SAMFUNNSØKONOMISKE STUDIER

43

MODIS IV

A MODEL FOR ECONOMIC ANALYSIS AND NATIONAL PLANNING
MODELL FOR ØKONOMISK ANALYSE OG NASJONAL PLANLEGGING

By/Av
Olav Bjerkholt and/og Svein Longva

STATISTISK SENTRALBYRÅ
CENTRAL BUREAU OF STATISTICS OF NORWAY
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FORORD

Et hovedsiktemål for den økonomiske forskningsvirksomhet i Statistisk Sentralbyrå har vært å utvikle hensiktsmessig analyseverktøy for politikkanalyse og planlegging. I om lag 20 år har MODIS-modellen - fra MODIS I til MODIS IV - vært det viktigste og mest brukte analyseverktøy for økonomiske problemstillinger utviklet i Statistisk Sentralbyrå. Et stort antall av Byråets medarbeidere har gjennom årene bidratt til utvikling og bruk av modellen. Gjennom bruken av modellen utenfor Byrået, i første rekke Finansdepartementet, har det også kommet impulser og forslag til forbedringer og videreutvikling.

MODIS-modellen har svært nær sammenheng med nasjonalregnskapet både i begreper og definisjoner og ved bruken av regnskapet som datakilde. Arbeidet med MODIS-modellen representerer derved en naturlig videreføring av Statistisk Sentralbyrås arbeid med nasjonalregnskapet.

Arbeidet med å utvikle MODIS IV ble forberedt for henimot ti år siden, bl.a. fordi nasjonalregnskapsstandarden på denne tid ble avgjørende endret. MODIS IV har viktige trekk til felles med sine forgjengere, men har omfattende utvidelser og nye egenskaper sammenliknet med disse. Siden siktemålet med MODIS IV i første rekke har vært å utvikle et verktøy for praktiske analyseformål snarere enn å gi økt innsikt innenfor et avgrenset forskningsmiljø, ble det lagt stor vekt på erfaringer med de tidligere modellversjoner og på tilpasningen av modellen til dens ytre omgivelser og forventede bruk.

Hovedansvarlige for utarbeiding, planlegging og gjennomføring av MODIS IV har vært forskningssjef Olav Bjerkholt og forsker Svein Longva. Modellen ble tatt i bruk i 1973 og er dokumentert i en rekke arbeidsnotater. Resultater fra analyser utført ved hjelp av modellen har blitt publisert i mange sammenhenger. En samlet og inngående beskrivelse av oppbygning og utforming av MODIS IV har imidlertid ikke blitt publisert før nå.

Statistisk Sentralbyrå, Oslo, 13. august 1979

Petter Jakob Bjerve

PREFACE

The economic research activity of the Central Bureau of Statistics has been directed to a considerable extent towards developing appropriate tools for economic policy analysis and planning. For about twenty years the MODIS model - from MODIS I to MODIS IV - has been the most prominent and also the most widely applied result of this development. Several of the employees of the Central Bureau of Statistics have contributed to the development and use of the model.

This publication presents a thorough and detailed study of the structure and design of MODIS IV. The publication has been written by Mr. Olav Bjerkholt and Mr. Svein Longva.

Central Bureau of Statistics, Oslo, 13 August 1979

Petter Jakob Bjerve

INNHOLD

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1. INTRODUCTION

1.1. The background of Norwegian model building

The construction and use of the MODIS models has been an ongoing project for nearly twenty years. For a long time before the first MODIS model was constructed in 1960, stretching all the way back to the period immediately after the Second World War, a comprehensive model for national economic planning was envisaged as a future development and the foundations for such a model were laid.

Over the years since 1960 the successive versions of MODIS have become useful tools for the management of the Norwegian economy. The use of the model has been integrated in the day-to-day tasks of the Ministry of Finance. Changes in the model structure have induced altered routines in the preparation of economic policies and, reversely, changes in user needs have thoroughly influenced the development of the model. Other models have been developed and put to regular use in the wake of MODIS.

Some parts of this increasing reliance on model tools in economic policy-making had been foreseen at an early stage as a possible and attractive future development. It was, of course, not possible for anyone to foresee the tremendous advance in computer speed and capacity and the ensuing magnitude of computer print-outs of model results. On the other hand it may have been imagined that models would have become problem solvers to a higher degree. It cannot be said with any factual pretence whatsoever that MODIS actually governs economic policy. Earlier visions may have implied a more ambitious role for the model in that respect.

The origin of Norwegian planning models

The MODIS model was built as a tool for economic planning to fill a specific need within a historical, political and administrative context. The environment of the model today is basically the same as it was in 1960 with some new features added. The economic problems faced by Norway have changed considerably but the approach and methods of dealing with them within a comprehensive "national budgeting" framework is not fundamentally different.

In fact, the use of macro-economic models for policy purposes in Norway is closely related to the reliance upon national budgeting in the management of economic policy. The national budget was introduced as a conceptual framework as well as a quantitative instrument of planning in the early postwar period. The first Norwegian national budget was presented to the Storting (Norwegian Parliament) in February 1947 as the national budget for 1947.[1] The national budget was introduced by the government as a tool for the formulation and execution of an economic policy with clearly defined targets and instruments. Underlying the national budget was the system of accounts of national budgeting and, equally important, elements of a theory of economic policy. From the beginning, the national budget was worked out on a detailed level, or more correctly, with very detailed considerations of some aspects and a more crude treatment of others. This is still a predominant feature, with some shifts over time in areas of detailed consideration.

The emergence of the national budget as a planning tool can be traced to contributing factors of political, theoretical and economic origin. The parliamentary elections after the war gave the Labour party for the first time a comfortable parliamentary majority and freedom to put into effect its ideas of a planned economy. On the theoretical level came the influence from the Keynesian revolution and the appearance of macro-economic theory, as well as the concepts of national accounts. The economic tasks of postwar reconstruction coupled with the uncertainty of the international development and the government's ambitious aims towards growth and development of the Norwegian economy stressed the need for a comprehensive framework to secure consistency in the detailed quantitative planning measures as well as provide a macro-economic picture for overall analysis.

The first national budget, presented to the Storting by the Minister of Finance Erik Brofoss, discusses at some length three alternative principles of national budgeting.[2] The first alternative, called a diagnostic budget in the document, is simply an accumulation of plans and expectations. The diagnostic budget will in general not be consistent with the balance equations of the national accounts. The value of a diagnostic budget lies in its ability to reveal gaps and inconsistencies. The second alternative, called a prognostic budget, has the shape of a complete and consistent budget and is a prognosis based on assumptions about the behaviour of the economic agents and estimates of exogenous influences. Both these alternatives were discarded in the document in

1) St.meld. (1947). A preliminary national budget for 1946 had been worked out and presented as an annex to the fiscal budget for 1946/47. This, however, served mainly as an exemplification of the new approach rather than as an actual tool. 2) St.meld. (1947), in particular pp. 10-11.

favour of the programmatic budget. In the programmatic budget the targets of economic policy are embedded in the budget. The programmatic budget must be accompanied by a set of instruments which can make the targets achievable under the assumptions of exogenous influences. Some instruments are specified within the budget, i.e. as items directly or almost directly controlled by some branch of government. Other instruments influence the national budget items via monetary policy, fiscal policy, or other regulations.

The document argues against what it calls the common misunderstanding that a programmatic budget is of interest only in an economy with extensive direct goverment control. It is argued that a programmatic budget will be useful whenever the government intends to pursue a rational economic policy and quite independent of what instruments might be at its disposal. This somewhat general statement is borne out by later developments, namely that the national budget has been found useful also in other circumstances than in the immediate postwar period with its tight constraints and special problems. As the domestic economy recovered from the abnormal postwar situation and the external environment changed the set of instruments was gradually changed, the rationing devices for crucial commodities were to a great extent abandoned and replaced by the use of new instruments. We shall not go into the economic problems of Norway in the postwar period and how the national budget approach coped with them, but refer to the existing literature.[1)]

The introduction of national budgeting in Norway can also be seen as an exercise in political education. The national budget implied an extension of the political responsibility for the economic development. The national budget has never been actually voted upon and thus never formally approved by the Storting as has been the case for the fiscal budget. This did not diminish its role as a political programme nor the government's responsibility for its content. The national budget also implied much wider terms of reference for the political discussion of economic policy than had been the case before the war. It was necessary to rid the political debate of the false analogies between sound government economy policy on the one hand and prudence in private

1) See e.g. Statistisk Sentralbyrå (1965), Bjerve (1959) and (1976).

households and enterprises on the other. This had been achieved on the theoretical level by Keynes and followers but as Keynes pointed out: politicians and civil servants above thirty years of age are usually uninfluenced by new theories (Keynes (1936), p. 383-384). The Norwegian Minister of Finance was bent on not letting this happen to the Norwegian parliamentarians! The first national budgets met with some political opposition but not overly much. The principles of national budgeting and the implied scope of government economic policy were never seriously challenged by a strong political opposition; thus the national budget was never contested by later non-Labour governments.

The national budget put the fiscal budget in its proper perspective as an important, but nevertheless subordinate, part of the government's scope of economic policy. The national budget was logically prior to the fiscal budget which could be viewed as a supervisory instrument to achieve the goals of the national budget. The fiscal budget was, however, surrounded by rules, laws, political traditions and prestige which to some extent counteracted its subordinate role. It did not make the co-ordination between the national budget and the fiscal budget any easier that the fiscal year was different from the calendar year. This implied that the preparation of fiscal policy and sometimes the final decisions had to be undertaken prior to the national budget analysis. The change to calendar fiscal years in 1961 made things easier but it was the use of models from 1960 onwards which started a process of more comprehensive national budgets and better integration of the fiscal budget and other institutional structures within the framework of the national budget.

The first national budget also discusses two other general problems of particular interest from a model point of view. The first is the problem of the time horizon. The national budget was a programme for one year ahead. A rational economic policy required a more distant horizon. In the short run there were more constraints and less freedom of action. The national budget was thus later supplemented by a four-year programme, in Norway called a long-term programme. The first long-term programme was prepared on the initiative of the Organization for European Economic Cooperation (OEEC) in 1948 and covered the period 1949 - 1952. This was succeeded by other long-term programmes at regular four-year intervals coinciding with election periods. The National Budget 1947 expressed the need for a long-term programme and stressed the importance of a full co-ordination between this and the annual national budgets. Until the arrival of model tools this proved difficult to achieve in full for various reasons.

The difficulty was mainly that the various assumptions of the long-term programme had to be revised and updated to be on the same footing as the individual national budgets of the programme period. Otherwise the national budget would lose touch with the long-term programme and the influence of the latter on the political decisions would fade. Problems of this sort were present until both the national budget and the long-term programme could be dealt with by an appropriate model which could also serve as an instrument for updating the long-term programme on a "rolling" basis. Although the problem was diagnosed at an early stage, a satisfactory solution was not found until the 1970's.

Another problem or, rather, set of problems, touched upon in the first national budget was the question of the realism of the budget. The budget was never meant to be a set of consistent forecasts worked out by the Labour Government. The value of the budget was its role as an integrating tool linking various government and semi-official bodies in the process of working out the economic prospects and the economic policy for the coming year. Realistic assessments and assumptions were of crucial importance for the final results. This was not only a question of prudent realism on the part of the participants in the national budgeting process. To abstain from wishful thinking was not enough. The various sub-budgets had to be tied together in a way which took care of the interrelations of the economy and the actual functioning of the economic mechanisms. To deal with this on the chosen level of disaggregation in a time when national accounts, computers and econometric models were still in their infancy, was to try to manage the unmanageable. Even with national accounts, computers, econometric models and decades of experience we are still far from fully satisfactory solutions of the day-to-day problems of national budgeting. However, it was the introduction of the national budget and its success as a vehicle of economic policy which led the way for the controlled invasion, after 1960, of macro-economic models into the inner life of the policy-making departments of the Ministry of Finance.

The general philosophy of model building

The community of Norwegian economists is perhaps more homogeneous with regard to basic conceptions than its counterpart in many other countries. This may have to do with the size of the country and the fact that the majority of Norwegian economists received their higher education at the same institute. But it certainly also has to do with the preponderance of Professor Ragnar Frisch at that institute. His general philosophy on economics as a science has left its mark on a full generation of Norwegian economists, also on those who did not share his particular views on economic and political issues.

Professor Ragnar Frisch's contributions to the theory of economic planning and econometric model building were pioneering and far-reaching. This is not the place, however, to present a full appraisal of his contributions.[1] Frisch's contributions to the construction and use of the MODIS model has been manifest in several ways, foremost by his all-pervasive influence on economic theorizing in Norway, in general, and by his central role as teacher of numerous vintages of economists at the Institute of Economics, University of Oslo. Of more direct interest for our exposition are his contributions to the philosophy and methodology of macro-economic model building, as set forth in lectures and a great number of mimeographed memoranda from the Institute of Economics, of which comparatively few have been given a wider circulation. His efforts in this field were not limited to theoretical studies, he inititated a number of pioneering attempts to build models of the Norwegian economy at a time when the available data and computing equipment could not do justice to his ambitious aims. Some of these early models of the 1950's, in particular the Oslo Median Model, see Frisch (1956), are direct precursors of the first MODIS model.

To Frisch the basic and all important rationale of economic model building was the need for and use of models as tools for a comprehensive national and international economic planning. In a classification well-known among his students he distinguished between four stages in economic forecasting: the on-looker approach, the ad hoc instrument approach, the feasible instrument approach and the optimalization approach (Frisch (1961), pp. 1-6). These are stages of attitudes rather than of methods.

The <u>on-looker approach</u> may cover a wide range of methods from mechanical trend extrapolation to refined econometric models. The common feature is that the on-looker analyst "simply tries to guess at what will happen without making any systematic attempt at finding out what somebody - the Government or a private organization or a coalition of private organizations - <u>ought to</u> do if they want to influence the course of affairs" (ibid. p. 2).

1) See e.g. Johansen (1969) and Edvardsen (1970).

In the second stage, the ad hoc instrument approach, it has dawned upon the analyst that there are in the economy certain instruments or decisional elements which may be changed at will to induce changes in the course of affairs. His understanding of the interrelations of the economy has not reached the stage, however, where it can be formulated as a complete model with a definite number of degrees of freedom. To follow the advice of the ad hoc instrument analyst one runs the risk of arriving at "... quite unexpected, even chaotic, results, producing extreme tensions and contradictions in the economic structure" (ibid. p. 3). The ad hoc instrument approach is thus an intermediary stage, "... a very first and tentative preparation for a further analysis that does lead to a precise dynamic model with a well defined number of degrees of freedom" (ibid. p. 3).

In the feasible instrument approach the analyst has reached a stage where he thinks in terms of a complete model where the degrees of freedom correspond to instrument and truly exogenous (uncontrollable) variables. For each set of guesses at values for the exogenous variables there is a whole range of alternative fixations of the instruments which span the feasibility space. At this stage the analyst has to co-operate with the decision-makers. "Only through such a co-operation with demonstration of alternatives will it be possible to map out to the authorities the feasible alternatives and to help them understand which one - or which ones - amongst the feasible alternatives are the most desirable from their own viewpoint. To develop a technique of discussing feasible policy alternatives in such a scientific way is one of the most burning needs in economic policy-making today" (ibid. p. 4).

But even the feasible instrument approach is not sufficient for a rational approach to economic policy. "When the effort to map out a spectrum of feasible alternatives has gone on for a while, the conclusion will inevitably force itself upon the public and the authorities that the number of feasible alternatives is so great that it is impossible to keep track of them simply by listing them and looking at them" (ibid. p.5). The fourth and final stage is the optimalization approach which includes a preference function and a mathematical programming technique for locating the most preferred solutions among the feasible policy alternatives.

Frisch never lost this perspective on the future of model building. In fact, when macro-economic model building had got well under way and been put to practical use in Norway as well as in other countries, Frisch devoted a great part of his energy and ingenuity to attack the crucial problems of the final stage in model building, namely, how to deduce and establish preference functions and how to solve the ensuing problems of mathematical programming.

On occasions Frisch could let out considerable scorn over model builders and users who - in his view - did not have a proper understanding and perspective of what they were doing. He had a never faltering and strong belief in the possibilities for improving the material conditions of mankind as well as promoting a true democracy by appropriate use of scientific economic programming at the national and international level.

The macro-economic model building work in Norway has never reached higher than a moderately reasonable satisfaction of the feasible instrument approach. A formal approach to preference functions in the context of overall macro-economic models has been tried in very few places around the world except as merely academic exercises. The identification and estimation of observed preference functions of various interest groups - as attempted by Frisch - involve methodological questions of an exceedingly intricate nature. Rational discussion of postulated preference functions seems to be a much harder task than discussion of policy alternatives. On the other hand, in the absence of an explicit preference function one is left with precisely the problem expressed by Frisch in the quotation above, that a successfully implemented feasible instrument model may be used to generate too many feasible alternatives to be sorted out and evaluated in a wholly intuitive manner in the minds of the planners. Paradoxically, the better the model is for generating feasible alternatives the more difficult it may seem to choose one amongst them. With regard to MODIS IV this problem has been dealt with on the basis of a close collaboration between the planners and the model building unit. Great effort has been put into achieving a user-oriented model, especially with regard to the two-way communication between the planners and the model. A basic idea has been that the shortcomings of the model with regard to the theoretical content as well as the lack of formal procedures for evaluation of alternative results and other weaknesses have to be compensated for in some way or other within the administrative environment of the model. The close collaboration between economic theoreticians, statisticians, model builders and planners and the integration of the model into the planning administration is certainly in the spirit of Frisch although his formal devices in terms of preference functions and optimalization are lagging far behind.

National accounts and input-output analysis

The MODIS models are rooted in national accounting both historically and by the internal structure of the models. After the Second World War a comprehensive political effort was made to build up a planning apparatus in Norway to deal with reconstruction and other post-war economic problems. At the time planning models in the modern sense were only to be seen in the distant horizon by the most farsighted planners and theoreticians. The immediate task on the research front was to collect and organize statistical data to give a coherent overall picture. National accounting was in an embryonic stage growing out of the pre-war discipline of measuring national income and early efforts to formulate comprehensive and consistent systems of concepts for national accounting.

The task of constructing national accounts for Norway was undertaken by the Central Bureau of Statistics. By the middle of the 1950's operational routines for the construction of annual national accounts were well established. Detailed accounts had been worked out for every year from 1949 and in a more aggregated form from 1930 (with war years excluded). The main responsibility for this work was carried by Odd Aukrust. The early work is summed up in the introduction to NOS (1952) and in Aukrust (1955).

In his discussion of the theoretical foundation of the national accounts Aukrust underlines the potential analytic use of the data thus organized. Rather than relying on some conventional accounting principles Aukrust stresses that "the main function of national accounting is to produce a well-organized system of economic statistics to meet the needs of economic policy and economic theory" (ibid. p. 103). This orientation of the national accounting work also makes clear its place within an overall effort towards developing analytic tools for economic analysis and policy formation.

Aukrust sees furthermore the accounting system he develops as a fruitful synthesis of three major influences, which can to a great extent be identified with those of Ragnar Frisch, Richard Stone, and Wassily Leontief. Ragnar Frisch had, since before 1940, been preoccupied with the idea of replacing the then common national income calculations with comprehensive national accounts based on an adequate and consistent system of concepts. The idea was worked out in some detail in the early 1940's, see e.g. Frisch (1942), and later systematized in an axiomatic form by Frisch and associates, see Aukrust, Bjerve and Frisch (1948). The contribution by Frisch et al. was centered on the conceptual problems of national accounting covering the logical structure between the concepts and the use of terminology and mathematical notation. The conceptual structure was called by Frisch "the eco-cirk system", a term which became common in Norway ("økosirksystemet") but never won international recognition.

Hardly less important than Frisch was the influence of Richard Stone. Aukrust in particular stresses Stone's influence on the Norwegian national accounts and on his own views in many matters. Aukrust gives credit to Stone as the leading theoretician behind the effort to achieve an international standardization of national accounting. One of Stone's first seminal contributions to the field was a report to a sub-committee of the League of Nations, see Stone (1947). Since then he has kept up an almost incessant activity in the field and played a major role in promoting national accounting as a discipline in its own right and in extending the applications of the basic concepts and ideas to new areas of social interaction. His model building activities as initiator and supervisor of the Cambridge Growth Project are no less notable.

Stone's representation of the economic system is in terms of a large number of accounting entities. The national accounts are defined as an appropriate aggregation of the accounts of individual entities. While Frisch draws a fundamental distinction between <u>real flows</u> and <u>financial flows</u>, Stone's accounting approach is in terms of payment flows.

Stone's early work is more pragmatic and empirically oriented than that of Frisch. Aukrust acknowledges the direct influence of Stone on the practical implementation of a national accounting system in Norway within a theoretical framework taken over from Frisch. Stone has also been instrumental in bringing about a revised version of United Nations Standard of National Accounts, see United Nations (1968a). He acted as chairman of the sessions of the Expert Group convened to assist and advice in preparation of the new standard. Norway adopted the new standard from 1970 and it will become clear from chapter 2 and 3 that the new standard has had great significance for the present version of the MODIS model.

The third influence named by Aukrust was that of Leontief. Although Leontief did not deal with the accounting problem per se, his work up to the early post-war period did significantly influence the Norwegian national accounting system in one important respect. The decision was taken at an early stage to include input-output tables as an integral part of the accounts. Aukrust says about this decision: "When the possibilities of input-output analysis had been demonstrated by Leontief it was reasonable to assume that an interest in such analyses would arise in Norway too. The Norwegian national accounting system was accordingly designed to supply data for such a purpose and it was one of very few accounting systems with this feature" (Aukrust (1955), p. 35). The subsequent use of input-output tables as soon as the computational facilities made them easy to handle, certainly proved the decision right. At the time, however, the use of input-output analysis was still at an illustrative stage. Leontief had worked for quite a long time with input-output tables of the American economy. His early publications, see Leontief (1936), (1941), were focused more on the structural and descriptive aspect of input-output tables. The operational use to which the tables could be put and the formulation of the simple Leontief model seem to have been conceived during the war. Some articles written by Leontief during the war were later added to his 1941 book and published in 1951, Leontief (1951). In 1953 he edited a seminal and influential volume, Leontief et al. (1953), which did a great deal to promote the use of input-output analysis within several related fields. (Unfortunately, the enthusiasm over this new tool and the tasks it could master backfired in the ruling political climate of the 1950's and in the first year of the Eisenhower administration the channels for financing the development of economic planning tools based on input-output analysis from defense research funds were cut off.[1] Nearly twenty-five years later several US Senators would be pursuing the aims of the Humphrey-Javits bill on economic planning by travelling all over the world studying i.a. the uses to which input-output analysis could be put to improve the performance of the national economy !)

The Leontief contribution to applied economic research is quite unique. It is in itself remarkable that the origin and furtherance of the simple and general idea underlying input-output analysis have been so closely associated with one person with hardly any predecessors apart from rather remote precursors in Quesnay and Marx.[2] Over the last two decades Leontief and associates have tilled new ground for applying input-output analysis in variety of fields including disarmament, pollution and energy and culminating in a UN study of 1977, Leontief et al. (1977).

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1) See Business Week, Aug. 29. 1953, p. 26. 2) Leontief (1941) recognizes and pays due respect to Quesnay as his predecessor in the field. The case for Marx is argued by Bródy (1970) who claims to have found the first proper interindustry table in the Grundrisse manuscript which became known in the West only two or three decades ago. To the history of the origin of the input-output analysis belongs also the assertion made by Oscar Lange (1963) that Leontief was introduced to the idea and concept of input-output analysis by Soviet economists while he was an employee of Gosplan. This seems to be incorrect, although Leontief may well have been influenced by the activity in the 1920's of Soviet economists whose work he knew well.

For Norwegians it is possible to take pride in a statement by Leontief from 1974. When he was asked in an interview whether the tool he created had been well used he quickly replied: "If I had to say which country makes the best use of it, I should probably say Norway".[1)] (Unfortunately, the statement is probably not true.) In his answer he also stressed the importance of providing an appropriate statistical data base. The mere formalism of input-output analysis has very limited value in empirical work unless it can be backed by a reasonably satisfactory body of compiled statistics. The framework in itself can serve as a guide to the collection of statistical data. The entries in an input-output table will typically be of highly varying quality ranging from direct observations with only a small margin of error to "best guesses" from very scanty data. He added somewhat reassuringly for input-output practitioners: "I do not think that the accuracy of statistics is infinitely important for the purposes of using the technique. Sometimes it may be so complicated to obtain highly accurate and refined statistics that it is not worth the effort involved".

Input-output tables at a fairly disaggregated level were included in the annual national accounts of Norway from 1949. By international comparison the Norwegian national accounts were brought up to a very high standard by the early 1950's.

On the basis of the first input-output tables prepared by the Central Bureau of Statistics model building efforts were exerted by Frisch and associates at the Institute of Economics. In this period there was close contact between the Institute and the research group of the Bureau with links to the policy-makers and planners. Towards the end of the 1950's the Bureau gained access to one of the first electronic computers installed in Norway, a British made DEUCE computer. In 1960 the first input-output model of the Bureau was worked out and with some ingenious programming effort it was made operational. The model was baptized MODIS, now known as MODIS I.

The choice of model structure

The original conception and later development of the model are rather different from the mainstream of short-run models in the sixties and seventies. This has been noted for instance by Waelbroeck (1975) in a survey of short-run model research outside the United States. "A completely different tradition in model building exists in Norway. Stemming

1) L'Expansion 1974. Quoted from Economic Impact, Number Nine, 1975, p.72.

from the model building work of Frisch and Johansen, the Central Statistical Bureau has, under the direction of Aukrust, built a series of "Modis" and "Prim" models, to predict industrial output, prices, and income distribution by means of input-output analyses. These models, in which the major final demand aggregates are predicted exogenously, and in which coefficients are not estimated econometrically, are completely different from the other models surveyed; no comparable work exists elsewhere: in other countries input-output has found applications in long-term rather than in short-term planning" (p. 425).

It is certainly misleading to speak of a "completely different tradition" as Waelbroeck does when referring to model building in Norway. There are marked differences in emphasis and environment rather than in basic methodology which makes the MODIS model stand apart from the bulk of models surveyed by Waelbroeck. In the modern fashion of characterizing a model by concatenation of the names of its inspirators and originators MODIS IV may be tagged as a Frisch-Stone-Leontief-Keynes-Aukrust model. We have already dwelt upon the contributions of these prominent economists in contributing to the background and foundation of the Norwegian macroeconomic model building.

The first MODIS model, MODIS I, was a simple Leontief model with an aggregate consumption function, see Sevaldson (1964). The number of industries was about 125. The model had variables in constant prices only and no price relations. The theory underlying the use of the model was that of Keynesian demand management. Final demand apart from private consumption was exogenously determined. A number of the production sectors had exogenous production levels on the assumption that other factors than demand were decisive in the short run. To obtain consistency between generated demand and exogenous production for these sectors endogenous adjustments were made in stocks, imports and input-output coefficients.

The next version, MODIS II, which arrived in 1965, had much more ambitious aims.[1)] This version included a complete set of input-output relations in prices as well as quantity relations. The number of production sectors was increased to about 140 and final demand and other variables were dealt with in a considerably more detailed way. The price model was based on the subdivision of the industries in sheltered vs. exposed industries, sometimes referred to as the Scandinavian model. The concepts and ideas of this distinction was to a great extent developed by Aukrust.[2)]

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1) See Øien (1966), Sevaldson (1968). 2) The distinction between sheltered and exposed industries has its forerunners in the history of economic analyses, however, see e.g. Harrod (1957), p. 53-56.

The first embodiment in an empirical model of these ideas was in MODIS II, although they became better known from a more aggregated model called PRIM originating in the report from 1966 of an expert committee on income settlements chaired by Aukrust.[1)]

By combining prices and quantities the results from MODIS II also displayed incomes. Furthermore, the model included relations for direct taxes and for indirect taxes and subsidies. The final results included a set of hierarchic accounts of disposable income. Disposable income for Norway was subdivided in government and private disposable income. The latter was subdivided further in disposable income for enterprises and disposable income for households which again was subdivided in disposable income for wage and salary earners and disposable income for self-employed.

The effort behind MODIS II was very ambitious with regard to completeness, in trying to build a model framework to cover the main areas of economic policy. Up to that time the whole national budget exercise was conducted in constant prices only. Prices were dealt with as a separate area of economic policy. As a matter of fact the national budget figures were not published in current prices as well as in constant prices until 1975 mainly out of reluctance with regard to publishing official forecasts of price increases. The inclusion of price relations in MODIS II paved the way for securing the consistency of the national budget in a general equilibrium sense just as MODIS I had been a tool to secure the consistency between final demand and the composition of production.

MODIS II had, however, its weaknesses both with regard to content and in terms of operationality and reliability. The model was improved and rebuilt as MODIS III in 1967.[2)] Throughout the period of MODIS III from 1967 until 1973 the use of the model by the Ministry of Finance increased tremendously. The model acquired its central role in the national budgeting process and was also used for other purposes such as medium term planning, ad hoc analysis of macro-economic problems and for the calculation of impact coefficients.

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1) Statsministeren (1966), Lønns- og prisdepartementet (1968) and Aukrust (1970). Later versions of the model were called PRIM II and III. It was decided, however, in connexion with the development of MODIS IV to merge the two model series as PRIM was nothing but a cruder and more aggregated version of important elements of MODIS. The importance PRIM acquired as a medium for discussing alternative income settlements was due in great extent to its formal simplicity. 2) See Bjerkholt (1968), Sevaldson (1971).

MODIS IV, which was completed in 1973, was planned to achieve the aims and ambitions of the preceding versions. In addition, the basic input-output structure was completely rebuilt, as will be discussed extensively in the following chapters. The weaknesses in content of the preceding versions were evaluated and efforts of improvement were exerted in the areas where it seemed to be feasible and most pressing. The experience from the use of MODIS III also played a major role in designing the user properties of the new model, mainly the system of communication between the model and its user, i.e. input forms, output tables, control of errors, consistency checks of input data etc.

As indicated above the use of models in policy-making had been envisaged at an early stage. The work on national accounts had included a conscious effort to lay the cornerstones for future model building. On the user side the model was expected to play a role primarily in the annual national budgeting process. The model was awaited to take over a task formerly performed by administrative routines. The first MODIS model could thus be put right into a planning context.

An important feature of the Norwegian model building effort has been the close co-operation between model builders, planners, data suppliers, and the main academic institution for economic research. The Central Bureau of Statistics has served in a double role within this co-operation, as the main model building agency as well as the data supplier. The Ministry of Finance which carries main responsibility for macro-economic planning has shown a very open-minded and positive attitude towards adapting its routines according to the requirements of the model. There has been a two-way channel of adaption. The Ministry of Finance has had ample opportunity to influence the model development, and the successive model versions have been more and more dedicated to the policy-making framework of the Ministry.

1.2. A brief summary of the model structure

The structure of the model may be outlined as in diagram 1.1. Full-drawn boxes indicate formalized parts of MODIS IV. Dotted boxes indicate still unformalized parts. Other informal models, e.g. sector models, might be added to the diagram. The connexion between informal models and MODIS IV is mediated through exogenous variables and parameter changes. The model is thus, at the present stage, "closed" at various points by exogenous assumption instead of appropriate additional models.

The central part of the model is the conceptual and accounting definitions and the basic relations representing the technological structure and the cost structure of the economy. The technological and cost structures are modelled by using a modified form of the input-output formulation of Leontief.

Apart from the accounting definitions and the basic structural relations the model consists of a number of parts, or submodels, the main ones being those belonging to the quantity model and the price model. At present there are in addition two submodels for direct and indirect taxes, respectively.

Indicated in diagram 1.1 are also additional submodels which at the present time are not formalized but which reside in the administrative environment. In the further development it is an aim to include in the formal framework all interrelations between the variables of the model. It is almost inevitable, however, that the full model of the functioning of the economy as seen by the user is for some parts too complex or too vague to be included in the computational set-up. The envisaged full model is referred to as the <u>outer model</u>, while the basic structural relations together with the "projections" of the outer model into the basic equations will be referred to as the <u>inner model</u>. The outer model thus includes the inner model as an embedded part, and the outer model may thus include parts which are not formalized for computational solution. The non-formalized parts of the outer model are represented in the formal structure as quasi-exogenous variables. It is important for the overall consistency of the model results that the logic underlying the estimation of these quasi-exogenous variables is consistent with the other assumptions of the model.

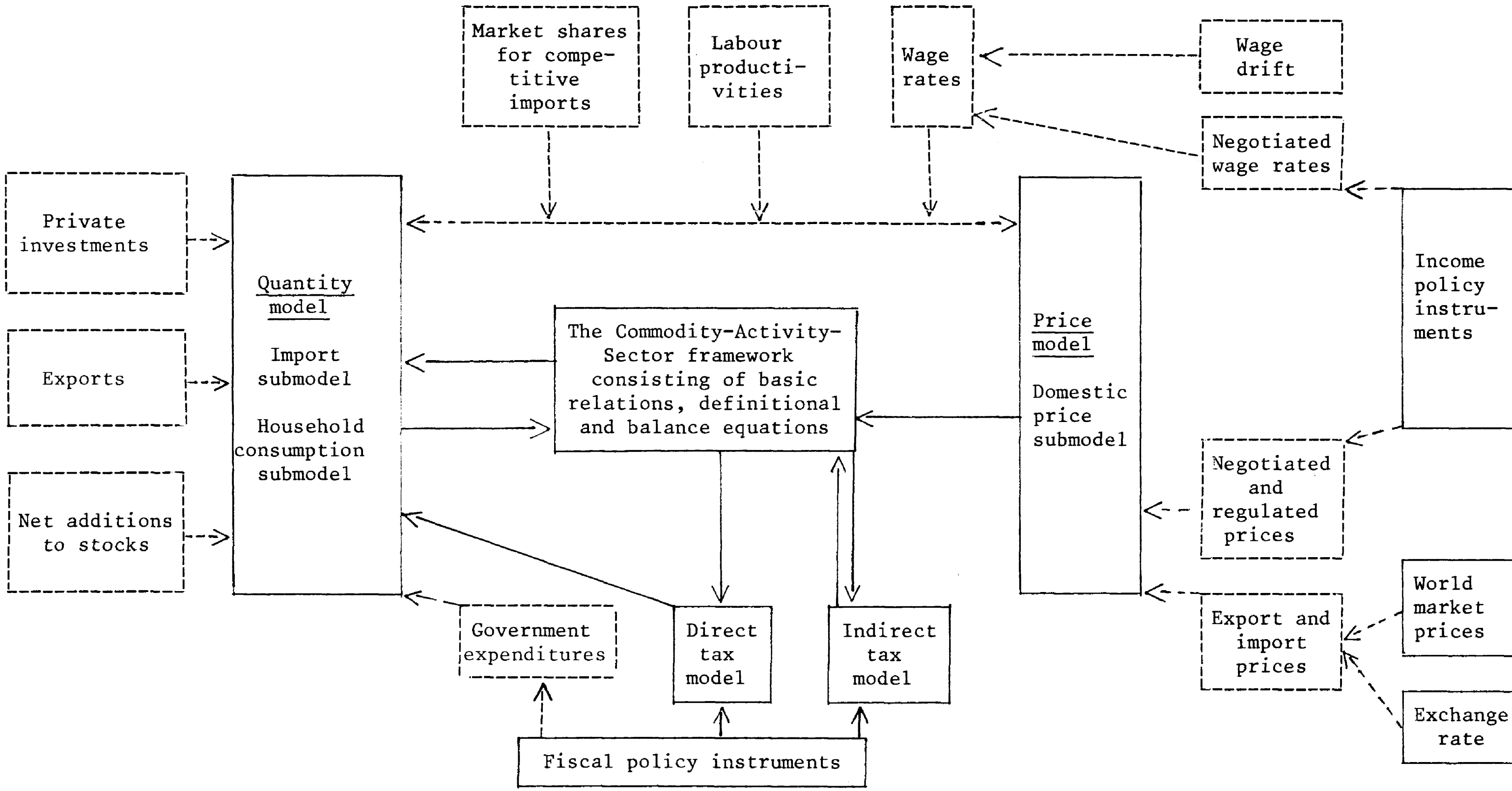

Diagram 1.1. Structural map of MODIS IV

The input-output framework

The core of the model is the input-output formulation of the technological and cost structures of the economy. MODIS IV has moved slightly away from its predecessors in that a distinction is now made between commodities, sectors and activities. By commodity is meant a grouping of goods and services, by sector a functional unit of the economy which takes part in the commodity circulation, and by activity a subdivision of sectors according to characteristic properties of the type of commodity generation, absorption or transformation which is taking place. The production activities have, normally, both input and output of commodities, while the import and final demand activities have only output and only input, respectively.

Within each activity fixed proportions are assumed between commodity inputs and commodity outputs. This means that, apart from the special treatment of some industries (especially ocean transport and crude oil extraction), the activity coefficients are assumed to remain constant irrespective of the levels of output (or input) and relative prices. The assumption of fixed coefficients in the original Leontief formulation is thus maintained, but only within each activity. On the production side this means that for a sector as a whole changes in the output mix will change the composition of input. On the final demand side the assumption of commodities in fixed proportions is applied to subcategories of the conventional sector classification.

The coefficients are estimated from commodity-by-sector input and output tables for the base year of the model (usually the year prior to the current year). The problem of allocating inputs to a production sector (an industry) among its activities is at present solved simply by classifying each production activity as having either a commodity technology, i.e. the same input structure as other activities with the same output, or a sector technology, i.e. the same input structure as other activities belonging to the same sector.

The commodity-activity-sector framework is a mapping of the commodity flows of the economy. The links between the sectors and economic entities outside the commodity sphere are provided by primary inputs and final outputs of the sectors. By definition, each sector is balanced in the sense that the value of primary input plus commodity input equals the value of final output plus commodity output.

A production sector has a primary input of labour services (wages), capital services (depreciation and operating surplus), which together with net indirect taxes, make up the difference between the value of commodity output and commodity input. The import and export sectors have primary inputs from and to the foreign account equal to the value of commodity output and input, respectively. The domestic final demand sectors have final outputs equal to the value of the various types of final demand.

The model results consists of sets of prices and quantities, complete and consistent in an accounting sense, with time reference to calendar years. Prices of commodity flows are indices of unit values, relative to a given base year, and quantities of commodity flows are measured in unit values of that year. The input-output structure of the economy is modelled both for quantities and prices, the input-output price relations being the dual counterpart to the input-output quantity relations. The rest of the model system is closely linked to and integrated through these basic structural relations.

The quantity model

The quantity model is mainly demand oriented and the supply side is assumed to respond to any real demand for commodities, labour and capital services etc. The model distinguishes between commodity demand through household consumption, private investments, government expenditures, exports and net additions in stocks. In addition, intermediate commodity demand plays an important role in a rather disaggregated model like MODIS IV. This demand is taken care of via the input-output quantity relations discussed above.

Household consumption is dealt with by a system of consumption relations. The main elements of the submodel for household consumption are an aggregate consumption function, and a set of distribution relations. The aggregate consumption function determines the total demand for household consumption as a function of real disposable income for three socio-economic groups, viz. (i) wage and salary earners, (ii) self-employed, and (iii) pensioners. The nominal incomes are made up of wages, profits of unincorporated enterprises (incl. agriculture) and government transfers distributed on the three socio-economic groups. After deducting direct taxes and deflating by an index of consumer prices real disposable incomes are arrived at. The distribution relations allocate the total demand for household consumption among the household consumption activities by means of income (Engel) and price (Cournot) derivatives.

The main groups of input variables of the submodel are (i) consumer good prices, including the appropriate consumer price index (determined in the price model), (ii) wage rates (exogenous), (iii) industrial employment (simultaneously determined) and government employment (exogenous), (iv) profits (simultaneously determined), (v) government transfers (exogenous), and (vi) direct taxes (simultaneously determined).

Private investments, exports, net additions to stocks and government expenditures are all exogenously given in the model.

