners is underpredicted by 3 per cent in 1993 in the simulation of historical data, this is however not presented in figure 6.5. Half of this discrepancy can be explained by to few old persons in the simulation of historical data, see chapter 4 for a discussion. The other half comprises NIS pensioners who died in November and December and still receive benefits at the end of the year and NIS pensioners inhabited abroad.

Figure 6.5. **Number of NIS pensioners. Actual level and projected with perspectives from 1993**



The number of old age pensioners will decrease slightly towards year 2005, until the large birth cohorts from the end of and after Second World War reach retirement age. Towards year 2040 the number of old age pensioners will increase with 80 per cent, and nearly half the increase is caused by lower mortality in the future than in 1993. With the perspectives from 1967 the number of old age pensioners would be half the level compared with the perspectives from 1993. Reduced mortality since 1967 explains 60 per cent of the difference, while lower retirement age in 1973 and net immigration explain the remaining difference.

Early retirement was introduced around 1989 as an agreement between the government and the labour market organisations, and approximately two-third of the labour force are entitled to early retirement at an age of 64 years. The benefit equals the old age pension benefit this person would have received from NIS if he or she had continued to work until general retirement age of 67 years. Less than 25 per cent of the entitled retired earlier than 67 years in 1993, but the arrangement is growing in popularity.

Widows and (widowers) may be granted a survivors' pension from NIS until they reach 67 years and enter old age pension. The benefit includes the late spouse's pension entitlement and is tested against the pensioner's own labour market earnings.

6.3. How pension entitlements are calculated

Pension benefits from the National Insurance Scheme (NIS) are based on entitlements each person achieves through his or her working years. These entitlements are guaranteed by the government and cannot be abandoned, although their value can be changed through adjustments of the Basic Pension Unit (see below). Existing pension entitlements will therefore have an impact on the Norwegian economy for decades ahead. A description of how pension entitlements are calculated is given in this section.

Basic Pension Unit

NIS has its own measuring unit called the *Basic Pension Unit* (BPU), with a value of 37 000 NOK in 1993. The BPU is used to calculate pension entitlements and adjust pension benefits according to inflation and general growth in wealth. How the BPU is indexed is important, and the consequences for the pension benefits are discussed in section 6.4. The intentions of the current laws regulating NIS are a BPU increasing with the same rate as the wage level[14]. The BPU grows with the same annual rate as wages in the projection with the perspectives from both 1967 and 1993, but of course with different initial values. Other rules for calculating entitlements and benefits are held constant as they were in respectively 1967 and 1993, including the Special Supplement.

Pension benefits

NIS benefits to old age, disability and survivor pensioners constitute of three elements:

(6.2) Pension benefit = Basic Pension + Maximum(Special Supplement, Supplementary Pension)

A pensioner married to a pensioner receives a Basic Pension of 0.75 BPUs, while other pensioners receive 1 BPU. The Special Supplement was 0.6 BPUs for most pensioners in 1995. The sum of the Basic Pension and the Special Supplement is the Minimum Pension Benefit all pensioners are guaranteed.

The Supplementary Pension is based on previous labour market earnings, including wages, income as self-employed, sick leave benefits, unemployment benefits and maternity leave benefits. Each year the person is 17 to 69 years old the labour market earnings are translated into Pension Points by using the BPU of the year the income was earned:

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[14] See law of 17th of June 1966 on the National Insurance Scheme, § 6.2 with supplement passed by the Norwegian parliament on 23rd of May 1993.

$$\text{(6.3) Pension Point} = \begin{cases} 0 & \text{If: Income} < 1\,\text{BPIU} \\ (\text{Income} - \text{BPU}) \,/\, \text{BPU} & \text{If: } 1\,\text{BPU} \leq \text{Income} < 6\,\text{BPU} \\ 5 + (\text{Income} - \text{BPU}) \,/\, (3\text{xBPU}) & \text{If: } 6\,\text{BPU} \leq \text{Income} < 12\,\text{BPU} \\ 7 & \text{If: Income} \geq 12\,\text{BPU} \end{cases}$$

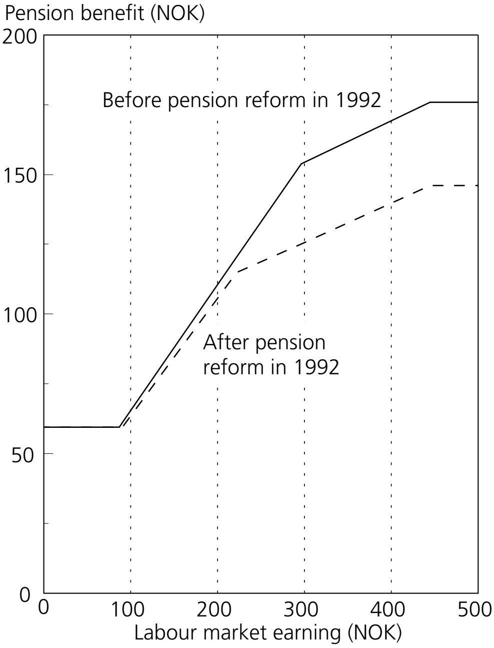
The main rule for calculating the Pension Point is that labour markets earnings exceeding 1 BPU, the Basic Pension, are divided by the BPU. Labour market earnings exceeding 6 BPU are divided by 3 BPU and labour market earnings exceeding 12 BPU is neglected, and this constitutes an income ceiling on the earnings of pension entitlements. The Final Pension Point is calculated as the average of the 20 largest positive Pension Points, while Pension Point Years is the number of years with labour market earnings above 1 BPU. The Supplementary Pension is calculated using the BPU at the time the pension benefit is received:

$$\begin{aligned} \text{(6.4) Supplementary Pension} = {} & \\ & \text{Supplementary Pension Rate} \times \\ & (\text{Minimum(Pension Point Years, 40)} \\ & /40) \times \\ & \text{BPU} \times \text{Final Pension Point} \end{aligned}$$

The Supplementary Pension Rate represents a (marginal) *benefit-wage ratio*, and its present value is 42 per cent. The second line, Pension Point Years divided by 40, represents the *earning time percentage*. The last line, BPU × Final Pension Point, represents an *income base*, and is very approximately the former income level as employee indexed with the growth in the BPU. This approximation is rough because income below 1 BPU and over 6 BPU weighs less or nothing. Figure 6.6 shows how pension benefits will vary with previous labour market earnings under certain strict conditions, especially that prices, labour market earnings and the BPU are held constant over the career. The figure also represents the situation if labour market earnings and the BPU grow at the same rate, and all nominal amounts are wage deflated.

Note that NIS is progressive, meaning that the ratio between benefits and wages decreases strictly with increasing income level. Average *full-time* earnings was roughly 210 000 NOK in 1993, implying that most persons are unaffected by the income ceiling of 6 and 12 BPUs in

Figure 6.6. **NIS pension benefits by previous income. A person with stable labour market earnings and at least 40 years of labour force participation, Basic Pension Unit = 37 033 NOK**



equation 6.3. The ratio between the NIS pension benefit and previous labour market earnings will be 50 to 55 per cent with an previous income of 180 000 NOK and 40 Pension Point Years. After taxation the ratio is roughly 60 to 65 per cent at this income level, depending on personal tax deductions.

The properties of the Supplementary Pension imply that the distribution of labour market earnings between persons and over the life course are important. An extreme example can visualize this. The Supplementary Pension based on annual labour market earnings of 2.44 BPUs from an age of 17 years until 69 years will be less than the Special Supplement. Supplementary Pension is maximised if this same life time labour market earnings are *redistributed* to 20 twenty years with 1 BPU and 20 twenty years with 5.47 BPUs. In this case the Supplementary Pension will exceed the Special Supplement with 1.27 BPU, or 47 000 NOK with the BPU from 1993. See chapter 7 for a discussion of the simulation and distribution of labour market earnings.

Other sources for pension entitlements

Disability pensioners are granted a Computed Pension Point for each year from disability occurred until retirement age as a compensation of lost opportunities of earning pension entitlements. The Computed Pension Point is either the average of the Pension Points in the three last years before disability occurred or the average of the best half of all Pension Point this person may have earned. This latter alternative is omitted in this version of the MOSART model. If disability occurs before an age of 23 years, the disability pensioner is granted a Computed Pension Point of 3.3 as born disabled.

Persons with children younger than 7 years or other extensive family-care obligations are each year granted a Pension Point of 3.0 if their own labour market earnings yield a lower Pension Point. The level corresponds roughly to the average Pension Point for unskilled women working full-time, and the Family-Care Pension Point was introduced in 1992.

Widows (and widowers) can choose between their own Supplementary Pension and 55 per cent of their own and the late spouse's Supplementary Pension. If the late spouse died before retirement age, the Supplementary Pension is calculated as if he (or she) had entered disability pension the same year.

Transitional rules

NIS was established in 1967, and it is only labour force participation after 1967 that yields entitlements for a Supplementary Pension. Persons born before 1937 are partly compensated for the missing opportunity of earning full pension entitlements, the same as earning 40 Pension Point Years. For these birth cohorts the Supplementary Pension in equation 6.4 is divided with the maximum possible number of Pension Point Years instead of the usual 40 years, however not less than 20 years, and only for the part of the Pension Point not exceeding a level of 4.

The 1992-reform

NIS was reformed in 1992 to make the system less favourable to those with high incomes. The income limit in equation 6.3 where labour market earnings are divided by three times the BPU was reduced from 8 BPUs to 6 BPUs in 1992, though not affecting already earned pension entitlements. The Supplementary Pension Rate in equation 6.4 was reduced from 0.45 to

0.42, without affecting already earned entitlements. Pension Point Years earned before 1992 is multiplied with 0.45, while the remaining years in equation 6.4 is multiplied with the new rate of 0.42. The reform also affected future Computed Pension Points of existing disability pensioners.

6.4. Pension benefits

The National Insurance Scheme (NIS) was as mentioned established in 1967, and the scheme needs several decades before the system of Supplementary Pensions is matured. The differences in supplementary pension by gender and birth year, excluded inherited entitlements, are reported in figure 6.7. This figure is based on a projection with a simulated initial population and a projection with the real initial population, and the discrepancies between the two projections are discussed later. Both projections use the perspectives from 1993. The Supplementary Pensions are decided by already earned entitlements for persons in the real initial population born until 1923. Supplementary Pensions for younger birth cohorts are to an increasing degree decided by the simulation, and birth cohorts born after 1972 are purely simulated.

The Supplementary Pensions are larger for younger birth cohorts because these birth cohorts have longer series of Pension Points in NIS. The transition rules give favourable pension benefits to persons born after 1917 who have worked every year from 1967 until they are 69 years old, while some years without labour market earnings can reduce the pension benefits very hard. Female labour participation was low in 1967, see figure 7.1 and 7.2. This explains why men born in the twenties receive near full pension benefits, while their contemporary women usually

Figure 6.7. **Average Supplementary Pension benefits, excluded inherited pension entitlements**

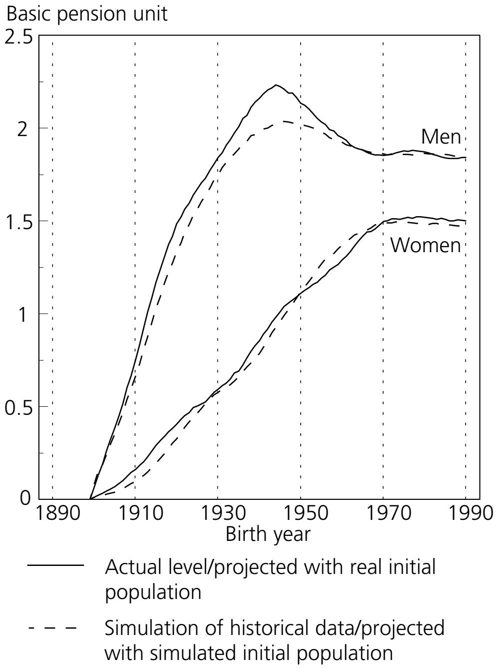
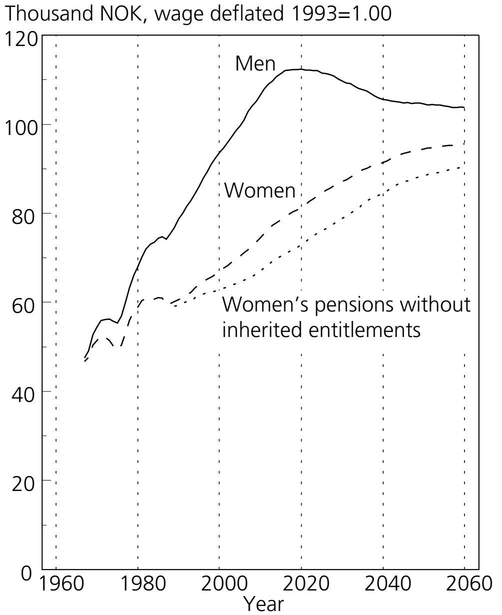


Figure 6.8. **Average old age pension benefits**



have Supplementary Pensions smaller than the Special Supplement.

