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Rolf Golombek and Arvid Raknerud

Exit Dynamics with Adjustment Costs

Abstract:
We use the Stock and Wise approximation of stochastic dynamic programming in order to identify the extent to which profitability can explain exit behavior. In our econometric model, heterogeneous firms engage in Bertrand (price) competition. Firms produce heterogeneous products, using labor, materials and capital as inputs. The stock of capital is changed through investments and disinvestments, where the firm incurs adjustment costs due to partial irreversibilities. The model is estimated for six manufacturing industries using Norwegian micro data for the period 1993-2002. We find that increased profitability lowers the exit probability, and this effect is statistically significant in all industries, while, ceteris paribus, high adjustment costs significantly decrease the probability of exit in five of the industries. Exiting firms are characterized by persistently, although only moderately higher, annual exit probabilities than the average firm. There is no tendency for exiting firms to have a high probability of exit just prior to exit.

Keywords: Firm exit, adjustment costs, Bertrand game, manufacturing firms, mixed logit, state space model.

JEL classification: C33, C51, C61, C72, D21

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Address: Rolf Golombek, The Ragnar Frisch Center for Economic Research, University of Oslo. E-mail: rolf.golombek@frisch.uio.no

Arvid Raknerud, Statistics Norway, Research Department. E-mail: rak@ssb.no

For printed Discussion Papers contact:

Statistics Norway
Sales- and subscription service
NO-2225 Kongsvinger

Telephone: +47 62 88 55 00
Telefax: +47 62 88 55 95
E-mail: Salg-abonnement@ssb.no

1 Introduction

An important challenge in economics is to explain what causes firms to exit. The standard answer is low profitability. This answer leaves, however, open a number of questions as profitability partly depends on factors that are difficult to measure, for example, demand conditions, efficiency and market structure. In addition, due to market imperfections and regulations, other variables may also play a role in explaining exits.

In the empirical literature, several variables have been used in order to explain exits. These include plant size, see Mata, Portugal and Guimarães (1995); capital intensity, see Boeri and Bellmann (1995); financial leverage, see Dimitras, Zanakis and Zopounidis (1996); productivity, see Olley and Pakes (1996); capital vintage, see Salvanes and Tveterås (2004), and demand shocks, see Foster, Haltiwanger and Syverson (2005). The purpose of the present paper is to identify, through a structural microeconometric model with heterogeneous firms, imperfect competition and adjustment costs, some key determinants of exit behavior.

While traditional exit rules were myopic, such as Marshall (1966), modern theories assume that the exit decision is based on both present and expected profits. In its most refined version, the exit rule is derived from stochastic dynamic programming (SDP): The firm will stay operative as long as the expected present value of continuing production exceeds the value of closing down; see e.g. Hopenhayn (1992). Although SDP provides an attractive general framework for interpreting and analyzing intertemporal decision problems, huge computational requirements put sever limits on the number of state and decision variables that can be included in the analysis. Because of the obstacles involved in applying SDP, the literature offers alternative approaches. It is an empirical question, however, whether the SDP exit rule – applied on a model of strictly simplifying assumptions – provides a better explanation of observed exits than exit rules based on alternative approaches.

For optimal stopping problems, a frequently used approach is that of Stock and Wise (1990), who analyze the retirement decisions of older employees. In our setting, the Stock and Wise rule says that a firm finds (at t) the exit time that maximizes expected profits, given the current information set of the firm. If the "optimal" time to exit is now, the firm exits. Alternatively, if the "optimal" time to exit is not now, the firm continues production and in the next period $(t+1)$ reexamines whether it should exit immediately or continue production, and so on. In the present paper we use the Stock and Wise approach because it enables us to specify a much richer model of firm behavior than if we had relied on SDP. Furthermore, it incorporates the important forward-looking aspect of the exit decision problem.

In our model, each firm produces a version of a differentiated good. The demand function depends on the prices of the firm's product and those of the competitors. Firms set prices simultaneously in a non-cooperative game (Bertrand competition). Production requires input of labor, materials and capital. The latter is a quasi-fixed factor. The production function incorporates both Hicks- and non-neutral technological progress.

We decompose unobserved differences in productivity and demand conditions into a firm-specific permanent effect ("initial condition") and cumulated innovations. The innovations are independent and capture neutral and non-neutral technological change. Our model is consistent with Gibrat's law that firms' growth rates are independent of firm size (see Sutton, 1997)[1]. Recent studies that model firm growth as a stochastic process with a high degree of persistence comprise Klepper (1996), Klette and Griliches (2000) and Klette and Kortum (2004).

While materials and labor are assumed to be fully flexible production factors, changes in the stock of capital are subject to adjustment costs. We build on the theory of partial irreversibilities, where the resale price of capital is assumed to be lower than the purchaser price of new capital; see Grossman and Laroque (1990), Caballero, Engel, and Haltiwanger (1995), Abel and Eberly (1996) and Bloom (2000). This theory allows for asymmetries between investment and disinvestment.

We use our theoretical model to derive explicit relations between explanatory variables and firm exit. Specifically, we estimate a mixed logit model where the two explanatory variables are (i) profits net of adjustment costs and (ii) the stock of capital. Neither of these variables are directly observed due to errors. Because the full system of supply and factor demand can be formulated in state-space form, the conditional expectation of the firm-specific unobserved (latent) variables, given the observed variables, can be calcu-

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[1] The empirical literature suggests that Gibrat's law is valid for large and medium sized firms. The validity of Gibrat's law for smaller firms depends on whether the analysis is limited to surviving firms. See Sutton (1997) and Caves (1998) for discussions and further references.

lated. The unknown parameters are estimated by maximum likelihood using a computer algorithm written in GAUSS.

Our model is estimated for six export oriented manufacturing industries: Wood products, Rubber and plastic products, Metal products, Machinery, Electronic equipment and Transport equipment. We use a new and detailed Norwegian micro data set for manufacturing firms for the period 1993-2002. In order to avoid problems related to multi-plant firms, we restrict the data set to single-plant firms. Moreover, firms that disappear for other reasons than "real" exit, for example due to a merger or takeover, are treated as randomly missing after the last observation year.

We find that increased profitability reduces the exit probability, and this effect is statistically significant in all industries. Also the impact of adjustment costs is significant. Our results suggest that the main characteristics of an exiting firm is not that its annual exit probability is much higher than that of a surviving firm, but rather that the difference in annual exit probabilities is highly persistent. It is the cumulated effect of moderately higher risk of exit over several years – compared with the average firm – which causes exits.

The rest of the paper is organized as follows. In Section 2 the theoretical model of the firms' operating decisions is presented when capital is quasi-fixed. Section 3 describes the structure of capital formation, focusing on implications of adjustment costs. In Section 4 we explain the exit decision rule, which is based on Stock and Wise, and specify a mixed logit model for exit. The model contains explanatory variables derived from the structural model in Sections 2 and 3. Section 5 specifies the econometric model of supply and factor demand and discusses identifying restrictions. The applied data set is presented in Section 6, whereas results are discussed in Section 7. Finally, Section 8 concludes.

2 Price competition and production

We consider a Bertrand game between n producers of a differentiated good. Each producer is faced with a demand function of the following form:

$$Q_{it}^{D} = \Phi_{it} P_{it}^{-e} \left(\prod_{k \neq i} P_{kt}^{\frac{1}{n-1}} \right)^{\sigma}, \tag{1}$$

where Q_{it}^D is the demand of output from firm i at time t, P_{it} is the output price and Φ_{it} is an exogenous demand-shift parameter. Furthermore, $e > 1$ is the absolute value of the direct price elasticity and σ is the cross-price elasticity of Q_{it}^D with respect to the geometric average of the prices of the $n-1$ other producers. As indicated by the notation, the demand shift parameters are firm-year specific, while the price elasticities are common to all firms.

The production function of producer i is assumed to be:

$$Q_{it} = A_{it} K_{i,t-1}^{\gamma} \left[(w_{it} L_{it})^{\rho} + M_{it}^{\rho} \right]^{\frac{\varepsilon}{\rho}}, \tag{2}$$

with elasticity of scale equal to $\varepsilon + \gamma$ and substitution parameter $\rho < 1$. In (2), L_{it} is labor input, M_{it} is material input, K_{it} is capital, and w_{it} is a distribution parameter determining the marginal product of labor relative to materials. Capital is assumed to be quasi-fixed; capital services in year t are determined by the capital stock at the end of $t-1$, i.e., $K_{i,t-1}$. In equilibrium

$$Q_{it} = Q_{it}^D. \tag{3}$$

Our production function can be seen as a Cobb-Douglas function defined over capital and an aggregate variable input obtained using a CES-aggregation function. The specification (2) allows heterogeneity in labor productivity through w_{it}: A positive change in w_{it} can be interpreted as a labor-augmenting innovation. On the other hand, Hicks-neutral changes in efficiency are picked up by A_{it}. Both A_{it} and w_{it} may shift over time and vary across firms.

