

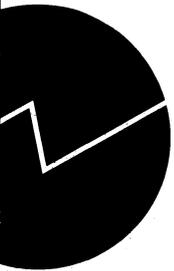
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Rapport

Stochastic Simulation of
KVARTS91

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Stochastic Simulation of KVARTS91

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Preface

This report documents stochastic simulations of the quarterly macroeconomic model KVARTS91, implemented in the TROLL software system.

By means of stochastic simulation we want to see whether the standard deterministic model solution approximates the expectation of a stochastic model solution, and to quantify (some of) the econometric uncertainty in that stochastic solution. A built-in Stochastic Simulator is used to simulate the model with stochastic residuals and stochastic parameter estimates, assuming normal distributions for the stochastic input. The model is simulated ex ante through 1993 and 1994. The results show less than 1 percent deterministic bias in the endogenous variables. On the other hand do the widths of the simulated (95 percent) prediction intervals vary a lot. But, for most variables the interval widths stay below 10 percent of the level of the variable. The uncertainty in the model solutions imply that an analysis based on stochastic rather than deterministic model simulations may lead to more subtle conclusions.

To make stochastic simulation a feasible and realistic alternative to standard deterministic simulation, commands that prepare and govern the stochastic simulation by TROLL's Stochastic Simulator have been collected into macros. Some effort has also been put into the writing of small programs that make the documentation of a stochastic simulation experiment a swift semi-automatic procedure.

Statistics Norway, Oslo, 22 December 1993

Svein Longva

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1 Introduction

Solving an econometric model with stochastic variables by stochastic simulation is not yet a common procedure. The main reasons behind this fact may be:

1. A standard deterministic simulation with only expectation values of the stochastic input variables is viewed to be sufficient. The reasoning is that the deterministic solution value approximates well the expectation value of the solution of the stochastic model.
2. A stochastic simulation is (thought to be prohibitively) more demanding than a deterministic simulation, both on human resources and on computer resources, as well as being a lot more time consuming.
3. It is easier to relate to the familiar solution point or trajectory than to an interval or distributional statistics. Quantitative measures of uncertainty in a simulated model solution is perceived as not very “useful” ...

Not even when building and testing an operative-to-be model does one seek to reveal stochastic simulation properties of the model. Hence, it seems that the traditional approach to building and using econometric models is basically deterministic. The above suggestive reasons 1.–3. used to be quite valid from a practical or operational point of view. But recent developments in economics, numerical algorithms, computer hardware and software have changed the situation somewhat:

1. Even though the majority of operational macroeconometric models are linear or only weakly non-linear, increasingly more non-linearity is being built into the models. And since more models are given a highly dynamic specification, one cannot simply assume an insignificant deterministic bias anymore.
2. Recent advances in computer hardware and software has to some degree invalidated point 2 above. Stochastic simulation of large scale macroeconometric models is today a (very) feasible task, even though it inherently is and will always be more demanding than a deterministic simulation.
3. The previous two points imply that the stochastic properties of a model ought to be of more practical interest. Simulating solution samples for relevant and interesting endogenous variables may reveal distributional properties that can be used to improve forecasts or influence on policy analysis. Ignoring this kind of information may lead to sub-optimal results, and in extreme cases of policy experiments even wrong conclusions.

This report confirms that the feasibility of stochastic simulation really does influence on the interest in stochastic simulation (it may be the main explanatory variable). One goal

of the work documented in this report was to write small programs that make stochastic simulation of the KVARTS91 model a practical procedure and a realistic alternative, or at least a supplement, to the standard deterministic simulation. Another goal was to estimate prediction intervals that reflect some of the econometric uncertainty in the model solutions. This report shows how to do a simple stochastic simulation of a macroeconomic model implemented in the TROLL software system [14]. It also presents results from *ex ante* stochastic simulation of the quarterly macroeconomic model KVARTS91, which is implemented in mainframe TROLL. A built-in *Stochastic Simulator* [15] is used to simulate the model with stochastic residuals and stochastic parameter estimates conditional on extrapolated (expectation) values of the model exogenous variables. We find for the forecast periods 1993:1 – 1994:4 that there is indeed *no significant bias in deterministic ex ante simulations relative to the mean of stochastic simulations*. But, on the other hand, we estimate *conditional 95 percent prediction intervals that are wide (up to 100 percent) relative to the level of the variables*. Believing that a somewhat simplistic stochastic simulation is more informative than a deterministic simulation, we focus on how to do a rather practical though very simple stochastic simulation. Besides that, we aim at no more than revealing some of the simulation properties of the model KVARTS91. These properties depend on assumptions underlying the specification and estimation of the model equations. But we do not venture into the more committing task of testing and evaluating these properties in light of the specification and estimation assumptions.

The report is organized as follows. We start in section 2 with a brief motivation for the undertaking of a stochastic rather than a deterministic simulation. Stochastic simulation of an econometric model is essentially a sampling of the distribution(s) of the model's endogenous variables. In section 3 we look at a few easy ways to do this which are consistent with the estimation methods applied in the modelling. Section 4 explains some basic sample statistics used in summarizing the simulation results besides graphic plots. The simulation model KVARTS91 and the simulation setup are sketched in section 5. Section 6 displays the simulation results for 18 selected endogenous variables. There are no analysis or economic interpretation of the displayed simulation results. That constitutes a considerable amount of work that goes beyond the limited intentions of this work. While the body of this report presents the most straight forward method of stochastic simulation of KVARTS91 — suitable for “automatic” inclusion in common forecasting work — appendix A contains a short discussion of a few alternative ways to perform stochastic simulation, mainly within the abilities of TROLL's Stochastic Simulator. Appendix B explains the very simple stochastic simulation procedure implemented for KVARTS91 in the TROLL system. Appendix C shows how a semi-automatic simulation–documentation system is put together by pipelining simulation results from mainframe TROLL to text and graphics formatting programs on PC.

Several papers and books may serve as an introductory text to the subject of stochastic simulation. Among them are [1, 9, 10, 18, 19, 26].

2 The stochastic econometric model

A modern macroeconometric model, like KVARTS91, is usually stated in a structural form as a system of interdependent, non-linear and dynamic equations. In a mathematical notation the model may be written as a relation between vector/matrix valued functions:

$$\mathbf{f}(\mathbf{y}_t) = \mathbf{F}(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t) \boldsymbol{\theta} + \mathbf{u}_t. \quad (1)$$

Boldface symbols denote vectors in low case and matrices in upper case. The symbols of the model are:

- \mathbf{y}_t : column vector of current endogenous variables,
- \mathbf{y}_{t-1} : column vector of lagged endogenous variables,
- \mathbf{x}_t : column vector of model exogenous variables,
- $\boldsymbol{\theta}$: column vector of structural parameters,
- \mathbf{u}_t : column vector of residual shocks,
- \mathbf{f} : column vector of identity, lag and/or log operators,
- \mathbf{F} : matrix of identity, lag, log, null and/or multiplication operators.

The simultaneity of the model is expressed by the vector \mathbf{y}_t of current endogenous variables appearing on the right as well as the left hand side of the equation system (1). The dynamic aspect is explicitly represented by the vector \mathbf{y}_{t-1} of lagged endogenous variables. To simplify the notation, without loss of generality, we assume only one period lag. The model is non-linear in the variables $\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t$, but linear in the parameters $\boldsymbol{\theta}$ (except for a few equations, cf. section 3). This is the reason why (1) is written in the form of a matrix equation, with the linear parameters gathered in the vector $\boldsymbol{\theta}$. The residual \mathbf{u}_t is typically additive to the structural form of the model. Definitional equations are irrelevant for the discussion¹ and are excluded from the stylized model. In the following we are going to use the word *variable(s)* in a broad sense to denote any function argument or model input/output, i.e. variable, parameter or residual.

Fitting the structural model (2 below) to observations $\mathbf{x}_t, \mathbf{y}_t, \mathbf{y}_{t-1}$ by methods of single equation or/and system estimation methods, returns the model (3) with the estimated parameter values $\hat{\boldsymbol{\theta}}$ and the estimated empirical residuals $\hat{\mathbf{u}}_t$. The estimated model (3) may then be solved numerically (simulated) to yield solutions for the endogenous variables. The solutions are conditional on the estimated parameter values and the historic or anticipated values of the model exogenous variables. The common way to solve the estimated model is to perform a simple deterministic dynamic simulation (4), with zero expectation values for the stochastic residuals. This pipeline procedure of specification,

¹An endogenous variable that is defined as a difference or product of two modelled stochastic variables could as such be very uncertain, besides possibly being an important variable (e.g. unemployment). But it has no influence on its own in the stochastic model.

estimation and simulation of the econometric model (1) can then be expressed by

Data, theory and methodology

↓ *specification*

$$\mathbf{f}(\mathbf{y}_t) = \mathbf{F}(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t) \boldsymbol{\theta} + \mathbf{u}_t, \quad \mathbf{u}_t \sim \mathcal{IN}(\mathbf{0}, \boldsymbol{\Sigma}), \quad (2)$$

↓ *estimation*

$$\mathbf{f}(\mathbf{y}_t) = \mathbf{F}(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t) \hat{\boldsymbol{\theta}} + \hat{\mathbf{u}}_t, \quad \hat{\boldsymbol{\theta}} \sim \mathcal{N}(\boldsymbol{\theta}, \hat{\boldsymbol{\Theta}}), \quad \hat{\mathbf{u}}_t \sim \mathcal{IN}(\mathbf{0}, \hat{\boldsymbol{\Sigma}}), \quad (3)$$

↓ *deterministic dynamic simulation*

$$\mathbf{f}(\hat{\mathbf{y}}_s) = \mathbf{F}(\hat{\mathbf{y}}_s, \hat{\mathbf{y}}_{s-1}, \mathbf{x}_s) \hat{\boldsymbol{\theta}}, \quad (4)$$

where the time index $t = 1, \dots, T$, runs through the sample periods, while the index $s = S_1, \dots, S_2$, denotes any simulation period. Hats denote estimated or simulated values. No serial correlation is allowed for in the residual process \mathbf{u}_t , so that $\hat{\boldsymbol{\Sigma}}$ is the estimated contemporaneous variance-covariance matrix. $\hat{\boldsymbol{\Theta}}$ is the variance-covariance matrix for the vector of estimated parameters, and the “true” value $\boldsymbol{\theta}$ is the expectation of an unbiased estimator in a well specified model.

The specification of the model (2) explicitly states a stochastic residual process \mathbf{u}_t , i.e. an unexplained random part of the endogenous variables \mathbf{y}_t . Since any estimator is a function of the stochastic residuals, the parameter estimates $\hat{\boldsymbol{\theta}}$ become stochastic variables, too. Since we are going to simulate the estimated model (3) — not the specified model (2) which has constant parameters — we *do* take into account this source of uncertainty in the model solutions². Of the input variables, only the model exogenous variables \mathbf{x}_t are treated as deterministic. Since the model maps the stochastic input variables onto the output variables, the endogenous variables are also stochastic. We do not consider other sources of uncertainty in the simulated model solutions, like unknown future values of the exogenous variables, the uncomplete model specification, unrevised historic data values³, unknown residual distributions. These sources are not as easily quantified, but confer e.g. [8, 9, 10, 12] for discussions. In this paper we only deal with some of the econometric uncertainty in the model solutions that come from the model input being stochastic variables.

The estimated model (3) is stochastic and should be simulated as such, not as the model (4) with deterministic variables. The reason for this is that the stochastic endogenous variables \mathbf{y}_t have unknown distributions which are not well represented by

²The premise of parameter constancy is not violated, since it is the parameter *estimates* $\hat{\boldsymbol{\theta}}$ that are stochastic due to an uncomplete specification. The “true” parameters $\boldsymbol{\theta}$ are still perceived as economic constants. An alternative (Bayesian) view on stochastic parameters is discussed in [5].

³The short term solutions of *dynamic* deterministic simulations are sometimes found to be sensitive to the choice of a starting date. Hence, there may well be additional uncertainty in the short term stochastic simulations, especially when starting from recent (unrevised) dates. This uncertainty gradually disappears as the dynamic simulation “forgets” the initial conditions and, when stable, converge to the long term equilibrium.

a single deterministic simulation. The deterministic point solution do not necessarily coincide with the expectation, the mean, the mode or other likely values of the endogenous distributions. Nor do they contain any information on the higher order moments of the distributions, like dispersion, symmetry or normality. Thus, it is impossible to quantify any confidence measures for the simulated solution of the stochastic model (3).

A non-linear and dynamic simultaneous equation system does not have a general analytical reduced form solution in terms of predetermined input variables. Still, solving the model (3) by numerical simulation let us perceive an implicit reduced form solution for the current endogenous vector in terms of predetermined variables:

$$\mathbf{y}_t = \mathbf{g}(\mathbf{y}_{t-1}, \mathbf{x}_t, \hat{\boldsymbol{\theta}}, \mathbf{u}_t).$$

We have dropped the hat on the residual \mathbf{u}_t to simplify the notation. On the other hand we have kept the hat on the parameter vector $\hat{\boldsymbol{\theta}}$ to denote the estimates. In the literature the main motivation for stochastic simulation of a non-linear model seems to be the fact that the expectation values of the endogenous variables are generally not equal to the deterministic solution, since we have that

$$\begin{aligned} \mathbf{E}[\mathbf{y}_t] &= \mathbf{E}[\mathbf{g}(\mathbf{y}_{t-1}, \mathbf{x}_t, \hat{\boldsymbol{\theta}}, \mathbf{u}_t)] && (\mathbf{x} \text{ is treated as deterministic}) \\ &\neq \mathbf{g}(\mathbf{E}[\mathbf{y}_{t-1}], \mathbf{x}_t, \mathbf{E}[\hat{\boldsymbol{\theta}}], \mathbf{E}[\mathbf{u}_t]) && (\text{as } \mathbf{g} \text{ is non-linear}) \\ &\neq \mathbf{g}(\hat{\mathbf{y}}_{t-1}, \mathbf{x}_t, \hat{\boldsymbol{\theta}}, \mathbf{0}) = \hat{\mathbf{y}}_t && (\text{deterministic simulation}) \end{aligned}$$

with an unbiased parameter estimator. The estimated deterministic bias $\hat{\mathbf{y}}_t - \hat{\mathbf{E}}[\mathbf{y}_t]$ in (mildly) non-linear operative macroeconomic models tends be small and without serious implications, cf. [13] for a summary of simulations with mostly stochastic residuals only, and [19] for results from a Norwegian model similar to KVARTS91. These results are not surprising; a fairly linear response to the input is to be expected as most models are (mis-)specified towards linearity, according to [11].

The model (3) maps the predetermined input variables onto the output variables, i.e. current endogenous values. Consequently, the model also maps the distributions p of the stochastic input variables onto the distribution q of its output variables:

$$\mathbf{g}: \hat{\boldsymbol{\theta}}, \mathbf{u}_t, \mathbf{y}_{t-1} \mid \mathbf{x}_t \rightarrow \mathbf{y}_t \quad \Rightarrow \quad \mathbf{g}: p(\hat{\boldsymbol{\theta}}), p(\mathbf{u}_t), q(\mathbf{y}_{t-1}) \mid \mathbf{x}_t \rightarrow q(\mathbf{y}_t). \quad (5)$$

Since the mapping \mathbf{g} of the input *variables* is only implicitly known through the structural model (3), the mapping of the variables' *distributions* is also unknown. Hence, for any input distributions p the output distributions q remain unknown — even though the central limit theorem suggests *approximate* normality⁴, ($q \approx \mathcal{N}$).

⁴The *central limit theorem* implies under general conditions (applicable in nearly every situation that occurs in practice) that a sum of a large number of independent random variables converges to a normal distribution. The *approximate* normality is due to lack of independence among the limited number of right hand side variables because of the lag structure and constant stochastic parameter estimates through a replication.

3 Stochastic simulation

The discussion in section 2 suggests an alternative to the traditional deterministic simulation. Simulating the stochastic model (3) with stochastic variables is a way of solving the estimated model that is more consistent with the assumptions underlying the specification and estimation of the model. The stochastic simulation is done simply by replicating a single simulation $n = 1, \dots, N$ times, each time n with new values of the stochastic variables drawn from their respective distributions p :

$$\mathbf{f}(\mathbf{y}_s^{(n)}) = \mathbf{F}(\mathbf{y}_s^{(n)}, \mathbf{y}_{s-1}^{(n)}, \mathbf{x}_s, \hat{\boldsymbol{\theta}}^{(n)}) + \mathbf{u}_s^{(n)}, \quad n = 1, \dots, N,$$

By stochastic simulation the computer solves the model numerically to yield one piece of information $\mathbf{y}_s^{(n)}$ on the unknown and period- s -specific q from each replication n . The superscript (n) denotes the n 'th replication of a single dynamic simulation through the simulation periods $s = S_1, \dots, S_2$. N is the (large) number of replications, say 1000.

The rest of this section presents the method by which the model KVARTS91 was simulated. It is a somewhat simplistic method, but it is well justified as both a first and a practical way of doing stochastic simulation of a simultaneous equation system implemented in the TROLL system. More sophisticated methods are discussed in appendix A. First we have to decide which input distributions p to sample from, and then find out a way to do it. The specification of the model (2) explicitly states the structure of the residual process \mathbf{u}_t . It is generally assumed to be independent and multivariate normal, with a diagonal or block-diagonal contemporaneous covariance matrix $\boldsymbol{\Sigma}$. This has implications for the method(s) of estimation, which also depends on whether there are current endogenous variables on the right hand sides of the equations, in \mathbf{F} . Ordinary least squares (OLS) can safely be applied to estimate a single equation where (1) there is no correlation between the residual and any right hand side current model endogenous variables and (2) there is no correlation between the residuals of any right hand side endogenous variables. If condition (1) does not hold — so that two or more variables are simultaneously (interdependently) determined (cf. [21] for the model KVARTS91) — instrumental variable estimation (IV) may be called for to secure consistency. If condition (2) does not hold, system estimation methods like full information maximum likelihood (FIML) takes into account the information on residual correlation to yield efficient as well as consistent estimates. Another reason for system estimation, which applies to one block (subsystem) in the KVARTS91 model, is that the same parameters may appear in several (different) equations, cf. [22]. Then they cannot be estimated by single equation methods. The three estimation methods — OLS, IV and FIML — represent increasingly more sophisticated “levels” of parameter estimation, and imply different ways of estimating the variance-covariance matrices of the parameter estimates $\hat{\boldsymbol{\Theta}}$ as well as the residuals' variance-covariance matrix $\hat{\boldsymbol{\Sigma}}$.

As indicated by (2) and (3), the assumption of multivariate normal residuals implies multivariate normal parameter estimates, hence we are to generate pseudo-random

“draws” form the multivariate normal distributions, such that

$$\hat{\mathbf{u}}_s^{(n)} \sim \mathcal{IN}(\mathbf{0}, \hat{\Sigma}) \text{ and } \hat{\boldsymbol{\theta}}^{(n)} \sim \mathcal{N}(\hat{\boldsymbol{\theta}}, \hat{\Theta}), \quad (6)$$

where the parameter distribution is centered on $\hat{\boldsymbol{\theta}}$, the unbiased estimate of the unknown “true” parameters $\boldsymbol{\theta}$. Once the variance-covariance matrices $\hat{\Sigma}$ and $\hat{\Theta}$ are estimated, the pseudo-random values are constructed as

$$\hat{\mathbf{u}}_s^{(n)} = \mathbf{S}'\mathbf{v}_s^{(n)} \text{ and } \hat{\boldsymbol{\theta}}^{(n)} = \hat{\boldsymbol{\theta}} + \mathbf{P}'\mathbf{v}^{(n)}, \quad (7)$$

where the square matrices \mathbf{S} and \mathbf{P} are Cholesky factorizations such that $\mathbf{S}\mathbf{S}' = \hat{\Sigma}$ and $\mathbf{P}\mathbf{P}' = \hat{\Theta}$. The unit normal vectors $\mathbf{v}_s^{(n)}$, $\mathbf{v}^{(n)} \sim \mathcal{IN}(\mathbf{0}, \mathbf{I})$ are the only random elements⁵. Note that while new residuals are generated each simulation period, the concept of parameter constancy implies that the parameter estimates stay fixed through all the simulation periods of each replication n . See [20] on how to generate random numbers with certain distributions, and [19, 24], for more details on the formula (7).

Generating the input sample distributions (6) by direct or so-called naive sampling of the unit normal distribution $\mathcal{IN}(\mathbf{0}, \mathbf{I})$, introduces small sample errors in any sample statistic. The mean, variance and higher order central moments of a unit sample $\{\mathbf{v}_s^{(n)}\}$, $n = 1, \dots, N$, converge slowly to those of the normal parent distribution (i.e. $\mathbf{0}, \mathbf{I}, \mathbf{0}, \dots$) as $N \rightarrow \infty$. A more efficient way of improving the sample representation of a normal distribution than increasing the sample size, is based on the symmetry of the normal distribution. The symmetry is crucial to the central moments, and it is easily maintained by generating *antithetic* variates:

$$\mathbf{v}_s^{(n)} \begin{cases} \sim \mathcal{IN}(\mathbf{0}, \mathbf{I}) & \text{if } n \text{ is an odd integer,} \\ = -\mathbf{v}_s^{(n)} & \text{if } n \text{ is an even integer.} \end{cases} \quad (8)$$

The accuracy in the sample statistics may improve dramatically by antithetic sampling, as reported in [6]. Other variance reducing techniques are discussed in [25, chapter 10]. Antithetic variates is an automatic option in TROLL’s Stochastic Simulator, cf. [15]. We are going to simulate 1000 replications⁶ with antithetic variates, which means that only $n = 1, \dots, 500$ pseudo-random vectors $\mathbf{v}_s^{(n)}$ have to be generated each simulation period s .

The Stochastic Simulator in TROLL automatically calculates the random variables given the variance-covariance matrices $\hat{\Sigma}$ and $\hat{\Theta}$. Using the Stochastic Simulator in

⁵The expectation and variance-covariance matrices of the stochastic variables $\hat{\mathbf{u}}_s^{(n)}$ and $\hat{\boldsymbol{\theta}}^{(n)}$ are

$$\begin{aligned} \mathbf{E}[\hat{\mathbf{u}}_s^{(n)}] &= \mathbf{E}[\mathbf{S}'\mathbf{v}_s^{(n)}] = \mathbf{S}'\mathbf{E}[\mathbf{v}_s^{(n)}] = \mathbf{0}, & \mathbf{C}[\hat{\mathbf{u}}_s^{(n)}] &= \mathbf{C}[\mathbf{S}'\mathbf{v}_s^{(n)}] = \mathbf{S}\mathbf{S}'\mathbf{C}[\mathbf{v}_s^{(n)}] = \mathbf{S}\mathbf{S}'\mathbf{I} = \hat{\Sigma}, \\ \mathbf{E}[\hat{\boldsymbol{\theta}}^{(n)}] &= \mathbf{E}[\hat{\boldsymbol{\theta}} + \mathbf{P}'\mathbf{v}_s^{(n)}] = \hat{\boldsymbol{\theta}} + \mathbf{P}'\mathbf{E}[\mathbf{v}_s^{(n)}] = \hat{\boldsymbol{\theta}}, & \mathbf{C}[\hat{\boldsymbol{\theta}}^{(n)}] &= \mathbf{C}[\hat{\boldsymbol{\theta}} + \mathbf{P}'\mathbf{v}_s^{(n)}] = \mathbf{P}\mathbf{P}'\mathbf{C}[\mathbf{v}_s^{(n)}] = \mathbf{P}\mathbf{P}'\mathbf{I} = \hat{\Theta}. \end{aligned}$$

⁶The “sufficient” number of replicated simulations depend on what kind of information one is seeking from the stochastic simulation. To asses (no) deterministic bias, a small number ($N=100$) may do. But to investigate the distribution of a simulated univariate sample, a large number ($N=1000$, or better 10000) may be necessary, cf. section 5.

TROLL and treating the exogenous variables as deterministic, we are left with the task of providing the variance-covariance matrices $\hat{\Sigma}$ and $\hat{\Theta}$. The estimation routines in TROLL can optionally return the estimation residuals and the estimated covariance matrix for single equations and subsystems. Then only the estimated residual variance-covariance matrix $\hat{\Sigma}$ needs to be calculated. The KVARTS91 model is mainly estimated by single equation OLS. Only 2 equations are estimated with instruments, and one block of 9 equations is estimated by FIML. We briefly review the different variance-covariance matrices following from these different estimation methods.

OLS: The assumptions underlying the OLS method allow us to deal with each OLS estimated equation in the model as if it was the only equation (and not a part of a simultaneous equation model). First we review the single equation model:

$$\begin{pmatrix} f(y_2) \\ \vdots \\ f(y_T) \end{pmatrix} = \begin{pmatrix} F_1(y_2, \mathbf{y}_1, \mathbf{x}_2) & \dots & F_K(y_2, \mathbf{y}_1, \mathbf{x}_2) \\ \vdots & \ddots & \vdots \\ F_1(y_T, \mathbf{y}_{T-1}, \mathbf{x}_T) & \dots & F_K(y_T, \mathbf{y}_{T-1}, \mathbf{x}_T) \end{pmatrix} \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_K \end{pmatrix} + \begin{pmatrix} u_2 \\ \vdots \\ u_T \end{pmatrix}, \quad (9)$$

where we assume K parameters in the equation. The functions $F_k(y_t, \mathbf{y}_{t-1}, \mathbf{x}_t)$ return a possibly transformed (lag, log, ...) single variable from its arguments. To contrast the OLS to the instrumental variables method underneath, we have, without any loss of generality, assumed no other *current* endogenous variables on the right hand side of the equation. Note that the first observation is lost to the lag, so that we are left with $T - 1$ observations. Since (9) is linear in the parameters, we may use the more compact (textbook) notation $\mathbf{z} = \mathbf{Z}\boldsymbol{\theta} + \mathbf{u}$, where \mathbf{z} and \mathbf{Z} contain the (transformed) variable and regressors, respectively. Here $\boldsymbol{\theta}$ denotes the vector of parameters in that single equation only. Likewise, \mathbf{u} denotes the time-vector of the residual in the same equation.

The following formulas are found in any econometrics textbook. The sample variance of the estimation residuals of the single equation is simply

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{T-1} = \frac{1}{T-1}(\mathbf{z} - \mathbf{Z}\hat{\boldsymbol{\theta}})'(\mathbf{z} - \mathbf{Z}\hat{\boldsymbol{\theta}}) = \frac{1}{T-1} \sum_{t=1}^T \hat{u}_t^2, \quad (10)$$

where $\hat{\boldsymbol{\theta}} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{z}$ the OLS estimator. Whenever in a situation where the expectation value is known à priori (it is zero by construction), the denominator $T - 2$ in the standard sample variance formula (when one observations is lagged, cf. (9)) should be changed to $T - 1$. The entries in the residual variance-covariance matrix $\hat{\Sigma}$ are the empirical estimate $\hat{\sigma}^2$ on the diagonal position and 0 in all off-diagonal positions of the row and the column corresponding to the single OLS estimated equation. The setting of off-diagonal elements (covariances) to zero reflects that since no covariances are accounted for in the estimation, we have no reason to do otherwise in the simulation, cf. Appendix A for a discussion.