Theoretically can persons born in 1937 achieve full pension entitlements. However, persons born in 1950 will be the first birth cohort were the full working career is included in the calculation of Supplementary Pensions, and the system will continue to mature until this. The mentioned 1992-reform explains why men born after 1950 will receive smaller Supplementary Pensions than men born in the forties. Additionally the gender differences in wages decrease in the projection period due to a stronger growth in educational level for women. When average wages per man-year are to remain constant, the income level decreases by 3 per cent for men. Supplementary Pensions continue to grow for women born after 1950, and the reason is the large increase in female labour force participation over the last 25 years, see also figure 7.1 and 7.2.

The pension benefits in figure 6.8 are roughly speaking a moving average of figure 6.7 plus the indexing of the BPU and the Special Supplement. The short term variations in average old age pension benefits over the last 25 years are caused by adjustments of the BPU and the Special Supplement. Even when birth cohorts with full entitlements reach retirement in year 2020, persons born in 1950, the average old age pension benefits continues to grow until all pensioners have full entitlements.

While NIS today is far more favourable to men's patterns of labour force participation, the matured NIS is clearly more favourable to women than men compared with an actuarially correct system. When NIS is fully matured, the average female old age pensioner will receive a benefit of 92 per cent of the average male pensioner, while her labour market earnings were only 67 per cent of his. Several aspects of NIS contribute to this equalisation. First, the system is progressive in the sense that the benefit/wage ratio decreases with increasing income. A variable income is also favoured because the system only considers the 20 best income years and do not give extra credit to working careers longer than 40 years. This implies that a limited period with reduced income has no direct effect on the pension entitlements, for example women working part-time during the years they have small children. Family-Care Pension Points and inherited entitlements are also important for women, see figure 6.8 for the effect of the latter. Fredriksen and Koren (1993) discuss the male-female aspects of NIS in more detail.

Simulation of historical data: Old age pension benefits

The projections with a simulated rather than the real initial population give systematically lower Supplementary Pensions for male birth cohorts born before 1960. For these birth cohorts already earned pension entitlements are important, and any discrepancies may be explained by shortcomings in the simulation of labour market earnings. The difference is roughly 5 per cent for male birth cohorts born around 1930 and 10 per cent for men born around 1945, see figure 6.7. The discussion in section 7.3 points out several possible reasons.

First, the distribution of labour market earnings across age groups is not consistent with the actual level in the simulation of historical data, see figure 7.4. Older persons receive too little labour market earnings, while persons younger than 30 years receive too much. Since labour market earnings before 1967 are neglected in NIS, this reduces the Supplementary Pensions for the older birth cohorts. For younger birth cohorts this effect is levelled out since the calculations include both the too high labour market earnings before an age of 30 years and the too low one's after an age of 30 years.

Second, the variation in labour market earning across the life course is to small for men in the simulation of historical data, see figure 7.6. Since only the 20 best Pension Points are used to calculate the level of the Supplementary Pension, this reduces the level of the Supplementary Pension for all male birth cohorts in the simulations. This will affect both old and young male birth cohorts, but this is not visible in figure 6.7 since the youngest birth cohorts are simulated also in the projections with a real initial population.

The two mentioned errors in the distribution of labour market earnings across age groups/the life course may explain the discrepancies in Supplementary Pensions for men born between 1910 and 1960. We believe the future effect to be that female pensions are relatively well predicted, while male Supplementary Pensions are underpredicted. The latter discrepancy may be approximately 4 per cent if the distribution of labour market earnings in the future equals the period 1967 to 1993.

Widows (and widowers) receive 55 per cent of the sum of their own and the late spouse's Supplementary Pension if this exceeds their own Supplementary Pension. This so-called inherited pension entitlements are systematically underpredicted in the projection with a simulated rather than the real initial population. This shows that matching of spouses and possible (negative) correlation in spouses' labour supply are important for the pension benefits. This matching is captured by the real initial population in the first decades of the projections, while the simulated initial population misses this matching.

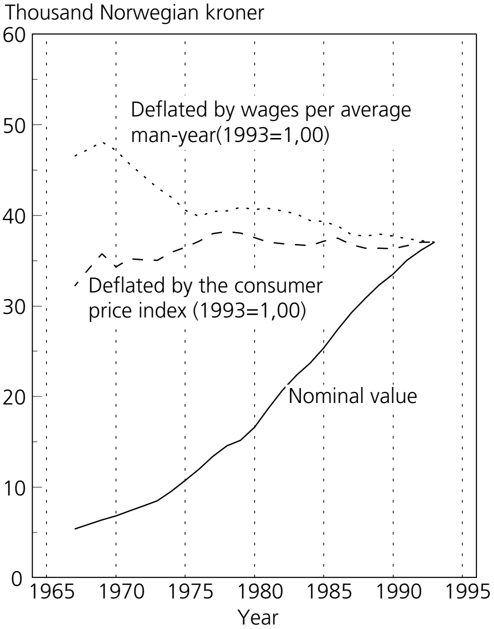
The simulation of historical data underpredicts the Basic Pension in 1967, and the reason must be a too low percentage of pensioners not married to another pensioner. The reason for this is too rough methods of generating the initial population in the base year 1960, probably with an underestimation of the age differences between spouses (see also appendix A).

Simulation of historical data: Disability pension benefits

Two components contribute to underpredict the average disability pension benefit in the simulation of historical data. Disability pension benefits are in the simulation calculated on basis of the labour market earnings in the last years before disability pension was granted, and this is also the main rule in the NIS. However, disability pension benefits will be calculated on basis of the best half of all possible income years if this is more favourable for the pensioner. This option is disregarded in this version of the MOSART model. Furthermore, rehabilitation benefits are not included in the simulation, and this arrangement is an important transition status between labour force participation and disability pension. Without rehabilitation as a characteristic, our transition probabilities make too many persons without any labour force experience enter disability pension, thus making Supplementary Pension lower for this group. These two simplifications are made visible by an overprediction of the percentage of disability pensioners without Supplementary Pensions.

However, average disability pension benefits are the same in the simulation of historical data as the actual level. This implies that other components contribute to an overestimation of disability pension benefits. One possible explanation is the exclusion of income level as a covariate for disability risk, while persons with labour market earnings above the average have a far smaller risk of entering disability pension. Income level is excluded as a covariate for the risk of disability and any other transitions in this version of the MOSART model, because the personal residuals ε_{it} in equation 7.3 else would be biased. See section 7.2 for details.

Figure 6.9. **Basic Pension Unit**



Indexing of the Basic Pension Unit

Changes in the value of the BPU will in effect be the indexing of all existing pension entitlements in NIS from the point of time the entitlements were earned. If the BPU and wages increase with the same rate, this implies roughly that persons in the labour force and existing NIS pensioners will have the same growth in wealth. Pensioners with only Minimum Pension Benefits may be compensated through an increase in the Special Supplement. The value of the Minimum Pension Benefit increased relative to wages per man-year until 1980, but have changed little relative to the wage level since then.

The BPU has historically grown less than wages as figure 6.9 shows. The development can be roughly divided into two periods. Until 1976 the BPU increased by 1.6 per cent per year measured in fixed prices, while wages (per standardised man-years) increased by 3.3 per cent per year. Since 1976 the BPU has remained at a constant level measured in fixed prices, while wages over the last ten years have grown with 0.7 per cent per year.

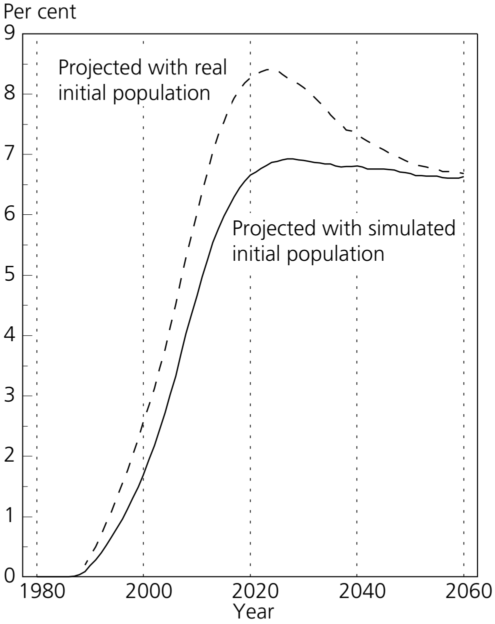
If the BPU is indexed less than wages every year, this will have large effects on the NIS with a time horizon of some decades. If the discrepancy is 1 per cent each year, the Pension Point earned at an age of 40 years will at an age of 75 years be 30 per cent less worth than the official policy implies. Future Pensions Points will with one exception not be affected by the *current* indexing of the BPU. A BPU that is 1 per cent less worth will yield 1 per cent larger Pension Points, thus being without effect. The exception is the income limits in equation 6.3, and a BPU that is less worth compared with wages will efficiently reduce the benefit/wage ratio. If the

Special Supplement at the same time increases to compensate pensioners with Minimum Pension Benefits, in the long run the maximum Supplementary Pension will be less worth than the Special Supplement.

An example can illustrate the significance, and all following nominal values are deflated with the wage level in 1993. The average old age pension benefit was in 1993 worth 70 500 NOK, and will stabilize at a level of 98 000 NOK after year 2035 if the BPU increases with the same rate as wages. In the calculated alternative the BPU grows one per cent less per year than wages and the Special Supplement is increased to compensate those who only receive the Minimum Pension Benefit. By year 2035 the average old age benefit will be worth roughly the same as in 1993, approximately 70 000 NOK. By year 2090 no Supplementary Pensions will exceed the Special Supplement, and the average old age pension benefits will be the average Minimum Pension Benefit, at that time 57 000 NOK.

Until recently most pensioners have only received the Minimum Pension Benefit, and therefore had little interest in the BPU as long as the Special Supplements have increased sufficiently. This is changing, and the interest for the BPU from pensioners and labour market organisations is increasing. The possibilities of continuing the historical trend of indexing the BPU less than the growth in wages, as shown in figure 6.9, might be diminished. The high and unstable inflation during the period 1970 to 1981 may also have complicated the indexing of the Basic Pension Unit.

Figure 6.10. **Effect on old age pension benefits. Reduction in average old age pension benefits by using 40 best Pension Points vs. 20 best Pension Points**



Comparative statics

The MOSART model is used for analyses where the underlying assumptions are changed, and figure 6.10 reports an analysis of a possible reform of NIS. Two analyses with respectively a simulated and the real initial population are compared. The Final Pension Points are calculated from the 40 best income years rather than the 20 best without any transitional rules. A real reform cannot affect already earned entitlements, and with proper transitional rules the full effect would not have taken place until year 2060, see Fredriksen and Koren (1993) for examples.

The effect of the reform starts in 1988 when the first pensioners with more than 20 Pension Point Years enters old age pension. The decreased effect after year 2020 is to some extent expected, since the effect of extending the average to 40 years is more harmful if you have only 40 pension points to choose between rather than 53 as the youngest birth cohorts will have. The simulated initial population systematically underpredicts the effect as long as the initial population is important. This indicates that the variations in labour market earnings across the career are too low in the simulation of historical data, as figure 7.6 also reports.

7. Labour force participation

Labour force participation is simulated in two steps in the MOSART model. First, the number of persons in the labour force and man-years are projected based on definitions and data from Statistics Norway's Labour Force Sample Surveys. Second, each person's labour market earnings are simulated in such a way that the number of income recipients and their labour market earnings are consistent with the above projected labour force. The difference between the labour force and the number of income recipients is caused by the first being an average across the year, while the latter include fully also those who have worked only parts of the year. Each of the two steps are described in the next two sections, while Fredriksen and Spurkland (1993) also describe the estimated parameters. The distribution of labour market earnings in the simulation of historical data is compared with actual panel data for the period 1967 to 1993 in section 7.3.

7.1. Labour force projections

The MOSART model projects the labour force and number of man-years by adding up each person's calculated labour force participation rate (LFPR) and expected working hours. The LFPR and working hours are estimated from the Labour Force Sample Surveys from 1991, and depend on gender, age, children, marital status, educational activities and attainment and pension status. Each person's calculated LFPR and working hours do not however depend on previous year's labour force participation. The projections only show the effect on the labour force of changes in population size and composition. In the projections the estimated LFPRs and working hours from 1991 are adjusted to the level in respectively 1967 and 1993, and then held constant.