The demand for each commodity is, on the supply side, met by domestic outputs and imports, the distribution between these two sources being dependent upon the demand composition for the commodity. Imports of commodities are divided more or less conventionally in competitive and non-competitive commodities. The non-competitive imports are directly determined by demand. The import relations for competitive commodities are built around a matrix of import shares for the input of each commodity to each activity. However, it is possible to change these import shares exogenously. The matrix of import shares (specified by commodity and receiving activity) reflects the fact that the import content of a given commodity will differ between receiving activities, especially between export and domestic demand. The distribution of domestic production of a given commodity among its various suppliers is dealt with through fixed market shares.

Value added production functions in inverted forms are used as labour requirement functions. In these functions industrial employment is linked to the domestic production level through exogenous productivity estimates. Industrial employment is thus also demand determined in MODIS IV. The numbers of self-employed and government employees are exogenously determined.

The price model

The price side of MODIS IV is, as in the predecessors, strongly supply (cost) oriented. The far-reaching changes in the actual formulation of the price relations are mainly a consequence of the new commodity-activity-sector approach, but some new elements have been added.

The commodity prices are the most important variables in the price model. The price of a given commodity flow is assumed to differ depending on whether it is imported or domestically produced and on whether it is exported or delivered to the domestic market. Each commodity mav. accordingly, have an import price, an export price, and a domestic price.

The import and export prices are exogenously given through forecasts for the world market prices and of the exchange rate.

Reflecting the openness of the Norwegian economy an important feature of the price model is the distinction between the exposed and the sheltered domestic prices. The exposed domestic prices are prices of commodities produced and marketed domestically under strong foreign competition. In the model it is assumed that the exposed domestic prices normally are adjusted to the corresponding import prices. The sheltered domestic prices on the other hand are prices of domestically produced dommodities sold in domestic markets sheltered to greater extent from foreign competition. For the latter commodities the model assumes two different kinds of price formation, namely regulated and negotiated prices and cost determined prices.

The regulated and negotiated prices are prices which are either fixed or regulated more or less completely by public bodies or determined through negotiations between the government and producer organizations (agricultural prices).

The cost determined prices are assumed to adjust to changes in the costs of producing the commodities. Wage costs per unit of production are given by the exogenous estimates for labour productivities and wage rates. In normal use of the model the exogenous mark-up rates are adjusted so that the share of gross profits (depreciation and profits) in factor income in the production for sheltered domestic markets is left more or less unaffected by changes in costs. The necessary parameters for the computation of the indirect tax costs are determined in the indirect tax model. The price propagation process which follows from the fact that higher output prices of commodities from one production sector means higher input prices, i.e. higher costs, in other, is dealt with by the input-output price relations discussed above.

The interactions between the relationships

From this short description of the formalized parts of the quantity and price sides of the model it follows that supply conditions determine prices independent of demand. It is possible to illustrate this by the simple supply-demand situation shown in diagram 1.2. The supply curves, with commodity prices as arguments, are infinitely elastic, i.e. horizontal, since the price model is solved independent of final demand. The demand curves have downward slopes due to the commodity price influence on household consumption. It follows from this that the price model can be solved before the quantity model. Apart from some minor obstacles, this is the actual solution procedure followed in MODIS IV.

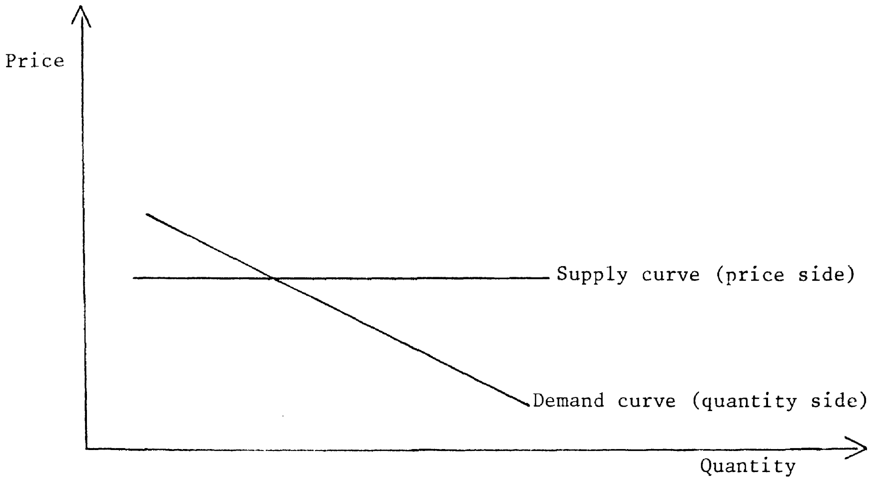

Diagram 1.2. The supply and demand curves of MODIS IV

This recursive structure of the price and quantity models of MODIS IV resembles that of a general equilibrium model in which production takes place under constant returns to scale and primary input prices are given. Both of these assumptions are also made in MODIS IV. It should also be noted that the horizontal supply curve is supposed to be a reasonable approximation to the actual one only in the area close to the desirable equilibrium point (near full employment).

In explaining the interactions within the quantity and price sides of MODIS IV we shall present the major closed loops of the model. Following Barker (1976), p. 21, we define a closed loop as one where the relationships feed back upon themselves so that any solution of the model satisfies the set of relationship in the loop.

As discussed above much of the interpretation and operational meaning of the model is provided by the unformalized parts of the outer model. In actual use the model thus also contains many open loops, not specified here, in which imbalances can arise. The imbalances may be due to differences between target values and model calculated values, in which case the policy instruments must be changed, or due to informal or formalized behavioural relationships not formally included in the model, in which case the model user must change the exogenous variables in such a way that the model results also are in accordance with these relationships.

The three main closed loops presented in the following are (i) the input-output domestic production loop, (ii) the household consumption-income loop, and (iii) the input-output domestic price loop. (i) and (ii) are both parts of the quantity model while (iii) forms the core of the price model.

The input-output domestic production loop is presented in diagram 1.3. Intermediate demand together with household consumption and the various exogenous final demand items add to commodity demand. The domestic part of commodity demand determines industry output. The link between industry output and intermediate commodity demand closes the loop.

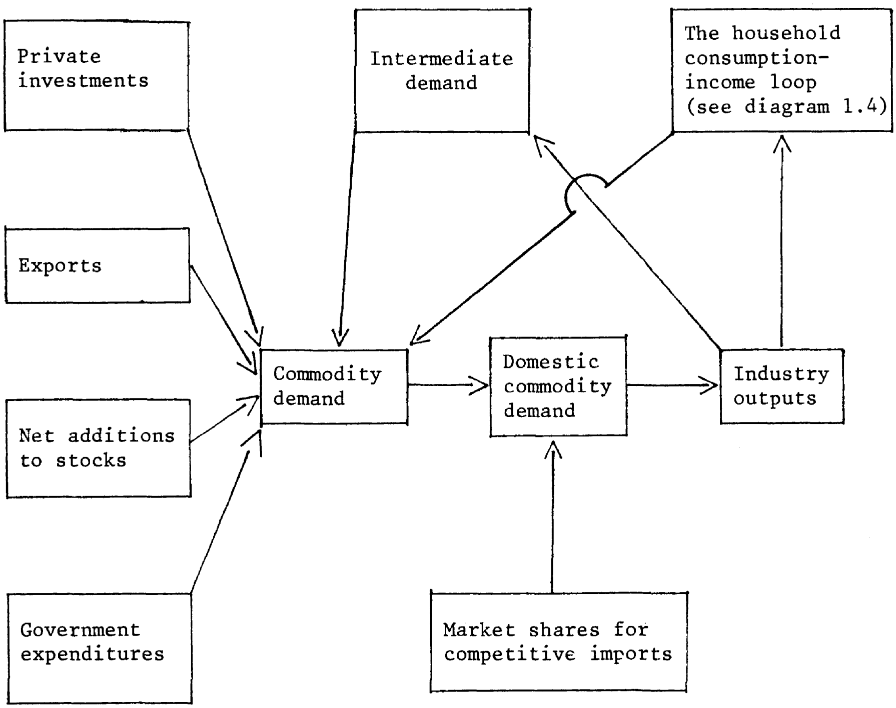

Diagram 1.3. The input-output domestic production loop

As indicated in diagram 1.3 there is also a link between industry outputs and household consumption. This household consumption-income loop is presented in diagram 1.4. Here household consumption is related to real disposable income for the various socio-economic groups and to relative consumer good prices. Household consumption results in commodity demand and industrial outputs. Through the wage relation, which includes exogenous estimates for industrial labour productivities, government employment and wage rates, and through the profit relation which includes results from the price model, industry outputs generate wage income and profits of unincorporated enterprises. These incomes, together with government transfers, are distributed to the various socio-economic groups. By deducting direct taxes (the parameters given from the direct tax model) and by deflating by the consumer price index (given from the price model) real disposable incomes are generated. This closes the loop.

The input-output domestic price loop (see diagram 1.5) is the counterpart on the price side to the input-output domestic production loop. Wage costs are determined by the exogenous wage rates and industrial labour productivities. Gross profit costs are given by the exogenous mark-up rates. The parameters necessary to determine the indirect tax costs are given from the indirect tax model. Intermediate inputs are partly import priced and partly domestically priced, the distribution being determined by the import market shares. Total unit costs generate domestic commodity prices and the loop is closed.

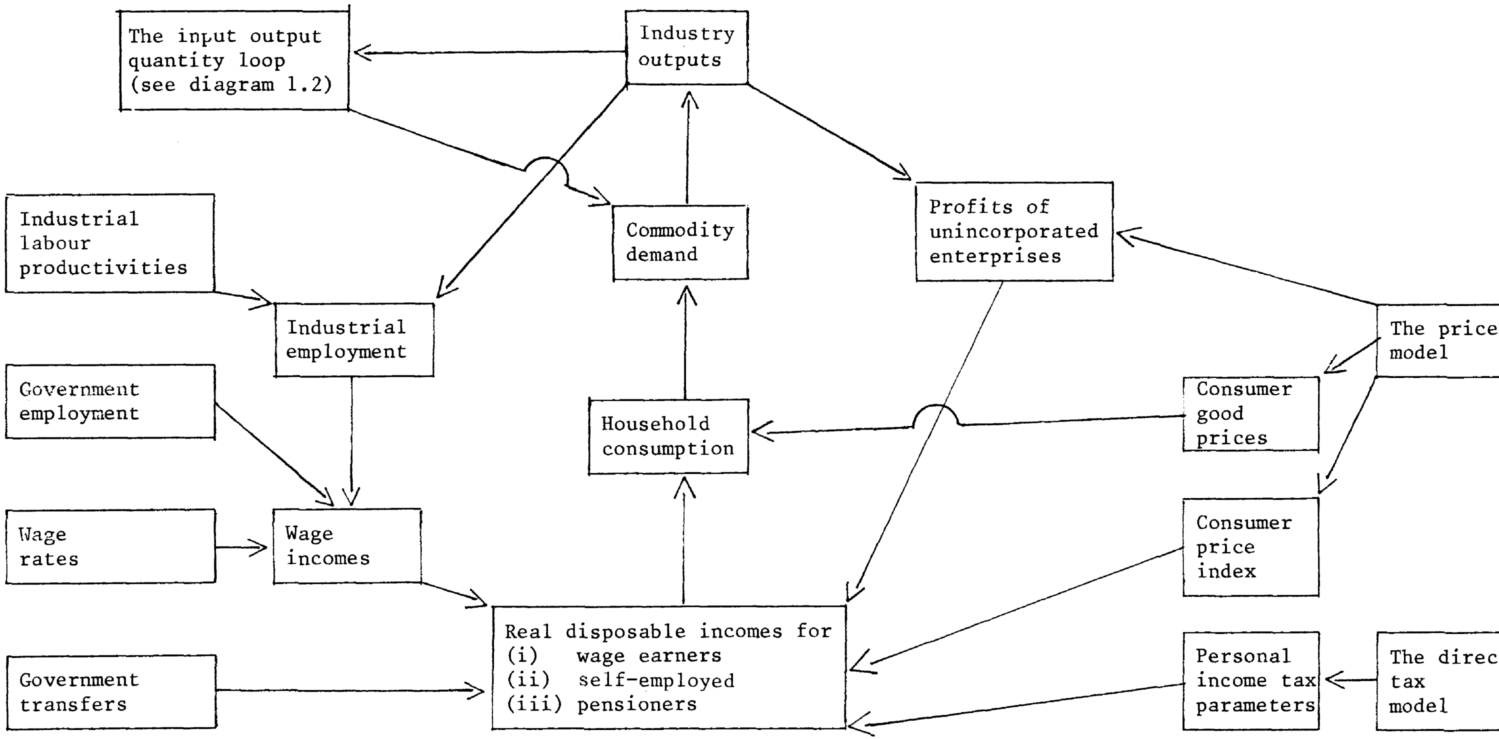

Diagram 1.4. The household consumption-income loop

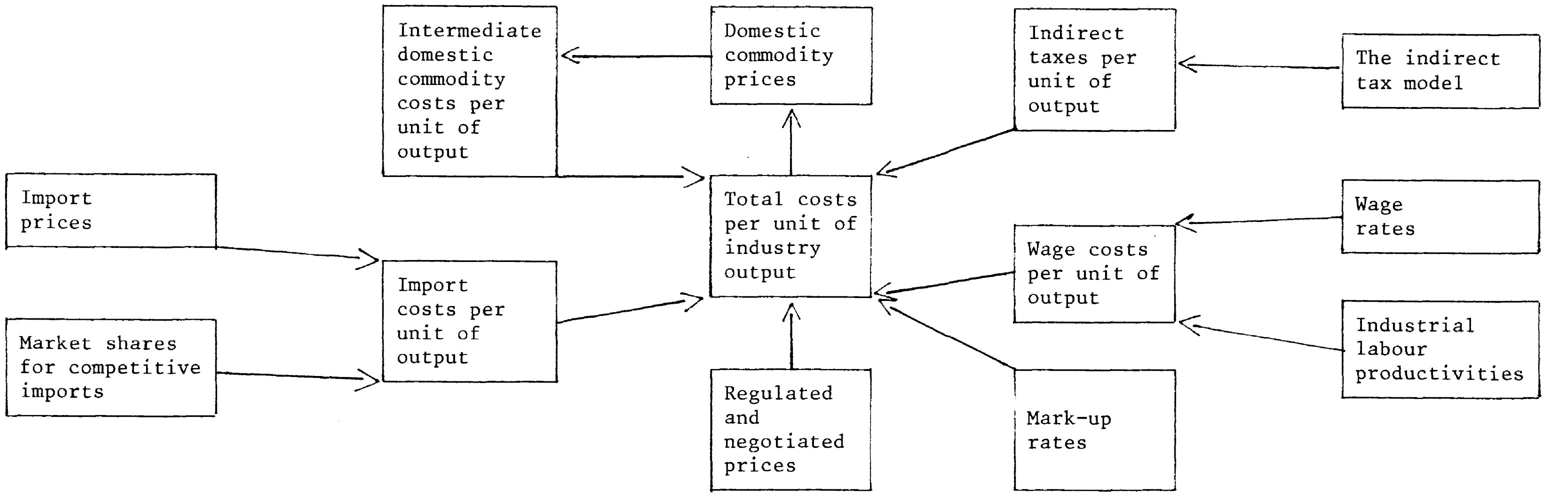

Diagram 1.5. The input-output domestic price loop

Taxes, transfers and fiscal budgeting

The treatment of fiscal items in MODIS IV is intended to serve a threefold purpose. First, while remaining within the macro-economic framework of the model, the specification of exogenous variables relative to fiscal budgets are made to correspond closely with fiscal instruments as determined by law or regulations. Second, for the purpose of fiscal budgeting it has been seen to that each fiscal item in the model belong to one and only one of the general government accounts. Third, arrangements have been made to allow a thorough treatment of the interrelations between the various fiscal items on the one hand and commodity flows and income categories on the other.

The direct tax model distinguishes between the three socio-economic groups of households introduced above, and the income distribution by income intervals within groups is represented in the model. The proceeds of some of the specified types of direct taxes are wholly exogenous. For the others the tax assessment rules are represented in a very detailed way in the non-stochastic tax functions in the micro part of the model. The micro part can be run as a separate model which requires input of forecasted growth of income, number of wage earners, self-employed, pensioners etc. As a part of MODIS IV the micro part is used to estimate parameters of macro tax functions which enter into the household consumption sub-model and therefore into the simultaneous solution of the quantity side of MODIS IV.

The design of the model for indirect taxes (and subsidies) is strongly influenced by the general framework of MODIS IV. The disaggregated representation of the commodity flows has opened up the possibility of establishing rather close connexions between the indirect tax parameters of the model and the kind of information contained in tax rules. Each indirect tax is classified as either a commodity tax or an industry tax. The proceeds from industry taxes are exogenous. The proceeds from commodity taxes are endogenous in the model. Each commodity tax is further classified according to the tax base and the tax payer. The tax base of a commodity tax is either the quantity or the current value of one or more commodities. The tax rate for each commodity tax is given by a vector. The vector gives the tax rate on commodity flows to each activity. In this way the model reflects the fact that the tax rate of a commodity tax may differ between the receivers of the commodities on which the tax is levied. Typically, the tax rate will be zero on deliveries to export, but the tax rate may be differentiated on deliveries to other receivers as well. As for direct taxes the assessment rules are thus represented by very detailed tax functions.

For use in fiscal budgeting the revenue models for direct and indirect taxes play a central role. There is no corresponding model covering the fiscal expenditures (apart from subsidies included in the indirect tax model). As a preliminary for such a model efforts have been made for a specification of fiscal expenditure items in the model to link the national accounts data to the fiscal budget. Expenditures for goods and services are specified (i) by government purposes, (ii) by government production sectors, (iii) by government account (institutional sector) and (iv) by type (wages, goods and services). Transfer expenditures are similarly classified by kind of government purposes, by type of government and by socio-economic groups. Sales of government services to the public are also specified to fit into this classification.

2. THE ACCOUNTING SYSTEM OF THE MODEL

MODIS IV is very closely linked to the Norwegian national accounting system.[1] The link comprises form as well as content. The model includes an accounting system which is derived from, and to a great extent identical with, that of the national accounts. The overwhelming part of statistical data required for estimation and base year values in the model is, of course, supplied by the national accounts. This close link between the model and the national accounts is a feature of utmost importance to the model builder as well as to the user. It is also a feature which distinguishes MODIS from most other input-output models.

The Norwegian national accounts are in adherence with the revised system of national accounts adopted by the Statistical Commission of the United Nations in 1968 and recommended for use by the national statistical authorities and in the international reporting of comparable national accounting data.[2] The parts of the national accounts which are integrated in the model are presented in this chapter in a very compact form as two arrays of accounting transactions, one covering the real flows in the economy (diagram 2.2) and one covering the income and capital finance flows (diagram 2.3). The complete national accounting system also include opening, revaluation and closing accounts for assets. MODIS IV, however, has only flow accounts. The two arrays may be considered as consolidated accounts of the complete accounting system of the model, though they are rather simplified descriptions of the full set of accounts. The diagrams are explained and commented upon in section 2.1 and 2.2.

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1) For a general presentation, see Homb (1975), Fløttum (1980). 2) See United Nations (1968a), an instructive illustration of the complete system is given in table 2.1, pp. 18-19.

The heavy reliance of the model upon the national accounts means that nearly all price and quantity variables as well as income and outlay variables are defined in accordance with the national accounts. Balance equations and definitional relations in the model are identical or closely corresponding with those of the national accounts.

The accounts of the model may be described as flows between transactors. There are two main classes of transactors, functional sectors and institutional sectors. Three groups of flows may be distinguished, commodity flows, non-commodity real flows and financial flows. The central part of the model is the commodity flow structure.

2.1. Real flows

The real flows are flows between functional sectors. The functional sectors are usually referred to just as sectors, and are generally defined as functional units of the economy like groups of establishments or similar economic units. The production sectors are aggregates of establishments and similar economic units defined in accordance with the principles of the International Standard Industrial Classification.[1] The production sectors include both industries and general government. These two groups of sectors are distinguished in diagram 2.2 and 2.3.

The sector concept is also applied to categories of goods and services classified by origin or use. This extended use of the sector concept has perhaps less intuitive appeal, but helps to provide an overall unity in the conceptual structure of the model. The other sector groups are defined as follows:

The import and export sectors represent the main categories in the trade statistics, including goods, services, shipping etc. The consumption sectors comprise the household consumption sectors representing various categories of household wants, and include also a breakdown of total general government consumption by categories of consumption purposes. The gross investment sectors represent different types of capital goods while the real capital formation sectors, like the production sectors, are aggregates of establishments and similar economic units and include subdivisions for industries and general government.

The real flows may be depicted as in diagram 2.1 with a sphere of commodity flows within a wider real sphere. The commodity flows are indicated by fully drawn arrows and non-commodity flows by broken arrows.

1) See United Nations (1968b) and Statistisk Sentralbyrå (1978).

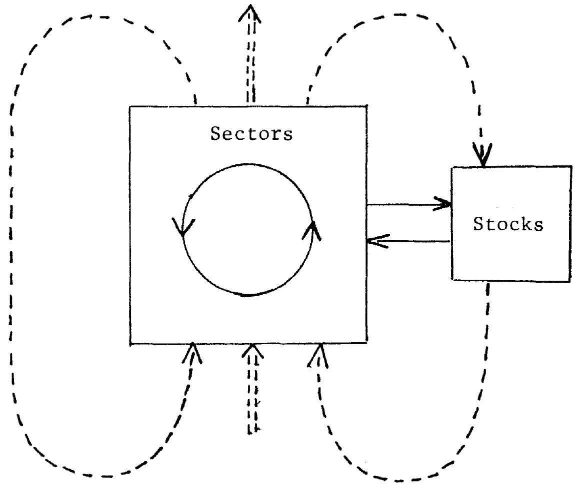

Diagram 2.1. The real flows of the economy.

The commodities flow between sectors, including a stock sector. The stock sector is kept apart from the others in view of its special role as inter-temporary storage.

There are two major subdivisions of commodities. The industry commodities include all commodities of the Bruxelles nomenclature[1)] and in addition groups of services (and some artificial commodities for accounting purposes). The marketed government services include all services produced by the general government and sold in a market.

The non-commodity real flows are subdivided in primary inputs, final outputs and non-commodity real transfers. The primary inputs and final outputs are indicated in diagram 2.1 by double broken arrows.

The single broken arrows indicate non-commodity real transfers. These cover various accounting conventions which do not always correspond to actual transactions and physical transfers. The broken arrow on the left in diagram 2.1 represents for instance capital goods credited or "delivered from" gross investment sectors and debited or "received by" real capital formation sectors. Non-marketed outputs are in a similar way "transferred" from general government production sectors to general government consumption sectors.

1) See Customs Co-operation Council (1972).

The upper broken arrow on the right in diagram 2.1 represents unfinished goods which by national accounting conventions are held as stocks. In the national accounts as well as in the model only certain products like ships and some others are accounted for as unfinished goods. For reasons of convenience this type of non-commodity transfer is dealt with formally as a commodity transaction which is why it does not appear explicitly in diagram 2.2. The lower broken arrow on the right represents net additions to stocks over all commodities debited to capital formation sectors. The two arrows mentioned in this paragraph may both be reversed.

The real sphere corresponds to the array in diagram 2.2. The primary inputs and final outputs connect the real sphere with the outer institutional and financial sphere. The primary inputs go to production sectors and import sectors. The final outputs come from consumption sectors, real capital formation sectors and export sectors.

All sectors are balanced in real flows in the sense that commodity plus non-commodity input, in value terms, equals commodity plus non-commodity output for each sector. After accounting for the commodity transactions there is in most sectors at a disaggregate level one residual non-commodity item required for balancing the sector. This residual takes the form of primary input, final output or non-commodity real transfer. A closer study of diagram 2.2 will reveal how each of the sector groups is balanced.

In the diagram the main groups of accounts are numbered from 1 to 13 with groups 1-2 for commodity accounts, groups 3-12 for the sector accounts and group 13 as the balancing account representing primary input as well as final output. In the entries the type of flow is indicated by 'C' for commodity flows, 'N' for non-commodity real transfers, 'P' for primary inputs and 'F' for final outputs.1) The double subscripts of C's and N's identify the credit (delivering) and debit (receiving) side of the transactions. The single subscripts of P's and F's indicate the sector receiving the primary input or delivering the final output, respectively. All other entries are zero by definition.

The commodity accounts are, of course, balanced for each commodity by total input of commodities being equal to total output of commodities. As the sector accounts are all balanced, as mentioned above, it follows therefore that total primary inputs equal total final outputs.

1) This notation is used in this chapter. In the formal presentation of the model in later chapters parts of flows indicated in diagram 2.2 are, to avoid confusion, given a different notation.

A SIMPLIFIED DESCRIPTION OF THE REAL FLOWS IN THE NATIONAL ACCOUNTS

Delivering accounts \ Receiving accounts			Commodities		Production sectors				Consumption sectors			Real capital formation sect.			
			Industry commodities	Marketed government services	Industries	General government	Import sectors	Export sectors	Households	General government	Gross investment sectors	Industries	General government	Net additions to stocks	Final outputs
			1	2	3	4	5	6	7	8	9	10	11	12	13
Commodities	Industry commodities	1			$C_{1,3}$	$C_{1,4}$		$C_{1,6}$	$C_{1,7}$		$C_{1,9}$			$C_{1,12}$	
	Marketed government services	2			$C_{2,3}$			$C_{2,6}$	$C_{2,7}$						
Production sectors	Industries	3	$C_{3,1}$												
	General government	4		$C_{4,2}$						$N_{4,8}$					
Import sectors		5	$C_{5,1}$												
Export sectors		6													F_6
Consumption sectors	Households	7													F_7
	General government	8													F_8
Gross investment sectors		9										$N_{9,10}$	$N_{9,11}$		
Real capital formation sect.	Industries	10													F_{10}
	General government	11													F_{11}
Net additions to stocks		12										$N_{12,10}$			
Primary inputs		13			P_3	P_4	P_5								

Diagram 2.2. Explanation of flow symbols: C = commodity flows, N = non-commodity real flows, P = primary inputs, and F = final outputs

As seen from diagram 2.2 the industry production sectors absorb commodities ($C_{1,3}$and $C_{2,3}$) and generate commodities ($C_{3,1}$). An industry production sector has primary inputs or value added (P_3) consisting of services of labour and capital. Net indirect taxes are also part of the primary inputs which make up the difference between the value of commodity outputs and commodity inputs.

The various general government production sectors absorb commodities ($C_{1,4}$) for the production of the different categories of government services, like administration, defence, health, education etc. The output of government production sectors consists of two parts. A minor part is commodities (marketed government services) ($C_{4,2}$). The major part, however, is not defined as commodity outputs but as non-commodity real transfers ($N_{4,8}$). The non-commodity real transfers are transferred from government sectors to government consumption sectors. A government production sector has an output of non-commodity real transfers and an output of commodities equal to the input of commodities and the sum of primary inputs of labour and capital (P_4).

The import sectors have primary inputs from foreign accounts (P_5) equal to the value of commodity outputs ($C_{5,1}$). The export sectors have final outputs to foreign account (F_6) equal to the value of commodity inputs ($C_{1,6}$ and $C_{2,6}$).

The household consumption sectors have final outputs of household consumption (F_7) equal to the value of commodity inputs ($C_{1,7}$ and $C_{2,7}$). The government consumption sectors have final outputs of government consumption (F_8) equal to the value of input of non-commodity real transfers ($N_{4,8}$).

The gross investment sectors absorb commodities ($C_{1,9}$) and total outputs are "transferred" as non-commodity real transfers to the real capital formation sectors ($N_{9,10}$and $N_{9,11}$). A gross investment sector has outputs of direct real transfers equal to the value of commodity inputs. The real capital formation sectors have final outputs (F_{10} and F_{11}) equal to the value of input of non-commodity real transfers ($N_{9,10}$, $N_{12,10}$ and $N_{9,11}$, respectively). The sector for net additions to stocks absorbs commodities ($C_{1,12}$) and transfers them to real capital formation in the various industries ($N_{12,10}$).

As mentioned above, diagram 2.2 gives a rather simplified picture of the real flows in the model accounting system. In addition to the types of sectors included in the diagram, there are also a number of <u>transformation sectors</u> and <u>transfer sectors</u> in the accounts. For the <u>transformation</u>

sectors, which are included for convenience and accounting purposes, the value of commodity output equals the value of commodity inputs. In the following these sectors are formally included among the industry production sectors.[1] The transfer sectors, which are included to obtain completeness and accounting consistency, are used to account for certain categories of non-commodity real transfers between sectors, for instance used fixed capital and such items as foreigners' consumption in Norway.

The real flows as depicted in diagram 2.2 constitute a semi-closed subsystem of flows between functional sectors. The primary inputs and the final outputs are the open ends of the system. These accounts form the bridge between the real flow accounts and the income and capital finance flow accounts.

The distinctive feature of the real flows is that they may be measured in constant as well as in current unit values (prices). The basis for an evaluation of the real flows in constant values is the availability of price indices for all commodities.[2] Non-commodity real flows are evaluated in constant values either by direct assessment as for unfinished goods or as a consequence of the evaluation of commodity flows. Some non-commodity items, like capital goods, are simply defined as a bundle of commodities and, accordingly, the constant value of the bundle is equal to the sum of the constant values of the commodities it consists of. Other non-commodity items like primary inputs are evaluated as residual items for balancing the sector accounts in constant values.

In the national accounts and in the model all flows depicted in diagram 2.2 are measured both in constant and current values. Furthermore, each commodity flow is measured in three different value concepts, basic values, producers' values and purchasers' values, to allow complete accounts in each of these value sets. The value concepts of the model are defined and discussed in section 3.4. See also the discussion in section 6.2 about how the indirect taxes are connected with the commodity flows.

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1) The accounts also define some transformation commodities simply as the output from the transformation sectors (one commodity for each sector). These are counted among the industry commodities. 2) The occurrence of price differentiation causes theoretical and practical problems for accounts in constant values. This is dealt with in Sevaldson (1973a).

2.2. Financial flows

The concept of "financial flows" is here introduced in contradistinction to "real flows" as short for "income and capital finance flows". The inclusion of financial flows may be viewed as an extension of the real flow system towards a complete system of accounts. What is still missing is opening accounts, revaluations and closing accounts for assets. The accounting system of MODIS IV includes only flow accounts.

The financial flows of MODIS IV are set out in a consolidated form as an array in diagram 2.3.[1] The transactors of the financial flows are the institutional sectors. By an institutional sector is meant a group of institutional units. In diagram 2.3 there are three groups of institutional sectors, private sectors, general government sectors and the foreign sector (the rest of the world). Each institutional sector has two groups of accounts, income accounts and capital finance accounts. In the model, and in the national accounts, the private sector is actually subdivided into four sectors: corporations, wage and salary earners, owners of unincorporated enterprises, and pensioners (see the discussion in section 4.4). The model and the national accounts distinguish between five general government sectors, namely the treasury, other central government, social security funds, local government and tax collectors (see section 6.3).

The link between the real flow accounts and the financial flow accounts is provided by the primary inputs and final outputs. In diagram 2.3 the final output (final expenditure category) rows numbered 6, 7, 8, 10, and 11 correspond to the functional sector rows with final outputs of diagram 2.2. The row totals of diagram 2.3 are identical with the final outputs of the corresponding rows of diagram 2.2. Likewise the primary input (primary income) columns numbered 3, 4 and 5 correspond to the functional sector columns with primary inputs of diagram 2.2. The column totals of diagram 2.3 are identical to the primary inputs of the corresponding columns of diagram 2.2.

In addition there are in diagram 2.3 thirteen other groups of accounts with rows and columns numbered from 14 to 26, of these 14-19 are income categories, 20 is an account for increases in financial assets/liabilities, 21-23 are income accounts of institutional sectors and 24-26 are capital finance accounts of institutional sectors. In the entries the type of flow is indicated by 'Y' for income and outlay

1) Compare table 2.1 of United Nations (1968a). The accounting system here is somewhat simplified compared with the UN System of National Accounts.

A SIMPLIFIED DESCRIPTION OF THE INCOME AND CAPITAL FINANCE FLOWS IN THE NATIONAL ACCOUNTS

Institutional sectors: Capital finance: Foreign	Institutional sectors: Capital finance: Government	Institutional sectors: Capital finance: Private	Institutional sectors: Income: Foreign	Institutional sectors: Income: Government	Institutional sectors: Income: Private	Increases in financial assets	Income categories: Direct taxes	Income categories: Government transfers	Income categories: Components in value added: Indirect taxes, net	Income categories: Components in value added: Depreciation	Income categories: Components in value added: Operating surplus	Income categories: Components in value added: Wage costs	Final output: Real capital formation sectors: General government	Final output: Real capital formation sectors: Industries	Final output: Consumption sectors: General government	Final output: Consumption sectors: Households	Export sectors	Delivering accounts / Receiving accounts		
26	25	24	23	22	21	20	19	18	17	16	15	14	11	10	8	7	6			
									$Y_{17,3}$	$Y_{16,3}$	$Y_{15,3}$	$Y_{14,3}$						3	Industries	Primary input: Production sectors
										$Y_{16,4}$		$Y_{14,4}$						4	General government	Primary input: Production sectors
			$Y_{23,5}$															5	Import sectors	Primary input
					$Y_{21,14}$													14	Wage costs	Income categories: Components in value added
				$Y_{22,15}$	$Y_{21,15}$													15	Operating surplus	Income categories: Components in value added
	$Y_{25,16}$	$Y_{24,16}$																16	Depreciation	Income categories: Components in value added
				$Y_{22,17}$														17	Indirect taxes, net	Income categories: Components in value added
					$Y_{21,18}$													18	Government transfers	Income categories
				$Y_{22,19}$														19	Direct taxes	Income categories
$H_{26,20}$	$H_{25,20}$	$H_{24,20}$																20	Increases in financial liabilities	
		$G_{24,21}$					$Y_{19,21}$									$Y_{7,21}$		21	Private	Institutional sectors: Income
	$G_{25,22}$							$Y_{18,22}$							$Y_{8,22}$			22	Government	Institutional sectors: Income
$G_{26,23}$																	$Y_{6,23}$	23	Foreign	Institutional sectors: Income
						$H_{20,24}$								$Y_{10,24}$				24	Private	Institutional sectors: Capital finance
						$H_{20,25}$							$Y_{11,25}$	$Y_{10,25}$				25	Government	Institutional sectors: Capital finance
						$H_{20,26}$												26	Foreign	Institutional sectors: Capital finance

Diagram 2.3. Explanation of flow symbols: Y = income and outlay (expenditure) flows, H = financial flows, and G = savings.

(expenditure) flows, 'H' for increases in financial assets and liabilities and 'G' for savings.[1] As in diagram 2.2 the double subscripts identify the credit and debit side of the transactions, respectively. All other entries are zero by definition.

By accounting consistency, each of these additional accounts are also balanced. For each income category the value of total outlay (a row sum) equals the value of total income (a column sum). The total value of increases in financial assets equals the total value of increases in financial liabilities. Furthermore, the grand total of gross real capital formation equals the grand total of saving plus depreciation.

The income categories in diagram 2.3 include four value added components, namely, wage costs, operating surplus, depreciation and net indirect taxes and in addition, government transfers and direct taxes. In the model, and in the accounts, wage costs are decomposed into wages and employers' contribution to social securities (see sections 5.6 and 6.1) and indirect taxes and subsidies are specified by kind (see section 6.2).

The logic of the financial flow accounts is briefly as follows: The primary input of the production sectors which is equal to value added is credited to the institutional sectors via the income categories. Wages are entered as part of private income. The operating surplus is distributed between private income and government income according to ownership. Depreciation is entered in both private and government capital finance accounts. Indirect taxes, net, is entered as government income. The primary input of the import sectors is transferred to the foreign income account. The final output is similarly debited as outlays of the respective institutional sectors. Government transfers are debited to the government income account and credited to the private income account, while direct taxes on the other hand are debited to the private income account and credited to the government income accounts. The balances of the income accounts are transferred to the respective capital finance accounts.

The institutional income accounts show the receipts and expenditures of the institutional income sectors. As seen from diagram 2.3 the private income sectors (row and column 21) have incomes from wages ($Y_{21,14}$), operating surplus ($Y_{21,15}$) and government transfers ($Y_{21,18}$) and outlays to consumption ($Y_{7,21}$) and direct taxes ($Y_{19,21}$). Savings ($G_{24,21}$) make up the difference between incomes and outlays.

1) This notation is used only in this chapter.

The government income sectors (row and column 22) have incomes from operating surplus ($Y_{22,15}$), indirect taxes, net ($Y_{22,17}$) and direct taxes ($Y_{22,19}$) and outlays to consumption ($Y_{8,22}$) and government transfers ($Y_{18,22}$). Savings ($G_{25,22}$) make up the difference.

The foreign income sector (row and column 23) has income from import ($Y_{23,5}$) and outlays to export ($Y_{6,23}$). Savings for the foreign sector ($G_{26,23}$) are equal to the deficit on the national account for current transactions.

The capital finance accounts show the forms in which the capital finance sectors accumulate capital and how they finance this accumulation. The private capital finance sectors (row and column 24) accumulate real capital ($Y_{10,24}$) and financial assets ($H_{20,24}$). The accumulation is financed from depreciation allowances ($Y_{24,16}$), savings ($G_{24,21}$) and financial liabilities ($H_{24,20}$). The same applies to the government capital finance sectors (row and column 25). The accumulation of real capital ($Y_{10,25}$ and $Y_{11,25}$) and financial assets ($H_{20,25}$) is financed by depreciation allowances ($Y_{25,16}$), savings ($G_{25,22}$) and increases in financial liabilities ($H_{25,20}$).

The foreign capital finance sector (row and column 26) has increases in financial assets ($H_{20,26}$) equal to the value of savings ($G_{26,23}$) plus increases in financial liabilities ($H_{26,20}$).

For simplification purposes some items have been omitted in the array. Most of the omitted items play no substantial role in the relations of the model.

In the national accounts and in the model all flows depicted in diagram 2.3 are measured in current values only. All current value flows, both in diagram 2.2 and 2.3, are recorded on an accrual basis, i.e. the items are recorded as of the time at which it is incurred or earned.[1] However, both direct and indirect taxes (incl. subsidies) in the general government fiscal budgets are recorded on a cash basis, i.e. the taxes are recorded as of the time at which payment is actually received or made. To link the national accounts and the model to the fiscal accounts and budgets a special institutional government sector, tax collectors, holds the margin between taxes on an accrual and on a cash basis (see further discussion in chapter 6).

1) See United Nations (1968a) p. 230.

3. INPUT-OUTPUT FRAMEWORK

MODIS IV is basically an input-output model. Its input-output framework differs, however, from the traditional Leontief type square matrix of input-output coefficients for industry-by-industry transactions with a set of conversion coefficients connecting industry flows and final demand categories.

As can be seen from diagram 2.2 in the preceding chapter the Norwegian national accounts do not include as part of the accounts an input-output table of flows of intermediate inputs between industries and a table of industry flows to final commodity demand categories. The traditional square input-output table has been replaced by a pair of rectangular commodity-by-industry tables, one for input of industry commodities into industries and one for output of industry commodities from industries ($C_{1,3}$ and $C_{3,1}$ in diagram 2.2).[1] The table of industry flows to final demand has been replaced by a table of input of industry commodities into final commodity demand categories ($C_{1,4}$, $C_{1,6}$, $C_{1,7}$, $C_{1,9}$ and $C_{1,12}$ in diagram 2.2).

The commodity-by-industry tables are set out in matrix form in diagram 3.1. The input table (represented by $C_{1,3}$ in diagram 2.2) shows the input of industry commodities to the various industries. The output table (represented by $C_{3,1}$ in diagram 2.2) displays the output of industry commodities from the various industries.

Commodity-by-industry tables

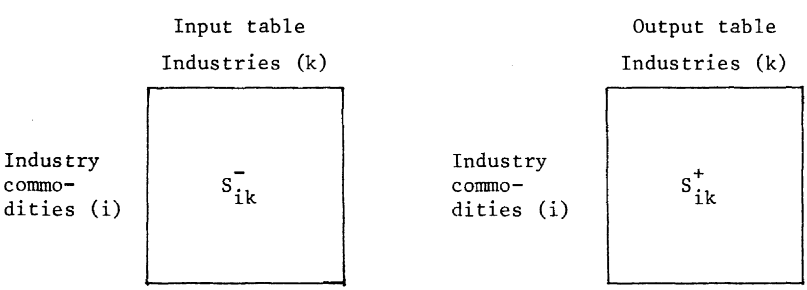

Diagram 3.1

1) The first and preliminary presentation of the input-output framework of MODIS IV was given in Bjerkholt and Longva (1970), (1971). For more complete updated presentations, see Bjerkholt (1974) and Furunes and Longva (1976).