The parameter variance-covariance matrix $\hat{\Theta}$ is constructed to be block-diagonal for the OLS estimated equations, i.e. $\hat{\Theta}$ has a block on the diagonal for each OLS estimated

equation. The block is the variance-covariance matrix of the parameter estimates of one single OLS estimated equation. If all parameters are unique and confined to one single equation, and assuming no residual covariances among the OLS estimated equations, there is no covariance between any parameter estimates of different blocks. The standard estimated variance-covariance matrix of the linear OLS estimates is

$$\widehat{\Theta} = \hat{\sigma}^2(\mathbf{Z}'\mathbf{Z})^{-1}. \quad (11)$$

In the TROLL system, this variance-covariance matrix is optionally returned along with the parameter estimates by the OLS estimation routine. Likewise, the return of the estimation residuals $\hat{\mathbf{u}} = \mathbf{z} - \mathbf{Z}\hat{\boldsymbol{\theta}}$ used for calculating the variance (10) is also an option.

IV: If there are other current model endogenous variables on the right hand side (r.h.s.) of the equation (9), the function $F_k(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t)$ should be changed to $F_k(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t)$ to denote the possibility of other current endogenous variables in the explanatory functions for $f(\mathbf{y}_t)$. If two or more endogenous variables are simultaneously determined by each other, single equation OLS will result in biased parameter estimates since the interdependence of the variables is “ignored”. In the case of two variables y_g and y_j , say $f(y_{g,t}) = F(y_{j,t})\theta_g + \dots + u_{g,t}$ and $f(y_{j,t}) = F(y_{g,t})\theta_j + \dots + u_{j,t}$, substitution shows that the regressor $F(y_{.,t})$ and the residual $u_{.,t}$ in each equation will be (non-linearly) correlated. To eliminate any simultaneity bias caused by correlations between current regressors and current residuals, the regressors may be replaced with instruments. An instrument should be uncorrelated with the residual, and be increasingly more efficient the more highly correlated it is with the regressor. If we let $\widehat{\mathbf{Z}}$ denote the matrix of the original regressors where at least one is replaced by an instrument⁷ \widehat{F}_k , the standard IV regressor is $\hat{\boldsymbol{\theta}}_{IV} = (\widehat{\mathbf{Z}}'\widehat{\mathbf{Z}})^{-1}\widehat{\mathbf{Z}}'\mathbf{z}$. In close resemblance to the OLS case we have the variance-covariance matrix of the IV estimator

$$\widehat{\Theta} = \hat{\sigma}^2(\widehat{\mathbf{Z}}'\widehat{\mathbf{Z}})^{-1}, \quad (12)$$

where $\hat{\sigma}^2 = (\mathbf{z} - \mathbf{Z}\hat{\boldsymbol{\theta}})'(\mathbf{z} - \mathbf{Z}\hat{\boldsymbol{\theta}})/(T - 1)$ is calculated from the original (*uninstrumented*) right hand side variables. Doing IV estimation by feeding preconstructed instruments into a computer program for OLS, the automatically estimated variance-covariance matrix is not the correct one, since $\hat{\sigma}^2 = (\mathbf{z} - \widehat{\mathbf{Z}}\hat{\boldsymbol{\theta}})'(\mathbf{z} - \widehat{\mathbf{Z}}\hat{\boldsymbol{\theta}})/(T - 1)$ is then used for its calculation. Consequently, the same holds for the returned estimation residuals. Hence, both the empirical residuals and the variance-covariance matrix of the parameter estimates have to be calculated manually after the estimation.

⁷Obtaining a good instrument, or first-stage regressor $\widehat{F}(y_t)$ is not trivial when F is a non-linear function. It is important to regress the non-linear function of the endogenous variable rather than the variable itself, since $\widehat{F}(y_t) \neq F(\hat{y}_t) = r(\hat{\varepsilon}(u_t))$, where \hat{y}_t is reduced form regression and $\hat{\varepsilon}_t$ the reduced form residuals. The last equality says that substituting a regressed variable \hat{y}_t into the right hand side non-linear functions F would make the instrument $F(\hat{y}_t)$ correlated with the residual u_t , since the reduced form residual $\varepsilon_t = \varepsilon(u_t)$ is a function of the structural residuals, cf. [3].

The single equation estimation with instruments is justified by the assumption of uncorrelated residual processes. This implies zero covariances between parameters in different IV equations, and consequently $\widehat{\Theta}$ is constructed to be block-diagonal for the IV estimated equations, just like the OLS equations. The variance of the IV residual is calculated like the sample variance of the OLS residuals (10), but with the IV estimates substituted for the OLS estimates. The IV routine in TROLL optionally returns the estimated variance-covariance matrix of the parameter estimates, and the empirical residuals $\mathbf{z} - \mathbf{Z}\hat{\theta}$ used for calculating the variance.

FIML: The final estimator used for the estimation of the model KVARTS91 is a full information system method, which is applied to a block of equations in the model. It is based on the explicit specification of a multivariate normal residual distribution for the interdependent variables in that block. When the residuals are correlated the FIML estimator is more efficient (less variance) than limited information methods like IV and OLS. We may use the model notation (2) to denote the sub-model, the FIML block: $\mathbf{f}_t = \mathbf{F}_t\theta + \mathbf{u}_t$, where $\mathbf{u}_t \sim \mathcal{IN}(\mathbf{0}, \Sigma)$ is non-diagonal, reflecting contemporaneous (only) residual correlations. The maximum likelihood method finds the most likely parameter values, given the model, the observations and the normality of the residuals, cf. [7, 9, 16, 19]. The estimates $\hat{\theta}$ are found by numerical maximization of the (concentrated) log-likelihood

$$\ell(\theta) = \sum_{t=2}^T \ln |\det \mathbf{J}_t| - \frac{T-1}{2} \ln(\det \widehat{\Sigma}),$$

where the Jacobian $\mathbf{J}_t = \partial \mathbf{u}_t / \partial \mathbf{y}_t$ is a matrix that varies over time for a non-linear model. The variance-covariance matrix of the FIML estimates $\hat{\theta}$ is estimated by

$$\widehat{\Theta} = - \left(\frac{\partial \ell(\theta)}{\partial \theta \partial \theta'} \right)_{\hat{\theta}}^{-1}. \quad (13)$$

System estimation by the FIML routine in TROLL's GREMLIN package optionally returns the estimated variance-covariance matrix (13) of the parameter estimates, as well as a matrix of estimation residuals (with the first observation lost to the lag):

$$\widehat{\mathbf{U}} = (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \dots, \hat{\mathbf{u}}_G) = \begin{pmatrix} \hat{u}_{1,2} & \hat{u}_{2,2} & \cdots & \hat{u}_{G,2} \\ \hat{u}_{1,3} & \hat{u}_{2,3} & \cdots & \hat{u}_{G,3} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{1,T} & \hat{u}_{2,T} & \cdots & \hat{u}_{G,T} \end{pmatrix}.$$

Each column \mathbf{u}_g contains the empirical residuals for a single endogenous variable $y_{g,t}$, i.e. one of the $g = 1, \dots, G$ equations in the FIML block. The contemporaneous covariance matrix Σ is estimated by the empirical covariance matrix

$$\widehat{\Sigma} = \frac{\widehat{\mathbf{U}}' \widehat{\mathbf{U}}}{T-1} = \left(\frac{\hat{\mathbf{u}}_g' \hat{\mathbf{u}}_j}{T-1} \right), \quad g, j \in \{1, \dots, G\}. \quad (14)$$

4 Simulation sample statistics

There are numerous existing statistics to choose from when we want to sum up the results of deterministic and stochastic model simulations. We shall use a few descriptive sample statistics to summarize the distribution properties of some important endogenous variables in the model KVARTS91. The results are conditional on the limited stochasticity imposed on the dynamic ex ante simulations (without any historical data values for exogenous and lagged endogenous variables).

Stochastic simulations produce a lot of data. Only for a very small model is it practical to analyse all the simulated data thoroughly. In this report we only use within-variable sample statistics to extract the most relevant information on the distribution of a number of single endogenous variables $y_{g,s}$ from their $s = T+1, \dots, T+S$ ex ante simulated samples $\{y_{g,s}^{(n)}\}$. To simplify the notation we drop the equation number footscript g on the single endogenous variable. First we look at a number of static or within-period s statistics, and start with three measures of the central tendency in a simulated sample. A consistent estimator of the expectation $E[y_t]$ is \tilde{y}_t , the mean of N stochastically simulated values $y_s^{(n)}$:

$$\tilde{y}_s = \frac{1}{N} \sum_{n=1}^N y_s^{(n)}, \quad (= \hat{y}_s \text{ for a linear model}). \quad (15)$$

The bias in the deterministic solution \hat{y}_s relative to the mean stochastic simulation \tilde{y}_s is a dimensionless measure of the error in the deterministic simulation relative to the mean stochastic simulation. Measured in percent, we have:

$$b_s = 100 \frac{\hat{y}_s - \tilde{y}_s}{\tilde{y}_s}, \quad (= 0 \text{ for a linear model}). \quad (16)$$

Many operative macro models show a fairly linear response to residual shocks and perturbations of parameter estimates, cf. e.g. [11, 13, 19]. Since KVARTS91 is mostly a log-linear model (with small curvature in the vicinity of the output values) we expect only minor deterministic biases, say maximum a few percent.

The next three second-order statistics measure the dispersion of a simulated sample. A consistent estimator of the variance of an endogenous variable is the “mean”⁸ of the squared deviations of the stochastic solutions $y_s^{(n)}$ from their sample mean \tilde{y}_s :

$$\tilde{\sigma}_s^2 = \frac{1}{N-1} \sum_{n=1}^N (y_s^{(n)} - \tilde{y}_s)^2. \quad (17)$$

An approximate 95 percent prediction interval for a (close-to-)normal sample distribution is spanned by $y_s \pm 2\tilde{\sigma}_s$. The width of a normality interval relative to the level of the stochastic mean \tilde{y}_s is a dimensionless measure of “uncertainty” in the model solution

⁸Consult any good statistics text for the story about why the denominator of (17) is $N-1$ instead of N . Anyway, if the difference between the two matters, then the number of replications N is too low!

conditional on the imposed stochasticity (i.e. the distributions of the stochastic input variables). Again measured in percent, we have

$$n_s = 100 \frac{4\tilde{\sigma}_s}{\tilde{y}_s}. \quad (18)$$

Alternatively, a 95 percent prediction interval for a non-normal distribution is approximated by a percentile interval $(y_s^{(0.025)}, y_s^{(0.975)})$ which includes the *central* 95 percent of the simulated values. The relative width of this interval, in percent of the variable's level, is

$$q_s = 100 \frac{y_s^{(0.975)} - y_s^{(0.025)}}{\tilde{y}_s}, \quad (\approx n_s \text{ for a normal distribution}). \quad (19)$$

For a skewed distribution (19) is not the narrowest interval containing 95 percent of the simulated values.

In the case of an approximately normal sample distribution, we should have $n_s \approx q_s$. But the percentile width q_s has large small sample variation, so when doing a limited number of replications (e.g. 1000 or even 10000 simulated endogenous values), $n_s \approx q_s$ should not be interpreted as normality of the distribution. To confirm possible normality, higher order statistics are necessary.

To check for symmetry, fat tails and normality of the simulated samples, three higher-order central moment statistics — skewness, excess kurtosis, and their joint test, the Jarque-Bera normality statistic [17] — are useful:

$$s_s = \frac{1}{N} \sum_{n=1}^N \left(\frac{y_s^{(n)} - \tilde{y}_s}{\tilde{\sigma}_s} \right)^3, \quad (= 0 \text{ for a symmetric distribution}), \quad (20)$$

$$k_s = \frac{1}{N} \sum_{n=1}^N \left(\frac{y_s^{(n)} - \tilde{y}_s}{\tilde{\sigma}_s} \right)^4 - 3, \quad (= 0 \text{ with normal distribution tails}), \quad (21)$$

$$jb_s = N \left(\frac{s_s^2}{6} + \frac{k_s^2}{24} \right), \quad (= 0 \text{ if the distribution is normal}). \quad (22)$$

A sample distribution is downward asymmetric if $s_s < 0$ and, accordingly, upward asymmetric if $s_s > 0$. In case of a true normal distribution, s_s has the approximate variation $V[s_s] = 6/N$. The sample distribution has positive kurtosis ($k_s > 0$) if it has leaner tails than a normal distribution, and negative kurtosis ($k_s < 0$) if the tails are fatter. In case of a true normal distribution, k_s has the approximate variation $V[k_s] = 24/N$. These variations are used to “normalize” the normality statistic jb_s , which is a joint skewness-kurtosis statistic that is distributed χ_2^2 . In small samples such higher order statistics (20)–(22) should be used with caution since they are not very robust. They are sensitive to small sample anomalies and outliers. In section 6, before looking at the simulation results, we return to the variance and confidence of the higher order statistics (20)–(22)

In section 6, the eight statistics (15)–(22) are used to sum up the simulation results in the form of tables to accompany the graphic plots.

5 Simulating the KVARTS91 model

KVARTS91 is a medium-size quarterly macroeconometric model. The model is relatively disaggregated, with many sectors and commodities. A previous version is documented in [4]. The model contains more than 1600 equations, out of which 108 are structural econometric equations. The remaining are definitions, mostly input-output equations. The 108 econometric equations are estimated by different methods. 97 equations are estimated individually by single equation OLS. Two current endogenous variables appear on the right hand side of each other's equation, hence the 2 equations are estimated with instruments. Due to some parameters appearing not just in one but in several equations, a system of 9 equations are estimated by FIML system estimation method. Altogether this amounts to more than 1000 estimated parameters.

The econometric equations are given a dynamic specification, with long memory in terms of lags (≤ 36 quarters). This dependence on the past along with the lack of data in future ex ante simulations, imply dynamic simulations where previously simulated values are fed back into the simulations as lagged values. The simulations start in 1993 1 and end eight quarters later in 1994 4. For lagged endogenous variables, observations are used up to and including 1992 4. Thereafter simulated values ($y_s^{(n)}$, $s = 1993\ 1$ to $1994\ 4$) are fed into the model as values of the lagged variables. The observations from the two or three first quarters of 1993 (y_t , $t = 1993\ 1$ to $1993\ 3$, in the tables of section 6) are not used in the simulations. They are only displayed as reference points. Extrapolated (expectation) values, and no observations, are used for the exogenous variables. The lag structure causes the prediction intervals (i.e. the uncertainty) to keep unfolding during the simulation (cf. n_s or q_s in the tables of section 6), though there are some ("stationary") exceptions.

The model is simulated 4 times. The first is a deterministic simulation \hat{y}_s that serves as a reference simulation for the bias statistic b_s . Then follows three stochastic simulations. One is with stochastic residuals only, and another is with stochastic parameter estimates only. Finally the two are combined into a simulation with both stochastic residuals and stochastic parameter estimates. For all the equations estimated by single equation methods like OLS or IV, each equation is shocked individually each simulation period by an independent and normally distributed additive residual with the same variance as the estimation residual, cf. (7), (10). The estimated parameter values of an equation are perturbed only once each replication (and not each period) according to a multivariate normal distribution with zero mean and the same variance-covariance matrix as its single-equation parameter estimates, cf. (7), (11), (12). The block of FIML estimated equations are shocked each period by adding a multivariate normal vector of residuals with the same variance-covariance matrix as the estimation residuals, cf. (7), (14). All the estimated parameter values in the block of equations are perturbed once each replication according to a multivariate normal distribution with zero mean and the same variance-covariance matrix as the FIML parameter estimates, cf. (7), (13).

There is one major exception to the above outlined scheme of perturbing the parameter estimates. The model KVARTS91 was (once) big enough to just break the TROLL limit of $2^{12} = 4096$ coefficients. By defining the estimated parameters of three blocks of equations (5 export equations, 3 import equations and 9 investment equations) to be so-called Almon coefficients, the total number of coefficients in the model was kept below 2^{12} . Unfortunately, there is a trade-off. Currently, TROLL's Stochastic Simulator cannot shock Almon coefficients. The results for several of the variables reported in section 6 are influenced by this. In the next section we return to this problem.

To maintain symmetry in the sample distributions of the generated normal residuals and parameter estimates, we use the option of antithetic variates in TROLL's Stochastic Simulator, cf. formulas (7), (8), and the reference [15]. When the (antithetically) improved sampling of the input distributions carries through the model's mapping to the simulated output distributions, the variance of the output sample statistics may decrease dramatically. Such an effect on the estimated deterministic bias b_s is reported in [6]. The improvements from a variance reducing technique like antithetic sampling may be especially noticeable when doing only a limited number of replications. A (very) large number of replications is necessary for making reliable histograms of the simulated sample distributions. But for lower order moments like the sample statistics of section 4, a smaller number may be acceptable — especially when the model is only mildly non-linear. In light of all the omitted uncertainty due to modelling assumptions and conditioning, demanding very high accuracy in measures of econometric uncertainty does not make sense. Due to the size of the model we only do a limited number of replications. Repeating the simulations with 100, 200, 500 and 1000 replications, we note minimal changes in the results and statistics by stepping from 500 up to 1000 antithetic replications. Hence, we go no further. The results documented in this report is from 1000 antithetic replications.

The numerical solution of the model is found by an iterative procedure. When simulating a non-linear model the iterations do not always converge to a solution within the iteration limit. Or the solution procedure stops when facing numerical problems such as taking the logarithmic value of a negative number. The former problem may occur in a deterministic simulation due to unfortunate exogenous values and/or the dynamics of the model, while the latter problem is generated by “extreme” values of the stochastic perturbations of the input variables. The failing to solve problem is mainly due to the fact that we are simulating an estimation model rather than what we can call a simulation model. The specification of the model is aimed at the estimation of the parameters from the observed data (cf. (2)–(4)). Hence, the structure of the model equations reflects the procedure of fitting the equations to the observations. At the same time the equation structure reflects the ignorance of simulation properties. When attempting stochastic simulation of the model, the lack of simulation robustness becomes evident as the model fails to solve. For KVARTS91 this problem occurs often, up to 1/3 of the replications failed to solve. Consequently, the prediction intervals may be biased and underestimated.

6 Stochastic simulation results

This section presents simulation results for selected endogenous variables of interest. The variable presented are (in this order):

- C: Private consumption, fixed 1991 prices, billion NOK,
- G: Public consumption, fixed 1991 prices, billion NOK,
- JK: Gross fixed capital formation, fixed 1991 prices, billion NOK,
- JK6: Gross investments, mainland, fixed 1991 prices, billion NOK,
- A: Total exports, fixed 1991 prices, billion NOK,
- A4: Export, traditional goods, fixed 1991 prices, billion NOK,
- I: Total imports, fixed 1991 prices, billion NOK,
- I4: Imports, traditional goods, fixed 1991 prices, billion NOK,
- Q: GDP, fixed 1991 prices, billion NOK,
- Q6: GDP, mainland, fixed 1991 prices, billion NOK,
- LW: Man hours, million hours,
- NW: Employed wage earners, million persons,
- NT: Labour force, million persons,
- UR: Unemployment rate, percentage,
- KPI: Consumer price index, 1991 = 1,
- PA4: Export deflator, traditional goods, 1991 = 1,
- WW: Average wage rate, NOK,
- RS500: Current account, billion NOK.

Each variable occupies a spread (two opposing pages), with three tables of simulation statistics on the left page and three graphic plots on the right page. The tables and the plots show, from the top down, results for simulations with stochastic residuals only (top), stochastic parameter estimates only (middle), and both stochastic residuals and stochastic parameter estimates (bottom). The plots on the right page correspond horizontally to the tables on the left page. The tables display the simulation sample statistics of section 4, while the graphic plots reveal the dynamics of the simulated developments in the endogenous variables. The solid graph in the plots is the mean of the stochastic solutions. It is enveloped between the broken graphs of plus/minus two standard deviations. Observations are plotted dot-and-dashed for the first three periods of 1993.

In the previous section we noted the problem that the current version of TROLL's Stochastic Simulator cannot shock the Almon coefficients. The obvious consequences of this "omission" (which cannot be mended since the Stochastic Simulator is precoded and closed module) is that the estimated prediction intervals most likely are too narrow in the cases of simulations with stochastic parameter estimates, alone or in combination with stochastic residuals. Of the reported variables, the prediction intervals for the following variables are most certainly too narrow, since these variables are directly affected by the "deterministic" parameter estimates in three blocks of 9, 5 and 3 equations:

JK, JK6: aggregates over the 9 investment equations,

A, A4: aggregates over the 5 export equations,

I, I4: aggregates over the 3 import equations.

The prediction intervals estimated by stochastic simulations are highly conditional on modelling assumptions and the structure of the stochastic simulation input. If the widths of the simulated prediction intervals are viewed as lower bounds on the uncertainty in the model predictions, the omissions above and the model's failing to solve are quite tolerable. They constitute minor problems that can be avoided by making the necessary changes in the TROLL implementation of the next version of the model KVARTS.

To facilitate the interpretation of the up-coming tables of simulation statistics we note that for 1000 replication a 95 percent confidence interval for the symmetry statistic s_s under true normality is $\pm 2\sigma(s_s) = \pm 2\sqrt{6/1000} \approx \pm 0.155$. The corresponding interval for the kurtosis k_s is $\pm 2\sigma(k_s) = \pm 2\sqrt{24/1000} \approx \pm 0.31$. We also tabulate some critical values of the chi-square distribution χ^2 to aid the interpretation of the Jarque-Bera normality statistic jb_s :

$Pr(\text{normality}) \geq$	99%	95%	90%	75%	50%	25%	10%	5%	1%
Value of $jb_s \leq$	0.020	0.103	0.211	0.575	1.39	2.77	4.61	5.99	9.21

As previously noted, distributional properties inferred from a small sample are not robust due to the higher moments involved. Does an extreme sample statistic value that is still within a *wide* small sample 95 percent confidence interval represent a large deviation from the expectation value? Or does it merely reflect non-normality? One may easily reject normality due to "outliers" (extreme values seem to occur more frequently in a computer generated random sequence than in the real world ... ?) On the other hand, a normal-like — or obviously not normal — classification may be completely sufficient for the intended level of precision in the inference.

From the up-coming tables, we preview a few obvious and general simulation results for the KVARTS91 model:

- Hardly any bias in the deterministic simulation relative to the mean stochastic simulation, i.e. < 1 percent of the level of the variable for all variables reported (cf. b_s in the tables).
- The width of the approximate 95 percent prediction interval relative to the level of the variable varies from less than 1 percent (in the case of G, public consumption) up to more than 100 percent (RS500, current account). For most variables the uncertainty (interval widths) are less than 10 percent of the level of the variable (cf. n_s or q_s in the tables).
- The simulated small samples (1000 replications) suggest approximately symmetric univariate distributions that have mostly slightly fatter tails than a normal distribution ($k_s > 0$). But for most variables and most simulation periods normality cannot be rejected at the 5 percent level (since the jb_s statistic is less than 5.99).
- Simulating with only stochastic residuals, and not stochastic parameter estimates in addition, captures a (very) large majority of the uncertainty in the solution measured by the relative width of the prediction intervals.

From the results we make only the following two conclusions for KVARTS91:

- The practically non-existent deterministic bias reveals no “asymmetric” effect of non-linearity in the mapping of stochastic residuals and parameter estimates onto the endogenous variables. The expectations of the input variables are mapped onto the expectations of the endogenous variables. A deterministic simulation is an “optimal” point estimate of the stochastic model solution.
- The (wide) dispersion of the simulated model solutions — reflected by the widths of the prediction intervals — show that the model predictions are imprecise or uncertain. When comparing simulations that differ only in certain exogenous assumptions, e.g. when evaluating different policy alternatives, the simulated solutions should be viewed in light of the uncertainty in the simulations. Or even better, the analysis should be stochastic rather than deterministic. The benefits of stochastic simulations may be more obvious when the notion of probability is brought into the context of an economic analysis, allowing one to arrive at more subtle and realistic conclusions.