The effects of educational activities and disability pension on LFPR and working hours are estimated by coupling the Labour Force Sample Survey with administrative registers. The Labour Force Sample Survey in Norway has only conditional questions on educational activity or disability, and misses therefore that some persons working (part-time) may also be a student or disability pensioner. After the coupling we find that disability pensioners have an average LFPR of 16 per cent, while pupils and students have an average LFPR of 39 per cent. The effect on the labour force of one more person entering disability

Table 7.1. **Some labour market projections. LFPRs in per cent, other in thousand**

Year/assumptions	Population 16-74 years	Labour force	LFPRs	Man-years	Students and pupils	Disability pensioners
1967						
Actual level[1]	2628	1639	62.4	1751	155	96
1993						
Actual level[1]	3119	2132	68.4	1670	427	227
LFPRs as in 1987	3119	2192	70.3	1788	427	229
Propensity to study as in 1987	3119	2187	70.1	1723	293	231
Women's LFPRs equal men's	3119	2268	72.7	2042	428	231
Perspectives from 1967	3140	1999	63.7	2141	227	120
Year 2000						
Perspectives from 1993	3188	2225	69.8	1764	447	259
LFPRs as in 1987	3187	2296	72.0	1887	444	251
Entry of disability pensioners as in 1993	3188	2249	70.6	1784	448	217
Propensity to study as in 1987	3186	2286	71.9	1819	285	260
Women's LFPRs equal men's	3186	2360	74.1	2144	446	253
Perspectives from 1967	3362	2196	65.3	2354	244	118
Year 2030						
Perspectives from 1993	3587	2454	68.4	1967	513	271
LFPRs as in 1987	3587	2543	70.9	2101	508	259
Entry of disability pensioners as in 1993	3590	2504	69.7	2012	509	196
Propensity to study as in 1987	3585	2480	69.2	1976	308	299
Women's LFPRs equal men's	3588	2565	71.5	2331	508	267
Perspectives from 1967	4738	3113	65.7	3338	334	161

[1] Projection results from the projections with perspectives from respectively 1967 and 1993.

pension is the difference in LFPR before and after disability occurred, roughly 65 percentage points in the MOSART model. For example, if the risk of entering disability pension increases, 100 more disability pensioners reduce the labour force with approximately 65 persons. The same figure for pupils and students is 40 persons in the short run, and in long run none because persons with more educational attainment remain longer in the labour force. See also table 7.1 for the effect of disability pension and educational activities.

Labour force participation rates

The development in LFPRs and working hours over the last 30 years and in the projection with the perspectives from 1993 is reported in figures 7.1 and 7.2.

Labour force participation has decreased among men over the last 30 years, but still consistent with the increased number of pupils, students and disability pensioners (see table 3.2). Female labour force participation has increased in spite of the changes in educational activities and disability pension. The gender differences in LFPRs have decreased dramatically, but are still larger than in the other Nordic countries.

The LFPRs in the projections increase because the age composition of the population changes and the educational level increases. The educational level increases more for women than for men, and this explains that gender difference decrease in the projections.

Figure 7.1. **Labour force participation rates. Actual level and projected with perspectives from 1993**

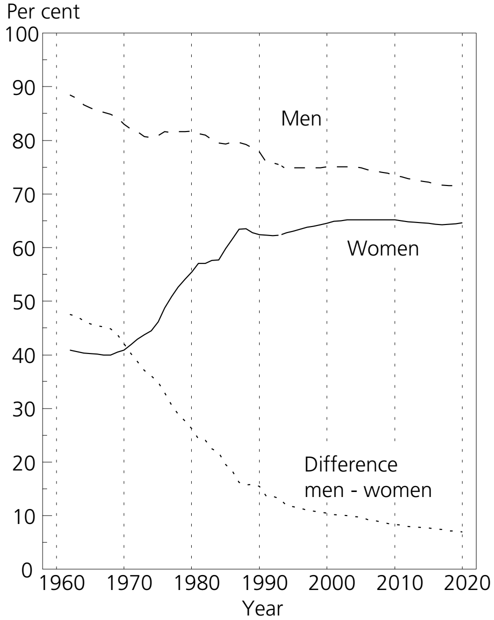
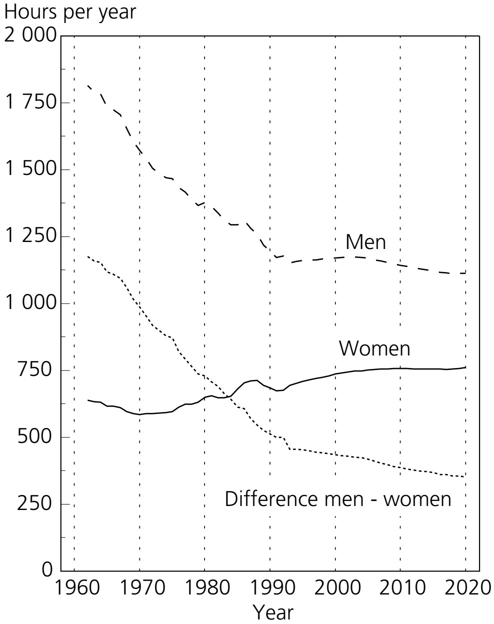


Figure 7.2. **Average working hours. Actual level and projected with perspectives from 1993**



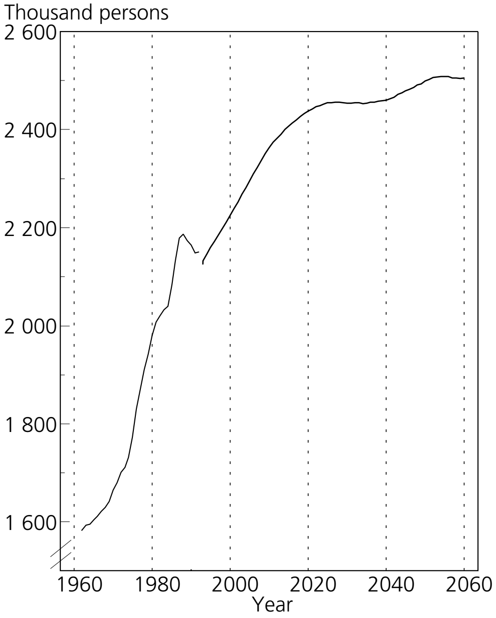
The simulation of historical data overpredicts the number of persons in the labour force in 1993 by approximately 2 per cent due to a similar overprediction of the number of persons in age groups with a high LFPR. Younger immigrants in the simulation of historical data than the actual level may explain why the number of persons in these age groups is overpredicted. The aggregated LFPR is consistent with the actual level in 1993 in the simulation of historical data, indicating that the covariates deciding the labour force participation are well predicted. This comprises variables such as children, education and disability pension.

Labour supply

The development in the labour force throughout the last 30 years and in the projection with the perspectives from 1993 is reported in figure 7.3. The strong increase in the seventies was an effect of the mentioned increase in female labour force participation. The eighties are characterised by increasing unemployment, especially from 1988. This has affected entry into the labour force, and 1987 was a peak year both regarding the number of persons and the LFPRs. The problems in the labour market since 1988 have especially affected persons 16 to 24 years old, leading to 12 percentage points lower LFPRs and a corresponding increase in the propensity to study.

The labour force increases with 300 000 persons towards year 2020 with the perspectives from 1993, and two-third of the increase is due to more persons in the age group 25 to 66 years and the remaining is explained by an increasing educational level. Some key figures for the

Figure 7.3. **Labour force. Actual level and projected with perspectives from 1993**



labour market until year 2030 with different underlying assumptions are reported in table 7.1.

The reduction in LFPRs from 1987 to 1993 partly corresponds to an increase in the number of pupils and students since 1987. The effect on the labour force in 1993 by using LFPRs from 1987 is modest. The effect in the long run of more pupils and students is however small since persons with higher educational level remain longer in the labour force and have higher LFPRs when approaching the retirement age. If female labour force participation increases to men's level, the effect on the labour force and especially man-years is substantial. A projection with the perspectives from 1967 overpredicts the number of man-years in 1993 because full-time working hours have decreased since 1967 due to several labour market reforms.

Labour force by educational attainment

The educational level in the labour force has increased and will continue to increase substantially in the coming decades. Differences in educational level between the young persons entering the labour force and the older persons retiring are the main reason for the growth in educational level. With the high propensity to study in 1993 also some persons older than 30 years are expected to increase their educational level. Table 7.2 reports the educational attainment of the labour force in 1993 and year 2030, in the latter year the present propensity to study has reached almost full effect on the educational attainment of the labour force. The propensity to be either a pupil or students corresponds to the level in 1993, but the relative distribution between educational subjects is the same as in 1987. The increase in pupils and students has *not* been equal between educational subjects, and for example the number of pupils and students in crafts, tecnology and economics have increased less than the average. See also section 5.1.

Persons with unknown education comprise mainly new immigrants, and this percentage will remain stable. The percentage with only primary school decreases substantially, while the percentage with higher education increases correspondingly. We believe that the projection may overpredict the percentage with higher education compared with the current propensity to study, but the projection indicates that more than 40 per cent of the labour force will have a higher education in year 2030. The educational level will be higher among women than among men, and especially nursing and teaching will contribute to this. Gender differences will

Table 7.2. **Labour force by educational level, actual level in 1993 and projected with the perspectives from 1993**

	Men		Women	
	1993	2030	1993	2030
	Thousand persons			
All	1166	1314	970	1168
	Per cent			
All	100.0	100.0	100.0	100.0
Educational level:				
Unknown	4.1	4.6	2.9	3.3
Primary school	18.4	6.8	18.7	5.2
Secondary school, 1 year	20.0	8.4	28.2	7.4
Crafts	10.1	6.1	2.1	1.0
Economics and administration	3.0	0.6	9.8	2.6
Others	6.8	1.8	16.4	3.8
Secondary school, 2-3 years	35.6	39.1	28.0	36.8
Crafts	17.5	21.4	1.5	1.7
Economics and administration	4.5	3.9	6.9	15.7
Auxiliary nursing	0.1	0.1	4.5	3.3
Grammar school	6.3	10.3	8.5	8.4
Others	7.1	3.4	6.6	7.7
Higher education, 1-4 years	15.5	27.8	20.3	38.6
Technology	4.2	8.1	0.8	1.7
Economics and administration	3.1	7.0	3.7	9.9
Nursing	0.3	0.6	4.7	7.1
Teaching	3.2	4.6	6.1	11.0
Others	4.7	7.4	4.9	8.9
Higher education, 5 years and more	6.4	13.3	1.9	8.7
Technology	2.1	4.8	0.2	1.3
Natural sciences	1.1	1.6	0.3	1.2
Law	0.7	1.8	0.3	1.8
Social sciences	0.5	1.4	0.3	1.7
Medicine	0.7	0.8	0.3	0.6
Humanities	0.6	0.9	0.3	1.3
Others	0.8	2.1	0.2	0.8

however remain, with care professions dominated by women and crafts and technology dominated by men. Within secondary school the percentage with three year courses (skilled craft) will increase at the expense of the percentage with one year courses (basic education).

The growth in the percentage with certain educations is so strong that we believe that either will the number of new candidates be lower in the future, or the candidates must find new types of jobs. An example may be economy and administration. Cappelen and Stølen (1994) match the labour supply by educational attainment from the MOSART model with labour demand from the macroeconometric model MODAG. This is used to analyse possible future mismatches in the Norwegian labour market.

7.2. Labour market earnings

The simulation of labour market earnings is divided into two sub-steps. The first step simulates if a person receives labour market earnings or not, and the second step decides how large this income is. Labour market earnings comprise wages, incomes as self-employed, sick leave benefits, unemployment benefits and maternity leave benefits. The probabilities of receiving labour market earnings depend on the same characteristics as the LFPRs, that will say gender, age, children, marital status, educational activities and attainment and pension status. Labour market earnings also depend on previous years' labour market earnings, and the simulation describes in this sense gross flows into and out of the labour force. The probabilities are estimated using maximum-likelihood over a logit function with microdata for the period 1985 to 1988:

(7.1) $p_{it} = \exp(X_{it}\beta+\delta_t)/(1+\exp(X_{it}\beta+\delta_t))$

Where:

- p_{it} is the probability that person 'i' will receive labour market earnings in year 't'
- X_{it} is a row-vector of characteristics for person 'i' in year 't'
- β is a column-vector with covariates for each characteristic
- δ_t is an adjustment factor/dummy variable for each year 't'

The probabilities in equation 7.1 are adjusted each year by changing the "constant" term δ_t, such that the number of recipients of labour market earnings in the simulation is consistent with the projected labour force in section 7.1[15]. Each person is assigned a probability r_i of being in the labour force if he or she receives labour market earnings, and this probability can be interpreted as the percentage of the year this person is working. For most persons r_i is 100 per cent, while pupils, students, disability pensioners and those beginning or quitting to work during the year have lower percentages. Consistency is achieved when:

(7.2) Projected labour force year 't'
$= \Sigma_i \, p_{it} r_i$

The value of δ_t is found by an iteration over a first order Taylor polynomial with p_{it} as a function of δ_t. With the drawing method in the MOSART model the simulated number of income recipients also equal the adjusted number, see appendix B. If the model draws that a person receives labour market earnings, the income level is simulated by:

(7.3) $Y_{it}= \exp(X_{it}\alpha+\varepsilon_{it}+\gamma_t)$

Where:

- Y_{it} is labour market earnings for person 'i' in year 't'
- X_{it} is a row-vector of characteristics for person 'i' in year 't'
- α is a column-vector with covariates for each characteristic
- ε_{it} is a residual for person 'i' in year 't'

---

[15] We have found that with exact adjustment the process becomes unstable, with large streams respectively into and out of the labour force every second year. We start with the adjustment factor δ from the previous year and computes what the δ is for this year, and for persons inside the labour force we use a weighted average of last year's δ (two-third) and this year's δ (one-third), while persons outside the labour force use this year's δ.