In the Norwegian accounts there are about 320 industry commodities and 150 industries[1] and each industry has at least one output commodity. A row in the output table - corresponding to a certain commodity - will show the industries of origin of the domestic supply of that commodity. A column in the table corresponds to a certain industry and shows the commodity composition of the output of that industry.

If the output table contains only one non-zero element on each row the output commodities of each industry can be neatly aggregated so as to ensure a one-to-one correspondence between aggregated commodities and industries. A corresponding aggregation over commodities in the input table will result in the traditional square input-output table.

When the relation between industries and commodities in the output table is a one-to-one correspondence there is also a natural characterization of technology by the columns of commodity inputs in the input table. The traditional input-output model is thus based on a single-valued correspondence between industries as groups of establishments, commodities as the product of those industries, and technologies as the composition of the commodity inputs of the industries.

3.1. The traditional Leontief input-output model

The simplicity of the traditional input-output model including the duality between price and quantity relations is derived directly from an assumed one-to-one correspondence between industry output commodities and industries at some level of aggregation. When the world is not so simple as to allow a one-to-one correspondence between proper commodities and industries, the correspondence can be saved by auxiliary assumptions, e.g. that the commodities produced by a certain industry partake in all flows of the same commodities in proportions given by the total market share of the industry for each of the commodities. This particular assumption with some modifications has been used in Norway up to 1970 for producing industry-by-industry input tables from primary data in the form of the commodity-by-industry tables of diagram 3.1.[2] More sophisticated assumptions may be applied to achieve similar results.[3]

1) This refers to the preliminary accounts with the most recent data. In the revised and final national accounts there are about 1 750 commodities. 2) When such a table is used as the basis for the computation of input-output coefficients in an input-output model these assumptions imply what will later be referred to as a general assumption of sector technology (see section 3.3) combined with fixed market shares (see section 4.2).
3) For a discussion see for instance United Nations (1968a), pp. 48-51 and Gigantes (1970). A more practical oriented presentation is given in United Nations (1973).

At the level of aggregation selected for MODIS IV it is most certainly the case in the Norwegian national accounts that the non-zero off-diagonal elements of the commodity-by-industry output table are non-negligible in number as well as in magnitude. Through some kind of more or less mechanical transformation of the commodity-by-industry tables, e.g. as described above, one may arrive at an industry-by-industry input table. The traditional input-output matrix is derived from such a table by division of the columns with industry totals. Writing Λ for this matrix, X for a vector of industry totals, and Y for a vector of final demand by industry the basic equation for the quantity side of the traditional Leontief model can be written as

(3.1) $X = \Lambda X + Y$

X and Y are here assumed to be measured in a set of constant prices. The corresponding basic equation for the price side can be written as

(3.2) $p = \Lambda' p + q$

where p is a vector of price indices of industry outputs, and

q is a vector of value added in current values per unit of industry output.

In the most common form of the model the final demand vector (Y) is exogenous in the quantity equation and the value added per unit (q) in the price equation. It is trivial to prove from (3.1) and (3.2) that the total value of final demand is equal to the total value added, i.e.

(3.3) $p'Y = q'X$

Most input-output models are based on some version of (3.1) and (3.2) or both. Extensions of this basic framework may include (i) an interchange of exogenous and endogenous variables, (ii) further specifications of the variables for final demand and value added per unit, (iii) modification of the assumption of constant input-output coefficients, and (iv) integration of the input-output model in a wider model framework.

It is pertinent to our discussion of the input-output framework of MODIS IV that the basic assumption to validate the traditional input-output model, as set out above, is not only the constancy of the input-output coefficients as most textbooks contend. Underlying the matrix Λ is, as we have seen, a set of primary data of a different format than the industry-by-industry table from which Λ is derived. What is lacking in most textbooks is a discussion of how this table is arrived at. At stake is the observability of the industry-by-industry table as well as the meaningfulness of the price and quantity variables. We hope that the remainder of this chapter will clarify how these problems have been tackled in MODIS IV.

3.2. The commodity-activity-sector approach and the basic quantity equation

Transforming the commodity-by-industry tables of the new SNA to an industry-by-industry table is not the only way of obtaining a simpler framework. By making other assumptions one may alternatively arrive at a commodity-by-commodity table, for instance by aggregating the commodities to the same number as there are industries and assuming that each aggregated commodity is produced with the same technology regardless of which industry it is produced by.[1)]

For some problems the commodity-by-commodity table may give a better representation of the underlying structural relationships than an industry-by-industry table, and vice versa. In the quantity relations the focus is on the repercussions for intermediate demand of a given final demand by commodity. Hence a commodity-by-commodity table seems to be the best choice. However, the direct link between production by industry and final demand is severed in the commodity-by-commodity table. Links between production by industry and production by commodity must therefore be added if explicit quantity relations between final demand items and value added of industries, for instance consumption functions, are to be included in the model.

In the price relations the crucial link is between primary costs, known from the statistical data by industry and not by commodity, and intermediate input costs. Hence the price relations seem to favour the choice of an industry-by-industry table. On the other hand, the treatment of intermediate input costs may become inferior compared with the use of a commodity-by-commodity table.

The aggregation level of MODIS IV is such that the elements outside the diagonal of the output table are not negligible. However, the model, which include both quantity and price input-output relations, is not based on a prior transformation of the input and output tables into an industry-by-industry table or, alternatively, a commodity-by-commodity table. The elimination of the degrees of freedom stemming from the fact that more than one industry produce a certain commodity is, as will be

1) This is the same assumption as what will later be referred to as a general assumption of commodity technology (see section 3.3). See also United Nations (1968a), pp. 48-51 and Gigantes (1970), pp. 280-284.

discussed below, considered as part and parcel of the model, not as a constraint on the data requirements at the outset of constructing the model. The framework adopted is formally somewhat related to that of the general linear activity analysis, see e.g. Koopmans (1951).

The three central concepts of the input-output framework of the model are commodity, sector and activity. These concepts distinguish between three different aspects of absorption and generation of goods and services, namely what is absorbed or generated (the commodity concept), where the absorption or generation is taking place (the sector concept), and how the goods and services are absorbed or generated (the activity concept).

The number of commodities of the model, which are aggregates of the commodities of the national accounts, is about 200. Nearly 190 of these are industry commodities while 10 are marketed government services.[1)]

As can be seen from diagram 2.2 in the preceding chapter a sector may generate commodities or absorb commodities, or both. The most important group of sectors are the industries which, together with the general government production units, form the production sectors. The production sectors transform input flows of commodities into output flows of commodities and thereby absorb some commodities while generating others. The other main sector groups, which either generate or absorb commodities, are the import sectors, export sectors, the household consumption sectors and the gross investment sectors.

By activity is meant a subdivision of sectors according to characteristic properties of the type of commodity generation, absorption or transformation which are taking place. The subdivision of sectors into activities carries a different meaning for each type of sector. The commodity flows between activities include all generation and absorption of commodities except changes in stocks. Within each activity there are assumed fixed proportions between commodity inputs and commodity outputs.

The main purpose of subdividing sectors into activities is to avoid having to assume fixed proportions between commodity inputs and commodity outputs for the sector as a whole. The subdivision also makes it possible to distinguish between different ways of generating or absorbing a certain commodity within the same sector.

1) The commodity specification and aggregation are further discussed in section 3.3.

To describe the commodity-by-sector flows of the economy we introduce the following definitions and balance equations (all entities are measured in a consistent set of values, see section 3.4):

S^{+}_{ij} = output of commodity i from sector j

S^{-}_{ij} = input of commodity i to sector j

S_j = net output of commodities in sector j = sector level of sector j.

(3.4) $$S_j = \sum_i S^{+}_{ij} - \sum_i S^{-}_{ij}$$

X_i = net addition to stocks of commodity i

(3.5) $$X_i = \sum_j S^{+}_{ij} - \sum_j S^{-}_{ij}$$

Balance equations similar to those given for the sectors in equations (3.4) and (3.5) will also hold for commodity flows into and out of activities:

A^{+}_{ij} = output of commodity i from activity j

A^{-}_{ij} = input of commodity i to activity j

A_j = net output of commodities in activity j = activity level of activity j.

(3.6) $$A_j = \sum_i A^{+}_{ij} - \sum_i A^{-}_{ij}$$

(3.7) $$X_i = \sum_j A^{+}_{ij} - \sum_j A^{-}_{ij}$$

By the concepts of sector and activity levels introduced above is meant a measure of the net commodity generation and absorption that take place in a sector or an activity, respectively. The values of the activity levels within each sector add up to the sector level. As for the sector and activity concepts themselves the activity level and sector level carry a different meaning for each type of sector.

The basic quantity equation of the input-output framework is

(3.8) $\Lambda A = X$

where $\Lambda = \{\lambda_{ij}\}$ is a commodity-by-activity coefficient matrix in which the element $\lambda_{ij} = (A^{+}_{ij} - A^{-}_{ij})/Aj$ (positive or negative) gives net output of commodity i per unit of activity level j,

A is a vector of activity levels, and

X is a vector of net additions to stocks (by commodity).

Equation (3.8) follows directly from manipulations of (3.7) by inserting the expression for λ_{ij}.

The basic assumption of the quantity input-output model of MODIS IV is that the quantities of commodity inputs to and outputs from an activity are related by fixed proportions, i.e. that all elements in Λ are constants. This assumption has been thoroughly investigated on Norwegian data by Sevaldson.[1] In general, the elements in Λ are estimated from the national accounts for the base year of the model, usually the year prior to the current year. This means that quantities of commodity flows are measured in unit values (prices) of the base year, constant values. The value concepts of the model will be further discussed in section 3.4. The system of commodity flows is closed with regard to all generation and absorption of commodities except changes in stocks. The excess of commodities generated over commodities absorbed is thus defined as net additions to stocks.

The commodity-activity-sector framework presented above and formalized in equation (3.8) provides a representation only of the commodity flows of the economy. All activities and sectors are assumed to have either commodity inputs or outputs or both. However, as seen from diagram 2.2 in chapter 2, the accounting system of the real flows of the economy also includes non-commodity real transfers specified as intersectorial flows. The non-commodity real transfers typically represent accounting conventions rather than actual transactions.

1) See Sevaldson (1970), (1972), (1973b), (1974a), (1974b). See also Frenger (1978).

For completeness and accounting consistency all sectors in the real flow accounts are included in the model framework. In addition to the internal activities, which have commodity inputs and/or outputs, the subdivision of sectors into activities therefore also include external book-keeping acitivites. These activities "generate" or "absorb" the non-commodity real transfers which are taking place, if any. In general, each external activity covers only one type of non-commodity real transfer to or from a sector.

When the non-commodity real transfers are included in the model framework the sector level has to be redefined as the difference between input and output of commodities and non-commodity real transfers. The sector level thus becomes equal to the difference between final outputs and primary inputs in the sector (see diagram 2.2). The activity level for an internal and an external activity is defined as net output of commodities and of non-commodity real transfers, respectively.

Together, the internal and external activities include all commodity transactions and non-commodity real transfers in the accounting system and the values of external and internal activity levels within each sector add to the (redefined) sector level.

The number of industry production sectors is about 125. The various sectors absorb commodities (both industry commodities and marketed government services) and primary inputs and use them for the production of industry commodities.[1] About 60 industry production sectors encompass only one production activity each while the rest have more than one production activity. All these activities are internal. The industry production activities can be interpreted as macro processes aggregated across establishments within the same production sector. Altogether there are about 220 industry activities.[2]

There are about 20 general government production sectors including separate sectors for central and local government. The various sectors absorb commodities and use them for the production of different categories of government services, like administration, defence, health, education, etc. Each general government production sector has been divided into three activities, one internal for commodity absorption in government production, one internal for the production of commodities (marketed government services), if any, and one external for the accounting of non-commodity real transfers to consumption sectors (see below). The purpose of this separation is to make the model more useful for fiscal budgeting (see chapter 6).

1) The classification of industry production sectors and industry commodities is discussed in section 3.3. 2) The production structure is further discussed in section 3.3.

The import sectors have separate activities for all imported commodities. Likewise, the export sectors have separate activities for nearly all exported commodities. (Import and export activities might conceivable be used to distinguish also between imports and exports of identical commodities from and to different foreign markets.) There are about 130 internal import activities and 110 internal export activities in the model.

The export sectors also encompass some external activities covering the non-commodity real transfers like used fixed capital from the gross capital formation sectors and foreigners' consumption from the household consumption sectors.

Most of the household consumption sectors contain only one internal activity. Altogether there are about 50 such household consumption activities. In addition there is one external activity for the non-commodity real transfers of foreigners' consumption to export sectors.

Each general government consumption sector has external activities by type of government services (non-commodity transfers from the government sectors). There are about 100 general government consumption activities in the model.

Each of the gross investment sectors is divided into one or more internal activities, altogether about 35. To each internal activity there is a corresponding activity covering the non-commodity real transfers to the real capital formation sectors. The real capital formation sectors have external activities by category of capital goods (non-commodity transfers from the gross investment sectors). There are about 150 such capital formation activities in the model, close to 100 for industries and about 50 for general government. In addition there are some external activities covering the non-commodity real transfers of used fixed capital from the capital formation sectors to export sectors.

For the transformation sectors and associated internal activities (15 of each) the value of commodity output equals the value of commodity input. The output of each transformation sector is defined as a separate commodity and included among the industry commodities. For transformation activities the activity level is defined as the total value sum of commodity output. In the following these sectors and activities are included among the industry production sectors and activities.

All the internal activities in the model may conveniently be divided into two groups. The final demand activities are those activities which only absorb commodities, i.e. withdraw commodities from the flow circulation without producing new commodities. These activities include the household consumption, gross investment, and export activities and the general government production activities for absorption of commodities. The remaining internal activities, supply activities, which add to the commodity flow circulation, include import and industry production activities and general government production activities for production of commodities (marketed government services). By the general definition of internal activity level stated earlier the activity level of final demand activities will be negative while the activity level of supply activities normally will be positive.

3.3. The specification of the production structure and the industry commodities

The multisectoral production structure of the economy as it is described in MODIS IV can be characterized in general terms as a multiple commodity output technology. At the chosen level of industry aggregation the same commodity can, and often will , be produced in more than one industry production sector and each sector will, therefore, normally have more than one commodity output. Altogether there are about 175 industry commodities in the model.[1] On this aggregation level it is reasonable to assume that most commodity outputs from an industry production sector are produced non-jointly, e.g. with separate production functions. The assumption of a non-joint production structure rules out the possibility of having separability in the different industries between commodity inputs (materials) and primary inputs (value added) on one side and commodity outputs (gross output) on the other.[2] An assumption of separability is equivalent to requiring that all the marginal rates of substitution between the commodity outputs are independent of commodity and primary inputs. From non-jointness it therefore follows that changes in the commodity output composition of an industry entail changes both in the composition and level of commodity and primary inputs of that industry.

It may be argued that not all commodity and primary inputs can be allocated among the different output commodities by means of separate, and in general different, production functions because some inputs serve a "general" purpose in the production process. This can be the case both for parts of materials and primary inputs. However, for the sake of

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1) Not including the transformation commodities. 2) For a discussion of the use of separability and (non-) jointness restrictions in connection with characterizations of technologies, see for instance Hall (1973). As pointed out by Hall the only overlap between the restrictions of non-jointness and separability for a multiple output technology with constant returns to scale is the case where the individual production functions are identical except for scalar multiples.

simplicity and due to lack of data we are at present adhering to the assumption of non-jointness.[1)]

It follows from this discussion that the main principle of subdividing an industry production sector is to let each important output commodity of the sector be produced by a separate activity. The activity with the greatest share of total output in the sector is called the main activity of the sector. Minor output commodities, if any, are lumped together and included in the main activity as joint products. The subdivision does imply a problem of estimation from observable data since inputs to production activities are not directly observable when there are more than one activity in the industry production sector. The data source is the commodity-by-industry tables for the base year of the model. The number of linearly independent commodity input structures can thus not exceed the number of industries.

The problem of allocating commodity inputs of a production sector among its activities is at present solved in a simple way by classifying each of the industry production activities as having either a commodity technology or a sector technology. An activity with a commodity technology is assumed to have the same input structure, i.e. commodity input composition, and the same commodity productivity, i.e. proportion between total commodity output and total commodity input, as other commodity technology activities with the same output commodity. An activity with a sector technology is assumed to have the same input structure and commodity productivity as other sector technology activities which belong to the same sector. If all activities within a sector are classified as having a sector technology they will all have the same input structure and it will be equal to the input structure of the sector itself.[2)] With few exceptions the assumption of commodity technology is used wherever possible, i.e. in cases where there are two or more activities with the same output commodity in different sectors. In some cases extraneous information, e.g. engineering data, are used to determine the input structure.[3)]

1) As will be further discussed in section 4.1 we are assuming that primary input has the same composition in all activities belonging to the same industry production sector even though the proportions between primary input and gross output may differ between activities. To some extent this will accomodate the argument above. 2) This situation corresponds to the case with overlap between the restrictions of non-jointness and separability, see footnote 2, p. 54. 3) More details on the procedure outlined above can be found in appendix 1 and in Furunes and Longva (1976), see also Bjerkholt and Longva (1970).

The activity level of an industry production activity is defined as the difference in value terms between commodity outputs (gross output) and commodity inputs (materials). When materials are in a constant ratio to output in real terms, as is the case for the industry production activities, primary input (real value added) is defined up to a factor of proportionality, i.e. as being in proportion to materials or to gross output. Since the activity level of an industry production activity is proportional to both materials and gross output it can be interpreted as a measure of total primary input to that activity. The factor of proportionality is conventionally determined by the use of base year prices as constant values.

While the proportion between total commodity input and total primary input to an activity is determined by the commodity or sector technology assumption the composition of total primary input is always assumed to be the same for all activities belonging to the same sector i.e. a sector technology assumption. This is further discussed in section 4.1.

The classification of the industry commodities is central to the model and it is closely linked to the classification of industries. Theoretically, a group of micro commodities (i.e. BTN-commodities and groups of services) can be safely aggregated to an industry commodity in the model if these micro commodities are separable from all other commodities, i.e. that the marginal rates of substitution between the micro commodities that make up the industry commodity are independent of the quantities of all other commodities, see e.g. Green (1964), p. 12. Since the model is economy-wide we must add the condition that the marginal rates of substitution between any two micro commodities within the aggregate are identical both in production and consumption, i.e. in all markets.

As mentioned above, the quantities of commodity flows in the model are measured in base year unit values. This means that the micro commodities are aggregated to industry commodities of the model in the base year by adding the values of the micro commodities. In principle, we are therefore adding the physical quantities with the prices of the base year as weights. This is a reasonable procedure on the assumptions that there is one and only one price for each micro commodity in the base year (no price differentiation) and that the marginal rates of substitution between any two micro commodities in the aggregate are the same both in production and consumption, and equal to the relative price.[1)] The marginal rates of substitution indicate the rate at which micro commodities can be interchanged without changing the quantity of the aggregate (movements along an isoquant). By using the undifferentiated prices as weights when aggregating the micro commodities in the base year, a unit of each industry commodity will therefore have the same quantitative interpretation in all submarkets. Since the industry commodities also must have the same quantitative interpretation in all markets in the projection period, the micro commodities that make up each aggregate must have constant relative prices within each submarket.

From this discussion we conclude that given the assumptions of no price differentiation in the base year and equality between marginal rates of substitution and relative prices, a group of micro commodities which has constant relative prices within each submarket and thereby constant marginal rates of substitution, i.e. that they are separable from all other commodities, can be aggregated to an industry commodity with the base year prices as weights.

The actual classification of industry commodities in the model is done by adopting the main producer principle, that is letting all BTN-commodities and groups of services with the same industry as the main producer form one industry commodity. Strictly followed, this will give the same number of industry commodities as the number of industries, i.e. about 125. In the quantity analysis the focus is on the repercussions for intermediate demand of a given final demand by commodity. As long as the number of linearly independent production structures is equal to the number of industries nothing can be gained by having more industry commodities than industries.[2)] At a given level of aggregation it seems reasonable to assume that the chosen procedure will result in the aggregation of BTN-commodities with sufficiently stable relative prices for the commodities of the model to be interpreted as homogenous commodities (see above).

Trade margins, which in the national accounts are distributed on each commodity flow, are aggregated over commodity flows and treated as an ordinary industry commodity in MODIS IV. Since we are assuming that commodity inputs to and outputs from an activity are related by fixed

1) For a discussion of this see e.g. Sevaldson (1975). 2) Another way of saying this is to point out that a consequence of having more industry commodities than industries will be that different commodities will have to be assigned the same sector technology, i.e. identical input-structure in the production process, implying that there is effectively only a single commodity from the production point of view. See also footnote 2, p. 54.

proportions nothing could be gained in the quantity model computations by having separate and fixed trade margin rates for each commodity flow.

The method for classifying industry commodities, as it is outlined above, is critically dependent on the way the industry production sectors are classified. At the given level of aggregation we have tried to maximize the degree of specialization, i.e. maximize the share of each sector's total output that is made up of micro commodities of which the sector is the main producer. The use of the main producer principle in the classification of industry commodities will maximize the degree of coverage, i.e. maximize the main producer's share of total production of the industry commodity. Together these two procedures will lead to a minimization of elements outside the diagonal in the commodity-by-industry output table in diagram 3.1. This again will keep at a low level the number of production activities necessary to cope with the byproduct problem and thereby minimize the number of activities with identical input structures, since the number of different input structures at most will be equal to the number of industry production sectors.

In some cases the outlined procedure for classification of industry commodities is not strictly adhered to. First of all there are specified about 20 industry commodities with no domestic production (non-competitive imports) and about 15 commodities are defined by the transformation sectors. In addition, some commodities with the same main producer are kept separate due to differences in demand structure, indirect tax structure or assumed differences in price development. Altogether there are about 190 industry commodities. More details on the classification of industry commodities and production sectors are given in Furunes and Longva (1976).

3.4. Value concepts and the basic price equation

The principal concept for evaluating commodity flows in the model is basic values. In terms of the more commonly used concepts of producers' value and purchasers' value the basic value of a commodity flow is defined as the producers' value less commodity taxes, net, in respect of production, or the purchasers' value less trade margins and commodity taxes, net, in respect of production and trade.[1] In the national accounts, each commodity flow is decomposed into several different value components, i.e. basic value and commodity taxes, net, in respect of production, which add up to producers' value, and trade margin (in basic value) and commodity

1) A more comprehensive discussion is given in Furunes and Longva (1976), pp. 22-25. This definition of basic value corresponds to that of approximate basic value of United Nations (1968a), p. 230.

taxes, net, in respect of trade, which together with the producers' value add up to purchasers' value. In addition, the commodity taxes, net, for each commodity flow, both in respect of production and trade, are further split between general purchase tax (VAT), other commodity taxes, and commodity subsidies.[1)]

The basic value concept is preferred to producers' value or purchasers' value because the trade margins (which include transport charges) and commodity tax rates may vary between receiving sectors for some commodities and may change over time. (This will be the case even with a more disaggregated commodity classification than that of MODIS IV.) As pointed out in section 3.3 the aggregation of micro commodities to industry commodities of the model implies the assumption of one and only one price for each commodity in the base year. The choice of basic values as the concept for evaluating commodity flows increases the realism of this assumption. The importance of this for the model computations in constant values is easily seen. If the relative shares of buyers of a commodity with differentiated trade margins or commodity tax rates deviate from those of the base year this will in itself appear as a change in demand of the commodity as measured in producers' or purchasers' values. In input-output models it seems therefore advantageous to use the basic value concept for measurement of commodity flows.[2)]

It is important to have in mind that apart from trade margins and commodity taxes there may be genuine price differentiations in the base year. This implies a bias in the base year values used as weights for quantity summation and thereby a source of errors in the model computations

All model computations start from a chosen base year. Quantities of commodity flows are measured in the unit basic values of that year. Prices of commodity flows are indices of unit basic values relative to the base year.

The activity levels for internal activities were defined in section 3.2 as the value of net commodity output of each activity. The activity levels are not, however, evaluated in basic values but in market values. The use of market values in the definitions of activity levels gives the valuation scheme of the model framework a hybrid form combining commodity flows in basic values with activity levels in market values. Activity or sector levels measured in basic values is a concept which can hardly be said to exist outside the model. It seems desirable to remain with the

1) Customs duties are included in basic values. 2) See furthermore United Nations (1968a), p. 40 and pp. 53-56.

accounting framework of the environment at the expense of some awkwardness in the price relations. The market value of internal activity levels is computed as producers' value of commodity outputs less purchasers' value of commodity inputs. The addition to stocks (by commodity) is on the other hand evaluated, as in the national accounts, in basic values.

In accordance with the principles of value evaluation outlined above the vector of internal activity levels may be written as

$$(3.9) \quad A = A_X + A_T$$

where A_X is a vector of net output of commodities by activity in constant values computed as basic value of commodity outputs less basic value of commodity inputs, and

A_T is a vector of commodity taxes, net, by activity in constant values, computed as commodity taxes, net, on commodity outputs less commodity taxes, net, on commodity inputs.

Both A_X and X represent the basic value of net output of commodities from the internal activities by activity and by commodity, respectively. The totals of these two vectors must accordingly be the same, i.e.

$$(3.10) \quad e'A_X = e'X$$

By combining (3.8) and (3.9) the basic quantity equation may be written as

$$(3.11) \quad \Lambda(A_X + A_T) = X$$

Since the input-output coefficients for commodities (the elements of Λ) are assumed constant, it follows that the proportions between corresponding elements in A and A_X and thereby also in A and A_T will be fixed. This means that the constant values of commodity taxes, net, by activity are computed as fixed shares of the activity levels.

Conceptually, this result may be arrived at in two steps. The first step is the computation of commodity taxes, net, by commodity and sector by means of tax rates given from the base year. This is in accordance with the definition and the computation of commodity taxes in constant values in the national accounts and it involves no assumption of fixed coefficients. The second step is the combination of the tax rates from the base year and a set of input-output (activity) coefficients which together imply that the constant values of commodity taxes, net, by activity are in a fixed proportion to the activity levels in constant market values. The constancy of the input-output coefficients is, of course, a specific model assumption.

As discussed above prices of activity levels are indices of unit market values, relative to the base year. Corresponding to the decomposition of activity levels in constant market values in (3.9) the prices of activity levels may be written as

$$(3.12) \quad p_A = b_{AX} + b_{AT}$$

where p_A is a vector of price indices of activity levels in market values,

b_{AX} is a vector of net output of commodities in current basic values by activity per unit of activity level, and

b_{AT} is a vector of commodity taxes, net, in current values by activity per unit of activity level.

With the prices of commodity flows defined as basic values relative to the base year it follows from the definitions of b_{AX} and the activity coefficients Λ that

$$(3.13) \quad \Lambda' b_X = b_{AX}$$

where b_X is a vector of commodity price indices in basic values.

By combining (3.12) and (3.13) the basic price equation, which is the dual of (3.8), can be written as

$$(3.14) \quad \Lambda' b_X = p_A - b_{AT}$$

In section 3.1 it was shown that the basic price and quantity equations of the traditional Leontief model fulfill the overall equilibrium condition that the total value of final demand is equal to the total value added which again is equal to the sum of total wages, total operating surplus, total depreciations, and total net indirect taxes. This condition followed directly from the basic equations. The corresponding condition in the more elaborate input-output framework outlined above is that the total (algebraic) value of all activity levels is equal to the total value of net additions in stocks. By evaluation in basic values the equilibrium condition follows directly from the basic equations.

More relevant is evaluation in market values. It is easily derived from the basic equations (3.8) and 3.14) that

$$(3.15)\quad p_A^{'}A = b_X^{'}\Lambda A + b_{AT}^{'}A = b_X^{'}X + b_{AT}^{'}A$$

The overall equilibrium condition is thus satisfied if and only if

$$(3.16)\quad b_{AT}^{'}A = 0$$

i.e. the net value of commodity taxes over all activities is equal to zero. In other words commodity taxes imposed on the products of some sector must eventually be paid by purchasers of the commodities. However, this is not a trivial condition within a formal model structure because tax rates may differ for different purchasers of the same commodity. Thus the prices of activity levels cannot be obtained until the quantity solution is known. The indirect tax relations of the model, see section 6.2, ensure that (3.16) holds and hence that the overall equilibrium condition is satisfied.

In the traditional Leontief model as set out in section 3.1 the absence of price differentiation between purchasers implies that the equality of the values of total demand and total supply holds for current as well as for constant market values. Within the commodity-activity-sector framework the corresponding equality will in general not be satisfied for constant values (except for the base year when current and constant values are identical). With the constant values of net commodity taxes calculated as fixed shares of the activity levels there will be a difference between total constant market value of primary inputs and total constant market value of final demand (incl. changes in stocks) whenever the relative shares of purchasers of commodities with differentiated tax rates deviate from the base year shares. This difference, which will be identical to $e^{'}A_T$, may be interpreted as a gain (or loss) in the overall constant value of demand caused by changes in demand composition. To achieve formally consistent model results, in accordance with the solution of very much the same problem in the national accounts, the constant value discrepancy ($e^{'}A_T$) is inserted as primary input in a dummy production sector. Changing the base year of the model means zeroing the discrepancy, in the same way as a change in reference year for constant value calculations in the national accounts does away with the accumulated discrepancy in the constant value accounts.[1)]

1) For a discussion underlying the treatment of this problem in the Norwegian national accounts see Sevaldson (1973a).

4. MAIN STRUCTURE OF THE MODEL

In this chapter the two main parts of MODIS IV will be presented with particular regard to central concepts and the structure of equations. An outline of the solution procedure in successive stages is also given. In chapter 5 most of the submodels are dealt with and discussed in a fairly large amount of detail although empirical specifications and estimation problems have not been elaborated upon. The submodels for direct and indirect taxes are presented in chapter 6 in the context of the use of the model in fiscal budgeting. The submodels discussed in chapters 5 and 6 are in general not self-contained in the sense that they exist as separate operational tools. They are submodels primarily in a conceptual sense. All the submodels presented are completely integrated in the central fabric of the model. The submodel for direct taxes is, however, nearly identical with a separate model employed for tax revenue calculations.

A brief general characteristic of MODIS IV could run as follows. The model is an input-output model for short-to-medium term macro-economic planning and policy-making using a fairly disaggregate set of national accounts in the description of the economy. The input-output framework is rectangular with explicit consideration of commodity flows as well as of sectors, and of activities within sectors. Most quantities in the model are demand determined. All final demand components except household consumption are exogenously given. There is a two-stage procedure for determining household consumption as a function of disposable household income, relative prices and population changes. The price side of the model uses the same detailed input-output framework as the quantity side. A central feature of the price side is the distinction between sheltered and exposed prices. The sheltered prices are mostly cost (supply) determined while the exposed prices are assumed to be determined on the world market. The specification of variables pertaining to government policy instruments is very detailed, in particular with regard to taxes, transfers and subsidies. The model is thus to a considerable degree a model dedicated to the tasks and problems of the Ministry of Finance. The considerable openness of the model in the sense that it has many exogenous variables apart from clearly defined policy instruments is to some extent an expression of the present division of labour between the model and the policy-maker using the model.

The representation within the model of a detailed set of national accounts - as set out in chapter 2 - serves as a unifying feature of the model. The close link between the model and the national accounts has been made possible by the close co-operation between the model building group and the national accounts division of the Central Bureau. The detailed information in the national accounts has been exploited in modelling many of the relations of the model.

The input-output framework is built around input-output tables fully integrated in the national accounts. The input-output relations are - as discussed in chapter 3 - in accordance with the basic ideas of the original Leontief scheme, but have been given a more general formulation which in our view gives a more adequate representation of the underlying empirical data and also a better foundation for building a broadbased model like MODIS IV.

As in the original Leontief scheme the model has a major subdivision in a quantity side and a price side. The basic quantity equation is as defined and introduced in (3.8)

$$(4.1) \quad \Lambda A = X,$$

with Λ an activity coefficient matrix, A a vector of activity levels and X a vector of net additions to stocks.

The basic price equation is as given in (3.14)

$$(4.2) \quad \Lambda' b_X = p_A - b_{AT},$$

with b_X a vector of commodity prices (in basic values), p_A a vector of market prices of activity levels, and b_{AT} a vector of commodity taxes per unit of activity level.

The concepts and assumptions underlying the two basic equations have been discussed in chapter 3. The two basic equations may be said to represent the <u>technological structure</u> and the (unit) <u>cost structure</u>, respectively. (4.1) and (4.2) are, at the present stage, closed partly by including appropriate submodels and partly by assuming that some variables are exogenous. In the presentation of the model below the quantity side is discussed in section 4.1 and the price side in section 4.2. The quantity side and the price side are here given a certain compact form which we shall call the <u>inner model</u>. The inner model has a clearcut formal structure given by a set of "unknown" variables, a set of "given" variables, a set of relations equal in number to the number of unknown variables, and a set of coefficients and parameters entering into the relations.

From the user's point of view the inner model is a condensed representation of a complete and comprehensive outer model of the interrelations of the economy. The basic equations together with the "projection", so to say, of the outer model into the basic equations determine the structure of the inner model.

It is hardly obtainable to have the outer model fully formalized. Insufficient data may be an obstacle to the estimation of complex relationships. Often the theoretical content of some parts of the outer model is not precise enough to warrant econometric treatment.

In MODIS IV the outer model is thus only partly formalized. The formalized parts take the form of submodels to be described in chapters 5 and 6. For the interrelation between the inner and the outer model it should be noted that the outer model includes the inner model as an embedded part. Given variables of the inner model may be either exogenous in the outer model or determined by relations of the outer model not represented in the inner model. Even coefficients and parameters of the inner model may be considered as determined in the context of the outer model. From this follows that the outer model provides much of the interpretation and operational meaning of the inner model.

In section 4.3 the solution of the model in successive stages is discussed. The discussion of the solution procedure is based only on the inner price and quantity models. In chapter 5 and chapter 6 the formalized submodels and the assumptions of exogenous variables are presented and discussed and their connexion with the inner model spelt out.

4.1. The quantity side of the inner model[1)]

In equation (4.1) the activity coefficient matrix Λ and the vector of activity levels A may be partitioned by type of internal activity. The equation can then be rewritten as

$$(4.3) \quad \Lambda_B A_B + \Lambda_P A_P + \Lambda_C A_C + \Lambda_I A_I + \Lambda_E A_E = X$$

where P = production (including general government production as well as commodity transformation activities)

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1) A more complete presentation of the quantity side of the inner model is given in Longva (1975a).

B = imports

C = household consumption

I = investments

E = exports

The import activity matrix Λ_B and the export activity matrix Λ_E have typically columns with only one non-zero element each. To each imported and exported commodity except minor items there corresponds at least one activity with a non-zero element on the row of the commodity in question.[1)]

The columns of the production activity matrix Λ_P have much in common with the columns of the traditional input-output matrix. The fixed ratios between input commodities are maintained. The production activity matrix differ, however, from the columns of the traditional input-output matrix in four respects. First, inputs and outputs are defined by a classification of commodities, not by a classification of sectors of origin. Second, output of more than one commodity from a single activity are allowed.[2)] Third, the same commodity may be produced in different activities, and fourth, the activity coefficients are normalized by value added (gross product), not gross production.

The household consumption activities are typically activities with only a small number of input commodities, input from trade (trade margins) frequently among them. The columns of the household consumption activity matrix Λ_C will consequently have only a few non-zero elements each. The same is true for the investment activity matrix Λ_I.

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1) The element may deviate from one due to customs duties on imports and trade margins and export taxes (subsidies) on exports. This is because commodity flows are recorded in basic values (which include customs duties and exclude export taxes and trade margins) while the market values of import and export activity levels, in accordance with the principles of the foreign trade statistics and national accounts, exclude customs duties on imports and include export taxes and trade margins on exports. If there is a trade margin associated with the commodity delivery to export there will actually be two non-zero elements in the column of Λ in question, one for the exported commodity and one for the trade margin commodity. This is because trade margins are treated as a separate industry commodity in the model (see section 3.3). 2) Since we in general assume non-jointness (see section 3.3), the occurrence of multiple output in activities must be viewed as a convenient way to take care of minor commodity outputs and not as an essential part of the activity structure.

The supply structure

For exogenously given additions to stocks (4.1) (or (4.3)) has a number of degrees of freedom equal to the excess of the number of activities over the number of commodities. The degrees of freedom have to be eliminated by introducing additional relations between activity levels, or by singling out a sufficient number of given activity levels. In the complete model some of the degrees of freedom are eliminated through determination of most of the private consumption activity levels by demand relationships and exogenous fixation of many activity levels including all the export and investment activity levels. The remaining degrees of freedom correspond mostly to alternatives with regard to sources of supply for the various commodities. Commodities may be brought into circulation either by imports or by production in one of the activities which has the commodity in question as one of its outputs. In this respect the basic equation of the model has more degrees of freedom than the corresponding equation in the traditional Leontief model where a given demand in most formulations can be supplied from one source only. The structure is more similar to the general activity analysis allocation model. A straightforward computation of an optimal choice of non-zero activity levels is possible if a preference function can be defined but the "either-or answers" of linear programming may not carry much meaning except as a hypothetical reference point. At least for the time being, other ways of closing the model in regard to the sources of supply are therefore preferred. For most commodities the commodity market share approach (to be explained below) is adhered to. However, in determining the sources of supply it must be considered that the aggregation of the set of all goods and services in the economy into nearly 200 commodities means that many of these commodities are quite inhomogeneous. Use of the same label for corresponding groups of imported and domestically produced goods may obscure the fact that these commodities may not be fully substitutable, that they may be highly different with regard to technological properties and want satisfaction and that, accordingly, they may be marketed at different prices.

For the elimination of the degrees of freedom of (4.1) the approach taken is to wrap up the formal structure in a compact set of equations consisting of (4.1) together with (4.4) and (4.5) below. The content of the latter two equations may be given different interpretations depending on the relations of the outer model.

In an attempt to simplify the presentation of the formal structure we shall adhere to one specific interpretation and later indicate where and in what way the existing model differs from this particular interpretation. For this purpose we shall assume that all final demand activities and the net additions to stocks have given levels. For each commodity we designate a main producer among the supply activities, that is, a production or import activity, having the commodity as its main product, producing and supplying more of the given commodity in the base year than any other activity. The remaining degrees of freedom can then be eliminated by letting each of the remaining import and production activities change proportionally with the main producer of its output commodity, i.e. by forming supply activity groups in a one-to-one correspondence with commodities.

The grouping of activities can be introduced in a way which is more symmetric with regard to the partaking activities than by first designating a main producer. For this purpose we introduce a vector Z of auxiliary variables. The dimension of Z is equal to the number of supply activity groups, i.e. the number of commodities. In general there is a corresponding element of Z for each supply group representing some measure of the activity level of the group as a whole, e.g. total commodity supply or total activity level.

Each activity level is now either given or related to the element of Z corresponding to its supply activity group. We can thus write

$$(4.4) \qquad A = \Pi_A Z + A^*$$

For activity levels with given values, that is all the final demand activities, the corresponding rows of Π_A are equal to zero and the values are given in A^*. For the other activity levels, i.e. all the supply activities, the elements of A^* are zero and the elements in each column of Π_A are the shares each partaking activity has in the aggregate measure of the supply group represented by the elements of Z. Each row of Π_A will contain at most one non-zero element because each supply activity is included in one and only one supply group.

Given the condition that (4.1) and (4.4) shall form a determined system of equations, it is easily seen from (4.4) that Z, and thereby Π_A, may be specified in several different ways. In general, the purpose of Z and Π_A is to link supply activities together in such a way that they form so many linearly independent groups as there are commodities in the model.