C: Private consumption, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	84.4831	84.5592	84.5738	-0.02	0.8614	4.07	4.01	0.05	-0.05	0.49
9	2	87.6510	87.4589	87.4804	-0.02	1.0367	4.74	4.53	0.11	0.04	1.99
9	3	91.5091	89.4024	89.4414	-0.04	1.0562	4.72	4.67	0.07	0.14	1.56
9	4	NA	98.8091	98.8603	-0.05	1.2703	5.14	4.85	0.04	-0.06	0.48
1	1	NA	87.2134	87.2467	-0.04	1.3060	5.99	5.83	0.04	-0.16	1.37
9	2	NA	90.5296	90.5687	-0.04	1.3754	6.07	6.03	0.05	-0.04	0.47
9	3	NA	92.0535	92.1444	-0.10	1.4397	6.25	6.32	0.09	0.03	1.33
4	4	NA	101.8470	101.9740	-0.13	1.6087	6.31	6.06	0.09	0.05	1.32
<i>Mean</i>		87.8810	91.4840	91.5362	-0.06	1.2443	5.41	5.29	0.07	-0.01	1.13

Table 1.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	84.4831	84.5592	84.5397	0.02	0.6179	2.92	2.90	0.13	0.27	5.92
9	2	87.6510	87.4589	87.4565	0.00	0.7730	3.54	3.48	0.18	0.02	5.61
9	3	91.5091	89.4024	89.3957	0.01	0.8706	3.90	3.72	0.19	-0.06	6.20
9	4	NA	98.8091	98.8161	-0.01	1.0160	4.11	3.95	0.25	-0.03	10.27
1	1	NA	87.2134	87.2239	-0.01	1.1641	5.34	5.18	0.27	0.20	13.75
9	2	NA	90.5296	90.5594	-0.03	1.2106	5.35	5.23	0.23	0.05	8.98
9	3	NA	92.0535	92.0598	-0.01	1.2101	5.26	5.00	0.22	0.20	9.92
4	4	NA	101.8470	101.8620	-0.02	1.3586	5.34	5.17	0.23	0.25	11.26
<i>Mean</i>		87.8810	91.4840	91.4891	-0.01	1.0276	4.47	4.33	0.21	0.11	8.99

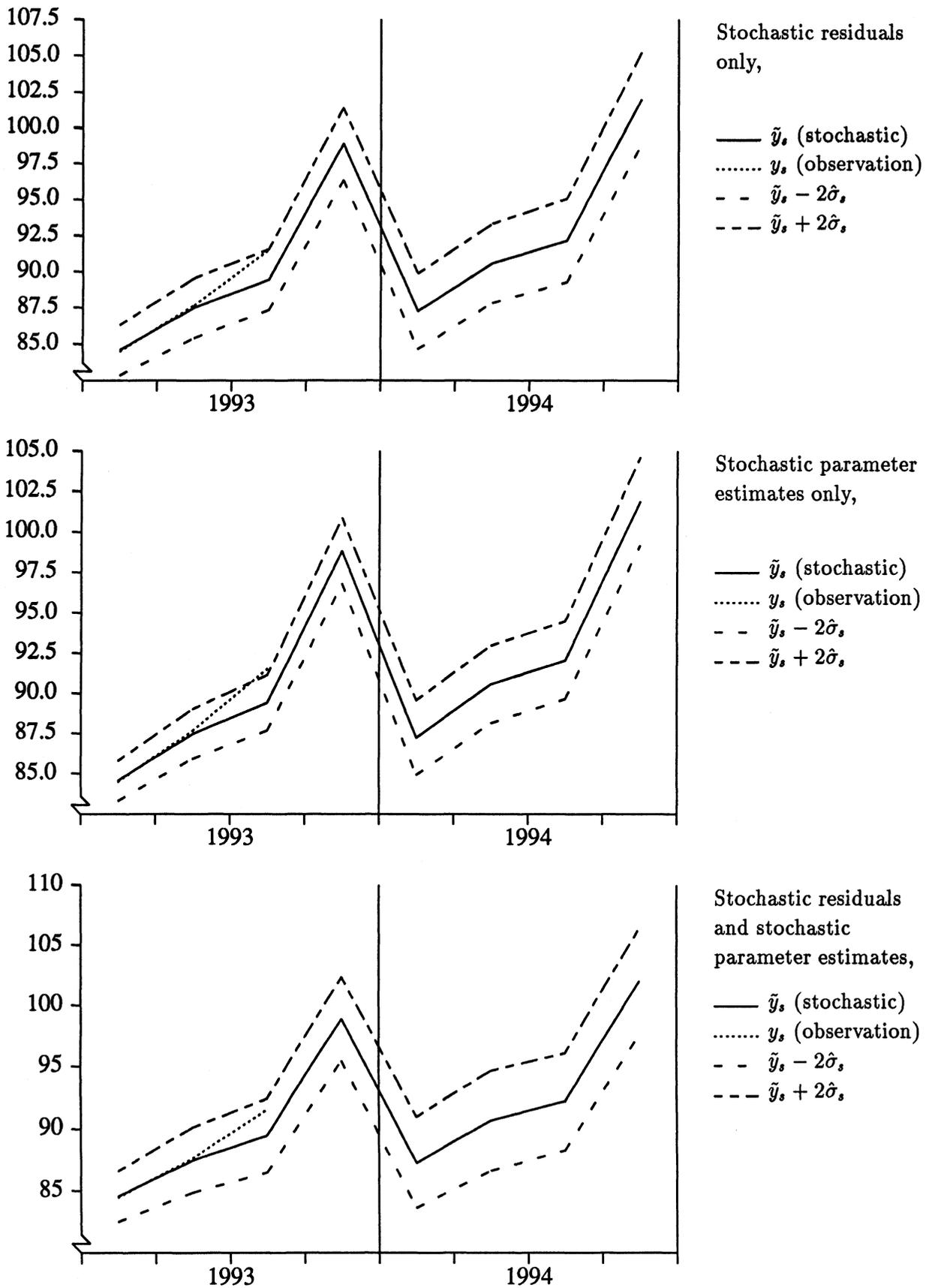
Table 1.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	84.4831	84.5592	84.5630	-0.00	1.0261	4.85	4.70	0.09	-0.11	1.99
9	2	87.6510	87.4589	87.4820	-0.03	1.3038	5.96	5.92	0.16	0.42	11.64
9	3	91.5091	89.4024	89.4445	-0.05	1.4691	6.57	6.42	0.24	0.17	11.24
9	4	NA	98.8091	98.8989	-0.09	1.7239	6.97	7.08	0.25	0.32	14.98
1	1	NA	87.2134	87.2945	-0.09	1.8208	8.34	8.04	0.25	0.31	14.18
9	2	NA	90.5296	90.6583	-0.14	2.0047	8.84	8.74	0.37	0.79	49.74
9	3	NA	92.0535	92.1937	-0.15	1.9594	8.50	8.51	0.35	0.56	33.63
4	4	NA	101.8470	102.0460	-0.20	2.2183	8.70	8.55	0.30	0.14	16.04
<i>Mean</i>		87.8810	91.4840	91.5726	-0.09	1.6908	7.34	7.25	0.25	0.32	19.18

Table 1.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 1. Graphic plots of C corresponding to the tables on the facing page.



G: Public consumption, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	37.5280	37.4333	37.4304	0.01	0.0292	0.31	0.29	0.26	-0.18	12.55
9	2	38.8082	38.5821	38.5790	0.01	0.0360	0.37	0.37	0.19	-0.05	6.39
9	3	38.9570	39.1745	39.1703	0.01	0.0410	0.42	0.40	0.06	0.02	0.59
9	4	NA	42.0384	42.0329	0.01	0.0440	0.42	0.41	0.00	0.06	0.13
1	1	NA	38.5544	38.5470	0.02	0.0440	0.46	0.45	0.12	-0.30	5.96
9	2	NA	39.4019	39.3918	0.03	0.0496	0.50	0.48	0.13	-0.21	4.58
9	3	NA	39.7764	39.7636	0.03	0.0560	0.56	0.53	0.05	-0.19	1.99
4	4	NA	42.7300	42.7138	0.04	0.0561	0.52	0.51	0.00	0.13	0.66
<i>Mean</i>		38.4310	39.7113	39.7036	0.02	0.0445	0.45	0.43	0.10	-0.09	4.11

Table 2.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	37.5280	37.4333	37.4312	0.01	0.0126	0.13	0.14	0.34	0.22	21.81
9	2	38.8082	38.5821	38.5795	0.01	0.0172	0.18	0.18	0.20	0.07	6.82
9	3	38.9570	39.1745	39.1720	0.01	0.0222	0.23	0.23	0.20	0.03	6.48
9	4	NA	42.0384	42.0355	0.01	0.0283	0.27	0.28	0.11	0.02	2.05
1	1	NA	38.5544	38.5513	0.01	0.0337	0.35	0.36	0.07	0.04	0.97
9	2	NA	39.4019	39.3986	0.01	0.0393	0.40	0.41	0.11	0.10	2.33
9	3	NA	39.7764	39.7738	0.01	0.0445	0.45	0.45	0.15	0.06	4.06
4	4	NA	42.7300	42.7276	0.01	0.0516	0.48	0.50	0.16	0.12	5.14
<i>Mean</i>		38.4310	39.7113	39.7087	0.01	0.0312	0.31	0.32	0.17	0.08	6.21

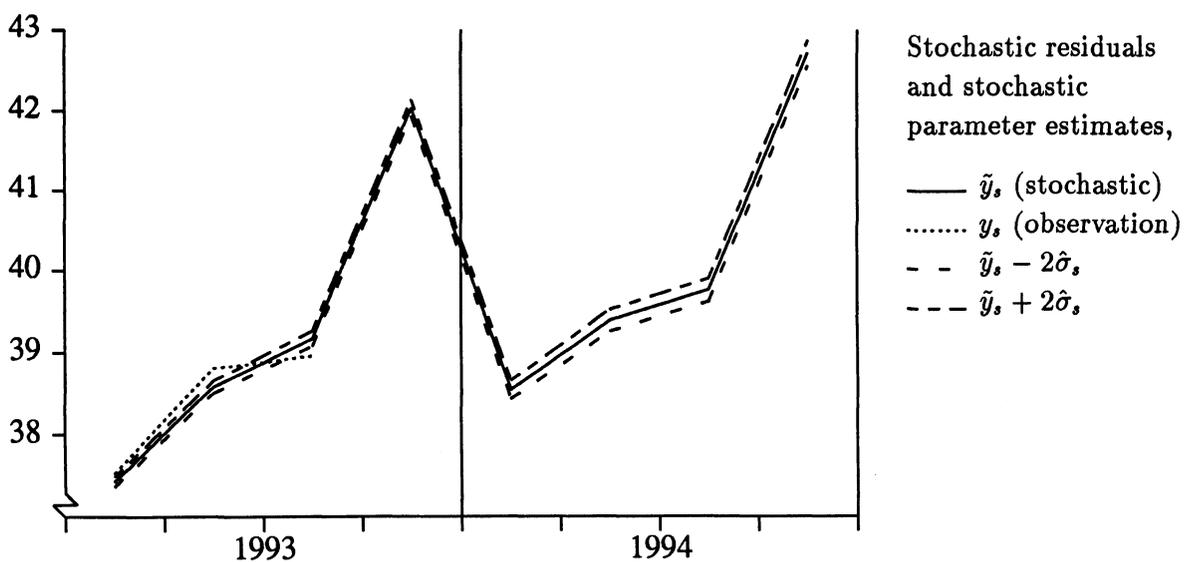
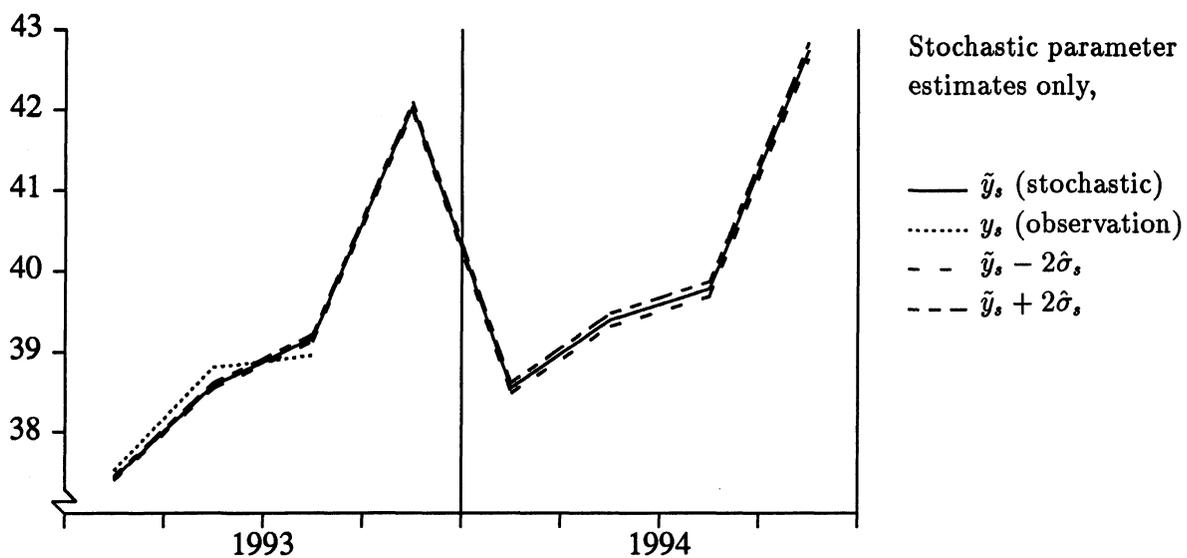
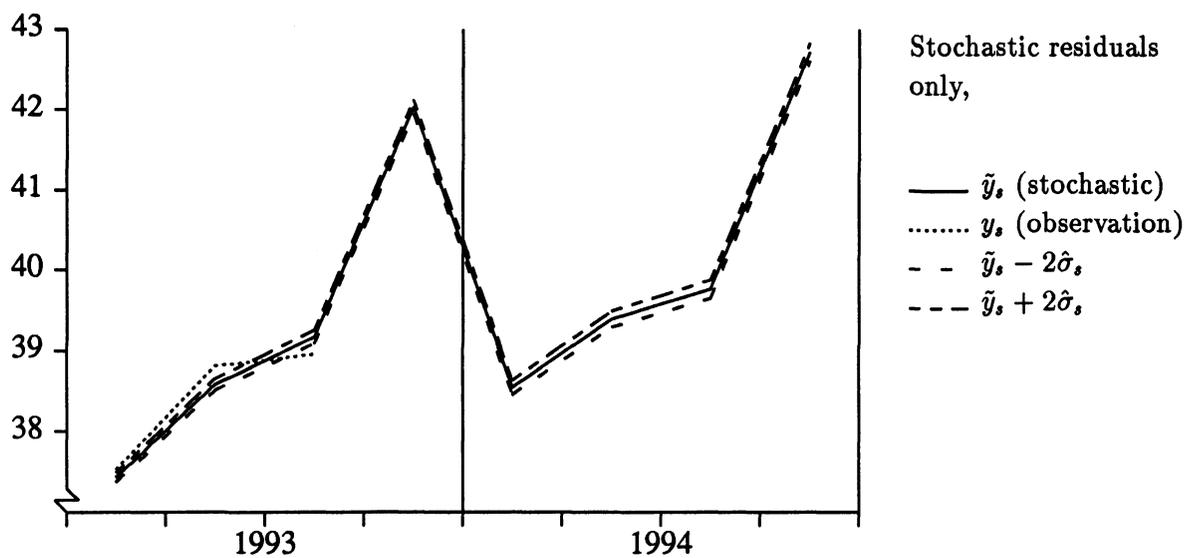
Table 2.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	37.5280	37.4333	37.4288	0.01	0.0305	0.33	0.33	0.12	0.05	2.41
9	2	38.8082	38.5821	38.5769	0.01	0.0383	0.40	0.40	0.09	0.12	2.02
9	3	38.9570	39.1745	39.1692	0.01	0.0463	0.47	0.45	0.07	-0.35	6.14
9	4	NA	42.0384	42.0315	0.02	0.0518	0.49	0.49	0.05	0.21	2.14
1	1	NA	38.5544	38.5448	0.02	0.0560	0.58	0.55	-0.01	0.09	0.38
9	2	NA	39.4019	39.3896	0.03	0.0652	0.66	0.64	-0.06	0.31	4.68
9	3	NA	39.7764	39.7638	0.03	0.0714	0.72	0.72	0.02	0.17	1.29
4	4	NA	42.7300	42.7130	0.04	0.0781	0.73	0.74	-0.02	0.30	3.74
<i>Mean</i>		38.4310	39.7113	39.7022	0.02	0.0547	0.55	0.54	0.03	0.11	2.85

Table 2.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 2. Graphic plots of G corresponding to the tables on the facing page.



JK: Gross fixed capital formation, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	23.7431	24.1577	24.1511	0.03	0.5872	9.73	9.73	0.06	-0.07	0.89
9	2	36.9860	41.5109	41.5123	-0.00	0.7168	6.91	6.53	-0.02	-0.10	0.43
9	3	55.8284	55.0390	55.0710	-0.06	0.8234	5.98	5.93	0.00	0.09	0.34
3	4	NA	37.0594	37.1890	-0.35	0.9387	10.10	9.66	0.01	0.12	0.62
1	1	NA	29.8047	30.0412	-0.79	1.0189	13.57	13.04	0.02	-0.25	2.70
9	2	NA	38.0537	38.4079	-0.92	1.0977	11.43	11.12	-0.04	-0.21	2.12
9	3	NA	33.4458	33.8655	-1.24	1.1072	13.08	12.83	-0.01	-0.20	1.68
4	4	NA	36.5219	37.0556	-1.44	1.2103	13.06	13.21	0.01	0.12	0.65
Mean		38.8525	36.9491	37.1617	-0.60	0.9375	10.48	10.26	0.00	-0.06	1.18

Table 3.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	23.7431	24.1577	24.1619	-0.02	0.2432	4.03	3.98	0.08	-0.00	1.01
9	2	36.9860	41.5109	41.5161	-0.01	0.4739	4.57	4.29	0.11	0.02	1.99
9	3	55.8284	55.0390	55.0472	-0.01	0.6483	4.71	4.44	0.08	0.02	0.99
3	4	NA	37.0594	37.0695	-0.03	0.8341	9.00	8.50	-0.07	0.02	0.84
1	1	NA	29.8047	29.8210	-0.05	1.0597	14.21	13.31	-0.19	0.11	6.77
9	2	NA	38.0537	38.0568	-0.01	1.2756	13.41	12.58	-0.26	0.17	12.61
9	3	NA	33.4458	33.4237	0.07	1.4717	17.61	16.52	-0.33	0.25	20.66
4	4	NA	36.5219	36.4742	0.13	1.6682	18.29	17.15	-0.43	0.40	37.92
Mean		38.8525	36.9491	36.9463	0.01	0.9593	10.73	10.10	-0.13	0.12	10.35

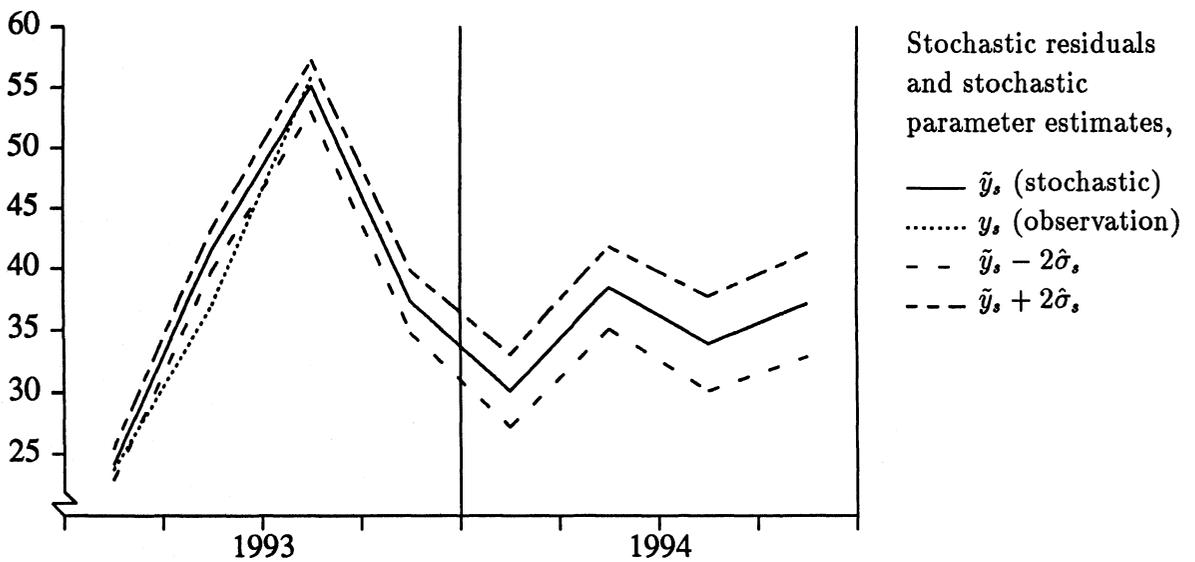
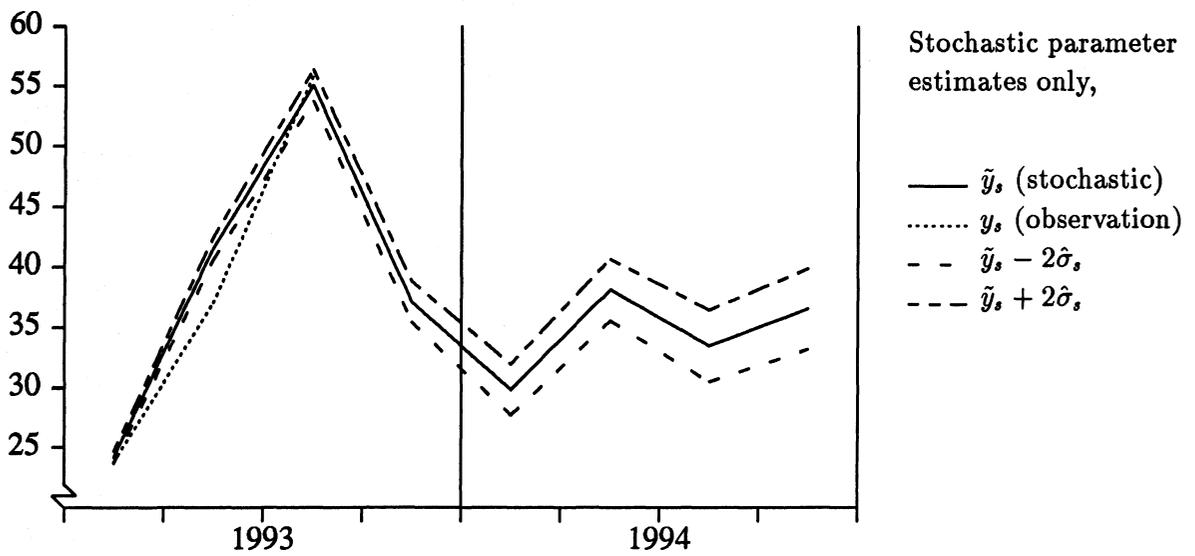
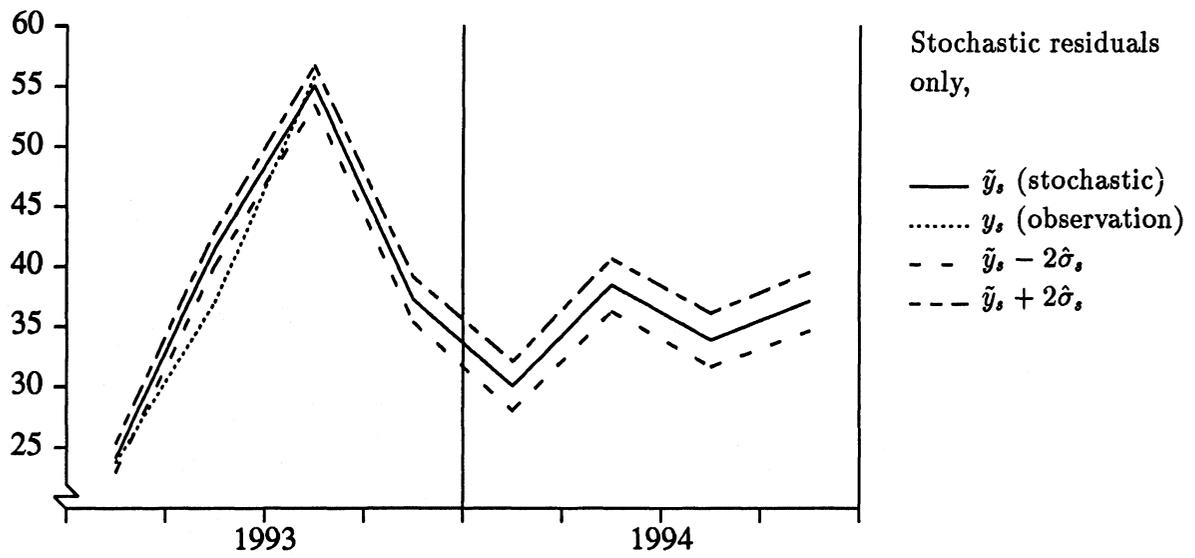
Table 3.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	23.7431	24.1577	24.1722	-0.06	0.6313	10.45	10.13	-0.02	-0.20	1.84
9	2	36.9860	41.5109	41.5274	-0.04	0.8734	8.41	7.92	-0.02	-0.27	3.08
9	3	55.8284	55.0390	55.1138	-0.14	1.0553	7.66	7.45	-0.02	-0.12	0.75
3	4	NA	37.0594	37.2325	-0.46	1.2616	13.55	13.13	-0.05	-0.04	0.47
1	1	NA	29.8047	30.0926	-0.96	1.4513	19.29	19.17	-0.13	0.35	7.88
9	2	NA	38.0537	38.4229	-0.96	1.6775	17.46	17.35	-0.11	0.24	4.23
9	3	NA	33.4458	33.8889	-1.31	1.8939	22.35	22.04	-0.16	0.59	18.66
4	4	NA	36.5219	37.0556	-1.44	2.1012	22.68	22.28	-0.14	0.53	15.38
Mean		38.8525	36.9491	37.1882	-0.67	1.3682	15.23	14.93	-0.08	0.13	6.54

Table 3.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 3. Graphic plots of JK corresponding to the tables on the facing page.