- γ_t is an adjustment factor/dummy variable for each year ‘t’

The level of the labour market earnings depends on the same characteristics as the probability of receiving labour market earnings, and the covariates in α are estimated using GLS on microdata for the period 1985 to 1988. An exponential function is chosen to avoid non-positive income amounts. Total labour market earnings is restricted to equal the product of the projected man-years M_t from section 7.1 and an exogenous wage W_t per man-year:

$$(7.4)\ \Sigma_i Y_{it} = M_t \cdot W_t$$

The adjustment factor γ_t in equation 7.3 is found by first calculating the income for each income recipient without γ_t:

$$(7.5)\ \gamma_t = \mathrm{Ln}(M_t \cdot W_t / \Sigma_i Y_{it}^{\text{without } \gamma})$$

The labour market earnings for each income recipient are then adjusted by multiplying with the exponent of γ_t. The estimation of α in equation 7.3 included a so-called disturbance term, and this disturbance term is included in the simulation by the personal residuals ε_{it}. The intention is to capture variations in labour market earnings not explained by the model $X_{it}\alpha$. In the real initial population the residuals ε_{it} are calculated for each person from the difference between the actual labour market earnings in 1988 and the expected income by equation 7.3. Residuals ε_{it} are drawn for new persons and persons without stable labour market earnings, using the observed distribution of the disturbance term from the estimation of equation 7.3. Each person keeps the same residual ε_{it} through his or her entire career in the base line alternative. The distribution of labour market earnings from the simulation of historical data and the actual distribution are compared in section 7.3. This comparison also includes a simulation where new residuals ε_{it} are drawn every year for each person.

In the projections with the perspectives from 1967 and 1993 the exogenous wage per standardised man-year is held constant at respectively the level in 1967 and 1993. This is a trivial assumption as long as income growth do not influence the distribution of labour market earnings and the Basic Pension Unit is adjusted relative to wages. Any other assumption on income growth can be implemented afterwards by multiplying *all* labour markets earnings and pension benefits each year by any index.

Simulation of historical data: Labour market earnings

Labour market earnings from equations 7.1 and 7.3 are in the simulation of historical data adjusted towards the actual level for the labour force and total labour market earnings. The adjustment factors δ_t and γ_t are calculated separately for men and women. We want primarily to test the distribution of labour market earnings, and the figures on the actual labour force are increased by 2 per cent to match the overprediction of the number of persons in the age group 16 to 74 years. On average the probabilities of receiving labour market earnings are adjusted down by 4 per cent in the estimation period 1985 to 1988. Labour market earnings are adjusted down by 6 per cent for men and 11 per cent for women in the same period. Note that the definition of labour market earnings has been extended, for example with unemployment benefits, during the period 1967 to 1993.

7.3. Distribution of labour market earnings in a "life course" context

Supplementary Pensions from the National Insurance Scheme depend in a complicated manner on labour market earnings during the full working career, see section 6.3 for details. In the future Supplementary Pensions will be calculated from the 20 to 40 best income years during the full possible working career of 53 years. This makes the distribution of labour market earnings across the life course important with respect to the National Insurance Scheme. We want therefore to test the "life course" proprieties of the distribution of labour market earnings in the MOSART model.

However, we only have access to personal histories of labour market earnings from the period 1967 to 1993. Some measures on income distribution with this time span are reported in figures 7.4 to 7.7. Each of the four aspects of the income distribution reported in these figures is discussed below. Box 7.1 gives a compact description of the contents of these figures. Average labour market earnings during the period and standard deviation on the income series are calculated for each person in the sample:

$$
\begin{aligned}
(7.6)\quad A_i &= \text{Average}(I_{it} \mid t \in T) \\
S_i &= \text{Standard deviation}(I_{it} \mid t \in T)
\end{aligned}
$$

Where:
- i denotes person and t denotes year
- A_i is average annual labour market earnings for person 'i' during the period 1967 to 1993
- S_i is the standard deviation on the same income series for each person
- I_{it} is labour market earnings for person 'i' in year 't', inclusive years with zero income
- T is the period 1967 to 1993

The average and standard deviation for A_i and S_i are calculated for each birth cohort by gender, and interpreted as the following:

$$
\begin{aligned}
(7.7)\quad D^1_{bg} &= \text{Average}(A_i \mid i \in P_{bg}) \\
D^2_{bg} &= \text{Standard deviation}\ (A_i \mid i \in P_{bg}) / D^1_{bg} \\
D^3_{bg} &= \text{Average}(S_i \mid i \in P_{bg}) / D^1_{bg} \\
D^4_{bg} &= \text{Standard deviation}\ (S_i \mid i \in P_{bg}) / D^1_{bg}
\end{aligned}
$$

Where:
- b denotes birth year, g denotes gender and i denotes person
- P_{bg} is all persons with birth year 'b' and gender 'g'
- D^2, D^3 and D^4 are normalised with average income within each sub-population

Interpretation:
- D^1 measures distribution between generations
- D^2 measures distribution within generations
- D^3 measures distribution across the "life course"
- D^4 measures the distribution of distribution across the "life course"

Two simulations of the period 1967 to 1993 are compared with the actual level. The first simulation is the simulation of historical data with fixed personal income residuals ε_{it} across the career, see equation 7.3. The second simulation is identical, except that new income residuals ε_{it} are drawn every year for each person.

Fredriksen and Spurkland (1993) have a comparison similar to section 7.3, except that the simulated income series are from the future instead of the period with historical data. This study uses simulated income series from the same period as the period with historical data, thus using the same underlying assumptions for especially disability pension and female labour force participation. This reveals some weaknesses in the simulation of labour market earnings that were blurred in Fredriksen and Spurkland (1993) by differences in male and female labour force participation in the past and in the future.

Discrepancies between the actual level and the simulation of historical data may be explained by errors in the simulation of other characteristics as well as the simulation of labour market earnings. Basis for the comparison is all persons inhabited in Norway in the period 1967 to 1993. Persons who have immigrated, emigrated or died during the analysis period are excluded from the analysis, thus giving all persons an equal number of possible working years. For the same reason are persons who where younger than 17 years in 1967 or older than 69 years in 1993 excluded from the analysis. The simulation of historical data comprises more persons than the actual level in figures 7.4 to 7.7, because the MOSART model only simulates net migration.

Distribution between generations

Each birth cohort goes through different age groups during the period 1967 to 1993, and this explains much of the shape of the figures. The title *distribution between generations* is for this reason somewhat misleading, at least for men. Anyhow, figure 7.4 reports average annual labour market earnings for each birth cohort during the period 1967 to 1993 (D^1 from equation 7.7). Labour market earnings are underpredicted with approximately 5 per cent for all birth cohorts born before 1948, while labour market earnings are overpredicted for later birth cohorts (not shown in the figure). Changes in the distribution of labour market earnings across age groups may explain this.

The calculated personal income residuals ε_{it} for the real initial population in 1988 depends on birth year, and this indicates that the age effect is insufficiently represented. The age effects in equations 7.1 and 7.3 are represented simple with quadratic variables (age, age×age), and this may explain why the simulation do not fit the actual age variations in labour market earnings.

Distribution within generations

Distribution of labour market earnings within generations is measured with D^2 from equation 7.7. Each birth cohort is represented by the standard deviation across each person's average labour market earnings. This distribution-measure is reported in figure 7.5 as a percentage of average annual labour market earnings by gender and birth year.

The simulation of historical data predicts the distribution of labour market earnings between women relatively well. The actual income distribution between men lies somewhere between the simulation with fixed residuals ε_{it} and new residuals ε_{it} every year. The income distribution within generations may not affect average Supplementary Pensions, but may contribute to understand the other errors in the distribution of labour market earnings.

Box 7.1. Distribution of labour market earnings in a "life course" context, 1967 to 1993

The population in figures 7.4 to 7.7 are persons who have inhabited Norway every year during the period 1967 to 1993. The table shows how the different measures are calculated and interpreted. The results in figures 7.5 to 7.7 are reported as the percentage of average labour market earnings from figure 7.4. The age of each birth cohort during the period 1967 to 1993 is shown in brackets. The distribution-measures are reported for the actual level, the simulation of historical data and a simulation where new income residuals eit are drawn every year (see equation 7.3).

	Then calculate within each birth cohort by gender:	
First calculate for each person:	Average	Standard deviation
Average labour market earnings during the period 1967 to 1993	Figure 7.4. Distribution between generations	Figure 7.5. Distribution within generations
Standard deviation on the history of labour market earnings during the period 1967 to 1993	Figure 7.6. Distribution across the "life" course "life" course	Figure 7.7. Distribution of distribution across the

Figure 7.4. **Distribution across generations**

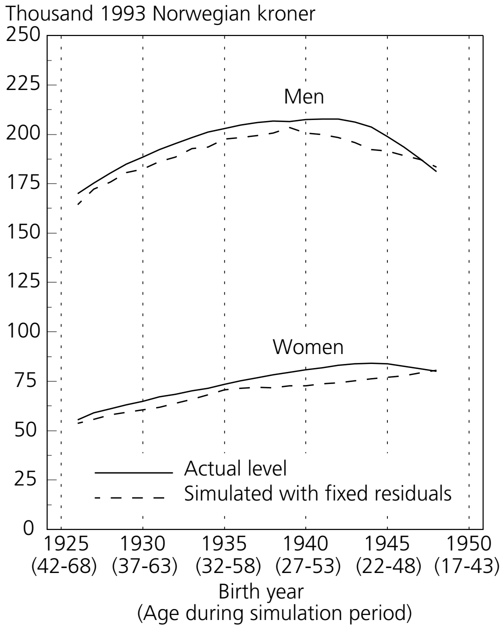


Figure 7.5. **Distribution within generations**

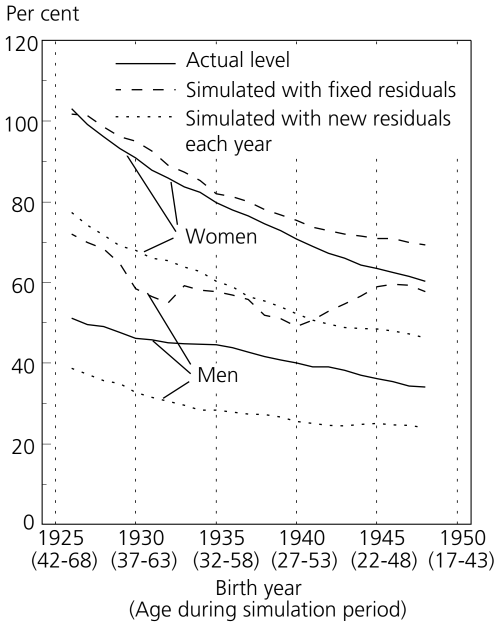


Figure 7.6. **Distribution across "life"**

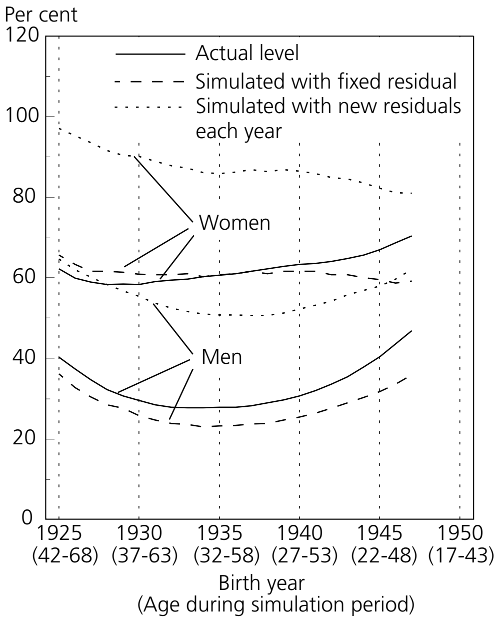
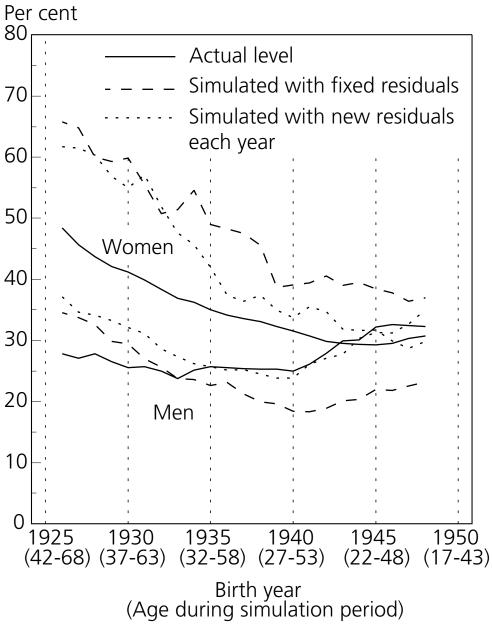


Figure 7.7. **Distribution of distribution across "life"**



Distribution across the "life-course"

Distribution of labour market earnings across the life-course is measured with D^3 from equation 7.7. Each birth cohort is represented with the average across each person's standard deviation on their income series. This distribution-measure is reported in figure 7.6 as a percentage of average annual labour market earnings by gender and birth year. Increased variation in labour market earnings across the life course will contribute with few exceptions to larger pension benefits, and this aspect of the income distribution is important.

