



**ESTIMATING PRODUCTION FUNCTIONS  
AND  
TECHNICAL CHANGE FROM MICRO DATA**

**An Exploratory Study of Individual Establishment Time-  
Series from Norwegian Mining and Manufacturing  
1959—1967**

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OG  
TEKNISKE ENDRINGER FRA MIKRO DATA**

**Analyser på grunnlag av tidsrekker for individuelle  
bedrifter fra norsk bergverk og industri  
1959—1967**

By/Av  
Vidar Ringstad

STATISTISK SENTRALBYRÅ  
CENTRAL BUREAU OF STATISTICS OF NORWAY  
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**SAMFUNNSØKONOMISKE STUDIER NR. 21**



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## FORORD

Det arbeid som her legges fram, ble utført mens forfatteren var universitetsstipendiat ved Sosialøkonomisk institutt, Universitetet i Oslo. Når Statistisk Sentralbyrå sender analysen ut i serien Samfunnsøkonomiske studier, har dette følgende årsaker: Arbeidet bygger helt ut på Statistisk Sentralbyrås tallmateriale, vesentlig data for bedrifter til utvalgte store foretak i bergverk og industri, og forfatterens bearbeiding av dette har krav på videre interesse. Det gir eksempler på bruk av statistiske metoder som hittil ikke har vært brukt i vesentlig grad i anvendt økonometrisk forskning, klargjør svakheter ved datamaterialet og gir enkelte forslag til å eliminere disse. Analysen viser også interessante trekk ved produksjonsstrukturen og endringene i denne i de bergverks- og industri-bransjer som analysen omfatter. Dessuten gir analysen et bidrag til arbeidet med å utnytte Statistisk Sentralbyrås datamateriale til å kærtlegge sammenhenger som kan bygges inn i planleggings- og prognosemodeller for den norske økonomien.

Statistisk Sentralbyrå har finansiert analysen, og alle beregningene er utført ved Byråets regneanlegg. Forfatteren har selv stått for programmeringsarbeidet.

Statistisk Sentralbyrå, Oslo, 18. november 1970

Petter Jakob Bjerve

## PREFACE

The present study was carried out while the author held a scholarship at the Institute of Economics, University of Oslo. It is published in the series "Samfunnsøkonomiske studier" (Studies in National Economy) because of its close relationship to the regular work of the Central Bureau of Statistics. Thus, the study is based entirely on official statistical data, mainly data from establishments of large mining and manufacturing firms and illustrates uses to which such data may be put. Certain weaknesses in the data are unveiled by the study and proposals are made for their possible elimination in future censuses. From a methodological point of view the study exemplifies uses of statistical methods which have barely been tried in applied econometric research. For the mining and manufacturing industries covered by the study conclusions are reached about production structure and technical change which may turn out to be useful in future work of the Central Bureau of Statistics in constructing planning and forecasting models for the Norwegian economy.

The study was financed by the Central Bureau of Statistics. All computations were carried out at the Bureau's computer department according to computer programs written by the author.

Central Bureau of Statistics, Oslo, 18 November 1970

Petter Jakob Bjerve

## FORFATTERENS MERKNADER

Som den engelske tittelen på analysen antyder, er den av sonderende karakter, hvor en rekke forskjellige problemer i forbindelse med estimering av produktfunksjoner og tekniske endringer er tatt opp. Analysen er derfor blitt noe uensartet, med nokså svake bånd mellom de forskjellige delene av den. En kan kanskje si at det heller er en samling av mindre analyser enn én analyse. Og det var opprinnelig min hensikt å presentere resultatene i en serie mer eller mindre uavhengige artikler; om beregning av manglende observasjoner i økonometriske modeller; om målefeil i simultane likningssystemer; om multipl tippel testing i økonometriske modeller; og om visse problemer i forbindelse med måling av tekniske endringer. En viktig innvending kan imidlertid reises mot en slik måte å presentere resultatene på, nemlig at det empiriske grunnlaget er felles for alle del-analyser. Det ble derfor bestemt at de skulle presenteres samlet.

Selv om analysen tar for seg et betydelig antall spørsmål, er det uten tvil en rekke løse ender. Dette er imidlertid en studie i anvendt økonometri, og derfor har jeg valgt å se bort fra alle problemer som ikke på en eller annen måte kan belyses ved hjelp av det tilgjengelige datamaterialet. For eksempel ville jeg i Kapittel III ha foretrukket å bruke en dynamisk modell heller enn en statisk. En løs ende (eller kanskje heller en klasse av løse ender) er derfor tolkningen av resultater oppnådd ved en statisk modell når den "sanne" modellen er av dynamisk natur. Det ble faktisk gjort forsøk på å undersøke dette spørsmålet, men datagrunnlaget viste seg å være for dårlig til at resultatene kunne bli av særlig interesse. Under alle omstendigheter, selv om data var av god kvalitet, kan bare enkle dynamiske modeller bli analysert ved hjelp av den type data som er brukt.

Jeg har hatt meget god støtte i råd og veiledning fra en rekke personer. Dosent Herdis Thorén Amundsen, professor Zvi Griliches, professor Trygve Haavelmo, professor Leif Johansen, forsker Arne Amundsen, konsulent Karl Erik Biørn, vitenskapelig assistent Harald Goldstein og vitenskapelig assistent Steinar Strøm har lest mindre eller større deler av forskjellige utkast til analysen og gitt verdifulle merknader og forslag til forbedringer. Fru Janet Aagenæs har rettet opp og forbedret språket i det endelige utkastet. Jeg vil rette en takk til alle disse. Gjenværende feil og mangler er selvsagt jeg selv ansvarlig for.

Jeg vil også rette en takk til ansatte ved Sosialøkonomisk institutt, Universitetet i Oslo og Statistisk Sentralbyrå for dyktig maskinskriving av mine uryddige manuskripter.

Oslo, 6. november 1970

*Vidar Ringstad*

*AUTHOR'S NOTE*

As the title suggests this study is of an exploratory nature where a variety of problems concerning the estimation of production function parameters and technical change are considered. The study has thus become slightly heterogeneous with somewhat weak links between its various parts. One might say that it is a collection of smaller studies rather than one study. It was, in fact, my original intention to present the results obtained in a series of more or less independent articles: on the calculation of missing observations in econometric models; on measurement errors in simultaneous equations models; on multiple testing in econometric models; and on certain problems concerning the measurement of technical change. However, one important objection could be raised against this manner of presenting the results, namely that the empirical basis is common to all of them. It was thus finally decided to present them in one study.

Even though the study deals with a considerable number of issues there are unquestionably a number of loose ends. However, this is a study in applied econometrics and thus all problems that cannot in some way be illuminated by the data available are ignored. For example, in Chapter III I would have preferred to use a dynamic model rather than a static one. Thus one loose end (or perhaps rather a class of loose ends) is the interpretation of the results of a static model when the "true" model is of a dynamic nature. Attempts were made in fact to explore this issue but the data turned out to be too poor for the results to be of much interest. In any case, even though the data of the type used were of quite good quality only very simple dynamic models could be investigated.

I have benefitted greatly from the assistance and advice of a number of people. Professor Herdis Thorén Amundsen, Professor Zvi Griliches, Professor Trygve Haavelmo, Professor Leif Johansen, Mr. Arne Amundsen, Mr. Karl Erik Biørn, Mr. Harald Goldstein and Mr. Steinar Strøm have read smaller or larger parts of various drafts of the study and provided valuable comments and proposals for improvements. Mrs. Janet Aagenæs has done a good job in improving the language of the final draft. My gratitude goes to all of them. The remaining errors and shortcomings are, of course, my own.

I am also deeply indebted to members of the staff at the Institute of Economics, University of Oslo, and the Central Bureau of Statistics of Norway for the efficient typing of my unwieldy manuscripts.

Oslo, 6 November 1970

*Vidar Ringstad*

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## CHAPTER I. INTRODUCTION

During the sixties there has been a revival in the interest in production function estimation, apparently initiated through the famous study by K. Arrow, M.B. Chenery, B. Minhas and R. Solow (1961): "Capital-labour substitution and economic efficiency". In contrast to most other previous studies this one focused on the substitution dimension of the production function, deriving a function where the form of the isoquants was subject to estimation. This function, the CES production function<sup>1)</sup> has become quite popular and today there are numerous studies on the estimation of that function, particularly the key parameter of it, the elasticity of substitution.<sup>2)</sup> A number of attempts have also been made to develop new production functions with different properties, both concerning substitution and scale.<sup>3)</sup>

If nothing else, one important conclusion could be drawn from these studies, namely that it is very difficult to estimate higher order properties of production functions, such as the elasticity of substitution, with any reasonable degree of accuracy by means of data usually available for econometric production function studies. However, the opposite would in fact be quite surprising since we still experience serious difficulties in estimating such first order properties as marginal productivities and marginal elasticities. Some important reasons for this will be dealt with in the present study.

Thus new and constantly more refined production functions can hardly solve any of the basic problems present in production function estimation. Rather, there is a need for better theories concerning the behaviour of the

---

 Notes :

- 1) CES is an abbreviation for Constant Elasticity of Substitution.
- 2) Having the two factor production function  $y = F(x_1, x_2)$ , the elasticity of substitution is defined as

$$b(x_1, x_2) = \frac{\frac{x_2}{x_1} d\left(\frac{x_1}{x_2}\right)}{\frac{f_1}{f_2} d\left(\frac{f_2}{f_1}\right)} \Bigg|_{y = \text{constant}}$$

where  $f_i = \partial F / \partial x_i$  is the marginal productivity of the  $i$ -th factor.

- 3) Cf. for example: NERLOVE (1967): Recent Empirical Studies of the CES and related Production Functions in BROWN (ed.): The Theory and Empirical Analysis of Production.



production units, and more than anything else there is a need for better data. Limitations in the data are likely to be the more efficient constraint in the development of production function estimation.

Most empirical studies of production functions are based on more or less aggregate data, such as for industries or for a country. Very few are based on data at the micro level, for establishments or firms.<sup>1)</sup> In fact, a quite recent study seems to be the first one that is based on micro data as well as covering most activities of a main production sector of a country, namely manufacturing of Norway.<sup>2)</sup>

As compared to aggregate data the most apparent virtues of micro data from sources like Censuses of Establishments and Industrial Production Statistics are the vast number of observations and the fact that relevant explanatory variables show a much wider variation making it easier, in principle, to estimate more accurately the parameters of interest. However, such data have some serious problems of their own. For example, it has been shown that errors in variables are very serious in such data, probably much more serious than in aggregates, and that there are generally serious missing observation problems.<sup>3)</sup>

Some of the data problems are due to the fact that such sources of micro data are not designed to be used as empirical bases for econometric studies. Thus when actually using them in econometric studies one must expect to encounter problems of various kinds. On the other hand, if it is accepted that such sources are actually or potentially useful for econometric production function studies, at least some of their more serious weaknesses could be eliminated in future vintages of such statistics. We must then, however, try to determine which weaknesses are the more serious ones. One apparent, and probably the most efficient way of doing this is just to carry out econometric studies on such data. And in fact, an important result of the study based on Norwegian Census of Establishment data referred to is the unveiling of weaknesses of these data from a production function estimation point of view.<sup>4)</sup>

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N o t e s :

- 1) A recent exception is: KRISHNA (1967): Production Relations in Manufacturing Plants: An Exploratory Study.
- 2) GRILICHES and RINGSTAD (1971): Economies of Scale in Manufacturing and the Form of the Production Function: An Econometric Study of Norwegian Establishment Data. This study is based on data from the Census of Establishments 1963 and covers 5,361 establishments.
- 3) Ibid., Ch.s III and IV.
- 4) Ibid., Ch. VI.

The present study is in this respect an extension of that study, since we here will try to determine the virtues and weaknesses of another but related body of data, namely the establishments of large firms in Norwegian mining and manufacturing with data for the period 1959-1967. However, the scope of the study is wider than that. Basically, this is a study in applied econometric methods where some well-known and some not so well-known tools are used to squeeze information from the data available, both concerning the properties of the data and the production structure of the industries concerned.

We are thus also interested in the results per se and since this study and the study of the Census of Establishment data referred to above cover roughly the same industries, the results of the two studies will be compared whenever this is possible.

The theoretical framework of this study is rather simple. Thus, in contrast to most related studies no separate chapter is devoted to a discussion of theoretical issues. Instead, the theoretical tools needed are derived and discussed in the context they are used. To some extent we will also refer to other studies where the relevant theoretical issues are discussed.

In most parts of the study, however, one and the same "model" is used. We will therefore briefly explain the contents of it here.

We assume that the following CES production function is a valid representation of the production structure of the establishments to be analysed:

$$(1) \quad V = \gamma(\delta L^{-\rho} + (1-\delta)K^{-\rho})^{-\frac{\epsilon}{\rho}} e^{u'}$$

where  $V$ ,  $K$  and  $L$  are value added, labour input and capital input respectively, and  $u'$  is a stochastic residual variable.<sup>1)</sup> We also assume that profit is maximized with respect to labour, with perfect competition both in the output and labour markets.

This yields the following behaviour relation

$$(2) \quad \frac{V}{L} = a W^{\mu} L^{(1-\mu)(1-\epsilon)} e^{v'}$$

where  $W$  is the wage rate-output price ratio,  $v'$  is a stochastic residual error and  $\mu = \epsilon/(\epsilon+\rho)$ .<sup>2)</sup>

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#### N o t e s :

- 1)  $\gamma$  is often denoted as the efficiency parameter,  $\delta$  the distribution parameter,  $\rho$  the substitution parameter since  $b = 1/(1+\rho)$  is the elasticity of substitution, and  $\epsilon$  the scale parameter since it is equal to the elasticity of scale.
- 2) We could also write the behaviour relation as  $V/L = a_2 W^{b_V} L^{(1-b_V)((\epsilon-1)/\epsilon)} e^{v'}$  where  $b = 1/(1+\rho)$  is the elasticity of substitution. Cf. NERLOVE (1967), op.cit., and GRILICHES and RINGSTAD (1971), op.cit., Ch. II.

The model (1) and (2) is not easy to use, however, particularly since the production function is not, and cannot be transformed to, a relation linear in the parameters. To obtain a model that is easier to handle we utilize two results obtained in the study of the Census of Establishment data: a) The level of the output elasticities of labour and capital is fairly well determined by a Cobb-Douglas relation, which is a CES relation with an elasticity of substitution;  $b = 1$ , even in cases where the results of a CES relation suggest that the elasticity of substitution is in fact different from one. b) The estimates on the elasticity of substitution obtained from the behaviour relation (2) are not sensitive to an assumption of constant returns to scale.<sup>1)</sup>

Thus instead of using (1) and (2) we use the following relations:<sup>2)</sup>

$$(3) \quad V = a_0 L^{\alpha} K^{\beta} e^u$$

$$(4) \quad \frac{V}{L} = a_1 W^b e^v$$

or written in logs:

$$(5) \quad \ln V = \ln a_0 + \alpha \ln L + \beta \ln K + u$$

$$(6) \quad \ln \frac{V}{L} = \ln a_1 + b \ln W + v$$

So far we have said nothing about the error terms. We can think of four reasons for introducing error terms in econometric relations:

- 1) Incorrect specification of functional forms
- 2) Left-out variables
- 3) Errors of measurement
- 4) Non-constant parameters

For (5) and (6) there are reasons to believe that all four types of error term components are present, and in fact most of the study concerns the analysis of these "causes" of residual errors.

If we really believe that the CES relation (1) is the "true" production function and that both  $\epsilon$  and  $b$  are different from one,  $u$  and  $v$  must necessarily contain approximation errors. But, as has been argued above, this should not raise serious difficulties in the estimation of the parameters. The three other types of error term components turn out to be more

---

N o t e s :

- 1) GRILICHES and RINGSTAD (1971), op.cit., Ch. IV. Cf. also RINGSTAD (1967): Econometric Analyses Based on a Production Function with Neutrally Variable Scale-Elasticity.
- 2) Related models are discussed in: MADDALA and KADANE (1966): Some Notes on the Estimation of the Constant Elasticity of Substitution Production Function, and GRILICHES and RINGSTAD (1971), op.cit.

serious, particularly the errors present in the measures of the variables used. Thus in this study much of the discussion will concern the properties of the error terms and the problems they raise concerning the estimation of the production function parameters.

The plan of the study is as follows: In the next chapter we review the empirical basis of the study, the information available, the definition of variables and the classification of industries. In a separate section of that chapter we deal with a problem quite common in micro econometric studies, namely incomplete sets of data or missing observations. This section is also intended to be of some methodological interest since it discusses methods for calculating the observations missing, which in our case concern the capital variable.

In chapter II we also present a few tables with some sample statistics of the main variables used in this study. We are in this context mainly interested in their variation along the three main dimensions of the observations: establishment, time and size.

Finally, we have in that chapter a short evaluation of the data, indicating which of the data errors are likely to cause the more serious difficulties in our attempts to estimate the production function parameters.

These difficulties are mainly encountered in chapter III where we first show that due to the main data errors, methods of estimation often applied on a simultaneous equations system like (5) and (6) do not work. Instead we use other methods, evaluating their properties in the present context; we finally end up with a method for estimating the factor elasticities used in a related study, concluding that this method seems to be the best given the data we have.<sup>1)</sup>

Since there are reasons to believe that the ordinary least square (OLS) estimator for the elasticity of substitution from (6) is strongly biased towards one, we try to estimate this parameter by means of OLS on the so-called Kmenta relation, which is a Taylor expansion of the CES relation around the value of  $b = 1$  (or  $\rho = 0$ ), ignoring terms of third and higher orders.<sup>2)</sup> We then also try to evaluate the effects of simultaneity and errors of measurement on the estimates obtained.

There are a few by-products in our search for proper methods of estimation. We show that we must pay a very high price, in terms of highly biased estimators for the factor elasticities, for eliminating the serial

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N o t e s :

1) Cf. GRILICHES and RINGSTAD (1971), op.cit., Ch. IV.

2) As pointed out the CES relation is a Cobb-Douglas relation when  $b = 1$ .

correlation present in the error term of the production function. We also show that with a cross section of time series data the simultaneous equations problem in the production function is, in fact, likely to be mainly due to variations in management, or efficiency in general between units. This has been asserted, but not shown to hold true, by others using cross sections of time series data to estimate production functions.

The data used in this study are fairly well-suited as a basis for an investigation of a statistical tool that has barely been tried in econometric analyses, namely multiple testing. The outcome of a few experiments with this tool is presented in chapter IV. We use it to explore the variation of the means of the error terms between establishments and over time, and in a particular context also the variation of the main production function parameters over the same dimensions in data.

Chapter V deals with problems concerning the measurement of technical change, its importance and nature. First, we try to determine the rate of technical change and indicate an aggregation problem present when using cross sections of time series data. Second, we explore issues concerning the nature of technical change, investigating especially whether it has been neutral or not, and if not whether it has been labour or capital saving. In this context multiple tests are also used. We also present a test of the embodiment hypothesis and investigate the role of materials and semi-products in a technical change process. In an appendix to chapter V we present a few results of calculations carried out to investigate whether there are transitory variations in demand and costs of change.

In chapter VI we summarize what we seem to have learned from this study. In the main chapters we do not discuss in detail the results obtained for the various industries. Thus, in an appendix to the concluding chapter we include a summary of our findings by industry, with emphasis on differences between the industries concerned.

## CHAPTER II. THE EMPIRICAL BASIS OF THE STUDY

This chapter describes the empirical framework within which we shall work in this study. In the first section, the data sources, the sample selected and the industries to be analysed are presented. Section 2 presents the information available together with the measures applied for the main variables.

In Section 3 we consider the problems encountered, and how we have attempted to solve them when trying to obtain observations on our capital input measure for all years for the units of our study. This section is also a case study of the calculation of missing observations of a variable entering an econometric model, with capital as the variable with observations missing.

Section 4 contains some tables with a few comments on the "behaviour" of the main variables entering the models analysed in the following chapters. This section is intended to be a useful supplement to the results presented later. Finally, in Section 5, an attempt is made to evaluate the quality of the data.

In Appendix 1 of this chapter we present the composition of the industries of this study. Appendix 2 deals with two data problems we encounter. In the first section of this appendix we consider various causes of births and deaths of establishments, and in the second we explain how missing values for subsidies and duties are calculated.

In Appendix 3 of this chapter the analysis of variance statistics applied in Section 4 are derived, and in Appendix 4 we present a method of analysing the consistency of time series for capital and investment.

## 1. The Units to be Studied

## a. The Data Sources

The units of this study are the establishments of "large" Norwegian firms in mining and manufacturing for the nine years 1959 through 1967. A large firm in this context is defined as one having an average of at least 100 employees in 1963 according to the Census of Establishments for that year.<sup>1)</sup> Approximately 600 firms with about 1,300 establishments in mining and manufacturing industries satisfy this criterion. The data for these establishments for 1963 are also obtained from the Census. Information based

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N o t e :

- 1) Number of employees is defined as wage-earners (production workers) + salaried workers (non-production workers) + owners and unpaid family members working daily in the establishment. Cf. Section 2.a.

on the Annual Industrial Production Statistics is used for the other years. In addition, price data for gross production and materials are obtained from the national accounts system.<sup>1)</sup> A price index based on current information on prices of new capital goods is applied to deflate the capital stock data.

#### b. The Sample Selected

In this study we will concentrate our efforts on complete time-series.<sup>2)</sup> Thus, those establishments which according to their identification number did not exist in one or more of the years 1959-1967 have been excluded.<sup>3)</sup> Since we would like to include only production establishments, auxiliary units and so-called investment establishments are also excluded.<sup>4)</sup>

Excluding incomplete time-series, auxiliary units and investment establishments, we have 913 complete time-series for the production units remaining. For various reasons six of these were also excluded.<sup>5)</sup> The remaining 907 establishments are therefore the units selected for further analysis in this study.<sup>6)</sup>

#### Notes :

- 1) There are about 85 sectors in this accounting system for mining and manufacturing. By using the data for gross production and materials in current and constant f.o.b. and c.i.f. prices respectively, we have implicit price indices to deflate the corresponding variables of the individual production units of our study.
- 2) In some contexts incomplete time-series are as interesting as complete ones. Cf. WEDERVANG (1965): Development of a Population of Industrial Firms. However, since the high number of incomplete time-series in the present context seems to be a result of artificial births and deaths of establishments no attempt is made to analyse the structure of these units. Cf. Section a of Appendix II.2.
- 3) Cf. Section a of Appendix II.2.
- 4) Investment establishments are new production units which have not yet started production in the year for which the information is reported. Most such units are, however, excluded as incomplete time-series. Should one wish to analyse questions concerning "natural" births of establishments by means of this body of data, a look at these investment establishments seems to be the best point of departure.
- 5) Two of these were excluded because they obviously were investment establishments during 1959, even though they were reported to be ordinary production units for the entire period. Three establishments were excluded because they reported having no employees for one or more years. The remaining unit was excluded because of a complete break in production during one year.
- 6) Some of the time-series that appear as complete are in fact incomplete due to identification numbers referring to different production units in different years. However, we have not been able to do anything with this problem. Cf. Section a of Appendix II.2.

### c. The Industries

The 907 units selected are divided into 15 "industries" for which results are reported separately during most parts of this study.

In Appendix II.1 a table on the composition of these 15 industries is presented.<sup>1)</sup> Although the presentation is by four-digit industry groups, the base unit of the industry construction is the two-digit group.<sup>2)</sup> The division between the industries may in some cases appear somewhat arbitrary. However, if we are not going to rearrange two-digit industry groups by our industry construction it seems for example more convenient to "merge" industry groups 21 and 22 together with the main one, 20, than to merge them with the following, industry group 23. Obviously then, the notation "Food Products" is only approximate. The same is true for Basic Chemicals with two units of the 29 industry group, Leather products and 6 from the 30 industry group, Rubber products.<sup>3)</sup>

## 2. The Choice of Operational Definitions for the Main Variables

### a. The Characteristics Reported

In addition to such general characteristics as industry group, location, type of ownership, the following information with the exceptions pointed out below, is obtained for each establishment for each of the nine years. This information will in some way or another be applied in the study, mainly in the construction of the variables on which the principal part of the analysis is based:

#### Notes :

- 1) A few establishments were classified in different industry groups in different years. To avoid ambiguity in the industry group classification these units were classified in the industry group to which they belonged in 1963. This is clearly a rather arbitrary procedure, but it only relates to a small number of units.
- 2) In other contexts this detailed presentation of the composition of industries is more important. Cf. Chapter IV.
- 3) It is not possible to construct very homogeneous industries if we are to cover all industry groups. We could, for instance, have group 2311, Spinning and weaving of wool, and group 2710, Manufacture of mechanical pulp as two of our industries, but what would we then do with such groups as 2313, Spinning and weaving of hamp, jute and linen, and 2722, Manufacture of sulphate pulp? We could exclude them or merge them with the remaining groups of their respective two-digit industries. The first approach leads to a substantial reduction of units, and the second does not solve our problem of heterogeneous industries. We choose then to go ahead with the industry construction presented in Appendix II.1.



X <sub>1</sub>	Production on own account
X <sub>2</sub>	Repairs
X <sub>3</sub>	Contract work
M <sub>1</sub>	Raw materials
M <sub>2</sub>	Packing
M <sub>3</sub>	Fuel
M <sub>4</sub>	Auxiliary materials <sup>1)</sup>
M <sub>5</sub>	Contract work
n <sub>1</sub>	Number of wage-earners (production workers)
n <sub>2</sub>	Number of salaried employees (non-production workers)
n <sub>3</sub>	Number of owners and family members
h	Hours worked (in 1,000) by wage-earners
W <sub>1</sub>	Wages, wage-earners
W <sub>2</sub>	Wages, salaried employees
W <sub>3</sub>	Wages, home workers <sup>2)</sup>
U <sub>1</sub>	Duties
U <sub>2</sub>	Subsidies <sup>3)</sup>
I <sub>1</sub>	Investments, purchased capital goods
I <sub>2</sub>	Investments, repairs and maintenance

In addition to this information, we also have for the years 1959 and 1963 information on:

K <sub>1</sub>	Full fire insurance value of buildings
K <sub>2</sub>	Full fire insurance value of machinery <sup>4)</sup>

By means of the characteristics above we will attempt to construct the variables needed for the present analysis.<sup>5)</sup>

#### Notes:

- 1) For the years 1959 and 1960 M<sub>3</sub> + M<sub>4</sub> is reported instead of each component separately, and for the years 1965-67 M<sub>4</sub> is included in M<sub>1</sub>. Thus, only for the years 1961-64 do we obtain separate information on each of the components M<sub>1</sub> - M<sub>5</sub>.
- 2) Home workers are those who do not work on the premises of the establishment.
- 3) Information on duties and subsidies is not reported for 1959 and 1960. See Section b of Appendix II.2 for the calculation of this information.
- 4) For 1959, but not for 1963, we also have information about "other property".
- 5) Except for h, n<sub>1</sub>, n<sub>2</sub> and n<sub>3</sub> all numbers are in 1,000 (current) Norwegian kroner. For price data, see below.

b. Gross Production, Materials and Value Added

Since we would like to compare the results of this study with those of a related study we try to let the definitions of the main variables conform as closely as possible to those of that study.<sup>1)</sup>

First, we define gross production in current "factor prices" as:

$$(1) \quad Y' = X_1 + X_2 + X_3 + U_2 - U_1$$

The input of materials is defined as all inputs "from the outside" in buyers' prices.

$$(2) \quad M' = M_1 + M_2 + M_3 + M_4 + M_5 + W_3$$

Since both  $Y'$  and  $M'$  are in current prices they are deflated with price indices obtained from the national accounts system.<sup>2)</sup> Thus we obtain gross production and materials in constant prices as<sup>3)</sup>:

$$(3) \quad Y = \frac{Y'}{P_Y}$$

$$(4) \quad M = \frac{M'}{P_M}$$

where  $P_Y$  and  $P_M$  are the two price indices for gross production and materials respectively.<sup>4)</sup>

We thus have value added in current prices as:

$$(5) \quad V' = Y' - M'$$

and in constant (1961) prices as:<sup>5)</sup>

$$(6) \quad V = Y - M$$

Notes:

1) Cf. GRILICHES and RINGSTAD (1971), op. cit.

2) The basis year of the national accounts system is now 1961, while it was previously 1955. By simple chaining we obtain indices with 1961 as the basis year for 1959 and 1960.

3) The price index of output for some industries is, however, quite misleading, and this turns out to have serious effects on some of the results of the industries concerned. For quite a few national accounts sectors the output price data are very spotty or generally of poor quality. For these sectors price indices for output are computed by means of price data for the inputs, i.e. deliveries from other sectors and labour. Since increased wages due to improved labour productivity are not eliminated from the input price data, the price increase for the industries concerned is overstated and thus the growth in output "in constant prices" is understated. In Appendix II.1 industry groups for which the output price indices are computed in this way are marked with an asterisk. A further discussion of the particular problems this price index computation causes for the interpretation of some of the results is presented in Chapter V.

4) Cf. footnote 1 on p. 26.

5) These definitions of gross production, materials and value added are generally the same as those applied in: GRILICHES and RINGSTAD (1971), op. cit. Cf. Ch. III.

Thus, we have implicitly a price index with 1961 as the basis year for value added as:

$$(7) \quad P_V = \frac{V'}{V} = \frac{P_Y Y - P_M M}{Y - M}$$

#### c. The Labour Input Measure

The labour input measure to be applied is the following<sup>1)</sup>:

$$(8) \quad L = \frac{h(W_1 + W_2)}{W_1} + 2 n_3$$

This measure implies that we calculate the number of hours worked by salaried employees in production workers' equivalents. We also assume that owners and unpaid family members work 2,000 hours a year. This is approximately the average for production workers in 1963.

In a related study both total number of employees  $N$ , and the two variables  $h$  and  $n_2 + n_3$  together were tried out as labour input measures.<sup>2)</sup> Some experiments showed that  $L$  as defined in (8) was generally superior to both of these alternatives. In light of these results we chose to go ahead with our labour input measure without further investigation of the validity of the aggregation used.

#### d. The "Real" Wage Rate

The price of labour input is measured as average wages per hour for production workers. That is:

$$(9) \quad W' = \frac{W_1}{h}$$

However, since in the present study we are more interested in the price of labour - price of output ratio, we apply the following "real" wage rate:<sup>3)</sup>

$$(10) \quad W = \frac{W'}{P_V}$$

where  $P_V$  is defined in (7) above.

By means of the information available we could have constructed other wage-rate measures, but neither these nor the one to be used are particularly good as measures of the price of labour as a factor of production.

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#### Notes :

- 1) Ibid., Ch. III.
- 2) Ibid., Appendix B.
- 3) Ibid., Ch. III.

The main drawback of our wage-rate measure, as well as of the labour input measure, is that they both refer to the quantity component of labour. This property of the wage-rate and labour input variables is discussed in detail in Chapter III.

#### e. The Capital Input Measure

The information available for capital is, as indicated above, full fire insurance values for two categories, namely buildings and machinery, but only for the years 1959 and 1963.<sup>1)</sup> On the other hand, we have information on two kinds of gross investment for all years: for purchased capital goods and for repairs and maintenance. Thus, in principle it is possible to obtain some kind of a capital measure for the remaining years as well.

We will not consider the conceptual problems present when trying to measure the productive performance of capital.<sup>2)</sup> We will, instead, accept the following measure:<sup>3)</sup>

$$(11) \quad K = \frac{K_1 + K_2}{P_K}$$

where  $P_K$  is a price index for total capital (buildings and machinery) based on price data for new capital for mining and manufacturing, and concentrate our efforts on how to obtain data for  $K$  for all units for all years.<sup>4)</sup> The results of our efforts are presented in the following section.

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#### Notes :

- 1) Ibid., Ch. III contains a fairly detailed discussion of the contents of "full fire insurance values" of capital.
- 2) See for example: GRILICHES (1963,I):Capital Measures in Investment Functions in CHRIST (ed.): Measurement in Economics.
- 3) In GRILICHES and RINGSTAD (1971) op. cit. a basically different measure was also applied, albeit without much success, namely the horse-power of the installed equipment as a measure of the capital's production capacity and the energy consumption (mainly electricity) per horse-power installation as a measure of the utilization of this capacity.
- 4) As in the case of gross production and materials the indices available for gross investments for 1959 and 1960 have 1955 as a basis year. In this case as well, by the simple chaining of corresponding indices with 1955 and 1961 as basis years we obtain price indices for gross investments with a basis year in 1961 for 1959 and 1960.

### 3. The Treatment of Missing Observations for Capital

A very common empirical problem of econometric research is that of incomplete sets of data, or missing observations. This is particularly true for studies based on micro-economic data, since the missing values usually disappear in aggregates. On the other hand, such aggregates constructed of incomplete data at the micro-level may be subject to serious errors of measurement. This problem is well-known among the main collectors of micro-economic data, the various national bureaus of statistics. They have control and revision procedures on the current statistics by means of which obviously inaccurate data, including missing observations, are detected and corrected. The correction of them seems to be done mainly by obtaining correct information from the economic units concerned, at least for the more important characteristics. There seems, however, to be some amount of "guessing" with a considerable amount of leeway for judging "reasonable" values. "Guessing" is, after all, generally better than doing nothing at all. Quite probably, the aggregates which usually are the output of such statistics become more reliable through such corrections.

Nevertheless, for some reason or another, after the controls and corrections have taken place, there are quite often a number of missing observations on important variables left. If an econometrician is interested in analysing these data at the micro-level, he has to do something with them. Usually this problem is solved by excluding the units concerned. Not so often he "guestimates" the missing observations on an ad hoc basis.

Obviously one should not be too satisfied with such ad hoc solutions, even though it may be impossible to obtain more satisfactory methods that are generally applicable on the whole range of missing observation problems in micro-economic data. On the other hand, methods do exist which can be of use in some missing observation situations.

One important property of such methods must be that it makes the "guestimation" look more like true estimation: That economic theory and statistical methods are applied to make the calculation of missing observations more systematic. By putting such computations into an econometric framework it may also be easier to evaluate what really happens to the data, and eventually to the results of analyses carried out on data with missing observations calculated by means of the observations reported.

In this section we report on some attempts made to calculate missing observations for capital. Even though we are not very successful in these attempts, they seem to be interesting enough to deserve a fairly detailed presentation and discussion.

#### a. T h e C a p i t a l D a t a M i s s i n g

By inspecting the capital numbers reported for 1959 and 1963 a substantial portion of the establishments was found to report no buildings or no machinery for one or both of the years.

This suggests that the capital data are rather poor. They may, however, look somewhat poorer than they really are. First, by examining the numbers more closely, it turns out that most of those establishments which reported one of the components of capital zero for one of the years must have lumped together both categories of capital and reported it as either buildings or machinery. This conclusion is based on the capital reported for the other year when the categories were reported separately, the investments in the period between the two years under consideration and price movements for that period. Second, some of those establishments which reported only buildings or only machinery for both years seemed to have lumped together the two categories for capital for both years and have reported it either as buildings or as machinery. This conclusion is also based on investments and price movements of capital, but also on the level of employment and value added of the units under consideration. For these units we accept the capital reported as representing total capital stock according to the definition in (11) above.

There are then 60 units remaining with missing or obviously incomplete information on capital for 1959, and 37 in 1963. The net number of units with incomplete information on capital is somewhat less than the sum of these numbers, about 85, since there are 12 units with missing or incomplete information for both years.

As pointed out in Section II.1 our 907 units are divided into 15 industries.<sup>1)</sup> In Table II.1. the number of units of each industry is presented together with the number of missing observations for capital for each of the years 1959 and 1963.

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#### N o t e :

1) Appendix II.1 provides the composition of the industries.

Table II.1. The Number of Establishments and the Number of Missing Capital Values for 1959 and 1963 by Industry

Industry	Number of establishments	Number of missing capital values	
		1959	1963
Mining and Quarrying .....	26	0	1
Food Products .....	164	9	10
Textiles .....	58	0	2
Clothing .....	67	6	6
Wood Products .....	45	1	1
Pulp and Paper .....	103	8	1
Printing .....	63	10	1
Basic Chemicals .....	72	4	3
Mineral Products .....	36	1	0
Basic Steel .....	42	1	3
Metal Products .....	60	3	3
Non-Electrical Machinery .....	37	4	1
Electrical Machinery .....	34	4	2
Transport Equipment .....	87	6	2
Misc. Products .....	13	3	1
<b>Total Mining and Manufacturing .....</b>	<b>907</b>	<b>60</b>	<b>37</b>

We note that the relative number of missing capital values is quite different for different industries. None is missing for Mining and Quarrying and Textiles in 1959 and Mineral Products in 1963, while almost 25 % are missing for Misc. Products and about 16 % for Printing in 1959.<sup>1)</sup>

Since we accept only complete time-series, we see from Table II.1 that it would, at least for some industries, imply a substantial loss in number of degrees of freedom to exclude the units with missing observations for capital. On the average the loss is almost 10% of the total number of degrees of freedom. If we, on the other hand, in some way managed to calculate the missing observations we "lose" less than 2 % of the total numbers of degree of freedom. Thus, there is a strong argument for adopting the calculation approach in this case.

            
N o t e :

- 1) The difference in the total number of missing capital observations for the two years suggests that there has either been an improvement in the reporting and/or the control of the data, or that the quality of Census of Establishment data is better than that of the Annual Industrial Production Statistics.

b. The Model and the Basic Properties of the Method Applied

The model we use in our attempts to calculate missing capital observations is<sup>1)</sup>

$$(12) \quad \begin{aligned} y &= a_0 + \alpha x + \beta z + u \\ y - x &= b_0 + bw + v \end{aligned}$$

Where  $y = \ln V$ ,  $x = \ln L$ ,  $z = \ln K$  and  $w = \ln W$ . In this sub-section the errors  $u$  and  $v$  are assumed to have zero means; in addition, we assume that they have constant variances, that they are distributed independently and show no serial correlation.

In the literature concerning how to treat missing observations in statistical research a number of methods are proposed.<sup>2)</sup> Among these methods only one will be considered, namely the one that presumably is the more appealing intuitively. The contents and implications of it are also easy to understand. To be sure a fairly detailed derivation of it is presented below, since this also clearly shows under what conditions a couple of ad hoc methods do not work, under what conditions they may work, and also under what conditions one of them may be better than the more "refined" one.

To illustrate the basic properties of the method we will use, let us for a moment assume that labour input is not endogenous i.e. the behaviour relation in (12) is invalid and also that the variables are correctly measured so that  $x$  and  $z$  are two true exogenous variables in the production relation. We know then that the ordinary least square method on this relation gives the best linear unbiased estimators for  $\alpha$  and  $\beta$ .

We have  $n$  sets of observations of which  $n_1 \leq n$  are complete. Thus there are  $n - n_1$  unknown values of  $z$ , and we will calculate these values along the same lines as for  $\alpha$  and  $\beta$ . We can write the sum of squares function to be minimized as:

$$(13) \quad \begin{aligned} U^2 &= \sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta(z_{1i} - \bar{z}))^2 \\ &+ \sum_{i=n_1+1}^n (y_{2i} - \bar{y} - \alpha(x_{2i} - \bar{x}) - \beta(z_{2i} - \bar{z}))^2 \end{aligned}$$

where the subscripts 1 and 2 of the variables refer to complete and

Notes :

1) Cf. Chapter I and Chapter III.

2) For a survey of the literature and a discussion of the different methods cf. ELASHOFF and AFIFI (1966, 1967, 1969): Missing Observations in Multivariate Statistics, Journal of the American Statistical Association. Part I; Review of the Literature, Part II; Point Estimation in Simple Linear Regression, Part III; Large Sample Analysis of Simple Linear Regression, and Part IV; A Note on Simple Linear Regression.



incomplete sets of observations respectively and the averages  $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ ,  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$  and  $\bar{z} = \frac{1}{n} \sum_{i=1}^n z_i$  refer to all sets of observations, which implies that for  $\bar{z}$  also the  $n-n_1$  unknown values of  $z$  are included.

Minimizing (13) with respect to the unknown  $z$ -values gives the  $n-n_1$  first order conditions for minimum as:

$$(14) \quad \begin{aligned} \frac{\partial U^2}{\partial z_j} &= \frac{2\beta}{n} \sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta(z_{1i} - \bar{z})) \\ &+ \frac{2\beta}{n} \sum_{i=n_1+1}^n (y_{2i} - \bar{y} - \alpha(x_{2i} - \bar{x}) - \beta(z_{2i} - \bar{z})) \\ &- 2\beta (y_{2j} - \bar{y} - \alpha(x_{2j} - \bar{x}) - \beta(z_{2j} - \bar{z})) = 0 \\ &\quad (j = n_1 + 1, \dots, n) \end{aligned}$$

Since the sum of the two first terms of (14) is zero we get:

$$(15) \quad (y_{2j} - \bar{y}) - \alpha(x_{2j} - \bar{x}) - \beta(z_{2j} - \bar{z}) = 0 \quad (j = n_1 + 1, \dots, n)$$

That is, each unit with a missing observation gets a value of  $z$  which implies a zero error of relation for the unit concerned, or in other words the error is "absorbed" in the calculated value of  $z$ . This is a property of the method subject to further comments below.

The formula in (15) cannot be used directly to calculate the missing  $z$ -values since it includes  $\bar{z}$ . However,  $\bar{z}$  is found in the following way:

From (15) we have:

$$(16) \quad \sum_{j=n_1+1}^n (y_{2j} - \bar{y} - \alpha(x_{2j} - \bar{x}) - \beta(z_{2j} - \bar{z})) = 0$$

This implies that

$$(17) \quad \sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta(z_{1i} - \bar{z})) = 0$$

And so we get:

$$(18) \quad \bar{z} = -\frac{1}{\beta n_1} \sum_{i=1}^{n_1} (y_{1i} - \bar{y} - \alpha(x_{1i} - \bar{x}) - \beta z_{1i})$$

or

$$(19) \quad \bar{z} = -\frac{1}{\beta} (\bar{y}_1 - \bar{y} - \alpha(\bar{x}_1 - \bar{x}) - \beta \bar{z}_1)$$

$$\text{where } \bar{k}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} k_{1i} \quad (k_1 = y_1, x_1, z_1)$$

Inserting (19) into (15) yields;

$$(20) \quad (y_{2j} - \bar{y}_1) - \alpha(x_{2j} - \bar{x}_1) - \beta(z_{2j} - \bar{z}_1) = 0 \quad (j = n_1 + 1, \dots, n)$$

Calculating the  $n - n_1$  missing capital values by means of (20) also implies that the second term of (13) disappears.<sup>1)</sup> Thus to estimate  $\alpha$  and  $\beta$  we are left to minimize the sum of squares of the complete sets of data. In this expression  $\bar{z}$  enters also. Inserting (19) into (13) yields:

$$(21) \quad U^2 = \sum_{i=1}^{n_1} (y_{1i} - \bar{y}_1 - \alpha(x_{1i} - \bar{x}_1) - \beta(z_{1i} - \bar{z}_1))^2$$

Thus this least square method of calculating missing observations is, not surprisingly, separable in the sense that we can first estimate the parameters of the relation concerned by means of the complete sets of data and then use a relation like (20) to calculate the missing observations.

### c. Modifications of the Method, I

The procedure for calculating missing values for capital derived in the previous section is based on assumptions that imply consistent estimators for  $\alpha$  and  $\beta$  by the ordinary least square method on the production relation. The model used tells us, however, that profit is maximized with respect to labour. We know too that the observed capital data are of rather poor quality, containing a substantial, but presumably random error-component. As shown in Chapter III this implies inconsistent estimators on the factor elasticities when applying the ordinary least square method. From (20) we see that this also implies "inconsistent estimators" on the missing capital values.<sup>2)</sup>

We need therefore a method which will take care of both the simultaneity of  $y$  and  $x$  and the errors of measurement of  $z$ . Such a method is discussed in Chapter III. It implies that the elasticity of labour is estimated by a particular factor share method, assuming the elasticity of substitution equal to unity, and that the elasticity of capital is estimated by a size-dummy instrumental variable method.<sup>3)</sup>

#### Notes:

- 1) There are "no degrees of freedom left" for this part of the sum of squares function.
- 2) Given  $y_1, \bar{x}_1, z_1, y_{2j}$  and  $x_{2j}$  we get the probability limit of  $\hat{z}_{2j}$  as:
 
$$\text{plim } \hat{z}_{2j} = \bar{z}_1 + \frac{\alpha + \text{bias } \hat{\alpha}}{\beta + \text{bias } \hat{\beta}} (y_{2j} - \bar{y}_1) - \frac{\alpha + \text{bias } \hat{\alpha}}{\beta + \text{bias } \hat{\beta}} (x_{2j} - \bar{x}_1)$$
 after having estimated  $\alpha$  and  $\beta$  by ordinary least squares. Under reasonable assumptions we have  $\text{bias } \hat{\alpha} > 0$  and  $\text{bias } \hat{\beta} < 0$ . This implies that we overstate the importance of the deviations of  $y_{2j}$  from the mean of this variable for the complete observations and thus also the "transitory" components in output. We also overstate the importance of the deviation of labour from its mean of the complete observations.
- 3) Cf. Section III.3.

The adoption of an estimation method other than simple least squares to estimate  $\alpha$  and  $\beta$  has no consequences for the "algebra" of calculating the missing capital values as derived above. To estimate  $\alpha$  we can now apply all sets of data, while the complete sets only enter when  $\beta$  is estimated. We then proceed to calculate the capital values missing by means of (20).

d. Two ad hoc Methods to Compute Missing Capital Values

Having estimated  $\alpha$  and  $\beta$  we obtain the capital estimates from (20)

as:

$$(22) \quad \hat{z}_{2j} = \bar{z}_1 + \frac{1}{\beta} (y_{2j} - \bar{y}_1) - \frac{\hat{\alpha}}{\beta} (x_{2j} - \bar{x}_1)$$

When working with incomplete data one may be tempted to calculate the missing values of a variable by means of the average of this variable for the complete sets of data. That would in our case be equivalent to ignoring the two last terms on the right side of (22). However, even if there may be substantial transitory variation in  $y_{2j}$ , this method is not recommendable in the present case since it completely ignores differences in the size of the units.

We can, however, write (22) as:

$$(23) \quad z_{2j} - x_{2j} = \bar{z}_1 - \bar{x}_1 + \frac{1}{\beta} (y_{2j} - x_{2j} - (\bar{y}_1 - \bar{x}_1)) + \frac{(1 - \hat{\alpha} - \hat{\beta})}{\beta} (x_{2j} - \bar{x}_1).$$

We see from (23) that another ad hoc method may work fairly well provided we have approximately constant returns to scale, namely by using the geometric mean of the capital-labour ratio for the complete sets of observation to compute the capital-labour ratio for the incomplete sets of data. In that case the last term of (23) can be ignored and the difference between this ad hoc method and the least square method is that the latter takes care of the difference between the average productivity of labour for each of the units with incomplete data and the average for the complete sets of data. Thus, in cases of large transitory variation in output between units, or in other words a large standard deviation of the residual, the ad hoc procedure may give more reasonable results than the least square method.

e. Modifications of the Method, II

Since we may expect a rather poor fit for our kind of data, we should adopt a mixed method of calculation: We calculate the missing capital values by means of the "consistent" method described in Section c above. If the values calculated are within certain limits, they are accepted. If not,

a modified version of the average capital-labour ratio for complete sets of data-method is applied.

The limits of "the region of acceptance" are determined by the average capital-labour ratio for the industry concerned so that the lower limit is one third of the average and the upper is three times the average. For units with missing capital values for only one year calculated values outside the region are accepted, provided that the observed capital-labour ratios for the other year are also outside the corresponding region for that year, and outside on the same side as the values calculated.

With the exception mentioned, values below the lower limit or above the upper limit are set equal to the corresponding limits. This seems to be better than to calculate them by means of the average capital-labour ratio since extreme values may be "true". In a sense this last step in our procedure corresponds to the method of "Wisorizing" samples in errors of variables situations.<sup>1)</sup>

Thus, the main part of this method for calculating missing observations is theoretically fairly well-founded, but "the empirical reality" forces us to adopt ad hoc-coloured modifications. The results of these experiments also show that this is necessary.

#### f. T h e R e s u l t s

In Table II.2. the estimates of  $\alpha$  and  $\beta$  are presented as well as estimates of their standard deviations based on formulas presented in Section III.3. The mean square of the estimated residual error obtained from ordinary least squares on the Cobb-Douglas relation applied on the complete sets of data is also presented to give some idea of the fit, or rather the lack of fit.<sup>2)</sup>

As we see from this table, the fit is poor and we have not obtained a sharp determination of the parameter values. Thus, as expected, we get a number of "wild shots" when calculating missing capital values by applying our method. A total number of 21 out of 93 values are outside "the regions of acceptance" discussed above. This is not an unreasonable

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#### N o t e s :

1. Cf. TUKEY (1962): The Future of Data Analysis, pp. 17-19.
2. No results are presented for Misc. Products since the method could not be applied on this industry due to problems related to the degrees of freedom. Instead, the missing capital values are calculated by extrapolations, using the information on the capital-labour ratio (for the year for which capital is not missing), investments and price movements during the period 1959-63 and the depreciation ratio estimated (cf. Section 4.g.iii. below).

number judged by the contents of Table II.2. Four of these values are not necessarily as wild as they may look since the capital-labour ratio for the other year for which capital is reported is also outside the region. Thus these values are also accepted. For the 17 remaining "wild shots" Table II.3. provides their distribution by industry and year and whether they are "too low" or "too high".

Table II.2. Output Elasticities of Labour and Capital Estimated by the Complete Sets of Data in 1959 and 1963\*

Industry	1959				1963			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}+\hat{\beta}$	MSE	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}+\hat{\beta}$	MSE
Mining and Quarrying	-	-	-	-	0.605 (0.047)	0.367 (0.035)	0.972	0.163
Food Products .....	0.641 (0.034)	0.324 (0.037)	0.965	0.367	0.649 (0.036)	0.243 (0.037)	0.892	0.373
Textiles .....	-	-	-	-	0.587 (0.025)	0.278 (0.043)	0.865	0.114
Clothing .....	0.625 (0.029)	0.322 (0.060)	0.947	0.112	0.615 (0.022)	0.346 (0.067)	0.961	0.118
Wood Products .....	0.749 (0.045)	0.296 (0.052)	1.045	0.173	0.753 (0.051)	0.364 (0.055)	1.116	0.223
Pulp and Paper .....	0.526 (0.020)	0.373 (0.030)	0.899	0.120	0.635 (0.025)	0.233 (0.032)	0.868	0.144
Printing .....	0.721 (0.034)	0.367 (0.060)	1.088	0.147	0.770 (0.029)	0.232 (0.036)	1.002	0.133
Basic Chemicals ....	0.568 (0.045)	0.431 (0.053)	0.999	0.390	0.657 (0.061)	0.327 (0.054)	0.984	0.510
Mineral Products ...	0.643 (0.050)	0.410 (0.058)	1.053	0.241	-	-	-	-
Basic Steel .....	0.547 (0.034)	0.604 (0.043)	1.151	0.146	0.621 (0.041)	0.420 (0.048)	1.041	0.171
Metal Products .....	0.680 (0.035)	0.286 (0.045)	0.966	0.144	0.608 (0.028)	0.382 (0.041)	0.990	0.107
Non-El. Machinery ..	0.686 (0.037)	0.371 (0.039)	1.057	0.089	0.726 (0.053)	0.319 (0.059)	1.045	0.166
El. Machinery .....	0.687 (0.047)	0.267 (0.058)	0.954	0.148	0.655 (0.049)	0.576 (0.072)	1.231	0.197
Transport Equipment.	0.858 (0.037)	0.227 (0.022)	1.085	0.124	0.778 (0.028)	0.280 (0.019)	1.058	0.089

\* Cf. Section III.3.d. concerning the method of estimation applied.  $\alpha$  is the elasticity of labour and  $\beta$  the elasticity of capital. MSE is the mean square of the estimated residual error. The MSE-values presented are obtained from the ordinary least square method on the Cobb-Douglas relation applied on the complete sets of data.

Table II.3. Calculated Missing Capital Values Outside the "Region of Acceptance"

Industry	Number of "wild shots"				Total
	1959		1963		
	Too low	Too high	Too low	Too high	
Mining and Quarrying .....	-	-	0	0	0
Food Products .....	0	2	0	0	2
Textiles .....	-	-	0	0	0
Clothing .....	0	1	0	0	1
Wood Products .....	0	0	0	0	0
Pulp and Paper .....	0	1	1	0	2
Printing .....	2	0	0	0	2
Basic Chemicals .....	0	2	2	0	4
Mineral Products .....	0	0	-	-	0
Basic Steel .....	0	0	0	0	0
Metal Products .....	0	1	1	0	2 <sup>1)</sup>
Non-El. Machinery .....	0	0	1	0	1
El. Machinery .....	0	1	0	0	1
Transport Equipment .....	1	1	0	0	2
<b>Total .....</b>	<b>3</b>	<b>9</b>	<b>5</b>	<b>0</b>	<b>17</b>

1) Refers to the same unit.

We note that Basic Chemicals has 4 "wild shots". This is a rather poor result inasmuch as this industry only has 7 missing capital values. However, since it also has the highest mean square error among our industries, this result is not too surprising. On the other hand, Food Products which also has a high mean square error behaves fairly well since only 2 of the 19 values are wild. In the case of Pulp and Paper and Non-El. Machinery in 1963 there is only one missing observation, but the value calculated is wild for both, even though the mean square errors of these industries are relatively low.

As pointed out, the missing capital values for those units for which we get "wild shots" are calculated by setting them equal to the upper or lower limit of the "region of acceptance" depending on whether the "wild shot" is above or below this region. This implies, inter alia, that the capital values thus calculated are in some cases quite inconsistent with information on the other year for which capital is reported, taking investments, price movements and depreciation into consideration. This may clearly also be the case for values within the "region of acceptance". However, such obvious

inconsistencies can also be observed quite frequently among those units with complete sets of data.

g. Calculation of Capital Values for  
Years Other than 1959 and 1963

g.i. The Information Required

We have, in principle, information on all characteristics but one, of those necessary for the calculation of capital values for years other than 1959 and 1963. We have capital values for 1959 and 1963 and we have gross investments for all years. We also have a price index that makes it possible to eliminate, albeit in a rather approximate way, the price movements over time in these two variables. What we need in addition is information on depreciation.

We assume that the capital stock as measured by us is reduced by a constant fraction during one year due to depreciation.<sup>1)</sup> We can then either adopt the "official" depreciation ratios calculated by the Central Bureau of Statistics, or we could attempt to estimate them. The simplest alternative would clearly be to accept the former, but since they seem unreasonably low, about 5% - 6% on the average, the latter approach is preferable since it may also serve as a check on the validity of the former.

g.i.i. On the Consistency of the Capital and Investment Information:  
A Digression

Before trying to estimate the depreciation rate, there is a particular issue that deserves a few comments, namely the consistency of the capital and investment data applied.

We know that the capital measure of 1959 and 1963 refers to full fire insurance values at the end of these years, while the investment measure is the accumulated flows of repairs and maintenance and of purchased capital goods during the year. The question is now whether, or to what extent, the investment during one year is reported as part of the capital stock at the end of the year. The results of a recent study suggest that there is in fact, and not unexpectedly so, a lag or kind of sluggishness

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N o t e :

- 1) It is evident that a constant depreciation ratio, in the sense that the initial value of the capital is reduced by a constant fraction each year, is preferable. However, since we then would need to know the age distribution of the capital stock to compute the depreciation each year, this concept is not operational in the present case.

between investments reported and capital reported.<sup>1)</sup> The lag may occur because current outlays on investments are reported even if some investment projects are not completed before the next year (or even later), while the capital stock reported is adjusted only for completed investment projects. In addition, there may be a sluggishness in adjusting the full fire insurance values even for completed investment projects.

If the lag-hypothesis is valid, this is in fact an improvement of our capital measure, since incompleting investment projects do not usually add to the production capacity of capital. On the other hand, when computing capital values by means of current investments, we obviously should know how much the outlays on incompleting investment projects constitute of the total outlays on investments. Such information is not available. Any sluggishness in adjusting the fire insurance values for new capital goods makes matters difficult in another way since this implies that the reported capital values of 1959 and 1963 are generally too low.