JK6: Gross investments, mainland, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	18.3674	18.6728	18.6662	0.03	0.5872	12.58	12.59	0.06	-0.07	0.88
9	2	20.9865	21.0967	21.0983	-0.01	0.7168	13.59	12.84	-0.02	-0.10	0.44
9	3	21.4242	22.5380	22.5701	-0.14	0.8234	14.59	14.47	0.00	0.09	0.34
9	4	NA	27.4383	27.5680	-0.47	0.9387	13.62	13.03	0.01	0.12	0.62
1	1	NA	20.4679	20.7045	-1.14	1.0189	19.68	18.92	0.02	-0.25	2.69
9	2	NA	22.5581	22.9125	-1.55	1.0977	19.16	18.63	-0.05	-0.21	2.12
9	3	NA	23.0497	23.4695	-1.79	1.1072	18.87	18.51	-0.01	-0.20	1.68
4	4	NA	28.5291	29.0629	-1.84	1.2103	16.66	16.85	0.01	0.12	0.65
Mean		20.2594	23.0438	23.2565	-0.86	0.9375	16.10	15.73	0.00	-0.06	1.18

Table 4.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993 1, \dots, 1994 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	18.3674	18.6728	18.6771	-0.02	0.2432	5.21	5.14	0.08	-0.00	0.97
9	2	20.9865	21.0967	21.1020	-0.03	0.4739	8.98	8.44	0.11	0.02	1.96
9	3	21.4242	22.5380	22.5464	-0.04	0.6483	11.50	10.84	0.08	0.02	0.96
9	4	NA	27.4383	27.4484	-0.04	0.8341	12.16	11.48	-0.07	0.03	0.85
1	1	NA	20.4679	20.4844	-0.08	1.0598	20.69	19.38	-0.19	0.11	6.79
9	2	NA	22.5581	22.5613	-0.01	1.2756	22.62	21.22	-0.26	0.17	12.64
9	3	NA	23.0497	23.0276	0.10	1.4717	25.56	23.97	-0.33	0.25	20.68
4	4	NA	28.5291	28.4813	0.17	1.6682	23.43	21.96	-0.43	0.40	37.92
Mean		20.2594	23.0438	23.0411	0.01	0.9593	16.27	15.30	-0.13	0.12	10.35

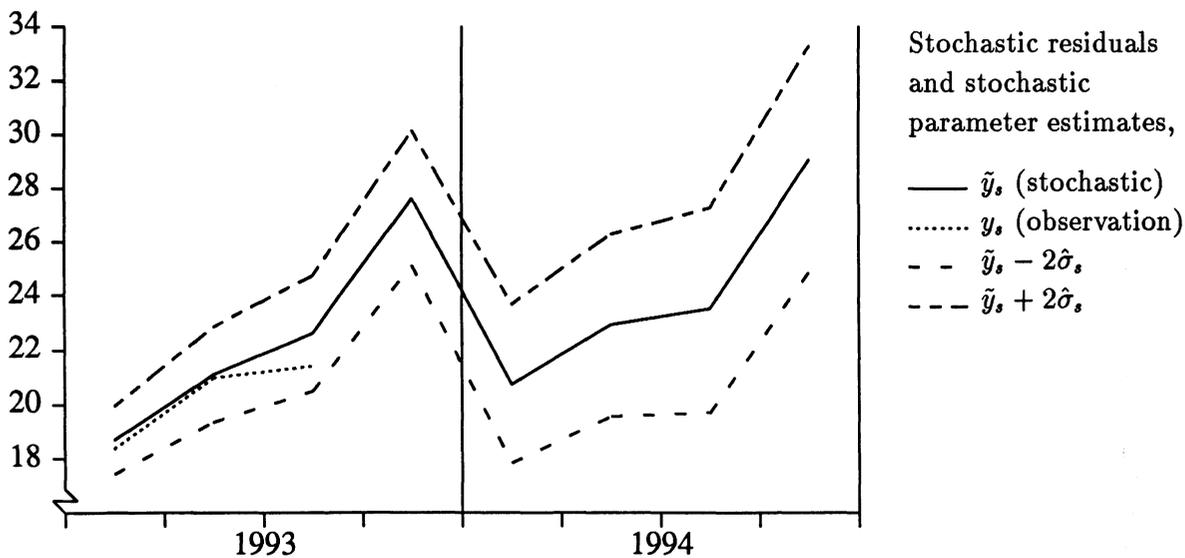
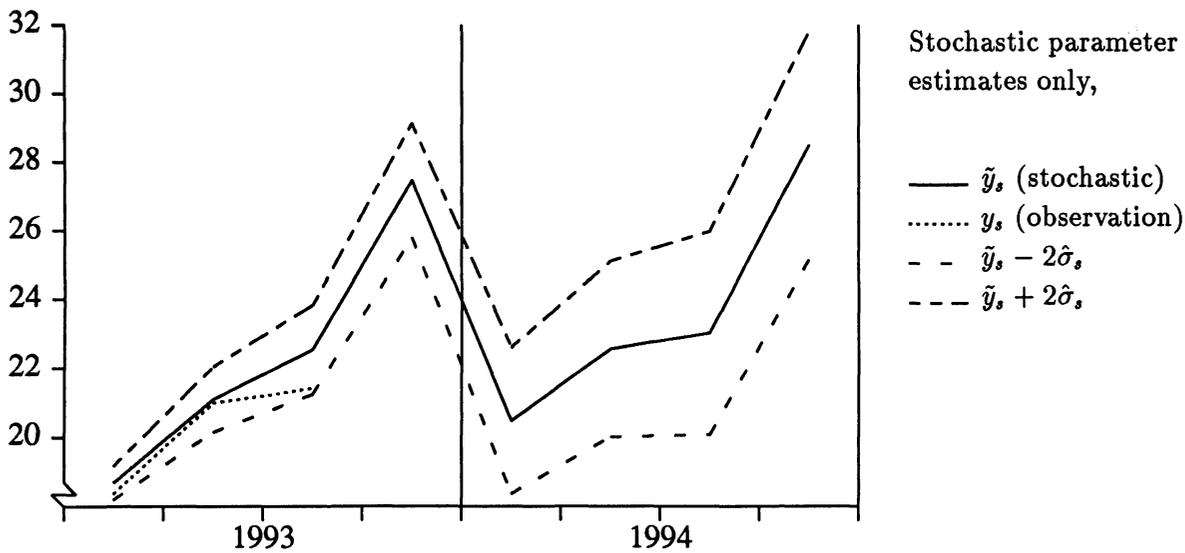
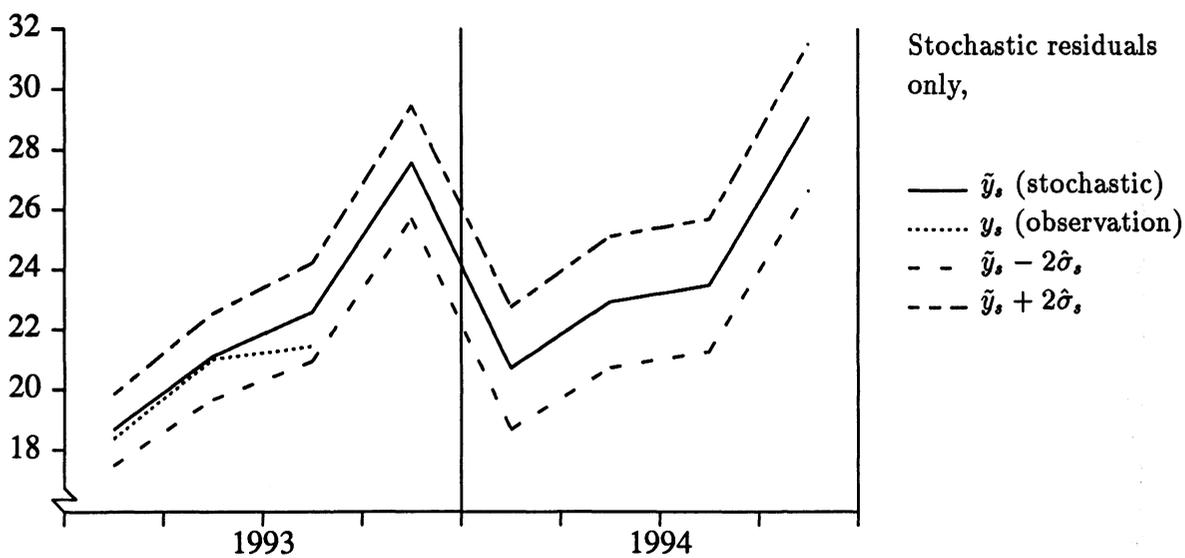
Table 4.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	18.3674	18.6728	18.6873	-0.08	0.6313	13.51	13.10	-0.02	-0.20	1.84
9	2	20.9865	21.0967	21.1135	-0.08	0.8734	16.55	15.57	-0.02	-0.27	3.08
9	3	21.4242	22.5380	22.6129	-0.33	1.0553	18.67	18.15	-0.03	-0.12	0.76
9	4	NA	27.4383	27.6114	-0.63	1.2616	18.28	17.70	-0.05	-0.04	0.47
1	1	NA	20.4679	20.7561	-1.39	1.4513	27.97	27.79	-0.13	0.35	7.90
9	2	NA	22.5581	22.9274	-1.61	1.6775	29.27	29.08	-0.11	0.24	4.23
9	3	NA	23.0497	23.4928	-1.89	1.8939	32.25	31.80	-0.16	0.59	18.67
4	4	NA	28.5291	29.0629	-1.84	2.1012	28.92	28.41	-0.14	0.53	15.39
Mean		20.2594	23.0438	23.2831	-0.98	1.3682	23.18	22.70	-0.08	0.13	6.54

Table 4.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 4. Graphic plots of JK6 corresponding to the tables on the facing page.



A: Total exports, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	78.4038	78.2838	78.3144	-0.04	0.7215	3.69	3.56	0.11	-0.19	3.61
9	2	84.0686	81.4279	81.4574	-0.04	0.7774	3.82	3.72	0.05	0.05	0.51
9	3	82.2897	82.4727	82.5171	-0.05	0.8707	4.22	4.15	0.11	-0.11	2.57
9	4	NA	85.1033	85.1677	-0.08	0.8468	3.98	3.84	0.13	-0.05	2.98
1	1	NA	82.9964	83.0366	-0.05	0.8461	4.08	4.02	0.04	-0.29	3.90
9	2	NA	86.0063	86.0393	-0.04	0.9039	4.20	4.25	0.10	0.20	3.35
9	3	NA	86.8970	86.9386	-0.05	0.9226	4.24	4.14	0.13	-0.17	4.21
4	4	NA	87.8515	87.9032	-0.06	0.9181	4.18	4.21	0.12	-0.02	2.36
Mean		81.5874	83.8798	83.9217	-0.05	0.8509	4.05	3.99	0.10	-0.07	2.94

Table 5.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s , and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	78.4038	78.2838	78.2768	0.01	0.0298	0.15	0.15	0.83	0.50	127.10
9	2	84.0686	81.4279	81.4176	0.01	0.0749	0.37	0.36	0.39	0.14	26.88
9	3	82.2897	82.4727	82.4597	0.02	0.1218	0.59	0.57	0.17	0.02	4.78
9	4	NA	85.1033	85.0930	0.01	0.1563	0.73	0.73	0.20	-0.14	7.44
1	1	NA	82.9964	82.9915	0.01	0.1854	0.89	0.92	0.25	-0.13	10.83
9	2	NA	86.0063	86.0055	0.00	0.2061	0.96	0.96	0.30	-0.15	15.86
9	3	NA	86.8970	86.9002	-0.00	0.2172	1.00	1.03	0.30	-0.02	14.73
4	4	NA	87.8515	87.8590	-0.01	0.2304	1.05	1.05	0.32	0.06	17.55
Mean		81.5874	83.8798	83.8754	0.01	0.1528	0.72	0.72	0.34	0.03	28.15

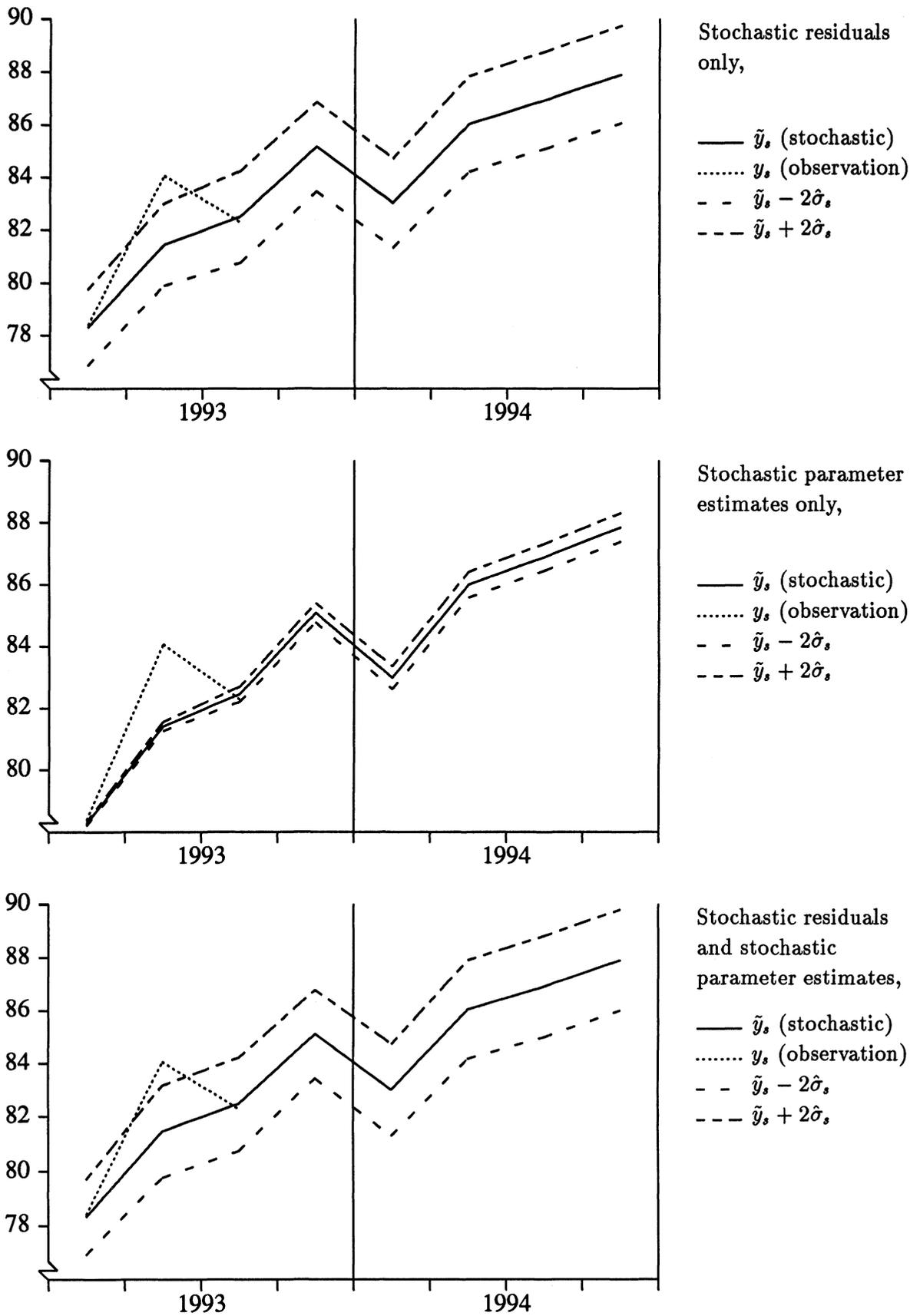
Table 5.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	78.4038	78.2838	78.3194	-0.05	0.6966	3.56	3.41	0.08	-0.44	9.49
9	2	84.0686	81.4279	81.4806	-0.06	0.8522	4.18	4.06	0.11	-0.30	5.72
9	3	82.2897	82.4727	82.4922	-0.02	0.8670	4.20	4.08	0.09	-0.19	2.78
9	4	NA	85.1033	85.1136	-0.01	0.8345	3.92	3.70	0.04	-0.31	4.27
1	1	NA	82.9964	83.0234	-0.03	0.8587	4.14	3.94	0.14	-0.10	3.60
9	2	NA	86.0063	86.0480	-0.05	0.9308	4.33	4.42	0.19	0.10	6.23
9	3	NA	86.8970	86.9078	-0.01	0.9477	4.36	4.21	0.11	-0.05	2.13
4	4	NA	87.8515	87.8966	-0.05	0.9463	4.31	4.24	0.15	-0.23	5.85
Mean		81.5874	83.8798	83.9101	-0.04	0.8667	4.12	4.01	0.11	-0.19	5.01

Table 5.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 5. Graphic plots of A corresponding to the tables on the facing page.



A4: Export, traditional goods, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	28.2287	28.2140	28.2405	-0.09	0.6788	9.61	9.40	0.07	-0.23	2.98
9	2	30.6964	30.5257	30.5540	-0.09	0.7462	9.77	9.66	0.07	0.00	0.71
9	3	28.4482	28.2793	28.3220	-0.15	0.7465	10.54	10.21	0.12	-0.07	2.47
3	4	NA	30.6708	30.7307	-0.19	0.8085	10.52	10.28	0.11	-0.08	2.45
1	1	NA	29.2932	29.3306	-0.13	0.7950	10.84	10.85	0.01	-0.28	3.23
9	2	NA	30.7369	30.7631	-0.09	0.8263	10.74	10.87	0.11	0.37	7.72
9	3	NA	29.1792	29.2153	-0.12	0.7714	10.56	10.40	0.06	-0.07	0.86
4	4	NA	30.9277	30.9632	-0.11	0.8447	10.91	10.68	0.13	-0.00	2.81
Mean		29.1244	29.7283	29.7649	-0.12	0.7772	10.44	10.29	0.09	-0.04	2.90

Table 6.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	28.2287	28.2140	28.2122	0.01	0.0298	0.42	0.41	0.30	0.11	15.28
9	2	30.6964	30.5257	30.5215	0.01	0.0740	0.97	0.92	0.15	0.03	3.74
9	3	28.4482	28.2793	28.2731	0.02	0.1193	1.69	1.65	0.01	-0.04	0.09
3	4	NA	30.6708	30.6684	0.01	0.1552	2.02	2.00	0.06	-0.15	1.58
1	1	NA	29.2932	29.2952	-0.01	0.1844	2.52	2.57	0.15	-0.15	4.58
9	2	NA	30.7369	30.7439	-0.02	0.2037	2.65	2.68	0.22	-0.19	9.49
9	3	NA	29.1792	29.1913	-0.04	0.2103	2.88	2.96	0.24	-0.05	9.76
4	4	NA	30.9277	30.9445	-0.05	0.2263	2.93	2.94	0.26	0.10	11.38
Mean		29.1244	29.7283	29.7312	-0.01	0.1504	2.01	2.02	0.17	-0.04	6.99

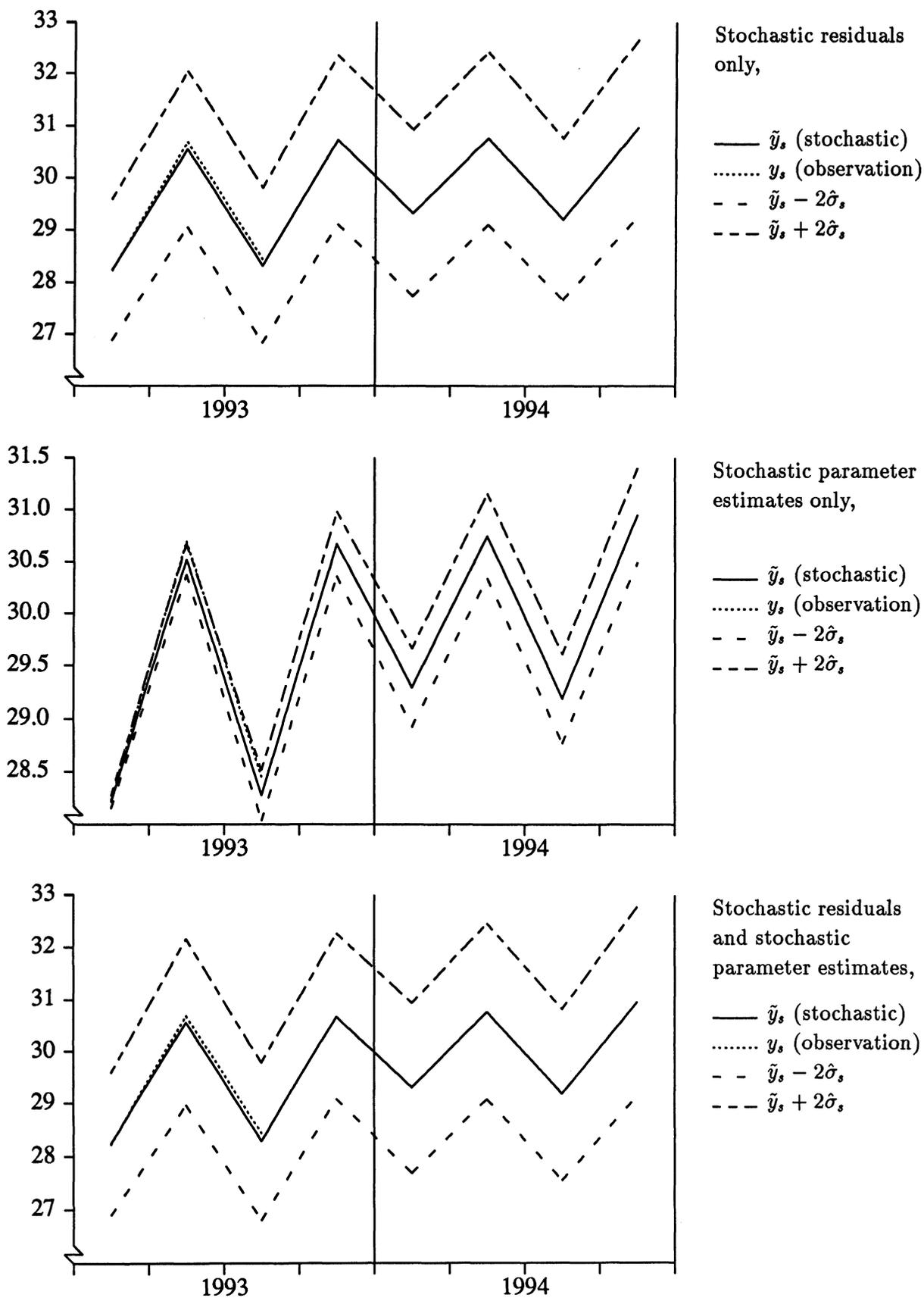
Table 6.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	28.2287	28.2140	28.2449	-0.11	0.6735	9.54	9.07	0.06	-0.48	10.24
9	2	30.6964	30.5257	30.5676	-0.14	0.7962	10.42	10.12	0.12	-0.05	2.49
9	3	28.4482	28.2793	28.2963	-0.06	0.7461	10.55	10.51	0.03	-0.21	1.98
3	4	NA	30.6708	30.6809	-0.03	0.7915	10.32	9.56	0.00	-0.27	3.14
1	1	NA	29.2932	29.3191	-0.09	0.8130	11.09	10.70	0.10	-0.16	2.67
9	2	NA	30.7369	30.7757	-0.13	0.8397	10.91	11.10	0.15	0.05	3.76
9	3	NA	29.1792	29.1988	-0.07	0.8171	11.19	11.10	0.11	-0.21	3.79
4	4	NA	30.9277	30.9631	-0.11	0.9066	11.71	11.07	0.11	-0.28	5.54
Mean		29.1244	29.7283	29.7558	-0.09	0.7980	10.72	10.40	0.08	-0.20	4.20

Table 6.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 6. Graphic plots of A4 corresponding to the tables on the facing page.



I: Total imports, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	60.2696	60.4076	60.4370	-0.05	0.7715	5.11	5.06	0.08	0.46	10.14
9	2	63.4352	59.6191	59.6596	-0.07	0.9270	6.22	6.20	0.01	-0.23	2.23
9	3	70.2299	68.6054	68.6809	-0.11	1.0733	6.25	6.22	0.02	0.21	1.96
3	4	NA	67.6104	67.7309	-0.18	1.2239	7.23	6.82	0.08	-0.12	1.68
1	1	NA	65.9249	66.0921	-0.25	1.2286	7.44	7.15	-0.02	-0.26	2.90
9	2	NA	64.7922	65.0143	-0.34	1.2617	7.76	7.60	0.06	-0.14	1.49
9	3	NA	69.6520	69.8989	-0.35	1.2854	7.36	7.51	0.11	-0.03	1.98
4	4	NA	70.3290	70.6796	-0.50	1.4631	8.28	8.10	0.15	0.08	3.83
<i>Mean</i>		64.6449	65.8675	66.0241	-0.23	1.1543	6.95	6.84	0.06	-0.00	3.28

Table 7.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	60.2696	60.4076	60.4056	0.00	0.2936	1.94	1.88	0.09	0.10	1.85
9	2	63.4352	59.6191	59.6319	-0.02	0.3977	2.67	2.65	0.16	0.25	6.82
9	3	70.2299	68.6054	68.6144	-0.01	0.5082	2.96	3.00	0.19	0.07	6.03
3	4	NA	67.6104	67.6269	-0.02	0.6624	3.92	3.82	0.14	-0.06	3.56
1	1	NA	65.9249	65.9422	-0.03	0.7546	4.58	4.49	0.12	0.22	4.59
9	2	NA	64.7922	64.8262	-0.05	0.8489	5.24	5.06	0.07	0.30	4.55
9	3	NA	69.6520	69.6795	-0.04	0.9307	5.34	5.21	0.03	0.25	2.75
4	4	NA	70.3290	70.3558	-0.04	1.0792	6.14	6.05	-0.03	0.18	1.52
<i>Mean</i>		64.6449	65.8675	65.8853	-0.03	0.6844	4.10	4.02	0.10	0.16	3.96

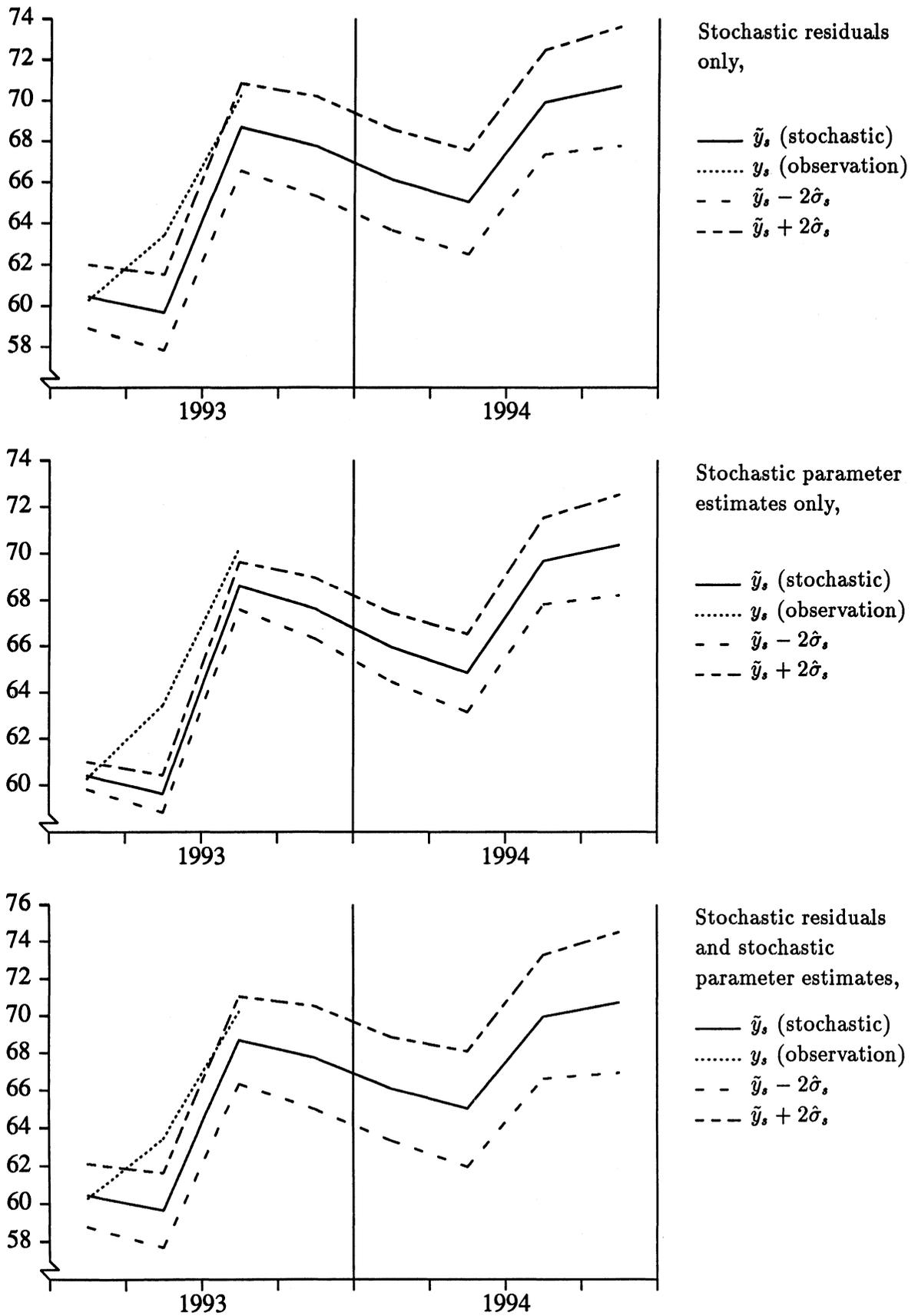
Table 7.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	60.2696	60.4076	60.4495	-0.07	0.8366	5.54	5.38	-0.00	-0.05	0.11
9	2	63.4352	59.6191	59.6673	-0.08	0.9877	6.62	6.54	0.06	0.02	0.60
9	3	70.2299	68.6054	68.6936	-0.13	1.1745	6.84	6.86	0.07	-0.04	0.94
3	4	NA	67.6104	67.7529	-0.21	1.3792	8.14	8.18	0.04	0.02	0.26
1	1	NA	65.9249	66.0986	-0.26	1.3815	8.36	8.28	0.04	-0.06	0.39
9	2	NA	64.7922	65.0447	-0.39	1.5328	9.43	8.91	0.13	0.12	3.38
9	3	NA	69.6520	69.9618	-0.44	1.6628	9.51	9.47	0.16	0.26	7.09
4	4	NA	70.3290	70.7327	-0.57	1.8869	10.67	10.68	0.19	0.04	6.00
<i>Mean</i>		64.6449	65.8675	66.0501	-0.27	1.3553	8.14	8.04	0.08	0.04	2.35

Table 7.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 7. Graphic plots of I corresponding to the tables on the facing page.