Let q_{Lt} and q_{Mt} be the unit price of labor and materials, respectively, and define $\mathbf{q}_t = (q_{Lt}, q_{Mt})$. The short-run cost function, i.e., when capital is quasi-fixed, corresponding to (2) is

$$C(q_t, K_{i,t-1}, Q_{it}) = c_{it} \left(\frac{Q_{it}}{A_{it} K_{i,t-1}^{\gamma}} \right)^{\frac{1}{\varepsilon}}, \tag{4}$$

where

$$c_{it} = \left[(q_{Lt}/w_{it})^{\frac{\rho}{\rho-1}} + q_{Mt}^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}, \tag{5}$$

which can be interpreted as a firm-specific price index of the variable inputs. Note that c_{it} depends on the distribution parameter w_{it}.

The conditional (short-run) factor demand functions can be derived from (4) by Shepards lemma:

$$\begin{aligned}
\ln L_{it} &= \frac{1}{\varepsilon}(\ln Q_{it} - \ln A_{it} - \gamma \ln K_{i,t-1}) + \frac{1}{1-\rho}\ln c_{it} - \frac{1}{1-\rho}\ln q_{Lt} + \frac{\rho}{1-\rho}\ln w_{it} \\
\ln M_{it} &= \frac{1}{\varepsilon}(\ln Q_{it} - \ln A_{it} - \gamma \ln K_{i,t-1}) + \frac{1}{1-\rho}\ln c_{it} - \frac{1}{1-\rho}\ln q_{Mt}. \qquad (6)
\end{aligned}$$

The short-run optimization problem of firm i, when capital is quasi-fixed, is to choose the price that maximizes the operating surplus, i.e., revenue less variable factor costs:

$$\max_{P_{it}} \left\{ \Phi_{it} P_{it}^{1-e} \prod_{k \neq i} P_{kt}^{\frac{\sigma}{n-1}} - c_{it} \left(\frac{\Phi_{it} P_{it}^{-e}}{A_{it} K_{i,t-1}^{\gamma}} \prod_{k \neq i} P_{kt}^{\frac{\sigma}{n-1}} \right)^{\frac{1}{\varepsilon}} \right\}. \qquad (7)$$

In the non-cooperative equilibrium, the first-order condition for firm i, given the prices of the other firms (ignoring additive constants), is

$$\begin{aligned}
\ln P_{it} &= \frac{\varepsilon}{\varepsilon + e - e\varepsilon}\ln c_{it} - \frac{1}{\varepsilon + e - e\varepsilon}\ln A_{it} - \frac{\gamma}{\varepsilon + e - e\varepsilon}\ln K_{i,t-1} \\
&\quad - \frac{\varepsilon - 1}{\varepsilon + e - e\varepsilon}\ln \Phi_{it} + \frac{\sigma(1-\varepsilon)}{\varepsilon + e - e\varepsilon}\left(\frac{1}{n-1}\sum_{k \neq i} \ln P_{kt} \right). \qquad (8)
\end{aligned}$$

Solving the first-order-conditions (8), using (1)-(6), yields a solution with the following structure in terms of revenue, $R_{it} \equiv P_{it}Q_{it}$, and the short-run factor costs $q_{Mt}M_{it}$ and $q_{Lt}L_{it}$:

$$\begin{bmatrix} \ln R_{it} \\ \ln(q_{Mt}M_{it}) \\ \ln(q_{Lt}L_{it}) \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ a & b + \frac{\rho}{1-\rho} & 0 \\ a & b + \frac{\rho}{1-\rho} & \frac{\rho}{1-\rho} \end{bmatrix} \boldsymbol{\alpha}_{it} + \begin{bmatrix} \gamma a \\ \gamma a \\ \gamma a \end{bmatrix} \ln K_{i,t-1} + \boldsymbol{\nu}(\mathbf{q}_t), \qquad (9)$$

where $\boldsymbol{\nu}(\mathbf{q}_t)$ is a vector function of prices common to all firms and

$$\boldsymbol{\alpha}_{it} = \begin{bmatrix} \ln A_{it}^{*} \\ \ln c_{it} \\ \ln w_{it} \end{bmatrix}, \qquad (10)$$

with

$$\ln A_{it}^{*} = \ln A_{it} + \lambda \ln \Phi_{it} + \frac{1}{n-1}\sum_{k \neq i} \left(d_1 \left(\ln A_{kt} + \gamma \ln K_{kt} - \ln c_{kt} \right) + d_2 \ln \Phi_{kt} \right). \qquad (11)$$

The coefficients a, b, d_1, d_2, and λ in (9)-(11) are functions of the parameters σ, ε, and e. In general, the loading coefficient a is positive, while b is negative.[2]

Several special cases can be derived from (9)-(11). When $\sigma = 0$ we have monopolistic competition, with $a = (e-1)/(\varepsilon + e - e\varepsilon)$, $b = -\varepsilon(e-1)/(\varepsilon + e - e\varepsilon)$, $\lambda = 1/(e-1)$ and $A^*_{it} = A_{it}\Phi^{\lambda}_{it}$. Furthermore, when $e \to \infty$ we obtain a model with a competitive market and a homogeneous good: $a = 1/(1-\varepsilon)$, $b = -\varepsilon/(1-\varepsilon)$ and $A^*_{it} = A_{it}$.

We see that a is a common loading coefficient of $\ln A^*_{it}$ in all the three equations in (9). This component comprises a linear combination of the Hicks-neutral efficiency term, $\ln A_{it}$, the demand shift parameter, $\ln \Phi_{it}$, and the other firms' state variables; i.e., their efficiency, capital, cost and demand terms. On the other hand, a change in $\ln c_{it}$ will have a different impact on revenues and factor costs: the loading coefficient of $\ln c_{it}$ is b (< 0) in the first equation and $b + \rho/(1-\rho)$ in the material and labor demand equations.

An increase in w_{it}, i.e., a labor-augmenting innovation, reduces c_{it}. Thus, ceteris paribus, both revenue and factor costs will be increased if $b + \rho/(1-\rho) < 0$, while the factor cost share of labor will be reduced if and only if $\rho < 0$.

In Section 5 we will present an econometric specification of (9). This specification accounts for firm-specific initial conditions, stochastic trends, industry-wide effects and various types of errors. The latter should be interpreted in a broad sense, so as to include any transient deviation between the actual realizations of revenue, factor costs and capital (including pure data errors) and their corresponding equilibrium level as defined in the theoretical model (9). Such deviations are both realistic and important in practice.

3 Capital formation

We now turn to the structure of capital formation. We allow partial irreversibilities, so that the resale price of capital, q_{St}, may be lower than the purchaser price of capital, q_{Kt}. Let $S_{it} \equiv s_{it}K_{i,t-1}$ denote total sales (disinvestment) of capital during year t, where s_{it} is total sales as a share of the capital stock at the end of the previous year. Thus, $s_{it} < 0$

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[2] In the special case of $n = 2$ we obtain:

$$\begin{aligned}
m &= \varepsilon^2 + 2e\varepsilon - 2\varepsilon^2 e + e^2 - 2e^2\varepsilon + e^2\varepsilon^2 - \sigma^2 + 2\sigma^2\varepsilon - \sigma^2\varepsilon^2, \\
a &= -m^{-1}(\varepsilon + e - 2e\varepsilon - e^2 + e^2\varepsilon - \sigma^2\varepsilon + \sigma^2),\ b = -\varepsilon a, \\
\lambda &= (-\varepsilon - e + e\varepsilon)/(\varepsilon + e - 2e\varepsilon - e^2 + e^2\varepsilon - \sigma^2\varepsilon + \sigma^2),\ d_1 = -m^{-1}\varepsilon\sigma/a,\ d_2 = m^{-1}(1-\varepsilon)\sigma/a.
\end{aligned}$$

can be interpreted as representing (positive) investments. The adjustment cost function can then be written as:

$$D(s_{it}) = \begin{cases} s_{it}\xi_t\overline{K}_{i,t-1} & s_{it} \geq 0 \\ 0 & s_{it} < 0, \end{cases} \tag{12}$$

where

$$\overline{K}_{i,t-1} \equiv q_{Kt}K_{i,t-1}$$

is the net capital stock at the beginning of year t (end of $t-1$) in year-t prices and

$$\xi_t \equiv \frac{q_{Kt} - q_{St}}{q_{Kt}}$$

is an expression for the relative wedge between the purchasing price of new capital and the selling price of used capital. Thus ξ_t is an exogenous time-specific variable. Note that the adjustment-cost function (12) is weakly convex and kinked at zero: there are no adjustment costs related to purchases of capital, i.e., when $s_{it} < 0$.

We see from (9) that the firm's operating surplus, Π_{it}, is homogeneous in capital, $K_{i,t-1}$:

$$\begin{aligned} \Pi_{it} &\equiv R_{it} - q_{Lt}L_{it} - q_{Mt}M_{it} \\ &= \widetilde{\pi}_{it}K_{i,t-1}^{\gamma a},\ \gamma a < 1, \end{aligned} \tag{13}$$

for a random variable $\widetilde{\pi}_{it}$, that depends on $\boldsymbol{\alpha}_{it}$ and $\mathbf{q}_t$. We can then utilize a result from Bloom (2000), which says that if $\widetilde{\pi}_{it}$ is a Markovian stochastic process and adjustment costs are weakly convex, then the actual capital stock, $K_{i,t-1}$, and the hypotethical frictionless capital stock, $K_{i,t-1}^*$ (which can be adjusted freely at beginning of year t) will have the same long run growth rate. Following Bloom, Bond and Van Reenen (2001), we operationalize this as:

$$\ln K_{i,t-1} = \ln K_{i,t-1}^* + \text{error},$$

where the error term is stationary.

