SAMFUNNSØKONOMISKE STUDIER

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ANALYSIS OF SUPPLY AND DEMAND OF ELECTRICITY IN THE NORWEGIAN ECONOMY

ANALYSE AV TILBUD OG ETTERSPØRSEL ETTER ELEKTRISITET I NORSK ØKONOMI

EDITED BY/REDIGERT AV
OLAV BJERKHOLT, SVEIN LONGVA, ØYSTEIN OLSEN
AND/OG STEINAR STRØM

STATISTISK SENTRALBYRÅ
CENTRAL BUREAU OF STATISTICS OF NORWAY

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STATISTISK SENTRALBYRÅ
Oslo—Kongsvinger 1983

ISBN 82-537-1815-2
ISSN 0085-4344

ISBN: 82-00-09540-1

Distributing offices:

Norway:
Universitetsforlaget
Box 2959, Tøyen
Oslo 6

United Kingdom:
Global Book Resources Ltd.
109 Great Russel Street
London WC 1B 3NA

United States and Canada:
Columbia University Press
136 South Broadway
Irvington-on-Hudson
New York 10533

FORORD

Et hovedsiktemål for den økonomiske forskningsvirksomhet i Statistisk Sentralbyrå har vært å utvikle hensiktsmessig analyseverktøy for politikkanalyse og planlegging. I de seinere år har planleggingen av elektrisitetsforsyningen i Norge vært sterkt i fokus. Dette har sammenheng med flere forhold. De gjenstående vannkraftreserver blir stadig mindre og relativt dyrere å bygge ut. Det har blitt større oppmerksomhet om miljøverdier som går tapt ved vannkraftutbygging. Internasjonalt har energiprisene gått sterkt opp. Usikkerhet om den økonomiske utvikling har gjort det vanskeligere å anslå framtidig elektrisitetsbehov.

Dette er noe av bakgrunnen for at Byrået har engasjert seg sterkt i forskning og analyse av energiøkonomiske problemstillinger. Denne boka presenterer resultatene fra et avsluttet prosjekt som har tatt sikte på å forbedre metodegrunnlaget for analyse av tilbud og etterspørsel av elektrisitet i norsk økonomi, særlig i et noe langsiktig perspektiv. Hoveddelen av prosjektet har tatt sikte på å videreutvikle MSG-modellen til å bli bedre egnet for energiøkonomiske studier. Dette har resultert i en ny versjon av modellen, MSG-4, som første gang ble tatt i bruk ved utarbeiding av regjeringens Langtidsprogram 1982 - 1985. Andre deler av prosjektet har omfattet studier av etterspørsel etter elektrisitet, kostnadsstruktur i kraftforsyningen, investerings- og prissettingskriterier, behandling av usikkerhet o.a.

Prosjektet har vært delvis finansiert av NAVF i 1978 - 1980 og Byrået er svært takknemlig for denne støtten. Prosjektet har omfattet en rekke medarbeidere i Byrået og har også hatt deltakelse av forskere ved Sosialøkonomisk institutt.

Statistisk Sentralbyrå, Oslo, 6. august 1982

Arne Øien

PREFACE

The economic research activity of the Central Bureau of Statistics has been directed to a considerable extent towards developing appropriate tools for policy analysis and planning. In recent years the planning of electricity supply has for a number of reasons been of central economic concern: the remaining hydroelectric power reserves in Norway have diminished quickly; there is more concern with the environmental consequences; international prices of energy have increased sharply; and uncertainty about the economic development has made the forecasting of energy demand more difficult.

On this background the Central Bureau has given priority to energy economic research. This book presents the results from a project undertaken to improve methods of analysing supply and demand of electricity in the Norwegian economy. The main part of the project had been the further development for energy economic analysis of the MSG model, which is used for long-term economic planning. Other parts of the project have included studies of electricity demand, cost structure in electricity supply, investment and pricing criteria, uncertainty in supply etc.

The Central Bureau of Statistics acknowledges the generous support given to the project by the Norwegian Research Council for Science and Humanities. The project has been a cooperation between the Research Department of the Central Bureau and faculty members of the Institute of Economics at the University of Oslo.

Central Bureau of Statistics, Oslo, 6 August 1982

Arne Øien

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EDITORS' NOTE OF ACKNOWLEDGEMENT

This book has been a cooperative effort involving ten authors. Six of the authors - Olav Bjerkholt, Eilev S. Jansen, Svein Longva, Lorents Lorentsen, Øystein Olsen and Jon Rinde - are staff members of the Research Department of the Central Bureau of Statistics, three - Finn R. Førsund, Asbjørn Rødseth and Steinar Strøm - are faculty members and one - Alette Schreiner - is a student at the Institute of Economics at the University of Oslo. Several others have contributed to the final result in various ways. Jørgen Ouren and Inger Holm have been members of the project team along with the authors. Jørgen Ouren has been responsible for the computer programming of the MSG model, and Inger Holm has given valuable assistance in data preparation and data handling in the project.

The editors would like to thank the Norwegian Research Council for Science and Humanities for its interest and financial support of the project. The Norwegian Water Resources and Electricity Board, in particular dr.ing. Arne Fagerberg, has taken a keen interest in the project and offered valuable advice as well as micro data on the cost structure of electricity supply. The Bank of Norway operates, under a contract with M.I.T. computing center, the software system TROLL which has been used to implement the MSG model and for much of the estimation work. The bank has, in particular, sponsored a project to improve the equation solving algorithm of TROLL with results that turned out to be of very great value for the practical use of the model.

Through four years of work the project team has drawn on the resources of various departments of the Central Bureau and on the secretarial and typing assistance of the Research Department, in particular. The editors would like to thank Kari Jensen, Sylvia Staalesen and Solveig Wiig for their patience and skill in transforming innumerable (and sometimes illegible) drafts into readable manuscripts.

I. INTRODUCTION

Energy problems have become a pressing economic issue in many countries in recent years. Energy has come on the agenda in most countries first of all because of the change in the supply situation of crude oil and gas brought forth by the OPEC embargo and the ensuing price increases in 1973-74 and 1979-80. Although the timing and the course of action decided upon by the OPEC countries may have caused increased problems of adjustment for the industrialized countries, the underlying inevitable fact is that the global supplies of crude oil and gas are limited; hence the development since 1950, with increased worldwide petroleum production combined with falling real prices, could not have endured for very long.

In Norway and other countries higher oil prices have implied changes in relative prices for factors of production and for consumer goods. The overall impact of higher energy prices is mediated through the substitution, in myriads of firms and households, from high-priced energy and goods with high energy content to cheaper energy sources and less energy intensive technologies and commodities. Economists and econometricians have jumped at the chance of proving their worth.

Within the OECD area the impact of higher worldwide energy prices has also been felt through its recessionary effects. Yet there has been less concern in Norway over higher oil prices than in many other countries, mainly for three reasons. First, Norway has felt less compelled to adopt contractive policies and has throughout the 1970s maintained high growth rates and avoided a surge in unemployment. Second, Norway was at the time of the OPEC embargo already on its way to become an oil exporting country. Third, Norway relies for more than 99 per cent of its electricity production on hydro power, and it is this fact that provides the background for the studies in the other chapters of this volume.

The electricity production in Norway is not only based on a renewable energy source, it is also exceedingly high in per capita terms. There are still quite considerable reserves of hydro power in Norway. However, the fast increase in electricity production over the last two decades - from 30 TWh to 90 TWh - means that the end of the expansion of the hydroelectric power supply has been brought within the planning horizon.

This volume contains results of a three-year research project on electricity supply and demand in Norway. The project has focused on the long-term macroeconomic impacts of energy policy. The main outcome of the project has been a new energy oriented version of the MSG model. This model is a disaggregate general equilibrium model of the Norwegian economy constructed originally by professor Leif Johansen of the Institute of Economics at the University of Oslo and further developed and used for many years in cooperation between the Ministry of Finance and the Central Bureau of Statistics.

The outcome of this project has been two versions of the model, MSG-4 and MSG-4E, respectively. The two versions have the same theoretical content but differ with regard to how the model is closed. The MSG-4E version, with the appended 'E' for Energy, is specifically intended for simulation studies of energy policy. In this version, the model has been used for forecasting energy demand, and for studying the interrelation between the production and use of energy and the overall economic development. The model is described, and the distinction between the two model versions are explained, in chapter II of this volume. The main advantage of MSG-4 and MSG-4E over their predecessors is that they allow more substitution to take place subsequently to energy price changes. This has, however, called for a number of changes of the specification of the variables of the model. It has also required a major effort in estimating relations representing producer and consumer behaviour.

1. The electricity supply system in Norway

In contrast to most other countries in the world the electricity supply system in Norway consists, as mentioned above, almost exclusively of hydro power. The abundant access to watercourses, which have been developed over the years at very low costs compared with thermal power, has furthermore implied that a relatively large share of total energy consumption in Norway is covered by electricity.

The supply of cheap electrical energy from hydro power projects was an important factor behind the rapid industrial development in Norway at the beginning of this century. Foremost in this development was the establishment of electrochemical and electrometallurgical manufacturing plants in remote areas near the source of hydro power. Furthermore, relatively low electricity prices have motivated consumers to use electricity for heating purposes to a larger extent than may be observed in other countries.

A brief description of the supply and demand pattern for the most important energy carriers in Norway is given in table 1 below. From the table it appears that about 39 per cent of total domestic energy demand in Norway is provided by electricity. For comparison the "electricity share" for Sweden is 22, for Denmark 10, for Great Britain 11 and for United States 11.

Table 1. The supply and demand for energy in Norway. 1980. All figures in TWh

	Electricity	Oil products	Other fuels
Production	82.5	73.9	9.6
Imports	1.8	46.7[1]	13.1
Transmission losses[2]	7.4	-	-
Domestic net demand	74.7	95.0[1]	20.8
Energy intensive industries ..	28.1	15.6	10.8
Other manufacturing industries	11.1	17.2	5.0
Other industries and government	13.0	40.3	0.0
Households	22.5	21.9	5.0
Exports	2.2	25.6	1.9

1) Exclusive of the use of fuels in ocean transport.
2) Losses in the electricity transmission and distribution network.

Source: Energy accounts of Norway 1980.

The role of public authorities in electricity supply

The electrification of the Norwegian economy started about 100 years ago and developed rapidly after the turn of century. In the early period the rights to development were often acquired by private - usually foreign owned - companies. However, it was soon decided that the exploitation of hydro power and supply of electricity was a task for public bodies. In 1917 special laws were enacted, stating that all regulations and exploitation of watercourses must be subject to concession and, furthermore, that privately owned power plants should be handed over to the State after 50 - 60 years of operation. Municipalities also started to engage directly in hydro power projecting in the beginning of this century. After the World War II a large share of the increase of the capacity of electricity production has been undertaken by the Central government through its company the State Power Plants (Statskraftverkene). Table 2 below shows gross production of electricity in some selected years and the shares provided by private companies, municipalities and the State Power Plants.

Table 2. Gross production of electricity by ownership. TWh. Relative shares in parenthesis

	1960	1970	1980
Total capacity	31.1 (100%)	57.6 (100%)	84.1 (100%)
Private companies	9.7 (31.2%)	12.6 (21.9%)	11.6 (13.7%)
Municipalities	13.5 (43.4%)	25.8 (44.8%)	42.3 (50.3%)
The State Power Plants	7.9 (25.4%)	19.2 (33.3%)	30.2 (36.0%)

Parallel to the expansion of the production capacity of the hydro power system a countrywide transmission and distribution network has been constructed, and today the whole country, apart from some areas in the north, is interconnected through a central transmission system. Consequently, demand requirements of the various regions can be met by transmission and exchange of power among companies. As a general rule the producers are responsible for the transmission of power from production units to distribution centres, while the local distribution of electricity is provided by utilities formed as municipal or intermunicipal companies.

The electricity supply system thus consists of a large number of production and distribution units. To a certain degree these units act as independent companies maximizing some measure of net benefit, but with strong regulations imposed on their decisions by government regulations. In addition to the restrictions on investments implied by the concession laws local distribution utilities are also required to follow uniform accounting procedures. In most cases prices are set so that they cover historical costs.

Even though many decisions in the electricity system are decentralized, the overall responsibility for the supply of electricity rests with the Central government. In 1978 a specific Energy Ministry was established. However, the actual operation and the preparation of plans for the electricity sector takes place in the Norwegian Water Resources and Electricity Board, in short NVE, which is a directorate subordinate to the Energy Ministry. NVE has the role as adviser to the Central government in all questions concerning the electricity sector, including evaluation of water power projects. By means of rather detailed and sophisticated models each potential water power project's contribution to the benefits of the whole system is calculated.

In the planning of the electricity supply system the authorities are faced with two main problems:

i) The operation of the supply system: how should the existing power plants and the distribution network be operated and how should the supply be allocated between consumers, so that total benefits to society are maximized?

ii) The dimensioning of the supply system: at what rate should the system be expanded, and which projects should be included in the plans at every point of time?

As stressed above the research project presented in this volume has focused on the long-term interactions between energy production and use and the overall economic development. Consequently, in most of the following chapters the focus is on the dimensioning problem. However, before turning to a brief discussion of the long-term planning aspects, we will touch upon a couple of problems concerning the actual operation of the electricity system.

Pricing policies

The prices on electricity from the State Power Plants are decided by the Storting (Parliament). Electricity deliveries are also subject to a specific electricity tax, which to some extent is used by the Central government to regulate and differentiate the prices paid by different consumers. However, as indicated above, for a large share of the deliveries the price decisions are left to local utilities, which as a rule set prices equal to historical costs. This decentralization of decisions has led to considerable price differences between different consuming groups and regions in the electricity market. As the costs of production vary considerably between different hydro power projects, some consumers receive power from old, amortized establishments at very low prices, while others pay a price which is closer to the average costs in new projects, i.e. the long run marginal cost. Price differentiation in the electricity market is also caused by the special position of energy intensive industries. The deliveries to these industries from the State Power Plants are based on long-term contracts which traditionally have been very favourable with low initial prices and mild index adjustments. In addition many power plants are directly owned by establishments in the energy intensive industries. In table 3 prices of electricity paid by different categories of electricity users are given with the electricity tax included but exclusive of the general value added tax.

Table 3. Average electricity prices paid by different groups of consumers. 1980

Sector group	Øre/kWh
Energy intensive industries	5.3
Other manufacturing industries	12.7
Other industries and government	16.4
Households	15.2
Total domestic demand	11.2
Exports	11.8

The price differentials apparent from table 3 are not solely the result of price discrimination in the electricity market. These price differences reflect also that the real costs of transmitting and distributing electricity differ considerably between consumer groups. The significance of real distribution cost and the existence of price differentiation is discussed by Longva and Olsen in chapter V. The calculations presented there indicate that a considerable part of the differences in the observed market prices may be accounted for by differences in real distribution costs. However, there are still residual price differences left "unexplained" which indicate that total electricity supply is not allocated among consumers in the most efficient way.

The existence of uncertainty

So far we have not mentioned the phenomenon of uncertainty on the supply side which is an essential characteristic of a hydro power system. This kind of uncertainty could be dealt with within a general equilibrium framework through contingent markets for electricity, i.e. contingent with regard to the certainty of delivery (see e.g. Malinvaud (1972)). An optimal organization of the electricity market could in this case imply some degree of differing prices (see also Serck-Hanssen (1981)).

The actual planning of the Norwegian electricity system distingquishes between two main categories of power: firm power and surplus power. Most of the production is classified as firm power, which is assumed to have a high degree of certainty. The brief description of pricing policies above, stressing the influence of public authorities, referred to the deliveries of firm power. Surplus power on the other hand is traded in a short-term market, and the prices equilibrating supply and demand may vary by month, day and hour. There may, however, be some doubt whether firm power and surplus power can be regarded as "contingent goods" in a theoretical sense. Few consumers in Norway actually have the opportunity to choose between deliveries with different degrees of certainty, so that electricity on the demand side just as well may be treated as a homogeneous product. The distinction between firm power and surplus power will then have to be regarded as a specific institutional arrangement.

This interpretation is adopted in the article written by Bjerkholt and Olsen in chapter XIII, where the significance of uncertainty for the planning of the electricity sector is discussed. A simplified theoretical model is developed where a central element is the distinction between the consumers short-term and long-term demand for electricity. The latter concept is assumed to be synonymous with the demand for firm power. It is then shown how the optimal sale of firm power relative to the capacity depends on the shape of short-term and long-term demand curves. In the actual planning of Norwegian electricity supply special consideration is given to the fact that uncertainty prevails both on the supply side and demand side in the electricity market, and the strategy pursued has led to an "overcapacity" in the supply system compared to expected demand. With reasonable assumptions regarding the form of the demand structure the theoretical discussion may support this kind of strategy. The significance of uncertainty on the demand side of the electricity market is also briefly analysed within the same framework.

2. The long-term planning problem

The traditional planning system

The building of hydro power projects has a construction period of about 4 - 5 years. In addition the advance planning and concession treatment of hydro power projects will typically stretch over several years. Projections of future demand are therefore needed for the dimensioning of the electricity system. These demand projections have, until few years ago, been constructed by means of rather simple methods based on official forecasts of the overall economic development. In these projections the effects of prices on the demand for electricity, in particular the industrial demand, were almost neglected. Another weakness of the projection method was that the feedbacks from changes in energy demand to the rest of the economy were not taken into account in a satisfactory manner. On this background it may be justified to characterize the traditional planning process in Norway as a strategy of choosing a mix of technology that minimize total system costs with the development of demand taken as given rather than as aiming at maximization of social benefits. Following Joskow (1976) this planning procedure - which was originally developed by Turvey (1968) - may be labelled the "British approach". A more technical and detailed description of the planning system in Norway is given in Hveding (1968).

The long run marginal cost criterion

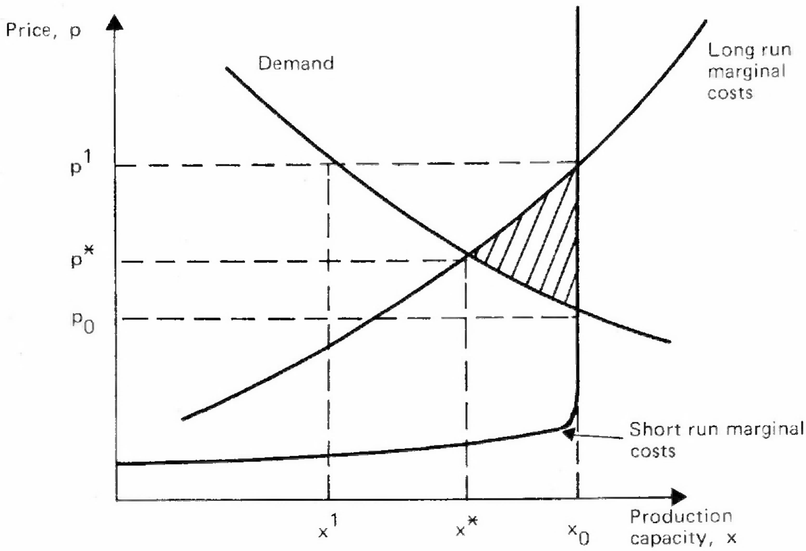
The objection against this kind of planning policy is naturally that the costs of expanding the production capacity are not properly weighed against the benefits to society resulting from the increased supply of electricity. A commonly accepted decision rule for the evaluation of investment projects is the present value criterion, which states that a project should be implemented if the discounted value of all cost and income streams exceed zero. The application of the present value criterion on investments in electricity supply is discussed by Strøm in chapter XII. It is shown that the comparison of incomes and costs may be carried out on an annual basis. Transformed to this dimension the criterion expresses that a project should be undertaken if the average of the market prices for electricity exceeds long run marginal costs in electricity supply put on an annual basis.

As a result of economic growth and changes in prices over time the question of optimal timing of investments in hydro power projects also becomes important. This aspect is discussed in some detail by Rødseth in chapter XIV, as well as by Strøm. With certain assumptions regarding the development of prices the long run marginal cost criterion is still valid, i.e. in the evaluation of projects the incomes from the first year of operation should be compared with annual costs.

Thus, from an economic point of view an optimal expansion path of the production capacity in electricity supply is characterized by the fact that the average price of electricity equals long run marginal costs. During the last decade economists in particular have argued that this long run marginal cost criterion should be applied in the planning of electricity supply in Norway. Referring again to Joskow (1976) this would imply to change the planning strategy towards what may be called the "French approach".

As shown by Strøm in chapter XII applications of the marginal cost criterion on hydro power supply indicate that the rate of expansion of the production capacity in this sector has been too high, i.e. the prices paid by consumers are generally lower than the marginal costs of expanding the hydro power system. This implies that too much capital has been allocated to electricity production, and that further investments in hydro power production will give a smaller return to capital than investments in other sectors of the economy. A simple illustration of the present situation in the Norwegian electricity supply is given in figure 1.

Figure 1. A graphical illustration of the Norwegian electricity market



In this figure a partial demand curve of "today" is drawn together with a curve indicating long run marginal costs as a function of the production capacity. In a hydro power system a typical feature is that this latter relation is increasing, i.e. decreasing returns to scale prevail. The level of optimal capacity is indicated in this figure by x^*. This quantity could have been sold to the consumers at an equilibrium price, p^*, which is equal to long run marginal cost. As mentioned above calculations indicate that the actual production capacity in Norway today exceeds this optimal level; in the figure the actual capacity is indicated by x_0. The loss to society of having "over-invested" in electricity production is indicated by the shaded area in figure 1.

We have already touched upon the main causes which may be assumed to have "generated" the overinvestment in the elctricity supply system. The fact that decisions are decentralized to local authorities and that these do not behave as profit maximizers has led to differentiated and generally low prices. Furthermore the formal model framework available for the construction of demand projections has been rather weak.

Some further remarks should be made as regards the interpretation of the long run marginal cost criterion and its implications for actual pricing policies. As stressed by Strøm in chapter XII the "rule" that the prices should equal long run marginal costs is an investment criterion derived from the present value criterion. A common misinterpretation is that the actual prices should be equal to long run marginal costs at every point of time. From figure 1 it is easy to see that this would lead to unreasonable results. If the market prices are increased to p^1, and if demand as presumed is elastic, actual demand for electricity would be reduced correspondingly, to x^1 as depicted in the figure. This would imply idle capacity in the production system and a waste of resources. The capital invested in electricity supply should more correctly be regarded as "sunk costs", and the relevant criteria for pricing policy is therefore short run marginal costs, which increase rapidly when the production approaches the capacity of the supply system. Given the position of the demand and cost curves in figure 1 the optimal equilibrium price is thus p_0, i.e. the price at which the given capacity is fully utilized.

An argument that has been raised against the use of the long run marginal cost criterion and the conclusion that the capacity in the electricity supply is too high, is the possibility of exporting firm power at a large scale (today only surplus power is exported). However, it should be emphasized that this is not an objection against the marginal cost criterion itself. The argument only concerns which prices and cost components that should be used in the cost-income comparison. Generally the electricity should be sold in the market that gives the highest net profit. This may surely be the export market. However, the present transmission capacity for electricity from Norway to other countries is limited. In a cost-benefit evaluation the costs of expanding this capacity must obviously be included. Furthermore, it may be claimed that economic growth will imply that the domestic willingness to pay for electricity will, within few years, equal the world market price. The length of the period when it will be beneficial to export hydro power may thus be rather short.

Application of the MSG-4E model in the long-term planning of electricity supply

If the situation in the Norwegian electricity market is as depicted in figure 1, the optimal strategy for the planning authorities would be to stop further investments in the electricity supply system until the consumers' willingness to pay for electricity, as a result of economic growth, has reached the long run marginal cost. The further planning of the capacity of the electricity sector and the simultaneous determination of prices may obviously be a complicated task for the authorities, which may be solved only by studying the linkages between the electricity sector and the overall economy. For analysing this kind of problems a model like MSG-4E may be a useful planning tool.

The main structure and functioning of the MSG-4E model is described by Longva, Lorentsen and Olsen in chapter II. As outlined there the model is intended primarily to be used for analysing changes over time in the allocation of production by industries and their use of resources, the consumption pattern, and the development of the corresponding equilibrium prices. Particular attention is paid to the electricity flows in the model, for instance the demand for electricity may be calculated in physical units. A basic property characterizing these demand projections, is that they are consistent with the overall development of the economy. Estimates of future demand for electricity calculated by the MSG model are presented by Longva, Lorentsen, Rinde and Strøm in chapter IX. In the same chapter the impacts on the economy of an energy policy that aims at equalizing prices and long run marginal costs are analysed.

When using the MSG model to analyse this kind of problems the included relations both on the demand side and the supply side of the electricity market are obviously of crucial importance. It should be clear from the graphical illustration that the application of the long run marginal cost criterion in the evaluation of the capacity and investments in hydro power supply depends heavily on the elasticity of demand. The fact that the actual prices of electricity are lower than long run marginal costs does not disturb the resource allocation of the economy, if electricity demand is completely inelastic. In figure 1 the shaded area then vanishes. In this case pricing policy would only be a question of evaluating the effects on the income distribution. However, recent econometric studies, among which those presented in this volume, indicate that demand is elastic. An outline of the production structure and estimates of price elasticities in Norwegian industries as specified in the MSG model are presented by Longva and Olsen in chapter III. Rather flexible production functions are estimated with labour, capital, energy and materials as inputs. For one of the energy intensive industries of the model - Production of Metals - a specific sector study is carried out, and the results are presented in chapter XI by Førsund and Jansen.

The specification of the demand structure for private households is described by Bjerkholt and Rinde in chapter IV. A complete system of demand functions for the 18 consumption goods of the MSG model is estimated. A sector model for household consumption is also developed and presented by Rødseth in chapter X. In this sector model, which at present is not integrated in the main model, the close relationship between energy demand and the use of consumer durables is taken explicitly care of.

The price sensitivity of energy demand is "summed up" in chapter VIII written by Longva, Olsen and Rinde. In this contribution total price elasticities are calculated by simulating the whole MSG model, so that all substitution and income effects on demand are taken into account simultaneously. For the aggregate sector groups Primary industries, Energy intensive industries, Other manufacturing industries, Service industries and Households the own price elasticities for electricity are estimated to -0.4, -0.7, -0.6, -0.7 and -0.6, respectively. For the total economy the direct price elasticity for electricity is estimated to -0.5.

The arguments that were mentioned above regarding the optimal capacity of the Norwegian electricity supply system are naturally also dependent on the shape and the level of the long run marginal cost curve depicted in figure 1 or, more generally, the production structure of electricity supply. In the MSG model production and distribution of electricity are treated as two separate activities. The cost structure of the production part of the supply system, i.e. the power plants, is described by Rinde and Strøm in chapter VI. As hydro power production is based on the extraction of natural resources, decreasing returns to scale is commonly assumed in this activity. A cost function is estimated which confirms this hypothesis, but the authors also show that decreasing returns may not occur if the projects are included in the system in correspondance with the succession in the official plans.

The production structure of the distribution part of the supply system, comprising both the transmission and local distribution network, is estimated and the results presented by Schreiner and Strøm in chapter VII: Real capital and physical power losses are assumed to be substitutes in the production process. The estimation results indicate, in accordance with other studies, that increasing returns to scale prevail in the distribution of electricity.

3. Some concluding remarks

The main purpose of the research project presented in this book has been to provide better tools in the long-term planning of the Norwegian electricity supply system and to integrate energy sector planning and overall macroeconomic planning into the same framework. The MSG-4E model, which is the main outcome of the research project, has already begun to serve this purpose. It was the main model tool in preparing the projections of the macroeconomic development to year 2000 in the Long-Term Programme (Langtidsprogrammet) 1982 - 1985 presented to the Parliament by the Ministry of Finance in spring 1981. At the same time MSG-4E has been used by the energy planning authorities to make consistent electricity demand forecast.

Even though MSG-4E is operational and intensively used it should not be forgotten that the model, of course, contains several weaknesses and undeveloped parts (see chapter II for a further discussion). There is still much work to be done before the model, if ever, can be said to be a fully satisfactory tool for the long-term planning of the electricity supply system.

II. ENERGY IN THE MULTI-SECTORAL GROWTH MODEL MSG

by

Svein Longva, Lorents Lorentsen and Øystein Olsen

1. A brief outline of the model

The pattern of direct energy use in Norway is rather simple. Oil products are used for transport and heating, while hydro electric power covers most of the remaining energy demand from industries and households. This pattern has long traditions; both the industrial development at the beginning of this century and the post-war economic reconstruction programs promoted the expansion of heavy industries based on hydro electric power. The government has a decisive influence in the development and operation of electricity production in Norway. However, the analytical tools provided for analysing the supply and demand of energy in a macroeconomic context in Norway have so far been rather unsatisfactory. Existing macroeconomic models have included aggregate descriptions of energy in value terms only, while sector models for energy supply have not taken sufficient account of the overall economic development.

The Central Bureau of Statistics is the main supplier of operational models for macroeconomic planning and analysis in Norway. The Bureau is also responsible for the preparation of national accounts and energy accounts. One obvious task in the development of better tools for analysing energy development is to integrate energy flows, in physical as well as value terms, in operational macroeconomic planning models. It is thereby possible to forge energy sector planning and overall macroeconomic analysis into the same framework.

In one of the projects in the Central Bureau of Statistics the emphasis is on the short to medium term relationship between energy demand and economic development (see Hervik (1980)). This support model to the national budgeting and planning model MODIS IV (see Bjerkholt and Longva (1980)) translates energy flows in constant values into physical units in great detail. The support model is used to provide short to medium term forecasts for energy consumption and to check the consistency between the overall economic plan and the existing sector plans for energy supply.

The aim of the model described below is to study the long-term interaction between economic growth and energy production and use. The point of departure has been an existing multi-sectoral growth model, called MSG. The model orginated as an empirical study of the growth potential of the Norwegian economy in Johansen (1960 and 1974). It was later turned into an operational model, mainly for the use of the Ministry of Finance. The latest version was completed in 1975 (see Lorentsen and Skoglund (1976)). Compared with that version, known as MSG-3, several parts have been modified or added to provide the energy oriented version of the model described in this chapter (and also in Longva, Lorentsen and Olsen (1980)).

A model suited for the analysis of alternative energy policies has to be a disaggregate, comprehensive model where the substitution and scale effects of policy changes are well taken care of. Some of the calculated macro effects of different energy policies are basically dependent upon how the labour and capital markets are treated in the model, see Hogan (1979). In the MSG model, like in most other economic growth models, the total supply of labour is exogenous, or inelastic. Hence, a change in the use of materials, energy or capital must change the equilibrium price of labour in real terms. This approach seems appropriate as an approximation to the long-run equilibrium in the Norwegian labour market, or in any economy where full employment is the first priority target.

The choice of an approximation for the long-run equilibrium in the capital market is less obvious. Two extreme aternatives offer themselves as convenient simplifications:

i) A fixed total input of capital, i.e. inelastic supply.
ii) Fixed real rate of return to capital, i.e. perfectly elastic supply.

In case i) changes in other inputs - materials, labour and energy - will change the marginal productivity of capital. With a given total stock of capital the equilibrium rate of return to capital in real terms must also change. This may, over time, affect the willingness to save and invest, and the approximation of inelastic supply of capital may turn out to be implausible without some compensating capital policy or without some iterative mechanisms back to capital supply. The interplay of energy and capital at the macro level will thus be trivial or arbitrary.

In case ii) capital input is adjusted to changes in materials, labour and energy inputs so that the marginal productivity of capital is maintained. With this approximation of the long-run equilibrium of the capital market, a change in the price of energy will change the total use of capital, materials and energy; the real price of labour and energy; and the gross output.

The two extreme ways of modelling the capital market have been embedded in two versions of the new model. Since exogenous total supply of capital has been a feature of previous MSG models, the version with inelastic supply of capital is called MSG-4. The version with elastic supply of capital is called MSG-4E, where E is short for energy. Except for this difference in the philosophy and modelling of the capital market, the two MSG versions are identical. The description which follows thus applies to both versions.

The industry classification of the previous MSG model has been revised to improve the modelling of energy flows and the generation and absorption of energy. The input-output part traces flows of energy and non-energy commodities measured in constant prices as inputs to industries and final demand. To identify the flows of energy in physical terms different distribution costs and the occurrence of price differentiation are accounted for.

On the demand side for energy, the production model for each industry has been developed to allow for substitution between various energy inputs and between energy, materials, capital and labour. For most industries the specification of the production structure is based on the neoclassical theory of production and the assumption of constant returns to scale. The household consumption model has been developed to include effects of energy demand of changes in stocks of consumer durables and to give a proper representation of substitution possibilities between different types of energy.

The energy supply is elaborated in some detail for the production of electricity. In its treatment of the supply of hydro electric power, the model benefits from calculations carried out by the Norwegian Water Resources and Electricity Board. The results are used to estimate a cost function for the electricity producing sector. The model specification allows for differentiated resource use in the distribution of electricity from power stations to the various users. For the other main energy supply sector - North Sea oil and gas production - production, prices and investments are all exogenously determined.

The MSG model traces out the long-term growth paths of the economy, especially the distribution of labour, capital and production over a disaggregated set of industries, changes in the household consumption patterns, and the development in corresponding equilibrium prices. A system of partly non-linear, simultaneous equations forms the core of the model. It is often a difficult task to explain the economic logic of a simultaneous system in an explicit and yet comprehensive way. One can start at some point and follow the chain of causes and effects, but inevitably the reasoning loops back since a simultaneous model has no obvious beginning or end. A simplified structure of the MSG-4E version of the model is depicted in figure 1. The MSG-4E version is easier to explain than MSG-4, since the outside assessment of both wages and returns to capital and the assumption of constant returns to scale makes the model neatly recursive in a price model and a quantity model.

For a guidance through figure 1 assume for simplicity that all industries produce at constant returns to scale, minimize costs, and set prices equal to unit costs. Start in the upper part of the diagram and assume given wage rates, returns to capital, trends of technical change and capacity utilization indices. The intersectoral price-cost relations and the price dependent input demand functions then simultaneously determine the cost minimizing techniques in terms of input coefficients for labour, capital, materials and energy, and determine the commodity prices that cover calculated costs adjusted by the given mark-up indices. The capacity utilization and mark-up indices are used to adjust for short-term deviations from normal or long-run equilibrium behaviour.

For given final demand the scale of production by industry is determined as in simple traditional input-output models. Industry demand for capital and labour services is also derived. Imports are calculated from import shares, differentiated by commodity and by purchasing sector. Actually, final demand is partly exogenous, such as exports and government expenditures, and partly endogenous, such as private gross investments and household consumption.

Private gross investments are determined in a closed loop with the scale of production by industry. The scale of production by industry determines the demand for capital services and thereby capital stock by industry and by kind of capital good. This again determines private gross investments by commodity.

For given prices the commodity composition of household consumption depends only upon total household consumption, which is determined in such a way that full employment is ensured. The total productive capacity for the economy as a whole is determined by the exogenous total labour force, technical change, and the capital stock consistent with the exogenously determined rates of return to capital.

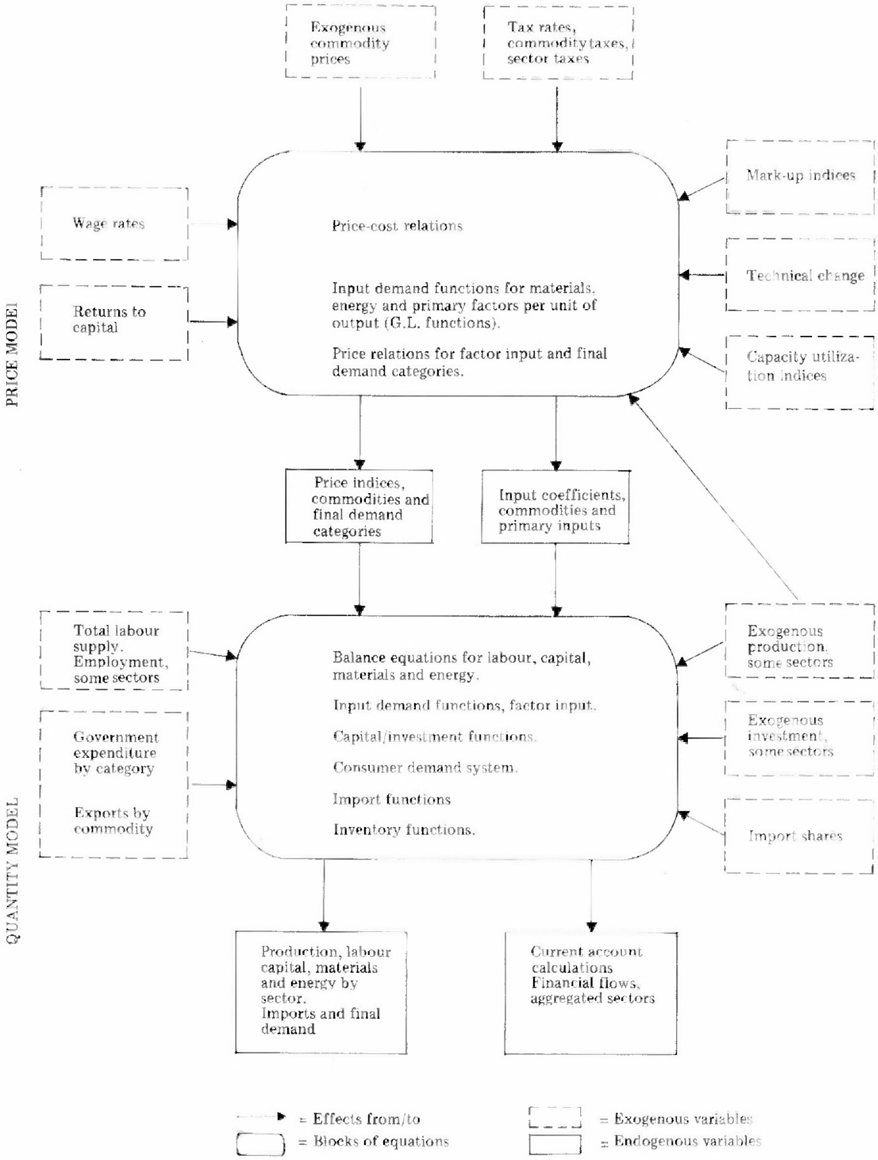
The MSG model also includes calculations not indicated in the figure such as submodels for capital depreciation, indirect taxes, government consumption etc. Special options to "control" the model's results for the balance of trade (e.g. adjusting the import shares or the export estimates) are introduced. In a few sectors, mainly primary industries, decreasing returns to scale are assumed. This impose links between prices and quantities since unit costs in these sectors depend upon the scale of production. To avoid these links and for other reasons, some commodity prices are given outside the model, and the production levels and/or investments of a few industries are exogenous. Some of these special cases referred to above (decreasing returns to scale, exogenous prices, exogenous estimates for production and investment) apply to the energy producing sectors (see section 4 below).

In the MSG-4 version of the model, with inelastic supply of capital, there is a crucial link between the price and quantity side of the model represented by the overall level of returns to capital. Given the resource restriction for capital, the level of returns to capital has to be endogenously determined. The equation system remains unaltered, but is simultaneous in prices and quantities.

A complete representation of technological and behavioural relations within households and industries would exceed the limits of a manageable model. In MSG the interplay of sectors in a growth process is focused; behaviour and technology within sectors are given a rather simple representation. It should also be noted that the use of energy in some sectors is more or less directly determined outside the model. For instance, all exports are exogenous. Energy intensive industries are major export industries, which means that the assessments of exports directly influence the development of energy use. Several ongoing projects are aiming at modifying and improving various parts of the model. Results from these projects will gradually be implemented.

A number of support routines and models are linked to MSG. These models are either pre-calculations to provide exogenous estimates (labour force, population growth, oil investment and production profiles etc.) or post-calculations to provide consequences of model results (current account calculations as indicated in figure 1, demand for different types of skilled labour, industry pollution etc.)

Figure 1. Structure of MSG–4E.



2. Basic concepts[1)]

The Norwegian national accounting system, which is in very close adherence to the revised SNA (see the United Nations (1968)) forms the conceptual framework of the MSG model. The model includes an accounting system, i.e. balance equations and definitional relations, which to a great extent are identical with the real flows of the national accounts. The financial flows are not included except for some post-calculations of aggregated current account figures. A major part of the statistical data required for estimation, including base year values, is supplied by the national accounts.

Sectors, commodities and primary factors

The inter-industry transactions of the economy form a central component of the MSG model. The commodity flows of the model may be described as flows between (functional) sectors. The sector concept is first of all used for the classification of establishments and similar economic units into production sectors (industries). The model has 32 production sectors, including five general government production sectors. Special attention is paid to the specification of energy producing and energy consuming industries. The major energy producing industries include sectors for the production of electricity, the production of crude oil and natural gas, and the refining of crude oil. The major energy consuming industries (energy intensive industries) include sectors for the production of pulp and paper, the production of metals, and the production of chemicals.

In addition to a classification of establishments, the sector concept is also applied to broad categories of goods and services classified by origin or use, i.e. sectors for imports, exports, household consumption, general government consumption, private investments, and general government investments.

The commodity classification is arrived at by adopting the "main producer" principle, i.e. letting all goods and services with the same industry (production sector) as the main producer form one commodity. The classifications of production sectors and commodities are thus closely related. If strictly followed, this procedure will give the same number of commodities as the number of industries, i.e. square commodity-

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1) A more comprehensive discussion of these concepts is given in Bjerkholt and Longva (1980).

by-industry matrices[1]. However, in a couple of cases energy commodities have been separated from other commodities with the same main producer. Also commodities representing imports for which there is no domestic production (non-competitive imports) are included as separate commodities. Altogether there are 42 commodities in the model. Six of these may be characterized as energy commodities, namely electricity, crude oil, natural gas, coal, petrol and fuel oil. The production sector for refining of crude oil has both petrol and fuel oil as separate output commodities while coal is a separate output commodity in the production sector for mining.

In addition to commodities, each production sector has input of primary factors, i.e. of labour and capital services. In the model there is just one category of labour input, while the model distinguishes between three categories of capital goods ("buildings and constructions", "machinery" and "transportation equipment")[2].

Activities

The rather disaggregate representation of the commodity-by-sector flows makes it possible to focus both on the industrial structure and the industrial interdependence in a growth process, both important aspects when analysing the links between economic growth and energy use. Disaggregation also makes it possible to give the energy commodities and the energy producing and consuming sectors a proper representation. The detailed input-output description makes it hardly possible, nor essential for the quality of the model results, to introduce substitution possibilities between all inputs and outputs of each sector. To simplify, we have therefore partitioned the set of detailed commodity and primary input flows of each sector into mutually exclusive and exhaustive subsets. We have a priori imposed the restriction that the production (or utility) function is separable in these subsets.[3] Each subset defines an aggregate of input or output commodities or of primary inputs.

1) This does not mean that there is a one-to-one correspondence between commodities and industry outputs. At the chosen level of aggregation there will still be significant non-zero off-diagonal elements in the commodity-by-industry output matrix, i.e. multiple output in industries.
2) In addition, capital in shipping and capital in crude oil production form separate categories.
3) Sufficient conditions and implications for a production function having this property are discussed by Berndt and Christensen (1973). Subsets for primary inputs and commodity outputs are of course only relevant for the production sectors.

Substitution possibilities are introduced only between these aggregates. Within each aggregate we assume fixed proportions, i.e. the aggregator functions are simple Leontief functions. In the following these fixed coefficient aggregates are called activities. The actual specification of activities in the producer and consumer submodels will be further discussed in sections 3 and 5.

Formally, the subdivision of sectors into activities is also extended to the import and final demand sectors others than household consumption. For imports and exports there are one activity for each commodity flow, while general government consumption activities represent types of government services. The private and general government investment activities correspond to categories of capital goods.

In the model we distinguish between commodity activities, i.e. an aggregate of commodity flows in fixed proportions, and primary activities, i.e. an aggregate of primary inputs in fixed proportions. The commodity flows between commodity activities include all generation and absorption of commodities in the economy except changes in commodity stocks.

Value concepts

The different value concepts adopted in the model are essential in the modelling of the interindustry transactions and in the modelling of substitution induced by changes in relative prices.

In general, the commodity activity coefficients are estimated from the national accounts for the base year of the model. This means that quantities of commodity flows are measured in unit prices of the base year, i.e. constant unit values. The principal concept for evaluating commodity flows in the model is (approximate) basic values.[1] The basic value concept is preferred to producers' value or purchasers' value because the trade margins (including transport charges) and commodity tax rates may vary between receiving sectors of the same commodity and thus may cause a discrepancy between total supply and total demand in constant unit values.[2]

In the MSG model the activities are evaluated in market values,

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1) The Norwegian national accounting system includes a set of value notions, as recommended in A System of National Accounts, United Nations (1968).

2) Note that, apart from trade margins and commodity taxes, there may be genuine price differentiation in the base year. This bias in the base year weights may be a source of error in the model computations. As will be discussed later, price differentiation will be explicitly corrected for in the case of electricity.

computed as producers' value of commodity outputs and as purchasers' value of commodity inputs or primary inputs. The rationale behind this choice is first of all that the substitution possibilities within each sector is specified between activities, not between commodities. Market prices are then the relevant price concept in modelling the producers' and consumers' behaviour.

Energy flows in physical and value terms[1)]

The model outlined in this chapter is designed to be used inter alia as a tool for planning the capacity of the electricity sector. It is therefore particularly important that the supply and demand of electricity in constant value terms in the model can be correctly "translated" into physical units, i.e. kWh. For other energy flows the translation from value terms to physical units is considerably simpler.

As mentioned above, constant basic values are adopted as volume measures for commodity flows. In the case of electricity, the basic value flows recorded in the national accounts are revised in two respects to provide proper volume measures in the model. First, the single electricity flow in the national accounts is divided into the two model commodities electricity and distribution services with two corresponding production sectors. The two commodities are constructed by deducting calculated, user differentiated distribution costs from the recorded single flow in the accounts. Second, price differentiation terms are deducted from the remaining electricity flow to yield a constant value flow for electricity which is proportionate to kWh for all users.

The resulting constant value flow defines the volume concept for electricity in the model, referred to as "constant standard value". The price differentiation terms are specified explicitly in the model as artificial "taxes" or "subsidies" with differentiated rates.[2)] On the demand side of the model the two commodities, electricity and distribution services, are assumed to be used in fixed but purchaser differentiated proportions. In model language they thus constitute one commodity activity in each sector.

1) A more thorough discussion is given by Longva and Olsen in chapter V of this volume.
2) Total net price differentiation is conventionally normalized to zero in the base year.

The restructuring of the national accounting figures elaborated above is of course motivated by the need to specify energy flows adequately in the model. Distribution costs per kWh received vary considerably between users. Energy intensive industries receive electricity directly from the high voltage transmission system, hence both capital costs and transmission losses are relatively low. Households receive electricity transmitted through all stages of the power grid, labour and capital costs and power losses per kWh received are thus relatively high. On the macro level, the distribution services needed per kWh are therefore dependent on the composition of demand. Since the input structures and scale properties of the production and distribution sectors are rather different, the total resource use on the supply side is highly dependent on the composition of electricity demand. Specifying two sectors for electricity supply pays attention to these dependencies.

Due to the rather detailed treatment of electricity demand and supply in the model the time series data from the national accounts have to be supplemented with information from other data sources. The model structure outlined above thus requires a decomposition of value flows in the national accounts. The necessary information to estimate this breakdown empirically is provided by electricity and industrial statistics and by the Norwegian energy accounts, which are closely related to the national accounting system.

3. The sector model of production

Several recent studies of energy demand indicate, for aggregated sectors, considerable substitution possibilities between different energy commodities, and also between energy and other aggregate inputs.[1] In MSG such substitution possibilities are integrated in the model.

The formal specification of the production structure is similar for most of the 27 industry sectors of the model. The neoclassical theory of production, formalized by Generalized Leontief cost functions and Hicks neutral technical change, is chosen as the approximation of producers' behaviour. For the energy producing sectors and for some of the main energy using sectors this approach has several limitations. For these sectors the neoclassical approach will therefore be supplemented, or replaced, by separate process oriented sector models. For the electricity supply sector such a sector model is from the outset integrated in the main model.

1) For a survey of such studies, see Blaalid and Olsen (1978).

General model

A primary objective of the model for producer behaviour is to derive demand functions for each commodity and primary input of each industry and supply functions for each output commodity. To simplify we have, as discussed above, imposed a two-tier structure on inputs and outputs of each sector: Within each sector the substitution possibilities are introduced only between aggregates of commodity and primary inputs, i.e. between activities.

On the input side, commodities and primary inputs are aggregated into five input activities, namely one for capital services (three types of capital goods), one for labour (one type only), one for materials (all non-energy commodities), one for electricity (electricity and distribution services) and one for other energy inputs (petrol and fuel oil), for short called fuels. On the output side commodities supplied by a sector are, with three exceptions discussed later in this section, aggregated into one output activity.

The model for producer behaviour is defined in terms of these input and output activities. Since there are fixed proportions between the flows (commodities or primary inputs) composing each activity, the commodity supply and demand, and primary input demand, are easily derived once the activity levels are determined.

The neoclassical theory of production can be represented in two ways, as shown e.g. by Diewert (1971); either by postulating production functions and necessary conditions for producer equilibrium or, alternatively, by directly specifying the cost functions of the model. Under certain assumptions the two procedures will give an equivalent description of the production structure. Cost functions are convenient since input demand functions (following Shephard (1953)) can be derived simply as partial derivatives of the cost functions with respect to the corresponding input prices. Furthermore, this specification can facilitate both the estimation and the solution of large equation systems. Such considerations form the background for our choice of cost functions.

Below a brief outline of the relations between the specified activities and the corresponding price indices is given. On the demand side of the producer behaviour model we can, for an arbitrary sector, write

$$(1)\quad A_i = Z_i A_X \qquad i = \begin{cases} M & \text{Materials} \\ E & \text{Electricity} \\ F & \text{Fuels} \\ L & \text{Labour} \\ K & \text{Capital} \end{cases}$$

where A_i are activity levels for aggregated inputs,

A_X is the output activity level (total output, assuming only one output activity in each sector), and

Z_i are input-output coefficients for the various input activities.

The Z-coefficients are endogenously determined by assuming, first, the existence of a "well-behaved" homogeneous production function of degree one and, second, that factor demand is determined by cost minimization. The producers are assumed to be profit maximizers which includes the assumption of cost minimization. The input-output coefficients for activities in the sector can then be written as a function of input prices and technical change, i.e. as

(2) $$Z_i = f_i\ (P_M, P_E, P_F, P_L, P_K, t)$$

where P_M, P_E, P_F, P_L, and P_K are price indices for commodity and primary input activities, and

t represents technical change.

When the production function is homogeneous of degree one the profit maximization fails to determine a unique supply curve for sector output. On the supply side of the producer behaviour model we instead assume that in each industry the output will be priced such that the price covers average costs, i.e. zero excess profit.

We can then write

(3) $$P_X = c$$

where c is total unit costs, and

P_X is the price of the output activity.

Total unit costs can be written as

(4) $$c = Z_M P_M + Z_E P_E + Z_F P_F + Z_L P_L + Z_K P_K + T_S$$

where T_S represents net indirect taxes per unit of total output.

For given input prices the output price is thus independent of the output level and the producer supplies what is demanded without any changes in prices, i.e. the supply curve is infinitely elastic.

The principal features of the production structure outlined above are similar to the model of producer behaviour in the energy model for the American economy developed by Hudson and Jorgenson (1974). In that model the cost functions are represented by Translog price possibility frontiers. The point of departure for our energy model for the Norwegian economy is the MSG model where the production structure is represented by fixed coefficients or Leontief technology for input of commodities and a Cobb-Douglas technology for labour and capital inputs. A generalized representation of this rather rigid structure is provided by the Generalized Leontief (GL) cost function, first introduced by Diewert (1971).

In estimating these cost functions an additional a priori hypothesis of separability is introduced by assuming that the two specified energy activities can only be substituted against other inputs via an aggregate for total energy input. Thus, on the most aggregate input level, only four inputs are specified in the production functions; labour (L), capital (K), materials (M) and total energy (U). While the aggregator functions for each of the first three of these inputs are described simply by fixed coefficient activities, a GL functional form is chosen for the energy activity aggregate to allow for substitution between electricity (E) and fuels (F).

When a production function is separable in some aggregate inputs, the corresponding cost function will be separable in the respective price indices. The dual to the energy activity aggregate can thus be thought of as an aggregate price index for energy. Furthermore, assuming that the energy activity aggregate is linearly homogeneous in its components, the price index in market equilibrium will equal average energy costs. This opens for a two-stage optimization procedure; first optimize the mix of activities within the energy aggregate and then optimize the level of the four aggregate inputs.[1)]

Assuming that the production function of an arbitrary sector is homogeneous of degree one, the GL cost functions can be set out as

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1) Homotheticity is a sufficient condition for the validity of the two-stage procedure. The further restriction of linear homogeneity is required to ensure that the product of the aggregate price and quantity indices equals total energy cost. Since the activities themselves are commodity and primary input aggregates the cost minimization can actually be viewed as a three-stage procedure. However, because of the assumption of simple Leontief technology the "optimization" in the first stage degenerates to provide that there are no waste of resources.

$$(5)\quad C = H(t)A_X\sum_i\sum_j c_{ij}(P_iP_j)^{\frac{1}{2}} \qquad i,j = L,K,U,M$$

where the P's are price indices for the four aggregated inputs and A_X is the output activity level as defined above. The term H(t) represents an assumption of Hicks neutral technical change.

Differentiating the cost function with respect to the input price, P_i, gives the demand function for the corresponding input aggregate. Dividing by the output activity level the input-output coefficients are derived as

$$(6)\quad Z_i = H(t)\sum_j c_{ij}P_i^{-\frac{1}{2}}P_j^{\frac{1}{2}} \qquad i,j = L,K,U,M$$

The price indices for total energy, specified as GL unit cost functions, are defined by

$$(7)\quad P_U = \sum_i\sum_j b_{ij}(P_iP_j)^{\frac{1}{2}} \qquad i,j = E,F$$

Equivalent to (6), the demand for the energy activities relative to total energy input is derived as functions of the corresponding activity prices by differentiating the system (7). Multiplying these energy coefficients by the input coefficient Z_U in (6) gives the input coefficients for the two energy activities relative to total output as postulated in general form by relation (2).

The estimation of cost functions is based on national accounting figures for the five aggregate inputs labour, capital, materials, electricity and fuels, and price indices of the same inputs. A further discussion of the model specification, estimation procedure and empirical results is given by Longva and Olsen in chapter III of this volume.

4. Energy production

The most important energy producing sectors in the Norwegian economy are the electricity supply sector and the sectors for extraction and refining of crude oil and gas. In addition to the energy outputs from these sectors coal is explicitly specified as an output commodity of the mining sector of the model. Some minor products which might be characterized as energy goods are included in non-energy commodities of the model. As will be described below the electricity sector in MSG is analysed in a rather detailed way, while the production of the other energy commodities is given a rather simple treatment in the present version of the model.

Production of electricity

In Norway the electricity supply system is based on hydroelectric power. Public authorities have a decisive influence on the planning and operation of the sector.

The electricity sector in the MSG model is subdivided in two parts, with separate production functions for the production of electricity and for the production of electricity distribution services. The production part of a hydro power system has the following characteristic features:

i) Nearly all costs can be considered as fixed costs consisting of capital outlays, while variable costs (wages and material costs) are low. This cost structure is very different from that of thermal power plants, where variable costs dominate.

ii) The capacity of a hydro electric power system has two main dimensions. As in a thermal power system, it is necessary to provide sufficient plant capacity to meet peak load demand. In addition, the hydro electric system must provide a sufficient amount of primary energy by water storage facilities with a capacity sufficient to meet normal variations in annual energy demand.[1)]

iii) A common feature of production based on extraction of natural resources is decreasing returns to scale. Thus, in modelling the production structure of a hydroelectric power system it is essential to allow for increasing marginal costs.

The production model for electricity is based on relationships and data on the micro level. In this chapter only the derived macro relationships included in the MSG model will be indicated; for a complete presentation of the production model for electricity, see Rinde and Strøm in chapter VI of this volume.

A given water storage represents a certain volume of potential

1) Runoff to the reservoirs varies from period to period. For any given storage capacity there is therefore a certain risk that deficiencies may occur. This uncertainty must be handled outside the MSG model in the actual planning of the supply system. For an analysis of this problem see the article of Bjerkholt and Olsen in chapter XIII of this volume.

energy (kWh). The load capacity (in kW) of the hydro power system will depend on the capacity of the waterways (tunnels) from the reservoirs to the power stations and on the efficiency of turbins and generators in the hydro plants.

The demand for electricity varies during the day and over the year. Integration of the actual load curve gives the total annual electricity consumption. To simplify the MSG model we assume that the proportion between load capacity in the hydro power system and the annual production of electric energy is given. Accordingly, the planning of the hydro power system is in the model reduced to one dimension, namely to determine annual electricity production.

In MSG the minor inputs in the production process, labour and materials, are assumed to be proportionate to the output level. By the assumption of proportionality between the volume of machine installations and the load capacity and the exogenously given proportion between load capacity and energy production, the amount of machinery (turbins and generators) also becomes proportionate to the output level. A more flexible function is specified for input of constructions and buildings, estimated to give decreasing returns to scale in electricity production.

Assuming cost minimization, and specifying the function for constructions and buildings as a constant elasticity function, the cost function may be derived as[1)]

$$(8)\quad C = \left[\frac{P_{LP}}{\alpha_{LP}} + \frac{P_{MP}}{\alpha_{MP}} + \frac{P_{GP}}{\alpha_{GP}}\right] A_{XP} + \frac{P_{CP}}{\alpha_{CP}} A_{XP}^{\gamma}$$

where P_{LP} and P_{MP} are price indices for labour and material inputs,

P_{GP} and P_{CP} denote indices for user costs of capital for machinery and construction, respectively,[2)]

A_{XP} is the production of electricity, and the

α's and γ are parameters.

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1) In the operational MSG model the cost function (8) is actually related to changes in the capacity from a chosen base year. Factor augmenting technical change is also included in the cost function.

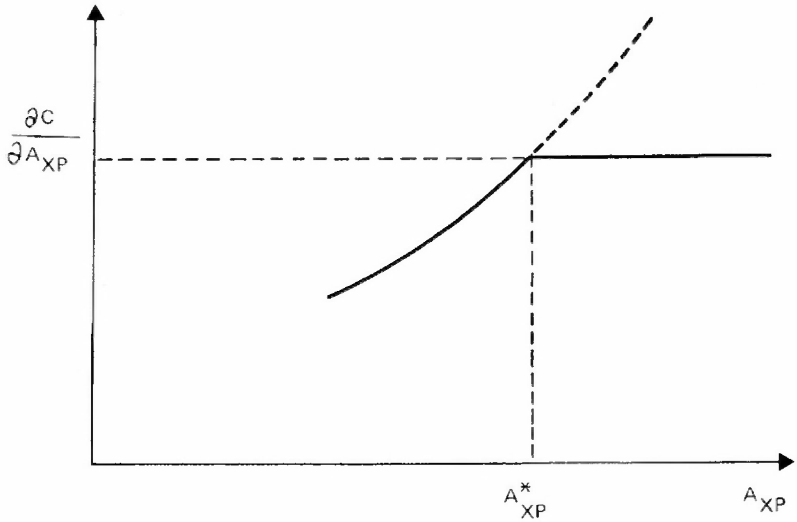
2) The user costs in the electricity sector are defined by the relation $P_{kP} = \nu(R,t) \cdot P_{kI}$ (k=C,G) where ν is an annuity factor depending on a marginal rate of return (R) imposed on public investments and the economic life time of the projects (t), and P_{kI} (k=C,G) are price indices of the investment activities for the two categories of real capital. This way of calculating user cost of capital implies net profits in sectors with decreasing returns to scale.

The estimation of the cost structure (8) is based on time series of calculated costs of present and potential water power projects. In the planning of the electricity sector the Norwegian Water Resources and Electricity Board has developed methods to calculate and rank all known water power projects according to average costs at constant prices.[1] Average cost in a "marginal plant" can be considered as an approximation to the long-term marginal cost of the production system. The schedule of water power projects is therefore used to estimate the γ-coefficient in the marginal cost function specified as[2]

$$(9) \quad \frac{\partial C}{\partial A_{XP}} = \left(\frac{P_{LP}}{\alpha_{LP}} + \frac{P_{MP}}{\alpha_{MP}} + \frac{P_{GP}}{\alpha_{GP}}\right) + \gamma \frac{P_{CP}}{\alpha_{CP}} A_{XP}^{\gamma-1}$$

The derived cost curve will have to be somewhat modified allowing for the fact that the Norwegian electricity supply system at a certain level of energy capacity will be supplemented by thermal power. In a supply system based for example on oil, gas or coal, constant returns to scale will typically be the case; hence the marginal cost will be constant. The complete marginal cost function for the production of electricity is illustrated in figure 2.

Figure 2. Long run marginal costs in electricity supply



1) See Statens Energiråd (1969) and Fagerberg (1978). Of course these cost calculations take into consideration only techniques that are available today. Thus, in the estimation of the cost structure (8) on these data technical change should be neglected.
2) Note that all prices are kept constant in the estimation of γ.

At the production level A^{*}_{XP}, the costs of increasing the water power production just equal the marginal cost of introducing thermal power in the supply system. From that point the increasing marginal cost curve for hydro electric power is therefore irrelevant.

As outlined above, the relations to be specified in the MSG model are the relative demand function (2) for input activities. The demand function for the various inputs is obtained as the partial derivative of (8) with respect to the relevant price index. For materials, labour and machinery the corresponding input coefficients (the Z's) equal the inverse of the α-parameters, hence the input coefficients of the electricity producing sector are independent of relative input prices. The input coefficient for construction will depend on the output level A_{XP}, but not on relative prices. Thus, marginal costs will in general not be equal to average costs, and a pricing principle other than (3) is introduced. In the use of the model it is most convenient to assume that the price of electricity is exogenously given. However, by iterative solutions the model can also be utilized to trace out optimal investment paths for the electricity sector in the sense that the price of electricity in the long run should equal marginal costs. The specification and estimation of the production model for electricity is discussed in more detail by Rinde and Strøm in chapter VI of this volume.

Distribution of electricity

In the distribution part of the electricity supply sector total physical power losses are regarded as inputs in the production process, in addition to the inputs of labour, capital and materials.[1] As in the production part, input of labour and materials are assumed to be proportionate to the output volume. Between the two production factors capital and power losses we assume substitution possibilities since engineering studies suggest that power losses can be reduced considerably by reinforcement of the distribution network. There are some indications of increasing returns to scale in the production of distribution services. Some of the construction costs may be less than proportionate to the number of kWh distributed and the marginal costs

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1) The capital input in the distribution network is related both to load capacity and to the volume of annual energy deliveries. However, as in the production part, the planning problem is simplified by the assumption of proportionality between load capacity and the volumes of distribution services.

following a future increase in electricity demand may accordingly be exceeded by average costs.[1)]

Assuming the input function for capital and power losses to be homothetic with a constant elasticity of scale, the cost function may be written as[2)]

$$(10)\quad C_D = [\frac{P_{LD}}{\alpha_{LD}} + \frac{P_{MD}}{\alpha_{MD}}]\, A_{XD} + A_{XD}^{\mu}\, h_D(P_{KD}, P_{ED})$$

where P_{LD} and P_{MD} are price indices for labour and material inputs,

P_{KD} is a price index for the user costs of capital,

P_{ED} is a price index for input of electricity (power losses),

A_{XD} is total output of distribution services, and the

α's and μ are parameters.

The h_D-function is specified as a Generalized Leontief unit cost function.

The relations explicitly specified in the MSG model are again the input coefficients for the production factors. The Z-coefficients for labour and materials are defined simply as the inverse values of the corresponding α-parameters. However, due to the general formulation of the h_D-function, the Z-coefficients for the capital and electricity inputs are functions of the output level of distribution services and the prices of the two inputs.

The estimation of the input demand functions is mainly based on data drawn from electricity statistics of Norway, which provide information of labour input, material costs, and physical power losses. Investments in the electricity sector are subdivided in production plants and distribution network, respectively; the statistics thus provide information

1) Increasing returns to scale in the production of distribution services must not be confounded with the effect on production caused by changes on the demand side of the model. The coefficients of the input activities for electricity, which include both electricity and distribution services as separate commodities, vary between the various receiving sectors. Changes in the composition of electricity demand may therefore change the relation between the production of distribution services and the total volume of kWh produced.

2) In the operational model factor augmenting technical change is included.

for the calculation of capital in both sectors.[1] The model for distribution of electricity is further discussed by Schreiner and Strøm in chapter VII of this volume.

Production of other energy commodities

In addition to electricity the energy commodities of the model include crude oil, natural gas, petrol, fuel oil and coal. Compared with the detailed representation of the production structure for electricity, the production of other energy commodities are treated in a rather simple way.

For the extraction of crude oil and natural gas, which is an important and mainly export oriented off-shore industry, separate estimates of oil and gas production and the input activities for materials, energy, and labour services are exogenously given. In addition, gross investments are also exogenous. With constant rates of depreciation the gross investment estimates determine the development of the capital stock. This means that both the production level and the input structure are determined outside the model. The prices of crude oil and natural gas are exogenously given, which implies that return to capital in the oil sector is endogenous (determined as a residual). This straight forward and exogenous treatment of the oil sector is due to the fact that oil and gas production is under tight government control. The interactions between the oil sector and the rest of the economy are reasonably well depicted, but the description is clearly deficient for analysing the resource allocation within the oil sector. Several rather detailed sector models are developed for that purpose.

The oil refinery sector is the main producer of both fuel oil and petrol. The output mix of fuel oil and petrol can be varied by refining different qualities of crude oil. We are therefore assuming that the two commodities are produced non-jointly, i.e. with separate production functions. By adding an assumption of separability in inputs and outputs of the production sector as a whole it is implied that the two individual production functions are identical (see Hall (1973)). Two additive output activities, one for each output commodity, are therefore specified. The price indices of fuel oil and petrol are set equal and determined in

1) The user cost of capital in the distribution network is calculated in the same way as for the cost structure of the production part, i.e. by applying the rate of interest imposed on public investments and by specifying a relevant depreciation formula. The relevant price for power losses is the long-term marginal cost in the production of electricity. The costs of producing an additional kWh should be balanced against the costs of reducing the power losses with a kWh by reinforcement or rebuilding of the distribution network.

the way indicated by equation (3), i.e. as cost determined prices.

Even though coal at present is a minor domestic product, it is, due to possible future imports for the production of electricity, given a separate treatment. The domestic production is included in the mining industry as a separate non-joint output activity. Both production and price are exogeneously given.

For all energy sectors where production is determined exogenously, imports is determined as the difference between production and domestic use plus exports.

5. Household consumption

As on the production side of the model a multi-tier structure is imposed on the inputs to the household consumption sector. The individual input commodities are aggregated into 18 activities. Once the levels of these activities are determined, the demand for each commodity can be derived from the assumption of fixed proportions within each activity. The household demand system included in the core of the MSG model may be viewed an approximation to a more elaborate sector model for household consumption. The sector model, developed by Rødseth and presented in chapter X of this volume, integrates some of the impacts of consumer durables on energy demand into a system of demand functions. The amount of details in this sector model makes it too cumbersome to be formally integrated in the main equation system of MSG. The household consumption sector model may be used instead to estimate parameters of the approximate demand system of the main model. In the present version of the MSG model the estimation procedure is directly related to empirical specification of the approximate household demand system. Below we shall first describe the main features of the approximate demand system of the main model and then give a brief outline of the sector model for household consumption.

The demand system of the main model

In the MSG model total consumption expenditure is defined in two alternative ways. One is the national accounting concept, defined as the value of commodity purchases, including durable goods. Imputed costs (interest and depreciation) are used only for housing services. The alternative concept of total household expenditure differs from the national accounting concept by having purchases of cars deducted and imputed costs of car services (interest and depreciation) added. Since household consumption is a flow the latter concept is theoretically more appropriate in a consumption model. For instance, it is more reasonable to assume direct substitution between the use of private cars and public transportation than between purchases of cars and use of public transportation. But, to provide the link between household consumption and production and imports it is necessary to calculate the purchases (gross investments) of cars by households.

The budget constraint of households is written

$$(11) \quad V_C = \sum_j P_{Cj} A_{Cj} \qquad (j=1,\ldots,18)$$

where V_C is total household expenditure (with imputed costs of car services)[1],

A_{Cj} is the activity level of consumer activity j, and

P_{Cj} denotes the price index of consumer activity j.

The total household expenditure is distributed between the 18 consumer activities according to a system of demand functions written as

$$(12) \quad A_{Ci} = \alpha_{Ci}(\theta_C V_C)^{\xi_i} \prod_j P_{Cj}^{\kappa_{ij}} \qquad (i,j=1,\ldots,18)$$

where α_{Ci}, ξ_i, κ_{ij} are parameters, and

θ_C is an endogenous variable necessary to ensure that the demand functions are consistent with the budget constraint at any point of time ("horizontal adjustment of Engel curves").

The relations (11) and (12) may be viewed as a local approximation to an arbitrary and more complicated system of demand equations.

It should be noted that if the variable θ_C in (12) is equal to one, as will be the case in the base year, the parameters ξ_i and κ_{ij} can in interpreted as total expenditure and price elasticities, respectively Estimates of the parameters in the equation (12) may thus be price and income elasticities in the base year. A discussion of the estimation procedure and empirical results is given by Bjerkholt and Rinde in chapter IV of this volume.

The sector model for household consumption

The sector model is based on the "new" approach to consumer theory where households combine commodities in consumption technology functions to produce the consumption "goods" that enter the utility function. The

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1) All consumption activities are calculated in per capita terms.

system of demand functions is derived by specifying the indirect utility function of the quadratic expenditure system, see Pollak and Wales (1978). The utility function of "the representative consumer" is assumed to be separable in the consumption goods (activities and activity aggregates). Assuming that the households are minimizing costs in "producing" these consumer goods the indirect utility function is separable in the corresponding price indices.

Households are divided into eight groups according to (i) four types of dwelling and heating equipment and (ii) whether the household owns a private car or not.[1] While the utility functions for the various groups of households in the sector model are assumed to be the same, the technology relations differ between groups. With this specification it is possible to include effects on aggregate demand of changes in stocks of heating and transportation equipments. While most consumption production functions are simply activities (commodities in fixed proportions), two of the consumption goods, "light and heating", and "transportation", are defined as activity aggregates being produced with more flexible technologies.

6. Use of the model

The aim of the project is to design and make operational a model suited for analysing alternative energy policies. The model emphasizes the description of energy supply and demand while the rest of the economy is also given a relatively disaggregate description in MSG. The main advantage of using MSG is its ability to trace out coherent and consistent alternative paths of development. In the overall planning of the economy energy production and use play a central role, mainly as instruments in achieving overall goals rather than as targets by themselves. If the model fails to predict the development of the total economy, it will also fail to predict supply and demand of energy, even if energy relations are correctly represented in the model.

Studying alternative energy policies requires that actual policy instruments can be translated into model parameters. For instance, indirect taxes on electricity by user are explicitly specified in the model, which allows for analyses of impacts of changed electricity prices via indirect taxation. Different development programs for electricity supply can be "controlled" via the specified sector investments.

1) The model includes relations describing the distribution of households between the different groups (see Rinde (1979)). In later versions of the main model these relations may be integrated in the consumer demand system to facilitate the updating of the parameters.

With respect to energy analyses the model is intended to be used in three major areas:

i) Planning of the electricity sector. In the production of hydro electric power the time lag between investments and new production capacity is 4-6 years. In the short run the principal problem is to regulate demand for a given capacity. The long run problem also includes determining the growth path for the optimal capacity and the break-even point between long-term marginal costs in production of hydro electric power and production of electricity based on oil or coal. Examples of using the MSG model for the planning of the electricity sector are given by Lorentsen, Strøm and Østby (1979) and in chapter IX of this volume.

ii) Demand analyses, i.e. effects of changed demand patterns of industries and households. For example the model may be used to calculate the impacts of eliminating price discrimination in the electricity market or the effects of changes in the relative prices for electricity and fuels. Another example could be to calculate effects of energy conservation programs imposed on the consumers.

iii) Analyses of resource allocation. Alternative energy policies would mean different allocations of labour, capital and production between industries and regions. In the model, all production factors are assumed to be freely moveable. Since both labour and capital, particularly in energy intensive industries, could be regarded as local resources, any considerable reshuffling of labour and capital should be checked for realism. An example of using the MSG model for demand management and resource allocation problems is provided by Bjerkholt, Lorentsen and Strøm (1980 and 1981).