There are two contrary ways of grouping activities into supply structures, namely the activity sector share approach and the commodity market share approach. Adopting the activity sector share approach essentially implies assuming constant product mix within each sector. In this approach the supply activities are grouped such that all activities in the same group belong to the same supply sector. Activities with different main products are then linked together and each group of activities will cover total supply of commodities from the sector to which the group belongs. A convenient specification will be to let each element in Z represent total supply of commodities from a sector. The elements of Π_A will then be activity sector shares. By an activity sector share is meant the share an activity level has in the total level of the sector, i.e. sector level, to which it belongs. This procedure will give us as many linearly independent supply structures as there are supply sectors. If we add the simplifying assumption that we have as many supply sectors as commodities (4.1) and (4.4), specified as indicated above, will form a determined system of equations.

The actual formulation of the supply source relations of MODIS IV is, however, based upon a commodity market share approach. Here all supply activities with the same main product are grouped together. Each element in Z then represents total supply of each commodity and the elements of Π_A are commodity market shares normalized by activity level per unit of main product.1) This implies given market shares among suppliers which supply the same commodity as their main product. From the assumption of constant activity coefficients it follows that by-products of a supply activity are in fixed proportions to the main product. By-products are thus treated formally as negative inputs.

In the present version of MODIS IV these commodity market shares are assumed to be constant and estimated from the base year of the model2). The implicit assumption that the commodity market share matrix is more stable over time than the activity sector share matrix, at least in the short and medium run, is only based upon an a priori assessment and an evaluation of some scattered time series. A thorough investigation has not yet been undertaken.

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1) To be more precise the corresponding element of Z represents net supply for the commodity with net supply defined as the excess of total supply over the supply of the commodity as by-product, i.e. total supply of the commodity as a main product. 2) This is somewhat modified for the import activities, see the discussion below.

Ideally the matrices Λ in (4.1) and Π_A in (4.4) should have been derived from a producer behaviour model rather than assumed to have constant elements from the outset. The quantity side of such a model should include that the production sectors are faced with a set of technological possibilities represented by alternative activities for the same product and that the production sectors select between the possible technologies in such a way that least cost production patterns are undertaken. The producer behaviour model should also include assumptions about how each sector select its product line, for example by letting changes in the sector activity composition, i.e. the product mix, depend upon changes in some measure of relative profitability.[1] The price side of such a model is discussed in section 4.2 but so far no quantity counterpart is introduced.

The commodity market shares and the sector activity shares are linked to the concepts of degree of coverage and degree of specialization, respectively (see section 3.3). To assume that the market shares are more stable than the activity sector shares is the same as assuming that the relative distribution along the rows in a commodity-by-industry output table are more stable than the distribution along the columns. The principles used in the classification of commodities and production sectors lead to a minimization of elements outside the diagonal in the commodity-by-industry output table. The importance of the choice between the assumptions of constant market shares and constant activity sector shares is thereby reduced as far as possible, given the level of aggregation.

It should also be noted that it is possible to make a differentiated choice between constant market shares and constant activity sector shares simply by including the product mix assumptions in the specification of production activities. If we want to assume constant proportions between two or more output commodities in the same sector this can be done by specifying one activity for these commodities, i.e. link them together in fixed proportions. This means that there is an element of arbitrariness between the assumptions of constant activity sector shares and the specification of activities. Actually, the activity sector share approach, using base year shares, will give the

1) If relative profitability are introduced the reduced form of such a model may take the form of (4.4) with Π_A specified according to the sector activity share approach.

same results in model computations as with only one supply activity in each production sector, i.e. joint production in sectors. Within a constant coefficient framework, i.e. either constant commodity market shares or constant activity sector shares, the real choice is therefore between joint and non-joint production.[1)]

If we more explicitly include the import activities among the supply activities it is easily seen that a market share model for imports similar to that of production activities may be applied. Within a simplified interpretation of the quantity side this means that the import activities change proportionally with the main domestic supplier of the activity's main product.

Changes in stocks as a source of supply

In a slightly extended interpretation of the market share approach as introduced above we shall allow some of the supply activities to have given activity levels. The corresponding rows of Π_A are then set equal to zero and the given levels included in A*. There may be various reasons for assuming that supply activities have given levels in the inner model. In general, it implies that some or all of the supply of a commodity is determined by other factors than total demand for the commodity in question. The interpretation of the elements in the Z vector must now be modified so as to represent the net supply for each commodity defined as the excess of total supply over given supply[2)].

If the total supply of some commodity as a main product is given, something on the demand side has to be "loosened" to prevent the system from being overdetermined. In this case we shall assume that the net addition to stocks of the commodity in question is changed from being a given to an unknown entity in the inner model. The system of equations will be modified to accommodate this assumption by adding

$$(4.5) \quad X = -\Pi_X Z + X^*$$

The whole X vector of (4.1) is no longer assumed to be a given vector. In (4.5) some elements of X are given, implying rows of zeros in Π_X, and

1) Although we in general assume non-joint production and constant market shares this is somewhat modified for commodities connected with own investment and repair work which are assumed to be produced jointly with the main commodity in the production sectors where they appear. Constant activity sector shares seem to be a more appropriate assumption for these commodities than constant market shares. 2) See also note 1, p. 69 for the modification in the interpretation of the elements in the Z vector due to by-products in activities.

some are determined as a share of net supply. The matrices Π_A and Π_X have to be related so that all of net supply for each commodity is distributed among sources of supply including stocks. In the model the matrix Π_X will contain non-zero elements only when all supply activities having the same commodity as a main product have given values. The share of net supply from stocks will then be equal to one. The treatment of net additions to stocks are further discussed in section 5.1.

Above we have outlined one particular heuristic interpretation. This will be further modified in the subsequent chapters on the outer model. While equation (4.1) above was said to represent the technological structure of the economy, (4.4) and (4.5) can heuristically be said to represent the market structure.

The demand structure

The quantity model is mainly demand oriented and the supply side is, as discussed above, assumed to respond only to real demand for commodities. The model distinguishes between commodity demand through intermediate demand, household consumption, government consumption expenditures (commodity absorption in general government production), private and government investments, and exports. The same commodity can be, and normally are, included in more than one demand activity. A given commodity demand may thus be the aggregate of commodity demand from several sources. However, this property of the model is implicit in the basic equation (4.1) and no special formulations are needed in the demand relations.

Submodels of the quantity side

The matrices Π_A and Π_X may be interpreted to include linear specifications of different types of relations. Some of the most common extensions of the traditional input-output model may be incorporated within this scheme.

Equation (4.4) can be subdivided by type of activity. Using the same notation as in (4.3) we have

$$(4.6) \quad A_B = \Pi_{A_B} Z + A_B^*,$$

$$(4.7) \quad A_P = \Pi_{A_P} Z + A_P^*,$$

$$(4.8) \quad A_C = \Pi_{A_C} Z + A_C^*,$$

(4.9) $A_I = \Pi_{A_I} Z + A_I^*$, and

(4.10) $A_E = \Pi_{A_E} Z + A_E^*$.

In the present version of MODIS IV most of the net additions to stocks and all the activity levels for final demand activities except household consumption are exogenous. The supply activities, including the government production of marketed government services, are, with some exceptions, endogenous. The same is true also for the household consumption activities. If the whole or part of an endogenous activity level does not depend upon the "solution vector" Z, it will show up in the inner model as given rather than unknown.

The quantity side of the producer model of MODIS IV, which is called the submodel for production, is described by the production functions and by the domestic supply structure of the economy. The production functions are given by the base year coefficients of the production part of the activity matrix, i.e. Λ_P (see (4.3)), while the supply structure is given by (4.7). The matrix Π_{A_P} in (4.7) is, as has been indicated above, essentially a matrix of constant commodity market shares. The given production activity levels A_P^* has non-zero elements for commodity absorption in general government production which is fully exogenous and also for some industry production activities which are determined exogenously independent of demand. There are thus no explicitly behavioural elements on the quantity side of the production model. The submodel for production is further discussed in section 5.2. The labour demand model, which takes the form of value added production functions in inverted form, is also presented in that section.

The submodel for household consumption takes in reduced form the shape of (4.8). The conversion matrix between the consumption activities and commodity flows, represented by Λ_C (see (4.3)), may also be regarded as part of the submodel. The main elements of the submodel are an aggregate consumption function, and a set of distributional relations. The aggregate consumption function determines the total demand for household consumption as a function of real disposable income for (i) wage and salary earners, (ii) self-employed, and (iii) pensioners. The gross incomes are made up of wages, profits of unincorporated enterprises (incl. agriculture) and government transfers distributed to the three socio-economic groups. By subtracting direct taxes and deflating by a consumer price index real disposable incomes are arrived at.

The set of distributional relations allocates the total demand for household consumption among the household consumption activities by means of income and price derivatives. Apart from the given coefficients, the main group of given variables of the submodel are (i) exogenously given wage rates and industrial labour productivities, (ii) government employment, (iii) consumption activity prices, the consumer price index, and profit rates from the price side, (iv) personal income tax parameters from the submodel for direct taxes, (v) government transfers, and (vi) some exogenously given levels of consumption activities. In addition to the exogenous consumption items the A* of (4.8) includes consumption which is independent of the simultaneous quantity solution, i.e. of the "solution" vector Z. The coefficients of the matrix Π_{A_C} will be reduced form expressions for relations which connect household consumption activity levels with demand dependent income components. The links here are the industrial labour productivities and profit rates, all related to the production sector levels. The production sector levels are added up from production activity levels. The endogenously determined sector activity composition, i.e. the activity sector shares, is influenced by the constant commodity market shares and the Π_{A_P} of (4.7) thus actually enters into the expression for Π_{A_C}. The submodel for household consumption, including the income formation, is further dealt with in sections 5.3 and 5.4.

Both investments and exports are fully exogenous in the present version of MODIS IV. In (4.9) and (4.10) above the matrices Π_{A_I} and Π_{A_E} are zero and the vectors of given values A_I^* and A_E^* contain the actual exogenous estimates for gross investments and exports. This is further discussed in section 5.1.

In (4.6) a market share model for imports similar to that of production activities might have been applied. The actual submodel for imports is, however, based on demand differentiated market shares for imported commodities and not on global market shares. This means that for each commodity there is a differentiated import share for each receiving actitity. The connexion between import activity levels and endogenous supply represented by the solution vector Z is thus indirect and goes via all demand influenced receiving activities. The Π_{A_P} of (4.7) and Π_{A_C} of (4.8) thus enter into the expression Π_{A_B} of (4.6). The non-zero elements of A_B^* are, in addition to a small number of exogenous import activity levels, mostly derived from the given (parts of) activity levels for the other groups of activities.

The solution of the quantity side

The formal structure of the quantity side of the inner model is given by (4.1), (4.4) and (4.5). The explicit solution of A in terms of X* and A* (eliminating Z) using (4.1), (4.4) and (4.5) is

$$(4.11)\quad A = \Pi_A(\Lambda\Pi_A + \Pi_X)^{-1}X^* + (I-\Pi_A(\Lambda\Pi_A + \Pi_X)^{-1}\Lambda)A^*$$

The solution of X is easily found by inserting the result for A into (4.1).

For this system to be uniquely determined it is required that the vector Z is of dimension equal to the number of commodities, and that $(\Lambda\Pi_A + \Pi_X)$ is a non-singular matrix. We shall say that the coefficient matrices Π_A and Π_X are imposed on the inner model by the outer model as a general reduced form expression. The precise content and interpretation of Z and the matrices Π_A and Π_X will depend upon the outer model. The non-singularity condition will similarly depend on the actual content given to Z, Π_A and Π_X. Singularity will normally indicate a faulty logic in the specification of Π_A and Π_X. By aggregating activity levels for activities belonging to the same sector we get sector levels. The activity sector shares, which can be derived from these results, are therefore endogenously determined on the quantity side.

In diagrams 4.1 and 4.2 the quantity side of the inner model of MODIS IV has been given a pictorial representation. Diagram 4.1 shows how the given variables and coefficients of the various subparts of the inner quantity model are determined. In diagram 4.2 the subparts are shown to form a simultaneous equation system which can be solved for A and X.

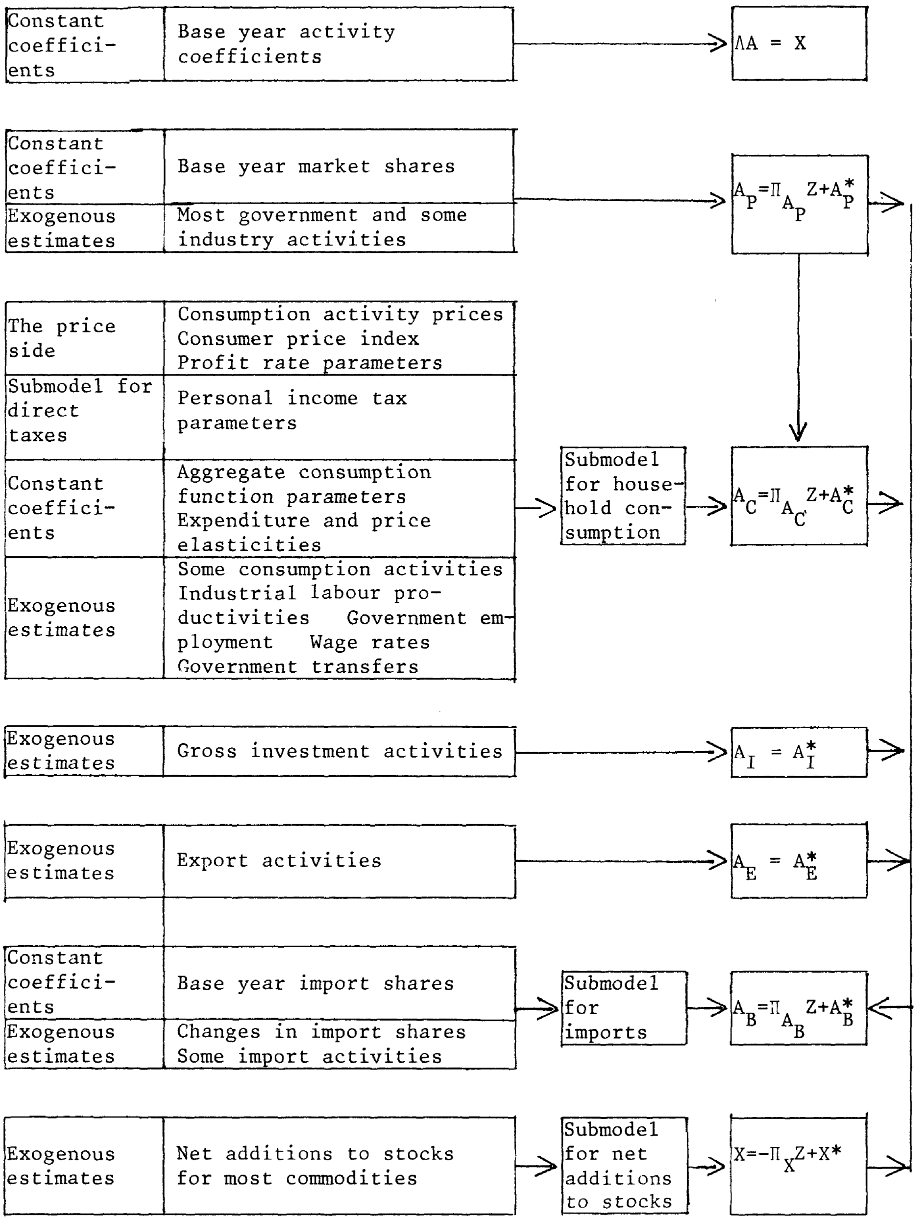

Diagram 4.1. The generation of the inner quantity side of MODIS IV

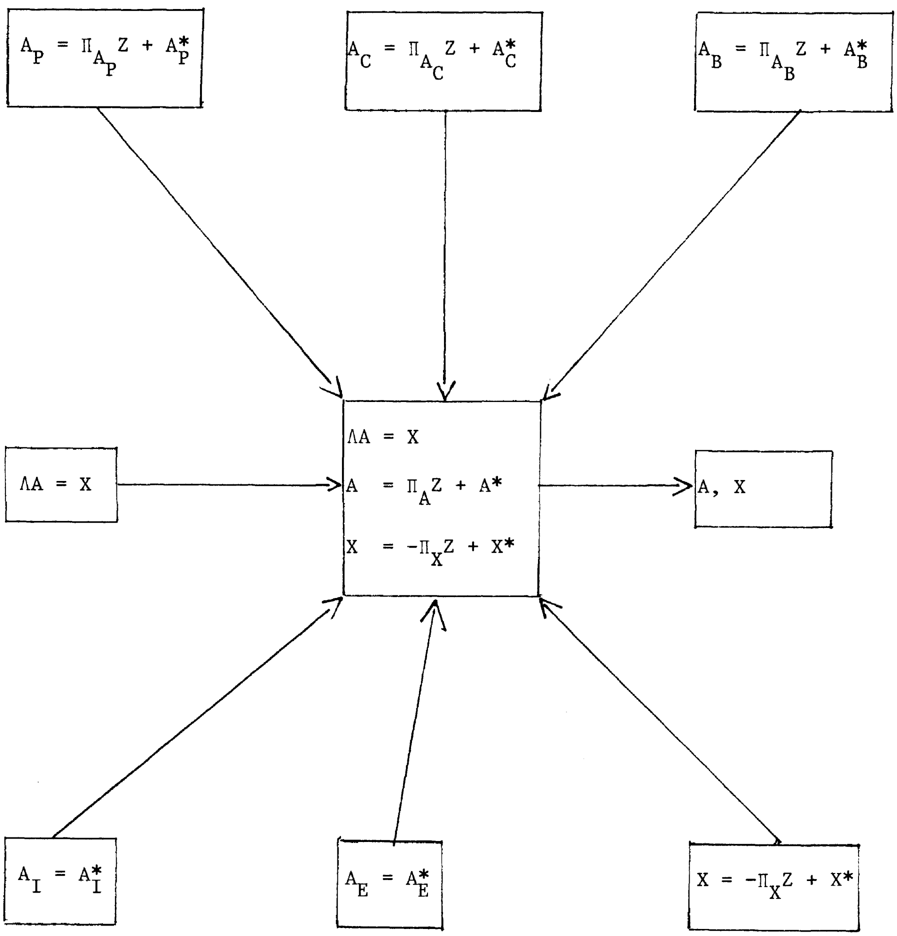

Diagram 4.2. The solution of the inner quantity side of MODIS IV

4.2. The price side of the inner model[1)]

As discussed in section 4.1 the quantity side of the inner model is "closed" by the introduction of projections from the outer model into the basic quantity equation (4.1). In general form these projections are given by (4.4) and (4.5). The formal structure of the price side is similar to that of the quantity side by also being "closed" by projections from the outer model into the basic price equation (4.2). The formal presentation of the price relations is, however, more complicated than for the quantity side. The main reason for this is the introduction of price differentiation by markets for the same commodity and the implication of this for the cost calculation.

In an attempt to simplify the presentation of the formal structure we shall at the outset ignore some of these complicating assumptions. As in the presentation of the quantity side we shall adhere to one fairly simple interpretation and later indicate where and in what way the existing model differs from this simplification.

We shall at the outset treat the difference between activity level price (the elements of p_A) and commodity taxes, net, per unit of activity level (the elements of b_{AT}) for each activity as one variable. The vector of these variables can be interpreted as prices of activity levels in basic values, shortened in the following to activity basic prices and denoted by b_{AX} (see (3.13)). As in (3.14) the basic price equation (4.2) can therefore be written as

$$(4.12) \quad \Lambda' b_X = b_{AX} (= p_A - b_{AT})$$

The cost structure

For endogenously determined commodity prices (4.12) has a number of degrees of freedom equal to the number of commodities. The degrees of freedom have to be eliminated by introducing additional relations between activity basic prices or by singling out a sufficient number of exogenously given activity basic prices.

The starting point in the simplified discussion of the quantity side was to assume that final demand and net additions to stocks were given. The solution of the quantity side was thus only dependent upon demand considerations. If we on the price side assume that all commodity prices

1) A more complete presentation of the price side is given in Longva and Tveitereid (1975).

b_X are endogenous the commodity price formation may conversely be assumed to be dependent only upon supply considerations represented by given primary costs. A simulation of the traditional Leontief price model by exogenous determination of the activity basic prices, i.e. the elements of b_{AX}, will, however, give us an overdetermined system of equations. The economic content of this inconsistency is that identical commodities are produced in different production activities. There is no reason to expect prices of the same commodity calculated by cost addition from independently given primary costs to be identical. In addition, the commodity input-output coefficient structure will be different if sector technology is assumed. Compared with the corresponding equation in the traditional Leontief model, where given primary costs in each source of supply generate a consistent set of prices, the basic cost equation in MODIS IV has less degrees of freedom.

A simple way of solving this problem is to designate a price leader for each commodity among the supply activities. The activities in this preselected set of supply activities have a one-to-one correspondence with the set of commodities, each activity having the corresponding commodity as its main output. A natural way of selecting these activities is to assume that the main supplier of the commodity is the price leader.

For the price leaders the unit primary basic costs, which are equal to the activity basic prices, are then assumed to be given. This will eliminate the degrees of freedom of (4.12). The commodity prices will be equal to the unit production costs in the respective price leading activities. By-products in activities are again treated as negative inputs.

Instead of designating one supply activity as price leader for each commodity, we may form groups of activities and assume that a weighted average of the activity cost structure in each group determines the price of the corresponding commodity. The unit primary cost for each group average (unit primary basic cost), i.e. the weighted sum of activity basic prices within each group, is assumed to be given. The weighted cost structure can, of course, be specified in several different ways, some of which will be discussed below.

In the algebraic formulation of the relations discussed above we shall use a similar formal device as for the quantity side, namely an auxiliary vector b_Z. The dimension of b_Z is equal to the number of price leading activity groups, i.e. the number of commodities. In general there is in b_Z an element corresponding to each commodity price and representing some measure of the cost structure of the relevant activity group as a whole, e.g. unit primary basic cost or total cost per unit of output.

In the simplified interpretation we assume that unit primary basic costs by price leading activity group, for short, unit primary costs, have given values. We can then write

$$(4.13) \quad \Pi_{b_A} b_{AX} = \Pi_{b_Z} b_Z + b_Z^*$$

The values for unit primary costs by price leading activity group are given in b_Z^*. b_Z^* may be interpreted as a given component of the auxiliary vector b_Z. Π_{b_Z} are all zero; this matrix is used in extended interpretations when unit primary costs of some activity groups are assumed to be endogenously determined.

Given the condition that (4.12) and (4.13) shall form a determined system of equations, it can be seen from (4.13) that b_Z^* and Π_{b_A} may be specified in several alternative ways. In general Π_{b_A} serves to link supply activities together in such a way that they form as many linearly independent groups as there are designated price leaders or cost structures in the model.

There are two contrary ways of grouping activities into price leading cost structures, namely by following the activity calculation approach or the sector calculation approach. In the activity calculation approach it is assumed that the cost calculations are performed in single or aggregated activities with the same main product in all activities belonging to the same cost structure. The weighted cost structure can be specified in several different ways, one obvious possibility is to use the share the main product of each activity has in total output of the cost structure to which they belong. The elements of Π_{b_A} will then be commodity market shares.[1)2)]

1) To be more precise the elements of b_Z^* then represent net unit primary costs with net unit primary costs defined as the excess of total unit primary costs over the unit primary costs for commodities as by-products, i.e. total unit primary costs for commodities as a main product. 2) Compare this specification of Π_{b_A} with that of Π_A of (4.4) when the commodity market share approach is applied on the quantity side.

The actual formulation of the price formation relations of MODIS IV is, however, based upon a sector calculation approach. The basic idea is that the commodity prices are assumed to appear as cost prices from a calculation of total incomes and outlays in supply sectors. In this approach all activities in the same supply sector are grouped together and the weighted sums of activity basic prices within each sector, are assumed to be given as elements of b_Z^*. The weights used, i.e. the elements of Π_{b_A}, are the shares each activity level has in the level of the supply sector to which they belong, i.e. activity sector shares.[1)] The unit primary costs of these separate cost structures may then be interpreted as sector basic prices. The procedure will give us as many linearly independent cost structures as there are supply sectors. If we add the simplifying assumption that we have as many supply sectors as commodities (4.12) and (4.13), specified as indicated above, will form a determined system of equations.

In principle, a choice between the activity and the sector cost calculation approaches within a producer behaviour model should depend upon how the production is organized in those establishments that form a sector. If the technological and organizational structure is such that the primary inputs can be associated with each commodity produced in the sector, or with each activity, an activity calculation approach ought to be chosen. If this is not the case, a sector calculation approach should be chosen (see Frisch (1962), pp. 29-30).

In MODIS IV the activities must be regarded as technical transformation functions. More specifically, the production activities can be interpreted as macro processes aggregated across establishments within the same sector. As to be elaborated upon in section 5.1 we are assuming sector technology for value added by activities, i.e. that primary input is identically composed for all activities within the same sector and thereby may be regarded as a common "pool". Within a producer behaviour model it seems therefore reasonable to treat the sectors as the decision making units and to base the price side of such a model upon the sector calculation approach outlined above.

The general idea behind the price side of the present production model of MODIS IV is that each supply sector is faced with its technological environment given by the activity coefficients, its market environment given by the commodity market shares and commodity demand, the unit

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1) Compare this specification of Π_{b_A} with that of Π_A if the activity sector share approach were applied on the quantity side.

primary basic costs, the prices charged for material (intermediate) inputs, and the prices obtained for its by-products. It then acts to set the price of the commodity for which it is the price leader in such a way that incomes cover costs. The price relations of the sector calculation model presented above satisfy these requirements since the solution of these relations yields commodity prices that permit all suppliers to just cover all costs, including primary costs, when account is taken of the fact that cost determined output prices in one supply sector also may represent prices on material inputs and by-products faced by others.

The present version of MODIS IV applies base year activity coefficients for describing the technology and base year commodity market shares for describing the market structure (see section 4.1). The activity sector shares are endogenously determined as part of the solution of the quantity side. This procedure is, as discussed in section 4.1, simply based on the judgement that the market shares for commodities are relatively more stable over time than the activity sector shares or the product mix.[1)]

More commodities than supply sectors

In the simplified sector calculation formulation presented above the prices are determined, as in the traditional input-output model, from exogenously given unit primary costs by supply sector. However, in MODIS IV we need additional assumptions to close the model since it has more commodities than supply sectors.

For each commodity we designate a price leader among the supply sectors, normally the production or import sector supplying more of the given commodity in the base year than any other sector. Each supply sector is assumed to be the price leader for at least one commodity. The remaining degrees of freedom can thus be eliminated by letting the prices for commodities with the same sector as a designated price leader be related to each other, for example by letting them change proportionally (identical price indices). The commodity prices will then be equal to

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1) If we instead of the commodity market share approach had used the activity sector share approach on the quantity side it is easily seen that the link from the quantity side to the price side had been completely avoided. However, the activity sector share approach must, at present, be built upon the assumption of a constant share matrix, which seems to be empirically less acceptable. A third possibility within a fixed coefficient framework is to combine the market share approach on the quantity side with the activity calculation approach on the price side. However, compared with the sector calculation approach the activity calculation approach seems in general to be a less appropriate specification of the price formation.

unit production costs in the corresponding price leading sectors. In this way we can close the price side of the inner model when we have more commodities than supply sectors.

In the algebraic formulation of the relations discussed above we shall use the auxiliary vector b_Z. For each supply sector there is now a corresponding element of b_Z representing total costs per unit of output of the commodities for which the sector is the price leader.[1] For short we shall call these elements unit supply costs.

By setting each commodity price equal to unit supply costs in the price leading sector we can write

$$(4.14) \quad b_X = \Pi_{b_X} b_Z + b_X^*$$

In (4.14) Π_{b_X} consists of ones and zeroes so that each commodity price in b_X is set equal to the unit cost in the relevant price leading sector. Each row of Π_{b_X} will thus contain only one unit element because each commodity has one and only one price leader. In this simplified interpretation the elements of b_X^* are all zero; this vector is used in extended interpretations when some commodity prices are assumed to be given or have given components.

Exogenous prices

Above we have given a simplified description of the inner price model based upon the sector calculation approach with all commodity prices endogenous and all sector basic prices (unit primary basic costs) given. In a slightly extended interpretation we shall allow some of the commodities to have given price indices. In (4.14) the corresponding rows of Π_{b_X} are then set equal to zero and the given values of the price indices are included in b_X^*. There may be various reasons for assuming that some commodities have given price indices. In general, it means that the price index of a commodity is determined by other factors than cost considerations in a supply sector.

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1) To be more precise the corresponding element of b_Z represents <u>net</u> cost per unit of total commodity output, with net cost defined as the excess of total cost over the incomes from output of commodities for which the sector is not price leader.

In the preceding discussion of the inner price model the main supplying sector of each commodity was supposed to be the price leader of that commodity, and each supply sector was supposed to be price leader for at least one commodity. However, if all commodities for which a supply sector is the main supplier have given price indices something on the cost side has to be "loosened" to prevent the system from being overdetermined. In this case we shall assume that the sector basic price (unit primary basic costs) is changed from a given to an unknown entity in the inner model. For supply sectors with given unit primary costs, i.e. for the price leading supply sectors, the elements of the corresponding rows of Π_{b_Z} in (4.13) are equal to zero and the values are given in the vector b^*_Z. For the other supply sectors, which we may call <u>price takers</u>, the elements of b^* are zero and there is a unit element on the relevant row (and column) in the square matrix Π_{b_Z}, i.e. on the diagonal. For these supply sectors the interpretation of the elements of the b_Z vector is modified to represent unit primary basic costs (sector basic price).

The matrices Π_{b_X} and Π_{b_Z} have to be related so that columns in Π_{b_X} with one or more unit elements correspond to zero rows in Π_{b_Z}. Rows with a unit element in Π_{b_Z} appear as zero columns in Π_{b_X}.

The system of price relations presented above has a similar form as the quantity side discussed in section 4.1, and it may be given various interpretations depending on the precise content of the Π-matrices. To summarise, in the interpretation given above the basic assumption is that each commodity price is either given or endogenously determined by cost considerations. There is a preselected set of supply sectors each of which is a price leader for one or more of the commodities with endogenous prices. The rest of the supply sectors are price takers, i.e. all output prices are either given from the outer model or given by cost considerations in the price leading supply sectors. For the price leaders the unit primary basic costs are given while they are endogenously (residually) determined for the price takers. While (4.12) (or (4.2)) above was said to represent the (unit) cost structure of the economy, (4.13) and (4.14) can heuristically be said to represent the <u>price formation structure</u>.

The solution of a simplified version of the price side

The formal structure of the version of the price side presented above is given by (4.12), (4.13) and (4.14). The explicit solution of b_X in terms of b_X^* and b_Z^* (eliminating b_Z) using (4.12), (4.13), (4.14) is

$$(4.15) \quad b_X = \Pi_{b_X}(\Pi_{b_A}\Lambda'\Pi_{b_X} - \Pi_{b_Z})^{-1}b_Z^* + (I-\Pi_{b_X}(\Pi_{b_A}\Lambda'\Pi_{b_X}-\Pi_{b_Z})^{-1}\Pi_{b_A})b_X^*$$

The solution of b_{AX} is easily formed by inserting the result for b_X into (4.12).

The distinction between import, domestic and export prices

In MODIS IV special attention has been given to the treatment of prices on exports and imports. In traditional input-output models, like MODIS II and III, the formulation of the price side is normally based upon the assumption that price factors on the same row change proportionally, i.e. that a sector delivery has the same price index for all purchasing sectors or final demand categories. In the basic price equation of MODIS IV as specified in (4.12) the corresponding assumption has seemingly been made as the price index vector b_X contains one and only one element for each commodity. Within this framework a possible way of specifying the impact of world market prices, is to let import sectors (and possibly also export sectors) be price leaders of commodities for which imports or exports play a major role.

However, an important feature of the actual price relations of MODIS IV is that the import price index as well as the export price index may, and generally will, differ from the domestic price for the same commodity. From the national accounts time series data there is clear evidence for a large number of the commodities specified within the model against assuming that price indices for imported and exported commodities are identical with those for domestically supplied and demanded commodities (see Ringstad (1974), Appendices A and B). This is the case even when basic values are used to evaluate commodity flows. The main reason for this price index differentiation is believed to be inhomogeneity with regard to the underlying micro commodities in the composition of domestic, imported, and exported commodities, respectively.

As the import shares of commodity inputs as well as the export shares of commodity outputs may be open to change with corresponding changes in domestic shares, it is necessary for an adequate treatment to split the activity coefficient matrix Λ in (4.12) into three separate matrices, i.e.

$$(4.16) \quad \Lambda = B + P + E$$

where B, P and E represent imports, domestic production and use, and exports, respectively

When this decomposition is introduced in (4.12) with the appropriate commodity price index vector for each coefficient matrix we arrive at

$$(4.17) \quad B' b^*_{XB} + P' b_{XP} + E' b^*_{XE} = b_{AX} \; (= p_A - b_{AT})$$

where b^*_{XB}, b_{XP} and b^*_{XE} represent <u>import prices</u>, <u>domestic prices</u>, and <u>export prices</u>, respectively.[1)]

The decomposition of Λ into separate matrices for the three components of each commodity flow can be performed on the basis of the solution of the quantity side. This will be further discussed in section 4.3.

In the quantity side of the inner model the activity coefficient matrix Λ and the vector of activity levels A were partitioned by type of internal activity and the basic quantity equation rewritten as in (4.3). In a similar way the activity coefficient matrices B, P and E, and the activity basic price vector b_{AX} in the basic price equation (4.17) can be partitioned by type of internal activity and rewritten in five equations as

$$(4.18) \quad B_B' \, b^*_{XB} = b_{A_BX} \; (= p_{A_B} - b_{A_BT})$$

$$(4.19) \quad B_P' \, b^*_{XB} + P_P' \, b_{XP} + E_P' b^*_{XE} = b_{A_PX} (= p_{A_P} - b_{A_PT})$$

$$(4.20) \quad B_C' \, b^*_{XB} + P_C' b_{XP} = b_{A_CX} \; (= p_{A_C} - b_{A_CT})$$

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1) The superscript star in b^*_{XB} and b^*_{XE} indicates that these prices are exogenously given. We shall return to this below. Note also that b^*_{XB}, b_{XP} and b^*_{XE} are all indices of unit values relative to the base year. This follows from the assumption of no price differentiation in the base year.

(4.21) $B_I^{'} b_{XB}^{*} + P_I^{'} b_{XP} = b_{A_I X} \; (= p_{A_I} - b_{A_I T})$

(4.22) $E_E^{'} b_{XE}^{*} = b_{A_E X} \; (= p_{A_E} - b_{A_E T})$

The definitions of the subscripts are given after equation (4.3). Some of the partitioned matrices are for obvious reasons identically zero, namely P_B, E_B, E_C, E_I, B_E and P_E, and are therefore left out in the equations (4.18)-(4.22).[1] It should be noted that the partitioning of the basic quantity equation by type of internal activity resulted in an additive form of the equation while the same operation on the dual basic price equation gives separate sets of equations.

When separate import, export and domestic prices are introduced with (4.12) replaced by (4.17) (or (4.18)-(4.22)) the price formation equations (4.13) and (4.14) must also be rewritten to be in accordance with the new formulation. The most general way of doing this will be to replace (4.14) with three equations, one for each set of commodity prices. Here the imports, domestic and export prices of each commodity will be associated with the unit costs of the relevant designated price leaders. The group of possible price leaders must be extended to include the export sectors.

In MODIS IV the import sectors and export sectors are, with a few minor exceptions, designated as price leaders for imports and exports of each commodity, respectively. This is based upon the assumption that these prices are determined in the world market, by and large independently of domestic production costs. Instead of introducing these assumptions into the general framework sketched above, the import and export prices of each commodity are simply treated as given variables in the inner price model. Formally the equations similar to (4.13) and (4.14) are therefore, for sake of simplicity, only applied for prices of domestic production and use.

(4.14) is thus replaced by

(4.23) $b_{XP} = \Pi_{b_{XP}} b_Z + b_{XP}^{*}$

1) We are suppressing here some minor occurrences of re-export, i.e. non-zero elements in B_E.

In (4.23) the dimension of b_Z is equal to the number of production sectors. This means that among the possible price leading sectors for domestic production and use, only the production sectors are included in $\Pi_{b_{XP}}$. If an export or import sector is assumed to be the actual price leader for domestic production and use of a given commodity it is more convenient to set the relevant elements of b^*_{XP} equal to the corresponding elements of b^*_{XB} or b^*_{XE}.

With these simplified specifications of $\Pi_{b_{XP}}$ and b_Z (4.13) is replaced by

$$(4.24) \quad \Pi_{b_{AP}} b_{A_P} X = \Pi_{b_Z} b_Z Z + b^*_Z$$

The elements of $\Pi_{b_{AP}}$ are now production activity sector shares and b^*_Z unit primary basic costs.

A reformulation of the cost calculation

Since all production sectors are price takers for imports and exports it follows that imports enter as outlays and exports as incomes in the domestic price calculation of a price leading production sector. A change in the price of an exported commodity would therefore, cet. par., result in a change in the opposite direction of the corresponding domestic price. In this respect the result is quite similar to the effect of a change in the price of a commodity which the sector is producing as a by-product. While we accept this effect on the price calculation for commodities sold within the same market, i.e. that a domestic price of a commodity influences domestic prices of other commodities through the income side, the incomes from exports are not taken into account in the domestic price calculation in the present version of MODIS IV. This complete separation between markets (and no separation between commodities (activities)) in the domestic price formation is mainly based upon evidence in available data that the price leading sectors do not significantly "compensate" on the domestic markets for changes in the export prices (cp. Ringstad (1974), chapter 4). It is also much easier to "guide" or "control" the model with separation between the markets because we will not get "unexpected" effects on the domestic prices through the exogenous export prices.

If the domestic price formation of MODIS IV is to be built upon a separate price calculation for the domestic market, the exogenous estimates given in b_Z^* must be interpreted as unit primary basic cost in the production for the domestic market only and the activity basic prices (b_{A_PX}) of (4.24) have to be corrected for net income gain/loss due to higher/lower export than domestic price indices. (4.24) is accordingly replaced by

$$(4.25) \quad \Pi_{b_{AP}} [b_{A_PX} - E_P' (b_{XE}^* - b_{XP})] = \Pi_{b_Z} b_Z + b_Z^* \;^{1)}$$

The submodel for domestic commodity prices

If we ignore the submodel for indirect taxes the submodel for domestic commodity prices is the only submodel of the price side of MODIS IV. The submodel for domestic prices takes in reduced form the shape of (4.19), (4.23) and (4.25). The general specification of these equations are discussed at length above. The precise content follows from the specification of the matrices $\Pi_{b_{XP}}$ and Π_{b_Z} and the vector b_Z^*. The specification is adapted to a small open economy where export and import prices in general are assumed to be determined in the world market. A distinction is drawn between the exposed and sheltered domestic prices. The exposed domestic prices are prices of commodities sold under strong foreign competition while sheltered domestic prices are prices of commodities relatively sheltered from foreign competition. In the model it is assumed that the exposed domestic prices adjust to the corresponding import prices. The regulated and negotiated sheltered domestic prices are assumed to be exogenously given while the rest, i.e. the cost determined sheltered domestic prices, are assumed to be adjusted to changes in the costs of producing the commodities. This classification determines the elements of $\Pi_{b_{XP}}$ and Π_{b_Z}. The primary unit cost for the cost determined sheltered domestic prices, i.e. the elements of b_Z^*, are determined through exogenous estimates for industrial labour productivities and

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1) As pointed out in chapter 3 both the price and volume concepts of the model are based upon the perhaps doubtful assumption of no commodity price differentiation in the base year. Prices of commodity flows are indices of base year values relative to the base year. It follows from this that all weights in the primary cost structure for deliveries of the same commodities to the domestic markets and the export markets are stipulated to be identical in the base year.

wage rates, exogenous mark-up rates, and tax rates for commodity taxes and subsidies on commodity inputs given from the submodel for indirect taxes.[1] The submodel for domestic prices is further dealt with in section 5.6.