I4: Imports, traditional goods, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	38.0800	38.1643	38.1730	-0.02	0.6262	6.56	6.32	0.02	0.02	0.08
	2	37.8421	38.0926	38.1189	-0.07	0.7469	7.84	7.65	-0.01	-0.08	0.33
9	3	39.5958	41.8220	41.8691	-0.11	0.8453	8.08	7.90	0.00	0.25	2.53
	4	NA	43.1119	43.1994	-0.20	0.9821	9.09	8.83	0.05	-0.32	4.66
1	1	NA	41.2624	41.3792	-0.28	0.9696	9.37	8.60	0.04	-0.35	5.44
	2	NA	39.6066	39.7692	-0.41	0.9853	9.91	9.77	0.08	0.25	3.81
9	3	NA	42.0383	42.2226	-0.44	1.0183	9.65	9.67	0.06	-0.01	0.52
	4	NA	44.1069	44.3743	-0.60	1.1733	10.58	10.47	0.06	0.07	0.81
<i>Mean</i>		38.5060	41.0256	41.1382	-0.27	0.9184	8.88	8.65	0.04	-0.02	2.27

Table 8.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	38.0800	38.1643	38.1579	0.02	0.2119	2.22	2.22	0.10	0.31	5.91
	2	37.8421	38.0926	38.0907	0.01	0.2923	3.07	3.02	0.14	0.46	12.27
9	3	39.5958	41.8220	41.8182	0.01	0.3879	3.71	3.55	0.13	0.11	3.25
	4	NA	43.1119	43.1143	-0.01	0.5124	4.75	4.49	0.06	0.03	0.68
1	1	NA	41.2624	41.2675	-0.01	0.5814	5.64	5.32	0.04	0.18	1.72
	2	NA	39.6066	39.6180	-0.03	0.6704	6.77	6.42	-0.02	0.34	5.00
9	3	NA	42.0383	42.0405	-0.01	0.7516	7.15	6.76	-0.07	0.20	2.68
	4	NA	44.1069	44.1134	-0.01	0.8807	7.99	7.72	-0.15	0.28	7.24
<i>Mean</i>		38.5060	41.0256	41.0275	-0.00	0.5361	5.16	4.94	0.03	0.24	4.84

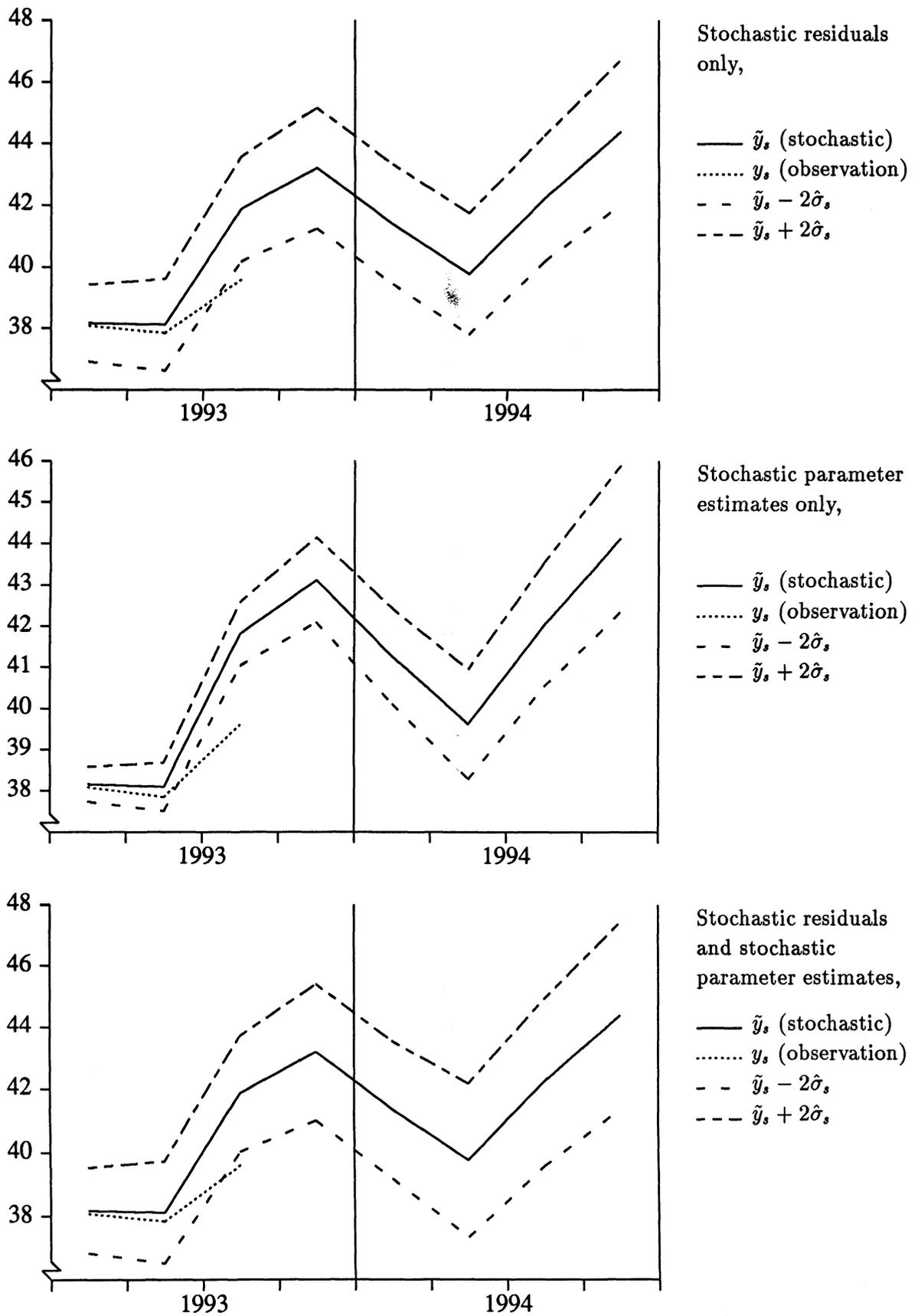
Table 8.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	38.0800	38.1643	38.1807	-0.04	0.6714	7.03	6.93	0.01	-0.29	3.69
	2	37.8421	38.0926	38.1151	-0.06	0.7994	8.39	7.94	0.07	-0.10	1.19
9	3	39.5958	41.8220	41.8662	-0.11	0.9234	8.82	8.93	0.06	-0.05	0.83
	4	NA	43.1119	43.1951	-0.19	1.1009	10.19	10.11	0.03	0.11	0.73
1	1	NA	41.2624	41.3575	-0.23	1.0933	10.57	10.28	0.07	0.27	3.87
	2	NA	39.6066	39.7596	-0.38	1.2086	12.16	11.69	0.15	0.56	16.77
9	3	NA	42.0383	42.2493	-0.50	1.3397	12.68	12.97	0.14	0.42	10.59
	4	NA	44.1069	44.3932	-0.64	1.5242	13.73	13.42	0.15	0.19	5.36
<i>Mean</i>		38.5060	41.0256	41.1395	-0.27	1.0826	10.45	10.28	0.09	0.14	5.38

Table 8.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 8. Graphic plots of I4 corresponding to the tables on the facing page.



Q: GDP, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

<i>s</i>		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	175.5770	175.1400	175.1430	-0.00	0.9665	2.21	2.14	0.12	-0.01	2.30
9	2	175.2080	174.4130	174.4210	-0.00	1.1794	2.70	2.70	0.03	-0.06	0.28
9	3	182.6780	182.9010	182.9350	-0.02	1.2658	2.77	2.67	0.11	0.38	8.20
9	4	NA	191.3040	191.4240	-0.06	1.3968	2.92	2.92	0.03	0.07	0.29
1	1	NA	182.7500	182.8850	-0.07	1.5128	3.31	3.26	-0.08	-0.18	2.32
9	2	NA	181.3310	181.5250	-0.11	1.5431	3.40	3.39	-0.06	0.38	6.53
9	3	NA	187.5910	187.8800	-0.15	1.6368	3.48	3.45	0.03	0.20	1.85
4	4	NA	195.3250	195.6700	-0.18	1.8526	3.79	3.76	0.02	0.70	20.62
<i>Mean</i>		177.8210	183.8440	183.9850	-0.07	1.4192	3.07	3.04	0.02	0.18	5.30

Table 9.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

<i>s</i>		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	175.5770	175.1400	175.1160	0.01	0.4177	0.95	0.95	0.22	0.17	9.66
9	2	175.2080	174.4130	174.3890	0.01	0.5930	1.36	1.38	0.24	0.22	11.77
9	3	182.6780	182.9010	182.8740	0.01	0.7297	1.60	1.54	0.19	-0.04	6.25
9	4	NA	191.3040	191.2910	0.01	0.8933	1.87	1.89	0.15	0.18	5.01
1	1	NA	182.7500	182.7500	0.00	1.0847	2.37	2.39	0.09	0.22	3.42
9	2	NA	181.3310	181.3250	0.00	1.2563	2.77	2.82	0.01	0.17	1.31
9	3	NA	187.5910	187.5470	0.02	1.3892	2.96	2.96	-0.03	0.08	0.39
4	4	NA	195.3250	195.2710	0.03	1.5874	3.25	3.37	-0.10	0.28	4.90
<i>Mean</i>		177.8210	183.8440	183.8200	0.01	0.9939	2.14	2.16	0.10	0.16	5.34

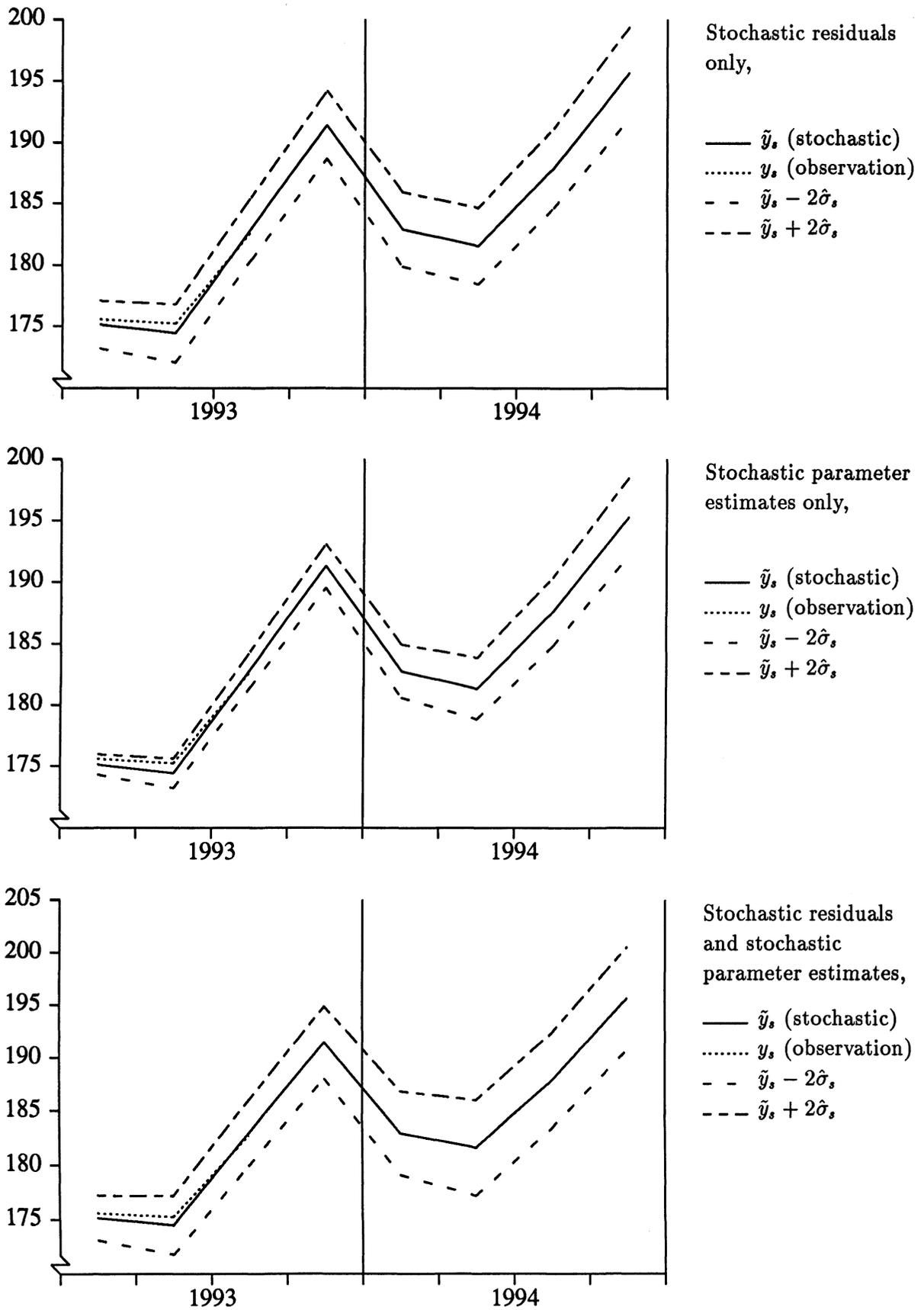
Table 9.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

<i>s</i>		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	175.5770	175.1400	175.1450	-0.00	1.0360	2.37	2.32	0.09	0.10	1.63
9	2	175.2080	174.4130	174.4520	-0.02	1.3529	3.10	3.07	0.09	0.01	1.38
9	3	182.6780	182.9010	182.9410	-0.02	1.5202	3.32	3.17	0.10	0.45	10.20
9	4	NA	191.3040	191.4280	-0.06	1.7101	3.57	3.55	0.10	0.37	7.60
1	1	NA	182.7500	182.9620	-0.12	1.9467	4.26	4.08	0.06	-0.02	0.67
9	2	NA	181.3310	181.6060	-0.15	2.2096	4.87	4.90	0.11	0.19	3.49
9	3	NA	187.5910	187.8610	-0.14	2.2487	4.79	4.65	0.08	0.50	11.79
4	4	NA	195.3250	195.6820	-0.18	2.4447	5.00	4.96	0.05	0.10	0.86
<i>Mean</i>		177.8210	183.8440	184.0100	-0.09	1.8086	3.91	3.84	0.09	0.21	4.70

Table 9.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 9. Graphic plots of Q corresponding to the tables on the facing page.



Q6: GDP, mainland, fixed 1991 prices, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	142.1730	141.7570	141.7830	-0.02	0.9159	2.58	2.47	0.12	0.18	3.61
9	2	141.8840	141.1310	141.1560	-0.02	1.1442	3.24	3.07	0.03	-0.02	0.17
9	3	148.2650	147.8190	147.8820	-0.04	1.2227	3.31	3.29	0.17	0.44	12.87
3	4	NA	153.1410	153.2970	-0.10	1.3434	3.51	3.49	0.03	0.23	2.29
1	1	NA	145.7520	145.9250	-0.12	1.4107	3.87	3.87	-0.03	-0.14	0.97
9	2	NA	144.7800	145.0160	-0.16	1.4546	4.01	3.98	-0.09	-0.00	1.21
9	3	NA	150.1260	150.4590	-0.22	1.4836	3.94	3.76	0.10	0.00	1.61
4	4	NA	155.7910	156.1920	-0.26	1.6673	4.27	4.14	0.03	0.21	2.01
<i>Mean</i>		144.1070	147.5370	147.7140	-0.12	1.3303	3.59	3.51	0.04	0.11	3.09

Table 10.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	142.1730	141.7570	141.7350	0.02	0.4167	1.18	1.16	0.22	0.26	11.30
9	2	141.8840	141.1310	141.1130	0.01	0.5920	1.68	1.71	0.23	0.21	10.72
9	3	148.2650	147.8190	147.7960	0.02	0.7224	1.96	1.88	0.19	-0.07	6.32
3	4	NA	153.1410	153.1330	0.00	0.8824	2.30	2.28	0.15	0.12	4.30
1	1	NA	145.7520	145.7530	-0.00	1.0750	2.95	2.91	0.09	0.19	2.74
9	2	NA	144.7800	144.7760	0.00	1.2424	3.43	3.51	0.01	0.14	0.85
9	3	NA	150.1260	150.0830	0.03	1.3689	3.65	3.59	-0.04	0.02	0.23
4	4	NA	155.7910	155.7380	0.03	1.5660	4.02	4.02	-0.11	0.20	3.74
<i>Mean</i>		144.1070	147.5370	147.5160	0.01	0.9832	2.65	2.63	0.09	0.13	5.03

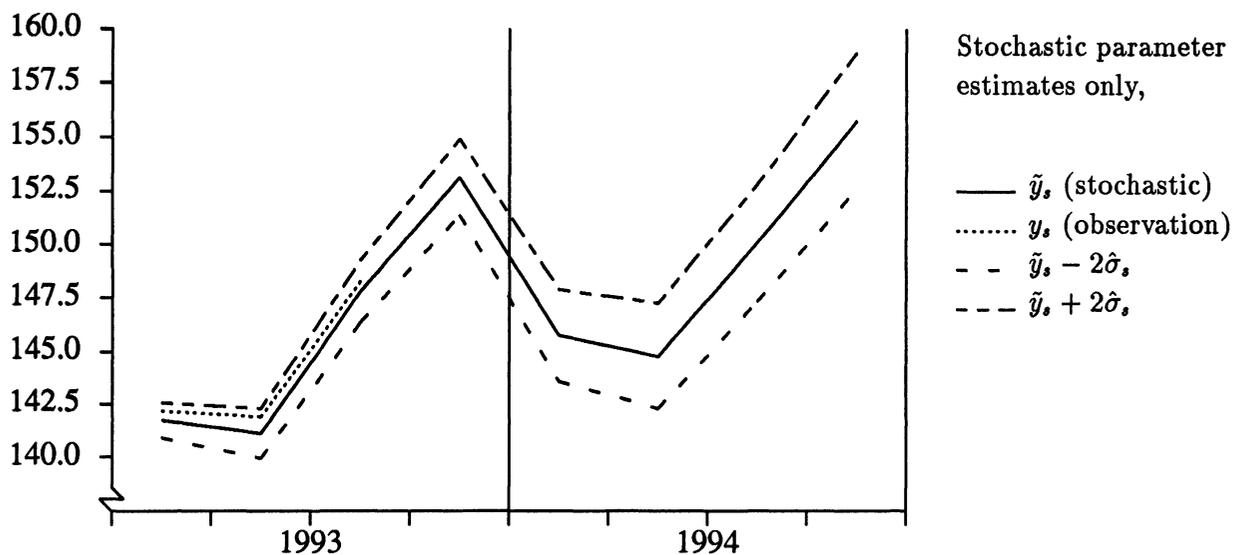
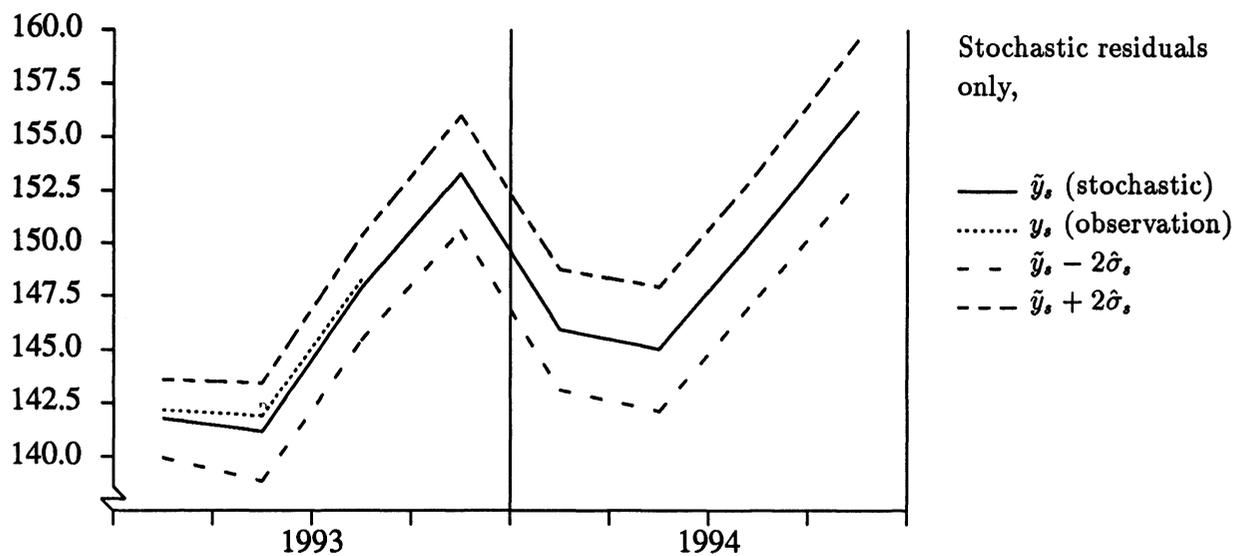
Table 10.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	142.1730	141.7570	141.7740	-0.01	0.9851	2.78	2.75	0.07	-0.13	1.49
9	2	141.8840	141.1310	141.1830	-0.04	1.2936	3.67	3.66	0.07	0.00	0.80
9	3	148.2650	147.8190	147.8800	-0.04	1.4639	3.96	3.83	0.11	0.31	6.19
3	4	NA	153.1410	153.2970	-0.10	1.6476	4.30	4.29	0.08	0.10	1.66
1	1	NA	145.7520	146.0050	-0.17	1.8842	5.16	4.88	0.08	-0.06	1.20
9	2	NA	144.7800	145.1020	-0.22	2.1329	5.88	5.79	0.13	0.26	5.91
9	3	NA	150.1260	150.4340	-0.20	2.1764	5.79	5.68	0.07	0.57	14.56
4	4	NA	155.7910	156.1930	-0.26	2.3519	6.02	6.04	0.01	0.18	1.37
<i>Mean</i>		144.1070	147.5370	147.7330	-0.13	1.7419	4.69	4.62	0.08	0.16	4.15

Table 10.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 10. Graphic plots of Q6 corresponding to the tables on the facing page.



LW: Man hours, million hours

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	630.5820	630.9470	631.0880	-0.02	3.7288	2.36	2.34	0.05	-0.23	2.62
9	2	610.7400	613.8120	614.0270	-0.04	4.3488	2.83	2.69	0.06	-0.17	1.93
9	3	576.9500	582.7820	583.0150	-0.04	4.3992	3.02	2.95	0.10	-0.29	5.42
3	4	NA	652.3260	652.8010	-0.07	5.8367	3.58	3.53	0.07	0.27	3.99
1	1	NA	641.0750	641.6620	-0.09	6.1752	3.85	3.74	0.10	0.04	1.79
9	2	NA	617.8420	618.6450	-0.13	6.1877	4.00	3.82	0.02	-0.22	2.02
9	3	NA	586.3520	587.3890	-0.18	6.2490	4.26	4.23	-0.02	0.40	6.69
4	4	NA	659.6190	661.0330	-0.21	7.5778	4.59	4.56	0.06	0.03	0.72
<i>Mean</i>		606.0910	623.0940	623.7070	-0.10	5.5629	3.56	3.48	0.06	-0.02	3.15

Table 11.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s, n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993 1, \dots, 1994 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	630.5820	630.9470	630.9200	0.00	1.6861	1.07	1.01	-0.03	0.23	2.25
9	2	610.7400	613.8120	613.8400	-0.00	1.7163	1.12	1.10	0.09	0.18	2.86
9	3	576.9500	582.7820	582.8520	-0.01	1.9162	1.32	1.28	0.09	-0.12	1.98
3	4	NA	652.3260	652.2940	0.01	2.8806	1.77	1.73	0.05	-0.18	1.74
1	1	NA	641.0750	641.2540	-0.03	3.4084	2.13	2.06	0.03	-0.32	4.38
9	2	NA	617.8420	617.9090	-0.01	3.7239	2.41	2.39	0.07	0.11	1.39
9	3	NA	586.3520	586.4130	-0.01	3.9939	2.72	2.74	0.06	-0.07	0.78
4	4	NA	659.6190	659.5830	0.01	5.4894	3.33	3.26	0.01	-0.05	0.11
<i>Mean</i>		606.0910	623.0940	623.1330	-0.01	3.1019	1.98	1.95	0.05	-0.03	1.94