For analyses mentioned above the relevant scope of the model would be 10 - 20 years, long enough to allow for changes in economic structure and short enough to assume technology roughly predictable.

III. PRODUCER BEHAVIOUR IN THE MSG MODEL

by

Svein Longva and Øystein Olsen

1. Introduction

In analysing the energy-economic interface in a long-term macroeconomic context the substitution possibilities between various kinds of energy and other inputs (materials, labour and capital) within the production processes are of crucial importance. An important element in the development of the long-term growth model MSG to cover the field of energy analysis has therefore been to introduce production functions that do not place (strong) a priori restrictions on the substitution possibilities[1].

The study presented below outlines the main elements of the model of producer behaviour of MSG, emphasizing the presentation of substitution possibilities. The study also reports on the empirical findings concerning price sensitivity of energy demand in Norwegian industries.

The formal specification of the production structure and producer behaviour for the majority of industries in the MSG model is based on the neoclassical theory of production. This theory has been criticized from several points of view. Many authors, see e.g. Fisher (1969), have pointed out the serious aggregation problems in empirical applications of the theory. More realism could be given to empirical applications by distinguishing ex ante decisions from ex post adjustments. However, the data situation makes the estimation of any sort of vintage model rather troublesome. It should also be emphasized that the model of producer behaviour is included in a macroeconomic model designed to trace out the long-term growth paths of the economy and the interactions between the various sectors without spelling out the timing of the adjustment processes. For this purpose the neoclassical production model may be palatable as an approximation to the actual production structure and producer behaviour.

Altogether the model includes 27 production sectors for private industries. For some of these industries one may, on a priori grounds, doubt whether the crucial assumptions of a "well-behaved" production function and maximizing behaviour underlying neoclassical theory of

1) In the previous versions of the MSG model the production model imbedded the assumption of Leontief technology (fixed coefficients) for commodity inputs and a Cobb-Douglas technology for the primary inputs (labour and capital) of each industry.

production are likely to be fulfilled, even as a rough approximation for long-term analysis. This applies for example to Agriculture, Fishing and Domestic transportation, where capital formation and labour demand are strongly influenced by the government via subsidies and direct regulations. Accordingly, it is doubtful whether relations derived from rather simplified neoclassical theory and estimated solely from observed market data are of much value. In the present version of the MSG model the parameters of the input demand functions for these three industries may be characterized as "guesstimates"[1]. Furthermore, the four energy producing industries (Production and Distribution of electricity, Extraction and Refining of crude oil) and Ocean transport are treated separately[2].

The modelling of energy demand and estimation of substitution possibilities for the remaining 19 industries is the topic of this study. In section 2 the general framework is discussed. The deterministic model and a discussion of price and substitution responses are presented in sections 3 and 4, respectively. An overview of the data used is given in section 5, while sections 6 and 7 comprise the method of estimation and empirical results and the derived demand elasticities for the specified inputs. In section 8 the results for the aggregate manufacturing industry are compared with similar estimates from other studies.

2. The general framework

By choosing a sufficiently flexible functional form within each industry it is possible to introduce substitution possibilities between every detailed commodity and primary input specified in the MSG model. However, this will imply a large number of parameters to be estimated, and will complicate the simulations of the complete model. In an energy oriented model it is hardly essential for the quality of the model results to introduce substitution possibilities between all non-energy inputs of each sector. We have therefore restricted the substitution

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1) It may be reasonable to regard capital input in Domestic transportation and labour input in Agriculture as fixed production factors, and assume that the producers minimize costs only with respect to the variable inputs. As well known this behaviour leads to a set of demand functions with the fixed factor as argument in addition to the prices of the variable inputs. In later versions of the model this type of "restricted" demand relations for the industries in question may be introduced.

2) The sector models and the estimated relations for the Production and Distribution of electricity are presented in chapters VI and VII, respectively. The production structures for Extraction and Refining of crude oil and Ocean transport are completely exogenous in the present version of the model.

possibilities within each industry by a priori assuming that the production structure is weakly separable in five aggregate inputs: capital (A_K), labour (A_L), electricity (A_E), fuels (A_F) and materials or non-energy commodity inputs (A_M). The aggregates are called activities. Within each activity fixed proportions are assumed, i.e. the aggregator functions are simple Leontief functions.

By aggregating individual commodity flows and primary inputs we have implicitly assumed that the production function is weakly separable in the specified subsets of inputs. An important implication of the separability assumption is that all elasticities of substitution between the inputs belonging to an aggregate and an arbitrary variable outside this subset are identical[1].

In addition to the separation of the industry inputs into activities a further separability condition is introduced restricting the substitution properties of the two energy inputs. Electricity and fuels are assumed to be weakly separable from the other aggregate inputs, implying that the energy goods are only substituted against other inputs via an aggregate for total energy input, in the following denoted by A_U. This specification is partly motivated by the fact that it reduces the number of parameters to be estimated. Moreover, this approach is comparable with recent studies of energy demand, e.g. Fuss (1977) and Pindyck (1979).

The production function for each industry can thus be written as

$$(1) \quad A_X = F(A_K, A_L, A_U(A_E, A_F), A_M; t)$$

where A_X denotes total output and t technical change.

The producers are assumed to be profit maximizers which includes the assumption that the composition of inputs is determined so that least cost production pattern is undertaken, i.e. cost minimizing. If the A_U-function in (1) is homothetic, the optimal proportion between the two energy activities is independent of the level of the energy aggregate A_U. Consequently the producer may reach the cost minimizing factor input composition in two steps; first by optimizing the mix of activities A_E and A_F within the energy aggregate and then optimizing the mix of the four aggregate inputs A_K, A_L, A_U and A_M. A corresponding two-stage procedure may be applied in the estimation of the relations describing producer behaviour. In our study the A_U-functions are assumed to be line-

1) For a discussion of functional separability and elasticities of substitution see Berndt and Christensen (1973).

arly homogeneous. This is required to ensure that the product of the aggregate price and quantity indices equals total energy costs.

When cost minimizing behaviour is assumed it is well known that the neoclassical theory of production can be represented in two ways; either by postulating production functions and the necessary conditions for producer equilibrium or, alternatively, by directly specifying the dual cost functions. Under certain assumptions the two procedures will give an equivalent description of the production structure and producer behaviour. In MSG we have chosen the specification of cost functions. Our motive is first of all that it facilitates the estimation and the simulation of the model since the input demand functions may be derived simply as partial derivatives of the cost functions with regard to the corresponding input prices (Shephard's lemma, Shephard (1953)).

When a production function is separable in certain subsets of inputs and each of the aggregator functions is linearly homogeneous, the cost function will be separable in the corresponding aggregate price indices (Berndt and Christensen (1973)). Especially, the dual of the energy activity aggregate can be thought of as an aggregate price index for energy. The cost structure of an industry can thus be represented by a cost function of the form

$$(2) \quad C = G(P_K, P_L, P_U(P_E, P_F), P_M, A_X; t)$$

where C is total cost and the P's are factor prices.

In accordance with the formulation above, P_K, P_L, P_E, P_F and P_M are simply activity price indices while the aggregate price index for energy, P_U, is a flexible dual unit cost function for the energy activity aggregate.

Following "Shephard's lemma" the system of demand functions related to the cost structure (2) may be derived as

$$(3) \quad A_i = g_i(P_K, P_L, P_U, P_M, A_X; t) \equiv \frac{\partial C}{\partial P_i} \qquad i = K,L,U,M$$

$$(4) \quad A_i = A_U \cdot f_i(P_E, P_F) \equiv A_U \cdot \frac{\partial P_U}{\partial P_i} \qquad i = E,F$$

Theoretically, the assumption of profit maximization furthermore implies that marginal cost equals the output price, i.e.

$$(5) \quad \frac{\partial C}{\partial A_X} = P_X$$

where P_X is the price of the output activity. (5) uniquely determines the supply function.

However, if the production function is linearly homogeneous, profit maximization fails to determine a unique supply curve. For most industries the assumption of constant returns to scale, i.e. linearly homogeneous production functions, are imposed. In these industries it is assumed that output is priced in such a way that the output price just covers average costs, which equal marginal costs. This means that (5) is still valid, but must be interpreted as a restriction on the output price rather than as a supply function. Cost minimization is then sufficient as a description of producer behaviour.

3. The measurement of price effects

Within the model of producer behaviour presented above it is possible to separate the total effect of a change in an energy price into two steps. The first step is related to the inter energy substitution. Electricity and fuels can be substituted against each other in order to "produce" the same amount of energy input, A_U. Accordingly, from the relation (4) one can define the elasticity of energy good i with respect to the price of energy good j, η_{ij}, holding total energy input constant as

$$(6) \quad \eta_{ij} = \frac{\partial \log f_i}{\partial \log P_j} ; \qquad i,j = E,F$$

Adopting the terminology used in Berndt and Wood (1979) these partial elasticities may be called gross price elasticities.

This inter energy substitution represents a movement along a certain A_U-isoquant in the $A_E - A_F$ diagram (see figure 1a). For example, if the price of electricity is increased, the optimal composition of the energy inputs may, as a first partial effect, shift from S to M along the A_U^S-isoquant.

The increased electricity price will lead to an increase in the price index for total energy, P_U (assuming that the partial derivatives of the P_U-function are positive). If the same production level is to be maintained it will in general also be beneficial for the producer to change the composition of the aggregate inputs, reducing the input of total energy. This second type of price effect may be measured by the set of partial price elasticities, ε_{ij}, defined from the demand func-

Figure 1. Effects of an increase in the price of electricity. A simplified example

Figure 1 a. Effects on the use of electricity. The use of electricity is reduced by an inter energy substitution effect (the electricity price is increased) and by scale effects following the lower demand for total energy.

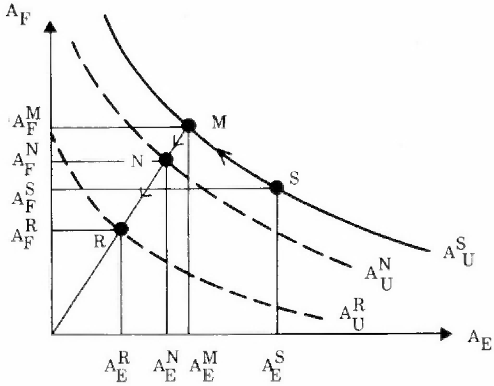


Figure 1 b. Effects on total energy use. The use of energy is reduced by a substitution effect (the price of total energy is increased) and a scale effect (the demand for output is reduced).

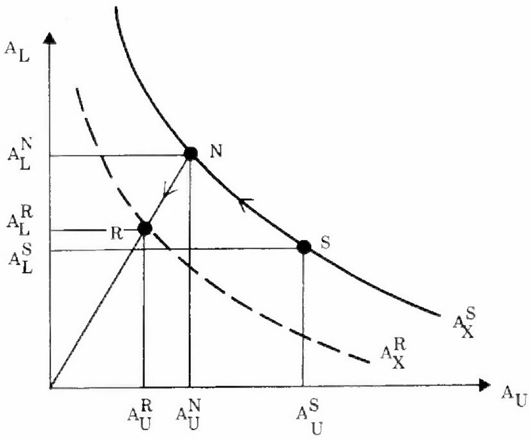
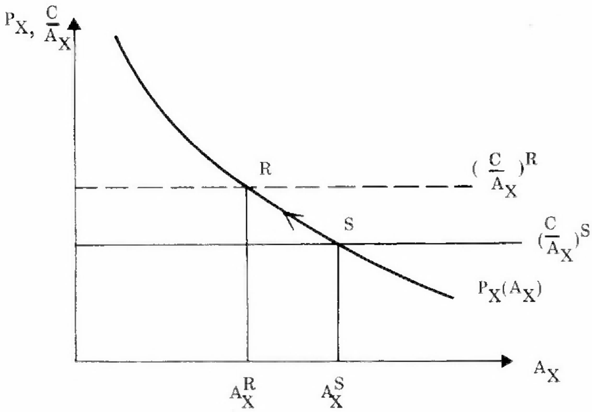


Figure 1 c. Effects on the output level. The output level is (or may be) reduced as a result of increased average costs and output price.



tions (3) as

$$(7) \quad \varepsilon_{ij} = \frac{\partial \log g_i}{\partial \log P_j} ; \qquad i,j = K,L,U,M$$

In the two-factor case (specifying only energy and labour as inputs) the substitution effect between aggregate inputs may be illustrated as in figure 1b. Under the restriction that the same production level, A_X^S, is going to be supplied, the increased energy price may lead the producers to change the input mix from S to N. Assuming a "well-behaved" cost (production) function the direct effect on the use of energy is negative.

The reduced level of total energy input has additional effects on the demand for electricity and fuels. As the energy price function is homogeneous of degree one in the "output" level (A_U), the inputs of the two energy goods will be reduced in proportion to the change in total energy use. Returning again to figure 1a this internal "scale" effect is represented by a movement from M to N along the ray passing through origo and the point M.

Combining the substitution and scale effects on the demand for electricity and fuels the <u>net price elasticities</u> for the two energy goods may be defined as

$$(8) \quad \varepsilon_{ij} = \eta_{ij} + \varepsilon_{UU} \cdot \frac{\partial \log P_U}{\partial \log P_j} = \eta_{ij} + \varepsilon_{UU} \cdot S_{Uj} \qquad i,j = E,F$$

where S_{Uj} is the fitted cost share of the energy good j within the energy aggregate.

The gross and net price elasticities defined above may all be regarded as measures of substitution possibilities in production, i.e. the effects on the demand for the various inputs in production, process following a change in energy prices when <u>total output and all other prices are assumed to be constant</u>.

If also the demand structure for the sector output is taken into consideration, a third effect in the adjustment of the production factors to an initial price increase (in this example on electricity) may come through changes in the output level. A partial reasoning may be carried out in connection with figure 1c. A partial demand curve for the sector output, P_X (A_X), is drawn in the P_X-A_X diagram. Furthermore, average cost curves are specified as horizontal lines, assuming that constant returns to scale prevail in the production process. An increase in the electricity price will result in an upward shift in the average cost curve; from $(C/A_X)^S$ to $(C/A_X)^R$ in figure 1c. If the position of the demand curve is unchanged, the same shift must take place in the equilibrium price. The result will be that the price increase causes a reduction in the output level. This lower level of production has further implications for the demand for factor inputs, as indicated in the figure. The optimal level of total energy input is reduced from A_U^N to A_U^R (proportional to the change in the output, see figure 1b), and this causes a further reduction in the use of both electricity and fuels (a movement from N to R in figure 1a).

However, it is important to notice that the different price effects we have described above are all partial, in the sence that they refer to the input demand in one single sector, assuming all other variables than the one (or those) in question constant. In particular it should be noticed that in a general equilibrium setting all commodity prices will be influenced by a change in energy prices. This will cause changes in relative prices for factor inputs and shift the demand curves for sector outputs. In addition the different output levels will not all be reduced as a result of an increased energy price. The demand for some products (probably the most energy intensive) will be reduced, while the production in other sectors may increase in order to secure full utilization of the resources labour and capital. In figure 1c this means that for some sectors the drawn demand curve in the new equilibrium situation will shift outward (assuming that the average cost curve shifts upward). The energy price effects working through changes in the output levels or through changes in other prices must therefore be studied within a complete model.[1)]

A commonly used measure of how the different inputs may be substituted against each other is the elasticity of substitution. In the two-factor case the elasticity of substitution concept has an unambigious interpretation; it measures the relative change in the proportion between the two inputs along an isoquant. In our model we denote the elasticity of substitution between A_E and A_F, holding A_U constant, by σ_{EF}. It has been shown e.g. by Uzawa (1962) that σ_{EF} is related to the gross price elasticities defined by relation (6) by

$$(9) \quad \sigma_{EF} = \eta_{EF}/S_{UF} = \eta_{FE}/S_{UE}$$

1) Effects of increased energy prices on the aggregate demand for inputs, taking all simultaneous relations in the MSG model into account are analyzed in chapter VIII.

Specifying more than two inputs in the production function several concepts of elasticity of substitution have been defined (see for example Sato and Koizumi (1973)), differing with respect to which variables are kept constant in the adjustment of inputs to price changes. From our overall cost function (2) Hicks-Allen elasticities of substitution between aggregate inputs may be defined in a similar way as (9), i.e.

$$(10) \quad \sigma_{ij} = \varepsilon_{ij}/S_j \qquad i,j = K,L,U,M$$

It is seen that the Hicks-Allen elasticities of substitution are in essence normalized price elasticities. A main effect of this normalization is to make the elasticities symmetric so that $\sigma_{ij} = \sigma_{ji}$, which may be rather convenient for presentation purposes. However, it is important to note that the Hicks-Allen elasticities of substitution defined by (10) implies that all inputs are adjusted to their new optimal levels. The interpretation is thus not as straightforward as in the two-factor case.

4. The deterministic model

By extending the production analysis to comprise more than two inputs, functional forms like Cobb-Douglas or CES place too strong a priori restrictions on the substitution properties. Recent econometric analysis of multi-input production processes have applied functional forms that are less restrictive in this sence. Among these are the Translog cost function introduced by Christensen, Jorgenson and Lau (1973) and the Generalized Leontief (GL) cost function introduced by Diewert (1971) the ones most frequently applied. Both of these functional forms may be regarded as general second-order approximations of the "true" structure. In previous versions of the MSG model (see Johansen (1960, 1974)) the production structure was based on fixed coefficients for commodity inputs and a Cobb-Douglas technology for labour and capital inputs. As the GL cost function provides a generalized representation of this rather rigid structure, it was a natural choice for an approximation of the production structure in the further development of the model.

Within the general formulation (2) the cost function is thus specified as

$$(11) \quad C = H(A_X,t) \quad \Sigma_i \Sigma_j \; c_{ij}(P_i P_j)^{\frac{1}{2}} \qquad i,j = K,L,U,M$$

This functional representation restricts the production function to be homothetic, i.e. the cost function is separable in a price and a quantity term which means that the scale and substitution properties may be studied independently. The assumption of homotheticity further implies that the expansion paths are straight lines through origo, which means that a change in the quantity of output will have the same relative effect on the demand for all specified inputs. From the relation (11) it is furthermore seen that technical change is assumed to be neutral.

It is a well known econometric problem to separate technical change from returns to scale. However, in this study we focus on the substitution possibilities within the production processes. In the estimation of the substitution properties, i.e. the parameters c_{ij} of the price term, the H-function is not specified explicitly. Consequently, no attempt is made to estimate the economics of scale and the rate of technical change. By this specification we get more degrees of freedom for the estimation of the production model. In return, we will have to face the problem that the quantity term $H(A_X,t)$ is unobservable in the estimation of the substitution parameters, c_{ij}, of the production model. This difficulty is overcome by constructing an index to be used as an instrument variable for the quantity term (see section 5).

In the equation system of the model the H-function is specified as

$$H(A_X,t) = A_X^{\frac{1}{\mu}} e^{\frac{-\varepsilon t}{\mu}}$$

For most industries constant returns to scale, i.e. $\mu = 1$, are assumed. With constant returns to scale the estimate of "total factor productivity", i.e. the estimated growth rate of the H-function, reported in section 6, may be used as an estimate for the rate of technical change. In some primary industries, where the production is based on the extraction of natural resources, decreasing returns, i.e. $\mu < 1$, are assumed. The scale paramenters in these industries are in the present version of the model determined by "reasonable guesses".

Applying Shephard's lemma we get the aggregate input demand functions for the GL-function as

$$(12) \quad A_i = H(A_X,t)\Sigma_j c_{ij} P_i^{-\frac{1}{2}} \; P_j^{\frac{1}{2}} \qquad i,j = K,L,U,M$$

It is seen from equation (12) that if all the off-diagonal elements of the matrix $[c_{ij}]$ are zero, the production model reduces to ordinary fixed-coefficient Leontief technology. It thus is possible to test for the existence of substitution among aggregate inputs.

The Generalized Leontief functional form is also used as an approximation to the price of the energy aggregate, i.e.

$$(13) \quad P_U = \Sigma_i \Sigma_j b_{ij} (P_i P_j)^{\frac{1}{2}} \qquad i,j=E,F.$$

In the same way as (12) the energy activities are related to the input prices and total energy by

$$(14) \quad A_i = A_U \Sigma_j b_{ij} P_i^{-\frac{1}{2}} P_j^{\frac{1}{2}} \qquad i,j=E,F.$$

The set of partial price elasticities, η_{ij} and ε_{ij}, are defined by (6) and (7), respectively. With a Generalized Leontief specification the η_{ij}-elasticities are derived from (14) as

$$(15) \quad \eta_{ij} = \tfrac{1}{2} \left(\frac{b_{ij} P_j^{\frac{1}{2}}}{\sum_k b_{ik} P_k^{\frac{1}{2}}} - \delta_{ij} \right) \qquad i,j,k=E,F$$

where δ_{ij} is the Kronecher delta.

From (12) the ε_{ij}-elasticities are derived as

$$(16) \quad \varepsilon_{ij} = \tfrac{1}{2} \left(\frac{c_{ij} P_j^{\frac{1}{2}}}{\sum_k c_{ik} P_k^{\frac{1}{2}}} - \delta_{ij} \right) \qquad i,j,k=K,L,U,M$$

5. Price and volume measures

The estimation of the parameters of the GL cost functions for the 19 industries is based on national accounting figures. The data are time series observations from 1962 to 1978.

Labour, material and energy inputs

Observations for the material and energy inputs (activities) and the corresponding price indices are constructed simply by aggregating commodity flows in the national accounts in current and constant prices. For the labour input the accounts provide time series of man-hours. The price index for the labour input is correspondingly defined as wage costs per man-hour.

The one-dimensional measurement of labour input is clearly unsatisfactory and should in later estimations of the cost functions be replaced by more differentiated input measures properly weighted. The same may be said for capital services discussed below. The Laspeyres-aggregates for material and energy inputs, implying infinite elasticities of substitution within each aggregate, should also be replaced by index aggregators which are approximations to more flexible functional forms.

Capital services and the user costs of capital

The national accounts also include figures for capital stock in constant prices for each industry. The real capital stock calculations are based upon vintage data for gross investments in constant prices. If we assume full capacity utilization capital service flows are proportional to real capital stocks. Furthermore, if constant returns to scale and competitive behaviour prevail, the returns to capital can be determined as the residual of factor income. The user cost of capital (price of capital services) can then be derived from the observed rate of return, capital prices, and depreciation rates.[1)]

However, costs of capital and capital service flows calculated in this way will be highly fluctuating. There are many reasons for such fluctuations, e.g. unfulfilled expectations, oligopolistic behaviour, and various external cyclical shocks. The MSG-4E model is designed to study long-term development with the economy running at full capacity. The estimation of the input demand functions is therefore based on data for real capital stocks and capital service prices corrected for such short-term fluctuations. In the estimation of the utilization rate of existing capital stock the Wharton trend-through-peaks technique is applied (see e.g. Klein and Summers (1966)).[2)]

The construction of relevant user costs in the various industries starts out with the estimation of a five year moving average of the actual rate of return of capital for the aggregate manufacturing industry.

1) Strictly this requires that depreciation (physical outwear) is calculated on the basis of exponentially declining survival functions for each capital asset (geometric depreciation). In fact depreciation in constant prices in the national accounts is calculated on a straight line basis. This may create some inconsistency in our calculations of capital stocks and user costs. However, since we are working on a rather aggregated level and with the rather stable growth in capital formation in the period of observation we believe that this is not a serious problem.
2) The results from using this method on Norwegian data are presented in Lesteberg (1979).

This measure, $\bar{R}$, is assumed to be an indicator of the average expected returns to capital in the economy as a whole.[1] The expected rate of return to capital in a specific industry, $\hat{R}_j$, is assumed to be proportionate to this indicator, i.e.

$$(17)\quad \hat{R}_j = \rho_j \cdot \bar{R}$$

where the ρ_j's are structural coefficients. This assumption of return differentials between industries may be explained by traditional differences in profit requirements, investment risks, average size of the firms, degree of monopolization etc. within the various industries (see Johansen (1960)).

For manufacturing industries the actual rates of return, R_j, are estimated on the base of data on operating surplus in the national accounts. Actual (ex post) relative rates of return in a specific industry may then be defined as

$$(18)\quad r_j = \frac{R_j}{\bar{R}}$$

Furthermore we are specifying a relation set out in Strøm (1967), which suggests the hypothesis that there is a convergent development in the actual relative rates of return. More precisely we formulate the scheme

$$(19)\quad r_j(t) = r_j(t-1) + k_j\,(r_j(t-1) - \rho_j)$$

The relations (19) are estimated by ordinary least squares regression. The results are presented in the Appendix. For all manufacturing industries these calculations indicate a convergent development in the r_j's with the corresponding ρ_j's as stationary levels[2]. The standard deviations of the parameter estimates are, however, very high in the sectors 18 Textiles and wearing apparel and 26 Wood and wood products where the actual rates of return have been low and steadily declining during the sample period.

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1) The average duration of Norwegian manufacturing cycles is close to five years (see Wettergreen (1978)). The estimation of rates of return based on observed interest rates in the credit market is not an attractive alternative as this market has been regulated by the Central government in the period of estimation.

2) A convergent development is secured if the k_j is estimated to be in the interval (-1,0).

For the majority of the manufacturing industries rates of return to capital to be used in the estimation of cost functions are calculated by the relations (17) and the estimated structural coefficients (ρ_j). For the sectors 18 and 26 the rates of return in total manufacturing, $\bar{R}$, is applied, representing the opportunity returns to capital.

For primary and service industries the reported operating surplus in the national accounts are highly fluctuating, and it is thus doubtful whether this measure reflects the producer's ex ante profit requirements for returns to real capital. Also for these industries the relevant user cost of capital is constructed by applying the indicator of rate of return for the aggregate manufacturing industry ($\bar{R}$).[1)]

An index for total factor input

As we have already noted, in the absence of an explicit formulation of the time function for technical change and the returns to scale in the estimation of the substitution parameters we need an instrument variable for the quantity term in the cost function (11). The common practice in empirical studies of this kind is to make use of an index aggregator in order to find an approximation to the flexible functional form for the price term. In Diewert (1976) a class of such index numbers (price indices) are defined as

$$(20) \quad P(q) = \left[\frac{\Sigma_i S_i^0 \left(\frac{P_i^1}{P_i^0} \right)^{\frac{q}{2}}}{\Sigma_i S_i^1 \left(\frac{P_i^0}{P_i^1} \right)^{\frac{q}{2}}} \right]^{\frac{1}{q}}$$

where S_i are the cost shares of the (aggregate) inputs, q is a dummy parameter, and the superscripts 1 and 0 denote two successive periods.

Diewert shows that under certain assumptions P(1) is an exact index for a GL unit cost function (denoted by c(P)), in the sense that[2)]

1) An exception from this rule is sector 83 Dwellings. This is a "constructed" sector in the national accounts, where output is calculated e.g. from imputed capital costs. For this industry the actual rate of return is applied in the estimation.

2) The proof is in fact carried out for the general quadratic mean of order q unit cost function,

$$c_q(P) = \left[\Sigma_i \Sigma_j c_{ij} (P_i \; P_j)^{\frac{q}{2}} \right]^{\frac{1}{q}}$$

For q = 1 we arrive at the GL unit cost function.

$$(21)\quad P(1) = \frac{c(P^1)}{c(P^0)}$$

Given the price index P(1) we can define a quantity index Q(1) for the quantity term of the cost function by

$$(22)\quad Q(1) = \Sigma_i P_i^1 A_i^1 \,/\, (\Sigma_i P_i^0 A_i^0)\, P(1)$$

According to (21) and (22) Q(1) can be interpreted as an index for total quantity input consistent with the GL aggregator function, and may thus be used as an instrument variable for the composite scale and technical change term $H(A_X,t)$ in our model.

6. Estimation techniques and results

In accordance with the separability restrictions imposed on our model the complete estimation of the cost structure for each industry is carried out in two steps:

(i) First the b_{ij}-coefficients of the energy submodel are estimated. By inserting these estimated parameters into the relation (13) an estimate of the price index for aggregate energy input, $\hat{P}_U$, is obtained.

(ii) The parameters of the overall cost function are then estimated by replacing the unobservable P_U by its instrument $\hat{P}_U$.

Given an appropriate stochastic specification this two-stage procedure will provide consistent parameter estimates both for the sub-model and for the overall model (see Fuss (1977)).

Estimation of the energy submodel

A common practice when working with a GL cost function is to base the estimation of the unknown parameters on a stochastic specification of the derived demand function (the relations (12) and (14)). However, the level of total energy input is unobservable. The relations (14) are therefore transformed to the corresponding expressions for the cost shares, S_i. Introducing additive stochastic error terms, U_k, the equations to be estimated are

$$(23)\quad S_k = P_k^{\frac{1}{2}} \frac{\Sigma_j b_{ij} P_j^{\frac{1}{2}}}{\Sigma_i \Sigma_j\, b_{ij}\, (P_i P_j)^{\frac{1}{2}}} + U_k \qquad i,j,k = E,\ F.$$

The residuals are assumed to be joint normally distributed with mean vector zero and variance-covariance matrix $\Omega \otimes I$, where Ω is a positive semi-definite matrix of order two.

It is seen from (23) that both observed and predicted cost shares sum identically to one; hence the matrix Ω is singular.[1)] The standard solution to this problem is to delete one equation from the system. In the calculations presented in this chapter the demand relations for fuels are deleted.

The levels of the b_{ij}-coefficients are conventionally determined by assuming that the aggregate price index for energy equals one in the base year, i.e.

(24) $\Sigma_i \Sigma_j b_{ij} = 1 \qquad i,j = E,F$

We also impose the symmetry restriction

(25) $b_{EF} = b_{FE}$

Under these restrictions the energy submodel is estimated by ordinary least squares (OLS), which gives consistent and efficient estimates of the parameters. The results are presented in table 1.

It is of major interest whether the calculations are at variance with the hypothesis that the cost function is concave, which is a necessary condition for its interpretation as a dual relation to a "well-behaved" production function and cost minimization behaviour. With a GL-specification of the cost structure a necessary and sufficient condition for this property being globally fulfilled is that all off-diagonal elements of the coefficient matrix $B = [b_{ij}]$ are non-negative. From the results presented in table 1 it is seen that for every industry, except 34 Paper and paper products and 37 Industrial chemicals the estimates of the b_{EF} (= b_{FE}) - parameters are positive. The calculations

1) The U's are restricted by

$$U_E + U_F = 0$$

The variance-covariance matrix, Ω, will then be of the form

$$\Omega = \begin{bmatrix} \sigma^2 & -\sigma^2 \\ -\sigma^2 & \sigma^2 \end{bmatrix}$$

where σ is the (common) standard deviation of the two error terms. This matrix is clearly singular.

Table 1. Parameter estimates and price and sub-

Industries[1)]
16 Manufacture of food ....
17 Manufacture of beverages and tobacco ....
18 Manufacture of textiles and wearing apparels ....
26 Manufacture of wood and wood products ....
27 Manufacture of non-industrial chemicals ....
28 Printing and publishing ....
31 Mining and quarrying ....
34 Manufacture of paper and paper products ....
37 Manufacture of industrial chemicals ....
43 Manufacture of metals ....
45 Manufacture of metal products, machinery and eq. ....
50 Building of ships and oil platforms ....
55 Construction ....
79 Repair of motor vehicles and household appliances ....
84 Other private services ....

1) For the industries 12, 81, 82 and 83 either only one or no energy activities are specified in the present version of the MSG model. These industries are therefore not included in the table.

2) Standard errors are given in parenthesis.

stitution elasticities for the energy submodel

Coefficients[2)]			Gross price elasticities		Elasticities of substitution
b_{EE}	b_{EF}	b_{FF}	η_{EE}	η_{FF}	σ_{EF}
0.153 (0.078)	0.285 (0.077)	0.277 (0.076)	-0.326	-0.253	0.579
-0.485 (0.259)	0.799 (0.222)	-0.113 (0.196)	-1.270	-0.583	1.853
-0.020 (0.135)	0.544 (0.124)	-0.067 (0.113)	-0.520	-0.570	1.090
0.294 (0.079)	0.315 (0.075)	0.076 (0.071)	-0.259	-0.403	0.661
0.237 (0.113)	0.132 (0.111)	0.499 (0.109)	-0.178	-0.104	0.233
0.351 (0.367)	0.203 (0.327)	0.243 (0.289)	-0.183	-0.227	0.410
0.505 (0.202)	0.056 (0.194)	0.383 (0.188)	-0.050	-0.064	0.113
0.573 (0.105)	-0.010 (0.111)	0.449 (0.119)	0.009	0.012	-0.022
0.973 (0.462)	-0.282 (0.439)	0.590 (0.419)	0.204	0.456	-0.660
0.889 (0.024)	0.015 (0.027)	0.080 (0.029)	-0.009	-0.080	0.089
-0.058 (0.207)	0.571 (0.190)	-0.085 (0.175)	-0.556	-0.587	1.143
0.340 (0.096)	0.220 (0.089)	0.220 (0.083)	-0.196	-0.250	0.447
-0.443 (0.922)	0.748 (0.890)	-0.054 (0.861)	-1.226	-0.539	1.764
0.203 (0.056)	0.384 (0.047)	0.029 (0.038)	-0.327	-0.465	0.792
0.258 (0.072)	0.482 (0.074)	-0.222 (0.077)	-0.326	-0.928	1.253

give thus in this respect some support to the neoclassical assumptions.[1] It is also seen from table 1, by considering the standard errors of the off-diagonal elements, that for the majority of the industries a hypothesis of simple Leontief technology within the energy aggregate may be rejected at a 10 per cent level of significance. However, for the sectors 27, 28, 31, 43 and 55 this hypothesis cannot be rejected.

In table 1 we also present estimates of direct gross price elasticities for energy activities and the substitution elasticities between the two types of energy inputs, as defined in section 3, relation (6) and (9), respectively. In the table the elasticities are calculated for the observed prices of the last year in our time series, 1978. We first note that all direct elasticities, except for the two industries with negative b_{EF}-coefficients (34 and 37), are estimated to be negative. In agreement with this result the elasticity of substitution between the two energy activities are positive for the majority of the sectors. Furthermore we notice that the magnitudes of the estimated elasticities vary considerably between industries. The substitution possibilities between different energy goods are according to our results quite small in the industries 31 Mining and quarrying and 43 Metals, with direct elasticities close to zero and low σ_{EF}-elasticities. On the higher side we find the two industries 17 Beverages and tobacco and 55 Construction where the demand for electricity is estimated to be rather elastic (with direct elasticities lower than -1). In the same industries we have estimated the substitution possibilities to be quite large (σ_{EF} as high as 1.8)[2]. The demand for fuels is most price elastic in industry 84 Other private services. For the rest of the industries the internal price sensitiveness for the energy goods may be characterized as moderate with demand elasticities varying between -0.2 and -0.6 and substitution elasticities between 0.3 and 1.1.[3]

Estimation of the aggregate model

As indicated in previous sections the estimation of the overall

1) It is not surprising that the neoclassical assumption fails in sectors 34 and 37. In the Chemical industry electricity power plants are to a large extent owned by the firms themselves and operated at very low marginal costs. In the Paper industry the existence of electrical boilers using surplus power favourably priced makes the inter energy substitution modelling rather complicated.
2) Note, however, that the standard errors for the parameters in sector 55 are large.
3) Apart from sectors 34 and 37 the estimates presented in table 1 are applied in the present version of the model. The results for sectors 34 and 37 are replaced by "guesstimates".

cost function is based upon a stochastic spesification of the demand system (12) using a constructed index Q as a measure of the quantity term and replacing P_U by its instrument $\hat{P}_U$. We then specify the stochastic relations as

$$(26) \quad A_i = Q \Sigma_j c_{ij} P_i^{-\frac{1}{2}} P_j^{\frac{1}{2}} + V_i \qquad i,j = K,L,U,M$$

Again, the error terms are assumed to be joint normally distributed and to have zero means and a variance - covariance matrix of the form $\Psi \otimes 1$, where Ψ is a positive semi-definite matrix of order four.

As in the energy submodel we impose a priori the restriction that the coefficient matrix is symmetric, i.e.

$$(27) \quad c_{ij} = c_{ji} \qquad i,j = K,L,U,M$$

The system of demand equations for each industry is estimated by Zellner's efficient iterative estimation method. Because of the stochastic specification and the parameter restrictions across the equations this method is more efficient than OLS. Moreover, Malinvaud (1970) has shown that using Zellner's method iteratively with respect to the variance - covariance matrix is equivalent to FIML-estimation of the same equation system. As opposed to the estimation of the energy relations, there are no linear restrictions between the stochastic error terms in the aggregate stochastic model (26); accordingly all the demand functions are included in the estimation.[1)]

The estimates of the 10 coefficients of the aggregate model for each industry are presented in table 2. By comparing the standard errors with the estimates we see that, as a general feature, a majority of the parameter estimates of the cost functions are significantly different from zero at a 10 per cent level.

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1) From (26) it follows that

$$\Sigma_i P_i A_i = Q \Sigma_i \Sigma_j c_{ij} (P_i P_j)^{\frac{1}{2}} + \Sigma_i P_i V_i \qquad i,j = K,L,U,M$$

Furthermore we assume that there is an additive error term in the overall cost function, i.e.

$$C = Q \Sigma_i \Sigma_j c_{ij} (P_i P_j)^{\frac{1}{2}} + W \qquad i,j = K,L,U,M$$

The relation between the V_i-terms and W is then given by

$$\Sigma_i P_i V_i = W \qquad i = K,L,U,M$$

Thus, we see that if the overall cost function is stochastic, there is a linear restriction on the error terms only if this relation is included in the system. If we neglect the stochastic specification of the cost function, the variance-covariance matrix Ψ of the "remaining" system (26) is non-singular.

Table 2. Parameter estimates

Industries	Coeffi-		
	c_{UU}	c_{UL}	c_{UK}
12 Forestry	0.003	-	-
	(0.008)	-	-
16 Manufacture of food	-0.003	-0.068	0.024
	(0.005)	(0.024)	(0.010)
17 Manufacture of beverages and tobacco .	-0.003	-0.032	0.038
	(0.016)	(0.036)	(0.040)
18 Manufacture of textiles and wearing apparels	0.016	0.153	-0.016
	(0.007)	(0.038)	(0.033)
26 Manufacture of wood and wood products	-0.027	0.058	0.024
	(0.005)	(0.046)	(0.041)
27 Manufacture of non-industrial chemicals	0.003	0.139	-0.151
	(0.019)	(0.054)	(0.033)
28 Printing and publishing	-0.020	0.094	0.002
	(0.027)	(0.049)	(0.010)
31 Mining and quarrying	-0.155	0.508	0.015
	(0.025)	(0.108)	(0.060)
34 Manufacture of paper and paper products	0.020	0.135	0.053
	(0.034)	(0.094)	(0.061)
37 Manufacture of industrial chemicals ..	0.022	0.300	-0.103
	(0.054)	(0.136)	(0.150)
43 Manufacture of metals	-0.068	0.227	-0.073
	(0.024)	(0.038)	(0.029)
45 Manufacture of metal products, machinery and eq.	0.004	0.010	0.032
	(0.004)	(0.024)	(0.026)
50 Building of ships and oil platforms ..	-0.011	-0.001	-0.003
	(0.005)	(0.019)	(0.015)
55 Construction	0.005	0.050	-0.002
	(0.003)	(0.010)	(0.006)
79 Repair of motor vehicles and household appliances	0.011	-0.070	-0.103
	(0.006)	(0.034)	(0.037)
81 Wholesale and retail trade	-0.010	0.018	0.019
	(0.002)	(0.014)	(0.006)
82 Financing and insurance services	-	-	-
83 Dwellings	-	-	-
84 Other private services	-0.010	0.271	0.051
	(0.008)	(0.044)	(0.026)

1) Standard errors are given in paranthesis.

of the overall cost functions

cients[1]						
c_{UM}	c_{LL}	c_{LK}	c_{LM}	c_{KK}	c_{KM}	c_{MM}
-	-6.724	10.287	-0.023	1.363	0.041	0.077
-	(4.220)	(2.393)	(0.121)	(1.414)	(0.074)	(0.022)
0.019	-2.033	0.449	0.994	0.414	-0.081	0.592
(0.004)	(0.272)	(0.091)	(0.041)	(0.081)	(0.018)	(0.007)
0.021	1.263	1.673	0.835	-0.388	-0.017	0.349
(0.015)	(0.173)	(0.106)	(0.059)	0.165	(0.054)	(0.022)
-0.020	-1.011	0.987	1.550	0.292	-0.067	0.299
(0.004)	(0.522)	(0.304)	(0.063)	(0.259)	(0.028)	(0.009)
0.018	-6.837	1.587	2.136	0.163	-0.209	0.237
(0.008)	(0.730)	(0.540)	(0.123)	(0.505)	(0.098)	(0.023)
0.058	-2.390	0.406	1.511	-0.679	0.571	0.008
(0.024)	(1.201)	(0.283)	(0.293)	(0.157)	(0.101)	(0.085)
0.005	2.816	1.726	0.153	0.907	-0.509	0.657
(0.015)	(0.755)	(0.151)	(0.169)	(0.260)	(0.109)	(0.062)
0.080	-6.153	2.146	1.669	0.984	-0.126	-0.057
(0.022)	(0.843)	(0.379)	(0.122)	(0.266)	(0.080)	(0.032)
-0.008	-2.697	1.226	1.019	-0.653	0.106	0.420
(0.025)	(0.410)	(0.302)	(0.088)	(0.333)	(0.068)	(0.025)
0.044	-0.230	1.107	0.415	0.165	0.431	0.271
(0.028)	(0.560)	(0.356)	(0.124)	(0.703)	(0.109)	(0.037)
0.119	-2.099	0.812	0.856	0.603	0.045	0.274
(0.026)	(0.305)	(0.069)	(0.073)	(0.070)	(0.033)	(0.037)
-0.001	-0.954	1.343	1.432	-0.107	-0.061	0.226
(0.005)	(0.464)	(0.335)	(0.077)	(0.438)	(0.073)	(0.015)
0.017	-8.233	-1.004	3.285	0.116	0.309	-0.164
(0.005)	(0.422)	(0.181)	(0.099)	(0.249)	(0.059)	(0.029)
-0.011	-7.256	1.123	2.547	-0.131	-0.112	0.095
(0.003)	(0.269)	(0.048)	(0.051)	(0.041)	(0.021)	(0.016)
0.032	1.679	5.779	0.842	3.238	-1.328	0.465
(0.005)	(1.444)	(0.833)	(0.242)	(0.954)	(0.139)	(0.049)
-0.007	-3.862	0.777	2.625	-0.080	0.050	-0.193
(0.004)	(0.530)	(0.095)	(0.123)	(0.066)	(0.025)	(0.029)
-	2.740	1.177	0.122	-0.226	-0.054	0.639
	(1.285)	(0.205)	(0.264)	(0.057)	(0.038)	(0.055)
-	-0.362	-0.219	0.217	16.530	0.270	0.113
	(0.049)	(0.036)	(0.015)	(0.152)	(0.035)	(0.009)
-0.003	4.411	1.204	0.736	0.912	-0.009	0.217
(0.013)	(0.561)	(0.203)	(0.113)	(0.148)	(0.048)	(0.032)

Table 3. Price and substitution elasticities for aggregate inputs

Industries	Price elasticities			
	ε_{UU}	ε_{LL}	ε_{KK}	ε_{MM}
12 Forestry	-	-0.917	-0.421	-0.041
16 Manufacture of food	-0.511	-0.806	-0.092	-0.122
17 Manufacture of beverages and tobacco	-0.567	-0.402	-0.646	-0.188
18 Manufacture of textiles and wearing apparels	-0.065	-0.551	-0.247	-0.228
26 Manufacture of wood and wood products	-1.597	-1.081	-0.369	-0.316
27 Manufacture of non-industrial chemicals	-0.464	-0.710	-0.883	-0.493
28 Printing and publishing	-1.753	-0.298	0.171	0.182
31 Mining and quarrying	-2.307	-0.920	-0.250	-0.583
34 Manufacture of paper and paper products	-0.321	-0.831	-0.876	-0.186
37 Manufacture of industrial chemicals	-0.405	-0.530	-0.469	-0.239
43 Manufacture of metals	-0.951	-0.755	-0.209	-0.270
45 Manufacture of metal products, machinery and eq. ...	-0.483	-0.565	-0.574	-0.287
50 Building of ships and oil platforms	-1.593	-1.407	-0.325	-0.614
55 Construction	0.095	-1.088	-0.846	-0.424
79 Repair of motor vehicles and household appliances	7.868	-0.440	0.238	0.310
81 Wholesale and retail trade .	-0.980	-0.682	-0.608	-0.747
82 Financing and insurance services	-	-0.210	-0.714	-0.010
83 Dwellings	-	-0.880	-0.031	-0.235
84 Other private services	-0.712	-0.308	-0.185	-0.197

1978. The rate of growth of total factor productivity in the sample period 1962 - 1978

Hicks-Allen elasticities of substitution						The rate of growth of total factor productivity
σ_{UL}	σ_{UK}	σ_{UM}	σ_{LK}	σ_{LM}	σ_{KM}	
-	-	-	1.463	-0.091	0.106	0.001
-3.675	4.818	0.892	1.827	0.940	-0.285	0.004
-0.540	3.044	0.939	1.769	0.500	-0.048	0.024
2.251	-2.077	-1.050	1.238	0.755	-0.287	0.006
1.889	4.624	1.163	2.953	1.312	-0.762	0.017
1.289	-6.087	1.159	0.560	1.026	1.689	0.009
3.533	0.335	0.698	1.820	0.097	-1.841	0.001
3.694	0.249	2.734	0.929	1.512	-0.256	0.012
1.298	1.872	-0.106	2.745	0.840	0.321	0.010
1.433	-0.718	0.378	0.953	0.437	0.658	0.010
1.651	-1.347	1.314	1.250	0.789	0.104	0.006
0.231	5.346	-0.056	1.678	0.809	-0.240	0.012
-0.065	-2.587	2.394	-4.169	2.201	1.868	0.008
4.044	-3.128	-2.103	4.547	1.428	-1.039	0.007
-19.354	-126.083	85.893	1.704	0.536	-3.731	0.026
1.699	4.747	-0.880	0.872	1.464	0.324	0.016
-	-	-	2.970	0.079	-0.248	-0.019
-	-	-	-0.305	5.208	0.172	0.002
2.497	-2.196	-0.193	0.541	0.450	-0.026	0.011

Another characteristic of the results is that for all industries at least one of the estimated c_{ij}-coefficients ($i \neq j$) turns out to be negative. As noted above we can then neither be sure that the estimated cost function is concave in the input prices nor that the fitted values of the demand equations are strictly positive. To investigate the concavity property one needs only consider the off-diagonal elements of the coefficient matrix. If some of these parameters are negative we can conclude that the estimated relation is not globally concave, as there will exist some combination of prices for which the Hessian matrix is not negative semi-definite. However, since the GL-function is interpreted as an approximation to the "real" cost structure, our main interest is whether the estimated relation is concave within the range of variation of observed prices. For this purpose we have, for each industry, checked the properties of the Hessian matrix at each point of observation by calculating the corresponding eigenvalues.[1)] For most industries the Hessian matrix of the cost function turned out to be negative semi-definite for all price vectors in the sample period. However, for the industries 16, 18, 28, 55 and 79 the estimated relations were not concave for several points of observation. For these industries the calculations indicate that we have not been able to estimate a cost function, though it is not determined whether or not the eigenvalues with incorrect signs are statistically significant. The curvature properties for the industries 16 and 18 were satisfied for the later years in the sample period. The estimated factor demand equations were not found to be negative at any point of observation.

As for the energy submodel we are interested in the price and substitution responses that can be derived from the estimated relations. Direct price elasticities for the aggregate inputs defined by relation (7) and Hicks-Allen elasticities of substitution defined by relation (10) are presented in table 3.

As we could expect from the concavity check most of the estimated own price elasticities have the correct sign, the only exceptions are the industries 28 Printing and publishing, 55 Construction and 79 Repair of motor vehicles and household appliances. We see from table 3 that the energy demand in some industries (26, 28, 31, 43 and 50) is estimated to be very elastic. However, except for industry 43 Metals,

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1) The system could in principle be estimated under the restriction that all c_{ij} ($i \neq j$) are non-negative, and thus secure the concavity property of the GL-function. However, it should be noted that this implies that all inputs must either be substitutes or independent. This restriction may be considered as even more unfortunate, as complementarity is believed to exist in the "real world".

the cost shares of energy in these industries are relatively low. In other energy intensive industries like 27 Non-industrial chemicals, 34 Paper and paper products and 37 Industrial chemicals, the price responses are estimated to be rather moderate. Regarding the own price elasticities for capital and labour the estimates in the table also vary considerably across industries, but still the range of variation must be characterized as reasonable. In general the own price elasticity for capital is rather low and lower than that for labour in most industries. As one might expect the own price sensitiveness of the material input is estimated to be relatively low.

Because of the broad economic and political implications the substitution properties of the inputs in the production functions are of great importance. In recent econometric studies of aggregate manufacturing the debate has concentrated in particular on the question whether energy and capital are substitutes or complements[1]. In our production study of industries at a relatively disaggregated level we find, not surprisingly, that in some industries energy and capital are estimated to be complements (σ_{UK} is negative) and in another group of industries these two inputs, according to our results, are substitutes. However, it may be noted that in the energy intensive sectors 27, 37 and 43 σ_{UK} is negative. Regarding the substitution between energy and labour we see from the table that they are estimated to be substitutes in 12 of 16 industries. We finally notice that our calculations for most industries have led to the reasonable and long accepted result that labour and capital are substitutes in the production processes.[2]

As stressed in section 4, this study emphasizes the estimation of "substitution parameters", i.e. coefficients in the price term of the chosen cost function (see equation (11)). The quantity term $H(A_X,t)$, including the effects of economics of scale and technical change, is not specified explicitly in the estimation. Inserting the observed prices in the estimated unit cost function, total quantity input $H(A_X,t)$ can be estimated for every year in the sample period. The difference between the observed rate of growth of production A_X and rate of growth of this estimate for total factor input may be called the rate of growth of

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1) See Griffin and Gregory (1976), Berndt and Wood (1979) and Pindyck (1979).

2) Apart from sectors 28, 55 and 79 the estimates presented in table 2 are applied in the present version of the model. The results for 28 Printing and publishing, 55 Construction and for 79 Repair of motor vehicles and household appliances are strongly influenced by poor data quality, especially for material and capital inputs. These results are therefore replaced by "guesstimates".

total factor productivity[1]. Average yearly rates of growth of total factor productivity are presented in the last column of table 3.[1] The growth rates are fairly moderate, reflecting that the functional forms are rather flexible and that materials are included as factor input.

7. Comparison between gross and net price elasticities for energy

Gross price elasticities for the two energy activities and the direct price elasticity for total energy for each sector are presented in tables 1 and 3, respectively. Given these estimates, net price elasticities for the two energy goods may be calculated as indicated in equation (8). As stressed in section 3 the net price elasticities may be said to represent the total price effect, i.e. the effect on the demand for electricity and fuels following a change in the price of one of these goods, when total output is assumed to be constant. The set of net price elasticities calculated for the observed prices of 1978 are presented in table 4.[2] For comparison the corresponding gross price elasticities reported in table 1 are also included.

When the direct price elasticity for total energy is negative, it follows from the relation between gross and net elasticities (equation (8)) that the internal "scale" effect on the demand for energy inputs increases the absolute magnitudes of the direct price elasticities and decreases the cross price elasticities.

For some sectors (16, 18, 27 and 45) the difference between net and gross direct price elasticities may be characterized as moderate (both for electricity and fuels) reflecting that the price elasticities for total energy in these industries are rather low. The scale effect is rather moderate also in 17 Beverages and tobacco but a relatively large cost share for fuels makes the scale effect on the demand for this energy good larger than for electricity. In 43 Manufacture of metals the situation is quite opposite; a dominating cost share of electricity and a rather elastic demand for total energy implies a net elasticity for electricity in absolute terms as high as 0.9, while the gross elasticity was estimated to be close to zero.

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1) As pointed out in section 5 the index Q (see equation (22)) is used as an instrument variable for $H(A_X,t)$ in the estimation. The yearly growth rates for Q and for the estimate of $H(A_X,t)$ are strongly correlated.

2) In this table we have omitted the sectors where either the estimated gross price elasticities (34 and 37) or the own price elasticity for total energy (55 and 79) have the wrong sign.

With respect to the cross price effect it is important to note that the scale effect may dominate over the internal energy substitution effect, so that the net elasticity becomes negative. Thus, an increase e.g. in the electricity price may actually decrease the use of fuels if the adjustment of total energy input is taken into account. From table 4 this is seen to be the case for the sectors 26, 27, 28, 31, 43 and 50. We recall from table 3 that, except for sector 27, these are all industries where the total energy demand is estimated to be elastic (one or higher in absolute value) so that the scale effects outweight the pure substitution effects between energy inputs. For these industries our results indicate that electricity and fuels are complements when all inputs are adjusted to their new optimal levels. For the other sectors in table 4 electricity and fuels are estimated to be substitutes also measured by the net elasticities.

Table 4. Gross and net price elasticities for energy goods (gross = η, net = ε)

Industries	Direct price elasticities				Cross price elasticities			
	η_{EE}	ε_{EE}	η_{FF}	ε_{FF}	η_{EF}	ε_{EF}	η_{FE}	ε_{FE}
16 Manufacture of food	-0.326	-0.549	-0.253	-0.541	0.326	0.038	0.253	0.030
17 Manufacture of beverages and tobacco	-1.270	-1.449	-0.583	-0.971	1.270	0.882	0.583	0.404
18 Manufacture of textiles and wearing apparels .	-0.520	-0.554	-0.570	-0.601	0.520	0.489	0.570	0.536
26 Manufacture of wood and wood products	-0.259	-1.231	-0.403	-1.027	0.259	-0.366	0.403	-0.570
27 Manufacture of non-industrial chemicals	-0.178	-0.350	-0.104	-0.397	0.178	-0.114	0.104	-0.070
28 Printing and publishing	-0.183	-1.154	-0.227	-1.009	0.183	-0.598	0.227	-0.744
31 Mining and quarrying	-0.050	-1.345	-0.064	-1.075	0.050	-0.962	0.064	-1.231
43 Manufacture of metals	-0.009	-0.869	-0.080	-0.172	0.009	-0.082	0.080	-0.780
45 Manufacture of metal products, machinery and eq.	-0.556	-0.804	-0.587	-0.822	0.556	0.321	0.587	0.339
50 Building of ships and oil platforms	-0.196	-1.089	-0.250	-0.951	0.196	-0.504	0.250	-0.642
84 Other private services	-0.326	-0.853	-0.928	-1.113	0.326	0.141	0.928	0.401

8. Comparison with results from other studies

It is difficult to compare our results with those of other studies mainly because there are relatively few published studies using data with a comparable industry classification, or even a corresponding level of aggregation. We have therefore chosen to make the comparison for the aggregate manufacturing industry, for which many empirical studies are undertaken.

An analysis of the price responses in the aggregate manufacturing industry may in our case be carried out along two different lines. One possibility is simply to construct weighted averages of the estimated elasticities for the individual manufacturing industries. An alternative procedure is to estimate an independent aggregate cost function for this subset of industries. In this study the latter method is chosen, in order to make a comparison of our results with similar econometric studies on the aggregate manufacturing level. It should be noted that by using this method the estimation results are influenced by changes in each industry's weight during the sample period as opposed to "pure" weighted elasticities which assume that output in each industry is held constant.

A GL cost function is thus estimated for the aggregate manufacturing industry, using the same data and estimation techniques as for the individual industries. The derived price elasticities for aggregated inputs are presented in table 5, together with similar estimates of price elasticities from the studies of Berndt and Wood (1975), Griffin and Gregory (1976), Fuss (1977) and Pindyck (1979). Berndt and Wood and Fuss are using U.S. and Canadian data, respectively, while Griffin and Gregory and Pindyck are using international data, including data for Norway.

Table 5. Price elasticities for the aggregate manufacturing industry[1)]

	ε_{UU}	ε_{LL}	ε_{KK}	ε_{MM}	ε_{UK}	ε_{UL}	ε_{LK}
Our estimates	-0.82	-0.70	-0.40	-0.20	-0.13	0.30	0.20
Berndt and Wood	-0.49	-0.45	-0.44	-0.24	-0.17	0.20	0,05
Fuss	-0.49	-0.49	-0.36	-0.76	-0.004	0.04	0.20
Pindyck[2)]	-0.84	-0.37	-0.41	-	0.25	0.60	0.30
Griffin and Gregory[2)] ..	-0.77	-0.27	-0.38	-	0.33	0.45	0.13

1) In this table we present a selection of cross price elasticities instead of the corresponding substitution elasticities because the latter are not presented in the study by Fuss.
2) Calculated for Norwegian prices.

From table 5 we conclude that our results for the aggregate manufacturing industry are in reasonable accordance with these previous studies. We notice in particular that our estimate of own price elasticity for energy is very close to those obtained by Pindyck and Griffin and Gregory. Furthermore the estimates of own price elasticity for capital are rather uniform in all the studies tabulated above, while we in our study have estimated the demand for labour to be a bit more elastic than indicated by previous analyses.

Regarding the cross price effects we see that in our study energy and capital are estimated to be complements. This result is supported by the studies of Berndt and Wood and Fuss, while Pindyck and Griffin and Gregory both found that these inputs are substitutes. As a possible explanation of these deviating results, Berndt and Wood in a later paper (Berndt and Wood (1978)) point at the fact that Griffin and Gregory (and this applies also to Pindyck), because of data problems, have omitted materials as specified input in the production function. As a consequence their estimates tend to underestimate the price effects. Taking all inputs into consideration the cross price elasticities may even change signs.

APPENDIX: The estimation of relative rates of return to capital for manufacturing industries.

For every manufacturing industry the following relation is estimated:

$$r_j(t) = r_j(t-1) + k_j(r(t-1)-\rho_j)$$

where $r_j(t)$ is the actual relative rate of return in sector j observed at time t, and k_j, ρ_j are coefficients. OLS-estimates of the coefficients are presented in table A1.

Table A1. Estimates of (ex ante) relative rates of returns (ρ_j) in manufacturing industries

Industry	k_j	ρ_j
16 Manufacture of food	-0.45 (0.25)	1.09 (0.16)
17 Manufacture of beverages and tobacco	-0.57 (0.21)	-0.08 (0.20)
18 Manufacture of textiles and wearing apparels ..	-0.13 (0.10)	0.46 (2.12)
26 Manufacture of wood and wood products	-0.05 (0.19)	0.58 (9.12)
27 Manufacture of non-industrial chemicals	-0.51 (0.25)	1.01 (0.19)
28 Printing and publishing	-0.30 (0.20)	1.81 (0.55)
34 Manufacture of paper and paper products	-0.54 (0.23)	0.42 (0.24)
37 Manufacture of industrial chemicals	-0.34 (0.27)	0.10 (0.22)
43 Manufacture of metals	-0.57 (0.26)	1.05 (0.22)
45 Manufacture of metal products, machinery and eq.	-0.36 (0.21)	0.94 (0.25)
50 Building of ships and oil platforms	-0.58 (0.24)	1.16 (0.29)

IV. CONSUMPTION DEMAND IN THE MSG MODEL

by

Olav Bjerkholt and Jon Rinde

In a multisectoral growth model disaggregate demand functions are needed for the allocation of consumer demand among commodities. A complete system of demand functions is a convenient way of combining a detailed treatment of household goods with overall simplicity. This chapter discusses the choice of a complete system of demand functions for the MSG model, the special attention paid to energy demand, the estimation procedure and empirical results, and some policy implications.

1. Introduction

The main features of the MSG model has been presented and discussed in chapter II. MSG is a disaggregate model of the long-term development of an economy working under equilibrium conditions with full employment and full capacity utilization. There is no aggregate consumption function in this model. Total consumption in determined residually as what is left of capacity output over gross investment, government consumption and net exports. The model is focused in particular on the industrial composition of the economy and on the demand for energy in industries and households. Primary factors of production, i.e. labour and capital, are assumed to be homogeneous and unconstrained in the allocation between industries. A household demand system determining the commodity composition of demand from relative prices and total expenditure is thus of central importance in the model. The demand functions determine the composition of consumption activities. Each consumption activity has fixed coefficient commodity inputs.

A complete system of demand functions ensures consistency and theoretically satisfying properties, but usually by a uniform treatment which pays less attention to intrinsic properties and relations of the categories of household goods. In our model we have chosen a rather simple complete system but with special consideration of energy as an input in household consumption. The special features characterizing the household demand for energy are i.a. the following:

- energy is always used in connection with durable goods,
- the scope for substitution between total energy and other goods, and
- lagged adjustments in stocks of appliances imply that demand elasticities are greater in the long run than in the short run.

The specification of these features within the proposed system of demand functions is highly simplified. A more comprehensive model of household demand, comprising explanatory variables in addition to total expenditure and prices such as demographic variables, types of dwelling and heating equipment, ownership of means of transportation etc., has been developed by Rødseth in chapter X. Rødseth's model is designed to fit into the specifications of the MSG model. A simpler demand system than that proposed by Rødseth has been chosen as part of the integrated MSG model mainly for computational reasons. The Rødseth model of chapter X is part of a wider MSG system of models and may be used in simulation in conjunction with MSG.

In section 2 below our complete system of demand functions is introduced, and in section 3 we discuss want relations between consumption activities following Frisch (1959). In sections 4 and 5 the estimation procedure is outlined and the estimation results are presented and evaluated. Section 6 summarizes the main findings and some policy implications are discussed.

2. The system of demand functions

The chosen system of demand functions has been directly specified rather than derived from an explicit specification of either the direct or the indirect utility function. It is important for the use within the context of the MSG model that the system has reasonable long run properties. For reasons of transparency it is advantageous that the parameters of the demand functions have fairly straightforward interpretations. Household consumption in the model consists of a mixture of 18 composite commodities (in the model's terminology called activities) representing categories of consumer expenditure. The demand for commodities is derived by an assumption of fixed commodity proportions within each activity. The starting point is the following system:

$$(1)\quad A_i = \alpha_i V^{\xi_i} \prod_j P_i^{\kappa_{ij}} \qquad i=1,\ldots,18$$

where A_i is the level of consumer activity i per capita,
P_i is the price index of consumer activity i,
V is total expenditure per capita, and
α_i, ξ_i and κ_{ij} are parameters.

The system (1) has straightforward interpretations of the ξ's and κ's as Engel and Cournot elasticities, respectively. The system can be interpreted as a first-order logarithmic approximation of any complete system of demand functions. However, the system (1) does not fulfill the adding-up condition which can be written as

$$(2) \quad \sum_i P_i A_i = V$$

We mend this shortcoming by introducing an auxiliary variable Θ which implies horizontal adjustment of the Engel curves, i.e. we replace (1) by

$$(3) \quad A_i = \alpha_i (V\Theta)^{\xi_i} \prod_j P_j^{\kappa_{ij}} \qquad i=1,\ldots,18$$

(2) and (3) together constitute our demand system. Elimination of Θ will give explicit demand functions. Certain restrictions on the parameters follow from the usual postulates of demand theory. Additional restrictions will be imposed as part of the estimation procedure.

By differentiating the demand system with regard to total expenditure and prices we arrive at the following expressions for the expenditure (E_i) and price (e_{ik}) elasticities:

$$(4) \quad E_i = \partial \ln A_i / \partial \ln V = \xi_i / \sum_j a_j \xi_j \qquad i=1,\ldots,18$$

$$(5) \quad e_{ik} = \partial \ln A_i / \partial \ln P_k = \kappa_{ik} - E_i (\sum_j a_j \kappa_{jk} + a_k) \quad i,k=1,\ldots,18$$

where $a_i = P_i A_i / V$ is the budget share of activity i.

The adding-up or consistency condition of a demand system is, of course, always fulfilled by the system (2)-(3). Homogeneity of the demand function in all prices and total expenditure requires $\sum_j e_{ij} = -E_i$ which implies

$$(6) \quad \sum_j \kappa_{ij} = -\xi_i \qquad i=1,\ldots,18$$

The symmetry property of the demand elasticities, i.e. the restrictions that must be fulfilled by the demand system for it to be derived from maximization of a utility function, requires that

$$(7) \quad a_i(\kappa_{ik}+\xi_i a_k) = a_k(\kappa_{ki}+\xi_k a_i) \qquad i,k=1,\ldots,18$$

If the demand system is adjusted to fit the data in a base year (with $\Theta=1$) the ξ's and κ's are then identical with the expenditure and price elasticities, respectively, and the adding-up, homogeneity and symmetry conditions will be satisfied in that year. Along a growth path adding-up and homogeneity conditions will be satisfied while the symmetry property cannot be maintained. All expenditure elasticities will change proportionately, i.e. such that

$$(8a) \quad E_i/E_k = \text{constant} \qquad i,k=1,\ldots,18$$

while all price elasticities will change so that

$$(8b) \quad e_{ik}/E_i - e_{jk}/E_j = \text{constant} \qquad i,j,k=1,\ldots,18$$

The close relation between the parameters of the demand system and the demand elasticities will facilitate the use of the demand system as a proxy for a more comprehensive model of household consumption such as Rødseth's model in chapter X.

The parameters of the demand system, i.e. the ξ's and the κ's, have the same degrees of freedom as a complete set of expenditure and price elasticities for a given year and can easily be derived from a set of elasticities. We shall place strong restrictions on the elasticities in the estimation, following Frisch (1959).

The 18 consumption activities represented in the model are the following:

1. Food
2. Beverages and tobacco
3. Clothing and footwear
4. Housing services
5. Electricity
6. Fuel
7. Furniture and electrical equipment
8. Other household goods
9. Insurance and domestic services
10. User cost of cars
11. Petrol and car maintenance
12. Public transport services
13. Durable recreation goods
14. Other recreation goods
15. Public entertainment and education
16. Other goods
17. Other services
18. Norwegians' consumption abroad

The level of activities correspond to items of household consumption in the national accounts with the exception of user cost of cars which comprises imputed costs (interest and depreciation) while the national accounts include car purchases as part of consumption expenditure. Total expenditure differs correspondingly from the concept used in the national accounts.

As can be seen from the list most of the activities comprise fairly broad categories of consumption expenditures. Of particular interest for energy studies are the substitution interrelations within the Housing and heating group, consisting of Housing services, Electricity and Fuel, and within the Transportation group, consisting of User cost of cars, Petrol and car maintenance and Public transport services.

Several partial studies of the households demand for energy have stressed the characteristic features of energy demand. For a survey of such studies see Taylor (1975), (1977) and Blaalid and Olsen (1978). However, to integrate all of these into the complete system's framework is a difficult task. Thus far only very aggregate models of this type have been estimated. Jorgenson (1974, 1977) estimated a demand system for three goods: Services from durables, energy, and other goods. Rødseth and Strøm (1976) estimated a system with four goods, and Pindyck (1980) had 6 goods. All these studies are based on the translog indirect utility function. With our time series data (only 17 observations) this approach is impossible with the given level of aggregation.

3. Want relations between consumption activities

In estimating the price elasticities we have relied upon the "complete scheme" approach of Frisch (1959). Frisch assumes want independence (additive utility function). This assumption is very restrictive in itself but may not be unreasonable for broad aggregates of consumer goods. However, since the MSG model will be used in analysing alternative energy policies, it is essential to take into account the want dependence of energy related activities.

By assuming separability between goods or groups of goods one can reduce the number of parameters to be estimated. In the general case with no separability restrictions on the utility function, the Cournot elasticities can be expressed as follows, see Frisch (1959):

$$(9) \quad e_{ik} = \eta_{ik} - \lambda E_i a_k E_k - E_i a_k \qquad i,k=1,\dots,18$$

Demand elasticities with regard to price (e_{ik}) and total expenditure (E_i) and budget shares (a_k) have been introduced above. λ is the inverse of the elasticity of marginal utility of total expenditure with regard to total expenditure, i.e.

$$(10) \quad 1/\lambda = \partial \ln \omega / \partial \ln V$$

where ω is the marginal utility of total expenditure.
η_{ik} is defined by

$$(11) \quad \eta_{ik} = u^{ik} \omega P_k / A_i \qquad i,k=1,\ldots,18$$

where u^{ik} is the typical element in the inverse of the Hessian of the utility function.

The η_{ik}'s are denoted "want elasticities" by Frisch. They can be interpreted as demand price elasticities with total expenditure varied so as to keep the marginal utility, ω, constant. It can be shown that

$$(12) \quad \sum_j \eta_{ij} = \lambda E_i \qquad i=1,\ldots,18$$

and

$$(13) \quad \sum_i a_i \eta_{ij} = \lambda a_j E_j \qquad j=1,\ldots,18$$

Complete want independence, i.e. additive utility function, implies that $\eta_{ik} = 0$ for $i \neq k$. The direct want elasticity η_{ii} is then equal to the right-hand side of (12) and the Cournot elasticities can be expressed as:

$$(9') \quad e_{ik} = E_i (\lambda \delta_{ik} - \lambda a_k E_k - a_k) \qquad i,k=1,\ldots,18$$

where δ_{ik} is the Kronecker delta.

Thus all price elasticities can under want independence be directly derived from estimates of expenditure elasticities and λ.

Our separability assumptions will be that there is want independence between all goods except within the Housing and heating group and within the Transportation group. This means that the utility function is additive in the utilities of the two groups and each of the other goods.

The matrix of η_{ik}'s is then block diagonal. In general, for any block representing a group of consumption activities, G, we assume

$$(14) \quad \eta_{ik} = \frac{\Lambda_G \, a_k}{a_G} \qquad (i,k\in G \quad i\neq k)$$

where $a_G = \sum_{j\in G} a_j$ and Λ_G is a parameter.

The elements on the main diagonal are derived from (12)

$$(15) \quad \eta_{ii} = \lambda E_i - \sum_{\substack{j\in G \\ j\neq i}} \eta_{ij} \qquad i\in G$$

A direct interpretation of Λ_G is somewhat difficult. Let us try in the following way. If there is complete want independence, then an increase in the price of any good can be compensated by a change in total expenditure such that the demand of all other goods are unaffected, as follows directly from the proportionality of Cournot and expenditure elasticities as stated in (9'). With groupwise want dependence this is no longer possible. An increase in any price can, however, be compensated so that the demand of all goods not belonging to the same group is left unaffected. Within the group the relative increase in the demand of other goods will be the same and equal to the product of Λ_G and the group budget share (a_k/a_G) of the good increasing in price.

Note that (14) implies that all goods within the same group have the same sign of the cross want elasticities. They are either positive ("want substitutes") or negative ("want complements"). That is hardly a convincing assumption for <u>any</u> group with want dependence. The method can be extended, however, to include subgroups of G, i.e. a block diagonal structure within the want elasticity matrix of group G. We may then have a group e.g. with want substitutability including a subgroup with want complementarity or vice versa.

Let K be any group or subgroup within the block diagonal structure and define the budget shares relative to this group or subgroup by

$$(16) \quad a_{Ki} = a_i / \sum_{j\in K} a_j \text{ for } i\in K \text{ and zero otherwise, } i=1,\dots,18$$

If good i belongs to subgroup S of group G then the want elasticity of good i with regard to the price of good k is simply

$$(17) \quad \eta_{ik} = a_{Gk}\, \Lambda_G + a_{Sk}\, \Lambda_S \qquad k=1,\dots,18$$

4. Estimation of expenditure and price elasticities

We start out with logarithmic differentiation of the demand functions for consumption activities:

$$(18)\quad \ln A_i = \beta_i + E_i \ln V + \sum_j e_{ij} \ln P_j \qquad i=1,\ldots,18$$

where β_i is a constant but E_i and e_{ij} are functions of parameters as defined in (4) - (5).

Substituting from (9) we arrive at

$$(18')\quad \ln A_i = \beta_i + E_i \ln \bar{V} + \sum_j \eta_{ij} \ln P_j - \lambda E_i \ln \bar{\bar{P}} \quad i=1,\ldots,18$$

where $\ln \bar{V} = \ln V - \ln \bar{P}$ and $\ln \bar{P} = \sum_j a_j \ln P_j$, and

$$\ln \bar{\bar{P}} = \sum_j a_j E_j \ln P_j$$

Applying (17) for a general block diagonal structure we get

$$(18'')\quad \ln A_i = \beta_i + E_i \ln \bar{V} + \lambda E_i \ln \bar{\bar{P}}_i - \sum_G \delta_{Gi} [\Lambda_G \ln \bar{P}_{Gi} + \sum_{S\subset G} \delta_{Si} \Lambda_S \ln \bar{P}_{Si}] \qquad i=1,\ldots,18$$

where $\ln \bar{\bar{P}}_i = \ln P_i - \ln \bar{\bar{P}}$,

$\ln \bar{P}_{Ki} = \ln P_i - \ln \bar{P}_K$ and $\ln \bar{P}_K = \sum_j a_{Kj} \ln P_j$ for any group or subgroup K, and $\delta_{Ki} = 1$ when $i \in K$ and zero otherwise for any group or subgroup K.