Clearly, there may also be substantial individual variations relating to "lags and sluggishness", but we can at best take into account the average of these effects. We could try to take care of them by adding lagged instead of current investments to depreciated capital of the previous year to obtain the capital value of a year.<sup>2)</sup> This is, however, rather arbitrary even though it seems to have some support in the study referred to above.<sup>3)</sup> An alternative is to weight lagged and current investment in the computations of capital and perhaps also attempt to find out something about the average lag. Such an approach, however, does not seem particularly promising in the present context.

Since a choice has to be made, the problem of possible "lags and sluggishness" is ignored. However, in a particular context below, we also refer to a few results obtained by using lagged instead of current investment.<sup>4)</sup>

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**N o t e s :**

- 1) RINGSTAD and GRILICHES (1968): A Method of Analyzing the Consistency of Time-Series for Capital and Investments.
- 2) Both in this context and later we speak of capital and investments data which have a common price base, namely 1961.
- 3) RINGSTAD and GRILICHES (1968), op.cit.
- 4) In the case of capital we would also like to have an average for the year instead of the stock at the end of the year. In this context this problem is of minor importance. It is also ignored since we would need information about capital at the end of 1958 to obtain an average for 1959; otherwise, we would have to exclude 1959 from the analysis.



## g.iii. Estimation of the Depreciation Ratio

By means of the information available on capital and investments, we obtain the capital values of the years 1960 through 1963 as:

$$\begin{aligned}
 K_{60} &= (1 - \Delta) K_{59} + I_{60} \\
 (24) \quad K_{61} &= (1 - \Delta)^2 K_{59} + (1 - \Delta) I_{60} + I_{61} \\
 K_{62} &= (1 - \Delta)^3 K_{59} + (1 - \Delta)^2 I_{60} + (1 - \Delta) I_{61} + I_{62} \\
 K_{63} &= (1 - \Delta)^4 K_{59} + (1 - \Delta)^3 I_{60} + (1 - \Delta)^2 I_{61} + (1 - \Delta) I_{62} + I_{63}
 \end{aligned}$$

where  $\Delta$  is the depreciation ratio and  $K$  and  $I$  are capital and investment respectively in constant 1961-prices.

The last relation will be used to estimate the depreciation ratio by fitting it to the data involved. This may seem like a rather complex optimization problem since there are non-linear constraints on the parameters. However, since the relation considered has no intercept, the following relation must hold for an optimal value of  $\Delta$ .

$$(25) \quad \bar{K}_{63} - \bar{I}_{63} = (1 - \Delta)^4 \bar{K}_{59} + (1 - \Delta)^3 \bar{I}_{60} + (1 - \Delta)^2 \bar{I}_{61} + (1 - \Delta) \bar{I}_{62}$$

where the barred variables are averages per establishment. Since for the present data it does not make much sense to apply an expensive optimization method to obtain an estimate on  $\Delta$  with many decimal places, we use instead a "scanning" procedure to obtain a much cheaper, but also somewhat rougher estimate.

For different values of  $\Delta$  the difference between the left and the right side of (25) was computed and the value of  $\Delta$  that gave the lowest absolute value of this difference was chosen as the optimum value. For a relation like (25) there is clearly no problem of local optimums, since the difference between the left and the right side increases monotonically from negative to positive values with an increasing  $\Delta$ .<sup>1)</sup>

The search was made for values of  $\Delta$  between - 10 % and 20 % with steps of 0.1 %. For Total Mining and Manufacturing we obtained an optimum value of  $\Delta$  of 7.7%.<sup>2)3)</sup> This estimate seems quite reasonable and it also suggests that the CBS depreciation ratios are somewhat low.

#### Notes :

- 1) Thus, the mean square error has the absolute and only minimum of zero when this difference is zero.
- 2) Clearly by "scanning" the region 7.6-7.8 % in one or more stages using smaller steps we could get as many decimals in our estimate as we liked. However, as pointed out above this does not seem to be worthwhile.
- 3) An attempt to apply this method on the individual industries did not work very well since we obtained unreasonably large variations in the optimum value of  $\Delta$  between industries with about 18 % for Mining and Quarrying and 2 % for Basic Steel as extremes. These results provide additional evidence for the poor quality of the data involved and that we in fact need averages for a fairly large number of units to obtain reasonable answers on the kind of questions we ask.

A related scanning procedure for the same values of  $\Delta$  as above on the last relation of (24) when all constraints, except that the intercept is zero, were taken into account gave an estimate on  $\Delta$  of 5 %.<sup>1)</sup> This implies that the intercept estimate is negative. It may also lend some support to the "lags and sluggishness" hypothesis. Using lagged instead of current investments in the sense outlined above on a relation related to (25) we obtained an estimate on  $\Delta$  of 6.8 % through our scanning procedure.

g.iv. Calculation of Capital Values by Means of Investments and the Estimated Depreciation Ratio

Even though there is some evidence of a lag between the investments reported and capital data, we choose to compute the missing capital data using the depreciation ratio estimated by means of (25) with current and not lagged investments. Since the results for the individual industries for  $\Delta$  appear rather unreliable, it seems better to use the result obtained for Total Mining and Manufacturing for all industries.

Thus for  $\Delta = 0.077$  we compute capital data for 1960, 1961 and 1962 by means of the first three relations of (24) and correspondingly, we obtain the estimates on capital for the years after 1963 as:

$$\begin{aligned}
 (26) \quad K_{64} &= 0.923 K_{63} + I_{64} \\
 K_{65} &= (0.923)^2 K_{63} + 0.923 I_{64} + I_{65} \\
 K_{66} &= (0.923)^3 K_{63} + (0.923)^2 I_{64} + 0.923 I_{65} + I_{66} \\
 K_{67} &= (0.923)^4 K_{63} + (0.923)^3 I_{64} + (0.923)^2 I_{65} + 0.923 I_{66} + I_{67}
 \end{aligned}$$

Thus, in this manner we obtain capital data for all 907 establishments for the nine years 1959-1967. However, whatever standard is used for judging the quality of these data, the conclusion must be that they are quite poor. The consequences of this fact when they are applied in econometric analysis are, however, subject to investigation in another context.<sup>2)</sup>

h. Some Concluding Remarks

Even though we in these attempts to calculate missing capital values have tried to apply systematic analysis, they are strongly coloured by ad hoc procedures based on personal judgement, taste and intuition. In econometric research one can probably never expect to become completely independent of ad hoc solutions to empirical problems. An attempt should be made, however, to use more satisfactory solutions whenever possible, i.e. solutions based

Notes:

- 1) In this case we use explicitly the mean square error as the criterion of fit.
- 2) Cf. Chapter III.

on well-founded econometric methods. The area under discussion in this section has thus far been highly dominated by such ad hoc solutions, but our attempts to systematize the calculation of missing observations have not been very successful. The quality of the data constitutes one obvious reason for this. The missing values for capital are quite clearly one among several indications that the information for this variable is generally poor. To some extent this may also be true for the other variables entering the production function. Since the quality of the reported investment data is generally considered to be of an even more inferior quality than the capital data, the second stage of our calculation of capital values also becomes difficult. Thus, the main conclusion of this excursion is quite obvious: Shaky reported observations imply shaky calculated values on missing observations whatever method is applied. On the other hand, even if this is true, calculation is better than exclusion of the units concerned for reasons already indicated.<sup>1)</sup>

#### 4. Some Basic Characteristics of the Main Variables

This section includes a series of tables containing some characteristics of the main variables of this study. The variables are converted in a way that should make the contents of these tables more easily comparable to the results obtained in the following chapters.

The variables for which characteristics are presented are labour input, the average "value added" productivity of labour, the capital-labour ratio, the materials-labour ratio, the "real" wage rate, i.e. the current wage rate divided by the price index for value added, the share of labour in value added, and finally materials' share in gross production. All, except the two latter variables, are converted to logs.

In addition to the mean and standard deviation, we are basically interested in the variation of these variables along the main dimensions of our data; between establishments, over time and with size.

In order to determine the significance of the systematic variation of the variables along the two former dimensions we use the analysis of variance approach.<sup>2)</sup> We also run regressions with time as the independent variable to get some idea of the average growth rates of these variables. It should be noted that the standard deviation presented relates to variation of growth

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#### Notes :

1) Cf. Section 3.a. above.

2) The statistics applied are derived in Appendix II.3.

rates both between establishments and over time.<sup>1)</sup> We try to determine the importance of the size by running regressions on  $\ln N$ , where  $N$  = total number of employees is the criterion for size. This is done, both when imposing a common intercept for all units for all years, and when allowing different intercepts for different units and years.

Even though the contents of the tables speak for themselves, it may be worthwhile to summarize the main findings.<sup>2)</sup>

#### a. Labour Input and Total Number of Employees

Table II.4. tells us that even though the units selected are those of large firms (i.e. with 100 employees or more in 1963), there are quite a few small establishments in our sample. The median value of  $N$  for all units is, we note, only slightly above 100, and one third of the units has 67 employees or less. As could be expected such industries as Food Products, Wood Products and Printing have mainly small establishments. At the other end of the scale we have the more heavy industries such as Pulp and Paper and Basic Steel. More surprisingly, the samples for both Textiles and Clothing contain mostly large units. We also note that industries like Mining and Quarrying, Basic Chemicals, Mineral Products and Transport Equipment cover a rather wide range of size.

Judged by the analysis of variance statistics there is for labour input a marked difference between the significance of the variation between

#### Notes :

1) Later, the variation over time for average (per establishment) growth rates is considered. (Cf. Table V.2.)

The OLS method on  $X_{it} = a + b_x t$  must necessarily yield the same estimate on  $b_x$  as the OLS method on  $\bar{X}_t = a + b_x t$  where

$\bar{X}_t = \frac{1}{I} \sum_{i=1}^I X_{it}$  For the first relation we have:

$$\hat{b}_x = \frac{\sum_{i=1}^I \sum_{t=1}^T (X_{it} - \bar{X})(t-\bar{t})}{\sum_{i=1}^I \sum_{t=1}^T (t-\bar{t})^2} = \frac{\sum_{i=1}^I \sum_{t=1}^T (X_{it} - \bar{X}_t)(t-\bar{t}) + I \sum_{t=1}^T (\bar{X}_t - \bar{X})(t-\bar{t})}{I \sum_{t=1}^T (t-\bar{t})^2}$$

But the first term of the numerator must be zero as  $\sum_{i=1}^I (X_{it} - \bar{X}_t) = 0$  for each  $t$ , and therefore

$$\hat{b}_x = \frac{\sum_{t=1}^T (\bar{X}_t - \bar{X})(t-\bar{t})}{\sum_{t=1}^T (t-\bar{t})^2}$$

which is also the OLS estimate on  $b_x$  from the second relation above. The standard deviation of the estimate on  $b_x$  will, however, generally be different for the two relations.

2) The level of the tests carried out in this section is 5 % assuming tentatively the error terms to be normally distributed, with constant variances and no serial correlation.

establishments and over time. The results of the regression of  $\ln L$  on  $t$  also tell us that labour input is on the average fairly stable over time. However, the large standard errors of the estimates of the growth rates suggest that there are probably large individual differences concerning the growth of labour.

b. Average "Value Added" Productivity  
of Labour

The analysis of variance statistics of Table II.5. tell us that there are significant variations in the average productivity of labour for all industries both between establishments and over time. The growth rates must be fairly uniform between establishments since the standard deviation of the estimated growth rate is fairly low for most industries, and the growth rate is significant for all industries. For some industries the growth rate of the average productivity of labour is quite probably underrated. This is at least evident for Printing where the growth rate is significantly negative.<sup>1)</sup>

There is a substantial difference between industries concerning the variation of the average productivity of labour with size. Imposing the same intercept for all units for all years in the regression on  $\ln N$ , we get a significantly positive slope-coefficient for six industries and a significantly negative one for three. We obtain widely differing results when we allow the intercept to vary between units, or both between units and over time. Generally, the estimate on the slope-coefficient becomes lower. In the latter case there are now only one significantly positive slope-coefficient (for Textiles) and eight significantly negative ones. This finding strongly smacks of errors of measurement. However, it is probably not due to errors of reporting, but rather is an effect of transitory variation in labour input.  $N$  instead of  $L$  was used as the size variable in order to avoid distorted slope-coefficients due to errors of measurement in labour input. It turns out, however, that these two variables do not yield very different results when used as measures of size. Having eliminated the systematic variation of both average productivity of labour and number of employees both between establishments and over time, the slope-coefficient is dominated by the negative correlation between the non-systematic components of  $-\ln L$  and  $\ln N$ .

A related argument seems to be valid for the other ratio variables as well, where  $\ln L$  enters.

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Note:

1) Cf. Section II.2.b. and Appendix II.1.

Table II.4. Basic Characteristics for lnL and N\*

Industry	Mean and st. dev. for lnL	Growth rate for lnL	Analysis of Variance for lnL		Median value for N	1/3-Fractiles for N	
			F <sub>C</sub>	F <sub>T</sub>		Lower	Upper
Total Mining and Manufacturing ..	5.309 (1.218)	0.0058 (0.0052)	228.45	11.97	113	67	160
Mining and Quarrying .....	5.144 (1.383)	-0.0161 (0.0351)	470.15	1.82	105	33	183
Food Products ..	4.729 (1.245)	0.0089 (0.0126)	214.36	4.73	60	32	102
Textiles .....	5.763 (0.795)	-0.0001 (0.0135)	126.33	3.56	159	119	218
Clothing .....	5.446 (0.713)	-0.0073 (0.0113)	65.55	2.30	121	102	147
Wood Products ..	4.615 (1.102)	0.0126 (0.0212)	160.82	2.01	50	34	79
Pulp and Paper .	5.657 (0.943)	-0.0160 (0.0120)	308.72	10.42	144	106	191
Printing .....	4.607 (0.866)	0.0021 (0.0141)	170.35	1.29	43	30	70
Basic Chemicals.	5.213 (1.457)	-0.0018 (0.0222)	230.81	1.48	105	45	168
Mineral Products	5.430 (1.168)	-0.0033 (0.0252)	236.36	0.82	135	78	214
Basic Steel ....	6.475 (0.858)	0.0257 (0.0171)	151.66	5.25	294	195	418
Metal Products..	5.577 (0.977)	0.0221 (0.0163)	172.15	4.92	125	98	155
Non-El. Machinery .....	5.563 (0.984)	0.0038 (0.0209)	139.03	2.42	130	91	182
El. Machinery ..	5.884 (0.986)	0.0296 (0.0218)	191.84	6.71	128	105	183
Transport Equipment .....	5.531 (1.443)	0.0178 (0.0200)	260.77	3.51	143	96	247
Misc. Products .	4.768 (1.290)	0.0623 (0.0460)	120.20	4.24	75	35	138

\* Cf. notes on p. 56.

Table II.5. Basic Characteristics for  $\ln \frac{V}{L}$  \*

Industry	Mean and st. dev.	Growth rate	Slope-coefficients from regressions on $\ln N$			Analysis of Variance	
			A.	B.	C.	F <sub>C</sub>	F <sub>T</sub>
Total Mining and Manufacturing..	2.612 (0.576)	0.0421 (0.0024)	-0.0054 (0.0052)	-0.0655 (0.0172)	-0.1288 (0.0165)	16.13	104.16
Mining and Quarrying .....	2.821 (0.477)	0.0613 (0.0114)	0.0406 (0.0226)	-0.4859 (0.0968)	-0.3331 (0.0856)	20.20	11.64
Food Products .....	2.726 (0.740)	0.0625 (0.0073)	-0.1477 (0.0149)	-0.0951 (0.0494)	-0.2070 (0.0466)	16.65	27.69
Textiles .....	2.414 (0.422)	0.0285 (0.0070)	-0.0253 (0.0232)	0.1620 (0.0672)	0.1461 (0.0674)	9.13	4.19
Clothing .....	2.311 (0.372)	0.0215 (0.0058)	0.0012 (0.0223)	-0.3182 (0.0452)	-0.3062 (0.0447)	9.25	4.92
Wood Products .	2.388 (0.558)	0.0274 (0.0107)	0.0959 (0.0246)	0.1927 (0.0708)	0.1326 (0.0697)	14.12	5.10
Pulp and Paper.	2.751 (0.451)	0.0733 (0.0052)	-0.0478 (0.0157)	-0.5362 (0.0712)	-0.3850 (0.0601)	10.94	54.41
Printing .....	2.407 (0.362)	-0.0173 (0.0059)	0.0604 (0.0173)	-0.2898 (0.0551)	-0.2562 (0.0551)	10.51	3.98
Basic Chemicals .....	2.940 (0.725)	0.0671 (0.0107)	-0.0333 (0.0196)	0.1026 (0.0619)	0.0809 (0.0570)	19.74	15.50
Mineral Products .....	2.741 (0.534)	0.0245 (0.0114)	0.1625 (0.0243)	-0.1938 (0.0647)	-0.1885 (0.0633)	27.01	2.95
Basic Steel ...	2.979 (0.476)	0.0607 (0.0090)	0.1266 (0.0280)	0.0323 (0.0789)	-0.2500 (0.0716)	14.35	15.11
Metal Products .....	2.587 (0.424)	0.0314 (0.0070)	-0.0633 (0.0185)	-0.0435 (0.0661)	-0.1784 (0.0673)	6.44	4.52
Non-El. Machinery .....	2.565 (0.379)	0.0302 (0.0079)	0.0862 (0.0207)	0.0911 (0.0619)	0.0773 (0.0610)	10.18	5.02
El. Machinery .	2.585 (0.497)	0.0415 (0.0108)	0.0523 (0.0288)	0.0824 (0.0826)	-0.1525 (0.0864)	14.54	5.50
Transport Equipment .....	2.337 (0.400)	0.0194 (0.0055)	0.0662 (0.0097)	-0.0482 (0.0439)	-0.0958 (0.0447)	6.00	2.84
Misc. Products .....	2.637 (0.737)	0.0883 (0.0252)	-0.0751 (0.0516)	0.1379 (0.1345)	-0.1851 (0.1440)	8.63	3.38

\* Cf. notes on p. 56.

Table II.6. Basic Characteristics for  $\ln \frac{K}{L}$  \*

Industry	Mean and st. dev.	Growth rate	Slope-coefficients from regressions on $\ln N$			Analysis of Variance	
			A.	B.	C.	F <sub>C</sub>	F <sub>T</sub>
Total Mining and Manufacturing..	3.288 (0.823)	0.0266 (0.0035)	-0.0098 (0.0075)	-0.5291 (0.0145)	-0.5777 (0.0145)	47.12	49.34
Mining and Quarrying .....	3.366 (0.751)	0.0566 (0.0187)	0.1845 (0.0336)	-0.9020 (0.1142)	-0.8112 (0.1091)	31.85	5.77
Food Products .....	3.424 (0.705)	0.0225 (0.0071)	-0.1233 (0.0143)	-0.5859 (0.0326)	-0.6346 (0.0321)	31.35	6.90
Textiles .....	3.198 (0.506)	0.0285 (0.0085)	0.0624 (0.0277)	-0.4103 (0.0517)	-0.4328 (0.0493)	32.61	8.85
Clothing .....	2.148 (0.621)	0.0301 (0.0097)	-0.0281 (0.0371)	-0.7885 (0.0477)	-0.7906 (0.0466)	24.53	4.94
Wood Products .	3.061 (0.703)	0.0280 (0.0135)	0.0039 (0.0316)	-0.4305 (0.0687)	-0.4828 (0.0676)	26.17	3.25
Pulp and Paper.	4.030 (0.594)	0.0491 (0.0074)	0.1555 (0.0201)	-0.6309 (0.0697)	-0.5493 (0.0660)	22.69	21.46
Printing .....	3.342 (0.641)	0.0065 (0.0104)	0.1198 (0.0305)	-0.5395 (0.0719)	-0.5666 (0.0734)	23.53	0.35
Basic Chemicals .....	3.923 (0.762)	0.0160 (0.0116)	-0.0914 (0.0203)	-0.4409 (0.0469)	-0.4369 (0.0470)	34.15	2.17
Mineral Products .....	3.399 (0.679)	0.0341 (0.0145)	0.1798 (0.0315)	-0.3690 (0.0635)	-0.3617 (0.0602)	46.63	5.04
Basic Steel ...	3.765 (0.646)	0.0354 (0.0128)	0.2632 (0.0366)	-0.4280 (0.0786)	-0.6553 (0.0770)	26.09	3.82
Metal Products .....	3.113 (0.532)	0.0326 (0.0088)	-0.0417 (0.0233)	-0.5451 (0.0612)	-0.7195 (0.0574)	16.36	8.21
Non-El. Machinery .....	3.018 (0.554)	0.0438 (0.0115)	0.0747 (0.0308)	-0.6055 (0.0651)	-0.6678 (0.0580)	22.63	8.67
El. Machinery .	2.823 (0.668)	0.0221 (0.0148)	0.0895 (0.0386)	-0.4909 (0.0689)	-0.7191 (0.0693)	40.89	2.83
Transport Equipment .....	2.795 (0.661)	0.0051 (0.0092)	-0.0782 (0.0162)	-0.4080 (0.0473)	-0.4407 (0.0487)	21.72	0.31
Misc. Products .....	3.072 (1.019)	-0.0083 (0.0366)	-0.0496 (0.0717)	-0.5133 (0.0935)	-0.5930 (0.1098)	40.96	0.75

\* Cf. notes on p. 56.



Table II.7. Basic Characteristics for  $\ln \frac{M}{L}$  \*

Industry	Mean and st. dev.	Growth rate	Slope-coefficients from regressions on $\ln N$			Analysis of Variance	
			A.	B.	C.	$F_C$	$F_T$
Total Mining and Manufacturing..	2.720 (1.233)	0.0403 (0.0053)	-0.0564 (0.0111)	-0.0692 (0.0187)	-0.1328 (0.0182)	85.92	77.14
Mining and Quarrying .....	1.024 (0.654)	0.0213 (0.0165)	0.0451 (0.0310)	-0.6409 (0.1406)	-0.6316 (0.1440)	12.40	1.39
Food Products .....	3.639 (1.216)	0.0293 (0.0122)	-0.4786 (0.0219)	-0.3243 (0.0277)	-0.3879 (0.0265)	182.51	15.39
Textiles .....	2.417 (1.007)	0.0265 (0.0171)	0.2447 (0.0544)	0.1072 (0.0765)	0.1122 (0.0774)	68.56	2.98
Clothing .....	2.181 (1.460)	0.0124 (0.0230)	0.5262 (0.0845)	-0.2768 (0.1039)	-0.2888 (0.1060)	43.89	0.37
Wood Products .	3.046 (0.600)	0.0169 (0.0115)	-0.1056 (0.0265)	0.0381 (0.0422)	0.0046 (0.0422)	64.85	3.18
Pulp and Paper.	3.424 (0.621)	0.0456 (0.0078)	0.0676 (0.0216)	-0.4325 (0.0439)	-0.3568 (0.0373)	96.65	50.42
Printing .....	1.889 (0.860)	0.0408 (0.0139)	0.2833 (0.0398)	-0.1914 (0.0832)	-0.2852 (0.0807)	42.11	6.73
Basic Chemicals .....	2.940 (0.921)	0.0808 (0.0137)	0.0826 (0.0248)	-0.0529 (0.0607)	-0.0880 (0.0529)	44.52	25.44
Mineral Products .....	1.980 (1.231)	0.0414 (0.0264)	0.1681 (0.0592)	-0.3618 (0.0933)	-0.3638 (0.0909)	81.49	3.22
Basic Steel ...	3.153 (1.001)	0.0624 (0.0197)	0.2359 (0.0593)	0.3069 (0.0727)	0.0356 (0.0666)	121.79	18.39
Metal Products .....	2.486 (0.993)	0.0267 (0.0165)	-0.0211 (0.0437)	0.2034 (0.0850)	0.0958 (0.0885)	39.81	2.96
Non-El. Machinery .....	2.324 (0.866)	0.0520 (0.0182)	0.3539 (0.0445)	0.1491 (0.0945)	0.0956 (0.0928)	34.53	5.36
El. Machinery .	2.592 (0.770)	0.0628 (0.0167)	0.0921 (0.0446)	0.3811 (0.0848)	0.0666 (0.0859)	47.80	11.03
Transport Equipment .....	1.983 (1.202)	0.0507 (0.0166)	0.1873 (0.0292)	0.2944 (0.0784)	0.1832 (0.0793)	32.25	5.37
Misc. Products .....	2.864 (0.687)	0.0646 (0.0240)	-0.3030 (0.0394)	-0.1322 (0.0751)	-0.4630 (0.0610)	48.30	5.95

\* Cf. notes on p. 56.

Table II.8. Basic Characteristics for  $\ln W^*$ 

Industry	Mean and st. dev.	Growth rate	Slope-coefficients from regressions on $\ln N$			Analysis of Variance	
			A.	B.	C.	$F_C$	$F_T$
Total Mining and Manufacturing..	1.984 (0.300)	0.0458 (0.0012)	0.0308 (0.0026)	0.0130 (0.0111)	-0.0544 (0.0095)	9.33	362.89
Mining and Quarrying .....	2.162 (0.191)	0.0454 (0.0038)	0.0225 (0.0090)	-0.0961 (0.0527)	0.0295 (0.0338)	13.37	42.80
Food Products .....	1.888 (0.363)	0.0645 (0.0033)	-0.0031 (0.0075)	-0.0077 (0.0336)	-0.1134 (0.0281)	5.34	78.72
Textiles .....	1.787 (0.191)	0.0344 (0.0029)	0.0062 (0.0105)	-0.0923 (0.0330)	-0.0958 (0.0258)	10.43	42.06
Clothing .....	1.748 (0.220)	0.0308 (0.0032)	0.0065 (0.0131)	-0.1017 (0.0314)	-0.0774 (0.0278)	6.46	22.72
Wood Products .	1.949 (0.188)	0.0360 (0.0032)	0.0341 (0.0083)	0.1129 (0.0289)	0.0383 (0.0223)	10.87	37.87
Pulp and Paper.	2.098 (0.337)	0.0960 (0.0029)	0.0400 (0.0117)	-0.2751 (0.0681)	-0.0734 (0.0410)	4.89	205.28
Printing .....	2.010 (0.218)	-0.0133 (0.0035)	-0.0062 (0.0105)	-0.0330 (0.0300)	-0.0118 (0.0289)	17.02	8.67
Basic Chemicals .....	2.041 (0.345)	0.0705 (0.0045)	0.0357 (0.0093)	0.0328 (0.0419)	0.0070 (0.0319)	7.18	56.00
Mineral Products .....	2.067 (0.207)	0.0292 (0.0042)	0.0431 (0.0098)	0.1111 (0.0363)	0.1276 (0.0306)	10.97	14.97
Basic Steel ...	2.214 (0.214)	0.0593 (0.0030)	0.0521 (0.0127)	0.1874 (0.0460)	-0.0679 (0.0265)	9.30	106.16
Metal Products .....	2.027 (0.191)	0.0321 (0.0029)	0.0243 (0.0083)	0.0541 (0.0292)	-0.0747 (0.0246)	10.12	33.54
Non-El. Machinery .....	2.058 (0.163)	0.0312 (0.0030)	0.0081 (0.0091)	-0.0080 (0.0297)	-0.0196 (0.0215)	12.05	39.45
El. Machinery .	2.062 (0.295)	0.0475 (0.0059)	0.0632 (0.0168)	0.2796 (0.0543)	0.0720 (0.0510)	11.58	18.95
Transport Equipment .....	2.008 (0.224)	0.0083 (0.0031)	0.0094 (0.0056)	-0.0224 (0.0202)	-0.0441 (0.0205)	12.85	3.64
Misc. Products .....	1.995 (0.375)	0.0767 (0.0114)	0.0606 (0.0259)	0.2806 (0.0688)	0.0378 (0.0608)	10.45	13.89

\* Cf. notes on p. 56.

Table II.9. Basic Characteristics for  $\frac{WL}{V}$ \*

Industry	Mean and st. dev.	Trend	Slope-coefficients from regressions on lnN			Analysis of Variance	
			A.	B.	C.	F <sub>C</sub>	F <sub>T</sub>
Total Mining and Manufacturing ..	(0.6026) (0.3867)	0.0017 (0.0018)	-0.0066 (0.0035)	-0.0230 (0.0152)	-0.0270 (0.0154)	5.09	4.58
Mining and Quarrying .....	0.5688 (0.2980)	-0.0101 (0.0075)	-0.0064 (0.0142)	0.3091 (0.0650)	0.2936 (0.0670)	11.73	1.28
Food Products .....	0.5246 (0.4466)	-0.0009 (0.0045)	0.0336 (0.0092)	-0.0995 (0.0392)	-0.1037 (0.0401)	4.86	0.38
Textiles .....	0.5709 (0.2425)	0.0041 (0.0041)	-0.0061 (0.0134)	-0.3252 (0.0418)	-0.3148 (0.0430)	4.92	1.50
Clothing .....	0.5997 (0.2017)	0.0047 (0.0032)	-0.0204 (0.0120)	0.1013 (0.0251)	0.1097 (0.0254)	8.75	1.15
Wood Products ..	0.7492 (0.8751)	0.0281 (0.0168)	-0.1033 (0.0390)	-0.3750 (0.1586)	-0.4400 (0.1610)	2.27	1.52
Pulp and Paper .	0.5567 (0.2287)	0.0117 (0.0029)	0.0415 (0.0079)	0.1130 (0.0346)	0.1387 (0.0341)	10.23	10.04
Printing .....	0.7051 (0.2203)	0.0001 (0.0036)	-0.0435 (0.0105)	0.1217 (0.0352)	0.1187 (0.0358)	8.86	1.50
Basic Chemicals .....	0.4961 (0.4044)	0.0017 (0.0061)	0.0200 (0.0109)	-0.0324 (0.0429)	-0.0307 (0.0433)	7.86	0.85
Mineral Products .....	0.5559 (0.2247)	0.0033 (0.0048)	-0.0737 (0.0101)	0.1347 (0.0290)	0.1417 (0.0291)	20.45	0.99
Basic Steel ....	0.5122 (0.2206)	-0.0005 (0.0044)	-0.0419 (0.0132)	0.0889 (0.0423)	0.1020 (0.0467)	5.88	1.65
Metal Products .....	0.6203 (0.4293)	-0.0028 (0.0072)	0.0371 (0.0188)	0.0504 (0.0760)	0.0572 (0.0802)	2.88	0.78
Non -El. Machinery .....	0.6405 (0.2109)	0.0011 (0.0045)	-0.0486 (0.0115)	-0.0499 (0.0335)	-0.0548 (0.0347)	10.24	0.83
El. Machinery ..	0.6428 (0.3396)	0.0006 (0.0075)	-0.0140 (0.0198)	0.1034 (0.0654)	0.1164 (0.0730)	7.39	1.05
Transport Equipment .....	0.7718 (0.3518)	-0.0111 (0.0049)	0.0518 (0.0086)	0.0147 (0.0419)	0.0378 (0.0429)	3.79	2.21
Misc. Products .....	0.6137 (0.3816)	-0.0138 (0.0137)	0.0354 (0.0267)	-0.0461 (0.0817)	-0.0070 (0.0979)	2.87	0.38

\* Cf. notes on p. 56.

Table II.10. Basic Characteristics for  $\frac{M'}{Y}$  \*

Industry	Mean and st. dev.	Trend	Slope-coefficients from regressions on lnN			Analysis of Variance	
			A.	B.	C.	F <sub>C</sub>	F <sub>T</sub>
Total Mining and Manufacturing..	0.5197 (0.2120)	-0.0039 (0.0009)	-0.0111 (0.0019)	-0.0085 (0.0037)	-0.0031 (0.0037)	59.08	17.99
Mining and Quarrying .....	0.1611 (0.0995)	-0.0064 (0.0025)	0.0025 (0.0047)	0.0042 (0.0225)	-0.0171 (0.0223)	12.84	3.23
Food Products .....	0.6780 (0.2141)	-0.0057 (0.0022)	-0.0682 (0.0041)	-0.0357 (0.0074)	-0.0265 (0.0074)	78.71	9.32
Textiles .....	0.4998 (0.1456)	-0.0104 (0.0024)	0.0286 (0.0079)	-0.0372 (0.0187)	-0.0248 (0.0183)	19.76	7.60
Clothing .....	0.4962 (0.1753)	-0.0090 (0.0027)	0.0603 (0.0101)	-0.0121 (0.0144)	-0.0169 (0.0140)	34.63	7.02
Wood Products .	0.6293 (0.1362)	-0.0051 (0.0026)	-0.0341 (0.0059)	-0.0327 (0.0148)	-0.0254 (0.0149)	21.82	2.92
Pulp and Paper.	0.6585 (0.1338)	0.0003 (0.0017)	0.0317 (0.0046)	0.0046 (0.0108)	-0.0022 (0.0110)	55.30	3.74
Printing .....	0.3586 (0.1545)	-0.0028 (0.0025)	0.0392 (0.0073)	0.0020 (0.0149)	0.0079 (0.0150)	40.30	2.64
Basic Chemicals .....	0.4977 (0.1920)	-0.0009 (0.0029)	0.0257 (0.0051)	-0.0284 (0.0104)	-0.0310 (0.0105)	52.34	0.84
Mineral Products .....	0.3402 (0.1735)	-0.0038 (0.0037)	-0.0049 (0.0084)	-0.0363 (0.0177)	-0.0342 (0.0177)	40.68	1.22
Basic Steel ...	0.5448 (0.1984)	0.0004 (0.0040)	0.0254 (0.0119)	0.0371 (0.0178)	0.0383 (0.0191)	56.11	1.35
Metal Products .....	0.4686 (0.1555)	-0.0083 (0.0026)	-0.0112 (0.0068)	0.0141 (0.0175)	0.0473 (0.0177)	20.44	5.62
Non-El. Machinery .....	0.4395 (0.1568)	-0.0019 (0.0033)	0.0485 (0.0084)	-0.0037 (0.0215)	-0.0023 (0.0223)	16.63	0.73
El. Machinery .	0.4997 (0.1423)	0.0015 (0.0032)	0.0067 (0.0083)	0.0649 (0.0183)	0.0684 (0.0205)	25.47	0.70
Transport Equipment .....	0.4138 (0.1740)	-0.0003 (0.0024)	0.0161 (0.0043)	0.0482 (0.0141)	0.0516 (0.0146)	17.04	0.27
Misc. Products .....	0.5172 (0.1214)	-0.0071 (0.0043)	-0.0452 (0.0075)	-0.0653 (0.0189)	-0.0655 (0.0225)	11.13	0.83

\* Cf. notes on p. 56.

## NOTES TO TABLES II. 4-10

- a) The growth rates are determined as the OLS estimates on  $b$  from the relation:

$$X_{it} = a + bt \quad (X = \ln L, \ln \frac{V}{L}, \ln \frac{K}{L}, \ln \frac{M}{L}, \ln W, \frac{WL}{V}, \frac{M'}{Y'})$$

The variable measures are presented in (1), (2), (4), (6), (8), (10) and (11) above.

- b) The slope-coefficients from regressions on  $\ln N$  are determined as the estimates on  $c_1$ ,  $c_2$  and  $c_3$  from the relations:

A.  $X_{it} = a + c_1 \ln N_{it}$

B.  $X_{it} = a_i + c_2 \ln N_{it} \quad (X = \ln \frac{V}{L}, \ln \frac{K}{L}, \ln \frac{M}{L}, \ln W, \frac{WL}{V}, \frac{M'}{Y'})$

C.  $X_{it} = a_i + b_t + c_3 \ln N_{it}$

where  $a_i$  are establishment-specific parameters and  $b_t$  time-specific parameters.

- c)  $F_C$  and  $F_T$  are defined in Appendix II.3.

### c. The Capital - Labour Ratio

There are significant differences in the capital-labour ratio between establishments for all industries, judged by the  $F_C$ -statistics. The  $F_T$ -statistics also tell us that except for four industries, Printing, Basic Chemicals, Transport Equipment and Misc. Products, there are also significant differences in this variable over time. These four also rank lowest with respect to growth rate over time. The latter industry is the only one with a negative growth rate.<sup>1)</sup> Among the remaining eleven industries all except one have a significantly positive growth rate. The more heavy industries such as Mining and Quarrying, Pulp and Paper and Non-El. Machinery are those with the most rapid growth in the capital-labour ratio.

There are eight industries with a significantly positive slope-coefficient in the regression of  $\ln \frac{K}{L}$  on the size-variable  $\ln N$ , when imposing the same intercept for all units for all years, while there are three industries with a significantly negative one. However, when allowing the intercept to vary between units, or both between units and over time, the slope-coefficient shows an even sharper drop than for  $\ln \frac{V}{L}$ .<sup>2)</sup>

### d. The Materials - Labour Ratio

The systematic variation of the materials-labour ratio is somewhat "more significant" than for the capital-labour ratio both between establishments and over time. All  $F_C$ -statistics are above the corresponding upper 5 % fractile, and the same is true for all but one of the  $F_T$ -statistics. The exception is Mining and Quarrying, which is also the one among our industries with the lower value of the  $F_C$ -statistics.<sup>3)</sup>

The trend of the materials-labour ratio is positive for all industries, and it is significantly positive for nine. There are substantial

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#### Notes:

- 1) This industry has, however, a substantial growth in both factors. From Table II.4. we know that the growth in labour input is 6.2 %. Thus the growth rate of capital input is 5.4 %. Both growth rates, particularly the one for labour, are substantially above the average for our industries.
- 2) Cf. the last part of Section 4.b. above.
- 3) Strictly speaking, a comparison of F-statistics between industries is not directly possible due to different degrees of freedom. However, these differences are not very important for the fractiles of interest of the corresponding F-distributions. Cf. Appendix II.3.

differences between industries for variation with size. When imposing the same intercept for all units and all years, there are ten industries with a significantly positive slope-coefficient in the regression on  $\ln N$ , while there are three industries with a significantly negative one. Thus, there are only two industries with a slope-coefficient not significantly different from zero, namely Mining and Quarrying and Metal Products. The extremes are Food Products and Clothing with coefficients of approximately minus and plus 0.5 respectively.

The drop in the slope-coefficient when allowing the intercept to vary between units, or both between units and over time is substantially less pronounced and uniform for the materials-labour ratio than for the capital-labour ratio. In fact some of the industries have a higher slope-coefficient for the former variable in those cases. However, there are only two industries with a significantly positive slope-coefficient while there are now seven industries with a significantly negative one. It is evident then that the correlation between the "transitory" components of  $\ln L$  and  $\ln N$  seems to play an important role for these results as well.<sup>1)</sup>

#### e. The "Real" Wage Rate

Not unexpectedly the main dimension of the variation of the "real" wage rate is over time. The  $F_T$ -values are quite high for most industries. However, since the wage rate as defined by us is deflated with the price index for value added, its growth is underrated for some industries in the same way as the growth in value added (or average value added productivity of labour) is underrated.<sup>2)</sup> This seems to be more serious for Printing which has a significantly negative growth rate, and Transport Equipment which has a positive but not significant growth rate. The growth rate for the other industries is significantly positive.

According to the results of Table II.8. large production units seem in general to pay a higher "real" wage rate than smaller ones. The coefficient of  $\ln N$  when a common intercept is imposed for all units for all years is significantly positive for nine industries. It is negative, but not significant for two, namely Food Products and Printing. It is somewhat more difficult to explain the general drop in the slope-coefficient for this variable than for the previous ones when allowing the intercept to vary between units and over time. The explanation may be the way the wage rate is defined, i.e. as wages paid to production workers divided by total number

#### Notes :

- 1) Cf. Section 4.b. above.
- 2) Cf. Section II.2.b. and Appendix II.1.

of hours worked by this type of employee. The denominator is clearly positively correlated with the systematic parts of  $N$ , and presumably also with the more "transitory" part of this variable. This seems to be true at least for some of the industries, since there are six industries with a significantly negative slope-coefficient of  $\ln N$  when allowing the intercept to vary between units and over time, and only one industry with a significantly positive one, namely Mineral Products.

#### f. Labour's Share in Value Added

According to the analysis of variance results there are for all industries significant differences between establishments in labour's share in value added. However, there are only two industries with significant differences over time, namely Pulp and Paper and Transport Equipment. These two industries are also the only ones with significant trend coefficients,<sup>1)</sup> a positive one for Pulp and Paper and a negative one for Transport Equipment.

Not surprisingly there are also some differences between industries for the level of labour's share. For Basic Chemicals and Basic Steel it is about 0.5, while for Wood Products and Transport Equipment it is about 0.75 and 0.77 respectively.

Labour's share also shows a significant variation with size for some industries. For three it has a significantly negative slope-coefficient and for five a significantly positive slope-coefficient in the regression on  $\ln N$  with a common intercept for all units for all years. When the intercept is allowed to differ, the results are rather puzzling with a change of sign for a number of industries, etc.

Generally, the results suggest that our samples for the different industries are rather heterogenous, that labour's role in production may be widely different even for units belonging to the same two-digit industry group.<sup>2)</sup>

#### g. Materials' Share in Gross Production

The heterogeneity of the samples is still more apparent in the results of materials' share in gross production presented in Table II.10. The  $F_C$ -statistics are quite high for most industries and the inter-establishment

#### Notes:

- 1) In contrast to the previous variables absolute and not relative changes are studied for the two share-variables. The term "growth rate" is therefore avoided.
- 2) Examining the composition of our industries we see that this is not very surprising. Cf. Appendix II.1.



differences are significant for all industries, while there are significant differences over time for eight industries. There is a quite uniform downward trend in materials' share over time.<sup>1)</sup> For the five industries, Mining and Quarrying through Wood Products and for Metal Products, the trend coefficient is significantly negative. There are also substantial differences between industries in the level of materials' share. For Mining and Quarrying it is as low as 0.16, while at the other extreme it is about 0.68 for Food Products and about 0.66 for Pulp and Paper.

The heterogeneity of the samples is underlined by the results of the regressions on  $\ln N$ . For eight industries the coefficient of  $\ln N$  is significantly positive while it is significantly negative for three when a common intercept is imposed for all units for all years. However, the results turn out to be quite different when allowing the intercept to vary between units, or between units and over time.

#### 5. A Summary Evaluation of the Data

The main sources of data applied in this study, the Census of Establishments for 1963 and the Annual Production Statistics, are intended to cover other needs than the one of empirical bases of econometric studies. They may be good enough for the computation of sums and means of various central economic magnitudes such as production, materials, number of employees etc. However, serious difficulties are encountered when trying to use the data in estimating production and behaviour relations, which is the main purpose of the present study. This should be evident from the contents of this chapter. It was also clearly demonstrated in a related study.<sup>2)</sup>

In the latter study some efforts were concentrated on analysing the effects of two types of errors that were considered to be the more important, i.e. errors of measurement in the capital input measure and the lack of a quality component of labour input.

These two types of errors also appear to be among the main ones in the present study. We must therefore pay proper attention to them. Indeed, an entire chapter, the following one, is devoted to the analysis of these errors with the model presented in Chapter I as the framework.

In addition, there are a few other errors which may at least have a serious impact on the results concerning technical change. The main one is

#### N o t e s :

1) Since we consider absolute changes in materials' share, the term "growth rate" is avoided.

2) GRILICHES and RINGSTAD (1971), op. cit.

the deflator used for output. When discussing the importance and nature of technical change we must also try to ascertain in what way this error has affected our findings.

We will not argue that these errors are the only ones present or even that these are the only ones that may have a significant impact on the results. They are clearly, however, among the more serious. They will also be discussed explicitly since these are the errors we may be able to say something more about than just that they are present in our data.

Appendix II.1. Composition of the Industries, by Four-Digit Industry Groups<sup>1)</sup>

Industry group	Name of industry group	Number of units
1100	Coal mining .....	1
1210	Iron ore mining .....	4
1220	Pyrites and copper ore mining .....	5
1290	Metal mining not elsewhere classified .....	3
1410*	Stone quarrying .....	3
1510*	Limestone quarrying .....	5
1520*	Quartz and felspar quarrying .....	3
1590*	Mineral quarrying not elsewhere classified .....	2
----- Total for Mining and Quarrying .....		26
2010	Slaughtering and preparation of meat .....	38
2021	Dairies .....	22
2022	Manufacture of condensed and dried milk .....	2
2023	Manufacture of ice-cream .....	2
2029	Milk collecting stations .....	9
2031	Canning of fruits and vegetables .....	1
2039	Other preserving of fruits and vegetables .....	3
2040	Canning of fish and meat .....	17
2051	Frozen fish .....	14
2052	Manufacture of prepared fish dishes, etc. ....	3
2061	Local grain mills .....	1
2062	Commercial grain mills .....	8
2069	Other grain processing .....	1
2071	Manufacture of perishable bakery products .....	5
2072	Manufacture of bisquits, etc. ....	2
2080	Manufacture of cocoa, chocolate and sugar confectionary .....	6
2091	Manufacture of margarine .....	5
2093	Manufacture of livestock feeds .....	3
2099	Manufacture of other food preparations .....	6
2110	Distilling, rectifying and blending og spirits .....	1
2130	Breweries and manufacturing of malt .....	9
2140	Soft drinks and carbonated water industries .....	2
2200	Tobacco manufactures .....	4
----- Total for Food Products .....		164

## N o t e :

- 1) For those industry groups marked with an asterisk (\*) output is deflated by means of an index for inputs of materials, semi-products and labour.

Industry group	Name of industry group	Number of units
2311	Spinning and weaving of wool .....	17
2312	Spinning and weaving of cotton and rayon .....	11
2313	Spinning and weaving of hemp, jute and linen .....	2
2314	Manufacture of narrow fabrics .....	3
2321	Manufacture of hosiery .....	2
2329	Other knitting mills .....	14
2330	Cordage, rope and twine industries .....	7
2392	Manufacture of impregnated textiles etc. ....	2
<b>Total for Textiles .....</b>		<b>58</b>
2410	Manufacture of footwear .....	13
2431	Manufacture of garments of waterproof material .....	5
2432	Manufacture of work clothing .....	1
2433	Manufacture of men's and boys' garments .....	26
2434	Manufacture of women's, girls' and infants' garments .	13
2443	Manufacture of hats and caps .....	3
2491	Manufacture of furnishings, etc. ....	5
2499	Manufacture of other made-up textile goods .....	1
<b>Total for Clothing .....</b>		<b>67</b>
2510	Sawmills and planing mills .....	25
2521	Wood preserving industries .....	4
2523	Prefabrication of wooden houses and structures .....	1
2525	Manufacture of wood-wool cement products .....	4
2529	Manufacture of other building material of wood, etc. .	2
2532	Manufacture of casks .....	1
2599	Manufacture of wooden articles not elsewhere classified	1
2611	Manufacture of wooden furniture .....	5
2512	Manufacture of metal furniture .....	2
<b>Total for Wood Products .....</b>		<b>45</b>
2710	Manufacture of mechanical pulp .....	22
2721	Manufacture of sulphite pulp .....	13
2722	Manufacture of sulphate pulp .....	5
2730	Manufacture of paper, paperboard and cardboard .....	40
2740	Manufacture of wallboards etc. ....	5
2751	Manufacture of paper and paperboard container .....	13
2759	Manufacture of other paper and paperboard products ...	5
<b>Total for Pulp and Paper .....</b>		<b>103</b>

Industry group	Name of industry group	Number of units
2821 <sup>**</sup>	Printing of newspapers .....	23
2822 <sup>**</sup>	Printing of books .....	5
2823 <sup>**</sup>	Printing of commercial matter .....	6
2829 <sup>**</sup>	Other printing activity .....	14
2830 <sup>**</sup>	Bookbinding .....	9
2891 <sup>**</sup>	Electrotyping and stereotyping .....	5
2899 <sup>**</sup>	Other services incidental to printing .....	1
----- Total for Printing .....		63
2910	Tanneries and leather finishing plants .....	1
2930	Manufacture of leather products, except footwear, etc. ....	1
3010	Manufacture of rubber products .....	6
3111	Manufacture of calcium carbide and cyanamide .....	5
3112	Manufacture of other fertilizers .....	4
3113	Manufacture of explosives .....	3
3114	Manufacture of synthetic fibres, resins, etc. ....	3
3119	Manufacture of other basic industrial chemicals .....	19
3122	Herring oil and fish-meal factories .....	9
3123	Vegetable oil mills .....	1
3129	Other oil refineries, etc. ....	3
3130	Manufacture of paints, varnishes and lacquers .....	5
3191	Manufacture of pharmaceutical preparations .....	3
3192	Manufacture of soap .....	4
3193	Manufacture of cosmetics, etc. ....	1
3194	Manufacture of candles .....	1
3199	Manufacture of other chemical products .....	3
----- Total for Basic Chemicals .....		72
3210	Manufacture of asphaltic felt .....	1
3290	Other coal and mineral oil processing .....	1
3310	Manufacture of structural clay products .....	4
3321	Manufacture of glass and glass products from raw materials .....	3
3329	Manufacture of glass products from purchased glass ..	1
3331	Manufacture of china and fine earthenware .....	5
3339	Manufacture of pottery and other earthenware .....	1
3340	Manufacture of cement (hydraulic) .....	3
3350	Manufacture of cement products .....	2
3391	Manufacture of abrasives .....	1
3393	Grinding of other non-metallic minerals .....	5

Industry group	Name of industry group	Number of units
3394	Manufacture of cut-stone and stone products .....	3
3399	Manufacture of other non-metallic mineral products ...	6
----- Total for Mineral Products .....		36
3411	Manufacture of ferro-alloys .....	9
3412	Iron and steel works and rolling mills .....	6
3413	Iron and steel foundries .....	12
3420	Refining of aluminium .....	6
3430	Manufacture of crude metals, not elsewhere classified.	4
3491	Non-ferrous metal rolling mills .....	3
3492	Smelting and refining of metals .....	2
----- Total for Basic Steel .....		42
3511	Manufacture of wire and wire products .....	8
3512 <sup>x</sup>	Manufacture of other metal building articles .....	5
3513 <sup>x</sup>	Manufacture of steel structural parts .....	13
3520	Manufacture of metal shipping containers, etc. ....	8
3530	Manufacture of metal household articles .....	5
3591 <sup>x</sup>	Manufacture of metal equipment for offices and shops .	4
3592 <sup>x</sup>	Manufacture of lighting fixtures .....	3
3593 <sup>x</sup>	Manufacture of hand tools and implements .....	2
3594	Manufacture of metal fittings .....	3
3595 <sup>x</sup>	Manufacture of arms and ammunition .....	4
3599 <sup>x</sup>	Manufacture of other metal products not elsewhere classified .....	5
----- Total for Metal Products .....		60
3610 <sup>x</sup>	Manufacture of mining and industrial machinery .....	11
3620 <sup>x</sup>	Manufacture of agricultural and forestry machinery ...	3
3680 <sup>x</sup>	Machinery repair shops .....	4
3691 <sup>x</sup>	Manufacture of household, office and shop machinery ..	5
3699 <sup>x</sup>	Manufacture of other machinery .....	14
----- Total for Non-Electrical Machinery .....		37
3711 <sup>x</sup>	Manufacture of accumulators and batteries .....	2
3712	Manufacture of wires and cables .....	3
3713 <sup>x</sup>	Manufacture of transformers, generators and electric motors .....	3
3719 <sup>x</sup>	Manufacture of other distribution equipment .....	7
3720	Manufacture of signalling, radio and other tele- communication equipment .....	11
3780 <sup>x</sup>	Electro-technical repair shops .....	2

Industry group	Name of industry group	Number of units
3791 <sup>x</sup>	Manufacture of electric lamps .....	2
3799	Manufacture of other electrical products .....	4
<u>Total for Electrical Machinery .....</u>		<u>34</u>
3811 <sup>x</sup>	Building and repairing of steel ships .....	33
3813 <sup>x</sup>	Building and repairing of wooden ships .....	7
3814 <sup>x</sup>	Manufacture of other marine machinery .....	2
3819 <sup>x</sup>	Other services for ships .....	1
3821 <sup>x</sup>	Manufacture of railroad cars and locomotives .....	3
3822 <sup>x</sup>	Repairing of railroad cars and locomotives .....	13
3831 <sup>x</sup>	Manufacture of bodies for motor vehicles .....	2
3839 <sup>x</sup>	Manufacture of motor vehicles and parts not elsewhere classified .....	2
3840 <sup>x</sup>	Repair of motor vehicles .....	17
3851 <sup>x</sup>	Manufacture of motor-cycles and bicycles .....	1
3860 <sup>x</sup>	Manufacture of aircraft .....	3
3890 <sup>x</sup>	Manufacture of transport equipment not elsewhere classified .....	3
<u>Total for Transport Equipment .....</u>		<u>87</u>
3940	Manufacture of jewellery and related products .....	3
3991	Manufacture of brooms and brushes .....	1
3994	Manufacture of plastic products not elsewhere classified .....	8
3999	Manufacture of other products not elsewhere classified .....	1
<u>Total for Miscellaneous Products .....</u>		<u>13</u>
<u>Total for Mining and Manufacturing .....</u>		<u>907</u>

## Appendix II.2. Two Data Problems

a. On Natural and Artificial Births  
and Deaths of Establishments

There are significant movements in the reported number of establishments during the period covered by this study judged by the identification numbers of the establishments. These movements can be divided into "natural" births and deaths of establishments and obvious "artificial" ones. Natural births and deaths include the establishment of a completely new production unit and the closing of a production unit previously in operation. More doubtful cases are movements into and out of the sample of establishments due to the buying and selling of production units. Artificial births and deaths include those due to slightly different definitions over time of an establishment.<sup>1)</sup>

Generally, however, this does not seem to entail serious difficulties. A more disturbing cause of artificial movements in the number of establishments is the following: Due to a widening of the range of goods produced by an establishment it is partitioned according to kind of activity, and the parts are classified into different industry groups. The opposite also seems to have taken place to some extent, i.e. that two (or more) establishments of a firm are merged into one.

According to the identification number system we have in an "unmerging" case one complete time-series, since one of the branches (usually the "main branch") inherits the identification number of the unit subject to unmerging. Thus we also get some "new" establishments the year the unmerging takes place. In the case of a merger of two or more establishments of a firm, the merged unit usually gets the identification number of the more important of those establishments subject to merging. In this case one or more establishments "die" since their identification numbers disappear.

A change in location (municipality) may also lead to a change in identification, and thus lead to a break in the time-series. This is, however, a "less artificial" cause of movements compared to those mentioned previously. In this case it is more reasonable to speak of a new production unit since a change in location probably also implies a basic change in economic environment. Why move otherwise?

---

 Note:

- 1) There does not seem to have been any basic change in the definition of an establishment during the period considered, but rather a somewhat varying usage of the definition.



Thus, according to the establishments' identification numbers there are substantial movements in the population of establishments during the period 1959-1967. However, due to the causes pointed out above, many of these movements are artificial. Or said in another way: The identification number is rather unreliable when tracing physical production units back into the past, trying to construct time-series for them. On the other hand, nothing better is available.

b. Calculation of Missing Values for Subsidies and Duties

As pointed out in Section II.2.a. we do not have information concerning subsidies and duties for 1959 and 1960. Thus in order to obtain a measure of output that is comparable over time, we must either compute it in market prices, or in some way calculate subsidies and duties for 1959 and 1960 to obtain a measure in factor prices. For most industries it does not matter much whether we use market or factor prices. For a few, however, particularly those using inputs from agriculture and fishing, there is a substantial difference between these two measures. Therefore the calculation approach is chosen.

We adopt an ad hoc procedure based on the assumption that there is a fixed ratio between subsidies and gross production and duties and gross production. We calculate the missing values of subsidies and duties as:

$$U_{i,t} = \frac{Y'_t}{2} \left( \frac{U_{i,61}}{Y'_{61}} + \frac{U_{i,62}}{Y'_{62}} \right) \quad \begin{cases} i = 1.2 \\ t = 59.60 \end{cases}$$

trying to reduce the effects of errors of observation by averaging the information for 1961 and 1962.

Three objections could be raised against this procedure:

First, for some types of activities duties or subsidies are rather determined by input of materials than by gross production. This is presumably not very serious since in the short run we would expect a fairly stable ratio between materials and gross production for each establishment. It should therefore not make much difference which one we use in the formulas above. Since we would like to use only one of these variables, gross production was chosen.