The solution of the price side

The formal structure of the price side of the inner model is now given by (4.18)-(4.22), (4.23) and (4.25). The explicit solution of b_{XP} in terms of b^*_{XP}, b^*_Z and b^*_{XB} can be expressed in similar form as the solution given for b_X in equation (4.15). However, a close inspection of the system of equations shows that (4.19), (4.23) and (4.25) form a determined subsystem. The explicit solution of b_{XP} in terms of b^*_{XB} and b^*_Z (eliminating b_Z) using (4.19), (4.23) and (4.25) is

$$(4.26)\quad b_{XP} = \Pi_{b_{XP}}[\Pi_{b_{AP}}(P_P' + E_P')\Pi_{b_{XP}} - \Pi_{b_Z}]^{-1}[b^*_Z - \Pi_{b_Z} B_P' b^*_{XB}] +$$

$$[I - \Pi_{b_{XP}}[\Pi_{b_{AP}}(P_P' + E_P')\Pi_{b_{XP}} - \Pi_{b_Z}]^{-1}\Pi_{b_{AP}}(P_P' + E_P')]b^*_{XP}{}^{2)}$$

The solution for b_{AX} is most easily found by inserting the result for b_{XP} in (4.17) or in (4.18) to (4.22). Within this formulation of the solution of the price side (4.18) to (4.22) may be interpreted as definitions of basic price indices for the different activity levels.

As noted in the beginning of this section we have treated the difference between activity level price (the elements of p_A) and commodity taxes, net, per unit of activity level (the elements of b_{AT}) for each activity as one variable denoted by b_{AX}. The solution of p_A, which is the ultimate result for the inner price side of the model, can therefore first be found after the submodel for indirect taxes have been solved. This will be further elaborated upon in sections 4.3 and 6.2.

For the price side to be uniquely determined it is required that the vector b_Z is of dimension equal to the number of production sectors, and that $[\Pi_{b_{AP}}(P_P' + E_P')\Pi_{b_{XP}} - \Pi_{b_Z}]$ is a non-singular matrix. We shall say that the coefficient matrices $\Pi_{b_{XP}}$ and Π_{b_Z} are imposed on the inner

---

1) The reasons why indirect taxes and subsidies enter the unit primary basic costs are explained in section 5.6. Note, however, that only the rates enter. 2) Note that b^*_{XE} does not enter in the determination of b_{XP}. This is, of course, due to the separation of the domestic and export markets in the cost calculation.

model by the outer model as a general reduced form expression. The precise content and interpretation of b_Z^* and the matrices $\Pi_{b_{XP}}$ and Π_{b_Z} will depend upon the outer model. The non-singularity condition will similarly depend upon the actual content given to b_Z^*, $\Pi_{b_{XP}}$ and Π_{b_Z}. Singularity will normally indicate a faulty logic in the specification of $\Pi_{b_{XP}}$ and Π_{b_Z}.

In diagrams 4.3 and 4.4 the structure of the price side of MODIS IV has been given a pictorial representation. Diagram 4.3 shows how the given variables and coefficients of the subparts of the inner price model are determined. In diagram 4.4 the solution of the inner price model in successive stages are indicated.

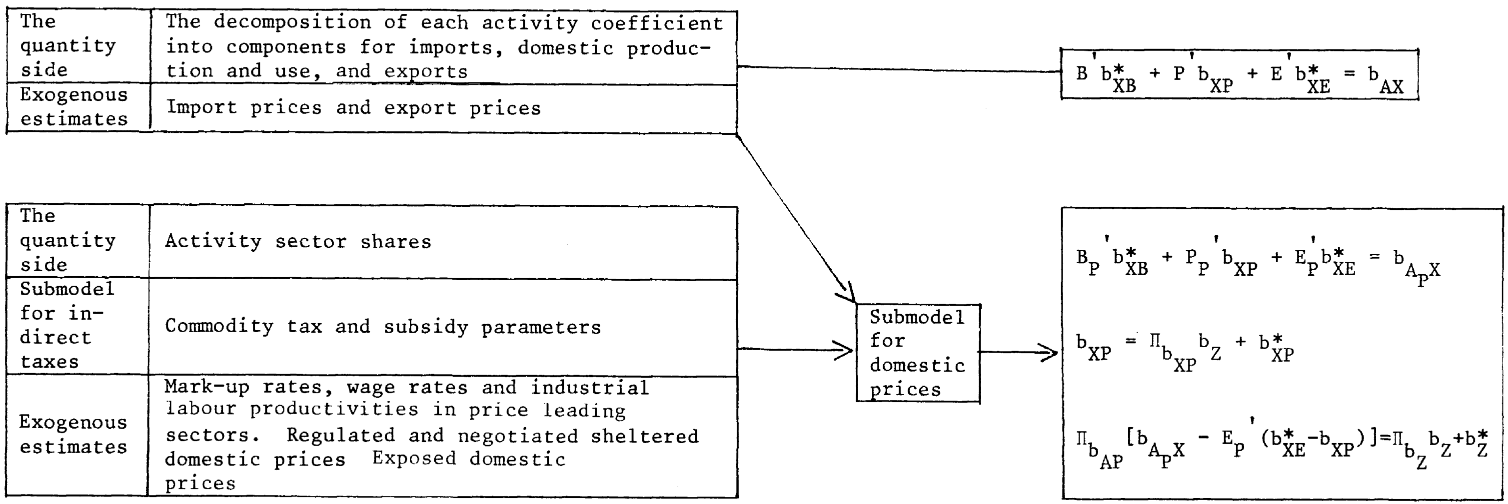

Diagram 4.3. The generation of the inner price side of MODIS IV

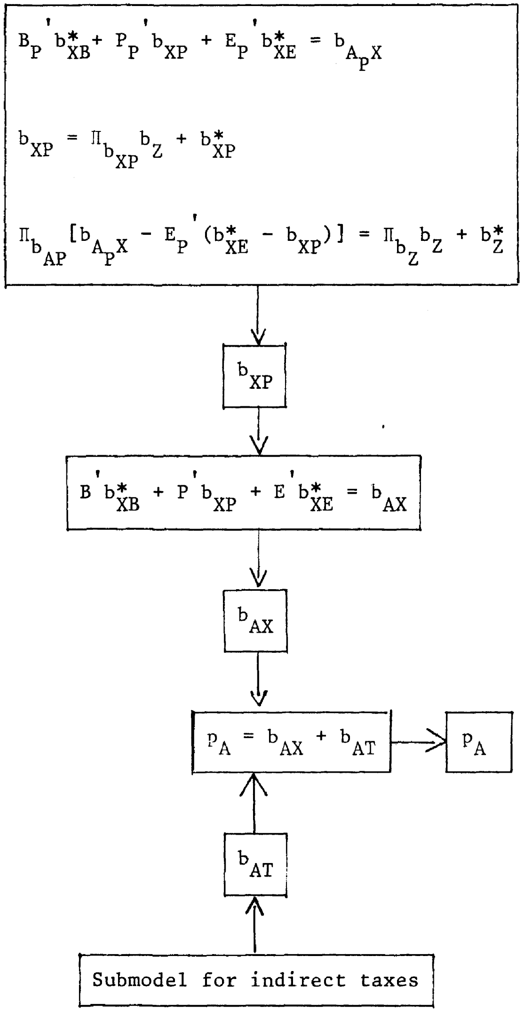

Diagram 4.4. The solution of the inner price side

4.3. The solution of the inner price and quantity models in successive stages

The inner price model of MODIS IV has the appearance of a dual of the inner quantity model. The duality is, however, somewhat superficial. There is not very much left of the intrinsic duality between price and quantity relations of the simple traditional input-output model. This duality as set out in section 3.1 was the basis for the separability of the price and quantity solutions as used for instance in MODIS II and III where the price side was solved prior to the quantity side. In those models (sector) prices were either exogenous or determined by cost considerations, independent of final demand. Given these horisontal supply curves the demand curves determined the quantities produced.

In MODIS IV, however, the fully consistent solution of the price side, and thereby also the calculation of the current values of the components of value added like wages, profits etc. by production sector, can be arrived at only after the quantities have been determined. This is due to three major deviations from the formulation of the simple input-output model:

First of all, the use of market prices for production activity levels, denoted by the elements of p_A (see (3.12) and (4.12)), makes the solution for these prices dependent upon the quantity solution whenever there are different commodity tax rates among the purchasers of a commocity. Commodity taxes on output from production activities per unit of activity level and, thereby in general the b_{AT} vector, are therefore dependent upon the quantity solution. We shall return to this in section 6.2.

Secondly, the differentiation between import, domestic and export price indices makes both the market and basic prices of activity levels dependent upon the quantity solution whenever these price indices for the same commodity are different (see (4.16) and (4.17)). However, the solution of the submodel for basic domestic commodity prices is only dependent upon the distribution between import and domestic supply of each commodity which appear as material input in the production sectors (see (4.26)).

Thirdly, the sector calculation approach in the submodel for domestic prices, which includes an activity sector share matrix $\Pi_{b_{AP}}$, makes the solution of this model dependent also on the activity distribution within each production sector, i.e. upon the quantity solution.

Altogether, this means that both the commodity prices and the activity prices, to a greater or lesser degree, are dependent upon the quantity solution. On the other hand, the solution of the quantity side is - via the submodel for household consumption - dependent upon consumption prices (household consumption activity prices p_{A_C}) which are assumed in the submodel to be given. Besides profit relations, with profit margins which are dependent upon the price solution, are also included in the consumption submodel. This is further discussed in sections 5.3 and 5.4.

There is thus from a formal point of view more simultaneity in MODIS IV than in the traditional input-output model. However, due to a few simplifications, this fact does not significantly impede the solution of the model in successive stages. In the present version of the model the submodel for domestic commodity prices (given by the equations (4.19), (4.23) and (4.25)) is solved prior to the quantity side by assuming that (i) the production activity sector shares represented by the elements of $\Pi_{b_{AP}}$, and (ii) the import and domestic supply distribution of each input commodity, indicated by the elements of B_P and $(E_P + P_P)$, are predetermined in each model computation.[1] These matrices appear in the submodel with a time lag of one year, which means that these weights in the production cost calculation are dated one year behind the computational year.[2] We believe that this simplification has only minor influence on the model result and it enables us to solve the model in successive stages. It may also be argued that historic weights are those which are actually applied in the cost calcualtions of the price leaders.

Since the submodel for domestic prices is solved independently of final demand, the supply curves, with basic commodity prices as arguments, are horisontal. However, the demand curves derived from the quantity side have market prices for household activities and basic prices for production activities as arguments. The household consumption activity prices enter explicitly as argument in the submodel for household consumption while the production activity prices enter indirectly through the coefficients of the profit relations. Prior to the solution of the quantity side, preliminary estimates for these activity prices

1) Only the sum of P_P and E_P enters (4.26) because of the separation of the domestic and export markets in the cost calculation. 2) At present we are actually simplifying further by using the base year weights.

are made by means of (4.19) and (4.20). These preliminary estimates come for several reasons very close to the final solution. For the household consumption activity prices p_{A_C}, which are the definitely most important prices for the quantity solution, the only influence from the quantity side is the distribution between import and domestic supply of each commodity. The matrices B_C and P_C representing the supply distribution are present both explicitly and implicitly (through $b_{A_C T}$) in equation (4.20). However, as will be further discussed in section 5.5 most of the changes in the supply distribution are handled through exogenous changes in the important shares. A very close approximation to the final p_{A_C} estimate may therefore be made simply by correcting the supply distribution in the previous year by means of the exogenous import share adjustments.

In the preliminary estimates for the production activity basic prices $b_{A_P X}$ the distribution between deliveries to the domestic and the export markets enters in addition to the import-domestic supply distribution.[1)] These distributions are represented by the elements of B_P, P_P and E_P in equation (4.19). The distribution between import and domestic supply are handled in the same way as discussed above for the consumption activity prices while the proportion between export and domestic deliveries for each commodity is regarded as given from the previous year in these preliminary computations. The effect of this last simplification is negligible since most of the profits that enters into the personal income relations of the submodel for household consumption are generated in domestically oriented production sectors (see the discussion in sector 5.4).[2)]

The fully consistent solution

After the submodel for domestic prices and the quantity side are solved the solution of the price side can be made by means of the price equation (4.17) (or (4.18) to (4.22)). The estimation of the elements of b_{AT}, which is necessary for the computation of p_A, can also be performed after the quantities are solved for. This is further discussed in section 6.2.

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1) Note that indirect taxes are not involved here. 2) Some minor problems in these preliminary computations for the profit relations are suppressed here, e.g. those connected with depreciation and activity sector shares. All details can be found in Longva (ed.) (1975), pp. 15-22.

So far we have not discussed how the market distributed activity coefficient matrices, B, P and E, are solved. B, P and E are defined in such a way that the conditions

(4.27) $BA = 0$,

(4.28) $PA = X$,

(4.29) $EA = 0$, and (4.16), i.e. $\Lambda = B + P + E$, are fulfilled.[1)]

The elements in B, P and E are determined by adjusting the corresponding base year coefficients by means of adjustment factors, one for each row (commodity) in each of the three matrices. The economic interpretation of these adjustment factors is that changes have occurred in the distribution between import and domestic production on the supply side and between export and domestic demand on the demand side for each commodity. The adjustment factors are estimated by means of (4.16), (4.27)-(4.29) after the quantities, i.e. the activity levels A, are solved for.[2)]

The decomposition of the activity coefficient matrix, Λ, on the price side of the model does not interfere with the overall equilibrium condition, as set out in section 3.4, as long as (4.27)-(4.29) hold. A more complete discussion of consistency problems in connexion with the solution of the inner price model is given in Longva and Tveitereid (1975).

The elements of B, P and E for the base year cannot directly be derived from the base year national accounts since the distribution by delivering and receiving sectors is not identified for each commodity flow in the commodity-by-industry output table (see diagrams 2.2 and 2.3). The base year elements of B are derived from an estimated matrix of market shares where each market share is defined as the import share of the demand for a given commodity in a given purchasing activity. The estimation of

1) In the consistency conditions as they are presented here it is implicitly assumed that all net additions to stocks are valued at domestic prices only. However, in principle all three price types may be applied. In the present version of MODIS IV we are distinguishing between domestically produced and imported net additions to stocks (see section 5.1). The actually used consistency equations include this modification. 2) The adjustment factors of B for the commodities with market share determined imports will coincide with the exogenous changes in the market shares which are introduced in the submodel for imports (see section 5.5).

this base year import market share matrix is discussed in section 5.5. The base year elements of E are derived from the assumption that, except for some minor reexport items, all exports are supplied by the production sectors and exports by commodity are distributed among the supplying production sectors in proportion with their market shares of each commodity. The elements of P are determined by equation (4.16) after the elements of B and E have been estimated.

In diagram 4.5 the solution of the price and quantity sides in successive stages has been given a pictorial representation. The indicated links to and from the submodels for direct and indirect taxes are discussed in sections 6.1 and 6.2. After the consistent solution of the price and quantity sides, several "post calculations" are usually made, especially for the components of value added and for government consumption. This is dealt with in section 5.7.

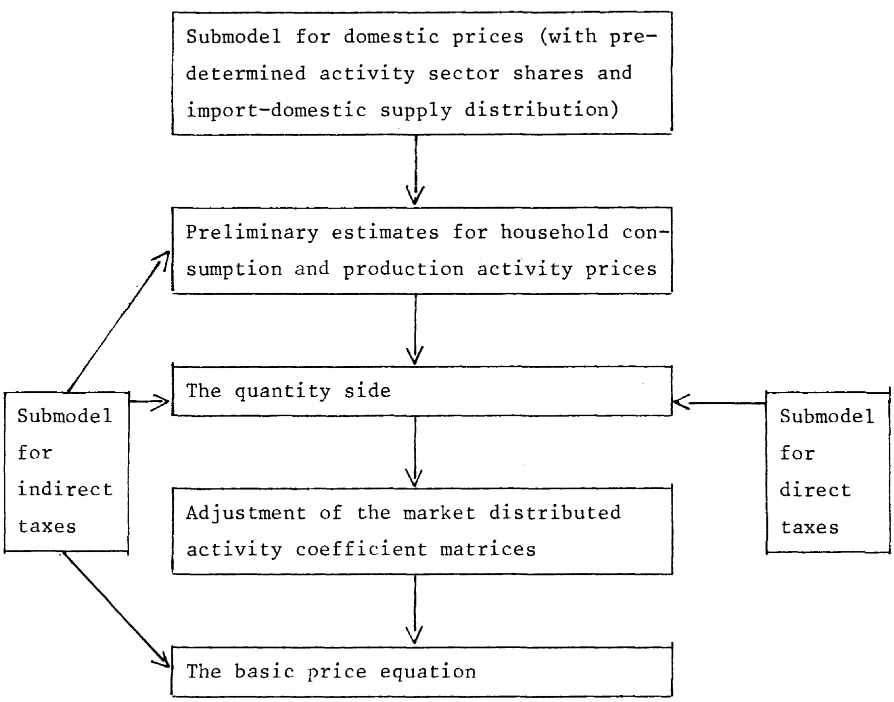

Diagram 4.5. The solution of the price and quantity sides in successive stages

5. SUBMODELS AND EXOGENOUS VARIABLES

In this chapter the submodels and exogenous variables introduced in chapter 4 are discussed in more detail.[1] Much of the interpretation and operational meaning of the inner model as it is presented above is provided through these submodels and exogenous variables.

Some of the exogenous variables may be regarded as surrogates for missing submodels (behavioural relationships) which have so far not been formally included in the model. In such cases the model user must adjust the estimates for the exogenous variables in such a way that the model results are in accordance with the relationships assumed by the user. The actual accomplishment of this procedure cannot be fully understood without some knowledge of the model environment and the administrative use of the model. This will be elaborated upon in chapter 7.

Another group of exogenous variables are exogenous in the usual sense; they represent non-controlled external forces influencing the economy as it is portrayed in the model.

A third group of exogenous variables are the policy instruments. In MODIS IV they are found in particular within the submodels for fiscal budgeting. Other policy instruments may, however, indirectly enter the model through for instance exogenous estimates of investment, domestic prices etc.

5.1. Investments, exports and net additions to stocks

Investments

The relations determining the levels of investments activities are given in (4.9) as

$$(5.1) \qquad A_I = \Pi_{A_I} Z + A_I^*$$

Since all gross investments in fixed capital are assumed to be exogenously given the matrix Π_{A_I} is zero while A_I^* contains the exogenous estimates. The estimates of gross investment in A_I^* are actually aggregates of exogenously given levels for capital formation activities, measured in constant market values.

1) The submodels for direct and indirect taxes are dealt with in chapter 6 in the context of fiscal budgeting.

The exogenous estimates of gross investments constitute one of the major open loops in MODIS IV. The development of an investment model within the MODIS framework is discussed in Amundsen and Biørn (1975), see also Biørn (1979). It should, however, be noted that the greater part of gross investments in Norway is either under direct or indirect control by the government, such as gross investments in general government, in oil activities and in dwellings and transportation, or determined mainly by external developments like gross investments in shipping.[1)] Gross investments in manufacturing industries, which also include government owned enterprises, constitute only a small fraction of total gross investments. For the one year projections estimates given by the Ministry of Finance for investments in manufacturing industries are to some extent based upon anticipation data, collected on a quarterly basis.[2)]

Various credit market regulations form a group of very important policy instruments in Norway. These instruments are not explicitly included in MODIS IV but are underlying the exogenous estimates in particular of some items of gross investments. A model developed by the Bank of Norway, called KRØSUS, is used by the Ministry of Finance to study the impact of these credit policy instruments.[3)] Among the exogenous variables of KRØSUS both exogenous and endogenous variables of MODIS IV are represented. The most important endogenous variables of MODIS IV which are treated as exogenous in KRØSUS are household consumption and industry profits. Since gross investments are completely exogenous in MODIS IV and partly endogenous in KRØSUS iterative runs of MODIS and KRØSUS are necessary to achieve a consistent solution.

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1) Gross investments in general government sectors are further discussed in section 6.3 in the context of fiscal budgeting. 2) In Andreassen (1969) these data are used in the estimation of investment demand functions for manufacturing industries. In addition to anticipated investment, production and profits are included as explanatory variables. 3) In KRØSUS the links between the banking sectors and an aggregated representation of the other institutional income and capital finance sectors are spelt out in some detail (see diagram 2.3 of chapter 2). The model includes a very disaggregated representation of the banking sectors and of financial assets and liabilities, i.e. the credit markets, and all major credit policy instruments are specified. The core of the model is a credit multiplier process determining bank loans and deposits endogenously. In addition the model includes an investment demand relation for gross investments in industries, except the oil, shipping and dwellings sectors. This investment demand is assumed to depend upon internal savings and depreciation allowances, supply of credit and anticipated investment, see Bank of Norway (1978).

Exports

The relations for determining the levels of exports activities are given in (4.10) as

$$(5.2) \quad A_E = \Pi_{A_E} Z + A_E^*$$

Since all exports are assumed to be exogenously given Π_{A_E} is a zero matrix while A_E^* contains the exogenous estimates. These estimates are given in constant market values.

Exports are of central importance in an open economy like the Norwegian. So far no complete formal support model for the fixation of the exogenous export estimates has been developed. Instead the Ministry of Finance relies on expert assessments both about the world market development (the demand side) and about the export oriented industries (the supply side) as a basis for exogenous export estimates.[1)]

Net additions to stocks

The relations for determining the net additions to stocks are given in (4.5) as

$$(5.3) \quad X = -\Pi_X Z + X^*$$

There is a distinction in the model between imported and domestically produced net additions to stocks. This is taken care of by two equations which add up to (5.3), one for imported and one for domestically produced net additions to stocks; these equations have the form

1) A submodel for manufactured goods, called MODEX, which may be viewed as a first stage in the development of a support model for the foreign sector of MODIS, has been developed (see Frenger, Jansen and Reymert (1979)). In the model the volume and price of Norwegian exports and the price of Norwegian imports are determined from variables representing costs (unit wage costs), domestic price levels, exchange rates, customs duties and production levels (GNP) of 14 OECD countries. The model has some similarities with the world trade model described in Samuelson (1973) and is separated in a price and a quantity part. The price part is a simultaneous system of price equations in which the export price of each country, including Norway, is determined as a function of an index of production costs and a competitive price which is a doubly weighted sum of all export prices. Changes in the exchange rates and customs duties for each country are also included. In the quantity part the volume of imports of each country is determined as a function of the GNP and the ratio of the domestic price level to an import price index. The import volumes are again weighted and summed to determine the size of the Norwegian export market. This together with the ratio of the Norwegian export price index and the competitive price index for Norwegian exports, exchange rates and customs duties determine the volume of Norwegian exports.

$$(5.4) \quad X_N = -\Pi_{X_N} Z + X_N^*$$

$$(5.5) \quad X_B = -\Pi_{X_B} Z + X_B^*$$

where X_N and X_B are net additions to stocks of domestically produced and imported goods, respectively. All imported and most domestically produced additions to stocks are considered as exogenously given in the model. The exogenous estimates are given in X_N^* and X_B^*. The matrix Π_{X_B} is identically zero, while Π_{X_N} has unit coefficients for those commodities for which the domestic main producers have exogenously given levels. These commodities consists mainly of primary industry products such as milk and dairy products, meat and fish. These additions to stocks are determined as residuals in the balance equations for the respective commodities. The net additions to stocks are thus largely exogenously determined. For short term planning purposes this obviously is quite unsatisfactory. The poor data base for commodity stocks in Norway is one of the reasons why attempts have not been made to model this highly fluctuating component of demand.

5.2. Production and employment

Production

The production structure of the model is characterized by the constant coefficients of the production part of the activity matrix Λ. The content of the submatrix Λ_P has been discussed above in section 3.2 and in section 4.1. The separation into production activities does, of course, play a very central role in the technological specification of the commodity flows of the economy (cp. (4.3)). The production activity levels for given commodity demand are not determined from the technological structure alone but also from the assumptions underlying relation (4.4) which we somewhat loosely have referred to as "the market structure". The production activity part of (4.4) can be written, cp. (4.7), as

$$(5.6) \quad A_P = \Pi_{A_P} Z + A_P^*$$

The matrix Π_{A_P} is, as discussed in section 4.1, essentially a matrix of market shares. However, some production activities are exogenous in the model. The exogenous estimates are given in A_P^* and the corresponding rows in Π_{A_P} have therefore zero-elements only. The exogenous production activities comprise all activities for commodity absorption in general government production and some industry production activities.[1] Among the latter are activities in primary industries like agriculture, forestry, fishing, crude oil and natural gas. Also included here are electricity supply and pipeline transport and refining of petroleum.[2] The exogenous estimates of commodity absorption in general government production are actually given in the same specification as the external activities for general government consumption (see section 3.2), but afterwards aggregated to activity levels for absorption of commodities in general government production. While the exogenous estimates of commodity absorption in general government can be linked to government budgets and thereby viewed as policy instruments,[3] the exogenous estimates of the industry production activities are based upon information about production capacities and utilization rates.

As stated in section 4.1, the quantity model is mainly demand oriented in the sense that total demand by commodity is determined through exogenous estimates and demand relationships. Except for the exogenous production activities, the market share model will determine the activity levels for production activities by distributing total demand for each commodity among its main producers. The rationale for this model was discussed rather extensively in section 4.1. The market share matrix Π_{A_P} imposes constant proportions between the main products of identical commodities from different activities. The constant market share estimates are, as a rule, taken from observed values in the base year. The market share coefficients may be changed as part of the model computations either exogenously or by means of relations connecting market shares with other variables of the model. Up to the present, however, the base year market shares have been adhered to.

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1) Note that the activity levels for marketed government services are endogenous. It follows that general government consumption is endogenous (see section 5.7). 2) For agriculture, shipping, shipbuilding, oilrig building and for various activities connected with the production of crude petroleum inputs and outputs of commodities have been specified in separate activities to account for the fact that the proportion between total input and total output may be highly varying. The "input activities" are all exogenous. 3) See the further discussion in section 6.3.

It should be noted that the assumption of constant market shares is imposed only for domestic production of commodities. It might have been quite easily extended to include imported commodities as well. This would have meant much simpler import relations than those actually specified in the model, but presumably less valid ones. The submodel for imports is presented in section 5.5.

Employment

The modelling of employment in MODIS IV is at present rather crude and provisional. The demand for labour is simply linked to the production activity levels through exogenous estimates of productivity change.

As explained in section 3.2 the elements of the vector A_P of production activity levels can be interpreted as measures of primary inputs (value added) by production activities. Value added production functions in inverted form are used in the model as labour requirement functions. The labour requirement functions are written as

$$(5.7) \qquad N_W = \hat{Q}_W S_P = \hat{Q}_W \Sigma_P A_P$$

where N_W is a vector of employment by production sector,

Q_W is a vector giving the number of wage and salary earners per unit of value added (inverse labour productivities),

S_P is a vector of value added by production sector (production sector levels), and

Σ_P is an aggregation matrix which adds up production activity levels to production sector levels.

In equation (5.7) total employment by sector is written as a (linear) function of total primary input in each production sector, i.e. we assume separability between labour input and total value added by sector. As discussed in section 3.2 we are in general assuming non-jointness in the production structure. This means that labour input can be allocated between the different activities within each sector. The combination of the restrictions of separability and non-jointness in a technology with constant returns to scale is only possible if the individual production functions in such a technology are identical except for a scalar multiple (see Hall (1973)). This implies that (5.7) is based upon the assumption that the labour share of value added is the same for all activities within the same sector. In other words we are assuming sector technology for value added by activity.

The estimates for labour productivity can be derived from an assumption about cost minimization for given total primary input by sector. Given the functional form and the parameters of the value added production functions, the wage rates and the user cost of capital, then the labour productivities follow. In addition, cyclical changes in labour productivities must be taken into consideration. However, the labour requirement model as it is here envisaged, remains in the unformalized part of the "outer model". At the present stage in the development of the MODIS model, the labour productivities are exogenously given, usually in the form of percentage changes from the preceding year.[1)]

In some sectors the number of wage and salary earners rather than productivities are exogenously given. We can thus, in general, write

$$(5.8) \qquad N_W = \hat{Q}^*_W \, \Sigma_P A_P + N^*_W \, ,$$

For sectors with exogenously given productivities the estimates are included in Q^*_W. For other sectors the numbers of wage and salary earners are given in N^*_W; the corresponding elements in Q^*_W are then zero. The sectors with exogenous employment include shipping, production of petroleum, and all general government production sectors; these are sectors where short-run productivity changes seem harder to assess than the number of employees. The estimates for the general government sectors can be linked to government budgets and can thereby be viewed

1) The multi-sectoral long term growth model MSG includes value added production functions by production sectors. In the present version of the MSG model, MSG-3, the sectors are aggregates of the MODIS sectors, see Johansen (1960), (1964), Lorentsen and Skoglund (1976). These value added production functions are represented by Cobb-Douglas functions in labour and capital and with Hicks-neutral technical change. In addition constant returns to scale is, in most cases, assumed. In the medium term (4-6) years programming process some efforts have been made by the Ministry of Finance to "exchange information" between the two models, for example for labour productivities. As part of an energy study, a new version of MSG has been developed, see Longva, Lorentsen and Olsen (1980). In this model Generalized Leontief cost functions describe the production structure with input prices for materials, energy, labour and capital as arguments, combined with Hicks-neutral technical change.

as policy instruments.[1)]

5.3. Household consumption

The household consumption part of (4.4) can be written, cp. (4.8), as

$$(5.9) \quad A_C = \Pi_{A_C} Z + A_C^*$$

This set of equations is a derived form of a somewhat more complete submodel for household consumption which belongs to the outer model. This derived form or, as we shall say, the projection of the more complete model into the quantity side of the inner model presents the activity level vector for household consumption as a sum of two terms, one term dependent upon the solution vector for commodity supply (Z) and one term considered as given in the quantity side of the inner model (A_C^*). We shall first present and discuss the submodel for household consumption in its own right and return towards the end of the section to the question of how the submodel is fitted into the model framework through the equation (5.9). The formal structure of the submodel is described in great detail in Longva (1975a). The theoretical content of the consumption model and estimation problems are discussed in Biørn (1972) and (1974a). A recent reestimation of the aggregate consumption function is presented in Cappelen (1978).

The concept of total consumption determined in the aggregate consumption function to be described below is not fully coinciding with total household consumption according to the national accounts. Household consumption in the national accounts includes some items which are

1) Aggregate unemployment appear in the model as the difference between labour force and total labour demand. In addition to wage and salary earners, total labour also includes self-employed. The number of self-employed by (industry) production sector is exogenously given in the model. Labour force is an exogenous variable and the estimate is mainly based upon demographical factors and participation rates by sex and age, see Hernæs, Ljones and Vannebo (1977). Even though unemployment is not explicitly included in any behavioural equation in the model, it does, of course, play a very important role in the overall assessment of the model results. An extremely low or negative unemployment rate is an indication that the general picture of the economy drawn by the model may be economically inconsistent, e.g. that total demand may be too high. Furthermore, the user of the model may draw upon unformalized or formalized ideas about the links between for instance wage drift and the unemployment rate when assessing the results (see section 5.6). Through an iterative process the user may build such "outside information" into the exogenous part of the model.

only in part paid for by private consumers. Two of the household consumption activities belong to this category, namely, "Medical expenditure" and "School fees". Following the UN System of National Accounts (United Nations (1968a) p. 103) households should be considered to be the purchasers of such services even though most of the actual costs are financed by government bodies, for instance by the government medical insurance scheme. In the household income accounts these government financed outlays are matched by government transfers. The activity levels of the two above mentioned activities are exogenous in the outer model at present. It is conceivable that they might be made endogenous in a future version of the model as functions of demographic factors, institutional rules etc. It is nevertheless necessary to keep them apart from the other household consumption activities for which personal real disposable income, with the relevant government transfers excluded, and relative prices are determining factors.

It has also been found necessary for a satisfactory treatment of household consumption to maintain a separation between foreigners' consumption and Norwegians' consumption of each consumption activity. The level of each consumption activity is a sum of foreigners' consumption and Norwegians' consumption. There are some external activities - one under household consumption and three under export - which effect a transfer of the total of foreigners' consumption from household consumption to exports. This treatment of foreigners' consumption is an almost direct consequence of the treatment in the national accounts.

The foreigners' consumption in Norway is taken as given in the inner model. In the outer model only the total of foreigners' consumption is at present exogenous while the composition is determined by a coefficient vector partly derived from a study of tourists' consumption.

The remaining part of household consumption to be dealt with in the proper submodel may then more precisely be called "personally financed Norwegians' consumption". In the following we shall try, however, to avoid this rather cumbersome term and simply use "total consumption" to refer to this total except towards the end of the section when we are putting the pieces together. We are here dealing with total consumption as a partial sum of activity levels <u>in base year</u> values and are thus concerned with a measure of the volume of consumption. The internal household consumption activities constitute a classification of consumer goods. In the following we shall refer mainly to (volume of) consumer goods rather than to activity levels.[1)]

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1) The volume of consumer goods is equal to the negative of the corresponding activity level, cp. chapter 3.

The submodel can be described as consisting of five parts:

(1) The aggregate consumption function determining total consumption expenditure in constant values as a function of real disposable personal income or, more precisely, as will be explained in section 5.4, as a function of real disposable consumption motivating income for three socio-economic groups or institutional private sectors.

(2) Distribution relations determining the allocation of the total expenditure between the various consumption goods.

(3) Auxiliary relations for ad hoc model user modifications of (1) and (2) by means of exogenous estimates for certain consumption items.

(4) The actual exogenous estimates of certain consumption items (cp. (3)).

(5) Adding-up constraints to ensure that the aggregate of consumer goods is consistent with total consumption as estimated from the aggregate consumption function.

These parts are discussed, one by one, in the following paragraphs.

The aggregate consumption function

The aggregate consumption function of MODIS IV has the form[1)]

$$(5.10)\quad \bar{C} = a_0 + a_{-1}\bar{C}_{-1} + a_{WE}\frac{\bar{V}_{WE}}{\bar{P}_C} + a_{ES}\frac{\bar{V}_{ES}}{\bar{P}_C} + a_{UP}\frac{\bar{V}_{UP}}{\bar{P}_C}$$

where $\bar{C}$ is total consumption in constant values (as defined in the text above),

$\bar{C}_{-1}$ is total consumption in the preceding year,

1) See Longva (1975a). The original estimates of the parameters of this relation and several alternatives can be found in Biørn (1972). Revised estimates are presented in Cappelen (1978). The present values of the estimates are: a_{-1}=0,4686, a_{WE}=0,5378, a_{ES}=0,2693 and a_{UP}=0,3273. The estimate for a_0 was 2 294 mill.kr i 1970-prices and is revalued for each change of price basis, i.e. for each new base year of the model.

$\bar{V}_{WE}$, $\bar{V}_{ES}$ and $\bar{V}_{UP}$ are disposable consumption motivating income for wage and salary earners (WE), self-employed (ES) and pensioners (UP), respectively, and

$\bar{P}_C$ is a Laspeyres price index for total consumption.

The aggregate consumption function with $\bar{C}_{-1}$ as an argument is a dynamic relation; this is the only dynamic element of importance in MODIS IV.

The disposable income variables are defined as

$$\text{(5.11)} \quad \bar{V}_i = \bar{G}_{Ci} - \bar{T}_i, \qquad i = \text{WE, ES, UP}$$

where $\bar{G}_{Ci}$ is consumption motivating income for socio-economic group i, and
$\bar{T}_i$ is direct taxes accrued on the incomes of socio-economic group i.

The income variables $\bar{G}_{Ci}$ used as arguments in the consumption function are very closely related to total income for the three socio-economic groups, the only deviation being the omission of some government transfers. We shall return to the underlying specifications and relations of the income variables in section 5.4 and of the direct taxes in section 6.1.

The distribution relations

The distribution relations consist of one relation for each consumption good. The relative change from the base year of per capita consumption of the good is a function of the changes in deflated total per capita expenditure and relative prices. For consumption good i the distribution relation is

$$\text{(5.12)} \quad \Delta c_i/c_i = E_i \frac{\Delta(y/\bar{p}_c)}{y/\bar{p}_c} + \sum_j e_{ij} \frac{\Delta(p_j/\bar{p}_c)}{p_j/\bar{p}_c}$$

where c_i is consumption good i per capita,

y is total expenditure (nominal) per capita,

p_i is the market price index for consumption good i,

$\bar{p}_c$ is a Laspeyres price index of the consumer goods with base year weights,

E_i is the expenditure (Engel) elasticity for good i, and

e_{ij} is the price (Cournot) elasticity of good i with respect to the price of good j.

The set of distribution relations given by (5.12) is not formulated as a complete system of demand functions in a strict sense but is rather the result of logarithmic differentiation of such a system in the base year, with nominal expenditure and absolute prices deflated by a consumption price index and with infinitesimal changes replaced by changes from the base year (cp. Biørn (1972), pp. 29-31).

A complete system of demand functions fullfils the formal requirements of homogeneity, symmetry and the adding-up property when the elasticities and the budget shares satisfy the following relations for all values of prices and total expenditure:

(5.13a) $\sum_j e_{ij} = -E_i$ for all i (homogeneity)

(5.13b) $\alpha_i e_{ij} + \alpha_i E_i \alpha_j = \alpha_j e_{ji} + \alpha_j E_j \alpha_i$ for all i and j (symmetry)

(5.13c) $\sum_i \alpha_i e_{ij} = -\alpha_j$ for all j (adding-up)

where $\alpha_i = p_i c_i / y$ is the budget share of good i.

Our set of distribution relations has constant elasticities. This is strictly at variance with the formal requirements of a complete system of demand functions. As long as the constant elasticities satisfy (5.13a), however, which they are constrained to do, the homogeneity property will be fulfilled. The elasticities of (5.12) also satisfy (5.13b) using base year budget shares. This means that the symmetry property is exactly fulfilled only in the base year. Finally the elasticities and base year budget shares fulfill (5.13c). This ensures that the consumption good changes measured in base year values add up to the value of the change in deflated expenditure, i.e.

(5.14) $\sum_i p_i \Delta c_i = \bar{p}_c \Delta (y/\bar{p}_c)$

The expenditure elasticities have been estimated by combining information from two different sets of data. National accounts data for the consumption activity levels of the model, which are aggregated from more detailed specifications in the accounts, have been used in conjunction with consumer survey data from a comprehensive Norwegian household budget survey. The actual values of the expenditure elasticities have been arrived at more or less discretionary by a comparison of regression results in the two sets of data, giving time series estimates the greatest weight.[1] For some goods the time series estimates are obviously strongly biased. In those cases the cross-section data were the main source for elasticity estimates.

The price elasticities are calculated from the expenditure elasticities using a method suggested by Frisch (1959) under the assumption of want independence.[2] The general idea behind the whole approach of the distribution relations as given in (5.12) is that of per capita demand represented by a typical consumer with demand functions presumably derived from an underlying utility function. The additional assumption of want independence means that the utility function which is assumed to be a static functions of c_i's only, can be written as

$$(5.15)\quad u = \sum_i u_i(c_i)$$

The utility of a bundle of goods can be written as a sum of the utility of each good. There is in other words no interdependence between the goods in the utilities derived from them.[3]

Under want independence all price elasticities can be expressed as a function of expenditure elasticities, budget shares and the income flexibility of the marginal utility of total expenditure (ω) according to the formula[4]

$$(5.16)\quad e_{ij} = \frac{E_i}{\omega}(\delta_{ij} - \alpha_j E_j) - \alpha_j E_i \qquad i,j = \text{all consumer goods}$$

where $\delta_{ij} = 0 (i \neq j)$ and $\delta_{ii} = 1$.

1) The time series estimation procedure is very nearly the same as in Amundsen (1963) whose results were used in MODIS II and III. 2) The original paper, in Norwegian, on which Frisch's 1959 paper was based can be found in Frisch (1946). 3) The definitions and discussion of want independence above may seem to have been phrased in a cardinalistic language. However, "want independence" can be defined as an ordinal concept (although Frisch adhered to a cardinal interpretation) and the definition would go like this: A utility function has the property of "want independence" if and only if there exists increasing transformation of the utility function which makes it additive, like in (5.15). When the utility function is not additive, but yet expresses "want independence", the parameter ω referred to below should be interpreted as a general measure of the curvature of the indifference hyperface rather than as the income flexibility of the marginal utility of total expenditure. 4) Cp. Frisch (1959), p. 186-187, Biørn (1972), p. 31 and Amundsen (1963), p. 35. The formula given by Amundsen has a misprint.