The frictionless capital stock, $K_{i,t-1}^*$, is the capital stock the firm would choose if the marginal revenue of capital is equal to the user cost; see e.g. Haavelmo (1960) and Jorgenson (1963). This can be found in two steps. First, we find $K_{i,t-1}^*$ conditional on output, Q_{it}, by solving the following cost minimization problem:

$$K_{i,t-1}^* = \arg\min_{K_{i,t-1}} (r+\delta)q_{Kt}K_{i,t-1} + C(q_t, K_{i,t-1}, Q_{it}),$$

where r is the real interest rate, δ is the depreciation rate and $C(\cdot)$ is defined in (4). This leads to the first order condition (ignoring additive constants)

$$\ln K^*_{i,t-1} = \frac{1}{\gamma+\varepsilon}\left(\ln Q_{it} - \ln A_{it}\right) + \frac{1}{\gamma+\varepsilon}\ln c_{it} - \frac{\varepsilon}{\gamma+\varepsilon}\ln(r+\delta)q_{Kt}. \tag{14}$$

Replacing $K_{i,t-1}$ with $K^*_{i,t-1}$ in (6) and (8), and solving (1), (6), (8) and (14) with respect to $K^*_{i,t-1}$, Q_{it}, L_{it}, M_{it} and P_{it}, we obtain the profit maximizing frictionless capital stock. The solution has the form:

$$\ln K^*_{i,t-1} = \left[\begin{array}{ccc} \kappa_a, & \kappa_c, & 0 \end{array}\right] \boldsymbol{\alpha}_{it} + \kappa_t \tag{15}$$

for fixed coefficients κ_a and κ_c and a time-varying intercept κ_t. In Section 5, we formulate a simple error correction model for the stochastic variation in K_{it} around K^*_{it}. Note that $K^*_{i,t-1}$ is the optimal frictionless capital stock at the *beginning* of year t (when A_{it} is known). In contrast, the actual capital stock at the beginning of t, $K_{i,t-1}$, is quasi-fixed.

4 The Exit Decision

At the beginning of year t, the value of firm i's capital stock is $\overline{K}_{i,t-1}$ in real year t prices. If the firm decides to continue production for (at least) one more year, this generates operating surplus, Π_{it}, as well as capital costs. To derive an expression for the *annualized* capital costs of an operating firm, we follow Caballero (1999) and write this as the sum of the net flow payment on capital in the absence of adjustment costs: $(r+\delta)\overline{K}_{i,t-1}$ and the adjustment costs: $s_{it}\xi_t\overline{K}_{i,t-1}$. Let π_{1it} denote profit in year t of an operative firm:

$$\pi_{1it} = \Omega_{it} - s_{it}\xi_t\overline{K}_{i,t-1},$$

where

$$\Omega_{it} = \Pi_{it} - (r+\delta)\overline{K}_{i,t-1} \tag{16}$$

is the operating profit *before* netting out adjustment costs and Π_{it} is the operating surplus defined in (13).

Furthermore, let π_{0it} be firm i's profit if it closes down at the beginning of year t and sells its entire capital stock. Then $s_{it} = 1$ and the firm faces adjustment costs $\xi_t\overline{K}_{i,t-1}$(see

(12)), but avoids the flow payment of capital, $(r+\delta)\overline{K}_{i,t-1}$. Hence the profit of a firm that closes down in year t is

$$\pi_{0it} = -\xi_t \overline{K}_{i,t-1}. \tag{17}$$

We shall assume that $(\pi_{0it},\ \Omega_{it})$ are martingale processes and, furthermore, that $E_t(s_{i,t+s}) = s_i$. Thus,

$$\begin{aligned} E_t\left(\pi_{0i,t+s}\right) &= \pi_{0it} \\ E_t\left(\pi_{1i,t+s}\right) &= E_t\left(\Omega_{i,t+s} + s_{i,t+s}\pi_{0i,t+s}\right) \\ &= \Omega_{it} + s_i\pi_{0it}. \end{aligned} \tag{18}$$

The variables on the right hand side of (18) are not directly observable, since our econometric model (see Section 5) distinguishes between the value of the variables in the *theoretical* equilibrium described in Sections 2 and 3 and their actual *realizations*. The discrepancies may be due to both transient fluctuations around the equilibrium levels and data errors.

We shall now obtain an expression for the present value of firm i based on the structural model above and the Stock and Wise optimal stopping criterion. First, we consider the expected present value $V_{it}(T)$ of producing until $t+T$ and then closing down – assuming no exit option:

$$\begin{aligned} V_{it}(T) &= \sum_{s=0}^{T-1} \varphi^s E_t(\pi_{1i,t+s}) + \varphi^T E_t(\pi_{0i,t+T}) \\ &= \frac{\left(1-\varphi^T\right)\left(\Omega_{it} + s_i\pi_{0it}\right)}{1-\varphi} + \varphi^T \pi_{0it} \\ &\simeq \frac{\Omega_{it} + s_i\pi_{0it}}{r} + \varphi^T\left(\pi_{0it} - \frac{\Omega_{it} + s_i\pi_{0it}}{r}\right), \end{aligned} \tag{19}$$

where $\varphi \approx 1-r$ is the discount factor. In the approach of Stock and Wise, the maximum of $V_{it}(T)$ is used to approximate the net present value of the firm at time t. Assuming an infinite planning horizon, we see from the last equation in (19) that the optimal solution is either $V_{it}(0) = \pi_{0it}$ or $V_{it}(\infty) = (\Omega_{it} + s_i\pi_{0it})/r$. That is,

$$\max_{T:T\geq 0} V_{it}(T) = \begin{cases} \pi_{0it} & \text{if } \pi_{0it} > (\Omega_{it} + s_i\pi_{0it})/r \\ \frac{\Omega_{it}+s_i\pi_{0it}}{r} & \text{otherwise.} \end{cases} \tag{20}$$

When $\pi_{0it} = 0$, (20) has a similar structure as the value function derived by Melitz (2003), who investigated monopolistic competition under exit options. In that article future profitability is known and identical in all years once the firm is established. Only shocks unrelated to operational profits may cause the firm to exit later. In our model, realized future profitability is uncertain, but, conditional on the information set at t, expected profits are identical in all future years.

As in Melitz (2003), we also allow the exit rule to depend on an error term that is uncorrelated with profitability, denoted χ_{it}. Specifically, we assume that the firms' decision rule can be described as follows:

$$\text{Close down production if and only if } V_{it}(0) > V_{it}(\infty) + \chi_{it}, \tag{21}$$

where

$$\chi_{it} = \tau_i(\eta_{it} - \beta_{0t}),$$

η_{it}, for convenience, has a logistic distribution, and τ_i and β_{0t} are unknown scale and location parameters, respectively. The scale parameter, τ_i, is assumed to be firm-specifc to incorporate heterogeneity across firms with respect to the absolute magnitude of the error term, while β_{0t} is a time-varying intercept common to all firms.

Using (17), (20) and (21), the firm will exit if and only if

$$\begin{aligned} -\xi_t\overline{K}_{i,t-1} &> \tfrac{\Omega_{it}}{r} - \tfrac{s_i}{r}\xi_t\overline{K}_{i,t-1} + \chi_{it} \\ \Leftrightarrow \eta_{it} &< \beta_{0t} + \beta_{1i}\Omega_{it} + \beta_{2i}\xi_t\overline{K}_{i,t-1}, \end{aligned} \tag{22}$$

where

$$\begin{aligned} \beta_{1i} &= -\frac{1}{r\tau_i} \\ \beta_{2i} &= \frac{s_i - r}{r\tau_i}. \end{aligned} \tag{23}$$

According to (22)-(23), positive discounted operating profits, Ω_{it}/r, suggest that the firm will continue production. Immediate exit will lead to adjustment costs, $\xi_t\overline{K}_{i,t-1}$, which also suggests that production should be continued. On the other hand, if production is continued the firm will expect adjustment costs in all future periods, $s_i\xi_t\overline{K}_{i,t-1}/r$. If $s_i > r$ it may be optimal to exit immediately in order to avoid these costs.

According to (23), β_{1i} and β_{2i} are random coefficients, possibly correlated, that vary across firms. The structure of (23) suggests that the following stochastic specification may be appropriate:

$$\begin{bmatrix} \beta_{1i} - E(\beta_{1i}) \\ \beta_{2i} - E(\beta_{2i}) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\right), \tag{24}$$

for unknown parameters $E(\beta_{1i}), E(\beta_{2i}), \sigma_1$, σ_2, and σ_{12}. Furthermore, when estimating the model we will treat the ξ_t as unknown parameters with $\xi_1 = 1$ as an identifying restriction. Thus, we cannot identify the estimated ξ_t as the price wedge between the resale and purchaser price of capital as in Section 3.