When the E_i's are replaced using (4) and the budget shares are written out explicitly, (18") is a set of simultaneous equations with unknown parameters β_i, ξ_i, λ, Λ_G, Λ_S and with observable endogenous variables A_i and observable exogenous variables P_i and V (i=1,...,18). In applying relation (18") for estimating expenditure and price elasticities with prior restrictions on want dependencies we introduce a very simple group structure. There are only two groups with more than one consumption activity:

The first group is Housing and heating (H) consisting of the three activities Housing services, Electricity, and Fuel. Within this group Electricity and Fuel are defined as a subgroup (HE). What we expect to find for this group is (strong) substitutability within the subgroup and complementarity in the group as a whole.

The second group is Transportation (T) consisting of the activities User cost of cars, Petrol and car maintenance, and Public transport services. The first two activities are defined as a subgroup (TE). What we expect to find here is clearly complementarity within the subgroup and substitutability in the group as a whole. For the remaining 12 activities we have thus assumed want independence from all other activities.

With this group structure we get the following equations for estimation:

$$(19a) \quad \ln A_i = \beta_i + \xi_i \ln \bar{V} + \lambda\xi_i \ln\bar{P}_i + U_i \qquad i=1-3,\ 7-9,\ 13-18$$

$$(19b) \quad \ln A_i = \beta_i + \xi_i \ln \bar{V} + \lambda\xi_i \ln\bar{P}_i - \Lambda_H \ln\bar{P}_{Hi} - \delta_{iHE}\Lambda_{HE}\ln\bar{P}_{HEi} + U_i \qquad i=4-6$$

$$(19c) \quad \ln A_i = \beta_i + \xi_i \ln \bar{V} + \lambda\xi_i \ln\bar{P}_i - \Lambda_T \ln\bar{P}_{Ti} - \delta_{iTE}\Lambda_{TE}\ln\bar{P}_{TEi} + U_i \qquad i=10-12$$

where the U_i's are the stochastic disturbance terms.

We have here simplified (18") somewhat by the following approximations assumed to be valid for the period of observation:

$$(20) \quad \sum_j a_j \xi_j \approx 1$$

$$(21) \quad \ln \bar{\bar{P}} = \sum_j a_j E_j \ln P_j \approx \sum_j a_j \ln P_j = \ln \bar{P}$$

With small variations in the E_j's over the period of observation (20) is a reasonable approximation (cf (4)). (21) is perhaps more doubtful. If average price increases are markedly correlated with expenditure elasticities, e.g. luxury goods getting more expensive, then (21) cannot be expected to hold.

For the disturbance terms we assume

$U(t) \sim N(0,\Omega)$

$E(U(t_1)U(t_2)') = 0$ when $t_1 \neq t_2$

where $U(t) = [U_1(t), U_2(t), \ldots, U_{18}(t)]'$

In the complete system of equations (19a) - (19c) some parameters appear in different equations, and endogenous variables are present on the right hand sides (in the budget shares). A satisfactory method to deal with these complications is the Full Information Maximum Likelihood method (FIML). As the data base is national accounts time series for the period 1962 - 1978, i.e. fewer observations than the number of activities, the estimated covariance matrix will be singular and the maximum likelihood estimators do not exist.

To solve this problem we have a priori imposed the restriction that Ω is block diagonal in the following groups of activities: Housing and heating, Transportation, and the remaining want independent activities. This implies that we can estimate the relations for these three groups separately and that the covariance of error terms from different groups are zero.

Our demand relations do not explicitly take into account stocks of durable goods. This is of particular importance for our two groups of energy related consumption activities. As noted earlier lagged adjustments in stocks of appliances have been found to imply that demand elasticities are greater in the long run than in the short run. In the estimation we have tried to take this into consideration in a rather ad hoc way by applying a partial adjustment model.

$$\text{(22a)} \quad \ln A_i^*(t) = B_i \cdot Z_i(t)$$

$$\text{(22b)} \quad A_i(t)/A_i(t-1) = [A_i^*(t)/A_i(t-1)]^{\gamma_i}$$

where B_i and $Z_i(t)$ are coefficients and right-hand variables of equation no. i, and

$A_i^*(t)$ is the optimal level for consumption activity no. i.

Combining (22a) and (22b) equation no. i in the system for estimation becomes

$$\text{(23)} \quad \ln A_i(t) = \gamma_i B_i Z_i(t) + (1-\gamma_i)\ln A_i(t-1) + U_i(t) \quad i=4\text{-}6 \text{ and } 10\text{-}12$$

The assumptions made about disturbances rule out autocorrelation. This may well be a too rigid assumption for time series data. We have hence estimated the equations also under the assumption that the disturbances follow a first order autoregressive scheme:

$$U_i(t) = \rho_i U_i(t-1) + \varepsilon_i(t)$$

$$\varepsilon(t) \sim N(0,\Omega) \text{ and } E(\varepsilon(t_1)\varepsilon(t_2)') = 0 \text{ when } t_1 \neq t_2$$

It follows then that equation no. i becomes

$$(24)\quad \ln A_i(t) = B_i Z_i(t) + \rho_i \ln A_i\,(t-1) - \rho_i B_i Z_i(t-1) + \varepsilon_i(t) \qquad i=1,\ldots,18$$

or when partial adjustment is also applied

$$(25)\quad \ln A_i(t) = \gamma_i B_i Z_i(t) + (1-\gamma_i)\ \ln A_i(t-1) - \rho_i \gamma_i B_i Z_i(t-1) - \rho_i (1-\gamma_i)\ \ln A_i(t-2) + \varepsilon_i(t) \qquad i=4-6$$

5. Empirical results

The results are presented for each of the three groups separately with a summing up and comparison with other studies at the end.

A. The twelve want independent consumption activities

For these activities we tried to estimate all coefficients simultaneously using FIML on (19a) assuming no autocorrelation. Convergence was not obtained. For three activities (Insurance and domestic services, Public entertainment and education, and Other services) single equation estimation gave positive values for λ. Somewhat arbitrarily these were excluded and for the subset of the remaining 9 equations we obtained convergence using FIML. The results for the ξ_i's are given in the first column of table 1. All coefficients have reasonable values with correct sign and small standard errors. The estimate for the inverse of the elasticity of the marginal utility of total expenditure (λ) is -0.679 (T-value 11.5) which implies $1/\lambda$ around -1.5. This result is in reasonable accordance with other studies.

All equations were also estimated by Ordinary Least Squares (OLS) with λ fixed at -0.679 and the results for the ξ_i's are presented with the values for R^2 and the Durbin-Watson observator (DW) in column three of table 1.

Small sample size and a quite high degree of autocorrelated disturbances, as indicated by the Durbin-Watson observator, indicate that the sampling variances are probably seriously underestimated. To take care of the autocorrelation we also estimated the equations by assuming the disturbances to follow a first-order autoregressive scheme as set out in (24).

Using FIML for the 9 previously estimated equations convergence was again not obtained. We then employed a Cochrane-Orcutt iterative (COI) process on each single equation with λ fixed at -0.679. Error correlations across equations are thus disregarded.

Table 1. Estimation of demand equations for twelve want independent activities (19a). (T-values in parentheses)

Consumption activities	FIML with no autocorrelation, three goods excluded	COI with $\lambda=-0.679$		OLS with $\lambda=-0.679$		
	ξ_i	ξ_i	ρ	ξ_i	R^2	DW
1 Food	0.498 (30.8)	0.504 (17.6)	0.50	0.498 (26.8)	0.98	1.01
2 Beverages and tobacco ...	0.875 (13.4)	0.773 (4.9)	0.88	0.868 (12.2)	0.91	0.43
3 Clothing and footwear	0.520 (12.3)	0.550 (7.9)	0.57	0.524 (11.9)	0.90	0.92
7 Furniture and electrical equipment	1.392 (24.5)	1.381 (6.5)	-0.09	1.377 (21.0)	0.97	1.52
8 Other household goods ...	1.234 (24.1)	1.154 (13.7)	0.55	1.230 (23.9)	0.97	1.09
9 Insurance and domestic services ...	-	0.195 (0.55)	0.95	0.511 (0.97)	0.07	0.24
13 Durable recreation goods ...	1.896 (28.7)	1.834 (16.7)	0.35	1.877 (23.0)	0.97	1.25
14 Other recreation goods ...	1.489 (38.5)	1.460 (27.3)	0.40	1.482 (38.7)	0.99	1.21
15 Public entertainment and education ..	-	1.119 (7.9)	0.96	1.192 (19.1)	0.96	0.34
16 Other goods ...	1.440 (29.9)	1.370 (13.3)	0.80	1.436 (28.5)	0.98	0.57
17 Other services	-	0.886 (4.4)	0.74	0.798 (7.1)	0.77	0.60
18 Norwegians' consumption abroad ..	1.605 (16.7)	1.636 (6.9)	0.93	1.628 (16.7)	0.94	0.38

The results from the three different methods FIML, COI and OLS (the latter two with λ fixed at -0.679) are shown in table 1. It is notable that the coefficient estimates are very little affected by the method of estimation, but the estimated T-values are approximately halved when autocorrelation is taken into account.

B. Housing and heating

In the estimation of the demand equations for the Housing and heating group we retained the fixation of λ at the previously estimated value (-0.679). Using FIML on the static model with no autocorrelation (19b) we obtained the results in table 2.

Table 2. Estimation of demand equations for Housing and heating. FIML with λ = -0.679. (T-values in parentheses)

Activities	ξ_i	Λ_{HE}	Λ_H	R^2	DW
4 Housing services	1.043 (12.0)	-	-0.336 (-0.9)	0.98	0.81
5 Electricity	1.339 (3.2)	0.715 (1.1)	-0.336 (-0.9)	0.92	1.44
6 Fuel	0.323 (0.62)	0.715 (1.1)	-0.336 (-0.9)	0.48	1.57

All estimates have expected signs but the standard errors are large for Λ_H, Λ_{HE} and ξ_6.

Multicollinearity in the time series data makes it difficult to determine the income and price effects in simultaneous estimation. In an attempt to improve the quality of our estimates we restricted the expenditure elasticities for Electricity and Fuel to be identical. A similar assumption has often been made in other studies. With this additional restriction we got the results in the upper left quarter of table 3. All estimates have expected sign and much smaller standard errors than the estimates presented in table 2. The very low value of R^2 for the Fuel equation is unsatisfactory and the DW observator indicates autocorrelated disturbances, especially for Housing services.

In the other quarters of table 3 the results from the estimations with first order autoregressive disturbances and partial adjustment model are presented, all obtained by FIML. A likelihood ratio test shows that the static model cannot be rejected even at 10 percent level in favour of the partial adjustment model. The models where first order autocorrelation is taken into account do not turn out significantly better than models with no autocorrelation.

In fact the introduction of autocorrelated disturbances gives wrong signs for some of the price and income elasticities. This is probably again due to problems of multicollinearity.

The coefficients presented in table 3 for the partial adjustment model must be interpreted as short run coefficients. The long run coefficients are obtained by dividing through with γ_i. In estimating the partial adjustment model all symmetry restrictions are imposed in the long run, but not in the short run. If all symmetry restrictions are imposed both in the short and the long run it requires the same adjustment coefficient for all goods belonging to the same group. This is a too rigid assumption. However, we have imposed the same adjustment coefficients for Electricity and Fuel estimated to about 0.5. This means that almost half of the adjustment takes place in the first year while for Housing services only 10 per cent of the adjustment takes place in the first year.

In table 4 the long run elasticities of the partial adjustment model with no autocorrelation are compared with those of the corresponding static model. The elasticities are calculated for the year 1979. In both cases we get the reasonable result that Electricity and Fuel are substitutes while both are complements to Housing services. For Electricity and Fuel the results obtained by the two methods are rather similar, while the demand for Housing services is more elastic in the partial adjustment model both with regard to own-price and total expenditure.

Table 3. Estimation of demand equations for Housing and heating.

Activities	No auto-	
	ξ_i	Λ_{HE}
4 Housing services	1.098	-
	(14.3)	
5 Electricity	1.100	0.90
	(3.6)	(1.9)
6 Fuel	1.100	0.90
	(3.6)	(1.9)

	Partial		
	No autocor-		
	ξ_i	Λ_{HE}	Λ_H
4 Housing services	0.177	-	-0.03
	(5.6)		(2.0)
5 Electricity	0.384	0.43	-0.03
	(1.6)	(1.6)	(2.0)
6 Fuel	0.384	0.43	-0.03
	(1.6)	(1.6)	(2.0)

FIML with $\lambda = -0.679$ and $\xi_5 = \xi_6$. (T-values in parentheses)

Static model							
correlation			First order autocorrelation				
Λ_H	R^2	DW	ξ_i	Λ_{HE}	Λ_H	ρ	R^2
-0.48	0.97	0.82	0.859	-	0.61	0.03	0.99
(1.8)			(12.9)		(2.1)	(0.5)	
-0.48	0.93	1.21	-0.46	0.15	0.61	1.00	0.95
(1.8)			(1.2)	(0.5)	(2.1)	339.8	
-0.48	0.20	1.48	-0.46	0.15	0.61	0.39	0.28
(1.8)			(1.2)	(0.5)	(2.1)	(3.7)	

adjustment model								
relation			First order autocorrelation					
γ_i	R^2	DW	ξ_i	Λ_{HE}	Λ_H	γ_i	ρ	R^2
0.10	1.00	2.2	0.213		-0.07	0.12	-0.04	1.00
(3.3)			(8.6)		(2.6)	(4.7)	(0.6)	
0.48	0.96	2.6	0.736	0.26	-0.07	0.58	-0.54	0.97
(4.3)			(8.1)	(1.12)	(2.6)	(4.1)	(1.9)	
0.48	0.37	1.8	0.736	0.26	-0.07	0.58	-0.35	0.26
(4.3)			(8.1)	(1.12)	(2.6)	(4.1)	(1.4)	

Table 4. Comparison of expenditure and price elasticities for Housing and heating in 1979 from the static model (S) with long-run elasticities from the partial adjustment model (P)

Activities	Expenditure elasticities		Matrix of Cournot elasticities: Housing services		Electricity		Fuel	
	S	P	S	P	S	P	S	P
4 Housing services	1.11	1.77	-0.62	-1.09	-0.13	-0.11	-0.05	-0.05
5 Electricity ..	1.11	0.80	-0.35	-0.21	-0.67	-0.57	0.22	0.23
6 Fuel	1.11	0.80	-0.35	-0.21	0.50	0.53	-0.96	-0.88

In figure 1 price and expenditure data for estimating the electricity and fuel demand are shown. In figure 2 the estimates from the static model have been used in a simulation of the electricity demand in the period 1962 - 1981. The simulation results are presented together with the actually observed figures. In most of the estimation period 1962 - 1978 the model fits the data quite well but in the four latest years of observation, 1978 - 1981, the model forecast of demand is much too low. The sharp decrease in electricity demand in the model from 1977 - 1978 is due to the large price increase on electricity in this period. From 1979 to 1980 the sharp increase in electricity demand in the model is due to the sharp increase in the fuel price and also to the decline in the electricity price. Figures 1 and 2 give a good illustration of how the model works. In the model the consumers adjust immediately to the new price and income situations. In real life these adjustments will naturally take some time. If we could believe completely in the model results we would say that the electricity demand was in equilibrium in 1977 while in 1981 the equilibrium demand was approximately 10 per cent lower than actually observed. However, from comparing the model results with the observed data in the period 1977 - 1981 one may doubt whether the model can be of any use in the short-to-medium term forecasting of electricity demand, at least when the price changes are as large as in these years. In most of the estimating period the real prices showed only small fluctuations. When the price changes are larger the price responses may not be as smooth and symmetric as the model assumes. In the model simulations presented in chapter IX the same elasticities have been used from an assumed equilibrium situation in 1979.

Figure 1. Deflated indices of energy prices and total expenditure. 1979 = 100

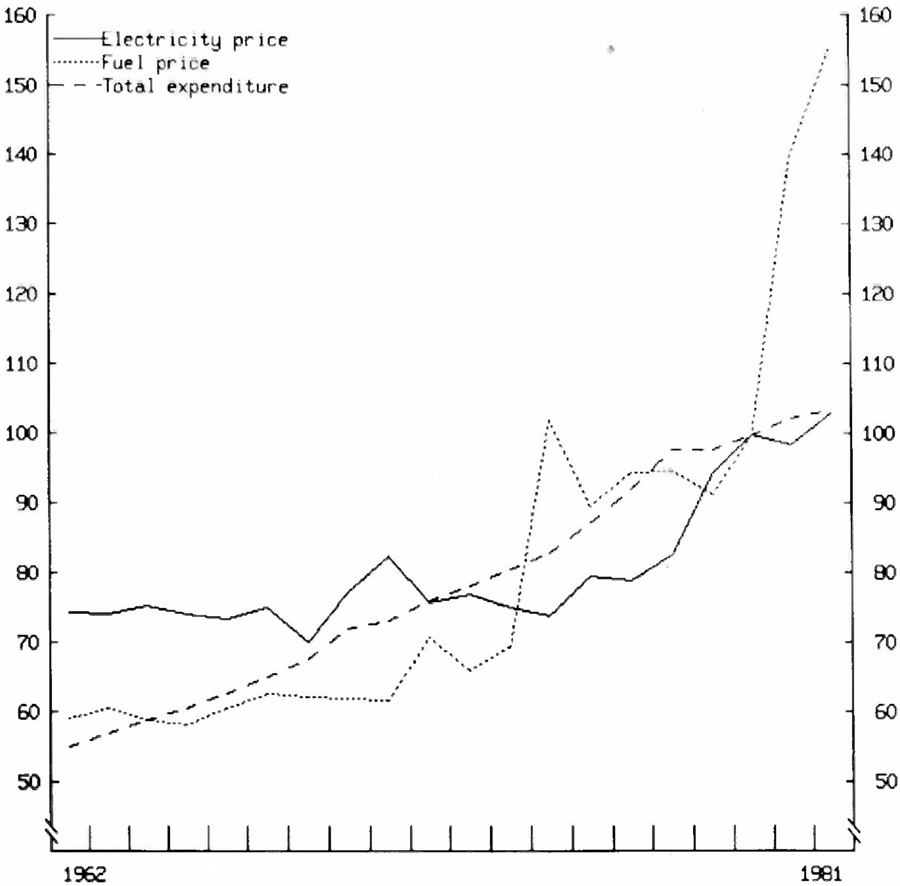


Figure 2. Electricity consumption in households. TWh

C. Transportation

For the Transportation group we estimated the static and partial adjustment model both by FIML and with λ fixed at -0.679 with results as given in table 5. For this group we did not take into account autocorrelated disturbances as the DW observator indicated no serious problem of autocorrelation. For the Transportation group the static model was rejected at 10 per cent level in favour of the partial adjustment model, but not at 5 per cent level.

Table 5. Estimation of demand equation for Transportation. FIML with $\lambda = -0.659$. (T-values in parentheses)

Activities	Static model				
	ξ_i	Λ_{TE}	Λ_T	R^2	DW
10 User cost of cars ..	1.457	-0.85	0.15	0.89	2.33
	(9.9)	(8.1)	(1.8)		
11 Petrol and car maintenance	1.585	-0.85	0.15	0.95	1.47
	(15.5)	(8.1)	(1.8)		
12 Public transport services	0.825	-	0.15	0.98	2.03
	(14.6)		(1.8)		

	Partial adjustment model					
	ξ_i	Λ_{TE}	Λ_T	γ_i	R^2	DW
10 User cost of cars	0.208	-0.18	0.02	0.17	1.00	2.07
	(10.1)	(4.9)	(1.5)	(12.8)		
11 Petrol and car maintenance	0.208	-0.18	0.02	0.17	0.98	2.20
	(10.1)	(4.9)	(1.5)	(12.8)		
12 Public transport services	0.599	-	0.02	0.70	0.98	1.98
	(5.6)		(1.5)	(4.9)		

In the partial adjustment model the same expenditure elasticities and adjustment coefficients were imposed on User cost of cars and Petrol and car maintenance. This assumption cannot be statistically rejected in favour of free estimation of these coefficients. In table 6 the long run elasticities from the two models are compared. The elasticities for Public transport services are almost identical for the two methods. The demand for the private transport activities is more elastic both with regard to total expenditure and to own-price in the static model than in the partial adjustment model.

Table 6. Comparison of expenditure and price elasticities for Transportation in 1979 from the static model (S) with long run elasticities from the partial adjustment model (P)

Activities	Expenditure elasticities		Matrix of Cournot elasticities: User cost of cars		Petrol and car maintenance		Public transport services	
	S	P	S	P	S	P	S	P
10 User cost of cars	1.47	1.22	-0.65	-0.43	-0.39	-0.44	0.02	0.01
11 Petrol and car maintenance ...	1.60	1.22	-0.35	-0.44	-0.78	-0.43	0.02	0.01
12 Public transport services .	0.83	0.86	0.05	0.03	0.05	0.03	-0.68	-0.66

D. Summary of estimation results

In table 7 a set of expenditure and direct price elasticities for all activities is presented. For the 12 want independent activities the calculations are based on the COI estimates presented in table 1. For Housing and heating and Transportation the elasticities are calculated from the static model without autocorrelation. The corresponding cross elasticities for the two energy related activity groups are presented earlier in tables 4 and 6.

The estimates in table 7 have been implemented in the current version of the MSG model. As discussed above the elasticities from the static model are by and large not significantly different from the long run elasticities from the partial adjustment model, and the latter model does not fit the data significantly better.

The MSG model is a long run equilibrium model and for many of the applications of the model only the equilibrium results matter and not the details and time structure of the particular equilibrating mechanisms at work. There are other applications for which the dynamic processes and time lags are of great concern. We have little belief, however, that the partial adjustment model outlined above is elaborate enough to catch the essentials of the adjustment over time in the use of energy. We have on this background and without strong conviction preferred the static model. As stated in the introduction to this chapter the model of Rødseth in chapter X is designed to be used for simulation in conjunction with MSG.

Table 7. Budget shares, Expenditure elasticities and Own-price elasticities. Static model, no autocorrelation

Activities	Budget shares	Expenditure elasticities	Own-price elasticities
1 Food	0.210	0.509	-0.416
2 Beverages and tobacco	0.071	0.781	-0.556
3 Clothing and footwear	0.087	0.555	-0.407
4 Housing services	0.110	1.109	-0.619
5 Electricity	0.040	1.111	-0.672
6 Fuel	0.017	1.111	-0.956
7 Furniture and electrical equipment	0.055	1.395	-0.951
8 Other household goods	0.019	1.165	-0.796
9 Insurance and domestic services	0.017	0.195	-0.356
10 User cost of cars	0.043	1.471	-0.649
11 Petrol and car maintenance	0.049	1.601	-0.778
12 Public transport services	0.044	0.833	-0.683
13 Durable recreation goods .	0.021	1.811	-1.221
14 Other recreation goods ...	0.041	1.474	-1.001
15 Public entertainment and education	0.023	1.130	-0.773
16 Other goods	0.036	1.384	-0.942
17 Other services	0.055	0,806	-0.576
18 Norwegians' consumption abroad	0.060	1.652	-1.110

E. Comparison with other studies

In table 8 we present a survey of recent estimates by others of residential energy demand elasticities. One could expect the elasticities estimated for different countries to vary because of the differences in the patterns of energy use. Differences in estimates between and within countries are also due to differences in model specifications, estimation methods, the quality of data and the time period.

Looking first at the long run own-price elasticity of total energy use, we find a range extending from -0.33 to -1.15. Only Pindyck (1980) obtained elasticities greater than one in absolute value. Our estimate, -0.45, is in the lower end of the spectrum.

Our estimate of long run own-price elasticity for electricity, -0.67, is very close to Pindyck's -0.65. The two other estimates for Norway, Blaalid and Log (1977) and Rødseth and Strøm (1976) are both approximately -0.3.

Table 8. Other estimates of long run residential energy demand elasticities

Study	Own-price elasticities		Income elasticities		Country
	Total energy	Electricity	Total energy	Electricity	
Our study	-0.45	-0.67	1.11	1.11	Norway
Fuss and Waverman (1975) ...	-0.33 to -0.56		0.83 to 1.26		Canada
Nordhaus (1975)	-0.71		1.09		6 countries pooled
Joskow and Baughman (1976)	-0.50	-1.00	0.60		USA
Taylor, Blattenberg and Verleger (1976)		-0.81		1.05	USA
FEA (1976) ...		-1.46		1.10	USA
Rødseth and Strøm (1976) .		-0.33			Norway
Mount and Chapman (1976)		-1.17		0.61	USA
Blaalid and Log (1977) ...		-0.29		1.04	Norway
Pindyck (1980)	-1.11	-0.65	1.00[1)]	1.00[1)]	Norway
Pindyck (1980)	-1.10	-0.30	1.00[1)]	1.00[1)]	USA
Pindyck (1980)	-1.15	-0.39	1.00[1)]	1.00[1)]	Canada

1) The income elasticities are equal to one by the assumption of homotheticity.

The income elasticities both for total energy and electricity 1.11 are quite similar to the results obtained in many other studies.

6. Conclusions and policy implications

In terms of policy implications, our most important findings are that the own-price elasticities of residential electricity and fuel are markedly larger than previously estimated by Norwegian reasearchers and what seem to be adhered to by government authorities. As pointed out in section 5B predictions from our model are somewhat uncertain especially for large and sudden increases in energy prices. However, our estimates are very close to what Pindyck (1980) obtained for Norway and also similar to what other international studies report.

Our results indicate that price oriented policies for energy can be quite effective given enough time to allow the price mechanism to work. However, even if the own-price elasticities for energy goods may seem high the real price increase must be quite large if the purpose is to stabilize the residential energy consumption, given a normal growth in total expenditure. An average annual growth rate for total expenditure of 2.5 per cent from now until the year 2000 which is somewhat lower than projected by the government would imply approximately 4 per cent annual average growth rate for the real price of electricity to keep electricity consumption constant. In order to stabilize the total residential energy consumption the total energy price must grow at annual average rate of 6 per cent. This means a doubling of the real price of energy every 12 years.

V. THE SPECIFICATION OF ELECTRICITY FLOWS IN THE MSG MODEL

by

Svein Longva and Øystein Olsen

1. Introduction

The MSG-4E model is designed to be used as a planning tool for the dimensioning of the capacity of the electricity sector in addition to its traditional usage in the long term economic planning of the Norwegian economy. As a consequence special attention is paid to the specification of relations describing the supply and demand for electricity. Furthermore, in order to secure the consistency with sector plans for the electricity sector it is of crucial importance that the model projections for the production of electricity in value terms easily can be translated into physical measures for electricity. The possibilities for doing this depend of course fundamentally on the characteristics of the electricity market and the data which serve as a basis for the model specifications. Following a long tradition in macro-economic model building in Norway the MSG-4E model is closely linked to the national accounting system. However, the treatment of the electricity flows in these accounts is rather unsatisfactory with respect to the model specification of MSG-4E; in particular the value flows of the accounts are not suitable as a basis for measuring electricity in physical terms. The national accounts therefore have to be supplemented with data from other sources.

The purpose of this chapter is to describe in detail the principal background for the specification of the electricity flows in the MSG-4E model and the methods of calculation which are actually used in implementing the theoretical scheme. In section 2 a brief overview of the electricity market in Norway is given, stressing the physical characteristics of the electricity supply and demand system and the actual pricing policy pursued by the public authorities. In section 3 some basic principles for aggregating micro commodities are discussed, and a specification of the electricity flows in the MSG-4E model is chosen. The actual arrangement of data and the calculations necessary for constructing an accounting framework for electricity flows are described in section 4. The final section contains some concluding remarks.

2. The electricity market in Norway

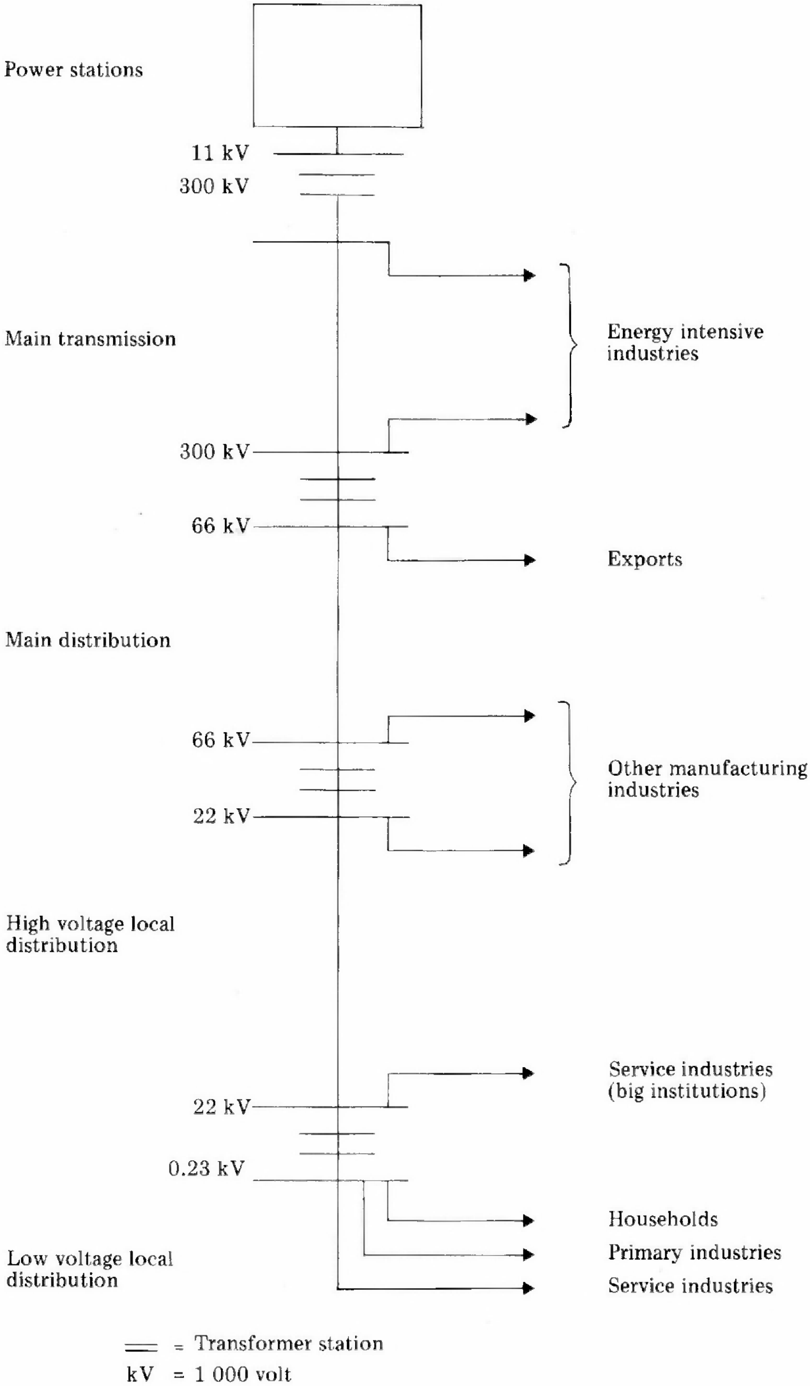
The electricity supply and demand system

The electricity supply system of Norway differs from that of most other countries in the world by beeing predominately a hydro power system. A brief and summary description of the supply system is given in figure 1. The different consumers are supplied with electricity at different voltage levels. In order to reduce the physical power losses by transmission, the "produced electricity" is up-transformed to 300 kV. The energy intensive industries receive this high-voltage electricity directly from the main transmission network. From the main distribution network high voltage power (66 kV) is delivered to manufacturing industries for production purposes (e.g. production of pulp and paper) and exported. From the local distribution network high voltage power (22 kV) is delivered to transportation purposes and is also applied in electric boilers in manufacturing and some large institutions (e.g. hospitals). Households, primary industries and the majority of service and smaller industries receive low voltage electricity (0.23 kV) mainly for light and heating purposes.

A central point to be noted from figure 1 is that the use of resources in the total supply system is dependent on the distribution of the electricity demand between consumers. Energy intensive industries are, as a general feature, located near the hydro electric power plants. The capital costs in the "transportation" of electricity to these industries are therefore relatively low, and the same applies to the physical power losses both because of the short transportation distance and because of the high voltage level of these deliveries. Electricity consumed by private households, on the other hand, is carried over longer distances and through all stages of the distribution network; both capital costs and physical power losses, in addition to costs of operating the system, are thus much higher.

From this brief overview of the electricity market it should be clear that a number of characteristics seem to be specific for each delivery of electricity to consuming sectors. A simplified description of the electricity system, which will serve as a point of departure for the specification in the MSG-4E model, may be the following: From the discussion above it may be useful to consider the electricity supply system as separated into two major parts, i.e. a production part, which covers the power stations, and a distribution part comprising the transmission and distribution system. Electricity, i.e. the quantities in physical terms, are produced in the production part, while the distribution

Figure 1. The electricity supply system[1)]

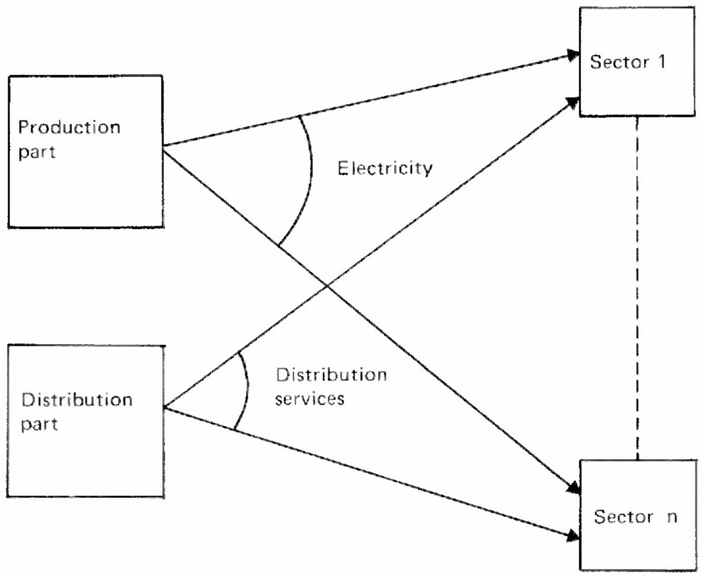


1) This figure is borrowed from Vinjar (1980).

part provides distribution services. Moreover, the discussion above also suggests that one ought to distinguish between several "types" of "electricity" depending on the voltage level and other characteristica. In the same way the "distribution service" associated with each delivery of "electricity" may in the outset be viewed as specific for each consumer.

A principle outline of this way of regarding and specifying the electricity market is given in figure 2.

Figure 2. A principal outline of the electricity market



If we assume that a uniform volume measure may be established for "distribution services" a main conclusion from the description above is that "electricity" and "distribution services" are demanded by the electricity consuming sectors in varying proportions, depending on the voltage level, the distance of transportation etc. Energy intensive industries need relatively little of input of "distribution services" per unit of "electricity", while the amount of "distribution services" needed per unit of "electricity" used is much higher in households and other consuming sectors using low voltage power.

The structure of technology in the hydro power system and the concept of electricity

It is essential to notice that there are important differences between the production structure of the power plants and the distribution network. As the production of electricity is based on extraction of natural resources it is reasonable to assume that decreasing returns to scale prevail in the "production sector". On the contrary there are some indications of increasing returns to scale in the activity of distributing electricity to the different consuming sectors; probably this applies in particular to the main transmission network.

A detailed discussion of the technology in the production part of the electricity supply system is given by Rinde and Strøm in chapter VI of this volume. As analyzed there the specification of the production structure in the power plants is highly connected with the problem of how to measure different quantities of electricity. As a general rule one may say that one should choose the volume measure which is most suitable in the production function for electricity; i.e. which is believed to be most directly related to the input side. In a hydro power system it is particularly important to distinguish between kW (a measure for load capacity) and kWh (an energy measure). As in a thermal power system the load dimension is related to capacity sufficient to meet peak load demand. Accordingly, the input of machine capital is directly related to the concept of kW. However, in a hydro power system there is also an energy dimension; whereas fuels for thermal plants can be bought as required, water supply to a hydro plant cannot be controlled except by storage. The capacity of the water reservoirs (construction capital) is related to the energy potential in the water storage necessary to coordinate supply and demand both within and between years.

An important implication of this structure of technology is that the total use of resources in the power plants is dependent on the distribution of the energy demand between consuming sectors. The main reason for this is that the instaneous electricity demand (measured in kW) in industries where the power is used for production purposes is rather uniform during the year, while the demand for e.g. heating purposes varies considerably. It is obvious that it is necessary with relatively larger machine installations in the production units to cover a given demand for energy for the latter type of use.

These characteristic features of the supply side of the electricity market could probably only be given a proper treatment within a rather complicated dynamic model, distinguishing between the instaneous flows and accumulated deliveries. For the specification of energy flows in the MSG-4E we have to decide upon a one-dimensional physical measure for electricity. Since MSG-4E is calculating on year-to-year basis kWh is a natural choice.

An analysis of the distribution part of the electricity system is presented by Schreiner and Strøm in chapter VII of this volume. Even though the output from this production activity is unobservable there exists some a priori information of the input structure in the production of "distribution services". More specifically, engineering studies suggest that power losses can be reduced considerably by reinforcement of the transmission and distribution network. Accordingly, physical power losses should be regarded as a separate input in the distribution of electricity in addition to labour, capital and materials.

The pricing of electricity

In Norway public authorities have a decisive influence on the operation of the electricity market in general and on the electricity prices in particular. About 80 per cent of all power plants and distribution establishments are owned by the Central government or municipal authorities. Furthermore, the Central government uses both the general value added tax and a specific commodity tax for electricity to regulate the prices paid by the consuming sectors.

In this section we shall avoid any detailed discussion of pricing principles for electricity, but only repeat a couple of important features regarding the actual pricing policy in Norway. The first fact to be noted is that the average price of electricity has been and still is considerably lower than long run marginal costs. Consequently, the production capacity is too high to ensure economic efficiency. The other important element of the Norwegian pricing policy to be mentioned here is the existence of price differentiation in the electricity market. Partly, this fact is a result of considerable regional differences in the prices, reflecting different production costs for electricity. However, it is commonly assumed that the most important element of price differentiation in the Norwegian electricity market exists between deliveries to energy intensive industries on the one hand, and deliveries to other electricity consumers on the other. Due to favourable long term contracts it is believed that prices paid by energy intensive industries are relatively low compared to the charges of other users even when prices are corrected for different distribution costs.

Some implications for the design of the model

As the MSG-4E model is designed with the specific purpose to be used as a planning tool for the electricity sector it follows that the following elements should be reflected or taken care of by the model specification:

i) Total demand for electricity measured in kWh should be easily derived from the model calculations

ii) The model should distinguish between production and distribution of electricity

iii) The model should contain relations describing the long run technology of producing and distributing electric power

iv) The model should include a representation of the indirect tax structure for electricity, and the existence of pure price differentiation should be accounted for.

3. The national accounts and the accounting system of the model

Traditionally the national accounts form the conceptual framework for both macroeconomic planning and macroeconomic models in Norway.[1)] This applies also to the MSG-4E model. Balance equations and definitional relations in the accounts and in the model are identical or correspond, and most of the statistical data required for estimation and base year values are supplied by the national accounts. However, as mentioned above, the MSG-4E model is designed to be used as a planning tool for the expansion of the electricity sector. It is therefore of particular importance to integrate the supply and demand for electricity, both in value terms and in physical units, into the accounting system of the model. Before looking more closely at the representation of the electricity flows we shall give a short introduction to the quantity concept of the commodity flows of the model.

The quantity concept of commodity flows

The commodity flows of the MSG-4E model are - like in other macroeconomic models - aggregates of the micro commodity flows of the economy. The quantities of commodity flows in the model are measured in base year values, i.e. constant prices. This means that the micro commodities are aggregated to commodities of the model by adding base year

1) For a discussion of this linkage see Bjerkholt and Longva (1980).

values of the micro commodities, i.e. the quantities are aggregated with base year prices as weights. As stressed in Bjerkholt and Longva (1980) this is a reasonable procedure if

i) the micro commodities constituting the aggregate are separable from all other commodities, i.e. marginal rates of substitution between any two commodities in the aggregate are independent of all other commodities, and equal both in production and consumption,

ii) the marginal rates of substitution are equal to the relative prices, and constant over time, and

iii) there is one and only one price for each commodity in the base year (no price differentiation).

The assumption of separability is a basic presupposition for the existence of aggregates (see e.g. Berndt and Christensen (1973)). Roughly speaking it means that the optimal mix of commodities within the aggregate is determined independently of quantities of commodities outside this group, i.e. the optimal quantities of commodities outside the separable group are only related to the level of the aggregate, not to the composition of commodities within the aggregate.

The assumption (ii) assures that the relative base year prices used as weights when aggregating micro commodities reflect the rate at which the commodities can be substituted (at the margin) without changing the quantity of the aggregate. When relative prices are constant over time, the quantitative interpretation of the aggregate will also be unchanged in the projection period.

The assumption of no price differentiation is necessary in the first place to secure that the micro commodities (measured in constant prices) have the same quantitative interpretation in all markets, i.e. the same use of real resources per unit. Furthermore, the existence of price differentiation will most commonly imply that relative prices differ between markets, i.e. the assumptions (i) and (ii) are not fulfilled.

From this discussion we conclude that, given the assumptions of no price differentiation and equality between marginal rates of substitution and relative prices, a group of micro commodities can be aggregated with the base year prices as weights and a unit of each aggregated commodity will have the same quantitative interpretation in all submarkets.

A proper representation of the electricity sector in the MSG-4E model

With the description of the electricity market in section 2 and the conditions for consistent aggregation discussed above as the point of departure we will discuss the specification of electricity flows in MSG-4E. It should be kept in mind that we are working with a macroeconomic model where the benefits of a detailed specification must be evaluated against the operationality of the model together with the possibilities of implementing the specification empirically. It is therefore impossible to work into the model all the details mentioned in section 2. As a first step in constructing an accounting framework for electricity flows we thus assume that all the micro commodities in the electricity market indicated in section 2 (see e.g. figure 2) may be aggregated to one commodity for electricity delivered from the production part and one commodity for distribution services produced in the distribution part. We will argue that these two commodities should be specified separately and thus that the production part and the distribution part of the electricity supply system should be treated as separate production sectors. The reason for this conclusion is that the two commodities are not separable on the production side (i.e. the condition (i) above is not fulfilled). In section 2 we argued that the two commodities are produced non-jointly, i.e. with separate production functions, which is just another way of saying that the commodities are not separable from the specified inputs.[1)] Furthermore, since electricity and distribution services are demanded in different proportions in the various submarkets the total use of real resources in the electricity supply system is dependent upon the composition of the demand.

We believe that the specification of only one commodity for electricity and thus a uniform technology in the production part is rather reasonable in relation to the intended use of the model. The aggregation of the different commodities representing distribution services to one aggregate commodity may, however, be more problematic. As mentioned in section 2 there are some indications that there are important differences in technology between various parts of the distribution network. On pure theoretical grounds it might therefore have been desirable with a further disaggregation of the electricity supply system in the MSG-4E model. However, the present data situation does not allow an "ideal" specification of the electricity supply system, and accordingly we have settled with the separation of the electricity supply system into one production part and only one distribution part.

1) See Hall (1973).

It is important to notice that it is not the existence of demand differentiated distribution costs in itself that make a disaggregate specification of the electricity sector necessary. If the separability condition on the production side had been fulfilled, the resource use could have been correctly predicted by the model even with an aggregate specification of the electricity sector. The remaining problem would then have been that the value flows of the aggregate commodity would not have had the same quantitative interpretation (in kWh) in all markets. However, the number of kWh demanded by the different consuming sectors might have been calculated consistently by dividing the value flows by the kWh-prices in the base year. A real aggregation problem arises only because we assume that electricity and distribution services are produced with separate technologies.

The existence of price differentiation in the electricity market pointed at in section 2 must also be given special attention in the specification of the electricity flows. There are several possible reasons why the electricity prices paid by the consuming sectors (the purchaser prices) differ. One is the existence of user-differentiated distribution costs. Another reason is that the specific commodity tax imposed on electricity also varies between consumers. Moreover, there are, as mentioned in section 2, important elements of what we may call pure price differentiation in the electricity market. By this we mean that even if we correct for real distribution costs and indirect taxes, the prices paid for electricity vary considerably between consuming sectors. As a consequence the "resource content" in the aggregated value flows are not the same in all markets; i.e. the conditions (ii) and (iii) above are not fulfilled. As will be shown in section 4 the specification and calculation of user-differentiated distribution costs make it possible to estimate the rates of pure price differentiation between sectors.

The accounting system of the model

In the present Norwegian national accounts the electricity supply system is represented by one sector, delivering one single commodity to the electricity consuming sectors. This means that if we regard the specification in figure 2 in section 2 as the basic description of the electricity system, we may say that in the national accounts the two (micro) commodities electricity and distribution services are aggregated to one national account commodity and that the production part and the distribution part of the system form one production sector. We argued above that, in the model context, this is not an acceptable procedure. In order to construct a suitable volume measure for electricity and distribution services it is therefore necessary to undertake a separation of the value flows in the national accounts.

The principal concept for evaluating commodity flows in the national accounts is (approximate) basic values, and constant basic value is also the general volume concept for commodity flows in the MSG-4E model.[1)] The basic value of a commodity flow is defined as the purchasers' value less trade margins and commodity taxes, net, in production and trade. For electricity flows in particular this means that in the present national accounts the purchasers' values (equal to producers' value because trade margins are zero) are decomposed into the following value components:

i) Basic value
ii) Value added tax
iii) Other commodity taxes (the electricity tax)

Differences in purchaser prices caused by demand differentiated tax rates are thus already taken care of by the calculations in the national accounts. However, as we have pointed out above, also the basic value flows for electricity as defined in the national accounts contain value components which are specific for each consuming sector. In order to construct a suitable volume measure for electricity it is therefore necessary to undertake a further separation of the basic value flows in the national accounts.

In the model the basic values are, according to the discussion above, divided into three components:

(a) standard value for electricity,
(b) real distribution costs, and
(c) pure price differentiation.

In figure 3 a simplified description of the accounting framework of the MSG-4E model is presented, emphasizing the specification of value flows related to electricity. The upper part of the table indicates

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1) The basic value concept is preferred to producers' value or purchasers' value because rates of trade margins and commodity taxes may vary between receiving sectors. Use of the latter value concepts as volume measures would therefore complicate the aggregation of micro commodities as discussed above and would also cause a discrepancy between total supply and total demand in constant values. (See e.g. Sevaldson (1973) for a further discussion of this point.)

input of commodities to production sectors and to final demand, and the lower part indicates the output structure of commodities from production sectors.

Standard value is the basic volume measure for the commodity electricity in the model. When deducting distribution costs and price differentiation from the basic values of electricity flows in the national accounts this value concept should in principle be proportional to kWh. Distribution costs are, as discussed above, represented by a separate commodity in the model, while price differentiation by definition does not represent any use of real resources, and is therefore specified in the model as an artificial "tax" (or "subsidy") with demand differentiated rates. By convention price differentiation is specified as an indirect tax on the commodity distribution services.

In addition to the separation of the basic values, the other value components that are attached to the electricity flows in the national accounts, i.e. value added tax and the specific commodity tax on electricity, must also be explicitely dispersed on the two commodities. The electricity tax is, as seems reasonable, related to electricity measured in standard value, while the value added tax is proportionally allocated between the two commodities.

Finally, because we distinguish explicitly on the supply side between the production and distribution of electricity it is necessary also to separate the inputs to the electricity sector in the national accounts between the two sectors in the model.

In the next section we will describe in more detail the calculation of the different electricity related flows indicated in figure 3.

Figure 3. A simplified presentation of the accounting system of the MSG-4E model

Delivering accounts		Receiving accounts		Commodities							Production sectors			Final demand	Value added
				1	2	3	4	5	6	7	8	9	10	11	12
Commodities	Electricity related value flows	Electricity in standard value	1									x	x	x	
		Electricity tax	2										x	x	
		Value added tax on electricity	3										x	x	
		Distribution services in standard value	4										x	x	
		Price differentiation	5										x	x	
		Value added tax on distribution services	6										x	x	
	Other commodity flows		7								x	x	x	x	
Production sectors	Electricity related sectors	Production of electricity	8	x	x	x									
		Distribution of electricity ..	9				x	x	x						
	Other production sectors		10							x					
Final demand			11												x
Value added (primary inputs)			12								x	x	x		

4. The accounting framework for the electricity demand and supply flows, empirical results

The implementation of the accounting framwork presented in figure 3 requires data only for the base year of the model. The reason is that the commodities electricity and distribution services are demanded in fixed (but different) proportions both by production sectors and by households (see Longva, Lorentsen and Olsen, chapter II of this volume for a further explanation). The same applies to commodity inputs both in the production sector and in the distribution sector for electricity.[1] Consequently, base year data are sufficient for the separation of national account figures. Since the base year of the MSG-model in this study is 1979 all figures in the following refer to 1979.

It is also important to notice that the calculation of the various value flows indicated in figure 3 always starts out from corresponding value flows of the national accounts. In general the number of independent estimates necessary to undertake the separation of value flows is therefore one less than the number of components. In the separation of the basic values for electricity it seems natural to calculate price differentiation as a residual.[2] This method is used in the following. The calculated rates of price differentiation therefore catch up all sorts of errors in measurement and lack of consistency between the estimated variables. The errors may be particularly serious for industries other than manufacturing where data sources are rather weak. The reader should keep this in mind.

The information about flows in physical units is provided by the energy accounts. During the last years an account for energy resources in physical units - including electricity measured in kWh - has been worked out in the Central Bureau of Statistics.[3] In addition to estimates of energy reserves and extraction these accounts show in detail

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1) However, as described by Schreiner and Strøm in chapter VII of this volume we have assumed that there are substitution possibilities between capital and physical power losses in the production of distribution services. For these two inputs we therefore need time series for estimation purposes. The same applies to capital input in the production of electricity (see Rinde and Strøm, chapter VI of this volume).
2) It may be interesting to notice that if price differentiation had been neglected in the model specification the distribution costs (a real cost component) could have been derived residually.
3) For a presentation of the energy accounts see e.g. Birkeland et. al. (1980).

the pattern of supply and demand of different energy resources in the Norwegian economy. The classification of sectors corresponds reasonably well to the detailed level of aggregation in the national accounts. The energy accounts may therefore easily be combined with the value flows in the national accounts. However, the correspondance is not complete and a "correction" of the energy accounts is necessary to achieve consistency between the physical flows in these accounts and the corresponding value flows of the national accounts. This applies in particular to value and physical flows to industries other than manufacturing.[1)]

The separation of the electricity sector of the national accounts into a production part and a distribution part

The input and output structure of the electricity sector of the national accounts, including figures for capital stock and labour input, are presented in the first column of table 1. To simplify we have excluded the value added tax. The results of the separation of the electricity sector into a production part and a distribution part are presented in the last two columns of table 1. The output (upper) part of these columns corresponds to rows 8 and 9 of figure 3 while the input (lower) part corresponds to columns 8 and 9.

1) In the national accounts the deliveries of a number of commodities including electricity to service industries are entered as intermediate input to the sector "Commercial buildings". In the energy accounts this construction is omitted, implying that all deliveries of electricity are traced directly as input to the final consuming sectors. As the model specification is based on the national accounts, we have corrected the energy accounts in accordance with the treatment of electricity consumption in the national accounts.

Table 1. The input and output structure of the electricity sector of the national accounts and the production and distribution sectors of the model. Figures for 1979 in million kroner

	The national accounts	The model accounts	
		The production sector	The distribution sector
Output			
Production of electricity and distribution services in standard/basic value	6 421.9	1 936.9	4 673.2
Electricity tax	1 241.6	1 241.6	-
Price differentiation	0	0	0
Other commodity output in basic value1)	839.6	839.6	0
Input			
Material (commodity) input in net purchasers' value	1 073.1	204.3	868.8
Power losses in standard value	0	0	188.2
Value added in net market value	7 430.0	3 813.9	3 616.1
Wages	1 677.5	0	1 677.5
Depreciation	2 166.0	1 236.4	929.6
Operating surplus	2 350.9	1 341.9	1 009.0
Electricity tax	1 241.6	1 241.6	0
Sector taxes	-6.0	-6.0	0
Labour input (in man-years) ..	16 800.0	0	16 800.0
Capital stock	77 419.9	44 191.2	33 228.7

1) These commodity outputs include mainly own-account capital formations (construction work).

(i) The separation of materials, labour and capital inputs

The inputs to the electricity sector of the national accounts, i.e. materials, labour and real capital, have to be distributed between the two production sectors of the model. The necessary information to carry out this breakdown empirically is provided by the Norwegian electricity statistics.[1] Materials, which is a minor input in electricity supply, is dispersed on production plants and the distribution network

1) The calculations are presented in detail in Myklestu (1979).

respectively. The labour force in the electricity sector is mainly engaged in the administration and operation of the supply system. By convention the labour input is in its entirety allocated to the distribution sector.

The most important production factor in electricity supply is of course the capital stock. The electricity statistics do not contain information of capital stock, only of gross investments in the different parts of the supply system. However, by accumulating gross investments separately for the production and distribution parts of the supply system, and taking depreciation into account (applying the perpetual inventory method), we have estimated the capital stock separately for the two sectors.[1)]

(ii) Production of electricity in standard value and the electricity tax

Flows of electricity in standard value should be proportional to the physical measure for electricity. As the energy accounts[2)] provide us with information of deliveries in kWh to the different consumers, the calculation of standard values may be seen as a pure normalization of these physical flows. The standard price for electricity in kWh may in this respect be determined arbitrarily. However, the level of the standard price will obviously determine the dispersion of operating surplus (defined as the residual of factor income) on the production and distribution part, respectively. A reasonable procedure, which is followed in the present version of the MSG-4E model, is to let the standard price be calculated in such a way that the operating surplus is distributed between the two parts of the electricity sector in accordance with the relative proportions of capital stock. Given the separation of the input structure in the electricity sector described above the total output measured in standard value in the production part of the supply system, E^P, can be estimated by:

$$(1) \quad E^P = M^P + \frac{K^P}{K}(Y_K + Y_S)$$

where K^P is capital stock in the production sector of the model,

K is capital stock in the electricity sector of the national accounts,

Y_K is depreciation in the electricity sector of the national accounts,

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1) Investments are accumulated for the period 1967 to 1977.
2) See table 2 below.

Y_S is operating surplus in the electricity sector of the national accounts, and

M^P is material inputs in the production sector of the model (measured in net purchasers' value)

The standard price of electricity, P_S, calculated for the base year is then easily derived by dividing the estimated production value with the total production measured in kWh, X^P, given from the resource accounts[1], i.e.

$$(2) \quad P_S = \frac{E^P}{X^P}$$

For 1979 (the base year of the model in this study) the standard price is estimated by this method to 2.32 øre/kWh.

As discussed above the electricity tax is related to electricity measured in standard values. The total of the electricity tax is therefore allocated to the production sector.

iii) Valuation of distribution services and price differentiation

The fundamental problem regarding the calculation of flows of distribution services is of course that these deliveries are not directly observable. As a first step in constructing these value flows the total output of distribution services is estimated from the cost side analogous to the calculation of total standard value of electricity flows. Total net costs in the distribution part, D_N, is calculated by

$$(3) \quad D_N = (M-M^P) + Y_L + \frac{K-K^P}{K}(Y_K + Y_S) = B^P - E^P$$

where M is input of materials in the electricity sector of the national accounts

Y_L is labour costs in the electricity sector of the national accounts, and

B^P is total production in the electricity sector of the national accounts, measured in basic values.

In order to derive the production value for distribution services from net costs estimated by (3) two corrections must be made. Firstly, it should be remembered that physical power losses, X^D, is treated as an input in the distribution part and therefore must be taken

1) See table 2 below.

into account as a real cost component.[1)2)] Secondly, before reaching total standard value for distribution services which is to be used as a basis for the calculation of commodity flows in the model, total costs must be corrected for the level of net price differentiation (the sum of price differentiation over receiving sectors), W.[3)] Total distribution services in standard values, D, is then calculated by

$$(4) \quad D = D_N + p_S X^D - W$$

The level of net price differentiation may be determined independently of national account figures; the relation (4) will always provide that the sum of net output of electricity, $E^P - p_S X^D$, output of distribution services, D, and net price differentiation, W, equals total output of electricity in the national accounts, B^P. For simplicity net price differentiation in the present MSG-4E version is normalized to zero in the base year of the model, i.e. $D = D_N + p_S X^D$.[4)]

1) The figure for total physical power loss in kWh is given from the resource accounts, see table 2 below.

2) In general the separation of a sector where internal deliveries are not specified will "blow up" the total production value, leaving, however, value added unchanged, see table 1.

3) As mentioned in section 3 price differentiation is introduced as a commodity "tax" on distribution services. However, it is clear that price differentiation is not an ordinary tax in the sense that it is collected and "passed over" to public authorities. Accordingly, price differentiation has no "counter part" on the cost side. On this background net price differentiation may rather be regarded as an explicit specification of the level of over-/underpricing of distribution services. The calculation of price differentiation by receiving sector is discussed below.

4) An additional problem relevant for this discussion should be mentioned, namely the fact that import prices for electricity may differ from the standard price calculated for domestic production by (2). In that case the procedure of calculation described above will imply that net price differentiation calculated from the demand side will deviate from zero. In the program for calculating these flows for the MSG-4E model this problem is avoided by replacing the relation (2) by

$$(2)' \quad p_S' = \frac{B^P + B^I - D_N}{X^P + X^I} = \frac{E^P + B^I}{X^P + X^I}$$

where B^I is the import value of electricity, and

X^I is the import of electricity measured in kWh.

It is seen from this relation that the two ways of estimating the standard price of electricity are identical (i.e. $p_S = p_S'$) if and only if $\frac{E^P}{X^P} = \frac{B^I}{X^I}$.

Electricity related flows by consuming sectors

The demand and supply of electricity of the resource accounts and the national accounts are presented in table 2. In column 1 the accounts for electricity demand and supply in physical units are given. In the second column the national accounts figures for electricity measured in net market values (the purchasers' or sellers' values excluding value added tax) are given[1]. In the third and fourth column the value figures of column 2 are broken down into electricity flows in basic values and electricity tax.

The results of the separation of the commodity electricity of the national accounts into one commodity for electricity delivered from the production part and one commodity for distribution services produced in the distribution part are presented in the last four columns of table 2. The demand (lower) part of these columns corresponds to rows 1, 2, 4 and 5 of figure 3.

(i) Deliveries from the production part

The flows of electricity to the different sectors measured in standard values, E_j, (the elements in row 1 in figure 3) may be easily calculated as

$$(5) \quad E_j = p_S X_j \qquad j=1,\ldots, n_S$$

where X_j is the delivery of electricity in physical units to sector j and n_S is the number of electricity receiving sectors.

The electricity tax is, as mentioned, conventially treated as a delivery from the production part.

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1) For industries other than manufacturing the presented net market values are not actual national account figures but adjusted figures based on Norwegian electricity statistics. The estimated value flows for electricity in the national accounts for these industries are at present subject to serious errors of measurement and registration.

Table 2. Electricity related flows in physical and value units in the resource accounts, the national accounts and in the model. Figures for 1979. Value flows in million kroner

	The resource accounts	The national accounts			The model accounts: Deliveries from imports and the production sector		The model accounts: Deliveries from the distribution sector	
	Electricity in GWh ($=10^6$ kWh)	Electricity in net market values	Electricity in basic values	Electricity tax	Electricity in standard values	Electricity tax	Distribution services in standard values	Price differentiation
	1	2	3	4	5	6	7	8
Supply								
Imports	846	96.3	84.3	12.0	84.3	12.0	-	-
Production[1)]	87 769	7 663.5	6 421.9	1 241.6	1 936.9	1 241.6	-	-
Distribution	-				-	-	4 673.2	0
Demand								
Domestic gross demand ...	83 128	7 343.2	6 089.6	1 253.6	1 896.07	1 253.6	4 476.53	0
Private industries	56 307	3 517.0	2 799.8	717.2	1 096.17	717.2	1 751.4	-47.65
11 Agriculture	682	95.7	82.1	13.6	15.56	13.6	69.29	-2.75
12 Forestry	0	0	0	0	0	0	0	0
13 Fishing	40	5.6	4.8	0.8	0.91	0.8	4.06	-0.18
16 Manuf. of food	1 451	166.5	137.5	29.0	33.1	29.0	121.41	-17.00
17 Manuf. of beverages and tobacco	162	12.1	8.9	3.2	3.7	3.2	13.55	-8.35
18 Manuf. of textiles and wearing apparel .	220	27.9	23.5	4.4	5.02	4.4	18.41	0.07
26 Manuf. of wood and wood products	651	87.7	74.7	13.0	14.85	13.0	54.47	5.38
27 Manuf. of non-industrial chemicals	1 508	138.8	108.6	30.2	34.4	30.2	18.03	56.18
28 Printing and publishing	221	36.9	32.5	4.4	5.04	4.4	18.49	8.97
31 Mining and quarrying	874	70.1	52.6	17.5	19.94	17.5	73.13	-40.46

	1	2	3	4	5	6	7	8
34 Manuf. of paper and paper products	3 457	227.5	191.5	36.0	78.85	36.0	123.97	-11.32
37 Manuf. of industrial chemicals	5 518	305.8	225.4	80.4	125.86	80.4	65.96	33.58
40 Refining of crude oil .	186	16.0	12.3	3.7	4.24	3.7	2.22	5.83
43 Manuf. of metals	24 106	1 054.1	752.8	301.3	549.83	301.3	288.14	-85.18
45 Manuf. of metal products, machinery and equipment	1 284	145.2	119.5	25.7	29.29	25.7	107.44	-17.22
50 Building of ships and oil platforms	518	48.9	38.5	10.4	11.82	10.4	43.34	-16.66
55 Construction	608	90.5	78.3	12.2	13.87	12.2	61.77	2.66
60 Ocean transport	0	0	0	0	0	0	0	0
65 Extraction of crude oil	0	0	0	0	0	0	0	0
72 Production of electricity	-	-	-	-	0	-	-	-
73 Distribution of electricity	8 250	-	-	-	188.17	-	-	-
74 Domestic transportation	842	82.7	65.9	16.8	19.21	16.8	85.55	-38.85
79 Repair of motor vehicles and household appliances	13	2.1	1.8	0.3	0.3	0.3	1.32	0.18
81 Wholesale and retail trade	-	-	-	-	0	-	0	0
82 Financing and insurance services	220	34.8	30.4	4.4	5.02	4.4	22.35	3.03
83 Dwellings	-	-	-	-	0	-	0	0
84 Other private services	5 496	868.1	758.2	109.9	125.36	109.9	558.41	74.44
General government	4 334	667.1	580.4	86.7	98.86	86.7	440.4	41.2
91 Public administration .	288	44.3	38.5	5.8	6.57	5.8	29.26	2.67
92 Defence	409	63.1	54.9	8.2	9.33	8.2	41.56	4.02
93 Education and research	1 712	263.4	229.2	34.2	39.05	34.2	173.94	16.21
94 Health services	1 446	222.5	193.6	28.9	32.98	28.9	146.92	13.7
95 Other public services .	479	73.8	64.2	9.6	10.93	9.6	48.67	4.61
Households	22 487	3 159.1	2 709.4	449.7	512.90	449.7	2 284.73	-88.24
Exports	5 487	416.6	416.6	0	125.15	0	196.76	94.69

1) Note that physical power losses are treated as electricity input in the distribution part and included in the output of the production part in the model accounts while this internal delivery is netted out in the electricity sector of the national accounts.

(ii) Deliveries from the distribution part

Above we described how we have defined and calculated the total standard value of production in the distribution sector. The problem then remains to determine deliveries of distribution services to the various consuming sectors. Since distribution services are not sold separately in a market, the available information for carrying out this separation is rather scarce, and our method of calculation will therefore have to be based on rather rough assumptions.

Our objective is to estimate the differences in distribution costs per kWh between sectors. For this purpose we divide the consuming sectors in the MSG-4E model into five groups (see table 3). Within each of these groups we assume that the input of distribution services per kWh is identical between sectors. Between these five groups we assume that the relative differences in input of distribution services per kWh are reflected in relative differences in physical power losses per kWh received. Estimates of physical power losses in per cent, α_i, can be derived from engineering studies, and are presented in table 3 for the five sector groups.[1)]

Table 3. Physical power losses in per cent[2)]

Sector groups	α_i
1. Energy intensive industries	2
2. Production of pulp and paper	6
3. Other manufacturing	14
4. Other industries, government and households	17
5. Exports	6

With the above assumptions the physical power losses related to the different deliveries of electricity to sectors, X_j^D, may be calculated as

$$(6)\quad X_j^D = \lambda_{ij} \cdot \alpha_i \cdot X_j, \qquad (i=1,\ldots,5;\ j=1,\ldots,n_S)$$

where the λ_{ij}'s are dummy variables; i.e. λ_{ij} is either one or zero depending on whether sector j belongs to the sector group i or not.

1) Information of physical power losses can e.g. be found in Krokan (1977).
2) These figures do not comprise use of electricity in power stations or pumping plants.

Flows of distribution services to the different consuming sectors, D_j, may then be estimated as

$$(7)\quad D_j = \frac{X_j^D}{\sum_{k=1}^{n_S} X_k^D} \cdot D \qquad j=1,\ldots,n_S$$

As described in section 3 price differentiation is defined and calculated as the residual component of the basic value flows of electricity to different sectors, i.e.

$$(8)\quad W_j = B_j - E_j - D_j, \qquad j=1,\ldots,n_S$$

where B_j is the basic value of electricity demand in sector j, given from the national accounts.

Value added taxes imposed on electricity and distribution services respectively are calculated using value added tax rates estimated from the appropriate value flows in the national accounts. These calculations are not presented here.

5. Some concluding remarks

In interpreting the results presented above it should be emphasized that a positive sign on the price differentiation component (the figures in the column 8 of table 2) means that the deliveries are discriminately "taxed", i.e. the sector in question pays a higher price for electricity compared to other consumers when real costs in production and distribution are accounted for. On the contrary, when the price differentiation component is negative the sector receives a relative "subsidy" on the electricity deliveries. From table 2 it is seen that positive price differentiation appears for the multitude of service sectors (private and public) and some manufacturing industries. Among these is, perhaps a bit surprisingly, the energy intensive industry Manufacture of industrial chemicals (sector 37)[1]. "Subsidies" are, according to our results, imposed on the deliveries to Households, Domestic transportation (sector 74) and the majority of the manufacturing industries, including such large consumers of electricity as Manufacture of metals (sector 43) and Manufacture of paper and paper products (sector 34).

1) The "tax" paid by this sector is, however, rather small compared to the level of electricity consumption.

In order to make some further comments it may be useful to normalize the figures of the model accounts of table 2 with the electricity consumption in kWh's for the different sectors. Price figures and rates of electricity tax, distribution costs and price differentiation for aggregate sector groups are presented in table 4.

Table 4. Electricity prices, rates of electricity tax, rates of distribution costs and rates of price differentiation for groups of consuming sectors. All figures in øre/kWh. 1979

	Net market prices for electricity	Standard price for electricity	Rates of electricity tax	Rates of distribution costs	Rates of price discrimination
Energy intensive industries	4.6	2.3	1.3	1.2	-0.2
Other manufacturing industries	9.3	2.3	1.7	5.6	-0.3
Other industries and government	15.1	2.3	2.0	10.2	0.7
Households	14.0	2.3	2.0	10.2	-0.4
Exports	7.6	2.3	0	3.6	1.7
Total demand ..	8.8	2.3	1.5	5.4	0

In the first column of this table average net market prices (i.e. market prices less value added tax) for the various groups are presented. As mentioned earlier these prices are relatively low for energy intensive industries, and relatively high for service industries and households. It is indicated by the figures in table 4 that, with the assumptions and calculation methods outlined above, the differences in the observed market prices mainly reflect differences in real distribution costs. For households and industries other than manufacturing distribution costs amount to more than two thirds of the net market value. For energy intensive industries, at the other extreme, the relative share of distribution costs is only about one forth. Still our calculations indicate that the prices paid by power intensive industries are lower than total real costs when a comparison with other consumers is undertaken. However, the subsidies received by other manufacturing industries and households are of the same magnitude according to our results. As mentioned above positive rates of price differentiation appear for the majority of service industries. Exports prices also exceed real costs of production and distribution.

The estimated standard price of electricity may seem very low compared to the rates of distribution costs. It should be emphasized that the estimated standard price obviously reflects the method by which total costs in electricity supply are separated between production and distribution respectively. As mentioned above, all labour costs are allocated to the distribution sector. Furthermore, total operating surplus in the national accounts is divided proportionally with the capital stock on the two activities. As decreasing returns to scale are commonly assumed to prevail in hydro power production while there are some indications of increasing returns in distribution (see chapter VII of this volume), it may be argued that a relatively larger share of total operating surplus should have been allocated to the production sector.

As stressed several times the figures in the last column of table 4 are measures only of relative price differentiation, as total price differentiation by convention is normalized to zero. In recent discussions of energy policy in Norway the question of "subsidies" on electricity is commonly related to the fact that the prices paid by consumers are considerably lower than long run marginal costs in electricity supply. In our calculations capital costs are assumed to be identical to operating surplus in the national accounts. The rate of return on capital calculated by dividing operating surplus with the capital stock (in current prices) is far below the real rate of return that is used in the calculations of long run marginal costs[1]. The existence of decreasing returns to scale in hydro power production may furthermore "push" downwards the actual marginal rate of return to capital implicitly used in our calculations.

1) By dividing the operating surplus calculated for the electricity sector in the national accounts with total capital stock we get 0.03. If we view the proceeds of the electricity tax as part of the capital income the real rate of return becomes 0.046. The real rate of return imposed on public investments in 1979 was 0.07.

VI. COST STRUCTURE OF ELECTRICITY PRODUCTION

by

Jon Rinde and Steinar Strøm

1. Theoretical considerations

Hydro electric power

In hydro electric power production we may distinguish between three main activities:

- Collecting and storing water
- Conducting the water to the power station machinery (turbines and generators)
- Carrying out the transformation of energy from kinetic energy to electricity.

As opposed to a thermal power system, where the supply of energy is only limited by the capacity of the power plants as long as sufficient fuels may be bought in the market, the energy capacity in a hydro power system can only be controlled by building water reservoirs. This means that the planning problem in a hydro power system includes a specific energy dimension, making the distinction between energy capacity and load capacity important in the description of the production structure.

The reservoirs collect precipitation from a catchment area which is planned and structured for this purpose. Thus, in a hydro electric power system real resources (labour, capital, etc.) must be employed in order to make the store of energy available. Furthermore, topography and precipitation factors will set a limit to the energy capacity. A given water storage represents a certain energy potential, in the sense that the reservoir capacity expresses how much energy can be produced in the power stations by one-time depletion of full reservoirs.

The energy capacity of a single power station can be expressed by the following physical law:

$$(1) \quad E_i = g\, M_i H_i ,$$

where E_i is the energy capacity (measured in Joule or kWh per annum),
g is the acceleration of gravity,
M_i is the expected quantity of water running through the

machines of the power station in the course of a year, and
H_i is the difference in altitude between the water reservoir and the turbines in the power station.

By aggregating over individual power plants we obtain from (1)

(2) $E = g\,\bar{H}\,M + N\,g\,\mathrm{Cov}\,(H,M)$

N is the total number of power stations. E is the energy capacity per annum in the total hydro power system. M is the total quantity of water running through the machines during a year. $\bar{H}$ is the average altitude between the water reservoirs and the turbines. The covariance between the altitudes, H_i, and the amount of water M_i, is presumably negative due to the topographical conditions. In order to simplify we have assumed an exogeneously given relationship between Cov (H,M) and the average energy capacity in the power stations. Moreover the constant g in (2) is transformed so that E is measured in TWh (terawatt hours). Thus, (2) can be written as

(3) $E = G\,\bar{H}\,M$

where G is a positive constant.