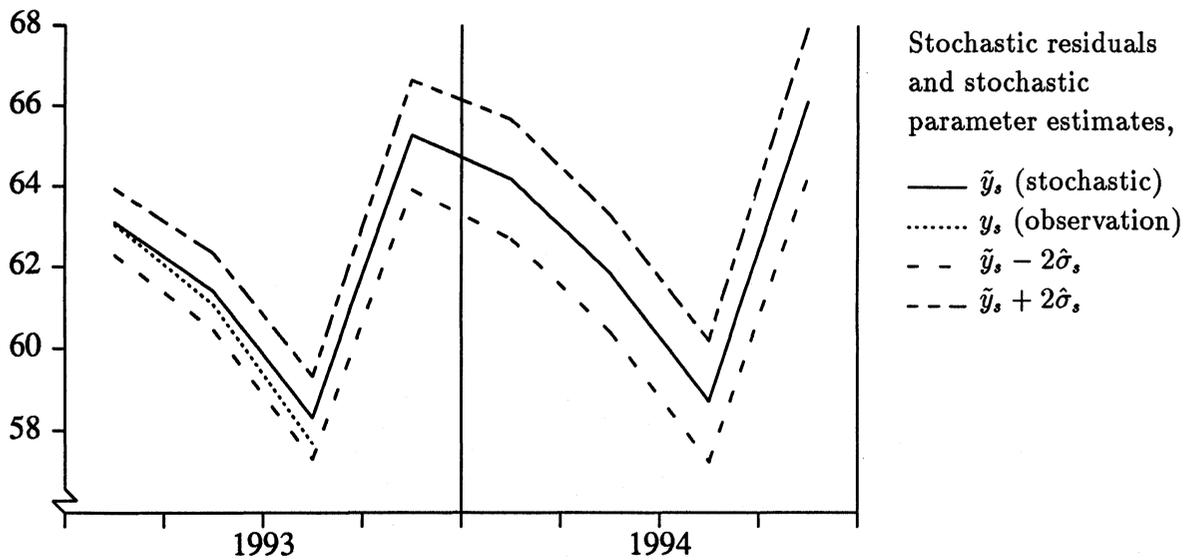
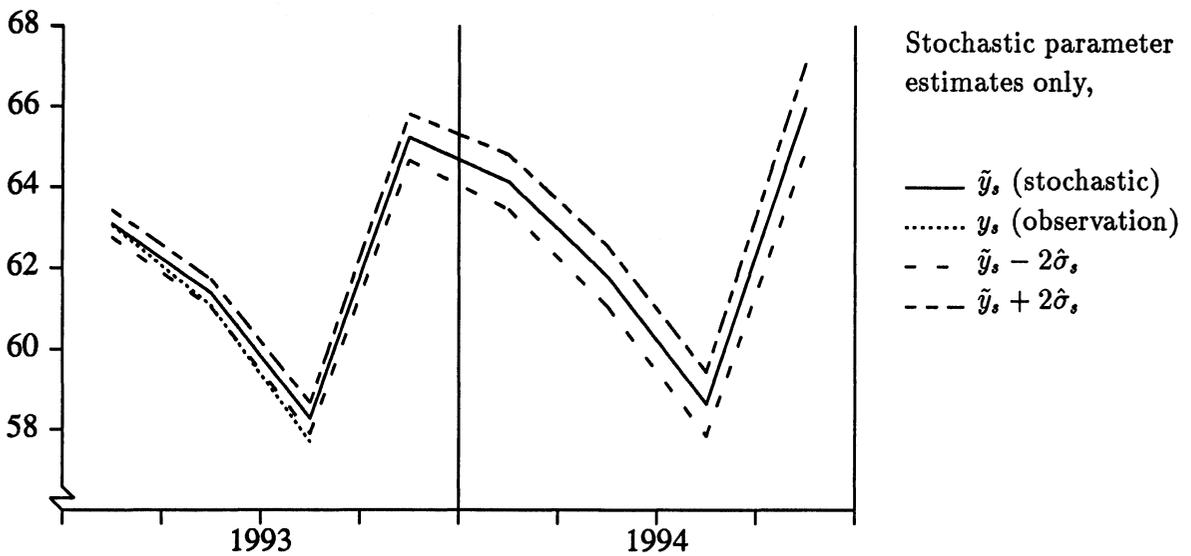
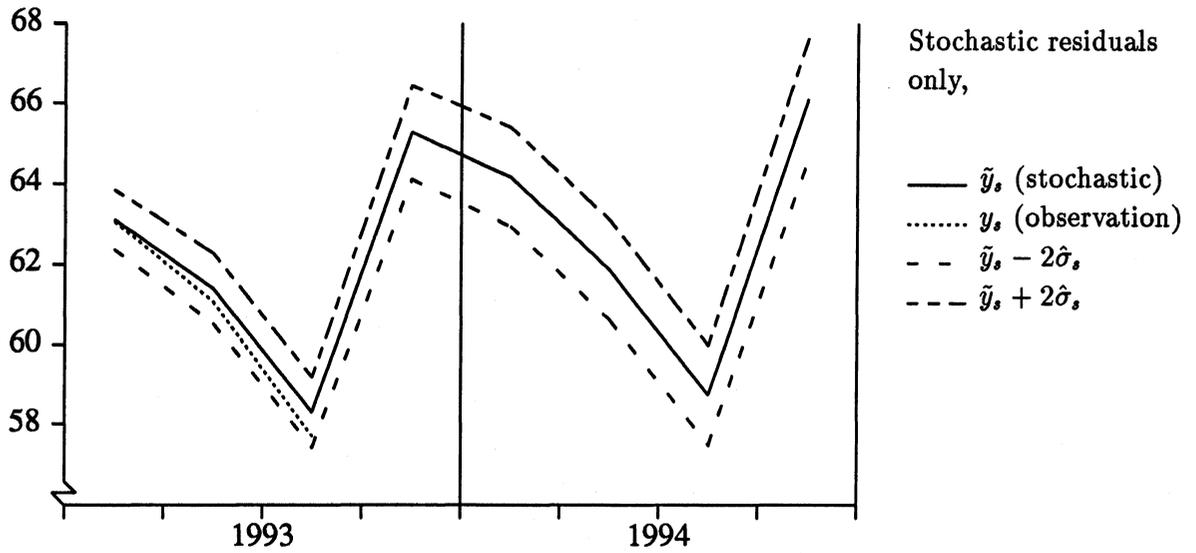
Table 11.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	630.5820	630.9470	631.1820	-0.04	4.0956	2.60	2.51	0.09	0.21	3.10
9	2	610.7400	613.8120	614.0500	-0.04	4.7157	3.07	3.01	0.05	-0.13	1.24
9	3	576.9500	582.7820	583.1880	-0.07	5.0262	3.45	3.35	0.07	0.08	1.02
3	4	NA	652.3260	652.8030	-0.07	6.8254	4.18	4.23	0.04	0.07	0.53
1	1	NA	641.0750	641.8960	-0.13	7.3945	4.61	4.32	-0.02	-0.16	1.08
9	2	NA	617.8420	618.7060	-0.14	7.1993	4.65	4.64	-0.14	0.01	3.46
9	3	NA	586.3520	587.3110	-0.16	7.4003	5.04	5.02	0.03	-0.16	1.25
4	4	NA	659.6190	661.0550	-0.22	9.2823	5.62	5.58	-0.04	0.06	0.44
<i>Mean</i>		606.0910	623.0940	623.7730	-0.11	6.4924	4.15	4.08	0.01	-0.00	1.52

Table 11.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 11. Graphic plots of LW corresponding to the tables on the facing page.



NW: Employed wage earners, million persons

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	1.7972	1.7972	1.7975	-0.02	0.0097	2.16	2.16	0.05	-0.21	2.33
9	2	1.8212	1.8294	1.8300	-0.03	0.0124	2.71	2.59	0.06	-0.17	1.67
9	3	1.8529	1.8715	1.8721	-0.04	0.0141	3.01	2.90	0.10	-0.28	4.99
9	4	NA	1.8359	1.8370	-0.06	0.0155	3.36	3.33	0.08	0.32	5.42
1	1	NA	1.8268	1.8283	-0.08	0.0165	3.61	3.44	0.12	0.05	2.37
9	2	NA	1.8453	1.8475	-0.11	0.0176	3.81	3.62	0.02	-0.14	0.92
9	3	NA	1.8857	1.8889	-0.17	0.0197	4.18	4.14	-0.01	0.26	2.94
4	4	NA	1.8545	1.8580	-0.19	0.0195	4.19	4.16	0.06	0.10	1.05
Mean		1.8238	1.8433	1.8449	-0.09	0.0156	3.38	3.29	0.06	-0.01	2.71

Table 12.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	1.7972	1.7972	1.7970	0.01	0.0050	1.12	1.08	0.06	0.17	1.74
9	2	1.8212	1.8294	1.8293	0.01	0.0057	1.25	1.23	0.02	0.05	0.19
9	3	1.8529	1.8715	1.8715	-0.00	0.0071	1.51	1.50	0.06	-0.03	0.57
9	4	NA	1.8359	1.8357	0.01	0.0076	1.66	1.61	0.09	-0.21	3.24
1	1	NA	1.8268	1.8272	-0.02	0.0092	2.01	1.94	0.06	-0.45	9.20
9	2	NA	1.8453	1.8455	-0.01	0.0105	2.28	2.24	0.10	0.07	1.98
9	3	NA	1.8857	1.8860	-0.01	0.0126	2.67	2.67	0.08	-0.06	1.28
4	4	NA	1.8545	1.8546	-0.01	0.0144	3.11	3.15	0.02	-0.02	0.11
Mean		1.8238	1.8433	1.8433	-0.00	0.0090	1.95	1.93	0.06	-0.06	2.29

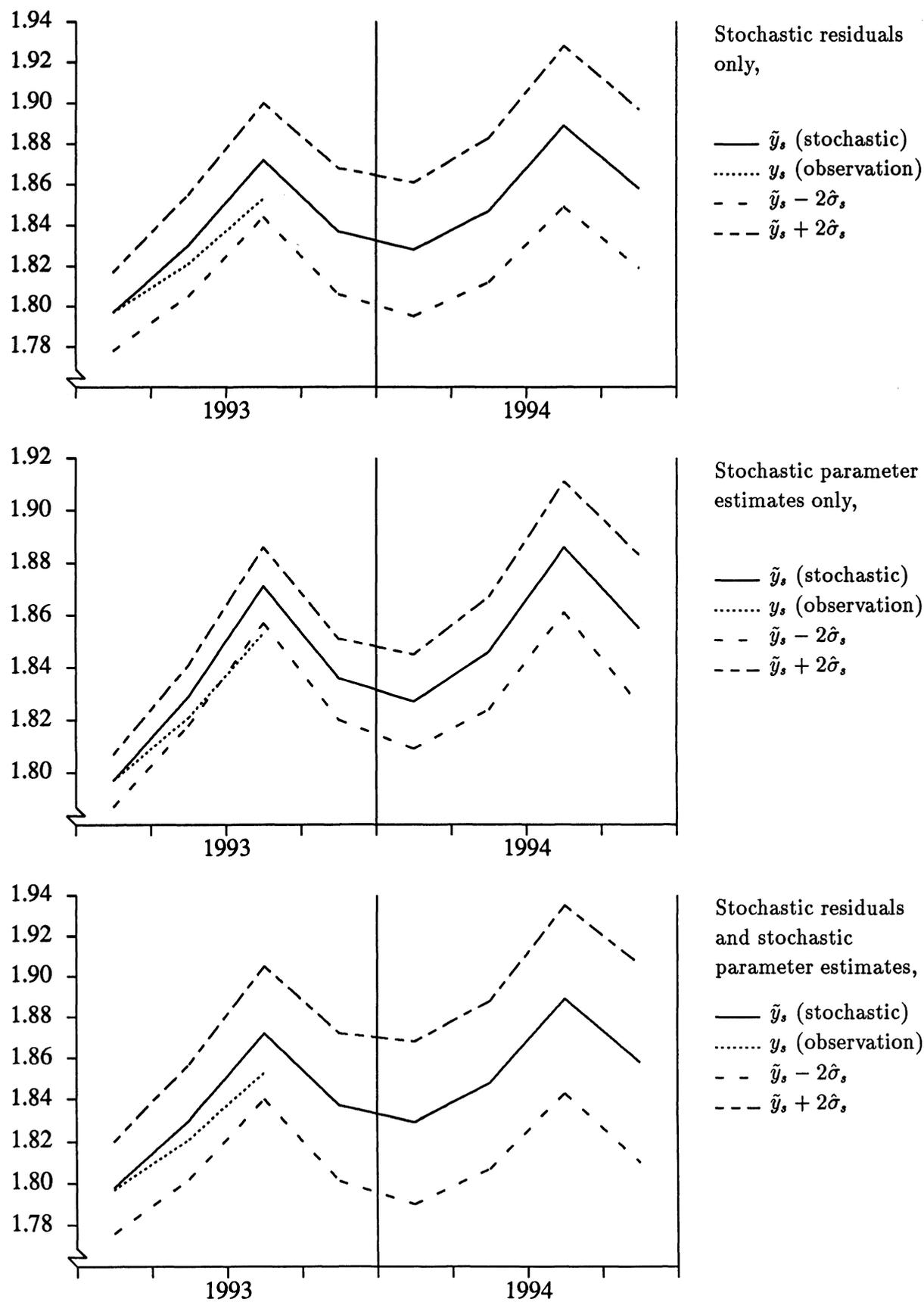
Table 12.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	1.7972	1.7972	1.7977	-0.03	0.0110	2.44	2.40	0.09	0.31	5.59
9	2	1.8212	1.8294	1.8298	-0.02	0.0137	2.99	2.99	0.07	-0.01	0.94
9	3	1.8529	1.8715	1.8725	-0.05	0.0164	3.50	3.35	0.08	0.20	2.84
9	4	NA	1.8359	1.8368	-0.05	0.0178	3.88	3.90	0.08	0.18	2.34
1	1	NA	1.8268	1.8287	-0.11	0.0194	4.25	4.12	-0.00	0.01	0.01
9	2	NA	1.8453	1.8476	-0.12	0.0203	4.39	4.34	-0.14	0.05	3.29
9	3	NA	1.8857	1.8886	-0.15	0.0230	4.87	4.81	0.04	-0.16	1.39
4	4	NA	1.8545	1.8582	-0.20	0.0240	5.16	4.99	-0.01	0.04	0.08
Mean		1.8238	1.8433	1.8450	-0.09	0.0182	3.93	3.86	0.03	0.08	2.06

Table 12.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 12. Graphic plots of NW corresponding to the tables on the facing page.



NT: Labour force, million persons

Stochastic residuals, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	2.1305	2.1320	2.1311	0.04	0.0169	3.18	3.08	-0.24	-0.01	10.02
9	2	2.1515	2.1613	2.1607	0.03	0.0154	2.85	2.80	-0.21	0.22	9.55
9	3	2.1943	2.2081	2.2074	0.03	0.0142	2.56	2.53	-0.10	-0.05	1.94
3	4	NA	2.1594	2.1589	0.02	0.0154	2.86	2.77	-0.09	-0.35	6.56
1	1	NA	2.1620	2.1619	0.00	0.0166	3.08	3.04	-0.14	-0.03	3.52
9	2	NA	2.1756	2.1751	0.03	0.0163	3.00	2.88	-0.16	-0.00	4.19
9	3	NA	2.2176	2.2182	-0.03	0.0151	2.73	2.72	-0.11	0.12	2.78
4	4	NA	2.1766	2.1768	-0.01	0.0176	3.24	3.34	-0.15	-0.07	3.74
Mean		2.1588	2.1741	2.1738	0.01	0.0160	2.94	2.90	-0.15	-0.02	5.29

Table 13.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993 1, \dots, 1994 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	2.1305	2.1320	2.1313	0.03	0.0095	1.77	1.72	-0.09	0.38	7.43
9	2	2.1515	2.1613	2.1603	0.05	0.0099	1.84	1.80	0.00	-0.01	0.01
9	3	2.1943	2.2081	2.2070	0.05	0.0093	1.68	1.70	-0.02	-0.00	0.06
3	4	NA	2.1594	2.1581	0.06	0.0108	2.00	1.91	-0.01	0.06	0.17
1	1	NA	2.1620	2.1611	0.04	0.0098	1.81	1.83	0.06	0.36	5.83
9	2	NA	2.1756	2.1744	0.06	0.0118	2.17	2.08	-0.18	0.47	14.51
9	3	NA	2.2176	2.2170	0.03	0.0097	1.75	1.77	0.04	0.06	0.44
4	4	NA	2.1766	2.1756	0.05	0.0117	2.14	2.07	0.03	0.24	2.55
Mean		2.1588	2.1741	2.1731	0.04	0.0103	1.90	1.86	-0.02	0.19	3.88

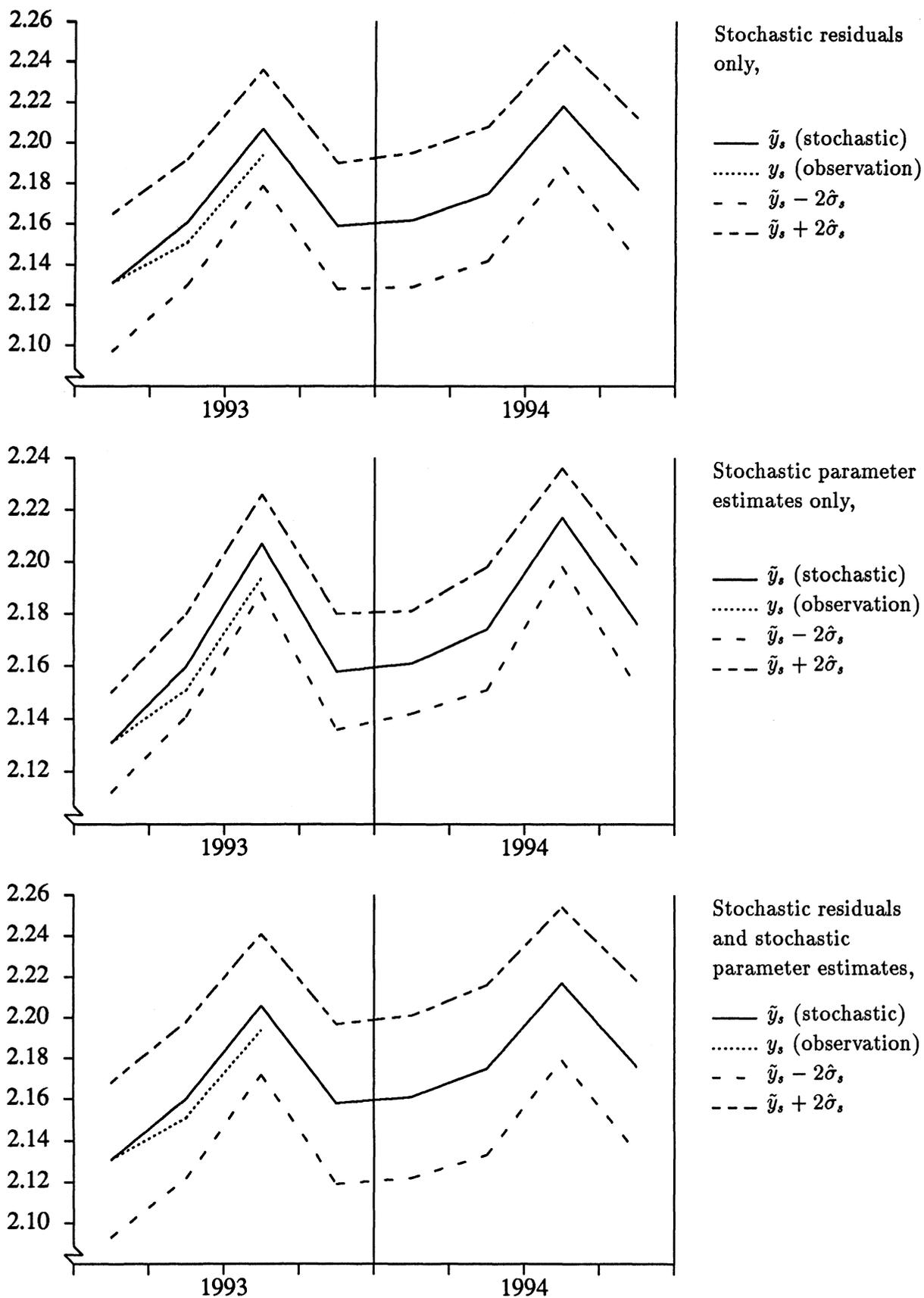
Table 13.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	2.1305	2.1320	2.1307	0.06	0.0186	3.49	3.33	-0.14	0.15	4.47
9	2	2.1515	2.1613	2.1598	0.07	0.0190	3.53	3.45	-0.10	-0.25	4.23
9	3	2.1943	2.2081	2.2063	0.08	0.0172	3.12	2.99	-0.20	0.44	14.51
3	4	NA	2.1594	2.1580	0.07	0.0196	3.64	3.63	-0.21	0.23	9.43
1	1	NA	2.1620	2.1615	0.03	0.0195	3.62	3.55	-0.10	0.32	6.12
9	2	NA	2.1756	2.1748	0.04	0.0208	3.82	3.67	-0.09	0.11	1.76
9	3	NA	2.2176	2.2168	0.04	0.0187	3.37	3.31	-0.21	0.28	10.44
4	4	NA	2.1766	2.1764	0.01	0.0210	3.86	3.84	-0.17	0.17	5.97
Mean		2.1588	2.1741	2.1730	0.05	0.0193	3.56	3.47	-0.15	0.18	7.12

Table 13.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 13. Graphic plots of NT corresponding to the tables on the facing page.



UR: Unemployment rate, percentage

Stochastic residuals, ex ante simulation, starting 1993 1:

<i>s</i>		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	6.4163	6.4178	6.3560	0.97	0.8616	54.22	53.26	-0.23	-0.02	8.47
	2	6.1119	6.0388	5.9795	0.99	0.7974	53.34	52.29	-0.10	-0.07	2.02
	3	6.2269	6.2499	6.1824	1.09	0.8134	52.63	51.87	-0.10	-0.00	1.73
	4	NA	5.6828	5.6048	1.39	0.8702	62.10	59.21	-0.04	0.31	4.38
9	1	NA	6.4099	6.3343	1.19	0.9023	56.98	55.47	-0.12	-0.21	4.37
	2	NA	5.8949	5.7690	2.18	0.9041	62.69	60.15	-0.08	0.14	1.82
	3	NA	6.0245	5.9051	2.02	0.8672	58.74	58.09	0.05	-0.16	1.37
	4	NA	5.6363	5.4802	2.85	0.9401	68.61	69.09	-0.10	0.12	2.11
Mean		6.2517	6.0444	5.9514	1.59	0.8695	58.66	57.43	-0.09	0.01	3.28

Table 14.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

<i>s</i>		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	6.4163	6.4178	6.3905	0.43	0.4733	29.63	28.92	-0.14	0.64	20.57
	2	6.1119	6.0388	5.9995	0.66	0.4944	32.97	31.93	-0.06	-0.01	0.59
	3	6.2269	6.2499	6.1996	0.81	0.4740	30.58	29.19	-0.07	-0.07	0.99
	4	NA	5.6828	5.6324	0.89	0.5325	37.82	36.20	0.02	-0.25	2.69
9	1	NA	6.4099	6.3526	0.90	0.4994	31.44	30.60	0.03	0.27	3.16
	2	NA	5.8949	5.8300	1.11	0.6236	42.79	43.60	-0.07	0.02	0.82
	3	NA	6.0245	5.9860	0.64	0.5617	37.54	36.96	0.04	-0.00	0.24
	4	NA	5.6363	5.5865	0.89	0.6652	47.63	45.64	0.13	0.03	2.99
Mean		6.2517	6.0444	5.9971	0.79	0.5405	36.30	35.38	-0.02	0.08	4.00

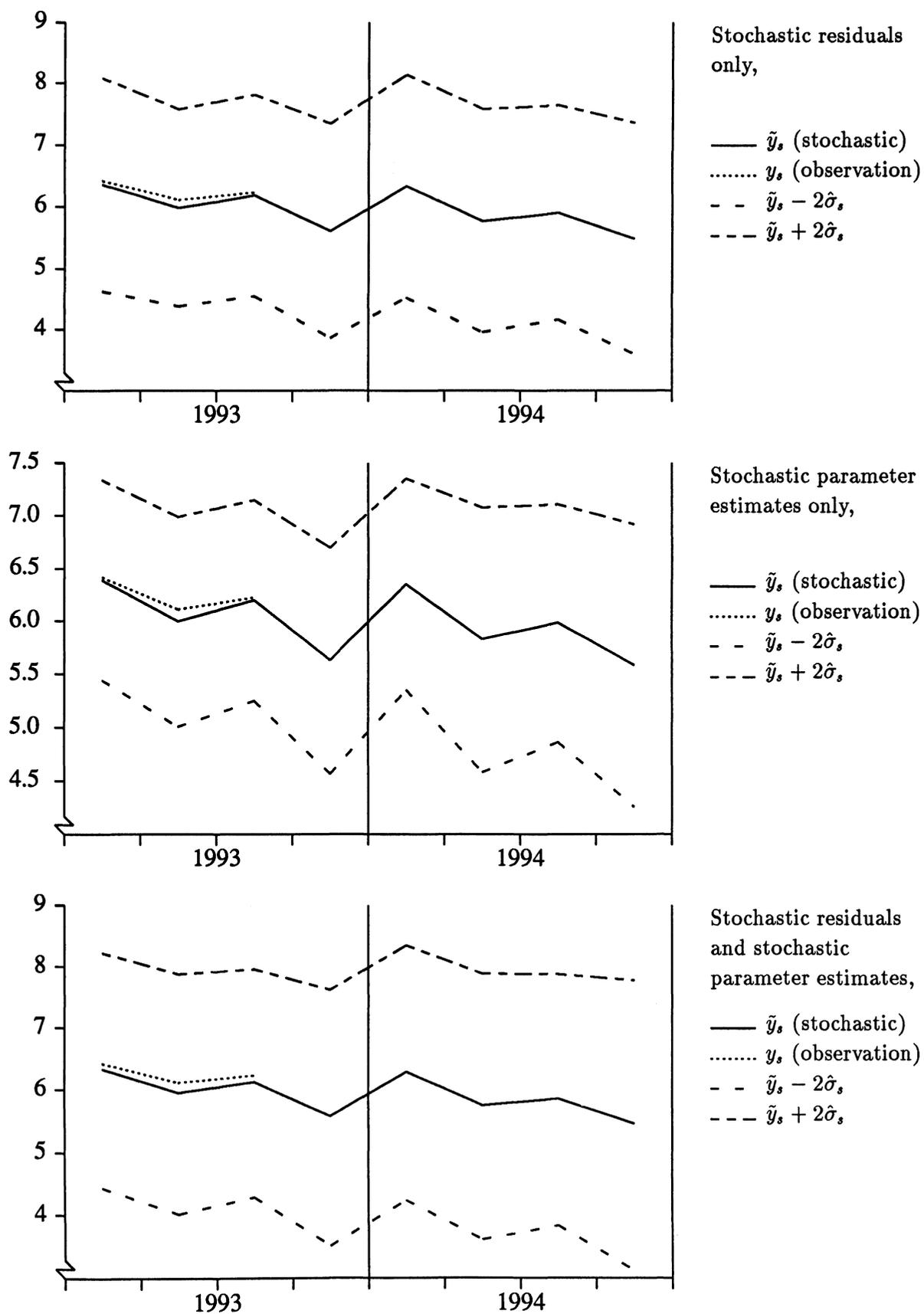
Table 14.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

<i>s</i>		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	6.4163	6.4178	6.3282	1.42	0.9462	59.81	59.59	-0.11	-0.12	2.81
	2	6.1119	6.0388	5.9473	1.54	0.9659	64.96	65.24	-0.12	0.03	2.52
	3	6.2269	6.2499	6.1198	2.13	0.9197	60.11	59.37	-0.15	-0.01	3.97
	4	NA	5.6828	5.5696	2.03	1.0288	73.89	70.84	-0.12	-0.29	5.91
9	1	NA	6.4099	6.2911	1.89	1.0286	65.40	64.85	-0.05	-0.25	3.12
	2	NA	5.8949	5.7499	2.52	1.0707	74.48	76.07	-0.02	0.04	0.12
	3	NA	6.0245	5.8566	2.87	1.0126	69.16	66.19	-0.07	-0.34	5.62
	4	NA	5.6363	5.4518	3.38	1.1653	85.50	88.79	-0.01	0.28	3.24
Mean		6.2517	6.0444	5.9143	2.22	1.0172	69.16	68.87	-0.08	-0.08	3.42

Table 14.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 14. Graphic plots of UR corresponding to the tables on the facing page.