Finally, let z_{it} be the indicator that firm i is operative in year t:

$$z_{it} = \begin{cases} 1 & \text{if } i \text{ is operative in year } t \\ 0 & \text{otherwise.} \end{cases}$$

We then obtain a mixed logit model of firm exit[3]:

$$\begin{aligned} & \Pr(z_{it} = 0 | \, \boldsymbol{\beta}_i, \Omega_{it}, \, \overline{K}_{i,t-1}, \, z_{i,t-1} = 1) \qquad (25) \\ = \; & \frac{1}{1 + e^{-\left\{\beta_{0t} + \beta_{1i}\Omega_{it} + \beta_{2i}\xi_t\overline{K}_{i,t-1}\right\}}}, \end{aligned}$$

where $\boldsymbol{\beta}_i = (\beta_{1i}, \beta_{2i})$ is a vector of random coefficients and $(\boldsymbol{\beta}_{0t}, \xi_t)$ are fixed parameters to be estimated.

5 Econometric specification

In this section we formulate a structural time series model in state space form that encompasses our behavioral model.

Supply and demand: Define:

$$\mathbf{y}_{it} = \left[\; \ln R_{it}, \;\; \ln(q_{Mt}M_{it}), \;\; \ln(q_{Lt}L_{it}) \; \right]', \qquad (26)$$

where all prices are real prices, i.e., all nominal prices have been deflated by the consumer price index. The endogenous variables R_{it}, M_{it} and L_{it} are given by the structural equations (9).

Combining (9) and (26) we obtain:

$$\mathbf{y}_{it} = \boldsymbol{\theta}_A \boldsymbol{\alpha}_{it} + \boldsymbol{\theta}_K \ln K_{i,t-1} + \boldsymbol{\nu}(\mathbf{q}_t), \; t = 1, ..., T. \qquad (27)$$

In order for (27) to represent (9), we impose the following restrictions:

$$\begin{aligned} \boldsymbol{\theta}_A &= \begin{bmatrix} a & b & 0 \\ a & b + \frac{\rho}{1-\rho} & 0 \\ a & b + \frac{\rho}{1-\rho} & \frac{\rho}{1-\rho} \end{bmatrix} \\ \boldsymbol{\theta}_K &= \left[\; \gamma a, \;\; \gamma a, \;\; \gamma a \; \right]'. \qquad (28) \end{aligned}$$

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[3] See Train (2003) for a discussion and overview of mixed logit models.

The latent exogenous variables of the model consist of the vector $\boldsymbol{\alpha}_{it}$ defined in (10), whereas $\mathbf{y}_{it}$ contains the endogenous variables.

The actual observations of revenue and factor demand may not be consistent with our theoretical model. For example, according to (13) operating surplus is strictly positive, whereas observations of negative operating surpluses are not uncommon in real data sets. Thus it is necessary to include error terms that can account for different types of discrepancies. Let $\widehat{\mathbf{y}}_{it}$ be the observed counterpart of $\mathbf{y}_{it}$, including measurement errors as well as (transient) deviations from the Bertrand equilibrium characterized by (13). We will assume the following relation:

$$\widehat{\mathbf{y}}_{it} = \mathbf{y}_{it} + \mathbf{e}_{it}, \;\; \mathbf{e}_{it} \sim \mathcal{IN}(\mathbf{0}, \mathbf{\Sigma}), \tag{29}$$

where $\mathbf{e}_{it} = \left[\begin{array}{ccc} e_{Rit}, & e_{Mit} & e_{Lit} \end{array} \right]'$ is a vector of transient errors and $\mathbf{0}$ denotes a matrix of zeros of appropriate dimension.

A consequence of (29) is that Ω_{it}, which is used as an explanatory variable in the exit probability (25), is not directly observable. Ω_{it} is the ex ante (expected) operating profit of a firm that is operative in period t, not the ex post (realized) profit. If the firm exits in year t, operating profits will never be realized.

Capital stock dynamics: We now turn to the econometric specification of capital adjustment. Despite partial irreversibilities, which we discussed in Section 3, investments tend to be relatively smooth at the firm level when one type of aggregate capital is considered[4]. This observation justifies the use of a linear error correction model as an approximation to the capital formation process with (15) as an equilibrium path. That is:

$$\begin{aligned} \Delta \ln K_{it} &= (\phi - 1)\left(\ln K_{i,t-1} - \ln K^*_{i,t-1}\right) + \varepsilon_{it} \qquad (30) \\ &= (\phi - 1)\left(\ln K_{i,t-1} - \left[\begin{array}{ccc} \kappa_a, & \kappa_c, & 0 \end{array} \right] \boldsymbol{\alpha}_{it}\right) + (1-\phi)\kappa_t + \varepsilon_{it}, \end{aligned}$$

where ε_{it} is an error term with variance $\sigma_{\varepsilon\varepsilon}$. We can rewrite (30) as:

$$\ln K_{it} = \boldsymbol{\kappa}'_K \boldsymbol{\alpha}_{it} + \phi \ln K_{i,t-1} + (1-\phi)\kappa_t + \varepsilon_{it} \tag{31}$$

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[4] See Bloom, Bond and Van Reenen (2001) and Nilsen and Schiantarelli (2003).

with

$$\boldsymbol{\kappa}_K = (1-\phi)\left[\begin{array}{ccc} \kappa_a, & \kappa_c, & 0 \end{array}\right]'.$$

The exogenous variables: Let us now consider the stochastic specification of $\boldsymbol{\alpha}_{it}$. We assume, provided the firm enters the sample at $t=1$, that

$$\begin{aligned} \boldsymbol{\alpha}_{it} &= \begin{cases} \boldsymbol{\alpha}_{i1} & t=1 \\ \boldsymbol{\alpha}_{i,t-1}+\boldsymbol{\eta}_{it} & t=2,...,T, \end{cases} \\ \boldsymbol{\alpha}_{i1} &\sim \mathcal{IN}(\mathbf{0},\boldsymbol{\Sigma}_1),\ \boldsymbol{\eta}_{it} \sim \mathcal{IN}\left(\mathbf{0},\boldsymbol{\Sigma}_\eta\right). \end{aligned} \tag{32}$$

The covariance matrix $\boldsymbol{\Sigma}_1$ of $\boldsymbol{\alpha}_{i1}$ characterizes the cross-sectional heterogeneity across firms in their first observation year, while heterogeneity between firms in any later year can be decomposed into the *initial condition*, $\boldsymbol{\alpha}_{i1}$, and the *cumulated innovations*, $\sum_{s=2}^{t}\boldsymbol{\eta}_{it}$, where the covariance matrix of $\boldsymbol{\eta}_{it}$ is $\boldsymbol{\Sigma}_\eta$. In order to obtain identification, both the initial condition, $\boldsymbol{\alpha}_{i1}$, and the subsequent innovations, $\boldsymbol{\eta}_{it}$, must have a mean of zero, since any non-zero mean will be indistinguishable from the industry-wide intercept $\boldsymbol{\nu}(\mathbf{q}_t)$ in (27) .

While it may be restrictive to assume that $\boldsymbol{\alpha}_{it}$ is a random walk, this assumption conveniently simplifies the interpretation and estimation of our model. Moreover, the random walk assumption is consistent with Gibrat's law that firms' growth rates are independent of firm size; cf. the discussion in Section 1[5]. Our empirical framework is also largely consistent with the elaborate model of firm evolution in Hopenhayn (1992)[6].

Combining (31) and (32), we obtain:

$$\left[\begin{array}{c} \boldsymbol{\alpha}_{it} \\ \ln K_{i,t-1} \end{array}\right] = \begin{cases} \left[\begin{array}{c} \boldsymbol{\alpha}_{i1} \\ \ln K_{i0} \end{array}\right] & t=1 \\ \left[\begin{array}{cc} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\kappa}_K' & \phi \end{array}\right]\left[\begin{array}{c} \boldsymbol{\alpha}_{i,t-1} \\ \ln K_{i,t-2} \end{array}\right] + \left[\begin{array}{c} \mathbf{0} \\ (1-\phi)\kappa_{t-1} \end{array}\right] + \left[\begin{array}{c} \boldsymbol{\eta}_{it} \\ \varepsilon_{i,t-1} \end{array}\right] & t=2,...,T, \end{cases} \tag{33}$$

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[5] This is not strictly true since it follows from (9) that:

$$\Delta \ln R_{it} = \left[\begin{array}{ccc} a & b & 0 \end{array}\right]\boldsymbol{\eta}_{it} + \gamma a \Delta \ln K_{i,t-1} + \cdots,$$

which indirectly depends on $R_{i,t-1}$ through $\Delta \ln K_{i,t-1}$. However, this link is weak when there are high adjustment costs of capital, as our results indicate.