The load capacity of a hydro power system is determined by the speed and maximum amount of water instantanously flowing through the machinery. Hence, the load capacity is determined by the dimension of the penstocks running from the reservoirs to the power stations and the dimension of the machinery. The two essential factors determining the dimension of a penstock are the altitude between the power station and the reservoir, and the diameter. The capacity of the penstock increases both with the altitude and the diameter. The relation between investments in penstocks and the load capacity is specified as

(4) $K_T = h(Q, \bar{H})$

where Q is the load capacity, and
K_T denotes capital embedded in penstocks.

The higher load capacity which is required the more has to be invested in penstocks. It should be noticed that a given load capacity could be achieved by different combinations of the altitude and the diameter of the penstocks. The partial derivative of the h-function with respect to the average altitude is positive if the costs of increasing the penstock by lengthening it exceed the costs of widening it, given the effect capacity.

The installations of machinery in the power stations are assumed to be proportionate to the load capacity, i.e.

(5) $K_M = a\ Q \qquad\qquad a > 0$

where K_M denotes stocks of turbines and generators.

The total amount of water flowing through the machinery in the hydro power system during a year depends on the investments in the reservoirs and on the load capacity. The following macro relationship is therefore assumed:

(6) $M = m(K_R, Q; \bar{H})$

where K_R is the capital stocks embedded in reservoirs.

m is assumed to be an increasing, strictly concave function of K_R. This property reflects the decreasing returns to scale of collecting water in a given environment. For obvious reasons a partial increase in Q increases the maximum amount of water which the stations can receive. Moreover, through a coordinated operation of the power plants, investments in reservoirs could be reduced while maintaining the same overall energy capacity. The reason why the average altitude is included is that the higher $\bar{H}$ is, given M and Q, the higher are the costs of collecting and storing water. Hence, the partial derivative of m with respect to $\bar{H}$ is negative.

It should be noted that technical progress is not included in the relationships specified above. This is due to the fact that our data of investments in machinery and construction equipment used to estimate the cost structure of the hydro power system are expressed in 1978-prices and based on 1978-technology.

In order to derive the cost structure in electricity production we shall transfer total investment expenses to annual capital costs. In this connection the expression capital rental price will be used.

Let us assume that we are about to undertake a project which involves extending the capacity of electricity production. The expansion implies an investment outlay "today" of

(7) $P_M \Delta K_M + P_B \Delta K_T + P_B \Delta K_R = P_K \Delta K$

where P_M is the purchasers' price of plant capital, and

P_B is the purchasers' price of constructions (tunnels and reservoirs)

Adding together investments in plant capital, ΔK_M, work on tunnels and conduits, ΔK_T, and work on reservoirs, ΔK_R, we get total capital input, ΔK, with corresponding price index P_K. For the sake of simplicity we assume for the moment that the extension will increase net profits by Nkr $\Delta\Pi$ annually. Revenue is calculated in terms of constant prices. If r is the social discount rate and T the life-span of the new plant, then the project has the following present value:

$$(8)\quad V = \sum_{t=1}^{T} \Delta\Pi(1+r)^{-t} - P_K \Delta K$$

The project should be accepted if the present value is non-negative.

Business economies in particular have often operated with calculations in which a comparison is made between income and costs designated in terms of Nkr per annum. Many people consider a comparison of this kind more convenient than a consideration of expressions of present value. If we are to arrive at the same conclusions as those based on expressions of present value, we must, however, ensure that there is a correspondence between the two ways of valuating costs against incomes. We can transform the present value expression V in (8) to an expression, V*, which gives us the difference between annual, current net income and a fixed, annual capital outlay. If the decision rule originally is that we should accept the project if the present value is positive, then the result of the transformation is that we instead must compare current net income with the annual capital outlay. Once we have then made our choice, either carrying out an investment or refraining from doing so, we no longer need to use magnitude V*.

$$(9)\quad V^* = \Delta\Pi - [\nu(r,T) P_K \cdot \Delta K]$$

where $$\nu(r,T) = \left[\sum_{t=1}^{T} (1+r)^{-t}\right]^{-1} = \frac{r}{1-e^{-rT}}$$

$\nu(r,T)$ is an annuity factor which transforms initial investment outlays to annual costs. With the aid of this factor we can therefore operate with a transformed present value expression. $\nu(r,T)P_K$ can be interpreted as the rental price for machinery and construction capital. It should be emphasized, however, that the fundamental method of calculation is based on the present value criterion. The current expenses incorporated in the net income $\Delta\Pi$ include costs of materials and repairs, labour expenses, and other current inputs. For the sake of simplification and in accordance with the way variable costs are treated in the data, we assume variable costs to be proportionate (with a factor b) to capital costs. The total costs of the power system on annual basis are then defined as

$$(10) \quad C = (1+b)[\nu P_M K_M + \nu P_B (K_R + K_T)]$$

To obtain the optimal cost structure of the hydro power system we minimize (10) given E, ν, P_M, P_B and the restrictions (3) - (6). In addition Q must fulfill the constraint

$$(11) \quad Q \geqq \bar{Q}d$$

where $\bar{Q}d$ is the peak load demand.[1] From this optimization it follows that K_M, K_T and K_R can be expressed as functions of E and the relative price P_M/P_B.

Since the purpose of this study is to obtain numerical estimates of the cost structure of the hydro power system, the functions must be fully specified. For K_R and K_T the data gives information only on the sum $K_T + K_R$. In Strøm (1979) it is shown that under reasonable assumptions, which also are supported by engineering data, cost minimization with respect to K_T and K_R implies that the ratio K_T/K_R is constant and independent of E.

We choose the following functional forms:

$$(12) \quad K_B = \varepsilon_0 E + \varepsilon_1 E^{\varepsilon_2} \qquad \varepsilon_0 \geqq 0 \quad \varepsilon_1 \geqq 0 \quad \varepsilon_2 \geqq 1$$

where $K_B = K_R + K_T$

$$(13) \quad K_M = \alpha E \qquad \alpha > 0$$

$\varepsilon_1 > 0$ and $\varepsilon_2 > 1$ imply decreasing returns to scale. As already noted this is considered as an essential feature of a hydro power system. $\alpha + \varepsilon_0 > 0$ implies a variable elasticity of scale starting at a level just below 1 and decreasing down to the level $1/\varepsilon_2$. $\varepsilon_0 > 0$ implies a

1) One way of implementing the optimum solution is through peak load pricing, see chapter XII of this volume.

similar development in the scale elasticity for the demand of K_B. If $\varepsilon_0 > 0$ this scale elasticity is constant and less than 1.

In the specification above we have ignored the relative price P_M/P_B. As the data of investment costs are only calculated in constant prices, the impact of changes in relative prices cannot be identified. Included in our specification of the cost structure is that the utilization time of the load capacity is constant. The utilization time, t_u, is defined as the number of hours it will take for the production at peak load to meet the energy capacity, i.e.

$$(14) \quad E = t_u \cdot Q$$

From (5), (13) and (14) we obtain

$$(15) \quad t_u = \frac{a}{\alpha}$$

Our specification therefore implies that t_u is determined independently of E.

Substituting (12) and (13) into (10) we get

$$(16) \quad C = (1+b)\ [\nu P_M\ \alpha\ E + \nu P_B(\varepsilon_0 E + \varepsilon_1 E^{\varepsilon_2})]$$

or

$$(16) \quad C = (1+b)\ [(\nu P_M\ \alpha + \nu P_B\ \varepsilon_0)E + \nu P_B\ \varepsilon_1\ E^{\varepsilon_2}]$$

The long run marginal cost of the hydro power production is then

$$(17) \quad C' = \beta_0 + \beta_1 E^{\beta_2}$$

where

$$(18) \quad \begin{cases} \beta_0 = (1+b)\nu[P_M\ \alpha + P_B\varepsilon_0] & \geqq 0 \\ \beta_1 = \varepsilon_1\varepsilon_2\ \nu P_B(1+b) & \geqq 0 \\ \beta_2 = \varepsilon_2 - 1 & \geqq 0 \end{cases}$$

Thermal Power

Although there is a rich supply of watercourses in Norway it is a question of relative costs when it will be beneficial to include thermal power in the power system. It has been argued that hydro power should be preferred whatsoever since it is based on the utilization of a renewable resource (water). This argument, however, neglects the fact that resources like labour, raw materials, transportation equipment etc. are needed to develop watercourses and to transform kinetic energy into electricity.

Let E_H and E_T be the amount of energy produced in hydro power plants and thermal plants respectively. The minimized cost function associated with the hydro power part of the system is given above by relation (16) and may be indicated as $C_H(E_H)$. In the same way the minimized cost function associated with the thermal part may be written $C_T(E_T)$.

A reasonable assumption is that there is a constant returns to scale in the thermal part of the system. Hence, the marginal cost in the thermal part is independent of E_T. Denoting this constant marginal cost by C_T' the conditions for the optimal mix of hydro/thermal power can be found by solving the following problem:

$$(19)\quad \begin{cases} \min C = C_H(E_H) + C_T \cdot E_T \\ \text{with respect to } E_H,\ E_T \\ \text{subject to } E_H + E_T = \bar{E} \end{cases}$$

The optimal production levels, $\tilde{E}_H$ and $\tilde{E}_T$, are determined by

$$(20)\quad \begin{cases} \tilde{E}_H = \bar{E} & \text{if } \bar{E} \leq E^* \\ \tilde{E}_H = E^*,\ \tilde{E}_T = \bar{E} - E^* & \text{if } \bar{E} > E^* \\ C_H'(E^*) = C_T' \end{cases}$$

In the solution of this problem it is tacitly assumed that there exists at least one new hydro power plant which produces power at lower costs than a thermal power plant. This is a condition which is met in Norway.

2. The data

Instead of using historical data, which would probably have given us a "wrong" picture of the future cost structure, we use project data covering an expansion of the system starting from the production level in 1978 and ending at a production level 50 per cent above the 1978-level.

The data, which are presented in Strøm (1979), consist of calculated costs and output of firm power from 176 different hydro power projects.[1] Taking the stochastic nature of the hydro power system into account firm power is usually interpreted as the production capacity which is available with a high degree of certainty - approximately in 9 of 10 years. Power production in excess of this limit is denoted surplus power.

In our data the expected income from the sale of surplus power is deducted from total costs. In MSG-4E, which is a deterministic model, the central capacity measure is mean production rather than the firm power level. To adjust our data for this inconsistency we simply divide all the firm power figures by 0.9. We furthermore assume that total costs divided by the energy capacity (energy capacity average cost) is equal to firm power average cost. This means that expected income per unit from the sale of surplus power is equal to firm power average cost.

As mentioned above the cost figures are calculated in 1978-prices and based on the technological knowledge in 1978.

The central planning authority for the electricity system in Norway (The Norwegian Water Resources and Electricity Board, NWE), which has provided us with the data, has ranked the projects according to the succession decided by the central and the local authorities. The projects may, however, also be ranked according to increasing costs, and thus we have two sets of data which can be used to estimate the cost function.

3. Empirical results

The thermal cost function

The most favourable alternative to hydro power plants is coal based plants (nuclear plants are excluded due to political reasons). Based on 1978-prices and technique C_T' is estimated to 0.151 kroner/kWh (roughly 3 cent/kWh).

1) The concept of firm power is further discussed in chapter XIII of this volume.

The hydro power cost function when projects are ranked according to increasing costs

We assume the following stochastic model:

$$(21) \quad C'_{H,i} = \beta_0 + \beta_1 e_{H,i}^{\beta_2} + U_i \qquad i = 1,\dots,176$$

U_i is a random disturbance term. $C'_{H,i}$ is the energy capacity average cost of project no. i which may be considered as an estimate of the marginal cost of the whole power system. The projects are ranked according to increasing energy capacity average cost. $e_{H,i}$ is the associated energy capacity of the hydro power system normalized to zero in 1978. Taking the initial capacity level into account, the total energy capacity in a future year, as long as this future capacity is solely based on hydro power, is $E_{78} + e_H$. In the estimation of coefficients in (21) $C'_{H,i}$ is measured in million kroner/TWh (1978-prices) and $e_{H,i}$ is measured in TWh.

The error terms are assumed to have the following properties:

$$(22) \quad \begin{cases} EU_i = 0 & \text{for } i = 1 \dots 176 \\ EU_iU_j = \sigma^2 & \text{for } i = j \\ EU_iU_j = 0 & \text{for } i \neq j \end{cases}$$

By minimizing

$$\sum_{i=1}^{176} (C'_{H,i} - \beta_0 - \beta_1 e_{H,i}^{\beta_2})^2$$ maximum likelihood estimates are

obtained.

The results were (t-values in the parentheses):

$$(23) \quad \begin{cases} \hat{\beta}_0 = 58.60 \ (33.42) \\ \hat{\beta}_1 = 0.18 \ (2.73) \\ \hat{\beta}_2 = 1.67 \ (16.99) \\ R^2 = 0.92 \end{cases}$$

All the parameter estimates are seen to be significantly different from zero at a 1 per cent level.

In figure 1 the estimated marginal cost functions are drawn. We observe that the marginal cost function in the hydro power system is increasing, implying, as expected a priori, decreasing returns to scale in the production of hydro power.

From (18) we observe that some of the coefficients in the structural model are not identified. Due to a priori information, however, this situation can be improved.

Based on engineering data on investments in machinery and on the corresponding increase in energy production, the ratio K_M/E may be estimated. The independent estimate on this ratio, $\hat{\alpha}$, is 450 million kroner/TWh in 1978-prices.

The calculations of annual costs are based on a social rate of discount equal to 0.07 and a time horizon of 40 years. This implies an annuity factor, ν, of 0.075.

Moreover the annual variable costs are set equal to 1 per cent of the investment outlays. Variable costs as a share of capital costs on annual basis will then be

$$(24) \quad b = \frac{0.01}{\nu} = 0.133.$$

Finally, it should be remembered that in the estimation of the β-s the cost data are in 1978-prices. The price indices may be normalized so that P_M and P_B both are equal to 1 in 1978.

Using this information we obtain the following estimates of the coefficients in relation (12):

$$(25) \begin{cases} \hat{\varepsilon}_0 = \dfrac{\hat{\beta}_0}{(1+b)\nu} - \hat{\alpha} = 689.6 - 450 = 239.6 \\ \hat{\varepsilon}_1 = \dfrac{\hat{\beta}_1}{(1+\hat{\beta}_2)} \dfrac{1}{(1+b)\nu} = 0.8 \\ \hat{\varepsilon}_2 = \hat{\beta}_2+1 = 2.67 \end{cases}$$

As noticed above $\alpha + \varepsilon_0 > 0$ means that the scale elasticity decreases with the production level and assymptotically approaching the level $1/\varepsilon_2$ estimated to be 0.375.

With respect to the implementation of the estimated demand functions in the MSG model it is important to notice that the coefficients are estimated based on data giving increments in energy capacity from the level in 1978. What we have estimated is therefore increments in capital stocks. Taking this into account, the following demand functions for capital stocks in a future year t are obtained by

$$(26)\quad K_{Mt} = K_{M1978} + \hat{\alpha}\,(E_t - E_{1978})$$

$$(27)\quad K_{Bt} = K_{B1978} + \hat{\varepsilon}_0(E_t - E_{1978}) + \hat{\varepsilon}_1(E_t - E_{1978})^{\hat{\varepsilon}_2}$$

The demand functions which are included in the MSG model are further discussed in the appendix to this chapter.

The hydro power cost function when projects are ranked according to existing plans

The cost function derived above presupposes that projects are selected in an order which minimizes total system costs. As already noted the local and central authorities generally do not take the projects in this succession. Figure 2 shows how the projects scatter in the marginal cost-capacity diagram when these are selected according to present official plans. Applying the same statistical methods as previously we obtain the following estimates (t-values in parenthesis):

$$(28)\quad \begin{cases} \tilde{\beta}_0 = 93.66 \quad (24.60) \\ \tilde{\beta}_1 = 10.52 \quad (1.25) \\ \tilde{\beta}_2 = -0.79 \quad (2.07) \\ R^2 = 0.066 \end{cases}$$

A linear approximation of the estimated marginal cost function is also shown in figure 2. The official plans seem to select the projects almost randomly with a weak tendency of choosing an expensive project before a cheap one. A chi-square-test shows that a hypothesis that the marginal cost is constant and independent of E can be rejected at a 0.5 per cent level. However, since the estimated function is not able to explain the variations in data (R^2 is very small), and because of the uncertainty regarding the question whether the official plans actually will be followed we have derived the demand functions based on a weighted average costs of all projects. The weights are the energy capacity of each project. Total investments per TWh is calculated to 1 072 million kroner, corresponding to an annual cost of

$$(29)\quad \hat{B} = 91.13 \text{ million kroner/TWh}$$
$$(0.091 \text{ kroner/KWh})$$

When the projects are selected in this rather arbitrary and inefficient way the long run marginal cost will first remain constant (0.091 kroner/KWh) (or weakly decline) and then it will jump up to the thermal cost level (0.151 kroner/KWh). This in contrast to the smooth marginal cost curve which is estimated when the projects are selected according to increasing costs (see figure 1). The factor demand functions for capital stocks which can be derived from the weighted average cost function are given by

$$(26)\quad K_{Mt} = K_{M1978} + \hat{\alpha}[E_t - E_{1978}]$$

$$(30)\quad K_{Bt} = K_{B1978} + \hat{\varepsilon}(E_t - E_{1978})$$

where

$$(31)\quad \hat{\varepsilon} = [\frac{\hat{B}}{(1+b)\nu} - \hat{\alpha}] = 622 \text{ million kroner/TWh}$$

4. Are the official plans as inefficient as they look?

There are several reasons why the official selection of projects differ from what seems to be the most rational choice:

(i). The cost figures do not include transmission or distribution costs. The possibility that some of the most favourable remaining watercourses imply relatively high distribution costs can not be disregarded. It is, however, rather doubtful whether this possibility can turn the apparent inefficient selection of projects into an optimal succession.

(ii). Another factor which may have influenced the official plans is that large projects with relatively low costs take much time to be discussed and licenced in political bodies. This may be explained by the fact that there are often specific environmental problems connected with the development of large watercourses. This being the case the reported cost figures may underestimate the social costs of some large projects. It is again rather doubtful whether this can justify the chosen succession of projects.

Figure 1. Long run marginal cost when the projects are selected accordning to increasing costs[1)]

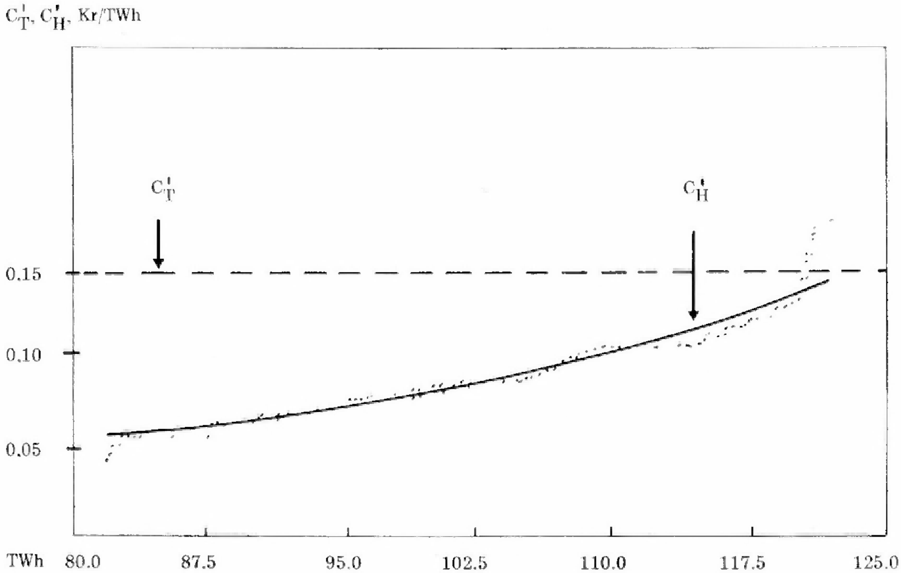
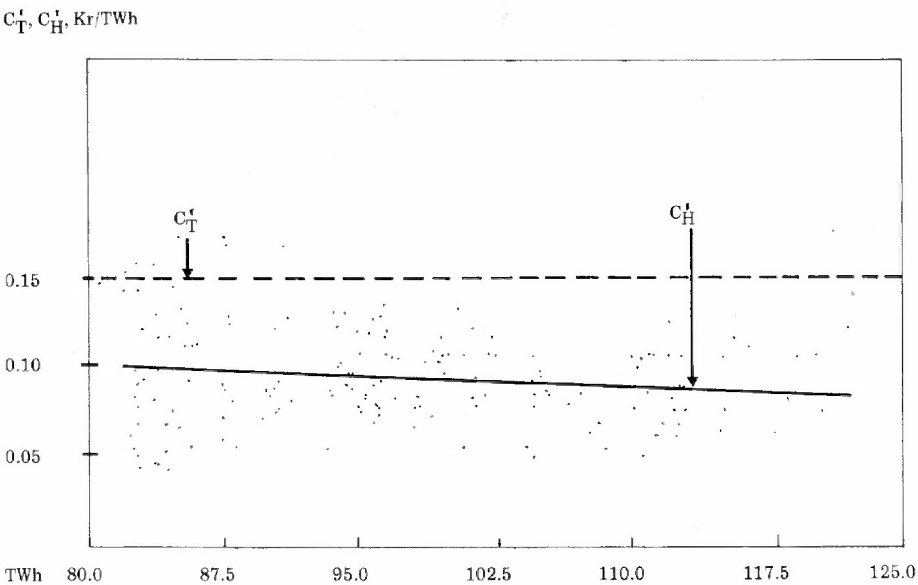


Figure 2. Long run marginal cost when projects are selected according to official plans



1) C'_H is the marginal cost in the hydro power. C'_T is the marginal cost in a coal-based thermal power alternative.

(iii). Thirdly, one may wonder whether the data of some projects are of a better quality than others. The cost figures for the projects which are considered to be taken first are possibly worked out more accurately than the calculations for projects which are supposed to be developed later. The cost figures will successively be revised. There is, however, the same tendency of choosing an expensive project before a cheap one in the development of watercourses in the past. This fact weighs heavily against the hypothesis suggested here.

(iv). In Norway the local authorities have a decisive influence on the selection and timing of water power projects. Since there is only a weak coordination of the development of the watercourses between regions, despite the existence of a national planning agency and a nation-wide transmission system, it is not really surprising that the selection of watercourses does not follow a least cost procedure. In fact, this may be the main reason why projects may be selected as shown above. If this is the case the society may gain from a reshuffling of plant selections, and from a removal of institutional barriers. The gains are great; assuming an annual growth of 2 per cent in electricity demand, the present value of the gain can be estimated to about 4 000 million N.kroner (1981-prices).

(v). Indivisible projects may imply that the optimal selection of plants involves a declining marginal cost function in some intervals. The argument can be outlined as follows: Let us consider two projects, a small one and a large one, and assume that the latter project has the lowest average cost. But in an optimal planned and operated system the prices have to respond to the expansion of the system. The large and indivisible project may cause the price to drop to such an extent that the small project with the relatively high average cost passes the long-run marginal cost criterion while the large project does not. Although we have not analysed this phenomenon in any detail, it seems unlikely that this can explain the findings reported above. It should also be noticed that all the remaining projects are small compared to the initial production level. Indivisibility is further/discussed in chapter XIV of this volume.

Appendix: Cost and demand functions in the MSG model

The MSG model is updated yearly on the base of national accounts figures. This has important implications for the demand functions estimated in section 3.

The capital stocks in the base year T are denoted K_{MT} and K_{BT}. These values will probably deviate somewhat from the values predicted by both (26) - (27) and (30) - (31). A main reason is that the hydro power projects actually chosen in the period 1978 (the base year of the estimation) will deviate from the most efficient selection, and probably also from the average cost of all projects. This does not imply that the estimated cost and demand functions are obsolete and useless, but some transformations and adjustments of the relations must be made.

All volume figures in the MSG model are measured in the prices of the base year. We therefore have to transform the energy capacity figures from TWh to constant prices of the base year. We also have to transform capital stock from 1978-prices to base year prices.

The national accounts figures for actual electricity production in the base year T may deviate from the energy capacity in the same year. Since the growth in energy capacity and its impact on factor demand are predicted by the model, the base year production figures have to be corrected for capacity utilisation. Another aspect that may affect the demand for capital inputs, is the fact that the base year capital stocks also include capital invested in projects not yet completed. The energy capacity of these projects is obviously not included in the base year figures. This argument means that there may be a tendency of overpredicting the future demand for K_B and K_M in a model simulation. The demand functions that are actually used in the MSG model are the following:

$$(26') \quad K_{Mt} = K_{MT} + \alpha_T \cdot \left[\frac{E_{OT}}{X_T}\left(\frac{X_t}{Y_t} - \frac{X_T}{Y_T}\right)\right]$$

$$(27') \quad K_{Bt} = K_{BT} + \varepsilon_{OT} \cdot \left[\frac{E_{OT}}{X_T}\left(\frac{X_t}{Y_t} - \frac{X_T}{Y_T}\right)\right] + \varepsilon_{1T} \cdot \left[\frac{E_{OT}}{X_T}\left(\frac{X_t}{Y_t} - \frac{X_T}{Y_T}\right)\right]^{\varepsilon_2}$$

$$(30') \quad K_{Bt} = K_{BT} + \varepsilon_T \left[\frac{E_{OT}}{X_T}\left(\frac{X_t}{Y_t} - \frac{X_T}{Y_T}\right)\right]$$

K_{ji} j = B,M i = T,t are measured in constant prices of the base year (T)

X_i i = T,t is the volume of electricity production in year i measured in constant T-prices

E_{0i} i = T,t is electricity production in year i measured in TWh

$\gamma_i = \frac{E_{0i}}{E_i}$ i = T,t measures the capacity utilisation in the electricity production, year i

γ_T may differ from 1 mainly because of weather conditions. γ_T may also be used to correct for amounts of capital invested in projects not yet completed in the base year. The capacity utilization in a future year t, γ_t, is usually set equal to 1.

The coefficients which appear in (26'), (27') and (30') may be calculated as

$$(32)\quad \begin{cases} \alpha_T = \hat{\alpha} \cdot \frac{P_{MT}}{P_{M1978}} \\ \varepsilon_{0T} = \hat{\varepsilon}_0 \cdot \frac{P_{BT}}{P_{B1978}} \\ \varepsilon_{1T} = \hat{\varepsilon}_1 \cdot \frac{P_{BT}}{P_{B1978}} \\ \varepsilon_T = \hat{\varepsilon} \cdot \frac{P_{BT}}{P_{B1978}} \end{cases}$$

The P_{ji}'s (j = M,B i = T,1978) are prices indicies for K_j in the year i.

These equations thus show the transformations of the estimated coefficients from 1978-prices to base year(T)-prices. Revised data from NWE indicate that average costs of new hydro power projects are considerably higher than listed in the data from 1978, implying a positive shift in the cost functions. At present we have no new information of single project data which could indicate whether the shape of the cost function is changed.

In 1978-prices the average total investment per TWh ($\alpha + \varepsilon_0$) is calculated to 1 500 million N.kroner which is 40 per cent higher than previously estimated. The revised data does not indicate an increase in α, the parameter determining investments in machinery. The revised estimate for ε is thus

(31') $\hat{\varepsilon}$ = 1 050 million kroner/TWh

The revised estimate for B, the average cost an annual basis, is

(29') $\hat{B}$ = 127.5 million kroner/TWh
(0.1275 kroner/KWh)

If the shape of the cost function is not changed, the estimate for β_2 in (23) is unaffected while $\hat{\beta}_0$ and $\hat{\beta}_1$ should be increased by 40 per cent, thus affecting the estimates for ε_0 and ε_1 in (25).

We have estimated two cost functions. Which one should be used in the MSG model? If the purpose of running the model is to make predictions of the Norwegian economy, given technical and institutional constraints, the cost and factor demand functions based on official plans have to be chosen, i.e. the relations (26') and (30'). If the purpose is to calculate the optimal development of the economy with respect to the allocation of resources, the relations (26') and (27') should be chosen. An interesting exercise (which could have political implications) would be to compare the results from simulations of the model with the two different cost structures estimated above. The impact on GDP or some other measure of the benefit to the economy of changing policies in the selection of hydro power plants could then be estimated.

In the estimation of the cost and factor demand functions the social rate of discount was kept constant and equal to 0.07. In the supply-oriented version of the MSG model (see chapter II) the overall return to capital in the economy is an endogeneous variable. A fall in the social rate of discount will produce a negative shift in the marginal cost function. This should influence the long run pricing policy for energy.

Since hydro power is more capital intensive than thermal power, a change in the social rate of discount will also change the optimal mix of hydro/thermal power. A fall in the social rate of discount will increase E^x in (20), the optimal production level of hydro power.

If a simulation of the MSG model implies a change in the relative capital price it could affect the demand functions for capital stocks. As previously mentioned we have not been able to estimate these effects.

VII. TRANSMISSION AND DISTRIBUTION OF ELECTRICITY

by

Alette Schreiner and Steinar Strøm

1. Introduction

In the preceding chapter the marginal costs and factor demand functions in the production of electricity were estimated. The production sector (P-sector) includes power plants, storage reservoirs, penstocks etc. In this chapter marginal costs and factor demand functions in transmitting and distributing electricity will be estimated. Transmission and distribution are grouped together and considered as one sector (D-sector). An alternative choice would have been to include transmission in the P-sector or even to deal with transmission and distribution separately.

The treatment of the D-sector is analogous to some extent with the treatment of the trade sector in the national accounts. Transmission and distribution services accompany deliveries of electric energy from the P-sector. The amount of distribution services per unit of electric energy varies considerably between users of electricity. This is a major reason why it is important to separate distribution from production of electricity within the model.[1] In the analysis of transmission and distribution services we distinguish between five user categories:

1) Power intensive industries.
2) Pulp and paper.
3) Other manufacturing industries and mining.
4) Export (of hydroelectric power).
5) Agriculture, forestry, fishing, wholesale and retail trade, other service industries and households.

2. The cost structure of the transmission and distribution of electricity

The inputs in the D-sector are transmission losses measured in kWh, capital and materials measured in constant prices, and labour

1) The aggregation of commodities and the grouping of the different parts of the electricity system was discussed in more detail in chapter V.

measured in hours worked. Input of labour and materials are assumed to be proportionate to the output level. Engineering studies suggest that power losses can be reduced through higher capital intensity in the distribution network. We therefore assume substitution between real capital and power losses. It is questionable to what extent this holds ex post. The MSG model assumes, however, moveable and malleable capital, i.e. an ex ante structure.

The production structure of the D-sector can be described as follows,

(1a) $$h(X_D) = f_D\ (E_D, K_D)$$

(1b) $$L_D = \frac{1}{\alpha_L} X_D$$

(1c) $$M_D = \frac{1}{\alpha_D} X_D$$

where X_D is the production of distribution services,

E_D is the energy losses in the transmission and distribution network

K_D is the capital of the D-sector, and

L_D and M_D are the inputs of labour and materials, respectively.

α_L and α_D are constants and f_D is assumed to be a quasi-concave, continuous function with positive partial derivatives and homogeneous of degree one.

In the distribution of electricity one cannot exclude the possibility of economies of scale. If the increase in electricity demand goes together with non-perfect correlation between consumers, the minimized marginal costs will tend to increase less than with output. The h-function will then imply increasing returns to scale in contrast to the decreasing returns to scale in the production of hydro power. Thus, it is open for empirical verification whether the <u>total</u> marginal costs in the supply of electricity will increase or decrease with the volume of electricity distributed to the consumers.

Technical progress is not specified in (1a) - (1c), but will be introduced below. Although it is difficult to distinguish empirically between pure economies of scale and scale-augmenting technical change we will try to estimate the returns to scale.

The total costs of production can be written as

(2) $$C_D = q_E E_D + q_K K_D + q_L L_D + q_M M_D,$$

where q_E is the price of power losses in transmission and distribution,

q_K is the user cost of capital,

q_L is the labour cost per hour, and

q_M is the price of materials.

Our distinction between a P-sector and a D-sector is defined in a purely functional way. Some of the companies in the electricity sector are mixed enterprises in the sense that they are producing as well as transmitting and distributing electricity. Others are just distribution plants. A majority of the companies are owned either by the central government or by local government at county or municipality level. The price of electricity differs between regions. In some cases the prices may be set by political bodies, e.g. municipality councils, with a view to financial needs as well as income distribution aspects. In other cases the prices are set so that the income from the electricity plants covers historical costs. Finally, the increase in the overall production and transmission of electricity is mainly decided by the Storting (Parliament). Since transmission is part of the D-sector, this means that part of the expansion of the D-sector follows from decisions in the Storting. In view of these scanty remarks on the influences exerted in the D-sector from the outside, a reasonable way of modelling the behaviour of this sector seems to be minimization of costs for given output level. Admittedly, in the cases in which prices are set according to historical costs, the cost minimizing assumption may seem less obvious.

Minimizing costs yield the following first order condition:

$$(3) \qquad \frac{\partial f_D}{\partial E_D} \Big/ \frac{\partial f_D}{\partial K_D} = \frac{q_E}{q_K}$$

which together with (1a) determines the factor demand as functions of relative prices and output.

$$(4a) \qquad E_D = g_E(X_D,\ q_E/q_K)$$

$$(4b) \qquad K_D = g_K(X_D,\ q_E/q_K)$$

The demand functions for labour and materials are independent of relative prices and are as given in (1b) - (1c).

The minimized cost function is

$$(5) \qquad C_D = q_E g_E(X_D,\ q_E/q_K) + q_K g_K\ (X_D, q_E/q_K) + q_L/\alpha_L X_D + q_M/\alpha_M X_D$$

or shorter

$$(5') \qquad C_D = g(X_D, q)$$

where q is the factor price vector.

Due to Shephard's duality-theorem we know that the minimized cost function contains all the information necessary to reconstruct the production structure, see Fuss & McFadden (1978). Thus, we will follow the common procedure of estimating the parameters of the production technology by estimating the parameters of the cost function.

The specified cost relationship we use is the Generalized Leontief cost function (GL) with scale augmenting technical progress.

$$C_D(X_D,q,t) = h(X_D,t)\Sigma_i\Sigma_j b_{ij}(q_i q_j)^{\frac{1}{2}} + q_L X_D/\alpha_L + q_M X_D/\alpha_M \quad (6)$$

$i,j = E,K$

We assume $b_{EK} = b_{KE}$ and we expect this coefficient to be non-negative so that the cost function is concave. $h(X_D,t)$ takes care of the economies of scale and technical progress. The way $h(X_D,t)$ appears in (6) implies that the production function is assumed to be homothetic and technical change is assumed to be scale augmenting only.

Following Shephard's lemma (Shephard (1953)) the corresponding demand functions are derived as

$$E_D = \frac{\partial C_D}{\partial q_E} = h(X_D,t)q_E^{-\frac{1}{2}}\ [b_{EE}\ q_E^{\frac{1}{2}} + b_{EK}\ q_K^{\frac{1}{2}}] \quad (7a)$$

$$K_D = \frac{\partial C_D}{\partial q_K} = h(X_D,t)q_K^{-\frac{1}{2}}\ [b_{KK}q_K^{\frac{1}{2}} + b_{EK}\ q_E^{\frac{1}{2}}] \quad (7b)$$

$$L_D = \frac{X_D}{\alpha_L} \quad (7c)$$

$$M_D = \frac{X_D}{\alpha_M} \quad (7d)$$

This yields the following partial demand elasticities. (Note that X_D is held constant.):

$$\varepsilon_{EE} = \frac{\partial \log E_D}{\partial \log q_E} = -\frac{1}{2} \cdot \frac{b_{EK}\ q_K^{\frac{1}{2}}}{b_{EE}q_E^{\frac{1}{2}} + b_{EK}q_K^{\frac{1}{2}}} \quad (8a)$$

$$\varepsilon_{KK} = \frac{\partial \log K_D}{\partial \log q_K} = -\frac{1}{2}\ \frac{b_{EK}\ q_E^{\frac{1}{2}}}{b_{KK}q_K^{\frac{1}{2}} + b_{EK}q_E^{\frac{1}{2}}} \quad (8b)$$

$$\varepsilon_{EK} = \frac{\partial \log E_D}{\partial \log q_K} = \frac{1}{2}\ \frac{b_{EK}q_K^{\frac{1}{2}}}{b_{EE}q_E^{\frac{1}{2}} + b_{EK}q_K^{\frac{1}{2}}} \quad (8c)$$

(8d) $$\varepsilon_{KE} = \frac{\partial \log K_D}{\partial \log q_E} = \frac{1}{2} \frac{b_{EK} q_E^{\frac{1}{2}}}{b_{KK} q_K^{\frac{1}{2}} + b_{EK} q_E^{\frac{1}{2}}}$$

We expect ε_{EE} and ε_{KK} to be negative and the two cross-price elasticities to be positive.

3. Data

The data on capital stock are derived from national account estimates. Real capital (K_D) is measured in constant prices with 1978 as a base year, see Myklestu (1979). We have not made any attempt to correct for the degree of capacity utilization although this might be important in the electricity sector. Capacity utilization will certainly vary through the year and between years due to weather conditions, the general economic situation in the economy etc. The price (p_K) and the user cost of capital ($q_K = (R+\delta)p_K$ where R is the rate of return and δ is the depreciation rate) are measured in the same way as in the preceding chapters.

Data on power losses (E_D) are again taken from Myklestu (1979). The price of the losses (q_E) is the wholesale price index for firm power sold by the State Power Plants.

The expenditures on raw materials and labour are taken from the national accounts.

We have no direct observations of $h(X_D,t)$ and X_D. Thus, it is necessary to construct time series for $h(X_D,t)$ in order to estimate the parameters of the cost function. Furthermore, data on X_D are needed if we want to separate the scale augmenting technical change from pure economies of scale. In contrast to Fuss (in Fuss and McFadden, Part II, 1978) we will try to estimate the two different scale augmenting factors.

In constructing time series for $h(X_D,t)$ we follow a procedure suggested in Diewert (1976) and define an index number which may be interpreted as an approximation to the price term of the cost function.

The index proposed by Diewert is

(9) $$Q_1 = \frac{\Sigma_i \alpha_i^o \left(\frac{q_i^1}{q_i^o}\right)^{\frac{1}{2}}}{\Sigma_i \alpha_i^1 \left(\frac{q_i^o}{q_i^1}\right)^{\frac{1}{2}}}$$

where the α_i's are cost shares and the superscripts 1 and 0 denote two successive periods.

According to Diewert (1976) the index Q_1 is an exact index for a GL cost function in the sense that

(10) $$Q_1 = \frac{\bar{C}(q^1)}{\bar{C}(q^0)}$$

where $\bar{C}$ is the unit cost function when raw materials and labour are ignored. A quantity index for $h(X_D,t)$ is then obtained by dividing power losses and capital costs by a price index constructed according to (10).

In order to construct time series for X_D we must introduce rather strong assumptions:

1. In the base year pure profit in the D-sector is zero,

$$P_D^{78} X_D^{78} = C_D^{78}$$

Moreover, in the base year the price index $P_D^{78} = 1$. Hence

(11) $$X_D^{78} = C_D^{78}$$

2. Within each user category the ratio between consumption of electricity (which can be observed) and consumption of D-services (which cannot be observed) is assumed to be constant through time.

(12) $$X_{Di} = a_i X_{Pi} \qquad i=1,\ldots,5$$ (Five user categories as introduced above.)

where X_{Pi} is the delivery of electricity (in kWh) to user no. i.

3. We next assume that the user category no i's share of total losses in the base year is equal to the share of D-services. This means

(13) $$\frac{X_{Di}^{78}}{X_D^{78}} = \frac{E_{Di}^{78}}{E_D^{78}} \qquad i=1,\ldots,5.$$

From (11) - (13) we get

(14) $$X_D^t = \sum_i a_i X_{Pi}^t = \frac{C_D^{78}}{E_D^{78}} \sum_i \frac{E_{Di}^{78}}{X_{Pi}^{78}} \cdot X_{Pi}^t$$

and we observe that $X_D^{78} = C_D^{78}$ since $E_D^{78} = \sum_i E_{Di}^{78}$. All the terms to the right in (14) can be observed.

4. Results

Adding normally distributed error terms with zero expectations, constant variances and covariances to the factor demand functions (7a) and (7b), the coefficients b_{KK}, b_{EE} and b_{EK} can be estimated by applying standard regression programs. In this case "Zellners method of estimating seemingly unrelated regressions" (Johnston (1960)) was used.

The results were

	b_{KK}	b_{EE}	b_{EK}
Estimates	7.09	0.17	0.01
Stand. dev.	0.23	0.01	0.05
t-values	30.4	12.0	0.1

The estimate for b_{EK} is not significantly different from zero. This implies that we cannot reject the hypothesis that substitution is non-existent. The GL-function then collapses into a simple Leontief-structure.

The demand elasticities, ε_{ij}, vary over time. We have estimated the elasticities in the base year, 1978. b_{EK} is set equal to the estimated value 0.01 but the uncertainty associated with this estimate should be kept in mind.

The following estimates were obtained for the elasticities in the base year 1978:

ε_{KK}	ε_{EE}	ε_{KE}	ε_{EK}
-0.003	-0.007	0.007	0.003

All the elasticities have the expected sign but they are all close to zero reflecting the low estimate of b_{EK}. The estimated value of the elasticity of substitution is close to zero as well,

$$\sigma_{EK} = 0.009.$$

Although engineering studies suggest stronger substitution possibilities between E_D and K_D in the f_D-function we cannot reject the hypothesis of no substitution possibilities.

There are several possible explanations why we have not been able to trace any substitution possibilities:

1. The "true" production structure in the D-sector in Norway is in fact of the simple Leontief-type.

2. There are substitution possibilities present, but they have not been used. The reason why might be too small changes in relative prices or that our period of observation is too short to identify these substitution adjustments.

3. The model might be wrong. We have followed a traditional neo-classical approach and used time series in estimating the parameters. An

alternative approach, with a distinction between ex post and ex ante substitution, might have changed our conclusions. The reason why this approach has not been adopted is the lack of data.

4. The behaviour assumption of minimization of costs for given output level might be wrong.

5. Errors in the measurement of the variables cannot be excluded.

Despite these objections the estimated structure of the D-sector is implemented in the MSG model. To summarize the result we therefore repeat the estimated demand functions for capital and electricity inputs

(15a) $K_D = h(X_D,t)\ q_K^{-\frac{1}{2}}\ [7.09\ q_K^{\frac{1}{2}} + 0.01\ q_E^{\frac{1}{2}}]$

(15b) $E_D = h(X_D,t)\ q_E^{-\frac{1}{2}}\ [0.17\ q_E^{\frac{1}{2}} + 0.01\ q_K^{\frac{1}{2}}]$

We then proceed to the estimation of the elasticity of scale and technical change. As already discussed we have constructed time series for X_D and $h(X_D,t)$. We now assume the following specification of h:

(16) $$h(X_D,t) = H \cdot X_D^{\frac{1}{\mu}}\ e^{-\gamma t}\ u_t$$

where H, μ and γ are coefficients all expected to be positive. Combining (1a) and (16) we observe that

(17) $$X_D = H^{-\mu} e^{\gamma\mu t}[f_D(E_D,K_D)]^{\mu}$$

$\gamma\mu$ will then be the rate of scale-augmenting technical change. Since f_D is homogeneous of degree one, μ will be the elasticity of scale.

In the lack of any spesific information on the rate of technical change in the D-sector we assume that it is equal to the weighted average of technical change in the manufacturing industry as a whole. In chapter III the technical change in the different manufacturing sectors has been estimated. A weighted average with gross production values in the base year 1978 as weights is 0.0084 (or 0.84 pct. per year). We take this as an estimate of γ. μ is unknown and in fact is the key parameter to be estimated below. If μ, however, deviate not too much from 1, γ in (17) can be approximated by the weighted average 0.0084.

Making this assumption (16) can be written to yield:

(18) $$[\ln h(X_D,t) + 0.0084 \cdot t] = \ln H + \frac{1}{\mu} \ln X_D + \ln u_t.$$

u_t is an error term with expectation 1 and constant variance.

Based on the constructed time series for $h(X_D,t)$ and X_D we obtained the following result.

Estimate on μ : 1.27

t-value : 28.4

The estimated $h(X_D,t)$-function is then

$$h(X_D,t) = 1.21\, X_D^{0.79}\, e^{-0.0084\, t} \tag{18}$$

The estimated value of μ implies increasing returns to scale in accordance with what we should expect for this sector.

From (6) we easily obtain the long run marginal cost (LMC)

$$C_D' = \frac{\partial h(X_D,t)}{\partial X_D} \cdot \sum_i \sum_j (q_i q_j)^{\frac{1}{2}} \cdot b_{ij} + \left(\frac{q_L}{\alpha_L} + \frac{q_M}{\alpha_M}\right) \tag{19}$$

LMC depends on time t and on the level of output since

$$\frac{\partial h(X_D,t)}{\partial X_D} = 0.96 \cdot e^{-0.0084\, t} \cdot X_D^{-0.21}$$

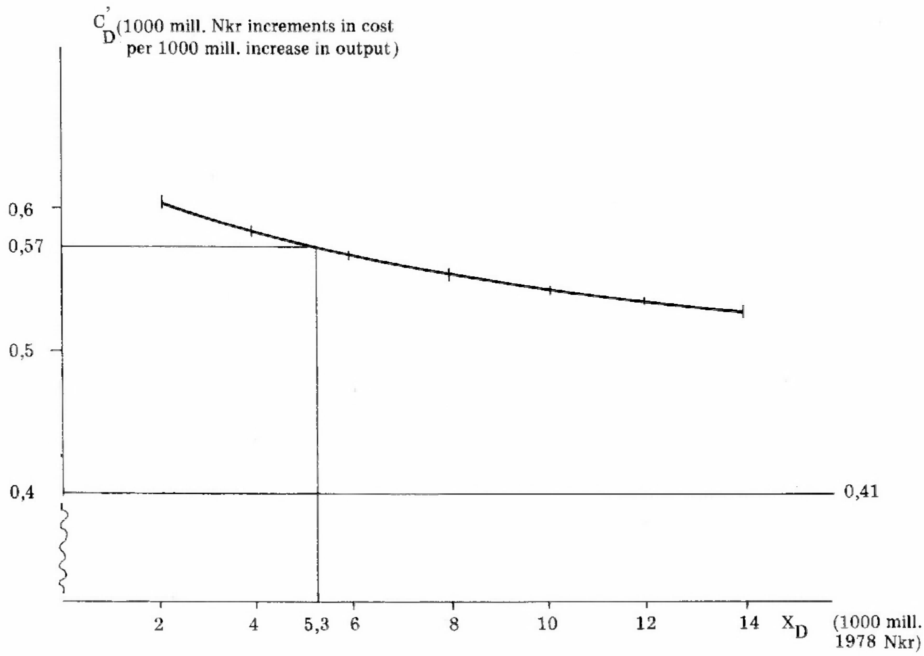
We also observe that LMC depends on the level of the factor prices, especially on the user cost of capital and therefore on the social rate of discount.

Inserting values for the base year 1978 and setting t=12 in 1978 we obtain the following estimate on C_D':

$$C_D' = 0.23\, X_D^{-0.21} + 0.41 \tag{20}$$

C_D' decreases asymptotically with output towards the lower level 0.41. This level is determined by $\left(\frac{q_L}{\alpha_L} + \frac{q_M}{\alpha_M}\right)$. In figure 1 we have shown the C_D'-function for the base year 1978. In that year X_D is 5 300 million N.kroner.

Figure 1. Long run marginal cost in the distribution and transmission sector. 1978-prices and 1978-technology



VIII. ENERGY PRICE SENSITIVITY OF THE NORWEGIAN ECONOMY

by

Svein Longva, Øystein Olsen and Jon Rinde

1. Introduction

To study how the economy responds to changes in energy prices is essential for the evaluation of energy policy in general and for the planning of the energy sector in particular. Of special interest is the effect of energy price changes on the demand for energy itself. The price sensitivity of energy demand is important for policy fields such as forecasts of future energy consumption, the possibilities of using prices or taxes to achieve energy conservation, and the evaluation of the optimal investment path in energy production. In addition, the energy price effects on demand for labour and capital are of crucial importance in forming an energy policy.

The response of demand to the change in a price is commonly stated in terms of a price elasticity. Roughly speaking the energy price elasticity of demand measures the percentage change in the quantity demanded, e.g. energy, labour or capital, in response to a one per cent change in the energy price. Since the concept of price elasticity is defined in terms of relative changes it is independent of the unit of measurement, and it it therefore possible to compare estimates of price elasticities for different sectors in the economy or for different countries. In short, energy price elasticities provide estimates of the energy price sensitivity of the economy in general and of energy demand in particular.

The significance of energy price elasticities has been especially emphasized after the first round of dramatic increase in oil prices in 1973, which led to a considerable deterioration in terms of trade and a slowdown of the economic growth in non-oil producing countries. The demand sensitivity of changes in energy prices influenced the impacts on employment, capital formation and production in the different countries. The higher the price elasticity of demand for energy, the smaller will be the impact on energy demand and the production level of a given increase in energy prices. The ratio between energy consumption and the production level of the economy is thus influenced by the ability of the economy to substitute capital and labour input for energy on the production side and the ability to substitute away from energy intensive goods on the consumption side.

The purpose of the present study is to provide numerical estimates of energy price elasticities for the Norwegian economy. The formal framework of our study is the MSG-4E model, described by Longva, Lorentsen and Olsen in chapter II of this volume. As discussed there MSG-4E is a disaggregate neoclassical growth model. The model contains 32 production sectors, of which 27 are private industries, and one sector for private households. 40 commodities are distinguished. Partial price elasticities for private production sectors and for households are presented in chapters III and IV of this volume, respectively. By partial price elasticities we mean measures of demand effects caused by a change in a given energy price when all other arguments in the demand relation are assumed to be constant. In this chapter we present estimates of long run price elasticities illustrating the energy price sensitiveness of the model as a whole, i.e. we are using the entire MSG-4E model to calculate demand responses with respect to energy prices. In these elasticities, which we call total price elasticities, indirect effects of the initial energy price change are also taken into account.

The next section contains a discussion of a number of conceptual and methodological problems related to the computation of aggregate total energy price elasticities within the MSG-4E framework. The main results of the study - estimates of energy price elasticities of demand for electricity, fuels, total energy, labour and capital - are presented in section 3. The elasticities are computed at an aggregate level, i.e. both for various groups of sectors and for the economy as a whole. A comparison with results from other studies is also included. A decomposition of the total elasticities and a study of income effects of an increase in the world market price of energy are presented in sections 4 and 5, respectively.

2. Conceptual and methodological problems in computing total elasticities at an aggregate level

In Energy Modelling Forum (1980) a number of methodological and conceptual problems related to the calculation of total energy demand elasticities at an aggregate level are discussed. In this section an evaluation of some of these issues with respect to the formulation of the MSG-4E model will be undertaken.

The concept of price elasticities in an aggregate setting

In general the concept of price elasticity may be related to each "micro unit", e.g. household or firm, of the economy, for which it is meaningful to specify separate demand relations. Starting out from this micro level the households or firms may be aggregated to groups or sectors. At this level the responses to price changes may be studied for each group as a whole. At the most aggregate level macro elasticities, which measure the demand responses for the economy as a whole following an energy price increase, may be calculated.

Obviously, partial price elasticities may be defined and calculated also for an aggregate of micro units, for example as an average of the partial elasticities of the micro units, using their relative shares of demand as weights. However, in the real world a change in e.g. the price of energy will also imply changes in other variables affecting the demand of the micro units, for example the output level and prices of other inputs, and these effects will in general be different for the various micro units within a group. The simple weighted partial elasticities may therefore be viewed as being of limited interest.

The policy relevant definition of the interplay between energy and the rest of the economy is the change in equilibrium demand induced by a change in the price of energy. It is this kind of price responses that will be focused in this study. Aggregate total price elasticities should reflect all simultaneous effects and feedbacks in the economy on prices and quantities which are induced by the initial price change. These elasticities are thus summary parameters which include the substitution possibilities both in production and consumption, i.e. they indicate the ability of various groups or sectors or of the economy as whole to adjust to changes in energy prices.

For analytical purposes it is, as stressed in Energy Modelling Forum (1980), useful to separate the various demand impacts into the following two components:

(i) A demand response to higher energy price with aggregate economic activity held constant.

(ii) A demand response to higher energy prices allowing for changes in aggregate economic activity.

In the present study our main focus is on the first of these components which corresponds to the concept of price elasticity studied in Energy Modelling Forum (1980). The level of economic activity must therefore be normalized in the calculations and these elasticities are in the following denoted output-constrained total price elasticities.

However, since Norway is a net exporter of energy a world wide increase in the price of energy will improve the terms of trade and, if the same balance of payment is to be maintained, increase income and aggregate economic activity. Such impacts are thus clearly related to the economic policy which accompanies the energy price increases. In the end of this chapter we also present estimates of aggregate elasticities calculated by the MSG-4E model when this kind of income effects are included. These estimates are denoted unconstrained total energy price elasticities.

The relevance of the MSG-4E model in the calculation of total elasticities

In MSG-4E four commodities which may be characterized as secondary energy commodities are specified: electricity (deliveries from power stations), electricity distribution services, petrol and other fuels. In each of the 27 private industries the two electricity goods - electricity and electricity distribution services - are assumed to compose an activity called "Electricity" where fixed coefficients between the two commodities are imposed. In the same way petrol and other fuels are aggregated to a fixed coefficient activity called "Fuels". These spesifications imply that there are no possibilities of substitution between the energy goods within the same activity. However, the MSG-4E model allows for substitution possibilities between the activities "Electricity" and "Fuels" and between "Energy", defined as the aggregate of these activities, and other aggregate inputs (materials, labour and capital). A parametric "production function" for aggregate energy is estimated for each industry.[1] The aggregate inputs are related to sector output by rather flexible production functions. The production functions are flexible in the sense that, except for the assumed separability conditions, the chosen functional forms do not place any a priori restrictions on the substitution properties. In chapter III of this volume two concepts of partial energy price elasticities for each industry are defined. The inter energy substitution, i.e. assuming total energy input constant, may be measured by gross elasticities, while net elasticities include the effect on the demand for electricity and fuels induced by changes in the level of aggregate energy input. Partial energy price elasticities of demand for labour and capital in each industry are also presented.

As described by Bjerkholt and Rinde in chapter IV of this volume total household consumption is divided into 18 activities defined as aggregates with fixed coefficients for commodity input, which represent

1) Actually, a unit cost function for total energy is estimated. The "volume" of total energy is, however, implicitly defined by dividing current energy costs by this aggregate price index.

categories of consumer expenditure. The submodel for household consumption consists of a system of demand relations for these specified consumption categories. Energy commodities are contained in the consumption activities "Electricity" (including production and distribution of electricity), "Petrol and car maintenance" (including petrol) and "Fuels" (including other fuels). In the econometric specification used in the estimation of the complete demand system, want dependence between energy and energy related goods is taken explicitly into consideration. Cross price elasticities (different from zero) are accordingly estimated between the consumption categories "Housing services", "Electricity" and "Fuels", and between the consumption categories "Use of cars", "Petrol and car maintenance" and "Public transport services". A complete set of Engel and Cournot elasticities is presented in chapter IV.

The price elasticities for the various industries and the household sector which are presented in chapters III and IV are clearly partial price elasticities since they are estimated for units with separate demand relations and since all other prices, each sector output and consumer expenditure are assumed to be constant. However, as pointed out above total energy price elasticities should be studied in a general equilibrium setting. The working of the complete MSG-4E model may be viewed as a reasonable approximation to the equilibrium behaviour of the Norwegian economy. A change in the price of an energy commodity will produce changes in a number of other prices and quantities in the economy; in the MSG-4E model all endogenous variables will be changed simultaneously: The input composition of each industry, i.e. the proportions between labour, capital, materials and energy, and the household consumption patterns will change together with the distribution of labour, capital and production between industries; a new set of equilibrium prices will be generated. As in most long run models the total supply of labour is exogenous. The total supply of capital is assumed to be perfectly elastic (exogenous real rate of return to capital) which makes the interplay between energy and capital non-trivial even on the macro level.

The resulting effects of an increase in the price of energy on energy demand and demand for capital and labour are measured by what we have called total price elasticities. When interpreting these total elasticities it is, however, important to have in mind that the results from the MSG model at best represent a rather crude approximation to the actual equilibrium solution. Several important components of demand are exogenous and thus considered independent of changes in energy prices.

The most important exogenous demand components are public consumption and exports. In addition the production of agricultural products, crude oil production and ocean transport services are exogenous in the model. This clearly biases the computed aggregate elasticities downward (in absolute value).

The aggregation problem and the units of measurement

As discussed above the price elasticities presented in this chapter, measuring the effect on demand when simultaneous effects in the economy are taken into account, are related to groups of micro units. Aggregate total price elasticities are in this study calculated for each of the following groups of sectors:

1. Primary industries.
2. Energy intensive industries.
3. Other manufacturing industries.
4. Service industries.
5. Private households.

In addition macro elasticities - measuring demand effects for the economy as a whole - are presented.

It should be noted that some of the production sectors in MSG-4E are not covered by the present study, i.e. they are not included in any of the five groups listed above. This applies firstly to the energy production sectors - the production and distribution of electricity, the production of crude oil and oil refineries - but also to ocean transport.[1)] However, in the estimates for the total economy all sectors with endogenous production are included.

The elasticities are related to the demand for energy, capital and labour. While there are no estimates for subgroups of labour and capital, energy demand elasticities are estimated separately for electricity and fuels (including petrol and other fuels) in addition to total energy use.

In Energy Modelling Forum (1980) it is stressed that for many purposes it is desirable to measure elasticities of demand as close to consumption as possible, i.e. at the retail level. This means that the elasticities measure the effects on the quantities relative to the purchasers' price for energy. The main argument for this procedure is that demand choices are made at the retail level.

1) The use of fuels in ocean transport is thus not included in this study.

This choice of point of measurement obviously influences the size of the calculated energy price elasticities. A characteristic feature of the energy system is the transformation of primary energy to secondary energy goods; the latter concept interpreted as commodities that are used as energy input in consumption and non-energy production sectors. An example will be the transformation of crude oil into petrol and other fuels in oil refineries. Furthermore, the transportation and distribution of energy products to final consumers must also be taken into consideration as the costs of these activities are included in the prices paid by the different consumers. In these purchasers' prices distribution costs appear as additive mark-ups, implying that a certain increase of the price of a primary energy commodity produces a smaller relative increase in the corresponding purchasers' price.[1)] Accordingly, elasticities measured at the retail level will in general be greater than elasticities at the wholesale level, and the latter will in general be greater than elasticities measured at the primary level.

Having chosen the retail level as the point of measurement, it should also be noticed that this represents the most straightforward way of calculating demand effects by means of the MSG-4E model. As mentioned above electricity and fuels are specified as separate activities on the production side, and the further aggregation of energy goods to total energy within each sector is also taken care of by the model specifications. Demand for capital and labour at the retail level correspond directly to the activities "Capital" and "Labour", respectively. On the consumption side there are no assumptions of separability of energy inputs in the underlying utility function parallel to the restrictions imposed on the production functions. As a consequence, in a strict sense neither fuels, i.e. the aggregate of the activities "Fuels" and "Petrol and car maintainance", nor total energy are meaningful concepts in the household sector. However, for the purpose of this study - both the comparison with demand effects on the production side and the calculation of macro elasticities - these aggregates will have to be defined.

When studying demand effects for groups of micro units we also face the question how the aggregation of sectors should be carried out. As was the case for the aggregation of energy goods in the household sector, a single-valued demand function for electricity, fuels or total

1) The purchasers' prices also include commodity taxes and subsidies. These are, however, specified as proportional mark-ups (value taxes and subsidies), and therefore do not create any deviations between price effects at different levels in the system.

energy may not even exist on a more aggregated level. For the calculation of demand for electricity, fuels and total energy in economic terms for each group of sectors, and for the economy as a whole, we have therefore simply added the demand within each sector measured in constant (purchasers') values. As is well known this procedure is equivalent to a Laspeyres aggregation of the individual quantities. The corresponding energy price indices are Paasche indices. Laspeyres quantity indices are also used to form aggregates of labour demand and capital demand for groups of sectors. As shown in Energy Modelling Forum (1980) the elasticities are not sensitive to the choice of economic indices, e.g. Laspeyres, Paasche or Fisher. Such indices provide approximations to quantities and average prices obtainable from aggregate demand functions.

In many energy studies energy demand is measured in physical units, e.g. heat content (BTU). In order to obtain an aggregate measure of energy in physical terms a BTU-weighted index is also applied, which means that the demand is aggregated with weights corresponding to the heat content of the group of energy commodities. In the specification of MSG-4E special efforts are made in order to establish proper volume concepts for energy flows. In particular the specification of the electricity system has been focused by distinguishing explicitly between production and distribution of electricity (see the discussion by Longva and Olsen in chapter V of this volume). Thus, when studying aggregate demand effects for energy goods the impact on the demand for electricity measured in kWh, and on the use of oil products measured in tons, both convertible to BTU, may be directly derived from the model calculations. This kind of quantity effects are obviously of interest, and are therefore included in the presentation below.

However, from an economic point of view one may argue that the effects on energy demand measured in physical units are not the most relevant type of "quantity effect" to be studied. On the demand side a BTU-weighted index is based on the rather unreasonable assumption that all energy forms are perfect substitutes on a BTU-basis. On the supply side a BTU-weighted index does not properly reflect the effects on the use of resources that are directly related to the demand changes. Let us as an example regard the electricity supply: Electricity measured in kWh is produced in power plants - composing the production sector for electricity of the MSG-4E model. The effect on "the demand for kWh" thus reflects the impact on the use of real resources in this sector. However, changes in the deliveries of kWh's to industries and households will also initiate changes in the deliveries from the production sector for electricity distribution services and therefore influence the resource use in this part of the supply system. In order to measure a demand effect which is related to the total use of resources in electricity supply we may study the effect on the electricity aggregates defined by the aggregate of the "Electricity" activities for the individual sectors that are included in the sector group in question. As mentioned above these activities are defined both in production and consumption. The same type of reasoning may be carried out for oil products: The demand for the activity "Fuels" reflects both the impact on the use of resources in the refinery sector and the necessary resource use in the trade sector. From this point of view aggregate demand measured in constant values is a more relevant measure of resource use than aggregate demand measured in BTU.

3. Output-constrained total energy price elasticities

As we concluded in section 2 the total elasticities should be calculated at the retail level, which means that the energy price changes correspond to changes in purchasers' prices. These variables are, however, endogenous in the model. In order to estimate total elasticities by means of MSG-4E we must introduce changes in energy prices that are defined as exogenous variables. In the MSG-4E model this applies to the following commodity prices:

1. The price of electricity delivered from the production sector.
2. The price of electricity distribution services.
3. The price of crude oil.

The estimation of total demand elasticities starts out from a scenario of the development of the Norwegian economy described in chapter IX of this volume. With this scenario as a reference projection aggregate elasticities are estimated by simulating the model with the following changes carried out one at the time:[1]

(i) A 10 per cent increase in the price of electricity and the price of electricity distribution services.

(ii) A 10 per cent increase in the price of crude oil.

(iii) A simultaneous increase in the two electricity prices and the crude oil price.

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1) Since MSG-4E basically is a static model a close approximation to the long run estimates for the aggregate elasticities was arrived at by simulating the model over a period of five years.

The first of these changes will lead to a 10 per cent increase in the purchasers' prices of electricity of all consumers, since all commodity prices included in the activity "Electricity" are increased and commodity taxes are specified as proportional mark-ups. The 10 per cent increase in the price of crude oil will, however, produce increases in the purchasers' prices for oil products less than 10 per cent because the latter prices also reflect refining costs (other than costs of crude oil) and trade margins. The motivation for the third type of initial price change above - a simultaneous increase in the electricity prices and the crude oil price - is to simulate the effects on demand of (approximate) proportional increases in the purchasers' prices of electricity and oil products, respectively. so that no "internal" substitution is motivated between electricity and fuels within each sector on the demand side. The calculations displayed that in order to produce the same changes in <u>average</u> purchasers' prices the crude oil price has to be increased with close to 12 per cent, when a 10 per cent increase in the electricity prices is imposed.

In interpreting the estimated elasticities it is important to remember that the elasticities are output-constrained elasticities, i.e. they are normalized for changes in the activity level. The precise content of this normalization varies among sectors for which results are presented. For groups of industries the price elasticities are defined by holding the output level of the group constant, while for the household sector total expenditure is constant. The estimated elasticities for the total economy are normalized against the change in gross domestic product.

<u>Total elasticities of energy demand</u>

Total price elasticities for energy in physical (BTU) and economic terms are presented in tables 1 and 2, respectively. As the energy commodities in the MSG-4E model are aggregated into energy activities by fixed proportions (see section 2) it follows that in each production sector of the model the elasticities in economic and physical terms are equal. The calculated elasticities for the groups of sectors specified above will be weighted averages of the similar elasticities in the production sector belonging to each group. Since the price per physical unit differs between production sectors, the two elasticity measures will also differ. The absolute value of the price elasticity in economic terms will be larger than the corresponding price elasticity in physical terms if there is a tendency that the production sectors within an aggregate which pay the highest energy prices also have the highest absolute values of the price elasticities. For electricity we see that this is the case for Primary industries and for Other manufacturing industries while the opposite is the case for Energy intensive industries and Service industries. However, as a general feature we observe that there are little difference between the price elasticity for electricity measured in physical and economic terms. For fuel the same is true, although this cannot be observed directly from the tables as the demand effects in physical terms are calculated separately for petrol and other fuels, while the effects in money terms are defined for the aggregate of these two energy commodities.

With respect to the magnitudes of the estimated elasticities we see from table 2 that the direct price elasticities for electricity (in economic terms) vary between -0.40 and -0.70. The direct price elasticities for fuels vary between -0.09 and -0.64. The direct price elasticities for total energy calculated by simultaneous and proportional increases in the purchasers' prices for electricity and fuels show elasticities between 0.0 and -0.65, while the estimate for the total economy is -0.32.[1)]

The estimated cross price elasticities are positive for some of the aggregate sectors, implying that electricity and fuels are substitutes when taking all simultaneous effects in the economy into account, and for other sectors negative, i.e. the two energy groups are estimated to be complements in this overall setting. A positive cross price elasticity is perhaps what should be expected. However, even if we restrict the discussion to partial net elasticities, negative cross price elasticities, i.e. complementarity between electricity and fuels, may occur. For production sectors in the MSG-4E model this will be the case if the positive internal energy substitution effect between electricity and fuels is lower than the direct price effect for total energy, i.e. the scale effect (see chapter III of this volume for an elaboration of this point). When calculating aggregate elasticities by means of the entire model complementarity between electricity and fuels may occur also as a result of simultaneous changes in other prices and quantities and in the composition of sectors within each group.

From table 2 it is seen that negative cross price elasticities between electricity and fuels have been estimated for the two groups of manufacturing industries. For Energy intensive industries this result is rather easy to explain. In these sectors the substitution possibilities

1) For Primary industries total energy input is assumed to be proportional to the output level. This explains why energy demand for Primary industries is inelastic in this case.

between electricity and fuels are estimated to be small (as measured by partial gross elasticities), while the elasticities of substitution between energy and other inputs are estimated to be quite large. For the larger group of Other manufacturing industries it seems less obvious why the two energy activities should be complements. For the total economy the estimated cross price elasticities are positive.

Table 1. Output-constrained, total energy price elasticities of energy demand, physical terms

Sector	Electricity price elasticities[1)]				Fuel price elasticities[2)]				Energy price elasticities[3)]			
	Electricity	Fuels		Total energy	Electricity	Fuels		Total energy	Electricity	Fuels		Total energy
		Petrol	Other fuels			Petrol	Other fuels			Petrol	Other fuels	
1. Primary industries	-.39	.16	.09	.0	.43	-.19	-.09	.0	.0	.0	.0	.0
2. Energy intensive industries	-.71	-.09	-.28	-.62	-.09	-.30	-.28	-.10	-.78	-.36	-.51	-.72
3. Other manufacturing industries	-.53	-.08	-.04	-.22	-.13	-.74	-.56	-.34	-.63	-.77	-.57	-.60
4. Service industries and transport	-.75	.10	.08	-.10	.25	-.37	-.50	-.18	-.54	-.25	-.29	-.34
5. Households	-.59	.07	.54	-.23	.31	-.48	-.91	-.07	-.39	-.18	-.23	-.29
Total economy	-.55	.10	.11	-.21	.05	-.38	-.45	-.14	-.50	-.21	-.26	-.37

1) The electricity price is increased.
2) The crude oil price is increased.
3) The electricity price and the crude oil price are increased simultaneously.

A recent study of the long run elasticity of total energy demand of five of the most well-known comprehensive economic models, utilizing historical data for the estimation of parameters, is presented in Energy Modelling Forum (1980). Models for both the U.S. and the rest of the OECD area are included. Long run energy price elasticity estimates for the total economy in these five models range between -0.3 and -0.7. The estimate of the MSG-4E model, using a comparable estimation method, is -0.32. The main reason why our result is on the lower side in absolute value is, as mentioned in section 2, that some of the energy-using sectors are exogenous in MSG-4E while the five models in the Energy Modelling Forum-study cover all energy-using sectors. In the Energy Modelling Forum-study the estimates for the household sector range between -0.5 and -1.0 and for the manufacturing and service industries between -0.3 and -0.7.

Table 2. Output-constrained total energy price elasticities of energy demand, economic terms

Sector	Electricity price elasticities[1)]			Fuel price elasticities[2)]			Energy price elasticities[3)]		
	Elec-tri-city	Fuels	Total ener-gy	Elec-tri-city	Fuels	Total ener-gy	Elec-tri-city	Fuels	Total ener-gy
1. Primary industries	-.40	.10	.0	.44	-.09	.0	.0	.0	.0
2. Energy intensive industries	-.69	-.20	-.53	-.09	-.29	-.15	-.76	-.44	-.65
3. Other manufacturing industries	-.58	-.05	-.12	-.13	-.60	-.39	-.69	-.61	-.64
4. Service industries and transport	-.70	.10	-.17	.24	-.40	-.16	-.50	-.28	-.32
5. Households	-.59	.18	-.10	.31	-.64	-.30	-.38	-.23	-.29
Total economy	-.53	.15	-.12	.14	-.45	-.25	-.43	-.22	-.32

1) The electricity price is increased.
2) The crude oil price is increased.
3) The electricity price and the crude oil price are increased simultaneously.

In St.meld. (1980), a governmental report to the Norwegian Parliament which include forecasts for future energy demand, the implicit elasticity of total energy demand is as low as -0.05 (see Longva (1980)). The direct elasticities for electricity and fuels are both -0.20, compared to -0.53 and -0.45 in our study. The main reason for the relatively price inelastic energy demand implicit in St.meld. (1980) is that the forecasts are based on subjectively determined low elasticities. Our study indicates that the scope of energy substitutions is quite substantial compared to the implicit assumptions in St.meld. (1980).

Total energy price elasticities of demand for labour and capital

The effects of increased energy prices on aggregate demand for energy - both in physical and economic terms - were discussed in the preceding subsection. However, when simulating the effects on the economy of increases in energy prices by means of a macroeconomic model, it is also of considerable interest to derive the effects on the demand for other key factors of production, in particular labour and capital. The results of these calculations are presented in table 3. In all groups of sectors included in this presentation elasticities of capital demand are negative, i.e. capital and energy are estimated to be complements in the production processes. For the total economy the energy price elasticity of capital is estimated to -0.08, which means that a ten per cent increase in the price of energy will lower the demand for capital with 0.8 per cent.

Table 3. Output-constrained total energy price elasticities of demand for labour and capital

Sector	Electricity price elasticities[1]		Fuel price elasticities[2]		Energy price elasticities[3]	
	Labour	Capital	Labour	Capital	Labour	Capital
1. Primary industries ..	.02	-.01	.02	.0	.04	-.02
2. Energy intensive industries	.14	-.06	.07	-.04	.20	-.09
3. Other manufacturing industries ..	.03	.0	.03	-.01	.07	-.01
4. Service industries and transport	.03	-.01	.03	.0	.05	-.01
Total economy ..	.02	-.08	.01	.01	.03	-.08

1) The electricity price is increased.
2) The crude oil price is increased.
3) The electricity price and the crude oil price are increased simultaneously.

Most econometric studies of the production structure of manufacturing industries display that energy and capital are complements. Our study extends this result to the economy as a whole.