At each change of base year the expenditure elasticities are usually adjusted proportionally to fulfill the following condition in the base year (easily derivable from (5.13)):

$$(5.17) \quad \sum_i \alpha_i E_i = 1$$

For any given value of the parameter ω (assumed negative) the price elasticities determined by (5.16) will satisfy (5.13a)-(5.13c). At each change of base year, i.e. in practice at least once a year, the expenditure elasticities are recalculated.

The parameter ω referred to above by its interpretation as the income flexibility of the marginal utility of total expenditure is assumed to have the value of -2. This parameter serves as a general measure of substitution propensity in consumption. High negative ω means low price sensitivity on the part of consumers and vice versa. The assumed value of -2 is derived from various attempts to assess the value of the parameter.[1)] Complete want independence cannot be considered as a very realistic assumption in view of the fairly detailed specification of consumption goods. Some modifications of this hypothesis have been dealt with in Biørn (1972).

Exogenous estimates

The user of the model has the option of modification of the submodel for household consumption. The reason why such flexibility has been provided is threefold. The user may decide at the outset that for some consumption good total consumption and relative prices are not nearly sufficient as factors of explanation, and that an exogenous assessment is preferable. Secondly, in some situations the user may possess information he judges to have considerable impact on the residual term of the aggregate consumption function. For short term estimates exogenous corrections of the residual term have been allowed. On the other hand such corrections are difficult to make in a fully consistent way and requires a thorough comparison of the present situation with the estimation period. Actual instances have been expiration of tax-free saving contracts and precipitated purchases prior to increased indirect taxes. Finally, there may for a certain use of the model be required that the consumption good variables have given values. The options for

1) See Biørn (1974b).

modification of the submodel may thus be used to put the submodel out of operation, for instance in a setting with target values for all consumption items.

The following options are available for the user: He may change the constant term of the aggregate consumption function by a specific amount. He may fill in a correction item for any of the consumption goods with a further specification as to whether the correction shall replace, or be an addition to, the outcome of the distribution relations. The user may also indicate whether correction items for individual goods shall be included in, or added to, the total consumption from the aggregate consumption function. Individual goods may furthermore be kept out of the adjustment of the adding-up constraint.

The adding-up constraint

The total consumption as estimated from (5.10) when divided by the mean population is taken to be the deflated total expenditure of (5.12). When the model is used for projections more than one year ahead from the base year the relation (5.12) is applied to changes from the base year rather than to annual changes. This implies that the annual changes have constant Engel and Cournot derivatives rather than constant elasticities. In principle, the adding-up property satisfied by the distribution relations should imply that the values of the consumption goods inflated by the mean population add up to total consumption, cf. (5.14). This will not usually be the case, however, for two reasons. The elasticities may have been incompletely adjusted at the change of base year, so that the adding-up property is incompletely fulfilled in the base year. The other reason is that the user choice with regard to exogenous estimates may upset the adding-up property of the elasticities.

The adding-up constraint is essentially of the form

$$(5.18) \quad \bar{C} = e'C$$

where $\bar{C}$ is total consumption as defined above, and

C is a vector of consumption demand, $C_i = c_i N$, N being the mean population.

This constraint is met by adjusting the (unadjusted) base year values of all goods proportionately with the marginal budget shares to add up to total consumption from the aggregate consumption function. The actual formulation of the adding-up constraint is a rather cumbersome expression due to the various options for user choice described above.

As can be seen from (5.12) the unadjusted household demand for the consumption goods are linear functions of total consumption (with the relative prices included in the constant term). Using the adding-up constraint the adjusted levels of consumption demand can still be expressed as linear functions of total consumption.

The complete model

The derived form of the distribution relations, referred to earlier, can be written as

$$(5.19) \quad C = F_1 + F_2\bar{C}$$

where F_1 and F_2 are matrices of composite parameters.[1)] In F_1 is included the effect both of relative prices and of population change. Together with (5.10) this is the core of the submodel for household consumption.

Equations (5.19) and (5.10) imply a two stage approach to the consumer's allocation problem. It is assumed here that the consumers first determine the amount of their real disposable income they are going to consume and then decide the allocation of this total over consumption goods.[2)]

The income variables of (5.11) are defined and discussed in sections 5.4 and 6.1. For our purpose in this section it is sufficient to note that disposable income for wage and salary earners (mostly wages) as well as for the self-employed (mostly profits) is linearly related to the sector level of the production sector of income origin. Certain components of the income variables will thus be proportional with sector levels. Since sector levels are aggregates of activity levels the income variables can be related to the vector of auxiliary variables for commodity supplies Z via the submodel for production. Combining this with

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1) Compare Longva (1975a), pp. 17-40, in particular equation (5.107), p. 35. 2) This is admittedly a debatable assumption. It seems preferable, as argued by Biørn (1972), to start out with a general dynamic theory of consumer behaviour in which consumption and saving decisions are simultaneously determined. The household consumption as defined in the national accounts may prove to be rather deficient for such a generalization, as noted by Biørn (1972), because the national accounts data refer to purchases of consumption goods rather than to actual consumption. The two stage approach thus may be viewed as a major simplification compared with a more comprehensive model of consumer behaviour.

(5.10) and (5.19) one finds that the levels of household demand are linear functions of Z. In this way one arrives, finally, at (5.9) where household consumption is represented as activity levels together with government financed items of household consumption. The matrix, Π_{A_C}, of (5.9) is a rather complicated expression of input-output coefficients, various parameters and variables. All variables included in Π_{A_C} are predetermined (at least preliminary) at the stage of solution of the quantity side. The given term, A_C^*, of (5.9) includes the exogenous estimates but also that part of the endogenous consumption which does not depend upon the solution for Z. Most of the pensioners' and government employees' consumption are for instance included in A_C^* and so is the shift in composition due to relative prices.

5.4. Personal incomes

As seen from diagram 2.3 in chapter 2 we distinguish in the national accounts and in the model between income categories, such as wages and operating surplus, and income by institutional sectors. In diagram 2.3 only the three main institutional sector groups are listed, namely the private, the government and the foreign sector group. The institutional sectors are actually far more detailed specified; the private sector is thus decomposed into (i) corporations, including non-financial enterprises, financial institutions and non-profit institutions, (ii) wage and salary earners, i.e. households with wages and salaries as their main source of income, (iii) owners of unincorporated enterprises (the self-employed), i.e. households with income from unincorporated enterprises as their main source of income, and (iv) pensioners, i.e. households with pensions as their main source of income.

The main division is between corporations and households (personal institutional sectors). Corporations have income only from operating surplus while households receive both wages, operating surplus and government transfers.[1] Household incomes are important for the determination of household consumption (see section 5.3. and below). On the other hand, in the present version of MODIS IV there are no formal relationships describing the influence of corporate income on other variables, notably

1) At present, mainly due to lack of available data, the model (and partly also the accounts) does not include property income variables (interest, dividends, rent) as income categories. Nor are stock account variables included. This obviously restricts the formulation of the consumption relations of section 5.3.

private investment. While the household consumption-income loop is closed the private investment-corporate income relationship is an open loop (see section 5.1).

Gross income by socio-economic group

Wages, operating surplus and government transfers are allocated among the three socio-economic household groups by means of a set of constant shares. The personal income relations for each socio-economic group can then be written as

$$(5.20)\quad \bar{G}_i = F'_{Wi}Y_W + F'_{Ei}Y_E + F'_{Ui}Y^*_U, \quad i = WE,ES,UP$$

where $\bar{G}_i$ is gross income for socio-economic group i,

Y_W, Y_E and Y^*_U are vectors of wages by production sector, operating surplus by production sector, and government transfers by kind, respectively, and

F_{Wi}, F_{Ei}, and F_{Ui} are distribution vectors giving the share of each element in the corresponding Y-vector allocated to socio-economic group i.

The F_{Wi} vectors distribute all wage income among the three socio-economic groups. The same applies to the F_{Ui} vectors and government transfers. The operating surplus is, however, distributed as entrepreneurial income between households and corporations, the shares going to each of the three socio-economic groups will therefore in general not add up to one.

The elements of the F-vectors are assumed to be constant, and they are estimated from the latest available income data.[1)] The coefficients have been found to be reasonably stable over time, possibly partly due to the disaggregated spesification of the coefficients, see Cappelen (1978). With regard to the distribution of operating surplus between corporations and households it should be noted that only a small proportion of operating surplus in manufacturing industries is distributed to households while the shares are fairly high in the primary and service industries.

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1) Government transfers are treated a bit more sophisticated in the current version of MODIS IV than indicated here.

Consumption motivating income

The personal income concepts introduced above are closely related to those of the national accounts. However, these income concepts are applied directly neither in the aggregate consumption function of the submodel for household consumption (see section 5.3) nor in the aggregate direct tax functions of the submodel for direct taxes (see section 6.1).

As indicated in section 5.3 consumption motivating incomes by socio-economic group, denoted by G_{Ci} (see (5.11)), differ from gross incomes as defined by (5.20) as government transfers directly related to government financed household consumption are excluded from the consumption motivating incomes. Formally, this is easily done by replacing F_{Ui} of (5.20) by an almost identical vector F_{CUi} with zero elements corresponding to these government transfers.[1] The relation for consumption motivating income $\bar{G}_{Ci}$ for each socio-economic group can then be written as

$$(5.21) \quad \bar{G}_{Ci} = F_{Wi}^{'}Y_W + F_{Ei}^{'}Y_E + F_{CUi}^{'}Y_U^*, \qquad i = WE,ES,UP$$

Gross taxable income

The personal income concepts of the national accounts are not directly suitable as approximations for gross taxable incomes in the aggregated direct tax functions of section 6.1, mainly because some government transfer items are not subject to taxation at present. In a similar way as for consumption motivating income the F_{Ui} vector of (5.20) is replaced by a vector F_{TUi} with zero elements for non-taxable government transfers.[2] The relation for gross taxable income $\bar{G}_{Ti}$ for each socio-economic group can then be written as

$$(5.22) \quad \bar{G}_{Ti} = F_{Wi}^{'}Y_W + F_{Ei}^{'}Y_E + F_{TUi}^{'}Y_U^*, \qquad i = WE,ES,UP.$$

Income categories

Both wages and operating surplus are current value concepts. The wage and profit relations discussed below are therefore formed by bringing together price relations from the price side and quantity relations from the quantity side. This means that the wage and profit relations do not exist as independent equations included in the

1) The F_{CUi} vectors will then not add to a unit vector. 2) The F_{TUi} vectors will not add to a unit vector.

structural specification of the model but in a derived form, i.e. as a stage in the solution procedure of the model.

The wage and profit relations as part of the quantity side of the model can be written as

(5.23) $Y_W = \Pi_W S_P + Y_W^*$

and

(5.24) $Y_E = \Pi_E S_P + Y_E^*$

respectively. In these equations each income vector is expressed as a sum of two terms, one term dependent upon the solution for production sector level (S_P) and one term considered as given in the quantity side of the model.

The wage relations (5.23) are particularly simple. As will be discussed in section 5.6 wage rates by production sector are exogenously given. By combining this with the labour requirement functions (5.7) of section 5.2 the wage relations can be expressed as (5.23) with all the elements of Π_W and Y_W^* directly derived from exogenous variables. The given term Y_W^* includes wages in sectors with exogenously given employment (mainly general government production sectors) while the exogenously given labour productivities and wage rates for the other sectors determine Π_W.

The parameters Π_E and Y_E^* of the profit relations (5.24) are formally much more complicated to derive than those of the wage relations. The basic argument is however fairly simple. As will be pointed out in section 5.7 operating surplus by production sector are determined as the difference between total value added in current market value and the sum of the value added components wages, employers' social security contributions, depreciation, commodity taxes and sector taxes. By inserting the relations for the various components of the definitional relationship stated above the profit relations (5.24) are derived.

Value added in current market value by production sector are given by the production activity market prices from the price side, the production activity sector share matrix[1] and the production sector levels. Wages and employers' contributions are given by (5.23)

1) To simplify the solution of the quantity side we assume this matrix to be predetermined in the computations. Tests have shown that the effect of this simplification is negligible for the use of the profit relations in the household consumption computations.

and the tax relation of section 6.1, respectively. The sector taxes are exogenously given while the derivation of the relation for commodity taxes are discussed in section 6.2.[1] The relations for depreciation are presented in section 5.7.

The matrix Π_E of (5.24) is a rather complicated expression including activity prices, wage rates, labour productivities and indirect tax rates. All variables included in Π_E are predetermined (at least preliminary) at the stage of solution of the quantity side. The given term Y_E^* of (5.24) is a function of a number variables including exogenous wages, sector taxes and depreciation related variables like base year capital stock, gross investments and gross investment activity prices. Details on the derivation of Π_E and Y_E^* are given in Longva (1975b).

Government transfer items, i.e. the elements of Y_U^*, are all exogenously given as an integrated part of the use of the model in fiscal budgeting (see section 6.3).

From (5.21), (5.23) and (5.24) it is easily seen that $\bar{G}_{Ci}$, as assumed in section 5.3, can be written as a linear function of the production sector levels.

5.5. Imports

The levels of the import activities are determined in the inner model. The relevant part of the market structure equation (4.4) is, as also given in (4.6),

$$(5.25)\quad A_B = \Pi_{A_B} Z + A_B^*$$

In section 4.1 the vector Z was given a preliminary interpretation as (simultaneously determined) total supply by commodity. This interpretation could have been maintained to a greater extent if the determination of imports had been based on global market shares. In such a case the elements of Π_{A_B} would have been the global market shares and A_B^* would have been zero.

1) By subtracting commodity taxes (see (3.12) and (3.14) in section 3.4), operating surplus by production sector can also be written as the difference between value added in current basic value and wages, employers' contributions, depreciation, not refunded commodity taxes on commodity inputs, and sector taxes. This simplifies the problem - without any loss of accuracy - since commodity taxes on commodity outputs, which are dependent upon the solution of the quantity side (see section 6.2) are then not included in the profit relations. The relations for not refunded commodity taxes are discussed in section 6.2, see also section 5.6.

The major part of the import activities, is, however, determined by demand differentiated market shares and not by global market shares. For each element in the commodity by activity input table there is a separate import share. That is to say, for each imported commodity there is no a priori global share of demand but a differentiated share for each receiving activity.

The reason for using the computationally more inconvenient differentiated shares rather than overall global shares is that the statistical data indicate clearly that for several imported commodities the market share is highly varying across receiving activities. The export activities naturally have very low import shares but also between and within the major groups of domestic demand the import shares of some commodities vary considerably. This is doubtlessly due to inhomogenous composition of domestically produced and imported commodities with regard to the underlying micro commodities, even with 200 commodities specified in the model.1)

The market shares are defined as a matrix of import shares of the demand for each commodity from each and every activity. The import shares thus constitute a commodity by activity matrix of full dimension. The market shares are estimated for the base year. When the base year is the preceding year, which normally is the case, this estimation is based on the latest set of final accounts, i.e. on production and trade statistics from the calendar year two years prior to the base year. These data are updated to fit with the base year totals. For the projection period the import shares are adjusted exogenously. For a given year in the projection period it is possible to apply an exogenous adjustment factor for each commodity, that is to say, the import shares for all receiving activities are adjusted proportionally.2)

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1) See Furunes (1978). 2) Instead of exogenous adjustments of the import shares for the projection periode, changes in these coefficients may be linked to other variables of the model, e.g. changes in relative prices of imported and domestically produced commodities. A submodel "explaining" the changes in the import shares has been developed and may later formally be included as an integrated part of MODIS IV (see Frenger (1979)). In this submodel it is assumed that each activity has a separable input technology with a constant return to scale CES input function for each commodity input where the inputs are domestic production and import of the commodity. In addition Hicks-neutral technical change is assumed. By combining this with cost minimization the changes in each import share will be a function of changes in relative price (the proportion between import and domestic price) and a trend. The parameter estimates of the model are based upon national accounts time series. A similar model is actually already applied in the updating of the import shares from the latest set of final accounts to the base year of MODIS IV.

Some import activities are not determined by means of market shares. These fall into two groups. One group consists of exogenous import activities. All of these are related either to shipping or to the exploration, production, and transportation of crude oil and gas. The other group is called residual import activities. These activities are supplying commodities for which there is no domestic production or the domestic production as main product is exogenous. The activity levels are thus determined as the excess of total demand over domestic production (if any). These activities are said to be residual import activities because they are determined as the residual term in the balance equations for the respective commodities.

The import activities are thus determined by three different sets of relations. All of these are brought together in (5.25) as explained below. The subdivision of import activities in three groups is coincidental with a subdivision of imported commodities. Some are demand determined by demand differentiated market shares, some are determined exogenously, and some are determined residually as defined above.

The use of demand differentiated import shares implies that the equation system becomes more complicated in presentation and understanding without really interfering with the solution. The demand differentiated import shares also affect the price side considerably as discussed in section 4.2. While (5.25) fitted nicely into the model with a global market share interpretation of Π_{A_B} the interpretation under the assumption of demand differentiated market shares is more subtle. Under this assumption the levels of import activities can be written as a set of homogenous linear equations of all other activity levels and net additions to stocks.[1] As these again can be expressed as functions of Z in the inner model, as stated in (4.7)-(4.10), the import activity levels can be written as (5.25) by substitution. (5.25) is thus a very compressed form of the submodel for imports.

The exogenous import activities will, of course, enter directly into the A_B^* term of (5.25). The levels for residual import activities will be identical with the excess demand as expressed by the corresponding elements of Z. All import activities can thus be expressed in the form (5.25). Details of the derivation of Π_{A_B} and A_B^* are given in Longva (1975a).

1) Note that the model distinguishes between imported and domestically produced net additions to stocks by commodity.

5.6. Commodity prices, wage rates and mark-up rates

In the preceding sections of this chapter submodels and exogenous variables connected with the quantity side of MODIS IV have been discussed. In this section the main submodel and exogenous variables on the price side will be presented.

The discussion of the inner price side in section 4.2 is rather formal and the economic content of the model is heavily dependent both upon the exact specification of the price formation structure, especially the classification of price taking and price leading production sectors, and upon the assumptions underlying the exogenous estimates, especially for unit primary costs (wage and mark-up rates). From a methodological point of view it may be argued that the critical part of the formulation of the price side of the present version of the model lies in the classification of endogenous and exogenous variables and in distinguishing correctly the type of disaggregation that is needed for this classification.

In the first part of this section a rather general discussion of the assumed cost and price formation process will be given. In the second part the only formalized submodel on the price side of MODIS IV, the submodel for domestic commodity prices, is presented and discussed.

General features of the price and cost formation

The main ideas of the price formation of MODIS IV are the same as those of MODIS II (see Øien (1966) and Sevaldson (1968)), MODIS III (see Bjerkholt (1968)) and PRIM (see Holte (1968) and Aukrust (1970)). These models contained the first explicit formulations of what has later been known as the Aukrust or the Scandinavian model of inflation. A general discussion and a critical examination of its analytical power is given in Aukrust (1977) where also other references are included.

Compared with its predecessors MODIS IV contains some new elements like the commodity-activity-sector approach and the distinction between export, import, and domestic price for the same commodity. The changes in the price formation structure of MODIS IV are mostly a consequence of these new model specifications, but also some new elements are included.

In MODIS IV all domestic sectors are assumed to be price takers for prices of imported and exported commodities. The export and import prices are assumed to be exogenously given based upon independent forecasts for world market prices and assumptions about the development in exchange rates.[1] As mentioned in section 4.2 it is assumed that domestic cost conditions have no direct influence on these prices.[2]

Reflecting the openness of the Norwegian economy a fundamental distinction is drawn between the exposed and the sheltered domestic commodity markets. In the exposed markets commodities are sold under strong foreign competition. In the sheltered markets commodities are marketed under conditions such as to leave them relatively sheltered from foreign competition, either because of the physical nature of their products or because of government protection. As pointed out by Aukrust (1977) this does not mean that the producers of these commodities do not compete on prices amongst themselves but that prices may be adjusted to domestic costs without having to fear a loss of markets to foreign producers. For short the domestic prices in the exposed and sheltered markets are referred to as exposed and sheltered prices.

Since the predecessors of MODIS IV were based on sector flows and not on commodity flows the distinction there was between sheltered and exposed production sectors. The two ways of classifying are, however, rather similar. The classification of commodity markets can be extended to sectors by saying that a production sector of MODIS IV is a sheltered sector if the commodities for which the sector is the main producer have sheltered prices and an exposed sector otherwise.

The distinction between commodities with exposed and sheltered domestic prices has much in common with a distinction between tradeable and non-tradeable goods. However, there is no clearcut line of division between exposed and sheltered commodities and the present classification of MODIS IV is based on an empirical assessment of the degree and nature of foreign competition for each sector.[3]

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1) For a discussion of a support model for export and import prices, see footnote 1) p. 101. 2) These assumptions may be questioned for commodities for which Norwegian exports have a significant market share in the world market. Similarly, some foreign producers may have a strong position on the domestic market. Such conditions must be taken directly into consideration through the exogenous estimates for export and import prices. Prices of some minor exports and imports are directly linked to the corresponding domestic prices. 3) See Jansen (1978). Empirical tests have shown this classification to be just as good or better than classifications based on more mechanical calcualtions of import and export shares (see Ringstad (1974), chapter 4).

The sheltered domestic prices fall into three broad categories, namely regulated prices, negotiated prices, and cost determined prices. Both the regulated and the negotiated prices are exogenously given in the model. Regulated prices include domestic prices on all marketed government services and prices of products from industries where the public bodies have a dominant market power (transportation, communication, electricity supply). The regulated prices also include prices subject to direct government control. The negotiated prices cover the prices determined through negotiations between the government and the producers' organizations as part of the income settlements. Most domestic prices on products from the agricultural and fishing production sectors are included here. The rather detailed specification and the exogenous treatment chosen for these prices reflect the extensive use of MODIS IV in connection with the income settlements (see chapter 7).

For the cost determined prices the basic assumption is that the domestic market is sufficiently free of foreign competition to allow domestic cost changes to be reflected in the prices. The idea is that firms can, as a group, raise prices when costs go up without losing market shares to imports. The price leader for a commodity is thus assumed to adapt to changes in the cost of producing the commodity by following a "cost plus" or "mark-up" pricing policy. Increases in costs are passed on quickly in such a way as to leave the share of gross profits (operating surplus, depreciation, and sector taxes) in factor income largely unaffected. Apart from the "nuisance" introduced by the sector taxes (which are unimportant in the designated price leading sectors) gross profits may be interpreted as the sum of gross capital incomes and incomes for the self-employed.

The assumed stability in the relationship between wage costs and gross profits in these price leading sectors are based upon empirical studies of Norwegian data (see e.g. Aukrust (1970)). It may be explained as the outcome of an income distribution struggle or as the (casual) result of a mark-up setting policy that meets the requirement for internally generated funds with which to maintain its growth. Under special parametric conditions the result may also be derived from neo-classical assumptions. However, in the present version of MODIS IV the observed relationship between wage costs and gross profits is not formally included in the model but is merely used as background information when fixing the value of the exogenous mark-up rates in the price leading sectors.

Time series data show, in general, that there have been fluctuations in the profit shares. The fluctuations may be due to variations in capacity utilization. It seems therefore reasonable to assume that the price behaviour can be described as "normal cost plus pricing" or just "normal pricing" (see e.g. Godley and Nordhaus (1972) and Coutts, Godley and Nordhaus (1978)). Prices are independent of quantities at least over a certain range; the assumption is that prices are set on the basis of costs normalized for the cycle.[1] Since commodity incomes and outlays and wage costs in the submodel for domestic prices, as it is formalized in MODIS IV, are actual incomes and costs for each price leading production sector, and not normalized costs, the estimate for the mark-up rate must reflect the deviation between "normal" and actual costs. Even though it is not a formalized part of the model it is thus recognized, by ad hoc reasoning, that the mark-up rate may depend on the general state of demand. When demand is low so that actual unit costs are above "normal", mainly due to labour hoarding, the mark-up rate is set below the "normal", while when demand is high and actual unit costs are below "normal" the mark-up rate is set above the "normal". However, the model is in general used to study development paths with full or close to full employment. The problems connected with normalization of costs for the cycle are thereby greatly reduced.

The cost determined sheltered prices are the only endogenously determined commodity prices of the model. The submodel for these prices will be further discussed at the end of this section.

The treatment of the exposed domestic prices in the predecessors to the price model of MODIS IV was based on the assumption that these prices normally follow movements of the corresponding import prices.[2] In the light of a study by Ringstad (see Ringstad (1974), especially chapter 4), indicating that production costs influence prices also for exposed commodities, this rather crude assumption is modified in MODIS IV. We distinguish between two groups of exposed domestic prices, fully exposed prices and partly exposed prices. The fully exposed domestic prices are normally assumed to follow the (exogenous) prices of similar imported or exported goods. The partly exposed domestic prices are assumed to depend partly on changes in the unit production costs and partly on changes in the corresponding import or export prices. The

1) Short term quantitative adjustments are assumed to be much more important than short term price adjustment. This will give a horizontal supply at least over a range of quantities as it was assumed in section 4.3. 2) This assumption was made for "import-competing" sectors in MODIS II and III (see Aukrust (1970)).

weights in the aggregation of the two influences are intuitive "best-guess" estimates and not formally included in the model. In the present version of MODIS IV all exposed domestic prices are therefore exogenous.

Also the wage rates by production sector are exogenous in the present version of MODIS IV. Conceptually the changes in the wage rates may be separated into wage drift and negotiated wage rate changes. This distinction plays a role in some uses of the model in connection with income policy proposals and income settlements.

The reasoning behind the exogenous wage rate estimates is that the long run national wage level is determined by the profitability of the exposed industries, defined as the ability of these industries to pay out wages and generate operating surplus, which in turn depends on export and import prices (world market prices and exhange rates), and labour productivities (see Aukrust (1977)). It is assumed to be a tendency for wages in the exposed industries to adjust so as to leave actual gross profits close to a "normal" level determined by the ability of these industries to compete on the markets for tradeable goods. In addition the relationship between wages in the sheltered and exposed industries is supposed to be relatively stable over time; empirical evidence supports this.

Several "correction mechanisms" may be mentioned which, at least in the long run, will give this result, the most important being the market forces which through the labour market influence both the wage negotiations and the wage drift. Incomes policy, in which the MODIS model plays a role as a tool for consequence and overall assessment studies, may be viewed as an attempt to reach a "feasible" solution without having to use the labour market as a regulator.

There are, however, important short run deviations from the long run wage rate development indicated above. In the short run (one year) the wage drift in manufacturing industries is likely to depend upon factors like consumer prices (lagged) and some measure of changes in relative wages, in addition to changes in export prices and the rate of unemployment. Wage drift relations based on such independent variables have been estimated (see NOU (1977)) but are so far not formally included in the model. They are, however, used by the Ministry of Finance in their one and two years assessments. Iterative runs of MODIS and the wage drift equations are needed to achieve a consistent solution since both the unemployment rate (see section 5.2) and the consumer prices are endogenous variables of the model.[1)]

For the understanding of the functioning of the price side of MODIS IV it is important to note that sectors which mainly produce for exports or with exposed prices are typically capital intensive and mass-producing with high efficiency gains. The sectors which mainly produce sheltered commodities (sheltered industries) are on the other hand dominated by service industries where efficiency gains are less pronounced. This means that wage increases are much more easily absorbed by the exposed industries without effecting prices and/or profits than by the sheltered industries.

The basic transmission mechanism of the inflationary process as it is represented in MODIS IV may thus briefly be outlined as follows: Given world market prices, currency rates, labour productivities, the outcome of the income settlements, and an estimate for the wage drift the profits in the exposed sectors are residually determined. In the sheltered sectors (formally only in the price leading sheltered sectors) the output prices are set so that the relationship between gross profits and wages is kept stable. Since the productivity growth normally is higher in the exposed sectors the growth rate of sheltered domestic prices must be higher than exposed domestic prices and export prices just to maintain a stable income distribution between (i) the owners (entrepreneurs) of the sheltered industries and the wage earners and (ii) the owners (entrepreneurs) of the exposed industries.

1) A quarterly model called KPM has been developed to study the short-run movements in wages and prices of consumer goods. This model includes both consumer prices and wage rates as endogenous variables. The submodel for domestic commodity prices of MODIS IV in reduced form is used to generate cost indices for 23 components of the consumer price index. An assumption of stability in the relationship between wage costs and gross profits is included. The main components in these cost indices are then unit wage costs, import prices, regulated prices and negotiated prices. Dynamic relations between the cost indices and the corresponding observed consumer price indices corrected for indirect taxes are estimated on a quarterly base. The wage costs in the cost indices are explained by a division of production sectors into wage leaders and wage followers. The wage costs for the leaders (mainly exposed industries) are explained i.a. by the consumer price index, export prices, productivities, capacity utilization rates, centrally negotiated wage changes and taxes. For a more detailed presentation of the model, see Tveitereid (1979).

The submodel for domestic commodity prices

The framework of the cost and price formation process discussed above is only partly formalized in MODIS IV, and the formal structure is called the submodel for domestic commodity prices. It is given in (4.19), (4.23) and (4.25) as

$$(5.26)\quad B_P^{'}b_{XB}^{*} + P_P^{'}b_{XP} + E_P^{'}b_{XE}^{*} = b_{A_P}X,$$

$$(5.27)\quad b_{XP} = \Pi_{b_{XP}}b_Z + b_{XP}^{*}, \text{ and}$$

$$(5.28)\quad \Pi_{b_{AP}}(b_{A_P}X - E_P^{'}(b_{XE}^{*} - b_{XP})) = \Pi_{b_Z}b_Z + b_Z^{*}$$

The explicit solution for b_{XP} is given by (4.26).

The submodel for domestic commodity prices is part of the outer model. The projection of this submodel into the price side of the inner model takes the shape of (5.26), (5.27) and (5.28).

The unit cost structure of the production for the domestic market is expressed by (5.26) while (5.27) and (5.28) represent the domestic price formation structure of the economy.[1] (5.26) is discussed in section 4.2 and says that the activity basic price of each production activity is determined as the weighted sum of import prices of commodity inputs, domestic prices of commodity inputs and outputs, and export prices of commodity outputs. As discussed above export prices, denoted by b_{XE}^{*}, and import prices, denoted by b_{XB}^{*}, are exogenously given.

(5.27) is introduced to handle the exogenous domestic prices. As already elaborated upon, exposed domestic prices, regulated domestic prices and negotiated domestic prices are all exogenous in the model. The estimates of exogenous domestic prices are given in b_{XP}^{*} and the corresponding rows of $\Pi_{b_{XP}}$ have only zero elements. For the endogenous domestic prices, i.e. the cost determined prices, the corresponding rows of $\Pi_{b_{XP}}$ have unit elements in the columns for the designated price leading production sectors. The cost determined prices are then set equal to the unit supply (production) costs of these sectors, denoted by the relevant elements of b_Z.

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1) Note that the domestic price formation is built upon a separate calculation for the domestic market (see section 4.2).

Equation (5.28) may be viewed as a derived form of a submodel of the outer model for unit basic primary costs in the production for the domestic market. In the discussion of (5.28) it is convenient to distinguish between the price taking and the price leading production sectors.

For those production sectors which are assumed to be price takers for all commodity outputs the corresponding elements in the diagonal of Π_{b_Z} are unit elements while the corresponding elements of b_Z^* are zero. This means that the unit basic primary costs in the production for the domestic market, which is the interpretation of b_Z for these sectors, are residually determined as the difference between given commodity incomes and outlays.

For the price leading production sectors the derived form of the submodel for unit basic primary costs in the production for the domestic market takes the shape of the linear equation (5.28). The unit basic primary costs for the price leading sectors are endogenously determined, and in reduced form they are dependent partly upon the solution for unit supply costs (represented by the relevant elements of b_Z) and partly on variables considered as given in the inner model (included in b_Z^*). However, only a minor part, connected with value based commodity taxes on material inputs, is dependent upon other endogenous variables of the inner model. Both wages and gross profits per unit of value added are included in the predetermined term. These items are directly derived from exogenous estimates of wage rates, productivities and mark-up rates. It is thus only from a purely formal point of view that these components are endogenously determined.

In the accounting framework of the model primary costs (value added) in current market values for each production sector are decomposed into wages and salaries, employers' social security contributions, operating surplus, depreciations, commodity taxes and subsidies, sector taxes and subsidies.[1] Value added in market value is computed as the difference between producers' value of commodity outputs and purchasers' value of commodity inputs. Commodity taxes and subsidies are therefore included among the primary cost (value added) components (see the discussion in section 3.4). For each sector the value added component

1) Compared with the similar listing in the simplified presentation of the accounting framework in chapter 2 more components are specified here. However, this is still simplified compared with the actual specification in MODIS IV (see Tveitereid and Longva (1975)).

for commodity taxes and subsidies is computed as the value of commodity taxes and subsidies on commodity outputs less the value of commodity taxes which are refunded the sector on commodity inputs.[1] This will be further discussed in section 6.2.

The primary costs (value added) in current basic value by production sector are defined as the difference between the primary costs in market value and commodity taxes. Commodity taxes as part of the value added of each production sector are computed as commodity taxes and subsidies on commodity outputs less commodity taxes and subsidies on commodity inputs (see (3.12) and (3.14)). This means that basic primary costs include only subsidies and those commodity taxes on commodity inputs which are not refunded.

In the relations for unit basic primary costs in production for the domestic market we distinguish between three main components of primary costs, namely wages, gross profits and not refunded commodity taxes. The right-hand side of (5.28) can be written as the sum of these components, i.e.

$$(5.29) \quad \Pi_{b_Z} b_Z + b_Z^* = b_{WF} + b_R + b_{IT}$$

where b_{WF} is a vector of unit wage costs (wages and salaries and employers' social security contributions) per unit of sector level,

b_R is a vector of gross profits (operating surplus, depreciations, sector taxes and subsidies) per unit of sector level, and

b_{IT} is a vector of subsidies and not refunded commodity taxes on commodity inputs per unit of sector level. All costs are in the production for the domestic market.

Since we here are discussing only the price leading production sectors the vector definitions above refer only to the rows for these sectors. For the price takers the unit basic primary costs will be residually determined, as discussed above.

In the present version of MODIS IV wages and salaries per wage earner (wage rates) by production sector are exogenously given, and the tax rates for employers' contributions to social securities are specified as a vector of rates on wages and salaries by production sector. If we

1) In the present Norwegian tax system only value added taxes on commodity inputs are refunded. Commodity taxes on commodity inputs which are not refunded are included in the basic value of commodity outputs. This is why United Nations (1968a) use approximate basic value to designate what we call basic value.

combine this with the labour requirement function (5.8) and normalize with the sector levels the unit wage costs may be written as

$$(5.30) \quad b_{WF} = (I + \hat{t}_F)\hat{w}^*Q^*_{WE}$$

where t_F is a vector of tax rates by production sectors for employers' contribution to social securities (see section 6.2 for a further discussion),

w^* is a vector of wage rates by production sectors defined as wages and salaries per wage earner, and

Q^*_{WE} is a vector giving the number of wage earners per unit of value added, see the discussion in section 5.2.

The labour requirement function (5.8) also includes an exogenous term for the employment in some production sectors. However, all production sectors where the employment is exogenously given are assumed to be price takers. This means that these exogenous estimates and thereby also the sector levels for these sectors do not enter the expression for b_{WF} above.

Since all components in (5.30) are independent of variables of the inner model - they are actually all exogenously given - unit wage costs are included in the predetermined term b^*_Z of equation (5.28).

Gross profits per unit of sector level, which we shall call the mark-up rate, in the production for the domestic market are exogenously given for all price leading production sectors. This means that the elements of the vector b_R for these sectors have given elements and they are included in the predetermined term b^*_Z of (5.28).[1)]

The elements in the vector b_{IT} for subsidies and not refunded commodity taxes on commodity inputs may in general be written as linear functions of the basic commodity prices. Since the elements of b_Z for the price leading production sectors are interpreted as unit production costs, i.e. as domestic commodity prices, for the commodities for which they are the price leaders these relations represent linear constraints on the inner model. The elements on the diagonal of Π_{b_Z} for the price

1) As noted in the first part of this section increases in costs are assumed to be passed on in such a way as to leave the share of gross profits in factor income largely unaffected. This is approximately equivalent to letting the unit wage costs of (5.30) and the mark-up rates change proportionally.

leaders are composites which depend on tax rates for value based commodity taxes.[1] The elements of b_Z^* connected with subsidies and not refunded commodity taxes are composites of exogenous variables like rates for commodity taxes and subsidies, import and export price indices, and given domestic price indices.

The *tax and subsidy rates on* commodity inputs, and thereby also the elements of b_{IT}, are independent of the quantity solution. This is the reason why we have chosen to specify the primary unit costs in basic value in the submodel for domestic basic prices. Commodity tax and subsidy rates are normally differentiated among the purchasers of the same commodity. Because of this commodity taxes and subsidy rates on commodity outputs, and thereby primary unit costs in market value, will be dependent upon the quantity solution. This will be further discussed in section 6.2.

If we summarize the discussion of the submodel for unit primary basic costs in production for the domestic market, given by (5.28), the diagonal of Π_{b_Z} contains unit elements and the vector b_Z^* zero elements on the rows for price taking production sector and the unit basic primary costs are residually determined. For the price leading production sectors the predetermined term b_Z^* of the unit basic primary costs consists of elements which for each sector are composites of given unit wage costs, given mark-up rates, and rates for subsidies and not refunded commodity taxes on commodity inputs, apart from value based commodity taxes on domestically produced inputs, per unit of sector level. In the present version of MODIS IV only value based commodity taxes on domestically produced commodity inputs (which are few and relatively minor) make the submodel for unit basic costs dependent upon other variables, i.e. b_Z and thereby b_{XP}, of the inner model. Details in the derivation of the price equations are given in Longva and Tveitereid (1975).

1) If all elements of b_Z for the price leading production sectors are predetermined, as it was assumed in the discussion in section 4.2, the corresponding elements on the diagonal of Π_{b_Z} are all zero. For the price takers the relevant elements are, as discussed above, unit elements.

5.7. Depreciation, government consumption and other post calculations

As pointed out in section 4.3 several "post calculation" are usually made on the basis of the solution of the price and quantity sides. Together, the model results form a set of accounts approximately identical to those of the national accounts presented in chapter 2. Most flows follows directly from the price and quantity solutions, only the calculations of depreciation and government consumption deserve a separate discussion.

Depreciation

Depreciation in current values is, in the model and in the national accounts, defined as the value, at current replacement cost, of the reproducible fixed capital used up in the process of production during a period of accounts as a result of normal wear and tear, and foreseen obsolescence.[1] The procedure followed in the depreciation calculations of the model is very closely related to that of the national accounts. Actually, the calculation routines of the national accounts are directly used in the model.

The depreciation calculations are based on long time series for annual gross investment in constant values. The gross investments are classified by categories of capital goods and by real capital formation sectors, i.e. by capital formation activities (altogether about 150), and the time series include the model calculation (forecasting) period. The depreciation calculations are therefore based upon vintage data for gross investments in constant values. Depreciation is compiled on a straight line basis with reference to the expected average length of economic service life of the individual asset of capital good. Each asset is fully depreciated also in the year the investment is undertaken and no corrections for changes in productivity and unforseen obsolescence are made at present.

The estimate of depreciation enters the model results as a component of value added by production sector in current values, i.e. as a component of current cost of production. Following the conventions of the national accounts (and that of United Nations (1968a)) the current depreciation costs are valued at acquisition prices of the corresponding new capital good (gross investment activity) for each asset, i.e. the estimates takes into account the cost replacing the asset in the periode for which the calculations are been made.

1) Our definition of depreciation corresponds to that of consumption of fixed capital of United Nations (1968a) p. 122.