[6] Hopenhayn's model accounts for differences in initial conditions as well as idiosyncratic innovations during the firms' life cycles.

where $\mathbf{I}$ is the identity matrix, $\mathbf{0}$ is a vector of zeros and

$$\begin{bmatrix} \boldsymbol{\alpha}_{i1} \\ \ln K_{i0} \end{bmatrix} \sim \mathcal{IN}\left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \varrho \end{pmatrix} \right)$$
$$\begin{bmatrix} \boldsymbol{\eta}_{it} \\ \varepsilon_{i,t-1} \end{bmatrix} \sim \mathcal{IN}\left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_\eta & \mathbf{0} \\ \mathbf{0} & \sigma_{\varepsilon\varepsilon} \end{pmatrix} \right) \quad t = 2, 3, ... \tag{34}$$

The scalar ϱ is not estimated, but is set arbitrarily large to reflect a diffuse prior distribution of the initial capital stock, $\ln K_{i0}$ (see de Jong, 1991).

Appendix A shows how our model can be represented in state-space form with $\mathbf{a}_{it} \equiv \begin{bmatrix} \boldsymbol{\alpha}'_{it}, & \ln K_{i,t-1} \end{bmatrix}'$ as the state vector. The state space formulation allows to incorporate transient errors in the capital data. Analagously to (29), we may observe K_{it} with measurement errors. That is, we only observe $\widehat{K}_{i,t}$, where

$$\ln \widehat{K}_{it} = \ln K_{it} + e_{Kit},$$

for a transient error term e_{Kit}, with variance σ_{KK}.

Even though the state vector is unobserved by the econometrician, we can make inference about its actual realization using the *conditional* distribution of the state vector given the observed data on the firm. This is the basis for estimating the firm-year specific exit probabilities of the mixed logit model (25). However, estimation of the model is far from trivial. A maximum likelihood algorithm implemented in GAUSS is outlined in Appendix B.

Identification: Since $\boldsymbol{\alpha}_{it}$ is unobservable it is not possible to identify the parameters of $\boldsymbol{\theta}_A$ defined in (28): The term $\boldsymbol{\theta}_A \boldsymbol{\alpha}_{it}$ in equation (27) is observationally equivalent to $\widetilde{\boldsymbol{\theta}} \widetilde{\boldsymbol{\alpha}}_{it}$, where $\widetilde{\boldsymbol{\theta}} = \boldsymbol{\theta}_A \mathbf{R}$ and $\widetilde{\boldsymbol{\alpha}}_{it} = \mathbf{R}^{-1} \boldsymbol{\alpha}_{it}$ for any invertible matrix $\mathbf{R}$. Our structural model does, however, imply a number of useful restrictions on $\mathbf{R}$, which we shall now examine. First, let

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}.$$

In order for $\widetilde{\boldsymbol{\theta}}$ to have the same structure as $\boldsymbol{\theta}_A$, it must be:

$$\widetilde{\boldsymbol{\theta}} = \begin{bmatrix} \widetilde{\theta}_1 & \widetilde{\theta}_2 & 0 \\ \widetilde{\theta}_1 & \widetilde{\theta}_3 & 0 \\ \widetilde{\theta}_1 & \widetilde{\theta}_3 & \widetilde{\theta}_4 \end{bmatrix}$$

with $\widetilde{\theta}_1 > 0$ and $\widetilde{\theta}_2 < 0$. By considering the equation $\widetilde{\boldsymbol{\theta}} = \boldsymbol{\theta}_A \mathbf{R}$, we easily derive the following zero restrictions on $\mathbf{R}$:

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & r_{33} \end{bmatrix}.$$

Furthermore, we will assume that $\ln A^*_{it}$ is independent of $(\ln c_{it}, \ln w_{it})$, i.e., neutral efficiency and demand shocks are independent of labor augmenting innovations. Then $r_{12} = 0$ and $\mathbf{R}$ becomes a diagonal matrix.[7]

The restriction $r_{12} = 0$ enables us to identify $\boldsymbol{\theta}_A$ up to an arbitrary proportionality factor for each of its columns. Equivalently, we can identify $\boldsymbol{\alpha}_{it}$ up to an arbitrary proportionality factor for each component of $\boldsymbol{\alpha}_{it}$. We choose to standardize $\boldsymbol{\alpha}_{it}$ by assuming that the innovations, $\boldsymbol{\eta}_{it}$, of the stochastic process $\boldsymbol{\alpha}_{it}$ have unit variance, i.e.,

$$\boldsymbol{\Sigma}_\eta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sigma_{cw} \\ 0 & \sigma_{cw} & 1 \end{bmatrix}, \qquad (35)$$

where σ_{cw} is the correlation between $\ln c_{it}$ and $\ln w_{it}$. We then obtain full identification from the restrictions $\widetilde{\theta}_1 > 0$, $\widetilde{\theta}_2 < 0$ and $\sigma_{cw} < 0$, where the last restriction follows because an increase in w_{it} reduces c_{it}; see (5).

6 Data and variable construction

Table 1: **Descriptive statistics**

Industry (NACE)	# of firms	mean # of employees	# of exits
Wood products (20)	947	14	236
Plastic products (25)	381	16	97
Metal products (28)	1498	16	384
Machinery (29)	1227	15	361
Electrical equipment (30-33)	785	17	232
Transport equipment (34-35)	795	33	242

We use a recently established database from Statistics Norway: the Capital database, which contains annual observations on fixed capital (tangible fixed assets), revenue, wage

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[7] Assume, conversely, that $r_{12} \neq 0$. Then the first component of $\widetilde{\boldsymbol{\alpha}}_{it}$ is correlated with the other components, while the first component of $\boldsymbol{\alpha}_{it}$ *is not* correlated with any of the other components (given the stated assumption that $\ln A^*_{it}$ is independent of $(\ln c_{it}, \ln w_{it})$).

costs, intermediates, and many other variables for all Norwegian joint stock (i.e., limited liability) manufacturing firms for the period 1993-2002[8]. The database combines information from two sources: (i) accounts statistics for all Norwegian joint-stock companies, and (ii) structural statistics for the manufacturing sector. The accounts statistics is of a very good quality as it contains the audited accounting figures of the firms. The structural statistics should also be of high quality, especially for firms with at least 10 employees, since these figures are revised by Statistics Norway using electronic tax return forms.

In general, all costs and revenues are measured in nominal prices, and incorporate taxes and subsidies, except VAT. Labor costs incorporate salaries and wages in cash and kind, social security and other costs incurred by the employer. In this paper we analyze six industries, which are relatively export oriented: Wood products (NACE 20), Rubber and plastic products (NACE 25), Metal products (NACE 28), Machinery (NACE 29), Electronic equipment (NACE 30-33) and Transport equipment (NACE 34-35). Table 1 presents some summary statistics for these industries. The numbers in the table imply that in each sector the average exit frequency is around 20 to 30 percent.

The main statistical unit in the database is the firm: A firm is defined as “the smallest legal unit comprising all economic activities engaged in by one and the same owner” and corresponds in general to the concept of a company. A firm may consist of one or more establishments. An establishment is a geographically local unit conducting economic activity within an industry class. Because our data is at the firm level, we can distinguish between single- and multi-plant firms. To avoid problems with the analysis of multi-plant firms (which may close down only some of their plants, see discussion in Section 8), we analyze only single-plant firms.[9] In the industries we consider, only 10-20 percent of the plants belong to multi-plant firms. When a single-plant firm is merged with or acquired by another firm, or acquires a new plant, it is counted as ”missing” from the data set from that year onwards. In a few cases, firms were excluded from the entire sample because the value of an endogenous variable was missing for two or more subsequent years and then reappeared.

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[8] See “Documentation of the capital database. A database with data for tangible fixed assets and other economic data at the firm level,” which can be downloaded from: http://www.ssb.no/english/subjects/10/90/doc_200416_en/doc_200416_en.pdf

[9] Caves (1998) points out that most results on firm growth and turnover are insensitive to the establishment-firm distinction.

A unique feature of the database is that it contains detailed measurement of the net capital stock in both current and fixed prices at the firm level. Furthermore, the data set distinguishes between two types of capital goods: (i) buildings and land, and (ii) other tangible fixed assets. The latter group consists of machinery, equipment, vehicles, movables, furniture, tools, etc. and is therefore quite heterogeneous. The method for calculating capital stocks in current prices is based on combining book values of the two categories of fixed tangible assets from the balance sheet and gross investment data. Detailed descriptions of the method and the data are found in Raknerud, Rønningen and Skjerpen (2003), including estimates of median depreciation rates for the different types of capital. Since our econometric model contains only a single aggregate capital variable, we have constructed this as a Törnqvist volume index, where each type of capital is proportional to the sum of: (i) the user cost of of capital owned by the firm, and (ii) the total operational leasing costs. This aggregation corresponds to a constant returns to scale Cobb-Douglas aggregation function for different types of capital (see OECD, 2001).

7 Results

7.1 Estimates of factor loadings and capital coefficients

First, recall that the vector of latent factors $\boldsymbol{\alpha}_{it}$ contains three components: (i) $\ln A^*_{it}$, which incorporates cumulated Hicks-neutral innovations and demand shocks, (ii) a firm-specific variable factor price index, $\ln c_{it}$, and (iii) the distribution parameter in the CES aggregation of labor and materials, $\ln w_{it}$. Under the identifying restrictions stated in Section 5, we are able to identify the loading coefficients of $\boldsymbol{\alpha}_{it}$ in each equation. The estimates for these loading coefficients are depicted in Table 2, where equations 1-3 refer to the structural equations (27) for revenue and the two types of variable factor demand, whereas equation 4 refers to the capital equation (31).