For all the sector groups included in the table an increase in the electricity price causes an increase in the demand for labour, i.e. these two inputs are substitutes. These results apply also to an increase in the price of crude oil and a simultaneous increase in both energy prices. Consequently, for the economy as a whole these elasticities are all positive.[1]

4. A comparison of partial and total output-constrained elasticities of energy demand

In section 2 we emphasized the distinction between the total elasticities presented above and partial price elasticities, and stated that

1) In the MSG-4E model the total labour force is exogenously given. However, it should be remembered that the elasticities presented above are defined as "output-constrained". The elasticities for labour input for the total economy in table 3 thus reflect the (negative) changes in GNP that are implied by the increased energy prices.

the latter may be regarded as the "first step effect" in the chain of price and quantity changes which finally are measured by total elasticities. To study the empirical importance of this distinction, partial and total elasticities are presented in table 4 for the two cases of separate price increases. The partial price elasticities discussed in section 2 are shown in columns marked I. They are simply weighted averages of the partial price elasticities for the individual sectors included in each sector group[1]. Columns marked II contain estimates of price elasticities when also all induced price effects on the demand composition within each individual sector is taken into account, while the composition of sector outputs within each sector group is unchanged.[2] The estimates of total elasticities of energy demand of table 2 are reproduced in columns marked III. The difference between columns I and II shows the effect of taking all induced price effects into account within each individual sector while the difference between columns II and III shows the effect of the induced change in the sector composition.

Table 4. A comparison of partial and output-constrained total energy price elasticities of energy demand

I: Partial price elasticities. No indirect effects are included
II: Price elasticities where all induced changes in prices are included but induced changes in the sector composition are excluded
III: Total price elasticities

Sector	Electricity price elasticities[1]						Fuel price elasticities[2]					
	Electricity			Fuels			Electricity			Fuels		
	I	II	III	I	II	III	I	II	III	I	II	III
1. Primary industries	-.39	-.41	-.40	.09	.09	.1	.42	.44	.44	-.09	-.11	-.09
2. Energy intensive industries	-.71	-.69	-.69	-.18	-.20	-.20	-.10	-.09	-.09	-.33	-.28	-.29
3. Other manufacturing industries	-.57	-.58	-.58	-.07	-.04	-.05	-.06	-.13	-.13	-.52	-.61	-.60
4. Service industries and transport	-.64	-.71	-.70	.13	.09	.10	.16	.24	.24	-.24	-.42	-.40
5. Households	-.61	-.59	-.59	.18	.18	.18	.29	.30	.30	-.68	-.64	-.64
Total economy	-.59	-.58	-.53	.11	.10	.15	.17	.16	.14	-.45	-.43	-.45

1) The electricity price is increased.
2) The crude oil price is increased.

1) For households an "income compensated" elasticity is presented in order to make it comparable with the output-constrained total elasticities.
2) This separation of effects is possible within the MSG-4E framework since the total model system may formally be divided into a price and quantity block (see chapter II of this volume).

A conclusion to be drawn from the results in table 4 is that most of the calculated total elasticities do not differ very much from the (weighted) partial elasticities. This may reflect both that the induced price and quantity changes in the economy are small compared to the initial effects of increases in energy prices, and the fact that the cross price elasticities in each individual sector are relatively low. However, it should be emphasized that when energy price elasticities are used in a long term planning context a difference of say one tenth in an estimated elasticity may have significant consequences for future levels of the variables in question. On this background the results in table 4 may still illustrate the importance of calculating energy price elasticities in a general equilibrium setting.

For energy intensive industries and for the household sector the direct total elasticities are lower (in absolute value) than the (weighted) partial elasticities while the opposite is the case for service industries. For the total economy the direct elasticities of energy demand are, as expected, reduced when the induced changes in other prices are taken into account. It is also seen from table 4 that changes in the sector composition utterly reduces the direct price elasticity for electricity. There are two reasons for this result, which may seem strange on the base of the changes in the elasticities for the subgroups in the table. Firstly, it should be emphasized that exports are exogenous in the model. This fact tend to maintain the production in energy intensive industries. Secondly, it may be noticed that an important effect of increasing the electricity price is a squeeze of investments, in particular in the electricity producing sector itself and, as a consequence, a reduction of the activity level in the construction sector. As full employment is assumed in the model, this is counteracted by an increase in household consumption and in the activity levels in sectors producing consumption goods. These industries are more "electricity-intensive" than the construction sector, and the isolated effect of the changes in the sector composition is thus an increase in electricity demand. The rise in household consumption caused by the change in the electricity price also implies an increase in the demand for fuels (the cross price elasticity is raised from 0.10 to 0.15).

5. Unconstrained total elasticities of energy demand

In the preceding sections we have presented estimates of output-constrained total elasticities, i.e. total elasticities normalized for changes in the activity levels. In this section we are presenting elasticity estimates where the changes in income or aggregate economic activity implied by energy price increases are taken into account. As already mentioned these effects may presuppose some kind of policy action from the central authorities.

Before presenting the estimates of the unconstrained total elasticities of energy demand we shall evaluate the relationship between energy prices and the aggregate economic activity in the MSG-4E model. In table 5 we present the impacts of energy price increases on main economic variables as estimated by simulations both with and without restrictions on the balance of payments. When no restriction on the balance of payments is imposed, i.e. when possible effects through changes in terms of trade are not accounted for, the overall energy capital complementarity causes a reduction in the production level of the economy. GDP is thus slightly reduced compared to the reference scenario in all the three alternative projections. Electricity has little weight both in exports and imports and the main effect of the partial increase in the electricity price is a fall in investment, in particular in the electricity sector itself. As a consequence more real resources are available for production of consumption goods. The increase in household consumption also leads to a small increase in total imports. Crude oil is both imported and exported. When the crude oil price is raised and the increased revenues from exports are not "used" in the economy, a negative real income effect causes a reduction in domestic demand and production, while the balance of payments is considerably improved.

In the model simulations with an unchanged balance of payments, i.e. effects through changes in terms of trade are included, all export and import volumes are adjusted proportionally. These rather mechanical adjustments are obviously only a crude representation of the changes which are actually taking place. The adjustments indicate, however, the direction of changes in exports and imports that must be the result when incomes from changes in terms of trade are channeled into the economy.

The estimates presented in table 5 show that effects of a change in the electricity price are approximately the same with and without restrictions on the balance of payments. This simply reflects the fact that changes in the electricity price does not significantly influence the terms of trade.[1] However, when the price of crude oil is increased, the terms of trade effects involve considerable changes in the total price

1) Electricity intensive products are important export commodities. However, the induced increases in the prices of these products are very moderate.

elasticities. This is due to the fact that crude oil and gas amount to about one third of the total value of Norwegian exports. The improvement in the terms of trade allows for an increase in domestic demand, and particularly household consumption is raised. This implies a reduction in export volumes and increased imports. The reallocation of resources is also seen to have a positive effect on GDP.

In table 6 we present estimates of total elasticities of energy demand when changes in income and aggregate economic activity stemming from energy price increases are taken into account. Columns marked II show the elasticities when induced changes in activity levels, apart from terms of trade effects, are included, while the elasticities reported in columns marked III also include the terms of trade effects of the increased energy prices. The estimates of output-constrained elasticities of energy demand of table 2 (where the elasticities are normalized against the changes in activity levels) are reproduced in columns marked I.

The general impression of the estimates of table 6 is that the unconstrained elasticities of columns II are not very different from those of columns I, reflecting the result that quantity changes caused by energy price increases are rather small when terms of trade effects are not accounted for.

Due to the very small terms of trade effect following an increase in the price of electricity the electricity price elasticities of columns II and III are also practically identical. The fuel price elasticities of columns III are, however, markedly different from the estimates in both columns I and II. The very strong terms of trade effects nearly reduce the absolute value of the elasticity of fuel demand for the total economy to one half (from -0.46 to -0.27) and double the elasticity of electricity demand (from 0.13 to 0.24). The terms of trade effects also induce changes in the production structure as resources are reallocated from the export oriented manufacturing industries to industries producing consumption goods and services. These effects are reflected in our calculations by a very strong increase in the absolute values of the elasticities for the manufacturing industries (groups 2 and 3).

The main conclusion to be drawn from table 6 is that the income effects caused by a change in the price of electricity are relatively minor, while the impacts from a change in the price of crude oil are very strong when the changes in incomes are channeled into the economy.

Table 5. Unconstrained total energy price elasticities of main economic variables

Main economic variables	Terms of trade effects excluded			Terms of trade effects accounted for		
	Electricity price elasticities 1)	Fuel price elasticities 2)	Energy price elasticities 3)	Electricity price elasticities 1)	Fuel price elasticities 2)	Energy price elasticities 3)
Gross domestic product	-.02	-.01	-.03	-.02	.01	.00
Imports	.01	-.03	-.03	.00	.13	.16
Exports	.00	.00	.00	.00	-.13	-.15
Total domestic use of goods and services ...	-.02	-.02	-.04	-.02	.13	.13
Household consumption	.06	-.04	.02	.05	.22	.39
Investment	-.13	-.03	-.16	-.13	.07	-.15
Capital stock	-.10	-.00	-.11	-.10	.03	-.04

1) The electricity price is increased.
2) The crude oil price is increased.
3) The electricity price and the crude oil price are increased simultaneously.

Table 6. Unconstrained total energy price elasticities of energy demand and a comparison with the output-constrained elasticities

I: Output-constrained total elasticities; induced changes in activity levels are excluded
II: Unconstrained total elasticities; induced changes in activity levels, apart from terms of trade effects, are accounted for
III: Unconstrained total elasticities; induced changes in activity levels, including the terms of trade effects, are accounted for

Sector	Electricity price elasticities 1)						Fuel price elasticities 2)					
	Electricity			Fuels			Electricity			Fuels		
	I	II	III	I	II	III	I	II	III	I	II	III
1. Primary industries	-.40	-.40	-.39	.09	.1	.09	.44	.43	.43	-.09	-.10	-.12
2. Energy intensive industries	-.69	-.70	-.69	-.20	-.21	-.20	-.09	-.09	-.64	-.29	-.29	-.91
3. Other manufacturing industries	-.58	-.60	-.59	-.05	-.07	-.07	-.13	-.14	-.39	-.60	-.61	-.83
4. Service industries and transport	-.70	-.69	-.70	.10	.11	.10	.24	.20	.28	-.40	-.44	-.39
5. Households	-.59	-.53	-.54	.18	.24	.23	.31	.27	.80	-.64	-.68	-.11
Total economy	-.53	-.55	-.56	.15	.13	.12	.14	.13	.24	-.45	-.46	-.27

1) The electricity price is increased.
2) The crude oil price is increased.

IX. USE OF THE MSG MODEL IN FORECASTING ELECTRICITY DEMAND

by

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1. Introduction

In Norway approximately half of the total energy consumption is electricity, produced and distributed through a decentralized, but centrally coordinated hydro power system. To plan and implement an expansion of this system is timeconsuming, often involving different political bodies in several rounds of discussions before new plans are approved, followed by a long period of construction. Normally, 6-10 years elapse from the first proposal is presented to local political bodies until a new plant is in operation. Investments in the electricity supply system are considerable; in the last five years they have amounted to between 50 and 85 per cent of total investments in manufacturing. Both the magnitude of the investments and the time lags make it highly desirable to integrate the forecasting of the future demand for electricity and the planning of the capacity expansion of the power system. Alternative energy policies may influence the performance of the economy, significant even on macroeconomic totals. In an economy aiming at efficiency the electricity production capacity should expand at a rate such that the marginal willingness to pay for power equals the long run marginal cost in electricity supply. A too rapid or too slow expansion of the hydro power system means inefficient allocation of scarce resources (labour, capital etc.). In addition, dependent upon the price policy, a too rapid or too slow expansion may create short term management problems in the electricity market, implying idle capacity or rationing.

In section 2 we present our reference scenario, a forecast of future electricity demand up to year 2000 based on simulations on the MSG-4E model. This model is described in chapter II of this volume. The general economic development in the reference scenario are close to the assessments made by the government in 1981 in the Long-Term Programme 1982 - 1985 (see St. meld. (1981)). The official plans for the expansion of the hydro power system and the proposed policy for pricing of electricity are also embedded in the reference scenario. The demand for electricity which follows from the reference scenario is compared with the official forecasts of electricity use and production as presented in the government's Energy Programme (see St.meld. (1980)). Our demand forecasts indicate a significant surplus production capacity in 1990 if the official plans for capacity expansion and pricing policy are followed. There are two reasons for the difference between our calculations and the official forecasts. First, our calculations are based on more recent and downward adjusted forecasts of the general economic development; second, the revised MSG model represents a further development and an improvement of the forecasting methods used in the official electricity forecasts. However, in view of the considerable uncertainty associated with the assessments of the overall economic development as well as with determination of the demand elasticities, it could still be argued that both our and the official electricity forecasts are within the range of possible future developments.

In section 3 some methodological aspects of using the MSG model in forecasting energy demand is discussed and in section 4 we present two alternatives to the reference scenario. As already noted the reference scenario gives a disequilibrium path for the domestic electricity market since the official plans for capacity expansion are not consistent with the price policy suggested by the authorities. In the first alternative we assume that the capacity of the electricity supply system is expanded as officially planned, but that prices are reduced to their equilibrium values at every point of time. In the second alternative the prices are increased at the rate suggested by the authorities, but part of the planned investments in the electricity supply system during the considered period is reallocated to investments in other industries, leaving the electricity market in balance at a lower capacity level than in the reference scenario. Section 5 contains comparisons and evaluations of results from all three scenarios.

2. The reference scenario

The simulation of macroeconomic development, on which our electricity demand forecast is based, covers the period 1979 - 2000, the same period as covered by the official documents referred to above. Table 1 displays some key results from the simulations.

Table 1. Macroeconomic development. Reference scenario. Average annual growth rates

	1979-1985	1985-1990	1979-1990	1990-2000
Gross domestic product	2.6	2.6	2.6	2.2
Gross domestic product excl. oil activities and shipping ...	2.2	2.5	2.3	2.0
Imports	3.4	2.9	3.2	2.6
Exports	2.3	1.9	2.1	1.9
Domestic use of goods and services	3.0	3.0	3.0	2.5
Household consumption	2.0	3.3	2.6	3.2
Public consumption	3.3	3.5	3.4	2.7
Investments	5.3	2.1	3.8	1.1

It is assumed that the economic growth, measured by the growth rate of GDP, will be moderate and declining towards the end of the century, reflecting a stagnating work force and a leveling off in the production of petroleum. Household consumption is expected to grow faster in the last part of the period than in the 1980's. The opposite holds for investments since oil investments are expected to decline.

The assumed macroeconomic development is in accordance with the medium alternative of the macroeconomic perspectives presented in the Long-Term Programme 1982 - 1985. In our simulations, however, the low growth performance of the Norwegian economy in 1981 and 1982 is incorporated so that the average growth rate of domestic use of goods and services in the period 1979 - 1985 is calculated to be 3.0 per cent while 3.7 per cent is expected in the Long-Term Programme.

Energy prices

The prices of electricity and oil products are essential for the development of electricity demand. Table 2 shows the assumptions of price development in real terms, i.e. deflated by the price index for domestic use of goods and services. For the period 1979 - 1981 the figures show the actual price development.

In accordance with the official plans the price of electricity is assumed to increase considerably until 1985. For domestic users, apart from energy intensive industries, the price is stipulated to reach long run marginal cost in that year. After 1985 the change in the real price of electricity is assumed to level off with only small increases through the 1990's reflecting small increases in the marginal costs of producing hydro power.

Table 2. Energy price assumptions. Reference scenario. Average annual growth rates

	1979-1981	1981-1985	1985-1990	1979-1990	1990-2000
Real price of electricity	1.5	3.4	0.0	1.5	1.0
Real price of crude oil .	18.5	-2.0	1.5	3.1	1.5
Real price of oil products	27.0	-2.3	0.7	3.9	0.7

The government controls the price of electricity through government owned power plants and through taxation. The assumed development of the electricity price is in accordance with the policy proposed in the government's Energy Programme and in the government's Long-Term Programme 1982 - 1985. In both documents the government commits itself to use the long run marginal cost as an investment criterion and thereby as a long run pricing principle. The price assumptions are thus based on calculations of long run marginal costs in electricity production and distribution[1]. Such calculations are, however, uncertain and disputable. Up to now, there seems to have been a tendency for construction costs to be consistently underestimated, which means that the assumed development of the electricity price in table 2 may turn out to be somewhat low compared to long run marginal costs revealed later on. On the other hand, a possible decline in the applied rate of real social discount rate is not taken into account.

Experience has shown that it is difficult to make reliable forecasts for the development of the crude oil price. As shown in table 2 the price of crude oil rose considerably from 1979 to 1981. The even larger increase in the price of oil products in the period 1979 - 1981 is partly due to the price adjustment for oil products that took place after the general price freeze in 1978 - 1979. Through 1981 and the first half of 1982 there has been a softening of the oil market and a fall in real prices. This is reflected in the figures for the period 1981 - 1985. After 1985 the real price of crude oil is assumed to increase again. The price development presented in table 2 reflects the common expectation

1) See the discussion of the calculation of long run marginal costs in Norwegian electricity supply by Strøm in chapter XII of this volume.

that the real price of crude oil will, after an adjustment away from the high level in 1981, increase towards the end of the century.

Electricity demand and supply

In table 3 electricity demand and supply in 1990 and 2000 in the reference scenario are presented. In addition, figures for production capacity are included.

Table 3. Electricity production and use[1]. Reference scenario

	TWh				Average annual growth rates	
	1979		1990	2000		
	Observed figures	Temperature corrected demand figures	Model fore-casts	Model fore-casts	1979-1990	1990-2000
Mean production capacity[2]	86.1	-	114.6	126.7	2.6	1.0
Idle capacity	-1.7	-	9.9	0.0	-	-
Production	87.8	-	104.7	126.7	1.6	1.9
Net export	4.7	-	6.2	7.7	2.4	2.4
Transmission losses	8.2	-	9.1	10.9	1.1	1.7
Domestic net demand	75.0	73.2	89.4	108.1	1.8	1.9
Energy intensive industries	29.6	29.6	33.7	38.2	1.2	1.3
Other manufacturing industries	10.9	10.9	9.8	9.0	-1.0	-0.9
Primary industries	0.7	0.7	0.9	0.9	2.0	0.5
Private service industries and transportation ...	7.2	6.9	10.3	13.6	3.7	2.8
Government service industries	4.3	4.1	5.9	7.9	3.4	3.0
Households	22.3	21.0	28.8	38.5	2.9	3.0

1) The figures for electricity use includes 1.5 TWh occasional power to electrical boilers.
2) Mean production capacity is defined as an average of the production potential at the beginning and at the end of the year, given normal water inflow.

In the 1980's the increase in total domestic net demand of electricity is calculated to be 16.2 TWh when temperature corrected figures are compared, while the increase in the period 1990 - 2000 is 18.5 TWh. The total growth is thus 34.9 TWh or 47.6 per cent from 1979 to 2000. The future domestic demand of electricity reflects the changes of industrial structure and consumption pattern associated with the expected growth of income. In Norway the growth of income is speeded up through the domestic use of oil and gas revenues. These structural changes are the main explanation behind the reduction in electricity demand in other manufacturing industries, and the reason why electricity demand in sectors like private and government services expands at a rate significantly above the average for the economy. The forecast also reflects the increase in the real price of electricity. This is indicated by the fact that total domestic electricity demand expands at a rate below the growth in domestic use of goods and services. On the other hand, the even larger increase in the real price of oil products pushes the electricity demand upwards[1].

The occurrence of net exports in the reference scenario is due to the fact that the supply system is completely hydro based. Optimal development of such a system implies some surplus power as a consequence of the uncertainty caused by annual variations in precipitation. At present the optimal surplus in a year with normal precipitation is calculated to be close to 12 per cent of the mean production capacity[2], i.e. 13.7 TWh and 15.2 TWh in 1990 and 2000, respectively, in the reference scenario. The optimal average supply surplus in the production system can be reduced by import contracts. Present import contracts cover 4 TWh. In our calculations we assume that the import contracts will be extended to 6 TWh during the 1980's, i.e. that the optimal average excess production capacities will be 7.7 TWh and 9.2 TWh in 1990 and 2000, respectively. The optimal excess supply can be sold domestically as occasional power or exported. In our assessment of net exports in the reference scenario we assume that net exports equal the calculated optimal excess supplies less 1.5 TWh auctioned domestically (the same amount as auctioned in 1979).

Production of electricity is given in the third row of table 3 and is defined as the sum of domestic net demand, transmission losses and net export. The first row contains figures for the development of mean production capacity. Up to 1990 we assume that the expansion of the production capacity follows the plans presented in the government's

1) The energy price sensitivity of the Norwegian economy, summarized by total energy price elasticities at an aggregated level, is discussed by Longva, Olsen and Rinde in chapter VIII of this volume.
2) These calculations are based on Statens Energiråd (1969). See also the theoretical discussion by Bjerkholt and Olsen in chapter XII of this volume and Raaholt et al. (1982).

Energy Programme. Our calculations indicate that this policy will lead to idle capacity, i.e. that the assumed macroeconomic development and electricity price increases will curb demand below the expanding capacity. The idle capacity is estimated to be 9.9 TWh in 1990. From 1990 to 2000 the expansion rate of mean production capacity in the reference scenario is adjusted in such a way that full capacity utilization is achieved in the year 2000[1].

In the reference scenario we thus assume that some of the investment in the electricity supply system accumulate as temporary idle capacity. However, it has been argued that the excess electricity could, if necessary, be exported. After scrutinizing this suggestion we find it hardly reasonable to assume that increased export of electricity is a feasible solution. In order to achieve full capacity utilization the net export of electricity would have to increase sharply, from 4.7 TWh in 1979 peaking in 1990 at 16 TWh. Net export of such quantities in 1990 is neither technically feasible (lack of transmission capacity) nor is it in accordance with market situations in our neighbouring countries, Denmark and Sweden, where the economic development has led to huge excess capacities in the electricity supply systems.

A comparison with official forecasts

The official plan for expansion of the hydro power system up to 1990 is based on the forecast for electricity demand presented in the government's Energy Programme. Compared with the reference scenario, this forecast gives about 6 TWh higher domestic electricity demand in 1990 for domestic users other than energy intensive industries. However, even if this forecast should turn out to be correct, the planned expansion of mean production capacity implies much larger quantities of surplus electricity than what follows from calculated optimal supply of surplus power and the import contracts. The reason is that the Energy Programme, on ad hoc basis, adds 2 TWh due to uncertainty on the demand side and 2 TWh due to "organizational conditions", in determining total optimal supply surplus in 1990.

Our forecast for electricity demand is lower than the official forecast mainly for two reasons. First, our expected annual growth rate of domestic use of goods and services is approximately 0.5 per cent lower than assumed in the Energy Programme. Second, our direct price

1) The Energy Programme does not contain any explicit plans for the development of production capacity after 1990.

elasticity for electricity is approximately -0.5 while only -0.2 is assumed in the official forecast (see Longva, Olsen and Rinde, chapter VIII of this volume, and Longva (1980)). The cross price elasticities from oil to electricity is lower in the MSG model than in the official forecast (0.13 and 0.20 respectively). The impact of different elasticities is modified since the real price increases of electricity in the Energy Programme is somewhat higher than in our forecast. The assumed long term increases in oil prices are on the other hand approximately the same in the two forecasts.

The growth of electricity use in the energy intensive industries is directly regulated by the government. For 1990 the government proposal for guaranteed deliveries (net of transmission losses) is 33 TWh. For the years after 1990 there are at present no publicly stated guaranteed deliveries. In the reference scenario the increase in electricity use in the energy intensive industries is rather modest and only slightly higher in 1990 than the government proposal. Our results reflect the exogenously given low export growth of energy intensive products and a relatively strong direct price sensitivity of energy demand (derived from a cost minimizing behaviour)[1]. The government proposals for the energy intensive industries thus seem to be consistent with the assumed expansion of these industries and the pricing policy for electricity.

Margins of errors

Our forecast of electricity demand is highly sensitive to some basic assumptions, in particular the assumptions about economic growth and development of energy prices. The sensitivity with respect to growth assessments may be illustrated by noting that our calculated net demand of 55.7 TWh in 1990 for domestic users other than energy intensive industries would instead have been close to the official forecast of 61.5 TWh if the annual economic growth were increased by 1 percentage point[2]. Our forecasts would also have been close to the official ones if the real price of electricity were kept constant and the real price of oil products were increased by 5-6 per cent, both annually from 1982 onwards.

1) The estimation of energy price elasticities for each production sector is discussed by Longva and Olsen in chapter III of this volume. See also chapter VIII.
2) To increase economic growth in the model we would have to increase technical change, labour participation etc.; GDP itself is endogenous in the model. See also chapter II of this volume.

3. Methodological aspects of electricity demand forecasts based on the MSG model

As indicated above the quality of the forecasts of electricity demand depends upon the price and income elasticities and on the type of model applied. In interpreting the model results it is therefore important to understand the nature of the model itself and the limitations of the methodological approach.

In estimating the model parameters we have used national accounting data for the last 20 years. During most of this sample period the real prices of energy were stable or falling, only the last few years show some fluctuations. The information one can draw from this sample might be of limited value in analysing different situations with sharp fluctuations in energy prices as indicated in table 2. The substitution possibilities between electricity and fuel oil for heating purposes are for this reason not easy to model adequately. It is conceivable that relative energy prices might lead to changes in demand which are asymmetric and stepwise, not symmetric and smooth as implied by the demand functions. This might lead to changes in energy use which will not be picked up by the model even if the assumed price development up to 1990 and 2000 on the average is not very different from the development in the sample period.

When interpreting our results, it is important to be aware of the equilibrium characteristics of the MSG model. In the real world, it necessarily takes time before firms and consumers have fully adjusted their energy use to changes in prices and other incentives. In the model, however, the agents react immediately to minimize their current costs. These equilibrium characteristics of the model effect calculations in two respects. First, the calculations are based on the assumption that the energy market was in balance in the base year 1979, i.e. that the users of energy had adjusted their consumption to the prices and incomes of that year. Some rough estimates of the error terms of the demand functions indicate that this may not have been the case. An equilibrium situation would have required that industries should have used more and households less electricity than actually observed in 1979. Second, the period from 1979 to 1985 might bring about some confusing and deceptive signals to energy consumers. As seen in table 2 prices of oil products increased sharply from 1979 to 1981, electricity prices only moderate. From 1981 to 1985 it is assumed that the oil prices will decrease whereas electricity prices will increase significantly. In the model this means a switch from fuel oil to electricity up to 1981 and from electricity to fuel oil in the period from 1981 to 1985. However, the energy consumers might be lagging behind. As should be expected, the model "forecast" of electricity demand in 1981 therefore seems to be too high[1]. For 1985 the results will probably turn out to be much too low, the firms and households will need more time to adjust to the substantial increase in electricity prices which is assumed to take place.

The lesson to be learned from the above reasoning over time lags is that the MSG model is not designed to predict year by year fluctuations in energy demand if prices change significantly. The purpose of our simulations is, however, to predict the expected development in electricity demand 10-20 years ahead. Up to 1990 and 2000 the calculations are based on moderate average increases in energy prices. Industries and households will then have had time enough to adjust to the relatively large changes in absolute and relative prices which take place up to 1985. The model results should be used and interpreted with caution, as estimates of expected electricity demand in 1990 and 2000, given the assumptions for activity levels, incomes and prices. The basic assumptions of full resource utilization and equilibrium development paths in the MSG model make it appropriate to interpret the results as expected potential demand.

4. Two alternative scenarios

The reference scenario shows the effects of pursuing a policy where both the expansion of the electricity system and the development of electricity prices are predetermined and in accordance with official plans. If our assessments of the future economic performance and energy elasticities are correct, the demand for electricity will not increase as rapidly as previously anticipated and hence this policy of capacity expansion and price increases will lead to an idle capacity of approximately 10 per cent in 1990, as elaborated in section 2. This "disequilibrium" in the electricity market will gradually become evident, and policies have to be adjusted. In the two scenarios below we analyse the effects of two likely adjustments, reducing prices and delaying investments in the electricity sector.

Alternative 1. Reduced electricity prices

In this scenario the capacity of the electricity supply system is expanded according to official plans, but the prices of electricity

1) The model result for 1981 is 1.5 TWh higher than preliminary statistics indicate.

are reduced compared to the reference scenario, such that from 1985 and onwards full capacity utilization is achieved. Purchaser prices are reduced proportionately for all users. The reduction necessary to balance off the market brings real prices of electricity down to the 1979 level in 1985. From 1985 the prices are increased gradually and reach the same real price level in the year 2000 as in the reference scenario, cf. table 4.

The reduction in the real prices of electricity affects the outcome of the model calculations in several ways. In the household sector, electricity is complementary to housing services, hence lower prices of electricity induce higher investments in dwellings. In most industries, electricity (or energy) is complementary to capital, hence lower prices of electricity lead to higher investments for given rates of return to capital. On the macro level this means higher total investments up to 1993, thereafter lower investments compared to the reference scenario. In alternative 1, therefore, the capital stock is always slightly larger than in the reference scenario except for the year 2000, when the stocks are equal. GDP is for all intermediate years higher in alternative 1 than in the reference scenario partly because the capital stock is higher and partly because there is no idle capacity or waste of resources in the electricity sector.

One important and interesting aspect of reduced electricity prices is the effects on costs and inflation and thereby on the competitive position of Norwegian manufacturing industries. In alternative 1 the balance of trade is year by year kept at the same level as in the reference scenario. Implicitly this means that a modest appreciation of the Norwegian currency is assumed to cancel out the effects of a somewhat lower growth rate of domestic inflation. With our assumptions of full employment in both scenarios and unchanged balance of trade, this seems to be a reasonable way to capture the cost-price-competitive effects of lower electricity prices.

Alternative 2. Reallocated investments

In this scenario the development of electricity prices is the same as in the reference scenario, but the expansion of the electricity supply system is slowed down. Clearly, it takes time to realize the mismatch, to adjust plans and to complete ongoing projects. We have therefore imposed the assumption that the electricity market will be in balance from 1987 and onwards. The investments which in the reference scenario were used to build up idle capacity in the electricity sector are reallocated to other industries. To achieve this reallocation in the model, the rates of return to capital in the manufacturing and service industries are reduced somewhat compared to the reference scenario up to the year 2000 when the rates are again equal. See table 4.

Table 4. Differences in assumptions between reference scenario, alternative 1 and alternative 2. Average annual growth rates. Per cent

	1979-1985	1985-1990	1990-2000
Real prices of electricity			
Reference scenario and Alternative 2	2.8	0.0	1.0
Alternative 1	0.0	0.2	2.6
Rates of return to capital			
Reference scenario and Alternative 1	-3.5	-3.5	-3.5
Alternative 2	-4.2	-4.0	-2.9

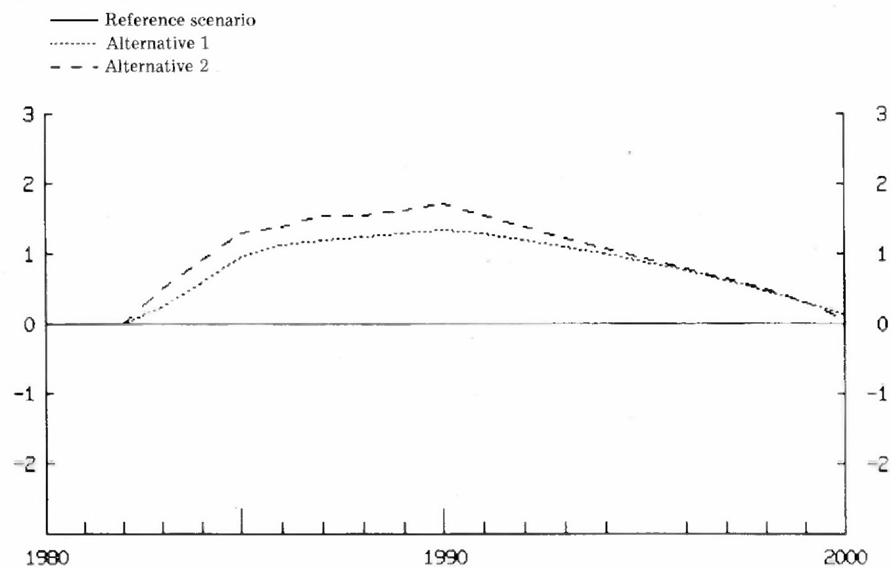
Compared to the reference scenario the investments in the electricity sector are reduced until 1990. In the last decade (1990 - 2000) the investments in the electricity sector is higher in alternative 2 than in the reference scenario, since in alternative 2 there is no idle capacity to draw from. Total gross investments are, except from some differences due to differences in depreciation rates among sectors, the same over the entire period since the terminal stocks of capital are equal. Productive investments in other industries than electricity supply are of course higher in the first part of the period, leading to a GDP which is significantly higher for all intermediate years between 1982 and 2000.

Also in alternative 2 the balance of trade is year by year kept at the same level as in the reference scenario. This means that the increase in production is - except for commodity switching through trade - assumed to be used domestically.

5. Macroeconomic comparisons of the three alternatives

Compared with the reference scenario the two alternative scenarios illustrate the gains which can be achieved from improving the resource allocation over a period of twenty years. At the terminal point of time, year 2000, all three scenarios are equal. The gains in GDP compared to the reference scenario are depicted in figure 1. It is easy

Figure 1. Deviation in GDP from reference scenario. 1 000 million 1979-kroner



to understand that there are gains to capture by using all available resources (alternative 1) or by reallocating investments to give better returns (alternative 2). It is still open to choose how and when these gains should materialize as increased household or public consumption. In the two alternative scenarios we have chosen to reap the benefits as increased household consumption, but the time profiles are different as shown in figures 2 and 3. In alternative 1, due to the energy capital

Figure 2. Deviation in total real investment from reference senario. 1 000 million 1979-kroner

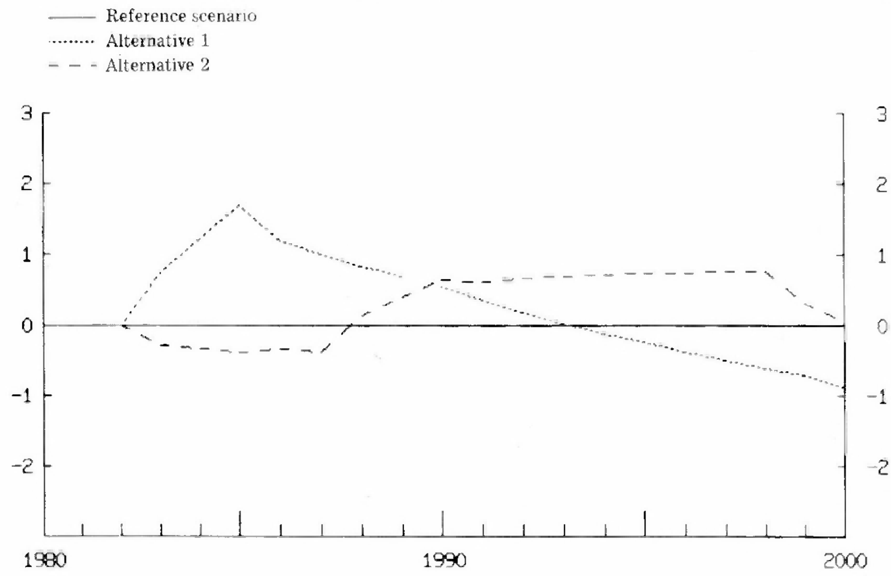
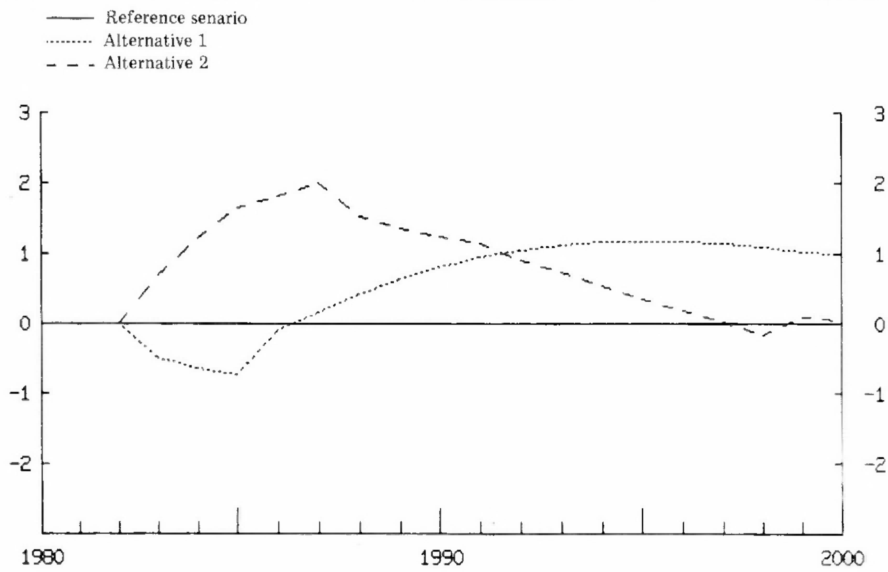


Figure 3. Deviation in household consumption from reference senario. 1 000 million 1979-kroner



complementarity, the total investments are higher in the first part of the period and the consumption gain is postponed to the 1990's. In alternative 2, the given total investments are reallocated from 1983 and onwards and the gain in production and income is currently harvested as increased household consumption. With a social real discount rate of 6 per cent this gives a present value gain in household consumption of 6 000 mill. Norwegian 1982-kroner in alternative 1 and 14 000 mill. Norwegian 1982-kroner in alternative 2 compared to the reference scenario[1]. The differences in present value gains between the two alternatives are clearly somewhat arbitrary, since an unlimited number of internally consistent consumption and investment profiles could be drawn in figures 2 and 3. But since the terminal stocks of capital are equal, the accumulated undiscounted net gains in consumption would be around 15 000 - 20 000 mill. Norwegian 1982-kroner over the twenty years period for all profiles.

In alternatives 1 and 2 the average saving and investment ratios to GDP are decreased compared to the reference scenario. The increments to GDP are consumed. If we instead of keeping the terminal stock of capital equal for all alternatives had chosen to keep the saving ratios equal, all end use categories could have been increased in alternatives 1 and 2 in the year 2000 compared with the reference scenario, leaving the economy potentially better off for all future years.

To summarize, elaborating plans for the expansion of the electricity sector is a difficult task since such plans have to be based on assessments of future demand derived from uncertain long-term forecasts of the economic development, the development of energy prices, and estimates of income and price elasticities. Our assessments of future electricity demand deviate from those implied by the present official plans for the expansion of the electricity sector up to 1990. Above, we have tried to calculate the gains from alternative policies, and found that the expected potential gains are significant even when measured as gains in macroeconomic totals.

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1) The net gain in alternative 2 is close to the discounted "normal return" to the idle capital in electricity supply in the reference scenario.

X. AN EXPENDITURE SYSTEM FOR ENERGY PLANNING

by

Asbjørn Rødseth*

1. Introduction

This study of consumer demand in Norway was part of a project for developing the Norwegian multi-sectoral growth model, MSG, into a more effective tool for energy planning. In a multi-sectoral growth model one will always need a complete system of consumer demand functions. In previous versions of the MSG model Frisch's "complete scheme" was applied, cf. Frisch (1959) and Johansen (1974). The commodities of the model were aggregated into consumption activities by an assumption of proportionality and the utility function was additive in the activity levels. The consumer demand functions of MSG-4, as described by Bjerkholt and Rinde in chapter IV of this volume, are based on the same ideas but pays more attention to energy demand. In this study we develop a more elaborate system designed to fit into MSG-4.

In the new demand system we wished to take account of some ideas and results from previous partial studies of the household's demand for energy (for a survey of these studies, see Taylor 1975, 1977 or Blaalid & Olsen, 1978):

- the scope for substitution between different categories of fuel is greater than for substitution between total energy and other goods.
- energy is always used in connection with durable goods to produce the goods that the consumers are really interested in.
- because of lagged adjustments in some of the stocks of appliances, demand elasticities are greater in the long run than in the short run.

To integrate all these ideas into the complete system framework is a difficult task. Jorgenson (1974, 1977) estimated a demand system for three goods: Services from durables, energy, and other non-durables. However, he seems to have neglected the findings and ideas from the partial studies: there are no lagged responses, and worse: services from durables

* I am grateful to other members of the project and to Arne Amundsen and Jørgen Aasness of the Institute of Economics for stimulating discussions and encouragement at various stages of the project. In addition I thank Tore Kristoffersen and Ivar Sand of the Central Bureau of Statistics for indispensable programming assistance. The many shortcomings which remain in the paper are of course my own responsibility.

are proportional to existing stocks. For energy consuming durables a better first approximation would be that services are proportional to energy consumption. The consumption of gasoline is probably a better proxy for the services from cars than the stock of automobile capital.

An attempt at integration was made by Rødseth & Strøm (1976, 1977) in a study using Norwegian data. The approach taken was, however, rather ad hoc. The present study may be seen as an attempt at a more rigorous development of ideas which we had then.

2. Individual demand

We assume that each household has a utility function

$$U = u(y,z) \tag{1}$$

where U is the utility level, $y' = (y_1,\ldots,y_L)$ a vector of (non-negative) consumption activity levels, and z a vector of exogenous variables affecting preferences. While z may take on different values for different households, the function u is the same for all households.

The existence of a household as opposed to an individual utility function independent of market prices is in itself a problematic assumption, cf. Manser and Brown (1980), but we shall stick to it. In the empirical part of the study the only variable in z is the number of heating degree-days, which of course is an important variable in explaining energy demand. Other variables which could have been included in z, are household composition and area of residence (which will influence the marginal rates of substitution between transport activities and other activities). Actually, the plan was to include these variables in a second stage of the project, if the first empirical results were promising. In the first stage we tried to keep the number of variables down to the minimum necessary to get a test of the practical usefullness of our approach.

As in the "new" approach to consumer theory developed by Morishima (1973) and Lancaster (1966), the commodities people buy in the market are inputs into the consumption activities which enter the utility function. For all but two of the activities we shall assume proportionality between commodity inputs and activity levels. We can then aggregate these commodities into composite commodities whose quantities are measured by the activity levels. Thus for these commodities we can write

$$y_i = x_{j(i)} \qquad i=1,2,\ldots,L \qquad i \neq e,t \tag{2}$$

where x_j is the quantity of the composite commodity; used as input in consumption activity i. We presuppose a list, such as that in table 1, telling which composite commodity corresponds to which activity (or, in other words, which index j corresponds to which index i). This list defines the function j(i).

The two exceptional activities are called "energy" (index e) and "transport" (index t). By "energy" we mean the activity of using energy for any stationary purpose in the household (such as space heating, water heating, cooking etc.). Inputs in the energy activity are the (composite) commodities electricity (el) and other fuels (of). These are combined in a production function which may contain substitution possibilities:

$$(3) \qquad y_e = f_k(x_{el}, x_{of}) \qquad\qquad k=1,2,\ldots,K$$

The index k tells us which energy technology the household has got. There are K different energy technologies available and each household applies one of them.

Ideally it would have been preferable to distinguish between several energy using activities (e.g. space heating, water heating, cooking etc.) specifying separate production functions for each activity. All the activities might then be assumed to make some contribution to space heating. However, data limitations forced us to merge everything into one activity. The grouping according to technology only takes account of differences related to space heating.

Before we proceed it should be noted that the commodity Other fuels consists mainly of heating oils. Wood also has a significant share, while coal, coke, and gas have been wholly insignificant over the observation period in this study, and are expected to remain so in the future.

Transport is produced with three inputs: Public transport, x_{pu}, private cars, x_I, and operation and maintenance of cars, x_{pr}. x_I is a dummy taking on the value 0 if the household has no car, and 1 if it has one or more cars. Fuel consumption is a part of the composite commodity operation and maintenance of cars. The production function is

$$(4) \qquad y_t = \varphi(x_I, x_{pr}, x_{pu})$$

Again we are guilty of a very crude simplification. The transport technology should not just depend on whether or not the household has a car, but also on how many cars it has. What we are actually going to do, is to neglect the possibility that a household can have more than one car.

We have neglected the differences between different models of automobiles. This means that we cannot trace in our model how increasing fuel prices induce people to choose cars with a lower specific fuel consumption. This choice was studied by Verleeger and Sheehan (1976) and by Crow and Ratchford (1977).

The budget constraint of the household is

$$(5) \qquad \sum_{i=1}^{I} q_i x_i = C$$

I is the total number of (composite) commodities included in our study. C is total consumption expenditure. The q_i's $(i=1,2,\ldots,I-1)$ are ordinary commodity prices. q_I is the imputed rental price of having a car for the decision period we imagine (in practice one year). For a justification of the use of the rental price of a durable good in complete systems of demand functions, cf. Diewert (1974b).

The commodity prices and total consumption are given exogenously. The consumer maximizes (1) subject to (2)-(5) and the restriction that x_I can only take on the values 0 and 1. It is convenient, however, to divide the maximization process into several stages.

First we notice that by substituting (2), (3) and (4) into (1) we can write the utility function in terms of commodities instead of in terms of activity levels. In this utility function (x_{el}, x_{of}) and (x_I, x_{pr}, x_{pu}) appear as two weakly separable groups of commodities, cf. Katzner (1970) or Deaton and Muellbauer (1980). The activity levels y_e and y_t are unobservable. The reason why we chose to formulate the model in terms of production functions instead of in terms of a separable utility function, was that the differences in f_k between households are best interpreted as differences in technology and not in preferences. The same applies to the differences in the transport technology (4) dependent on whether $x_I = 0$ or $x_I = 1$.

Bearing in mind that our model has an equivalent formulation in terms of a separable utility function, we know, e.g. from Katzner (1970) or Deaton and Muellbauer (1980), that if the functions f_k and φ are homothetic, the choice of the optimal commodity bundle can be divided into two stages. Since x_I is discrete, φ cannot be homothetic. But we shall make the following assumption:

(6) f_k is homogeneous of degree one for all k and φ is homogeneous of degree one in x_{pr} and x_{pu} both when $x_I = 0$ and $x_I = 1$.

If we take the value of x_I as given, the utility function defined over commodities appears with (x_{el}, x_{of}) and (x_{pr}, x_{pu}) as two homogeneously separable groups. Thus the maximization of utility can be conceived of as taking place in the following stages:

1. Recall that with a homogeneous production function average and marginal costs are equal. Given k we can find the combination of x_{el} and x_{of} which minimizes unit costs, and we can define a price of the energy activity, p_e, which equals this unit cost:

(7) $$p_e = p_e(q_{el}, q_{of}, k)$$

Similarly, when x_I is given, we can find the combination of x_{pr} and x_{pu} which minimizes unit <u>variable</u> costs $(q_{pr}x_{pr} + q_{pu}x_{pu})$, and define the price of the transport activity, p_t, as equal to unit variable cost:

(8) $$p_t = p_t(q_{pr}, q_{pu}, x_I)$$

The cost functions in (7) and (8) are of course both homogeneous of degree one in the two commodity prices. For the other activities we can define prices

(9) $$p_i = q_{j(i)} \qquad i=1,2,\ldots,L \qquad i \neq e,t$$

where again we presuppose the same list of corresponding indices i and j as in (2)

2. Now (1) can be maximized given the budget equation

(10) $$\sum_{i=1}^{I-1} p_i y_i = C - q_I x_I$$

where x_I is given. The result will be demand functions for the activities

(11) $$y_i = y_i(p, C - q_I x_I, z) \qquad i=1,2,\ldots,L$$

where p, the vector of price levels for the activities, depends on x_I and k. Sometimes we shall write $p = p(k, x_I)$.

3. (11) can be substituted back into (1). Then we get the indirect utility function

(12) $U = v(p(k,x_I), C - q_I x_I, z)$

Now the value of this can be compared for $x_I = 0$ and $x_I = 1$, and the result of that comparison will determine whether the consumer is going to have a private car or not. The resulting value of x_I can be substituted back into (11) to yield the actual demand for each activity.

4. In step one we already determined the optimal factor proportions. Generally we can write the factor shares as

(13)
$$\frac{q_{el}x_{el}}{p_e y_e} = A_{el}(q_{el}, q_{of}, k)$$

$$\frac{q_{of}x_{of}}{p_e y_e} = A_{of}(q_{el}, q_{of}, k)$$

$$\frac{q_{pr}x_{pr}}{p_t y_t} = A_{pr}(q_{pr}, q_{pu}, x_I)$$

$$\frac{q_{pu}x_{pu}}{p_t y_t} = A_{pu}(q_{pr}, q_{pu}, x_I)$$

From these and from (2) we determine the commodity demands given the demands for the various activities.

Carrying out this four-step procedure is under assumption (6) equivalent to solving the maximization problem we originally posed.

Some readers will notice that in order to be able to divide the maximization in stages, it is sufficient to assume homotheticity instead of homogeneity of degree one. y_e and y_t are unobservable, and we are free to choose their scales of measurement. By the appropriate choice of scales we can always transform a homothetic function into one which is homogeneous of degree one. Assumption (6) is still somewhat stronger than the assumption we would get if we substituted "homothetic" for "homogeneous of degree one". This is because we are measuring the activity levels in the same way irrespective of k and x_I. An equivalent assumption to (6) would begin:

> All f_k can be written as <u>the same</u> monotonically increasing transformation of functions which are homogeneous of degree one......

A similar formulation would have to be given for φ. In addition to homotheticity, which is necessary for dividing the maximization into stages, we have assumed that the returns to scale follow the same pattern irrespective of which values are given for k and x_I. This part of the assumption is made to simplify the derivation of aggregate demand. For transport it seems easy to accept, while for energy it may be more dubious. The returns to scale may be different in heating with electricity and fossil fuels.

Another point should be noted about (4) and (8). Having a car will increase the output of the transport activity "produced" by a given combination of x_{pr} and x_{pu} (provided x_{pr} is positive). This means that it lowers the price of operating the transport activity, p_t. At the same time it will obviously lead to a change in factor proportions or a substitution from x_{pu} to x_{pr}. To see what kind of theory of demand for transport commodities this involves, it is instructive to think of what happens to x_I, x_{pu} and x_{pr} as expenditure increases.

At a low level of total expenditure the demand for transport will be low and the possible productivity gain from having a car cannot justify incurring the relatively high fixed costs. As total expenditure increases, demand for transport, i.e. public transport, will increase until the consumer reaches the point where the gains from having a car are just so large that he is indifferent between having and not having a car. If he buys a car, he will, because of the substitution effect, consume more of transport and less of other goods. When total consumption passes the critical point mentioned above, the level of the transport activity will thus make a jump upwards. At the same time the share of public transport in the total expenditure on x_{pu} and x_{pr} will become less than one. It is reasonable to expect a fall in the absolute level of x_{pu}, although this does not follow with a priori necessity from the model. When total expenditure increases further, both x_{pr} and x_{pu} will increase, while the proportion between them will remain unchanged. Thus, for a household which already owns a car, the Engel elasticities for the use of private and public transport will be equal.

The discussion above reveals the strong assumptions which underlie our model of transport demand. No utility is attributed to the mere possession of a car. If the value of travelling time increases with increasing total expenditure, this does not lead to any change in the proportion between the use of private and public transport for a household which has a car. (Since a specific distance may be performed faster by public transport than by private car, it is not clear in which direction we should expect an increased value of time to work.) Furthermore the composition of the composite commodity public transport will normally change when the household gets a car. Still the present model must be considered an advancement over earlier models, which treated purchases of automobiles as part of current consumption. Our model includes three of the most important characteristics of the demand for private transport: The durability of the automobile, the discrete choice element and the jump in transport demand when the consumer gets a car.

3. Aggregate demand

Most students of aggregate demand using the complete system approach have assumed a representative consumer. One of the distinguishing features of our model is that consumers differ in an essential way. We therefore have to start with the individual consumer and consider the aggregation problem explicitly.

First we assume that all consumers face identical commodity prices, q. The number of degree-days (z) clearly varies from consumer to consumer, but we shall make the simplification that everybody experiences the national average of the year. (With the functional forms chosen later this is anyhow a rather innocent assumption.) What varies from household to household is then total expenditure C and energy technology (the value of the index k).

The shares of consumers who possess the different heating technologies, m(k) k=1,2,...,K, are predetermined. By definition

$$(14)\qquad \sum_{k=1}^{K} m(k) = 1$$

The change in these shares from year to year has been the subject of a special study in Rinde (1979).

The distribution of total expenditure within the group of consumers with energy technology k is given by a density function

$$(15)\qquad f(C;\bar{C},\ q,\ k,\ \theta)$$

$\bar{C}$ is average consumption in the population:

$$(16)\qquad \bar{C} = \sum_{k=1}^{K} m(k)\int_{0}^{\infty} Cf(C;\bar{C},\ q,\ k,\ \theta)dC$$

θ is a vector of parameters of the expenditure distribution other than the mean $\bar{C}$.

Let the critical level of total expenditure, $C^*(k)$, $(k=1,2,\ldots,K)$ be the solution for C to the equation

$$(17) \qquad v(p(k,\ 0),\ C,\ z) = v(p(k,\ 1),\ C - q_I,\ z) \qquad k=1,2,\ldots,K$$

Then, assuming that transport is always a normal good, consumers with heating technology k and total expenditure below $C^*(k)$ will not have a car, while those with total expenditure above $C^*(k)$ will possess a car.

Thus, for each k the share of consumers who have a car is

$$(18) \qquad a_I^*(k) = \int_{C^*(k)}^{\infty} f(C,\bar{C},\ q,\ k,\ \theta)dC$$

and the aggregate expenditure share on fixed automobile costs is

$$(19) \qquad a_I = \frac{q_I}{\bar{C}} \sum_{k=1}^{K} m(k)\ a_I^*(k)$$

Our model of the aggregate demand for automobiles is in fact very close to that of Aitchison and Brown (1957), although they were only concerned with cross-sections and did not include relative price effects.

In order to find the expenditure shares for the other commodities, we also have to integrate over total expenditure and then aggregate over the different energy technologies. For consumers with technology k who don't have a car, the average level of activity i is:

$$(20) \qquad \bar{y}_i(k,0) = \frac{1}{1-a_I^*(k)} \int_0^{C^*(k)} y_i(p(k,0),\ C,\ z)f(C)dC$$

For those who do have a car it is

$$(21) \qquad \bar{y}_i(k,1) = \frac{1}{a_I^*(k)} \int_{C^*(k)}^{\infty} y_i(p(k,1),\ C - q_I,\ z)f(C)dC$$

Thus, for the commodities for which there is a one-to-one correspondence to activities, the aggregate budget share is

$$(22) \qquad a_{j(i)} = \frac{q_{j(i)}}{\bar{C}} \sum_{k=1}^{K} m(k)[(1-a_I^*(k))\bar{y}_i(k,0) + a_I^*(k)\bar{y}_i(k,1)]$$

$$i=1,2,\ldots,L \qquad\qquad i \neq e,t$$

For the four remaining commodities we get the more complicated expressions

$$a_{el} = \frac{1}{\bar{C}} \sum_{k=1}^{K} m(k) A_{el}(q_{el}, q_{of}, k) p_e(k) [(1-a_I^*(k)) \bar{y}_e(k,0) + a_I^*(k) \bar{y}_e(k,1)]$$

$$a_{of} = \frac{1}{\bar{C}} \sum_{k=1}^{K} m(k) A_{of}(q_{el}, q_{of}, k) p_e(k) [(1-a_I^*(k)) \bar{y}_e(k,0) + a_I^*(k) \bar{y}_e(k,1)]$$

(23)

$$a_{pr} = \frac{1}{\bar{C}} \sum_{k=1}^{K} m(k) [A_{pr}(q_{pr}, q_{pu}, 0) p_t(0) (1-a_I^*(k)) \bar{y}_t(k,0) + A_{pr}(q_{pr}, q_{pu}, 1) p_t(1) a_I^*(k) \bar{y}_t(k,1)]$$

$$a_{pu} = \frac{1}{\bar{C}} \sum_{k=1}^{K} m(k) [A_{pu}(q_{pr}, q_{pu}, 0) p_t(0) (1-a_I^*(k)) \bar{y}_t(k,0) + A_{pu}(q_{pr}, q_{pu}, 1) p_t(1) a_I^*(k) \bar{y}_t(k,1)]$$

We have now derived all the aggregate budget shares. Needless to say, they add to one since they have been derived by consistent aggregation from individual utility maximization subject to a budget constraint.

4. Functional forms

In order to estimate the demand system, we have to assume explicit functional forms. We shall follow the duality approach as surveyed by Diewert (1974a) and Lau (1977). This means that we will choose functional forms for the indirect utility function (12) and the "cost" functions (7) and (8). In addition we shall need to assume explicitly a functional form for the distribution of total expenditure, (15). The demand functions (11), are easily derived from (12) by Roy's identity, while the factor share functions can be similarly derived from (7) and (8) by Shephard's lemma. We can also determine the critical level of total expenditure for the decision on automobile ownership. Given the functional form of the distribution function we can then by explicit integration derive the aggregate demand. We do not require that the aggregate demand functions can be expressed in a closed form.

Since it is quite straightforward to derive demand functions from any differentiable indirect utility function, it is easy to suggest new functional forms, and the literature is full of them, cf. the surveys referred to above, and Aasness and Rødseth (1981). Still it has been rather difficult to find a functional form which is suitable for our purposes.

We want to use the model for long-term planning purposes. Several indirect utility functions which can be regarded as second order approximations to any indirect utility function, have been proposed. The generality of these functions means that they can yield a wide range of different demand functions. However, the experiences with these demand functions are mixed. Estimation has often given demand elasticities which are in strong conflict both with budget surveys and partial studies of the demand for single goods. For example Jorgenson (1974) got a negative Engel elasticity and a positive direct price elasticity for energy within the sample period. Moreover, for these systems no restrictions on the parameters can be found that prevent the indirect utility function from being quasi-concave for some values of prices and total expenditure. This means that for some values of prices and total expenditure these functions will predict a behaviour inconsistent with utility maximization under a linear budget constraint.

To make long run projections from a demand system which is too flexible, seems to be a most dangerous procedure. In addition to the arguments above, consider also the following: Suppose we estimate a high order polynomial between consumption and income with observations over a certain time period. If we then make a projection to a level of income twice as high as the highest level observed, high order terms will increase enormously in importance, and depending on small errors in these, the predicted level of consumption can be anything. The conclusion is that for our purpose we should choose a functional form that is not too flexible. This means that the predicted long-term development will to a large extent be influenced by our choice of functional form. We have to recognize that what we can learn about long-term development from a relatively short period of observations, is not much, and that the interpretation of this information requires strong assumptions.

Another important consideration is aggregation. (18), (20) and (21) require us to compute the value of 21 integrals for each k. Computing integrals which cannot be represented in a closed form is a very costly affair even on an advanced computer. Therefore it was imperative to find a functional form where the commodity specific terms could be put outside integral signs, since then the number of integrals necessary to calculate can be reduced greatly.

These considerations lead us to choose the indirect utility functions of the quadratic expenditure system proposed in Pollak and Wales (1978):

$$(24) \qquad U = \frac{-\Pi\ p_i^{\alpha_i}}{C-q_I x_I-\Sigma\ p_i\beta_i} - \lambda\ \frac{\Pi\ p_i^{\alpha_i}}{\Pi\ p_i^{\nu_i}}$$

where

$$\Sigma\alpha_i = \Sigma\nu_i = 1$$

and

$$\beta_e = \bar{\beta}_e + \beta'_e\ z$$

The effect of a change in the number of heating degree-days thus works through the parameter β_e.

In order to assure that (17) has a unique solution we assumed $\nu_t = \alpha_t$.

The expenditure on each activity (for the transport activity: exclusive of fixed automobile costs) which results from (24) is:

$$(25) \qquad \begin{aligned} p_i y_i = {} & p_i\beta_i + \alpha_i(C-q_I x_I-\Sigma p_j\beta_j) \\ & + (\alpha_i-\nu_i)\lambda\Pi\ p_j^{-\nu_j}(C-q_I x_I-\Sigma p_j\beta_j)^2 \qquad i=1,2,\ldots,L \end{aligned}$$

One can easily verify that for $\lambda = 0$ we get the linear expenditure system. We considered it a great advantage to have this as a special case, because there is no problem in delimiting the area where it can be valid. Its simplicity and ease of interpretation made it very useful while we were testing out the working of the more exotic parts of the model. But it was considered too restrictive to be the only case studied. Especially we believed it important to allow for more flexible Engel curves in a model which is going to be used for long-term projections. The quadratic expenditure system allows for greatly increased flexibility in the Engel curves, while it is economic in the use of extra parameters. In Aasness and Rødseth (1981) it was confirmed that its Engel curves may fit observed behaviour in cross sections quite well. But the flexibility also means that we have to tolerate some of the problems mentioned above.

Flexible Engel curves and few parameters mean that the system is rather restrictive with respect to the substitution effects. This should be born in mind if the model is used to study the effects of alternative pricing policies for energy. As an afterthought it seems that we put too little emphasis on modelling the substitution possibilities appropriately.

In the linear expenditure system a consumer with a low price of energy, p_e, will have the same marginal budget share on energy as a consumer facing a higher price. This is quite unreasonable. In the quadratic expenditure system the marginal budget share will change, but not much. MacKinnon (1976) estimated another generalization of the linear expenditure system, which he called the S-branch system. That system is better suited for investigating substitution possibilities, but retains the assumption of linear Engel curves. A combination of the two systems may easily be obtained. Basically it would consist in making the α's in (24) specific functions of prices. However, the amount of computation work required have prevented us from incorporating this generalization.

The elasticities of substitution in (7) and (8) are of primary interest in our study. At the same time it is important for the functioning of the model and the algorithms used in its estimation that these functions are well behaved. Therefore we chose to represent the technologies by CES cost functions. Thus

$$(26) \qquad p_e(k) = \gamma_e(k) \left[\delta_e(k)\, q_{el}^{\,1-\sigma_e(k)} + (1-\delta_e(k))\, q_{of}^{\,1-\sigma_e(k)} \right]^{\frac{1}{1-\sigma_e(k)}}$$

$$(27) \qquad p_t = \begin{cases} q_{pu} & \text{for } x_I = 0 \\ \gamma_t \left[\delta_t\, q_{pu}^{\,1-\sigma_t} + (1-\delta_t)\, q_{pr}^{\,1-\sigma_t} \right]^{\frac{1}{1-\sigma_t}} & \text{for } x_I = 1 \end{cases}$$

$\sigma_e(k)$ and σ_t are the elasticities of substitution.

Regarding the functional form of the expenditure distribution (15) the lognormal distribution was a natural choice, since it has fitted actual distributions of total expenditure in cross sections quite well. Since the utility function (24) is only defined for levels of total expenditure above the "minimum consumption" $\Sigma p_i(k,0)\beta_i$, it is total expenditure in excess of this level which is assumed to be lognormally distributed.

When (25) is substituted into (20) and (21), we see that for each k we shall solve three integrals:

i) $\int_0^{C^*(k)} (C-\Sigma p_i\beta_i) f(C)dC$

ii) $\int_0^{C^*(k)} (C-\Sigma p_i\beta_i)^2 f(C)dC$

$$\text{iii)} \int_{C^*(k)}^{\infty} (C - q_I - \Sigma p_i \beta_i)^2 f(C)dC$$

(The fourth integral complementing i) can be derived directly from i) and (16) without using an integration algorithm.) In addition we have to solve the integrals in (16). This gives a total of 4·K integrals to be solved, and unfortunately the cost of computing all these integrals turned out to be too high at this stage of the project. Therefore a rather artificial simplifying assumption about the expenditure distribution between consumers with different heating technologies was adopted. As we know, the lognormal distribution has two parameters, μ and σ, which are, respectively, the expected mean and standard deviation of the logarithm of the variable in question, $C - \Sigma p_i \beta_i$. σ is assumed to be known a priori and equal for all k. μ is also assumed to be equal for all k and given by

$$(28) \qquad \mu = \ln[\bar{C} - \sum_{k=1}^{K} m(k) \sum_{i=1}^{L} \beta_i p_i(k,0)] - \frac{1}{2}\sigma^2$$

This means that average consumption in excess of the minimum level will be the same irrespective of k. One can check that this is consistent with average total expenditure taken over all consumers being $\bar{C}$. With assumption (28) the number of integrals which we need to evaluate is reduced from 4·K to 4.

(28) is very close to assuming that average total expenditure is independent of k. With the chosen classification this need not be very far from the truth. The problem connected with the assumption (28) is that when the costs of different heating technologies move differently, this is partly compensated by a change in the distribution of total expenditure towards groups with the most rapid increase in costs. Theoretically some change in the expenditure distribution may be conceived of as taking place through an effect of relative prices on saving rates. But this is outside the scope of our model, and we are running the risk of getting some bias in the estimates of the price elasticities of electricity and other fuels.

The demand system which comes out of our model may seem very complicated. The complete equation system, as it has been programmed, is given in Rødseth (1983), appendix 1. Simplicity is of course a desirable feature of econometric models. But what we, in my opinion, should strive at, is simplicity in the basic assumptions. If the predictive equations which result from these assumptions are complicated, that should not cause too much concern. So it is in other sciences, and so it should be also in economics. But, of course, even in the age of the computer, we have to worry about how our models can be computationally implemented and estimated. These are the questions to which we now turn.

5. Data

From the very start of the project of developing the MSG-4 model it was clear that the definitions of price and quantity variables as well as the classification of commodities had to be in accordance with the national accounts of Norway. Within the national accounting framework the commodity classification of consumption actually chosen for the MSG-4 model, was a compromise between the wishes expressed by those who worked on different parts of the model, by the potential model users, and the need for co-ordination with the other models of the Central Bureau of Statistics. In order to reduce the computational burden some of the consumption activities of MSG were merged together in the more experimental sectoral model of consumption, which we are describing here. A list of consumption activities and commodities of the present model is given in table 1. The relationships to MSG-4 and the national accounts are given in Rødseth (1983), table 1 of appendix 2.

Originally the intention was to use micro data for the estimation of as many as possible of the parameters of the model. However, this had to be given up mainly because the Consumer Surveys contain no information on heating technology. Also, households reporting that they had no car turned out to have fairly large average expenditure on operation and maintenance of automobiles! Therefore, we decided to rely entirely on micro data, mainly the national accounts. A detailed description of the data is given in Rødseth (1983), appendix 2. The observations are for the 17 years from 1962 to 1978. This was at the time the longest series of consistent and final national accounts figures which included the seventies, we could get.

On some points the data are clearly not satisfactory. Fixed automobile costs are defined as interest plus depreciation, which is a constant fraction of the value of a new car each year, minus the gain from the price increases on new cars. In reality depreciation will depend on distance driven, while some of the expenditure on Operation and maintenance, e.g. part of insurance cost, are fixed. To improve on this within the national accounting framework will require considerable efforts of work. Also in a more thorough study the commodity Public transport, postal and telephone services should be split into at least three categories: the commodities which are only distant substitutes for the private car (postal- and telephone services etc.), those which may be rather

Table 1. Commodities and activities

Commodity		Activity	
1.	Food	1.	Food
2.	Beverages and tobacco	2.	Beverages and tobacco
3.	Clothing and footwear	3.	Clothing and footwear
4.	Rent	4.	Rent
5.	Electricity	5.	Energy
6.	Other fuels		
7.	Durable household goods	6.	Durable household goods
8.	Other household goods	7.	Other household goods
9.	Private transport (operation and maintenance)	8.	Transport
10.	Public transport		
11.	Leisure and education	9.	Leisure and education
12.	Other goods and services	10.	Other goods and services
13.	Ownership of private cars	8.	Transport

close substitutes (e.g. railroad travel), and automobile ferries, which in Norway are mainly complementary to the private car.

Our measure of the rental price of automobiles is very crude, as it probably has to be in any aggregate study, since people are facing very different after-tax real interest rates. In the calculations we used the interest rates on the highest yielding regular time deposits in commercial banks and measured expected inflation rates with a three years moving average of actual inflation rates. No corrections were made for taxes. This certainly biased the measure of the rental price upwards. On the other hand some bank deposits were exempted from taxes on interest, and many consumers also had other investment opportunities e.g. in housing, which during the whole period probably yielded an after-tax real interest rate above the before-tax rate on time deposits. Some consumers were exposed to credit rationing. It is not possible to determine the direction of the overall bias, and one should be aware that the figures are very uncertain. Looking at the calculated series for the rental price, one will observe that towards the end of the sixties the rental price tended to fall because nominal interest rates lagged much behind the increase in inflation rates. In the seventies nominal interest rates started to catch up again making the average increases in the rental price not far from the general inflation rate. At the same time large fluctuations in the price increase on new cars, fluctuations mainly due to exchange rate instability, caused large year to year fluctuations in the rental price.

The households were classified into four categories according to energy technology:

1. Homes with electric heating only in houses with no more than four dwelling units.
2. Other homes in houses with no more than four dwelling units.
3. Homes with electric heating only in larger houses.
4. Other homes in larger houses.

The index k has to take on one of these four values. This was the most detailed breakdown for which we could obtain reasonably reliable time series data.

Electricity is in Norway sold to the households on several different tariffs, some of which are rather complicated. The price index for electricity in the national accounts is computed on the basis of these tariffs assuming some standard consumption patterns. Thus they do not represent average prices per kWh.

The parameter σ of the lognormal distribution was set to 1.0 on the basis of some preliminary calculations on Norwegian Consumer Surveys. Although it can hardly be very far off the mark, it would be desirable to investigate the sensitivity of the results to differing assumptions about σ.

6. Estimation

We assume that formulas (22) and (23) represent the conditional expectations of the commodity budget shares given the commodity prices, total expenditure, temperature and the distribution of heating technologies. We can write the equations for the budget shares with additive error terms:

$$(29) \qquad a_i^!(\tau) = a_i(\tau) + u_i(\tau) \qquad \tau=1,2,\ldots,T; \quad i=1,2,\ldots,I$$

$a_i^!(\tau)$ is the observed budget share of commodity i at time τ while $a_i(\tau)$ is the corresponding expected budget share. $u_i(\tau)$ is the error term, which has a conditional expectation of zero. Furthermore, we assume that the residuals $u_1(\tau),\ldots,u_I(\tau)$ are normally distributed with a constant contemporaneous variance-covariance matrix Ω, and no correlation between error terms for different years.

Since both the expected and average budget shares for the different commodities add up to unity, the errors will have to add up to zero:

$$(30) \qquad \sum_{i=1}^{I} u_i(\tau) = 0 \qquad\qquad \tau=1,2,\ldots,T$$

This means that the variance-covariance matrix Ω is singular and the joint density of the errors is not defined. But, as shown in Barten (1969), the joint densities for each subset of I-1 of the errors can in general be defined, and they will all be identical. Thus, without introducing any arbitrariness, we can drop out the equation for one of the budget shares and proceed to find maximum likelihood estimates by maximizing the joint density of the remaining I-1 commodities.

The parameters to be estimated are first those of the utility function: 9 α's, when one is eliminated by using the adding up restriction, 8 ν's, again using the adding up restriction and the assumption that $\alpha_t = \nu_t$, 11 β's and 1 λ. The cost function for transport contains 3 parameters. The cost functions for energy contain $3 \cdot K = 12$ parameters. However, since two of the household groups do not use other fuels than electricity, we can set $\delta_e(1) = \delta_e(3) = 1$ and $\sigma_e(1) = \sigma_e(3) = 0$. Furthermore we have to fix the scale by which we measure the energy good. That is done by setting $\gamma_e(1) = 1$. One normalization of this kind is necessary in order to make all the other parameters identifiable. Still 7 parameters remain for which the main information on their values has to come from the 17 observations on the distribution of energy expenditures between electricity and other fuels. This gave, we decided, a too small "number of degrees of freedom" in that area. Therefore we introduced the further simplifying assumptions that $\delta_e(2) = \delta_e(4) = \delta_e$, that $\sigma_e(2) = \sigma_e(4) = \sigma_e$, and that $\gamma_e(4) = \gamma_e(2) \cdot \gamma_e(3)$. This reduced the number of independent parameters to estimate to 4 in the cost functions for energy and a total of 36 for the whole model.