The actual depreciation calculations for the forecasting periode start out with a calculation in constant values. It is done in two steps, one covering gross investments made up to and including the base year of the model, i.e. depreciation on the capital stock existing in the base year, and the second step covering gross investments in the forecasting periode. The first step is usually performed once a year in connection with the establishment of the new base year of the model and gives, for each year in the forecasting periode, a vector of total depreciation in constant values by capital formation activities on the capital stock that existed in the base year. In these calculations the routines for depreciation calculation of the national accounts are directly applied, assuming zero gross investments for the forecasting period. These routines, which are part of the system for capital stock calculations, are based upon the work by Johansen and Sørsveen (1967) and Bjerke (1971). The results of the first step of the depreciation calculation are of course not influenced by any exogenous or endogenous variable of the model, apart from the exogenously given depreciation rates (the inverse of the expected economic lifetime of the various assets).

The second step is performed as a "post calculation" to the solution of the main model. The input data are the exogenous estimates for activity levels of real capital formation and the exogenously given depreciation rates. The computation gives, for each year in the forecasting periode, a vector of total depreciation in constant values by real capital formation activities on the capital stock accumulated after the base year. By adding these results to the calculated depreciation on the base year capital stock we get total depreciation in constant values by real capital formation activities.[1)]

Each depreciation item in constant values is then inflated separately. The relevant part of the activity price vector gives the price indices for each category of capital good (activity prices for gross investment, p_{A_I}) and by assuming that each capital good has the same price index in all real capital formation activities in which it is included, total depreciation in current values by real capital formation activities can be calculated.

The real capital formation sectors of the model (and the national accounts) are somewhat more aggregated than the production sectors. To get the depreciation by production sectors the result are therefore further distributed among the production sectors by means of the base year capital stock distribution.

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1) For more details see Furunes and Sand (1976).

Government consumption

As discussed in sections 2.1 and 3.2 general government consumption is defined as (i) commodity expenditures, plus (ii) wage and salary expenditures, plus (iii) depreciation of fixed capital, less (iv) revenues from marketed government services. Within the model general government consumption is specified by production sector, i.e. external activities for the production of real transfers to consumption sectors, and by consumption sector, which each is further decomposed into external activities by type of government services (direct transfers from each government production sector to each government consumption sector).

Commodity and wage and salary expenditures by production and by consumption sector, i.e. by general government consumption activity, are, through the fiscal budgets, exogenously given in the model (see the discussion in section 6.3). All the estimates are in real terms (constant market values).

Revenues from sales of marketed government services in constant values are given by the corresponding activity levels, one activity for each production sector. These activities are endogenously determined in the submodel for industry and general government production (see section 5.2). By means of the base year distribution for the revenues from marketed government services the activity levels for marketed government services are distributed among the general government consumption activities.

Depreciation by government production sector is determined in the submodel for depreciation (see above). These results are further distributed among the general government consumption activities by means of the base year capital stock distribution.

The estimations in current values are performed after the prices have been solved for. Each component of government consumption is inflated separately. The relevant parts of the activity price vector p_A give the price indices for commodity expenditures (activity prices for absorption of commodities in general government) and for commodity revenues (activity prices for marketed government services). The wage and salary expenditures (wage costs) by government production sector are given both in current and constant values and the price indices are therefore implicitly determined (see section 6.3). The determination of price indices for depreciation by government production sector is discussed above. The computation of the general government consumption activities in current values is based upon the assumption that each component has the same price index in all consumption activities.

The model results in the form of national accounts

The model results can be presented within the conceptual framework indicated by diagram 2.2 and 2.3 of chapter 2.

Nearly all real flows in current and constant values (see diagram 2.2) follows directly from the solution of the price and quantity sides. The only important exception is general government consumption discussed above.

After the prices and quantities are solved for, the income and capital finance flows, which are all in current values (see diagram 2.3) can be computed. Exports and imports are entered as outlays and incomes, respectively, in the foreign income accounts. Household consumption and general government consumption are outlays for the private sectors and the general government sectors in the respective income accounts. Real capital formation by functional sector is distributed between the institutional government and private sectors according to ownership.

The income categories, which somewhat simplified include the value added components wages, operating surplus, depreciation and indirect taxes in addition to government transfers and direct taxes, are mainly endogenously determined. Wages are determined by equation (5.23) in section 5.4[1], depreciation in the submodel discussed above, and indirect taxes and subsidies in the submodel for indirect taxes presented in section 6.2 (see equations (6.9) and (6.10)). Operating surplus are residually determined as the difference between total value added in current value and the value added components listed above (compare also equation (5.24) in section 5.4). Direct taxes are computed by equation (6.1) in section 6.1 and government transfers are exogenously given (see sections 5.4 and 6.3).

In the income accounts for the institutional sectors wages are entered as private income and operating surplus are distributed between private incomes and government incomes according to capital ownership. Wages and operating surplus as private income are further distributed on the three personal institutional sectors (socio-economic groups) as discussed in section 5.4. Indirect taxes are entered as government income while direct taxes are credited to the government income accounts

1) Employers' social security contributions, which are also a component of value added (and of wage costs), are determined by equation (6.2) in section 6.1.

and debited to the three personal institutional accounts (see sections 6.1 and 6.2). Government transfers are debited to the government account and credited to the three personal accounts (see sections 5.4 and 6.3).

Depreciation is distributed on the private and government capital finance accounts according to capital ownership. All increases in financial assets and liabilities which enter the capital finance accounts are exogenously given. The balances of the income accounts (savings) are transferred to the respective capital finance accounts. This completes the real flow, the income and outlay, and the capital finance accounts in which form the model results are presented.

6. TAXES, TRANSFERS AND FISCAL BUDGETING

A main use of the MODIS model is as a tool for "national budgeting", i.e. in the analysis and implementation of the short-to-medium term macro-economic policy. The fiscal budget may be considered as an important but subordinate part of the national budget. The fiscal instruments implied by the income and outlay accounts of the fiscal budget are of major importance in the model.

Two submodels, one for direct taxes and one for indirect taxes and subsidies, deal with the taxes and transfers. These submodels serve as a linkage between the detailed structure of taxation and the macro-economic variables. The submodels are used in two ways: (i) To derive certain aggregates required in the solution of the main parts of MODIS IV, and (ii) to calculate tax revenues after the solution of the price and quantity sides of the main model.

In section 6.1 below the submodel for direct taxes is presented in some detail. Section 6.2 gives an outline of the submodel for indirect taxes and subsidies. Section 6.3, which deals with the use of the model in fiscal budgeting, includes a summary of the specification of the model with regard to government incomes and outlays.

6.1. Direct taxes

The submodel for direct taxes existed as an independent model prior to MODIS IV. This direct tax model and earlier versions of MODIS gave frequently different and in a way competing forecasts of tax revenues. However, the two models had quite complementary advantages. The direct tax model combined a detailed representation of the institutional tax structure and the income distribution with exogenous assumptions about growth of incomes by socio-economic group. MODIS, on the other hand, determined growth and distribution of income within an input-output framework with the tax structure represented in a much broader way. Inconsistencies between the results of the two models led to numerous sessions of clarification and discussion as to which model was most reliable. This situation led typically to a separation of the general economic policy analysis which was based on MODIS and the fiscal budgeting based on the direct tax model. Hence the effort to integrate the direct tax model into the MODIS IV structure. The model is described in Engebretsen (1974).

The direct tax submodel of MODIS IV does not cover all direct taxes. It is restricted to personal income taxes (incl. social security contributions), which, however, add up to more than 90 per cent of total direct tax revenue. Other direct taxes, e.g. corporate taxes, are included as exogenous elements in the model.

Taxes on personal incomes are calculated separately for three socio-economic groups (personal institutional sectors): (i) Wage and salary earners, (ii) self-employed, and (iii) pensioners. These three groups correspond to the decomposition of gross income in the national accounts and also to the decomposition of "consumption motivating income" in the submodel for household consumption (see sections 5.4 and 5.3). Within each socio-economic group the tax-payers are divided into two tax classes in order to take into account that different tax schedules apply to taxpayers with different family situation.

The direct tax model consists of two main parts, a micro part and a macro part.

The micro part contains taxation rules for individual taxpayers in each of six taxpayer groups. The parameters of the micro part describe the taxation rules in a fairly detailed way. In addition, the micro part includes a size distribution of income for each of the six groups. The micro rules (micro tax functions) are applied along with the income distribution to calculate accrued tax revenue by income intervals and taxpayer group. This information is used to derive macro tax rates. For each kind of personal income tax specified in the model, and for each socio-economic group, two tax rates, the macro marginal tax rate and the macro average tax rate, are determined. The macro tax rates enter the macro tax functions which are wholly integrated in the main part of MODIS IV.

In addition to the two main parts of the submodel for direct taxes, there is also a third part computationally separate from the others. This part, called the accounting part, performs the transformation from taxes accrued, which is the concept used in the model as well as in the national accounts, to taxes paid which is the concept used in the government accounts including the fiscal budget (see also section 2.2).

The model specifies about 30 different types of personal income taxes (incl. social security contributions). The proceeds of some of these are wholly exogenous, mainly because the tax assessment rules for these taxes cannot be represented adequately on the basis of the other variables specified in the model. The total personal income tax revenue is, however, overwhelmingly dominated by the endogenous income taxes which are represented in great detail in the model.

Changes in the personal income taxation are of considerable importance for the development of consumption demand and, accordingly, for the short-term economic development in general. The parameters of the tax rules are thus among the most important instrument variables. To enable a user to integrate tax analysis in a macro-economic framework it has been found to be almost a necessity to represent the tax rules within the model in much detail and with a view to cover the more important changes in parameters and tax schedules. Changes in tax rules are channelled through the micro part before they reach the macro tax rates entering the main model. These macro tax rates are changed not only through changes in tax rules. They are also influenced by changes in demographic factors and the income distribution. Even with no changes in tax rules the macro tax rates will normally change from one year to the next.

The structure of the submodel for direct taxes is outlined in diagram 6.1.

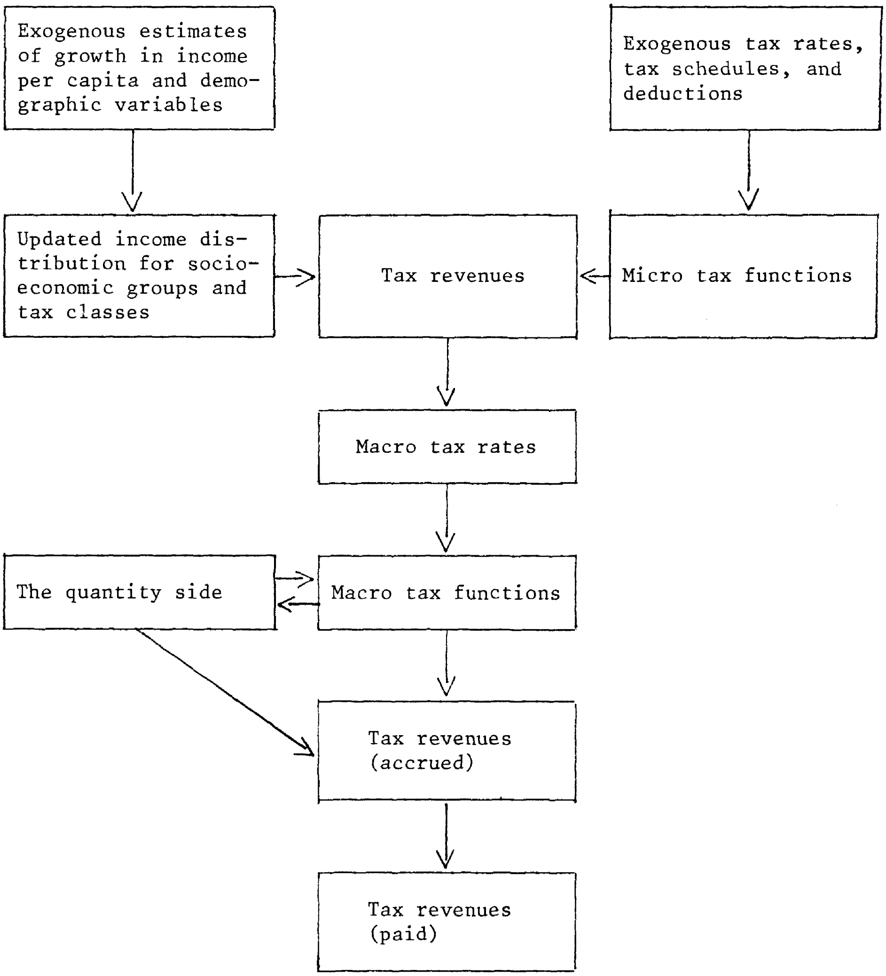

Diagram 6.1. Structural map of the submodel for direct taxes

Micro tax functions

The tax functions for the major direct income taxes are progressive but are basically of two different types. For some taxes, e.g. the municipal income tax, the tax amount is proportional to the excess of income over a certain level which depends upon the family situation and may vary between the various kinds of taxes.[1] The marginal tax rate is thus zero up to a certain level of income and has a positive constant value above that level (degressive tax functions). Other taxes, e.g. the central government income tax, have a more obvious progressive character with stepwise increasing marginal tax rates. While the municipal income tax function is characterized by very few parameters the central government income tax function has a sequence of interval limits and tax rates as parameters.

The income distribution size is specified by the number of taxpayers and their total income in discrete income intervals. The data on income distribution is taken from the latest available annual tax statistics and will normally be dated two years prior to the base year. It is thus necessary - outside the tax model - to update the income distribution to the base year by means of various assumptions which will not be discussed here, see Engebretsen (1974), pp. 16-18.

In the micro part of the model the income distribution is further updated to the years for which the model is to be solved. This updating procedure requires e.g. exogenous estimates of growth in per capita income and in the number of taxpayers for each socio-economic group. The size of the two tax classes of each socio-economic group is assumed to develop proportionately, but this assumption is at present under re-consideration. Some of the exogenous variables in the updating are closely related to endogenous variables in the main model. However, a difference between the exogenous estimates used in the updating and the endogenous estimates of MODIS will only have a second-order effect on the macro tax parameters calculated in the macro part. Too high a priori estimates of growth in income per capita will imply a positive bias in predicted tax revenue depending upon how progressive the tax schedule is. This could be easily corrected for, however, by iterative use of the model.

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1) If the level is set at zero the tax function will be strictly proportional.

The micro tax functions are combined with the income distribution in the revenue calculation. Numerical differentiation is required to evaluate the tax functions along the income scale when the cumulative income distribution is known only in discrete points. This is performed by approximating the income distribution density by the traditional trapeze methods. By this approximation any tax schedule can be applied and tax revenue calculated for incomes in any interval.

Macro tax functions

When the direct tax model is used separately it consists only of the micro part. Rather than integrating this fully into the simultaneous solution of the MODIS framework the macro part has been worked out to provide aggregated and linear specifications of the direct tax functions. Tax renevues will in general be dependent upon changes in the income distribution. However, if we assume proportional changes in incomes for all taxpayers within the same socio-economic group only changes in the income distribution between the groups matter. For each socio-economic group the macro tax functions will then be homogenous of degree one in the number of taxpayers.

Different linear specifications are used for each year (and each alternative set of tax rules) for which the model is to be solved. For each socio-economic group the tax functions for direct income tax (including personal social security contributions) have the simple form (cp. Engebretsen (1974), p. 31):

$$(6.1) \qquad T_i = t_{gi}\frac{\bar{G}^o_{Ti}}{\bar{N}^o_i}\bar{N}_i + t_{mi}\left(\frac{\bar{G}_{Ti}}{\bar{N}_i} - \frac{\bar{G}^o_{Ti}}{\bar{N}^o_i}\right)\bar{N}_i, \quad i = \text{WE, ES, UP}$$

where T_i is a vector of tax revenues by kind of income tax accrued on income of socio-economic group i (wage and salary earners (WE), self-employed (ES), pensioners (UP)),

$\frac{\bar{G}_{Ti}}{\bar{N}_i}$ and $\frac{\bar{G}^o_{Ti}}{\bar{N}^o_i}$ are gross taxable income per tax payer of socio-economic group i for a future and the base year, respectively,

$\bar{N}_i$ is the number of taxpayers in socio-economic group i, and

t_{gi} and t_{mi} are vectors of macro average and macro marginal tax rates.

The elements of the t_{gi} and t_{mi} vectors are estimated by linear interpolation between tax revenue estimates for the base year and for the future year, in both cases applying the future year tax rules. These tax revenue estimates are derived from the micro part of the submodel.

$\bar{G}_{Ti}$, gross taxable income for socio-economic group i, is already defined and discussed in section 5.4 (see equation (5.22)). The income concept used in the disaggregated part of the model is net taxable income. An underlying and quite crucial assumption in the modeling of income tax relations is that gross and net taxable incomes are strictly proportional. Deviations from proportionality caused by changes in the tax rules related to deductions are at present formally handled through exogenous correction factors of the income variables of the macro tax functions (not included in (6.1)).[1]

$\bar{N}_i$, the number of taxpayers in socio-economic group i, is exogenously given for the self-employed and for the pensioners.[2] For the wage and salary earners the number of taxpayers is simply derived as the aggregate of the number of wage earners by production sector, which is given by equation (5.8) of section 5.2.[3]

The tax functions (6.1) are integrated into the main part of MODIS. When the a priori estimates discussed above are consistent with those solved from MODIS, the linearized tax functions above are exact relative to the validity of the assumptions used in updating the income distribution derived from the tax statistics. The macro tax functions are used, first in the aggregate consumption function of the quantity side of the model[4] and, secondly, when the main model has been solved, in the calculation of tax revenue by kind of personal income tax.

Employers' social security contributions are treated separately from the other personal income taxes because of its separate tax base. The tax base for employers' contributions is wages and salaries, not gross taxable income. The tax functions for employers' contributions

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1) A special model for studying the effects of changes in deduction rules has been developed, see Garaas (1977). This model is, however, at present not integrated in the MODIS system. 2) The estimate for the self-employed is the same as the exogenous estimate for self-employed by (industry) production sector referred to in section 5.2. 3) Deviations between the number of persons and the number of tax-payers in each socio-economic group are corrected for by exogenous correction factors (not included in (6.1)). 4) $\underline{T}_i$ of (5.11) is simply the sum of the elements of T_i.

are written as

$$(6.2) \quad T_F = (I + \hat{t}_F)Y_W$$

where T_F is a vector of employers' social security contributions by production sector,

t_F is a vector of tax rates by production sector for employers' contributions (see also equation (5.30) of section 5.6)), and

Y_W is a vector of wages by production sector (see also equations (5.20) and (5.23) of section 5.4).

A separate tax rate is applied in each sector because of various exceptions from the general rule of one uniform rate.

The accounting part

The accounting part of the submodel for direct taxes comes into operation after the solution of the main model. This model embodies institutional and behavioural rules with regard to the payment of accrued taxes into the various government accounts. The basic structure of the relations of the accounting part is that taxes paid depend linearly on taxes accrued in the current and preceding year, with correction factors for changes in the institutional rules.

6.2. Indirect taxes[1)]

The design of the submodel for indirect taxes (incl. subsidies) of MODIS IV is strongly influenced by the general framework of the national accounts and of the main model. The disaggregated representation of the commodity flows, which is the core of the accounts and of the model, opens up the possibility of establishing rather close connections between indirect tax parameters of the model and information contained in legal tax rules. Indirect tax revenues are calculated in the minutest detail allowed by the specification of commodity flows. Given adequate specification of rates, the relevant tax bases and other variables the functional forms for indirect taxes which should be adopted actually

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1) The description of the submodel for indirect taxes given below must be regarded only as a survey of its general features. The actual model is too detailed and too complex to be presented in full here. A more complete description is given by Longva (1975b).

become identities.[1] The indirect tax functions have come a long way in this direction, may be as far as practically possible within a manageable comprehensive model.

Commodity taxes, sector taxes, and the accounting base

To achieve these goals a vast amount of information is needed about the tax rules themselves and about how the tax structure is connected with the commodity and income accounts of the national accounts. The accounts and the model specify about 85 different kinds of indirect taxes and subsidies. In general, indirect taxes include all taxes assessed on producers in respect of the production, sale, purchase or use of goods and services which they charge to the expenses of production. Subsidies include all grants on current account to producers to maintain prices at a level below costs of production.[2] Following the recommendations of United Nations (1968a) indirect taxes by kind are included among the components of value added for each production sector and thereby as income categories. On the income side the revenues from indirect taxes are distributed among the various general government accounts (see also diagram 2.3).

The indirect taxes (and subsidies) are classified in MODIS IV into the two broad categories commodity taxes and sector taxes. The commodity taxes are all proportional to the quantity or the value of commodities produced or sold, i.e. they are commodity based. Sector taxes, in contrast, are most often linked to the existence of the production itself without being associated with the quantity or the value of specific commodity flows.

In addition to being included as components of value added and thus as income categories the commodity taxes are also explicitly connected with the commodity flows of the accounts as separate value components. As pointed out in section 3.4 each commodity flow is decomposed in several different value components. Of central interest here is the distinction between (i) basic value, (ii) value added tax in respect of production, (iii) ordinary commodity taxes (subsidies) in respect of production, (iv) trade margins in basic values, (v) value added tax in respect of trade, and (vi) ordinary commodity taxes

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1) As pointed out by Davis (1976) the introduction of stochastic elements in the tax functions arises from a wish to keep the independent variables on an aggregated level and from a lack of adequate data rather than from uncertainty as to behavioural influences. 2) See United Nations (1968a), p. 234 and p. 237.

(subsidies) in respect of trade. This means that the national accounts as described in diagram 2.2 contain six complete commodity-by-sector input tables and six output tables, i.e. one for each value component of commodity flows as defined above.[1)]

The principal concept for evaluating commodity flows in the model is, as explained earlier, basic values. In terms of the more commonly used concepts of producers' value and purchasers' value the basic value of a commodity flow is defined as the producers' value less commodity taxes, net, in respect of production; or the purchasers' value less trade margins and commodity taxes, net, in respect of production and trade. The producers' value of commodity outputs of a production sector equals, by definition, the sum of commodity inputs in purchasers' value and value added in market value. Since value added includes both commodity and sector taxes, commodity taxes (and subsidies) can be viewed as not included in the commodity flows recorded in basic values, while the sector taxes are included.[2)]

The basic value concept is preferred to producers' value or purchasers' value because it is believed to increase the realism of the assumption of one and only one price for each commodity in the base year, an assumption which is essential for the interpretation of the commodities of the model as homogenous commodities (see section 3.4). If this assumption of no price differentiation for each commodity in the base year is to be valid in all markets and with prices measured as basic values, all commodity based taxes (and subsidies) with demand (purchaser) differentiated tax rates must be included among the commodity taxes and thereby excluded from the basic values while all commodity based taxes with supply (producer) differentiated tax rates, must be included among the sector taxes and thereby included in the basic values.[3)] Most of the commodity based taxes have demand differentiated tax rates (e.g. zero rates on deliveries to exports) and they are therefore included among the commodity taxes, i.e. in the difference between purchasers' values and basic values of commodity flows. However, some commodity

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1) It should be noted that, as recommended in United Nations (1968a), the national accounts and thereby also the model have adopted a full gross treatment of commodity tax and subsidy flows, including the value added tax (VAT). A complete presentation of the treatment of indirect taxes in the national accounts is given by Fløttum (1975). 2) Note, however, that commodity taxes on those commodity inputs which are not refunded are included in the basic value of commodity outputs (see section 5.6). 3) How to classify commodity based taxes with both supply and demand differentiated tax rates must be based upon the relative importance of the two types of differentiations. No "correct" classification can be made.

based taxes have tax rates differentiated between sources of supply. Most notably among these commodity based sector taxes are customs duties, investment taxes and subsidies, taxes on domestic produced crude oil and natural gas and some special subsidies on domestic agricultural products and fish.[1)]

The estimation in the national accounts of the commodity tax flows and the indirect taxes as components of value added and income categories is based upon a very detailed and comprehensive archive of tax rules and revenues and their connection with the commodity flows and sectors of the national accounts. The commodity-by-sector tables for commodity taxes and the indirect tax entries in the income accounts may be regarded as a condensed presentation of the information given in these background data files. The indirect tax model of MODIS IV is based on both the tax figures of the national accounts and on the information given in the tax archive.

Model calculations and the incidence of indirect taxation

The proceeds of all commodity taxes and subsidies, altogether nearly 40 in number, and of some of the commodity based sector taxes and subsidies are endogenous in the model, determined through tax functions which will be discussed below. For the other sector taxes and subsidies the proceeds are wholly exogenous, mainly because the tax assessment rules for these taxes and subsidies cannot be represented adequately on the basis of the model variables. These proceeds are specified by tax (subsidy) type and by production sector.

The total indirect tax revenue is overwhelmingly dominated by the endogenous taxes, with the value added tax (VAT) accounting for more than one half. This is in contrast to the subsidies where only somewhat more than fifty per cent of total outlay is endogenously determined.

The central link between the submodel for indirect taxes and the rest of the model system is the effects which changes in indirect taxation rules have on prices, i.e. the incidence of indirect taxation. The question is then how changes in tax and subsidy rates influence prices and how prices affect other variables of the model.

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1) Investment taxes and subsidies are obviously sector taxes because they influence the user cost of capital and thereby the domestic production costs. They are, however, commodity based because they are levied on commodities used for investment purposes.

In the price model domestic basic prices are either set as a mark-up over "normal" costs or they are exogenously given. This means that the supply curves, with basic commodity prices as arguments, are horisontal, i.e. infinitely elastic. It is a wellknown result in the standard static theory of tax incidence that changes in indirect taxes are fully passed on into the market prices of the commodities on which the taxes are levied when the supply curves are infinitely elastic. For the commodity taxes (and subsidies) we are therefore making the obvious assumption, given the formulation of the price model, that changes in the taxes are fully passed on into the relevant market prices. This means that the exogenous estimates for mark-up rates, exposed domestic basic prices, and export and import basic prices in the price model are assumed not to be influenced by changes in the commodity taxation system.

These assumptions for commodity tax incidence of MODIS IV cannot be applied directly to the sector taxes and subsidies. The mark-up estimates, and thereby the basic value of the commodity flows, include the sector taxes and subsidies. If the sector is a price leader we assume that changes in the sector taxes, net, are passed on into output prices, while profits are reduced if the sector is a price taker. This means that the exogenous estimates for mark-up rates and export and import basic prices in the price model are influenced by changes in the sector taxation system while the estimates for exposed domestic basic price are not.[1] In some sectors, e.g. agriculture, the output prices are exogenously given. Changes in sector taxes will therefore cet.par. change profits in these sectors.

As shown above, changes in the indirect taxation have an impact on both absolute and relative market prices. Through the consumption submodel these effects are transmitted to total household consumption and the composition of it and is thereby important for the economic picture in general.

The parameters of indirect taxation are important instrument variables of the model. However, if the policy-maker is interested in differential impacts of selective changes in indirect tax rates, household consumption must be specified on an appropriate disaggregated level. In MODIS IV there are nearly 50 different consumption activities as well as variables representing the income distribution (see section 5.3).

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1) Note that customs duties are included among the sector taxes.

Classification of commodity taxes and the specification of the input and output tax functions

If the indirect taxes are to serve as policy instruments it is also necessary to represent the taxation rules within the model in such a way that possible changes in the rules can be translated into parameter changes in the model. Each commodity tax and subsidy is therefore characterized by information about the tax base, the tax rate(s), and the tax payer. The tax base of a commodity tax is either the quantity or the current value of one or more commodities. A commodity tax or subsidy is therefore either a quantity tax or a value tax. The tax payers (collectors) are either the importing and producing sectors of the commodities on which the tax is levied, a production tax, or the trade sector, a trade tax. Commodity taxes on imports are, following the conventions of the national accounts, assumed to be paid (collected) by a special subsector of the trade sector.

The tax rate(s) for each commodity tax is given as a vector. In this way the model takes care of the fact that the tax rate of a commodity tax may differ between the receivers of the commodities on which the tax is levied. Typically, the tax rate will be zero on deliveries to exports, but the tax rate may be differentiated on deliveries to other receivers as well.

From the information given about the tax base and the tax payer the commodity taxes (and subsidies) may be classified as quantity production taxes, value production taxes, quantity trade taxes, value trade taxes, value added production tax and value added trade tax. The value added taxes are separated from the other value taxes because of the special revenue calculation for these taxes and because all other commodity taxes are included in their tax base. As will be discussed below the specification of the actual tax functions of the model is based upon these six classes of commodity taxes and subsidies.

In general form the input tax function for each commodity tax (or subsidy) is given by

(6.3) $\quad L_k^- = G_k \circ W_k^-$ [1)]

where L_k^- is an input matrix for commodity tax k which gives the value of tax k on input of each commodity to each activity,

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1) The sign o has the following definition: If two matrices X and Y are of the same order, we define X o Y = Z where $x_{ij} \cdot y_{ij} = z_{ij}$.

G_k is a matrix of tax coefficients for tax k which gives the value of tax k on input of each commodity to each activity per unit of each delivery measured in the value base of the tax, and

W_k^- is a commodity-by-activity matrix which gives the input of each commodity to each activity valued in the value base of tax k.

The elements of the tax coefficient matrix G_k can be derived from the specification of the tax base and the tax rate(s) of the commodity tax (or subsidy) in question. The connection between the tax coefficient matrix and the tax base and tax rate(s) is given by

(6.4) $$G_k = I_k T_k'$$

where I_k is a vector of commodity dimension with ones for commodities included in the tax base of tax k and zeroes elsewhere, and

T_k is a vector of activity dimension which gives the tax rate for tax k on commodity deliveries to each activity.

The vectors I_k and T_k are labeled the tax base vector and the tax rate vector of tax k, respectively. As seen from the above definitions the tax base vector includes information about which commodities are included in the tax base while the tax rate vector includes information about how the tax rate of commodity tax (or subsidy) differ between the receivers of the commodities on which the tax is levied. The tax rates are given exclusive of the tax itself, and with positive signs if it is a tax and with negative signs if it is a subsidy.

For the commodity taxes and subsidies, except the value added taxes, typically only one or a few commodities are included in the tax base of each tax. The matrix G_k, and accordingly also L_k^-, has therefore only a few rows with non-zero elements. The tax base of the value added taxes includes most commodities and the tax coefficient matrices and the input matrices for these commodity taxes have quite a few rows with zeroes only. Since most commodity taxes have zero tax rate on deliveries to export, the columns for export activities of L_k^- and G_k are normally empty.

The content of the matrix W_k^-, which may be labeled the <u>tax base matrix</u> for tax k, depends upon the value base on which the tax is levied. If it is a quantity tax (a quantity production tax or a quantity trade tax) the actual value base, as it is specified in the legal tax rules, will be the physical quantities of commodity flows. In MODIS IV the quantities are measured in constant basic values and the commodity flows so measured will represent the value base for the quantity taxes. The tax base matrix for each of these taxes (and subsidies) can therefore simply be written as

$$(6.5) \qquad W_k^- = \Lambda^- \hat{A}$$

where Λ^- is a commodity-by-activity input coefficient matrix in which element $\lambda_{ij} = \frac{A_{ij}^-}{A_j}$ gives input of commodity i per unit of activity level j, derived from the base year of the model (cp. equation (3.8)), and

A is a vector of activity levels (with $\hat{}$ indicating diagonalization).

For the value production taxes the value base is the current basic value of commodity flows. For each of these taxes (and subsidies) the tax base matrix then takes the form of

$$(6.6) \qquad W_k^- = (\hat{b}_{XB} B^- + \hat{b}_{XP} P^- + \hat{b}_{XE} E^-)\hat{A}$$

where b_{XB}, b_{XP} and b_{XE} are vectors of import, domestic, and export commodity basic prices, respectively (comp. equation (4.17)), and

B^-, P^- and E^- are commodity-by-activity input coefficient matrices for commodities which are imported, domestically produced and used, and exported, respectively, defined similarly to Λ^- above.[1)]

The tax base matrices for commodity taxes belonging to the other classes of value taxes, i.e. value trade taxes, value added production tax, and value added trade tax, are somewhat more complicated and cannot be expressed so straightforwardly as those presented above for the quantity taxes and the value production taxes. We shall not give the explicit expressions here but just give a description of the procedure in deriving these tax base matrices. More details are given in Longva (1975b).

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1) In the same way as B, P and E add to Λ (see equation (4.16)) B^-, P^- and E^- add to Λ^-.

The tax base for the value trade taxes is, in accordance with the legal tax rules, the current producers' value less value added production tax plus trade margins in current basic value of commodity flows. This is the same as current basic value plus quantity and value production taxes plus trade margins in current basic value of commodity flows. Compared with the tax base matrix for value production taxes, given in (6.6), we must therefore add the input matrices $\bar{L}_k$ for quantity and value production taxes and a matrix of trade margin flows in current basic value to get the tax base matrix for value trade taxes.

As discussed in section 3.3 trade margins, which in the national accounts are distributed on each commodity flow, form one separate industry commodity in MODIS IV. While nothing is gained in the quantity model by having separate and fixed trade margin rates compared with the chosen procedure, the trade margins must be distributed on their respective commodity flows in the computation of commodity value taxes in order to have the tax base of the value trade taxes defined in accordance with the legal tax rules. This is done by applying a commodity-by-activity dimensioned matrix of fixed trade margin rates on commodity inputs and by assuming that the trade margin on each commodity flow has the same basic price index as the commodity flow itself.[1)]

The tax base for the value added production tax is, according to the tax rules, the current producers' value less value added production tax of commodity flows, i.e. current basic value plus quantity and value production taxes. The tax base matrix for this tax is therefore derived by adding the input matrices $\bar{L}_k$ for quantity and value production taxes to the tax base matrix given in (6.6).

The tax base for the value added trade tax is the current producers' value of trade margins less value added trade tax on commodity flows, i.e. trade margins in current basic value plus quantity and value trade taxes. The calculation of trade margin flows in current basic value and the input matrices $\bar{L}_k$ for quantity and value trade taxes is discussed above.

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1) In the price model only one separate price index for the trade margin commodity is defined. However, this inconsistency between the price model and the indirect tax model simplifies the model calculations and is of minor importance for the model results.

As seen from this discussion, the commodity tax calculation starts out with the calculation of commodity tax flows on commodity inputs. This is the natural starting point since commodity taxes have demand (purchaser) differentiated tax rates.[1] The commodity tax flows on commodity outputs will thereby be dependent upon the demand composition. However, after the input matrices L_k^- for each commodity tax are estimated the tax output matrices can be computed by using the information given about the tax payers (collectors) in the tax rules. In general form the output tax function for tax k can be written as

$$(6.7) \qquad L_k^+ = \widehat{(L_k^- e)} M_k,$$

where L_k^+ is an output matrix for commodity tax k which gives the value of tax k on output of each commodity from each activity, and

M_k is a matrix of commodity-by-activity dimension which gives the proportion of total value of tax k which accrues on output of each commodity from each activity.

The vector $(L_k^- e)$ gives total value of the tax by commodity and the matrix M_k, which may be labeled the tax payer matrix of tax k, distributes the tax among the designated tax payers.

Commodity production taxes are collected and transferred to the government accounts by the importers and the producers of the taxed commodities while the commodity trade taxes are collected and transferred by the wholesalers and retailers, i.e. the trade sector. The tax payer matrices for the trade taxes, which are all identical, are therefore very simple. M_k for these taxes have unit elements in the column for the trade margin producing activity and zeroes elsewhere and the vector $(L_k^- e)$ appears as the trade margin activity column of L_k^+ for all trade taxes.

For the production taxes the total value of each tax is distributed according to the market shares of the taxed commodity deliveries (outputs). Since practically all deliveries to export are tax free this means that the tax payer matrices M_k for the production taxes, which are all assumed to be identical, will consist of market shares of deliveries to the domestic market. Production taxes on imports are, following the conventions of the national accounts, considered as paid by a subsector of the trade sector in the model. The market shares of import deliveries therefore appear in M_k in the activity column

1) Indirect taxes with supply differentiated tax rates are classified as sector taxes.

corresponding to this sub-sector.[1)]

Since the elements on each row of M_k add to one for all classes of commodity taxes all taxes on inputs of a given commodity are distributed on the designated tax payers. This simply reflects the fact that commodity taxes imposed in some sectors will have to be paid somewhere along the road.

Price and income computations for commodity taxes

All other model results related to commodity taxes are derived from the L_k^+ and L_k^- matrices. On the price side (see equation (4.17)) commodity taxes, net, per unit of activity level enter to make up the difference between activity levels in market value and net commodity outputs in basic value (activity levels in basic value). By computing the differences between commodity taxes on commodity outputs and inputs in each activity and normalizing by the activity level we arrive at

$$(6.8) \qquad b_{AT} = \hat{A}^{-1} \left[(L_1^+ - L_1^-) + \ldots + (L_{n_T}^+ - L_{n_T}^-) \right]' e$$

where b_{AT} is a vector of commodity taxes, net, per unit of activity level (see equation (3.12)), and

n_T is the number of commodity taxes and subsidies.[2)]

Another feature of the submodel for indirect taxes is that a breakdown of commodity taxes and subsidies is obtained, and entered as part of value added of production sectors in current value (see diagram 2.3). In the model (and in the national accounts) each commodity tax is distributed to the production sectors where it is imposed (collected). The distribution of each ordinary commodity tax or subsidy (commodity taxes and subsidies except the value added taxes) is easily derived by

$$(6.9) \qquad Y_k = \Sigma_P L_{k_P}^{+'} e$$

where Y_k is a vector of ordinary commodity tax k by production sector,

Σ_P is an aggregation matrix which add up production activities belonging to the same production sector (see equation (5.7)),

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1) Note that we are in general assuming no re-export. The market share for imports of a given commodity is therefore the ratio of total imports to domestic production less exports. 2) The overall equilibrium condition $b_{AT}'A = 0$ of equation (3.15) is fulfilled since each row sum of $(L_k^+ - L_k^-)$ is zero.

and

$L^+_{k_p}$ is the production activity part of L^+_k for the ordinary commodity tax k.

Value added taxes as a component of value added in each production sector are in general computed as the difference between value added taxes on commodity outputs less value added taxes on commodity inputs.[1] Value added taxes by production sector may then be computed by

$$(6.10) \quad Y_k = \Sigma_p (L^+_{k_p} - L^-_{k_p})'e$$

where Y_k is a vector of value added tax k (trade or production value added tax) by production sector, and

$L^+_{k_p}$ is the production activity part of L^+_k for the value added tax k.

In the procedure for commodity tax computations outlined above each of the nearly 40 commodity taxes and subsidies is treated separately. To cope with this rather unmanageable specification the actual tax computations of MODIS IV are based upon a class wise treatment of the commodity taxes and subsidies. The tax functions for a subset of commodity taxes and subsidies may be aggregated without any difference in the computational results if the taxes and the subsidies have identical tax base matrices (indicated by $\bar{W}_k$) and identical tax payer matrices (indicated by M_k) (see (6.3) and 6.7)). It is also possible to keep track of the individual taxes and subsidies as long as only one tax or subsidy in the subset is levied on the same commodity. Above, the commodity taxes and subsidies were classified as quantity production taxes, value production taxes, quantity trade taxes, value trade taxes, value added production tax and value added trade tax. All commodity taxes and subsidies belonging to the same tax class fulfill the requirement of common tax base and tax payer. The assumption of only one tax (or subsidy) on each commodity within a class is, with only minor exceptions, in accordance with the present Norwegian tax system. The actual commodity tax functions of MODIS IV are therefore based upon the six classes of taxes and subsidies referred to above. The tax coefficient matrix for each class is simply the sum of the individual ones (denoted by G_k, see (6.4)).

1) The actual tax rules are more complicated since value added taxes on commodity inputs in some production sectors are not or only partly refunded and value added taxes on some special commodities are not refunded. The tax functions of MODIS IV include these modifications of the general tax rule stated above.

The base year tax coefficient matrices are directly derived from the national accounts combined with information from the tax archives. As long as the changes in the tax rules for a given commodity tax are limited to proportional changes in the tax rates the updating may be performed directly on the base year tax coefficient matrices. Only with new taxes or more complicated changes in the tax rules of existing taxes, such as a differentiated change in the tax rate(s) or a change in the tax base, is it necessary to go back to the individual tax rate vectors T_k and tax base vectors I_k to form the tax coefficient matrix G_k.

After the calculations based upon the class wise tax functions are performed the results must be distributed on the individual taxes and subsidies included in the class. With a one to one correspondence between commodities and commodity taxes and subsidies this is relatively straightforward.[1)]

Revenue calculations for sector taxes

As mentioned in the beginning of this section the proceeds of most sector taxes and subsidies are exogenously given. However, some of the commodity based sector taxes and subsidies are endogenously determined through tax functions; of these the customs duties, the investment tax and the investment subsidy are most important.