KPI: Consumer price index, 1991 = 1

Stochastic residuals, ex ante simulation, starting 1993 1:

<i>s</i>	<i>y_t</i>	\hat{y}_s	\tilde{y}_s	<i>b_s</i> %	$\hat{\sigma}_s$	<i>n_s</i> %	<i>q_s</i> %	<i>s_s</i>	<i>k_s</i>	<i>jb_s</i>	
1	1	1.0389	1.0392	1.0393	-0.01	0.0053	2.04	2.00	0.09	-0.24	3.77
9	2	1.0489	1.0489	1.0489	-0.00	0.0070	2.67	2.66	0.08	0.51	11.99
9	3	1.0484	1.0487	1.0487	-0.00	0.0080	3.07	3.01	0.06	0.15	1.63
9	4	NA	1.0539	1.0540	-0.01	0.0089	3.40	3.36	0.02	-0.17	1.27
1	1	NA	1.0623	1.0627	-0.04	0.0100	3.75	3.63	0.03	-0.05	0.27
9	2	NA	1.0731	1.0737	-0.06	0.0112	4.18	3.84	0.08	-0.15	2.01
9	3	NA	1.0738	1.0746	-0.07	0.0117	4.37	4.16	0.08	-0.15	1.93
4	4	NA	1.0822	1.0831	-0.08	0.0126	4.67	4.53	0.08	0.01	1.11
<i>Mean</i>		1.0454	1.0602	1.0606	-0.04	0.0094	3.52	3.40	0.06	-0.01	3.00

Table 15.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

<i>s</i>	<i>y_t</i>	\hat{y}_s	\tilde{y}_s	<i>b_s</i> %	$\hat{\sigma}_s$	<i>n_s</i> %	<i>q_s</i> %	<i>s_s</i>	<i>k_s</i>	<i>jb_s</i>	
1	1	1.0389	1.0392	1.0391	0.01	0.0017	0.67	0.66	0.17	-0.15	5.98
9	2	1.0489	1.0489	1.0488	0.01	0.0031	1.18	1.17	0.14	-0.19	4.77
9	3	1.0484	1.0487	1.0488	-0.01	0.0043	1.63	1.59	0.12	-0.25	5.11
9	4	NA	1.0539	1.0540	-0.01	0.0055	2.08	2.07	0.09	-0.16	2.50
1	1	NA	1.0623	1.0625	-0.02	0.0069	2.61	2.50	0.09	-0.17	2.66
9	2	NA	1.0731	1.0734	-0.03	0.0085	3.16	3.06	0.11	-0.13	2.74
9	3	NA	1.0738	1.0744	-0.05	0.0097	3.62	3.53	0.15	-0.11	4.04
4	4	NA	1.0822	1.0829	-0.07	0.0112	4.12	4.02	0.17	-0.02	4.66
<i>Mean</i>		1.0454	1.0602	1.0605	-0.02	0.0064	2.38	2.33	0.13	-0.15	4.06

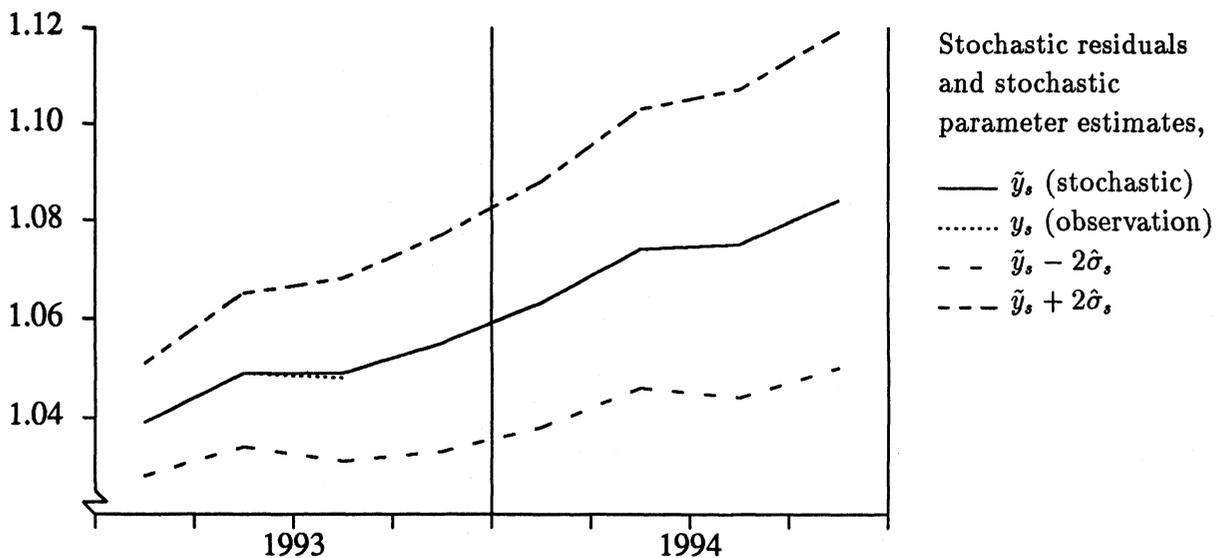
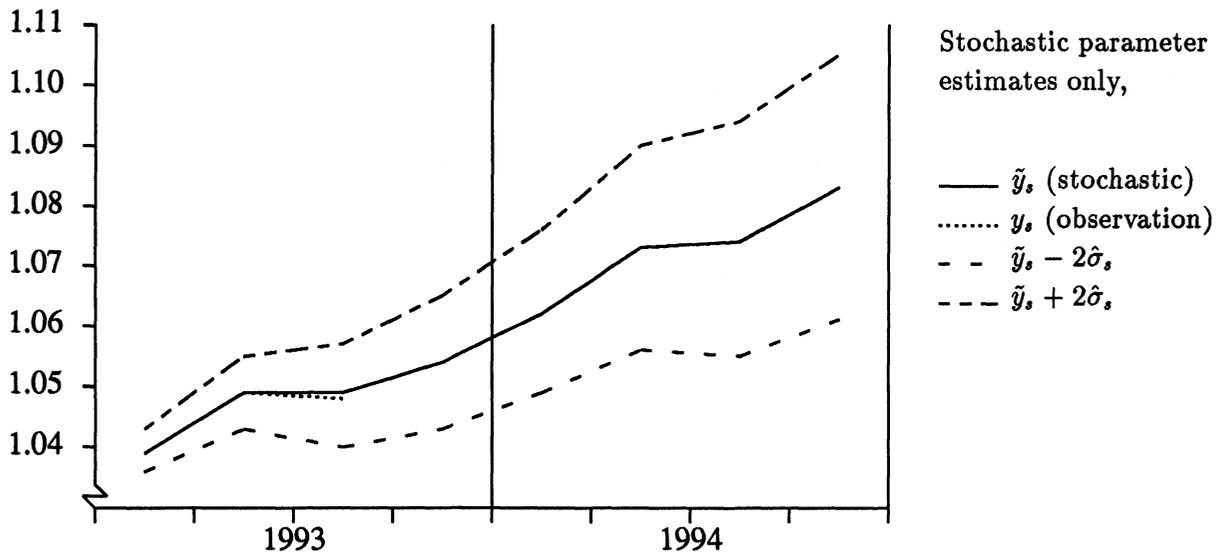
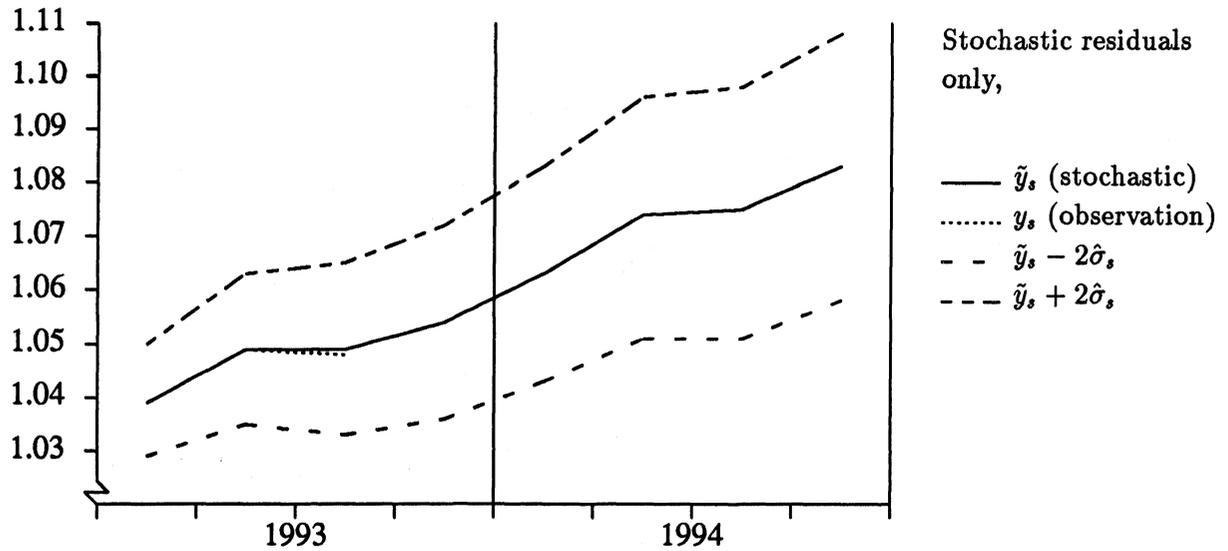
Table 15.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

<i>s</i>	<i>y_t</i>	\hat{y}_s	\tilde{y}_s	<i>b_s</i> %	$\hat{\sigma}_s$	<i>n_s</i> %	<i>q_s</i> %	<i>s_s</i>	<i>k_s</i>	<i>jb_s</i>	
1	1	1.0389	1.0392	1.0393	-0.01	0.0059	2.26	2.19	0.15	0.13	4.77
9	2	1.0489	1.0489	1.0493	-0.04	0.0078	2.96	2.90	0.14	0.32	7.48
9	3	1.0484	1.0487	1.0493	-0.06	0.0094	3.58	3.70	0.06	0.04	0.64
9	4	NA	1.0539	1.0547	-0.08	0.0110	4.16	4.10	0.06	-0.17	1.92
1	1	NA	1.0623	1.0634	-0.11	0.0125	4.71	4.88	0.12	0.14	3.47
9	2	NA	1.0731	1.0743	-0.12	0.0144	5.36	5.38	0.12	0.17	3.72
9	3	NA	1.0738	1.0755	-0.15	0.0159	5.90	5.93	0.07	-0.02	0.95
4	4	NA	1.0822	1.0841	-0.18	0.0173	6.38	6.53	0.04	0.07	0.55
<i>Mean</i>		1.0454	1.0602	1.0612	-0.09	0.0118	4.41	4.45	0.10	0.09	2.94

Table 15.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 15. Graphic plots of KPI corresponding to the tables on the facing page.



PA4: Export deflator, traditional goods, 1991 = 1

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	0.9407	0.9401	0.9407	-0.06	0.0121	5.13	5.01	0.04	0.50	10.57
9	2	0.9394	0.9303	0.9310	-0.07	0.0163	7.01	7.23	0.02	0.22	2.04
9	3	0.9341	0.9454	0.9461	-0.08	0.0159	6.74	6.41	0.01	-0.10	0.44
3	4	NA	0.9615	0.9627	-0.12	0.0198	8.21	8.02	0.09	-0.05	1.42
1	1	NA	0.9775	0.9792	-0.17	0.0214	8.74	8.18	0.07	-0.24	3.24
9	2	NA	0.9982	1.0003	-0.22	0.0220	8.78	8.60	0.09	-0.15	2.12
9	3	NA	1.0280	1.0303	-0.22	0.0227	8.80	8.46	0.11	-0.02	2.02
4	4	NA	1.0561	1.0587	-0.25	0.0247	9.32	8.96	0.14	0.06	3.45
<i>Mean</i>		0.9380	0.9796	0.9811	-0.15	0.0193	7.84	7.61	0.07	0.03	3.16

Table 16.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	0.9407	0.9401	0.9402	-0.01	0.0054	2.29	2.19	0.11	-0.23	4.32
9	2	0.9394	0.9303	0.9305	-0.02	0.0082	3.52	3.39	0.05	-0.10	0.78
9	3	0.9341	0.9454	0.9455	-0.01	0.0101	4.27	4.29	-0.03	-0.12	0.76
3	4	NA	0.9615	0.9615	0.01	0.0112	4.67	4.49	-0.10	-0.18	3.13
1	1	NA	0.9775	0.9770	0.05	0.0136	5.55	5.42	-0.23	-0.12	9.34
9	2	NA	0.9982	0.9975	0.07	0.0144	5.78	5.77	-0.26	0.04	11.66
9	3	NA	1.0280	1.0274	0.06	0.0145	5.65	5.45	-0.28	0.05	13.03
4	4	NA	1.0561	1.0559	0.02	0.0142	5.39	5.39	-0.13	-0.11	3.15
<i>Mean</i>		0.9380	0.9796	0.9794	0.02	0.0114	4.64	4.55	-0.11	-0.10	5.77

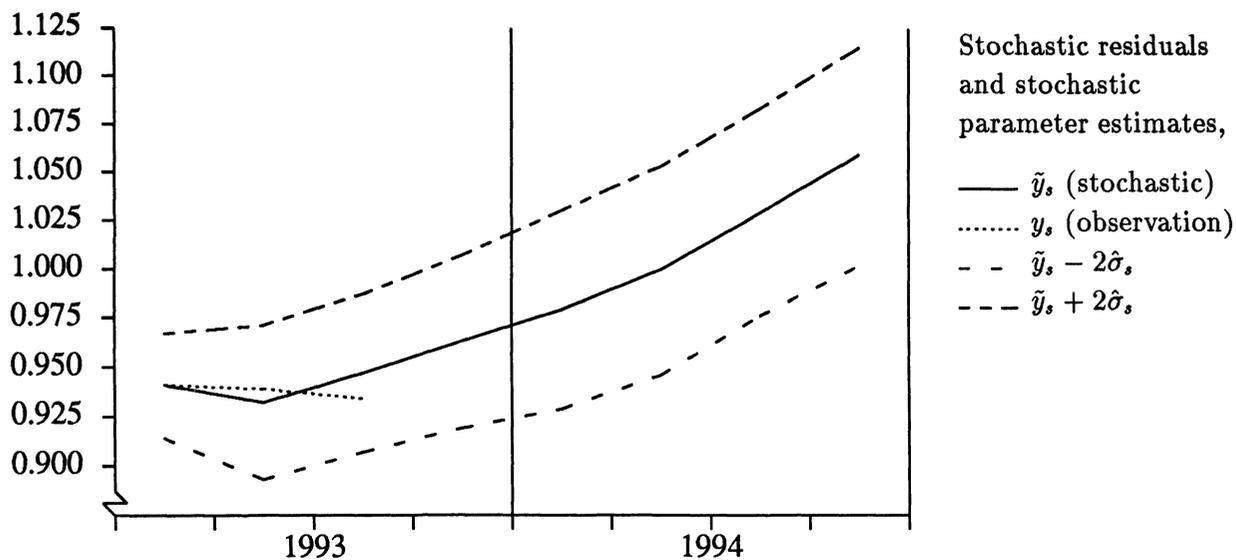
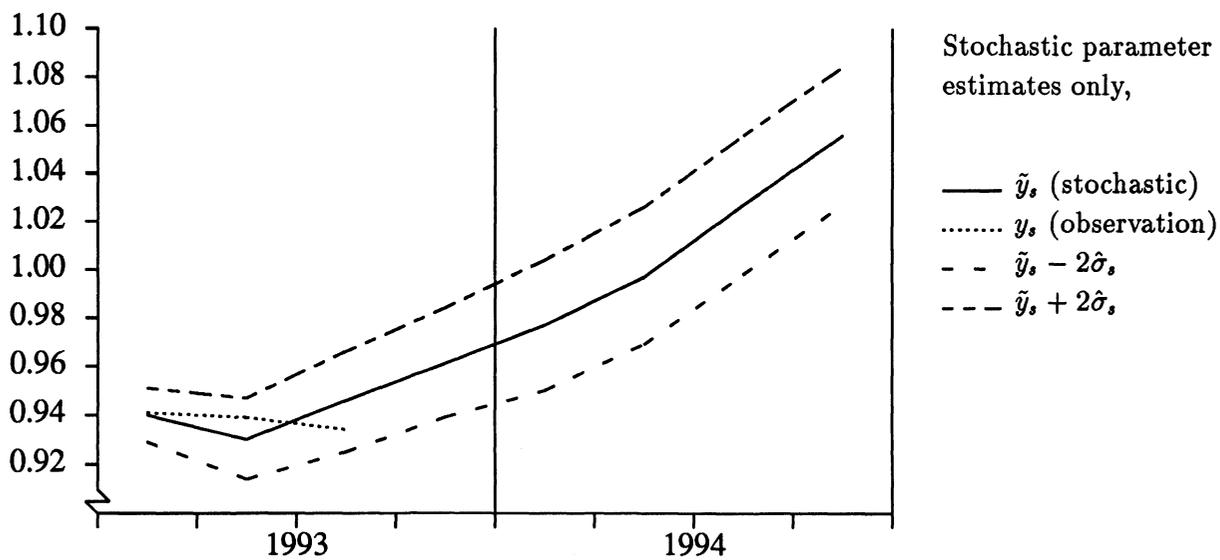
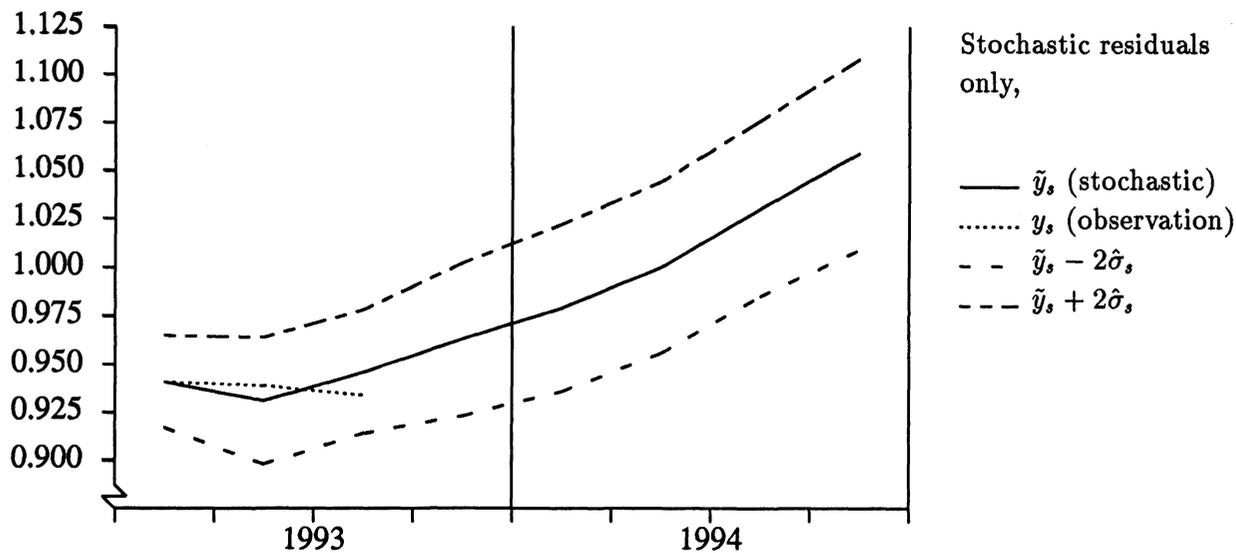
Table 16.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	0.9407	0.9401	0.9406	-0.05	0.0133	5.64	5.64	0.09	0.14	2.16
9	2	0.9394	0.9303	0.9321	-0.20	0.0196	8.40	8.27	0.28	0.82	41.94
9	3	0.9341	0.9454	0.9470	-0.17	0.0202	8.54	8.30	0.19	0.29	9.61
3	4	NA	0.9615	0.9631	-0.17	0.0221	9.16	9.08	0.14	0.23	5.77
1	1	NA	0.9775	0.9791	-0.17	0.0252	10.30	10.20	0.11	-0.00	2.13
9	2	NA	0.9982	0.9996	-0.14	0.0269	10.76	10.40	0.15	0.13	4.28
9	3	NA	1.0280	1.0294	-0.14	0.0269	10.46	10.64	0.24	0.41	16.54
4	4	NA	1.0561	1.0588	-0.25	0.0283	10.68	10.46	0.31	0.25	18.88
<i>Mean</i>		0.9380	0.9796	0.9812	-0.16	0.0228	9.24	9.12	0.19	0.28	12.66

Table 16.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 16. Graphic plots of PA4 corresponding to the tables on the facing page.



WW: Average wage rate in NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	127.9800	128.0600	128.0400	0.02	1.2653	3.95	3.82	0.05	-0.11	0.94
9	2	129.7460	129.8470	129.8510	-0.00	1.5831	4.88	4.76	0.11	0.04	1.92
9	3	129.5070	131.2240	131.2780	-0.04	1.8385	5.60	5.62	-0.02	0.32	4.33
3	4	NA	131.8420	131.8950	-0.04	2.0769	6.30	6.13	0.05	0.34	5.15
1	1	NA	131.7820	131.8560	-0.06	2.0791	6.31	6.07	0.08	0.13	1.95
9	2	NA	133.5330	133.6340	-0.08	2.5070	7.50	7.68	0.02	0.19	1.55
9	3	NA	135.0880	135.2810	-0.14	2.7852	8.24	8.11	0.07	0.22	2.74
4	4	NA	136.2850	136.4850	-0.15	2.9738	8.72	8.71	0.01	-0.12	0.64
<i>Mean</i>		129.0770	132.2080	132.2900	-0.06	2.1386	6.44	6.36	0.05	0.13	2.40

Table 17.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993 1, \dots, 1994 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	127.9800	128.0600	128.0270	0.03	0.4659	1.46	1.37	0.08	-0.06	1.09
9	2	129.7460	129.8470	129.8300	0.01	0.6735	2.08	2.01	0.12	-0.06	2.74
9	3	129.5070	131.2240	131.2220	0.00	0.8850	2.70	2.72	0.15	-0.12	4.51
3	4	NA	131.8420	131.8570	-0.01	1.0458	3.17	3.20	0.12	-0.05	2.69
1	1	NA	131.7820	131.7890	-0.01	1.2695	3.85	3.82	0.11	-0.04	2.15
9	2	NA	133.5330	133.5550	-0.02	1.5692	4.70	4.63	0.13	-0.04	2.98
9	3	NA	135.0880	135.1470	-0.04	1.9252	5.70	5.76	0.17	-0.13	5.46
4	4	NA	136.2850	136.3610	-0.06	2.2445	6.58	6.67	0.18	-0.06	5.41
<i>Mean</i>		129.0770	132.2080	132.2240	-0.01	1.2598	3.78	3.77	0.13	-0.07	3.38

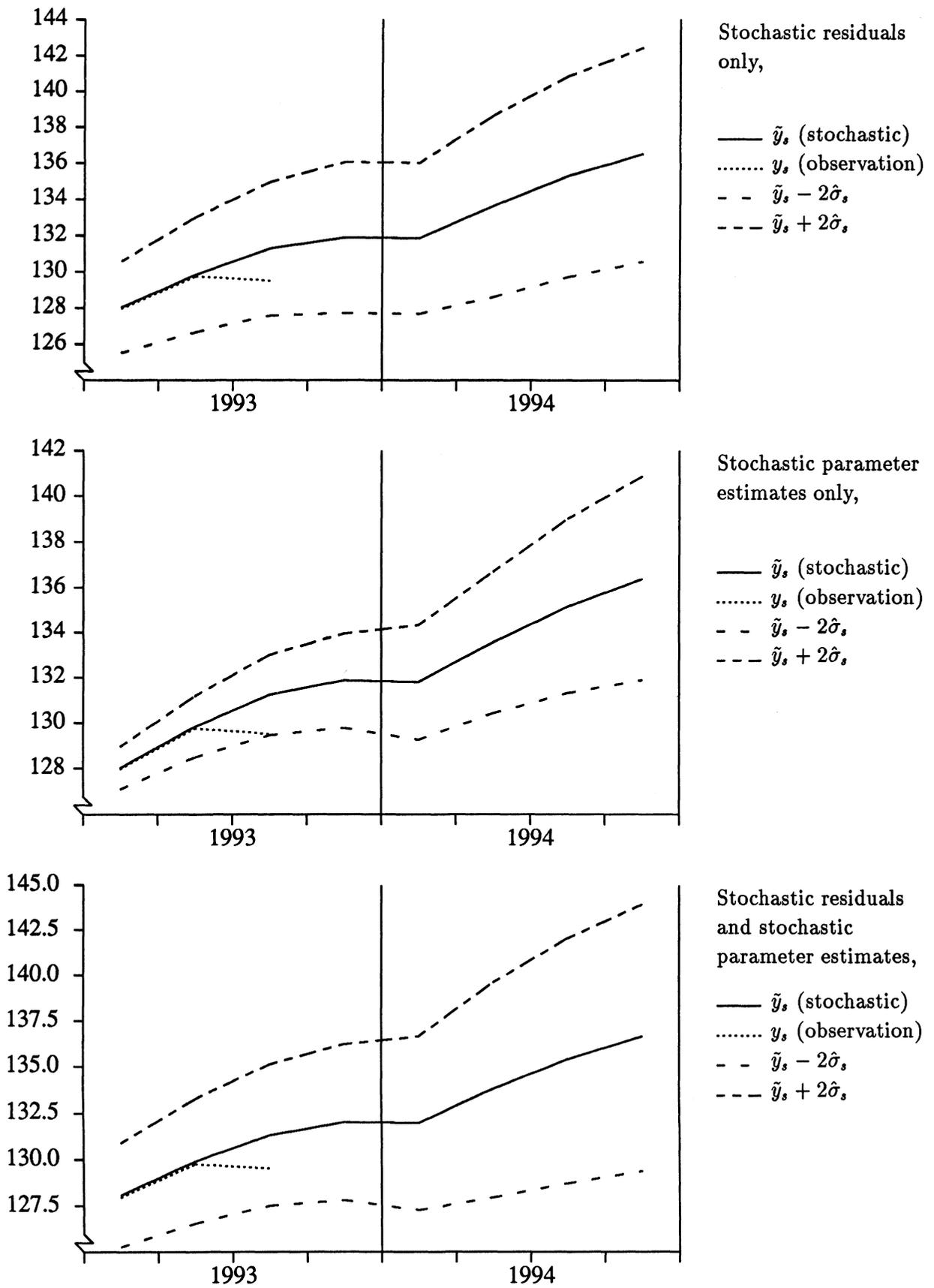
Table 17.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s		y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s
1	1	127.9800	128.0600	128.0860	-0.02	1.4062	4.39	4.19	0.04	0.10	0.72
9	2	129.7460	129.8470	129.9020	-0.04	1.6759	5.16	5.01	0.08	-0.13	1.84
9	3	129.5070	131.2240	131.3110	-0.07	1.8978	5.78	5.66	0.09	-0.06	1.65
3	4	NA	131.8420	132.0120	-0.13	2.1031	6.37	6.15	0.10	-0.10	2.13
1	1	NA	131.7820	131.9620	-0.14	2.3431	7.10	6.94	-0.05	-0.01	0.50
9	2	NA	133.5330	133.7890	-0.19	2.9162	8.72	8.49	0.07	0.27	3.93
9	3	NA	135.0880	135.3880	-0.22	3.3347	9.85	9.55	0.10	-0.07	2.04
4	4	NA	136.2850	136.6570	-0.27	3.6443	10.67	10.91	0.12	0.05	2.38
<i>Mean</i>		129.0770	132.2080	132.3880	-0.13	2.4152	7.26	7.11	0.07	0.01	1.90

Table 17.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 17. Graphic plots of WW corresponding to the tables on the facing page.