The equation which is dropped out contains no extra information which is not included in the observations on the other equations. Thus, with 17 observations on each of the remaining 12 equations we have a total of 204 observations and 168 degrees of freedom. In principle this should be enough for carrying out maximum likelihood estimation, cf. Dhrymes (1974), without further restrictions on the parameters. The estimates could then be calculated by minimizing the determinant of the matrix of the variances and covariances of the residuals from the estimation. This procedure breaks down, however, because, when we take the whole structure of the model into account, as many as 28 of the unknown parameters appear in most of the equations. With only 17 observations on each, all the residuals in one of these equations can be made equal to zero. Then the determinant of the variance-covariance matrix of the residuals is also zero, and it has several such minima at zero. Therefore, maximum likelihood estimation has no good meaning unless we place some restrictions on Ω.

The restrictions we chose were first applied by Deaton (1975). We assume that the variance-covariance matrix of the residuals has the following structure:

$$(31)\qquad \Omega = \sigma^2 \begin{bmatrix} \bar{a}_1 - \bar{a}_1^2 & -\bar{a}_1\bar{a}_2 & \cdots & -\bar{a}_1\bar{a}_I \\ -\bar{a}_2\bar{a}_1 & \bar{a}_2 - \bar{a}_2^2 & \cdots & -\bar{a}_2\bar{a}_I \\ \vdots & \vdots & & \vdots \\ -\bar{a}_I\bar{a}_1 & -\bar{a}_I\bar{a}_2 & & \bar{a}_I - \bar{a}_I^2 \end{bmatrix} = \sigma^2\,\Omega_o$$

where $\bar{a}_i = \frac{1}{T}\sum_{\tau=1}^{T} a_i^!(\tau) \qquad i=1,2,\ldots,I.$

This means that the covariances between two error terms are always negative and proportional to the product of the sample means of the budget shares of the corresponding commodities. The variances are roughly proportional to the sample means of the budget shares. The proportional factor σ^2 is the only unknown parameter in Ω. Define:

$$(32)\qquad V_o = \Omega_o + ii'$$

where i is an I-vector where each element is $1/\sqrt{I}$. Then Deaton (1975) has showed that the maximum likelihood estimators of the parameters in the demand system can be found by minimizing:

$$(33)\qquad \tilde{\sigma}^2 = \frac{1}{T(I-1)}\sum_{i=1}^{T} (a_i^!(\tau)-a_i(\tau))V_o^{-1}(a_i^!(\tau)-a_i(\tau))$$

The minimum value of this expression also provides the maximum likelihood estimate of σ^2.

The minimization of (33) with respect to the 36 unknown parameters has to be carried out numerically. With the complexities of the model this is a formidable task both for the computer and the programmer. Considerable run time was saved in the subprogram computing $\tilde{\sigma}$ by allowing the results from the previous call on that subprogram to be used over again in those parts of the program where no parameter values had changed, e.g.: The integrals i), ii) and iii) defined in section 4 depend neither on the α's (except α_t) nor on the ν's or λ. In calculating numerical approximations to the partial derivatives with respect to these parameters, the integrals are unchanged, and it is unnecessary to calculate them over again every time. This is avoided by the program.

It turned out that the program did not converge in a reasonable interval of time. The problem was mainly due to the two parameters $\gamma_e(2)$ and $\gamma_e(3)$, which soon went out of the region of reasonable values. With $\lambda=0$, convergence was obtained, but with unreasonable estimates of the two parameters. $\gamma_e(2)$ and $\gamma_e(3)$ are ratios between the base year energy costs of household group 2 and 3 respectively and group 1. Thus we could know fairly well that values around 5 or 0.05 were unreasonable. Therefore, we chose to use some extraneous information to fix $\gamma_e(2)$ and $\gamma_e(3)$. On the basis of some calculations in NOU (1975) $\gamma_e(3)$ was set equal to 0.625. The same source claimed that electric heating and heating with other fuels gave on average roughly the same costs in 1975. This lead us to impose the condition that

$$(34) \qquad p_e(q_{el}, q_{of}, 1, 1975) = p_e(q_{el}, q_{of}, 2, 1975)$$

This was used to eliminate $\gamma_e(2)$. The number of unknowns was then reduced to 34, and convergence was obtained.

7. Numerical results

The parameter estimates are given in table 2 both for the full "quadratic" model and for the more restricted model we get by imposing $\lambda=0$. Judged by the usual χ^2-test the improvement in goodness of fit by going from the linear to the quadratic expenditure system is considerable. The hypothesis that $\lambda=0$ is rejected even at the 0.001 level of significance. In table 3 the residual variance is examined equation by equation. Apparently the explanatory power of the model is fairly good for most commodities. Naturally the quadratic expenditure system leads to the greatest improvements in goodness of fit where the explanatory power of the linear expenditure system is the lowest. Judged from the coefficients of correlation, both systems seem to perform particularly badly for the

Table 2. Parameter estimates

Parameter	Linear	Quadratic
α_1	0.018	0.053
α_2	0.078	0.102
α_3	0.028	0.063
α_4	0.120	0.077
α_5	0.083	0.051
α_6	0.118	0.102
α_7	0.045	0.015
α_8	0.191	0.174
α_9	0.191	0.172
β_1	7.855	7.436
β_2	1.364	0.987
β_3	2.955	2.548
β_4	1.701	1.994
β_5'	0.533	0.924
$\bar{\beta}_5$	-0.494	-0.694
β_6	0.035	0.038
β_7	0.433	0.725
β_8	-0.237	-0.307
β_9	-0.172	-0.245
β_{10}	1.377	0.342
δ_e	0.604	0.593
σ_e	0.673	0.843
γ_t	0.156	0.161
δ_t	0.536	0.532
σ_t	1.164	1.317
$\lambda \cdot 1000$	.	0.097
ν_1	.	3.690
ν_2	.	2.712
ν_3	.	3.914
ν_4	.	-4.316
ν_5	.	-4.543
ν_6	.	-1.398
ν_7	.	-2.290
ν_9	0.156	-1.110
σ^2	0.00153	0.00117

two commodities Beverages and tobacco and Public transport. In the first case this may have been caused by the changing preferences for smoking. In the second case the budget share has been virtually constant through the whole period of observation. Even though the residual variance is larger than the gross variance, they are both very small.

Looking at the economic content of the results, the picture is not so bright. The quadratic expenditure system gives some positive direct Slutsky elasticities even in the middle of the sample; a clear sign that the indirect utility function is not quasi-convex. Also in other respects the computed elasticities are quite unreasonable.

Of course one cannot tell exactly why the results were such a failure, but one may have suspicions. My chief suspicion is the following: The quadratic expenditure system, which allow very flexible Engel curves, may be well suited for estimation in a sample which shows great variation in real income and small variation in relative prices. The system's strong interconnection between income and substitution effects will then mean that the substitution effects are mainly determined by the income effects. If, as in our sample, we have more variation also in real prices, it is revealed that the structure is too restrictive, and this results in biased estimates of both Slutsky and Engel elasticities.

In order to expose the ideas of the present study more fully and in order to show that they can be made to work, we have chosen to present some of the demand elasticities computed from the "linear" version of the model. Because of the restrictiveness of the assumptions implicit in the linear expenditure system, the numerical values of the elasticities should not be taken too seriously. The results based on the linear expenditure system look better than those based on the quadratic specification, just because the linear expenditure system is so restrictive that there is less which can go wrong.

The most interesting elasticities are given in tables 4-6. Compared to Aasness and Rødseth (1981) it seems that the Engel elasticities in the present study tend to be biased away from 1. Since we are primarily interested in the transport and energy activities, we shall confine the further comments to the commodities involved in these activities.

The estimated Engel elasticities for electricity and other fuels are considerably higher than in all other studies I know of, compare the surveys referred to in the introduction and the estimate of 1.10 for 1970 in Rødseth and Strøm (1976). The estimates of the direct price elasticities are also high in absolute values, around 0.8-0.9, compared to about

Table 3. Analysis of variance

Commodity no.	Average budget share	Standard deviation	Linear model: Residual variance	Linear model: Explained var. relative to total var.	Quadratic model: Residual variance	Quadratic model: Explained var. relative to total var.
1	0.2547	0.0161	$1.38\cdot10^{-5}$	0.947	$8.21\cdot10^{-6}$	0.997
2	0.0813	0.0034	$1.18\cdot10^{-5}$	-0.035	$8.31\cdot10^{-6}$	0.271
3	0.1087	0.0087	$1.27\cdot10^{-5}$	0.831	$8.89\cdot10^{-6}$	0.882
4	0.1104	0.0034	$7.49\cdot10^{-6}$	0.349	$2.78\cdot10^{-6}$	0.758
5	0.0277	0.0038	$2.52\cdot10^{-6}$	0.821	$2.39\cdot10^{-6}$	0.901
6	0.0121	0.0011	$1.00\cdot10^{-6}$	0.174	$4.65\cdot10^{-7}$	0.616
7	0.0567	0.0043	$3.15\cdot10^{-6}$	0.831	$2.64\cdot10^{-6}$	0.858
8	0.0353	0.0026	$1.26\cdot10^{-6}$	0.815	$4.30\cdot10^{-7}$	0.937
9	0.0394	0.0072	$8.33\cdot10^{-6}$	0.838	$7.75\cdot10^{-6}$	0.849
10	0.0462	0.0009	$1.32\cdot10^{-6}$	-0.559	$9.78\cdot10^{-7}$	-0.155
11	0.0843	0.0073	$3.44\cdot10^{-6}$	0.935	$2.70\cdot10^{-6}$	0.949
12	0.1031	0.0047	$1.23\cdot10^{-5}$	0.448	$7.81\cdot10^{-6}$	0.650
13	0.0404	0.0045	$9.64\cdot10^{-6}$	0.518	$1.00\cdot10^{-5}$	0.500

0.3 in Rødseth and Strøm (1976). In table 5 the price elasticities for the different consumer groups are given. The elasticities are computed for group total expenditure equal to the total population average expenditure in 1970. Surprisingly electricity demand is more elastic for those who only use electric heating than for those who have alternatives. An increase in q_{el} leads to a larger increase in p_e in groups 1 and 3 than in groups 2 and 4. Thus we get relatively stronger income effects and more substitution away from energy towards other goods in the former groups than in the latter. According to the estimates the income effects more than offset the small substitution effect we get between electricity and other fuels in groups 2 and 4. This effect is small because the estimate of σ_e is fairly low. The results actually say there is little scope for substitution between fuels but considerable substitution possibilities between energy and other goods, which is contrary to common beliefs. One should remember though, that while the former result probably is inherent in the data, the latter could be merely a consequence of our choice of functional form.

The cross-price elasticities are negative. This is true also for the compensated elasticities. The reason is again that a price increase causes much substitution away from energy and less substitution between fuels.

Looking at the results for transport we see how the Engel elasticity for having a car declines as that market is approaching saturation. Also the Engel elasticity for using the car is very high to begin with and falls sharply when the share of the population possessing a car grows. The Engel elasticity for public transport first increases and then falls. From the model it follows that the elasticity must be relatively low when the share of automobile owners increases most rapidly. After that period it increases, but falls again because in the linear expenditure system all Engel elasticities must converge towards 1 when real income grows.

From table 4 we can also notice that as the critical level, C*(k), moves into the lower tail of the distribution of total expenditure, the direct price elasticity for having a car declines considerably. The direct price elasticities for use of the two modes of transport remain high, though. That is partly due to the relatively high elasticity of substitution between the two transport commodities. With more people already possessing a car, this kind of substitution becomes more important. The cross-price elasticities in table 6 all show the expected

Table 4. Estimated Engel and direct price elasticities from the linear model

Commodity no.	Engel elasticity			Direct price elasticity		
	1962	1970	1978	1962	1970	1978
1	0.058	0.068	0.076	-0.043	-0.051	-0.058
2	0.849	0.929	0.924	-0.436	-0.504	-0.539
3	0.207	0.253	0.274	-0.119	-0.150	-0.172
4	0.937	1.031	1.073	-0.495	-0.570	-0.631
5	2.092	1.953	1.794	-0.884	-0.900	-0.912
6	2.093	1.954	1.794	-0.794	-0.794	-0.800
7	2.093	1.964	1.814	-0.974	-0.983	-0.987
8	1.224	1.224	1.145	-0.586	-0.628	-0.638
9	3.595	2.435	2.185	-1.191	-1.113	-1.107
10	1.751	1.852	1.700	-1.235	-1.166	-1.153
11	2.344	2.131	1.926	-1.073	-1.052	-1.037
12	1.173	1.164	1.124	-0.596	-0.632	-0.658
13	2.691	1.196	1.007	-1.175	-0.573	-0.530

Table 5. Energy price elasticities 1970

Commodity	Price	All	Consumer group			
			1	2	3	4
Electricity	Electricity	-0.900	-0.972	-0.857	-0.982	-0.862
Electricity	Other fuels	-0.077	*	-0.119	*	-0.123
Other fuels	Electricity	-0.183	*	-0.182	*	-0.187
Other fuels	Other fuels	-0.794	*	-0.794	*	-0.797

Table 6. Cross-price elasticities for the transport commodities

Commodity	Price	1962	1970	1978
Use of private t.	Use of public t.	0.184	0.100	0.096
" " " "	Automobiles	-0.590	-0.226	-0.205
Use of public t.	Use of privat t.	0.138	0.117	0.115
" " " "	Automobiles	0.313	0.108	0.093
Automobiles	Use of public t.	-0.297	-0.145	-0.131
"	Use of private t.	0.263	0.097	0.090

signs. We notice the strong decline in the effect of the rental price of automobiles on the use of the two modes of transport.

In the long run the distribution of the households on the four different categories for energy technology is endogenous. Changes in the shares, m(k), may have fairly strong effects on the demands for electricity and other fuels, while the effects on the demands for other goods are negligible. If 1 per cent of the population were moved from group 2 to group 1, the consumption of electricity in 1970 would increase by 0.6 per cent while the consumption of other fuels would be reduced by 1.3 per cent. A move from group 4 to group 1 gives roughly the same result. If 1 per cent of the population were moved from group 3 to group 1, the electricity consumption would increase with only 0.02 per cent. The reason why the increase in consumption is so small, in spite of the fairly large difference in efficiency ($\gamma(k)$) assumed, is that the more efficient households choose to have a much higher energy standard. This is result of the marginal budget share for energy (in this case electricity only) being constant by assumption and the estimate of $\bar{\beta}_e + \beta'_e$ being close to zero. Again the functional form has a rather unreasonable implication.

In 1970 21 per cent of the apartments in small houses (no more than four dwelling units) had only electric heating, while the same percentage for larger houses was 39. At the same time the shares of electric heating only in new houses were 55 per cent and 61 per cent, respectively. If these had been the actual shares for the whole population in 1970, then, according to the model estimates, the electricity consumption would have been 18 per cent higher and the consumption of other fuels 42 per cent lower. This would then have been the long run equilibrium, i.e. the state we would reach if the prices and total expenditure from 1970 remained constant forever. Long run elasticities tell how this equilibrium is changed when prices or total expenditure changes. These can be computed by combining the elasticities in the present model with the elasticities from the model of Rinde (1979), which determine the shares of electric heating in new houses. The long run elasticity with respect to the price of electricity was in 1970 -1.33 for electricity and 1.66 for other fuels. The corresponding elasticities with respect to the price of other fuels were 0.21 and -1.96. We see that according to these estimates the scope for substitution between fuels is much larger in the long run than in the short run.

As emphasized earlier, the empirical results of this paper must be interpreted with a considerable amount of sceptisism. Thus the results cannot provide much help for practical planners at the present stage. The contribution from the project has so far been primarily in the methodological field. We have shown how important ideas from the partial studies of energy demand can be incorporated in a complete system framework. We have also been able to utilise macro data without assuming a representative consumer. Further work should be directed at finding more suitable functional forms, incorporating demographic effects, allowing for more than one car in each household, and improving the data.

XI. ANALYSIS OF ENERGY INTENSIVE INDUSTRIES - THE CASE OF NORWEGIAN ALUMINIUM PRODUCTION 1966 - 1978

by

Finn R. Førsund and Eilev S. Jansen*

CONTENTS

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* We are indepted to student of economics Kjell Rosanoff for his valuable research assistance in the process of establishing the data base for the aluminium industry. In writing this paper we have also to a certain extent drawn on his work, Rosanoff (1981), which he carried out under our supervision. We would also like to thank Hans J. Bakke and Øyvind Eitrheim of the University of Oslo for computational assistance.

1. The short-run industry production function approach

The basic production unit in operational national planning models is a sector. The MSG-4 model includes 27 industry production sectors, each of which is endowed with a neoclassical aggregate production function. The model is intended for long-run projections and the production functions are long-run functions in the sense that capital is a variable factor of production along with labour and energy. Furthermore, the production functions of the model are designed to meet two important ends:

- to capture the actual production structure and producer behaviour in the sector, and
- to provide an approximation to this structure which is simple enough to be estimated given the available data (and the state of the art in econometrics).

In the MSG-4 model one has specified a separable homothetic production function for each sector with constant elasticity to scale, approximated by Generalized Leontief unit cost functions with Hicks neutral technical change.[1] This specification has desirable properties from an operational point of view. In order to evaluate its merits on the first account, we shall go beyond the sector level and use a micro-econometric approach to obtain insight into the structure of an industry.

This approach is to establish short-run industry production functions from micro observations following ideas developed by Johansen (1972) and Førsund and Hjalmarsson (1980). The key assumption - which makes the industrial structure an interesting concept - is that substitution possibilities ex post are more restricted than ex ante for each firm. Long-run aspects - which are particularly relevant in our case - are revealed by comparing such industry production functions over a sufficiently long period of time.

The industry production function is derived from ex post micro production functions. For each micro unit the input coefficients of short-run variable factors are assumed to be fixed ex post and independent of the rate of capacity utilization. The production possibilities of the industry as a whole are spanned by maximizing total output for alternative total levels of current inputs subject to the capacity constraint and the ex post technology of each unit. The resulting numerical production function is

1) This specification is discussed by Longva and Olsen in chapter III of this volume.

the short-run macro function of Johansen (1972). Under the assumptions above, this industry production function results from solving simple linear programming problems.

The connection between a series of short-run industry production functions over time goes through the ex ante production functions of the micro units in which the fixed factors are also variable. The ex ante function indicates the possible combinations of input coefficients to choose from for a new micro unit to be established. It summarizes the relevant technological knowledge at the moment of investment, conceivably in the form of a traditional production function with continuous substitution possibilities. Each production unit in operation has been established at some point in time from the ex ante function then in existence. The short-run industry production function thus reflects both the history of ex ante functions over time and the actual choices made from these ex ante functions.

In this chapter we shall explore the Norwegian aluminium industry, using data from the annual Industrial Statistics for the years 1966 to 1978. In 1978 this industry contributed with 37 per cent of the gross value of production in the MSG-4 sector 43 Manufacture of metals. The industry is extremely energy intensive, and the technology is largely based on electric power. The latter means that the data do not offer opportunities to study substitution possibilities between different types of energy inputs.

We shall, however, demonstrate how structural change and technical progress can be revealed by utilizing the short run industry production function and we shall focus interest on the conclusions we may draw from this on the realism of the MSG-4 assumptions for the long-run production function expressed in terms of the aggregate inputs (capital, labour, energy and materials).

Moreover, the study of structural changes in an industry is interesting in its own right. We shall display the production structure at four year intervals and we shall also try to trace out possible effects of changes in the relative prices during the period of observation. Special attention will be given to the years 1972, 1974 and 1975, in order to analyze the economic performance of the industry over a business cycle.

2. The aluminium industry in Norway. The data

The estimation of an industry production function ideally requires a sample of micro units which are producing a homogeneous output using homogeneous inputs, and it is preferable that all quantities be measureable in physical terms. The common denominator of the Norwegian aluminium works is no doubt the process of smelting aluminium oxide into primary aluminium. (A brief description of the production technology of the aluminium industry in Norway is given in the appendix.) For practical reasons we have chosen also to consider the processes up to and including casting as part of our industry production concept.

Our primary data source is the Industrial Statistics of Norway for the years 1966 - 1978, see e.g. NOS (1979). The unit of observation is the establishment, i.e. the information refers to the totals for all the production activities which take place at the same site.[1)] We have compiled extraneous information directly from those aluminium works which have diversified their production, in order to obtain comparable data for the combined process of smelting, refining and casting of primary aluminium.

The analysis comprises 9 aluminium works. One of these was shut down, while two works were established during the period of observation. Moreover, there has been major expansions at different points of time in several works (see table 1).

Total annual production is measured by the total volume of pure primary aluminium produced during one year. This volume includes the primary aluminium used internally as input for further processing. Resmelting of scrapped aluminium is not included.

We have also compiled data for the production capacity of each plant. Table 1 shows the total output in the industry as compared to the total capacity.

New capacity is added to the existing capacity at the time it is completed,[2)] using a simple interpolation method to construct an average capacity in the year of expansion. Moreover, the capacity utilization does not necessarily reflect market conditions as the production figures include deliveries to inventories. From a data point of view this crops

1) Since each establishment consists of several series of smelting cells of different vintages, the ideal micro unit might have been one series of smelting cells. However, our data source, the Industrial Statistics, contains no information at that level of aggregation.

2) This does not necessarily coincide with the time for the start of production, due to changing business cycles, etc.

Table 1. Total production and production capacity for pure primary aluminium (in tonnes) in the Norwegian aluminium industry. 1966 - 1978

Year	Production capacity (K)	Actual production (X)	Capacity utilization (X/K)·100	Number of establishments
1966	368 000	330 287	90	7
1967	368 000	349 112	95	7
1968	484 500	468 288	97	8
1969	537 000	505 475	94	8
1970	545 500	522 307	96	8
1971	607 250	531 371	88	9
1972	652 750	551 191	84	9
1973	661 500	620 897	94	9
1974	661 500	644 737	98	9
1975	646 600	609 461	94	8
1976	673 500	617 556	92	8
1977	675 000	622 730	92	8
1978	702 700	639 041	91	8

up as a problem when one tries to find the value of the annual production.[1)]

Labour input is measured in terms of blue-collar man-hours. The number of non-production workers varies a lot between the establishments, mainly because some of them belong to conglomerate corporations with a centralized administration. Information collected directly from the plants has enabled us to find the labour input for the smelting, refining and casting activities.

As far as the energy input is concerned, we have chosen to disregard energy inputs other than electric power. We have not deducted the electricity required for resmelting of scrapped aluminium, nor the electricity used in processing beyond the casting of aluminium.[2)] This causes the volume of electricity used in smelting and casting to be somewhat overestimated in the plants involved in such secondary activities.

1) The Industrial Statistics questionnaire asks for an estimate of this value at the current market price, which we have used to construct an average net price of primary aluminium for the industry, see below.
2) The latter phenomenon affects one establishment only: Such further processing of primary aluminium into semi-finished products (like wires, rods and profiles) is - with one exception - not located at the same site as the aluminium plants.

Real capital is treated as a fixed factor in short-run analysis. Nevertheless, we have compiled data for the real capital in each plant. The data are constructed by an elaborate method, which utilize information on fire insurance values[1] and current net investments in buildings and machinery for each year, see Rosanoff (1981). The method is inspired by Johansen and Sørsveen (1967) and by the methods employed by Ringstad (1971) and Jansen (1973).

Aluminium oxide is the dominating raw material input. Engineering information tells that this input can be treated as a shadow factor in production. The theoretical input coefficient is 1.88 tonnes of aluminium oxide per tonne of primary aluminium, but in practice one has observed a coefficient of approximately 1.93 in the industry.[2]

Other raw materials consist mainly of anode mass components. All Norwegian aluminium works - with one exception - have their own factory for producing anode mass. The method based on preburnt anodes requires less anode mass and electricity, but is also more capital intensive than the alternative method based on Søderberg anodes (see appendix to this chapter). Only one of the Norwegian aluminium works uses smelting cells with preburnt anodes exclusively, while another two have series of smelting cells of both types belonging to different vintages.

Some of the works employ various casting alloys for a part of their production. The input quantities of these alloys are small and they do not influence the volume of the production significantly.

Despite observed differences between works in the use of raw materials (other than aluminium oxide), we have chosen to treat these auxiliary inputs as shadow factors as well. Thus they are only considered when we construct a "net price" of aluminium:

We start by calculating the value of the aluminium production in each plant, by applying the average price of primary aluminium to the total volume of the plant's aluminium production (including internal deliveries for further manufacturing). Thus, this concept is comparable between plants since the existence of further processing within a work do not affect this measure of value. Then, we find the net value of production by deducting the value of all inputs which are not included

1) Considering the particular kind of production, one may question the relevance of the fire insurance value.

2) Information to the authors from professor J. Thonstad of the Institute of technical electrochemistry at the Norwegian Institute of Technology, University of Trondheim is gratefully acknowledged.

in our measures for labour, electricity or capital, that is the value of raw materials (including anode mass components[1]) and casting alloys), oil, salary to non-production workers (including social security expenditures) etc. *The net price* for each plant is then defined as the ratio between this saldo and the volume of the aluminium production. Moreover, we define *the average net price* of aluminium as the weighted average of these net prices, with weights equal to each plant's share of the total industry production. (The resulting average net price is found in figure 12 of section 5.)

Similarly, we define the average price of the electricity input as the total costs of electricity divided by the total volume of electricity in the industry. The average price of the labour input is defined in the same way as the total wages plus social security expenses divided by the total number of hours worked by production workers in the industry. The average input prices are stated in table 2.

Table 2. The average input prices for electricity and labour. 1966 - 1978*

Year	Average price of electricity Nkr/100 kWh (1)	Average wages (including social security expenses) Nkr/man-hour (2)	Relative price labour-electricity (2):(1)
1966	1.74	12.20	7.01
1967	1.68	13.27	7.90
1968	1.93	15.25	7.90
1969	1.97	16.23	8.24
1970	1.87	18.25	9.76
1971	2.16	21.14	9.78
1972	2.23	23.98	10.75
1973	2.33	25.50	10.94
1974	2.75	31.64	11.51
1975	3.04	40.87	13.44
1976	3.07	45.74	14.90
1977	3.87	51.02	13.18
1978	4.10	55.71	13.59

* Cf. the definition in the text.

1) The only aluminium plant without an internal factory for production of anode mass will register a systematically higher value for these items.

3. Structural changes in the aluminium industry

When describing the structural changes in the aluminium industry, we focus on four equidistant years: 1966, 1970, 1974 and 1978. The input coefficient distributions for labour and energy for these years are set out in Salter diagrams (cf. Salter (1960)) in figures 1 and 2.

The notation employed is:

Nl = labour (hours)

EL = electricity (kWh)

X = output (tonnes)

K-cum = cumulated shares of capacity

Nl/X, EL/X = input coefficients, measured by observed inputs and output

As regards the shape of the labour input coefficient distribution, as set out in figure 1, it has changed significantly from an even cumulative distribution to one with a constant level and a marked tail for about the last 10 per cent of industry capacity.

The downward movement over time, i.e. uniform increase of labour productivity, has almost come to a standstill between 1974 and 1978. The right-hand tail with small productivity improvement consists of units with very small capacity shares.

The relative downward change over time for the energy input coefficient distribution in figure 2 has been smaller than for labour. There is, however, a clear downward trend from 1966 to 1974, whereas the input coefficients are systematically higher in 1978 as compared to 1974 (except at the tails of the distributions). The distributions are all comparatively flat with tails from about the last 5-10 per cent of industry capacity. The range of variation is from about 16 000 to 23 000 kWh per tonne. (Excluding one extreme observation due to closure.) The overall shift of the distributions amounts to a reduction of about 1000 kWh per tonne from 1966 to 1974, except for the stable 5-10 per cent tail.

The observed input coefficients for labour and energy for the years 1966 and 1978, are put together in figure 3, which also shows the change in the capacity distribution. The size of the squares is proportional with capacity. The production capacity has generally been increasing for each unit except the smallest ones. The relatively much larger

Figure 1. The labour input coefficient distributions. 1966– 1978

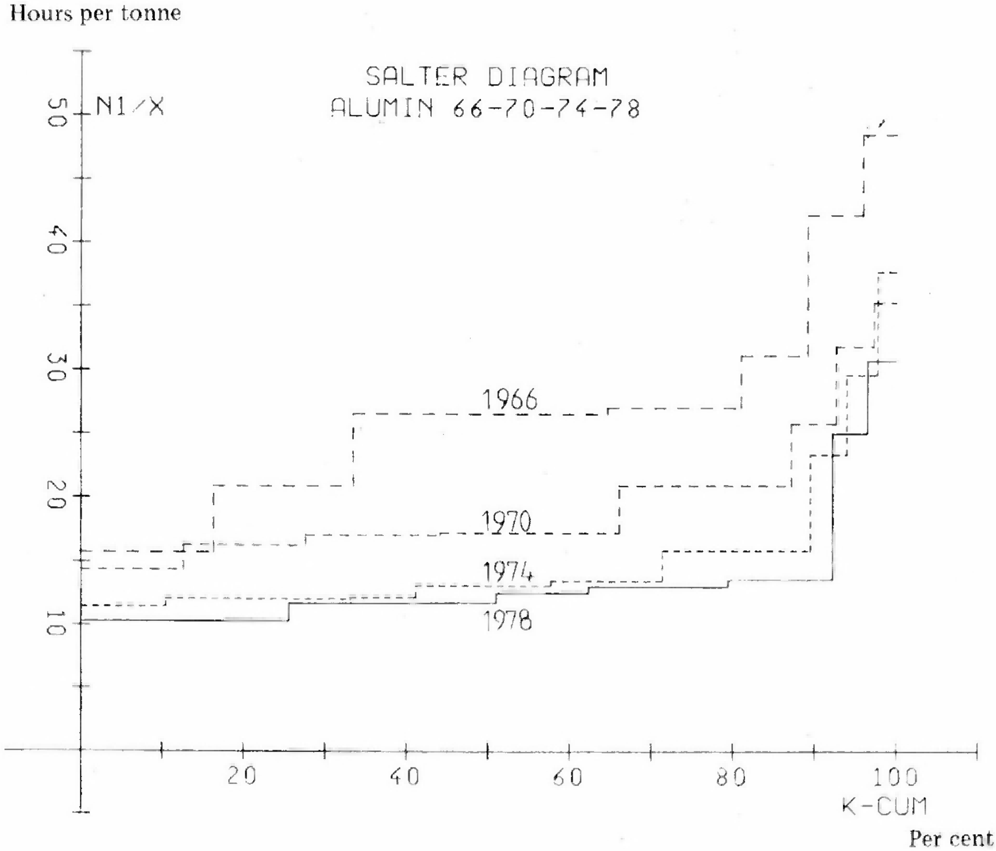
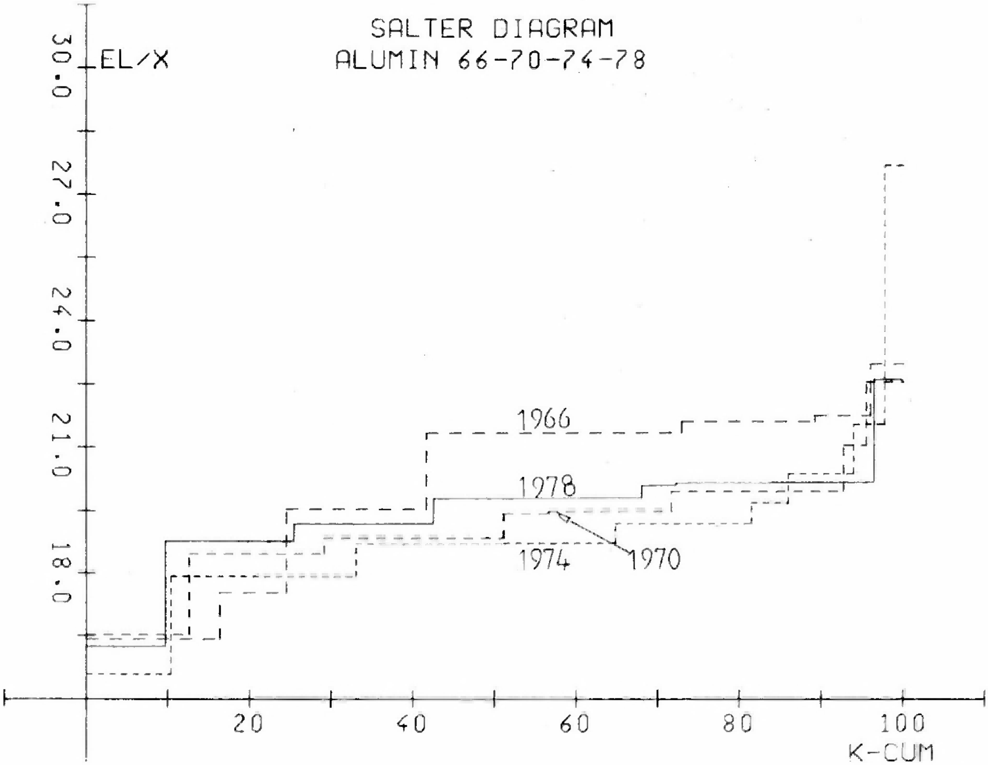


Figure 2. The energy input coefficient distributions. 1966—1978

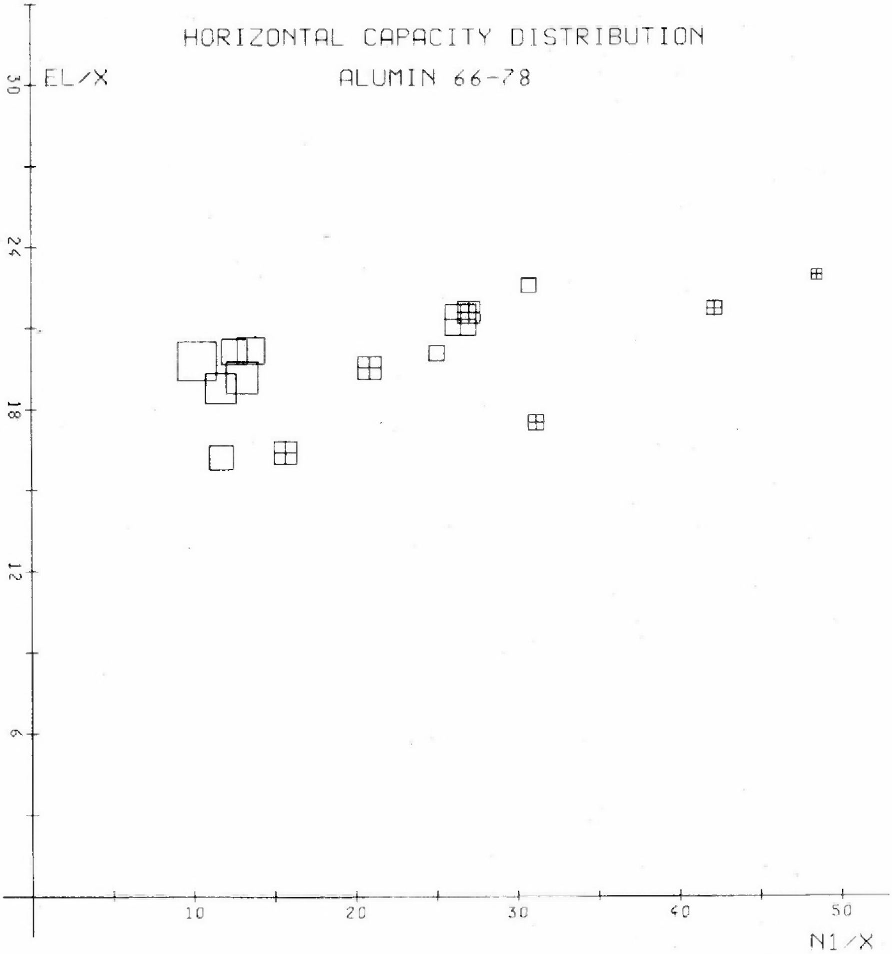
1 000 kWh per tonne



Per cent

Figure 3. The capacity distributions in 1966 (cross squares) and in 1978 (empty squares).

1 000 kWh per tonne



Hours per tonne

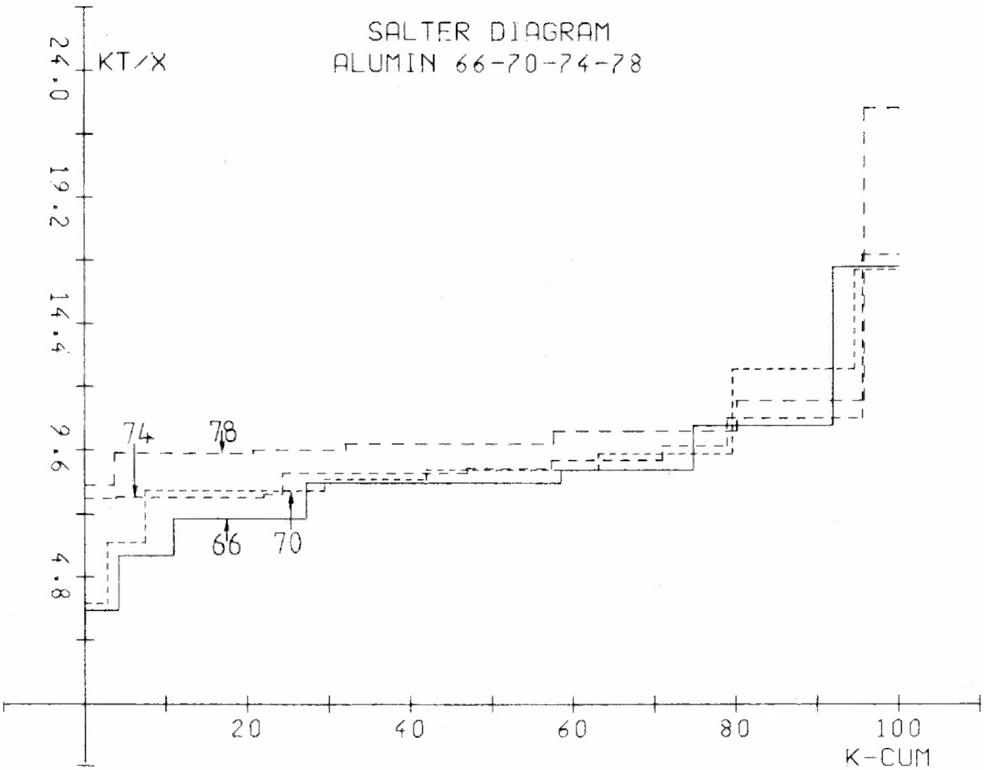
reduction in labour input coefficients than energy input coefficients is clearly depicted by the almost horizontal shift of the capacity distribution.

Structural changes and introduction of new production techniques are usually considered to be closely related to investment in new capital equipment. In the short run the real capital may be considered as fixed, but it may of course change over time. In order to give a crude picture of how the distribution of the real capital has changed among the establishments through the period of observation, we have constructed a Salter diagram for real capital per tonne aluminium in figure 4. KT is the value of machinery and buildings at constant 1975 prices.

We observe that there is a marked shift upwards over time in the distribution of real capital per tonne aluminium. This should be expected due to the vintage nature of the aluminium industry and *a priori* knowledge about long-run substitution possibilities between the variable inputs (labour and energy) and capital. Moreover, we find that the form of the capital/output distribution has changed over time in the same way as the labour input coefficient, i.e. from an even cumulative distribution to one with a constant level and a marked tail for the last percentages of the industry capacity. The correlation across firms between the capital/output coefficients and the input coefficients of energy and labour, respectively, have changed considerably over time. Both correlation coefficients were clearly negative in 1966 (see table 3). In 1978 the energy coefficient was uncorrelated with the capital/output ratio while the correlation between the labour input coefficient and the capital/output ratio was positive (0.34).

Figure 4. The capital/output distributions. 1966—1978

1 000 N.kr per tonne



Per cent

Table 3. The coefficient of correlation across firms between the input coefficients (per tonne aluminium) of labour, energy and real capital

Years	Coefficient of correlation between		
	Labour/ output and capital/ output	Electricity/ output and capital/ output	Electricity/ output and labour/ output
1966	-0.48	-0.74	0.71
1970	-0.40	-0.33	0.83
1974	-0.03	-0.14	0.87
1978	0.34	-0.02	0.68

4. Empirical results on the short-run industry production function

In order to derive the short-run macro function, information about the ex post micro production functions must be available. The production capacity of each unit is directly observed, and the fixed current input coefficients are simply calculated by using observed amounts of current inputs and output. This procedure is in accordance with the assumptions made about the ex post technology.

4.1. The region of substitution

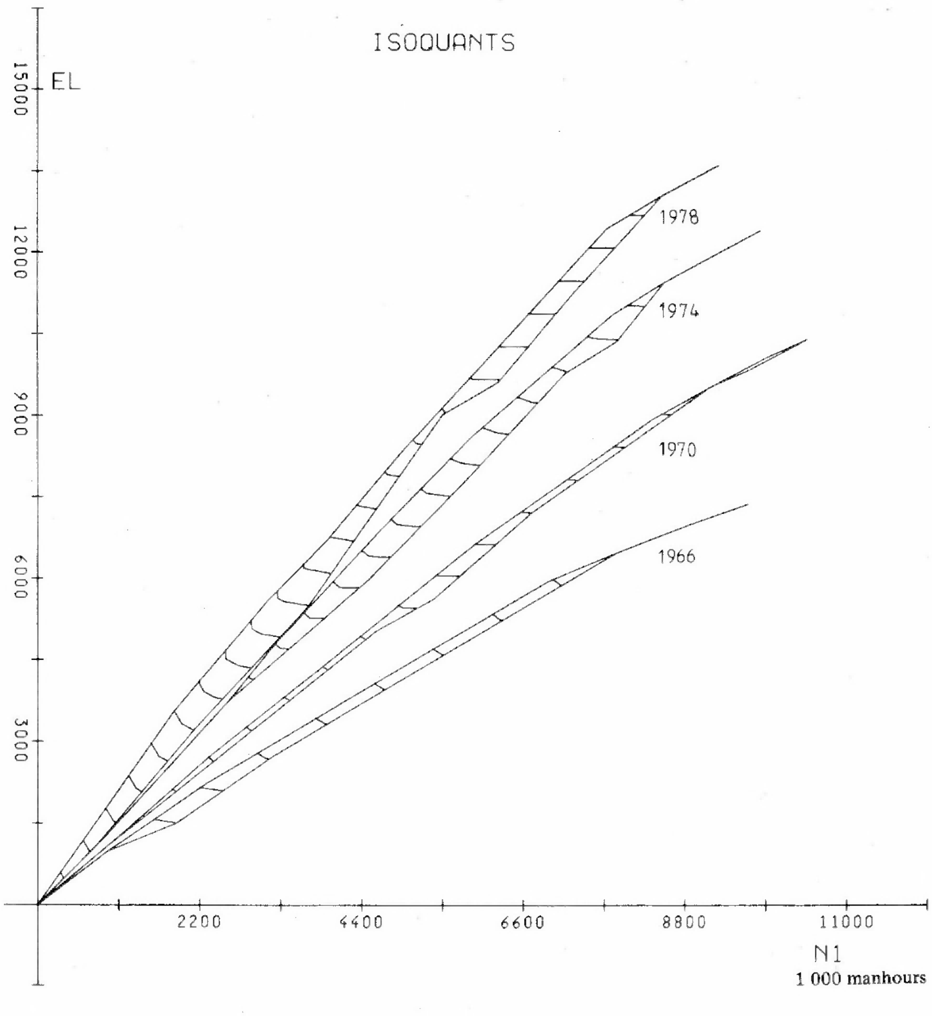
The region of substitution and isoquant map of the short-run industry production function for the selected years are set out in figure 5.[1)] The substitution regions are rather narrow for all years, reflecting the uniformity of the technique utilised in Norwegian aluminium plants. The collapsing of the substitution region into one line, as is the case for the tail end in 1966, 1974 and 1978, and for the front end in 1966, 1970 and 1974, corresponds to one unit obtaining the same rank number in the two partial input coefficient distributions in figures 1 and 2. (The probability for this to occur is, of course, higher the smaller is the number of production units. Recall that there are only between 7 and 9 units included in this study.) The remaining scope for substitution on the industry level is markedly largest for labour, as should be expected from the structural description provided in the previous section.

The last observation is also valid for the steady shift towards the energy axis revealed in figure 5. It should in this connection also be noted that there are strict physical limitations on the improvement

1) The algorithm behind the computer drawings is treated in detail in Førsund and Hjalmarsson (1980).

Figure 5. The development of the short run industry production function between 1966 and 1978. The interval between the isoquants are 30 000 tonnes.

100 000 kWh



1 000 manhours

in electricity productivity. According to Johansen and Thonstad (1979) there is - within the existing technology - very little feasible improvement left of the best practice electricity input coefficient at 1978 level, while the reduction in labour input coefficients does not run against any such physical law (except zero).

These shifts of the substitution region towards the energy axis are consistent with the changes observed in the relative input prices in table 2 (section 2). The development of the prices shows with few exceptions a steady increase in price of labour relative to the price of electricity, the relative price is nearly doubled during the period of observation.

We note that the isoquants are almost straight lines with only a few cornerpoints. Generally, the curvature of an isoquant is characterized by the *elasticity of substitution*. Short-run elasticities can be approximated by analogy with the definition in the case of smooth isoquants.[1] The change in the factor ratio relative to the average factor ratio measured for the extreme points of two consecutive isoquant segments is set in relation to the change in the marginal rate of substitution between the two segments relative to the average rate of substitution. Contrary to the visual impression of the isoquants approximating straight lines implying high values for the elasticity of substitution, we find rather low estimates for the elasticity of substitution between labour and electricity.[2]

4.2. The demand regions

What implications does the short-run industry production function have as regards industry demand for inputs? A simple transformation of the substitution regions shown in figure 5 yields the region within which the demand functions must lay for any set of input prices. Figures 6 and 7 show the demand regions for labour and electricity, respectively. The regions are constructed by plotting, for each output level, the minimal and maximal use of the input in question corresponding to the end points of the relevant isoquant in figure 5.

The upward shift of the labour demand regions is marked and corresponds to the productivity movement in figure 1. The demand regions for electricity are extremely narrow and ray-like, and stable over time; as should be expected on the basis of the Salter diagram in figure 1.

1) See table III in Førsund and Hjalmarsson (1983).
2) Low values for the elasticity of substitution are, however, in accordance with the conjectures in Hildenbrand (1981).

Figure 6. The demand region for labour. 1966, 1970, 1974 and 1978.

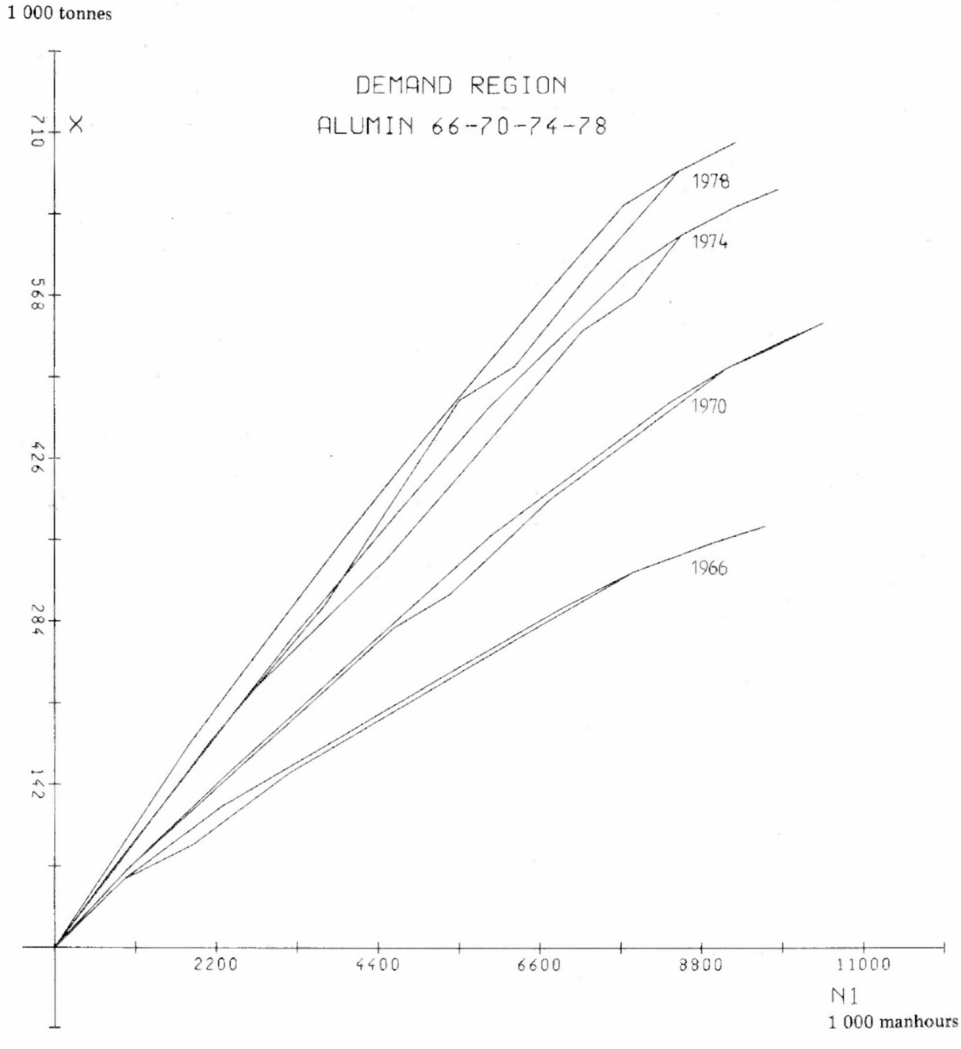
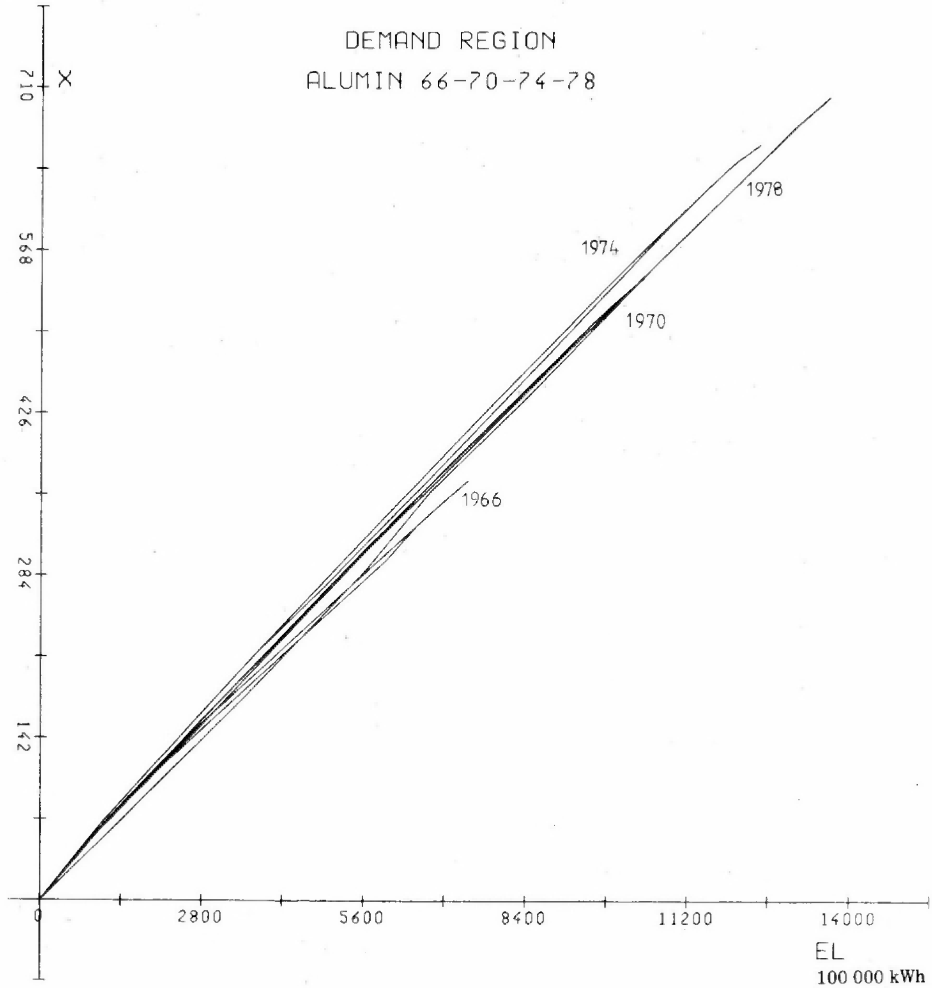


Figure 7. The demand region for electricity. 1966, 1970, 1974 and 1978.

1 000 tonnes



100 000 kWh

Demand for inputs can for each year be characterized by calculating the demand elasticities along isoclines (see Hildenbrand (1981)). Figure 8 shows the demand elasticities of labour (N1) and electricity (EL) for 1978 along the isocline corresponding to the observed average prices for that year. (The elasticities are calculated on a difference form.) Corresponding to the almost ray-like form of the demand region for electricity shown in figure 7 the demand elasticity is very close to one. There is an increase when best practice capacity is exhausted at the start in figure 8, and an increase again when taking into use the most inefficient capacity at the end. The variation in the demand elasticity for labour is greater with some marked peaks towards the end part of the demand elasticity curve in figure 8. Comparing figures 6 and 7 shows that the demand region for labour is wider than for electricity, providing greater scope for directional variations of the isocline in question. The peaks of the demand elasticity occur where the demand region contracts almost into a line and at the end when the most inefficient capacity is taken into use.

4.3. Productivity changes

The productivity improvements for various levels of output can be studied in figure 5 by following the movement of the isoquants in question. The interval length in figure 5 is 30 000 tonnes. The levels of 150 000, 300 000, 450 000 and 600 000 tonnes are shown separately in figure 9. The almost exclusively labour saving movement is clearly portrayed. The energy productivity has even decreased from the high capacity utilization year 1974 to the lower rate of capacity utilization year 1978 for all levels of output. Note from figure 2 that this feature for the industry as a whole does not only stem from the fact that one unit, the best practice unit in 1974, has the lowest electricity input coefficient in 1974 of all years, but that the electricity input coefficient curve for 1974 is lower than for 1978 except for the last 15 per cent of capacity.

The movement towards the electricity axis is also clearly portrayed by looking at the isoquant maps within the substitution regions transformed from the input space of figure 5 to the input coefficient space, carried out in figure 10. These transformations represent the feasible regions of input coefficients for the short-run industry function, and must then necessarily show more limited variations than the capacity distributions of individual units shown in figure 3.

As regards energy use figure 10 shows that the frontier values of the electricity input coefficients have been quite stable except for

Figure 8. The demand elasticities for labour (N1) and electricity (EL) in 1978, calculated along the isocline corresponding to the observed prices in that year.

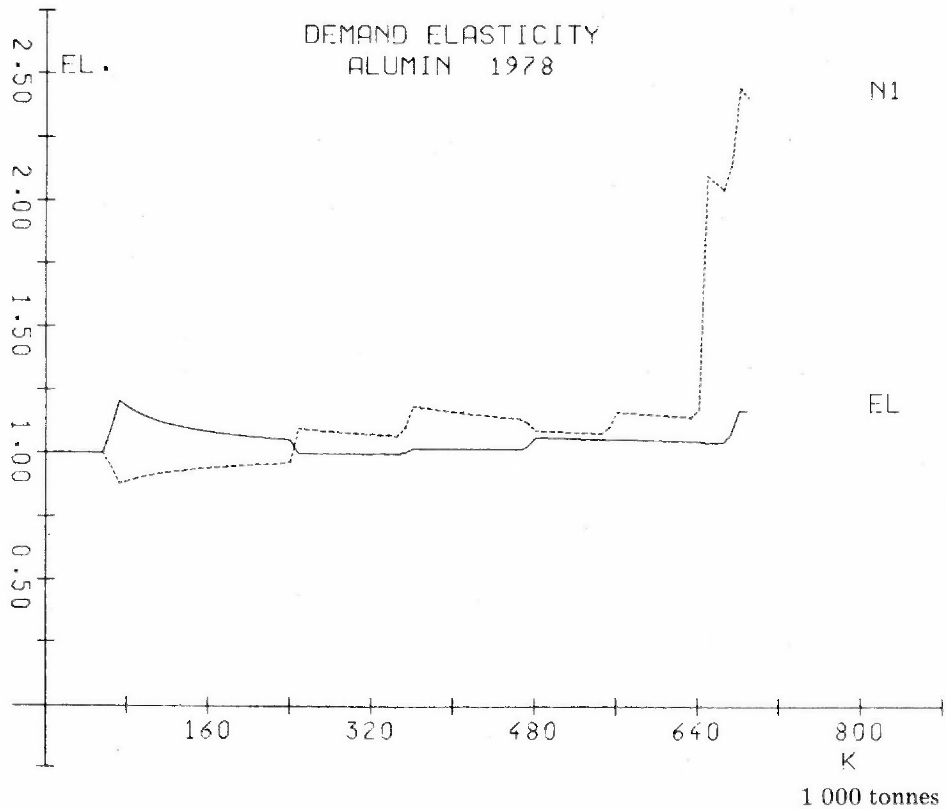
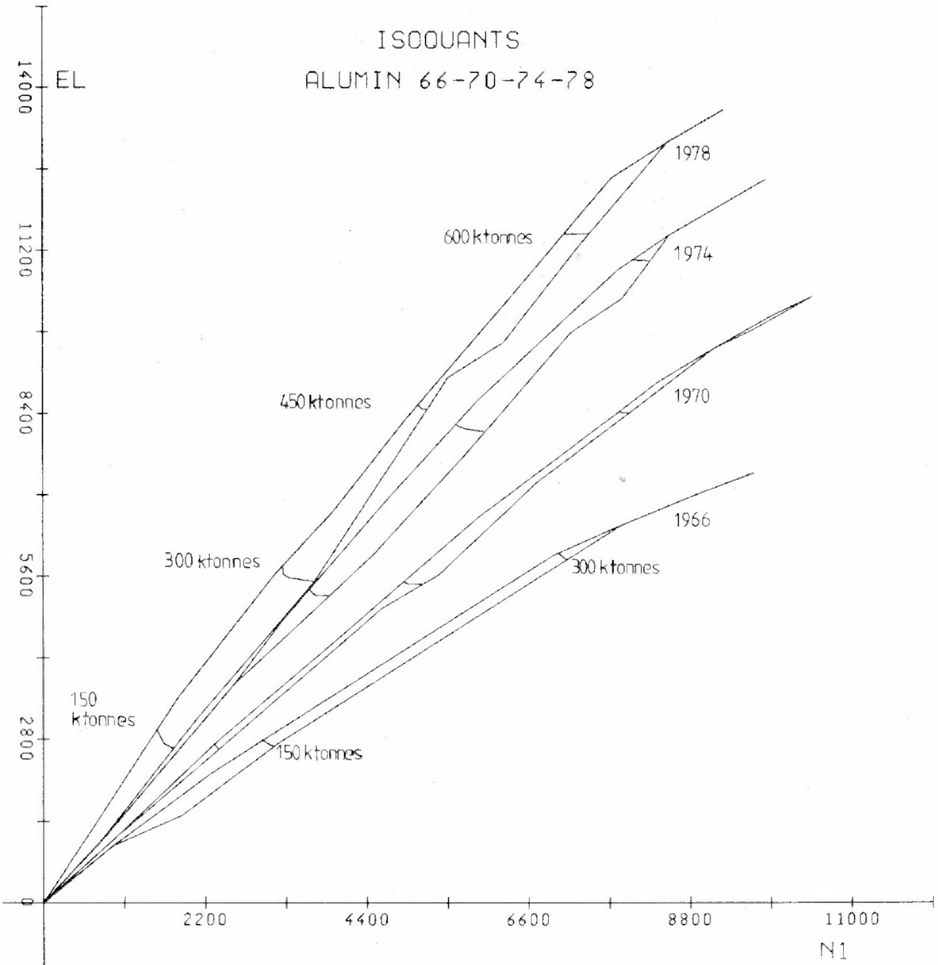


Figure 9. The short-run industry production functions for the years 1966, 1970, 1974 and 1978 with the isoquants for 150 000, 300 000, 450 000 and 600 000 tonnes.

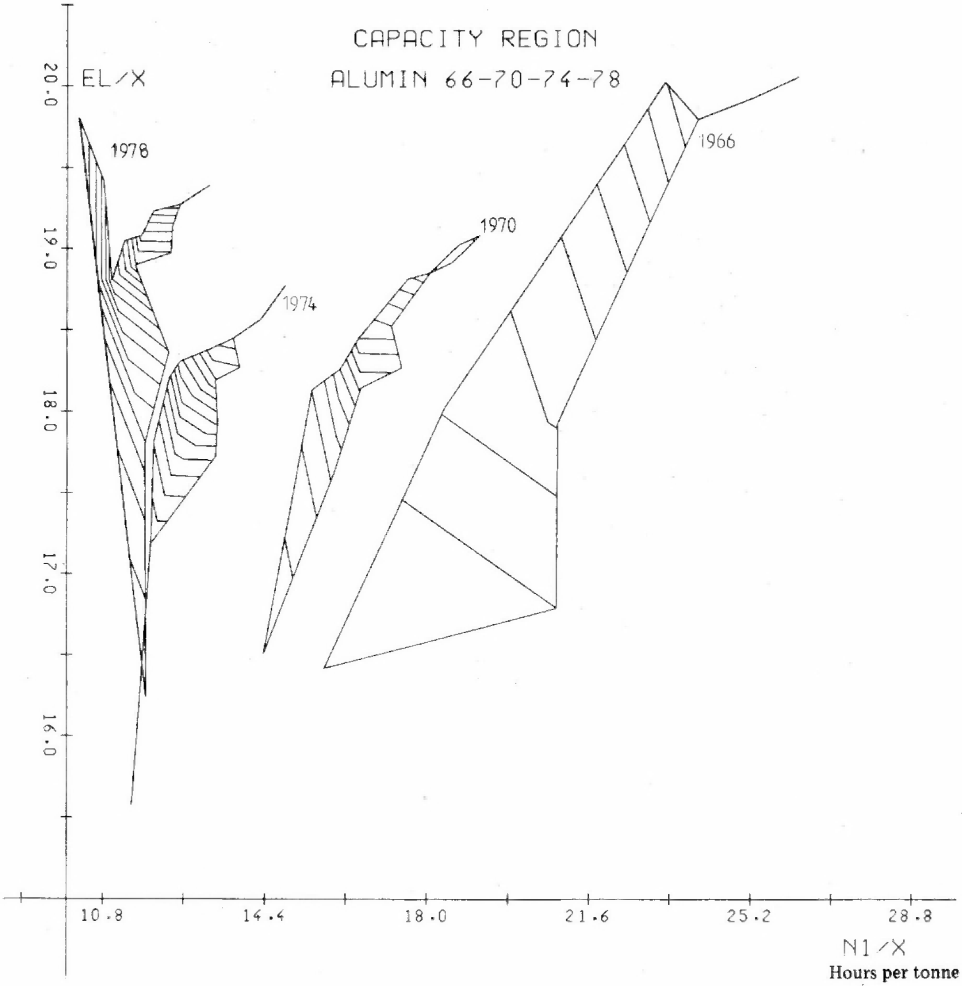
100 000 kWh



1 000 manhours

Figure 10. The development of the capacity region of the short-run industry production function. 1966, 1970, 1974 and 1978.

1 000 kWh per tonne



Hours per tonne

an extra low value for one unit in 1974. The industry improvement has consisted in the other units catching up with best practice performance. This trend has been weakened from 1974, the year with high capacity utilization, to 1978, a year with less than average rate of capacity utilization.

4.4. Measures of technical progress

Following Salter (1960), the significance of technical change can be assessed by computing the relative unit costs at constant input prices and output levels. We have chosen to use the average observed prices in the last year, 1978. The results for output intervals of 150 000 tonnes, including the frontier, i.e. the best practice performance, are set out in table 4.

Table 4. Technical progress. Relative unit costs. 1978 prices

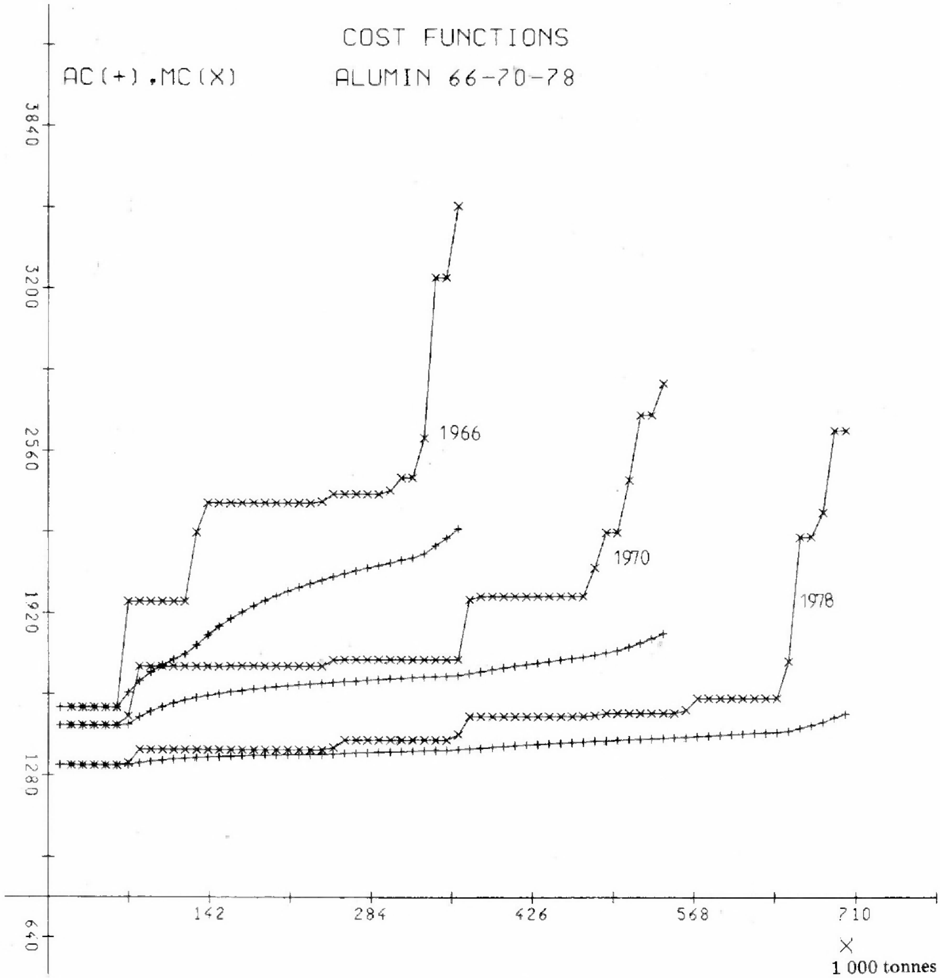
Years \ Output level	Frontier	150 000	300 000	450 000	600 000
1970 : 1966	0.96	0.86	0.78	-	-
1974 : 1970	0.86	0.84	0.85	0.83	-
1978 : 1974	1.04	0.99	0.98	0.98	0.96
1978 : 1966	0.85	0.72	0.65	-	-

The unit cost reduction from 1966 to 1978 varies significantly from the frontier at about 85 per cent to much higher reductions of unit costs at higher output levels, e.g. 65 per cent at 300 000 tonnes. Corresponding to what was revealed by figure 10, the only significant improvement of the frontier was from 1970 to 1974, but this was due to just one individual unit, and the performance slipped again resulting in an *increase* of unit costs at best practice from 1974 to 1978. The average catching up with best practice performance shows up in table 4 with the greatest unit cost reductions at higher output levels. The technical advance from 1974 to 1978 has been very small indeed; the reduction in labour input coefficients just offsetting *increases* in electricity input coefficients.

In table 4 only a few points on the average cost curves were utilized. The complete average cost curves for 1966, 1970 and 1978 are set out in figure 11, together with the marginal cost curves; all based on 1978 average observed input prices. (The curves for 1974 are excluded because they are so close to the 1978 curves, as is evident from table 4.)

Figure 11. The marginal and average cost-functions (MC and AC, respectively) for 1966, 1970 and 1978 in 1978 prices.

N.kr per tonne



Salter measures at various output levels may be calculated by comparing average costs in figure 11. As regards the shape of the average cost curve it has become flatter and flatter, as should be expected on the basis of the Salter diagrams (figures 1 and 2).

The form of the marginal cost curves add to the structural picture. They have become more and more like the average cost curves, and the tail on the J-shape apply to smaller and smaller shares of the output capacity. This development supports the impression of a greater and greater uniformity of the structure of aluminium smelters.

4.5. The elasticity of scale

Additional structural features can be brought out by looking at values of the elasticity of scale. In table 5 the development of the scale elasticity is shown for the average factor ratio. (When the factor ray is outside the substitution region the scale elasticity on the bordering isoquant segment in question is used.)

The maximal value of the scale elasticity in short-run industry functions of the type constructed is 1.0. The level of the elasticities has increased from 1966 to 1974. The high values in 1974 and 1978 reflect again the technical uniformity of the units. The extreme low value for the highest output level in 1978 is due to the fact that the least efficient unit is now utilized; corresponding to the top of the tail of the J-curved marginal cost curve for that year.

Table 5. The development of the scale elasticity along the average factor rays

Year	Output levels in 1 000 tonnes							Energy/labour
	100	200	300	400	500	600	700	Average factor ratio
1966	0.89	0.86	0.89	-	-	-	-	0.76
1970	0.91	0.94	0.94	0.92	0.94	-	-	1.00
1974	0.96	0.98	0.94	0.96	0.95	0.91	-	1.26
1978	0.94	0.93	0.95	0.97	0.94	0.95	0.42	1.47

5. Aspects of the business cycle

The aluminium industry in Norway is extremely exposed to impulses from abroad through international market fluctuations. As we observe in the appendix, the industry imports the bulk of its raw materials as alumina and 80 per cent of the output is exported as primary aluminium. It is well known that the international markets for raw materials and intermediate products of this type are very sensitive to cyclical movements.

In figure 12 we have depicted the average net price of primary aluminium, as reported by Norwegian aluminium producers. This average net price, which is defined in section 2, gives a crude, but informative picture of how the business cycles may have affected this branch of Norwegian industry. The changes in the (nominal) net price reflect the composite effect of the cyclical fluctuations in the market of primary aluminium as well as in the market for alumina. The decline of the dollar throughout the seventies has had a substantial effect.

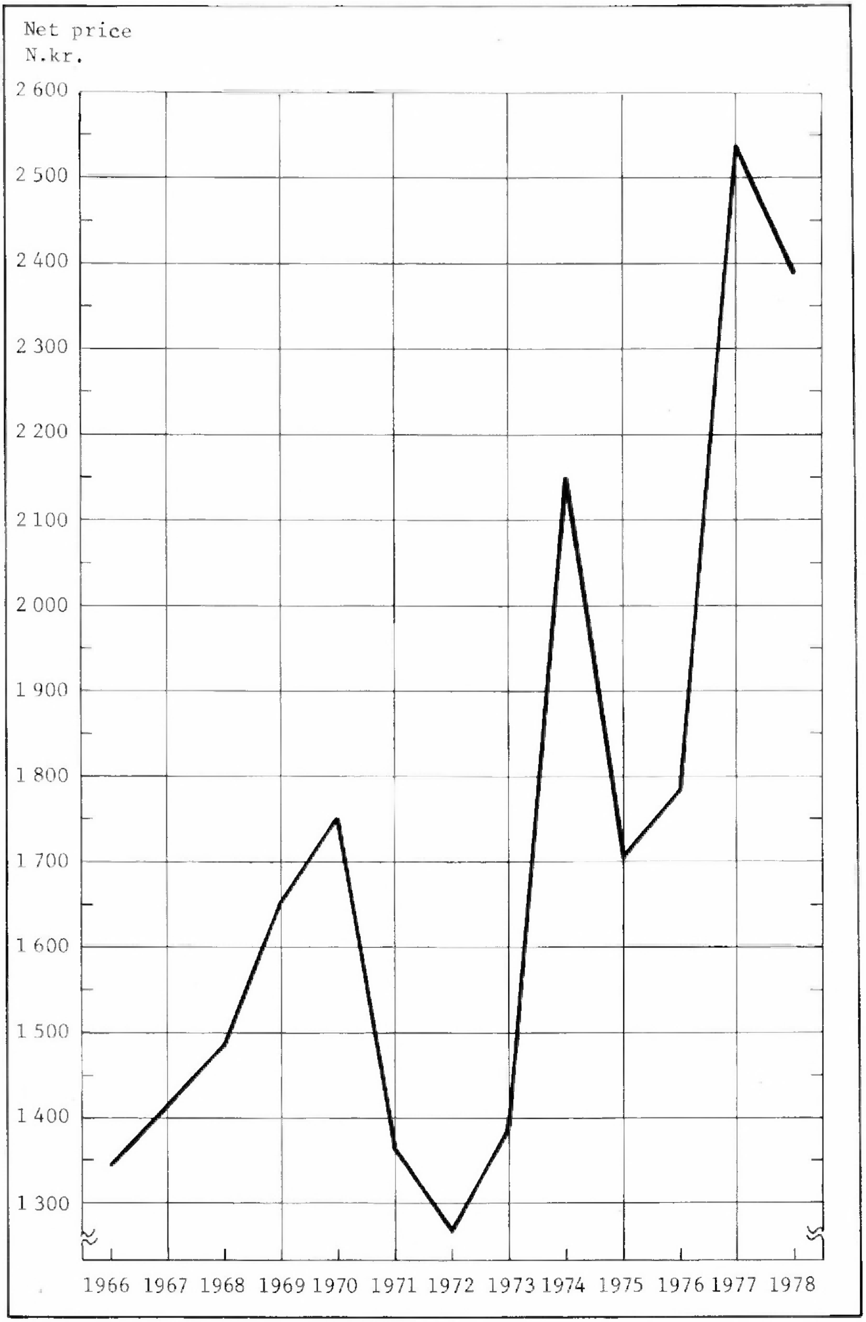
By comparing the graph of figure 12 with the figures for capacity utilization in table 1, we have chosen 1972, 1974 and 1975 as typical for a business cycle. We shall use the three years to investigate the robustness of the fixed coefficients assumption and what impacts a business cycle has on the short-run industry function.