Second, a change in the product mix (or materials input mix) may have taken place during the period considered. This is clearly of importance only if duties or subsidies depend on certain kinds of outputs or inputs. Again, however, the period under consideration is rather short so that serious errors due to this argument are unlikely.

Third, there may have been changes in the Government's policy on duties and subsidies affecting mining and manufacturing industries. There are always some minor changes and adjustments in this policy. For the period under consideration, however, there are no changes that can make the missing observation calculation above basically invalid. Thus, all in all, this method of obtaining subsidies and duties for 1959 and 1960 should not be too bad. Examining the values obtained, the method indeed seems to work quite well, confirming also an impression of a rather good quality in the reporting of subsidies and duties.

Appendix II.3. Analysis of Variance of Variables in Cross-Section Time-Series Data<sup>1)</sup>

With a stochastic variable  $y_{it}$ , where the subscripts represent establishment and time respectively, we may have the hypothesis that it shows systematic variation along the two dimensions in the following way:

$$(1) \quad y_{it} = \alpha + \beta_i + \gamma_t + \varepsilon_{it} \quad \begin{array}{l} (i = 1, \dots, I) \\ (t = 1, \dots, T) \end{array}$$

where  $\alpha$ ,  $\beta_i$  and  $\gamma_t$  are non-stochastic magnitudes while  $\varepsilon_{it}$  is a stochastic variable presumed to be normally distributed with zero mean and constant standard deviation  $\sigma$  and no serial correlation.  $I$  is the number of establishments and  $T$  is the number of years. This model corresponds to an analysis of variance model with a two-way classification without any interaction effect and with one unit per cell.

The total sum of squares of deviation from the mean can be decomposed in the following way:

$$(2) \quad S^2 = \sum_{i=1}^I \sum_{t=1}^T (y_{it} - \bar{y})^2 = \sum_{i=1}^I \sum_{t=1}^T (y_{it} - \bar{y}_{i.} - \bar{y}_{.t} + \bar{y})^2 + T \sum_{i=1}^I (\bar{y}_{i.} - \bar{y})^2 + I \sum_{t=1}^T (\bar{y}_{.t} - \bar{y})^2$$

or

$$(3) \quad S^2 = S_0^2 + S_C^2 + S_T^2$$

Provided that  $\sum_{i=1}^I \beta_i = \sum_{t=1}^T \gamma_t = 0$ , which implies no loss of generality, we have

$$\begin{aligned} \bar{y}_{i.} &= \frac{1}{T} \sum_{t=1}^T y_{it} = \alpha + \beta_i + \bar{\varepsilon}_{i.} \\ (4) \quad \bar{y}_{.t} &= \frac{1}{I} \sum_{i=1}^I y_{it} = \alpha + \gamma_t + \bar{\varepsilon}_{.t} \\ \bar{y} &= \frac{1}{I \cdot T} \sum_{i=1}^I \sum_{t=1}^T y_{it} = \alpha + \bar{\varepsilon} \end{aligned}$$

where

$$\begin{aligned} \bar{\varepsilon}_{i.} &\text{ is normally distributed } (0, \frac{\sigma}{\sqrt{T}}) \\ (5) \quad \bar{\varepsilon}_{.t} &\text{ " " " } (0, \frac{\sigma}{\sqrt{I}}) \\ \bar{\varepsilon} &\text{ " " " } (0, \frac{\sigma}{\sqrt{IT}}) \end{aligned}$$

Notes:

- 1) For a detailed discussion of analysis of variance models see: SCHEFFE (1959): The Analysis of Variance.

It can now be shown that under our assumptions we have:

$$\begin{aligned} E(S_0^2) &= (I - 1)(T - 1)\sigma^2 \\ (6) \quad E(S_C^2) &= T \sum_{i=1}^I \beta_i^2 + (I - 1)\sigma^2 \\ E(S_T^2) &= I \sum_{t=1}^T \gamma_t^2 + (T - 1)\sigma^2 \end{aligned}$$

And due to our assumptions  $\frac{S_0^2}{\sigma^2}$  is  $\chi^2$ -distributed with  $(I - 1)(T - 1)$  degrees of freedom, and  $S_0$ ,  $S_C$  and  $S_T$  are distributed independently.

And therefore, provided that  $\sum_{i=1}^I \beta_i^2 = 0$ , which implies that the establishment specific components of the error mean are zero, we have that

$$(7) \quad F_C = \frac{S_C^2}{S_0^2} (T - 1)$$

is F-distributed with  $(I - 1)$  and  $(T - 1)(I - 1)$  degrees of freedom.

Provided that  $\sum_{t=1}^T \gamma_t^2 = 0$ , which implies that the time specific components of the error mean are zero, we have

$$(8) \quad F_T = \frac{S_T^2}{S_0^2} (I - 1)$$

is F-distributed with  $(T - 1)$  and  $(I - 1)(T - 1)$  degrees of freedom.

Therefore, by means of (7) we can test the hypothesis:

$$H_{C0}: \sum_{i=1}^I \beta_i^2 = 0$$

against

$$H_{C1}: \sum_{i=1}^I \beta_i^2 > 0$$

By means of (8) we can correspondingly test the hypothesis:

$$H_{T0}: \sum_{t=1}^T \gamma_t^2 = 0$$

against

$$H_{T1}: \sum_{t=1}^T \gamma_t^2 > 0$$

We get tests with level  $\epsilon$  if we reject  $H_{C0}$  when we observe

$$F_C > F_{1-\epsilon, (I-1), (I-1)(T-1)} \quad \text{and reject } H_{T0} \text{ when we observe}$$

$$F_T > F_{1-\epsilon, (T-1), (I-1)(T-1)}. \quad \text{For our data with } T = 9 \text{ and } I = 13 \text{ at}$$

least and  $I = 164$  at most (except for Total Mining and Manufacturing where

$I = 907$ ) we have  $F_{0.95, (I-1), 8(I-1)}$  approximately between 1.90 and 1.25 and

$F_{0.95, (8, 8(I-1))}$  approximately between 2.05 and 1.95.

Appendix II.4. A Method of Analysing the Consistency of  
Time-Series for Capital and Investment<sup>\*</sup>

By Vidar Ringstad and Zvi Griliches<sup>\*\*\*</sup>

When independent time-series for capital and investment are used in econometric analyses it is important to know if the two sets of data are consistent, if the reported investments can "explain" the growth in capital when the other factors that also affect the capital stock are taken into account.<sup>1)</sup> This note presents a method for the analysis of such a question and applies it to capital and investment data for Norwegian Mining and Manufacturing at the two digit level and the years 1951-1959.

The change in capital value during a particular period can be thought of as consisting of three elements; gross investment, depreciation, and price change. We can, therefore, write:

$$(1) \quad K_{i,t} - K_{i,t-1} = J_{i,t} - \Delta_{i,t} K_{i,t-1} + \eta_{i,t} K_{i,t-1}$$

where  $K_{i,t-1}$  and  $K_{i,t}$  are the values of the capital stock at the beginning and at the end of the year respectively,  $J_{i,t}$  is gross investment during the year,  $\Delta_{i,t}$  is depreciation ratio,  $\eta_{i,t}$  is the price change ratio, and  $i$  and  $t$  are the industry and time subscripts respectively. If everything in this equation were measured correctly it would be an identity in all the variables.<sup>2)</sup>

If one had independent information about the appropriate depreciation and price-change ratios, one could compute the right side of relation (1) and thus have a direct check of the consistency of the two (capital and investment) sets of data provided, of course, that the depreciation and price change ratios were correct. Since this last requirement may not be fulfilled, one may prefer an approach which does not depend on *a priori* knowledge of these ratios, allowing the data to determine them instead.

\* This appendix is a slightly corrected reprint of a note with the same title that appeared in The Review of Income and Wealth, No. 4, 1968.

\*\*\* We are indebted to a number of the employees of the Central Bureau of Statistics of Norway for valuable assistance during the preparation of this analysis.

N o t e s :

- 1) This problem does not arise often. Usually one of the series, e.g., "capital", is "manufactured" from the investment data, as in the perpetual inventory approach, and the identities are satisfied provided no computational errors were made.
- 2) We presume that investment expenditures are reported on the basis of original costs, that is: No depreciation or price change on capital that is less than one year old. This seems to be the common way of measuring investment expenditures, and it corresponds to the definition of investment in the data we are going to use.

If the depreciation ratio and the price change ratio were to vary along both of the available sample dimensions - industry and time - we would not have enough degrees of freedom to compute all of the ratios on the basis of the data available to us. We make, therefore, what we believe are reasonable restrictions on these parameters and assume that: (a) depreciation ratios are independent of time but they may be different for different industries, and (b) price-change ratios are independent of industry but may be different for different years.

Dividing through by  $K_{i,t-1}$  and introducing the following dummy variables:

$$y_j = 1 \text{ when } j = i, y_j = 0 \text{ otherwise}$$

$$z_\tau = 1 \text{ when } \tau = t, z_\tau = 0 \text{ otherwise}$$

we can write relation(1) in the following way:

$$(2) \quad \frac{K_{i,t} - K_{i,t-1}}{K_{i,t-1}} = \alpha \frac{J_{it}}{K_{i,t-1}} - \sum_{j=1}^I \Delta_j y_j + \sum_{\tau=1}^T \eta_\tau z_\tau \quad \begin{array}{l} i = 1 \dots I \\ t = 1 \dots T \end{array}$$

where I is the number of industries and T is the number of years in our sample.

We have allowed the coefficient of  $J_{i,t}/K_{i,t-1}$  to differ from one in (2), both because we have made simplifying assumptions about the depreciation and price-change ratios and because there may be errors of measurement present in both the capital and investment data sets.

We shall estimate the parameters of this relation using ordinary least squares procedures.<sup>1)</sup> Since the simplifying assumptions we made are unlikely to lead to any systematic bias in the estimate of  $\alpha$ , we shall argue that the capital and investment data are inconsistent if  $\alpha$  is significantly different from one.

As mentioned above, we are applying this procedure to industry data in Norwegian Mining and Manufacturing. They are taken from the Central Bureau of Statistics' Industrial Production Statistics, Annual Survey. Between 1949 and 1950 there is a "break" in the data due to a revision of the lower bound for the size of the establishments included in the annual statistics, and 1959 was the last year in which the capital data were

        
N o t e :

- 1) We have to exclude one y-variable and one z-variable to avoid singularity. This implies that we cannot identify the different industry depreciation ratios or the price-change ratios of different years without additional information - such as the depreciation ratio of one industry or the price-change ratio for one year. But using the dummy-variables method we can detect and allow for *differences* in depreciation ratios between industries and in price-change ratios between years.

collected. We have then data for nine years; 1951 through 1959.<sup>1)</sup> In our analysis we have twenty-one industries, based on the two-digit ISIC code.<sup>2)</sup>

The Industrial Production Statistics for the years under consideration provide data on the full fire insurance value and on investment expenditures for three types of capital: Buildings, Other Construction, and Machinery. We have estimated relation (2) for Buildings and Machinery separately, and for Total Capital consisting of all three types of capital mentioned.

The estimates of  $\alpha$  are presented in Table 1.a. Since the results for the industry dummies indicated that there were few significant differences between the depreciation ratios for different industries, we also estimated relation (2) assuming the same depreciation ratio for all industries. The main effect of this is a reduction in the estimated standard deviation of  $\alpha$ .

The conclusion from both sets of results is that the capital and investment data are *not* consistent either for Buildings and Machinery or for Total Capital, since in all cases except one we can reject the hypothesis that  $\alpha = 1$ .<sup>3)</sup>

What, then, is wrong with these data? We know that there have been some minor changes in the lower bound on the size of establishments included in the annual survey, and also some regrouping between two-digit industry groups during the period under consideration. This is reflected in the relatively poor fit of the estimated relation and it might also have had a systematic effect on the estimate of  $\alpha$ . But it is difficult to believe that this is the only cause of our findings of inconsistency.

Since the capital stock data are "full fire insurance values", the inconsistency could be due to a "lag" effect; it may take some time before investment expenditures are "registered" as stocks of capital. If this conjecture is correct we would expect a positive and significant coefficient for lagged investment, both when it is included in relation (2) *together* with unlagged investment and when it is introduced *instead* of current investment. The results of these two tests are presented in Tables 1.b. and 1.c.,

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N o t e s :

- 1) Since the data on capital at the beginning of 1950 are before the "break", this year is dropped from the analysis. The data on capital at the end of 1950 (at the beginning of 1951) and investment during 1950 are after the "break" and hence usable.
- 2) Groups 11-19, Mining and Quarrying, are considered as one industry. The twenty two-digit manufacturing industry groups 20 through 39 are each considered as one industry.
- 3) At the 5 per cent level. The hypothesis is not rejected for Buildings when industry dummies are included. But since the hypothesis is rejected when these dummies are excluded and since the "acceptance margin" is very slight the conclusion of inconsistency appears to be valid also for Buildings.

respectively. They indicate rather clearly that the coefficient of the same year's investment is not significantly different from zero for any type of capital when lagged investment is included, and that the coefficient of lagged investment is not significantly different from one whether it is included alone or together with unlagged investment.<sup>1)</sup>

Table A.II.1. Estimates of a Relation Explaining the Relative Growth in Reported Capital Values\*

Table 1.a.	Buildings		Machinery		Total Capital	
$\frac{J_{i,t}}{K_{i,t-1}}$ .....	0.558 (0.243)	0.588 (0.162)	0.132 (0.063)	0.223 (0.052)	0.089 (0.053)	0.167 (0.046)
Dummies for years ....	Yes	Yes	Yes	Yes	Yes	Yes
Dummies for industries	Yes	No	Yes	No	Yes	No
Intercept .....	0.016	0.018	0.070	0.024	0.072	0.040
R .....	0.474	0.396	0.485	0.426	0.554	0.459
MSE .....	0.013	0.013	0.024	0.023	0.007	0.007

Table 1.b.	Buildings		Machinery		Total Capital	
$\frac{J_{i,t}}{K_{i,t-1}}$ .....	0.168 (0.240)	-0.026 (0.210)	0.008 (0.069)	0.043 (0.063)	-0.024 (0.055)	0.005 (0.051)
$\frac{J_{i,t-1}}{K_{i,t-1}}$ .....	1.360 (0.275)	1.033 (0.239)	1.240 (0.319)	0.884 (0.192)	1.005 (0.222)	0.826 (0.142)
Dummies for years ....	Yes	Yes	Yes	Yes	Yes	Yes
Dummies for industries	Yes	No	Yes	No	Yes	No
Intercept .....	-0.021	0.004	-0.014	-0.036	0.010	-0.008
R .....	0.573	0.486	0.549	0.519	0.621	0.580
MSE .....	0.012	0.012	0.022	0.020	0.006	0.006

\* Footnote overleaf.

N o t e :

1) Using F-statistics with 20 and 159 degrees of freedom based on the results of relation(2) and the results of this relation when assuming a common depreciation rate for all industries we cannot reject the hypothesis of a common depreciation rate at 5 per cent level, either for Buildings, Machinery or Total Capital. The results are the same when lagged investment is substituted for current investment. This corresponds quite well with other evidence on depreciation rates, suggesting that at the two-digit level and during this period the differences among such rates were rather insignificant in Norwegian Mining and Manufacturing industries.



Table 1.c.	<u>Buildings</u>		<u>Machinery</u>		<u>Total Capital</u>	
$\frac{J_{i,t-1}}{K_{i,t-1}}$ .....	1.423 (0.260)	1.013 (0.176)	1.257 (0.282)	0.966 (0.150)	0.963 (0.198)	0.834 (0.119)
Dummies for years ....	Yes	Yes	Yes	Yes	Yes	Yes
Dummies for industries	Yes	No	Yes	No	Yes	No
Intercept .....	-0.013	0.003	-0.014	-0.036	0.010	-0.008
R .....	0.571	0.486	0.549	0.517	0.621	0.580
MSE .....	0.011	0.011	0.022	0.020	0.006	0.006

\* Yes means that the dummy variables concerned are included in the regression. No means that the dummy variables concerned are not included in the regression.

The intercept is the sum of the coefficients of the two dummy variables excluded from the regression(see footnote 1 p.73), that is  $-\Delta_I + \eta_T$  where  $\Delta_I$  is the depreciation ratio of industry 39, Miscellaneous manufacturing industries, and  $\eta_T$  is the price change ratio of the year 1959. When industry dummies are not included in the regression the intercept is  $-\Delta + \eta_T$  where  $\Delta$  is the common depreciation ratio.

R is the multiple correlation coefficient and MSE is the mean square of the estimated residual error.

These findings imply strongly the existence of a lag between the purchase of a capital object and its emergence as a part of the capital stock. According to our results this lag is more than one year on the average.<sup>1)</sup>

Thus, we conclude that after all, the consistency between the capital and investment data sets is not as poor as the first results for relation (2) indicated. We do not have consistency between the change in capital in a particular year and the investment expenditures of the same year, but we have consistency between the change in capital and the investment expenditures of the previous year. Taking this into consideration when applying these data in contexts where consistency is important, they should for most purposes be as good as any other sets of data on capital and investment.

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N o t e :

- 1) There are probably two major sources of the observed lag between investments and growth in capital stock: (a) While all investment costs of a year are reported, the value of uncompleted investment projects at the end of the year is not reported as part of the capital stock. (b) There may be a general sluggishness in the adjustment of "full fire insurance value" which, as pointed out, is the measure of the current value of the capital stock. If the latter cause is dominating we would expect the estimated price-change ratios to show a lag also, compared with the price-change ratios implied by a current price index of capital.

To investigate this we computed the price-change ratios for Total Capital from the relation with lagged investments instead of current investments and a common depreciation rate for all industries. We cannot identify the *level* of the price-change ratios, by our method of estimation, but this does not matter in this context. These estimates were compared with the price-change ratios implied by a price index for Total Capital of the Mining and Manufacturing industries. The latter index is based on price indices for different categories of gross investment chained together with the amounts of corresponding categories of capital as weights. This comparison gives an indication of a lag of about one year between the two sets of price-change ratios in the period 1951 through 1953, while for the following years they have fairly similar movements year by year. Thus, this comparison does not provide particular support to either of the two main causes of lag mentioned. There is a slight suggestion of a twist of the relative importance over time of the two causes - the effects of "sluggishness" are reduced in relation to the effects of "incompleted investment projects". The basis for this suggestion is, however, rather weak and it is difficult to find any clear evidence of it from other sources.

CHAPTER III. ESTIMATING PRODUCTION FUNCTION  
PARAMETERS FROM SIMULTANEOUS EQUATIONS HAVING ERRORS OF MEASUREMENT

In applied econometric analyses there are generally two main problems. First, what is the proper specification of the model, particularly which variables are endogenous and which ones are exogenous. Second, how do the variable measures "behave" as compared to their theoretical counterparts, or in other words the problem of measurement errors in a broad sense. It is fair to say that considerably more attention has been devoted to the first problem than to the second, which is also usually disregarded when the former is discussed. On the other hand, any simultaneity problems are usually ignored when errors of measurement problems are handled.

In this chapter we will try to treat these problems in a more simultaneous way. The theoretical framework is the following model:<sup>1)</sup>

$$(1.a.) \quad y = \alpha x + \beta z + u$$

$$(1.b.) \quad y - x = bw + v$$

where  $y = \ln V$ ,  $x = \ln L$ ,  $z = \ln K$ ,  $w = \ln W$ .  $u$  and  $v$  are error terms, the properties of which will be subject to various assumptions throughout this chapter.<sup>2)</sup> In this model  $y$  and  $x$  are endogenous variables, while we assume that  $z$  and  $w$  would have been exogenous if they were correctly measured. However, we have argued in Chapter II that capital input contains a large, but presumably random error component, while  $w$  and also  $x$  are more systematically wrong as they both refer to the quantity component of labour input ignoring the quality component.

If the error terms of (1) had zero means for all observations, consistent estimators for the production function parameters could be obtained through simple textbook methods. In the first section of this chapter we show, however, that such methods may yield quite poor results having the two kinds of errors mentioned. It appears that they are not very robust against such errors.<sup>3)</sup> On the other hand, it is shown that ordinary

N o t e s:

1) Cf. Chapter I.

2) All variables are computed from their means.

3) A method is described as robust if the inferences are not seriously invalidated by the violation of the assumptions on which it is based. Thus, a method may be robust against some specification errors while it is little robust against others. Cf. BOX (1953): Non-normality and tests on variance, SCHEFFE (1959), op. cit., Section 10.6, and MALINVAUD (1966): Statistical Methods of Econometrics, Section 8.4.

In the present context the errors of measurement violate the assumption of zero (or constant) error term means, or more precisely  $E(u/z, w) = E(v/z, w) = c (=0)$ . Thus, some of the estimation methods to be considered are little robust against the violation of this assumption due to errors present in our data.

least squares on the production function, although it yields estimators for the factor elasticities that are subject to both simultaneous equations bias and errors of measurement biases, is generally preferable precisely because it is fairly robust against measurement errors. Thus, a main conclusion of this chapter is that having errors of measurement we may pay an unreasonably high price for the elimination of biases due to simultaneous equations.

With two or more cross-sections for the same units it has been argued that it is possible to reduce or eliminate the effects of simultaneity by means of covariance analysis.<sup>1)</sup> The argument runs as follows: in cross-section data the error term of a production function like (1.a.) has to catch differences in management and "environments" between units. Since more well-managed units or those with favourable environments tend to use more of the inputs than poorly managed ones or those with less favourable environments, there is a positive correlation between the error term and the inputs, and thus the OLS estimators are subject to a kind of simultaneous equations bias. Where we have more than one observation per unit we can eliminate the establishment-specific component of the error term by means of covariance analysis. If the time period covered by the data is not too long, the differences in management and environments between units are presumably fairly stable, and having eliminated them from the error term we have also presumably eliminated the main source of simultaneity bias of the OLS estimators. We show, however, in the second section of this chapter that this method of eliminating simultaneous equations bias is not very robust against measurement errors either. Thus, in this context as well we may pay an unreasonably high price for obtaining estimators "free" of simultaneous equations bias.<sup>2)</sup>

We therefore have to look for other methods of estimation. If we accept (1.a.) as the "true" production function and thus constrain the elasticity of substitution to unity, we may estimate the elasticity of labour as that factor's share in output. This is probably the best estimator for that parameter obtainable in the present context since neither of the two

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N o t e s :

- 1) See for example, MUNDLAK (1961): Empirical Production Function Free of Management Bias, HOCH (1957): Estimation of Agricultural Resource Productivities Combining Time Series and Cross Section Data, and HOCH (1962): Estimation of Production Function Parameters Combining Time Series and Cross Section Data.
- 2) In Section 2 of this chapter we also have a digression on auto-correlation and covariance analysis for relations in cross-section time series data.

kinds of errors mentioned affects it. Section 3 of this chapter includes a discussion of this subject as well as various possibilities for estimating the elasticity of capital given this particular estimator of the elasticity of labour. By constraining the elasticity of scale to unity one has an estimator for the elasticity of capital with the same properties as that for the elasticity of labour. However, as we would like the scale elasticity to be a free parameter a few other methods are tried. And we are finally converging towards a method of estimation that seems to be the best one given the kind of data on which we are working.

In Section 3 we also report on some further attempts made to estimate the elasticity of substitution. In a concluding section a short summary of the findings of the present chapter is presented.

### 1. The Properties of the ILS and OLS Methods of Estimation in the Present Context

#### a. The ILS Method

In this section we will try to determine how the two main errors in our data may affect the properties of two well-known methods of estimation, i.e. indirect least squares (ILS) and ordinary least squares (OLS). We will first consider the ILS method.

The reduced form of (1) consists of the second relation of that model together with:<sup>1)</sup>

$$(2) \quad x = \pi_1 w + \pi_2 z + r$$

Where

$$\pi_1 = -\frac{b}{1-\alpha}, \quad \pi_2 = \frac{\beta}{1-\alpha} \quad \text{and} \quad r = \frac{u-v}{1-\alpha}.$$

If there are no measurement errors and the errors of relations have zero (or constant) means, we can obtain unbiased estimators for the parameters of (1.b.) and (2) by OLS. Denoting these estimators by  $\hat{\delta}$ ,  $\hat{\pi}_1$  and  $\hat{\pi}_2$  we obtain the consistent ILS estimators for the factor elasticities as:<sup>2)</sup>

$$(3) \quad \hat{\alpha} = \frac{\hat{\pi}_1 + \hat{\delta}}{\hat{\pi}_1} \quad \hat{\beta} = -\frac{\hat{\delta} \hat{\pi}_2}{\hat{\pi}_1}$$

---

#### Notes :

- 1) We could solve the system with respect to  $y$  instead of  $x$ , but this does not make any difference.
- 2) Other methods like the two-stage least square method and the instrumental variable method do not yield exactly the same estimators for the factor elasticities as the ILS method. Asymptotically, however, they yield the same results both when the error terms are assumed to be uncorrelated with the exogenous variables and in the errors in variables cases discussed later.

We obtain the corresponding estimator for the scale elasticity as:

$$(4) \quad \hat{\varepsilon} = \frac{\hat{\pi}_1 + (1 - \hat{\pi}_2) \hat{\delta}}{\hat{\pi}_1}$$

We know, however, that there are particular types of errors of measurement present and we would like to know how they may affect the properties of the estimators above.

As pointed out, our labour input measure refers to the quantity of that factor. Now, there are obviously some variations in the quality of labour both between establishments and over time. Since the relevant measure of the productive performance of labour is "quantity times quality", variations in the quality component in our sample represent a potential cause of inconsistent estimators when labour input is measured by the quantity component alone.

The quality component of "total" labour input is not likely to show a completely random variation, since the observed wage rate, which also refers to the quantity component of labour, is likely to be positively correlated with it. In order to say something more about what can happen to our ILS estimators when there are such variations in labour quality, we adopt the rather extreme assumption of perfect correlation between that variable and the wage rate.<sup>1)</sup>

As shown in Section a of Appendix III.1. we then have:<sup>2)</sup>

$$(5) \quad \begin{aligned} \text{plim } \hat{\delta} &= 1 \\ \text{plim } \hat{\alpha} &= 0 \\ \text{plim } \hat{\beta} &= \frac{\beta}{1-\alpha} \end{aligned}$$

---

N o t e s :

- 1) This assumption conforms to the one made about quality differences between production and non-production workers when constructing the labour input measure applied. Cf. Section II.2.c. The rather approximate nature of the assumption of perfect correlation between the wage rate and quality of labour should be evident, however. For instance, since we apply a "real" wage rate, i.e. the ratio between the current wage rate and the price index of value added, our wage rate is clearly affected by the prices obtained on output (and also the prices of materials).
- 2) As usual "plim" denotes the probability limit. Basically it should not matter in our case whether this relates to the number of units (I) or the number of years (T). However, there may be arguments for the following kind of probability limit  $I \rightarrow \infty$  and  $T \rightarrow \infty$  but  $\frac{I}{T}$  is constant equal to the value of this ratio in the sample.  $\frac{I}{T}$

This is hardly a surprising result since there is by assumption no "real" variation in one of our identifying variables, namely  $w$ . Thus neither of the relations of our model is identifiable. What we manage to estimate is a ratio that may suggest whether there are decreasing, constant or increasing returns to scale in production.

Even if the assumptions by means of which we have derived (5) are approximate, we have demonstrated that the ILS method is quite sensitive to the type of error considered. There are undoubtedly differences in the quality of labour input correlated with the observed wage rate in Norwegian mining and manufacturing industries. Thus the ILS method is of little value in the present context.

Another reason why the ILS method does not work is the substantial errors of measurement in the capital data.<sup>1)</sup> Assuming that these errors are completely random we show in Section b of Appendix III.1. that the asymptotic biases of the ILS estimators for the factor elasticities are:

$$(6) \quad \begin{aligned} \text{bias } \hat{\alpha} &= (1-\alpha)b_{zw}B \\ \text{bias } \hat{\beta} &= (b-\beta b_{zw})B \end{aligned}$$

and the bias in the implied estimator for the elasticity of scale is:

$$(7) \quad \text{bias } \hat{\epsilon} = ((1-\epsilon)b_{zw} + b)B$$

or in the case of constant returns to scale:

$$(8) \quad \text{bias } \hat{\epsilon} = bB$$

where

$$B = \beta k^2 / (\beta b_{zw} k^2 - b(1-r_{zw}^2)), \quad k^2 = \left(\frac{\sigma_e}{\sigma_z}\right)^2 \text{ is the probability limit}$$

of the error to total variance ratio of the log-capital measure,  $b_{zw}$  is the probability limit of the slope coefficient from the auxiliary regression of  $z$  on  $w$ , and  $r_{zw}$  is the probability limit of the simple correlation coefficient between  $z$  and  $w$ .

Provided the elasticity of substitution is not too low both biases in (6) are presumably negative since it is reasonable to assume that  $b_{zw} > 0$ .<sup>2)</sup> We see, however, that the denominator of  $B$  may be positive implying a positive bias  $\hat{\alpha}$  while the sign of bias  $\hat{\beta}$  is undetermined, even if it is likely to be negative in this case as well.

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N o t e s :

1) Cf. Section II.3.

2) For Total Mining and Manufacturing we have that if  $\beta = 0.4$ ,  $b = 1$ ,  $k^2 = 0.25$  and sample statistics are substituted for the other parameters of the bias formulas that  $B = 0.25 \cdot 0.4 / (0.4 \cdot 1.24 \cdot 0.25 - 1(1-0.06)) \sim -0.125$ . This implies in the case of constant returns to scale that bias  $\hat{\alpha} \sim -0.062$  and bias  $\hat{\beta} \sim -0.063$  (and evidently bias  $\hat{\epsilon} = B \sim -0.125$ ). Cf. also the next section

In any event the nature of the denominator is such that even for moderate error-variance ratios the biases may be quite serious. Thus, in our case we must expect the present method of estimation to yield generally poor results, also because of errors of measurement in capital.

Since the two kinds of errors of measurement discussed are largely independent we could easily have analysed them simultaneously. However, this does not add anything new to the findings so far. From the above we have enough evidence for concluding that the present method of taking the simultaneity of the model into account is of little value due to errors of measurement.

#### b. The OLS Method

When using the OLS method instead of the ILS method on the production function there is an additional source of bias, namely simultaneous equations, or in our case, the endogeneity of labour input.

In a related study the biases of the OLS estimators of  $\alpha$  and  $\beta$  due to simultaneous equations, errors of measurement in labour input and errors of measurement in capital are derived and analysed.<sup>1)</sup>

Based on fairly general assumptions we can show that the asymptotic biases due to simultaneous equations, when there are no errors of measurement, are:

$$(9) \quad \text{bias } \hat{\alpha} = \frac{(1-\alpha) \sigma_u^2}{D_1}$$

$$\text{bias } \hat{\beta} = \frac{-(\beta - b_{wz}) \sigma_u^2}{D_1}$$

Thus the bias in the estimator for the elasticity of scale is:

$$(10) \quad \text{bias } \hat{\epsilon} = \frac{((1-\epsilon) + b_{wz}) \sigma_u^2}{D_1}$$

where  $b_{wz}$  is the probability limit of the coefficient of  $z$  in the auxiliary regression of  $w$  on  $z$  and:

$$(11) \quad D_1 = b_w^2 \sigma_w^2 (1 - r_{zw}^2) + \sigma_u^2 + \sigma_v^2$$

We note that the denominator will always be positive, and provided that  $\alpha < 1$  the bias in the estimator for  $\alpha$  due to simultaneous equations will also always be positive.  $b_{wz}$  is presumably also positive and we therefore cannot determine the sign of the bias of the estimator for  $\beta$ . It is,

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Note:

1) Cf. GRILICHES and RINGSTAD (1971), op.cit., Chapter IV and Appendix C.



however, likely to be negative. With constant or decreasing returns to scale, the estimator for the scale elasticity will be biased upwards. This is also true if we have slightly increasing returns to scale. We note that provided the "identifying" variables  $w$ ,  $z$  and  $v$  have large variances compared to the variance of the error term,  $u$  of the production function, the simultaneous equations biases need not be too bad. We also note that a large elasticity of substitution helps us to identify the parameters, and that this is particularly the case for  $\alpha$ .

With regard to errors of measurement in labour input, it could be shown that, using the same assumptions about the behaviour of these errors as we have adopted in the ILS case, we get the following asymptotic biases when applying the OLS method:

$$(12) \quad \begin{aligned} \text{bias } \hat{\alpha} &= \frac{\alpha(b_{wx} - b_{wz} b_{zx})}{D_2} \\ \text{bias } \hat{\beta} &= \frac{\alpha(b_{wz} - b_{wx} b_{xz})}{D_2} \\ \text{bias } \hat{\varepsilon} &= \frac{\alpha((1-b_{xz})b_{wx} + (1-b_{zx})b_{wz})}{D_2} \end{aligned}$$

where  $b_{wx}$ ,  $b_{wz}$ ,  $b_{xz}$  and  $b_{zx}$  are the probability limits of the slope coefficients of the simple auxiliary regressions of  $w$  on  $x$ ,  $w$  on  $z$ ,  $x$  on  $z$  and  $z$  on  $x$  respectively, and

$$(13) \quad D_2 = 1 - r_{xz}^2$$

where  $r_{xz}$  is the probability limit of the simple correlation coefficient between  $x$  and  $z$ . Correspondingly, we get for the case when having errors of measurement in capital only:<sup>1)</sup>

$$(14) \quad \begin{aligned} \text{bias } \hat{\alpha} &= \frac{\beta b_{zx}}{D_3} k_1^2 \\ \text{bias } \hat{\beta} &= \frac{-\beta}{D_3} k_1^2 \\ \text{bias } \hat{\varepsilon} &= \frac{\beta(b_{zx} - 1)}{D_3} k_1^2 \end{aligned}$$

where

$$(15) \quad D_3 = 1 - r_{x,z-x}^2$$

\_\_\_\_\_   
 N o t e :

1) Ibid., Appendix C.

with  $r_{x,z-x}$  as the probability limit of the correlation coefficient between the logs of labour and the capital-labour ratio, and  $k_1^2 = \left(\frac{\sigma_e}{\sigma_{z-x}}\right)^2$  is the probability limit of the ratio between variance of the error of the capital measure and the variance of the observed capital-labour ratio.

We note from (12) that we cannot determine the sign of the biases due to the kind of errors of measurement in labour input we are considering. It is somewhat easier to do this for the biases due to errors of measurement in capital. We note that the bias of  $\hat{\beta}$  is always negative and, except for rather unusual situations, the bias of  $\hat{\alpha}$  is always positive. If the coefficient  $b_{zx}$  is near one, we also note that the bias of  $\hat{\alpha}$  can be ignored.

Even if the biases derived are partial and asymptotic, they seem to provide sufficient evidence for concluding that in our case OLS is likely to be a better method of estimation than ILS due to the former's considerably greater degree of robustness against the errors present in the data. Later on we attempt to calculate these biases to get some idea of their magnitudes. However, even if OLS in contrast to ILS does not seem to give completely wild estimators, the biases of the former are presumably also of a magnitude that makes it rather poor in our case, indicating that we should look for something better. There is, however, one possibility for "saving" this method which should be investigated. That is to combine it with analysis of covariance.

## 2. Analysis of Covariance

In the previous section we demonstrated that using indirect least squares to take into account the simultaneity of the model considered will yield quite poor results. There are, however, other ways of doing this. One possibility is to use analysis of covariance.

The main cause of simultaneity is likely to be more or less stationary differences between production units relating to management, "environments" and efficiency in general. Having two or more observations per establishment we can use analysis of covariance to eliminate the establishment-specific components of the error term, and thus we presumably obtain estimators that are less biased due to simultaneity.<sup>1)</sup>

To explore this issue we specify the error term of the production function in the following way, also allowing a time-specific component:<sup>2)</sup>

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### Notes :

- 1) Cf. MUNDLAK (1961), op.cit., HOCH (1957), op.cit., HOCH (1962), op.cit. and Section 3.c. below.
- 2) Cf. SCHEFFE (1959), op.cit.

$$\begin{aligned}
 u_{it} &= a_i + b_t + c_{it} & (i = 1 \dots I) \\
 (16) \quad E u_{it} &= a_i + b_t & (t = 1 \dots T) \\
 \Omega_u &= \Omega_c = \sigma^2 J
 \end{aligned}$$

where  $\Omega$  is the variance covariance matrix,  $J$  a  $(I \times T) \times (I \times T)$  unit matrix and  $\sigma^2$  is the variance of the residual.

To remain flexible in our exploration we will consider four "cases". The parameters of the production function are estimated when:

- a) No components are eliminated from the error term
- b) The time-specific components are eliminated
- c) The establishment-specific components are eliminated
- d) Both time- and establishment-specific components are eliminated

Case a) corresponds to OLS discussed in the previous section, while case c) (and also d)) yield estimators free of simultaneous equations bias, or at least estimators that are less biased due to simultaneity.

However, by using this method for eliminating the simultaneous equations bias we encounter the following problem: Can we be sure that differences in the results obtained for case a) and case c) (or cases a) and b), and cases c) and d)) are due to simultaneity alone. The answer must be no, and there are at least two reasons for this. Since in case c) (and d)) the systematic variation of the variables between units (and over time) is used to eliminate the establishment-specific (and time-specific) components of the error term, estimates on the factor elasticities are of a more short-run nature than those obtained for case a) (and b)).<sup>1)</sup> For that reason we will expect the estimates obtained for cases c) and d) to be lower than those obtained for cases a) and b). We will argue, however, that the main reason why we must not expect differences in the results of the various cases to be due solely to differences in simultaneity bias is the errors present in the data; that they may be of different importance for the different cases.

Thus we will investigate the degree of robustness with respect to errors of measurement of the analysis of covariance method. This will be done first by estimating the slope coefficients of the production function

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**N o t e :**

- 1) For discussions of this and related issues, cf. KUH and MEYER (1957): How Extraneous are Extraneous Estimates? KUH (1963): Capital Stock Growth: A Micro-Econometric Approach and NERLOVE (1967), op. cit.

for the four cases, trying to determine whether or not the differences in the results could be explained by differences in the simultaneity bias and/or the short run - long run character of the parameters, and second by tentative calculations of the simultaneous equations and the errors of measurement biases derived for the OLS method in the previous section.

a. Analysis of Covariance of the Relations of the Model

Our main concern in this context is the production function. However, we have for two reasons also used analysis of covariance on the behaviour relation. First, these results may be of interest in themselves.<sup>1)</sup> Second, the results will be used shortly in the tentative bias calculations for the covariance analysis estimates on the factor elasticities.

The results for the two relations are presented in Table III.1 and Table III.2. For Total Mining and Manufacturing, where no components are eliminated from the error terms, they suggest that on the average there are constant returns to scale as well as an elasticity of substitution of unity for the industries concerned. The results are basically the same when eliminating the time-specific components from the error terms, while when eliminating the establishment-specific components they suggest that there are decreasing returns to scale as well as an elasticity of substitution below unity. This is also the main pattern of the individual industry results, even though there are some striking exceptions.

In general then our results seem to support our expectation concerning which cases would yield estimates on long-run and which ones on short-run parameters. And since we would expect the simultaneous equations bias in the OLS estimate on the scale elasticity to be positive, lower estimates on that parameter in cases c) and d) than in cases a) and b) are precisely what we should expect for that reason as well.<sup>2)</sup> It is much more difficult to explain the substantially lower estimates on the capital

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Notes :

1) For arguments supporting the use of covariance analysis on a behaviour relation like the present one, cf. MUNDLAK (1963): Estimation of Production and Behaviour Functions from a Combination of Cross-Section and Time-Series Data in CHRIST (ed.) Measurement in Economics.

2) Cf. (10) above.

Table III.1. Analysis of Covariance of the Cobb-Douglas Production Function.\*

Case: Industry	No components eliminated			Time-specific components eliminated			Establishment-specific components eliminated			Both time- and establishment-specific components eliminated		
	$\beta$	$\epsilon$	MSE	$\beta$	$\epsilon$	MSE	$\beta$	$\epsilon$	MSE	$\beta$	$\epsilon$	MSE
Total Mining and Manufacturing .....	0.272 (0.007)	0.994 (0.005)	0.281	0.263 (0.007)	0.993 (0.005)	0.273	0.178 (0.013)	0.895 (0.018)	0.113	0.076 (0.013)	0.799 (0.018)	0.102
Mining and Quarrying .....	0.281 (0.040)	0.988 (0.022)	0.187	0.247 (0.040)	0.997 (0.040)	0.176	0.211 (0.056)	0.739 (0.110)	0.072	0.079 (0.054)	0.773 (0.099)	0.056
Food Products .....	0.372 (0.025)	0.888 (0.014)	0.445	0.353 (0.025)	0.883 (0.014)	0.424	0.155 (0.041)	0.758 (0.055)	0.189	-0.020 (0.039)	0.557 (0.053)	0.159
Textiles .....	0.293 (0.034)	0.946 (0.022)	0.155	0.282 (0.035)	0.946 (0.022)	0.154	0.295 (0.060)	1.052 (0.071)	0.082	0.211 (0.065)	1.000 (0.074)	0.080
Clothing .....	0.080 (0.024)	0.993 (0.021)	0.137	0.070 (0.024)	0.995 (0.021)	0.134	0.060 (0.039)	0.663 (0.051)	0.058	0.017 (0.039)	0.648 (0.051)	0.056
Wood Products .....	0.188 (0.038)	1.092 (0.024)	0.285	0.179 (0.038)	1.089 (0.024)	0.281	0.192 (0.053)	1.194 (0.076)	0.117	0.123 (0.054)	1.102 (0.076)	0.109
Pulp and Paper .....	0.300 (0.023)	0.896 (0.015)	0.170	0.237 (0.023)	0.923 (0.014)	0.147	0.254 (0.032)	0.315 (0.066)	0.082	0.079 (0.029)	0.415 (0.058)	0.061
Printing .....	0.146 (0.023)	1.041 (0.017)	0.121	0.147 (0.023)	1.040 (0.017)	0.119	0.103 (0.032)	0.706 (0.057)	0.053	0.113 (0.032)	0.718 (0.056)	0.051
Basic Chemicals .....	0.195 (0.037)	0.893 (0.019)	0.503	0.182 (0.037)	0.982 (0.019)	0.481	0.077 (0.053)	1.020 (0.066)	0.174	0.001 (0.049)	0.999 (0.061)	0.145
Mineral Products .....	0.322 (0.039)	1.110 (0.023)	0.205	0.313 (0.040)	1.112 (0.023)	0.208	0.394 (0.054)	0.938 (0.068)	0.057	0.349 (0.058)	0.922 (0.068)	0.057
Basic Steel .....	0.200 (0.038)	1.074 (0.029)	0.201	0.173 (0.037)	1.069 (0.028)	0.185	0.109 (0.052)	1.041 (0.083)	0.097	-0.142 (0.048)	0.609 (0.078)	0.069
Metal Products .....	0.129 (0.034)	0.944 (0.018)	0.171	0.104 (0.034)	0.923 (0.018)	0.167	0.147 (0.048)	0.914 (0.071)	0.100	-0.009 (0.053)	0.690 (0.077)	0.094
Non-Electrical Machinery .....	0.028 (0.037)	1.078 (0.021)	0.138	-0.007 (0.037)	1.081 (0.021)	0.133	0.234 (0.054)	1.128 (0.070)	0.064	0.112 (0.062)	1.046 (0.074)	0.061
Electrical Machinery .....	0.111 (0.043)	1.023 (0.029)	0.242	0.098 (0.042)	1.017 (0.029)	0.237	0.238 (0.072)	1.060 (0.093)	0.094	-0.013 (0.077)	0.674 (0.105)	0.082
Transport Equipment..	0.091 (0.021)	1.070 (0.010)	0.149	0.089 (0.021)	1.069 (0.010)	0.148	0.107 (0.033)	0.944 (0.045)	0.091	0.089 (0.033)	0.899 (0.045)	0.089
Miscellaneous Products .....	0.355 (0.058)	0.938 (0.046)	0.410	0.367 (0.055)	0.911 (0.043)	0.354	0.319 (0.134)	1.243 (1.153)	0.285	0.238 (0.127)	0.877 (0.163)	0.236

\*  $\beta$  is the elasticity of capital,  $\epsilon$  is the elasticity of scale and MSE is the mean square of the estimated residual error.

Table III.2. Analysis of Covariance of the ACMS Relation.\*

Case: Industry	No components eliminated		Time-specific components eliminated		Establishment-specific components eliminated		Both time- and establishment-specific components eliminated	
	b	MSE	b	MSE	b	MSE	b	MSE
Total Mining and Manufacturing	0.992 (0.018)	0.243	1.006 (0.020)	0.243	0.894 (0.014)	0.079	0.885 (0.017)	0.078
Mining and Quarrying ....	0.989 (0.150)	0.193	0.811 (0.193)	0.194	1.008 (0.107)	0.061	0.563 (0.066)	0.057
Food Products	1.114 (0.045)	0.385	1.153 (0.051)	0.386	0.858 (0.031)	0.131	0.800 (0.038)	0.130
Textiles .....	1.082 (0.084)	0.135	1.185 (0.097)	0.135	0.950 (0.079)	0.067	1.088 (0.104)	0.067
Clothing .....	0.870 (0.059)	0.102	0.895 (0.065)	0.103	0.898 (0.048)	0.042	0.953 (0.056)	0.042
Wood Products.	1.433 (0.129)	0.239	1.641 (0.149)	0.234	0.974 (0.111)	0.101	1.069 (0.148)	0.098
Pulp and Paper	0.798 (0.035)	0.131	0.863 (0.053)	0.129	0.788 (0.024)	0.049	0.874 (0.040)	0.047
Printing .....	0.885 (0.056)	0.094	0.876 (0.060)	0.094	0.916 (0.069)	0.045	0.891 (0.073)	0.044
Basic Chemicals ....	1.047 (0.072)	0.396	1.098 (0.086)	0.399	0.892 (0.046)	0.111	0.868 (0.062)	0.111
Mineral Products .....	1.790 (0.104)	0.148	1.962 (0.111)	0.144	1.022 (0.081)	0.047	1.102 (0.097)	0.047
Basic Steel ..	0.906 (0.104)	0.189	0.840 (0.155)	0.190	0.910 (0.073)	0.069	0.779 (0.136)	0.067
Metal Products .....	0.714 (0.091)	0.162	0.658 (0.102)	0.163	1.091 (0.085)	0.079	1.178 (0.107)	0.079
Non-Electrical Machinery ....	0.853 (0.119)	0.124	0.807 (0.143)	0.126	1.119 (0.097)	0.048	1.296 (0.140)	0.048
Electrical Machinery ....	0.991 (0.078)	0.163	1.022 (0.088)	0.165	0.950 (0.064)	0.056	1.003 (0.080)	0.056
Transport Equipment ....	0.709 (0.059)	0.135	0.695 (0.059)	0.135	1.123 (0.066)	0.068	1.105 (0.068)	0.068
Miscellaneous Products .....	1.334 (0.135)	0.296	1.426 (0.170)	0.313	1.228 (0.127)	0.164	1.318 (0.192)	0.173

\* b is the elasticity of substitution and MSE is the mean square of the estimated residual error.

elasticity when the establishment-specific components of the error term are eliminated. As pointed out, we are then estimating a more short-run capital elasticity. On the other hand, we would expect the simultaneous equations bias to be negative,<sup>1)</sup> implying bigger estimates on that parameter for cases c) and d) than for cases a) and b). In addition, it is quite difficult to explain the differences between the industries concerning the impact on the results of various treatments of the establishment- and time-specific components of the error term. Thus we will seek an additional and presumably more compelling explanation to these findings, namely by the two main types of errors present in the data and differences in their importance in cases a) - d).

#### b. Bias Computations for the Analysis of Covariance Estimates

The analysis of covariance of the previous sub-section implies that the OLS method is used for the estimation of the various elasticities.<sup>2)</sup> Thus the estimates obtained are subject to the three types of biases discussed in Section 1.b. of this chapter. In this sub-section we will try to investigate to what extent these biases may explain the differences in the results obtained from the four "cases". We thus have to quantify the biases. We do this by using sample statistics for the various components entering these biases. This is clearly quite approximate since the biases derived are asymptotic. Such computations, however, may yield an indication of the importance of the various biases.<sup>3)</sup>

In the computation of the simultaneous equations biases we use, for each of the four cases, the corresponding estimates on the mean square errors from the production and behaviour relation of (1) for  $\sigma_u^2$  and  $\sigma_v^2$  respectively.<sup>4)</sup>

#### Notes :

- 1) Cf. (9) above and Section 2.b. below.
- 2) We need not necessarily use the OLS method for the estimation of the slope coefficients in a covariance analysis of the kind considered in this chapter. But in the present context it does not seem to be worthwhile to try alternatives to the OLS method.
- 3) Cf. GRILICHES and RINGSTAD (1971), op.cit. Ch. IV and Appendix C.
- 4) We know that due to simultaneous equations the OLS estimator for  $\sigma_u^2$  is biased downwards. On the other hand, this estimator for  $\sigma_u^2$  has a positive bias due to errors of measurement, and the latter bias is presumably more important than the former. Thus the way we estimate  $\sigma_u^2$  probably overstates the residual error of the production relation, and we therefore probably also overstate the simultaneous equations biases. Cf. Section 3.c. below.

Table III.3. Bias Computations of Covariance Analysis Estimates on the Capital and Scale Elasticities.\*

Case:	a. No components eliminated						b. Time-specific components eliminated					
	Simultaneous Equations		Errors in Labour		Errors in Capital		Simultaneous Equations		Errors in Labour		Errors in Capital	
Bias Due to:	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$
Industry												
Total Mining and Manufacturing ..	-0.160	0.024	0.066	0.016	-0.099	-0.001	-0.162	0.022	0.059	0.015	-0.099	-0.001
Mining and Quarrying .....	-0.185	0.010	0.026	0.007	-0.123	0.024	-0.191	0.007	0.004	0.013	-0.125	0.024
Food Products ..	-0.197	0.016	0.075	0.004	-0.125	-0.016	-0.201	0.013	0.065	0.001	-0.125	-0.016
Textiles .....	-0.182	0.019	0.083	-0.005	-0.108	0.005	-0.184	0.018	0.069	-0.003	-0.108	0.005
Clothing .....	-0.190	0.009	0.031	-0.002	-0.100	-0.002	-0.192	0.007	0.022	0.000	-0.100	-0.001
Wood Products ..	-0.098	0.022	0.035	0.020	-0.063	-0.001	-0.100	0.021	0.026	0.018	-0.063	-0.001
Pulp and Paper..	-0.185	0.018	0.081	0.003	-0.117	0.017	-0.194	0.015	0.027	0.017	-0.118	0.018
Printing .....	-0.136	0.006	0.055	-0.016	-0.075	0.008	-0.135	0.006	0.056	-0.017	-0.075	0.008
Basic Chemicals.	-0.227	0.021	0.029	0.018	-0.131	-0.013	-0.228	0.020	0.022	0.018	-0.131	-0.013
Mineral Products	-0.152	0.043	0.079	0.009	-0.122	0.021	-0.151	0.045	0.071	0.011	-0.123	0.022
Basic Steel ....	-0.211	0.020	0.034	0.017	-0.139	0.037	-0.220	0.012	0.017	0.014	-0.139	0.036
Metal Products..	-0.170	0.015	0.075	0.015	-0.096	-0.005	-0.175	0.011	0.062	0.011	-0.096	-0.006
Non -El.												
Machinery .....	-0.164	0.012	0.078	-0.003	-0.091	0.006	-0.167	0.009	0.059	-0.001	-0.091	0.006
El. Machinery ..	-0.172	0.004	-0.057	0.037	-0.090	0.007	-0.178	-0.001	-0.067	0.032	-0.090	0.007
Transport												
Equipment .....	-0.105	0.005	0.039	0.007	-0.059	-0.005	-0.105	0.005	0.034	0.006	-0.059	-0.005
Miscellaneous												
Products .....	-0.100	0.067	0.119	0.043	-0.097	-0.004	-0.102	0.068	0.126	0.030	-0.097	-0.004

\* Footnote overleaf.



Table III.3. (cont.). Bias Computations of Covariance Analysis Estimates on the Capital and Scale Elasticities\*

Case	c. Establishment-specific components eliminated						d. Both time-and establishment-specific components eliminated					
	Simultaneous Equations		Errors in Labour		Errors in Capital		Simultaneous Equations		Errors in Labour		Errors in Capital	
	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$	Bias $\hat{\beta}$	Bias $\hat{\epsilon}$
Total Mining and Manufacturing..	-0.149	0.046	0.084	-0.016	-0.123	-0.076	-0.197	0.002	0.017	-0.078	-0.127	-0.082
Mining and Quarrying .....	-0.131	0.070	0.085	0.002	-0.144	-0.143	-0.203	0.005	0.012	0.009	-0.139	-0.122
Food Products..	-0.207	0.025	0.057	-0.065	-0.164	-0.116	-0.277	-0.047	-0.037	-0.168	-0.170	-0.126
Textiles .....	-0.166	0.044	0.107	-0.050	-0.139	-0.080	-0.225	-0.013	0.027	-0.083	-0.145	-0.084
Clothing .....	-0.181	0.006	0.030	-0.089	-0.154	-0.120	-0.211	-0.022	-0.012	-0.099	-0.154	-0.118
Wood Products .	-0.072	0.052	0.087	0.060	-0.075	-0.043	-0.114	0.011	0.025	-0.021	-0.077	-0.047
Pulp and Paper.	-0.099	0.097	0.171	-0.235	-0.134	-0.115	-0.189	0.016	0.036	-0.135	-0.127	-0.090
Printing .....	-0.142	-0.002	0.008	-0.081	-0.085	-0.055	-0.140	0.000	0.013	-0.082	-0.085	-0.056
Basic Chemicals	-0.240	0.012	0.032	-0.024	-0.150	-0.074	-0.267	-0.015	-0.008	-0.037	-0.148	-0.072
Mineral Products	-0.105	0.100	0.133	0.085	-0.131	-0.065	-0.136	0.073	0.081	0.074	-0.132	-0.062
Basic Steel ...	-0.153	0.095	0.099	0.128	-0.133	-0.062	-0.250	-0.013	-0.011	-0.056	-0.149	-0.102
Metal Products.	-0.111	0.078	0.100	0.039	-0.117	-0.076	-0.150	0.008	0.024	-0.078	-0.136	-0.109
Non-El.												
Machinery .....	-0.076	0.103	0.142	0.041	-0.126	-0.087	-0.137	0.043	0.062	-0.007	-0.141	-0.102
El. Machinery..	-0.054	0.125	0.161	0.190	-0.116	-0.073	-0.157	0.018	0.029	-0.010	-0.142	-0.118
Transport												
Equipment .....	-0.116	-0.003	0.013	-0.045	-0.063	-0.027	-0.121	-0.009	0.006	-0.062	-0.064	-0.029
Miscellaneous												
Products .....	-0.051	0.142	0.102	0.208	-0.126	-0.069	-0.125	0.064	0.069	0.040	-0.128	-0.081

\* Formulas for the various types of biases are presented in (9), (10), (12) and (14) above.

We have not yet succeeded in identifying the production function parameters entering the bias-formulas. For the factor elasticities, however, we use factor share estimates to be considered later, since one of the conclusions of this chapter is that those are likely to be less biased than the estimates on those parameters considered so far.

For the elasticity of substitution we use the estimates presented in Table III.2. In the computation of the biases due to the errors of measurement in capital we need one additional piece of information, namely the ratio between the variance of the error component of the capital input measure and the observed capital-labour ratio.<sup>1)</sup> Such information is not available, however, and we therefore assume that  $k_1 = 0.5$  or  $k_1^2 = 0.25$  which seems to be reasonable on the average for the four "cases".<sup>2)</sup>

The results of the bias computations are presented in Table III.3.<sup>3)</sup> Considering the simultaneous equations bias first, the computations of case a) suggest that there is a positive bias in the OLS estimate on the elasticity of scale if the assumption of constant returns to scale is true. For most industries, however, it seems to be quite unimportant even though our computations understate this bias if we really have decreasing returns to scale. Only for Mineral Products and Miscellaneous Products is this bias of any magnitude, but we should note that as the former according to our OLS estimates seems to have increasing returns to scale, our computations may overstate this bias.<sup>4)</sup>

The simultaneous equations biases computed are of the same order of magnitude in the three other cases as well, even though it seems to be slightly more serious when only the establishment-specific components of the error term are eliminated. We should note, however, that what our

---

N o t e s :

- 1) We consider this ratio rather than the ratio between the error variance and the capital measure variance to avoid inconsistencies. Due to our assumptions we must have:

$$k_1^2 = \frac{\sigma_e^2}{\sigma_{z-x}^2} = \frac{\sigma_e^2}{\sigma_{z^*-x}^2 + \sigma_e^2} \leq 1. \quad \text{We have } k_1^2 = 1$$

only in the case when there is no variation in the "true" capital-labour ratio.