Variation across the life course is relatively well predicted for women in the simulation of historical data similar to the income distribution between women. Fixed personal residuals ε_{it} seem to give a satisfactory description of women's variations in labour market earnings across the life course. Covariates on educational activities, children and disability capture most of the female income variations both between women and across the life course.

Variations across the life course is under-predicted by approximately 5 per cent for men. The effect on the Supplementary Pensions depends on how labour market earnings are distributed across the career. Rough tests indicate that 5 per cent less standard deviation across the life course, as figure 7.6 report for men, will give 4 per cent lower Supplementary Pensions.

The simulation with new personal residuals ε_{it} every year presented in figure 7.6 show a far less realistic description of the distribution of labour market earnings across the life course. A more realistic simulation of labour market earnings for men seems however to require some minor variations in the personal residuals ε_{it} across the career. However, the alternative with fixed personal residuals ε_{it} is simple and makes the simulation results more transparent. We have therefore kept fixed individual residuals ε_{it} as the base line alternative until a better model is estimated.

Distribution of distribution across the "life-course"

The distribution of variations in labour market earnings is measured with D^4 from equation 7.7. Each birth cohort by gender is represented by the standard deviation across each person's standard deviation on their income series. This distribution-measure is reported in figure 7.7 as a percentage of average labour market earnings by gender and birth year. The predicted values are especially uncertain because the measure is the variance of a variance, and only large differences between the simulation of historical data and the actual level are worthwhile mentioning. Variations in labour market earning variations across the life course is to some degree overpredicted for women.

8. Contribution rates

A main purpose of the MOSART model is analyses of economic consequences of an ageing population, or more precisely how taxes will increase as a result of increasing public expenditures related to old persons. This version of the MOSART model excludes important parts of these expenditures, for example health care for old persons. Furthermore, the tax rules are represented in a very simple way as the MOSART model lacks important income components and excludes behavioural responses to changes in the tax level. However, we believe these analyses to be of interest since a large variety of sensitivity analyses are presented.

An analysis of the tax system and public pension benefits requires a clarification of how public pension benefits are financed and how pensioners are taxed. For simplicity we presume that current public pension benefits are financed through current taxes, a so-called pay-as-you-go system or PAYG. Furthermore, pensioners have special tax rules to compensate for lower income, old age and health problems, see Arneberg and Gravningsmyhr (1994) for details. The question here is how this tax gap will develop in the future. A reasonable approach may be that income after tax increases at the same rate for each pensioner as for wage earners[16]. This can be achieved through adjustments of the special tax rules for pensioners and the Special Supplement. The Basic Pension Unit is however an inappropriate mean of adjusting income after tax for pensioners, since this will affect the earnings of pension entitlements. In the absence of a more sophisticated representation of the tax rules, we presume that pensioners pay half the taxes of wage earners. Average tax rate caused by public pension benefits, the *contribution rate*, is then calculated by:

$$(8.1)\ \text{Contribution rate} = \Sigma_i\text{Pension benefits} / (\Sigma_i\text{Labour market earnings} + 0.5 \times \Sigma_i\text{Pension benefits})$$

The development of this contribution rate since 1967 and how it will develop with the perspectives from 1993 is discussed in the next section. Discrepancies between the simulation of historical data and the actual level in the period 1967 to 1993,

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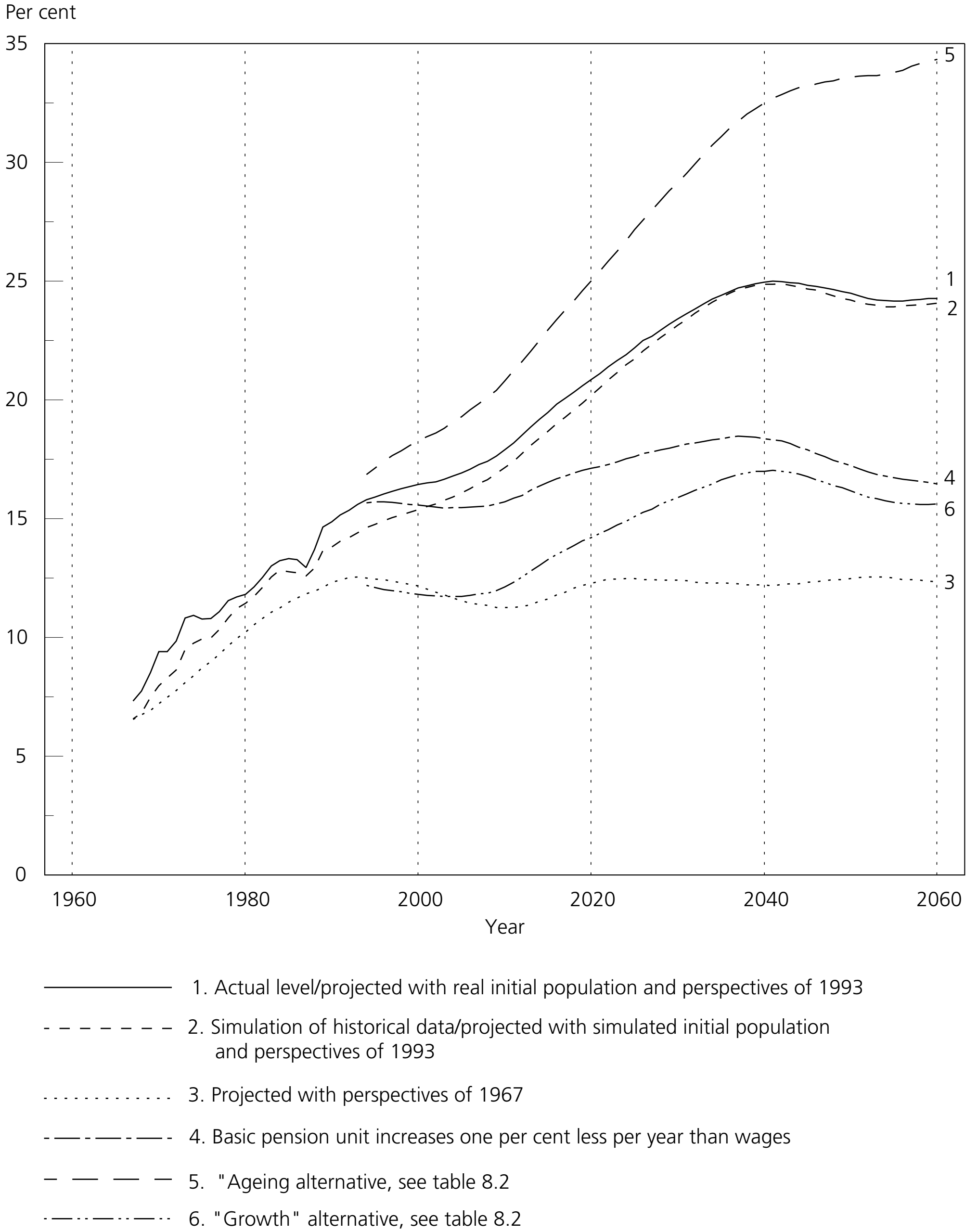
[16] The income level will thus grow for the average pensioner because new pensioners have larger Supplementary Pensions than present older pensioners.

Figure 8.1. **Contribution rates**



with emphasis on the year 1993, are discussed in section 8.2. A set of sensitivity analyses for the contribution rate is presented in section 8.3, similar to for example Fredriksen and Spurkland (1993). Finally the outlooks from 1967 and 1993 are compared in section 8.4, with emphasis on the effect changes in each underlying assumption may have. The contribution rates from different projections throughout chapter 8 are presented in figure 8.1.

8.1. Actual and expected development

The contribution rate in equation 8.1 may be reported as the result of the number of pensioners (figure 6.5), the number of persons in the labour force (figure 7.3) and the ratio between average pension benefits and wages (figure 6.8). The latter implies that the Basic Pension Unit and public pension benefits are indexed with the same rate as the wage growth. With this assumption economic growth will have no influence on the contribution rate unless economic growth affects the labour force participation rates or the retirement age. See table 3.2 for a presentation of the assumptions underlying the expected development; the projection with the perspectives from 1993.

The contribution rate increased from 7.3 per cent in 1967 to 15.6 per cent in 1993. A considerable increase in the number of pensioners and their benefits was partly counteracted by an increasing labour force. The number of pensioners increased due to lower retirement age, larger propensities to enter disability pension and lower mortality throughout the 20th century. The increasing benefits were an expected result of the introduction of the National Insurance Scheme. Underlying the increasing labour force in this period was a very strong increase in female labour force participation rates.

Towards year 2010 the contribution rate will increase less, especially if the entry of new disability pensioners remains at the low level in 1993. The number of pensioners is almost unchanged because new pensioners are recruited from the small birth cohorts between the two World Wars. Both the average pension benefit and number of persons in the labour force continue to increase, but since the growth is roughly equal and they occur in respectively the nominator and denominator of the contribution rate, the effect on the contribution rate is small.

The contribution rate increases substantially from year 2010 until year 2040, and then stabilizes at a level just below 25 per cent. This period is characterised by a strong increase in the number of pensioners as the large birth cohorts from the post-war period approach retirement age and are expected to attain a longer life expectancy than the preceding smaller birth cohorts. The average public pension benefit continues to increase until year 2035, when the National Insurance Scheme is “matured” also for the oldest pensioners and the increased female labour force participation has reached full effect on the pension benefits. The growth in the labour force diminishes after year 2020 as the size of the population also stabilizes.

Structural explanation

With focus on the transition probabilities, the contribution rate is a result of the population growth given by the number of persons in each birth cohort, and certain structural aspects of the life courses. These structural aspects comprise the average number of man-years through the working career, the average number of years as pensioner and the level of pension benefits measured relative to wages per man-year. The number of persons in each birth cohort is mainly influenced by fertility and migration, while the average number of years as pensioners is mainly affected by retirement age, the propensity to enter disability pension and life expectancy. The latter effect is demanding to identify precisely because reductions in mortality in the past affects the percentage of each *current* birth cohort that will survive until retirement age[17]. However, some rough estimates can be made by comparing the projections with the perspectives from 1967 (where fertility is set to reproduction level) and the perspective from 1993, see section 8.4.

The growth in the contribution rate from 1967 to year 2050 is thus mainly a result of increasing number of years as pensioner and increasing pension benefits measured relative to wages, and these two components explain very roughly half the growth each. The number of years as pensioners increases both due to lower mortality, lower retirement age and larger propensities to enter disability pension, see later table 8.3. The reductions in mortality prior to 1967, especially in first half of the 20th century, is not identified by this table. Changes in population growth make minor contributions, especially if the effects from the small birth cohorts between the two World Wars are disregarded[18]. The average number of man-years over the life course increases, and this reduces the growth in the contribution rate.

After year 2050 the population stabilizes both in number and age composition in the projections with the perspectives from 1993. With a stable population size, the contribution rate is the result of the above mentioned structural aspects of the life courses. Population growth or decrease will of course influence the contribution rate, but an increase in the population growth above zero that reduces the contribution rate significantly, cannot persist in the long run due to problems with overpopulation (see section 4.5). When the population size is stabilized and life expectancy for obvious reasons is not a *means* in public policy, any further permanent reductions in the contribution rate (from public policy) must arise from increased labour force participation rates, higher retirement age, less propensities to enter disability pension or reductions in the relative wealth of pensioners.

8.2. Simulation of historical data

The simulation of historical data systematically underpredicts the contribution rate, but without any trend towards a larger (relative) error, see figure 8.1. Most of the short term variations in the contribution rate are captured by the simulation of historical data, but most of these variations have simple explanations related to exogenously given assumptions in the projections. Examples are the lower retirement age in 1973, adjustments of

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[17] Formally this would require the age and gender specific mortality rates of each birth cohort in the period 1870 to year 2050, approximately 36 000 input parameters.

[18] Fertility has decreased substantially since 1967, but the effect on the number of person per birth cohort is however small due to increased immigration and the fact that the high fertility in the first decades after the Second World War was carried by the small birth cohorts from the period between the two World Wars. See figure 4.4 and add approximately 8 000 immigrants to each of the birth cohorts after 1950.

Table 8.1. **Decomposition of model error, 1967 and 1993. Per cent**

	Contribution rate		Relative error in each component	
	1967	1993	1967	1993
Actual level	7.3	15.6		
Simulation of historical data	6.6	14.4		
Discrepancy	0.7	1.2		
Caused by errors in:				
Labour force	0.1	0.3	2	2
Pension expenditure:	0.6	0.8	-8	-6
Old age pension	0.6	0.7	-12	-8
Number of pensioners	0.3	0.3	-7	-3
Benefits	0.3	0.4	-6	-4
Disability pension	0.1	0.1	-8	-3
Number of pensioners	0.0	0.2	-2	-4
Benefits	0.1	-0.0	-6	0
Survivors' pension	-0.1	0.0	22	-8
Number of pensioners	-0.1	0.2	19	-33
Benefits	-0.0	-0.1	2	37

the Basic Pension Unit and business cycles in the Norwegian labour market in the eighties.