With regard to equations 1-3, the estimated loading coefficients of $\ln A^*_{it}$ are very similar for all the sectors and lie between .24 and .29. Moreover, all the estimated loading coefficients of $\ln w_{it}$ – which, by assumption, is non-zero only for the third equation (labor) – lie between $-.22$ and $-.24$. As seen from (28), a negative loading coefficient is equivalent to $\rho < 0$. Thus, labor-augmenting innovations (i.e., positive increments in $\ln w_{it}$) reduce

Table 2: **Estimates of model for revenue and factor costs.** The standard errors in parentheses are obtained from the inverse Hessian of the log-likelihood function

Industry (NACE)	Eq. no.	Loading coefficient of:				R^2
		$\ln A_{it}^{*}$	$\ln c_{it}$	$\ln w_{it}$	$\ln\ K_{i,t-1}$	
Wood products (20)	1	.24 (.01)	$-.07$ (.02)	0	.09 (.02)	.90
	2	.24 (.01)	$-.14$ (.02)	0	.09 (.02)	
	3	.24 (.01)	$-.14$ (.02)	$-.23$ (.02)	.09 (.02)	
	4	.08 (.02)	$-.01$ (.02)	0	.72 (.03)	
Plastic products (25)	1	.26 (.01)	$-.05$ (.02)	0	.13 (.02)	.92
	2	.26 (.01)	$-.11$ (.02)	0	.13 (.02)	
	3	.26 (.01)	$-.11$ (.02)	$-.23$ (.02)	.13 (.02)	
	4	.09 (.02)	$-.00$ (.02)	0	.73 (.03)	
Metal products (28)	1	.26 (.01)	$-.02$ (.01)	0	.09 (.01)	.91
	2	.26 (.01)	$-.12$ (.01)	0	.09 (.01)	
	3	.26 (.01)	$-.12$ (.01)	$-.22$ (.01)	.09 (.01)	
	4	.08 (.01)	$-.00$ (.01)	0	.67 (.01)	
Machinery (29)	1	.28 (.01)	$-.04$ (.01)	0	.09 (.01)	.91
	2	.28 (.01)	$-.11$ (.02)	0	.09 (.01)	
	3	.28 (.01)	$-.11$ (.02)	$-.22$ (.01)	.09 (.01)	
	4	.10 (.01)	$-.00$ (.02)	0	.68 (.01)	
Electrical eq. (30-33)	1	.25 (.01)	$-.05$ (.01)	0	.06 (.01)	.92
	2	.25 (.01)	$-.10$ (.01)	0	.06 (.01)	
	3	.25 (.01)	$-.10$ (.01)	$-.22$ (.01)	.06 (.01)	
	4	.07 (.01)	$-.01$ (.01)	0	.64 (.01)	
Transport eq. (34-35)	1	.29 (.01)	$-.05$ (.02)	0	.14 (.01)	.92
	2	.29 (.01)	$-.14$ (.02)	0	.14 (.01)	
	3	.29 (.01)	$-.14$ (.02)	$-.24$ (.01)	.14 (.01)	
	4	.08 (.01)	$-.01$ (.02)	0	.72 (.01)	

the factor cost share of labor in all industries.

According to our structural model, the coefficient of lagged capital, $\ln K_{i,t-1}$, in equations 1-3 is equal to γa. Its estimates are depicted in the fourth column of Table 2, and vary between .06 and .14. This coefficient can be associated with the return to scale, which is clearly diminishing according to our results. Hence, either the elasticity of scale, $\gamma + \varepsilon$, is less than one or firms have market power, $e < \infty$; cf. the discussion following (11). Unfortunately, neither γ, ε or e are identifiable.

All the coefficients in Table 2 are highly significant, with standard errors between .01 and .03. Our model is parsimoniously parameterized relative to the amount of data, and we get a very high goodness of fit, as shown by our (pseudo) R^2 measure depicted in the last column of Table 2.[10] We find that R^2 varies between 90 and 92 percent for the different industries, which is a very good fit for panel data.

Let us turn to the results of equation 4, i.e., the capital accumulation equation. From the estimated loading coefficients, we see that capital accumulation is affected mainly by shocks in $\ln A^*_{it}$. For example, in Wood products we see that the estimated loading coefficient of $\ln A^*_{it}$ is .08 in the capital equation, which is 1/3 of the corresponding loading coefficients in equations 1-3. This indicates that if a Hicks-neutral innovation or a demand shock increases revenue and factor costs by 1 percent, the capital stock will increase by 1/3 percent by the end of the year and by $(1-\phi)^{-1}/3$ percent in the long run; cf. (30). Note that with $\phi = 2/3$ the latter expression becomes 1 percent. In fact, the estimates of ϕ are quite close to 2/3 in all the six industries, varying between .64 and .73. Thus, our results show a strong link between innovations and investments in the long run, although the speed of adjustment of capital towards the equilibrium path, K^*_{it}, is quite slow.

7.2 Estimates of exit probabilities

Table 3: **Exit probability estimates and likelihood ratio tests of parameter restrictions.** Standard errors of estimation in parenthesis

Industry	Coeff. of $\mathbf{\Omega}_{it}$		Test of: $\xi_t \equiv 1$	Coeff. of $\overline{K}_{i,t-1}$	
	$\mathrm{E}(\beta_{i1})$	σ_1	P-value	$\mathrm{E}(\beta_{i2})$	σ_2
Wood products	−.26 (.05)	.18 (.04)	.77	−2.19(.83)	2.20(.84)
Plastic products	−1.00 (.40)	.98 (.24)	.67	−.21 (.27)	.35 (.24)
Metal products	−.65 (.13)	.82 (.15)	.84	−1.38 (.47)	.92 (.24)
Machinery	−.84 (.20)	.67 (.16)	.18	−.26 (.11)	.12 (.04)
Electrical eq.	−.44 (.17)	.55 (.20)	.67	−.93 (.36)	.97 (.31)
Transport eq.	−.41 (.11)	.63 (.15)	.72	−.15 (.07)	.11 (.09)

Table 3 shows the parameter estimates of the mixed model of firm exit, with random coefficients $\boldsymbol{\beta}_i = (\beta_{1i}, \beta_{2i})$ and fixed coefficients (β_{0t}, ξ_t). From the second and third columns of the table, we see that profits (before netting out adjustment costs), Ω_{it}, have a

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[10] The pseudo R^2 measure is defined as:

$$R^2 = 1 - \frac{\mathrm{tr}\ \widehat{Var}(\mathbf{e}_{it})}{\mathrm{tr}\ \widehat{Var}(\widehat{\mathbf{y}}_{it} - \widehat{\mathbf{d}}_t)}$$

where tr denotes the trace, that is, the sum of the diagonal elements.

significant impact on the probability of exit. The coefficient estimate of $\mathrm{E}(\beta_{1i})$ is negative in all the industries. In Electrical Equipment and Plastic products, the estimate of $\mathrm{E}(\beta_{1i})$ is between two and three standard errors away from zero; in the other industries the estimates are between 3 and 5 standard errors away from zero. We also see that there is considerable variation in $\mathrm{E}(\beta_{1i})$ among the different firms within an industry: according to our estimates, the standard deviation of β_{1i}, i.e., σ_1, is of the same order of magnitude as $\mathrm{E}(\beta_{1i})$.

The effect of adjustment costs, which enter the logit model through the term $\beta_{2i}\xi_t\overline{K}_{i,t-1}$, is more complicated to analyze as it consists of a time-dependent industry-wide factor, ξ_t, and a firm-specific random factor, β_{2i}. When testing this specification empirically, we find that it can be simplified: the hypothesis that $\xi_t \equiv 1$ was not rejected in any industry, as seen from the likelihood ratio tests reported in column 4. Columns 5 and 6 show the results for the expected value, $\mathrm{E}(\beta_{2i})$ and the corresponding standard deviation of β_{2i}, σ_2, after the restriction $\xi_t \equiv 1$ is imposed.

According to (22), the exit decision depends, among other things, on the difference between adjustment costs due when the firm liquidates immediately and the discounted stream of all future adjustment costs. Table 3 shows that the estimates of $E(\beta_{2i})$ are negative in all industries and significantly different from zero at the 5 percent level, except for Plastic products. A negative value of β_{2i} means that high adjustment costs decrease the probability of exiting immediately. Moreover, the estimates of the dispersion parameter σ_2 are significantly different from zero in four of the six industries; the exceptions are Plastic products and Transport equipment.

In order to evaluate the aggregate performance of our model, in each year we divide firms into two groups: the *continuing* firms in any year t consist of the firms that did not exit during the observation period, while the *closing down* firms in year t consist of the firms which were operative at the end of year t but not in $t+1$.[11] For each firm we are able to estimate – for each year – the conditional exit probability in that year[12].

Figure 1 plots annual averages of the estimated conditional exit probabilities for the

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[11] Hence, firms exiting in $t+s$ $(s \neq 0)$ are not included in any of the two groups.