- 2) It may be too high when none of the systematic components of the error term is eliminated, but it is certainly too low when both the establishment- and time-specific components are eliminated. We shall have some comments on this later.
- 3) The estimates of the slope coefficients from the auxiliary regressions are presented in Appendix III.2.
- 4) Cf. formula (10) above.

calculations are telling us is the importance of the simultaneous equations bias when all components of the gross error term  $u$  are transferred to the behaviour relation and back into the production function via  $x$ . We have previously argued that this is likely to be true for the establishment-specific components only, or at least that this is the main cause of simultaneous equations bias. According to this the biases computed for cases c) and d) must be too high.<sup>1)</sup>

The bias due to errors of measurement in labour is related to the simultaneous equations bias due to the way in which it is computed, namely by assuming perfect correlation between the observed wage rate and the quality component of labour input. We note, however, that this bias behaves differently. Except for Electrical Machinery in cases a) and b) and four industries (but not Electrical Machinery) in case d), errors in labour seem to yield a positive bias in the estimate on the capital elasticity. The bias in the scale elasticity is positive for most industries in cases a) and b) but is in general not very serious. In cases c) and d) it is quite serious for a number of industries, and in the latter case mostly negative. A comparison of the errors in labour biases computed with the corresponding estimates of Table III.1. gives rather strong support to the conclusion that, at least for some industries, these errors must be a main cause of the differences in the results obtained for the different cases.

The errors in capital bias of the capital elasticity estimates are negative for all industries for all four cases, while the bias in the scale elasticity estimates is negative for all industries in cases c) and d). In general the biases of both estimates are more serious in those cases. However, we are likely to understate the differences between cases a) and b) and cases c) and d) with regard to the seriousness of the errors in capital. There may be doubts about the validity of the particular value of  $k_1^2 = 0.25$  used in the computations, but one can hardly doubt that it must be higher in cases c) and d) than in cases a) and b). After having eliminated the establishment-specific (and time-specific) differences in the capital-labour ratio, the errors are likely to be considerably more dominating. However, by comparing the errors in capital biases computed for the same value of  $k_1^2$  with the corresponding estimates of Table III.1. we find that also this type of error must be a main cause of differences between cases in the results for a number of industries.

---

N o t e :

1) Cf. Section 3.c. below.

For different reasons the bias computations presented must be rather tentative. Nevertheless, they should provide sufficient evidence for concluding that analysis of covariance is not sufficiently robust with respect to the errors present in our data to be of any use for our purpose. The analysis of covariance estimators may be "free" of management- or simultaneity-bias, but they are much more seriously biased due to errors of measurement than the OLS estimators.

c. A u t o - c o r r e l a t i o n   a n d   A n a l y s i s   o f  
C o v a r i a n c e :   A   D i g r e s s i o n

If there are establishment- and/or time-specific components of the error term, or in other words systematic variation of the error mean between units and/or over time, there are non-zero off-diagonal elements in what we (erroneously) consider to be the variance covariance matrix of the error term when the OLS method is applied. As shown in the previous section these off-diagonal elements can be eliminated by means of analysis of covariance.

There is, however, another possible cause of non-zero off-diagonal elements of the variance covariance matrix, namely auto-correlation. In fact, the establishment-specific components will necessarily imply auto-correlation since each of them is a constant common to the observations of each time series. Clearly the time-specific components may also cause auto-correlation. However, even when both the establishment- and time-specific components are eliminated, auto-correlation may still be present in the error term, i.e. we have true auto-correlation in the error term.

In this sub-section we would first like to explore to what extent a first-order auto-regressive scheme-specification of the error term can be a substitute for the components-specification of the previous sub-section; second, whether or nor the error term is auto-correlated when the establishment- and time-specific components are eliminated; and third, the robustness against measurement errors of a method for eliminating auto-correlation from the errors.

Since the establishment-specific components are likely to cause the most trouble, both with regard to auto-correlation of the gross error  $u$  and the endogeneity of  $x$ , they will be our main concern in this context. Thus the auto-correlation issue will be considered for two cases: when the establishment-specific components of the errors are not eliminated and when they are. In both cases the time-specific components are eliminated.<sup>1)</sup>

---

N o t e :

1) Our exploration is carried out for Total Mining and Manufacturing only.

In the first case we have the following specification of the error term of the production function:

$$(17) \quad \begin{aligned} u'_{it} &= u_{it} - b_t = a_i + c_{it} \\ u'_{it} &= \rho_1 u'_{i,t-1} + d_{it} = \rho_1 (a_i + c_{i,t-1}) + d_{it} \end{aligned}$$

which implies that:

$$(18) \quad d_{it} = (1-\rho_1)a_i + c_{it} - \rho_1 c_{i,t-1}$$

In the second case we obviously have:

$$(19) \quad d_{it} = c_{it} - \rho_1 c_{i,t-1}$$

A corresponding error-term specification is used for the behaviour relation.

By transforming the two relations of our main model so that they have non-autocorrelated errors we get:<sup>1)</sup>

$$(20) \quad \begin{aligned} y_{it} &= \epsilon x_{it} - \epsilon \rho_1 x_{i,t-1} + \beta (z-x)_{it} - \beta \rho_1 (z-x)_{i,t-1} \\ &+ \rho_1 y_{i,t-1} (+ a_i) + b_t + d_{it} \end{aligned}$$

$$(21) \quad (y-x)_{it} = b w_{it} - b \rho_2 w_{i,t-1} + \rho_2 (y-x)_{i,t-1} (+ a'_i) + b'_t + d'_{it}$$

To obtain unique estimates on the parameters, however, we apparently have to introduce constraints in the estimation procedure. Thus for the production function we use a scanning procedure for  $\rho_1$  when the relation is written as:<sup>2)</sup>

$$(22) \quad \begin{aligned} y_{it} - \rho_1 y_{i,t-1} &= \epsilon (x_{it} - \rho_1 x_{i,t-1}) \\ &+ \beta ((z-x)_{it} - \rho_1 (z-x)_{i,t-1}) (+ a_i) + b_t + d_{it} \end{aligned}$$

For the behaviour relation the two estimates of  $\rho_2$ , which we may get by unconstrained OLS on (21), came out to be very close. Thus the direct estimate on it (the estimate on the coefficient of  $(y-x)_{i,t-1}$ ) was accepted. The results for the production function and the behaviour relation are presented in Table III.4. and III.5. respectively.

By unconstrained regression on (20) we get the direct estimates on  $\rho_1$  (the coefficient of  $y_{i,t-1}$ )  $\hat{\rho}_1 = \frac{0.766}{(0.008)}$  and  $\hat{\rho}_1 = \frac{0.329}{(0.011)}$  when the establishment-specific components of the error are not eliminated and when

#### Notes :

- 1) The auto-correlation coefficient of the behaviour relation is  $\rho_2$ , and the establishment- and time-specific components of the gross error of that relation are denoted  $a'_i$  and  $b'_t$  respectively.  $d'_{it}$  corresponds to the  $d_{it}$  of the production function.
- 2) Cf. RAO and GRILICHES (1967): Small Sample Properties of Several Two-Stage Regression Methods in the Context of Auto-Correlated Errors.

they are respectively. Thus from the results of Table III.4. we note first that the direct estimates on  $\rho_1$  obtained by unconstrained regression on (20) are the same (when using two digits) as those obtained by our constrained scanning regression procedure on (22).<sup>1)</sup> Second, we see that auto-correlation is substantially and significantly reduced when eliminating the establishment-specific components from the residual. Third, our results strongly suggest that there is a positive auto-correlation in the

Table III.4. Results for the Production Function Obtained by Relation (22)\*

Establishment-specific components of the error: 1)	$\hat{\beta}$	$\hat{\epsilon}$	$\hat{\rho}_1$ 2)	MSE
Not eliminated .....	0.243 (0.013)	0.930 (0.008)	0.77	0.123
Eliminated .....	0.069 (0.016)	0.692 (0.020)	0.33	0.087

\*  $\beta$  is the elasticity of capital,  $\epsilon$  the elasticity of scale,  $\rho_1$  the auto-correlation coefficient and MSE is the mean square of the estimated residual error.

1) In both cases the time-specific components of the error are eliminated.

2) The scanning for  $\rho_1$  is carried out in the region  $0 \leq \rho_1 < 1$  with steps 0.01.

Table III.5. Results for the Behaviour Relation Obtained by Relation (21)\*

Establishment-specific components of the error: 1)	$\hat{\beta}$	$b\hat{\rho}_2$	$\hat{\rho}_2$	$b\hat{\rho}_2/\hat{\beta}$	MSE
Not eliminated .....	0.943 (0.019)	0.719 (0.020)	0.771 (0.007)	0.762	0.100
Eliminated .....	0.903 (0.019)	0.273 (0.021)	0.290 (0.011)	0.302	0.069

\*  $b$  is the elasticity of substitution,  $\rho_2$  the autocorrelation coefficient and MSE is the mean square of the estimated residual error.

1) Time-specific components are eliminated in both cases.

error even when the establishment- and time-specific components are eliminated. Fourth, it is easily confirmed that our auto-correlation scheme cannot be substituted for our components specification: Even in this case

#### N o t e :

1) This is a bit surprising since the two indirect estimates on  $\rho_1$  that we may derive from the unconstrained regression results are quite different from the direct one and the direct estimates on  $\epsilon$  and  $\beta$  from that regression are also somewhat different from those presented in Table III.4.

there is a significant reduction in the mean square error when eliminating the establishment-specific components.<sup>1)</sup> The results of  $\epsilon$  and  $\beta$  will be discussed presently.

From the results of the behaviour relation we can conclude first that the direct and the indirect estimates on the auto-correlation coefficient are of the same order of magnitude; second, that the degree of auto-correlation is about the same for the behaviour relation as for the production relation; third, as this is true for both cases analysed, the effect on the degree of auto-correlation of establishment-specific components is about the same in the two relations; and fourth, as could be easily verified even in this case, the establishment-specific components are significant.

To determine the robustness of the method used to obtain non-auto-correlated errors we have computed the biases of the estimates of

Table III.6. Auto-correlation, Analysis of Covariance and Biases due to Simultaneous Equations and Errors of Measurement\*

Treatment of: Est. specific components:1)	Bias due to: Auto- correlation:	Simultaneous equations		Errors in labour		Errors in capital	
		bias $\hat{\beta}$	bias $\hat{\epsilon}$	bias $\hat{\beta}$	bias $\hat{\epsilon}$	bias $\hat{\beta}$	bias $\hat{\epsilon}$
Not eliminated	Not eliminated	-0.162	0.022	0.059	0.015	-0.099	-0.001
	Eliminated ...	-0.174	0.017	0.078	-0.017	-0.106	-0.025
Eliminated	Not eliminated	-0.197	0.002	0.017	-0.078	-0.127	-0.082
	Eliminated ...	-0.197	-0.001	0.016	-0.202	-0.140	-0.107

\*  $\beta$  is the elasticity of capital and  $\epsilon$  is the elasticity of scale.

1) Time-specific components are eliminated in all cases.

Table III.4.<sup>2)</sup> These are presented in Table III.6. together with the biases of the corresponding estimates when auto-correlation is not eliminated.

#### Notes :

1) In this section our main concern is the properties of the estimators for the production function parameters when using analysis of covariance. The impact on the residual variance of different treatments of the systematic components is an issue dealt with in the following chapter. However, it could be easily shown by F-tests (assuming normally distributed errors) that there are significant establishment-specific components in the errors both when assuming the errors to be non-auto-correlated and when assuming them to follow a first-order auto-regressive scheme as specified above.

2) Cf. the previous sub-section about the way in which the biases are computed. Note that we are now using statistics of the "corrected" variables  $r_{it} - \hat{\rho}_r r_{i,t-1}$  ( $r = x, z, w$ ).

The contents of this table seem to suggest the following: When the establishment-specific components of the error terms are not eliminated, the effect of the three biases of the estimate on the capital elasticity is about the same whether auto-correlation is eliminated or not. On the other hand, we seem to introduce a non-ignorable negative bias in the estimate on the scale elasticity by taking the auto-correlation structure into account. If we try to eliminate both the establishment-specific components of the error term and the auto-correlation, the errors of measurement biases of the estimate on the scale elasticity become extremely serious. Since we for that case are likely to underrate the biases due to errors of measurement in capital, this is true for the estimate on the capital elasticity as well.<sup>1)</sup>

Thus when there are errors in data we may pay an unreasonably high price for obtaining well-behaved error terms. In our case it is definitely to high.

### 3. A Search for Estimation Methods that are Robust against Errors of Measurement in Simultaneous Equations

#### a. A Factor Share Estimator for the Elasticity of Labour

Thus far we have not had much success in our attempts to obtain consistent estimators on the production function parameters. In this section some other methods of estimation are considered.

In the bias computations of the previous section particular estimators for the factor elasticities were applied, namely factor shares. They are based on the assumptions of perfect competition, an elasticity of substitution of unity, and that profit is maximized on an arithmetic rather than a geometric average.<sup>2)</sup>

The behaviour relation is now

$$(23) \quad S = \frac{WL}{V} = \alpha R$$

where, according to our assumptions, the mean of the random term  $R$  is  $ER = 1$ . Thus the average of labour's share is  $\bar{S} = \alpha\bar{R}$ , and we get an unbiased estimator on  $\alpha$  as<sup>3)</sup>

$$(24) \quad \tilde{\alpha} = \bar{S}$$

#### Notes :

1) Cf. the previous sub-section.

2) Cf. GRILICHES and RINGSTAD (1971), op.cit. Ch.IV and KLEIN (1963): A Text-book of Econometrics.

3) The variance on this estimator is easily obtained as  $\text{var } \bar{S} = \frac{1}{n} \text{var } S$ .



We note that  $\tilde{\alpha}$  is not subject to any of the biases discussed in the previous section. We have taken the simultaneity into account, and errors of measurement in capital clearly do not matter for this estimator. Nor does the particular kind of error in labour input matter.<sup>1)</sup> Therefore, even if the assumptions on which this factor share estimator is based are not completely realistic, particularly the one of perfect competition, this estimator seems to be more reliable than those discussed previously.

b. Estimators for the Capital and Scale Elasticities Free of Simultaneous Equations Bias

There is now one obvious way of estimating the elasticity of capital, i.e. by assuming constant returns to scale, and thus obtaining:

$$(25) \quad \tilde{\beta} = 1 - \tilde{\alpha}$$

The results so far suggest that this assumption concerning an elasticity of scale of one is not too bad for most industries. Due to the errors present in the data, however, we should not rely too heavily on these results.

If we are not willing to accept the assumption of constant returns to scale, at least without further investigation, we may estimate  $\beta$  from the relation:

$$(26) \quad y - \tilde{\alpha}x = \beta z + u'$$

by means of ordinary least squares. The estimators for the capital and scale elasticities thus obtained are not subject to simultaneous equations bias, but they are subject to both errors of measurement biases previously discussed.

c. A Tentative Test of the Hoch-Mundlak Hypothesis: A Digression

By comparing the analysis of covariance results of (26) and those previously obtained for the production function, there is a possibility for exploring whether or not the hypothesis put forward by Hoch and Mundlak is true; that the main cause of simultaneity is the establishment-specific components of the error term.<sup>2)</sup>

Notes:

1) Having  $W^*L^*/V$  where  $L^* = LQ$  and  $W^* = W/Q$  where  $Q$  is the quality index for labour input we have clearly  $W^*L^*/V = WL/V$ . Correspondingly, if we forget to deflate the output and wage-rate variables, or if they are incompletely deflated, we have  $W'L'/V' = WL/V$  as  $W' = WP$  and  $V' = VP$  where  $P$  is the price of output.

2) Cf. HOCH (1962), op.cit. and MUNDLAK (1961), op.cit. Cf. also Section 2 above.

By OLS on (26) we should obtain a bigger estimate on  $\beta$  than when  $\alpha$  is also estimated by OLS, since the former one is not biased due to simultaneity while the latter is.<sup>1)</sup> On the other hand, when using analysis of covariance (i.e. when the establishment-specific components are eliminated from the error), the two estimates on  $\beta$  should be of the same order of magnitude if the Hoch-Mundlak hypothesis is true, since they are both free of simultaneous equations bias.

Table III.7. Analysis of Covariance Estimates on the Capital Elasticity Free of Simultaneous Equations Bias.\*

Case: Industry	No components eliminated		Time-specific components eliminated		Establishment-specific components eliminated		Both time- and establishment-specific components eliminated	
	$\beta$	MSE	$\beta$	MSE	$\beta$	MSE	$\beta$	MSE
Total Mining and Manufacturing ..	0.354 (0.004)	0.288	0.351 (0.004)	0.281	0.202 (0.012)	0.114	0.101 (0.012)	0.103
Mining and Quarrying .....	0.379 (0.016)	0.192	0.376 (0.016)	0.185	0.210 (0.056)	0.072	0.085 (0.054)	0.056
Food Products ..	0.365 (0.013)	0.445	0.358 (0.013)	0.424	0.171 (0.040)	0.189	-0.010 (0.038)	0.160
Textiles .....	0.351 (0.018)	0.156	0.348 (0.018)	0.155	0.357 (0.056)	0.083	0.291 (0.060)	0.081
Clothing .....	0.258 (0.017)	0.158	0.257 (0.017)	0.158	0.060 (0.038)	0.058	0.023 (0.038)	0.056
Wood Products ..	0.313 (0.021)	0.295	0.309 (0.021)	0.292	0.249 (0.051)	0.120	0.174 (0.052)	0.112
Pulp and Paper..	0.327 (0.011)	0.170	0.319 (0.010)	0.150	0.234 (0.033)	0.088	0.060 (0.029)	0.062
Printing .....	0.265 (0.013)	0.129	0.266 (0.013)	0.127	0.090 (0.032)	0.054	0.100 (0.031)	0.051
Basic Chemicals.	0.439 (0.019)	0.547	0.437 (0.019)	0.529	0.221 (0.050)	0.186	0.166 (0.047)	0.160
Mineral Products	0.483 (0.017)	0.218	0.483 (0.017)	0.222	0.390 (0.049)	0.057	0.355 (0.052)	0.056
Basic Steel ....	0.418 (0.020)	0.223	0.406 (0.019)	0.210	0.195 (0.050)	0.104	-0.109 (0.048)	0.071
Metal Products..	0.276 (0.017)	0.179	0.269 (0.017)	0.177	0.174 (0.046)	0.101	0.002 (0.052)	0.094
Non-El. Machinery .....	0.328 (0.019)	0.172	0.325 (0.019)	0.173	0.295 (0.053)	0.067	0.194 (0.061)	0.065
El. Machinery ..	0.291 (0.023)	0.261	0.283 (0.023)	0.257	0.285 (0.068)	0.095	-0.007 (0.076)	0.081
Transport Equipment .....	0.273 (0.010)	0.166	0.272 (0.010)	0.166	0.126 (0.030)	0.091	0.100 (0.030)	0.091
Miscellaneous Products .....	0.336 (0.037)	0.407	0.324 (0.035)	0.354	0.435 (0.124)	0.294	0.245 (0.120)	0.234

\* Cf. (26). MSE is the mean square of the estimated residual error.

Note: 1) Cf. Section 2 above.

Table III.8. Biases of the Capital Elasticity Estimates Due to Errors of Measurement in Labour and Capital

Industry	No components eliminated		Time-specific components eliminated		Establishment-specific components eliminated		Both time- and establishment-specific components eliminated	
	Bias $\hat{\beta}$ due to Error of labour	Error of capital	Bias $\hat{\beta}$ due to Error of labour	Error of capital	Bias $\hat{\beta}$ due to Error of labour	Error of capital	Bias $\hat{\beta}$ due to Error of labour	Error of capital
Total Mining and Manufacturing	0.031	-0.032	0.029	-0.031	0.063	-0.113	-0.003	-0.117
Mining and Quarrying ...	0.013	-0.019	0.010	-0.018	0.085	-0.144	0.011	-0.138
Food Products	0.017	-0.036	0.013	-0.035	0.032	-0.154	-0.063	-0.161
Textiles ....	0.022	-0.029	0.018	-0.028	0.054	-0.119	-0.013	-0.122
Clothing ....	0.012	-0.044	0.010	-0.043	0.008	-0.147	-0.029	-0.146
Wood Products	0.024	-0.018	0.020	-0.018	0.081	-0.067	0.015	-0.071
Pulp and Paper .....	0.028	-0.026	0.020	-0.025	0.154	-0.133	0.022	-0.124
Printing ....	0.010	-0.023	0.010	-0.023	-0.003	-0.082	0.001	-0.082
Basic Chemicals ...	0.020	-0.032	0.018	-0.032	0.014	-0.125	-0.017	-0.123
Mineral Products ....	0.030	-0.022	0.029	-0.022	0.117	-0.110	0.078	-0.107
Basic Steel..	0.023	-0.033	0.015	-0.033	0.105	-0.119	-0.018	-0.142
Metal Products ....	0.027	-0.024	0.021	-0.023	0.089	-0.110	0.011	-0.132
Non-El. Machinery ...	0.019	-0.020	0.014	-0.019	0.118	-0.116	0.042	-0.130
El. Machinery	0.006	-0.025	-0.001	-0.025	0.169	-0.105	0.023	-0.138
Transport Equipment ...	0.011	-0.011	0.010	-0.011	-0.004	-0.053	-0.012	-0.054
Miscellaneous Products ....	0.071	-0.039	0.066	-0.039	0.142	-0.104	0.061	-0.116

In Table III.7. the results of (26) for the four "cases" studied in the previous section are presented. By comparing the results for  $\beta$  in this table with the corresponding results in Table III.1. we see that fairly strong support is given to the hypothesis under investigation. The estimates on  $\beta$  from (26) when applying analysis of covariance are, however, somewhat bigger on the average than the corresponding ones in Table III.1. suggesting that the random component of the error also matters for the endogeneity of  $x$ .

Our findings may, however, be strongly affected by the errors in the data; the support to the Hoch-Mundlak hypothesis is false if in fact the errors of measurement biases for  $\beta$  are more serious when analysis of covariance is applied on (26) than in the case analysed in the previous section.

We cannot be very conclusive about this issue since the only tool available for its investigation is, as previously, the computation of the biases of interest.

It is easily shown that the bias of the estimate on  $\beta$  from (26) due to random errors of measurement in capital is:

$$(27) \quad \text{bias } \hat{\beta} = -\beta \frac{\sigma_e^2}{\sigma_z^2}$$

We can also show that the bias due to errors of measurement in labour input of the kind previously discussed is:

$$(28) \quad \text{bias } \hat{\beta} = \tilde{\alpha} b_{wz}$$

where  $b_{wz}$  is the auxiliary regression coefficient of  $w$  on  $z$ .<sup>1)</sup> Assuming, as previously, that the "true"  $\beta$  is equal to the capital's share in value added,  $1-\bar{S}$  and that  $\sigma_e^2 = 0.25 \sigma_{z-x}^2$ , we obtain the calculated values of the biases from (27) and (28) as presented in Table III.8.

We note from this table that when the establishment-specific components are not eliminated, the two biases, based on the assumptions made, tend to balance each other, in contrast to the corresponding biases presented in Table III.3. where the (negative) error in capital bias seems to be more serious than the (positive) errors in labour bias. Thus, since we now get a bigger estimate on  $\beta$  when the establishment-specific components are not eliminated, this may be a combined effect of the elimination of the

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**N o t e :**

- 1) Based on reasonable assumptions it can be shown that these two biases are additive, that is, the joint effect of the two kinds of errors of measurement is

$$\text{bias } \hat{\beta} = -\beta \frac{\sigma_e^2}{\sigma_z^2} + \tilde{\alpha} b_{wz}$$

The biases are computed separately, however, to permit a separate evaluation of them.

simultaneous equations bias and the reduced net effect of the errors of measurement biases.<sup>1)</sup>

When the establishment-specific components are eliminated from the residual we note that the biases are of the same orders of magnitude as the corresponding ones computed in the previous section. Thus the support provided by our results for the Hoch-Mundlak hypothesis seems to be genuine

d. Estimators for the Capital and Scale Elasticities Free of Both Simultaneous Equations - and Errors of Measurement in Capital Biases

In this section we will go a step further and try a method that is robust against one of the main types of errors present in the data, namely errors of measurement in capital.

In relation (26) we have, when ignoring errors of measurement in labour, a classical error of measurement problem as concerns  $z$ . Econometric literature provides a number of methods for "solving" this problem. We will consider only one category of methods, namely instrumental variables.<sup>2)</sup>

In the present case the number of potential instruments that could serve our purpose is, however, very limited. We should not use the wage rate since that variable is itself subject to a particular type of measurement error. We therefore turn to another method known as grouping of data,<sup>3)</sup> which is equal to a particular application of dummy-variables as an instrumental variable for a right-side variable subject to error.

Notes :

- 1) This also suggests that we by our previous calculations have strongly overstated the simultaneous equations bias in the OLS estimate on  $\beta$ . The difference between that estimate and the one obtained by (26) for Total Mining and Manufacturing is  $-0.082$  while we obtain a simultaneous equations bias of  $-0.160$  for the former. This is likely to be due to the fact that the computed mean square error of the residual applied in the bias calculations contains components of measurement errors as well as other components that do not cause simultaneity, and that these are only partly balanced by the negative bias in the estimate on the mean square error due to simultaneity.
- 2) Cf. for instance SARGAN (1958): The Estimation of Economic Relationships using Instrumental Variables.
- 3) Cf. WALD (1940): Fitting of straight lines if both variables are subject to error, and MADANSKY (1959): The fitting of straight lines when both variables are subject to error. The properties of this method in various applications are considered in GABRIELSEN (1969): Grupperingsmetoden (The Method of Grouping Data).

As capital input clearly is correlated with size, we rank the observations according to total employment as a criterion for size and define  $r_1 = 1$  for the lower third of the units and  $r_1 = 0$  otherwise, and  $r_2 = 1$  for the upper third and  $r_2 = 0$  otherwise.<sup>1)</sup> Using  $r_2 - r_1$  as an instrumental variable for  $z$  in (26), we get the corresponding estimator for  $\beta$  as:<sup>2)</sup>

$$(29) \quad \tilde{\beta} = \frac{\bar{y}_2 - \bar{y}_1 - \tilde{\alpha}(\bar{x}_2 - \bar{x}_1)}{\bar{z}_2 - \bar{z}_1}$$

where the bars indicate means and the subscripts indicate size-groups.

We get a standard error of this estimator as:<sup>3)</sup>

$$(30) \quad \sigma_{\tilde{\beta}} = \frac{\sigma_u}{\sqrt{\frac{n}{6} (\bar{z}_2 - \bar{z}_1)^2}}$$

where  $\sigma_u$  is the standard deviation of the error term and  $n$  is the number of observations of the sample.

Having estimated the elasticity of labour by means of the factor share method described in sub-section a above, we now have by means of (29) also an estimate on the elasticity of scale free of both simultaneous equations bias and errors of measurement in capital bias.<sup>4)</sup>

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#### N o t e s :

- 1) The findings of some studies indicate that given rather broad conditions the efficiency of the estimators obtained by the method of grouping is best when about one third of the units of each extreme of the observations are included in the manner done by us in the present context. Cf. BARTLETT (1949): Fitting a straight line when both variables are subject to error, GIBSON and JOWETT (1957): "Three-group" regression analysis, Part I, Simple regression analysis, NAIR and SHRIVASTAVA (1942): On a simple method of curve fitting, and THEIL and YZEREN (1956): On the efficiency of Wald's method of fitting straight lines.
- 2) A closely related method of estimation is applied in GRILICHES and RINGSTAD (1971), op.cit. Ch. IV.
- 3) Cf. GOLDBERGER (1964): Econometric Theory. Section 6.5.
- 4) This method of estimating the factor elasticities will be referred to as the Klein Wald method in the following (Klein for the factor share method and Wald for the grouping method).

e. A Comparison of the Various  
Estimates Obtained on the Capital  
and Scale Elasticities

The results for the method derived in the previous sub-section are presented in Table III.9. together with the results for the capital elasticity and the scale elasticity obtained by means of the methods previously discussed.<sup>1)</sup> The first set of estimates presented in this table is subject to all three kinds of biases under discussion. The second set of estimates (containing the estimates of  $\beta$  only) is free of all three kinds of biases, but may be subject to other errors due to the assumption made about the returns to scale. The third set is subject to biases due to errors of measurement, while the final one is subject to bias due to errors of measurement in labour input only.<sup>2)</sup> In light of the bias computations above the differences between the different sets of estimates are as expected.

On the average for our industries the factor share estimates on the capital elasticity are about 50% higher than the pure OLS estimates. However, for a number of industries, such as Clothing, Metal Products, Non-El. Machinery, El. Machinery and Transport Equipment, the difference between the two capital elasticity estimates is several hundred per cent.

As expected the factor share OLS method generally yields lower estimates on the capital elasticity than the pure factor share method. There are three industries for which the opposite is true, namely Wood Products, Mineral Products and Transport Equipment. Thus these also have

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Notes :

- 1) With regard to the Klein Wald method, the estimate on the standard error of the estimate on the capital elasticity is approximate as we use the mean square error obtained by means of the OLS method as an estimate on the variance of the error term ( $\sigma_u^2$ ). However, examination of this approximation for Total Mining and Manufacturing suggested that it does not understate unduly the "true" estimate of this variance (implied by the Klein Wald estimates of the factor elasticities,  $\tilde{\alpha}$  and  $\tilde{\beta}$ ).
- 2) Since the bias due to errors of measurement in capital of the factor-share OLS estimate on  $\beta(\hat{\beta})$  is  $-\beta\sigma_e^2/\sigma_z^2$  (cf. (27) above) and the Klein Wald estimate on  $\beta(\tilde{\beta})$  is free of errors of measurement in capital bias, we could estimate  $\sigma_e^2/\sigma_z^2$  as  $\frac{\sigma_e^2/\sigma_z^2}{\sigma_e^2/\sigma_z^2} = 1 - \hat{\beta}/\tilde{\beta}$ . For Total Mining and Manufacturing we have  $\frac{\sigma_e^2/\sigma_z^2}{\sigma_e^2/\sigma_z^2} = 0.182$  which implies that  $\frac{\sigma_e^2/\sigma_z^2}{\sigma_e^2/\sigma_{z-x}^2} = 0.323$  since in the sample  $\hat{\sigma}_z^2 = 1.774 \hat{\sigma}_{z-x}^2$ . This suggests that the measurement errors in capital are more serious than assumed by us in the bias computations above, particularly when we take into consideration that the estimate on the error variance ratio derived is likely to be biased downwards. Cf. CARTER and BLALOCK (1970): Underestimation of Error in Wald Bartlett Slope Estimation.

Table III.9. Estimates on the Capital Elasticity and the Elasticity of Scale from the Cobb-Douglas Relation.\*

Method of Estimation	OLS		Factor share for $\alpha$ Factor share for $\beta$		Factor share for $\alpha$ OLS for $\beta$		Klein Wald method	
	$\beta$	$\epsilon$	$\beta$	$\epsilon$ <sup>1)</sup>	$\beta$	$\epsilon$	$\beta$	$\epsilon$
Estimates on: Industry								
Total Mining and Manufacturing .....	0.272 (0.007)	0.994 (0.005)	0.397 (0.004)		0.354 (0.004)	0.957	0.433 (0.006)	1.036
Mining and Quarrying .....	0.281 (0.040)	0.988 (0.022)	0.431 (0.019)		0.379 (0.016)	0.924	0.389 (0.018)	0.958
Food Products .....	0.372 (0.025)	0.888 (0.014)	0.475 (0.012)		0.365 (0.013)	0.890	0.420 (0.018)	0.945
Textiles .....	0.293 (0.034)	0.946 (0.022)	0.429 (0.011)		0.351 (0.018)	0.922	0.380 (0.023)	0.951
Clothing .....	0.080 (0.024)	0.933 (0.021)	0.400 (0.008)		0.258 (0.017)	0.858	0.453 (0.030)	1.053
Wood Products .....	0.188 (0.038)	1.092 (0.024)	0.251 (0.043)		0.289 (0.021)	1.038	0.393 (0.027)	1.142
Pulp and Paper ....	0.300 (0.023)	0.896 (0.015)	0.443 (0.008)		0.327 (0.011)	0.884	0.367 (0.016)	0.924
Printing .....	0.146 (0.023)	1.041 (0.017)	0.295 (0.009)		0.265 (0.013)	0.970	0.336 (0.017)	1.041
Basic Chemicals ...	0.195 (0.037)	0.893 (0.019)	0.504 (0.016)		0.439 (0.019)	0.935	0.527 (0.024)	1.023
Mineral Products ..	0.322 (0.023)	1.110 (0.023)	0.444 (0.012)		0.483 (0.017)	1.039	0.520 (0.021)	1.076
Basic Steel .....	0.200 (0.029)	1.074 (0.029)	0.488 (0.011)		0.418 (0.020)	0.930	0.543 (0.024)	1.055
Metal Products ....	0.129 (0.034)	0.944 (0.018)	0.380 (0.018)		0.276 (0.017)	0.896	0.335 (0.023)	0.955
Non-El. Machinery .	0.028 (0.037)	1.078 (0.021)	0.359 (0.012)		0.328 (0.019)	0.969	0.393 (0.023)	1.034
El. Machinery .....	0.111 (0.043)	1.023 (0.029)	0.357 (0.019)		0.290 (0.023)	0.933	0.464 (0.033)	1.107
Transport Equipment .....	0.091 (0.021)	1.070 (0.010)	0.228 (0.013)		0.273 (0.010)	1.045	0.304 (0.011)	1.076
Miscellaneous Products .....	0.355 (0.046)	0.938 (0.046)	0.386 (0.035)		0.336 (0.037)	0.950	0.309 (0.059)	0.923

\*  $\beta$  is the elasticity of capital and  $\epsilon$  is the elasticity of scale.

1)  $\epsilon = 1$  per assumption.

a factor share OLS estimate on the elasticity of scale above one. However, when eliminating the errors of measurement in capital bias there are nine industries with an estimate on the scale elasticity above one. Seven of these also have an OLS estimate on the scale elasticity above one. For the



former method there are thus six industries with an estimate on the capital elasticity below the factor share estimate on that parameter.

f. An Attempt to Eliminate the Effects of Quality Variations in Labour Input

All types of estimates on the capital and scale elasticities, except those obtained by the pure factor share method, are subject to one or more of the three biases under discussion in the present chapter. The last set of estimates presented in Table III.9. is, however, subject to biases due to quality variations in the labour input measure only.

We have previously argued that the assumption made about the behaviour of this error is rather extreme, i.e. that the quality component of labour input is perfectly correlated with the observed wage rate. The results of the ACMS relation suggest that this cannot be true for all industries. However, having adopted this assumption in the bias computations we may take the full consequences of it and measure labour input correspondingly, since it implies that the proper labour input measure is WL and not L.<sup>1)</sup>

In Table III.10. the results of our methods of estimation when WL is applied as the labour input measure are presented. The first set of estimates is "free" of errors of measurement in labour bias only; in the second set the simultaneous equations bias is also eliminated, and the final set of estimates should be "free" of all three types of bias.<sup>2)</sup> However, due to the extreme assumption made about the error in the labour input measure, we will instead argue that the last set of estimates presented in Table III.10. represents lower limits of the unbiased estimates of the capital and scale elasticities that we could have obtained by the Klein Wald method if the labour input was correctly measured. In the same sense

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Notes :

- 1) Where we estimate  $\beta$  by OLS given  $\alpha = \tilde{\alpha}$ , we get an estimate on  $\beta$  when using WL as the labour input measure equal to the corresponding estimate on  $\beta$  when using L plus the bias computed for this estimate due to errors of measurement in labour.
- 2) The standard error of  $\hat{\beta}$  is computed by means of formula (30) using the estimated standard deviation of the error term obtained from the OLS regression when WL is applied as the labour input measure. Thus this standard error of  $\beta$  is approximate in the same way as the standard error of this estimate when L is applied as input measure.

Table III.10. Estimates on the Capital and Scale Elasticities when using Total Wages as the Labour Input Measure.\*

Method of Estimation	OLS		Factor share for $\alpha$ OLS for $\beta$		Klein Wald Method	
	$\beta$	$\epsilon$	$\beta$	$\epsilon$	$\beta$	$\epsilon$
Industry	Estimates on:					
Total Mining and Manufacturing .....	0.178 (0.007)	0.977 (0.004)	0.323 (0.004)	0.926	0.410 (0.004)	1.013
Mining and Quarrying .....	0.241 (0.038)	0.981 (0.020)	0.342 (0.015)	0.911	0.382 (0.017)	0.951
Food Products .....	0.199 (0.023)	0.899 (0.012)	0.348 (0.012)	0.873	0.422 (0.016)	0.947
Textiles .....	0.159 (0.034)	0.961 (0.019)	0.329 (0.017)	0.900	0.373 (0.021)	0.944
Clothing .....	0.042 (0.021)	0.987 (0.018)	0.246 (0.016)	0.846	0.436 (0.025)	1.036
Wood Products .....	0.120 (0.035)	1.076 (0.035)	0.289 (0.019)	1.018	0.394 (0.025)	1.123
Pulp and Paper .....	0.188 (0.018)	0.897 (0.011)	0.299 (0.009)	0.856	0.388 (0.013)	0.895
Printing .....	0.087 (0.020)	1.053 (0.014)	0.255 (0.012)	0.960	0.346 (0.014)	1.052
Basic Chemicals .....	0.108 (0.032)	0.959 (0.017)	0.420 (0.018)	0.916	0.505 (0.021)	1.001
Mineral Products .....	0.128 (0.036)	1.117 (0.018)	0.453 (0.016)	1.011	0.502 (0.017)	1.058
Basic Steel .....	0.161 (0.036)	1.034 (0.024)	0.395 (0.019)	0.907	0.524 (0.023)	1.036
Metal Products .....	0.043 (0.034)	0.909 (0.017)	0.250 (0.017)	0.870	0.319 (0.021)	0.939
Non-El. Machinery .....	-0.097 (0.037)	1.076 (0.019)	0.310 (0.019)	0.951	0.391 (0.021)	1.032
El. Machinery .....	0.144 (0.029)	0.996 (0.021)	0.286 (0.019)	0.929	0.398 (0.025)	1.041
Transport Equipment .....	0.076 (0.020)	1.057 (0.009)	0.262 (0.009)	1.034	0.297 (0.011)	1.069
Miscellaneous Products ...	0.152 (0.056)	0.917 (0.035)	0.264 (0.031)	0.878	0.287 (0.050)	0.901

\*  $\beta$  is the elasticity of capital and  $\epsilon$  is the elasticity of scale.

the corresponding set of estimates of table III.9. represents the upper limit of these estimates.<sup>1)</sup>

N o t e :

1) We should note, however, that for two industries this interpretation of the two sets of estimates does not hold, namely for Food Products and Printing. When using WL as the labour input measure we get somewhat bigger estimates than when using L.

g. Simultaneous Equations, Errors of Measurement and the Estimation of the Elasticity of Substitution from the Kmenta Relation

The estimates obtained for the elasticity of substitution by means of the behaviour relation of our model are seriously distorted due to quality variations in labour input.<sup>1)</sup> An alternative to this method worth considering is to use the so-called Kmenta relation, which is a Taylor expansion of the CES relation around the value of the elasticity of substitution of one which corresponds to the Cobb-Douglas case.<sup>2)</sup> Excluding terms of third and higher orders we have this approximation as:

$$(31) \quad y = \alpha x + \beta z + \gamma(z-x)^2$$

And it can be shown that the elasticity of substitution for the mean of the log capital-labour ratio is:<sup>3)</sup>

$$(32) \quad b = \frac{1}{1 - \frac{2\gamma(\alpha+\beta)}{\alpha\beta}}$$

However, since there are serious problems present when trying to obtain reliable estimators for the factor elasticities from the Cobb-Douglas relation, it must be even more difficult to obtain reliable estimators for the elasticity of substitution from (31) since this method implies both a squared variable in the regression equation and an indirect estimation by (32) of the parameter of interest.

We must therefore consider the effects of the three kinds of errors previously discussed on the estimation of the elasticity of substitution by means of (31) and (32). The simultaneous equations biases will primarily have the effect that the product of  $\hat{\alpha}$  and  $\hat{\beta}$  in formula (32) is biased

Notes:

- 1) It is shown in Appendix III.1. that given our assumptions this estimate is biased towards one.
- 2) Cf. KMENTA (1967): On the Estimation of the CES Production Function.
- 3) In contrast to the Cobb-Douglas relation, the factor elasticities of the Kmenta relation are not constant as they depend on the log of the capital-labour ratio. The same is true for the elasticity of substitution.

To have a basis for comparison with our previous results it is convenient to compute the elasticities for the sample mean of the variable on which they depend. Note that since the variables in (31) are computed from their means we get the estimates on the elasticities of labour and capital for the mean of the log of the capital-labour ratio directly as the estimates on  $\alpha$  and  $\beta$ .

downwards. As shown previously the estimator for the scale elasticity is fairly robust against errors due to simultaneous equations and is about one. However, while the product of the two factor share estimates for e.g. Total Mining and Manufacturing is  $\sim 0.24$  the product of the OLS estimates is  $\sim 0.14$ . Thus, even if the effect on the estimator for  $\gamma$  of simultaneous equations is rather unpredictable, this kind of error probably biases the estimator for the elasticity of substitution away from one. For the same reason this also seems to be the main effect of errors of measurement of labour input. For errors of measurement in capital input it has been shown that the OLS estimators both for  $\beta$  and particularly on  $\gamma$  are seriously biased downwards.<sup>1)</sup> Generally  $\hat{\alpha}$  is biased upwards which implies that this kind of error has two opposite effects on the estimator for the elasticity of substitution. It is biased towards one because  $\hat{\gamma}$  is biased towards zero, while it is biased away from one because  $\hat{\alpha}\hat{\beta}$  is biased towards zero. However, if we adopt the assumptions of the study referred to above, it can be shown that the net effect is a bias of  $b$  towards one. We have for large samples  $\hat{\beta} \sim \beta(1-\lambda)$  and as constant returns to scale is assumed,  $\hat{\alpha} \sim \alpha + \beta\lambda$ , and  $\hat{\gamma} \sim \gamma(1-\lambda)^2$ , where  $\lambda = \frac{\sigma_e^2}{\sigma_{z-x}^2}$ , the ratio of the error variance to the variance of the measured log capital-labour ratio. Therefore

$$(33) \quad \text{plim } \hat{b} = \frac{1}{1 - \frac{2\gamma(1-\lambda)}{\beta(\alpha+\beta\lambda)}}$$

which clearly implies that  $\hat{b}$  is biased towards one.

We try to investigate the importance of the different types of biases in a manner similar to that used for the factor elasticities. First, we estimate  $b$  by means of the OLS estimates of the parameters in (31). This is done both when the elasticity of scale is unconstrained and when it is constrained to one. It is also done when both  $L$  and  $WL$  are applied as the labour input measure. Second, we estimate  $\gamma$  by OLS when the means of the factor elasticities are estimated by the factor share method.

Thus we have:

$$(34) \quad y - \tilde{\alpha}x - \tilde{\beta}z = \gamma(z-x)^2$$

and

$$(35) \quad y - \tilde{\alpha}(w+x) - \tilde{\beta}z = \gamma(z-(w+x))^2$$

---

N o t e :

1) Cf. GRILICHES and RINGSTAD (1970): Error-in-the-Variables Bias in Non-Linear Contexts.

Third, the Wald method is applied on (34) by ranking the units according to the size of the right side variable. Thus, this is done with L as the labour input only.

In Table III.11. the results of these computations are presented for Total Mining and Manufacturing. We note first that all results from the

Table III.11. Estimates on the Elasticity of Substitution from the Kmenta Relation and The ACMS Relation for Total Mining and Manufacturing\*

	Method of Estimation	$\hat{\gamma}$	$\hat{\delta}$
Kmenta Relation	Unconstrained OLS (L) .....	0.054 (0.006)	2.144
	Constrained OLS (L) .....	0.054 (0.006)	2.140
	Unconstrained OLS (WL) .....	0.026 (0.005)	1.573
	Constrained OLS (WL) .....	0.029 (0.005)	1.641
	Factor share/OLS (L) .....	0.067 (0.006)	2.255
	Factor share/OLS (WL) .....	0.045 (0.005)	1.593
	Factor share/Wald (L) .....	0.070 (0.011)	2.387
ACMS Relation	OLS .....		0.992 (0.016)
	Wald .....		0.980 (0.016)

\* L refers to L as labour input measure while WL refers to WL as labour input measure. Wald refers to the size-dummies instrumental variable method (or Wald's method of grouping).  $\gamma$  is the coefficient of the second order term of the Kmenta relation and  $b$  is the elasticity of substitution.

Kmenta relation imply that the elasticity of substitution is above one. This does not correspond very well to the results of the ACMS relation which suggests that the elasticity of substitution is below one.<sup>1)</sup> This divergence

**N o t e :**

1) Nor do these results correspond very well to the results obtained for these two relations in GRILICHES and RINGSTAD (1971), op.cit. Ch. IV. Even though these two relations showed highly different results for most of the individual industries, the results for Total Manufacturing were approximately the same.  $\hat{b} = 0.871$  from the Kmenta relation and  $\hat{b} = 0.950$  from the ACMS relation.

leads us to try the Wald method also for the ACMS relation by ranking the units by the size of  $W$ . After all,  $W$  is computed as a ratio between two characteristics that both may be subject to errors, but according to our results this does not seem to matter. In fact the Wald estimate on  $b$  is smaller than the OLS estimate.

The constraining of the elasticity of scale to one does not matter for the results when  $L$  is used as the labour input measure; nor does it make much difference when  $WL$  is applied. We note that the effect of the elimination of the simultaneous equations bias on  $b$  depends on whether or not we have eliminated the errors of measurement in labour bias. If not,  $\hat{\gamma}$  is slightly bigger and if so it is slightly smaller compared to the constrained OLS estimate. The elimination of the errors of measurement in capital bias leads as expected to a somewhat higher value of  $b$ . But rather surprisingly, neither the simultaneous equations bias nor the error of measurement in capital bias seems to be very important for the estimation of  $b$  from the Kmenta relation.

The errors of measurement in labour bias seem to be more important, but again we should remember that we presumably overstate this type of bias by our computations.<sup>1)</sup>

#### 4. Main Conclusions

The findings of this chapter<sup>2)</sup> provide sufficient evidence for concluding that in econometric studies such as this, one should be very careful when interpreting the results without a thorough investigation of the "behaviour" of the variables involved.

To be more specific, the following conclusions seem to be apparent:

1. In general one should never ignore possible errors of measurement when trying to deal with the problem of simultaneous equations.
2. In particular, indirect least squares definitely does not work in our case due to errors of measurement.
3. Nor does analysis of covariance work since this method also is not very robust against errors of measurement.
4. Estimating the elasticity of labour by means of the factor-share method and the elasticity of capital by means of the size-dummies-instrumental variables method seems to yield the more reliable estimates for the present kind of data.
5. We have not found any satisfactory method for the estimation of the elasticity of substitution.

#### N o t e s :

- 1) The results for the individual industries came out to be very poor, frequently yielding an estimate on the elasticity of substitution implying the wrong curvature of the isoquants. Thus generally our results concerning the elasticity of substitution are inconclusive.
- 2) The main results of this chapter together with the main results of other chapters are reviewed by industry in Appendix VI.1.

Appendix III.1. Biases of the ILS Estimators of the Production Function Parameters in Cases of Errors of Measurement.

a. The Effects of Quality Variations in Labour Input

The model is;

$$y = \alpha x + \beta z + u$$

(1)

$$y-x = bw + v$$

where  $y = \ln V$ ,  $x = \ln L$ ,  $z = \ln K$  and  $w = \ln W$ .<sup>1)</sup> The "correct" model is, however:

$$y = \alpha x^* + \beta z + u'$$

(2)

$$y-x^* = bw^* + v'$$

where  $x^* = x+q$  and  $w^* = w-q$  where  $q = \ln Q$  and  $Q$  is a quality index of labour input. Thus, we get:

$$u = u' + \alpha q$$

(3)

$$v = v' + (1-b)q$$

We assume that indirect least squares when applied on (2) yields consistent estimators on the parameters, assuming  $y$  and  $x^*$  to be endogenous, and  $z$  and  $w^*$  exogenous.

Provided now that the wage rate is perfectly correlated with the quality index, we obtain the following results when indirect least squares is applied on (1):

$$(4) \quad \hat{b} = \frac{\Sigma(y-x)w}{\Sigma w^2} = b + \frac{\Sigma(v' + (1-b)q)w}{\Sigma w^2}$$

Due to our assumptions:

$$(5) \quad \text{plim } \hat{b} = 1$$

From the second reduced form equation:

$$(6) \quad x = -\frac{b}{1-\alpha} w + \frac{\beta}{1-\alpha} z + \frac{u-v}{1-\alpha}$$

or

$$(7) \quad x = \pi_1 w + \pi_2 z + r$$

we get

$$(8) \quad \begin{aligned} \hat{\pi}_1 &= \frac{\Sigma x w \Sigma z^2 - \Sigma x z \Sigma w z}{D_1} = \pi_1 + \frac{\Sigma r w \Sigma z^2 - \Sigma r z \Sigma w z}{D_1} \\ \hat{\pi}_2 &= \frac{\Sigma x z \Sigma w^2 - \Sigma x w \Sigma z w}{D_1} = \pi_2 + \frac{\Sigma r z \Sigma w^2 - \Sigma r w \Sigma z w}{D_1} \end{aligned}$$

Note :

1) The variables are computed as deviations from their means.

Where:

$$(9) \quad D_1 = \Sigma w^2 \Sigma z^2 - (\Sigma wz)^2$$

Since

$$(10) \quad r = \frac{u' - v' - (1-\alpha-b)q}{1-\alpha}$$

we get

$$(11) \quad \text{plim } \hat{\pi}_1 = -1$$

$$\text{plim } \hat{\pi}_2 = \pi_2 = \frac{\beta}{1-\alpha}$$

Thus:

$$\text{plim } \hat{\alpha} = 0$$

$$(12) \quad \text{plim } \hat{\beta} = \frac{\beta}{1-\alpha}$$

b. The Effects of Errors of Measurement in Capital Input

We apply the model in (1) while the "correct" model now is:

$$y = \alpha x + \beta z^* + u^*$$

$$(13) \quad y - x = b w + v$$

where the "true" measure of capital  $z^*$  is equal to our measure minus an error term  $e$ . That is:

$$(14) \quad z^* = z - e$$

$e$  is assumed to be a random variable with zero mean, constant variance and no serial correlation. It is also assumed to be uncorrelated with  $u^*$ ,  $v$ , and  $z^*$ .

Indirect least squares applied on (13) is assumed to give consistent estimators.

From (1), (13) and (14) we get that:

$$(15) \quad u = u^* - \beta e$$

We note that the estimation of  $b$  is not affected of the error  $e$ , and clearly we have in this case:

$$(16) \quad E\hat{b} = b$$

Since we now have

$$(17) \quad r = \frac{u^* - v - \beta e}{1 - \alpha}$$

we get, by means of ordinary least squares on (7), using formulas (8) and (9) and (17):



$$(18) \quad \hat{\pi}_1 = \pi_1 + \frac{\frac{1}{1-\alpha}(\Sigma(u^* - v - \beta e)w\Sigma z^2 - \Sigma(u^* - v - \beta e)z\Sigma zw)}{D_1}$$

$$\hat{\pi}_2 = \pi_2 + \frac{\frac{1}{1-\alpha}(\Sigma(u^* - v - \beta e)z\Sigma w^2 - \Sigma(u^* - v - \beta e)w\Sigma zw)}{D_1}$$

Thus

$$(19) \quad \text{plim } \hat{\pi}_1 = \pi_1 + \frac{\beta}{1-\alpha} \frac{b_{zw}}{1-r_{zw}^2} k^2$$

$$\text{plim } \hat{\pi}_2 = \pi_2 - \frac{\beta}{1-\alpha} \frac{1}{1-r_{zw}^2} k^2$$

where  $b_{zw} = \sigma_{zw}/\sigma_w^2$  with  $\sigma_{zw}$  and  $\sigma_w^2$  as the probability limits of the covariance between  $z$  and  $w$  and the variance of  $w$  respectively,  $r_{zw}$  is the probability limit of the simple correlation coefficient between  $z$  and  $w$  and  $(\sigma_e/\sigma_z)^2$  is the probability limit of the error to total variance ratio of the log-capital measure.

Thus we have that:

$$(20) \quad \text{plim } (\hat{\alpha}-\alpha) = (1-\alpha)b_{zw}B$$

$$\text{plim } (\hat{\beta}-\beta) = (b-\beta b_{zw})B$$

where

$$(21) \quad B = \beta k^2 / (\beta b_{zw} k^2 - b(1-r_{zw}^2))$$

## Appendix III.2.

Table A.III.1. Coefficients from Auxiliary Regressions Applied in the Bias Computations of Section III.2.\*

## a) No Components Eliminated

Industry	$b_{xz}$	$b_{wx}$	$b_{wz}$	$b_{zx}$	$b_{zw}$	$b_{x,z-x}$
Total Mining and Manufacturing .....	0.6893	0.0252	0.0521	0.9902	1.2365	-0.0214
Mining and Quarrying .	0.7102	0.0213	0.0221	1.1915	1.9373	0.6496
Food Products .....	0.8169	-0.0102	0.0324	0.8753	0.4081	-0.3885
Textiles .....	0.6990	-0.0014	0.0381	1.0462	0.9889	0.1142
Clothing .....	0.5702	-0.0047	0.0204	0.9819	0.3676	-0.0239
Wood Products .....	0.7137	0.0265	0.0326	0.9915	1.5542	-0.0208
Pulp and Paper .....	0.6792	0.0261	0.0501	1.1439	0.6617	0.3632
Printing .....	0.6290	-0.0152	0.0143	1.1015	0.3938	0.1853
Basic Chemicals .....	0.8372	0.0307	0.0400	0.9021	0.7666	-0.3580
Mineral Products .....	0.6962	0.0413	0.0546	1.1756	2.9454	0.5203
Basic Steel .....	0.6029	0.0499	0.0456	1.2668	1.5340	0.4701
Metal Products .....	0.7971	0.0169	0.0433	0.9434	1.3439	-0.1908
Non-El. Machinery ....	0.7344	0.0042	0.0292	1.0691	1.5491	0.2175
El. Machinery .....	0.6678	0.0504	0.0089	1.0784	0.1616	0.1711
Transport Equipment ..	0.8760	0.0050	0.0142	0.9202	0.6179	-0.3798
Miscellaneous Products	0.6229	0.0606	0.1164	0.9552	0.2119	-0.0719

## b) Time-Specific Components Eliminated

Industry	$b_{xz}$	$b_{wx}$	$b_{wz}$	$b_{zx}$	$b_{zw}$	$b_{x,z-x}$
Total Mining and Manufacturing .....	0.6911	0.0241	0.0477	0.9897	1.3359	-0.0228
Mining and Quarrying .	0.7147	0.0239	0.0181	1.1949	2.5898	0.6909
Food Products .....	0.8190	-0.0131	0.0242	0.8749	0.3919	-0.3928
Textiles .....	0.7035	0.0008	0.0322	1.0480	1.0955	0.1219
Clothing .....	0.5734	0.0000	0.0159	0.9863	0.3393	-0.0183
Wood Products .....	0.7170	0.0236	0.0268	0.9908	1.7292	-0.0229
Pulp and Paper .....	0.6858	0.0379	0.0361	1.1502	1.0651	0.3988
Printing .....	0.6291	-0.0153	0.0147	1.1020	0.9330	0.1862
Basic Chemicals .....	0.8378	0.0312	0.0370	0.9029	1.0028	-0.3572
Mineral Products .....	0.6985	0.0423	0.0521	1.1769	3.3085	0.5341
Basic Steel .....	0.6060	0.0364	0.0300	1.2603	2.1778	0.4656
Metal Products .....	0.8075	0.0119	0.0338	0.9390	1.2857	-0.2143
Non-El. Machinery ....	0.7430	0.0050	0.0226	1.0702	1.6947	0.2340
El. Machinery .....	0.6694	0.0420	-0.0011	1.0760	-0.0242	0.1668
Transport Equipment ..	0.8461	0.0045	0.0136	0.9200	0.6011	-0.3809
Miscellaneous Products	0.6191	0.0406	0.1080	0.9608	2.9144	-0.0621

\* Footnote overleaf.