The difference between the simulation of historical data and the actual level is 1.4 percentage points in 1993, or 9 per cent measured relatively. This *model error* is a compound result of several smaller errors in the projections, and table 8.1 decomposes the model error in 1967 and 1993. None of the main components deviates more than 5 per cent from the actual level in 1993, but when nominator and denominator are biased in different directions, the total error will be larger. The discrepancies in the number of pensioners and the labour force can be explained by changes in the age distribution of net immigrants, changes these projections do not capture. The discrepancy in average pension benefits can be explained by weaknesses in the distribution of labour market earnings between birth cohorts and over the life course. See also sections 4.5, 5.4 and 7.1.

The simulation of historical data and the base line alternative of the MOSART model converge towards each other, as shown in figure 8.1. This is reassuring since both projections use the same underlying assumptions; the perspectives from 1993.

8.3. Sensitivity analyses

In this section each underlying assumption or component is varied within a range that we consider to be reasonable related to the changes throughout the last decades, and the effects on the contribution rate are reported. Two extreme cases are considered where all components change in the same direction with respect to changing the contribution rate, a “growth” alternative and an “ageing” alternative. The analyses are an updated version of Fredriksen and Spurkland (1993), where table 8.2 use the same presentation form as Bowitz and Cappelen (1994).

Table 8.2. **Contribution rates with different underlying assumptions, 1993-2060**

	Year				
	1993	2000	2010	2030	2060
	Per cent				
A. Perspectives from 1993[1]	15.6	16.4	17.9	23.4	24.3
Comparative statics	Difference, percentage points				
B. Total fertility rate of 2.08	..	0.0	0.0	-0.8	-2.0
C. Total fertility rate of 1.70	..	0.0	-0.1	0.6	2.5
D. Life expectancy increases 1-2 years	..	-0.2	-0.6	-1.7	-2.5
E. Life expectancy increases 7-8 years	..	0.0	0.2	1.1	2.7
F. Mortality depends only on gender and age	..	0.7	0.9	0.8	0.8
G. No migration	..	0.2	0.6	1.7	2.1
H. Net immigration of 12 000 persons per year	..	-0.1	-0.4	-0.9	-0.8
I. Risk of disability as in 1989	..	1.5	1.9	1.8	1.7
J. Risk of disability as in 1993	..	-0.9	-1.6	-1.6	-1.4
K. Labour force participation rates as in 1987	..	-0.9	-1.1	-1.0	-0.9
L. Women's labour force participation equals men's	..	-2.5	-2.4	-2.0	-1.6
M. "Ageing alternative" (C+E+G+I)	..	1.8	2.9	6.1	12.0
N. "Growth alternative" (B+D+H+J+K+L)	..	-4.6	-5.8	-7.5	-8.7
O. "Sustainable growth alternative" (B+D+J+K+L)	..	-4.6	-5.5	-6.1	-7.0
P. One per cent less annual growth in the Basic Pension Unit	..	-0.9	-2.2	-5.4	-7.8
Q. Perspectives from 1967[2]	-1.8	-3.2	-5.9	-10.7	-11.7
R. Perspectives from 1967, total fertility rate of 2.08[2]	-1.3	-2.0	-3.9	-5.9	-7.1

[1] Perspectives from 1993: Total fertility rate of 1.89, life expectancy at birth increases 4-5 years, net immigration of 8 000 persons per year, risk of disability as the average of 1989-1993. See also table 3.2.

[2] Compared with simulation of historical data.

Demographic assumptions

The total fertility rate varies between the lowest level observed in Norway and the reproduction level, contrary to a total fertility rate of 1.89 with the perspectives from 1993. The lowest level, the total fertility rate of 1.70 observed in 1981, is substantially higher than the current level in several European countries. The effect on the contribution rate from changes in fertility starts to have an effect several decades later, because of the simple fact that current new-borns will enter the labour market after 25 to 30 years. The variations in total fertility rate changes the contribution rate with plus/minus 2 to 3 percentage points.

Net immigration varies between zero and 12 000 persons, contrary to 8 000 persons with the perspectives from 1993. New immigrants are mainly in their twenties or thirties, and are assigned "unknown" as educational attainment. In the period used for estimation of the transition probabilities the population with unknown educational attainment comprised immigrants after 1980 (60 per cent), earlier immigrants (20 per cent) and Norwegians who never have accomplished primary school (20 per cent). Unknown education thus implies higher risk of entering disability pension and lower labour force participation rates than the average Norwegian population. We disregard that a period of assimilation may decrease these differences. Still, the projection show that increased immigration of 4 000 persons per year decreases the contribution rate with 0.9 percentage points by year 2030. After year 2030 the extra immigrants at beginning of the projection period start to retire. The effect on the contribution rate will then stabilize at a level just below 1 percentage point for several decades as long as the population continues to grow due too the extra immigrants. In the very long run, in this case several centuries, the effect will go towards zero as the population size again stabilizes at a new and higher level (see section 4.5). No immigration and constant fertility below reproduction level will have a permanent effect on the contribution rate since the population will continue to decrease towards zero with a permanent rate.

The increase in life expectancy is varied between respectively 1-2 years and 7-8 years towards year 2050, contrary to the increase of 3-4 years with the perspectives from 1993. The alternative with low mortality implies a growth in life expectancy of 0.13 years per year, approximately the same as the average growth in life expectancy during the period 1968 to 1993. The current mortality below an age of 60 years is low, and increased life expectancy will thus have little effects on the labour force and strong effects on the number of pensioners (presumed that life expectancy and retirement age are independent). Three extra years as pensioner will increase the contribution rate with approximately 2.5 percentage points. The included differences in mortality by sosioeconomic characteristics reduce the contribution rate by almost 1 percentage point. Higher mortality among disability pensioners reduces the contribution rate, while lower mortality among persons with higher income increases the contribution rate.

Education

Changes in the number of pupils and students have different effects on the contribution rate depending on the time horizon, see for example table 7.1. In the short run more students will increase the contribution rate since the labour force decreases simultaneously. In the long run more students will lead to a higher educational level in the population. A cross-section of the population shows that those with a higher educational level have higher labour force participation rates and lower risk of entering disability pension. Higher educational level also affects the projected labour force and projected number of disability pensioners from the MOSART model. In the long run the educational “explosion” during the last decades will decrease the contribution rate by 2 percentage points as shown in table 8.3 below.

Labour market conditions, disability pension and retirement

The retirement age influences the contribution rate strongly, since changes here both affect the number of pensioners and the labour force. Especially “early retirement” through disability pension will have minor effects on average pension benefits since disability pensioners are compensated through special pension entitlements. Furthermore, the National Insurance Scheme is “progressive” in the sense that one per cent higher labour market earnings on average gives less than one per cent higher pension benefits. The effects of variations in labour force participation and entry of new disability pensioners throughout the last decade is reported in table 8.2. The difference between the best and worst labour market conditions changes the contribution rate by 4 percentage points.

An important aspect is the indexing of the Basic Pension Unit and the pension benefits, and both are presumed to be indexed with the same rate as the growth in wages per standard full-time man-year. Changes in *normal* full-time working hours will thus have no effect on the contribution rate unless the change in normal working hours influences the number of persons in the labour force or the percentage with part-time working hours.

A full "emancipation" where female labour force participation increases to the current level of men is hardly realistic since women do most of the housework. Projection L in table 8.2 can though present the potential of continued increasing female labour force participation, and measured in average working hours the gender differences are still large. Increased female labour force participation and labour market earnings have less effect on female pension benefits due to the "progressive" elements of the National Insurance Scheme. In alternative L women will also loose the advantages of the Family-Care Pension Point, inherited entitlements, the calculation of the Final Pension Point from the 20 best income years and the income ceilingfor calculating Pension Points. The effect from increased female labour force participation is therefore strong even in the long run, and may reduce the contribution rate by almost 2 percentage points.

Composite effects

Variations in *each single* component with respect to demography, number of pensioners or labour force participation as in table 8.2 do not change the conclusion of a strongly growing contribution rate after year 2010. However, systematic but reasonable variations in all of these components can change this conclusion or make it dramatically worse. Both figure 8.1 and table 8.2 shows the effect of the two composite alternatives, an "ageing" alternative and a "growth" alternative. In the "ageing" alternative the total fertility rate drops to 1.70, there is no migration, life expectancy increases with 7-8 years and the entry of new disability pensioners remains at the high level in 1989. Under these circumstances the contribution rate will pass 35 per cent by the middle of the 21st century, and expenditure on public pension benefits will amount to 45 per cent of total labour market earnings.

The "growth" alternative is based on a total fertility rate at reproduction level, immigration increases to 12 000 persons per year and life expectancy increases with only 1-2 years towards year 2050. This latter assumption may be replaced by a stronger increase in life expectancy and a similar increase in the retirement age. The entry of new disability pensioners is assumed to remain at the low level in 1993 and male labour force participation increases to the high level in 1987 and gender differences in labour force participation vanishes. In this case the contribution will remain at the level in 1993 around 16 per cent. The expected strong growth in the number of old persons after year 2010 will then constitute no problems for public finances with respect to public pension benefits. A similar "sustainable growth" alternative with either lower fertility or less immigration, such that the population stabilizes at just above 5 million persons, will lead to a contribution rate of 17 to 18 per cent.

Rules for calculating Supplementary Pensions

The Supplementary Pensions from the National Insurance Scheme can slowly be adjusted by the indexing of the Basic Pension Unit or by changing the rules for the future earnings of pension entitlements. Both means have been used through the history of the National Insurance Scheme, and the Basic Pension Unit has grown 20 per cent less than wages per normal man-year in the period 1967 to 1993. Changing the rules for earning pension entitlements in the future normally require several decades to have any effect, and the full effect will often change the average pensions by only a few per cent. See Fredriksen and Spurkland (1993) for a discussion of the pension reform in 1992 and Fredriksen and Koren (1993) for a discussion of the Family-Care Pension Point and the number of Pension Points used to calculate the Final Pension Point. These analyses disregard that smaller pension benefits may delay retirement, and this can change the conclusion with respect to a weak effect on the contribution rate from pension benefit reforms.

Adjustments of the Basic Pension Unit however has an immediate effect on pension benefits and a long term effect by lowering the income ceiling of earning Pension Points, see equation 6.3 and section 6.4. Through the last 20 years the Basic Pension Unit has roughly been adjusted with the inflation rate and the Special Supplement has been adjusted to assure an equal increase in the Minimum Pension Benefit and wages. Figure 8.1 and table 8.2 show the future effect of 1 per cent less annual growth in the Basic Pension Unit than wages, while the Special Supplement is increased to compensate those with Minimum Pension Benefits. This leads to smaller pension benefits than the projection with the perspectives from 1993, and by year 2035 the average old age pension benefit will be the same as in 1993. The average pension benefit will continue to decrease towards year 2090, when the maximum Supplementary Pension will be less worth than the Special Supplement. By this time the system of Supplementary Pensions is de facto replaced with an equal pension for every pensioner with the same value as the current Minimum Pension Benefit measured relative to wages. The effect on the contribution rate is strong, see figure 8.1 and table 8.2. The contribution can remain at the current level if the Supplementary Pensions are removed through the indexing of the Basic Pension Unit, and furthermore, the population size stabilizes and retirement age and life expectancy have a similar development.

8.4. Projections with perspectives from 1967

The arrangement for simulation of historical data gives the opportunity of making a projection with the perspectives from 1967, the year the National Insurance Scheme was established. This projection can be compared with actual development and the current projections, but we are reluctant to interpret this as a test of the prediction value of the MOSART model. The comparison can however be used to analyse the changes in the underlying assumptions since 1967, and may be used in a discussion of how well the National Insurance Scheme was funded at its beginning in 1967. A compact presentation of the assumption underlying the projection with the perspectives from respectively 1967 and 1993 is found in table 3.2.

Table 8.3. **Decomposition of projection errors and differences, 1993 and long run[1]. Per cent**

	1993	Long run[2]
Actual level	15.6	
Projected with perspectives from 1993		24.4
Projected with perspectives from 1967	12.6	12.4
Discrepancy	3.1	12.0
Caused by:		
Fertility	0.0	6.0
Life expectancy	1.0	7.2
Disability pension	4.0	5.7
Retirement age	0.6	1.6
Education	0.4	-2.1
Labour force participation	-2.7	-1.5
Pension benefits	-0.9	-3.0
Immigration	-0.5	-1.9
Model error	1.2	-

[1] The underlying assumptions in the projection with the perspectives from 1967 are one by one replaced with the correct development until 1993 and the perspectives from 1993 in the years after, in the order listed above.

[2] The same as year 2200.

The discrepancy between the projection with the perspectives from 1967 and the actual level is remarkably small until 1993, while the difference between the projections with the perspectives from 1967 and 1993 is large in the long run. These projection errors and differences are decomposed by each of the underlying assumptions in table 8.3. One assumption at a time is shifted from the perspectives from 1967 to the actual level until 1993, and with the perspectives from 1993 beyond 1993. The order of changing the assumptions is the same as in table 8.3, and this may affect the estimated contribution from each assumption[19].