[12] Note that the probability of closing down at the end of year t equals $\Pr(z_{i,t+1} = 0|\ \boldsymbol{\beta}_i, \Omega_{i,t+1}, \overline{K}_{i,t}, z_{it} = 1,)$ since $z_{i,t+1}$ is zero if the firm is not operative at the *end* of $t+1$. Thus, year t is the last year for which the firm is observable in our data set. The information set of the firm, before choosing $z_{i,t+1}$, is assumed to consist of the variables that are known at the beginning of $t+1$, i.e., the firm knows $\Omega_{i,t+1}$ and K_{it}.

two groups of firms.[13] Our model discriminates to some degree between the two categories: the annual exit probabilities of closing-down firms are consistently higher than those for continuing firms, but the differences between the groups vary considerably over time and across industries, from only a few percentage points to around 20 percentage points. The figure clearly illustrates the difficulty of predicting the *time* of exit. We also note that the changes in the average (ex ante) exit probabilities of continuing firms are small and show no clear pattern over time.

7.3 Simulation of survival functions

The interpretation of Figure 1 is not straightforward as the graphs conflate different effects. First, they reflect temporal variations in both firm-specific and industry-specific conditions. Second, the annual exit probabilities are affected by sample self-selection due to exits and entries. For example, some entrants will become continuing firms, others will exit during the observation period.

To obtain a more easily interpretable picture, we undertook dynamic simulations in order to estimate ex ante survival functions, conditional on each firm's vector of initial conditions $(\widehat{\boldsymbol{\alpha}}_{i1}, \widehat{K}_{i0}, \widehat{\boldsymbol{\beta}}_i)$. The "hat" notation indicates that these variables are not directly observed, but have to be estimated by their conditional expectations given the observed data. For firms entering after 1993, we undertook the simulations *as if* the firms were established in 1993.

We obtained simulated realizations of $(\boldsymbol{\eta}_{it}^{'}, \varepsilon_{i,t-1})$ by random draws *with replacement* from the actual realizations $(\widehat{\boldsymbol{\eta}}_{it}^{'}, \widehat{\varepsilon}_{i,t-1})$. Again, the "hat" notation denotes conditional expectations given the actual data for each firm. Using the estimated mixed logit model we first calculated exit probabilities for all firms in 1994. A proportion of the firms were then eliminated by random draws from the conditional exit probability of each firm. Technically, this was done by drawing a number from the uniform distribution on $[0, 1]$ for each firm and then removing the firms with numbers that were lower than their estimated exit probability. For the "surviving" firms, new exit probabilities were estimated for 1995, and so on. By repeating the simulations for each firm year after year, and averaging over

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[13] While our data period runs from 1993 to 2002, we cannot calculate the 1993 exit probabilities since that would require capital data for 1992 (In our model capital is lagged by one year). On the other hand, because we also have observations on exits for 2003, we can calculate the 2002 exit probabilities.

Figure 1: **Estimated aggregate exit probabilities for continuing firms and closing down firms**

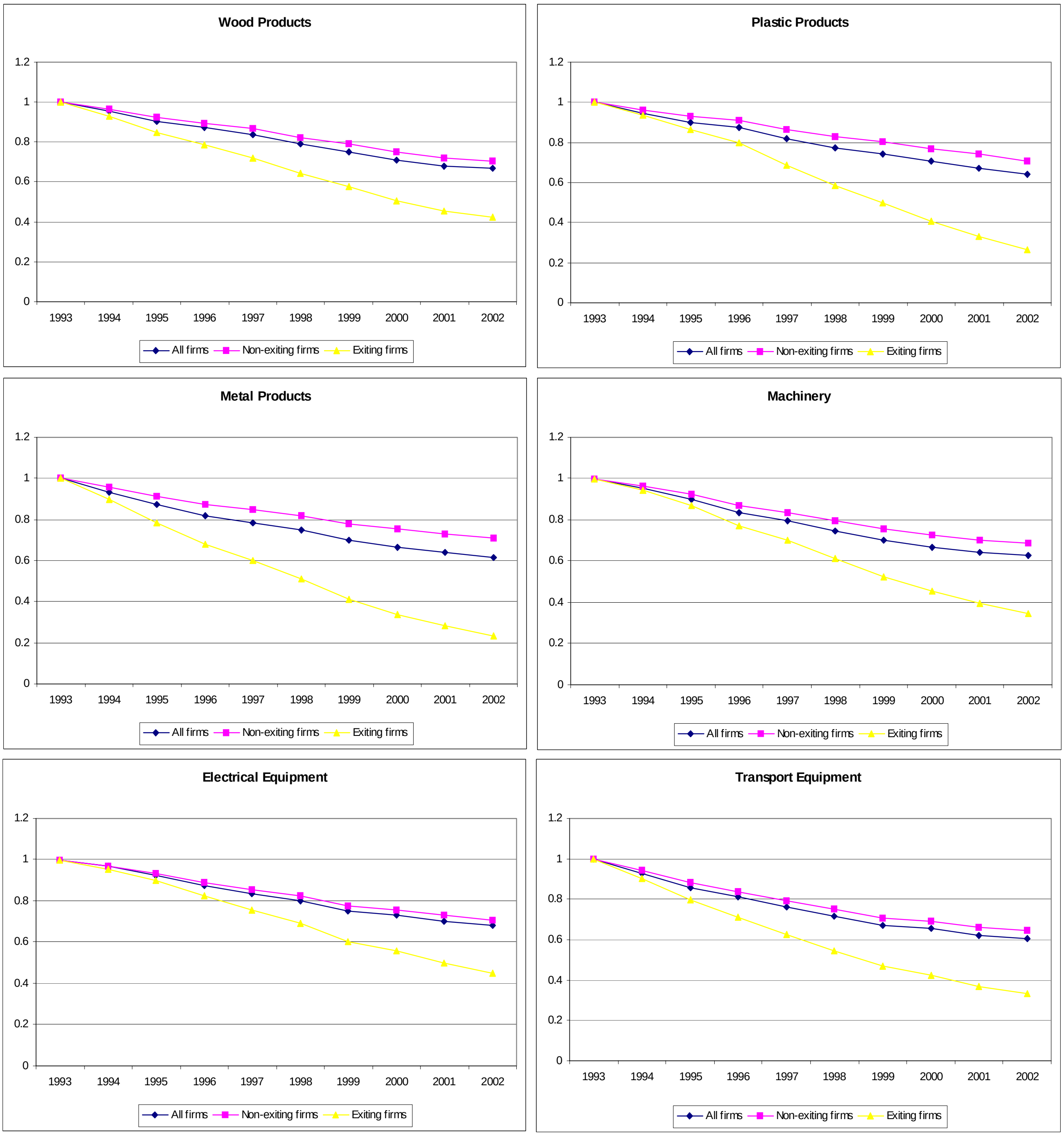

Figure 2: **Estimated ex ante survival functions for all firms, exiting firms and non-exiting firms**

the firms within the industry, we obtained industry specific aggregate survival functions; see Figure 2.

The figure contains three graphs for each industry: the (ex ante) survival functions for *all* firms, the (ex ante) survival functions for firms that did not exit in that period ("non-exiting firms") and the (ex ante) survival functions for firms that did exit during 1993-2002 ("exiting firms"). The latter function was estimated by continuing the simulations of the (annual) conditional exit probabilities *after* the exit time for these firms.

Figure 2 shows that in all industries the probability that a (representative) firm survives the whole 10-year period is quite similar across the industries; between 60 and 70 percent ("all firms"). This is consistent with the summary statistics in Table 1, which show average exit frequencies between 20 and 30 percent: since several firms were established after 1993, whereas the simulations assume that all firms were operative in 1993, the aggregate survival probabilities after 10 years (60-70 per cent) should be lower than those reported in Table 1 (70-80 pecent).

By comparing the survival functions for exiting and non-exiting firms, we can evaluate to what degree our model is able to "pick" firms that actually closed down during the observation period. Overall we find that our model distinctly discriminates between the two categories. For example, for Plastic products and Metal products we find that the *ex ante* survival probability of exiting firms is about 20 percent after 10 years, versus 70 percent for surviving firms. For the other industries, the gap between the two graphs is much narrower; about 30 percentage points (after 10 years). Our results suggest that the main characteristic of an exiting firm is not that its annual exit probability is much higher than that of a surviving firm, but rather that the difference in annual exit probabilities is highly persistent. Hence, it is the cumulated effect of somewhat higher annual exit probabilities over many years – compared with the average firm – that causes a firm to exit.

8 Conclusion

The purpose of the present study has been to examine the extent to which profitability can explain firm exit. Using a structural econometric model in combination with the Stock and Wise approximation of optimal stopping, we have derived explanatory variables from economic theory and estimated mixed logit models for six export-oriented manufacturing sectors. Our empirical model accommodates different types of latent variables that pick up unobserved heterogeneity between firms. The results show that increased profitability significantly lowers the exit probability, while, ceteris paribus, firms with a large capital stock tend to have lower probability of exit. According to our structural model, the latter result can be attributed to the impact of adjustment costs.