5.1. The validity of the production function assumptions

The key assumption of the production technology is the one of fixed input coefficients. Since our unit of time is one year, this can only hold as an approximation. Especially as regards labour, small scale productivity improvements take place more or less continuously. Improvements within the one year period may also occur concerning the running of the process, in addition to changes made when the smelting cells are relined. One way of checking the stability of the input coefficients is, of course, to investigate the substitution regions and isoquant maps for consecutive years.

Another assumption made is that the input coefficients are independent of the rate of capacity utilization. Even if this is correct technically, the way we estimate the input coefficients by current observed quantities may tend to make them instable. This may especially be the case for labour. Labour hoarding increases the labour input coefficients in periods of low capacity utilization. Since our unit of production contains several vintages of cells, low capacity utilization may lead to *lower* electricity input coefficients if the least efficient cells are taken out of production first.

Figure 12. The average net price of primary aluminium as defined in section 2. 1966–1978.



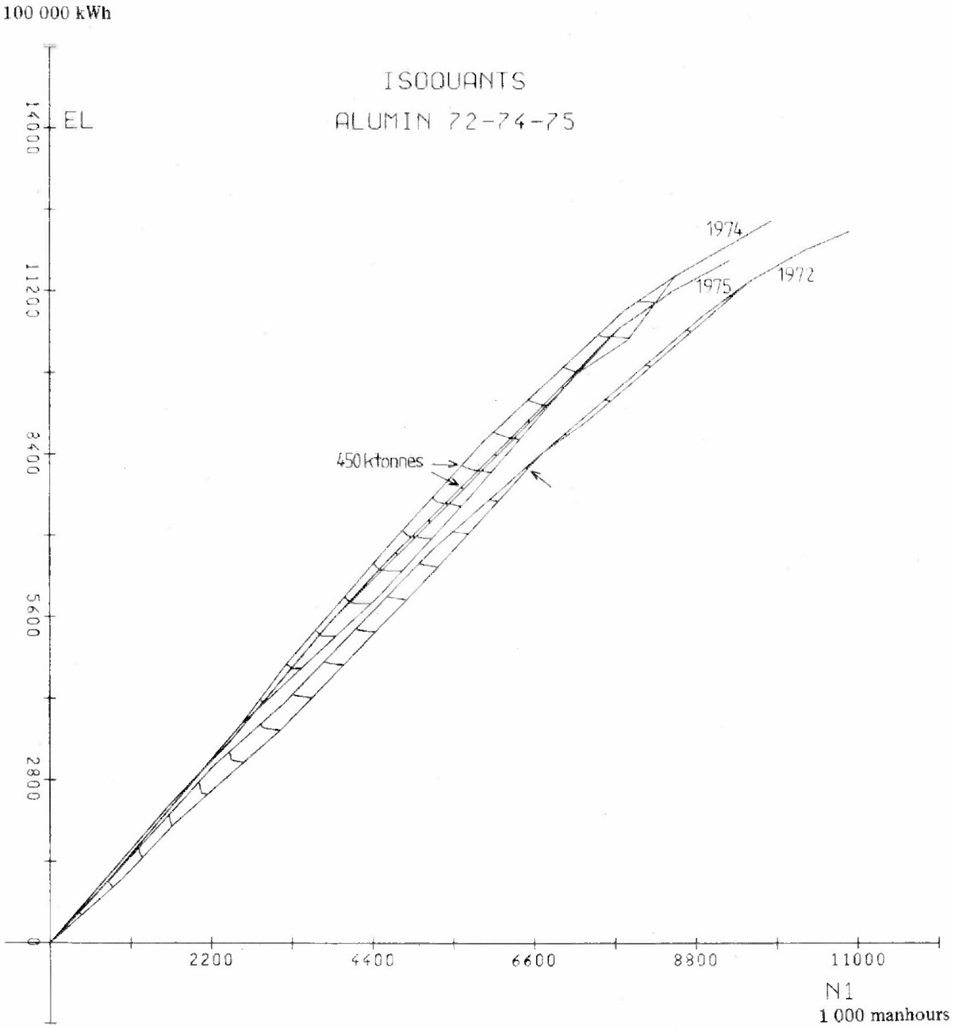
In order to reveal the impact of both types of effects mentioned above, we have put together the substitution regions and isoquant maps for the years 1972, 1974 and 1975 in figure 13. The year 1972 represents the trough of the business cycle with low rate of capacity utilization on the average and very low net price of aluminium. Comparing with 1970 and 1974 the short-run function for 1972 seems just like expected as regards the trend of factor substitution, shape of substitution region and isoquants, and productivity level. When moving to the boom year of 1974 with full capacity utilization and a record high net output price, there is a significant improvement in labour productivity, and a very slight, if any, improvement in electricity productivity. This is consistent with labour hoarding through the trough, but the factor substitution does not appear stronger than the underlying trend towards labour saving. As regards energy efficiency the very small improvement from 1972 to 1974 may be due to efficient contraction of output in 1972.

The shape of the substitution region corresponding to about 25 per cent of the most efficient units is much wider for 1972 than for 1974. While the best practice units in 1974 were very similar with respect to both input coefficients, we find that especially the energy coefficients differ for those units in 1972. The corresponding isoquants show a greater scope for energy coefficient variation than for the rest of the substitution region. The best practice plants reacted differently to output contraction.

In 1975 there was a slump again with a marked fall in net aluminium prices and a fall in the rate of capacity utilization. A striking feature of the substitution region of 1975 is that it has become very narrow indeed. The ranking of the units conforms for both electricity and labour, and the substitution region almost follows a factor ray, except for the tail, implying that the ratio of the coefficients are quite equal. The best practice part is almost identical to the previous year's substitution region, while the rest of the region lies inside that of 1974. The factor substitution has halted.

However, there is one disturbing feature of the 1975 short run function: There is a significant productivity improvement from 1974, as indicated in the figure by the locations of the 450 ktonnes isoquants for the three years. Closer analysis of the data reveals that this is due to one unit having a marked improvement in its electricity coefficient.

Figure 13. Changes in the short run industry production function through a business cycle. 1972, 1974 and 1975.



This improvement was not maintained in the years after, when the value of the coefficient went back to the previous level of 1974. The atypical value for 1975 may thus be due to a data error (in the firm's report to the Industrial Statistics). In any case it shows the vulnerability of the short-run function to best practice outliers.

5.2. The economic performance over the business cycle

The aluminium firms of Norway do not operate under a central authority as regards the decision which plants to cut back during downturns of the business cycle. The short-run industry production function offers a *normative* recommendation; provided that the firms face the same input prices. In practice the firms also get different prices for the output, even though aluminium is a quite homogeneous product. (There are also differences due to various alloys.) This may serve as an explanation, in addition to varying input prices, why the least cost solution of the short-run industry function is not realised.

Assuming fixed coefficients ex post for the firms, quasi rent development should indicate which firms have to cut back during a recession. The development for current costs and prices for the three years 1972, 1974 and 1975 is shown in figures 14-16 by means of *Heckscher diagrams*. Capacity is entered in the diagram according to increasing average current cost. The dotted lines show the split between labour costs (production workers) and electricity costs. Three price lines are entered in the diagram, the maximal, average and minimal net output price observed. The price is net of raw materials and white collar wage bill, confer the definition in section 2. The difference between the net price and current costs is the quasi rent per unit of output and shows the amount available for covering the return on capital. Combining the information in figures 4 and 14 we have that for the trough year 1972 the quasi rent is on the average too small to yield a normal rate of pre-tax return on capital. But the output price differences are so great that even for this bottom year some share of the total capacity generates a healthy yield.

The average net price increased considerably in the boom year of 1974. The minimum net price, on the other hand, was even at a *lower* level than in 1972, so that there was markedly greater price variation, and correspondingly quasi rent variations, in 1974 than in 1972. The end tail of the capacity was both in 1972 and 1974 characterized by negative quasi rents measured by the average net price line. (Actually, the minimal net price

Figure 14. Heckscher diagram showing the average current costs of labour (Q1) and electricity (Q2) in the aluminium industry in 1972. The three price lines are the maximal (p_{max}), average (p) and minimal (p_{min}) net output price observed.

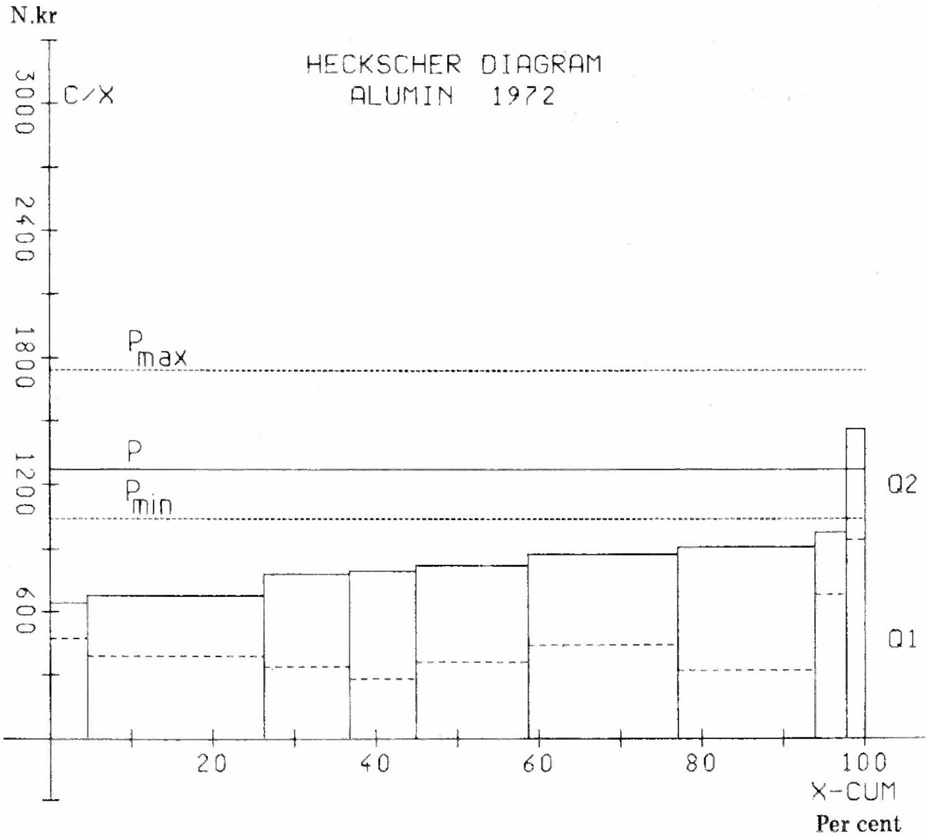


Figure 15. Heckscher diagram showing the average current costs of labour (Q1) and electricity (Q2) in the aluminium industry in 1974. The three price lines are the maximal (p_{max}), average (p) and minimal (p_{min}) net output price observed.

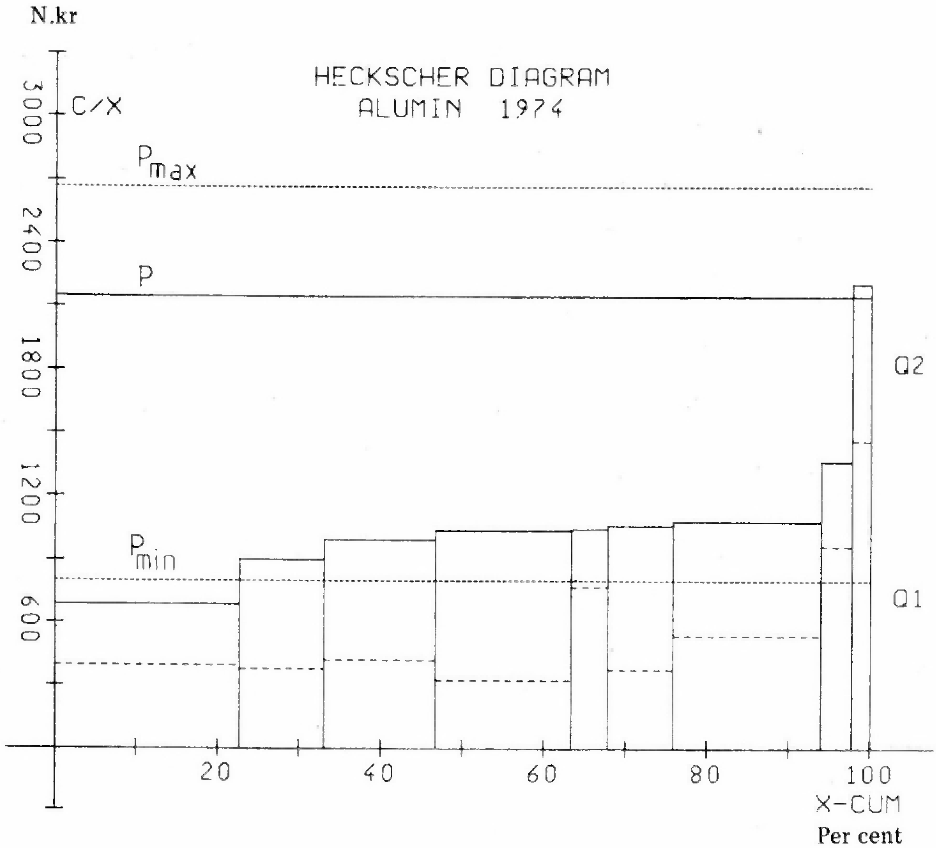
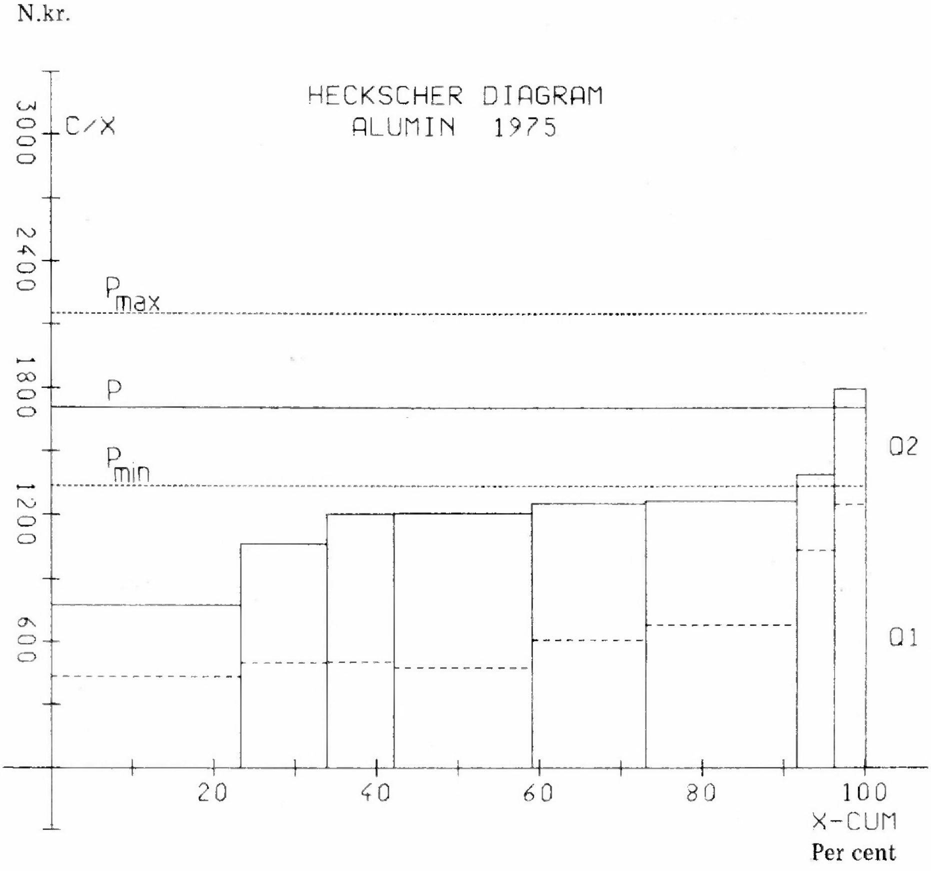


Figure 16. Heckscher diagram showing the average current costs of labour (Q1) and electricity (Q2) in the aluminium industry in 1975. The three price lines are the maximal (p_{max}), average (p) and minimal (p_{min}) net output price observed.



was relevant for this part of the capacity, worsening the picture considerably.) This part of the capacity was scrapped in 1975. The current average costs increased from 1972 to 1974, except for the 20-25 per cent of capacity with least average costs, due to both labour and electricity cost increases.

This cost increase continues into 1975 when the net output price falls again considerably, but now the lion's share of the increase is due to labour cost increases. This corresponds well with the fact that the general wage increase negotiated in 1974 was especially high for that decennium. The quasi rents are rolled back to almost the same low level as in 1972, with the tail of the capacity in 1975 earning negative quasi rent based on the average net price. The spread in net output prices is reduced again to about the same magnitude as in 1972.

Since, with the exception of a small tail, the quasi rents remained positive even over several recessions in the aluminium market, it is natural that the independent firms do not contract their output according to the least cost solution implied by the short run industry function.

6. Concluding remarks

Can the results obtained for the short-run industry production function of the Norwegian aluminium industry be said to be compatible with the assumptions underlying the production structure of the MSG model?

There has been a marked shift of the substitution region towards the electricity axis. Direct substitution between electricity and labour is possible only to a very limited extent, even when capital is a variable factor. Thus we interpret the result above as clear evidence of labour saving technical change over the period of observation. This phenomenon is probably induced by the rise in the relative price of labour compared to electricity and the technical possibilities for cost reductions. The MSG assumption of *Hicks-neutral* technical change is thus not supported by data for the aluminium industry.

The industry production function for aluminium is characterized by narrow substitution regions for all years, reflecting a high degree of technical uniformity between Norwegian aluminium smelters.

The straight and narrow regions of substitution indicate further that the short-run production function of the aluminium industry can be adequately represented by a simple Leontief function. It is, however, conceivable that inhomogeneity of the units within the MSG sector

"Manufacture of metals" will yield a substantially broader scope for substitution at that level of aggregation.

All kinds of neoclassical regularity which cannot be established from microeconomic studies like the present, must - if at all being present - be due to aggregation of inhomogeneous units within the model's broader production sectors.[1)]

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1) Recall that the aluminium industry accounts for one third of the gross value of production of MSG sector 43 Manufacture of metals, see section 1 above.

Appendix

A brief description of the production technology of the aluminium industry in Norway[1)]

The chain of production processes from the extraction of ore to the finished manufactured aluminium commodities, can conveniently be divided into five steps:

- i) Extraction of ore from bauxite mines.
- ii) Refining of bauxite to alumina (aluminium oxide).
- iii) Smelting of alumina to produce primary aluminium.
- iv) Refining and casting (with eventual use of various casting alloys).
- v) Further processing of aluminium into finished and semi-finished goods.

The Norwegian aluminium industry imports all its alumina and approximately 80 per cent of the production of primary aluminium are exported without being further processed in Norway. We shall therefore focus on the steps iii) and iv) as illustrated in figure A1.

The most widely used method for producing aluminium all over the world is the Hall-Hérould method. By this method alumina is dissolved in smelting cells - deep, rectangular steel shells lined with carbon - that are filled with a molten electrolyte, consisting mainly of cryolite and aluminium fluorite. By means of carbon anodes and a layer of molten aluminium at the bottom of the shell, which serves as cathode, direct current is passed through the electrolyte. The process electrolyzes the alumina: Molten metallic aluminium is deposited and siphoned off from the bottom, while the oxygen is released as carbon oxides from the anode. The Norwegian aluminium plants all use variants of this technique to produce primary aluminium[2)]. There are, however, two different kinds of carbon anodes in use: The carbon anodes, which basically are a mixture of pitch and coke (anode mass), must be prepared by burning. This may be done in a separate factory ("preburnt anodes") or the burning can take place in the smelting cells as part of the smelting process (Søderberg anodes). Preburnt anodes must be replaced every third week, while Søderberg anodes need a continuous supply of anode mass. If we consider

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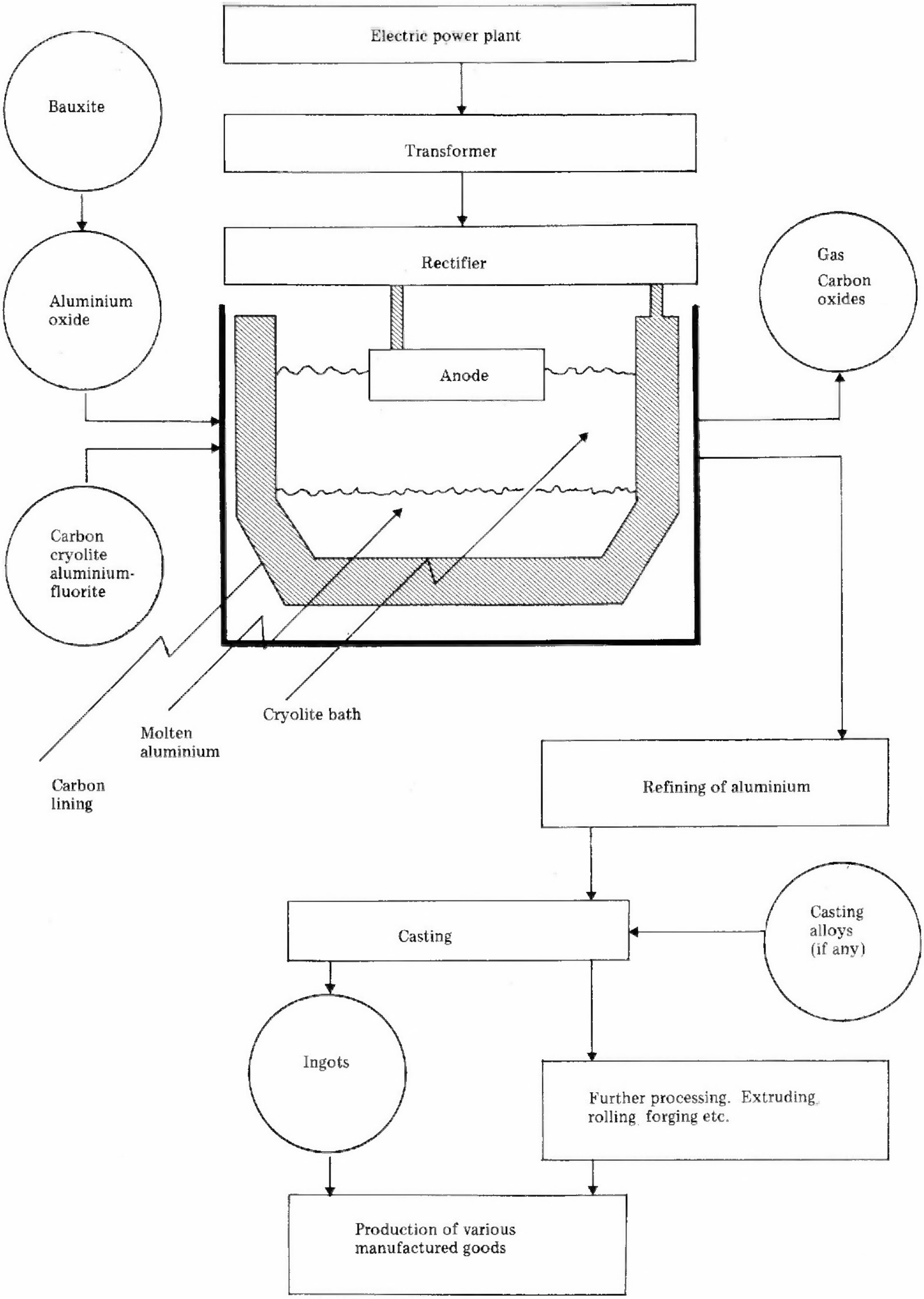
1) Our major source of information for this section has been Johansen and Thonstad (1979), see also Rosanoff (1981). The more technically oriented reader are referred to Grjotheim et al (1977).

2) The total number of plants varies from 7 to 9 in the period of observation, confer table 1 of section 2.

the production of "preburnt anodes" as part of the aluminium production, the economic implications of the different technologies are: The method based on preburnt anodes reduces the total input of anode mass and the use of electricity in the process as compared to the alternative, but it is also the more capital intensive method.

Figure A 1. A cross-section flow-chart for the chain of prosesses in the production of aluminium and aluminium products (Cf. Johansen and Thonstad (1979)).



XII. OPTIMAL PRICING AND INVESTMENT IN ELECTRICITY SUPPLY

by

Steinar Strøm

The supply of electric power involves four planning problems:

- Regulation of consumption so that the given capacities are not exceeded. (Optimal utilization of capacity.)
- Ensuring that the capacities at all times are sufficiently large. (Optimal energy and load capacity or optimal timing and dimensioning of hydro power projects.)
- Ensuring that a given amount of power is produced at the lowest possible cost. (Optimal structure of the power production system.)
- Ensuring that electricity is distributed among consumers so that redistribution would not involve any economic gain. (Optimal distribution among consumers.)

In this chapter we shall discuss these four problems as well as considering whether the Norwegian planning of the electric power supply satisfies the optimal conditions. Norwegian electricity production is based mainly on hydro power. The seminal work in this field is Turvey (1968).

1. The changing pattern of demand for electricity in Norway

Consumption of electric power will vary over time, also within a shorter span of time, in which maximum load and energy capacity can be considered as given.

Figure 1 shows monthly maximum load in Norway in 1975. Total consumption of electricity is the sum of the use of power in energy intensive industries, in electro-boilers, as pumping power, as net exports, and for all other purposes, in the following called general consumption. The consumption of power in the energy intensive industries is by far the greatest item after general consumption. The gap between general consumption and total consumption in figure 1 is practically the same all year round. This means that there are only slight seasonal variations in the consumption of power, apart from those in general consumption. The consumption of power in energy intensive industries is associated with the running of industrial processes. An important consumer item in general consumption is the electricity used for heating and lighting. In Norway there is practically no need for air conditioning in summer. For this reason general consumption - and also total consumption - registers a clear top load during the winter months.

Figure 2 shows consumption during the 24-hour period in 1975 in which maximum load was registered. A simple indicator of the difference in variation between general consumption and total consumption is the difference in utilization time. Utilization time is defined as the number of hours it will take for demand at maximum load to meet the observed energy consumption during a period. If the utilized effect is constant during the 8 760 hours of the year, the utilization time in the course of the year will be maximum and equal to 8 760 hours (100 per cent). The utilization time in energy intensive industries is very high, about 90 per cent. In general consumption utilization time was 5 545 hours in 1975, i.e. 63 per cent. The utilization time of total consumption was in 1975 6 479 hours, i.e. 73 per cent.

Utilization time, both in general and total consumption, has decreased over a period of time. This means that total consumption has become more effect-oriented and consequently less energy-oriented. This influences the kind of production methods, and consequently the types of power stations, that satisfy demand at the lowest possible costs.

An important reason for the decreasing utilization time is the change in composition of the total energy consumption. Table 1 shows how the structure of firm power consumption has changed in Norway from 1950 to 1980.

Table 1. Domestic net consumption of firm power by consumer category in Norway 1950 - 1980. Per cent

	1950	1960	1970	1975	1980
Energy intensive industries	46.3	47.3	45.6	42.7	37.7
Production of pulp and paper	6.9	7.2	7.1	4.8	4.4
Other manufacturing industries	9.8	9.8	10.1	12.7	10.7
Transport	1.5	1.3	1.0	0.9	0.9
Service industries and government ..		4.3	6.7	8.9	13.4
Primary industries	35.5	29.8	0.9	0.8	0.9
Households			28.7	28.7	31.1

Source: Electricity Statistics of Norway.

Figure 1. Monthly maximal load for general consumption and total consumption as registered at power stations, for Norway, 1975.

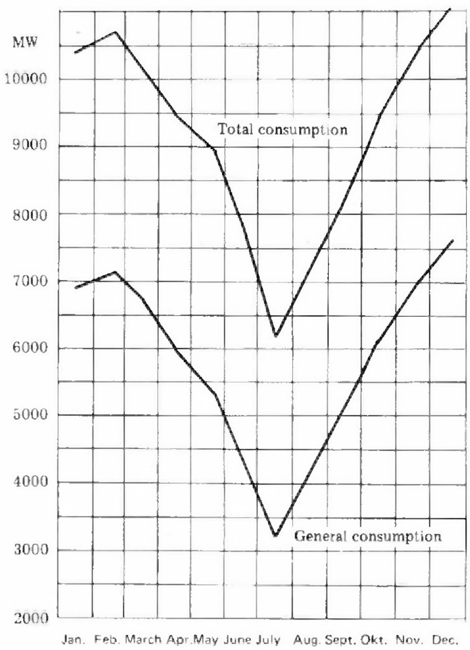
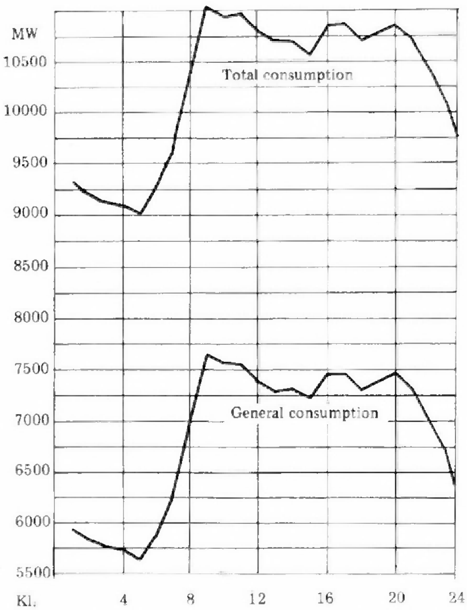


Figure 2. Consumption as registered at power stations during the 24—hour period of maximum load, for Norway, Friday, 19 December 1975.



Source: Norwegian National Power Stations Grid. Annual Report, 1975.

The most characteristic feature to be observed from table 1 is the marked decline, from 1970 to 1980, in the share of total consumption accounted for by energy intensive industries and pulp and paper. This is not only due to temporary market fluctuations. Figures for intervening years confirm the observed trend. It is also interesting to note the increased share of total consumption accounted for by the service industries.

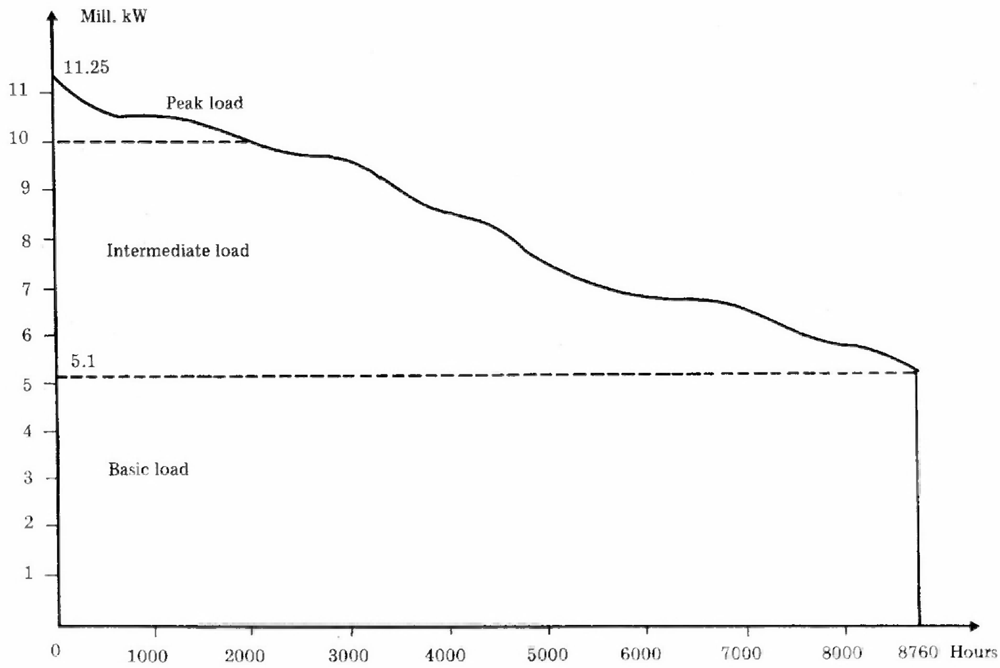
If we wish to find the reasons why the economy is becoming more effect-oriented and relatively less energy-oriented, we shall therefore have to trace the reasons that explain why sectors producing services, for example, are expanding more markedly than other sectors. The following factors are among the most important explanations for the observed development:

During periods of economic growth, in which real incomes are increasing, demand will, mutatis mutandis, increase most for goods that have the highest income elasticity. Several kinds of services have high income elasticities. This is also true for a number of industrial products associated with, for example, leisure pursuits, home furnishing, etc. However, the majority of these products are "light industrial products" which do not result from energy intensive processes. The conditions on the demand side combined with economic growth thereby result in an economic development in which the production of services and "light industrial products" increases steadily in scope. Activities of this kind require energy primarily for heating and lighting purposes. This development thus results in a demand for electricity that varies from one season to another and from one day to another.

In Norway this development has to a certain extent been slowed down by the fact that part of the increased demand for "light industrial products" with high income elasticities has been directed towards imports. The exports that have covered these imports have been of a kind demanding supplies of energy with an even effect load all year round (energy intensive industry). In the 1970s this pattern of development in Norway has been somewhat changed, as is confirmed too, by table 1. There is reason to expect that a development in which the traditional energy intensive industrial products play a relatively smaller role in the export picture will be intensified in Norway. In the first place, increased energy prices, and thereby electricity prices, will gradually reduce the share of total production accounted for by energy intensive industrial production. In the second place Norwegian revenue accruing from exports of oil and gas will result in a reduced need for the export of traditional export products of the energy intensive kind.

The conclusion, therefore, is that economic growth in Norway, combined with increasing exports of oil and gas, will render the Norwegian economy more and more effect-oriented, and less and less energy-oriented. For this reason it should no longer be so important to allow electricity development to be dominated by efforts to secure a large energy capacity.

Figure 3. Load curve for electricity consumption in Norway, 1975.



On the basis of data for effect load for every point of time in 1975 it is possible to construct the load curve in figure 3. Along the ordinate axis we measure effect load, e.g. in each of the 8 760 hours of the year. Hours are plotted along the abcissa axis. The load curve starts with the highest load, which in 1975 was 11.25 million kW. This load, according to figure 3 lasted for no more than a few hours. The lowest value on the load curve is the lowest load registered in 1975, viz. 5.1 million kW. The area under the curve measures total energy consumption that year.

2. The planning problem in electricity supply

Let us now assume a planning situation in which the problem involves determining optimal total energy consumption for a year and the maximum load the system can give. The production structure is described in terms of the cost function presented in chapter VI. The demand side is here dealt with somewhat summarily.

It is presupposed that demand varies systematically in the course of the year, but we shall ignore the fact that variations may also be the result of random circumstances. Consumers of electricity will be dealt with as one group; thus, we shall not distinguish between electricity demanded by households and electricity demanded by industrial firms. It is assumed that only the price of electricity plays any rôle in demand. This means that we shall apply a partial reasoning. We shall ignore the roles the prices of other goods and inputs may play in the demand for electricity. This means that we are assuming an upper price limit for electricity which makes consumers either stop using energy or change over to other sources of energy. Covered by this representation is the substantial potential for substitution between oil and electricity for heating purposes. We shall also ignore the role that income plays in electricity consumption. The load curve is approximated with a step function by dividing the year into n periods of varying length but with a constant load within each period. With an effect demand Q_i in period i of length t_i hours the energy consumption in this period is: $E_i = t_i Q_i$.

For every point of time within a period a demand function for effect in the market is given:

$$(1) \quad Q_i = f_i(P_i^*),$$

where Q_i is the effect demand, P_i^* the price of effect expressed in kr/kW, and $f_i(\cdot)$ the demand function. In order to obtain a price magnitude expressed in kr per kWh we can divide the effect price by the given number of hours in the period. We then get the demand function:

$$(2) \quad P_i \equiv \frac{P_i^*}{t_i} = \frac{f_i^{-1}(Q_i)}{t_i} \equiv P_i(Q_i), \; P_i' < 0$$

It is reasonable to assume that the demand function will vary in the course of the year; the willingness to pay is greater during the winter than in the summer.

In this partial set-up, consumers' total utility of the energy consumption is calculated as the time weighted sum of the area under each demand curve (2):

$$(3) \quad U = \sum_{i=1}^{n} t_i \int_{\zeta=0}^{Q_i} P_i(\zeta)d\zeta$$

With the same demand structure (2) from one year to the next, we only need to plan for one year.

As an objective function for the planning problem we use the total utility (3) minus minimized costs, see chapter VI. In that chapter the utilization time t_u for the <u>whole year</u> was given. In this chapter this is not assumed. The variables to be determined are total energy capacity, E, the maximum load which the system is capable of providing, Q, and the energy consumption and effect load in each subperiod. The length of the subperiod, t_i, is assumed to be exogenously given. The actual energy consumption is defined by $E^f \equiv \sum_{i=1}^{n} t_i Q_i$.

We get the following maximization problem:

(4)

$$\underset{E,Q_1,..,Q_n,Q}{\text{Maximize}} \sum_{i=1}^{n} t_i \int_{\zeta=0}^{Q_i} P_i(\zeta)d\zeta - C(E^f,E,Q)$$

under the conditions

a. $Q_i \leq Q, \quad i = 1,...n$

b. $\sum_{i=1}^{n} t_i Q_i \leq E$

c. $E,\ Q,\ Q_i(i=1,..,n),\ P_i(Q_i.)(i=1,..,n) \geq 0$

and where $E^f = \sum_{i=1}^{n} t_i Q_i$

The cost function $C(\cdot)$ is slightly different from the minimized cost function in chapter VI. C'_{E^f} takes care of marginal variable cost (assumed to be proportionate to capital cost in chapter VI). C'_Q expresses the marginal cost of expanding the load capacity. Transmission and distribution costs are suppressed for expository reasons. In chapter VI a given and constant utilization time, t_u, was assumed. Q could therefore be replaced by E/t_u.

(4.a) says that the output in each of the subperiods cannot be greater than the load capacity. (4.b) tells us that energy consumption in the course of the year, $E^f = \sum_{i=1}^{n} t_i Q_i$, cannot exceed the energy capacity. We note that no limit to energy consumption has been introduced in each of the subperiods, but only in the energy consumption on an annual basis. This means that it is presupposed that energy capacity from one subperiod can be transferred to the next. In a hydro power system this is realistic, since hydro power can be stored in reservoirs: Water from precipitation and the melting of ice/snow in spring, summer, and autumn can be stored for use during the winter. Water used for the production of energy consequently possesses in principle a shadow price, a water value. This is shown below. Storing and run-off are thus repeated from year to year. The stochastic aspect of this matter has been dealt with by Rødseth in chapter XIV. Other stochastic aspects are discussed by Bjerkholt and Olsen in chapter XIII.

The Lagrange function for the non-stochastic problem (4) is:

$$
\begin{aligned}
L = {} & \sum_{i=1}^{n} t_i \int_{\xi=0}^{Q_i} P_i(\xi)d\xi - C\Big(\sum_{i=1}^{n} t_i Q_i, E, Q\Big) \\
& - \sum_{i=1}^{n} \lambda_i \,(Q_i - Q) \\
& - \lambda_{n+1} \Big(\sum_{i=1}^{n} t_i Q_i - E\Big)
\end{aligned} \tag{5}
$$

If the cost function is assumed to be convex, the objective function is a concave function. The functions of the constraints are linear and consequently convex. Kuhn-Tucker's theorem for concave programming can therefore be used to give necessary and sufficient conditions for solution of problem (4).

Assume that $(Q_1, \ldots, Q_n, Q, E) \geq 0$ is a solution. There then exist numbers $\lambda_1, \ldots \lambda_n, \lambda_{n+1}$, all ≥ 0, so that:

$$
(6)\quad
\begin{array}{ll}
\text{a.} & t_i(P_i(Q_i) - C'_{Ef} - \lambda_{n+1}) - \lambda_i \le 0 \ (=0 \text{ or } Q_i = 0);\ i=1,\dots,n \\
\text{b.} & -C'_Q + \sum_{i=1}^{n} \lambda_i \le 0 \ (= 0 \text{ or } Q = 0) \\
\text{c.} & -C'_E + \lambda_{n+1} \le 0 (= 0 \text{ or } E = 0) \\
\text{d.} & Q_i \le Q;\quad i=1,\dots,n \\
\text{e.} & \sum_{i=1}^{n} t_i\, Q_i \le E \\
\text{f.} & \begin{cases} Q_i \ge 0 \ \ P_i(Q_i) \ge 0;\ i=1,\dots,n \\ Q \ge 0 \ \ \ E \ge 0 \end{cases} \\
\text{g.} & \lambda_i = 0 \text{ or } Q_i = Q \\
\text{h.} & \lambda_{n+1} = 0 \text{ or } \sum_{i=1}^{n} t_i\, Q_i = E
\end{array}
$$

The λ's can be given a price interpretation as objective function units (here kroner) per unit of the respective constraint variable (here kW for (4.a) and kWh for (4.b)).

In this non-stochastic case it would be optimal to utilize the energy capacity fully, i.e. (6.c) applies with equality. Obviously, it would also be optimal to utilize the maximum load for at least one period, i.e. (6.d) applies with equality for at least one i. We shall furthermore assume that we have an interior solution, i.e. (6.a-6.c) applies with equality.

From (6.a) we then get:

$$(7)\quad P_i(Q_i) = C'_{Ef} + \lambda_{n+1} + \frac{\lambda_i}{t_i} = C'_{Ef} + C'_E + \frac{\lambda_i}{t_i}$$

The term $(C'_{Ef} + \lambda_{n+1})$ takes care of energy shortage; C'_{Ef} is the current marginal outlay associated with energy production, whereas λ_{n+1} is a shadow price associated with the size of the energy store. The greater λ_{n+1} is, the greater is the value of an increase in the stored energy and consequently the scarcer is the energy. λ_{n+1} may be called the water value. The optimal water value in our non-stochastic model should at the point of adjustment be equal to the marginal cost associated with an extension of the water energy reservoir. $(C'_{E^f} + \lambda_{n+1})$ represents the energy part in an optimal price tariff for electricity. The last term, λ_i/t_i is a shadow price for load capacity and therefore reflects the shortage of the load capacity. λ_i/t_i represents the load part in a price tariff for electricity.

3. Optimal prices and optimal expansion in electricity supply

The price rule (7) tells us that an optimal price for electricity should consist of an energy term and a load term. A tariff that only penalizes the consumption of energy at a fixed rate is not an optimal rate. An optimal rate requires that electricity consumption in periods when the load capacity is fully utilized should be penalized extra.

In an optimal rate prices will vary in the course of the 8 760 hours of the year so that in the period, or periods (several may be involved), in which all capacities are fully utilized, the price is highest (peak-load pricing). From (6.g) it follows that if peak load in merely one period would be optimal, let us say period 1, then $\lambda_1 > 0$, whereas $\lambda_2 = \lambda_3 \ldots, \lambda_n = 0$. We can then see from (7) that prices will be equal in all periods in which $\lambda_i = 0$. It should be noted that in our one-period, non-stochastic framework λ_{n+1} is the shadow price of water used in energy production in each period. Since the supply of water is not assumed to vary over the year, the shadow price will be the same in all subperiods. In our framework the sum of C'_{E^f} and λ_{n+1} $(=C'_E)$ is therefore equal to short run marginal cost.

On the basis of equation (7) we can formulate the following general rule for pricing policies (which also applies in cases of stochastic demand and supply):

> Prices should be made equal to short run marginal costs during periods when capacity is not fully utilized. In peak-load periods prices should be increased sufficiently to keep demand within capacity limits (peak-load pricing).

The optimal prices can also be used to characterize optimal development of energy capacity and maximal effect. If the load capacity is fully utilized in one period only, say in period j, the optimal price in this period will be

$$P_j(Q_j) = [C'_{E^f} + C'_E] + \frac{C'_Q}{t_j},$$

i.e. equal to the sum of the short run marginal cost and the marginal cost of expanding the load capacity. This sum is the long run marginal cost. The optimal capacity is therefore characterized by the optimal peak load price being equal to long run marginal cost. Moreover, summing over i in (6.a) and employing (6.b) and (6.c), we get:

$$\sum_{i=1}^{n} t_i P_i(Q_i) = \sum_{i=1}^{n} t_i [C'_{E^f}(E^f,E,Q) + C'_E(E^f,E,Q)] + C'_Q(E^f,E,Q)$$

By dividing with the number of hours in the year on both sides of this relation we get the weighted average price in the course of the whole year on the left side and the system's total long run marginal costs, on the right side:

$$(8) \quad \bar{P} \equiv \sum_{i=1}^{n} \frac{t_i}{8760} P_i(Q_i) = C'_{E^f}(E^f,E,Q) + C'_E(E^f,E,Q) + \frac{C'_Q(E^f,E,Q)}{8760}$$

We can formulate the following rule for deciding the expansion of electricity production:

> Optimal development, i.e. optimal energy and effect capacity expansion, is characterized by the average price in the course of a year being equal to long run marginal costs put on an annual basis.

An essential point worth nothing is that planned development of energy and load capacity, capacity utilization in every period, and prices must be decided on simultaneously. Pricing is an integral part of investment planning. It should be noted that long run marginal cost (LMC) is dependent both on energy and load capacities and in optimum the value of LMC is obtained by inserting the optimal values for E and Q from the solution (6).

A characteristic feature of electricity development in Norway after the war is that maximum load capacity has remained at a level that is one to two times higher than maximum load. The development has been clearly energy-oriented. Even though in the above analysis we have ignored the stochastic aspects of rainfall and temperature, it looks as if planning has not taken sufficient account of the fact that the development of capacities and prices must be determined simultaneously. In particular it looks as though the load curve (see figure 3) has been taken as a given datum, and not as a magnitude that can be influenced by pricing.

In the (very) short run, energy and load capacities, E and Q, are given magnitudes. Is it then possible, with the aid of the analysis above, to state whether the energy and load capacities are sufficient, and whether the actual prices are optimal?

From our arguments above we concluded that capacities are optimal when long run marginal costs in the peak period are equal to the peak load price, or as a rule of thumb, when the average price is equal to the long run marginal costs put on an annual basis. If the average price is less than the long run marginal costs, too large capacity exists, and vice versa if the average price over the year is greater than long run marginal costs.

Let us assume that we observe the average price over the year to be less than long run marginal costs. This means that there is excess capacity.

How should prices be set, given that excess capacity exists? In this case too, optimal prices can be deduced from the price rule in equation (7). The objective function in (4) now has to be maximized with respect to $Q_1, \ldots Q_n$, with E and Q as given magnitudes. The prices, in other words, are to be equal to current marginal costs or higher, and if higher, equal to the prices that ensure that the given capacity is not exceeded. Since the existing power stations have no other alternative use than the production of electricity, and since all optimal prices, given a situation of excess capacity, cover short run marginal costs, full utilization of the capacity that has been developed would be optimal. Since the given installations have no other alternative use, it would not be optimal either to reduce capacity. Given the "mistakes" that have been committed, it is still necessary to arrange things optimally. The costs embedded in development are "sunk costs", and must not be allowed to influence any decision as to how and to what extent the given installations are to be utilized.

The question now is what further steps should be taken in a situation in which excessive capacity has been developed. As long as the demand curves are constant, not much can be done beyond what has been described above. A new situation arises if we assume changes in demand over time. This brings us to a dynamic analysis. Through economic growth the demand curves can move outwards, so that a future point of time exists at which the average prices over the year can once again equal long run marginal costs. Until this point of time is reached, existing power production should be rationed with the aid of optimal prices. Starting with this future point of time we can once again envisage a possible increase in production capacity.

In a dynamic analysis the two crucial issues are <u>when</u> and <u>how</u> the power system should be expanded. The static framework above does not include a distinction of this kind. The question will then be whether the rules for optimal prices and optimal capacities also apply in a more realistic case of this dynamic nature.

It is immediately obvious that the price rule applies. The problem of deriving the "once and for all" optimal capacity has to be replaced, however, by finding the optimal timing of new projects (the "when" issue) and the optimal dimensioning of each new plant (the "how" issue). A dynamic analysis of these problems is presented by Rødseth in chapter XIV, where necessary conditions are derived from the maximization of the present value of future consumers' surplus. It is shown that the necessary condition for an optimal timing of new projects is that the prices (averaged over the year) equals the long run marginal cost (LMC). Along the optimal expansion path new projects should be included at a rate or at dates such that price equals LMC, i.e. equal to the average cost in the last project included in the system. The necessary condition for an optimal dimensioning of each new plant is that marginal construction costs equal the discounted, future equilibrium prices put on an annual basis. Thus, the LMC criterion (8) in the static framework is replaced by these two conditions governing the timing and dimensioning of new projects.

The discussion of the four planning problems set out in the introduction to this chapter may be summarized as follows:

In an economy aiming at economic efficiency the optimal conditions governing pricing and investments follow from maximizing the discounted value of consumers surplus in all future time. Prices should be set equal to short run marginal cost and, if higher, equal to the prices which keeps demand within the given capacities. The condition for an optimal allocation of electricity consumption is that all consumers should be charged the same net price of electricity (purchasers' price less transmission and distribution costs).

The conditions for the optimal timing of the expansion of the power system, that is investment in electricity supply, say that project no. i should be put into operation at time τ_i if the average price (average over the year) is equal to or higher than the average cost of project no. i put on an annual basis. (See chapter XIV for further details.) The average cost of project no. i can be considered as an estimate of the long run marginal cost of the total power system. This is why the condition for the optimal timing of the expansion of the system has been labelled "The long run marginal cost criterion". In order to achieve optimality both in the short and in the long run future equilibrium prices and investments should be decided upon simultaneously.

Note that the long run marginal cost criterion should be used as a guidance for investments in the power system. It is not a pricing rule since prices should be set according to the rule mentioned above. The prices should at any moment of time clear the markets.

Finally, the condition for the optimal design of a particular plant (optimal dimensioning of each plant) says that the marginal construction cost of a particular plant should be equal to the discounted, future equilibrium prices.

An implication of the long run marginal cost criterion is that projects should be selected in an order which will minimize total costs. With the reservations made in chapter VI, this implies that the project with the lowest total average cost should be chosen first. The resulting long run marginal cost will therefore be increasing over time due to the fact that some watercourses are cheaper to develop than others (decreasing returns to scale).

4. Some observations on the electricity policy in Norway

Electricity prices

Electricity supply in Norway has been organized in such a way that the owners of power stations (municipalities, counties, private firms and the State, represented by the State Power Plants) separately have set prices for power so that they just cover historical costs. Since hydro power production requires a great deal of capital, this means that electricity produced by old power stations is offered at a very low price compared with electricity from new power stations. As a consequence, the price of electricity varies from one consumer to another, depending on the contract the consumer has secured for future supplies of electricity and what area he lives in. Consumers with favourable long term contracts are foremost to be found in the energy intensive industries. For other consumers, prices have been evened out to some extent in recent years by graduated consumer rates, graduated value-added tax, and improved facilities for transmitting electricity. There is, however, no doubt that the prices charged for electricity in Norway vary considerably from one consumer to another if we consider the entire power-supply situation under one heading. From an efficiency point of view the same commodity should not command a different price unless some economic reason for this price differential exists. The law of indifference does not seem to apply in the Norwegian electricity market.

In the retail market for firm power until recently the typical contract between the consumer and the power distributing firm is for a fixed amount per period (year) to be paid for the right to purchase electricity at a cheap rate when the load is below a certain limit and at a higher rate beyond this limit. To a certain extent this results in prices varying in the course of a day and a year, as described in the price rule (7). (This rate structure has, however, been changed in recent years. A flat energy tariff has been introduced in several regions.) In the wholesale market a similar system operates. The seller in this case is first and foremost the State Power Plants. Purchasers may be utility companies or power stations with insufficient production. The total agreed price consists of an energy term and a load term. The energy term varies in the course of the year, remaining at one level from 16 October and 15 May (winter and spring), followed by a lower level from 16 May to 15 October (summer). State supplies of power to major industrial undertakings comprise the bulk of firm power supplies. Most of this power is delivered under contracts from many years ago and to historical prices inflated by a somewhat unsatisfactory linking to the wholesale price index. At present there is also a power tax of 0.02 kroner per kWh.

Prices in short-term sales agreements, mostly what is called "non-guaranteed power" and "occasional power" (surplus power), show considerably more variation in the course of a year. In the case of non-guaranteed power the seller is not obliged to supply in the event of any restriction on supplies. The sale of occasional power is organized through the Norwegian Power Pool. The prices are set by the Board of the Norwegian Power Pool and reflect seasonal and daily variations in supply and demand. The price is highest during daytime in winter and lowest during summer. Altogether, the short-term prices reveal a pattern with many points of similarity with the pattern discussed above in connection with price rule (7).

The prices of electricity seem to work effectively as rationing factors. This may be concluded from the general observation that at current prices Norway has in recent years not suffered an unduly large number of cases of drops in voltage as a result of overloading, nor has there been an unduly large number of "save electricity" campaigns promted by rapidly diminishing reservoirs. This indicates that current prices have resulted in neither load nor energy capacity being exceeded. It is possible that capacities might have sufficed even with lower prices. This means that prices which ensure that capacity is not exceeded in Norway's power supplies are less than, or equal to, actual prices.

One reservation must be made, and this applies to the distribution of power to major industrial plants. It has been maintained that distribution is not rationed on the basis of price, but on the basis of quota regulations. By way of counter argument it may be maintained that industry has largely been allotted what it has asked for. It is doubtful whether the allotted power would have been demanded without regulations at a price equal to what this power can command in alternative uses.

Official investment calculations for power development projects

The revenue items in the authorities' calculations comprise the sale of firm power and occasional power or surplus power. The items that figure under expenses comprise investment costs (capital investments), compensation for environmental costs etc. and operating costs. It is by no means certain that it would be optimal to deal with the uncertainty arising in connection with electricity supplies by dividing power into only two categories, viz. firm power (with practically one hundred per cent certainty of supply guaranteed) and occasional power (with a very small degree of certainty for deliveries). Further grading of the extent of uncertainty (contingent commodities) might be more efficient, but we shall not pursue this possibility in what follows.

We shall draw attention to two weaknesses in the actual and official calculations of the present value of a power project. One involves the valuation of firm power. As a rule the price of firm power is set equal to LMC or, until a few years ago, set equal to the cost of the cheapest alternative thermal power station rather than the actual equilibrium price that one might expect to obtain during the life of the power project. The cost of thermal power is about twice as high as the actual market equilibrium price. By making a downward adjustment in the overall evaluation for firm power in a concrete planned project (the Upper Otta development project) towards the actual equilibrium price, and assuming that this price will rise at the same rate as the general price level, it has been shown by Olsen and Strøm (1978) that the present value will change its sign from positive to negative. Following this result the Upper Otta project ought not to be implemented until the actual price comes close to the long run marginal costs.

The other weakness involves the expense side. Environmental costs are only introduced in the case of compensation to farming and fishing interests and the like, which are able to prove direct economic losses. In many cases this will account for only a small share of what should be properly understood by environmental costs. Admittedly, costs of this nature are not easy to quantify in a manner acceptable to the parties concerned. A fairly useful calculation might be to decide the minimum environmental costs necessary to make the project unprofitable. A reasonable assumption is that damage to the environment will mean permanent reduction in alternative use of the areas affected. In so far as part of this alternative use is associated with recreational pursuits, it is also reasonable to assume that the willingness to pay will increase over time. For this reason it might reasonably be assumed that environmental costs are permanent, and that they will increase in real terms over time. If environmental costs rise at a fixed percentage rate annually, we get the following expression for the present value (PV) of a project:

$$(9)\quad PV = \sum_{t-1}^{T} (1+r)^{-t}\Delta\Pi - \sum_{t=1}^{\infty} (\frac{1+m}{1+r})^{t} \Delta M - P_K.\Delta K$$

$$= \Delta\Pi \frac{1-(1+r)^{-T}}{r} - \Delta M \frac{1+m}{r-m} - P_K\Delta K,$$

where T is the lifetime of the project, $\Delta\Pi$ represents the constant net income per annum, ΔM the initial environmental costs per annum, $P_K\Delta K$ the investment expenses, r the rate of discount, and m the growth rate for environmental costs.

Transformed to annual costs we get:

$$(10)\quad PV^* \equiv \nu\, PV = \Delta\Pi - \Delta M \frac{1+m}{r-m} \nu - \nu P_K\Delta K,$$

$$\nu = r/(1-(1+r)^{-T})$$

The critical value for the initial environmental cost can be found by solving (9) or (10) for Δ M with PV or PV^* equal to zero. Since the willingness to pay for the environment increases over time, annual costs will be greater than the initial costs. For $r = 0.1$, $m = 0.03$, and $T = 40$ the annual costs will be 1.6 times environmental costs in the first year, $\Delta M(1+m)/(1+r)$. If the growth rate for environmental costs is greater than the discount rate, an arbitrarily small initial environmental cost will be sufficient to render the present-day value of the project negative.

If the discount rate is reduced, annual capital costs will be reduced, but annual environmental costs will increase (future expences will carry greater weight). It is thus not obvious that a lower discount rate will make a power station project economically more profitable.

Is the expansion rate of Norwegian electricity supply too high?

The fact that the actual average annual price is considerably lower than long run marginal cost, while at the same time energy and load capacity have been ample, is a strong indication that the rate of investments in hydro supply has been too high.

Comparing prices and long run marginal cost in Norway in 1980 we will find that on the average the energy intensive industry pay a price less than 50 per cent of the long run marginal cost level. Other consumers pay on the average about 70 per cent of the long run marginal costs.

The present situation in Norway concerning prices and long run marginal costs is as described in table 2.

Table 2. Net consumption, prices and costs of electricity in Norway 1980

	Energy intensive industries, production of pulp & paper	Other industries and households
Net consumption per year, TWh	31.4	43.8
Prices[1)], øre/kWh		
Range	1.0 - 8.2	9.8 - 19.4
Average	5.6	15.4
Long run marginal costs[2)], øre/kWh	11.2	19.4

1) Inclusive the electricity tax, exclusive value added tax.
2) Official estimate, inclusive transmission costs. 7 per cent annual rate used as social rate of discount.

In August 1980 the government announced that the development of some hydro power projects will be postponed for some years. This change of attitude is consistent with a government Report to the Storting (Parliament) earlier that year in which the government declared that the real price of electricity should be increased to reach the long run marginal cost level within 8-10 years, with some vague exceptional clauses for parts of the energy intensive industries.

In the Storting the majority decided to tie the prices charged from the energy intensive industries to their product price. The implication of this index rule is, of course, that the price of electricity paid by the energy intensive industry will differ from the prices paid by others. Moreover, there are reasons to believe that these electricity prices will continue to be less than the LMC-levels.

The prices of electricity paid by the energy intensive industries in Norway may be among the lowest in the world. However, this is not due to low costs in expanding the system, but due to the internal pricing of energy. If the Central government succeeds in raising the price of electricity up to the long run marginal cost level, many of the Norwegian energy intensive firms will loose their present comparative advantage. Compared to other countries with still undeveloped hydro power potential the costs of expanding the power system are high in Norway. The investment costs per kWh for instance in a large scale hydro power project in Mexico in 1980 is around 16 cent/kWh (Barker and Brailovsky (1981)). Hence the Mexican social rate of discount could be allowed to be as high as 14.5 per cent per year before the long run marginal cost in Mexico exceeds the Norwegian cost level.

On the background of these observations it could be concluded that the timing of investments in electricity supply is not optimal. The Norwegian watercourses are developed too fast compared to the return to investments in other branches of the economy.

In the actual design of hydro power stations in Norway future long run marginal cost figures are used instead of future prices. Since prices differ from long run marginal cost and will continue to do so for some years ahead, the most efficient design is not achieved.

Second-best considerations in industrial policy

A view that has been advanced in the discussion of electricity supply is that since the price is not equal to long run marginal costs in other sectors of the economy, neither should this be the case in the electricity sector. The argument is based on the idea of the second-best solution. But it is by no means to be taken for granted that an optimal second-best strategy is that the price of electricity is set lower than long run marginal costs. On the contrary, there are good reasons for believing that the opposite might be the case.

In analytic descriptions of open economies like in Norway a common distinction is that of sheltered and exposed industries. The sheltered industries include those with protected markets, such as the eartbenware and stoneware industry, the production of beverages and tobacco, the printing industry, publishing etc. For political reasons the sheltered industries also include farming, the processing of agricultural and fish products, as well as the service sectors. If the price of the product is higher than the marginal cost in an industry, it is reaonable to assume that it belongs to the sheltered group. Sectors exposed to competition on the other hand must adjust to prices on the world market. There would consequently be little opportunity of deviating from a price equal to marginal costs (i.e. private marginal costs), assuming that the aim is to maximize profits. Among sectors exposed to competition are to be found those that are most polluting as well as the most energy-oriented. The chemical industry, the production of metals, and pulp and paper therefore, belong to the section of the economy in which the price is probably less than or equal to the social long run marginal costs. The supply of power is a sector in which the price is clearly less than long run marginal costs, though for other reasons than for the three exposed industrial sectors mentioned above.

The productive power of a country is not used in the most effective manner if prices are not equal to social marginal costs. Production in sectors sheltered from competition, or subject to positive indirect effects, is too small. It follows then that production in sectors with no deviation between price and marginal costs, or where negative indirect effects are involved, will be too large. The question then arises, what should be done to rectify the faulty allocation, and in particular whether "price equal to long run marginal costs" is useful as an investment criterion in electricity supply, when this criterion is not used in other sectors of the economy.

There are two possible answers to this question. The "first-best" answer is to intervene with charges, subsidies, and other measures vis-à-vis the industries with price different from social marginal costs. In this case a long-term balance in the energy market with the price equal to long run marginal costs should be sought.

The other way out might be called the "second-best" answer. This tacitly assumes that the authorities do not possess sufficient means for pursuing a "first-best" alternative. The authorities, however, have the final say in deciding electricity rates, and it is therefore maintained that in the second-best alternative electricity prices (on a long-run basis) should be set such that the erroneous allocations present in the rest of the economy can be counteracted. The question then is whether, in this second-best alternative, the price of electricity on a long run basis should be greater, equal to, or less than long run marginal costs.

In the second-best alternative the price of electricity should be set to counteract allocational inefficiencies in the total economy. This means that the price (on a long-term basis) should be such that resources are transferred from sectors where production is excessively great to sectors where it is too small. This means that the price of electricity should be set higher than marginal costs if a high electricity price induces a transfer of resources from those producing too much to those producing too little. This is precisely the case in Norway. An increase in the price of electricity penalizes the exposed, energy intensive, and polluting sectors, and provides sufficient resources for an expansion in the sheltered sectors. Higher electricity rates also reduce housing consumption, which is in accordance with the "second-best" philosophy as housing is another example of an industry with price less than long run marginal cost. A rise in the price of electricity in Norway thus implies a transfer of resources from sectors where the price is less than or equal to marginal costs to sectors where the price is greater than or equal to marginal costs.

XIII. FIRM POWER AND UNCERTAINTY IN HYDROELECTRIC POWER SUPPLY

by

Olav Bjerkholt and Øystein Olsen

1. Introduction

In an electricity supply system dominated by hydroelectric power, as in Norway, variations in natural conditions such as rainfall will to some extent be decisive for the production capacity in individual years. Even though there are ways of hedging against unfavourable conditions, e.g. by building larger reservoirs or by energy trade agreements with neighbouring countries, the inherent uncertainty is of central importance in the planning process. In Norway this has influenced both the type of contracts in the power market and the dimensioning of the supply system.

The power supply system in Norway is dominated by government owned companies, about 35-40 per cent of the total production of electric energy is produced by the Central government and an even greater share comes from companies owned by counties and municipalities. Only about 12-15 per cent of the production of electric energy comes from private companies. On this background the planning of the power supply system has naturally been a task for the Central government.

In the market for electric power one can distinguish between categories of electricity with differing degrees of certainty of delivery. With some simplification we shall consider only two categories: firm power and surplus power. Most of the production is delivered to power distribution companies and final users as firm power with high degree of certainty of delivery, while excess power is sold via the Norwegian Power Pool as surplus power. Among the final users of electricity only the power intensive industry has a contractual guarantee of power deliveries. This industry comprises Manufacture of industrial chemicals, Iron and steel works, Ferro-alloys works, Primary aluminium works and Other non-ferrous metal works and accounts for 35-40 per cent of total net consumption of firm power.

For other final users the guarantee of delivery rests on import possibilities being available and on the built-in "overcapacity" of the supply system, i.e. the difference between the average production potential and the firm power commitments. It is thus an important feature of the Norwegian electricity supply system that the expected production capability is considerably larger than the demand for firm power. The difference has recently been estimated by the Norwegian Water Resources and Electricity Board to be around 10 TWh or 12 per cent of total gross consumption of firm power. It is therefore clear that this way of securing a high degree of certainty of deliveries has considerable costs for society as a large amount of capital is tied up in providing the "overcapacity".

The purpose of the ensuing analysis is to discuss the principles of how uncertainty in supply should influence the planning of the expansion of the electricity supply system. We shall also deal briefly with uncertainty on the demand side and discuss how that should be taken into consideration in the planning process.

2. The concept of firm power

Actual production of a hydroelectric power supply system varies considerably within normal variations of rainfall. The variations can be considerably mitigated by the combined use of reservoirs and power grids which allows a surplus of power in one area to be transferred to cover a deficit in another part of the country. Total reservoirs in Norway normally hold a store of energy amounting to somewhat less than half of annual production. This may seem a comfortable amount relative to annual variations in production, which rarely exceed 10-15 per cent. The main function of the reservoirs is, however, to coordinate supply and demand within years.

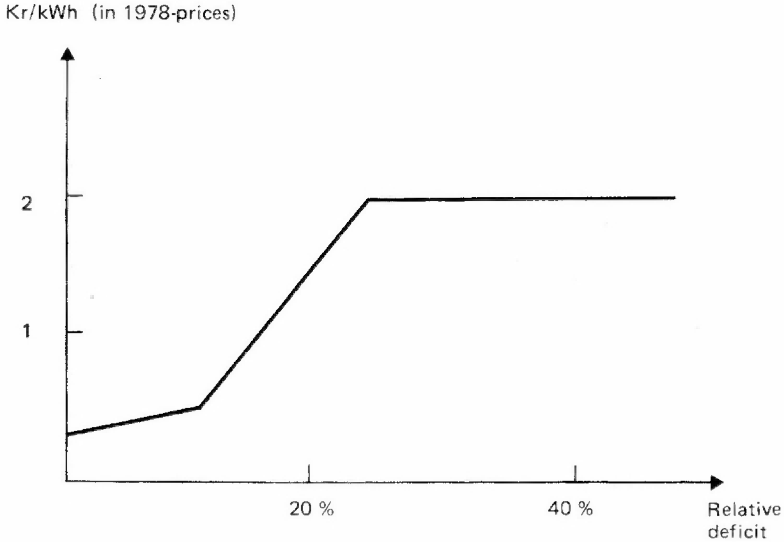
The satisfaction of a relatively stable demand with a stochastic supply could be attempted in several ways. One way would be to use the price mechanism to full extent to provide a clearing of the market every year. Another way would be rationing at fixed prices. In Norway an institutional organization of the market system has been chosen that avoids both of these extremes. The uncertainty on the supply side has been dealt with by introducing categories of power supply contracts with different degrees of certainty of delivery. As mentioned above the main distinction is between firm power and surplus power. One interpretation of the actual organization of the electricity market in Norway is that firm power and surplus power should be treated as different categories of power with different certainty of delivery. It may be shown (see e.g. Malinvaud (1972)) that the existence of such "contingent markets" is a necessary condition for maximum social benefits in a world where uncertainties prevail. It may be doubted, however, that firm power and surplus power in the Norwegian electricity market may actually be regarded as "contingent goods". An alternative interpretation is that the consumers in Norway do not actually choose between deliveries with different degrees of certainty, so that electricity on the demand side just as well may be regarded as a homogeneous product. The distinction between firm power and surplus power is, however, essential as regards the actual determination of prices. The price of firm power is determined by the Storting (Parliament) for rather long time intervals. Traditionally the price has been kept low and stable; recently the Storting has approved of a proposal to raise the price gradually toward long run marginal cost, presupposing that special consideration is given to power intensive export industries also in the future. The price of surplus power, on the other hand, is determined as an equilibrium price in the short-term market, with variations over the year as well as during the day. On the basis of these facts and the above considerations the demand for firm power will in the following be interpreted as the consumers' long-term demand for electricity at a price announced some time in advance. The demand for surplus power, on the other hand, will be associated with the short-term demand for electricity.

In earlier years the concept of firm power in the planning of the Norwegian electricity supply system, was tied to physical criteria of certainty of delivery: The firm power potential of a given system was defined as the production capability in the third worst year in a period of 30 years. The production potential, thus defined, was decisive for the amount of firm power commitments that would be made by power companies. The same consideration determined the dimensioning of the power supply system in a future year on the base of an assumed demand for firm power in that year.

In later years the concept of firm power is no longer tied to this physical definition. Using simulation models of the power supply system, the firm power potential of the system is determined by minimizing a short-term cost function. This function includes terms representing gain and loss to society when actual production is above or below the demand for firm power. While the gain of excess production is represented in the cost function by the income from the sale of surplus power, the loss from a deficit of firm power is determined from a curve estimated by a special committee (Tørrårskomitéen (1969)). The shape of this curve is as given in figure 1.[1)]

1) The curve is assumed to represent (marginal) costs due to lack of energy. This type of deficit may be announced to consumers some time in advance, as opposed to deficiences due to lack of system capacity which occur without prior notice.

Figure 1. Rationing costs used in the planning of the Norwegian electricity system: marginal costs as a function of relative deficit



We shall later comment upon the shape of this curve and the importance this has for the optimal determination of firm power commitments. The purpose of our discussion is to outline the proper framework and the principles for determining planned firm power commitments within the existing institutional arrangements as represented rather schematically in our formal framework. Our approach is not totally different from the one actually used by the Norwegian Water Resources and Electricity Board. Obviously in our theoretical analysis we must neglect many problems of the actual planning process, but we hope that we can offer a rather precise analytic statement of certain main economic aspects of the overall planning problem.

3. A simplified framework for the planning of hydroelectric power supply

The issue under consideration is the planning of the hydroelectric power supply in an economy with decentralized decisions on the demand side but with strong influence by a central authority over the supply side. The aim of the planning is two-fold:

(1) The dimensioning of the supply system: How much capacity to produce effect and energy should be provided for future years.

To deal with this problem it is advantageous to apply a criterion of optimal expansion. A criterion of this kind, called the long run marginal cost criterion, is derived in a very simplified form in section 4 and states roughly that the long run marginal cost should correspond to the price which the users of electricity are willing to pay. The dimensioning problem includes cost minimization with regard to providing new capacity. This involves the choice of sequence for construction of hydroelectric power projects under consideration, the introduction of thermal power, agreements on imports/exports of electricity, the construction of reservoirs to prevent energy losses through water overflow, and the construction of transmission lines to facilitate power pooling.