The projection with the perspectives from 1967 give a surprisingly good prediction on the actual contribution rate in 1993, but underlying this result are large changes in different underlying assumptions working in opposite directions. The actual retirement age given by disability pension and old age pension has decreased with approximately 4.3 years, and increased the contribution rate by 4.6 percentage points. This is counteracted by increased immigration, increased female labour force participation and less generous rules for calculating pension benefits, adding up to 4.1 percentage points.

The projections with the perspectives from both 1967 and 1993 have reached a stable level for the contribution rate by year 2040, and the difference between the two projections is large by this time. The expected contribution rate with our interpretation of the perspectives from 1967 is 12.4 per cent, approximately half the level expected with the current perspectives. The large changes are related to a dramatic reduction in fertility, the expected increase in life expectancy and reductions in the actual retirement age given by disability pension and old age pension. Changes in these components alone would have lead to a contribution rate above 30 per cent. The strong growth in pension

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[19] The cross-effect on the contribution rate from each component may be large. An extreme example is a projection where fertility, life expectancy and retirement age remain at the level in 1967 while immigration, educational activities, pension benefits and labour force participation shift to the level in 1993. By table 8.3 this would lead to a contribution rate of 3.9 per cent calculated by the level in the projections with the perspectives from 1967, 12.4 per cent, minus the effects of immigration, educational activities and labour force participation, a total of 8.5 per cent. A projection with these underlying assumptions gives however a contribution rate of 9.4 per cent. Table 8.2 is exposed to some of the same cross-effects, but somewhat weaker. The “ageing” alternative has an effect of 12.0 per cent, while the sum of each component is only 9.0 per cent. The “growth” alternative has an effect of -8.7 per cent, while the sum of each component is -9.2 per cent.

benefits should however have been anticipated with the perspectives from 1967, and the projection with the perspectives from 1993 actually gives lower benefits measured relative to wages. Increased immigration and female labour force participation have partly counteracted the growth in the contribution rate.

The total fertility rate of 2.8 in the perspectives from 1967 may seem unreasonable with the current focus on overpopulation and environmental problems. The projection with the perspectives from 1967 are therefore repeated, except that the total fertility rate is reduced to reproduction level, and this leads to a contribution rate of 16.6 per cent in the long run (see table 8.2). Compared with the current contribution rate we conclude that the National Insurance Scheme was well funded in 1967, and that changes since 1967 are the reason for the current worries about future public pension benefit expenditure. The combination of an unexpected increase in life expectancy, lower retirement age and larger propensities to enter disability pension is the main reason for the changing expectations. The expected number of years as pensioner at birth has increased from 11.3 years in 1967 to 20.6 years in 1993.

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Appendix A

Initial population in the simulation of historical data

We have generated a synthetic initial population from 1960 based on aggregate statistics from the same year and transition probabilities from the MOSART model. The cross distribution of the characteristics are calculated after making some simple, but probably realistic simplifications, and the different sources and methods are described below. This synthetic initial population was generated at a low cost, and the methods may be improved in coming versions.

Base year

The simulation of historical data has 1960 as a base year for several reasons. Those variables which are roughly initialized in 1960 have probably been reset by the simulation when the model reach 1967, the year the National Insurance Scheme was established. Furthermore, a population census from 1960 is available to support the creation of an initial population. The time series needed to adjust the transition probabilities to their actual level are fairly consistent back to 1960, but not before 1960.

Population by age and gender

The first step in simulating the initial population is to generate the numbers of persons by gender and age, and the numbers are calculated as 1 per cent of the Norwegian population at the end of 1960. The population figures are gathered from the population statistics presented in Statistics Norway (1976).

Birth histories

The model simulates birth histories until 1960 for women based on birth probabilities from 1989. The birth probabilities depend on age, number of children and age of the youngest child, and are adjusted proportionally each year to make them consistent with periodic total fertility rate each year.

Marital status and spouses

Marital status in 1960 is simulated based on information from the population census in 1960, see Statistics Norway (1963). We have used the distribution across marital status by gender and age at 1st of November 1960, and included the separated among the married. First the model draws which childless women who are unmarried (never married). If the number of unmarried women exceeds the number of childless women, the model draws some women with children to be unmarried. Women not assigned the status of unmarried draw whether they are married, divorced or widows, independent of the number of children. Married women draw the age of their husband depending on her own age, and a not married man with this age is chosen at random. The age distribution between existing couples in 1960 is for simplicity presumed to be the same as the age distribution between *new* spouses in 1984 as estimated in Kravdal (1986). Men who are not chosen as a spouse draw whether they are unmarried, divorced or widowers.

Educational attainment and activities

Educational characteristics in 1960 are simulated indirectly. First, the educational attainment in 1985 is simulated for the population in 1960, using the distribution across educational attainment in the initial population the current version of the MOSART model. The year each person accomplished the educational attainment in 1985 is then calculated as if he or she followed a stipulated progression through the educational system and did this successively after primary school. If the graduate year is before 1960, this person is assigned the same educational attainment in 1960 as in 1985. The person is also set to be a non-student. If the graduate year is after 1960, the person is assigned an educational activity in 1960 corresponding to the educational attainment in 1985 and his or her birth year. Pupils in secondary schools are assigned primary school as educational attainment, while students in higher education are assigned grammar school as educational attainment.

Pension status and pension entitlements

All persons 70 years and older are old age pensioners. Persons with ages from 16 to 69 draw if they are disability pensioners depending on gender, age and education. The age and gender covariates are based on the percentages of the population by gender and age that are disability pensioners in 1993. The educational covariates are based on the estimated risk of entering disability pension in the MOSART model, see equation 6.1. The risk level is adjusted so that the number of disability pensioners corresponds to the level in 1960.

The National Insurance Scheme was established in 1967, and consequently no pension entitlements existed in the National Insurance Scheme in 1960.

Labour market participation

Persons under education, retired or disabled do not receive labour market earnings, while other persons draw if they receive labour market earnings. Men's probability of receiving labour market earnings is 1.0, while women have a probability of just 0.4. The values 1.0 and 0.4 give a labour force by gender corresponding to the level in 1960. All persons in 1960 are assumed to have been either stable inside or stable outside the labour force in 1960.

Appendix B

Mean-constrained drawing method[20]

The main drawing method in the MOSART model is described in this appendix, while Fredriksen (1996B) gives a more detailed discussion of the method and its effects, and suggests how to solve the practical problems. The drawing method in the MOSART model does not complicate computer programmes and computer runtimes increase with less than one per cent in this version of the MOSART model. Standard deviations on projection results induced by stochastic drawing are 70 to 90 per cent lower compared with the random number method (see below).

The problems of stochastic drawing

Dynamic microsimulation models are generally solved by stochastic drawing, often called Monte Carlo technique. In the *random number* method the computer generates so-called random numbers with uniform distribution (0,1), and if the random number is smaller than the probability, the event occurs. This stochastic drawing generates an extra uncertainty in the projections often called Monte Carlo Variability (MCV). The MCV is measured in this study by relative empirical standard deviation across replicated simulations where the only difference is the sequence of random numbers.

The MCV is unwanted for most purposes in social planning, and often mentioned as an important critique against microsimulation. In the MOSART model we have found the MCV with the random number method troublesome when comparing two projections with different underlying assumptions. The MCV with the random number method is also troublesome when estimating year-to-year growth rates and for long run projections.

The idea of the drawing method in the MOSART model is to constrain the simulated number of events to its expectation value without changing each person's probability of experiencing the event. With exogenous probabilities the expectation value of the model is the sum of the transition probabilities. This constraining is easy when probabilities are (group-wise) homogeneous, and for example used when a stratified sample comprises a given number of persons from each stratum. The drawing method in the MOSART model handles heterogeneous binomial probabilities.

Some related methods

With a limited knowledge of statistical literature, we cannot exclude the existence of any similar and better method of reducing MCV than offered in equation B.8. However, the

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[20] The drawing method in the MOSART model has been slightly improved since the work on this study ended. We now add up the probabilities, and an event is assigned to every person that makes the sum of the probabilities enter a new number. This improved method is actually quite similar to the method in equation B.8, with a slight reduction in the variance and a more substantial reduction in complexity. This improved method is known in for example certain (newer) Canadian microsimulation models.

related methods discussed below do not offer any method that are able to both handle *heterogeneous* binomial probabilities, be unbiased, be easy to implement in a computer programme and require hardly any computer resources. Orcutt et al. (1986) is a major contribution on microsimulation in social planning, and the introduction by Orcutt (1986) includes a discussion of variance reduction methods. We have also found useful discussion in a preliminary version of Beckering (1995), a dissertation including one paper on variance reduction methods in microsimulation models. The discussion in Orcutt (1986) and Beckering (1995) is below related to the drawing method in the MOSART model.

The brute method is to increase the sample or use the average across replicated simulations. This implies that any interesting methods of reducing MCV must be efficient in the sense that they do not complicate the computer programmes too much or increase the computer runtimes too much. However, a four times larger sample normally reduces the standard deviation to only one-half.

The next idea is to present the sum of the transition probabilities as the simulation result, and this gives the expectation value of the simulation. This method is used in the MOSART model in the projection of labour force and man-hours. However, the method is insufficient in a model with multiple periods because the number of possible paths for each person will be too large. The process of ageing the population must draw what happens into the next period. If the MCV related to the number of births and persons who survive is large, this will also affect the sum of the probabilities in subsequent periods.

Two simulation can be executed with the same sequence of random numbers, mentioned as the method of *common random numbers*. This reduces the MCV on the difference between the two simulation, since only persons with changed probabilities can experience different events in the two simulations. Unfortunately, the method requires that the sequence of possible events is the same, and this is in general not true in dynamic microsimulation. Which events that may occur in one period depend heavily on other events in earlier periods, especially births and deaths. In this version of the MOSART model pension benefits do not affect any *transition* probabilities. An analysis where only rules for calculating pension benefits are changed can be carried out with the method of common random numbers in the MOSART model.

Orcutt (1986) focuses on a method called *alignment*, and implementations of the method can be found in Caldwell (1996) and Galen (1995). First all persons are assigned a personal random number, and the unadjusted number of events are counted as the number of persons with probabilities larger than the assigned random number. The discrepancy between the unadjusted number of events and the expected number is used to calculate an alignment factor, and this may require sorting or an iteration process. In the second round all probabilities are multiplied with the alignment factor, and if the adjusted probability is larger than the assigned random number an event occurs.:

(B1) An event occurs if: $p_i \cdot f > r_i$

Where:

- p_i the probability that person 'i' will experience the event
- r_i the random number with uniform distribution (0,1) assigned to person 'i'
- f the alignment factor

The task between the first and second round is to find the proper value of the alignment factor 'f' that will result in the expected number of events. The alignment method is similar to the drawing method in the MOSART model in the sense that alignment can be used to constrain the number of events to the expectation value of the simulation when probabilities are binomial and heterogeneous. However, the alignment method requires that the simulation model runs through the population at least twice, and perhaps more if the alignment factor 'f' is difficult to calculate. This implies that the alignment method complicates the computer programmes and increases computer runtimes more than the drawing method in the MOSART model.

More serious, the alignment technique introduces a (small) bias that may be visualized by a simple example. A person with a probability p_i should on average experience p_i events per drawing. If p_i is 1, the average number of events should also be 1. However, on some occasions will a small alignment factor 'f' and a large assigned random number r_i prevent persons with p_i equal to 1 from experiencing the event. Furthermore, the number of events per drawing will never exceed 1, and therefore will the average number of events with the alignment method be less than 1 for persons with p_i equal to 1.

The alignment method can also be used to *adjust* the probabilities so that the simulation is consistent with any reasonable exogenous constraint on the number of events. See Beckering (1995) and Galen (1995) for a discussion. In the MOSART model an adjustment against an exogenous number different from the sum of the (unadjusted) probabilities requires that the probabilities are adjusted before the simulation, and this may reduce the advantages of the drawing method in the MOSART model.

Beckering (1995) claims that most of the remaining methods requires that the transition probabilities are identical within a limited number of sub-populations, which we call group-wise homogeneous probabilities. Beckering (1995) also suggest and tests a new drawing method based on sorting that handles multinomial probabilities, but the method is according to the author "not easy to implement in a computer program".