We find that the differences in estimated annual exit probabilities between firms that exited in the data period (1993-2002) and firms that did not, is moderate. Exiting firms are not characterized by having a high probability of exit just prior to exiting. On the other hand, the differences in estimated annual exit probabilities between firms that exited in the data period and firms that did not is highly persistent. Consequently, there is a significant difference between the survival function of the two groups. Comparing results across the six industries, we find that the estimated survival functions of a representative firm for each industry over a 10-year period are quite similar, with a survival rate between 60 and 70 percent after 10 years.

While exit is a key issue in the theoretical literature, the econometric literature on exit is – to the best of our knowledge – rather scarse and often based on simplistic models.[14] Many contributions to the field are featured in the 1995 special issue of the International Journal of Industrial Organization. For example, Mata, Portugal and Guimarães (1995) use duration models to estimate the effect of plant size and of market dynamics variables on the survival of new plants. They find that for Portuguese firms, plant size exerts a negative effect on the instantaneous failure rate. Boeri and Bellmann (1995) find that exit among German firms is not responsive to the business cycle, while capital-intensive plants and plants using advanced technology are less likely to exit. Doms, Dunne and Roberts (1995) estimate exit probabilities for U.S. manufacturing firms, using plant characteristics (age, size, productivity, capital intensity and technology information) in one year as explanatory variables. A more structural approach, based on stochastic dynamic programming (SDP), is found in Das (1992), who considers the problem of whether to operate, hold idle, or close down production units in the cement industry. However, that

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[14] There are several descriptive studies on firm exit. For example, Dunne, Roberts and Samuelson (1988) provide summary measures of the patterns of entry, growth and exit of firms in the U.S. manufacturing industries over the period 1963-1982. They find substantial and persistent differences in exit rates across industries.

approach is hampered by the curse of dimensionality associated with SDP in multivariate decision problems, which has often led researchers to use over-simplified econometric models that do not fit the data well.

In the present study we have tried to integrate the study of supply, demand and exit at the firm level using a simplified, yet complex, decision theoretic framework. One issue that remains unresolved is the treatment of multi-plant firms, which we have removed from our analysis, since they might have a different relationship between plant profits and exit: a multi-plant firm may take into consideration that increased output from one plant will lower the output price, and hence hamper profitability of the other units. Due to this strategic consideration, for multi-plant firms there might not be a simple relationship between profitability and exits[15]. A topic for future research is to examine plant exit in multi-plant firms.

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[15] Recently, the largest Norwegian pulp and paper firm (Norske Skog), which owns several plants in Norway and abroad, announced a decision to close down one domestic unit. According to the management, although production at this unit was profitable, continued production would lower the profitability for the entire multi-plant firm. The management, later supported by the majority of the owners, was not even willing to sell the unit as they feared continued production under new owners would hamper the multi-plant firm.

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Appendix A: The state-space representation

The model (29)-(34) can be stated in the following state space form:

$$\begin{array}{l} \mathbf{y}^*_{it} = \mathbf{G}\mathbf{a}_{it} + \mathbf{d}_t + \mathbf{e}^*_{it} \\ \mathbf{a}_{it} = \mathbf{F}\,\mathbf{a}_{i,t-1} + \boldsymbol{\omega}_{it} \end{array} \quad t = 1, ..., T \tag{36}$$

where the observation and state vectors are, respectively,

$$\begin{array}{rcl} \mathbf{y}^*_{it} & = & \left[\begin{array}{cc} \widehat{\mathbf{y}}'_{it} & \ln \widehat{K}_{i,t-1} \end{array} \right]' \\ \mathbf{a}_{it} & = & \left[\begin{array}{cc} \boldsymbol{\alpha}'_{it} & \ln K_{i,t-1} \end{array} \right]', \end{array} \tag{37}$$

$\mathbf{d}_t$ is a time-varying intercept vector and

$$\begin{array}{l} \mathbf{G} = \left[\begin{array}{cc} \boldsymbol{\theta}_A & \boldsymbol{\theta}_K \\ \mathbf{0} & 1 \end{array} \right] \\ \mathbf{F} = \left[\begin{array}{cc} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\kappa}'_K & \phi \end{array} \right] \\ \mathbf{a}_{i1} \sim \mathcal{IN}\left(\mathbf{0}, \mathbf{R}_1\right), \, \boldsymbol{\omega}_{it} \sim \mathcal{IN}\left(\mathbf{0}, \mathbf{R}_t\right), \mathbf{e}^*_{it} \sim \mathcal{IN}\left(\mathbf{0}, \boldsymbol{\Sigma}^*_e\right) \\ \boldsymbol{\Sigma}^*_e = \left[\begin{array}{cc} \boldsymbol{\Sigma}_e & \mathbf{0} \\ \mathbf{0} & \sigma_{KK} \end{array} \right] \\ \mathbf{R}_t = \left\{ \begin{array}{ll} \left[\begin{array}{cc} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \varrho \end{array} \right] & t = 1 \\ \left[\begin{array}{cc} \boldsymbol{\Sigma}_\eta & \mathbf{0} \\ \mathbf{0} & \sigma_{\varepsilon\varepsilon} \end{array} \right] & t = 2, ... \end{array} \right. . \end{array}$$

($\mathbf{0}$ and $\mathbf{I}$ denote a matrix of zeros and the identity matrix, respectively, of approperiate dimension).

Appendix B: The likelihood function and the ML estimator

We will now outline the procedure for estimation of the parameters of the modified logistic model (25), β, and the remaining "time-series" parameters, θ. For notational simplicity, assume that all firms enter the sample at $t = 1$ (the general case is a straightforward extension). All probability statements will henceforth be conditional on the inital capital stock, K_{i0}, although for simplicity this conditioning is suppressed in the notation.

The observed data on firm i consist of $\{(z_{it}, \mathbf{y}_{it}^{*\prime}); t = 1, ..., T_i\}$ (see Appendix A). T_i is a realization of a random variable, say, τ_i. We will now establish the likelihood as a function of (β, θ).

Let $\nu^i = \{\mathbf{a}_{i1}, ..., \mathbf{a}_{iT_i}\}$ denote all the exogneous latent variables of the state space model (36)-(37). Let $f(\nu^i | Y_{T_i}^i; \theta)$ be the density of ν^i conditional on $Y_{T_i}^i \equiv (\mathbf{y}_{i1}^*, .., \mathbf{y}_{iT_i}^*)$, let $f(Y_{T_i}^i; \theta)$ be the marginal density of $Y_{T_i}^i$ and $f(\boldsymbol{\beta}_i; \beta)$ the marginal density function of $\boldsymbol{\beta}_i$; see (24). Then the joint log-likelihood function $l(\beta, \theta)$ becomes

$$l(\beta, \theta) = \sum_{i=1}^{N} l^i(\beta, \theta), \tag{38}$$

where N is the number of firms, and

$$\begin{aligned} l^i(\beta, \theta) = \ln \int \int \prod_{t=1}^{T_i} P\left(z_{it} | \boldsymbol{\beta}_i, \Omega_{it}, \overline{K}_{i,t-1}, z_{t-1}^i = 1\right) \; f(\boldsymbol{\beta}_i; \beta) \; f(\nu^i | Y_{T_i}^i; \theta) d\boldsymbol{\beta}_i d\nu^i \\ + \ln f(Y_{T_i}^i; \theta). \end{aligned} \tag{39}$$

A natural estimation strategy would be to maximize the log-likelihood with respect to the unknown parameters. We can then utilize the state space form of (36)-(37), to obtain $f(\nu^i | Y_{T_i}^i; \theta)$ by means of the Kalman filter and Kalman smoother. The integral in (39) can be evaluated by Monte Carlo simulations using the algorithm of Durbin and Koopman (2002). However, the estimation problem remains complex because $P\left(z_{it} | \boldsymbol{\beta}_i, \Omega_{it}, K_{i,t-1}, z_{t-1}^i = 1\right)$ depends in a complex way on the parameters θ through Ω_{it} and $K_{i,t-1}$.

In order to estimate β and θ we propose a two-step procedure, which we have implemented in GAUSS 7.0. In the first step, simple preliminary estimates $(\widetilde{\beta}, \widetilde{\theta})$ are obtained as follows:

$$\begin{aligned}
\widetilde{\theta} &= \arg\max_{\theta} \sum_{i=1}^{N} \ln f(Y_{T_i}^i;\theta) \\
\widetilde{\beta} &= \arg\max_{\beta} \sum_{i=1}^{N} \ln \int \prod_{t=1}^{T_i} P\left(z_{it}|\boldsymbol{\beta}_i, E\{\Omega_{it}|Y_{T_i}^i;\widetilde{\theta}\}, E\{\overline{K}_{i,t-1}|Y_{T_i}^i;\widetilde{\theta}\}, z_{t-1}^i = 1\right) f(\boldsymbol{\beta}_i;\beta)d\boldsymbol{\beta}_i.
\end{aligned}$$

These are then used as starting values when maximizing (39) jointly with respect to (β,θ).

We find that the final estimates, $(\widehat{\beta},\widehat{\theta})$, are close to the initial estimates, $(\widetilde{\beta},\widetilde{\theta})$, and that the method converges quite quickly. Visual inspection of the log-likelihood in orthogonal directions (corresponding to the eigenvectors of the estimated covariance matrix) confirms that we have found global maximizers.

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