Table A.III.1 (cont.). Coefficients from Auxiliary Regressions Applied in the Bias Computations of Section III.2.\*

## c) Establishment-Specific Components Eliminated

Industry	$b_{xz}$	$b_{wx}$	$b_{wz}$	$b_{zx}$	$b_{zw}$	$b_{x,z-x}$
Total Mining and Manufacturing .....	0.2124	-0.1133	0.1040	0.3772	0.1915	-0.3081
Mining and Quarrying .	0.0029	-0.1452	0.1493	0.0084	0.6551	-0.2554
Food Products .....	0.2075	-0.1998	0.0602	0.2919	0.0534	-0.3884
Textiles .....	0.3364	-0.1944	0.0946	0.4283	0.2304	-0.4036
Clothing .....	0.1827	-0.1871	0.0134	0.2175	0.0300	-0.4458
Wood Products .....	0.1827	0.0142	0.1075	0.4293	0.5961	-0.2811
Pulp and Paper .....	0.0404	-0.6840	0.2773	0.1435	0.2583	-0.2006
Printing .....	0.1279	-0.1232	-0.0045	0.3526	-0.0255	-0.2121
Basic Chemicals .....	0.3224	-0.0803	0.0274	0.5039	0.0392	-0.3190
Mineral Products .....	0.3200	0.0339	0.2113	0.5061	0.7003	-0.3147
Basic Steel .....	0.2055	0.1591	0.2053	0.5362	0.6378	-0.1828
Metal Products .....	0.1831	-0.0417	0.1437	0.3510	0.6256	-0.2930
Non-El. Machinery ....	0.2390	-0.0883	0.1839	0.3087	0.8589	-0.4129
El. Machinery .....	0.2621	0.1393	0.2622	0.3744	0.3849	-0.3724
Transport Equipment ..	0.2884	-0.0661	-0.0051	0.5672	-0.0330	-0.2362
Miscellaneous Products	0.3764	0.2475	0.2310	0.4516	0.4348	-0.4230

## d) Both Time- and Establishment-Specific Components Eliminated

Industry	$b_{xz}$	$b_{wx}$	$b_{wz}$	$b_{zx}$	$b_{zw}$	$b_{x,z-x}$
Total Mining and Manufacturing .....	0.2141	-0.1481	-0.0051	0.3528	-0.0120	-0.3333
Mining and Quarrying .	0.0476	-0.0029	0.0202	0.1255	0.2033	-0.2584
Food Products .....	0.1981	-0.2674	-0.1204	0.2624	-0.1450	-0.4098
Textiles .....	0.3676	-0.1729	-0.0233	0.4204	-0.0834	-0.4449
Clothing .....	0.2009	-0.1503	-0.0487	0.2318	-0.1373	-0.4546
Wood Products .....	0.2216	-0.0488	0.0202	0.3891	0.1851	-0.3089
Pulp and Paper .....	0.0837	-0.2890	0.0389	0.2903	0.0967	-0.1826
Printing .....	0.1244	-0.1275	0.0011	0.3476	0.0074	-0.2105
Basic Chemicals .....	0.3283	-0.0662	-0.0344	0.5169	-0.0866	-0.3136
Mineral Products .....	0.3618	0.0644	0.1406	0.5288	0.6007	-0.3356
Basic Steel .....	0.1391	-0.0950	-0.0343	0.3119	-0.2912	-0.2628
Metal Products .....	0.1339	-0.1575	0.0172	0.2025	0.0856	-0.3785
Non-El. Machinery ....	0.2791	-0.0808	0.0661	0.2778	0.4673	-0.5016
El. Machinery .....	0.1415	-0.0522	0.0362	0.1700	0.0584	-0.4458
Transport Equipment ..	0.2737	-0.0844	-0.0160	0.5445	-0.1042	-0.2397
Miscellaneous Products	0.2641	-0.0068	0.0997	0.3642	0.3441	-0.3852

\*  $x = \ln L$ ,  $z = \ln K$  and  $w = \ln W$ . Cf. Section II.2.

## CHAPTER IV. ON THE TESTING OF MULTIPLE HYPOTHESES

In econometric studies the testing of hypotheses is a valuable statistical tool for investigating the importance of various "causes", the validity of models specified, etc. Quite often, however, a fairly high number of tests are carried out, where subsequent tests are often directly or indirectly based on the outcome of former ones. This is, in particular, a common feature of exploratory studies based on data about which little is known a priori.

The tests thus carried out are usually partial, that is each test-situation is treated separately, and having carried out a series of data-snooping tests one may in fact question the value of the conclusions finally obtained. Generally the statistics of the final test(s) may be quite misleading. Clearly one should rather attempt to consider the multitude of tests as a whole. Moreover, by deciding a priori what to do with different outcomes of the individual tests, the prospects will be better for making a proper evaluation of the conclusions obtained.

There are, however, two basic problems when trying to apply such an approach. First, the issues subject to investigation may be of widely different natures. Thus one may be interested in testing a variety of hypotheses that are not all related in a manner that makes an overall multiple test procedure applicable. However, even in such cases something could be done if one managed to divide the hypotheses into groups so that multiple test procedures could be applied on each group separately.

Multiple testing is a fairly new branch of theoretical statistics<sup>1)</sup> and thus the second basic difficulty one encounters when trying to apply multiple test methods on particular problems in econometrics is just finding an appropriate method for which the properties are known. This is also a main problem in the present context. What we manage to do is to determine an upper limit of the level of the overall tests. This will be done in two ways through slightly different methods of testing.

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N o t e :

- 1) Cf. SCHEFFE (1959), op.cit., AITCHISON (1964): Confidence-region Tests, MILLER jr. (1966): Simultaneous Statistical Inference, GABRIEL (1969): Simultaneous test procedures - some theory of multiple comparisons, SCHEFFE (1970): Multiple testing versus multiple estimation. See also MALINVAUD (1966), op.cit. Ch. 7, § 3.

## 1. The Testing Scheme

The basis for our illustration of the application of multiple test methods in econometrics is the relations of the model used previously in this study, namely

$$(1a) \quad y_{it} = \alpha x_{it} + \beta z_{it} + u_{it} \quad (i = 1 \dots I)$$

$$(1b) \quad (y-x)_{it} = b w_{it} + v_{it} \quad (t = 1 \dots T)$$

where, at the moment, we assume the error terms to be distributed independently, with no serial correlation, with zero means and constant variances.

There are numerous possible errors of specification in this model. To mention a few:<sup>1)</sup> The functional form of the two relations may be wrong; perhaps we should have used gross production as the output measure instead of value added and with materials as an "independent" factor of production together with labour and capital; and perhaps also the specification of the error terms is wrong. Such questions and related ones could be investigated, but not easily at the same time. In this context we will consider only one, namely the specification of the error terms.<sup>2)</sup> This is clearly a "partial" analysis, as the other doubts we may have about the validity of 1a and 1b are not subject to discussion or investigation. More precisely, we shall study the assumption made above about the error means: that they are zero for all units of observation.

In the covariance-analysis of the previous chapter we asserted that the error means might vary both between establishments and over time. This assumption concerning the behaviour of the error means could clearly serve as one-hypotheses when testing the validity of the ones of zero means.

Thus we could have the following test-situation for the production relation<sup>3), 4)</sup>

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**N o t e s :**

- 1) We ignore the deliberate inconsistencies between 1a) and 1b) pointed out in Chapter I.
- 2) In Section 3 of this chapter we consider a multiple test situation concerning also the slope-coefficients of the relations above.
- 3) The test-situation for the behaviour relation is clearly the same.
- 4) The "contents" of the null-hypotheses is that the means are constant. Whether or not these constants are zero is trivial provided that we are not particularly interested in the identification of the intercept.

$$(2a) \quad H_0 : Eu_{it} = 0 \quad (i = 1 \dots I)$$

$$(2b) \quad H_1 : Eu_{it} = a_i + b_t \quad (t = 1 \dots T)$$

Assuming the error term is normally distributed, one can apply an ordinary F-statistics to test the null-hypothesis.<sup>1)</sup>

If  $H_0$  is rejected, however, we do not know whether it is due to the establishment-specific or the time-specific component of the mean, or both. In case we would like to know that, we should rather carry out two tests, with the null-hypotheses:

$$(3a) \quad H'_0 : Eu_{it} = a_i \quad (i = 1 \dots I)$$

$$(3b) \quad H''_0 : Eu_{it} = b_t \quad (t = 1 \dots T)$$

with (2b) as the common one-hypothesis.

In the present case, however, we are interested in an even further investigation of the nature of any variation of the error mean. In our two-way classification there is one observation per cell only. The one-hypothesis above implies that each cell may have its "own" error mean. If one or both of the null-hypotheses above are rejected, however, it may very well be due to a more "constrained" variation of the error mean. It could be true that for each of our industries the error mean varies between sub-industries while it is constant within sub-industries. And it could also be true that the variation of the error mean over time is equal to a trend.

This is the framework within which we will work in the search for the "nature" of any variation of the error mean. In Table IV.1. we present the various potential types of error mean variation implied by this framework, with an explanation of their contents and a notation to be used for them in the following.

\_\_\_\_\_

N o t e :

1) In this case the F-statistics would be:

$$F_{\text{obs}} = \frac{\frac{\sum \hat{u}_{it}^2 - \sum \hat{u}'_{it}{}^2}{\sum \hat{u}'_{it}{}^2}}{\frac{\sum \hat{u}_{it}^2}{\sum \hat{u}'_{it}{}^2}} \cdot \frac{(I-1)(T-1)-2}{I+T-1}$$

where  $\hat{u}_{it}$  is the estimated residual for the production function for "cell" (i,t) under the null-hypothesis and  $\hat{u}'_{it}$  the estimated residual under the one-hypothesis ( $u'_{it} = u_{it} - a_i - b_t$ ).

Table IV.1. Types of error Mean Variation\*.

No.	Type of Variation	Explanation	Notation
1.	$Eu_{it} = a_i + b_t$	Error mean may vary between establishments and show an unconstrained variation over time	E and T
2.	$Eu_{it} = c_j + b_t$ ( $c_1=a_1=a_2=\dots=a_{n_1}, \dots$ $\dots, c_J=a_{n_{J-1}+1}=\dots=a_{n_J}$ )	Error mean may vary between sub-industries but is constant within sub-industries, and it may show an unconstrained variation over time	J and T
3.	$Eu_{it} = a_i + bt$	Error mean may vary between establishments, and any variation over time is constrained to a trend	E and t
4.	$Eu_{it} = b_t$	No variation of the error mean between establishments while it may show an unconstrained variation over time	T
5.	$Eu_{it} = c_j + bt$ ( $c_1=a_1=a_2=\dots=a_{n_1}, \dots$ $\dots, c_J=a_{n_{J-1}+1}=\dots=a_{n_J}$ )	Error mean may vary between sub-industries but is constant within sub-industries, and any variation over time is constrained to a trend	J and t
6.	$Eu_{it} = a_i$	Error mean may vary between establishments but it shows no variation over time	E
7.	$Eu_{it} = bt$	No variation of the error mean between establishments and any variation over time is constrained to a trend	t
8.	$Eu_{it} = c_j$ ( $c_1=a_1=a_2=\dots=a_{n_1}, \dots$ $\dots, c_J=a_{n_{J-1}+1}=\dots=a_{n_J}$ )	Error mean may vary between sub-industries but is constant within sub-industries, and it shows no variation over time	J
9.	$Eu_{it} = 0$	No variation of the error mean either between establishments or over time	0

\*  $i=1\dots I$ ,  $I$  is the number of establishments.

$j=1\dots J$ ,  $J$  is the number of sub-industries (cf. Appendix II.1).

$t=1\dots T$ ,  $T$  is the number of years.

The "types of variation" presented in Table IV.1. form a hierarchy with the more general type (1) on top and the less general one, (9) - no variation - at the bottom. It is, however, not unique. In Fig. IV.1. we see that there are five "levels" in this hierarchy with (1) in the first, (2) and (3) in the second, (4), (5) and (6) in the third, (7) and (8) in the fourth, and (9) in the fifth.

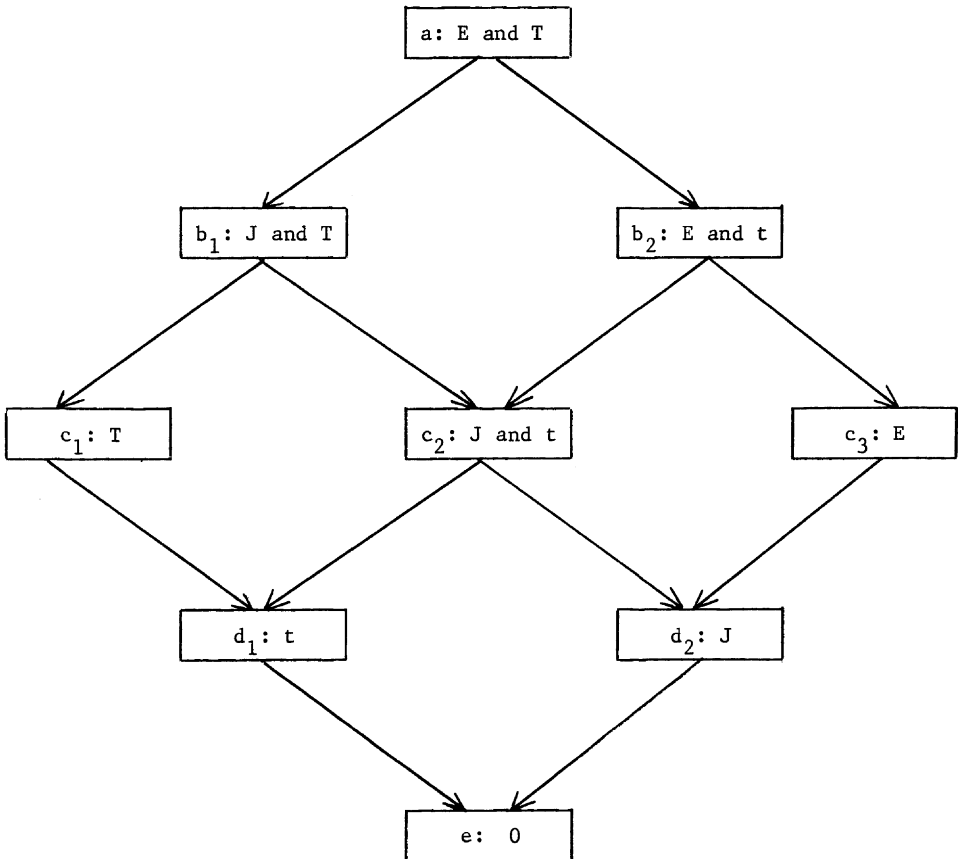


Fig. IV.1.

The Hierarchy of Types of Variation of Error Means

Fig. IV.1. will serve as a scheme for testing the nature of the error mean's variation. The strategy of the testing is as follows: In the first round the two types at the b-level ( $b_1$  and  $b_2$ ) serve as null-hypotheses and they are each tested with "a" as the one-hypothesis. If both are rejected, there is evidence of establishment-specific and year-specific



differences in the error mean. If either  $b_1$  or  $b_2$  or both cannot be rejected, the testing is continued, with the  $c$ -level types of variation as null-hypotheses. If, for example,  $b_1$  is rejected but not  $b_2$ , we test  $c_2$  and  $c_3$  against  $b_2$ .

If both are rejected,  $b_2$  is the "true" type of error mean variation. If, for example,  $c_2$  is not rejected while  $c_3$  is, we test  $d_2$  against  $c_2$ . If  $d_2$  is rejected,  $c_2$  is the "true" type of error mean variation. If not, a final test is carried out,  $e$  against  $d_2$ . And either  $d_2$  or  $e$  is the "true" type depending on the rejection or non-rejection of  $e$  by this test.

By this procedure we may encounter problems of interpretation since we may obtain more than one "true" type of error mean variation. However, we do not run into such problems in this context.<sup>1)</sup>

The individual tests are carried out by using F-statistics assuming the error terms to be normally distributed. As pointed out we do not manage to determine an exact level of the overall test.<sup>2)</sup> Instead, we apply two methods of testing for which it is possible to determine upper limits of the level. The first one implies the use of ordinary F-statistics for each of the individual tests, and the upper limit of the level is determined as the sum of the levels of the individual tests.<sup>3)</sup> If the number of individual tests is high, however, this upper limit is of little interest since it is presumably far from the true level.<sup>4)</sup> In this case we use a level of 0.5% for the individual tests, and since the potential number of tests is 12 we have an upper limit of the overall level of 6%.

The second test procedure is developed by E. Spjøtvoll and is also based on F-statistics.<sup>5)</sup> For each of the individual tests we use the "modified" F-statistics<sup>6)</sup>

$$F'_{\text{obs}} = \frac{Q_i^2 - Q_j^2}{Q_k^2} \frac{(I-1)(T-1)-a}{I+T-1}$$

where  $Q_i^2$  and  $Q_j^2$  are the sums of squares of the estimated residuals under the

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#### N o t e s :

- 1) They are, however, apparent in the next chapter where an attempt is made to determine the nature of technical change by a related scheme of testing. See Section V.2. Cf. also Section IV.3.
- 2) The overall level of the multiple test is the probability of accepting a particular type of error mean variation when any one of the other types specified in the testing scheme is right.
- 3) Cf. MALINVAUD (1966), op.cit. Ch. 7, § 3.
- 4) Cf. Section IV.3.
- 5) SPJØTVOLL (1969): Multiple Comparison of Regression Functions.
- 6)  $a$  is the number of slope-coefficients, and thus  $a = 2$  for the production relation and  $a = 1$  for the behaviour relation.

null-hypothesis and one-hypothesis respectively and  $Q_k^2$  is the sum of squares of the residual for the more general type of error mean variation, i.e. the one at the top of the hierarchy. Both  $Q_k^2$  and the degrees of freedom, which are those of the ordinary F-statistics of the tests of 2a) versus 2b) (or (9) versus (1) in Table IV.1.), are common for all individual tests.

Now, if we carry out the same test-procedure as for the first method of testing but reject each null-hypothesis for which  $F'_{\text{obs}} \geq F_{((1-\epsilon), (I+T-1), (I-1)(T-1)-a)}$ , we have an upper limit of the overall test of  $\epsilon$ . We choose  $\epsilon = 5\%$  to have an upper limit of the overall level of this test roughly comparable to the one discussed at first.

## 2. The Results

The test-procedures outlined above are applied on la) and lb) separately, and thus we ignore the simultaneous equations problem of the estimation of the parameters of la)<sup>1)</sup>. The outcome of the tests is presented by industry in Table IV.2. To give some idea of the magnitudes of the F-values computed we present for Food Products in Tables IV.3 and IV.4 the values of the ordinary F-values ( $F_{\text{obs}}$ ) as compared to the corresponding upper 0.5% fractiles ( $F_{0.995}$ ) as well as Spjøtvoll's F-statistics. ( $F'_{\text{obs}}$ .) In the latter case the fractile used (5%) is from the same F-distribution for all individual tests. Food Products is selected since this is the industry with the largest number of establishments as well as the largest number of sub-industries.

From Table IV.2. we note first that the two methods of testing yield somewhat different results. Generally, the Spjøtvoll method is rougher towards the time-components of the error mean, as compared to the ordinary F-statistics method. This is not so surprising inasmuch as the former method does not take into account the highly varying number of parameters involved in the individual tests. Thus, it does not seem to be very suitable for the test-situation considered in this section.

According to the results obtained by ordinary F-statistics we see for the Cobb-Douglas relation that the "true" type of error mean variation is the same for all except one of the fifteen industries, namely individual

---

N o t e :

1) As demonstrated in the previous chapter OLS, particularly when combined with analysis of covariance, is likely to yield quite poor estimates on the production function parameters. In this context, however, we are not interested in these parameters; we are interested only in determining the nature of any error mean variation. Cf., however, Section IV.3.

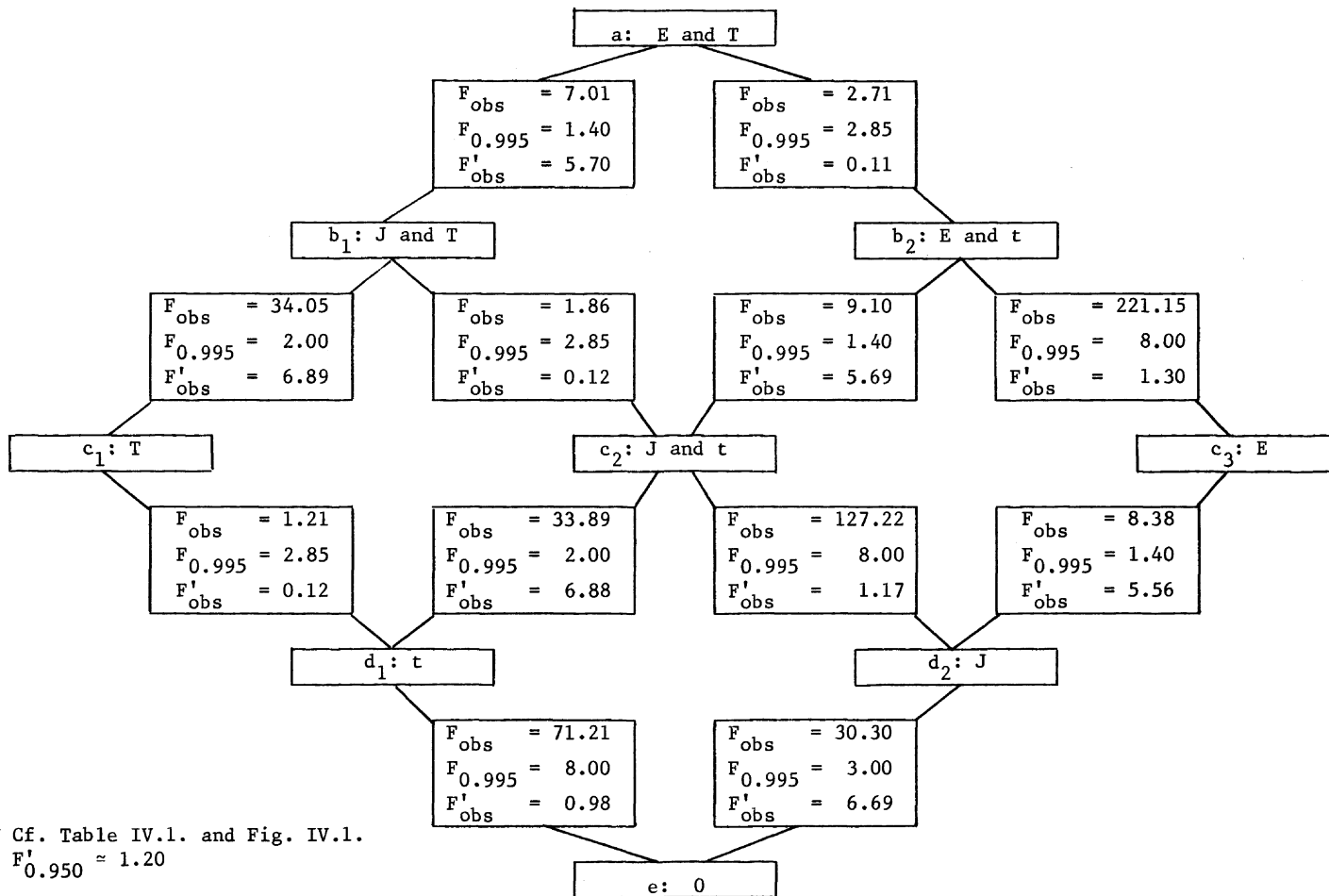
Table IV.2. The Results of Multiple Tests of the Error Means of the Cobb-Douglas and ACMS Relations\*

Industry	"True" Type of Error Mean Variation				No. of establish-ments	No. of sub-industries <sup>1)</sup>
	Cobb-Douglas		ACMS			
	Ordinary F-statistics	Spjøtvoll F-statistics	Ordinary F-statistics	Spjøtvoll F-statistics		
Mining and Quarrying .....	E and t	E and t	E and t	E	26	7
Food Products .....	E and t	E and t	E	E	164	22
Textiles .....	E and t	E	E	E	58	7
Clothing .....	E and t	E	E	E	67	7
Wood Products .....	E and t	E	E	E	45	8
Pulp and Paper .....	E and t	E and t	E and t	E	103	6
Printing .....	E and t	E	E	E	63	6
Basic Chemicals .....	E and t	E and t	E	E	72	16
Mineral Products .....	E	E	E	E	36	12
Basic Steel .....	E and t	E and t	E	E	42	6
Metal Products .....	E and t	E	E	E	60	10
Non-El. Machinery .....	E and t	E	E	E	37	4
El. Machinery .....	E and t	E	E	E	34	7
Transport Equipment .....	E and t	E	E	E	87	11
Miscellaneous Products .....	E and t	E	E	E	13	3

\* Cf. Table IV.1. and Fig. IV.1.

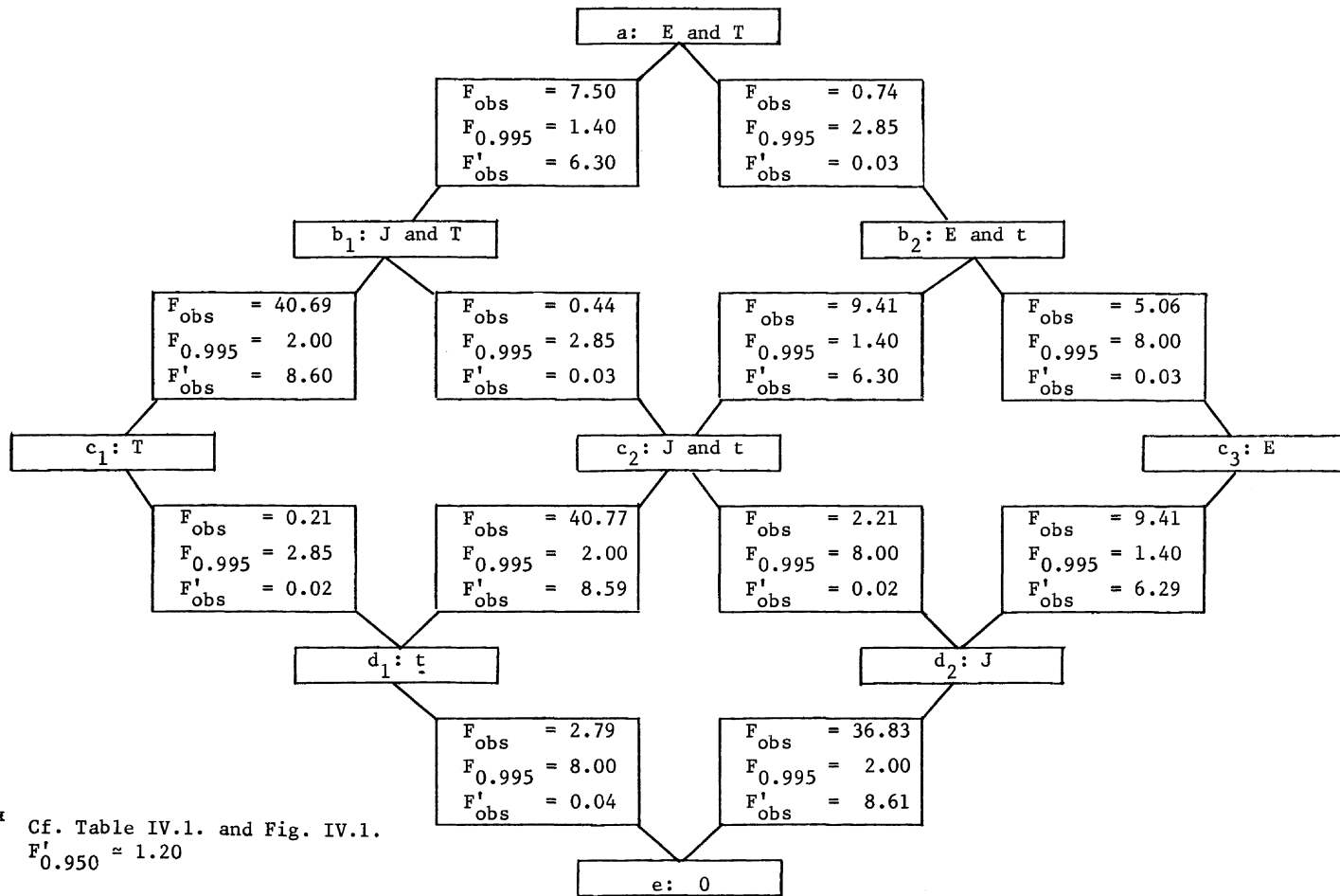
1) The sub-industries are two, three and four digit industry groups according to the Norwegian version of the ISIC-code. Cf. Appendix II.1.

Table IV.3. Testing Scheme for the Error Mean of the Cobb-Douglas Relation for Food Products\*



\* Cf. Table IV.1. and Fig. IV.1.  
 $F'_{0.950} \approx 1.20$

Table IV.4. Testing Scheme for the Error Mean of the ACMS Relation for Food Products\*



\* Cf. Table IV.1. and Fig. IV.1.  
 $F'_{0.950} \approx 1.20$

variation between establishments and a trend variation over time. For Mineral Products the "true" type is establishment-specific error means, with no variation over time.

For the ACMS relation the results are also the same for all but two industries, namely establishment-specific error means with no variation over time. Rather surprisingly the "true" type of error mean variation of the ACMS relation for Pulp and Paper is the more general one, while the "true" type of error mean variation for Mining and Quarrying is individual variations between establishments and a trend over time.

Thus, when applying the ordinary F-statistics method the results are fairly uniform for each relation. The difference between the relations is easily explained by the fact that if the elasticity of substitution is close to one, which is just the result obtained by the behaviour relation for most industries, the shifts over time in the intercept of this relation should not be significant. On the other hand, if there are technical changes of some importance, not accounted for by the input measures, we should have significant shifts over time in the production function.<sup>1)</sup> Using the Spjøtvoll method these shifts are significant only for the more heavy industries Mining and Quarrying, Pulp and Paper, Basic Chemicals and Basic Steel and for Food Products, while this method does not yield significant shifts over time in the behaviour relation for any industry. Various types of shifts over time in the two relations are subject to further discussion in the next chapter.

### 3. Results of a More Complex Test

In the multiple test schemes studied above even the more general type of error mean variation (E and T) is quite restrictive, since it presumes that the slope coefficients of the relations concerned are constant both between establishments and over time. We will in this section consider a more complex multiple test situation where differences both in error means and slope coefficients are permitted.

The analysis is carried out for Total Mining and Manufacturing only. For a given year we assume that all parameters (error means included) are

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N o t e :

1) Cf. Section V.2.

constant within each industry, while they may be different for different industries.<sup>1)</sup> Over time the parameters are allowed to have trends, common for all units.

Thus in the less restrictive case we have the two relations as:<sup>2)</sup>

$$(4a) \quad y_{it} = a_j + dt + (\alpha_j + \gamma t) x_{it} + (\beta_j + \mu t) z_{it} + r_{it}$$

$$(4b) \quad (y-x)_{it} = c_j + et + (b_j + \eta t) w_{it} + s_{it}$$

$$(i = 1 \dots 907)$$

$$(j = 1 \dots 13)$$

$$(t = 1 \dots 9)$$

where  $r_{it}$  and  $s_{it}$  are the "pure" error terms assumed to have zero means, constant variances and no serial correlation.

The production function (4a) is now on the top of the hierarchy of types of parameter variation, while at the bottom we have the same as the one in the case discussed in the previous section. In this case the number of types is, however, much higher. Since in (4a) there are six "effects",<sup>3)</sup> the first round of testing implies partial tests of six null-hypotheses against the common one-hypothesis in (4a).

These six types are in turn one-hypotheses for 15 null-hypotheses. In the third round these 15 are one-hypotheses for 20 null-hypotheses, which in turn are one-hypotheses for 15 null-hypotheses, which in turn are one-hypotheses for 6 null-hypotheses, which are one-hypotheses with a common null-hypothesis, namely when all parameters are constant over the sample. For the ACMS relation we have a related scheme.

It is easily shown that the number of types in testing schemes like the present ones is:

$$(5) \quad \sum_{i=0}^m \binom{m}{i} = 2^m$$

where  $m$  is the number of "effects" subject to testing, in this case six for the Cobb-Douglas relation and four for the ACMS relation. Thus we get

#### Notes :

- 1) The inter-industry variation is slightly more constrained than this as the coefficients are assumed to be the same for El. Machinery and Non-Electrical Machinery, and for Transport Equipment and Miscellaneous Products. This is done due to certain capacity problems of the program applied in the computations.
- 2) Note that we here redefine the error terms so that we get differences in the intercept instead of differences in error means. The contents of the relations are clearly not changed by this reformulation.
- 3) An industry and a time "effect" for each of the intercept, the labour elasticity and the capital elasticity.

Table IV.5. Results for the Cobb-Douglas Relation with "True" Types of Parameter Variation\*

Industry	Elasticities for t=63										
	Intercept	$\alpha_j$	$\beta_j$	$\mu$	El. of capital	El. of scale	Intercept	d	$\alpha_j$	$\beta_j$	El. of scale
Mining and Quarrying .....	-0.044 (0.189)	0.743 (0.060)	-0.012 (0.049)		0.253 (0.046)	0.996 (0.025)	-0.041 (0.189)		0.745 (0.060)	0.251 (0.046)	0.996 (0.025)
Food Products (Base) .....	2.036	0.525 (0.019)	0.095 (0.025)		0.360 (0.019)	0.885 (0.011)	-0.245		0.526 (0.019)	0.360 (0.019)	0.886 (0.011)
Textiles .....	-0.175 (0.222)	0.683 (0.053)	+0.000 <sup>1)</sup> (0.046)		0.265 (0.043)	0.948 (0.027)	-0.174 (0.222)		0.681 (0.053)	0.266 (0.043)	0.947 (0.027)
Clothing .....	0.159 (0.194)	0.929 (0.043)	-0.190 (0.036)		0.066 (0.033)	0.995 (0.028)	0.166 (0.194)		0.934 (0.043)	0.061 (0.033)	0.995 (0.028)
Wood Products ..	-0.591 (0.177)	0.915 (0.041)	-0.092 (0.039)		0.174 (0.035)	1.089 (0.023)	-0.595 (0.177)		0.915 (0.041)	0.175 (0.035)	1.091 (0.022)
Pulp and Paper .	0.201 (0.162)	0.649 (0.037)	-0.007 (0.033)	0.0042	0.258 (0.028)	0.907 (0.018)	0.186 (0.162)	0.0362	0.643 (0.037)	0.263 (0.028)	0.906 (0.018)
Printing .....	-0.283 (0.173)	0.898 (0.043)	-0.124 (0.036)	(0.0003)	(0.141) (0.033)	1.039 (0.024)	-0.290 (0.173)	(0.0021)	0.898 (0.043)	0.142 (0.033)	0.940 (0.024)
Basic Chemicals.	0.249 (0.163)	0.793 (0.027)	-0.076 (0.030)		0.190 (0.026)	0.983 (0.014)	0.254 (0.164)		0.794 (0.027)	0.188 (0.026)	0.982 (0.014)
Mineral Products .....	-0.925 (0.190)	0.814 (0.055)	0.034 (0.046)		0.299 (0.043)	1.113 (0.025)	-0.940 (0.190)		0.812 (0.055)	0.302 (0.043)	1.114 (0.025)
Basic Steel ....	-0.179 (0.231)	0.894 (0.061)	-0.089 (0.045)		0.176 (0.042)	1.070 (0.032)	-0.197 (0.231)		0.889 (0.061)	0.181 (0.042)	1.070 (0.032)
Metal Products .	0.644 (0.206)	0.826 (0.044)	-0.164 (0.044)		0.101 (0.040)	0.927 (0.022)	0.648 (0.206)		0.827 (0.044)	0.100 (0.040)	0.927 (0.022)
Non-El. Machinery) El. Machinery ...)	0.095 (0.168)	0.999 (0.039)	-0.214 (0.036)		0.052 (0.032)	1.051 (0.020)	0.098 (0.168)		0.998 (0.039)	0.052 (0.032)	1.050 (0.020)
Transport ) Equipment .... )	-0.396 (0.135)	0.874 (0.024)	-0.093 (0.028)		0.172 (0.023)	1.046 (0.012)	-0.396 (0.135)		0.875 (0.024)	0.171 (0.023)	1.046 (0.012)
Misc. Products . )											
	MSE = 0.2447						MSE = 0.2450				

\* Cf. relation (4a). Note that in the computations t = 59, 60...67.

1) Less than  $5 \cdot 10^{-6}$ .



64 types for the former and 16 for the latter relation. The number of tests is, however, substantially higher, as it is given by:

$$(6) \quad \sum_{i=0}^{m-1} \binom{m}{i} (m-i) = m2^{(m-1)}$$

which gives 192 tests for the Cobb-Douglas relation and 32 for the ACMS relation. Thus even with a level of 0.001 for the individual tests the upper limit of the level of the overall test when using the ordinary F-statistics method becomes quite high, at least for the former relation, namely 19.2 %.

Table IV.6. Results for the ACMS Relation with the "True" Type of Parameter Variation\*

Industry	Intercept	$b_j$
Mining and Quarrying .....	0.058 (0.348)	0.989 (0.157)
Food Products (Base) .....	0.624	1.114 (0.033)
Textiles .....	-0.143 (0.200)	1.082 (0.105)
Clothing .....	0.166 (0.163)	0.870 (0.085)
Wood Products .....	-1.029 (0.246)	1.433 (0.122)
Pulp and Paper .....	0.454 (0.115)	0.798 (0.045)
Printing .....	0.003 (0.190)	0.885 (0.089)
Basic Chemicals .....	0.179 (0.126)	1.047 (0.052)
Mineral Products .....	-1.583 (0.265)	1.790 (0.124)
Basic Steel Products .....	0.350 (0.254)	0.906 (0.110)
Metal Products .....	0.515 (0.220)	0.714 (0.104)
Non-El. Machinery .....	-0.020	0.957
El. Machinery .....	(0.173)	(0.077)
Transport Equipment .....	-0.024	0.885
Miscellaneous Products .....	(0.140)	(0.062)

MSE = 0.2122

\* Cf. relation (4b).

However, for the Spjøtvoll method we can, as previously, determine the level independently of the number of tests, and in this case we also use 5 %.

Carrying out the multiple test by the two methods we obtain two "true" types for the Cobb-Douglas relation; the same ones for both methods. Both types imply industry-specific intercepts and factor-elasticities, but while one implies a trend in the capital elasticity the other implies a trend in the intercept. Our testing procedures do not allow us to choose between them, but we note from Table IV.5. that the former one yields a slightly better fit.<sup>1)</sup>

We also note from table IV.5. that the two types yield approximately the same results concerning the factor- and scale-elasticities.<sup>2)</sup> Thus, in this respect as well, the two types are almost perfect substitutes.

For the ACMS relation we get, however, a unique "true" type. In this case as well the result is the same for the two methods. It implies that there are differences between industries both with regard to the intercept and the elasticity of substitution, while no trends are present (cf. Table IV.6.). Thus this finding supports the results of the previous section. In addition, this type suggests that there is no trend in the elasticity of substitution over time, at least not when imposing the same trend coefficient for all industries as done here.

#### 4. Concluding Remarks

The intention of this chapter on the application of multiple tests has been to show how this statistical tool can be used to analyse the nature of possible differences in parameters of structural relations along certain dimensions of a sample. As an illustration simple production and behaviour relations are used with combined cross-section time-series data as the empirical base.

Two methods of testing are tried, neither of which are quite satisfactory, however. The one based on ordinary F-statistics yields an upper limit of the overall test that is rather uninteresting in the case of a high number of tests. This is confirmed by formulas (5) and (6). Through the other one we are, in cases with a high number of tests at least, able to determine a less conservative level of the overall test. On the other hand, it has a basic weakness since it is not very suitable in situations where the number of parameters being tested is much different in the different parts of the testing scheme.

However, even if the methods applied are not quite satisfactory, we have dared to present some illustrations of the application of multiple tests in econometrics. The next chapter provides some additional examples of applications related to the ones presented in this chapter.

#### Notes :

1) Cf. Section V.2.

2) To make the results of the two types comparable we must, in the case of a trend in the capital-elasticity, compute the estimates for the average of  $t$ ,  $\bar{t}$  which is 63 in the computations.

CHAPTER V. ON THE ESTIMATION OF TECHNICAL  
CHANGE; SOME PROBLEMS OF METHOD AND MEASUREMENT

As should be fairly evident from the discussion in the previous chapters the present empirical basis is not particularly well suited for a discussion of the importance and nature of technical change and related issues. In addition to the more general weaknesses, such as errors of measurement and heterogeneous samples etc., there are three specific features which are of particular relevance in this context:

1. The price data applied have some apparent weaknesses that may affect quite strongly the conclusions reached, especially conclusions relating to the degree of technical change for some industries. 2. We have applied a common, and constant depreciation ratio for capital for all industries, and thus do not allow for differences due to the capital mix or the recentness of the capital stock. 3. We have no measure of the degree of utilization of the capital stock.

In spite of these data problems we will explore some issues concerning technical change, since we believe it is possible to shed some light at least on certain aspects of technical change in Norwegian mining and manufacturing by means of the present body of data.

The basic relation of all studies of technical change is, explicitly or implicitly, the production function, and technical change is usually defined in the following way. Having the production function:<sup>1)</sup>

$$(1) \quad Y_t = f_t(X_{1t}, \dots, X_{nt})$$

where  $Y$  is output and  $X_1, \dots, X_n$  are inputs and the index  $t$  denotes period of time, technical change is identified as shifts in the function "f<sub>t</sub>" over time in contrast to movements along the production function due to changes in the factors of production. Thus the nature of technical change can be identified by the way in which "f<sub>t</sub>" shifts.<sup>2)</sup> In this context there are three main problems. First, what is the proper specification of "f"; second, how should the output and the inputs be measured; and third, given a certain functional form of the production relation, what is the proper way of estimating the parameters.

Notes:

- 1) For studies on technical change cf., for example, BROWN (1966): On the Theory and Measurement of Technological Change, and SALTER (1966): Productivity and Technical Change. A recent econometric study that considers a number of different specifications of the nature of technical change is: BECKMANN and SATO (1969): Aggregate Production Functions and Types of Technical Change: A Statistical Analysis.
- 2) Cf. BECKMANN and SATO (1969), op.cit.

In the previous chapters we have used the Cobb-Douglas relation and to some extent the Kmenta relation as approximations to the CES relation. In this chapter we will use the former, but in a particular context a CES relation will be applied as well. By this choice of particular forms of the production function we may, however, have precluded a number of various types of technical change. On the other hand, the prospects for analysing more complex types of technical change by means of the present empirical basis are equally poor as for the analysis of more complex types of production functions. Instead, we use various "types" of Cobb-Douglas and CES relations allowing also for certain types of technical change.

The question of proper estimation of the parameters of a Cobb-Douglas relation is dealt with in Chapter III, and in the first section we attempt to determine the effects of improper parameter estimation on the estimate of the shift in the production function as well as on the estimates of the contributions to growth from labour and capital. In these calculations we use the measure of output, labour and capital as defined in Chapter II.

The use of a cross section of time series in log-linear relations implies a different type of aggregation than the use of pure time-series data of the usual type in that kind of relation. In the second part of the first section we consider how the results of the two methods of aggregation conform.

In the second section of this chapter various approaches are explored to determine the nature of technical change. We are particularly interested in the neutrality/non-neutrality aspect. First, we try a CES relation with factor-augmenting technical change for this purpose. Second, we apply a Cobb-Douglas relation allowing shifts in all parameters, and we use multiple tests related to those applied in the previous chapter to determine the "true" type of shift. Third, we explore the embodiment hypothesis by means of a tentative test. Fourth and finally, we present some calculations with materials entering more explicitly into the production function to ascertain whether this alters the conclusions previously reached concerning the importance and nature of technical change and also to investigate the role of materials in a technical change process. An appendix to this chapter contains some results of tentative calculations concerning two issues, namely transitory variations in demand and costs of change.

## 1. Separating Shifts in from Movements along the Production Function

Accepting the Cobb-Douglas relation, written in logs:

$$(2) \quad y_t = \alpha x_t + \beta z_t + u_t$$

we have the relative rate of growth in output as

$$(3) \quad \dot{y}_t = \alpha \dot{x}_t + \beta \dot{z}_t + \dot{u}_t$$

where  $\dot{x}_t$ ,  $\dot{z}_t$  and  $\dot{u}_t$  are the rates of growth of labour and capital and the change in the residual respectively. We rewrite this relation as

$$(4) \quad v_t = \alpha l_t + \beta k_t + \gamma_t$$

where  $\alpha l_t$  and  $\beta k_t$  are the contributions to growth in output from labour and capital respectively. Together they account for the movements along the production function while  $\gamma_t$  - the residual - represents the shift in the production function.

When trying to calculate these three components of growth and thus also separate the shifts in from movements along the production function we obviously encounter two basic problems:

What are the proper measures of the growth rates of output and the inputs and what is the more proper method for estimating the production function parameters.<sup>1)</sup>

To analyse the effects of biased estimation of parameters and growth rates, we write the contributions to growth of labour and capital in the following way:<sup>2)</sup>

$$(5a) \quad \alpha^{\#} l_t^{\#} = \alpha l_t + \alpha (l_t^{\#} - l_t) + (\alpha^{\#} - \alpha) l_t + (\alpha^{\#} - \alpha) (l_t^{\#} - l_t)$$

$$(5b) \quad \beta^{\#} k_t^{\#} = \beta k_t + \beta (k_t^{\#} - k_t) + (\beta^{\#} - \beta) k_t + (\beta^{\#} - \beta) (k_t^{\#} - k_t)$$

Consequently, we obtain the estimated contribution to growth from "other factors", or the shift in the production function as

$$(6a) \quad \gamma_t^{\#} = \gamma_t - [\alpha (l_t^{\#} - l_t) + (\alpha^{\#} - \alpha) l_t + (\alpha^{\#} - \alpha) (l_t^{\#} - l_t) + \beta (k_t^{\#} - k_t) + (\beta^{\#} - \beta) k_t + (\beta^{\#} - \beta) (k_t^{\#} - k_t)] + (v_t^{\#} - v_t)$$

or

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#### Notes :

- 1) The latter problem is not generally independent of the first inasmuch as "proper estimation" depends, inter alia, on the variable measures applied. (Cf. Chapter III.) Thus the measures may have two effects on the estimated contributions to growth; one direct via the growth rates and one indirect via the impact on the estimates of the factor elasticities.
- 2) For a related discussion of this issue see GRILLICHES (1967): Production Functions in Manufacturing. Some Preliminary Results. In BROWN (ed.), The Theory and Empirical Analysis of Production.

$$(6b) \quad \gamma_t^* = \gamma_t - (\alpha_t^* \lambda_t^* - \alpha_t) - (\beta_t^* k_t^* - \beta_t) + (v_t^* - v_t)$$

On the right side of (5a) the first term is the "true" contribution of growth from labour input; the second term is the bias of the estimate due to biased growth rate estimation of labour input; the third is the bias due to biased estimation of the labour elasticity and the fourth is the cross-effect of biased growth rate and labour elasticity estimation, a term that is zero if any one of the former two biases is zero. The interpretation of (5b.) for capital is similar. (6) tells us that the net effect of these biases has to appear in the estimate of the shift in the production function together with any bias in the measure of the growth rate of output.<sup>1)</sup>

#### a. The Effects of Biased Estimation of Factor Elasticities

In Chapter III we discussed the problem of consistent estimation of factor elasticities. It was shown that the ordinary least square method yields inconsistent estimators for the factor elasticities, while a mixed method with factor share estimation of the labour elasticity and a certain instrumental variable method for the estimation of the capital elasticity yields less biased estimators. We will now consider the results relating to estimated contributions to growth implied by these two methods of estimation.<sup>2)</sup>

The calculations are based on data for the period 1959-1967 as a whole, so that all growth rates (and thus also the various "contributions") are annual averages. By applying the OLS method on the Cobb-Douglas relation with a neutral trend we obtain the average percentage shift per year in the production function directly. In the case of the Klein Wald method the factor elasticities and the average shift cannot easily be estimated simultaneously. Instead, we accept the estimates of the factor

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#### Notes :

- 1) Cf. JORGENSON and GRILICHES (1967): The Explanation of Productivity Change.
- 2) It should be noted that both the OLS and the Klein Wald estimates obtained in the previous chapter are of a more long-run nature and thus they will tend to overstate the contributions to growth from the ordinary factors of production and understate the importance of technical change.

elasticities as previously obtained and estimate the trend from the estimated residual error values of the production function. 1),2)

As shown in Table V.1. we obtain by means of the OLS method a residual trend, or an average annual shift in the production function, of about 3.5 % for Total Mining and Manufacturing and the shift is significant according to a t-test at 5% level. Since the average annual growth in value added for the period under consideration is about 4.8%, we must conclude that according to the OLS results, shifts in the production function account for more than 70% of total growth in output. 3)

For 13 of the individual industries the residual trend is positive and significant, while it is positive but not significant in one case, i.e. Mineral Products. It is significantly negative for one, i.e. Printing. 4),5)

N o t e s :

1) That is, we estimate  $\gamma$  by means of the OLS method on

$y_{it} - \tilde{\alpha}x_{it} - \tilde{\beta}z_{it} = a + \gamma t + u'_{it}$  where  $\tilde{\alpha}$  and  $\tilde{\beta}$  are the Klein Wald estimates on the labour and capital elasticities respectively.

2) This non-symmetric estimation of the trend of the two methods may have some impact on the outcome of the comparison of the two methods' results concerning estimated contributions to growth. Comparing the results in Table V.1 for the factor elasticities when the OLS method is applied with the corresponding results of Table III.1, we find that including a trend the estimate on the capital elasticity becomes generally somewhat lower. It seems, however, to be quite unimportant except for a few industries such as Mining and Quarrying, Pulp and Paper and Non-El. Machinery, but for these industries at least the estimated "biases" presented later in Table V.2. are presumably too large.

3) The growth rates implied by this method of estimating the shift in the production function are unweighted means of the individual production units' growth rates, since they are equal to the regression coefficients  $b_x$  in the relation  $r_{it} = b_r t$  or equivalently  $r_t = b_r t$  where

$$\bar{r}_t = \frac{1}{I} \sum_{i=1}^I r_{it} \quad (r = y, x, z). \quad \text{Cf. Section II.4. and Section V.1.b.}$$

The latter relation is used in the growth rate calculations presented in Table V.2. and thus the standard deviations of the growth rates relate to their variation only over time, and not between establishments. In the growth rate calculations of Section II.4. the former relation is used and thus the standard deviations presented in that section refer to differences of growth rates both over time and between establishments.

4) Since the results are based on individual establishment data, the estimated standard deviation of  $\hat{\gamma}$  contains variations of growth rates between establishments as well.

5) This result for Printing, and probably also the result for Mineral Products, is likely to be caused by errors in the price index for output. Cf. Section II.2.b. and Appendix II.1.

Table V.1. Results for the Cobb-Douglas Relation with Disembodied Technical Change Using the OLS and the Klein Wald OLS Methods of Estimation\*

Industry	OLS				Klein Wald OLS <sup>1)</sup>			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	MSE	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\gamma}$	MSE
Total Mining and Manufacturing	0.730 (0.008)	0.263 (0.007)	0.03511 (0.00225)	0.272	0.603 (0.004)	0.433 (0.004)	0.03036 (0.00233)	0.294
Mining and Quarrying ..	0.756 (0.051)	0.242 (0.040)	0.04762 (0.01081)	0.173	0.569 (0.019)	0.389 (0.013)	0.03864 (0.01081)	0.182
Food Products ...	0.531 (0.026)	0.353 (0.025)	0.05564 (0.00660)	0.425	0.525 (0.012)	0.420 (0.013)	0.05360 (0.00661)	0.430
Textiles ...	0.669 (0.042)	0.278 (0.034)	0.02060 (0.00670)	0.153	0.571 (0.011)	0.380 (0.016)	0.01770 (0.00667)	0.155
Clothing ...	0.924 (0.032)	0.070 (0.024)	0.01935 (0.00583)	0.134	0.600 (0.008)	0.453 (0.021)	0.00826 (0.00690)	0.191
Wood Products ...	0.910 (0.044)	0.180 (0.038)	0.02122 (0.01029)	0.283	0.729 (0.043)	0.418 (0.019)	0.01385 (0.01076)	0.312
Pulp and Paper .....	0.674 (0.029)	0.238 (0.022)	0.06018 (0.00502)	0.147	0.557 (0.008)	0.367 (0.011)	0.05403 (0.00498)	0.153
Printing ...	0.893 (0.030)	0.147 (0.023)	-0.01833 (0.00560)	0.119	0.705 (0.009)	0.336 (0.012)	-0.01955 (0.00593)	0.133
Basic Chemicals ..	0.799 (0.038)	0.183 (0.036)	0.06417 (0.01052)	0.477	0.496 (0.016)	0.527 (0.017)	0.05875 (0.01120)	0.541
Mineral Products ...	0.797 (0.051)	0.314 (0.039)	0.01416 (0.00982)	0.204	0.556 (0.012)	0.520 (0.015)	0.00702 (0.01011)	0.221
Basic Steel.	0.896 (0.053)	0.173 (0.037)	0.05283 (0.00861)	0.183	0.512 (0.011)	0.543 (0.017)	0.04009 (0.00609)	0.237
Metal Products ...	0.823 (0.036)	0.105 (0.034)	0.02960 (0.00689)	0.166	0.620 (0.018)	0.335 (0.016)	0.02151 (0.00706)	0.180
Non-Electrical Machinery ..	1.080 (0.044)	-0.001 (0.020)	0.02991 (0.00789)	0.132	0.641 (0.012)	0.393 (0.016)	0.01282 (0.00893)	0.177
Electrical Machinery ..	0.916 (0.053)	0.100 (0.042)	0.03885 (0.01075)	0.233	0.643 (0.019)	0.464 (0.023)	0.02810 (0.01219)	0.303
Transport Equipment ..	0.980 (0.022)	0.089 (0.021)	0.01767 (0.00531)	0.147	0.772 (0.013)	0.304 (0.008)	0.01645 (0.00565)	0.166
Misc. Products ...	0.555 (0.067)	0.358 (0.054)	0.09661 (0.02136)	0.350	0.614 (0.035)	0.309 (0.042)	0.09561 (0.02109)	0.357

\*  $\alpha$  is the elasticity of labour,  $\beta$  the elasticity of capital and  $\gamma$  the degree of disembodied technical change. MSE is the mean square of the estimated residual error.