The idea of the drawing method in the MOSART model

Each person 'i' is assumed to have a probability p_i of experiencing the event. The population is then divided into drawing-groups where the sum of the transition probabilities within each drawing-group adds up to exactly one. The practical problems of making drawing groups and of rounding is addressed later. The model then draws for each conditionally on the events of the *previous* persons in the drawing group:

$$\text{(B2) } p_i^{\text{adjusted}} = \begin{cases} p_i \,/\, /1 - \sum_{j<i} p_j) & \text{if no event has occured in this drawing group yet} \\ 0 & \text{if an event has occured in this drawing group} \end{cases}$$

Where:

- 'i' is the person subject to drawing
- 'j < i' are all persons in the drawing-group that already have been exposed to B.2

The drawing is executed by comparing the adjusted probability with a random number with uniform distribution (0,1) generated by the computer. If the random number is smaller than the adjusted probability, the event occurs for person 'i'. The probability r_i of person 'i' experiencing an event by B.2 can be written as:

$$\text{(B3)}\quad r_i = E(p_i^{\text{adjusted}}) = (p_i / (1 - \Sigma_{j<i} p_j)) \cdot q_i + 0 \cdot (1 - q_i)$$

Where:

- q_i is the probability of no event before person 'i'
- $(1 - q_i)$ is the probability of an event before person 'i'

Two persons within the same drawing-group cannot experience an event by B.2, and the probability $(1 - q_i)$ of any of them experiencing an event is the sum of each' probability of experiencing the event. Using B.3 this gives:

$$\text{(B4)}\quad 1 - q_i = \Sigma_{j<i} r_j$$

Equation B.3 can now be simplified by substituting B.4 into B.3:

$$\text{(B5)}\quad r_i = (p_i / (1 - \Sigma_{j<i} p_j)) \cdot (1 - \Sigma_{j<i} r_j))$$

It is now easy to show by iteration that the probability r_i in equation B.5 is equal to the probability p_i:

(B6)

$$r_1 = (p_1 / (1 - 0)) \cdot (1 - 0) = p_1$$

$$r_2 = (p_2 / (1 - p_1)) \cdot (1 - r_1) = (p_2 / (1 - p_1)) \cdot (1 - p_1) = p_2$$

$$r_3 = (p_3 / (1 - p_1 - p_2)) \cdot (1 - r_1 - r_2) = (p_2 / (1 - p_1 - p_2)) \cdot (1 - p_1 - p_2) = p_3$$

....

$$r_i = (p_i / (1 - \Sigma_{j<i} p_j)) \cdot (1 - \Sigma_{j<i} r_j) = (p_i / (1 - \Sigma_{j<i} p_j)) \cdot (1 - \Sigma_{j<i} p_j) = p_i$$

The method does therefore not alter each person's probability of experiencing an event in this simple example. The last person within a drawing-group must experience the event if no one has previously in this group, since the probabilities add up to one:

$$\text{(B7)}\quad p_{\text{last}}^{\text{adjusted}} = p_{\text{last}} / (1 - \Sigma_{j<\text{last}} p_{\text{last}}) = p_{\text{last}} / (1 - (1 - p_{\text{last}})) = p_{\text{last}} / p_{\text{last}} = 1$$

Each drawing-group will therefore end up with one and only one event, which also is the expected number of events within each drawing-group. With the correct number within each drawing-group, the number of events for the total population is also correct.

Practical solutions of making drawing groups and rounding

The drawing-groups are formed as one person is simulated at the time by adding each new person to the present drawing-group. At the end of a drawing-group a rounding problem occurs, or more precisely the probabilities do generally not add up to exactly one. The transition probability of this last person is "divided" between the drawing-group that is to be closed and the new drawing-group by adjusting B.2:

$$\text{(B8) } p_i^{\text{adjusted}} = \begin{cases} \text{Max}(1, p_i \,/\, (1 - \sum_{j<i} p_j)) & \text{If event had occured in the present drawing} \\ & \text{group before person ' i'} \\ p_i^{\text{new}} \,/\, (1 - p_i^{\text{old}}) & \text{If an event had happened in the present drawing} \\ & \text{group before persons ' i' and } \sum_j p_j \geq 1 \\ 0 & \text{If an event had happened in the present drawing} \\ & \text{group before persons ' i' and } \sum_j p_j < 1 \end{cases}$$

Where:

- 'j' are all members in the present drawing-group
- $p_i^{old} = 1 - \Sigma_{j<i} p_j$
- $p_i^{new} = p_i - p_i^{old}$
- If $(\Sigma_j p_j \geq 1)$ open a new drawing-group and include in the new drawing-group the probability p_i^{new} as the first probability and also any event by $p_i^{new}/(1-p_i^{old})$

If the sum of the probabilities are less than one, the expected number of events by is the same as in equation B.3, else can the expected number of events by equation B.8 can be written as:

$$\begin{aligned} \text{(B9) } E(p_i^{adjusted} \mid \Sigma_j p_j \geq 1) &= s \cdot E(\text{Max}(1, p_i/(1-\Sigma_{j<i} p_j))) + (1-s) \cdot E(p_i^{new}/(1-p_i^{old})) \\ &= (1-\Sigma_{j<i} p_j) \cdot 1 + (\Sigma_{j<i} p_j) \cdot p_i^{new}/(1-p_i^{old}) = p_i^{old} + p_i^{new} = p_i \end{aligned}$$

Where:

- 's' is the probability of no events before person 'i' in the present drawing group

The expected number of events in the new drawing group for the person crossing two drawing groups are p_i^{new}, and this probability is also included in the new drawing group. Equation B.6 thus hold also in this case for the first member of the drawing group.

One sequence of drawing-groups is used for different groups of the population for each event to make each drawing group as homogeneous as possible. This eliminates not only the uncertainty on the total number of simulated events, but also the uncertainty on the number of events within each group of the population. This also contributes to reduce the uncertainty induced by which probabilities that "survive" to the next period.

Testing

Some brief tests of the suggested drawing method in equation B.8 will later be pubished in an extended version of Fredriksen (1996B). One test is performed by comparing the percentage entering disability pension sorted by the probability of experiencing this events. We find that the observed percentages fits the underlying probabilities, and the method is not biased towards small or large probabilities. Moreover, a simple model is presented with one event and 100 persons with transition probabilities randomly and uniformly distributed (0,1). Replicated simulations across this simple model indicates that the probability of experiencing the event is the same as the used probability independent of the size of the probability or the ordering of persons.

The drawing method in equation B.8 introduces a strong negative correlation of experiencing an event between members of the same drawing group. This requires some cautions to avoid that two persons enter the same drawing group where independence between the drawings are wanted. This may be two persons from the same household, or if a drawing group crosses two years, that this drawing do not include the same person twice.

Computer resources

The MOSART model is programmed in the object-oriented language Simula, see Kirkerud (1989) for details. The object-orientation simplifies the implementation of the drawing procedure in equation B.8. The drawing procedure is defined as a class including the necessary counting variables, and each type of event is assigned its own drawing object, and the procedure is invoked by writing:

(B.9) If name-of-drawing-object.event(probability) then ......

The counting variables are hidden within the object and properly updated. The drawing “procedure” with the random number method requires approximately the same amount of source code:

(B.10) If *probability* > Uniform(0,1,u) then ......

The effect on computer runtimes depends on the complexity of the model. The computer runtimes used for the comparison itself in respectively B.9 and B.10 is tripled, but these comparisons are seldom any significant part of the total computer runtime in a complex model. In the MOSART model the computer runtimes in most modules increase by roughly four per cent. At the other hand, the task of adjusting the number of recipients of labour market earnings to the projected labour force requires less computer runtime since the discrepancy between these two are on average less with a mean-constrained drawing method. We have compared 20 replicated simulations with the MOSART model without and with the mean-constrained drawing method, and found that total computer runtimes increases by less than one per cent.

Table B1. **Monte Carlo Variability (MCV), 1989-2200. MCV is measured by standard deviation on number of persons in the labour force as percentage of projected labour force**

Year	Projected level, thousand persons	Random number method, same 1-per cent sample	Mean-constrained drawing method		
			same 1-per cent sample	different 1-per cent samples	same 4-per cent sample
1989	2102	0.11	0.01	0.13	0.00
1993	2131	0.19	0.04	0.11	0.02
2000	2226	0.22	0.05	0.11	0.03
2010	2363	0.23	0.05	0.20	0.02
2030	2457	0.28	0.07	0.34	0.03
2060	2501	0.77	0.08	0.54	0.05
2100	2514	1.52	0.09	0.48	0.03
2200	2555	2.20	0.10	0.42	0.02
Average annual growth 1990-2010	12	28.6	5.9	8.4	2.6

The MCV with 4-per cent samples and different samples are based on 10 replicated simulations, while the two other are based on 20 replicated simulations

The effect on the Monte Carlo Variability

Some effects on the Monte Carlo Variability (MCV) after constraining all major events in the MOSART model are reported in table B1. Note that the labour force is projected as the sum of each person's calculated labour force participation rate, including the base year. The MCV related to the labour force projections is induced by variations in the number of persons and their covariates such as gender, age, pension status and educational activities and attainment. The level of the MCV and the effect of the mean-constrained drawing method depend on the number of events and the heterogeneity of the transition probabilities. With very heterogeneous transitions probabilities and some other extreme conditions, the mean-constrained drawing method may have no effect at all. However, in the MOSART model the effect on the MCV is seldom below 60 per cent.

The effect on the MCV in the base year from the mean-constrained drawing method is large because the transition probabilities are given with exception of explanatory variables not included in the initial population. The further development depends on which transition probabilities that "survive", and this is in general not constrained by the suggested drawing method in equation B8. The probabilities in any later periods are therefore stochastic, and the MCV increases throughout the projection period. However, in the example of the labour force projections it never exceeds 0.1 per cent of the projected level.

The MCV with the random number method increases very much throughout the projection period because the number of persons in each birth cohort is stochastic, and this MCV is accumulated as each birth cohort reproduces it self. The MCV on the projected labour force is reduced by at least 70 per cent with the mean-constrained drawing method compared with the random number method. The MCV can be reduced through a larger sample, but a reduction of 70 per cent would have required a sample that is at least 10 times larger, and 400 times larger with the reduction of 95 per cent in year 2200.

Uncertainty induced by the initial population is of course not removed by any drawing method, and this explains why the MCV with different samples is so large. We hope to reduce this uncertainty through better stratification of the sample, see also Fredriksen (1996A).

The labour force grows with 12 000 persons per year in the period from 1990 to year 2010. The relative MCV on the annual growth is 28 per cent with the random number method, making the estimated annual growth little informative. With the mean-constrained drawing method the relative MCV is reduced to 6 per cent with the same sample size, and this is near a satisfactory level for this type of results.

The combination of a larger sample and the mean-constrained drawing method has a positive cross-effect on the MCV as the example with a 4-per cent sample in table B.1 indicates (a four times larger sample would normally reduce the MCV to only one-half). The reason is probably that the MCV is so small that rounding errors related to drawing groups crossing two successive years is important (as each drawing group do not add up exactly to one, neither do the total expected number of events each year). With a larger sample, the relative significance of the rounding errors diminishes proportionally.

Experience has shown that an upper limit on the MCV on unconstrained events in the MOSART model can be approximated by the Poisson distribution when the projection period is less than 70 years. The Poisson distribution is generated by a large number of observations with an equal and small probability of experiencing the event, and the variance is equal to the expectation value. With a one per cent sample, the MCV or standard deviation for any projected number of persons is given as ten times the square root of the projected number. The NEDYMAS model does not include any variance reduction methods to our knowledge, and the reported MCVs in Nelissen (1993) are also of the same size as a Poisson distribution.

Appendix C

Classification of education

Below follows a description of how education is classified in the MOSART model. The level of an education is measured by the minimum number of years a full-time pupil or student must normally use to accomplish this education. The level of primary school is set to 9 years independent of the actual length. Secondary school comprises the levels 10 to 12 years, while higher education comprises the levels 13 to 19 years. Graduate level corresponds to levels of 17 to 18 years, while all post-graduate education is set to a level of 19 years. The levels each educational subject can be respectively studied or accomplished are shown in brackets. See Andreassen et al. (1993) for how the classification of education in the MOSART model corresponds with the official classification of education in Statistics Norway (1989). See Vassenden (1993) for a description of the educational registers used in the MOSART model.

Unknown education

The educational registers lack information on approximately 100 000 adults, mainly persons who did not report their own educational attainment in the population census in 1980. Roughly 60 per cent are persons who have immigrated after 1980, 20 per cent have immigrated before 1980 and 20 per cent are persons born in Norway. The latter group comprise probably the 0.5 per cent of each birth cohort that never accomplish primary school.

Primary school

All persons with only primary school are assigned an educational level of 9 years.

Secondary school

Secondary school comprises the three first years after primary school. Students in higher education are mainly recruited from grammar school, but formally most subjects give access to higher education. Approximately 10 per cent of the grammar school candidates participate in *diverse basic courses* (before they enter a higher education).

Grammar school (10-12,12), Administration and economics (10-12,10-12), Crafts (10-12,10-12), Diverse basic courses; domestic sciences, military subjects (not including compulsory military service), “folkehøyskole” (10,10), Other (10-12,10-12)

Higher education

Higher education comprises all education beyond secondary school, and requires an education from secondary school that entitles to higher education, for example grammar school.

Technology (13-19,13-17+19), Administration and economics (13-16,13-16), Nursing (13-16,15+16), Teaching (13-18,13-16+18), Preliminary tests (13,13), Humanities (13-19,13-19), Social sciences (13-19,13-19), Natural sciences (13-19,13-19), Law (15-19,15+17+19), Medicine (13-19,18+19), Dental care (13-19, 17+19), Other (13-19,13-19)

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