The tax base for customs duties is commodity flows from import activities. While commodity flows in basic value include customs duties, the market value of import activity levels exclude customs duties (in accordance with the national accounts and trade statistics conventions). The customs duties are therefore regarded as imposed (collected) by a special subsector of the trade sector.

The tax base for the investment tax is commodity flows, measured in basic value plus ordinary commodity taxes, to a selection of gross investment activities. In addition, value added tax on investment taxed deliveries are normally refunded. The tax base for the investment subsidy is commodity flows to a selection of gross investment activities in constant basic value. The market value of gross investment activity levels, in accordance with the national accounts, includes the investment

1) The problem of keeping track of the individual tax or subsidy in the few cases where two (or more) taxes of the same class are levied on the same commodity is solved by a system of updated proportional distributions.

tax and the investment subsidy and exclude refunded value added tax, i.e. the market value is here defined as "net" purchasers' value. Similar to the accounting convention adopted for the customs duties, the investment tax is also assumed to be imposed (collected), and the refunded value added tax and the investment subsidy to be received (granted), by special sub-sectors of the trade sector.

Indirect taxes and the solution of the inner model

As is easily seen from equations (6.3), (6.5), (6.6) and (6.7), the input and output matrices for commodity taxes are computed from given basic commodity prices and quantities. In general, the indirect tax computations therefore take place after the domestic basic price submodel and the inner quantity model have been solved. Then a fully consistent solution of the inner price side and thereby all real flows in current market values can be computed (see the discussion in section 4.3).

However, indirect tax parameters enter the solution procedure also at two earlier stages in the model solution. First of all the submodel for domestic basic prices includes the vector b_{IT} for subsidies and not refunded commodity taxes on commodity inputs, i.e. ordinary commodity taxes, per unit of sector level (see section (5.6)). The expression for b_{IT} is easily derived from the production part of b_{AT} (see (6.8)) by deleting all L_K^+ matrices, the L_k^- matrices for refunded commodity taxes, i.e. the value added taxes, and by aggregating over the production activities to production sectors. The activity levels then cancel out and the elements of b_{IT} are dependent only upon commodity basic prices and the distribution between import and domestic supply of each input commodity, apart from the specification of the tax rules and base year coefficients. As in the rest of the domestic price submodel, all market shares, including the supply distribution indicated by the elements of B_P^- and P_P^-, are predetermined (enter with a time lag) in each model computation (see section 4.3).[1] This makes the elements of b_{IT} independent of the quantity solution. This is necessary in order to solve for the domestic basic prices before the quantities. A detailed discussion of the derivation of the expression for b_{IT} is given in Longva and Tveitereid (1975).

1) The subscript P indicate the production part of the matrices. Since only deliveries to export have export prices the elements of E_P^- are all zero.

Secondly, the consumption submodel of the quantity side includes as arguments market prices for consumption activity levels. Prior to the solution of the quantity side, preliminary estimates of prices are made by means of (4.20) (see sections (4.2, 4.3 and 5.3)). The expression for $b_{A_C T}$ in these preliminary calculations is easily derived from the household consumption part of b_{AT} (see (6.8)). Since the household consumption activities have only commodity inputs all elements of the household consumption part of the L_k^+ matrices will be zero. The activity levels cancel out and the elements of $b_{A_C T}$ are therefore only dependent upon the distribution between import and domestic supply of each commodity, apart from the commodity basic prices, base year coefficients, and the specification of the tax rules. The supply distribution, represented by the elements of $\bar{B}_C$ and $\bar{P}_C$, then appears both explicitly and implicitly (through $b_{A_C T}$) in equation (4.20).[1] As discussed in section 5.5 most of the changes in the supply distribution are due to exogenous changes in the import shares. A very close approximation to the final $b_{A_C T}$ estimate is made by correcting the base year supply distribution by means of the exogenous import share adjustments.

As seen from the discussion above the expression for both b_{IT} and $b_{A_C T}$ are derived from the input tax function (6.3) on a price form (the activity levels cancel out). We may therefore label these functions <u>input tax price functions</u>.

The accounting part

As for direct taxes the value concept for indirect taxes in the model as well as in the national accounts is <u>taxes accrued</u>. There is an accounting part of the submodel for indirect taxes which transforms taxes accrued to <u>taxes paid</u>. Taxes paid is the concept used in the government income accounts (see diagram 2.3). This accounting part comes into operation after the solution of the main model and it is designed similarly to its counterpart in the submodel for direct taxes (see section 6.1).

The structure of the submodel for indirect taxes is outlined in diagram 6.2.

1) The subscript C indicate the household consumption part of the matrices.

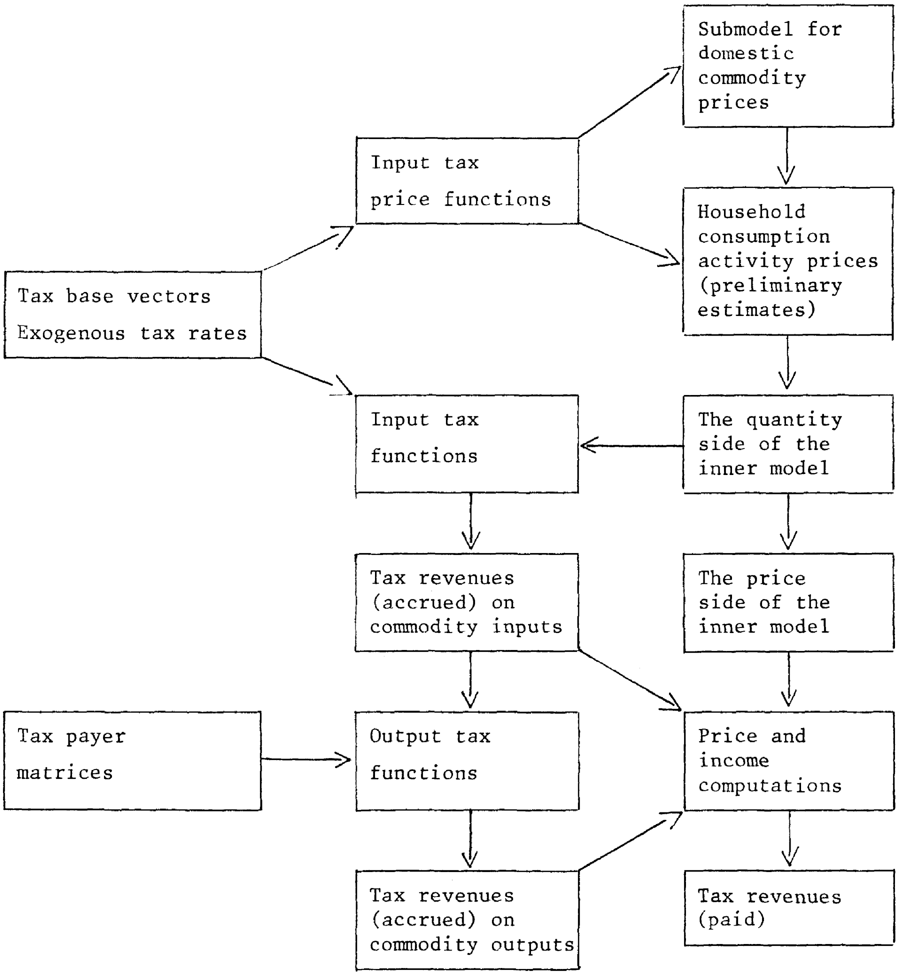

Diagram 6.2. Structural map of the submodel for indirect taxes

6.3. The interaction between the model and the fiscal budgets

Accounting framework

The fiscal budgets, and the corresponding accounts of the various institutional government sectors or bodies, consist of current expenditures and revenues on a cash basis. The revenue accounts include revenues from indirect taxes, direct taxes, sales of goods and services, operating surplus, rents etc., while the expenditure accounts include expenditures on goods and services (including labour), subsidies, transfer expenditures etc. In the national accounts, and in the model, the institutional government sectors are represented both by income and outlay and by capital finance accounts with flows on an accrual basis (see diagrams 2.2 and 2.3). The model and the national accounts distinguish between five institutional general government accounts or sectors, namely accounts for the treasury, other central government bodies, social security funds, local government and tax collectors (see below). The income accounts show the receipt (from direct and indirect taxes etc.) and disbursement (to consumption transfers, subsidies etc.) of income, with savings making up the difference. The capital finance accounts show the capital accumulation (real capital formation and increases in financial assets.)

The expenditures and revenues of the fiscal budgets are specified in several thousand items, each belonging to one and only one of the government institutional sectors. These items are classified in such a way as to link the government and national accounts together. The model specification of direct and indirect taxes, including subsidies, (discussed in sections 6.1 and 6.2 above) corresponds to a slight aggregation of the chapters covering these revenues and expenditures in the fiscal budgets. As discussed, the direct taxes are also classified by socio-economic groups (private institutional sectors). The links between taxes on an accrual and on a cash basis are provided by an institutional government sector called tax collectors which holds the margins or differences between taxes on an accrual and a cash basis.

Within the model there are specified about 20 different kinds of government transfers, a slight aggregation of the chapters covering these items in the fiscal budgets. As for direct taxes government transfers are classified by the socio-economic groups. In addition there are several kinds of transfers to and from abroad, between the government sectors etc. No correction is made, neither in the national accounts nor in the model, for the differences between transfers accrued and paid.

Each chapter of the fiscal budgets for goods and services (commodities and labour) includes in principle separate expenditure items for consumption purposes, investment purposes and for revenues (marketed government services). To serve as data input for the model and the national accounts government expenditures of goods and services for consumption purposes and revenues from sales of goods and services are given a four-way classification: (i) by institutional general government sectors (treasury, local government etc.). (ii) by general production sectors (a classification by functional units such as government administration sector, health sector, education sector etc.), (iii) by general government consumption sectors (by government consumption purposes such as health, education, defence etc.), and (iv) by type (that is expenditures for labour (wages) and for commodities bought, and revenues from sales of commodities). Similarly, government expenditures on goods and services for investment purposes are classified (i) by institutional general government sectors, (ii) by general government real capital formation sectors (a classification by functional units such as government administration sector, health sector, education sector etc.), and (iii) by gross investment sectors (categories of capital goods such as public buildings for educational purposes, power stations, etc.). Within the model there are specified close to 100 different kinds of commodity consumption expenditures, labour consumption expenditures and commodity sale revenues each (correspond to the specification of general government consumption activities) and about 50 different kinds of commodity investment expenditures (correspond to the specification of general government real capital formation activities). As for transfers no correction is made for the differences between accrued and paid government expenditures for goods and services.

The determination of items related to fiscal budgets

For use in fiscal budgeting the revenue models for direct and indirect taxes play a central role. There is no corresponding model covering the fiscal expenditures, apart from subsidies included in the indirect tax model. Government expenditures for goods and services are exogenously given, specified as discussed above. Commodity consumption expenditures by production sector serve as exogenous estimates for levels of activities for absorption of commodities in general government production (see section 5.2). Commodity and labour consumption expenditures by production and by consumption sector enter the submodel for government consumption (see section 5.7). Commodity investment expenditures by capital good categories serve as exogenous estimates for levels of general government gross investment activities (see section 5.1). Government transfers, rents and other items nessecary to complete the outlay side of the government sector accounts are also exogenously given. Revenues from sales of marketed government services are endogenously determined in the submodel for general government production (see section 5.2).[1)]

In contrast to the other items in the fiscal budgets the government expenditures on goods and services enter the model and the national accounts both in real and nominal terms. If we assume the relevant price indices to be endogenous then the government expenditures must be given either in current or in constant value. For one-year planning current value estimates may seem to be the most appropriate since these are the terms in which the fiscal budgets are actually formulated, while constant value estimates may be preferred for medium term planning reflecting the view that desired expenditures are specified in these terms.[2)]

However, in MODIS IV the prices are in general solved prior to the quantities and since the model is iteratively used by the central government to analyse the impact of fiscal policies it becomes mainly a question of conveniece whether the expenditures should be given in constant or current value. In the present version of MODIS IV all government expenditures for goods and services are given in real terms (constant value) but more flexible approaches are being considered.

The prices of government commodity expenditures are endogenously determined in the model; the part of p_{A_P} for activities for absorption of commodities in general government and the activity prices p_{A_I} for gross investment (see sections 3.2 and 4.2) give the market prices of commodity expenditures for consumption and investment purposes, respectively.

The estimates for labour expenditures differ from those of commodities in several respects. In real terms the estimates are given as Laspeyres indices with the base year occupational structures as weights. This is in accordance with the computational principles for these imputed figures in the national accounts. In addition to these

1) It should be noted that while the demand for marketed government services in constant value are endogenously determined, the prices of these commodities are included among the regulated prices and thereby exogenous (see section 5.6). 2) As pointed out by Davis (1976) exogenous government expenditures in money terms formally imply complete money illusion for the government bodies while exogenous specification in real terms imply a complete absence of money illusion.

real term estimates the wage rates and the number of wage earners by government production sectors are also exogenously given (see sections 5.6 and 5.2, respectively). This means that labour expenditures both in current and constant value are exogenously given, implying that the price indices are implicitly determined. Productivity change in each government sector will come as a result of shifts in the occupational structure.

7. THE PRESENT USE OF THE MODEL AND EXTENSIONS OF THE MODEL SYSTEM

7.1. The model environment and the administrative use of the model

The MODIS model is resident in the Central Bureau of Statistics. The construction, maintenance and use of the successive versions of the model have been major tasks of the Research Department of the Bureau since 1960. The official national accounts of Norway have been prepared and published by the Central Bureau of Statistics since 1946 and within the Research Department since it was established in 1950. The model building work has thus been performed in close coordination with the national accounting effort. On numerous occasions the detailed specifications and conventions in the national accounts have been changed to provide a better data base for the model. Such changes have, of course, mainly been related to details rather than to principles. The coordination with the national accounting work is of particular importance for the regular updating procedures of the model.

The other main link stretching out from the model is to the Ministry of Finance which has been and still is to an overwhelming extent the main user of the model. The decisions on the development and use of the MODIS model has been taken by the Bureau in close cooperation with the Ministry of Finance. The model building work has also been supervised - in particular with regard to the needs for further developments - by a special Model Committee ("Modellutvalget") with representatives from the main departments of the Ministry of Finance, the Central Bureau of Statistics, the Institute of Economics at the University of Oslo, and the Bank of Norway.

The national budget

The first MODIS model was conceived and designed to serve as a tool in the preparation of the macro-economic one-year plan called the national budget. The national budget is a government document containing a declaration of the policy which the government intends to pursue in the coming calendar year as well as a comprehensive description of the development in the economy which is expected to follow if the proposed policy is put into effect. The national budget is prepared during summer and presented to and discussed by the Parliament in the autumn of the year prior to the budget year but it is not formally acted upon by the Parliament. The policy recommendations implied in the national budget document are formally put before the Parliament for approval in separate documents. The fiscal budget is presented to the Parliament at the same time as the national budget. In the first half of the year the government prepares and presents to the Parliament a revised national budget for the current year.

The practice of national budgeting in Norway goes back to the early post-war period. The first national budget was prepared for 1947. The form and scope of the national budget has changed somewhat over the years. A survey of the background and development can be found in Bjerve (1959), (1968) and (1976). For a general survey of the present planning system, see Moe and Schreiner (1976), Strand (1980).

Macro-economic planning in Norway is part of the responsibility of ordinary government agencies. There are no separate planning institutions. Ministries, directorates and other administrative agencies take part in the planning process. For the short-term planning and the national budgeting the Economic Department of the Ministry of Finance coordinates the plan preparations and mediates between the political decision-making bodies and the agencies taking part in the planning process. The Ministry of Finance is thus the main user of the model.

The model plays a central role in the preparation of the national budgets. The flow of information in the planning process starts with the Ministry of Finance laying down general conditions for the plan in a circular to all agencies taking part in the planning preparation. The agencies (ministries and government directorates) return to the Ministry their proposals and estimates of plan figures based on their own assessment, taking into regard the directives of the circular. The Ministry is now ready to fill in the forms for exogenous variables of the model for the first round of model computations. In some cases the forms are filled in by the relevant agency, although for most of the input to the model the final judgement rests with the Ministry of Finance. Various submodels not formally included in the MODIS system are also used at this stage of the planning process.

After the first round of computations the results are analysed and proposals and estimates are revised by the Ministry or the relevant agency. The time schedule allows 4-6 rounds of model computations during the preparation of the national budget. Each round may include a number of alternatives. A similar process takes place in the winter as preparation of the revised national budget.

The four-year plan

After a few years experience with MODIS I the model was also used in the preparation of the four-year plan ("langtidsprogram"). The four-year plans in Norway were initiated at about the same time as the national budgets. They are rather similar to the national budgets in outline, somewhat broader in scope, but less specific with regard to policy proposals, see Moe and Schreiner (1976), pp. 70-84.

The model is used in the preparation of these plans in much the same way as for the national budgets. The four-year plan is presented to the Parliament every fourth year, and not as a rolling plan. In the later years the Ministry of Finance has for its own use as a background for the national budgets usually prepared five-to-seven year projections more than once a year by means of the model. The medium term projections are prepared by the Planning Department of the Ministry of Finance.[1) As a background for the medium-term projections perspective analysis covering 15-20 years are prepared by means of the MSG model.

Plan coordination

The MODIS model has been a great help in improving the coordination between the short-term and the medium-term plan. This rather common and often difficult practical problem usually arises when the original assumptions of the medium-term plan are revised during the plan period on the basis of more recent data, assessments and policy decisions. When revisions are comprehensive a proper updating of the plan may be cumbersome to undertake and the result may be that the medium-term plan fade into the background and only the short-term plan is used in the policy-making.

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1) From 1980 the Planning Department will be reorganized as a Planning Secretariat headed by a Minister and responsible for the medium-term plan coordination of all ministries.

A macro-economic model geared to deal with this problem may be a necessary requirement for achieving an appropriate coordination for short-term and medium-term planning. The MODIS models and especially MODIS III and IV have been designed to tackle this problem and make possible a recurrent updating of medium-term projections.

The national budgeting is the task the model was designed to tackle. The extension to cover also medium-term projections has been a natural development. In the later years the model has been used also on many other occasions than in the explicit plan preparation process. The model has been brought in on numerous occasions of ad hoc policy analyses, often of a very aggregate character relative to the specifications of the model. The advantage of the model in such situations is that the particular policy issues are analyzed within the framework of the current national budget or four-year plan. The disadvantages are basically that the model may have little to offer pertaining directly on the problem under consideration and it may seem a big apparatus to put into motion for a problem formulated in very aggregate terms.

Operationality

The demand from the administrative environment is for a highly operational model. Apart from being a computationally correct representation of the relations of the model, the model must be available for the user on short notice at almost any time. The time lapse between the user's final decision on input assumptions and the output of edited results should be minimized. At present the normal time lapse is from late afternoon till next morning. A shorter time lapse has been difficult to achieve with a model of this size.[1] During the user's intensive work sessions the input assumptions may be revised at very short intervals.

The model is basically a set of simultaneous relations between variables having the same date. There are, at present, only few occurrences of lagged variables in the model. The user's need for model results is more often than not related to a time path of variables rather than a single year. The length of the time path has varied from one to eight years. Often there is also a need for alternative assumptions in some or all of the years of a time path. The computer program for the

1) The technical possibilities are changing rapidly. We are now considering to adapt the model for a dedicated minicomputer or for interactive use.

model has been constructed to compute a number of time paths simultaneously. There is an upper limit on the product of the number of time paths and the maximum length of any time path. The limit, which has never been met in practical experience, is probably not less than 4-500. An ordinary run may consist of, say, two time paths of six years each and four two-year time paths, comprising altogether twenty solutions of the model. For special purposes runs with more than hundred alternatives have been tried.

There is also a strong need for repeated computations. Special arrangements have been made for making easy re-runs with only slight changes in input assumptions. The use of the model for one specific purpose, e.g. for working out the national budget for next year, will thus usually consist of a sequence of model runs each with a number of alternatives. The advantage of a sequence of runs rather than one single run are manifold. In a single run minor errors - and occasionally big ones - will invariably sneak in among the hundreds or thousands of input assumptions to be made. Throughout the sequence, with a scrutiny of the results between each run, most of them will be eliminated. Throughout the user's work session the input assumptions will undergo changes, partly as the result of decisions taken elsewhere or from reassessment of exogenous data like projections of world market prices. To some extent the input assumptions will also be changed on the basis of the insight gained through the model run sequence and results from submodels not integrated in the MODIS system.

The model has about 2000 exogenous variables. For a time path of six years there will be about 12 000 input assumptions to make. With alternative time paths and sequential runs the number of input assumptions in one single applications of the model will reach quite unmanageable heights even for the whole staff of a ministry department. In actual fact the number is not quite that overwhelming. The alternative time paths and the sequential runs will usually differ only slightly or to a limited extent from the "main alternative" in the first run of the model in terms of the number of exogenous variables that differ. To simplify the use of the model the set of input variables can be aggregated in alternative aggregations. The whole set of input variables is divided up into subsets by type of input assumptions. For each subset several level of aggregation can be defined in advance. To give a total set of input assumptions one may choose an appropriate level of aggregation for each subset. It is even possible to vary the level of aggregation for a given subset between years and alternatives, for instance by giving more detailed assumptions for the first year and more aggregate assumptions for the later years.

The possibility of alternative levels of aggregation of the input assumptions is wholly on the outside with no influence on the solution of the model and means very little for computing time and costs. The aggregate assumptions are spread out in full detail using base year data for the relative distribution.

On the output side the output from the model consists of values of about 5 000 variables for each year of each time path. The number of individual values may thus easily come in the range of hundreds of thousands. The whole set of values for output variables is passed on to a program package for tabulating national accounts data. The output variables from the model are given identification codes which correspond closely to those used within the national accounting system with identical codes when variable definitions coincide. The tabulating system includes a library with a wide range of edited tables. The user may choose tables from the library to fit the problems under consideration or even have special tables made to order if they can be constructed by manipulating the values of the 5 000 variables coming out of the model. The produced tables may also include historical values from the national accounts for the preceding years according to the user's wish.

In spite of various simplifying options for data input and output, the detailed specification of variables in the model is an obstacle for widespread use of the model. No potential user can sit down comfortably and make himself a national budget by means of the model without something like the administrative apparatus of the Economic Department of the Ministry of Finance. But it has never been supposed to be easy to make a national budget. The degree of detailed specification in the model today reflects to a great extent the requirements of the national budgeting process as it is presently performed. Over the period since MODIS I there have been requests for more detailed fragments of the macro-economic picture. MODIS IV is an attempt of coming to grips with such requests as far as the statistical data base allows.

The model is also used to produce impact tables with very detailed variable specifications.[1)] The impact tables are used by the Ministry of

1) The impact tables contain the partial derivatives, numerically calculated, of the reduced form of the model. The tables are constructed by solving the model for a grate number of alternatives, in each alternative a different group of exogenous variables is changed slightly to produce the impact on main categories of endogenous variables. The impact coefficients (the partial derivatives) are recalculated each time the model is updated, i.e. once a year, and normally the impact of separate changes in 100-150 groups of exogenous variables are computed.

Finance in the preparation of input for sequences of model runs and also for ad hoc analysis. The tables are freely available and are used for widely varying purposes when a full model run is not needed. The latest published impact tables are Cappelen and Sand (1980).

7.2. Further extentions of the model system

The MODIS model is a very incomplete representation of the interrelations of the economy. The inaccuracy of the coefficients of the model is one aspect of this, the unavoidable uncertainty of many of the model's relations is another. The model is also incomplete in the sense that important interrelations of the economy are completely left out of the model. This incompleteness makes the model very open. As discussed in chapter 5 the set of exogenous variables includes several variables that can hardly be regarded as independent relative to the whole set endogenous variables of the model. While the computer will take care of the consistency built into the relations of the model it is left to the user to check consistency in a wider sense, paying attention also to interrelations not covered by the model. The iterative use of the model through a sequence of runs is of great importance for this purpose.

The recognized incomplete character of the model naturally raises the question of extending the model to become a more self-contained representation of the economy. One main obstacle in pursuing this idea, is the sheer size of the model. An extension of the model with new relations will easily make the model unmanageable from an operational point of view without drastically reducing the dimensions of the model or reprogramming from scratch.

Another consideration preventing the extension of the model to include more behavioural relationships is the fact that model is meant to be, and actually is, deeply embedded in the administrative planning routines. If this is to continue the detailed links to the fiscal budget must be maintained and further developed. In addition, the model should not be developed to become a "black box" for the model users by introducing more or less wellfounded and maybe not generally accepted relationships into the model, however promising such experiments may seem from a purely exploratory point of view. All experiences with the MODIS model up to now have shown that if the model is to be used for useful purposes in the national budgeting and planning process it must be possible to have a full understanding of the logic of the model and to be able to "control" the running of it in such a way that the results can be accepted by the model users as a reasonable picture of the economy. This calls for a rather open model where even the rather few behavioural relationships can be modified at the user's choice.[1)]

In extending the MODIS system the strategy chosen is to leave the formal model as it is with only modest improvements in basic structure and improve the use and usefulness of the model through conjunction in use with other models. These models are of two kinds: General models considerably more aggregated than MODIS IV but covering the whole economy and special support models to improve deficient of simply non-existent parts of MODIS IV. A typical use of a support model is thus to provide exogenous estimates for MODIS IV. The purpose of the general models is to provide better and more manageable tools for analysing important economic issues such that the results can be "translated" into MODIS specifications for fully detailed breakdown using the latter model.

Several of the support models designed to provide exogenous estimates ("pre-models" in the current jargon) are discussed in chapter 5.[2)] Most of these pre-models include variables which are endogenous in the main model. This means that an iterative use of the system is necessary to achieve fully consistent solutions. With the more or less continuous use of the MODIS model satisfactory degrees of consistency are usually well within reach.

One pre-model called KONK has not been discussed in the preceding chapters. KONK is developed with the specific purpose to evaluate the consistency of exogenous estimates used in MODIS IV for cost and foreign trade variables. The model includes as endogenous variables all the crucial variables about the foreign trade that are exogenous in MODIS, namely export prices, domestic prices of exposed industries, and changes in market shares for imports as well as exports. Changes in wage rates, productivities and mark-up rates are exogenous as in MODIS IV. The development of this support model for MODIS should be considered as a

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1) For an example see the discussion in section 5.3. 2) The submodel for direct taxes (see section 6.1) may also be regarded as a pre-model.

way of systematizing the preparation of exogenous estimates.[1)]

Another group of support models uses results from the main model to further calculations ("post-models"). Some of these models are discussed in section 5.7. An "energy budgeting" model is an example of a somewhat more detached post-model. This model translates the MODIS results for energy flows in constant values into physical units in great detail, see Miljødepartementet (1979). The model is used to provide forecasts for future energy consumption and to check the consistency between the overall economic plans and the physical sector plans for energy supply (especially hydro electric power supply).

Other examples of such post-models which are using MODIS-results are a model for financial items of the foreign account (see Cappelen and Skoglund (1977)) and a multi-regional model (see Bjerkholt, Skoglund and Skomsvold (1978), Skoglund (1980)). The KRØSUS model as referred to in section 5.1 may also be considered as a post-model of MODIS IV.

Under construction at present is a comprehensive system of general models to surround MODIS IV rather than replace it. The central elements of this system will be three macro-economic models of different time horizon. One of these is a medium-term annual model with about 30 industries, the others are a quarterly short-term model with about 12 industries and a long-term model with about 30 industries.

The medium-term model will have strong similarities with MODIS IV and may be considered at the outset as a more aggregate version of MODIS. Plans for further development of this model, which has been baptized MODAG, will turn it into a more complete model of the economy. Results from exercises with the more aggregate model can be spread out in full MODIS IV detail by appropriate translation of assumptions and results into exogenous variables and parameters of MODIS IV. In this way one will try to combine the development of more complete models while still retaining the links with the details of MODIS IV.

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1) The KONK model has an input-output structure consisting of only four aggregate industries, three exposed industry groups and one industry aggregate for all sheltered industries. In the model the changes in the price indices of exposed industries, i.e. the export prices as well as the domestic importcompeting industries, are determined as weighted averages of unit costs and representative world market price indices. The cost structure of the industries are connected through the input-output structure of intermediate goods. On the basis of the price forecasts the changes in market shares both for exports and imports are derived straightforwardly as the product of estimated or assumed elasticities and time-weighted differences between Norwegian and world market prices. In MODIS the volume of exports is exogenous rather than the market share, hence it is in addition required with an estimate of overall growth of the export market. The model is also used to explore consequences of changes in competitiveness from a MODIS reference path. For a more detailed presentation of the model, see Hoel (1978).

As discussed at length in the preceding chapters, the detailed picture of the manifold economic transactions given by the national accounting system is carried over into the model. Until now the MODIS-system has mainly focused upon the horizontal aspect of the planning process. By the horizontal aspect is meant the connection and integration of plans for separate parts of the economy with only partly overlapping sets of variables. When more complete and aggregate models are introduced to provide key indices for MODIS IV the vertical aspect of the planning process will be given a more systematic treatment. This will extend the role of MODIS IV as an integrative framework for national budgeting and planning.

Appendix 1

THE ESTIMATION OF THE INPUT-OUTPUT COEFFICIENTS FOR THE INDUSTRY PRODUCTION ACTIVITIES[1)]

In this appendix we shall present the formal method applied in estimating the input-output or activity coefficients of the industry production activities. As discussed in section 3.2 the basic assumption of the quantity input-output model of MODIS IV is that the quantities of commodity inputs and outputs of an activity are related by fixed proportions and that these activity coefficients are estimated from the base year of the model (usually the year prior to the current year).

To describe the estimation procedure we introduce the following notation for base year commodity-activity flows:[2)]

A^{+}_{ij} = output of commodity i from activity j

W^{+}_{A} = activity output matrix with typical element A^{+}_{ij}

A^{-}_{ij} = input of commodity i to activity j

W^{-}_{A} = activity input matrix with typical element A^{-}_{ij}

A_{PP} = activity level vector with typical element A_{PPj}

The activity levels of industry production activities are defined by

(1) $$A_{PPj} = \sum_i A^{+}_{ij} - \sum_i A^{-}_{ij}$$

The activity coefficients are estimated by

(2) $$\Lambda_{PP} = (W^{+}_{A} - W^{-}_{A})\, \hat{A}_{PP}^{-1}$$

where Λ_{PP} is the industry production part of the activity coefficient

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1) A more complete and detailed presentation is given in Furunes and Longva (1976), see also Bjerkholt and Longva (1970). 2) In this appendix we use the term <u>activity</u> as short for <u>industry production activity</u> and, similarly <u>sector</u> for <u>industry production sector</u>.

matrix Λ in which each element λ_{ij} gives net output of commodity i per unit activity level.[1)]

As seen from (2) the production structure of a given activity is described by (i) the output structure, i.e. the commodity output composition, (ii) the input structure, i.e. the commodity input composition, and (iii) the commodity productivity, i.e. the proportion between total commodity output and total commodity input.

The elements of the W_A^+ and W_A^- matrices are in general not directly observable. The activities can be regarded as macro processes aggregated across establishments within the same industry (production sector). Establishment is the unit of observation in the production part of the national accounts. When there are more than one activity in a sector the subdivision thus imply a problem of estimation from observable data.

The data source used in the estimation of the activity input and output matrices for the base year is the corresponding commodity-by-industry matrices. For the base year commodity-sector flows, we introduce the following notation:

S_{ij}^+ = output of commodity i from sector j

S_{ij}^- = input of commodity i to sector j

The matrices of commodity-sector flows are denoted as W_S^+ and W_S^-.

W_S^+ = sector output matrix with typical element S_{ij}^+

W_S^- = sector input matrix with typical elements S_{ij}^-

The definitional relationship between W_A^+ and W_S^+ and between W_A^- and W_S^- are given by

(3) $$W_S^+ = W_A^+ \Sigma_{PP}' \quad \text{and}$$

(4) $$W_S^- = W_A^- \Sigma_{PP}'$$

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1) This presentation is somewhat simplified since the activity levels are evaluated in market values while the commodity flows are evaluated in basic values.

where Σ'_{PP} is an aggregation matrix which aggregate over activities belonging to the same sector.

At the chosen level of industry and commodity aggregation, as discussed in section 3.2 the same commodity can be, and often will be, produced in more than one sector and each sector will normally have more than one commodity output. The problem we are facing is thus to allocate the commodity inputs and outputs of each industry between the different activities specified within each industry. The derivation of the activity input and output matrices from the corresponding ones for sectors starts with an a priori specification of the distribution of sector outputs among activities, i.e. with a specification of the elements of W_A^+. This procedure is tantamount to a definition of the activities of the model.

It is assumed that most commodity outputs from a sector are produced with separate production functions, e.g. non-jointly. The main principle followed in subdividing a sector into activities is therefore to let each important output commodity be produced in a separate activity. Minor output commodities, if any, are lumped together and included as joint products in the activity with the biggest share of total output of the sector. This a priori allocation of all commodity outputs in each sector between activities gives us the elements of W_A^+ and Σ_{PP}, automatically fulfilling the restriction (3).

In distributing commodity inputs of a sector among its activities we use equation (4) as the starting point. Equation (4) says that the content of the columns of W_A^- is to be determined in such a way that the sum over all activities in a sector equals total sector input of each commodity. With n_A activities and n_S sectors (4) gives us thereby n_S independent equation between the n_A unknown columns of W_A^-. Equation (4) has thus $n_A - n_S$ degrees of freedom, i.e. so many as the excess of the number of activities over the number of sectors.

The degrees of freedom can be eliminated by imposing additional restrictions between the activity columns of W_A^-. An especially simple approach is to assume that all activities in the same sector have identical input structure and commodity productivity. This implies a general assumption of what we later shall refer to as sector technology.

Another possibility will be to assume that all activities with the same (main) output commodity have identical input structure and commodity productivity. This is straightforward if the number of commodities equals the number of sectors. However, if there are more commodities than sectors, as is the case in MODIS IV,[1] this pure commodity

1) See the discussion of the commodity classification in section 3.3.

technology approach must be modified for instance by assuming that all activities with output commodities which have the same sector as main supplier have identical input structure, i.e. a kind of sector technology. Such a mix of sector and commodity technology assumptions, is used in the present version of MODIS IV in estimating the elements of W_A^-.

It is important to note that, as long as the degrees of freedom of (4) are eliminated by assumptions of linear relationships between the input structures and commodity productivities of the various activities, the matrix W_A^- will at most have so many linearly independent columns as there are sectors. This is just a reflection of the fact that W_S^+ and W_S^- contains all our empirical information about the production structure in the base year of the model, i.e. about the available technologies.

The formal estimation procedure followed in MODIS IV starts out with a grouping of the n_A activities in n_S groups and by assuming that the input structure and commodity productivity are identical for all activities within the same group. This can be done by selecting a main activity in each group (for instance the one with the largest output) and linking the other activities in the group by proportionality to the main activity. The chosen procedure is, however, more symmetric with regard to the partaking activities. Each activity is linked to one of a set of n_S input structures, constituting a matrix T of dimension equal to the number of commodities (n_X) times the number of sectors (n_S). The element on row i and column j represent total input of commodity i to those activities that form activity group j. The matrix T may then be interpreted as the input matrix for a set of "macro" activities, each macro activity consisting of activities with identical input structure and commodity productivity.

Each activity input vector, i.e. each column of W_A^-, is now related to the column of T corresponding to the activity group it belongs to. We can thus write

(5) $$W_A^- = T\theta$$

The elements on each row of θ allocate the input vector of the corresponding "macro" activity between the activities belonging to the activity group. All activities with non-zero elements on the same row belong to the same group and activities within such a group will have identical input structure if there is one and only one element in each column of θ. With the interpretation of T given above the proportions between the elements different from zero on the same row of θ must be set equal to the proportions between the total commodity inputs of the activities in the group. These activities will have identical commodity productivities i.e. identical proportions between total commodity output and input, if the ratios between the elements on each row of θ are proportional to the ratios between total commodity outputs.[1)]

As will be easily seen from equation (6) below, the normalization of the rows of θ is irrelevant for the computation of the elements of W_A^-, it is only the ratios between the elements that count. The question of normalization is thus a question of convenience, although the actual interpretation of T will change. T is, however, only a matrix of additional variables without any separate interest. In the actual specification of θ in MODIS IV the elements on the same row are set equal to total commodity output of each activity belonging to the group. The total outputs are known from W_A^+.

When the elements of W_S^-, Σ_{PP} and θ are given (4) and (5) impose $n_S + n_A$ equations between the $n_S + n_A$ unknown columns of T and W_A^-. The explicit solution of W_S, θ and Σ_{PP} using (4) and (5) are

$$(6) \qquad W_A^- = W_S^- \, (\theta \Sigma_{PP}^{'})^{-1} \theta$$

For this system to be uniquely determined it is required that $(\theta \Sigma_{PP}^{'})$ is a non-singular matrix. The non-singularity condition will depend upon the actual content given to θ, singularity normally indicating a faulty logic in the specification.

In the model we have specified about 200 industry production activities. About 20 of these are assumed to have no commodity input and for 10, mostly covering oil and shipping, the input and productivity structure are based upon exogenous information, such as engineering data etc. The rest, about 170, are grouped in about 125 groups of activities with different input and productivity structures, i.e. equal to the number of industry production sectors.

1) It is possible to eliminate the degrees of freedom of (4) without assuming that the input structures and commodity productivities are identical for all activities within the same group. This can be done by allowing for more than one element in each column of θ. The columns of T may then be interpreted as proportional to the input structures in a set of n_S different technologies available in the base year of the model. The elements of θ then determine the input structure and the commodity productivity of each activity by combining these input technologies and by relating them to the commodity output in the way desired.

It is possible to distinguish between two main categories of activity groups in the specification of the elements of the 125 columns of θ, namely groups with commodity technology and groups with sector (industry) technology.

In a commodity technology group all activities have the same main output commodity. All the activities have the same input and productivity structure irrespective of which sectors the activities belong to. We are here assuming that the commodity is produced by a characteristic technology.

In a sector technology group all activities belong to the same sector. All the activities have the same input and productivity structure irrespective of which commodities the activities produce. We are here assuming that different commodities are produced by the same technology, and that the same commodity produced in different sectors are produced by different technologies.

In the present version of MODIS IV the assumption of commodity technology is, with few exceptions, made whenever possible, i.e. we have normally grouped together activities with the same main output commodity. Altogether we have formed more than 20 such groups, covering about 50 activities.

Many commodities are the main product in only one activity and about 80 of these activities form an activity group each. It is a matter of terminology whether to characterize each of these "groups" as having a sector or a commodity technology.

For the rest, nearly 25 groups covering about 50 activities, the assumption of sector technology is made, often more out of necessity due to the data sources available than due to a belief in the correctness of the assumption made. In most cases the output commodities of the activities within such a group have the same sector as main producer. Normally this means that no other activities have these commodities as their main output. All information we have about the input and productivity structures is then limited to the sector they belong to. However, this does not mean that the input and productivity structures of these activities are always identical to that of the sector they belong to because in many cases there are other activities in the sector for which we have assumed commodity technology.

Appendix 2

SUMMARY OF MAIN DIMENSIONS[1)]

Main groups of sectors	Number
Import sectors	20
Export sectors	14
Industry production sectors	122
General government production sectors	17
Household consumption sectors	48
General government consumption sectors	65
Gross investment sectors	31
Industry real capital formation sectors	27
General government real capital formation sectors	18
Commodities	
Industry commodities	187
Marketed government services	10
Main groups of internal activities	
Import activities	130
Export activities	111
Industry production activities	217
General government production activities	32
Household consumption activities	48
Gross investment activities	35
Main groups of external activities	
General government consumption activities	96
Industry real capital formation activities	93
General government real capital formation activities	52
Main groups of income categories	
Components in value added	11
Government transfers	21
Direct taxes	28
Indirect taxes	85

1) The indicated dimensions are for the 1979 version of the model. There are normally some changes in the specification of variables each time the model is updated, i.e. once a year.

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