1)  $\alpha$  and  $\beta$  are estimated by the Klein Wald method. (Cf. Section III.3).  $\gamma$  is estimated by OLS on the residual  $y_{it} - \tilde{\alpha}x_{it} - \tilde{\beta}z_{it} = a + \gamma t + u_{it}$



Table V.2. Growth Rates for Value Added, Labour and Capital. Estimated Contributions to Growth from Labour, Capital and Technical Change <sup>x</sup>

Industry	Unweighted growth rates			OLS				Klein Wald OLS				OLS-biases		
	v	ℓ	k	$\hat{\alpha}\ell$	$\hat{\beta}k$	$\hat{\gamma}$	$\frac{\hat{\gamma}100}{V}$	$\tilde{\alpha}\ell$	$\tilde{\beta}k$	$\tilde{\gamma}$	$\frac{\tilde{\gamma}100}{V}$	$(\hat{\alpha}-\tilde{\alpha})\ell$	$(\hat{\beta}-\tilde{\beta})k$	$\hat{\gamma}-\tilde{\gamma}^1$
Total Mining and Manufacturing ...	4.79 (0.38)	0.58 (0.30)	3.23 (0.37)	0.42	0.85	3.51	73	0.35	1.40	3.04	63	0.07	-0.55	0.47
Mining and Quarrying .....	4.52 (0.65)	-1.61 (-0.35)	4.05 (0.95)	-1.22	0.98	4.76	105	-0.92	1.58	3.86	85	-0.30	-0.60	0.90
Food Products ...	7.14 (0.89)	0.89 (0.49)	3.13 (0.26)	0.47	1.10	5.56	78	0.47	1.31	5.36	75	0.01	-0.21	0.20
Textiles .....	2.85 (0.77)	-0.01 (0.71)	2.84 (0.73)	-0.01	0.79	2.06	72	-0.01	1.08	1.77	62	-0.00	-0.29	0.29
Clothing .....	1.42 (0.61)	-0.73 (0.58)	2.28 (0.43)	-0.67	0.16	1.94	137	-0.44	1.03	0.83	58	-0.23	-0.87	1.11
Wood Products ...	4.00 (1.57)	1.26 (0.58)	4.06 (0.59)	1.15	0.73	2.12	53	0.92	1.70	1.39	35	0.23	-0.97	0.73
Pulp and Paper ..	5.73 (0.65)	-1.60 (0.35)	3.31 (0.78)	-1.08	0.79	6.02	105	-0.89	1.21	5.40	94	-0.19	-0.93	0.62
Printing .....	-1.52 (0.63)	0.21 (0.38)	0.86 (0.22)	0.19	0.13	-1.83	-	0.15	0.29	-1.96	-	0.04	-0.16	0.13
Basic Chemicals .	6.53 (0.61)	-0.18 (0.56)	1.42 (0.22)	-0.14	0.26	6.42	98	-0.09	0.75	5.88	90	-0.05	-0.49	0.54
Mineral Products.	2.12 (0.60)	-0.33 (0.46)	3.08 (0.38)	-0.26	0.97	1.42	67	-0.18	1.60	0.70	33	-0.08	-0.63	0.72
Basic Steel .....	8.64 (0.64)	2.57 (0.25)	6.11 (0.41)	2.30	1.06	5.28	61	1.32	3.32	4.01	46	0.99	-2.26	1.27
Metal Products ..	5.35 (0.40)	2.21 (0.23)	5.47 (0.91)	1.82	0.57	2.96	55	1.37	1.83	2.15	40	0.45	-1.26	0.81
Non-El. Machinery	3.40 (0.91)	0.38 (0.86)	4.76 (1.03)	0.41	-0.00	2.99	88	0.24	1.87	1.28	38	0.17	-1.87	1.71
El. Machinery ...	7.11 (0.92)	2.96 (0.66)	5.17 (0.67)	2.71	0.52	3.89	55	1.90	2.40	2.81	40	0.81	-1.88	1.08
Transport Equipment .....	3.71 (0.35)	1.78 (0.30)	2.29 (0.12)	1.74	0.20	1.77	48	1.37	0.70	1.65	44	0.37	-0.49	0.12
Misc. Products ..	15.06 (2.20)	6.23 (1.44)	5.40 (1.05)	3.46	1.93	9.66	64	3.83	1.67	9.56	63	-0.37	0.26	0.10

\* All numbers are percentages. For growth rates cf. footnote 1 on p. 136.

1) We should have  $(\hat{\alpha}-\tilde{\alpha})\ell + (\hat{\beta}-\tilde{\beta})k = -(\hat{\gamma}-\tilde{\gamma})$ , but as the right and left side are computed independently there are small differences due to rounding errors.

Apart from the two industries mentioned above the residual trend varies from about 9.7% for Miscellaneous Products to about 1.8% for Transport Equipment. It is also quite high for the three more heavy industries Pulp and Paper, Basic Chemicals and Basic Steel. It is also rather high for Mining and Quarrying and Food Products.

From Table V.2. we learn that there are six industries with a negative unweighted growth rate of labour input over the period considered. For three of these the residual trend is greater than the growth rate of output, namely for Mining and Quarrying, Clothing and Pulp and Paper. For Basic Chemicals, which is also among those with a drop in labour input over time, almost all of the growth in output is accounted for by the residual trend. In fact, only in the case of one industry can movements along the production function explain more than half of the growth in output, namely Transport Equipment for which shifts in the production function account for "only" 48 %.<sup>1)</sup>

These are the main findings concerning shifts in and movements along the production function when the OLS method is applied. Turning now to the Klein Wald OLS method of estimation, we know that the results must be somewhat different. As shown in Chapter III this method as compared to the OLS method yields smaller estimates on the labour elasticity and larger estimates on the capital elasticity.<sup>2)</sup> This must necessarily lead to a generally smaller estimate on technical change when using unweighted growth rates since according to that kind of growth rate capital has grown faster than labour for all but one industry.

We note, however, from the results of the Klein Wald OLS method presented in Table V.1. that for most industries the estimate on technical change is not much lower than the pure OLS estimate. For Total Mining and Manufacturing technical change is 3 % annually while it is 3.5 % according to the OLS method. These represent 63 % and 73 % respectively of the growth rate of output. For a few industries there are, however, quite notable differences between the results obtained by the Klein Wald OLS method and those by the OLS method. For Non-El. Machinery the share of technical change

#### Notes :

- 1) Note, however, that our estimate on technical change for this industry may be biased downwards due to a positive bias in the price index for output used. Cf. Section II.2.b. and Appendix II.1.
- 2) There is also another difference between these two methods of estimation, since the Klein Wald method in general yields a slightly larger estimate on the scale elasticity. The effects of this difference are, however, small.

in the output growth drops from 88 % to 38 %. For Clothing the corresponding percentages are 137 and 58. The drop is also substantial for Mineral Products. On the other hand, the differences are rather unimportant for Food Products, Textiles, Basic Chemicals, Transport Equipment and Miscellaneous Products.<sup>1)</sup> Even though the drop in the estimated shift is quite low or moderate for most industries there are now, at least, seven industries for which movements along the production function account for more than half of the growth in output.

If we believe the latter set of estimates to be a consistent one, we have the OLS biases in the estimated contributions to growth of labour and capital as  $(\hat{\alpha} - \tilde{\alpha})l$  and  $(\hat{\beta} - \tilde{\beta})k$ . They are presented in columns 12 and 13 of Table V.2. As the growth in labour input has been quite low, the bias in the estimated contributions to growth due to inconsistent estimation of the labour elasticity is also fairly low for most industries. It is more important for the two industries Basic Steel and Electrical Machinery, which rank third and second respectively with respect to growth in labour input.<sup>2)</sup>

The bias due to inconsistent estimation of the capital elasticity is generally much more important. This is particularly the case for the industry groups 34-37, or the industries Basic Steel, Metal Products, El.- and Non-El. Machinery.<sup>3)</sup> Thus, it appears that consistent estimation of the factor elasticities is of decisive importance for a correct evaluation of the contributions to growth from labour and capital. In our case it is

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N o t e s :

- 1) According to the conventional t-test at 5 % level we have now that the residual trend is significantly positive for 10 industries, and positive but not significant for 4. This latter group includes Mineral Products, Clothing, Wood Products and Non-El. Machinery. The shift is still negative for Printing.
- 2) The industry that ranks highest, Miscellaneous Products, has a negative bias due to the fact that the "consistent" method of estimation leads to a larger estimate on the labour elasticity than the OLS method.
- 3) At least for Non-El. Machinery this bias may be overstated due to the non-symmetrical estimation of the residual trend. Cf. footnote 2) page 138.

somewhat less important for the problem of separating shifts in from movements along the production function.<sup>1)</sup>

#### b. A Problem of Aggregation

The way in which the calculations of the previous section are carried out implies a particular type of aggregate, namely geometric means. Thus the implied aggregate production function is:

$$(7) \quad \bar{y}_t = \alpha \bar{x}_t + \beta \bar{z}_t + \bar{u}_t$$

$$\text{where } \bar{r}_t = \frac{1}{I} \sum_{i=1}^I r_{it} \quad (r = y, x, z)$$

---

#### Note :

1) There are substantial differences between units concerning the estimated shift in the production function. Various attempts were made to "explain" these differences.

Estimating the shift in the production function for each unit as  $\hat{\gamma}_i = v_i - \tilde{\alpha}x_i - \tilde{\beta}k_i$ , where  $\tilde{\alpha}$  and  $\tilde{\beta}$  are the Klein Wald estimates on  $\alpha$  and  $\beta$  we carried out the following three regressions:

$$1) \quad \hat{\gamma}_i = a_0 + a_1 \hat{\eta}_i$$

where  $\hat{\eta}_i$  is the estimated level of efficiency computed as

$$\hat{\eta}_i = \bar{y}_i - \bar{\alpha}x_i - \bar{\beta}z_i \quad (\bar{r}_i = \frac{1}{T} \sum_{t=1}^T r_{it}, \quad r = y, x, z)$$

$$2) \quad \hat{\gamma}_i = b_0 + b_1 \tilde{\mu}_i + b_2 \hat{\mu}_i$$

where  $\tilde{\mu}_i$  and  $\hat{\mu}_i$  are the level and trend of materials' share in gross production, and correspondingly.

$$3) \quad \hat{\gamma}_i = c_0 + c_1 \tilde{\alpha}_i + c_2 \hat{\alpha}_i$$

For Total Mining and Manufacturing we obtain by OLS:  $\hat{a}_1 = \frac{0.0124}{(0.0064)}$ ,

suggesting that units with high efficiency tend to become more efficient than those with lower efficiency;  $\hat{b}_1 = \frac{0.0739}{(0.0127)}$  and  $\hat{b}_2 = \frac{-1.7275}{(0.1477)}$

suggesting that units with a high share of materials in gross production also have a higher growth in efficiency, while as expected units with a decreasing share have a much higher shift in the value added production function;  $\hat{c}_1 = \frac{-0.0455}{(0.0057)}$  and  $\hat{c}_2 = \frac{-0.3643}{(0.0164)}$  suggesting that some of the differences in the shift in the production function between units are due to the fact that we have imposed the same factor elasticities for all units and for all years when computing  $\gamma_i$ , while there are in fact differences along both dimensions in these elasticities.

Some of the results of the next section of this chapter are related to the results of 2) and 3) presented here.

The aggregate growth rates used above are thus unweighted means of the growth rates of the individual production units;

$$(8) \quad \dot{r}_t = \frac{1}{I} \sum_{i=1}^I \dot{r}_{it} \quad (\dot{r}_t = v_t, l_t, k_t)$$

(7) is, however, a rather unusual aggregate production function. Using instead the more common method of aggregation, namely arithmetic sums over units, we obtain the aggregate growth rates as weighted means of the individual growth rates, i.e.:

$$(9) \quad \dot{r}'_t = \frac{\sum_{i=1}^I R_{it} \dot{r}_{it}}{\sum_{i=1}^I R_{it}} \quad (R = \exp(r), \quad r = y, x, z)$$

These growth rates then correspond to the growth rates we usually obtain from pure time-series data.

If larger production units tend to grow faster than smaller ones the unweighted growth rates  $\dot{r}_t$  will underrate the total growth of  $R_t = \sum_{i=1}^I R_{it}$  in the sample and overrate it if smaller units have the higher growth rates.

This method of aggregation also has some effects on the price index for output. Measuring aggregate real output as  $V_t = \sum_{i=1}^I V_{it}$  and having correspondingly output in current prices as  $V'_t = \sum_{i=1}^I V'_{it}$ , we have an aggregate price index that, to be consistent, must be equal to  $P'_{Vt} = \frac{V'_t}{V_t}$ , which corresponds to using a Paasche price index formula.<sup>1)</sup>

The separation of the price and quantity components when using weighted indices is the same for gross production and materials as for value added. To determine the price movements of gross production and materials for the different sectors in mining and manufacturing, the weighted price indices and their trends are presented in Table V.3.<sup>2)</sup> In Table V.4. the weighted index of value added in constant prices and the corresponding price index are presented, together with the trends of the unweighted and weighted price indices.<sup>3)</sup>

#### Notes :

- 1) This is evident also because the computation of  $V_t$  corresponds to using the Laspeyre quantity index.
- 2) The price indices of gross production and materials for mining and manufacturing are 114 and 110 respectively in 1967 according to the national accounts aggregates. Thus they are somewhat higher than those computed by us for Total Mining and Manufacturing.
- 3) According to the national accounts data the volume and price indices for value added of mining and manufacturing are 139 and 120 in 1967. The volume index is very close to the one we have computed for value added, while the price index is somewhat higher.

Table V.3. Price Indices for Gross Production and Materials (Basis year = 1961)\*

Industry	Gross Production									Materials									Growth rates in % p.a.	
	59	60	61	62	63	64	65	66	67	59	60	61	62	63	64	65	66	67	Gr.Pr.	Mat.s.
Total Mining and Manufacturing	98	98	100	101	100	104	107	109	110	100	100	100	100	100	103	106	107	107	1.54	1.05
Mining and Quarrying ...	101	103	100	101	100	106	112	115	110	105	111	100	100	102	103	104	105	108	(0.16)	(0.20)
Food Products .	102	100	100	105	102	109	110	113	116	103	100	100	102	103	111	112	114	116	1.60	0.09
Textiles .....	95	95	100	102	104	107	108	108	108	99	101	100	100	101	103	100	98	96	(0.43)	(0.46)
Clothing .....	95	95	100	103	105	107	111	113	113	100	97	100	100	101	103	105	105	104	1.84	2.00
Wood Products .	93	94	100	102	100	106	112	114	115	90	94	100	101	101	104	112	112	110	(0.29)	(0.34)
Pulp and Paper.	99	100	100	99	98	100	103	102	101	100	99	100	99	100	101	107	106	105	1.82	-0.29
Printing .....	92	93	100	105	113	119	127	136	145	97	98	100	103	105	106	110	112	115	(0.23)	(0.25)
Basic Chemicals	101	101	100	98	99	103	105	106	99	105	103	100	95	98	104	108	110	104	2.42	0.87
Mineral Products ....	99	95	100	101	101	101	101	102	104	109	108	100	100	101	101	98	96	100	(0.18)	(0.18)
Basic Steel ...	96	99	100	98	91	98	102	103	104	98	99	100	99	97	101	102	104	105	2.80	2.66
Metal Products.	95	96	100	102	102	106	110	111	108	97	100	100	98	96	99	102	103	103	(0.25)	(0.31)
Non-El. Machinery ...	93	92	100	105	105	107	110	114	116	97	97	100	101	99	102	104	105	106	0.36	1.01
El. Machinery .	101	97	100	102	107	108	112	118	116	101	106	100	100	101	104	109	116	113	(0.15)	(0.25)
Transport Equipment ...	97	97	100	105	109	113	120	122	125	95	96	100	102	100	103	107	108	109	5.92	2.15
Miscellaneous Products ....	117	112	100	104	105	105	106	107	109	113	109	100	98	94	98	99	100	98	(0.22)	(0.08)
																			0.29	0.63
																			(0.36)	(0.59)
																			0.80	-1.20
																			(0.21)	(0.35)
																			0.81	0.82
																			(0.45)	(0.21)
																			1.97	0.60
																			(0.21)	(0.27)
																			2.91	1.13
																			(0.27)	(0.14)
																			2.37	1.65
																			(0.31)	(0.44)
																			3.58	1.75
																			(0.19)	(0.17)
																			-0.50	-1.37
																			(0.61)	(0.58)

\* The price index for gross production is computed as  $100 \frac{\sum_{i=1}^n Y_{it}}{\sum_{i=1}^n Y_{it}}$  where  $Y_{it}$  is in current prices and  $Y_{it}$  is in constant prices. The price index for materials is computed correspondingly. Cf. Section II.2.

Table V.4. Volume and Price Indices for Value Added. (Basis year = 1961)

Industry	Volume indices of $V_t = \frac{1}{I} \sum V_{it}$									Price indices computed as $P'_{vt} = V'_t/V_t$									Growth rates in % p.a.	
	59	60	61	62	63	64	65	66	67	59	60	61	62	63	64	65	66	67	$P'_V$ <sup>1)</sup>	$P'_V$
Total Mining and Manufacturing	90	97	100	103	112	120	126	131	138	96	97	100	103	101	105	109	112	114	2.97	2.24
Mining and Quarrying ...	96	100	100	113	121	126	132	136	140	100	101	100	101	100	107	114	117	110	(0.22)	(0.17)
Food Products .	74	80	100	96	105	104	116	127	124	101	101	100	111	101	105	106	110	117	2.87	1.89
Textiles .....	95	107	100	108	107	113	114	112	114	101	88	100	104	107	112	117	120	122	(0.22)	(0.46)
Clothing .....	98	105	100	107	108	114	108	115	123	90	93	100	105	109	113	119	123	123	1.31	1.54
Wood Products .	74	100	100	101	122	131	145	142	138	101	93	100	105	100	108	112	117	124	(0.65)	(0.49)
Pulp and Paper.	100	104	100	98	106	125	134	132	135	98	103	100	97	95	97	93	92	93	4.54	4.17
Printing .....	98	107	100	107	98	109	104	96	96	89	90	100	107	121	130	142	161	179	(0.37)	(0.37)
Basic Chemicals	95	96	100	101	119	132	139	146	161	97	98	100	102	100	100	101	101	93	4.54	4.25
Mineral Products ....	91	105	100	107	112	132	138	138	158	91	87	100	102	101	102	106	111	111	(0.37)	(0.27)
Basic Steel ...	93	106	100	104	120	129	143	145	152	91	100	100	96	83	92	101	101	102	3.68	2.89
Metal Products. Non-El.	99	92	100	111	131	120	119	140	148	93	92	100	106	108	115	119	120	125	(0.53)	(0.53)
Machinery ...	85	99	100	110	109	119	124	117	124	90	87	100	109	110	112	115	125	128	-2.21	-1.11
El. Machinery .	75	94	100	101	111	121	120	121	130	102	90	100	103	113	111	116	120	120	(0.43)	(0.27)
Transport Equipment ...	90	89	100	100	99	105	106	116	132	99	98	100	109	117	124	136	141	147	8.88	9.07
Miscellaneous Products ....	62	72	100	105	105	122	158	155	163	121	115	100	110	117	113	114	115	118	(0.44)	(0.41)
																			(0.60)	(0.38)
																			(0.32)	(0.45)
																			0.81	0.80
																			(0.60)	(0.92)
																			4.05	4.01
																			(0.25)	(0.25)
																			4.76	4.67
																			(0.50)	(0.48)
																			3.03	3.19
																			(0.49)	(0.62)
																			6.78	5.73
																			(0.32)	(0.42)
																			-0.43	0.28
																			(1.16)	(0.26)

1)  $P'_V$  is the implied price index for output when using  $\dot{v}_t = \frac{1}{I} \sum_{i=1}^I \dot{v}_{it}$  as the measure for aggregate output growth. Cf. (8) above.

Table V.5. Weighted Growth Rates for Value Added, Labour and Capital, Estimated Contributions to Growth from Labour, Capital and Technical Change\*

Industry	Weighted growth rates			Contributions to growth				Biases due to un-weighted growth rates		
	v'	ℓ'	k'	$\tilde{\alpha}\ell'$	$\tilde{\beta}k'$	$\tilde{\gamma}'$	$\frac{\tilde{\gamma}'}{v'}100$	$\tilde{\alpha}(\ell-\ell')$	$\tilde{\beta}(k-k')$	$\tilde{\gamma}-\tilde{\gamma}'1$
Total Mining and Manufacturing	5.33 (0.21)	0.70 (0.15)	3.50 (0.41)	0.42	1.52	3.39	64	-0.07	-0.12	-0.35
Mining and Quarrying ...	5.11 (0.38)	-1.50 (0.38)	2.78 (0.59)	-0.85	1.08	4.88	96	-0.07	0.50	-1.02
Food Products ....	6.39 (0.78)	0.93 (0.47)	3.77 (0.04)	0.49	1.58	4.32	68	-0.02	-0.27	1.04
Textiles ....	1.88 (0.46)	-0.50 (0.55)	1.98 (0.71)	-0.29	0.75	1.42	76	0.28	0.33	0.35
Clothing ....	2.31 (0.41)	-0.63 (0.32)	0.93 (0.33)	-0.38	0.42	2.27	98	-0.06	0.61	-1.44
Wood Products ....	7.57 (1.19)	1.69 (0.14)	6.68 (0.81)	1.23	2.79	3.55	47	-0.31	-1.09	-2.16
Pulp and Paper .....	4.54 (0.84)	-1.60 (0.39)	3.42 (0.84)	-0.89	1.26	4.17	92	0.00	-0.05	1.23
Printing ....	-0.53 (0.64)	0.77 (0.46)	3.31 (0.68)	0.54	1.11	-2.18	-	-0.39	-0.82	0.22
Basic Chemicals ...	7.16 (0.57)	0.53 (0.23)	-1.10 (0.28)	0.26	-0.58	7.48	105	-0.35	1.33	-1.60
Mineral Products ....	6.46 (0.61)	0.37 (0.29)	6.39 (0.51)	0.21	3.32	2.93	45	-0.39	-1.72	-2.23
Basic Steel .....	6.36 (0.63)	1.73 (0.16)	6.29 (0.90)	0.89	3.42	2.05	32	0.43	-0.10	-1.96
Metal Products ....	5.50 (0.87)	2.62 (0.26)	5.51 (0.84)	1.62	1.85	2.03	37	-0.25	-0.02	0.12
Non-Electrical Machinery ...	4.19 (0.67)	2.93 (1.14)	3.51 (0.80)	1.88	1.38	0.93	22	-1.66	0.39	0.35
Electrical Machinery ...	5.82 (0.85)	0.28 (0.79)	4.57 (0.27)	0.18	2.12	3.52	61	1.72	0.28	-0.71
Transport Equipment ...	4.15 (0.61)	2.18 (0.28)	2.81 (0.26)	1.68	0.85	1.62	39	-0.31	-0.15	0.03
Miscellaneous Products ....	12.04 (1.24)	6.07 (1.22)	6.14 (0.97)	3.73	1.90	6.41	53	0.10	-0.23	3.15

\* The factor elasticity estimates are those obtained by the Klein Wald method (Cf. Section III.3.d.). For weighted growth rates cf. formula (9) of this chapter.

$$1) \quad \tilde{\gamma} - \tilde{\gamma}' = (v - v') - \tilde{\alpha}(\ell - \ell') - \tilde{\beta}(k - k').$$



For Total Mining and Manufacturing the trend of the weighted price index is somewhat lower than that of the unweighted one, implying that on the average smaller units have a somewhat more rapid price growth than larger ones.<sup>1)</sup> There are, however, substantial differences between industries in this respect, but generally the difference between the two price trends goes in the same direction as for the total.<sup>2)</sup>

In Table V.5. the weighted growth rates of value added in constant prices, labour and capital are presented. By comparing them to the unweighted growth rates presented in Table V.2. we find for Total Mining and Manufacturing that the individual growth rates for all three variables must be positively correlated with their weights, or in other words the level of the corresponding variables. However, since the differences of the weighted and unweighted growth rates go in the same direction for both output and inputs, it has little impact on the relative position of the computed contributions to growth from the three sources, labour, capital and technical change. Technical change accounts for 64 % of the growth in output using the Klein Wald method of estimation, while this percentage is 63 % or approximately the same when using unweighted growth rates.<sup>3)</sup>

Even though it makes relatively little difference what kind of aggregates we use for the total, it makes a substantial difference for some of the individual industries. The more notable differences occur for Clothing, due to substantially lower weighted than unweighted capital growth; Basic Chemicals, which has a negative growth in labour input and a positive growth in capital input when using unweighted growth rates, while the opposite is the case when using weighted growth rates; for Non-El. Machinery, with a substantially higher weighted than unweighted growth rate for labour; and for El. Machinery for which the reverse is true. These differences can also be read from the biases presented in Table V.5.

Notes :

- 1) Cf. (9) above.
- 2) There are also substantial differences between industries with regard to the level of the price trend as to whether this is based on a weighted or an unweighted price index. Some of these differences are, however, presumably a result of the way in which the price indices for output of some of the national accounts sectors are computed. Cf. Section II.2.b. and Appendix II.1.
- 3) In fact the percentages are 63.60 and 63.47 respectively. For labour we have 7.9 and 7.3 and for capital 28.5 and 29.2.

The main conclusion of this section is therefore that when calculating the contributions to growth from labour, capital and technical change for an industry by means of our data, we should use the Klein Wald estimates on the factor elasticities and the weighted growth rates of output and inputs. This seems to be the best we can manage to do. However, as pointed out, even the calculated "contributions" thus obtained are for some industries rather misleading due to problems in separating the price and quantity components of output in current prices.

## 2. On the Nature of Technical Change

Even though the direct results of our regressions based on combined cross-section time-series data may be misleading with regard to the importance of technical change when identified as shifts in the production function, this kind of result may be useful when trying to analyse the nature of technical change. In this second main section of this chapter we will, inter alia, try to analyse the nature of technical change in Norwegian mining and manufacturing through some additional regression results.

In this analysis we will concentrate our efforts on whether technical change is neutral or non-neutral. Adopting the Hicksian definition, we must have that the marginal rate of technical substitution

$$(10) \quad \frac{\frac{\partial V_t}{\partial L_t}}{\frac{\partial V_t}{\partial K_t}} = \frac{m_L}{m_K}$$

is constant over time in the case of neutrality. That is:

$$(11) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = 0$$

where the dots indicate partial derivatives with respect to time.

It is easily shown that this is the case for a Cobb-Douglas relation with a "traditional" residual trend: Technical change is neutral or purely product-augmenting. If (11) is negative, technical change is non-neutral and of the labour-saving type, since the marginal productivity of capital has increased as compared to that of labour. And if (10) is positive we have correspondingly non-neutral and capital-saving technical change.

We try two different approaches to analyse this issue. First, we apply a CES-function, without much success, however. Second, we apply a generalized Cobb-Douglas relation with trends both in the residual and in the factor elasticities. In this context we also try multiple test

procedures to determine the "true" type of shift in the production function.

We also try to investigate the relevance of the embodiment hypothesis by an ad hoc method of testing. Finally, we have a separate sub-section dealing with the role of materials concerning technical change, where we try among other things to ascertain whether technical change has been materials- or value added-saving.

#### a. Technical Change and the CES Relation

Assuming that both labour and capital consist of a quality and a quantity component and that the latter is properly measured by L and K respectively, and denoting the quality components as  $Q_L$  and  $Q_K$ , we have the CES relation:<sup>1)</sup>

$$(12) \quad V_t = \left[ (Q_{Lt} L_t)^{-\rho} + (Q_{Kt} K_t)^{-\rho} \right]^{-\frac{1}{\rho}}$$

Assuming that the quality components grow exponentially over time, we have:

$$(13) \quad V_t = \left[ (Q_{LO} e^{q_L t} L_t)^{-\rho} + (Q_{KO} e^{q_K t} K_t)^{-\rho} \right]^{-\frac{1}{\rho}}$$

and assuming, in addition, that profit is maximized with respect to both factors we get:<sup>2)</sup>

$$(14) \quad \frac{K_t}{L_t} = \frac{Q_{LO}}{Q_{KO}} \left( \frac{S_{Lt}}{S_{Kt}} \right)^{b/(1-b)} e^{(q_L - q_K)t}$$

or in logs:

$$(15) \quad \ln \frac{K_t}{L_t} = \ln \frac{Q_{LO}}{Q_{KO}} + b/(1-b) \ln \left( \frac{S_{Lt}}{S_{Kt}} \right) + (q_L - q_K)t$$

where b is the elasticity of substitution and  $S_L$  and  $S_K$  are the shares in value added of labour and capital respectively.<sup>3)</sup>

The partial relative change over time of the marginal rate of technical substitution is now:

$$(16) \quad \left( \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} \right) = \left[ (b-1)/b \right] (q_L - q_K)$$

#### Notes :

- 1) Cf. DAVID and van de KLUNDERT (1965): Biased Efficiency Growth and Capital-Labour Substitution in the U.S. 1899-1960.
- 2) With constant returns to scale this assumption clearly does not hold true if there is perfect competition in all markets. There are, however, various ways of "saving" this assumption, for example by claiming that the elasticity of scale is in fact below one and therefore (12) is an approximation to the true production function.
- 3) We should note that this relation breaks down if the production function is of a Cobb-Douglas type, that is  $b = 1/(1+\rho) = 1$ .

This implies that if the rate of growth in labour quality is higher than that of capital, technical change is of the labour-saving type provided the elasticity of substitution is below one. Technical change is also labour-saving if the growth rate of the quality of capital is above that of labour and the elasticity of substitution is above one. Thus technical change is capital-saving if the rate of growth in labour quality is higher than that of capital and the elasticity of substitution is above one, or if the growth rate of the quality of capital is above that of labour and the elasticity of substitution is below one.

We will try to determine the sign and size of (16) by estimating the parameters of relation (15). The basic assumptions for obtaining unbiased estimates on the parameters of this relation by means of the OLS method are, however, not fulfilled. By the assumptions made,  $\ln \frac{S_L}{S_K}$  is not an exogenous variable since  $S_L$  is equal to  $\frac{WL}{V}$  and thus  $S_K = 1 - \frac{WL}{V} S_K$  where both  $V$  and  $L$  are endogenous. In addition, the estimate of  $b/(1-b)$  may be distorted by spurious correlation due to errors of measurement in labour quantity input,  $L$ .<sup>1)</sup>

There are various ways to reduce the effects of these errors, and in order to investigate the performance of (15) and try to determine the importance of the errors involved, the OLS method is applied using the following kinds of data:

- a) Pooled cross-section time-series
- b) First differences
- c) Pure time-series

In addition, the Wald method is applied on (15) without a trend, for the pooled cross-section time-series data.<sup>2)</sup> Both  $L$  and  $N$ , the number of employees, are applied as the input measure in all of these four cases.

The results of these experiments can be summarized in the following way. All types of data gave generally negative point-estimates on  $b/(1-b)$ , but not less than minus 1. This implies that the point-estimate on  $b$  is negative. Clearly a negative  $b$  does not make much sense, and as expected the pooled cross-section time-series data gave the poorest results. First differences behaved much better, particularly when  $N$  was applied as the

#### Notes :

- 1) These errors of measurement will tend to bias the estimate on  $b/(1-b)$  downwards, but the magnitude of the bias is not easily determined due to the rather complex way  $L$  enters  $\ln \frac{S_L}{S_K}$ .
- 2) In this case, as well as in the case when first differences were applied, the effect of the  $t$ -variable was computed from the residuals.

labour input measure. Thus it "helps" both to eliminate the cross-sectional level of the variables and to introduce a labour input variable that is measured independently of labour's share. We did not gain anything by using the Wald method on the pooled data, either when L or N was used, as compared to the use of first differences. Finally, pure time-series gave very shaky results, and even for that method the point estimate on b is negative for Total Mining and Manufacturing, when using L as the labour input measure.

However, for those kinds of data where the errors of different kinds are less important (first differences and pure time-series with N as labour input measure),  $b/(1-b)$  is not significantly different from zero at 5 % level for most industries.<sup>1)</sup> This may allow us to conclude that the short-run elasticity of substitution is in fact very low. This is supported by the results of the ACMS relation implied by (12).

$$(17) \quad \ln \frac{V}{L} = a + b \ln W + (1-b)q_L t$$

which for pure time-series data for Total Mining and Manufacturing yields estimates on b of 0.075, and on  $(1-b)q_L$  of 0.0387. However, none of the parameters is significantly positive at 5 % level.

The results for the trend of (15) are not seriously affected by the kind of data applied. Its coefficient is, with a few exceptions, significantly positive. For the pure time-series data when L is applied as the labour input measure we get for Total Mining and Manufacturing  $q_L - q_K = 0.0278$ . This result together with the result of the trend of (17) implies that  $q_L = 4.17\%$  and  $q_K = 1.39\%$ . The total growth of labour and capital input is according to these results  $l + q_L = 0.58\% + 4.17\% = 4.75\%$  and  $k + q_K = 3.23\% + 1.39\% = 4.62\%$  respectively, for Total Mining and Manufacturing and with constant returns to scale these growth rates of "total" factor input account for approximately all of the growth in output.

Our results suggest that technical change is of the labour-augmenting type. All in all, we also have evidence of an elasticity of substitution below one for Total Mining and Manufacturing, even though our results do not allow us to determine more exactly the probable level of that parameter. This implies that the labour-augmenting technical change is also of the labour-saving type. We obtain further evidence for this last finding below.

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N o t e :

1) For pure time-series it was significantly different from zero for none. Clearly, however, the estimate concerned is not very efficient due to the low number of degrees of freedom.

b. Multiple Tests of Types of Shift  
in the Production Function

The calculations in Section 1 of this chapter are based on the assumption of constant factor elasticities. If we adopt the following generalized Cobb-Douglas relation

$$(18) \quad y = (\alpha + \gamma_1 t) x + (\beta + \gamma_2 t) z + u$$

it is possible to study more complex types of shifts in the production function. This will be done by estimating the parameters by means of OLS allowing explicitly, as previously, a trend in the residual as well, i.e.

$$(19) \quad y = (\alpha + \gamma_1 t) x + (\beta + \gamma_2 t) z + \gamma_0 t + u'$$

where  $u'$  is the net residual.

The partial relative change in the rate of technical substitution is now:

$$(20) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = \frac{\gamma_1 \beta - \gamma_2 \alpha}{(\alpha + \gamma_1 t)(\beta + \gamma_2 t)}$$

The type of non-neutrality of technical change is determined by the sign of  $\gamma_1 \beta - \gamma_2 \alpha$ . If it is positive, technical change is capital-saving and if it is negative, technical change is labour-saving. We have neutral technical change if  $\gamma_1 = \gamma_2 = 0$ , or the ordinary Cobb-Douglas case with (or without) a residual trend.<sup>1)</sup>

An analysis of the nature of technical change by means of (19) is not straightforward, however. For example, by the estimates on the parameters of (19) we are almost certain to obtain a value for (20) different from zero, even if we cannot reject the hypothesis of neutral technical change at any reasonable level of significance. Alternatively, we could, before computing (20), reject all trend components with non-significant coefficients. However, using ordinary t-tests for this purpose (as we would normally do) may lead us into statistical problems in cases where there are more than one non-significant coefficient. Thus we use instead a multiple test procedure related to the one discussed in Chapter IV.

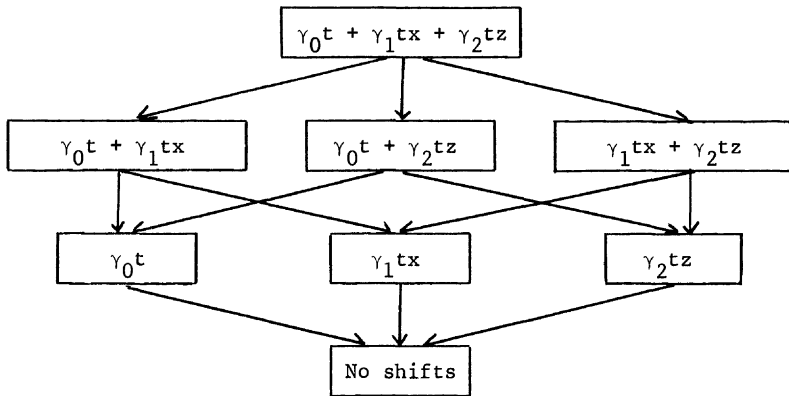
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N o t e :

- 1) There are clearly an infinite number of other parameter values that yield neutral technical change, but if we are to have positive factor elasticities they must satisfy the following condition:  
 $a(\alpha + \gamma_1 t) = \beta + \gamma_2 t > 0$  where  $a$  is any positive number.

Fig. V.1.

A Scheme for Multiple Tests of Types of Shifts in the Production Function



In Fig. V.1. we present the testing scheme to be used. As previously, we start testing from the top with three null-hypotheses (either  $\gamma_0$ ,  $\gamma_1$  or  $\gamma_2$  is zero) and we continue testing, with one-hypotheses, those null-hypotheses which are not rejected. This procedure is continued until we have a one-hypothesis for which all corresponding null-hypotheses are rejected. We accept that type of shift as the "true" one, with a probability of being wrong equal to the level of the multiple test.

In this case, when the number of parameters being tested is the same in all stages, namely one, the ordinary F-statistics method and the Spjøtvoll F-statistics method yield the same results, given roughly comparable levels of the two tests. For the first method we choose 1 % level of the individual tests and thus the corresponding upper limit of the overall test is 12 %. We then obtain the same results as for the second method with level 10 %.<sup>1)</sup>

This is not a robust procedure for analysing the nature of technical change, however, since we obtain a unique "true" type of shift for only 6 of the individual industries in addition to Total Mining and Manufacturing. The results of the testing are presented in Table V.6., where we see that for most industries different types of shifts implying different conclusions concerning the nature of technical change are equivalent or almost equivalent with regard to the fit to the data. Accepting the one that yields the lowest mean square error where there is more than one "true" type, we have in Table V.7.

Note :

1) Cf. Chapter IV.

summarized the findings on the nature of technical change obtained from the multiple test procedure. In this table the results obtained from the Cobb-Douglas relation with the more general type of shift are also presented.

Even though the uniqueness of the results is not too apparent, the results strongly suggest that the shift in the production function is substantially more complex than assumed for the previous computations when analysing the residual factor. Neutrality receives some support from the present computations only for a few industries.

The results for Total Mining and Manufacturing suggest that technical change is of the labour-saving type, but since the results are obtained by means of the OLS method there are some important biases in these results.<sup>1)</sup> We know from Chapter III that the OLS estimates of  $\alpha$  and  $\beta$  are generally biased upwards and downwards respectively. Thus  $\gamma_1\beta - \gamma_2\alpha$  is presumably biased downwards. That is, when the shift is truly labour-saving it is overstated. Also if it is neutral, or in fact capital-saving, we may estimate it to be labour-saving. The denominator of (20) is also biased downwards as the OLS method implies that the product of the factor elasticities, and in our case also their sum, is biased downwards. Apparently this tends to make the biases even more serious, except in the case where the estimated shift in the production function is capital-saving.

As is also shown in Chapter III we can reduce the biases substantially by estimating the labour elasticity by the factor share method and the capital elasticity by OLS. This leads us to try the following relation for Total Mining and Manufacturing:

$$(21) \quad y - \tilde{\alpha}_t x = (\beta + \gamma_2 t)z + \gamma_0 t + u$$

where  $\alpha$  is estimated separately for each year. There is, however, no significant trend in  $\alpha_t$  over time,<sup>2)</sup> and the nature of technical change is therefore completely determined by the sign and significance of  $\gamma_2$ .

$\gamma_2$  comes out to be significantly positive and we thus have evidence for concluding that technical change "on the average" in fact is labour-saving in Norwegian mining and manufacturing industries.

N o t e s :

1) The results for Total Mining and Manufacturing ( $-0.0108 tx + 0.0108 tz$ ) look suspiciously close to what could have been the results of a Kmenta term (cf. Section III.3.g.) since  $t(z-x)$  and  $(z-x)^2$  are likely to be highly correlated. Our results with regard to technical change are, however, not very sensitive to the introduction of a Kmenta term as we then obtain  $-0.0086 tx + 0.0093 tz$ .

(0.0032)      (0.0020)

2) Cf. Table II.9.



Table V.6. Results of Multiple Tests of Types of Shifts in the Production Function over Time

Industry	"True" Types of Shifts in the Production Function <sup>1)</sup>			
Total Mining and Manufacturing ..	-0.0108 tx (0.0032)	+0.0108 tz (0.0020)		
Mining and Quarrying .....	0.0476 t (0.0108)		0.0091 tx* (0.0020)	0.0055 tz (0.0012)
Food Products ..	0.0556 t (0.0066)			
Textiles .....	0.0206 t* (0.0067)		0.0034 tx (0.0012)	0.0021 tz (0.0008)
Clothing .....	0.0703 tx (0.0243)		0.0039 tz* (0.0011)	
Wood Products ..	-0.1117 t (0.0426)	+0.0290 tx* (0.0090)	-0.1586 t (0.0614)	+0.0235 tz (0.0079)
Pulp and Paper .	0.1599 t (0.0394)	-0.0103 tz (0.0040)		
Printing .....	0.0365 tx (0.0115)	-0.0236 tz (0.0067)		
Basic Chemicals.	0.0071 tz (0.0011)			
Mineral Products	No shifts			
Basic Steel ....	-0.0707 tx (0.0201)	+0.0497 tz (0.0127)		
Metal Products .	0.0296 t (0.0069)		0.0054 tx* (0.0012)	0.0035 tz (0.0008)
Non-Electrical Machinery .....	0.0299 t (0.0079)		0.0051 tx (0.0014)	0.0035 tz* (0.0009)
Electrical Machinery .....	0.0389 t (0.0108)		0.0068 tx* (0.0018)	
Transport Equipment .....	0.0177 t (0.0053)		0.0032 tx* (0.0009)	0.0021 tz (0.0006)
Misc. Products .	0.0966 t* (0.0214)		0.0114 tz (0.0027)	

1) When more than one type of shift is reported, the one that has the lowest MSE is marked with an asterisk. Cf. relation (19). (MSE denotes the mean square of the estimated residual error).

Table V.7. The Nature of Technical Change According to the Results Obtained by a Cobb-Douglas Relation with a General and "True" Type of Shift

Industry	Nature of Shift	
	General Type	"True" Type
Total Mining and Manufacturing	Labour Saving	Labour Saving
Mining and Quarrying .....	Capital "	Capital " *
Food Products .....	Labour " 1)	Neutral
Textiles .....	Capital "	Neutral*
Clothing .....	Capital "	Labour Saving*
Wood Products .....	Labour "	Capital " *
Pulp and Paper .....	Capital "	Capital "
Printing .....	Capital "	Capital "
Basic Chemicals .....	Labour "	Labour "
Mineral Products .....	Labour "	No shift
Basic Steel .....	Labour "	Labour Saving
Metal Products .....	Labour "	Capital " *
Non-Electrical Machinery .....	Labour "	Labour " *
Electrical Machinery .....	Capital "	Capital " *
Transport Equipment .....	Capital "	Capital " *
Misc. Products .....	Labour "	Neutral*

\* Not a unique "true" type. Cf. Table V.6.

1)  $\gamma_1^\beta - \gamma_2^\alpha$  is very low, -0.00073, while for Total Mining and Manufacturing, for example, we obtain -0.01310.

c. A Tentative Test of the Embodiment Hypothesis

To some extent the analysis of the nature of technical change is related to the quality components of the inputs and their behaviour over time.<sup>1)</sup> This is also true for the so-called embodiment hypothesis advanced by R. Solow.<sup>2)</sup> The basic idea of this hypothesis is that capital of recent vintages is more productive than capital of older ones, due to technical progress "embodied" in new capital goods.

One way of exploring the validity of the embodiment hypothesis with the kind of data available in the present study is to analyse the performance of variables expressing the recentness of capital. We do this by introducing the following quality or recentness variable into the Cobb-Douglas relation.<sup>3)</sup>

$$(22) \quad E_t = \frac{(1-\Delta)^3 I_{t-3} + (1-\Delta)^2 I_{t-2} + (1-\Delta) I_{t-1}}{K_t}$$

where the numerator expresses what is assumed by us to be left in year t of the most recent three vintages.<sup>4)</sup>

If the embodiment hypothesis is true, i.e. that capital goods of recent vintages are more productive than those of older vintages, it should show up in the results as a significantly positive coefficient of E. There are, however, a number of reasons why this must be a rather weak test. To point out three of the more important ones. First, we have assumed a declining balance depreciation formula to be valid in the computations of the physical deterioration of the capital goods. If, for example, the production

Notes :

- 1) Cf. Section a above.
- 2) Cf. SOLOW (1960): Investment and Technical Progress in ARROW, KARLIN and SAPPORS (editors), Mathematical Methods in Social Sciences. Cf. also BROWN (1966), op.cit., pp. 77-81.
- 3) On the application of such quality variables cf. GRILICHES and RINGSTAD (1971), op.cit., Section III.5. For studies where the embodiment hypothesis is analysed in a similar way cf. BERGLAS (1965): Investment and Technological Change, and KRISHNA (1967), op.cit. See also GRILICHES (1967), op.cit.
- 4) Since embodied technical change is initiated through purchased investment goods, only this category of investments is included in E. We have, however, not included current investments in E. This is done for two reasons. First, incompleted investment projects may be reported while these do not add to the production performance of the capital of that period. Cf. Section II.3.g.ii. Second, current investments may reflect "costs of change", and thus have a negative impact on output. (Cf. Appendix V.1.)

performance of capital goods less than four years old is unchanged, the undepreciated values of  $I_{t-i}$   $i = 1, 2, 3$  should enter the capital measure. If we in that case include  $E$  in the production function with  $K$  as the capital input measure, we may get a significantly positive coefficient of  $E$  even if the embodiment hypothesis is invalid. Second, we must be aware of the poor quality of the data. There is a rather high portion of  $E_t = 0$  in our sample and this is likely to be mainly a result of bad reporting. To avoid distorted estimates on the coefficient of  $E$  due to this fact, we introduce the following dummy-variable into the production function together with  $E$ .

$$(23) \quad F_E = \begin{cases} 1 & \text{where } E = 0 \\ 0 & \text{where } E > 0 \end{cases}$$

We will expect the coefficient of  $F_E$  to be negative for two reasons: First, because poorly managed units are likely to have a poor quality in the reporting of their activities, and second because  $F_E$  picks up stagnant and presumably less efficient units. If the embodiment hypothesis is true, there is a third reason, since we should then have a negative coefficient of  $F_E$ .

The first two reasons we have for expecting a negative coefficient of  $F_E$  suggest, however, that there may be difficulties in the interpretation of  $E$  itself;  $E$  may be positively correlated with the establishment-specific components of the error means and thus our results will be likely to overstate the importance of recent vintages in the capital stock.<sup>1)</sup> This is the third and probably the more important reason why our test of the embodiment hypothesis must be rather tentative.

All in all, the interpretation of the results of both  $E$  and  $F_E$  is rather difficult. On the other hand, they are both related to misspecifications and mismeasurement and as such, an analysis of their performance and effects is interesting. There are three aspects of these variables we would like to investigate: First, whether their effect on output is the one expected and whether their coefficients are significant, that is, whether the coefficients of  $E$  and  $F_E$  are significantly positive and negative respectively. Second, we would like to examine whether  $E$  in particular leads to a reduction of the residual trend, or the disembodied technical change; and third, whether their presence in the production function leads to substantially different estimates on the factor elasticities. In the case of the first question not much more can be said

N o t e :

1) Cf. Section III.2.

Table V.8. Results of Cobb-Douglas Relations with Embodied and Disembodied Technical Change.\*

Variable	z-x	x	t	MSE	z-x	x	E	F <sub>E</sub>	MSE
Total Mining and Manufacturing	0.280 (0.009)	-0.007 (0.006)	0.0382 (0.0042)	0.282	0.285 (0.009)	-0.021 (0.006)	0.213 (0.038)	-0.105 (0.025)	0.283
Mining and Quarrying ...	0.211 (0.050)	0.022 (0.026)	0.0613 (0.0202)	0.182	0.245 (0.053)	0.026 (0.028)	0.091 (0.184)	0.114 (0.128)	0.193
Food Products ....	0.354 (0.031)	-0.137 (0.018)	0.0579 (0.0125)	0.448	0.335 (0.033)	-0.160 (0.019)	0.046 (0.093)	-0.231 (0.069)	0.452
Textiles ....	0.253 (0.043)	-0.065 (0.028)	0.0158 (0.0128)	0.162	0.253 (0.043)	-0.111 (0.031)	0.921 (0.192)	-0.119 (0.085)	0.149
Clothing ....	0.056 (0.033)	0.027 (0.027)	0.0263 (0.0113)	0.147	0.050 (0.033)	-0.027 (0.030)	0.100 (0.126)	-0.213 (0.065)	0.144
Wood Products ....	0.196 (0.045)	0.134 (0.031)	0.0154 (0.0199)	0.308	0.191 (0.044)	0.109 (0.033)	0.663 (0.185)	-0.003 (0.092)	0.294
Pulp and Paper .....	0.178 (0.023)	-0.105 (0.014)	0.0701 (0.0078)	0.108	0.221 (0.025)	-0.129 (0.017)	0.060 (0.077)	-0.067 (0.048)	0.121
Printing ....	0.098 (0.027)	0.050 (0.020)	-0.0117 (0.0100)	0.109	0.095 (0.027)	0.050 (0.020)	-0.116 (0.063)	-0.014 (0.060)	0.109
Basic Chemicals ...	0.235 (0.044)	-0.016 (0.023)	0.0625 (0.0189)	0.449	0.225 (0.044)	-0.052 (0.024)	0.130 (0.238)	-0.397 (0.098)	0.440
Mineral Products ....	0.318 (0.046)	0.145 (0.027)	0.0188 (0.0178)	0.169	0.314 (0.044)	0.152 (0.027)	0.710 (0.168)	0.238 (0.161)	0.182
Basic Steel .	0.234 (0.045)	0.004 (0.035)	0.0653 (0.0158)	0.181	0.264 (0.046)	-0.008 (0.037)	0.278 (0.106)	-0.149 (0.311)	0.189
Metal Products ....	0.123 (0.041)	-0.053 (0.021)	0.0244 (0.0120)	0.149	0.176 (0.041)	-0.032 (0.021)	0.567 (0.145)	0.241 (0.072)	0.143
Non-El. Machinery ...	0.016 (0.049)	0.078 (0.027)	0.0388 (0.0155)	0.151	0.073 (0.051)	0.045 (0.030)	0.468 (0.198)	0.146 (0.126)	0.150
Electrical Machinery ...	0.052 (0.056)	0.041 (0.039)	0.0438 (0.0214)	0.271	0.037 (0.059)	0.060 (0.043)	-0.294 (0.240)	-0.004 (0.272)	0.276
Transport Equipment ...	0.079 (0.027)	0.077 (0.012)	0.0263 (0.0100)	0.152	0.102 (0.028)	0.083 (0.014)	0.359 (0.122)	0.123 (0.056)	0.152
Miscellaneous Products ....	0.387 (0.068)	-0.160 (0.048)	0.0793 (0.0362)	0.297	0.358 (0.076)	-0.158 (0.049)	1.295 (0.533)	0.234 (0.242)	0.296

\* The coefficient of z-x is the elasticity of capital and the coefficient of x is  $\epsilon-1$  where  $\epsilon$  is the elasticity of scale. E and F<sub>E</sub> are defined in (22) and (23). MSE is the mean square of the estimated residual error. Method of estimation: Ordinary least squares.

Table V.8 (cont.), Results of Cobb-Douglas Relations with Embodied and Dis-embodied Technical Change.\*

Variable Industry	z-x	x	t	E	F <sub>E</sub>	MSE	Number of F <sub>E</sub> = 1 in %
Total Mining and Manufacturing	0.280 (0.009)	-0.021 (0.006)	0.0379 (0.0042)	0.216 (0.038)	-0.100 (0.024)	0.279	12.2
Mining and Quarrying .....	0.223 (0.052)	0.028 (0.028)	0.0608 (0.0203)	0.109 (0.179)	0.091 (0.125)	0.184	12.8
Food Products .....	0.319 (0.032)	-0.162 (0.018)	0.0603 (0.0125)	0.034 (0.092)	-0.250 (0.068)	0.442	14.3
Textiles .....	0.244 (0.044)	-0.111 (0.031)	0.0159 (0.0123)	0.914 (0.192)	-0.125 (0.085)	0.149	11.2
Clothing .....	0.043 (0.033)	-0.023 (0.027)	0.0252 (0.0111)	0.117 (0.125)	-0.204 (0.064)	0.143	13.4
Wood Products .....	0.188 (0.045)	0.109 (0.033)	0.0137 (0.0194)	0.661 (0.185)	-0.003 (0.092)	0.294	23.0
Pulp and Paper .....	0.191 (0.024)	-0.111 (0.016)	0.0697 (0.0079)	0.070 (0.072)	-0.019 (0.045)	0.108	12.9
Printing .....	0.096 (0.027)	0.050 (0.020)	-0.0128 (0.0099)	-0.120 (0.063)	-0.013 (0.060)	0.108	9.8
Basic Chemicals .....	0.227 (0.043)	-0.051 (0.024)	0.0543 (0.0186)	0.128 (0.236)	-0.369 (0.097)	0.432	16.4
Mineral Products .....	0.308 (0.044)	0.153 (0.027)	0.0153 (0.0172)	0.704 (0.169)	0.232 (0.161)	0.183	4.2
Basic Steel .....	0.252 (0.045)	-0.009 (0.035)	0.0654 (0.0156)	0.283 (0.102)	-0.026 (0.303)	0.177	0.8
Metal Products .....	0.169 (0.041)	-0.034 (0.021)	0.0233 (0.0117)	0.560 (0.145)	0.240 (0.072)	0.142	10.3
Non-El. Machinery .....	0.053 (0.051)	0.052 (0.030)	0.0370 (0.0154)	0.476 (0.196)	-0.115 (0.125)	0.147	5.9
Electrical Machinery .....	0.027 (0.059)	0.061 (0.043)	0.0448 (0.0214)	-0.309 (0.238)	0.015 (0.270)	0.271	2.0
Transport Equipment .....	0.099 (0.028)	0.083 (0.014)	0.0266 (0.0099)	0.361 (0.122)	0.124 (0.056)	0.150	17.1
Miscellaneous Products .....	0.369 (0.074)	-0.153 (0.048)	0.0791 (0.0370)	1.141 (0.525)	0.359 (0.243)	0.282	9.0

\* See page 160.

a priori. For the second aspect it is reasonable to believe that since there is presumably little variation of  $E$  along the time-dimension, it cannot pick up much of the effect of  $t$ ; in our sample the coefficient of  $E$  must be determined mainly by the establishment dimension. For the third aspect one can at least "predict" that if the coefficient of  $E$  has the expected sign, the estimate on the capital elasticity will be lower, since there is then a positive effect of parts of the capital stock in addition to the "main" capital input variable.

Turning now to the empirical findings we should note that due to the way in which our recentness variable is constructed we "lose" one third of the degrees of freedoms available. Thus to carry out a complete analysis of the effects of  $E$  and  $F_E$  we re-run the Cobb-Douglas relation with purely disembodied technical change for the truncated sample. In addition, the results of two other regressions are presented, where only the recentness variables are included together with the ordinary factors of production, and where they are included together with the residual trend. The results of these three regressions are presented in Table V.8.<sup>1)</sup>

By comparing the results of the first regression of Table V.8. with the first one in Table V.1. we get an impression of the effects of the sample truncation, since the first one is based on data for the years 1962-67 while the second is based on data for the entire period 1959-1967. The main difference between the two sets of results is that the trend seems to be of greater importance for the truncated sample, suggesting that the trend is not constant but increasing over time. This effect is more notable for Mining and Quarrying, Pulp and Paper, Basic Steel, Non-El. Machinery and Transport Equipment, or generally rather heavy industries. The level of the capital elasticity is somewhat reduced for Total Mining and Manufacturing, but this is "compensated" for by a bigger elasticity of labour.

N o t e :

- 1) Another regression was also run, namely  $y - \tilde{\alpha}x - \tilde{\beta}z = a + \gamma t + \mu_1 E + \mu_2 F_E$ , where  $\alpha$  and  $\beta$  are the Klein Wald estimates obtained from the complete sample. (Cf. Chapter III.) This relation provides a test of the performance of the trend and the recentness variables when imposing presumably more consistent estimates on the factor elasticities than those implied by the OLS method. This approach, however, does not take care of the sample truncation, nor the possible effects of the technical change variables on the estimates of the factor elasticities. In spite of this the relation above did not yield results for  $E$  and  $F_E$  basically different from those obtained by means of the OLS method. For the effects of  $t$  of consistent estimation of the factor elasticities, cf. Section 1.a. above.