RS500: Current account, billion NOK

Stochastic residuals, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	5.1880	4.9479	4.9954	-0.95	0.9151	73.27	72.92	-0.03	0.02	0.14
9	2	6.8665	9.1137	9.1675	-0.59	1.1734	51.20	48.10	0.07	0.09	1.10
9	3	2.7836	5.3222	5.3510	-0.54	1.2423	92.86	89.94	0.04	0.05	0.30
9	4	NA	6.8388	6.8631	-0.35	1.4021	81.72	82.63	-0.03	0.07	0.39
1	1	NA	7.7864	7.7485	0.49	1.4774	76.27	73.40	-0.02	-0.26	2.80
9	2	NA	14.8836	14.8088	0.50	1.5779	42.62	40.88	0.01	0.07	0.21
9	3	NA	12.4046	12.3099	0.77	1.6654	54.11	53.57	0.01	0.40	6.57
4	4	NA	10.5190	10.3329	1.80	1.7714	68.57	69.80	-0.04	0.13	1.08
Mean		4.9460	8.9770	8.9471	0.14	1.4031	67.58	66.40	-0.00	0.07	1.57

Table 18.1. Observation y_t , deterministic simulation \hat{y}_s , mean stochastic simulation \tilde{y}_s , and within-period sample statistics for simulations with stochastic residuals only (The simulation setup is: 1000 antithetic replications, no correlations between equations except for a 9×9 FIML block). The statistics b_s , n_s , and q_s is in % of the level of \tilde{y}_s . The bottom row contains the mean statistic value of the simulated 8 periods of $s = 1993\ 1, \dots, 1994\ 4$.

Stochastic parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	5.1880	4.9479	4.9531	-0.11	0.3295	26.61	26.66	-0.07	-0.09	1.09
9	2	6.8665	9.1137	9.1090	0.05	0.4771	20.95	20.19	-0.13	-0.12	3.34
9	3	2.7836	5.3222	5.3185	0.07	0.5994	45.08	45.52	-0.19	0.06	6.10
9	4	NA	6.8388	6.8279	0.16	0.7932	46.47	45.47	-0.13	0.11	3.37
1	1	NA	7.7864	7.7699	0.21	0.8900	45.82	43.95	-0.14	0.26	6.09
9	2	NA	14.8836	14.8462	0.25	1.0072	27.14	27.40	-0.11	0.20	3.84
9	3	NA	12.4046	12.3882	0.13	1.0912	35.23	33.11	-0.09	0.13	2.16
4	4	NA	10.5190	10.5234	-0.04	1.2921	49.11	48.14	-0.01	0.20	1.69
Mean		4.9460	8.9770	8.9671	0.09	0.8100	37.05	36.30	-0.11	0.09	3.46

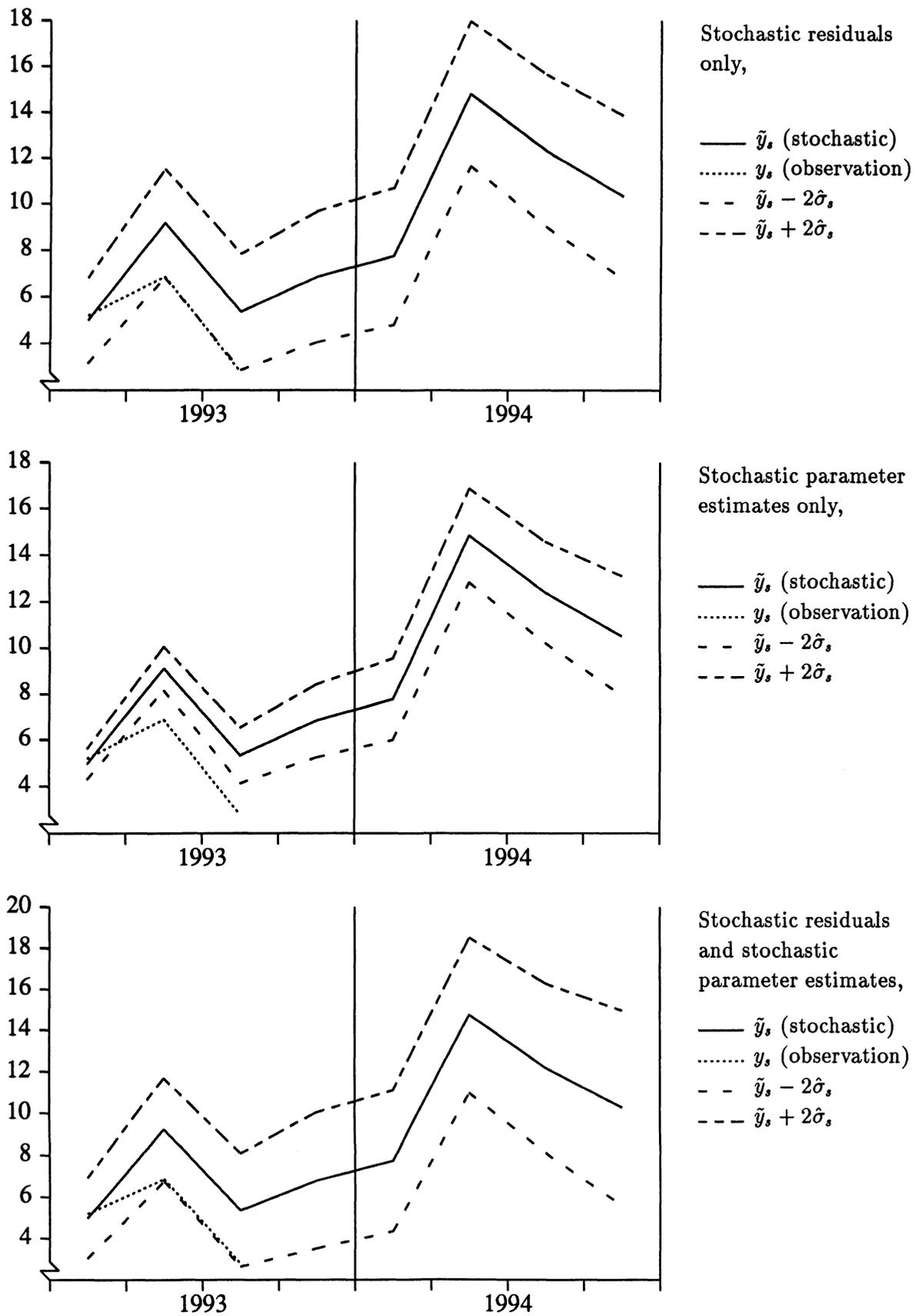
Table 18.2. Same statistics as in the table above, but for simulations with stochastic parameter estimates only.

Stochastic residuals and parameter estimates, ex ante simulation, starting 1993 1:

s	y_t	\hat{y}_s	\tilde{y}_s	$b_s\%$	$\hat{\sigma}_s$	$n_s\%$	$q_s\%$	s_s	k_s	jb_s	
1	1	5.1880	4.9479	4.9873	-0.79	0.9733	78.06	75.43	-0.01	-0.08	0.29
9	2	6.8665	9.1137	9.2268	-1.23	1.2353	53.55	54.52	0.03	0.00	0.16
9	3	2.7836	5.3222	5.3506	-0.53	1.3578	101.51	100.46	-0.09	-0.04	1.36
9	4	NA	6.8388	6.7807	0.86	1.6319	96.27	89.59	-0.07	-0.26	3.65
1	1	NA	7.7864	7.7271	0.77	1.6892	87.44	86.60	-0.01	0.08	0.33
9	2	NA	14.8836	14.7530	0.88	1.8856	51.13	49.78	-0.05	0.45	8.87
9	3	NA	12.4046	12.1867	1.79	2.0504	67.30	66.34	-0.13	0.15	3.85
4	4	NA	10.5190	10.2734	2.39	2.3456	91.33	89.79	-0.02	-0.01	0.10
Mean		4.9460	8.9770	8.9107	0.52	1.6461	78.32	76.56	-0.04	0.04	2.33

Table 18.3. Same statistics as in the tables above, but for simulations with both stochastic residuals and stochastic parameter estimates.

Figure 18. Graphic plots of RS500 corresponding to the tables on the facing page.



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Appendix A: Stochastic simulation alternatives

The pictorial equation (5) may be a suitable starting point for a brief discussion of alternative ways to perform stochastic simulations. A given model maps the distributions of its stochastic input variables onto the distribution(s) of its stochastic output variables, conditional on given exogenous values. We confine our discussion to the choice of input distributions p , and how to sample from them. More precisely, to maintain practical (and realistic) relevance, we limit our discussion to the alternatives that TROLL's Stochastic Simulator can handle. We ignore other sources of uncertainty, and refer to [8, 9, 10, 12] for more general views on the econometric uncertainty in model solutions.

When the econometric (structural) equations of a model are well specified, the empirical variance-covariance matrix $\hat{\Sigma}$ of all the estimation residuals should be consistent with the specified residual variance-covariance matrix Σ . In light of the picture (2)–(4) this amounts to whether $\hat{\Sigma}$ is diagonal or not for the single estimated OLS and IV equations. In the KVARTS91 simulations we have simply constructed the residual variance-covariance matrix of the single estimated equations to be diagonal, arguing that we ought to impose a stochastic structure that is consistent with the modelling assumption. Alternatively, we could have calculated a residual variance-covariance matrix for all the econometric equations. This could be done according to formula (14) if the number of equations does not exceed the number of common observation periods. In case of an undersized sample, formula (14) would yield a singular empirical variance-covariance matrix $\hat{\Sigma}$ that could not be factorized into $\mathbf{S}\mathbf{S}'$, cf. (7). In such a case simulation residuals $\hat{\mathbf{u}}_s^{(n)} \sim \mathcal{IN}(\mathbf{0}, \hat{\Sigma})$ could be constructed as a randomly weighted “mean” of the estimation residuals, according to McCarthy's method [23]:

$$\hat{\mathbf{u}}_s^{(n)} = \frac{1}{T-1} \widehat{\mathbf{U}} \mathbf{v}_s^{(n)}.$$

The paper [19] reports minimal differences in the simulated sample distributions from using a non-diagonal empirical residual variance-covariance matrix (with very small off-diagonal entries, though) for a model estimated by single equation OLS. It may perhaps not be worth the considerable extra simulation effort it is to use a full covariance matrix rather than a diagonal. This reservation is even more relevant when it comes to perturbing the stochastic parameter estimates, due to their considerably larger number (say a factor of 10). On the other hand, it may make a difference in the the case of simultaneous IV estimated equations, as indicated below.

Resampling the vectors of estimation residuals is another alternative. Not having to make any parametric assumptions on behalf the residual process(es), makes this an appealing method. A drawback of randomly reusing the residual vectors $\hat{\mathbf{u}}_t$ is that the number of vectors is limited. Thus, in many applications only very small samples can be generated before simulating the same values over. Fortunately, for a dynamic simulation of a model with a lag structure this problem quickly ceases with the simulation periods. The different trajectories simulated each replication n , i.e. $(y_{S_1}^{(n)}, \dots, y_{S_2}^{(n)})$, $n = 1, \dots, N$, may

overlap the first period(s), but will soon “diverge” due to the memory of different residual sequences. These alternative ways to generate residual variance-covariance matrices and stochastic residuals are implemented in TROLL’s Stochastic Simulator. Hence, they are easy to apply, cf. [15, 19] for more details.

The parameter estimators are functions of the stochastic residuals, hence the parameter estimates become stochastic variables. The variance-covariance matrix of all the parameter estimates is constructed to be block-diagonal on the assumption of no residual correlations, like [9, 13]. If we relax this assumption, we can estimate inter-equation covariances between vectors of single equation IV parameter estimates — as simultaneity is recognised by that estimation method. If we estimate inter-equation covariances between the IV estimated parameter vector $\hat{\theta}_g = \theta_g + (\hat{\mathbf{Z}}_g' \hat{\mathbf{Z}}_g)^{-1} \hat{\mathbf{Z}}_g' \mathbf{u}_g$ of equation g and the IV estimated parameter vector $\hat{\theta}_j = \theta_j + (\hat{\mathbf{Z}}_j' \hat{\mathbf{Z}}_j)^{-1} \hat{\mathbf{Z}}_j' \mathbf{u}_j$ of equation j , we get

$$\begin{aligned} \mathbf{C}[\hat{\theta}_g, \hat{\theta}_j] &= \mathbf{E}[(\hat{\theta}_g - \theta_g)(\hat{\theta}_j - \theta_j)'] \\ &= \mathbf{E}[(\hat{\mathbf{Z}}_g' \hat{\mathbf{Z}}_g)^{-1} \hat{\mathbf{Z}}_g' \mathbf{u}_g (\mathbf{u}_j' \hat{\mathbf{Z}}_j (\hat{\mathbf{Z}}_j' \hat{\mathbf{Z}}_j)^{-1})] \\ &= (\hat{\mathbf{Z}}_g' \hat{\mathbf{Z}}_g)^{-1} \hat{\mathbf{Z}}_g' \mathbf{E}[\mathbf{u}_g \mathbf{u}_j'] \hat{\mathbf{Z}}_j (\hat{\mathbf{Z}}_j' \hat{\mathbf{Z}}_j)^{-1} \\ &= \sigma_{gj} (\hat{\mathbf{Z}}_g' \hat{\mathbf{Z}}_g)^{-1} \hat{\mathbf{Z}}_g' \hat{\mathbf{Z}}_j (\hat{\mathbf{Z}}_j' \hat{\mathbf{Z}}_j)^{-1}, \end{aligned}$$

where $\mathbf{E}[\mathbf{u}_g \mathbf{u}_j'] = \sigma_{gj} \mathbf{I}$. The covariance will depend on the size of the covariance between the estimation residuals. Most likely it will be significantly different from zero (but small). If there are many IV equations/parameters in the model, if the few IV equations are important or of particular interest, it may be worth the extra effort to calculate the inter-equation covariances $\mathbf{C}[\hat{\theta}_g, \hat{\theta}_j]$ and to feed them into the simulation. If there are no memory problems due to a large non block-diagonal covariance matrix for several or all equations, TROLL’s Stochastic Simulator can handle full covariance matrices.

For realistic models of medium to large size, (efficient) simultaneous estimation of all the parameters in the econometric equations, e.g. by FIML, is not possible due to undersized sample and large model problems. An alternative method that seems fairly practical — though it is not applicable with the Stochastic Simulator — is applied by [27]. The author uses a method of structural instrumental variable estimation, followed by stochastic simulation and re-estimation from simulated samples to avoid simultaneous equation bias in the parameter estimates. At the same time the forecast uncertainty due to stochastic model input is estimated without having to impose any parametric assumptions on the distribution of the parameter estimates. The method is basically an iterative procedure, which is applied until convergence. The method outperforms the single equation and normality based procedure (which is used in this work and most others reported). The most surprising result reported in [27] is that parameter uncertainty tends to compensate for residual uncertainty due to negative correlations (which is lost in the single equation approach). If that result is representative for most operative models, the rationale for single equation modelling should be seriously reconsidered.

Appendix B: Stochastic simulation of KVARTS91 in TROLL

The Stochastic Simulator is a parallel to the standard deterministic simulator in main-frame TROLL. The Stochastic Simulator consists of several functions, which mirror the corresponding simulation functions in the deterministic context. The main difference is the need to prepare data on the stochastic structure(s) to impose on the simulation, and then to specify those data to the Stochastic Simulator routine. Basically, no changes to the model is needed. When familiar with standard (deterministic) TROLL simulation, stochastic simulation should come easily. The procedure of simulating KVARTS91 is split between three TROLL macros:

kopier_rescov: The empirical residuals and the estimated variance-covariance matrices from single equation and system estimation of the various econometric equations of the model are copied from the different estimating persons' archives into two special archives: *res_eq.equation number* and *cov_eq.equation number*. The equation number corresponds to the number of the equation in the system of collected equations which constitutes the model KVARTS91. The macro contains only the following kind of commands: *copy user name data file of either residuals or parameter covariance matrix res(or cov)_eq.equation number in the model*.

Previous to running this macro, the estimation residuals and the variance-covariance matrices of the parameter estimates must be stored after (re)estimation by issuing *fileresult res* and *fileresult covar* in case of single equation estimation (cf. [14, pages 4-1-77]), and by issuing *fileresid* and *filecovar* in case of system estimation (cf. [16, pages 47 and 50]) — for both cases cf. [15, page 5]. The appropriate pair of commands should be added to the various estimation macros.

beregn_res_stdev: The standard deviation of the estimation residuals of each single equation is calculated. It is stored in a vector together with standard deviations of other equations estimated by the same person. The vectors are named *data_res_stdev_the estimating person's initials*. The econometric equations estimated by a single person are grouped in one or a few blocks of equations in the KVARTS91 system of equations. The motivation for grouping the standard deviations into vectors, is that the standard deviations of each block can then be accessed by an index loop⁹. The variance-covariance matrix of the residuals from the FIML estimation is calculated according to (14), and stored as a data matrix *data_res_fiml_cov*.

stoksim: Simulates the model stochastically, calculates the simulation statistics and stores the statistics in tables at the level of the operating system CMS. This macro is explained in more detail on the following page.

⁹One single index loop would suffice if the econometric equations were gathered, say in the beginning of the model. This would be more practical in all aspects.

The simulation macro `stksim` has the following outline:

1. print information and simulation options;
2. read simulation choices;
 set up search paths, link model, data and file with estimated parameters;
 if label files do not exist
3. read selected endogenous names from the model into label files;
4. read endogenous names from label files into arguments;
 if deterministic simulation
5. simulate deterministically,
6. store simulation results only for the selected endogenous names in arguments;
 if stochastic simulation
 if stochastic residuals
7. read values from files and store specification of stochastic structure in arguments,
 if stochastic parameter estimates
8. read names and values from files and store stochastic specification in arguments,
9. simulate stochastically with specification from indexed arguments,
10. store simulation results only for the selected endogenous names in arguments,
11. calculate and store simulation statistics,
12. store table of statistics at the operating system level (for transfer to PC);

We comment on the numbered actions:

1.-2. The user can choose between deterministic and/or stochastic simulation. In the latter case there is a choice between stochastic residuals and/or stochastic parameter estimates. The final choices are the number of replications and the simulation periods. Antithetic variates are generated to reduce sample variance in the simulation statistics.

3.-4. All endogenous variable names are read from the model specification, and a selected number of them are stored in a label file to save a lot of explicit name listing.

5.-6. Dynamic deterministic simulation is used in the simulation statistics, cf. section 4. It is only necessary to run a deterministic simulation once before stochastic simulations, or after the simulation periods have been altered.

7.-8. The Stochastic Simulator needs detailed descriptions of the stochastic structures of the residuals and the parameter estimates. For each single estimated equation the syntax

`resid sdev stdev eqnnumber`

and

`coef covar pmatrix parameters`

is used. *stdev* is the estimated standard deviation of the residuals in equation number *eqnnumber*. *pmatrix* is a TROLL file containing the estimated variance-covariance matrix of the parameter estimates in the equation, and *parameters* is an alphabetic list of names of the parameters in the equation. The whole of the first command is put into `cifarg(eqnnum)`, while the second command is split between several `cifargs` due to all the parameter names to be listed. In both cases the structures of the stochastic residuals

and parameter estimates in the OLS and IV equations can be specified by simple index loops. For the FIML equations the syntax

```
resid covar rmatrix eqn1 to eqn2
```

and

```
coef covar pmatrix parameters
```

is used. *rmatrix* specifies the variance-covariance matrix of the FIML residuals, while *eqn1 to eqn2* specify the equations in the FIML block. The latter command is unaltered, as far as the syntax goes, from the single equation case (on the previous page).

9. A dynamic stochastic simulation with antithetic variates and the specified stochasticity is performed over the simulation periods requested.

10. The full replication matrix $(y_{g,s}^{(n)})$, where s runs through the simulation periods and n counts the replications, is saved for each selected endogenous variable y_g .

11. The simulation statistics are calculated on basis of the replication matrices (one for each variable) according to the formulas in section 4. Thereafter the replication matrices are deleted to save disk space. Only the statistics are stored. The deletion of the replication matrices can easily be cancelled by commenting out one line in the macro `stoksim`.

12. The statistics are collected in tables, which are then copied from TROLL to the main-frame operating system CMS. Since the Stochastic Simulator is not (yet) implemented in Portable TROLL, this step is necessary to facilitate transfer of simulation results to PC. The TROLL simulation data not needed for graphic plotting have been deleted to save disk space (cf. action 11 above).

The first time, when no preparation has been done, or when (relevant and significant) changes have been made to the equations in the model or when the some equations have been re-estimated, only then should it be necessary to run `kopier_rescov` and `beregn_res_stdev` before the `stoksim` macro. In those cases the pipeline is:

1. Store the empirical estimation residuals and the estimated variance-covariance matrix of the parameter estimates when (re)estimating.
2. Run the macro `kopier_rescov` to copy (all) the residuals and variance-covariance matrices to conveniently named archives and files.
3. Run the macro `beregn_res_stdev` to calculate the standard deviation in the estimation residuals (and the estimated variance-covariance matrix of the FIML residuals), and to store the results in certain data vectors.
4. Run the macro `stoksim` to do the simulation(s) and to store the results necessary for graphic plots and data transfer to PC.

No TROLL macro is displayed in this appendix since it is not easily read nor of general interest. The macros are well commented, and comprehensible to the TROLL knowledgeable.

Appendix C: Semi-automatic documentation of simulation results

The Stochastic Simulator (so far) runs only on mainframe computers. Documents are produced on PC's. Hence, simulation data has to be transferred from mainframe to PC for preparation and inclusion in reports. This appendix shows how some of this work can be automated by small routines that (1) generate batch files of commands, (2) performs the copying of data from mainframe to PC, (3) generate PostScript graphics and format T_EX tables, and (4) include the graphics and the tables in this report (cf. section 6). Statistics Norway has some software useful for some of the tasks, but a few small batch files and AWK programs [2] are written to make the transfer and preparation more automatic. The parenthesised numbers in this paragraph refer to the individual tasks of that procedure. The different AWK programs are explained briefly underneath, while the next page shows the procedure (1)–(4) of transferring and processing files by using these programs.

(1) The batch file `makefile.bat` lets three AWK programs read a list of names of KVARTS91 endogenous variables. For each name in the list the single AWK program writes the same command(s) to a batch file. The three batch files so generated, are used for transferring table data (`transfer.bat`), generating PostScript plots of the simulation series (`plot.bat`), editing table data into T_EX tables, and changing plot looks (`editfile.bat`) by editing the PostScript files.

(2) The transfer of table data from CMS by `transfer.bat` is done by repeating the `receive` command for each variable on the list. The graphic plots of simulation series are generated by calling `trplott` (cf. PC PERMEN, chapter 6) for each variable on the list. `plot.bat` saves the plots as PostScript files. Due to error in DOS the `copy` command is not recognized when the 3270 emulator occupies memory. Hence, the plots cannot be saved directly into subdirectories. To separate a variable that has been simulated with stochastic residuals from the same variable simulated with stochastic parameters (in addition), a prefix (`r-`, `p-` or `rp-`) is added to the variable name.

(3) This report is written in a document preparation system called T_EX. It is easy to combine tables of simulation results and graphic plots in a T_EX document when the tables are in T_EX code (ASCII text with control sequences) and the graphic plots are in PostScript. The program `trplott` saves the plots as PostScript code. The AWK program `editplot.awk` only makes a few cosmetic changes to the plot by editing the PostScript code. T_EX tables are basically matrices of data, with certain symbols separating the columns and the rows. The AWK program `edittab.awk` inserts T_EX formatting codes into the ASCII matrices transferred from CMS, such that the resulting tables look like the ones in section 6.

(4) The AWK program `editpage.awk` collects all the edited files and lays them out on the pages in such a way that the result becomes the spreads of section 6.

No AWK program is displayed in this appendix, since AWK is somewhat cryptic and since some programs require T_EX knowledge (the programs are not of general interest). The AWK codes are well commented. The next page show how to use these programs.

1. On the PC directory `d:\kv91trol` run the batch file `makebats.bat` by issuing the command `makebats`, which results in the processing:

input	processing	output
<code>endoname.asc</code>	$\left\{ \begin{array}{l} \text{transfer.awk} \\ \text{plot.awk} \\ \text{editfile.awk} \end{array} \right.$	$\rightarrow \begin{array}{l} \text{transfer.bat} \\ \text{plot.bat} \\ \text{editfile.bat} \end{array}$

where all the files are in the directory `d:\kv91trol`. This generation of `bat` files are only necessary a first time and after changes are made to the layout.

2. Log onto the mainframe computer (where TROLL is) via the 3270 emulator on PC, and go to CMS. Switch to the PC, go to `d:\kv91trol` and run a batch file by issuing the command `transfer prefix`. This results in the file copying:

CMS	PC
<code>variable prefixsim</code>	<code>prefix_variable.tab</code>

where the argument `prefix` is either `r` for simulation with stochastic residuals, `p` for simulation with stochastic parameter estimates or `rp` for simulation with both. The files are stored on the PC directory `d:\kv91trol`.

Switch to the mainframe, start TROLL and search the right data archives. Go to `d:\kv91trol` on the PC and run a batch file by issuing the command `plot prefix`. This results in the data files in TROLL being plotted and saved as PostScript files on PC:

TROLL	PC
<code>data_kv91_prefixsim_variable</code>	<code>prefix_variable.eps</code>

where the archives in TROLL are: `rsim` for simulation with stochastic residuals, `psim` for simulation with stochastic parameter estimates or `rpsim` for simulation with both. The files are stored on the PC directory `d:\kv91trol`. Due to an error in MS DOS, the files cannot be copied directly into a wanted directory, e.g. `d:\kv91trol\prefixsim`. Instead the prefix `prefix_` to the variable name is used.

3. Log off TROLL and the mainframe. On the PC directory `d:\kv91trol` run a batch file by issuing the command `editfile prefix`. This results in the processing:

input	processing	output
<code>prefix_variable.tab</code>	<code>→ edittab.awk</code>	<code>→ d:\kv91\prefix_variable.tex</code>
<code>prefix_variable.eps</code>	<code>→ editplot.awk</code>	<code>→ d:\kv91\prefix_variable.eps</code>

The input files are stored on the PC directory `d:\kv91trol`.

4. On directory `d:\kv91` run `editpage`, which results in the processing:

input	processing	TeXfile
<code>endoname.asc</code>	<code>→ editpage.awk</code>	<code>→ kv91page.tex</code>

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