(2) The capacity utilization of the existing system: How should the existing reservoirs, waterways, capacity of installed machinery and transmission lines be utilized to prevent overloading in any part of the system but at the same time provide an optimal use of the existing production capacity. One solution to the capacity utilization problem is to establish a price structure that will provide an equilibrium in the market for electric power at any point of time. In the absence of a perfect price structure the capacity utilization problem may also include how to deal with rationing situations caused by insufficient expansion or by an abnormally low production in a drought year.

In our treatment we will focus on just one or two aspects of the planning of the hydroelectric power supply that seem to be of particular importance from an economic point of view and ignore completely (and for good reason!) the engineering aspects of planning.[1)]

The dimensioning problem and the capacity utilization problem cannot be dealt with independently. The use of the long run marginal cost criterion presupposes that the prices used are equilibrium prices with regard to capacity utilization. The dimensioning of the system will on the other hand determine the future equilibrium prices. A third interrelation between the two planning problems is caused by the uncer-

1) The planning of hydroelectric power supply is also the main issue of the contributions of Steinar Strøm and Asbjørn Rødseth in chapters XII and XIV, respectively, of this volume.

tainty both on the supply and on the demand side. The uncertainty represented by the probability distribution of the production of electricity and the consumers' short-term reactions to price changes will determine how much it is economical to increase capacity and capacity utilization.

Our analysis is confirmed to the institutional framework described in section 2, with hydroelectric power marketed either as firm power with guaranteed delivery at a price announced some time in advance, or as surplus power with price set in a short-term equilibrium market.

One basic assumption underlying this framework is that the long-term demand for electric power or, equivalently, the demand for firm power, is a meaningful and operational concept that can also be empirically estimated as a function of price. We shall write this function:

$$(1)\quad \bar{p} = f(\bar{x}) \quad ; \qquad f' = \frac{df}{d\bar{x}} < 0$$

where $\bar{p}$ is the price of firm power and
$\bar{x}$ is the demand for firm power.

The interpretation of this curve as a long-term demand function means that it represents the consumers' demand for electricity when they have adjusted their stocks of production equipment, heating implements etc. to the announced price of firm power.

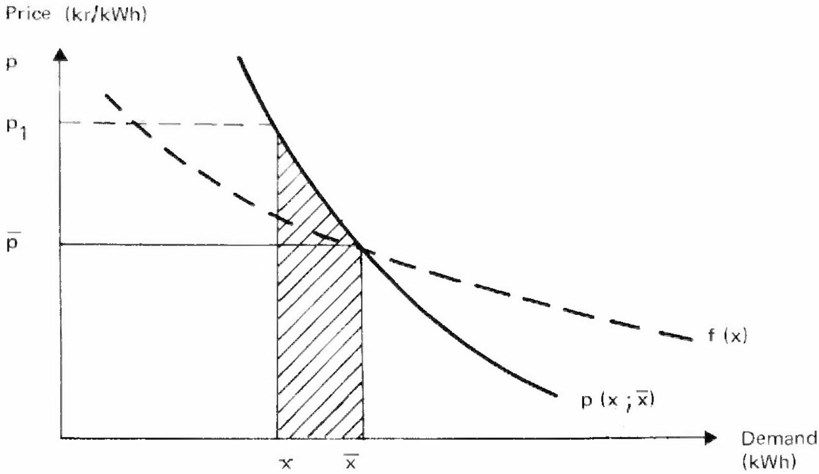
Although a relationship like (1) may be said to represent the most important aspect of the consumers' behaviour with regard to electricity demand, we shall see that the short-term behaviour is no less important with regard to optimal planning decisions when the existence of uncertainty in supply (or demand) is taken into consideration. It is intuitively clear that the loss to society indicated in figure 1 is closely connected with the consumers' short-term demand for electricity. We assume that the short-term behaviour can be represented by a short-term demand function

$$(2)\quad p = p(x;\bar{x})$$

expressing the consumers' more immediate reactions to changes in the price of electricity.

We have in figure 2 made a sketch of the demand structure for electricity which will serve as a basic model framework for the following analysis.

Figure 2. The demand structure for electricity



In figure 2 the dotted curve, $f(\bar{x})$, denotes the aggregate demand for firm power. If in figure 2 the price of firm power is stipulated to $\bar{p}$, the long-term demand for electricity, $\bar{x}$, is determined by the $f(\bar{x})$-curve. But if the consumers who have adjusted their capital stocks to $\bar{p}$ actually are faced with another electricity price in the market, e.g. $p_1 > \bar{p}$ in figure 2, the actual demand for electricity, x_1, is determined by the short-term demand relation, $p(x;\bar{x})$. It may reasonably be assumed that the latter demand function is steeper than $f(\bar{x})$; the demand is more elastic in the long run than in the short run.

Furthermore we have assumed that the consumers' short-term reactions on price changes depend on their "planned" demand for firm power, so that in the specified demand structure in figure 2 $\bar{x}$ is an argument in the short-term function.

It is perhaps less obvious how the short-term curve to the left of $\bar{x}$ of the curve can be interpreted as short-term demand and the interpretation is indeed dependent upon institutional arrangements provided to handle the rationing situation that emerges when firm power demand is larger than the available supply.

One interpretation could be that there are rationing procedures administrated by authorities so that the curve expresses the implicit utility losses to society of having consumers whose demand for electricity at the given price of firm power cannot be satisfied. Another interpretation is that rationing takes place via a market in which the consumers can trade the rights to firm power between themselves. In the latter interpretation consumers with smaller marginal utility losses from unsatisfied demand may be better off by trading their rights to firm power to consumers with bigger losses. In this rather idealized way everyone may be better off than a proportionate rationing of all consumers.

Gossen's law derived from the traditional theory of demand implies that if consumers are free to adjust their demand to a set of prices, marginal utilities will be proportionate with commodity prices, with the marginal utility of income as the common factor. If the marginal utility of income is constant, the demand function for electricity may be interpreted as a relation expressing the change in the utility of the consumers following a small (marginal) change in their electricity consumption. With respect to changes in the consumption of electricity caused by stochastic variations on the supply side it is obvious that it is the short-term demand function which is relevant for expressing marginal utility. Disregarding the possible existence of externalities in production or consumption we can measure gains or losses to society from variations in electricity supply by the consumers' marginal changes in utility.

On the basis of the demand structure (1) - (2), as represented in figure 2 and the interpretation of the demand curves as expressing marginal utilities, we define the utility for the society as a whole as a function of actual electricity production x_p as:

$$(3) \quad U(x_p;\bar{x}) = \int_0^{\bar{x}} f(x)\,dx + \int_{\bar{x}}^{x_p} p(x;\bar{x})\,dx$$

The first term in (3) represents the utility of consuming a certain quantity of firm power, $\bar{x}$. When the demand function $f(x)$ is assumed to express marginal utility, total utility of a specific quantity of electricity is calculated by integrating this curve. Now, if the actual supply of electricity, x_p, for some reason or another, deviates from $\bar{x}$ (the quantity which the consumers expect in their long-term adjustment), the stocks of production equipment in firms and durable goods in households will at the same time deviate from their optimal levels. In this case the consumers' willingness to pay for electricity is determined by the short-term curve, and the gain or loss in utility caused by the fact that $x_p \neq \bar{x}$ is thus determined by integrating the $p(x;\bar{x})$ curve from $\bar{x}$. This explains the specification of the second term in (3).

In the optimizations pursued in sections 4 and 5 below an expression identical or similar to (3) is differentiated with regard to $\bar{x}$ and set equal to zero. It can then be seen that the shape of f(x) to the left of $\bar{x}$ has no importance for the optimal solution. Only the value of f(x) at $\bar{x}$, i.e. $\bar{p}$, and its derivative at this point, $f'(\bar{x})$, is of importance.

A "society loss curve" like the one in figure 1 may be constructed on the base of the part of the short-term demand curve which lies to the left of $\bar{x}$. Using the utility function (3) in the analysis of how uncertainty should influence the planning of the electricity supply system may therefore be regarded as a transformation of the planning problem into a formal framework that is well known and founded in economic theory.

The <u>production structure</u> in electricity supply will, in our simplified model, be represented by a long-term cost function

$$(4) \quad C = G(x_M)$$

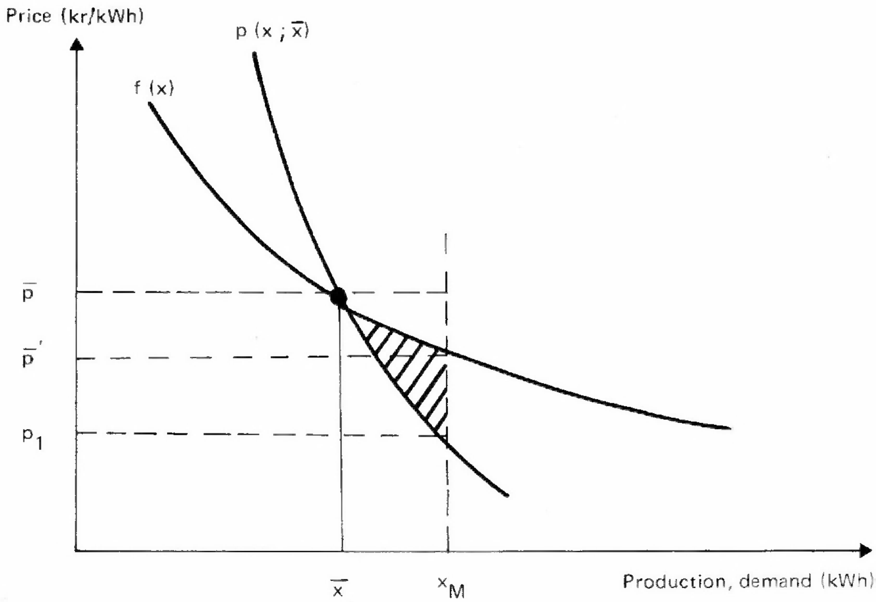
where C denotes total costs and x_M denotes mean production in the hydro power system. As mentioned in the introduction, the production of electric power will in the following analysis mostly be regarded as stochastic, in the sense that it may be represented by a density function $\phi(x_p;x_M,\sigma)$ with mean x_M and variance σ^2. By the specification (4), x_M in our model is regarded as a measure of the capacity of the hydro power system.

It should be admitted that several aspects and difficulties relevant for a discussion of limitations in electricity supply are neglected in the simplified model which we have postulated above. The loss for society following an "underproduction" ($x_p < \bar{x}$) will e.g. by highly dependent on how the limitations in supply affect different groups of consumers. Furthermore, in any situation with limitations in the electricity supply the duration of the period of "rationing" will be decisive for the costs to society. However, the purpose of the following analysis is to help to clarify some basic principles regarding how uncertainty should influence the planning of the electricity sector, and we also believe that many of the results would be valid in a more complex model.

4. Optimal firm power commitments with non-stochastic demand

In this section we shall discuss the determination of optimal firm power commitments when the demand structure, represented by the long-term and the short-term demand curves, is regarded as non-stochastic and fully known. As an introduction it may be useful to look at the case when neither demand nor electricity production is stochastic, i.e. when actual production equals the production capacity as measured by the annual mean production at anyone time.

Figure 3. A simplified case of non-stochastic demand: losses to society when firm power demand is too low.



$\bar{p}$ = the given price of firm power to which the consumers have adjusted their long-term demand

$\bar{x}$ = the demand for firm power associated with $\bar{p}$

x_M = the given production capacity

p_1 = the equilibrium price in the short-term market

$\bar{p}'$ = efficient price of firm power

In the situation depicted by figure 3 we have incorporated our institutional and behavioural assumptions of a demand for firm power identical with long-term demand and a short-term market for electricity when the supply deviates from firm power demand. The purchasers of electricity have adjusted their stocks of capital equipments to the firm power price $\bar{p}$. Actual production is x_M, larger than the firm power demand. The capacity of the supply system will be fully used when the excess supply is offered at price p_1.

Not to use the full capacity of the supply system would be a waste of resources. The price of the excess supply over firm power demand must be set as low as necessary for the short-term demand to absorb what is available. This may imply that the equilibrium price in the short-term market is below long run marginal costs of providing electricity. The utility of power produced is in this case measured by the area under $f(x)$ up to $\bar{x}$ plus the area under $p(x;\bar{x})$ between $\bar{x}$ and x_M.

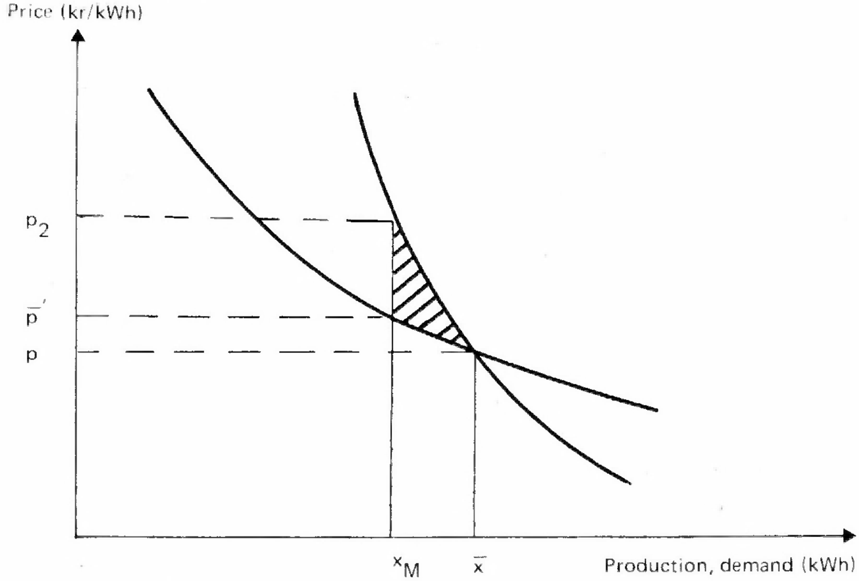
The depicted situation in figure 3 implies, however, that resources are underutilized even when the full capacity is used to satisfy the short-term demand. When production as well as demand is non-stochastic as we assume here for the sake of the argument, the price of firm power should be set so that there would be no need for a short-term market, i.e. equal to $\bar{p}'$. The firm power demand would then be equal to x_M. The loss to society from too high prices of firm power is measured by the shaded area.

We could on the other hand have the situation that the firm power commitments supercede the supply. This is depicted in figure 4. In this case it is the short-term curve to the left of $\bar{x}$ that is brought into operation. The choice of a rationing scheme whether by fiat or market will determine the shape of the short-term demand curve to the left of $\bar{x}$. The curve depicted in figure 1 represents one attempt of estimating the loss to society from rationing.

The symbols are as in figure 3 except that the equilibrium price in the short-term market - denoted by p_2 - is <u>higher</u> than the price of firm power, $\bar{p}$. The utility of the power produced is in this case measured by the area under the demand curve for firm power up to $\bar{x}$ <u>minus</u> the area under the short-term curve between $\bar{x}_M$ and $\bar{x}$. Also in this case it is easy to see that utility could have been increased if the consumers had adjusted to a different, namely higher, price of firm power $\bar{\bar{p}}'$. The loss to society is in this case measured by the shaded area in figure 4.

The need for a short-term market for electricity is a consequence of the stochastic elements in the electricity market. The main focus in this study is on the supply side, but also factors on the demand side may necessitate a short-term market for the capacity to be fully used. Shifts or stochastic variations in demand will imply that the consumers temporarily are not able to benefit fully from the available supply.

Figure 4. A simplified case of non-stochastic demand: losses to society when firm power demand is to high.



Given annual mean production

When uncertainty in actual production is taken into consideration utility losses by supply deficit and utility gains by supply surplus are stochastic. We assume that the aim of the planning is to maximize expected utility and that the planning authorities know the demand structure as well as the probability distribution of the actual production. The shape of the demand curves and the properties of the probability distribution of the actual production will be decisive for the optimal determination of planned firm power demand.

The utility or benefit from an actual production x_p is given by relation (3).

As x_p is stochastic with probability density $\phi(x_p, x_M, \sigma)$ the expected utility is

$$(5) \quad EU = \int_0^{\bar{x}} f(x)dx + \int_0^{\infty} \int_{\bar{x}}^{x_p} p(x;\bar{x})dx \;\; \phi(x_p;x_M,\sigma)dx_p$$

The annual mean production, x_M, is here taken as given. The expected utility will be maximized with regard to the volume of firm power, $\bar{x}$. (We might as well have maximized with regard to the price of firm power.) We shall assume for analytical convenience that the short-term demand price measured as a deviation from the firm power price is independent of the firm power demand, i.e. we assume

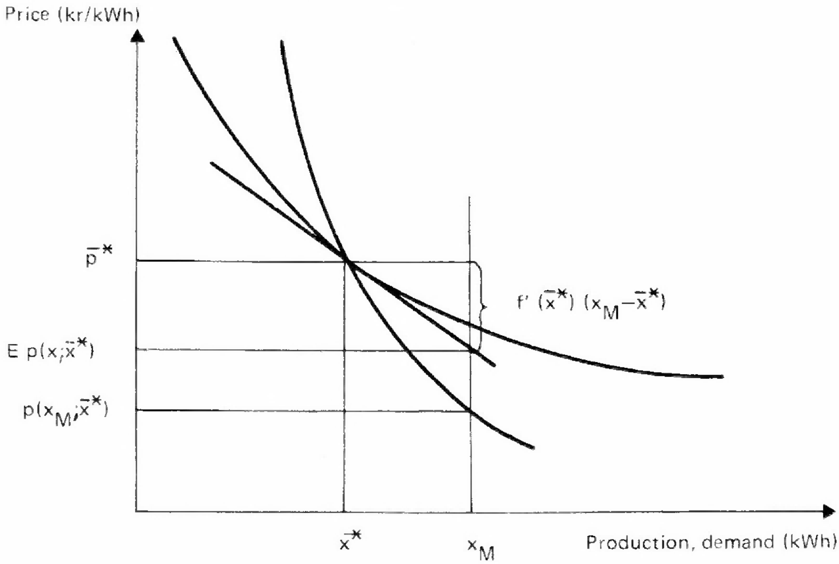
(6) $p(x;\bar{x}) - f(\bar{x}) = g(x-\bar{x}); \quad g(0) = 0$

Using this assumption we find that the maximum of (5) is achieved for $\bar{x} = \bar{x}^*$ satisfying

(7) $f(\bar{x}^*) + f'(\bar{x}^*)(x_M-\bar{x}^*) = Ep(x_p;\bar{x}^*)$

The situation is depicted in figure 5. The tangent of the firm power demand curve in the optimal point takes the value $Ep(x_p;\bar{x}^*)$ for $x = x_M$. If the firm power demand curve is linear or approximately linear, then (7) states that the expected equilibrium price in the short-term market, $Ep(x_p;\bar{x}^*)$ is equal to $f(x_M)$.

Figure 5. The determination of optimal firm power commitments with stochastic supply



If the firm power demand curve is horizontal, the planning problem is to decide on the optimal amount of firm power commitments at a given price. The condition (7) is in this case reduced to

(8) $Ep(x_p;\bar{x}^*) = \bar{p}$.

If the short-term demand curve furthermore is linear, it is easy to see that we must have $\bar{x}^* = x_M$. The optimality condition is thus in this case exactly the same as in the non-stochastic case: firm power commitments should be equal to annual mean production. The same conclusion will apply even when the firm power demand curve is not horizontal.

In this sense a linear short-term demand curve can be said to have a neutral effect on the optimal determination of firm power commitments.

A convex short-term demand curve seems to be a more realistic assumption. This implies that the marginal utility loss when production falls short of firm power demand is greater than the utility gain from a surplus over firm power demand. We therefore have

$$(9)\quad Ep(x_p;x_M) > \bar{p}$$

and $\bar{x}^*$ cannot be equal to x_M. As the expected short-term price, Ep, may be reduced by lowering the firm power commitments (9) reveals that $\bar{x}^*$ has to be less than x_M for (8) to hold. The conclusion is thus that an optimal determination for firm power commitments implies that these should be less than annual mean production. Concavity in the short-term demand, which seems a less likely assumption, would give the oposite conclusion.

When the firm power demand curve is falling, the same conclusion will hold under reasonable conditions: convexity in the short-term demand implies optimal firm power commitments less than annual mean production. In this situation the production capacity in the supply system, x_M, may be divided into two parts

- a firm power potential determined by $\bar{x}^*$, and
- an expected quantity of surplus power defined as $x_M-\bar{x}^*$

The expression (7) is simpler to interpret when rewritten in a slightly different form. Let ε_L be the demand elasticity of firm power. Then (7) is equivalent to

$$(10)\quad \frac{x_M-\bar{x}^*}{\bar{x}^*} = \varepsilon_L \cdot \frac{Ep(x;\bar{x}^*) - \bar{p}^*}{\bar{p}^*}$$

which says that the difference between the annual mean production and optimal firm power commitments shall be such that the effect of lowering the firm power price to the expected short-term equilibrium price would raise the firm power demand to become equal to the annual mean production. Relation (10) also expresses the close correspondence between the optimal overcapacity of the power supply system and the (expected) price differential between the firm power price and the clearing price in the short-term market. It is thus the shape of the short-term demand curve, in particular its convexity, that determines the optimal "overcapacity" of the system, i.e. the optimal difference between the annual mean production and firm power commitments.

Simultaneous determination of optimal capacity

In the preceding section we analyzed the determination of the firm power potential in a hydroelectric power supply system with given production capacity. The firm power potential was there defined as the amount which maximized the utility of the purchasers of power with a known probability distribution of actual power production. The next step is naturally to determine the dimensioning of the supply system within the same framework of optimization. This implies a simultaneous determination of production capacity, measured as annual mean production, and the firm power potential at this capacity.

The utility maximizing problem as formulated in the preceding section has to be modified by subtracting the annual costs of providing a certain production capacity from the utility value of electricity consumption. The annual costs, $G(x_M)$, are as discussed in section 3 assumed to be a function of annual mean production. Our optimization problem is thus to maximize

$$(11)\quad EU = \int_0^{\bar{x}} f(x)\,dx + E\left[\int_{\underline{x}}^{x_p} p(x;\bar{x})\,dx\right] - G(x_M)$$

with regard to $\bar{x}$ and x_M.

We shall now have two conditions of optimality by differentiating (11) with regard to $\bar{x}$ and x_M, respectively. It is easily seen that the first condition must be identical with (7). The second condition, from differentiating (11) with regard to x_M, is:

$$(12)\quad G'(x_M) = \int_0^{\infty} \int_{\underline{x}}^{x_p} p(x;\bar{x})\,dx \,\frac{\partial \Phi(x_p; x_M, \sigma)}{\partial x_M}\, dx_p$$

This equation can be interpreted as a criterion for optimal expansion of the hydroelectric power supply system. The left hand side of (12) expresses the long run marginal cost of power supply.

We shall make certain, not totally unreasonable, assumptions about the probability distribution Φ that will simplify (12). We shall say that Φ is invariant with regard to x_M if

$$(13) \quad \frac{\partial \phi(x_p;x_M,\sigma)}{\partial x_p} = -\frac{\partial \phi(x_p;x_M,\sigma)}{\partial x_M}$$

This implies that the probability of x_p deviating from x_M with a given amount is independent of x_M. This means in particular that the variance of the hydroelectric power production is independent of the annual mean production. This may seem unreasonable if one thinks of the power supply system as a collection of independent power supply units. Extensive use of regulation reservoirs and power pooling administered by a central authority, as is the case in Norway, may imply that constant variance is a reasonable approximation.

The assumption (13) can be used to integrate (12) by partial integration to give

$$(14) \quad G'(x_M) = Ep(x_p;\bar{x})$$

Assuming that the second order conditions are fulfilled we find that the criterion for optimal capacity is that the long run marginal cost is equal to the expected short-term equilibrium price. The short-term market includes, as explained in section 3, situations of shortage where rationing or reallocation of firm power rights apply.

The optimal capacity criterion (14) presupposes that the amount of firm power is determined according to (7). Combining (7) and (14) we can express the long run marginal cost criterion of optimal capacity as

$$(15) \quad G'(x_M) = f(\bar{x}) + f'(\bar{x})(x_M-\bar{x})$$

We found above that if the short-term demand curve is linear, optimal firm power supply coincides with annual mean production. From (15) it follows immediately that optimal capacity expansion in this case implies that the long run marginal cost is equal to the firm power price. If the short-term demand is convex, however, the power supply system should be dimensioned so that annual mean production is larger than the amount of firm power and, accordingly, with the long run marginal cost lower than the price of firm power.

The deviations in optimum between annual mean production and the amount of firm power on the one hand and between the price of firm power and the long run marginal cost on the other are thus positive if the short-term demand curve is convex, zero if linear and negative if concave. The relative size of the deviations in price and quantity is dependent upon the shape of the long-term demand curve. Rewriting (10) using (14) we get

$$(16) \quad \frac{(x_M^* - \bar{x}^*)/\bar{x}^*}{(\bar{p}^* - G'(x_M^*))/\bar{p}^*} = -\varepsilon_L$$

where x_M^*, $\bar{x}^*$ and $\bar{p}^*$ are the optimal values and ε_L is the elasticity of firm power demand with regard to the firm power price. The equation (16) says that the proportion of the excess capacity, given by the numerator in (16), to the excess price, given by the denominator of (16), shall be equal to the absolute value of the long-term elasticity of firm power demand. This may be regarded as a criterion of the overall optimality of the power supply system.

The excess capacity with accompanying excess price that follows from optimizing the hydroelectric power supply with a convex short-term demand curve can be considered as an expression of the social costs of providing the kind of certainty that the firm power market gives in a stochastic supply situation. These results, pointing at the importance of the curvature of the short-term demand curve, can be indicated by approximating the short-term demand curve by a second-order polynominal:

$$(17) \quad p = p(x;\bar{x}) = f(\bar{x}) + a(x-\bar{x}) + b(x-\bar{x})^2$$

with

$$a < 0, \; b > 0, \; a + 2b(x-\bar{x}) < 0$$

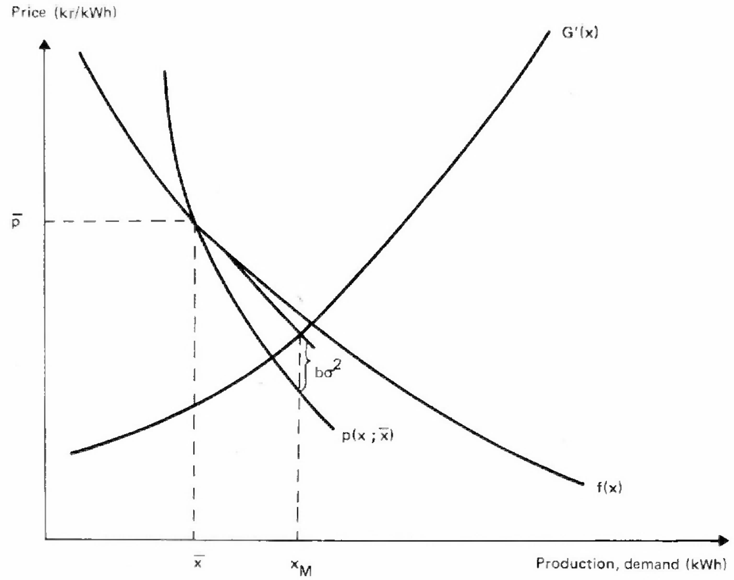
It follows then that

$$(18) \quad Ep(x;\bar{x}) = p(x_M;\bar{x}) + b\sigma^2$$

In figure 6 is shown how the optimal solution will be in this case.

When there is no uncertainty in the power supply we shall, of course, have no excess capacity as well as no excess price. We will have the same solution, as we have seen, when there is uncertainty and linearity in the short-term demand. The uncertainty will still have social cost, however, in a reduced utility value of the power consumed. In Bjerkholt and Olsen (1980) it is shown that the loss is proportionate to the variance in the power production and inversely proportionate to the absolute value of the short-term elasticity of demand. Hence, the importance of reducing the variance by supply measures, as for instance by re-

Figure 6. The simultaneous determination of optimal firm power commitments and capacity with stochastic supply



servoirs and power pooling, and "flattening" the short-term demand curve by efficient rationing procedures, market organisation or other measures is obvious.

Bjerkholt and Olsen (1980) discuss furthermore how the optimal solution depends upon the variance, σ^2, of the hydroelectric power production. The main result is that the partial effect of a higher variance is higher excess capacity and higher excess price. When the variance of power production increases with higher annual mean production the result seem to be that both the annual mean production and the amount of firm power will be lower.

5. Optimal firm power commitments with stochastic demand

Within the framework of our simplified model we attempt in this section to introduce uncertainty on the demand side of the electricity market. As in the preceding sections the focus of the discussion is how uncertainty should affect the planning of the hydroelectric power supply. In the following discussion we assume for simplicity that while the demand is stochastic the supply is given. We discuss briefly at the end of the section the interaction of uncertainty both on the demand side and on the supply side. As in preceding sections we disregard for simplicity the possibilities of using imports/exports as a buffer for covering deviations between demand and domestic production.

In Norway uncertainty in the demand for electricity have significant consequences for the actual planning of the supply system. In a recent report to the Storting (Parliament) the government suggests that 2 TWh are added to the (expected) demand for electricity in 1985 as a result of uncertainty regarding the general economic development and variations in temperature.

We introduce uncertainty on the demand side in our formal model framework by reinterpreting the firm power demand curve (1), $\bar{p} = f(\bar{x})$, as a relationship between the firm power price, $\bar{p}$, and the expected demand for firm power, $\bar{x}$, at that price. The actual demand for firm power, x_d, is assumed to be a stochastic variable with probability density given by $\Psi(x_d;\bar{x},\sigma_d)$

$$(19) \quad Ex_d = \bar{x} = f^{-1}(\bar{p}), \ \text{var}\ x_d = \sigma_d^2$$

Uncertainty on the demand side could surely have been introduced in other ways than as specified in (19)[1]. Neither is it obvious what kind of uncertainty may be properly represented by formulations like (19). We assume that consumers of electricity, firms and households, adjust their long-term demand of firm power to the given price, $\bar{p}$, but that the actual demand has stochastic variations around the mean, $\bar{x} = f^{-1}(\bar{p})$. One reason for such variations is annual changes in temperature which cause variations in the amount of electricity to provide a satisfying room temperature. Another reason is current business conditions which may cause variations in demand from firms using electricity in industrial processing. For electricity intensive industries such variations may be quite substantial. A third reason for specifying uncertainty in the demand for electricity is the lack of precision in the planning authorities' estimation of firm power demand. The modeling of uncertainty in (19) may not be equally well suited for all of these and other possible sources of variations in demand. It may for example be claimed that with respect to uncertainties in current business conditions variations around the mean of a long-term adjustment may not in practice be easy to distinguish from changes in the long-term adjustment.

The principal problem is then assumed to be the same as in section 4, i.e. determining the level of firm power commitments which maxi-

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1) The stochastic variations in the demand for firm power could e.g. have been formally represented by specifying a shift parameter, α, in the f(x)-function, i.e. $\bar{p} = f(\bar{x};\alpha)$.

mixes expected utility when the production capacity of the supply system, x_M, is given. However, in the present discussion the demand for firm power is regarded as stochastic as specified above, while we, as a first step, neglect the uncertainty on the supply side analyzed in section 4.

The short-term demand function for electricity, at price of firm power, $\bar{p}$, is assumed to be related to the actual demand for firm power, x_d, in a way corresponding to relation (2) for the deterministic case, and we furthermore assume that the short-term willingness to pay measured as a deviation from the firm power demand is independent of the stochastic variations in firm power demand:

$$(20)\quad p(x;x_d) = \bar{p} + g(x-x) = f(\bar{x}) + g(x-x_d)$$

The objective of the planning authorities is still to determine the expected firm power supply so as to maximize expected utility of the consumers as defined in section 3.

The maximum is achieved when the following relation is satisfied:

$$(21)\quad \frac{dEU}{d\bar{x}} = \bar{p} + \frac{d}{d\bar{x}} \int_0^\infty \int_{x_d}^{x_M} (f(\bar{x}) + g(x-x_d))\,dx\,\psi(x_d;\bar{x},\sigma_d)\,dx_d$$

$$= \bar{p} + f'(\bar{x})(x_M-\bar{x}) + \int_0^\infty \int_{x_d}^{x_M} (f(\bar{x}) + g(x-x_d)\,dx \frac{d}{d\bar{x}}\,\psi(x_d;\bar{x},\sigma_d)\,dx_d$$

$$= 0$$

In order to reach a form of this solution which is easier to interpret we assume that the probability density of x_d is dependent upon $\bar{x}$ only through the difference $x_d-\bar{x}$.[1] This means in particular that the variance σ_d is independent of x_d, as for instance in the normal distribution. Then we get from partial integration of (21):

$$(22)\quad \bar{p} + f'(\bar{x})\,(x_M-\bar{x}) = E\,p(x_M;x_d)$$

This is a condition which is strikingly similar to the condition (7) which determines the optimal firm power commitments in section 4. The expectations on the right-hand side of these two expressions have, of course, wholly different meaning. In (7) the supply, x_p, is stochastic with probability density given by ϕ, while in (22) the firm power

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1) This is equivalent to assuming that the firm power demand curve shifts horizontally.

demand, x_d, is stochastic with probability density ψ.

The interpretation of the condition (22) and the implication with respect to the utilization of the given capacity, x_M, are still parallell to those discussed in section 4: A linear short-term demand function implies that the authorities should determine the price of firm power so that the corresponding expected demand for firm power is equal to the given production capacity, x_M. With a convex short-term curve the firm power price should on the other hand be adjusted so that $\bar{x} < x_M$. The reason is again that in this case marginal utility losses when firm power demand exceeds the production capacity are greater than the gains in utility when there are surplus power in the system. In the same way as in section 4 $\bar{x}$ may in this case be defined as the firm power potential in the supply system. When the short-term demand curve is convex, which may be regarded as a more realistic assumption than linearity, the existence of a stochastic demand for electricity therefore implies that there should be a positive potential of surplus power, $x_M-\bar{x}$, in the supply system.

So far in this section we have analysed the effects of a stochastic demand for electricity isolated; i.e. we have neglected the existence of uncertainty on the supply side. It may be interesting to discuss briefly the implications for the decision regarding the optimal level of firm power commitments when both types of uncertainty are present.

Our results from (7) and (22) can be written as

$$(23) \quad f'(\bar{x})(x_M-\bar{x}) = E[g(x_p-x_d) \mid x_d = \bar{x}] \text{ and}$$

$$(24) \quad f'(\bar{x})(x_M-\bar{x}) = E[g(x_p-x_d) \mid x_p = x_M]$$

Combining the results for the case when both productions and demand are stochastic we get

$$(25) \quad f'(\bar{x})(x_M-\bar{x}) = E\, g(x_p-x_d)$$

When the short-term curve is convex it follows from this relation that the potential of firm power in the system, $\bar{x}$, should again be determined so that $\bar{x} < x_M$. It may furthermore be shown (see Bjerkholt and Olsen (1980)) that the difference between the production capacity and the optimal level of firm power is increased when the variance of the stochastic variable is higher. Normally there should not be any reason to assume (strong) negative correlation between the variations in x_p and x_d. We may therefore most reasonably assume that the variance of

$(x_p - x_d)$ is higher than σ^2 and σ_d^2, so that the two types of uncertainties we have considered will reinforce each other with respect to their implications for the optimal level of firm power commitments.

6. Conclusions

In this chapter we have discussed some central aspects of the dimensioning and capacity utilization of a hydroelectric power system when uncertainty in supply and demand is taken explicitly into consideration in the planning process. The analysis is carried out within the framework of a simplified theoretical model where an important element is the distinction between the short-term and long-term demand for electricity. The latter is interpreted to be equivalent to the demand for firm power which is a central concept in the planning of the Norwegian power supply system.

In section 2 the concept of firm power is discussed, while the formal model is presented in section 3. In section 4 we first use this model to analyse the capacity utilization problem, i.e. the determination of the optimal level of firm power commitments when the production system is given. It is shown how the optimal sales of firm power - the firm power potential - depend on properties of the long-term and short-term demand curves. A main conclusion is that when the short-term demand function is convex, the firm power potential is less than the production capacity so that there is a positive potential of surplus power in the supply system.

The dimensioning of the hydro power system - the determination of the optimal capacity - is then analysed. A long run marginal cost criterion for the case of uncertainty is derived, expressing that the long run marginal cost should be equal to the expected price - i.e. the willingness to pay - in the short-term market. Combining this result with the condition for optimal capacity utilization leads to the conclusion that if the optimal solution implies an excess capacity in the supply system in the sence that expected production exceeds the firm power potential then the demand for firm power should be adjusted to a firm power price which is higher than marginal costs.

In section 5 the existence of uncertainty on the demand side of the electricity market is analysed within the same formal framework. As might be expected the implications with respect to the utilization of a given capacity are quite parallell to the case of stochastic supply. In particular a convex short-term curve for electricity implies that the price of firm power should be determined so that expected firm power demand is lower than the capacity of the supply system. Taking uncertainties both on the supply side and the demand side of the electricity market into consideration simultaneously the two effects will, under reasonable assumptions, reinforce each other with respect to their implications for the excess capacity in the hydro power system.

The purpose of the analysis in this chapter has been on pure theoretical grounds to clarify some basic principles regarding the planning of a hydro power system under uncertainty. A more complex model, distinguishing e.g. between different groups of consumers, may perhaps have modified some of the results, but we still believe that the simplified analysis above may help to gain some new insight into this problem. From the derived results we may conclude that in order to reach an optimal utilization and expansion of the electricity supply it is necessary with rather detailed knowledge to the demand structure of electricity. In recent years many empirical studies of the demand for electricity have been carried out (see e.g. Taylor (1975)), but few - if any - of these are adjusted to the problem we have discussed above. There is thus obviously need for further empirical work in this field.

XIV. OPTIMAL TIMING AND DIMENSIONING OF HYDRO POWER PROJECTS

by

Asbjørn Rødseth

In the Norwegian debate about the future development of electricity supply various cost-benefit criteria have played an important part. The purpose of this paper is to throw some light on those criteria by deriving them within an explicit optimization framework. My hope is that this will reduce some of the confusion we have seen in the recent debate.

The order of exposition is as follows: In section 1 we will set out a highly simplified model of a pure hydro power system. The model is too crude to be of much help in actual planning, but it will serve to illustrate the justification for and interpretation of some investment criteria. These are derived in section 2 as necessary conditions for an optimal development of the water power system. The optimal path may deviate from what could be expected from Ricardian theory of exploitation of natural resources. In section 3, we introduce uncertainty about weather conditions. A formal derivation of the conditions for optimal reservoir management and optimal timing of new projects for the case with uncertainty is given in section 4. In section 5 we discuss briefly the extensions needed in the model in order to solve the planning problem we meet in practice. We conclude with some remarks about centralization vs. decentralization of the administration of electricity supply.

1. A model of a pure hydro power system

We assume that there are n different hydro power projects under consideration. Each project has a cost function:

$$(1) \qquad k_i = b_i(y_i) \quad b_i' > 0 \qquad\qquad i=1,2,\dots,n$$

y_i is the project's production capacity for electricity, k_i is total construction costs. For each project we are going to determine the optimal level of capacity, y_i, and the optimal time for completion of its construction, t_i. The total production of electricity at each point in time, t, is:

(2) $$y(t) = \sum_{i=1}^{n} m_i(t) y_i$$

where:

(3) $$m_i(t) = \begin{cases} 0 \text{ for } t < t_i \\ 1 \text{ for } t \geq t_i \end{cases} \qquad i=1,2,\ldots,n$$

The objective is to maximize the discounted value of consumers' surplus. The Marshallian demand curve for electricity is:

(4) $$p = p(x,t) \qquad p_x' < 0$$

x is the quantity consumed.

The objective function can thus be written:

(5) $$F = \int_o^\infty e^{-rt} \int_o^y p(x,t)dxdt - \sum_{i=1}^{n} k_i e^{-rt_i}$$

The rate of interest, r, is assumed to be constant.

The following points should be noted about the objective function:

1. *The costs of construction do not depend on time.*

This is equivalent to choosing construction costs as numeraire. The underlying assumption is that the real rate of interest is constant in terms of construction costs.

2. *Costs of operating the system have been ignored.*

This could be defended on the grounds that they are negligible in comparison with construction costs. If the operational costs depend only on capacity and not on the actual level of operations, they can then be discounted and included in k_i. This is not too unrealistic.

3. *Environmental costs are not included.*

We may, however, regard environmental costs as discounted and included in k_i.

4. *The construction period can be of any length.*

We assume that interest during the construction period is computed at rate r and included in k_i. When projects require different construction periods, this creates some extra problems in choosing the first project. We will ignore these problems.

5. *F must have a finite maximum.*

That can be guaranteed by the following assumptions:

a. $r > 0$

b. For all i, $b_i'(y_i) \to \infty$ for a finite level of y_i.

c. For all t there exists an upper bound for p(x,t) (a reservation price), and if this upper bound grows with t, it does so at a rate lower than r.

We shall denote the integral under the demand curve, representing consumers' surplus, f(y,t):

$$(6) \qquad f(y,t) = \int_o^y p(x,t)dx, \quad f_y'(y,t) = p(y,t)$$

Substitution from (1), (2), and (6) in the objective function gives:

$$(7) \qquad F = \int_o^\infty e^{-rt} f(\sum_{i=1}^n m_i(t)y_i, t)dt - \sum_{i=1}^n b_i(y_i)e^{-rt_i}$$

This we are going to maximize with respect to $t_1, t_2, \ldots, t_n$ and $y_1, y_2, \ldots, y_n$ taking account of (3). We assume that p(x,t), and thereby f(y,t), is continuous in the second argument. Taking also the first argument into account, it is clearly piecewise continuous in t. F can be looked upon as a function of the variables we are maximizing with respect to. In the next section we shall discuss some necessary conditions for a maximum of F.

2. Necessary conditions for an optimal plan

We have the usual conditions on the partial derivatives of the function we are maximizing:

$$(8a) \qquad \frac{\partial F}{\partial t_i} = e^{-rt_i}\left[f(y(t_i)-y_i, t_i) - f(y(t_i), t_i)\right] + b_i(y_i)re^{-rt_i} \leq 0$$

$$= 0 \text{ for those } t_i > 0 \qquad i=1,2,\ldots,n$$

$$(8b) \qquad \frac{\partial F}{\partial y_i} = \int_{t_i}^\infty e^{-rt} f_y'(y,t)dt - b_i'(y_i)e^{-rt_i} \leq 0 \qquad i=1,2,\ldots,n$$

$$= 0 \qquad \text{for those } y_i > 0$$

The first set of conditions can for project no. i be written

$$\text{(9)} \qquad \frac{f(y(t_i),t_i) - f(y(t_i)-y_i,t_i)}{y_i} \geq r\frac{b_i(y_i)}{y_i} \qquad (= \text{if } t_i > 0)$$

This can suitably be labeled the *timing condition.* The right hand side of (9) corresponds to what is often called the "long run marginal cost", namely the average cost of producing electricity in a new plant computed on an annuity basis. The timing condition thus says: *For t_i to be the optimal time of completion for project i, the average gain in consumers' surplus at time t_i from producing the additional quantity y_i must be equal to the long run marginal cost.*

We can go one step further. The average price of power in the supply interval covered by project no. i at the time of installation can be defined as

$$p_i^* = \frac{1}{y_i} \int_{y(t_i)-y_i}^{y(t_i)} p(x;t_i)dx = \frac{1}{y_i} [f(y(t_i),t_i) - f(y(t_i) - y_i,t_i)]$$

It follows that condition (9) becomes:

$$\text{(9')} \qquad p_i^* \geq r\frac{b_i(y_i)}{y_i} \qquad (=\text{if } t_i > 0)$$

A necessary condition for correct timing is thus that the appropriate average price, p_i^*, is equal to the long run marginal cost.

The second set of conditions can for project no. i be written

$$\text{(10)} \qquad b_i'(y_i) \geq \int_{t_i}^{\infty} p(y,t)e^{-r(t - t_i)}dt \qquad (=\text{if } y_i > 0)$$

This can suitably be labeled the *dimensioning condition.* For $y_i > 0$, it says: A necessary condition for a project completed at time t_i to be of the right size, is that *the project's marginal cost is equal to the discounted value of the future time path of the price which will equilibrate demand for electricity with the production planned.*

Naturally all the conditions are interrelated. The right dimensioning of a project depends on when it is going to be completed, as well as on when the other projects are finished and which sizes are chosen for them. Similarly, right timing depend on what dimensions are chosen.

It may occur that the whole system of necessary conditions have no solution for some of the t_is. One obvious case where this can happen. is if for a particular project the "long run marginal cost" can never become lower than the reservation price. It does not mean, however, that the optimization problem has no solution. It just means that at least one project should never be carried out (i.e. $t_i = \infty$ for at least one i).

What we have called the timing condition, is often found in the cost-benefit literature and sometimes called the *long run marginal cost criterion*. In Rødseth (1975) it was used for checking whether the completion of hydro power projects in Norway followed an optimal plan. The criterion is convenient, because it requires no information about electricity demand and construction plans after t_i. If the criterion is not satisfied, we can only state that we are not following an optimal path. By invoking three new assumptions we can, however, draw the more specific conclusion that the project should be postponed. These assumptions are:

1. All projects are of the correct size, i.e. in accordance with the dimensioning condition.
2. The sequence of the projects is the same as in the optimal solution.
3. Demand is increasing over time.

The dimensioning criterion is useful because it can be used for deciding on the detailed design of each project in two level planning procedures. It is important to notice that the design should be based on the whole time path of prices after completion of the project. If prices are increasing along the optimal path, and if the design is based on prices at completion time only, too low levels of production will tend to be chosen.

Both the dimensioning and timing conditions are derived by considering possible marginal changes from the optimal plan. The requirement is that such changes should not increase the value of the objective function. There are also numerous possible non-marginal changes from the optimal plan which we may consider. These should not increase the value of the objective function either. Thus such considerations will give rise to new necessary conditions.

Suppose project no. i is in the optimal plan. The change in the objective function from taking project no. i out of this plan will then be:

$$\Delta F = -\int_{t_i}^{\infty}\left[f(y,t) - f(y-y_i,t)\right]e^{-rt}dt + b_i(y_i)e^{-rt_i}$$

A necessary condition for optimum is then:

$$(11) \qquad \int_{t_i}^{\infty} [f(y,t) - f(y-y_i,t)] e^{-r(t - t_i)} dt - b_i(y_i) \geq 0$$

This can be interpreted as a *profitability condition.* It says that the discounted value of future consumers' surplus must exceed construction costs. If there is perfect price discrimination, this is equivalent to saying that the net discounted profits from the project must be positive.

Another change we can consider is this: Take two projects, i and k, and let them change places. Complete project i at time t_k and project k at time t_i. Assuming $t_i < t_k$ we can write the resulting change in the objective function:

$$\Delta F = \int_{t_i}^{t_k} [f(y - y_i + y_k, t) - f(y,t)] e^{-rt} dt$$

$$- b_k(y_k)(e^{-rt_i} - e^{-rt_k}) + b_i(y_i)(e^{-rt_i} - e^{-rt_k}) \leq 0$$

If the original plan was optimal, ΔF cannot be positive. After a little rearrangement this gives us the condition:

$$(12) \qquad \int_{t_i}^{t_k} [f(y - y_i + y_k, t) - f(y,t)] e^{-r(t - t_i)} dt$$

$$- [b_k(y_k) - b_i(y_i)] (1 - e^{-r(t_k - t_i)}) \leq 0$$

This we have named the *sequence condition.* It has the following implication: If a large project follows a small, then the larger project must have higher total costs. To see this, assume $y_k > y_i$. Then the integral above is clearly positive. This implies that the term in brackets must also be positive.

A possible way of solving the maximization problem is to find *all* solutions to (9) and (10) and compare the resulting values of the objective function. To solve (9) and (10) we are forced to assume a given sequence of projects first. Thus the system has to be solved for all sequences which are possible candidates for an optimal path. The sequence condition may help us to sort out some sequences which are not such candidates.

There are intuitive notions about properties of the optimal path some of which we will discuss briefly in this section, e.g. the notion that one should use the cheapest resources first. The question can be phrased:

a) Will the average cost, $b_i(y_i)/y_i$, rise for projects which succeed each other in time?

Another notion is that as time goes on, one will exploit the resources more intensively. This leads to the question:

b) Will $b_i'(y_i)$ rise for projects which succeed each other in time?

Furthermore, there is an expectation that as one proceeds to use more expensive resources, the equilibrium price will have to increase. Now, clearly in the interval between two projects production is constant and the price increases if and only if demand is increasing. We should therefore ask if the equilibrium prices at the time of completion of two successive projects will increase:

c) Will $p(y(t_i),t_i)$ rise for successive t_is?

Alternatively we may ask the same question about the hypothetical equilibrium price that would prevail at t_i if the production from the project just completed had not been available:

d) Will $p(y(t_i)-y_i,t_i)$ rise for successive t_is?

It turns out that there are no clearcut answers to these four questions, even if we assume that demand is increasing in the sense that $p(x,t) \geq p(x,t')$ for $t > t'$ and all x and t'. The average as well as marginal cost of new projects may fall through time. And completion of a new project may make prices fall deeper than ever before. Then reason why we cannot tell more about the optimal path, is the inevitable element of *indivisibilities* in hydro power projects.

As mentioned in the introduction, these results are in contrast with what one would expect from the Ricardian theory of exploitation of natural resources. In the Ricardian theory, when demand increases one will always go on to exploit resources which have higher costs. The reason for the difference in conclusions is, as already hinted at, that while the Ricardian tradition assumes that marginal increases in output can always be accomplished by marginal additions to costs, we have assumed that there is a finite number of projects which in some (weak) sense are indivisible. More details are given in Rødseth (1980).

The discussion above may have given the impression that the optimal development can look almost like anything. That is of course not true. The implication is only that in judging a specific plan answers to questions a-d are of little or no help. One should instead look at criteria like (9) and (10). It should be emphasized that the non-Ricardian result as regards the sequence of projects with different average costs require that the indivisible projects are *large* compared to the production capacity in the projects already developed, cf. Rinde and Strøm in chapter VI of this volume where it is discussed whether indivisibility can explain the tendency of decreasing marginal costs over time in the Norwegian hydro power system.

3. Incorporating uncertainty

The model discussed so far is mainly useful for illustrative purposes. Several practical issues have been ignored and most serious is probably the neglect of taking into account the uncertainties created by both supply and demand depending on the weather. When these uncertainties are taken account of, the production capacity of a project cannot be expressed as a single number.

It is not obvious how the objective function should be modified to take into account uncertainty. One possibility is to maximize the expected value of discounted consumers' surplus minus the (non-stochastic) value of discounted construction costs.

Consumers' surplus will be stochastic both because the decision about production levels will depend on stochastic variables and because demand itself is stochastic. This leads us into a formidable maximization problem. Obviously the production possibilities at each point in time not only depends on the existing levels of capacity, but also on how we in previous periods have decided to operate the system, and on previous as well as present realizations of the stochastic variables. This means that in deciding on the level of production at each point in time, we must take into account the effects this decision will have on future production possibilities. The decision should be based on the information available at that point in time. The information will include the present levels of capacity as well as those planned for the future, and it will include the whole past history of the electricity supply system. It will include the probability distribution of future weather conditions, but not the time path which is going to be realized.

We can imagine that the solution of our optimization problem take place in two steps. First, for each possible development of capacity, we determine strategies for the operation of the system contingent on the information available at each point of time. For each development of capacity we can then compute the value of the objective function, and in the last stage we compare these values to find the optimum. The first stage can be considerably simplified if we assume that weather conditions are serially independent. Then all relevant information about the past is summarized in the existing levels of water in the reservoirs and in the existing capacity levels. In addition to these variables the optimal strategies will in general also depend on the planned future developments of capacity as well as on the expected future shifts in the demand function. Thus, we still have a formidable problem in working out the optimal strategies. Part of the solution process will normally be to derive shadow prices for water in storage, often called *water values*.

A formal derivation of some necessary conditions for optimal reservoir management strategies and the associated water values is given in the next section. There is also given a necessary condition comparable to (9) for the model with uncertainty. Since the next section is technically rather difficult we shall here give a verbal discussion of the timing problem in the present context.

We can still imagine that we start from the optimal plan and then make a marginal delay of one project. This should not increase the value of the objective function. We can imagine that each project's capacity levels are kept constant. What happens then to production? Since the optimal production strategies at all previous points in time depend on when this project is to be completed, they will all change. The production strategies at the time of the change will of course also change. All this may lead to changes in the amount of water stored in different reservoirs and thus have consequences for the future. We realize that the actual level of production may change at all times and for all possible states of the world. It is the expected value of the differences in consumers' surplus caused by these changes that we should compare to the gains from postponing the construction costs. We see that this is not an easy task. In the simple model the change in the level of production was determined by the capacity of the project under consideration and nothing else. In the real world the relationship between the capacities and measured production of one project, and the changes in total production caused by a postponement may be very loose. An example will show that this is not a theoretical subtlety. For a newly completed project the optimal strategy will often be to keep a low level of production in order to build up reservoirs. The consequences of a postponement can then not be judged only from the planned level of production (or more correctly: The planned probability distribution of production) at the time of completion.

Intuition together with the previous analysis suggests - and the formal derivation in the next section confirms - the following necessary condition for optimal timing: *The expected addition to consumers' surplus from the production in this particular project plus the expected value of the addition of water to the reservoir of the project (in kr) should be equal to the long run marginal cost*, i.e. rk_i. The problem with this condition is that the expected values involved will in practice be hard to calculate. Some ways of simplifying the problem are discussed in Rødseth (1980).

All discussion so far has been in terms of maximizing expected consumers' surplus. Implicitly we have assumed that electricity is always sold in a spot market at equilibrating prices. In practice electricity is sold on contracts with various kinds of guarantees against interruption in the service, and only a residual is eventually sold in an auction market, where normally only a limited number of customers are able to participate. The optimality of existing arrangements may be called into question. But clearly having all customers participate in complete auction markets for each point in time would lead to overwhelming transaction costs. Some compromise has to be struck, and one should preferably take account of this in formulating the objective function. Whether or not this makes solution more difficult, is hard to say. An analysis based on existing contractual arrangements in the Norwegian electricity supply system is given by Bjerkholt and Olsen in chapter XIII of this volume.

Hveding (1968) presented a stochastic simulation model which can be used to determine investments in a combined hydro and thermal power system. The description of the production system is more detailed than in section 4. The model uses optimal production strategies and is able to handle multi-year storage. However, the model essentially assumes a stationary environment. Demand, construction costs, and the rate of interest are all stationary. Thus the timing problem is entirely absent. Plants should be constructed as soon as possible, or never. Furthermore, the demand for contracts for delivery of so-called firm power, electricity with a certain guarantee for delivery and first priority in case of shortage, is given exogenously and independently of prices. Thus the scale of the system is also essentially determined outside the model. Modified versions of the Hveding-model are still used by the Norwegian Water Resources and Electricity Board in making plans for future electricity supply. The model can be used to generate a time path by assuming different levels of demand at different points of time, though this is subject to the criticism that the dynamic relationships are really neglected. And future conditions will not be taken account of in the dimensioning, contrary to what we would recommend.

Will the existence of uncertainty change our conclusions in section 2 about the optimal path and the Ricardian theory? In general we are still left with the same possibilities as before. But clearly the opportunity for storing water will contribute to smooth and even out the movements in expected equilibrium prices. It also makes it easier to absorb large projects, since the project can pay off not only through increased present consumption but also through more water in the reservoirs. On the other hand the correlation between the precipitation falling on the different projects becomes important. If the objective function exhibits risk aversion, one may prefer a more expensive project provided it is less correlated with the other projects in the plan. Similarly, one may prefer two small projects to one twice as large, provided the precipitation falling on the two projects is not perfectly correlated. This will reinforce the tendency to choose relatively small projects in the initial stages of water power development.

4. Water values and optimal timing

We shall now develop in a formal way the optimizing procedure outlined in the preceding section. For an individual project we shall distinguish between three different kinds of capacity:

- the generating capacity measured in kW, z_{i1}.
- the capacity for storing energy in reservoirs measured in kWh, z_{i2}.
- the capacity for collecting water for the reservoirs, which (as an approximation) can be measured by the area from which the water is collected, z_{i3}.

A simplified description of the technical characteristics of the production system is then:

(13) $$\dot{w}_i = \alpha_i(z_{i3},\tau) + \Theta_i(\tau) - y_i - v_i \qquad i=1,\ldots,n$$

(14) $$y = \sum_{i=1}^{n} m_i(\tau)y_i \qquad (\lambda)$$

(15) $$w_i y_i \geqq 0 \qquad (\mu_i)$$

(16) $$(v_i - \alpha_i(z_{i3},\tau) + y_i)^2 \cdot (m_i(\tau)z_{i2} - w_i) \geqq 0 \qquad (\rho_i)$$

(17) $$v_i \geqq 0$$

(18) $$0 \leqq y_i \leqq z_{i1}$$

(The symbols in parentheses to the right are associated Lagrange multipliers to be used in the optimizing process.)

$w_i(t)$ is the amount of energy actually stored in the reservoirs of project i (measured in terms of potential electricity production). (13) is giving the time rate of change of the quantity of energy in storage, $\dot{w}_i(t) = dw_i/dt$. The inflow into the reservoirs is determined by the normal accumulation of water in the reservoirs, $\alpha_i(z_{i3},\tau)$, which depends on the capacity for collecting water, and by the stochastic variable $\theta_i(\tau)$, representing the deviation of precipitation from its expected value. From the inflow is subtracted two different kinds of outflow:

y_i - the outflow through the power stations, measured by the rate of electricity production.

v_i - the overflow

(14) is merely a restatement of (2). The inequality (15) tells that no production can take place without a positive level of energy stored in the reservoirs. w_i may drop below zero, meaning, of course, not that the lake is empty, but that the level of water is too low for electricity production to continue. A heavy rainfall may temporarily bring the level of water above the mark where it is allowed to flow over. (16) says that if the level of water increases above the capacity level of the reservoir, the overflow has to be equal to the expected inflow minus the outflow used in current electricity production. (17) and (18) define the admissible regions for v_i and y_i.

The assumptions imply that the absolute upper or lower limits of the reservoirs are never reached. This simplifies the analytical treatment considerably.

The cost functions will now depend on the three different capacity levels:

$$(19) \qquad k_i = b_i(z_{i1}, z_{i2}, z_{i3}) \qquad\qquad i=1,2,\ldots,n$$

Our description of the production structure, though complicated, is still grossly simplified from what may be deemed desirable in an actual planning model. One may want a further disaggregation of each project into individual reservoirs and power stations. And one may want to consider investment in pumping facilities.

We are going to apply the two-step procedure suggested in the previous section. First we take all capacity levels and completion times as given, and we derive optimal strategies for reservoir management. Then we derive the new timing condition. The first step is to maximize the expected benefit from operating the system, G, with respect to the time paths of y, $y_1,\dots,y_n$ and $v_1,\dots,v_n$ under the restrictions given by (13) - (18).

$$(20) \qquad G = E\left[\int_0^{\infty} e^{-r\tau} f(y,\tau)\,d\tau\right]$$

The optimal values at each point of time will be conditional on the past development of the stochastic variables $\Theta_1,\dots,\Theta_n$, and thus be stochastic variables themselves.

The stochastic variables Θ_i are assumed to follow a Brownian-motion stochastic process with variances and covariances $h_{ij} = h_{ij}(w_1,\dots,w_n,t)$ $i,j=1,2,\dots,n$. This implies that the random variations in additions to the reservoirs are not serially correlated, an assumption which does not correspond exactly to reality, but serves well for illustrative purposes. Note that in our formulation we have not taken account of the possibility for using weather forecasts as information.

Conditions for a solution

The problem we have posed is a *stochastic optimal control* problem. Methods for solving it are described in Dreyfus (1965), chapter VII. In order to solve the problem we have to know the initial levels of water in the reservoirs. We can imagine solving the problem for different initial levels of water and for an arbitrary starting point in time.

$S(w_1,\dots,w_n,t)$ = the value of the objective function G when the integral is taken from t instead of from 0, the system starts in a state $w_1,\dots,w_n$, and optimal strategies are used.

This function is called the *optimal expected value function*.

Dreyfus shows that the solution of our problem must satisfy the following partial differential equation:

$$(21) \qquad \max\left[e^{-rt} f(y,t) + \sum_{i=1}^{n} S_{w_i}(\alpha_i(z_{i3},t) - y_i - v_i) + S_t + \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} S_{w_i} S_{w_j} h_{ij}\right] = 0$$

where $S_{w_i} = \dfrac{\partial S}{\partial w_i}$ and $S_t = \dfrac{\partial S}{\partial t}$.

The maximization is with respect to $y, y_1, .., y_n$ and $v_1, ..., v_n$. (13) - (18) are to be regarded as constraints. We notice that the expression in brackets is concave and the constraints are all quasi-concave in the variables we are maximizing with respect to. We assume that in the optimum y is always strictly positive.

The derivatives with respect to y_i and v_i of the Lagrangean associated with the maximization problem in (21) can be written:

$$M_i = -S_{w_i} + \lambda\, m_i(t) + \mu_i w_i + \rho_i^2(m_i(t)z_{i2} - w_i)(v_i - \alpha_i(z_{i3},t) + y_i)$$

$i=1,...,n$

$$K_i = -S_{w_i} + \rho_i^2(m_i(t)z_{i2} - w_i)(v_i - \alpha_i(z_{i3},t) + y_i)$$

$i=1,...,n$

A set of necessary and sufficient conditions for a maximum of the expression in brackets in (21) are now given by the constraints together with the following statements:

i) $e^{-rt}f'_y - \lambda = 0$

ii) If $M_i < 0$, then $y_i = 0$

iii) If $M_i > 0$, then $y_i = z_{i1}$

iv) If $y_i = 0$, then $M_i \leq 0$

v) If $0 < y_i < z_{i1}$, then $M_i = 0$

vi) If $y_i = z_{i1}$, then $M_i \geq 0$

vii) $K_i \leq 0$

viii) If $v_i > 0$, then $K_i = 0$

ix) If $\rho_i > 0$, then constraint (16) is binding for this i.

x) If $\mu_i > 0$, then $w_i y_i = 0$

From condition (ix) it follows that we shall always have:

$$\rho_i(v_i - \alpha_i(z_{i3},t) + y_i)(m_i(t)z_{i2} - w_i) = 0$$

This simplifies the expressions for M_i and K_i in optimum to:

$$M_i = -S_{w_i} + \lambda\, m_i(t) + \mu_i w_i$$

$$K_i = -S_{w_i}$$

The interpretation of condition i) is that the Lagrange multiplier, λ, shall always be equal to the price of electricity discounted to the present.

Water values

The S_{w_i} s can be interpreted as discounted *water values*. They show the present value of a marginal addition to each reservoir at a future point in time. Condition vii) tells us that the water values will never become negative.

If $m_i(t) = 0$, no production can take place in plant i at this time. The interpretation of the conditions is thus interesting only when $m_i(t) = 1$. It is also useful to distinguish between cases where the water level is too low for production to take place, and cases where it is above this level. We shall interpret the remaining conditions for the latter case first.

Condition x) tells us that if $y_i > 0$, then $\mu_i w_i = 0$. Conditions v) and vi) can be simplified accordingly. From ii) - vi) we then get the following rule for operating the system:
If the water value for a reservoir exceeds the price of electricity, then no electricity should be produced from this reservoir. If the price of electricity exceeds the water value, electricity production should be at the full capacity level. If the water value equals the price, production can be at any level between zero and the full capacity level. (But production in different plants must still add up to the optimal level of aggregate supply.)

viii) tells us that there should be an overflow only when the water value is zero. With a positive price of electricity, this means that an overflow will always be accompanied by production at the full capacity level.

If the water level is below zero, there will of course be no production. A negative value of $\mu_i w_i$ permits this to happen even if the water value is less than the price of electricity.

Introduction of thermal power

The derivation above was for a pure hydro power system. To see the relation between fuel prices and water values in a combined system, we can introduce a thermal plant as plant no. 0. The objective function then has to be modified to include the discounted cost of fuels:

$$(20') \quad G = E[\int_0^\infty e^{-rt}(f(y,t) - qy_0)dt]$$

y_0 is the amount of electricity produced by thermal power, and q is the fuel cost per unit of electricity produced.

We must add y_0 to total production in (14) and we get a new constraint:

$$(22) \quad 0 \leq y_0 \leq z_{01}$$

z_{01} is the capacity for producing electricity from thermal power. Maximization can proceed in the same way as before. In addition to the conditions in the last section, we get some new conditions similar to ii) - vi) for thermal power. The strategy will be:
If the price of electricity is smaller than the proportional fuel cost, then do not operate the thermal power stations. If the price of electricity is above the fuel costs, then run the thermal plants at the full capacity level. If the price is equal to the fuel cost, then the thermal plants can be operated at any level consistent with overall demand.

From the strategies we have derived, it is clear that a quantity of energy stored in the form of water will not in general be equal in value to its energy equivalent in other fuels. The limited production possibilities in each plant together with the unpredictability of the weather will normally prevent such equalization.

Optimal timing

Suppose we start with an optimal plan for the development of the whole supply system. Then any *feasible* change from that plan should lead to a reduction in the value of the objective function. One feasible change is to postpone project no. i from t_i to $t_i + \Delta t_i$, operate the other plants during this interval as if nothing had happened, and take the resulting water levels as the starting point for determination of future strategies. This leads to the following change in the value of the objective function:

$$\Delta G = E\left[\int_{t_i}^{t_i+\Delta t_i} f(y,t)e^{-rt}dt - \int_{t_i}^{t_i+\Delta t_i} f(y - y_i,t)e^{-rt}dt\right.$$

$$\left. + S(w_1,\ldots,w_n,t_i + \Delta t_i) - S(w_1,\ldots,w_{i-1},0,w_{i+1},\ldots,w_n,t_i + \Delta t_i)\right]$$

$$- k_i e^{-rt_i} + k_i e^{-r(t_i+\Delta t_i)}.$$

(ΔG is the difference between the value of the objective function when project i is postponed and when it is carried out as planned.) By taking a Taylor-series expansion of S, assuming that higher order terms vanish, and dividing through by Δt_i, we get:

$$\frac{\Delta G}{\Delta t_i} \approx E\left[\frac{1}{\Delta t_i}\int_{t_i}^{t_i+\Delta t_i} f(y,t)e^{-rt}dt - \frac{1}{\Delta t_i}\int_{t_i}^{t_i+\Delta t_i} f(y - y_i,t)e^{-rt}dt\right.$$

$$\left. + S_{w_i}\frac{w_i(t_i + \Delta t_i) - w_i(t_i)}{\Delta t_i}\right] + k_i\frac{e^{-r(t_i + \Delta t_i)} - e^{-rt_i}}{\Delta t_i}$$

By taking the limit of this expression as Δt_i goes towards zero, we get the following condition, which is necessary if it shall not pay to postpone the project:

$$\lim_{\Delta t_i \to 0}\frac{\Delta G}{\Delta t_i} = E\left[f(y,t_i)e^{-rt_i} - f(y - y_i,t_i)e^{-rt_i} + S_{w_i}\dot{w}_i\right] - k_i re^{-rt_i} \geq 0$$

Or equivalently:

$$E\left[f(y,t_i) - f(y - y_i,t_i)\right] + E\left[S_{w_i}\dot{w}_i e^{rt_i}\right] \geq rk_i$$

We have shown that for the plan to be optimal, a necessary condition is that a project's expected addition to consumers' surplus plus its expected addition to the value of water in the reservoirs at the time of completion must be at least equal to the "long run marginal cost". By the same kind of argument for an earlier completion of the project, we can show that it must be less than or equal to the long run marginal cost. Thus, along an optimal plan, at the time each project is completed the sum of its expected contribution to consumers' surplus and its expected addition to the value of water in the reservoirs must be equal to the long run marginal cost.

5. The planning problem in practice

Practical planners cannot escape the following issues which we have neglected:

1. *The daily and seasonal variations in demand.*
2. *The regional dimension of demand and production.*
3. *Time-interdependencies in demand.*
4. *Uncertainty about the future development of demand and costs.*

Most of these points have been treated quite extensively in the literature, cf. Turvey (1968) and Turvey and Anderson (1977), Bessière and Morlat (1971), and Massé (1962). Still it seems worthwhile to make some brief comments on them in relation to the model we have presented.

1. Formally the *daily and seasonal variations in demand* can be incorporated in the model of section 2 without changing the demand function. What is not incorporated in section 2 is the response to this on the supply side. Varying demand is another reason besides the varying precipitation for building reservoirs. The modifications needed in the model are already incorporated in section 4.

One characteristic of water-power is that the costs of machinery are low compared to the costs associated with building the reservoirs. Thus a large generating capacity will often be installed just to permit an efficient management of the reservoirs. This means that the peak-load problem is often less serious in a water-power than in a thermal system (this holds true in production, not necessarily if we take production and distribution together). Significant simplifications may be achieved if one can assume one out of two extremes:

- total generating capacity is always determined by peak-load demand.
- peak-load demand will never need all the generating capacity installed for other reasons.

In the first case some possibilities for separating the maximization problem in two parts arise. In the second case the discrete approximations we will eventually use for computing actual solutions, can be made less detailed.

2. The *regional dimension of demand and production* gives rise to transmission costs, which the practical planner has to consider. For a demonstration of how this can be done for a thermal power system, see Turvey (1968) and Uri (1975). The possibilities for transmission makes each project smaller compared to its potential market. But it also means that each project will often be quite large relatively to the local market. In general it is thus hard to say what this means for the optimal path. If weather conditions and peak-loads are highly uncorrelated (or negatively correlated) geographically, it will probably pay to build a relatively large transmission system anyway. Then this may also contribute to smooth the development of prices and to that the cheapest projects are taken first, even if they are large.

3. All the way we have assumed that demand depends only on the current price, i.e. we have ignored *time-interdependencies in demand*. In investment decisions expected future as well as current prices should be taken into account. And investment decisions, especially in housing and in the electricity intensive industries, will obviously affect electricity demand. If expectations are based on planned future prices, they will enter the current demand function. All this casts doubts on our objective function, as it presupposes a kind of separability in time. This is a rather fundamental objection to the approach we have taken.

4. *Uncertainty about the future development of demand and costs* (in addition to that caused by the weather) is a common problem to most economic planning and we shall only refer to the treatment in textbooks like Massé (1962) and Johansen (1978).

6. Conclusions

In this chapter we have been more concerned with developing necessary conditions for an optimal plan than in deriving algorithms for actually carrying out the optimization. The latter problem is obviously very difficult if we are going to take into account all the complications we have mentioned. The first problem is important just because of that. The actual making of the plan must in practice take place through the use of one or more highly simplified formal models together with more informal procedures. To check whether the final plan can possibly be optimal, it would be nice to have some simple criteria to apply. And necessary conditions can provide such criteria, although, as we have seen, it may be hard to find conditions which are simple yet realistic enough. Furthermore, all the detailed plans for each project can probably never be derived from one formal model for the whole system. The necessary conditions can provide criteria for planning of the details.

We have shown that because of indivisibilities the development of electricity supply, prices and cost during a period of growing demand will in general not be as smooth as the Ricardian theory predicts. Finding the optimal path (or even finding a path which with high probability is close to the optimal path) may require a very large amount of labour. This is an argument in favour of centralizing the planning of electricity supply.

Suppose central planners have worked out a plan by the help of a formal model. Could they then announce future prices such that further decisions can be left to the individual utilities? This turns out to be very difficult. First we have the familiar problem of indivisibilities. This means that each utility's timing decisions should be based on consumers' surplus and not on equilibrium prices. We might get around this problem by using some subsidies and taxes. But then we create other problems, since the decision about timing is interrelated with the decisions about dimensioning and operational strategies. For the latter equilibrium prices are relevant. Thus we must induce the utilities to simultaneously take account of consumers' surplus and equilibrium prices. Even if we managed this, there is still another problem. How shall we induce the utilities to complete their projects in the right sequence? The addition to consumers' surplus depends on which projects are already in operation. If investment is rewarded according to this addition, there will be an incentive for everybody to try to be the first to complete a project. Thus, if decentralization through the announcement of prices, taxes, and subsidies is possible, the scheme is bound to be very complicated. All this suggests that the central authorities instead should try to exercise some direct control on when each project is going to be completed. After the timing is decided, the individual units may be allowed to decide the dimensioning based on maximization of expected profits at planned equilibrium prices.

In actual Norwegian planning detailed design is not based on an expected path of future equilibrium prices, but on a criterion related to expected "long run marginal cost" at a specific point of time. This is not consistent with our optimization approach, but it is difficult to see what differences it leads to.

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