The results of the second regression tell us that, at least for Total Mining and Manufacturing, the coefficients of both  $E$  and  $F_E$  have the expected signs and that they are both significant at conventional levels. Thus the embodiment hypothesis seems to receive some support from these results. The findings are not equally uniform for any of the individual industries, however. The coefficient of  $E$  is significantly positive for eight of the fifteen industries. The coefficient of  $F_E$  is significantly negative for only three, and none of these is among those with a significantly positive coefficient of  $E$ .<sup>1)</sup> On the other hand, for two industries, Metal Products and Transport Equipment, we have the rather strange result that both coefficients are significantly positive.

The third regression of Table V.8. tells us that our variables expressing embodied and disembodied technical change are largely independent. When compared to the results of the first regression we see that the residual trend is approximately of the same magnitude, and when compared to the results of the second regression we can conclude that the estimates of the recentness variables are also virtually unaffected by introducing a trend.

This confirms our a priori "predictions" of the results. Our recentness variables are, as pointed out, mainly determined by the establishment dimension and they therefore work more or less like dummy-variables for establishments. This is probably also the main reason why the labour elasticity seems to be more affected by these variables than the capital elasticity. The former is almost solely determined by the establishment dimension while for the latter the time dimension is of somewhat greater importance.<sup>2)</sup> All in all, however, our recentness variables do not have any serious impact on the estimates of the factor elasticities. Thus the fact that we have ignored them in the previous analysis of the levels of these parameters does not make this analysis basically invalid.<sup>3)</sup>

N o t e s :

- 1) The last column of Table V.8. shows the percentage of observations with  $F_E = 1$  (or  $E = 0$ ). We note that this percentage varies widely between industries, also suggesting that the quality of the reporting is substantially different. We should also note that since  $E$  covers a period of three years, the percentages of zeros reported on purchased investment goods are much higher than those presented in Table V.8.
- 2) Cf. Tables II.4. and II.6.
- 3) In a similar manner as for the embodiment hypothesis, attempts were made to investigate two other hypotheses, namely "costs of change" and "transitory variation in demand". The results were rather inconclusive with regard to the importance and validity of these hypotheses. On the other hand, we found, as for the embodiment hypothesis, that the results of the "main factors" were virtually unchanged. A summary of these computations is presented in Appendix V.1.



The main conclusion of this section is therefore that the embodiment hypothesis seems to gain some support in our data. However, the introduction of variables taking care of the quality component of capital has little impact on the main production function parameters; nor do they affect the residual trend significantly.

d. Technical Change and the Role of Materials

Basically there are three factors of production (or rather three groups of factors) in operation when manufacturing a final product, namely labour, capital and materials. The treatment of these is, however, generally rather asymmetrical, since the last one is usually subtracted from output to obtain a net output measure, value added.<sup>1)</sup>

So far this approach has also been adopted in this study. However, in this section we will analyse whether a more symmetrical treatment of the three factors of production leads to different conclusions concerning the importance and nature of technical change. Most of the analysis is carried out for Total Mining and Manufacturing only.

Since we have previously mainly used a Cobb-Douglas type of production function in labour and capital, one obvious way of treating all factors symmetrically is to adopt a three-factor Cobb-Douglas relation with gross production as the output measure. Assuming purely neutral technical change we have:

$$(24) \quad g = \alpha'x + \beta'z + \mu m + \gamma't + u'$$

where  $g = \ln Y$ ,  $m = \ln M$ ,  $\alpha'$ ,  $\beta'$ ,  $\mu$  are the gross production elasticities of labour, capital and materials respectively,  $\gamma'$  is the rate of technical change and  $u'$  is an error term tentatively assumed to be distributed randomly with zero mean, constant variance and no serial correlation.

Rewriting (24) as

$$(25) \quad g - x = (\alpha' + \beta' + \mu - 1)x + \beta'(z - x) + \mu(m - x) + \gamma't + u'$$

we obtain for Total Mining and Manufacturing by means of OLS:

$$(26) \quad g - x = -0.055x + 0.132(z - x) + 0.491(m - x) + 0.01764t$$

(0.033)    (0.005)            (0.003)            (0.00141)    MSE = 0.107

Both the labour and the capital elasticities as well as the residual trend are much lower for this relation than those obtained for the value added Cobb-Douglas relation.<sup>2), 3)</sup> However, since we now have a different output measure, these are not comparable.

N o t e s :

1) Cf. GRILICHES and RINGSTAD (1971): Op.cit., Chapter V.

2) The estimate on  $\alpha'$  implied by the estimates of (26) is 0.322.

3) Cf. Table V.1.

We can obtain some kind of comparability by rewriting (24) as:

$$(27) \quad g = (\alpha^x x + \beta^x z + \gamma^x t)(1-\mu) + \mu m + u'$$

where

$$(28) \quad s^x = s'/(1-\mu) \quad (s = \alpha, \beta, \gamma)$$

and we obtain a geometric value added Cobb-Douglas relation as<sup>1)</sup>

$$(29) \quad (g-\mu m)/(1-\mu) = \alpha^x x + \beta^x z + \gamma^x t + u^x$$

or writing it in a way that corresponds to (25):

$$(30) \quad (g-x-\mu(m-x))/(1-\mu) = (\alpha^x + \beta^x - 1)x + \beta^x(z-x) + \gamma^x t + u^x$$

There are now two ways of estimating the parameters on the right side of this relation: either by using the results of (25) together with (28) or by an independent estimate on  $\mu$  using (30).<sup>2)</sup>

The former yields  $\hat{\alpha}^x = 0.634$ ,  $\hat{\beta}^x = 0.259$  and  $\hat{\gamma}^x = 0.03466$ , while the latter using materials share in gross production as an estimate on  $\mu$ ,<sup>3)</sup>  $\hat{\mu} = 0.520$ , implies the following results by OLS on (30)

$$(31) \quad (g-m-\mu(m-x))/(1-\mu) = -0.111x + 0.243(z-x) + 0.03509t$$

(0.011)    (0.009)            (0.00295)            MSE = 0.470

Thus the importance of technical change measured by the residual trend is approximately the same whether ordinary value added or the geometric value added measure is applied. In the case of the factor elasticities, however, there is a striking difference between the results obtained by means of (30) and the ordinary value added relation. The estimate on  $\alpha^x$  implied by (31) is 0.646, while  $\hat{\beta}^x$  is as we note 0.243. The corresponding estimates from the ordinary value added relation are 0.730 and 0.263. Thus, the labour elasticity in particular is substantially lower when using the geometric value added measure. This also implies as we see from (31) that we have significantly decreasing returns to scale.<sup>4), 5)</sup>

#### N o t e s :

- 1) Cf. DOMAR (1961): On the Measurement of Technological Change.
- 2) We could also clearly estimate all parameters of (30) simultaneously using a non-linear estimation method.
- 3) Cf. Table V.9.
- 4) This finding is quite different from the one obtained in GRILICHES and RINGSTAD (1971), op.cit., where almost the same estimate on the scale elasticity was obtained for Total Manufacturing when using the geometric value added measure as when using the ordinary value added measure. Cf. Chapter V.
- 5) Constraining the labour elasticity to its share in (ordinary) value added, and using the size-dummies-instrumental variable method to estimate  $\beta^x$  leaves the estimate on the scale elasticity virtually unchanged. We obtained  $\hat{\beta}^x = 0.304$ , and since  $\hat{\alpha} = 0.603$  we have the estimate on the scale elasticity as 0.907 as compared to 0.889 obtained by OLS on (30).

Even when using the geometric value added measure it may be convenient, as done here, to use the share of labour in ordinary value added as an estimate on  $\alpha^x$ . Alternatively, we could have used  $\tilde{\alpha}^x = S_{LY}/(1-S_{MY})$  where  $S_{LY}$  and  $S_{MY}$  are the shares of labour and materials respectively in gross production.

The virtually unchanged trend estimate together with reduced factor elasticity estimates implies that when using the geometric value added measure, shifts in the production function account for a higher fraction of growth in output than when using the ordinary value added measure. The growth rate of the geometric value added measure is 4.67 % as compared to 4.79 % for ordinary value added. In the present case movements along the production function account for 1.15 % or 25 % of the growth in output, while it was 1.27 % or 27 % of the growth where ordinary value added was applied.<sup>1)</sup>

Therefore, it is misleading to conclude from the results in (26) that we succeed in explaining more of the growth in net output by means of movements along the production function by treating materials as a factor of production in the same way as the other two factors. It is true, of course, that the shifts in the production function are less important both absolutely and relatively for the gross production function with all three factors of production than for the ordinary value added relation. The point is, however, that the importance of the shift of the value added relation implied by our gross production function is equally large or larger than for the ordinary value added relation.

This conclusion is obtained, however, by assuming the elasticity of materials to be constant over time. We have in the above estimated this elasticity by materials' share in gross production, and the calculations presented in Table II.10. suggest that this share is decreasing over time for most industries. In Table V.9. we present the factor share estimates on the elasticity of materials and the trend in the share of materials over time. There is a negative and significant trend for eight of the individual industries as well as for Total Mining and Manufacturing. This suggests that for the industries concerned there is a kind of non-neutral technical change. We will return to this issue later.

When  $\mu$  is constant over time, we have the growth rate of geometric value added as  $(\dot{g} - \mu \dot{m}) / (1 - \mu)$  where  $\dot{g}$  and  $\dot{m}$  are the growth rates of gross production and materials respectively. Allowing a trend in  $\mu$  we have the growth rate as:

$$(32) \quad (\dot{g} - \mu \dot{m}) / (1 - \mu) + (\dot{\mu} / (1 - \mu)^2) (g - m)$$

which obviously must be lower than the one previously computed when  $\dot{\mu} < 0$ .

Note:

- 1) The use of a presumably more consistent method of estimation leads to a greater difference. Accepting the Klein Wald estimates referred to in footnote 5) page 165, we have that 29 % of the growth in output can be explained by movements along the production function as opposed to 37 % when ordinary value added is applied.

Tabel V.9. Level and Trend of Factor Share Estimates on the Elasticity of Materials\*

Industry	$\tilde{\mu}$	OLS on $\tilde{\mu} = a + bt$	
		$\hat{b}$	MSE <sup>1)</sup>
Total Mining and Manufacturing .....	0.520 (0.002)	-0.00387 (0.00039)	0.001
Mining and Quarrying .....	0.161 (0.007)	-0.00603 (0.00202)	0.024
Food Products .....	0.678 (0.006)	-0.00572 (0.00071)	0.003
Textiles .....	0.500 (0.006)	-0.00983 (0.00255)	0.039
Clothing .....	0.496 (0.007)	-0.00893 (0.00111)	0.007
Wood Products .....	0.629 (0.007)	-0.00513 (0.00179)	0.019
Pulp and Paper .....	0.659 (0.004)	0.00023 (0.00134)	0.011
Printing .....	0.359 (0.006)	-0.00285 (0.00152)	0.014
Basic Chemicals .....	0.498 (0.008)	-0.00095 (0.00108)	0.007
Mineral Products .....	0.340 (0.010)	-0.00385 (0.00124)	0.009
Basic Steel .....	0.545 (0.010)	0.00037 (0.00117)	0.008
Metal Products .....	0.469 (0.007)	-0.00832 (0.00189)	0.021
Non-Electrical Machinery .....	0.440 (0.009)	-0.00192 (0.00173)	0.018
Electrical Machinery .....	0.500 (0.008)	0.00148 (0.00138)	0.011
Transport Equipment .....	0.414 (0.006)	-0.00023 (0.00075)	0.003
Misc. Products .....	0.517 (0.011)	-0.00717 (0.00116)	0.008

\* The elasticity of materials is estimated as  $\tilde{\mu} = \frac{1}{IT} \sum_{i=1}^I \sum_{t=1}^T \left( \frac{M'}{Y'} \right)_{it}$  and the standard error of this estimate is estimated as  $\frac{1}{\sqrt{IT}} \hat{\sigma}_{\frac{M'}{Y'}}$ . Cf. Table II.10 and Section III.3.a. The trend is computed for  $\tilde{\mu}_t = \frac{1}{I} \sum_{i=1}^I \left( \frac{M'}{Y'} \right)_{it}$ . MSE is the mean square of the estimated residual error.

1) Multiply these entries by  $10^{-2}$ .

For Total Mining and Manufacturing we have found that  $\tilde{\mu} = 0.520$ ,  $\tilde{\mu} = 0.00387$  and  $\bar{g} - \bar{m} = 0.7756$  and thus the mean of the last term of (32) is -1.30 %. And since the first term was found to be 4.67 % we obtain (the mean of) the growth rate for geometric value added allowing a trend in the elasticity of materials as 3.37 %.

Using this "new" geometric value added measure in (30) we obtain:

$$(33) \quad (g - m - \tilde{\mu}_t(m-x)) / (1 - \tilde{\mu}_t) = \begin{matrix} -0.111x & + & 0.244(z-x) & + & 0.02200t \\ (0.006) & & (0.009) & & (0.00294) \end{matrix}$$

MSE = 0.468

Thus the labour and capital elasticities are unaffected by this new output measure, while the trend as could be expected is lower. This implies that the relative position of the trend is also somewhat reduced. Now it accounts for 65 % of the growth in output while using the previous geometric value added measure it accounted for 75 %. All in all, however, the results obtained concerning shifts in and movements along the production function are not much different when using the geometric value added measures and ordinary value added.

As pointed out, the trend in the factor share estimate on the elasticity of materials suggests that technical change is non-neutral. If the trend is significantly negative as it came out to be for eight of the individual industries, we have evidence of value added using or materials-saving technical change. In the last part of this section we would first like to explore whether the OLS results for Total Mining and Manufacturing on a three factor production function with trends in all coefficients support this finding. Second, we would like to investigate whether we obtain the same results concerning the nature of technical change as previously, namely that it is labour-saving.

We have now:

$$(34) \quad g = (\alpha + \gamma_1 t)x + (\beta + \gamma_2 t)z + (\mu + \gamma_3 t)m + \gamma_0 t + u'$$

The partial relative change over time in the rates of technical substitution between labour and capital and capital and materials is:

$$(35) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = \frac{\gamma_1 \beta - \gamma_2 \alpha}{(\alpha + \gamma_1 t)(\beta + \gamma_2 t)}$$

$$\frac{\dot{m}_K}{m_K} - \frac{\dot{m}_M}{m_M} = \frac{\gamma_2 \mu - \gamma_3 \beta}{(\beta + \gamma_2 t)(\mu + \gamma_3 t)}$$

Estimating the parameters of (34) by means of ordinary least squares for Total Mining and Manufacturing we have that<sup>1)</sup>

N o t e :

1)  $\gamma_3$  is significantly negative at 1 % level, while  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  are positive but not significant at that level.

$$(36) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -0.02472$$

$$\frac{\dot{m}_K}{m_K} - \frac{\dot{m}_M}{m_M} = 0.03235$$

which implies that

$$(37) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_M}{m_M} = 0.00763$$

Using a multiple test procedure similar to the one used previously for the value added Cobb-Douglas relation we have the "true" type of shift as:<sup>1)</sup>

$$(38) \quad \begin{array}{cc} 0.00475tz & - 0.00295tm \\ (0.00109) & (0.00106) \end{array}$$

This yields:

$$(39) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -\frac{\dot{m}_K}{m_K} = -0.00601$$

$$\frac{\dot{m}_K}{m_K} - \frac{\dot{m}_M}{m_M} = 0.04208$$

And thus:

$$(40) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_M}{m_M} = -\frac{\dot{m}_M}{m_M} = 0.03607$$

Thus, the previous finding of value added using or materials-saving technical change is supported by these direct production function regression results. They also suggest that technical change is more labour-saving than capital-saving, thus supporting the findings of Section V.2.b. The latter finding is also supported by the results obtained by means of (34) when constraining the elasticity of materials to its share in gross production. Assuming it to be constant over time we find by means of unconstrained estimation of the trend parameters  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  that

$$(41) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -0.00239$$

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N o t e :

1) Cf. Section V.2.b. and Chapter IV. The number of individual tests is 20 and thus we get an upper limit of the level of the overall test when using the ordinary F-statistics method of 20 % as we choose a level of the individual tests of 1 %. The Spjøtvoll F-statistics method yields the same result as the other one both when choosing a level of 10 % and of 25 %. (A tabulation of the upper 20 % fractiles of the F-distributions was not available.)

Allowing materials share to vary over time we have that unconstrained estimation of  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  yields

$$(42) \quad \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} = -0.00421$$

All in all, there is sufficient evidence for concluding that treating all three factors symmetrically does not alter the main findings obtained previously concerning the importance and nature of technical change at the value added level.

### 3. Concluding Remarks

As pointed out at the beginning of this chapter, there are certain shortcomings in the data which make our analysis rather tentative. First, the manner in which the price index for output is computed for some industries makes it impossible to determine the role of technical change. This is apparent in our results for Printing, but obviously the results for other industries are also affected. Second, since the price of capital refers to prices of new capital goods we may tend to underrate the growth in capital stock over time. Third, since we have used a depreciation rate and a price index for capital common for all units, there may be differences in the growth rate of capital between industries which we have not taken into account. Fourth and finally, we know nothing about the variation in the capacity utilization of the production units.

Thus serious objections could be raised against the validity of most of our findings in this chapter. However, they are the best answers we are able to provide on the issues raised, by means of the data available.

Appendix V.1. Tentative Tests of Transitory Variation in Demand and Costs of Change.

Basically the two issues considered in this appendix have to do with proper specification of the model. We have made a number of simplifying assumptions when constructing the model subject to analysis. This is primarily a result of "empirical necessity" since the possibilities for investigating empirically the performance of more complex models are quite limited.

If possible, we would have analysed the importance of transitory variation in demand and costs of change by means of a model specification taking them explicitly into account. Instead, we have to rely on an ad hoc procedure of the same kind as the one applied in the analysis of the embodiment hypothesis in section V.2.c, namely by adding presumably relevant variables to the production relation and estimating the parameters of that relation by ordinary least squares.

Two aspects of these variables are of particular interest: first, whether their coefficients have the expected signs and are significant, and second whether their presence in the production function alters the estimates on the main coefficients. In a sense the latter aspect is the more important as it indicates the seriousness of the specification errors in the main model due to the presence of any transitory variations in demand or of costs of change.

a. On Transitory Variation in Demand

We may expect that the establishments have adjusted themselves to what they consider to be the normal or "permanent" demand for goods.<sup>1)</sup> The actual demand may, however, show short-run variation which is not easily predictable. To some extent inventories can serve as a buffer towards such variations, but its absorbing capacity is generally limited. If a slack in demand cannot be absorbed by inventories, it must necessarily result in a

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Note :

- 1) Some establishments may rather have adjusted themselves to a normal supply of materials. This is presumably true for units which receive materials from primary production, or such industries as Slaughtering and Preparation of Meat, Dairies, Canning of Fruit and Vegetables, Canning of Fish and Meat, Fish and Herring Oil and Meal Factories, etc.



reduction of the capacity utilization.<sup>1)</sup>

No information is available to us, however, concerning the differences in the capacity utilization, either between establishments or over time. The question is, therefore, whether any of those characteristics actually available are affected by transitory variation in demand so that any variations in the capacity utilization could be traced indirectly. Clearly, as pointed out, inventories of finished goods are affected by such variations. This is of minor interest in this context, however, since it reflects that part of transitory variation in demand which does not imply variations in the capacity utilization. However, other information is available on repairs and maintenance, and this may tell us something about variations in demand which cannot be absorbed by inventories.

Some current repairs and maintenance must always be carried out to "keep the wheels going". These will be assumed to be proportional to the capital stock.<sup>2)</sup> For some of the repairs and maintenance, however, there is some flexibility as to when to carry them out. When, in particular, these repairs lead to a break in production, it will be profitable to carry them out, where possible, during a recession so that current demand can temporarily be dealt with by means of inventories. This is a fortiori true as establishments often prefer to have these repairs done by their own labour force which otherwise is engaged in pure production activities.

This leads us to try repairs and maintenance, or more precisely  $T = I_R/K$ , as a variable taking care of variations in the capacity utilization due to transitory variations in demand. Provided that the assumption concerning the role of this variable is true, and provided that it does not reflect other misspecifications, we will expect it to have a significantly negative coefficient. Even if the first assumption is correct, however, this is a very weak test since the second assumption quite probably does not hold true. This is discussed in section c of this appendix where the results of our experiments of the repairs and maintenance variable are presented.

#### N o t e s :

- 1) Variations in the capacity utilization due to variations in the demand are usually considered to be a time-series phenomena. No doubt, however, having production units with different locations, we may very well have differences in the capacity utilization between units due to factors that affect the net price of their goods differently. This is obviously true for those units which have adjusted themselves to a normal supply of materials such as Fish and Herring Oil and Meal Factories.
- 2) It is quite probable, however, that they depend on the age distribution of the capital stock, but this effect cannot be properly taken care of by the present kind of data.

## b. O n C o s t s o f C h a n g e

If an establishment wishes to hire additional workers, or in particular to expand the capital stock (or both), resources such as organization and administration, etc. have to be allocated for this purpose, resources which otherwise could have been used for current production. This is roughly the basic idea of the theory of adjustment costs or costs of change: that there are particular costs connected to a change in the scale of operation.<sup>1)</sup>

To investigate the importance of this theory for our model specification we introduce into the production function ratio-variables expressing change in the scale of operation. The variables that we can think of in this context are  $(I_p - \Delta K)/K$  and  $(N_t - N_{t-1})/N_{t-1}$ , where  $K$  and  $N$  are capital stock and number of employees respectively.  $I_p$  is new investment goods and  $\Delta$  is the estimated depreciation ratio.<sup>2)</sup> If the costs of change hypothesis is true, we will expect this to show up as significantly negative coefficients of these variables when estimating the production function with these variables included.

The results are presented in the next section.

## c. T h e R e s u l t s

To reduce the distortion of the estimates of the parameters of the variables under consideration due to the poor quality of the reporting, we introduce two dummy-variables. When analysing the effect of  $T = I_R/K$  we also include:

$$(1) \quad F_T = \begin{cases} 1 & \text{when } I_R = 0 \\ 0 & \text{when } I_R > 0 \end{cases}$$

In Table A.V.1, where the results of these variables are presented, the percentage of  $F_T = 1$  (or  $I_R = 0$ ) is also reported. In the same manner, when analysing the effects of  $C = \frac{I_p - \Delta K}{K}$ , we introduce the dummy-variable:

$$(2) \quad F_C = \begin{cases} 1 & \text{when } I_p = 0 \\ 0 & \text{when } I_p > 0 \end{cases}$$

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### N o t e s :

1) Cf. LUCAS (1967): Adjustment Costs and the Theory of Supply. See also NERLOVE (1965): Estimation and Identification of Cobb-Douglas Production Functions, and HODGINS (1968): On Estimating the Economics of Large Scale Production, Some Tests on Data for the Canadian Manufacturing Sector.

2) Cf. Section II.3.

The findings of these variables can be summarized in the following way: We receive little support for the transitory variation in demand hypothesis, and little or no support for the costs of change hypothesis. We find in fact that the coefficient of T is significantly positive for eight of the individual industries as well as for Total Mining and Manufacturing. It is negative and significant for one industry only, namely Electrical Machinery. On the other hand,  $F_T$  is significantly positive for seven industries and for Total Mining and Manufacturing. The results of that variable thus give a slight indication that there is some variation in the capacity utilization due to transitory variation in demand that can be traced through repairs and maintenance.

The results of T are rather puzzling since we in addition to a negative effect due to transitory variation in demand, also would expect a negative effect because units with predominantly old capital have more costs for repairs and maintenance than those with predominantly new capital; that is, a type of adverse embodiment effect. Presumably, the positive coefficients of T reflect a positive correlation between good management and good maintenance and this effect completely overshadows any negative effects due to transitory variations in demand.

The results of C are basically the same as those for T. There are three industries with a significantly positive coefficient of C, and there are four industries with a significantly negative coefficient of  $F_C$ . Therefore, except perhaps for Basic Steel and Mineral Products which have a significantly positive coefficient of  $F_C$ , there is no support whatsoever for the costs of change hypothesis.<sup>1)</sup>

The generally positive effect on output of C may be explained in two ways. First, there may be an embodiment effect of current investment, even if it is true as argued previously that some, perhaps most, of the investments carried out during a year do not add to the production performance of capital.<sup>2)</sup> Second, our costs of change variable may be positively correlated with the establishment-specific component of the error, i.e., units with good management and high efficiency are also the more expansive ones.

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Notes :

1) The results of  $\frac{N_t - N_{t-1}}{N_{t-1}}$  are not presented here since the performance of that variable is even poorer than that of C and  $F_C$ .

2) Cf. Section II.3.g.ii.

Table A.V.1. Results for the Cobb-Douglas Relation with Variables Presumed to Reflect Transitory Variation in Demand\*

Variable Industry	$z-x$	$x$	$T$	$F_T$	MSE	Number of $F_T = 1, \text{in } \%$
Total Mining and Manufacturing .....	0.286 (0.007)	-0.009 (0.005)	1.722 (0.146)	0.077 (0.016)	0.276	20.9
Mining and Quarrying	0.309 (0.041)	-0.001 (0.022)	0.794 (0.417)	0.242 (0.086)	0.182	19.2
Food Products .....	0.416 (0.027)	-0.112 (0.015)	2.632 (0.450)	0.148 (0.047)	0.435	24.6
Textiles .....	0.280 (0.035)	-0.085 (0.022)	3.767 (0.832)	-0.062 (0.054)	0.146	19.3
Clothing .....	0.097 (0.024)	-0.033 (0.022)	1.770 (0.430)	-0.059 (0.041)	0.131	20.6
Wood Products .....	0.223 (0.040)	0.121 (0.025)	1.047 (0.784)	0.268 (0.073)	0.277	26.2
Pulp and Paper .....	0.336 (0.025)	-0.115 (0.015)	1.035 (0.219)	0.006 (0.044)	0.166	11.9
Printing .....	0.153 (0.024)	0.029 (0.017)	1.985 (0.898)	-0.030 (0.040)	0.119	25.0
Basic Chemicals ....	0.237 (0.038)	-0.027 (0.019)	4.339 (0.916)	0.105 (0.087)	0.487	14.7
Mineral Products ...	0.339 (0.037)	0.104 (0.022)	3.772 (0.650)	0.239 (0.077)	0.187	15.7
Basic Steel .....	0.314 (0.039)	0.052 (0.027)	4.950 (0.688)	0.434 (0.072)	0.174	13.2
Metal Products .....	0.134 (0.034)	-0.047 (0.019)	0.167 (0.629)	0.139 (0.051)	0.169	22.8
Non -Electrical Machinery .....	0.037 (0.041)	0.080 (0.021)	0.134 (0.836)	0.056 (0.061)	0.138	18.3
Electrical Machinery	0.037 (0.044)	0.062 (0.028)	-4.184 (0.839)	0.079 (0.080)	0.219	16.0
Transport Equipment.	0.089 (0.021)	0.084 (0.012)	0.065 (0.408)	0.082 (0.040)	0.148	31.8
Misc. Products .....	0.376 (0.062)	-0.057 (0.048)	5.140 (3.392)	0.269 (0.187)	0.408	34.2

\* The coefficient of  $z-x$  is the elasticity of capital and the coefficient of  $x$  is  $\epsilon-1$  where  $\epsilon$  is the elasticity of scale.  $T$  is defined in section a and  $F_T$  in (1) above. MSE is the mean square of the estimated residual error. Method of estimation: Ordinary least squares.

Table A.V.2. Results for the Cobb-Douglas Relation with Variables Presumed to Reflect Costs of Change\*

Variable Industry	z-x	x	C	F <sub>C</sub>	MSE	Number of F <sub>C</sub> = 1, in %
Total Mining and Manufacturing .....	0.272 (0.007)	-0.014 (0.005)	0.175 (0.042)	-0.042 (0.016)	0.280	22.1
Mining and Quarrying	0.280 (0.040)	-0.006 (0.026)	0.097 (0.296)	0.039 (0.087)	0.188	22.7
Food Products .....	0.375 (0.026)	-0.112 (0.016)	0.105 (0.064)	0.006 (0.045)	0.445	26.6
Textiles .....	0.281 (0.034)	-0.081 (0.024)	0.465 (0.241)	-0.137 (0.055)	0.152	16.3
Clothing .....	0.069 (0.024)	-0.053 (0.024)	0.054 (0.173)	-0.161 (0.044)	0.134	20.4
Wood Products .....	0.178 (0.039)	0.097 (0.028)	0.719 (0.288)	0.072 (0.070)	0.282	38.8
Pulp and Paper .....	0.291 (0.023)	-0.130 (0.017)	-0.164 (0.181)	-0.131 (0.039)	0.168	23.4
Printing .....	0.144 (0.023)	0.036 (0.018)	0.050 (0.151)	-0.025 (0.039)	0.121	24.5
Basic Chemicals ....	0.188 (0.037)	-0.063 (0.022)	0.488 (0.292)	-0.272 (0.074)	0.489	27.6
Mineral Products ...	0.303 (0.039)	0.104 (0.027)	0.983 (0.297)	0.017 (0.090)	0.199	15.7
Basic Steel .....	0.204 (0.037)	0.105 (0.029)	0.338 (0.242)	0.452 (0.102)	0.192	5.8
Metal Products .....	0.137 (0.034)	-0.044 (0.020)	0.396 (0.192)	0.141 (0.053)	0.169	18.7
Non -Electrical Machinery .....	0.044 (0.038)	0.062 (0.024)	0.503 (0.267)	-0.060 (0.071)	0.136	13.8
Electrical Machinery	0.108 (0.043)	0.028 (0.030)	-0.214 (0.367)	0.001 (0.127)	0.244	5.6
Transport Equipment.	0.090 (0.022)	0.082 (0.011)	0.054 (0.155)	0.076 (0.040)	0.149	25.0
Misc. Products .....	0.394 (0.063)	-0.023 (0.052)	0.054 (0.770)	0.283 (0.127)	0.407	23.9

\* The coefficient of z-x is the elasticity of capital and the coefficient of x is  $\epsilon-1$  where  $\epsilon$  is the elasticity of scale. C is defined in section b and F<sub>C</sub> in (2) above. MSE is the mean square of the estimated residual. Method of estimation: Ordinary least squares.

By comparing the results of Tables A.V.1. and A.V.2. with the OLS method results of Table III.9. we can conclude that the variables introduced into the production function for analysing transitory variation in demand and costs of change have very little impact on the estimates of the factor elasticities. The main effect of these variables seems to be that  $T$  and  $F_T$  twist the estimates slightly; for most industries the estimate on the capital elasticity is somewhat higher, but the estimate on the elasticity of labour is correspondingly lower, leaving the elasticity of scale approximately unaffected. Therefore, whatever the proper interpretation of the variables analysed may be, they seem at least to have little or no importance for the results of our model previously obtained.

## CHAPTER VI SUMMARY AND CONCLUSIONS

In almost all parts of this study we have been faced with serious limitations in the data. Since one of the main aims of this study is to determine the weaknesses of the empirical basis used so that, if possible, they can be eliminated in future vintages of the Annual Industrial Production Statistics, we first in this concluding chapter review what appear to be the main errors in data and present some proposals for what could be done with them.

Already in Chapter II we encountered problems with the capital data. The fact that they are missing for years other than 1959 and 1963 is not necessarily so bad since we have data for gross investments for all years. Worse are the observations missing for 1959 and 1963, the poor quality of the capital data actually reported and the even poorer quality of the investment data. The attempts to calculate the capital data missing are not very successful because of the large residual error of the production relation used for this purpose. This is partly due to the poor quality of the capital data reported, and the interpolation and extrapolation to obtain capital observations for the remaining years are quite rough due to the poor quality of the investment data.

Problems with the capital data are encountered again in Chapter III where still another measurement error is discussed, namely that both the labour input and the wage rate refer to the quantity of labour input, ignoring the quality of labour. Since there are obviously some differences in the quality of labour, both between establishments and over time, we are in trouble with our production function estimation because these differences are likely to be strongly and positively correlated with the observed wage rate which, according to our model, should be one of the identifying variables.

We cannot expect to eliminate completely the two errors, which we maintain are the main errors encountered in this study, in future vintages of the Annual Industrial Production Statistics. Obviously, however, some improvements could be made. For example, it is likely that some of the errors in the capital data could be eliminated by a better check on the reporting. This is particularly important for the capital data missing

since missing observations at the micro level may appear as quite serious measurement errors in the corresponding aggregates.<sup>1)</sup>

There is also a need for additional information on the capital stock of the establishments, or alternatively different questions concerning that variable.<sup>2)</sup> What is in fact reported is the capital owned and not the capital used. One should thus either ask for capital actually used, or for rental costs and receipts of capital goods in addition to capital owned.<sup>3)</sup>

There is a need for information on capacity utilization, particularly for estimating technical change and the contribution to growth from capital. As pointed out in a related study, this is likely to be most easily obtained by asking for the total number of hours or days the establishment was in operation during the year.<sup>4)</sup>

We would also liked to have had a better price index for capital. The one used is, as indicated, a price index for new capital goods. However, since an improvement of this index would require information on the rate of quality improvement of new capital goods as well as of the age distribution of the capital stock, we are not likely to make much progress in this direction in the immediate future.

For labour input we need information of some kind on the quality of the labour stock. A good quality measure would, however, require rather detailed information, and experience seems to suggest, not surprisingly, that the more detailed the questions are, the poorer the quality of the answers. However, something is better than nothing and we would probably be able to construct a better labour input measure if, for instance, a rough distribution of the labour stock on education was available.<sup>5)</sup>

#### Notes :

- 1) From a production function estimation point of view this is especially serious if the ratio of missing capital values varies between years. This seems to be true for our data since there are 60 missing capital values in 1959 and 37 in 1963. Cf. Table II.1.
- 2) There is one item of information available but not used in this study, namely the composition of the capital stock. The main reason why we have not used it is that it is available only for the two years for which capital is reported. Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter III.
- 3) Ibid., Chapter III and Chapter VI.
- 4) Ibid., Chapter VI.
- 5) For attempts to construct labour quality indices, cf. GRILICHES (1967) op. cit., GRILICHES (1963 II): Estimates on the Aggregate Agricultural Production Function from Cross-Sectional Data. GRILICHES (1963 III): The Sources of Measured Productivity Growth: United States Agriculture 1940-60, and JORGENSON and GRILICHES (1967), op. cit.



In addition to the problems we have with our input variables there are a few others of apparent importance. As mentioned several places in this study the output price index is misleading for some industries, since for these industries it is based on prices for materials and labour without taking into consideration improvements in the labour stock. In this context as well we would thus liked to have had a quality index for labour, here to correct the price of labour in price of output computations.<sup>1)</sup>

As pointed out in Appendix II.2 there are likely to be a number of mongrel time series in our data due to identification numbers referring to different physical units over time. A simple way of avoiding this in the future is to revise the identification number system slightly so that merged or unmerged production units are given identification numbers not previously used. Alternatively, one could have an additional digit in the identification number indicating whether the establishment is a branch of a previous larger unit, whether it consists of two or more previous units or whether it is, in this respect, an ordinary establishment.

The second main aim of this study has been to explore the performance of various econometric tools. Some of these tools are used because of problems rather special to micro data of the type used in this study. For example, we encountered missing observation problems for capital. In a related study serious missing observation problems were also present.<sup>2)</sup> In that study these problems were solved simply by excluding the units with observations missing. There are arguments for trying an alternative to this approach and in the present study we have calculated the observations missing by means of a modified least square calculation technique. We are not very successful in our missing observation calculation; the samples are too heterogeneous and the measurement errors too serious for that. We will argue, however, that it is worth while to use such methods in certain situations even if the individual estimates are likely to be rather poor. For example, after the data have been controlled and revised any remaining missing observations should, if possible, be calculated by means of a missing observation calculation method. If nothing else, the aggregates of the variables concerned, e.g. the capital stock, are likely to be much more reliable if this is done.

#### N o t e s :

- 1) With no such quality index of labour input it might have been better simply to use the price index for materials as the price index for gross production, and thus also for value added. Cf. formula (7) of Chapter II. We have, however, not tried this in our computations.
- 2) GRILICHES and RINGSTAD (1971), op. cit., Chapter III.

We have also used some statistical tools for investigating the behaviour of the main variables of the study. We have used analysis of variance and regression techniques in order to determine the variation of these variables over the three main dimensions in the data, i.e. between establishments, over time and with size. There is virtually no alternative to analysis of variance for evaluating the variation of the variables along the establishment dimension. For the time and the size dimensions we could, instead of regressing the variables on a time and size variable, compute averages of the variables for years (or groups of years) and size classes. The latter method of presenting the data is, however, more unwieldy and it is more difficult to determine any systematic variation in the variables along the two dimensions concerned. Thus our way of presenting the empirical basis of this study is likely to be the more efficient one.

There is hardly any good substitute for good data in applied econometric studies. However, if the data are poor, and we cannot easily obtain anything better, we have to take the weaknesses of the data explicitly into account to reduce as far as possible their impact on the results. This is the main subject of Chapter III where we first show how errors of measurement in the inputs affect the properties of some well-known methods of estimation and second, try other and less well-known methods that are more robust against the measurement errors considered.

In the first part of Chapter III we show that the indirect least square method breaks down due to its sensitivity to measurement errors. This is likely to be the main reason why frequently unreasonable results are obtained when using this and related methods.<sup>1)</sup> We conclude that with errors of measurement it may be better to estimate the parameters by single equation least squares, even if the model specified implies that we have simultaneous equations.

We also show that the errors in data are too serious for the analysis of covariance to be a useful method for taking the simultaneity into account. It has been asserted by others, and shown tentatively by us to hold true, that the establishment-specific components of the error term of the production function are likely to be the main cause of simultaneity. Eliminating these components by means of analysis of covariance leads to estimates on the factor elasticities "free" of simultaneous equations bias. We show, however, that they are on the whole more seriously biased due to the strongly increased importance of the errors of measurement biases.

        
N o t e :

1) GRILICHES (1967), op. cit., pp. 276-277.

It is also shown that if we eliminate (or reduce) the autocorrelation of the error term of the production function, the errors of measurement biases become more serious than when autocorrelation is not eliminated. And the errors of measurement biases are extremely serious if we apply a two-stage method to eliminate serial correlation in the error term of the production function: that is to use analysis of covariance to eliminate the establishment- and time-specific components of the error term and a non-linear OLS method to eliminate the autocorrelation of the net residual (i.e. net of establishment- and time-specific components). We therefore conclude that even though a well-behaved error term is good, we are not willing to pay the price for it in the present context.

We have tried some other methods, taking the various types of biases into account. The method for estimating factor elasticities that seems to be the best one is a combined factor share instrumental variable method which is used in a related study in a slightly different fashion.<sup>1)</sup> The elasticity of labour is estimated by the (arithmetic) average of that factor's share in value added. Given this estimate on the labour elasticity we estimate the capital elasticity by using the difference between dummy-variables for the upper and lower third of the units when they are ranked according to size, as an instrumental variable for capital input. This does not, however, eliminate the biases due to errors in labour, but we show that they are likely to be rather small after having eliminated the simultaneity and errors of measurement in capital biases.

Finally, in Chapter III we have tried to estimate the elasticity of substitution from the so-called Kmenta relation, attempting to evaluate the importance of the biases in the estimator of that parameter due to simultaneous equations and errors of measurement. The biases do not seem to be very serious and the estimates obtained for Total Mining and Manufacturing are reasonable even though they do not correspond very well with other estimates obtained on the elasticity of substitution in this study. The results for the different industries, however, turned out to be quite poor, frequently implying the wrong curvature of the isoquants. Thus, only the results for the "Total" are reported.

In Chapter IV we have demonstrated how multiple tests can be used to analyse the nature of any variations of the parameters of a relation, i.e. the error mean and the slope coefficients. Even though it is difficult, probably impossible, to use this tool for an overall search for the "right"

N o t e :

1) GRILICHES and RINGSTAD (1971), op. cit., Chapter IV.

model, we argue that on a more narrow class of models this tool may be used with some success.

In the attempts made to explore issues concerning technical change we have obtained a few conclusions of interest, although we might concede that some of them may be quite sensitive to particular errors in the data pointed out. We show first that the OLS method generally leads to biases of some importance for the estimates of technical change and particularly for the contributions to growth from labour and capital. Second, we focus on a particular aggregation problem present in the estimation of technical change when having a cross section of time series. In this context we derive estimates on technical change as well as on contributions to growth from labour and capital that are comparable to corresponding estimates which could be obtained from pure time series data.

Most of the issues raised about technical change concern its nature. Various approaches are tried, with mixed success, however. The attempts made to estimate the parameters of a CES relation with factor augmenting technical change gave rather poor results. Nor are we very successful in our experiments with a Cobb-Douglas relation with trends both in the error mean (or equivalently, in the intercept) and the slope coefficients using a multiple test procedure to find out which trends are significant. However, for some industries, as well as for the "Total", the findings are rather conclusive and our results in general suggest that for most industries technical change is not neutral.

The results of a tentative test of the embodiment hypothesis suggest that this hypothesis is valid for most industries.

Finally, we have analysed the role of materials as well as the changes in the role of this factor of production over time. We find first that the conclusions previously drawn on the level and the nature of technical change are not basically different when allowing materials to enter explicitly into a production function, instead of subtracting it from gross output to obtain a net output measure, value added, which is used in all other parts of the study. Second, we find that the elasticity of materials seems to be falling over time for most industries, suggesting that the technical change taking place in these industries is of the materials saving type.

The third aim of this study concerns inferences about the production structure of Norwegian Mining and Manufacturing industries. This is left to an appendix to this chapter.

### Appendix VI.1 Summary of the Main Findings by Industry

Since we have carried out most parts of our investigations for fifteen individual industries, there may be some need for a summary of the findings by industry. It is, however, rather difficult to present the results in tables by industry as the calculations carried out are of widely different natures. Thus, another method of presentation is tried.

In this appendix our main interest is to shed some further light on the differences between the industries. Thus, issues explored which lead to largely similar results for the various industries, such as the outcome of the multiple tests in Chapter IV, are ignored in this context, and instead of reproducing the estimates or various numbers calculated we rank them, from 1 through 15. Where the estimates or numbers themselves are of particular interest they will be referred to in the text.

Rankings of the results deemed to be of most interest are presented in five tables. In this way we summarize in Table A.VI.1 some of the results of Section II.4, namely the mean values, growth rates and slope coefficients from regressions on  $\ln N$  of the seven main variables of this study. In Table A.VI.2 we have a corresponding ranking of the estimates on the capital and scale elasticities of the Cobb-Douglas relation obtained by the OLS method and the Klein Wald method of estimation. In the same manner Table A.VI.3 shows a ranking of the estimates on the elasticity of substitution obtained by covariance analysis of the ACMS relation. These are, after all, the only estimates obtained on that parameter that make some sense since the results of the Kmenta relation turned out to be generally very poor. We will not argue that any one of the four sets of estimates reported is "better" than the others. An evaluation of the four estimates together, however, may allow us to conclude something for some industries concerning the probable level of the elasticity of substitution.<sup>1)</sup>

The two concluding tables refer to results obtained in Chapter V. The first one, Table A.VI.4, presents a ranking of the unweighted and weighted growth rates of value added, labour and capital, and in the final one, Table A.VI.5, we have a ranking of the calculated contributions to growth from labour, capital and technical change according to the results obtained when using unweighted growth rates and the OLS and Klein Wald methods of estimation, and when using weighted growth rates and the latter method of estimation.

#### N o t e :

- 1) If nothing else is indicated the level of the tests carried out in this appendix is 5%.

Table A.VI.1. Ranking of Some Main Characteristics.<sup>1)</sup>

Characteristic Industry	Variable	Mean Value						Slope-coeff. from regr. on t <sup>2)</sup>				Slope-coeff. from regr. on lnN						
		lnL	ln $\frac{V}{L}$	ln $\frac{K}{L}$	ln $\frac{M}{L}$	lnW	$\frac{WL}{V}$	$\frac{M'}{Y'}$	ln $\frac{M}{L}$	lnW	$\frac{WL}{V}$	$\frac{M'}{Y'}$	ln $\frac{V}{L}$	ln $\frac{K}{L}$	ln $\frac{M}{L}$	lnW	$\frac{WL}{V}$	$\frac{M'}{Y'}$
Mining and Quarrying		10	3	6	15	2	10	15	13	7	13	11	8	2	11	9	8	10
Food Products .....		13	6	4	1	13	13	1	10	4	11	10	15	15	15	14*	5	15
Textiles .....		3	11	8	9	14	9	6	12	9	4	15	10*	8	4	13	7*	5
Clothing .....		8	15	15	11	15	8	9	15	12	3	14	9	10*	1	12	10	1
Wood Products .....		14	13	11	4	12	2	3	14	8	1	9	3	9	13	7	15	13
Pulp and Paper .....		4	4	1	2	3	11	2	7	1	2	3	12	4	10	5	2	4
Printing .....		15	12	7	14	9	3	13	9	15*	9	7	6	5	3	15	12	3
Basic Chemicals ....		11	2	2	5	7	15	8	1	3	6	5	11	14	9	6	6	6
Mineral Products ...		9	5	5	13	4	12	14	8	13	5	8	1	3	7	4	14	11*
Basic Steel .....		1	1	3	3	1	14	4	4	5	10*	2	2	1	5	3	11	7
Metal Products .....		5	8	9	8	8	6	10	11	10	12	13	13	11	12*	8	3	12
Non-El. Machinery ..		6	10	12	10	6	5	11	5	11	7	6	4	7	2	11	13	2
El. Machinery .....		2	9	14	7	5	4	7	3	6	8	1	7	6	8	1	9	9
Transport Equipment.		7	14	13	12	10	1	12	6	14	14	4*	5	13	6	10	1	8
Misc. Products .....		12	7	10	6	11	7	5	2	2	15	12	14	12	14	2	4	14

1) Cf. Tables A. II.4-10.

2) Rankings of the growth rates of value added, labour and capital are presented in Table A.VI.4.

\*) First negative number in the rank.

Table A.VI.2. Ranking of OLS and Klein Wald Estimates on the Capital and Scale Elasticities.<sup>1)</sup>

Method of Estimation Parameter Industry	OLS est.		Klein Wald est.	
	Capital- elasticity	Scale- elasticity	Capital- elasticity	Scale- elasticity
Mining and Quarrying ..	6	8*	9	10*
Food Products .....	1	15	6	13
Textiles .....	5	9	10	12
Clothing .....	14	12	5	6
Wood Products .....	9	2	7	1
Pulp and Paper .....	4	13	11	14
Printing .....	10	6	12	7
Basic Chemicals .....	8	14	2	9
Mineral Products .....	3	1	3	3
Basic Steel .....	7	4	1	5
Metal Products .....	11	10	13	11
Non-El. Machinery .....	15	3	8	8
El. Machinery .....	12	7	4	2
Transport Equipment ...	13	5	15	3
Misc. Products .....	2	11	14	15

1) Cf. Table III.9.

\* First estimate in the rank below one.

Table A.VI.3. Ranking of the Covariance Analysis Estimates on the Elasticity of Substitution from the ACMS Relation.<sup>1)</sup>

Industry	Case <sup>2)</sup>			
	A	B	C	D
Mining and Quarrying ...	8	12	6	15
Food Products .....		5	14	13
Textiles .....		4	8	6
Clothing .....	11	8*	12	9*
Wood Products .....	2	2	7*	7
Pulp and Paper .....	13	10	15	11
Printing .....	10	9	10	10
Basic Chemicals .....	6	6	13	12
Mineral Products .....	1	1	5	5
Basic Steel .....	9	11	11	14
Metal Products .....	14	15	4	3
Non-El. Machinery .....	12	13	3	2
El. Machinery .....	7*	7	8	8
Transport Equipment ....	15	14	2	4
Misc. Products .....	3	3	1	1

1) Cf. Table III.2.

2) A: No components eliminated from error mean.

B: Time-specific components eliminated from error mean.

C: Establishment-specific components eliminated from error mean.

D: Both time-and establishment-specific components eliminated from error mean.

\*) First estimate in the rank below one.



Table A.VI.4. Ranking of Unweighted and Weighted Growth Rates for Value Added, Labour and Capital.1)

Type of Growth Rates Variable Industry	Unweighted			Weighted		
	Value added	Labour input	Capital input	Value added	Labour input	Capital input
Mining and Quarrying.	8	15	7	9	14	12
Food Products .....	3	7	9	5	7	7
Textiles .....	12	10*	11	14	12*	13
Clothing .....	14	13	13	13	13	14
Wood Products .....	9	6	6	2	6	1
Pulp and Paper .....	6	14	8	10	15	9
Printing .....	15*	9	15	15*	8	10
Basic Chemicals .....	5	11	14	3	9	15*
Mineral Products ....	13	12	10	4	10	2
Basic Steel .....	2	3	1	6	5	3
Metal Products .....	7	4	2	8	3	5
Non-El. Machinery ...	11	8	5	11	2	8
El. Machinery .....	4	2	4	7	11	6
Transport Equipment..	10	5	12	12	4	11
Misc. Products .....	1	1	3	1	1	4

1) The unweighted growth rate is computed as the OLS estimate on  $b$  from the regression  $\ln X_{it} = a_0 + bt$ , while the weighted growth rates are computed as the OLS estimate on  $b'$  from the regression  $\ln(\sum_i X_{it}) = b_0 + b't$ . Cf. Section V.1.

\* First negative number in the rank.

Table A.VI.5. Ranking of Calculated Contributions to Growth from Labour, Capital and Technical Change<sup>1)</sup>

Type of Growth Rates	Unweighted						Weighted		
	OLS			Klein Wald			Klein Wald		
Method of Estimation	Lab- our	Capi- tal	Tech. Change	Lab- our	Capi- tal	Tech. Change	Lab- our	Capi- tal	Tech. Change
Contribution from: Industry									
Mining and Quarrying.	15	4	6	15	8	6	14	11	3
Food Products .....	7	2	4	7	9	4	8	7	4
Textiles .....	10*	6	11	10*	11	9	12*	13	13
Clothing .....	13	13	12	13	12	13	13	14	9
Wood Products .....	6	8	10	6	5	11	5	3	6
Pulp and Paper .....	14	6	3	14	10	3	15	9	5
Printing .....	9	14	15*	9	15	15*	7	10	15*
Basic Chemicals .....	11	11	2	11	13	2	9	15*	1
Mineral Products ....	12	5	14	12	7	14	10	2	8
Basic Steel .....	3	3	5	5	1	5	6	1	10
Metal Products .....	4	9	9	3	4	8	4	6	11
Non-El. Machinery ...	8	15*	8	8	3	12	2	8	14
El. Machinery .....	2	10	7	2	2	7	11	4	7
Transport Equipment..	5	12	13	3	14	10	3	12	12
Misc. Products .....	1	1	1	1	6	1	1	5	2

1) Cf. Tables V.1-2 and V.5.

\* First negative number in the rank.

In addition, we will also refer to the findings concerning the nature of technical change.

To some extent our findings concerning the production function parameters will be compared with the corresponding results of a related study for industries covering approximately the same activities in the two studies.<sup>1)</sup>

#### a. Mining and Quarrying

Of our fifteen industries Mining and Quarrying is the one which has the lowest mean value of the materials-labour ratio and materials' share in gross production. On the other hand, it ranks third with regard to average value added productivity of labour and second for wages. It has also the lowest (and negative) unweighted growth rate of labour input while only one other industry has a lower weighted growth rate of that variable. The growth rate of capital input drops from rank 7 to rank 12 when turning from unweighted to weighted growth rates. The (unweighted) growth rates of the materials-labour ratio, labour's share in value added and materials' share in gross production are also quite low, with rank 13, 13 and 11 respectively. We also note that Mining and Quarrying ranks second with reference to the growth of the capital-labour ratio with size. There is probably a basic difference between Mining on the one side and Quarrying on the other not accounted for in our analysis.

The OLS estimate of the capital elasticity has a rank slightly below the mean while the Klein Wald estimate has a higher rank. The estimate on the scale elasticity is slightly below one for both methods.

The covariance analysis estimates of the elasticity of substitution suggest that this parameter is below one. When eliminating time components the estimates are among the lower ones obtained for any one industry. In fact, when both time- and establishment-components are eliminated, it ranks lowest. In that case the elasticity of substitution is also significantly less than one.

The calculated contributions to growth imply that labour has the lowest rank when using unweighted growth rates and the second lowest when using weighted growth rates. Capital's contribution has a fairly high rank when using the OLS method of estimation and unweighted growth rates. It is lower using the Klein Wald method of estimation for the same growth rates,

#### N o t e :

1) Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter IV.

and it has a fairly low rank when using that method of estimation and weighted growth rates. In the latter case contributions from shifts rank third among our fifteen industries.

The significantly negative trend of materials' share in gross production suggests that there is a materials-saving type of technical change in this industry. At the "value added level" the results concerning the nature of technical change do not tell us much, although there is a slight suggestion that it is capital-saving. The embodiment hypothesis receives no support in our computations for this industry.

#### b. F o o d P r o d u c t s

This industry is rather heterogeneous, covering widely different activities. In a related study seven of the twenty-seven industries were from the 20- and 21-industry groups.<sup>1)</sup> The results for these industries were rather different, and since we also have a few units from group 22 in our Food Products industry, we should expect our relations to give a rather poor fit. This is proved by the computations carried out. Only one industry, Basic Chemicals, has higher mean square errors of the two main relations, the Cobb-Douglas production function and the ACMS behaviour relation.

Table A.VI.1 tells us that this industry consists of mostly small units; it pays low wages and has a low share of labour in value added. On the other hand, it is the industry having the highest average materials-labour ratio and the highest share of materials in gross production.

From the growth rates computed we note that if paying low wages this industry ranks fourth in terms of growth of wages over time, and also that it ranks third with regard to unweighted growth rate of value added and fifth in the case of the weighted growth rate of that variable.

The variation of the main variables along the size-dimension is rather peculiar, except in the case of labour's share in value added. The average productivity of labour, the capital-labour ratio, the wage rate and materials' share in gross production vary inversely with size, and as we see from Table A.VI.1 the slope coefficient for the wage rate ranks 14 while the others rank 15. This also suggests that our Food Products industry is quite heterogeneous.

N o t e :

1) Cf. Ibid., Chapters III and IV.

Rather surprisingly, this industry has the highest estimate of the capital elasticity when using the OLS method. The estimate is also high, but has a lower rank when using the Klein Wald method. The estimates on the scale elasticity suggest that decreasing returns to scale rules in this industry. The OLS estimate ranks 15, and the elasticity of scale is significantly below one according to the results of that method. The Klein Wald estimate on that parameter also has a low rank, namely 13.

There is a basic difference in the level of the covariance analysis estimates on the elasticity of substitution when eliminating establishment-specific components and when not. In the first case the estimates have a very low rank and the elasticity of substitution is according to both sets of results significantly below one, while in the second case the estimates have a fairly high rank and the elasticity of substitution is significantly above one.

As pointed out, this industry is rather heterogeneous and it is thus reasonable to believe that eliminating establishment-specific components from the error is likely to yield the more reliable estimates. Thus, our results suggest, if anything, that the elasticity of substitution of this industry is below one.

Due to the high OLS estimate on the capital elasticity the contribution to growth of capital using this method of estimation ranks second. We also note from Table A.VI.5 that irrespective of the method of estimation and the type of growth rates, contribution to growth from technical change has a high rank.

For this industry as well there are suggestions in our results of materials-saving technical change, while there is no strong evidence against neutrality of technical change at the value added level. There is some support, however, for the embodiment hypothesis in the results for this industry.

### c. Textiles

On the average Textiles has fairly large units since the mean value of labour input ranks third among our fifteen industries. The average productivity of labour is low, however, and the mean value of the wage rate is, in fact, the second lowest. In spite of the low wages in this industry the growth in the wage rate is moderate. On the other hand, labour's share in value added shows a fairly strong positive trend.

Textiles also shows the sharpest decrease in materials' share in

gross production over time. This is also reflected in the low rank of the growth rate of the materials-labour ratio. These two variables also show a rather strong positive covariation with size.

Using the OLS method the estimate on the capital elasticity ranks fairly high, while the estimate on the scale elasticity has a rank below the average. According to the OLS results the elasticity of scale is significantly below one, and the Klein Wald method yields almost the same point-estimate. Thus, there is evidence of decreasing returns to scale in this industry. This does not correspond very well to the results of a related study which suggested increasing returns to scale for this industry.<sup>1)</sup>

Three of the four covariance analysis estimates on the elasticity of substitution are above one, but none of the results implies that the elasticity of substitution is significantly different from one at conventional levels of the tests. These results at least correspond quite well to those of the study referred to above.

The contents of Table A.VI.4 tell us that Textiles is a stagnant industry since all the growth rates computed for this industry have a low rank. This is also true for the contributions to growth of Table A.VI.5.

As pointed out, there is a sharp decrease over time in materials' share in gross production suggesting materials-saving technical change also for this industry. At the value added level the findings concerning the nature of technical change are ambiguous. The results of the multiple test of Section V.2.b indicate, if anything, that technical change is neutral, while the results of Section V.2.c lend fairly strong support to the embodiment hypothesis.

#### d. C l o t h i n g

Like Textiles, Clothing is also a low-wage industry. In fact, according to our computations it is the industry having the lowest average wage rate. It also has the lowest average productivity of labour and the lowest average capital-labour ratio. Like Textiles, Clothing shows a sharp decline over time in materials' share in gross production and a low (the lowest) growth rate of the materials-labour ratio.

There is no tendency towards an equalization over time of the wages in this industry and the other industries since it ranks 12 in terms of the growth of the wage rate. In spite of this, Clothing ranks third concerning the growth of labour's share in value added.

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N o t e :

1) Ibid., Chapter IV.

This industry is also somewhat peculiar in another way, since it ranks first concerning the slope coefficient of the size variable  $\ln N$  both for the materials-labour ratio and materials' share in gross production.

According to the results of the Cobb-Douglas relation when applying the OLS method, this industry has very low elasticities of capital and scale as the estimates on them rank second and fourth lowest respectively. This is, however, one of the industries for which the net effect of the OLS biases discussed in Chapter III seems to be most important. When using the Klein Wald method the estimates on the elasticities are substantially bigger. While the results of the OLS method imply that the elasticity of scale is significantly below one, the results of the latter method suggest that it is above one. The latter results correspond better to those of a related study.<sup>1)</sup>

The four covariance analysis estimates of the elasticity of substitution are all below one, and when no components or establishment-specific components are eliminated this elasticity is significantly below one. Thus these results lend relatively strong support to the conclusion that in Clothing the elasticity of substitution is fairly low. The study referred to, however, yields quite different results on this point.

Table A.VI.4 tells us that like Textiles, Clothing is a stagnant industry. There is no basic difference between unweighted and weighted growth rates, and Table A.VI.5 tells us that the calculated contributions to growth are low irrespective of the method of estimation and type of growth rates used.

The findings concerning the nature of technical change issue strongly suggest that it is materials-saving, while at the value added level they are largely ambiguous, although there is slight support for the embodiment hypothesis.

#### e. W o o d P r o d u c t s

This industry consists of mostly small units. It also has a low average productivity for labour, a low capital-labour ratio and fairly low wages on the average. However, it ranks first in terms of the growth over time in labour's share in value added. On the other hand, the materials-labour ratio shows a fairly stable pattern over time since the growth rate of this variable is the second lowest.

N o t e :

1) Ibid., Chapter IV.

From the regressions of the main variables on the size variable we note that both the materials-labour ratio and materials' share in gross production rank third lowest, while labour's share in value added ranks lowest. The findings of the two share variables suggest that there is some heterogeneity along the size dimension in this industry, both at the gross production level and at the value added level.

The estimates of the capital elasticities have medium ranks while the estimates on the scale elasticity rank very high. Using the OLS method we find that the scale elasticity is significantly above one. The corresponding estimate ranks second and using the Klein Wald method it ranks first. These findings correspond fairly well to those of a related study.<sup>1)</sup>

From the covariance analysis results of the ACMS relation we note that there is a basic difference between the estimates on the elasticity of substitution when establishment-specific components in the error term are eliminated and when they are not. In the first case the estimates are close to one and the elasticity of substitution is not significantly different from one. In the second case the estimates are much bigger and the elasticity of substitution is significantly above one. Thus these results suggest, if anything, that the elasticity of substitution is fairly high. On the other hand, this does not correspond very well to the results obtained for the corresponding industries of the study referred to above.

The unweighted growth rates of value added and capital and labour input have medium ranks, while the weighted growth rate of value added ranks second and the corresponding growth rate of capital ranks first. This entails, as we note from Table A.VI.5, that the calculated contribution to growth from capital has a higher rank when using weighted growth rates, but due to the higher weighted growth rate of value added, the rank of technical change is higher when using that type of growth rate. The findings concerning the nature of technical change suggest that it is also materials-saving in this industry. The results are inconclusive with regard to the issue of capital or labour-saving, or neutral technical change. Finally, we note that there is evidence from our results of embodied technical change in Wood Products.

Notes:

1) Ibid., Chapter IV.



## f. P u l p   a n d   P a p e r

This is the first heavy industry of those considered so far. It ranks fourth in terms of average labour input and also for average productivity of labour. It ranks first concerning the average capital-labour ratio and third concerning average wages. It is fairly materials-intensive as both the materials-labour ratio and materials' share in gross production rank second, and it is also rather capital-intensive as labour's share in value added has a rather low rank.

Wages have a high growth rate. In fact, this industry has the highest growth rate of that variable. We should note, however, that this is partly due to decreasing prices of output.<sup>1)</sup> Pulp and Paper is one of the few industries with a fairly stable share of materials in gross production over time. On the other hand, the share of labour in value added has a relatively strong growth over time as its trend coefficient ranks second. Rather surprisingly, Pulp and Paper ranks second also in terms of the growth with size of labour's share in value added. This is rather odd, since this industry has a high rank for the capital-labour ratio's growth with size. It is also somewhat peculiar that this industry ranks as low as 12 for the growth of average productivity of labour with size and as high as 4 for the growth of materials' share in gross production with size. All in all, these computations suggest that the sample is rather heterogeneous. This was also the conclusion reached by a related study, particularly that there are basic differences between small and large units of this industry. Perhaps we could have reduced heterogeneity by splitting up this industry into the Pulp industry and the Paper industry.<sup>2)</sup>

The capital elasticity is fairly high according to the OLS method as the estimate on this parameter ranks fourth. The Klein Wald estimate is somewhat bigger but has a substitutionally lower rank. The estimates on the scale elasticity are low for both methods. The OLS estimate is the third lowest obtained for any of the fifteen industries and the Klein Wald estimate ranks as the second lowest. They are both below one, and according to the results of the former method the elasticity of scale is significantly below one at any reasonable levels of the test. Thus there is evidence of decreasing returns to scale for this industry. This finding is supported by the results of the study referred to above.

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 N o t e s :

1) Cf. Section II.2.d and Table V.4.

2) Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter IV and Appendix A.

The estimates on the elasticity of substitution are also rather low. According to all four sets of results the elasticity of substitution is significantly below one at 5% level. On the other hand, the results of the study referred to above suggested that this parameter is above one for Pulp and Paper.

This industry has a relatively sharp decline in labour input over time. The unweighted growth rate for this variable ranks second lowest while the weighted one ranks lowest. The corresponding computed contributions to growth from labour have the same ranks, and in both cases the contributions to growth from technical change rank high: as third when unweighted growth rates are used and as fifth when weighted ones are used. This difference is mainly due to a lower weighted than unweighted growth rate of value added.

In contrast to the industries mentioned above there are no indications of materials-saving technical change in this industry. On the other hand, there are fairly strong indications of a capital-saving type of technical change. There is no support for the embodiment hypothesis in this industry.

#### g. P r i n t i n g

As pointed out in the previous chapters, we have a serious problem with data for this industry due to an overrating of the price change over time and a corresponding underrating of the growth in output.<sup>1)</sup> This makes some of our results virtually worthless. For example, the growth rates of output and wages are negative and both rank last. Thus we are also likely to underrate the contribution to growth from technical change. There are, however, a few other results that should not be seriously affected by the particular data problem for this industry.

Printing is the industry with the lowest average size of units. Thus it also has the lowest rank in terms of labour input. It also has a low materials-labour ratio and a low share of materials in gross production. It is, however, rather labour-intensive as labour's share in value added is the third highest. The growth rates over time are fairly "normal" except those depending on the price index of output.

From the results of the regressions on the size-variable,  $\ln N$ , we note that the larger units of this industry tend to be more materials-

#### N o t e :

1) Cf. Section II.2.b and Appendix II.1.

intensive than smaller ones, while the smaller units seem to be less labour-intensive. The wage rate tends to be slightly lower for large units.

Printing seems to have a rather low capital elasticity. The two estimates on the scale elasticity are exactly the same and slightly above one, but according to the OLS results this parameter is not significantly above one at any reasonable levels of the test. These results conform fairly well with those of a related study.<sup>1)</sup>

The estimates on the elasticity of substitution are all below one, but only one set of results (when no components are eliminated) implies an elasticity of substitution significantly below one. In general, the results suggest relatively strongly that this parameter is below one for Printing, and this conforms quite well with the results of the study referred to above.

Printing has a very low growth in capital input judged by the unweighted growth rates. The weighted growth rate for this variable is somewhat higher and when using the Klein Wald method of estimation capital's contribution to growth in Printing rank as ten, while when using unweighted growth rates it ranks last using the same method of estimation, and next to last using the OLS method and unweighted growth rates. Both the growth rates of labour and this factor's contribution to growth are fairly "normal" as compared to the other industries.

There is no support for the embodiment hypothesis in the results for this industry. There are also no indications of materials-saving technical change. There is some evidence of capital-saving technical change in the Printing industry.

#### h. B a s i c C h e m i c a l s

This industry has a lower average size of units than one might expect, as one would usually consider it to be a rather heavy industry. Average productivity of labour and the capital-labour ratio are quite high, however, as they both rank second. Their high rank conforms quite well with the low (in fact the lowest) rank of labour's share in value added. Basic Chemicals has a relatively high growth rate of wages and it ranks first in terms of the growth in the materials' labour ratio.

Even though the mean square errors of the relations estimated are quite high, there are no strong suggestions of a heterogeneous sample along the size-dimension. The more surprising finding from the regressions of

N o t e :

1) Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter IV.

the main variables on the size criterion  $\ln N$  is the negative growth rate with size of the capital-labour ratio.

The OLS biases of the Cobb-Douglas production function seem to be quite serious for this industry. The OLS estimate on the capital elasticity ranks eighth while the Klein Wald estimate ranks second. The OLS estimate on the scale elasticity ranks next to last and this method of estimation yields an elasticity of scale significantly below one. The Klein Wald estimate has a much higher rank and it is slightly above one. Thus in this case the OLS results are quite misleading, and we should not conclude that the elasticity of scale is really below one.

When not eliminating establishment-specific components the covariance analysis estimates on the elasticity of substitution are slightly above one, while the elasticity of substitution is significantly below one when these components are eliminated. Thus there is a very slight suggestion of an elasticity of substitution below one in these results.

While the growth rates of value added are fairly high for Basic Chemicals, the growth rates of labour are rather low and those of the capital input are very low; in fact, the weighted growth rate of that variable is negative. This implies low ranks for the calculated contributions to growth from the ordinary factors of production and a high rank for technical change. This is confirmed by the contents of Table A.VI.5.

Our findings suggest that technical change is labour-saving, and there is a slight support for the embodiment hypothesis.

#### i. M i n e r a l P r o d u c t s

According to the mean values computed this industry seems to be rather capital-intensive with low ranks both for labour's share in value added and for materials' share in gross production. It also pays fairly high wages, but the growth rate of this variable is rather low. From the results of the regressions of the main variables on  $\ln N$  we note that large units are more capital-intensive than smaller ones as the regression coefficient of the size regression of labour's share in value added is significantly negative and ranks as next to last. This conforms quite well with the high rank of the "size"-regression coefficient of the capital-labour ratio and it may be the main explanation of the high "size"-regression coefficient of the average productivity of labour, which in fact ranks first.

Mineral Products seems to have both a high elasticity of capital and a high elasticity of scale. The OLS estimate on the latter has rank one, and the elasticity of scale is significantly above one according to these results. The Klein Wald estimate on that parameter is also quite high. The estimate on the capital elasticity ranks third for both methods.

The elasticity of substitution also seems to be quite high. When the establishment-specific components are not eliminated the estimates in fact rank first, while when they are eliminated the estimates are lower but still above one. According to the two former sets of results the elasticity of substitution is significantly above one.

The results concerning both the elasticity of scale and the elasticity of substitution conform fairly well with those of a related study.<sup>1)</sup>

There is a basic difference between the unweighted and weighted growth rates of value added and capital for Mineral Products. The unweighted ones are rather low while the weighted ones rank fourth and second respectively. This implies that the large establishments also have larger growth rates of these two variables than small ones. This may suggest that what we estimate as increasing returns to scale is a basic difference between small and large units with regard to the level of the scale elasticity due to a difference in the level of the capital elasticity; that large units also tend to have a large capital elasticity and scale elasticity. This is also suggested by the "size"-regressions, but this issue has not been subject to further investigation.

The calculated contribution to growth from capital when using weighted growth rates ranks second. However, due to the high weighted growth rate of value added, technical change also has a much higher rank than when unweighted growth rates are used.

Our results suggest that technical change is materials-saving, and that at the value added level it is, if anything, labour-saving. We have some support for the embodiment hypothesis since the coefficient of the "main embodiment variable",  $E$ , is significantly positive as it should be if the embodiment hypothesis is true. On the other hand, the coefficient of the dummy-variable,  $F_E$ , is also significantly positive while it should rather be negative to be "consistent" with the results obtained for  $E$ .

N o t e :

1) Ibid., Chapter IV.

## j. Basic Steel

This is a typically heavy industry, and it ranks first among our fifteen industries both with regard to average hours worked per establishment and the average productivity of labour. It also ranks first in terms of the level of wages and it has a high rank for the capital-labour ratio. In addition, we note from the mean values computed that this industry is rather capital-intensive and also fairly materials-intensive.

There are no significant trends in the share variables while there is a significantly positive growth with size of materials' share in gross production and a significantly negative growth with size of labour's share in value added. Large units also tend to pay higher wages and they have definitely higher capital-labour ratios on the average as well as higher average productivity of labour.

It makes a substantial difference as to which method of estimation is applied on the production function for this industry. Using the OLS method the estimate on the capital elasticity ranks seventh, while using the Klein Wald method it ranks first. The difference between the two estimates on the scale elasticity is substantially less. It is above one for both methods and according to the OLS method the elasticity of scale is significantly above one. Thus, even if the Klein Wald estimate is slightly lower, there is evidence of increasing returns to scale in Basic Steel.

The covariance analysis estimates of the elasticity of substitution suggest that this parameter is fairly low, but due to large standard errors we cannot reject the hypothesis of an elasticity of substitution of one for this industry.

The unweighted growth rates of value added, labour and capital are quite high as they rank second, third, and first respectively. The weighted growth rates are also high but with a somewhat lower rank. Using the Klein Wald method of estimation the calculated contribution to growth from capital ranks as one both when unweighted and when weighted growth rates are used. The contribution from labour also has a fairly high rank for Basic Steel.

Our results suggest that technical change is labour-saving, and there is also some support for the embodiment hypothesis in the results for Basic Steel.

## k. M e t a l P r o d u c t s

Judged by the mean values computed Metal Products is a rather "normal" industry. We note that the average size of the units is slightly above the average for our fifteen industries and that materials' share in value added is somewhat below. The results of the size-regressions suggest that there are only small differences between small and large establishments.

Both the capital elasticity and the scale elasticity seem to be rather low for this industry. The estimate of the latter is less than one both when using the OLS and the Klein Wald method, and according to the results of the former the elasticity of scale is significantly less than one. The Klein Wald estimate is only slightly higher, and thus there is some evidence of decreasing returns to scale in this industry. On the other hand, the results of a related study suggested that there are increasing returns to scale in the Metal Products industry.<sup>1)</sup>

With regard to the analysis of covariance estimates of the elasticity of substitution it makes a basic difference whether establishment-specific components are eliminated or not. When eliminated the estimates are above one but the elasticity of substitution is not significantly above one according to these results. When these components are not eliminated the estimates are below one and the elasticity of substitution is according to these results significantly less than one. The results of the study referred to above lend support to the latter results. Thus, if anything, the results indicate that the elasticity of substitution is rather low for Metal Products.

The growth rates for value added are moderate while those of the inputs are fairly high. The calculated contribution to growth from labour has a fairly high rank irrespective of the method of estimation and type of growth rates applied. Technical change has a low rank when we use the Klein Wald method of estimation and weighted growth rates. We should note, however, that for this industry we probably underrate the growth in output due to an overrating of the growth in prices.<sup>2)</sup> Thus we also underrate the contribution to growth from technical change.

### N o t e s :

- 1) Ibid., Chapter IV.
- 2) Cf. Section II.2.b and Appendix II.1.

The rather sharp decrease of materials' share in gross production over time suggests that technical change is materials-saving in Metal Products. The results of the value added relations are, however, inconclusive as to whether technical change is labour- or capital-saving. With regard to the embodiment hypothesis we obtain basically the same results for this industry as for Mineral Products. The coefficient of the main "embodiment variable",  $E$ , is significantly positive as it should be if the embodiment hypothesis is valid. However, the coefficient of the dummy-variable  $F_E$  is also significantly positive which should not be the case if the embodiment hypothesis is true. Even though the results are "inconsistent" there is, all in all, more evidence for than against the embodiment hypothesis.

#### 1. N o n - E l e c t r i c a l M a c h i n e r y

Judged by the mean values there are minor differences between this industry and the preceding one. The main difference is that the average productivity of labour and the capital-labour ratio are somewhat lower for Non-Electrical Machinery. The differences in growth rates are somewhat more apparent as this industry has a stronger positive trend in the materials-labour ratio, a positive trend in labour's share in value added and a less pronounced negative trend in materials' share in gross production. The size-regressions suggest that larger units are more materials-intensive and capital-intensive than smaller ones, that large units pay about the same wages as small ones and that average productivity of labour increases with size.

For this industry as well it matters considerably as to which method of estimation is applied on the production function. According to the OLS method the capital elasticity is not significantly different from zero, and its estimate is the lowest obtained for any industry. The Klein Wald estimate on this parameter is much higher and also has a much higher rank. According to the OLS method there are increasing returns to scale as the scale parameter is significantly above one. The Klein Wald estimate is somewhat lower, and also has a lower rank, but it is still above one. Thus, if anything, the results suggest that there are in fact increasing returns to scale in this industry. This finding is supported by the results of a related study.<sup>1)</sup>

N o t e :

1) Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter IV.



The covariance analysis results of the ACMS relation are very ambiguous concerning the level of the elasticity of substitution. When the establishment-specific components are not eliminated the estimates on that parameter are below one and when they are eliminated they are above one, but in no case do the results yield an elasticity of substitution significantly different from one. The results of the study referred to above conform more to the former results.

There is a basic difference between the unweighted and weighted growth rate of labour input, the latter being much larger than the former. The former has rank 2, the same rank as the calculated contribution to growth when that kind of growth rate is used together with the Klein Wald method of estimation. We should also note that the calculated contribution to growth from technical change has a low rank irrespective of the type of growth rate when the Klein Wald method of estimation is used. This may, however, partly be a result of an upward bias in the price index computed and a corresponding downward bias in the growth rate of output.<sup>1)</sup>

There are indications of a labour saving type of technical change. We also note that the embodiment hypothesis seems to receive some support from our calculations.

#### m. Electrical Machinery

Electrical Machinery ranks second concerning the average labour input. This does not conform very well with the median of number of employees of this industry where it ranks seventh. There are a number of possible reasons for this discrepancy, and probably the more important is wage differences between production and non-production workers and the role they play in our computations of hours worked by non-production workers.

We note that the share variables are fairly stable over time and that they do not vary much with size either.

As for Non-Electrical Machinery it makes a considerable difference whether we use the OLS or Klein Wald method of estimation. The estimates both on the capital and the scale elasticity are much larger and have much higher ranks when the latter method is applied. In fact, among our fifteen industries there is only one that has a higher Klein Wald estimate on the scale elasticity. Thus even if the OLS results do not yield an elasticity of scale significantly above one, there seems to be enough evidence for

N o t e :

1) Cf. Section II.2.b and Appendix II.1.

concluding that there are increasing returns to scale in this industry. The results of a related study seem to support this finding.<sup>1)</sup>

The covariance analysis estimates on the elasticity of substitution are about one. The results of the study referred to above lend some support to the conclusion of a rather low elasticity of substitution, but the standard error of the estimates are large and the elasticity of substitution is not significantly below one according to those results.

There are some differences between unweighted and weighted growth rates of output and labour input as the latter are substantially lower, particularly for labour input. The weighted growth rate of capital is also somewhat lower than the unweighted one. This also affects the rank of the calculated contribution to growth from labour as shown in Table A.VI.5. We also note from this table that the rank of the contribution to growth from technical change is not affected, either by method of estimation or type of growth rate.

There are some indications of a capital-saving type of technical change, while there is no support for the embodiment hypothesis in our results.

#### n. T r a n s p o r t   E q u i p m e n t

This industry has a low average productivity of labour, low capital-labour and materials-labour ratios and a rather low wage rate. In addition, we learn from the mean values computed that Transport Equipment is labour-intensive as it has the highest share of labour in value added among our industries, while there are only three other industries with a lower share of materials in gross production.

This industry also has a very low growth rate of wages, but we should note that this finding and partly also the mean values of the average productivity of labour and the wage rate are affected by the underrating of the growth in output, due to an overrating of the growth in prices.<sup>2)</sup> The share variables are not affected by these errors in data, and we note that neither of them shows any substantive trend-like variation over time.

The results of the regression of labour's share of value added on our size variable  $\ln N$  unveil a rather surprising difference between large

#### N o t e s :

- 1) Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter IV.
- 2) Cf. Section II.2.b and Appendix II.1.

and small units as the former evidently tend to be more labour-intensive than the latter. In fact, the slope coefficient of this regression for Transport Equipment ranks first. This greater labour intensity of large units is also evident in the negative slope coefficient from the size-regression of the capital-labour ratio.

According to both OLS and the Klein Wald methods of estimation the capital elasticity is rather low while the elasticity of scale is fairly high. The Klein Wald estimate of the capital elasticity for Transport Equipment is the lowest among the fifteen industries while there are only two industries with a higher Klein Wald estimate on the scale elasticity. Since the OLS results yield an elasticity of scale significantly above one, there is evidence of increasing returns to scale for Transport Equipment. This finding is supported by the results of a related study.<sup>1)</sup>

The covariance analysis results of the ACMS relation are rather ambiguous with regard to the elasticity of substitution. When the establishment-specific components are not eliminated it is significantly below one, while when they are, we get estimates above one. The standard deviations are rather large, however, so that we do not get an elasticity of substitution significantly above one. The results of the study referred to above give strong support to the former results, and there is thus more evidence for than against an elasticity of substitution below one for Transport Equipment.

There are no basic differences between unweighted and weighted growth rates for this industry; those of value added and capital are rather low while those of labour are relatively higher. Labour also has a fairly high rank concerning calculated contributions to growth, while capital's contribution has a low rank irrespective of the method of estimation and type of growth rate. The low rank of technical change, as well as the low rank of the growth rate of output, may be due to the errors of data pointed out previously, that is: the price growth is overvalued implying a corresponding undervaluation of growth of output in constant prices.<sup>2)</sup>

The results concerning the nature of technical change suggest, if anything, that it is of a capital-saving type. There is also some evidence for the validity of the embodiment hypothesis. However, the results are not unambiguous as the coefficient of the dummy-variable  $F_E$  is significantly

N o t e s :

1) Cf. GRILICHES and RINGSTAD (1971), op. cit., Chapter IV.

2) Cf. Section II.2.b and Appendix II.1.

positive, while it should be significantly negative to be consistent with the finding of a significantly positive coefficient of the "main" embodiment variable, E.

#### o. Miscellaneous Products

The size of the units of Misc. Products is rather low on the average. It is also by far the smallest industry measured by number of units.

It covers 13 units, 8 of which are engaged in various plastic products activities. These 8 units are also responsible for some of the rather strange findings of this industry, particularly those relating to growth rates.

The mean values computed are fairly normal while some of the growth rates computed are widely different from those of the other industries in this study. Misc. Products ranks first concerning the growth rates of both value added and labour input, irrespective of whether weighted or unweighted growth rates are used. In spite of this it ranks as high as second concerning the growth rate of materials-labour ratio. It also ranks second in terms of the growth rate of wages, but due to the high growth rate of output, the growth rate of labour's share in value added is negative and it ranks lowest. We also note that even though the value of materials must be growing quite rapidly, gross production is growing even faster leading to a fairly strong negative trend in materials' share in gross production.

The size-regression results tell us that the average productivity of labour, the capital-labour ratio, the materials-labour ratio and materials' share in gross production are lower for large units than for smaller ones. On the other hand, large units seem to pay higher wages and also seem to be more labour-intensive than smaller units.

In contrast to other industries the Klein Wald method yields a lower estimate on the capital elasticity than the OLS method. Thus, as we note there is an almost maximum possible difference in rank between these two estimates. The Klein Wald estimate on the scale elasticity is also somewhat lower than the OLS estimate. Both estimates are below one, but according to the latter method the scale elasticity is not significantly below one at conventional levels of the test.

The results from the covariance analysis of the ACMS relation suggest that the elasticity of substitution is above one. When establishment-specific components are not eliminated the elasticity of substitution is

significantly above one. When these components are eliminated the estimates are still above one, but in spite of the fact that they rank first we cannot reject the hypothesis of an elasticity of substitution of one at conventional levels of the tests. All in all, however, the results suggest that the elasticity of substitution is rather high in this industry.

Due to the high growth rates both of output and inputs, the ranks of the calculated contributions to growth are very high. All rank first except capital when the Klein Wald method is applied and except technical change when using that method of estimation and weighted growth rates.

Finally, we note that for this industry we have evidence of materials-saving technical change, while the results at the value added level are ambiguous. There is, however, some support for the embodiment hypothesis in our computations for Misc. Products.

## SAMMENDRAG

Det er tre hovedformål med denne analysen:

- 1) Å undersøke om den store mengde mikrodata som en har i Statistisk Sentralbyrås industristatistikk egner seg som empirisk grunnlag for økonometriske analyser av produktfunksjoner.
- 2) Å undersøke nytten av en del økonometriske metoder til å få ut informasjon om produksjonsstruktur og tekniske endringer fra et datamateriale av den type som brukes.
- 3) Å finne ut hva vi alt i alt kan lære om produksjonsstruktur og tekniske endringer i norsk bergverk og industri ved hjelp av det datamaterialet og de metoder som brukes.

De enheter som analysen omfatter, er 907 bedrifter til såkalte store foretak innenfor bergverk og industri. Et stort foretak er her definert som ett som i 1963 hadde en total sysselsetting på minst 100 personer. Disse enheter har en data for i en ni-årsperiode, fra 1959 til 1967.<sup>1)</sup> De bedriftene som er valgt ut, er delt opp i 15 bergverks- og industrinæringer, og i de fleste delene av analysen gis det resultater for hver av disse næringene.<sup>2)</sup> I enkelte tilfelle begrenses analysen til bare resultater som framkommer når alle 907 bedrifter betraktes under ett.

Denne analysen er nær beslektet med en annen analyse som er gjennomført på materiale fra Bedriftstillingen 1963 for industri.<sup>3)</sup> Variabeldefinisjonene er søkt lagt så nær som mulig opp til dem som der er brukt slik at det kan være et visst grunnlag for sammenlikning av resultatene.

I et sammendrag av analysen er det naturlig å følge de tre hovedformålene som er nevnt ovenfor. I det følgende vil vi derfor foreta en oppsummering av de tre punktene hver for seg.

N o t e r :

- 1) I avsnitt a av Appendiks II.2 er det gjort rede for problemer i forbindelse med det å indentifisere bedrifter over tiden i den perioden som betraktes.
- 2) I Appendiks II.1 er det gitt en oversikt over sammensetningen av disse næringene.
- 3) GRILICHES og RINGSTAD (1971): Economies of Scale in Manufacturing and the Form of the Production Function: An Econometric Study of Norwegian Establishment Data.

## 1. Industristatistikken som empirisk grunnlag for økonometriske produktfunksjons-analyser

Selv om de 907 enhetene som analysen omfatter, bare utgjør en liten del av det totale antall bedrifter som dekkes av Industristatistikken<sup>1)</sup>, gir analysen en god indikasjon på hvilke sterke og svake sider denne datakilden har som empirisk grunnlag for økonometriske produktfunksjons-analyser. De vesentligste fordelene ved denne datakilden, sammenliknet med f.eks. rene tidsrekke-data for aggregerte størrelser er den store mengde av observasjoner og at relevante forklaringsvariable (for produksjon og produktivitet) viser mye større variasjoner. Begge disse forhold skulle gjøre det lettere i prinsippet å estimere strukturparametre i produktfunksjoner med større presisjon. Det datamateriale som brukes, viser seg imidlertid å ha en rekke alvorlige svakheter som langt på vei oppveier fordelene. Det er særlig to som bør påpekes, nemlig at det mangler observasjoner for sentrale variable for en del bedrifter, og at enkelte variable er beheftet med betydelige målefeil.

En del av svakheterne i data bør imidlertid kunne elimineres i framtidige årganger av Industristatistikken. I noen grad er dette avhengig av hvilke ressurser en vil sette inn for et slikt formål.

Den variabel som vel skaper de største problemer i analysen, er kapitalinnsatsen. Den er målt som full brannforsikringsverdi for bygninger og maskiner. Nå har en observasjoner for denne variabel bare for 1959 og 1963, og selv for disse år har en ikke kapitaltall for alle bedrifter. De manglende kapitaltall for 1959 og 1963 er anslått ved hjelp av en spesiell metode som er gjort rede for i Avsnitt II.3<sup>2)</sup>. For de andre årene har en anslått kapitaltall ved hjelp av kapitaltallene for 1959 og 1963, brutto-investeringer som det er gitt opplysninger om for alle år og en estimert depresieringsrate. Dette er gjort rede for i Avsnitt II.3.g.

I tillegg til problemer på grunn av manglende observasjoner har en for kapital også store problemer fordi det mål en bruker av forskjellige grunner er beheftet med betydelige feil. Disse er særlig diskutert i Kapittel III, hvor en har lagt vekt på å finne fram til metoder som gjør at resultatene for sentrale strukturparametre i produktfunksjonen ikke i vesentlig grad blir påvirket av målefeilene.

### N o t e r :

- 1) De 907 bedriftene har imidlertid om lag halvparten av total produksjon, total sysselsetting etc. i bergverk og industri.
- 2) For 1959 og 1960 mangler også opplysninger for subsidier og avgifter. Disse er også anslått, noe som er gjort rede for i Avsnitt b i Appendiks II.2.

Men selv om det i en viss grad er mulig ved hjelp av spesielle estimeringsmetoder å eliminere virkningene på resultatene av et svakt data-materiale, må det slås fast at det neppe eksisterer noen god erstatning for gode data i en økonometrisk analyse. Og den framtidige nytten av Industri-statistikken i denne forbindelse er sterkt avhengig av bedre kapitaltall.

I noen grad skulle det være mulig å få bedre opplysninger ved bedre kontroll og revisjon av datamaterialet. Kommer ressursbeskrankninger inn i denne forbindelse, bør en gjøre et utvalg fortrinnsvis av store bedrifter (eller foretak) som blir gjenstand for spesielt nitid kontroll.

De opplysninger som gis, er for kapital som eies av bedriften. Kapital som leies, har en ingen informasjon om. Det er derfor et behov for en ny type spørsmål for kapital, alternativt spørsmål i tillegg til de en nå har. I stedet for å spørre om kapital som eies av bedriften, bør en spørre om kapital som virkelig brukes av bedriften, alternativt at en i tillegg til opplysninger om kapital bedriften eier får opplysninger om leieinntekter/leieutgifter for kapital.

I Industristatistikken har en intet mål for graden av kapasitets-utnyttelse. Det skulle imidlertid være mulig å lage et grovt mål for denne ved å spørre etter antall timer eller dager bedriften var i virksomhet i løpet av året.

Kapitaltallene som brukes, er deflatert med en prisindeks for bruttoinvesteringer for å få et mål for kapitalen "i faste priser". Hele kapitalen er således deflatert med en prisindeks for ny kapital. Dette er nok en årsak til at det kapitalmål vi bruker, er meget upålitelig. Men for å kunne danne oss en i denne forbindelse bedre prisindeks måtte en ha informasjoner både om kvalitetsforbedringer av ny kapital og fordeling av kapitalen på årgang. Det er derfor lite trolig at det kan gjøres vesentlige framskritt på dette punkt i den nærmeste framtid.

Det mål vi har for arbeidskraftsinnsatsen er også beheftet med feil. Det samme gjelder for målet for prisen på arbeidskraft, idet begge variable refererer seg til mengde av arbeidskraftsinnsats. En ser altså bort fra kvalitetsforskjeller både over tid og mellom bedrifter. Vi har hevdet i analysen at de forskjeller som kan observeres i lønnsatsene (gjennomsnittslønn for arbeidere) både over tiden og mellom bedrifter i vesentlig grad må antas å skyldes forskjeller i kvalitet på arbeidskraften. Som det vises i Kapittel III, har dette bestemte virkninger på resultatene, og det synes vanskelig å finne estimeringsmetoder som gir resultater som ikke er påvirket av at vi ikke eksplisitt tar hensyn til slike kvalitetsforskjeller i våre variabel-mål.



Et godt kvalitetsmål for arbeidskraft er det imidlertid ikke enkelt å få tak i. Vi ville trenge nokså detaljerte informasjoner, og erfaringen synes å vise at dess mer detaljerte spørsmålene er dess mer upålitelige er svarene. Men noe er bedre enn ingenting, og det skulle være mulig å lage seg et bedre mål for arbeidskraftsinnsatsen enn det vi har brukt i denne analysen dersom en hadde en grov fordeling av arbeidskraften (og lønn) etter utdanning.

Idet en gjerne vil ha produksjonen målt i faste priser, har en i denne analysen deflatert bruttoproduksjon og bearbeidingsverdi (og da også råvareinnsatsen) med prisindekser hentet fra nasjonalregnskapet. For noen sektorer har imidlertid prisindeksen for bruttoproduksjon, og dermed den avledede prisindeksen for bearbeidingsverdi en systematisk skjevhet fordi den er utregnet på grunnlag av prisene på råvarer og arbeidskraft, uten at det er tatt hensyn til kvalitetsforbedringer i arbeidskraften.<sup>1)</sup> For disse sektorene vil prisindeksen ha en tendens til å ligge for høyt, og dermed vil en undervurdere veksten i produksjonen regnet "i faste priser". Dette har særlig betydning for estimatene på nivået for de tekniske endringene.

Det er to måter å forbedre prisindeksen på dette punkt. Enten må en basere seg på prisobservasjoner for produksjonen, eller en får korrigere prisen på arbeidskraft for kvalitetsforbedringer (forutsatt at en aksepterer den prisdannelsesmekanisme som antas å gjelde for de sektorer hvor slike produksjonsprisberegninger blir foretatt).<sup>2)</sup> I siste tilfelle trenger en en kvalitetsindeks for arbeidskraft av liknende type som er antydnet ovenfor.

Som påpekt i Appendiks II.2, er det grunn til å tro at en i datamaterialet har en del tilfelle av tidsserier som refererer seg til forskjellige bedrifter på forskjellige tidspunkter. Dette er et resultat av at enkelte bedrifter er blitt delt på grunn av en utvidelse av spektret av produkter som blir framstilt og også i noen grad et resultat av sammenslåing av bedrifter. Problemet oppstår fordi en i første tilfelle lar identifikasjonsnummeret til den bedriften som blir delt, følge den største av de bedrifter som er resultatet av oppdelingen,

#### N o t e r :

- 1) I Appendiks II.1 er de næringsgruppene hvor dette blir gjort, merket med en stjerne (\*).
- 2) Grunnen til at en gjennomfører slike beregninger i det hele tatt, er at det produksjonsprismateriale en har for disse sektorene er meget spredt og dårlig. Men jeg ville tro at beste måten å forbedre prisindeksen for disse sektorene på, er ved et forbedret prismateriale for produksjonen, slik at en kan beregne prisindekser uavhengig av prisene på kostnadsfaktorene.

mens en i siste tilfelle lar den bedrift som er resultat av sammenslåingen få det identifikasjonsnummeret som den største av de bedrifter som er gjenstand for sammenslåing hadde.

Dette problemet kan unngås dersom en reviderer identifikasjons-systemet slik at ingen av de bedrifter som blir delt opp eller slått sammen gis identifikasjonsnummer som tidligere har vært brukt. Alternativt kunne en ha et tilleggssiffer i identifikasjonsnummeret som indikerer om bedriften er en del av en tidligere større bedrift, om den består av to eller flere tidligere bedrifter eller om den i dette henseende er en vanlig bedrift.

Selv om der er vesentlige svakheter ved det datamaterialet som er brukt, er svaret på spørsmålet om data av den typen som er brukt egner seg som empirisk grunnlag for økonomiske produktfunksjons-analyser et betinget ja. Denne konklusjon er i noen grad påvirket av det faktum at en som regel er nødt til å arbeide med datamaterialer med store svakheter. Som nevnt, bør en kunne eliminere, eller redusere betydningen av noen av svakhetene som er påpekt i framtidige årganger av Industristatistikken. Dersom det blir gjort, bør denne datakilden i framtida spille en rolle også som empirisk grunnlag for produktfunksjons-analyser.

## 2. Nyttan av en del økonomiske metoder

En del av de metodene som brukes, er betinget av svakheter i data-materialet. Vi har f.eks. brukt en bestemt metode for å beregne manglende kapitaltall, som det er gjort rede for i Avsnitt II.3. Et alternativ til dette er å utelukke fra samplet de enheter hvor kapitaltall mangler. Dette ville imidlertid i vårt tilfelle innebære at for hver bedrift som manglet kapitaltall for 1959 eller 1963 (eller for begge år) måtte hele tidsrekken med i alt ni observasjoner gå ut. For hvert kapitaltall vi anslår, "tjener" vi altså en hel tidsrekke. Selv om vi bruker informasjon ved å anslå manglende kapitaltall, får vi mer informasjon enn vi taper, slik at vi alt i alt har en nettogevinst.

Metoden går i korthet ut på at en anslår manglende observasjoner på samme måte som en estimerer strukturparametre i produksjonsmodellen. Men vesentlig på grunn av målefeil og inhomogene sampel for de forskjellige næringer gir ikke metoden særlig gode resultater, bedømt etter rimeligheten av de individuelle kapitaltall-anslag. Det innføres derfor visse modifikasjoner i beregningsmetoden. Men selv om metoden ikke gir helt gode resultater, kan den med fordel brukes i situasjoner analoge med den som her er betraktet. Den bør også kunne brukes i andre situasjoner, idet de aggregatstørrelser som ofte er det viktigste publiserte resultat av f.eks. Industristatistikken, trolig blir mer pålitelige når så blir gjort.

Visse økonometriske metoder er også brukt for å danne seg et bilde av hvordan de sentrale variable oppfører seg i de sampele vi har for de forskjellige næringene. Det er brukt variansanalyse og regresjonsmetoder for å finne ut hvordan disse variable varierer langs de tre hoveddimensjonene i data; over bedrifter, over tiden og med størrelse. Dette er gjort rede for i Avsnitt II.4, og resultatene er presentert i tabellene II.4-10. Der er alternative måter å danne seg et konsentrert bilde av sampelegenskapene til de variable på, men den måten som er valgt synes å være den mest hensiktsmessige.

I Kapittel III diskuteres forskjellige metoder som kan tenkes brukt for å estimere strukturparametre i en enkel produksjonsmodell. Vi er spesielt interessert i å finne estimeringsmetoder som er robuste overfor målefeil i et simultant likningssystem. Det vises først at indirekte minste kvadraters metode, som det er naturlig å bruke for den modellen som betraktes, er meget sårbar overfor de to målefeilene som antas å være de viktigste i denne forbindelse, nemlig målefeil i kapitalen og feilen i arbeidskraft- og lønnsvarsvariablene på grunn av at en ikke har tatt hensyn til kvalitetsforskjeller i målene for disse variable.

Det vises videre at minste kvadraters metode direkte på produkt-funksjonen gir bedre resultater for faktorelastisitetene og dermed passuskoeffisienten på tross av at denne metoden ikke tar hensyn til simultaniteten i modellen som er spesifisert. Årsaken til dette ligger i at ordinær minste kvadraters metode er mer robust overfor målefeil enn førstnevnte metode.

Men det vises at skjevhetene på grunn av simultanitet og målefeil i de estimatene som er oppnådd ved ordinær minste kvadraters metode også kan være betydelige. Som alternativ estimeringsmetode blir kovariansanalyse prøvd, men det vises at også den metoden er mer sårbar overfor målefeil enn ordinær minste kvadraters metode. Det blir derfor søkt etter andre metoder, og en ender opp med en metode som går ut på å estimere arbeidskrafts-elastisiteten ved en spesiell faktorandelsmetode, og gitt det estimatet en da får på denne parameteren estimerer en så kapitalelastisiteten ved hjelp av en spesiell instrumentvariabelmetode, også kjent som grupperingsmetoden.

Denne metoden tar hensyn til simultaniteten i modellen og målefeil i kapitalen, men ikke til den spesielle målefeilen vi har i arbeidskraften. Det vises imidlertid tentativt ved beregninger av skjevheter som skyldes målefeil i arbeidskraften at disse er små når skjevheter som skyldes simultanitet og målefeil i kapitalen er eliminert.

De resultater som er oppnådd for en annen sentral parameter, nemlig substitusjonselastisiteten, som er definert i Kapittel I, er heller dårlige. De mest pålitelige resultatene synes å bli oppnådd ved hjelp av atferdsrelasjonen i modellen, på tross av at disse antakelig er sterkt påvirket av at vi har sett bort fra kvalitetsforskjeller for arbeidskraften.

I Kapittel IV vises det hvordan testing av multiple hypoteser kan brukes til å "lete seg fram" til arten av systematiske variasjoner i parametrene i en relasjon, dvs. forventningsverdien til restleddet og koeffisientene til de ordinære variable i relasjonen. Denne metoden kan være til nytte hvis en ved hjelp av data vil lete seg fram til den "sanne" modellen innenfor en gitt klasse av modeller. Og de eksemplene som gis, indikerer at metoden i hvert fall er nyttig når letingen foregår innenfor relativt snevre klasser av modeller.

I Kapittel V, hvor visse problemer omkring estimering av tekniske endringer er diskutert, er også testing av multiple hypoteser brukt, her for å lete seg fram til den "sanne" type tekniske endringer blant et nærmere spesifisert sett av typer. I den forbindelse kommer en bestemt svakhet ved metoden fram, nemlig at en ikke alltid kan bestemme entydig den "sanne" type av tekniske endringer. I Kapittel V diskuteres for øvrig virkningen av å bruke skjeve estimater på strukturparametrene i beregningen av vekstbidragene fra arbeidskraft, kapital og tekniske endringer.

Et spesielt aggregeringsproblem i forbindelse med bruk av kombinert tverrsnitts-tidsrekke materiale tas også opp, og det utledes estimater på vekstbidrag fra arbeidskraft, kapital og tekniske endringer som er sammenliknbare med dem en får når en bruker rene tidsrekke data. Dette er gjort rede for i Avsnitt V.1.b.

I tillegg til anvendelsen av multippel testing for å bestemme arten av tekniske endringer er det brukt en del andre metoder. Det er blitt gjort forsøk på å anvende en CES produktfunksjon der det antas at kvaliteten på hver innsatsfaktor stiger med en konstant prosent over tiden. Typen av tekniske endringer er da avhengig av om kvalitetsendringene i arbeidskraften er større eller mindre enn de for kapitalen, og dessuten av nivået på substitusjonselastisiteten (om den er større eller mindre enn 1). Hovedsakelig på grunn av målefeil blir resultatene nokså upålitelige, men resultatene for totalen (for alle 907 bedriftene) synes å støtte resultatet av en multippel test på en produktfunksjon av annen type. Ved begge framgangsmåter får en at de tekniske endringer er arbeidskraftsparende, dvs. de tekniske endringer er ikke-nøytrale og fører til en øking over tiden i forholdet mellom grenseproduktiviteten til kapitalen og grenseproduktiviteten til arbeidskraften.

En annen måte å analysere arten av tekniske endringer på er å teste den såkalte embodiment hypotesen som sier at de tekniske endringer helt eller delvis skyldes at nyere kapital er av bedre kvalitet, har høyere produktivitet, enn eldre kapital. Denne testen er gjennomført ved å teste koeffisienten til en variabel som uttrykker hvor ny kapitalen er. Av flere grunner som er påpekt i Avsnitt V.2.c, hvor embodiment hypotesen behandles, er denne testen nokså tentativ.

I Avsnitt V.2.d trekkes råvarene mer eksplisitt inn i analysen ved å postulere en produktfunksjon med bruttoproduksjon som produksjonsmål og produksjonsfaktorene arbeidskraft, kapital og råvarer. Det vises i dette avsnittet at dersom en ved transformasjon av produktfunksjonen sikrer sammenliknbarhet med tidligere resultater for tekniske endringer, så er ikke resultatene som oppnås vesentlig forskjellige. Det vises dessuten i dette avsnittet et interessant trekk ved råvarenes stilling i produksjonsprosessen for de fleste næringer. Vi får nemlig at for 8 av 15 næringer er der råvare-sparende tekniske endringer, dvs. at grenseproduktiviteten til råvarer faller over tiden i forhold til grenseproduktiviteten for arbeidskraft og kapital.

I et Appendiks til Kapittel V er det gjort rede for forsøk på å spore virkninger av tilfeldige variasjoner i etterspørsel etter bedriftenes produkter og omkostninger ved nivåendring i produksjonen. Resultatene er stort sett negative, og det er angitt en del mulige forklaringer på dette.

### 3. Hva vi har lært av analysen om produksjonsstruktur og tekniske endringer i norsk bergverk og industri

Når en skal trekke slutninger fra de resultater som er oppnådd i denne analysen om produksjonsstruktur og tekniske endringer i norsk bergverk og industri, bør en ha i mente at analysen bygger på et utvalg av enheter. Utvalget er på ingen måte representativt idet hovedsakelig bare større bedrifter er med. Men idet de utvalgte bedrifter står for ca. halvparten av produksjonen og har tilnærmet en tilsvarende andel av total sysselsetting og kapital etc., må en kunne si at resultatene, i den grad de sier noe interessant om samplene, også må si noe vesentlig om de næringer samplene representerer.

I Appendiks VI.1 er det foretatt en oppsummering etter næring av det som synes å være de mest interessante resultatene, med vekt på å få fram forskjeller mellom næringene. Vi skal her gi et kort sammendrag av de resultatene som angår kapitalelastisiteten, passuskoeffisienten, substitusjonselastisiteten, vekstbidragene fra arbeidskraft, kapital og tekniske endringer og resultater vedrørende type av tekniske endringer.

Resultatene for vekstbidragene er basert på estimater for faktor-elastisitetene som ikke er skjeve på grunn av simultanitet og målefeil i kapitalen og på vekstrater som er analoge med dem som en får fra rene tidsrekke-data. Jfr. Avsnitt V.1.

Estimater vil ofte bli karakteriserte som "høye", "middels" eller "lave" og er da sett i relasjon til nivået for tilsvarende estimater for alle næringer under ett. Dette er i tråd med den måten resultatene er presentert på i Appendiks IV.1.

#### a. B e r g v e r k

Kapitalelastisiteten er av middels størrelse mens passuskoeffisienten synes å ligge i underkant av 1 som er noe under middels for alle næringer under ett. Substitusjonselastisiteten er i henhold til resultatene lav og synes å være mindre enn 1.

Vekstbidragene både fra arbeidskraft og kapital er lave, mens der bare er to andre næringer hvor tekniske endringer betyr mer for produksjonsveksten.

De tekniske endringene er av råvaresparende type.

#### b. M a t v a r e r

Kapitalelastisiteten er høy, mens resultatene for passuskoeffisienten gir relativt klare indikasjoner på at denne parameteren er mindre enn 1. Resultatene for substitusjonselastisiteten er tvetydige, men gir en svak antydning av at denne parameteren også er mindre enn 1.

Vekstbidragene fra arbeidskraft og kapital er middels, mens vekstbidraget fra tekniske endringer er relativt høyt.

De tekniske endringer er av råvaresparende type. Resultatene gir en svak støtte til embodiment hypotesen.

#### c. T e k s t i l e r

Kapasitetelastisiteten er noe i underkant av gjennomsnittet for alle næringer, og resultatene indikerer at passuskoeffisientene er mindre enn 1. Resultatene for øvrig gir en svak indikasjon på at substitusjonselastisiteten er større enn 1.

Vekstbidragene både fra arbeidskraft, kapital og tekniske endringer er lave sammenliknet med de fleste andre næringer.

Der er en klar indikasjon på at de tekniske endringer er av råvaresparende type. Embodiment hypotesen får også klar støtte i resultatene.

#### d. B e k l e d n i n g

Både kapitalelastisiteten og passuskoeffisienten synes å ligge noe over middels. Resultatene for substitusjonselastisiteten antyder at denne parameteren er mindre enn 1.

Alle tre typer av vekstbidrag er lave sammenliknet med de fleste andre næringer. Resultatene gir en sterk indikasjon på at de tekniske endringer er råvaresparende. Embodiment hypotesen gis svak støtte i beregningene.

#### e. T r e v a r e p r o d u k t e r

Kapitalelastisiteten er av middels størrelse, mens denne næringen synes å ha den høyeste passuskoeffisienten. Resultatene for substitusjonselastisiteten er tvetydige, men de gir en svak indikasjon på at den er over 1.

Vekstbidraget fra kapitalen er høyt mens vekstbidragene fra arbeidskraft og tekniske endringer er noe lavere, men også disse er over middels. Resultatene gir en klar indikasjon på at de tekniske endringene er av råvaresparende type, og de gir støtte til embodiment hypotesen.

#### f. T r e m a s s e o g p a p i r

Kapitalelastisiteten er relativt lav, og passuskoeffisienten er meget lav. Resultatene indikerer at den er mindre enn 1. Resultatene for substitusjonselastisiteten antyder relativt entydig at denne parameteren også er mindre enn 1.

Det er en markert nedgang i sysselsetting i denne næringen og arbeidskraftens vekstbidrag er derfor negativt. Vekstbidragene fra kapitalen og de tekniske endringer er henholdsvis noe under og noe over middels.

Resultatene gir en nokså klar indikasjon på at de tekniske endringene er av kapitalsparende type.

#### g. T r y k k e r i e r

Kapitalelastisiteten er relativt lav, og passuskoeffisienten er i henhold til våre resultater rundt 1. Substitusjonselastisiteten synes å være mindre enn 1 i denne næringen.

Vekstbidraget fra arbeidskraft er nær middels, mens vekstbidraget fra kapital er litt under middels.<sup>1)</sup>

Resultatene indikerer at de tekniske endringene er av kapital-sparende type.

#### h. K j e m i s k g r u n n i n d u s t r i

Kapitalelastisiteten er høy i denne næringen mens passuskoeffisienten synes å ligge i nærheten av 1. Resultatene for substitusjonselastisiteten er tvetydige, men de gir en svak indikasjon på at den er mindre enn 1.

Vekstbidraget fra arbeidskraft er litt under middels mens vekstbidraget fra kapitalen er negativt idet vekstraten for denne faktoren er negativ. Vekstbidraget fra tekniske endringer er høyere enn for noen av de andre næringene.

Resultatene indikerer at de tekniske endringer er arbeidskraft-sparende, og der er en svak støtte for embodiment hypotesen.

#### i. M i n e r a l p r o d u k t e r

Både kapitalelastisiteten og passuskoeffisienten er høye i denne næringen i henhold til våre resultater. Det samme er tilfelle for substitusjonselastisiteten.

Vekstbidraget fra kapitalen er høyt mens vekstbidragene fra arbeidskraften og tekniske endringer er under middels.

Våre resultater indikerer at de tekniske endringer er av råvare-sparende type. De gir dessuten en svak støtte til embodiment hypotesen.

#### j. S t å l i n d u s t r i

Kapitalelastisiteten er høy, og passuskoeffisienten synes å være noe over middels. Substitusjonselastisiteten synes å være lav i denne næringen, men de estimater vi har på denne parameteren har store standardavvik, og de er derfor nokså usikre.

Vekstbidraget fra kapitalen er høyere enn for noen av de andre næringene. Vekstbidraget fra arbeidskraften er over middels mens vekstbidraget fra tekniske endringer er relativt lavt.

Våre resultater indikerer at de tekniske endringer er av arbeidskraftsparende type. De gir også noe støtte til embodiment hypotesen.

#### N o t e :

1) P.g.a. problemer med prisindeksen for produksjonen i denne næringen er våre resultater for vekstbidraget fra tekniske endringer villedende. Jfr. punkt 1 ovenfor.



#### k. Metallvareindustri

Både kapitalelastisiteten og passuskoeffisienten er relativt lave i denne næringen, og sistnevnte parameter synes å være under 1. Resultatene for substitusjonselastisiteten er tvetydige, men de gir en svak indikasjon på at denne parameteren er mindre enn 1.

Vekstbidraget fra arbeidskraften er høyt mens vekstbidraget fra kapitalen er middels. Betydningen av tekniske endringer er relativt lav sammenliknet med de fleste andre næringer.<sup>1)</sup>

Våre resultater indikerer at de tekniske endringene er råvare-sparende, og de gir i noen grad støtte til embodiment hypotesen.

#### l. Ikke-elektriske maskiner

Både kapitalelastisiteten og passuskoeffisienten er av middels størrelse, og resultatene gir en svak indikasjon på at passuskoeffisienten er større enn 1. Resultatene for substitusjonselastisiteten er tvetydige, og det er ikke mulig å trekke noen slutninger av interesse fra dem.

Vekstbidraget fra arbeidskraft er meget høyt mens vekstbidraget fra kapital er litt under middels, og vekstbidraget fra tekniske endringer er lavt.<sup>2)</sup>

Embodiment hypotesen har noe støtte i våre beregninger.

#### m. Elektriske maskiner

Næringen har en høy kapitalelastisitet, og det er bare en næring som har en høyere passuskoeffisient. Også for denne næringen er resultatene for substitusjonselastisiteten tvetydige.

Vekstbidraget fra arbeidskraft er relativt lavt mens det er noe over middels for kapital og middels for tekniske endringer.

Det er ikke mulig å slutte noe om typen av tekniske endringer fra våre resultater.

#### n. Transportutstyr

Kapitalelastisiteten er relativt lav i denne næringen mens der er bare to næringer som har et høyere estimat på passuskoeffisienten, og vi har klar støtte i våre resultater for at denne parameteren er større enn 1.

#### Noter:

1) Dette resultatet kan imidlertid være påvirket av at stigningen i prisindeksen for produksjonen er overvurdert. Jfr. punkt l ovenfor.

2) Det siste resultatet kan imidlertid være påvirket av at stigningen i prisindeksen for produksjonen er overvurdert. Jfr. punkt l ovenfor.

Også for denne næringen er resultatene for substitusjonselastisiteten tvetydige, og vi kan ikke trekke noen interessante slutninger fra dem.

Vekstbidraget fra arbeidskraft er relativt høyt, mens vekstbidragene for kapital og tekniske endringer begge er relativt lave.

Våre resultater gir noe støtte til embodiment hypotesen.

#### o. D i v e r s e i n d u s t r i

Både kapitalelastisiteten og passuskoeffisienten synes å være lave i denne næringen, mens substitusjonselastisiteten synes å være større enn 1.

På grunn av høy vekstrate for arbeidskraft er vekstbidraget fra denne faktoren meget høyt. Det samme er tilfelle for vekstbidraget fra tekniske endringer mens vekstbidraget fra kapital er noe over middels.

For denne næringen indikerer resultatene at tekniske endringer er av råvaresparende type. Resultatene gir også noe støtte til embodiment hypotesen.

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